A Python Adaptation of the FRBNY Krusell-Smith Continuous Time Model

A Paper for ECO529

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1 The FRBNY DSGE Model

The New York Fed has developed several dynamic static general equilibrium models for estimation (eg, for the current neutral rate of interest) and forecasting [1]. Written in Julia, they are available online for the public in the DSGE.jl package [2]. They range in complexity and assumptions, for instance with some using heterogenous agents and some using representative agents. Of these models, the one most similar to what we worked with in class was the adaptation of the Krusell-Smith model [3] in continuous time, based on the work of Achdou et al [4].

1.1 The Krusell-Smith Model

The Krusell-Smith model assumes an economy of consumer agents with varying income and wealth; however, it also assumes consumer utilities have properties (as described in Altug and Labadie [5]) to make them aggregatable. Agents can trade riskless bonds and rent capital (with a borrowing constraint); the model assumes there are no other financial instruments. These bonds are useful since agents can face both idiosyncratic and aggregate unemployment shocks; these are discussed more thoroughly in the original discrete time paper [3]. Under shocks, employment status for each agent can be modelled as a two-state Markov chain whose probabilities depend on the state of the aggregate economy, which can itself be modelled as a separate two-state Markov chain.

The appendix of Achdou et al [4] outlines the equations for approximating both the HJB and the KFE, and for handling boundary conditions. Their equation for applying the Euler method to the HJB equation included typos, at least in the version of the paper I had access to, so I have included it below with those typos corrected.

$$
\begin{aligned}
0 = \frac{v_{i,j}^{n+1} - v_{i,j}^{n}}{\Delta t} - \rho v_{i,j}^{n} - \frac{\sigma_a^2(a_i)}{2} \frac{2v_{i,j}^{n} - v_{i+1,j}^{n} - v_{i-1,j}^{n}}{h_a^2} \\
- \frac{\sigma_z^2}{2} \frac{2v_{i,j}^{n} - v_{i,j+1}^{n} - v_{i,j-1}^{n}}{h_z^2} \\
- \mathcal{G}\Big(\frac{v_{i+1,j}^{n} - v_{i,j}^{n}}{h_a}, \frac{v_{i,j}^{n} - v_{i-1,j}^{n}}{h_a}, w^{n+1}(z_j) + a_i r^{n+1}\Big)
\end{aligned}
$$

2 My Process

As stated in my proposal, my goal was to translate at least some of the FRBNY model from Julia (which I am not familiar with) to Python (which I am familiar with). I originally hoped to translate the "Model1002" that serves as the authoritative FRBNY DSGE model. But after familiarizing myself with the documentation [6] and GitHub repository [2] for the entirety of DSGE.jl, I decided against Model1002 given its hundred-plus parameters, convoluted settings, and numerous dependencies across the DSGE object environment. As a discrete-time representative agent model, it was also less closely aligned with the models we worked with in class. While still more convoluted than some of the other FRBNY models, the Krusell-Smith-CT model was the only continuous time model in the repository and thus I could focus on the techniques from class. There was a significant downside, however, in that this model was not discussed in the DSGE.jl documentation. Instead, I relied on the appendix of Achdou et al [4] for the numerical algorithms - which were indeed similar to those used in class.

I was unfortunately limited by being unable to install the actual DSGE.jl package on my own computer; after several attempts at untangling version conflicts, I concluded that I lacked the experience in Julia to justify continued effort. This meant that I couldn't inspect, trace, or experiment with the working Julia package as I wrote and then debugged my Python version. While I was able to follow the original Julia code and use it to guide for my Python version, I had hoped to be able to inspect the data structures of the Julia model in order to trace any errors that appeared in mine.

In particular, I was having an issue where every repetition of the upwind step of the HJB approximation caused the data matrix to lose more and more values to "not a number" errors. Beyond ruining the actual data, this broke the numpy method used to invert the matrix, and thus (in order to test that the rest of the code would even run). I was ultimately unable to resolve this issue in the time I had, and so the code in my repository only runs the upwind step a single time. Obviously, this is insufficient to get anything close to a convergent steady state - this should be immediately apparent in the graph of the value function approximation at the end of file.

3 References

[1] Marco Del Negro et al. *The FRBNY DSGE Model Meets Julia*. Dec. 2015. URL: `https://libertystreeteconomics.newyorkfed.org/2015/12/the-frbny-dsge-model-meets-julia/`.

[2] DSGE.jl (Version 1.2.1)[Source code], https://github.com/PSLmodels/DSGE.jl.

[3] Per Krusell and Anthony A Smith. "Quantitative macroeconomic models with heterogeneous agents". In: *Econometric Society Monographs* 41 (Mar. 2006), p. 298. URL: `http://www.econ.yale.edu/smith/paper15.pdf`.

[4] Yves Achdou et al. "Heterogeneous agent models in continuous time". In: *Preprint* (Nov. 2014). URL: `https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=53a0c41bf9c5b9de0c9f763111948a6a41b824ae`.

[5] Sumru Altuğ and Pamela H Labadie. "Dynamic choice and asset markets". In: (1994).

[6] DSGE.jl (Version 1.2.1)[Documentation], https://frbny-dsge.github.io/DSGE.jl/stable/.