QBUS3830 Group Assignment (30%)

October 18, 2020

1 Background

Volatility, defined as the variance of a financial return such as stock returns or exchange rates, is of high interest in finance and banking as volatility is often used to characterize financial risk. More specifically, let $\{P_t,\ t = 0, 1, ...\}$ be a time series of the prices of an asset at the end of each time interval t, the return at time t is defined as

$$y_t = \log \frac{P_t}{P_{t-1}}, t = 1, 2, ...$$

The conditional variance $\sigma_t^2 = \mathbb{V}(y_t|y_{1:t-1})$ is the volatility, given the information up to time $t-1$. As the volatility is unobserved, econometricians often use mathematical models in order to understand the volatility dynamics and produce its prediction. One of the widely-used volatility models is the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model (Engle, 1982; Bollerslev, 1986), which formulates σ_t^2 as a linear combination of the previous squared return and conditional variance.

$$\begin{aligned} y_t &= \sigma_t \epsilon_t, \quad \epsilon_t \overset{i.i.d.}{\sim} \mathcal{N}(0, 1), \quad t = 1, 2, ..., T \\ \sigma_t^2 &= w + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2, \quad t = 2, ..., T, \end{aligned}$$

where $w,\alpha,\beta \geq 0$ and $\alpha+\beta<1$ to impose the stationarity of the process y_t. The initial σ_1^2 is often set to the sample variance of the time series y_t. The model parameters θ include w, α and β.

Suppose that Bayesian data analysis is used. Then some prior distribution for θ is needed. As $0<\alpha,\beta<1$ and $w>0$, suitable priors for $\theta=(\alpha,\beta,w)$ are

$$\alpha \sim \text{Beta}(1.5, 10), \quad \beta \sim \text{Beta}(10, 1.5), \quad w \propto 1.$$

The hyperparameters in the Beta distributions are selected to encourage α and β to be close to 0 and 1, respectively, as this has been long observed in the financial volatility modelling literature.

The data set `Price_History_Commonwealth_bank`, available on Canvas, contains the historical stock prices (the column named `Close`) of the Commonwealth Bank of Australia (CBA) from 03-Jan-2012 to 11-Sep-2020.

2 Assignment task

Your task is to analysis the volatility of the CBA stock returns.

- Perform an exploratory data analysis for the return data (calculate and report min, max, skewness, kurtosis, etc.). Comment on the findings. Financial time series data have some unique characteristics that do not share with other type of time series, you're encouraged to incorporate the domain knowledge into your discussion.

- Estimate the posterior distribution of θ, $p(\theta|y)$ for the GARCH model. You have two options: use MCMC or VB. **Please report the estimates of the posterior mean and posterior standard deviation of θ.**

  Hint: To impose the stationarity condition $\alpha+\beta<1$ and the positiveness constraint, the following transformation can be used

  $$\alpha=\psi_1\psi_2, \qquad \beta=\psi_1(1-\psi_2), \qquad 0<\psi_1,\psi_2<1,$$
  $$\widetilde{\theta}_1=\log\frac{\psi_1}{1-\psi_1}, \qquad \widetilde{\theta}_2=\log\frac{\psi_2}{1-\psi_2}, \qquad \widetilde{\theta}_3=\log w.$$

  Then, the transformed parameters $\widetilde{\theta} \coloneqq (\widetilde{\theta}_1,\widetilde{\theta}_2,\widetilde{\theta}_3)^\top \in \mathbb{R}^3$ are unconstrained and therefore it is easier to estimate the posterior $p(\widetilde{\theta}|y)$ using MCMC or Gaussian VB. However, you must report the results in the original scale of θ, i.e. transform back from $\widetilde{\theta}$ to θ.

- Report your forecast of the CBA return volatility for 12-Sep-2020.

3 Submission instructions

1. Each group needs to submit TWO files (or more if necessary) via the link in the Canvas site.

- A document file, named **Group_xxx_document.pdf**, that reports your data analysis procedure and results. You should replace the xxx in the file name with your group ID.
- The Matlab file, named **Group_xxx_implementation.m**, that implements your data analysis procedure. You might submit additional files that are needed for your implementation, the names of these files must follow the same format **Group_xxx_<name>**. You should replace the xxx in the file name with your group ID.

2. The document file **Group_xxx_document.pdf** is a report that describes and summarises your data analysis problem:

   - Describe your data analysis procedure and report the results in detail. The report should be detailed enough so that other business analysts, who are supposed to have background in your field, understand and are able to implement the task. All the numerical results are reported up to four decimal places.
   - There is no unique or best way of presentation. It's your opportunity to be creative.
   - Clearly and appropriately present any relevant graphs and tables.
   - The page limit is 25 pages including EVERYTHING: appendix, computer output, graphs, tables, etc.

3. The random seed in **Group_xxx_implementation.m** must be fixed, so that the marker expects to have the same results as you had.

4. Your group is required to submit meeting minutes. You may use the templates provided for preparing agendas and meeting minutes. The more detailed the meeting minutes, the better (who does what next, what has been done by whom, etc.). In case of a dispute within a group, I will use the meeting minutes and/or request for more information to make adjustment to the individual marks. **Should a dispute occurs, please treat each other in a professional and respectful manner**.

5. Each member of the group is also required to submit a peer assessment, named **Group_xxx_Peer_Assessment_<your SID>**, via the link on

Canvas under Assignments/Peer-review submission (this is for submitting the peer assessment only). Please use the peer criteria sheet *Peer Assessment Criteria* and assessment form *Peer Assessment of Team Members*, provided on Canvas, for this purpose. **It's important to have a fair and objective assessment of your team members**.

4 Marking Criteria

This assignment weighs 30 marks in total.

1. Marks will be allocated based on appropriateness and correctness of exploratory data analysis, modelling and estimation methods, accuracy of your estimation, appropriate discussion and conclusions.

2. As the technical nature of this unit, bonus marks might be awarded to projects that have in-depth technicality.

5 Errors

If you believe there are errors with this assignment please let me know immediately.

References

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3):307 – 327.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50(4):987–1007.