Group: Nathan Sanders, Devon Doud, and Sam Weston
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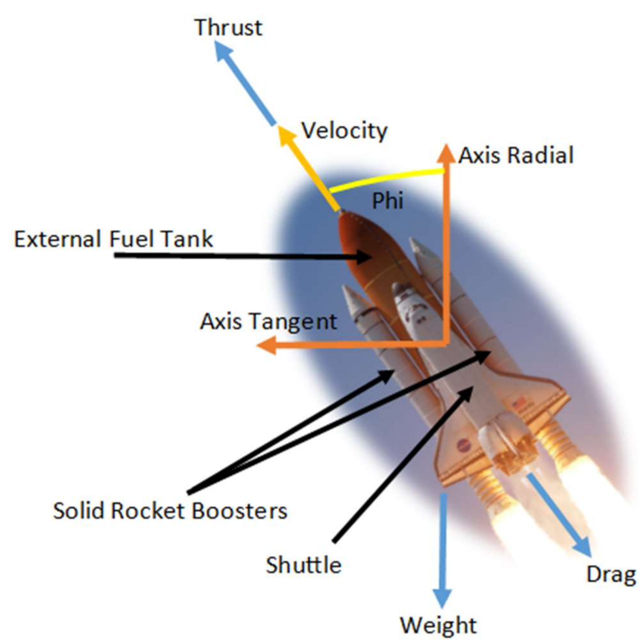
EMEC 303-01
Project 1: Modeling of Space Shuttle Launch

Introduction

The purpose of this project was to solve the trajectory of a space shuttle reaching orbit equal to that of the ISS using numerical analyses of differential equations. The shuttles goal is to assume an orbit at 400 km and velocity of 7,743 m/s parallel to the surface of the earth. Several simplifications and assumptions were made that produced error in the numerical analysis compared to an accurate shuttle launch. All other parameters were retrieved from the D2L handout[1] and the designated Wikipedia page[2].

Assumptions

1. Atmospheric behavior, such as wind and humidity, were disregarded, and air density was calculated using a provided MATLAB function[3].
2. The shuttle rockets and booster engines maintained full thrust during the entire launch sequence, and all the fuel is burned during launch.
3. The drag coefficient is constant across all surfaces of the shuttle and other components.
4. When the booster engines are jettisoned the shuttle is pulled to a new thrust angle.



Description of Model

This numerically analyzed shuttle launch was modeled using newton's second law in polar form as seen in *Eqn. 1* and *Eqn. 2*.

$$F_r = m * (\frac{d^2r}{dt^2} - r * (\frac{d\theta}{dt})^2) \qquad (Eqn.\ 1)$$

$$F_\theta = m * (r * \frac{d^2\theta}{dt^2} + 2 * \frac{dr}{dt} * \frac{d\theta}{dt}) (Eqn.\ 2)$$

From these equations we solved for the second order differentials in terms of the remaining variables. These second order differentials were broken up into 2 first order differentials, *Eqns. 3-6*, that could each be solved using an Euler's update solving method. The final ordinary differential equations used to solve the shuttle problem are:

$$\frac{d^2r}{dt^2} = \frac{F_r}{m} + r * (\frac{d\theta}{dt})^2$$

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[1] Loribeth Evertz and Mark Owkes. "Project 1-Shuttle". Montana State University D2L. Retrieved from ecat.montana.edu/d2l/le/content.
[2] Wikipedia. "Space Shuttle". Wikipedia. Retrieved from wikipedia.org/wiki/Space_Shuttle#Specifications.
[3] Owkes, Mark. "rhoe.m". Montana State University D2L. Retrieved from ecat.montana.edu/d2l/le/content.

(*Eqn. 3*)

$\frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = (\frac{F_\theta}{m} - 2 * \frac{dr}{dt} * \frac{d\theta}{dt}) * \frac{1}{r}$ (*Eqn. 4*)

$\frac{d\theta}{dt} = \omega$ (*Eqn. 5*)

$\frac{dr}{dt} = V_r$ (*Eqn. 6*)

These equations were updated using the shooting method and an Euler update. The shooting method was chosen because the initial launch angle is required to meet the desired orbit of 400 km and velocity of 7,743 m/s was unknown, and the Euler's method was chosen because of the simplicity of coding and debugging.

The launch happened in three stages as outlined in the D2L handout and the designated Wikipedia pages.

Stage 1: Both the boosters and main engines provide thrust.

Transition: Solid rocket boosters run out of fuel and are jettisoned.

Stage 2: Main engines continue and provide thrust.

Transition: External fuel tank empties and is jettisoned, main engines turn off.

Stage 3: Shuttle circles the earth in orbit without thrust.

Several parameters of the shuttle changed throughout these stages of the flight, including total mass, gravitational acceleration, drag force, thrust, and flight angle. The weight force changes as the fuel is consumed, boosters and fuel tank are jettisoned, and gravitational pull of the earth depletes with altitude. Gravity was modeled using *Eqn. 7,* using the mass of Earth, the changing mass of the shuttle, the gravitational constant, and the distance between the center of mass of the objects.

Stage 1: $m_{total} = m_{shuttle} + m_{boosters} + m_{liq.fuel} * (1 - \frac{t}{t_{tank}}) + m_{boosters} + m_{sol.fuel} * (1 - \frac{t}{t_{boosters}})$

Stage 2: $m_{total} = m_{shutt} \quad + m_{boosters} + m_{liq.fuel} * (1 - \frac{t}{t_{tank}})$

Stage 3: $m_{total} = m_{shuttle}$

$F_g = G * \frac{m_{total} + m_{Earth}}{r^2}$ (*Eqn. 7*)

The drag force was estimated using *Eqn. 8* and is assumed to always point in the opposite direction of the shuttle velocity. The drag force equation uses the density of air (ρ) which changes as the height increases, the velocity of the shuttle (V), the coefficient of drag (C_d) which is estimated to be 0.5, and the cross-sectional area (A) estimated by the radii of the components, which changes as the components are jettisoned.

$F_{Drag} = \frac{1}{2} * \rho * V^2 * C_d * A$ (*Eqn. 8*)

Thrust was assumed to be at constant maxima during each stage of the launch but changed in magnitude as the boosters were jettisoned. Thrusts values were obtained from the Wikipedia page.

Stage 1: $F_{Thrust} = F_{shutt} \quad + F_{boosters}$

Stage 2: $F_{Thrust} = F_{shuttle}$

Stage 3: $F_{Thrust} = 0$

The thrust angle (from the tangent of the theta axis) was calculated using the arctangent of the radial velocity divided by the tangential velocity.

Results

The launch scenario achieved the orbit depicted in *Figure 1* using the parameters, restrictions, and assumptions explained above. The best launch was with initial launch angle of 73.8 degrees, as seen in *Figure 3,* and with the angle of thrust continuously adjusted by an elliptical equation. The shuttle achieved an oblong orbit as it fluctuates from approximately 350 km to 450 km above the Earth's surface, as seen in *Figure 2B*.

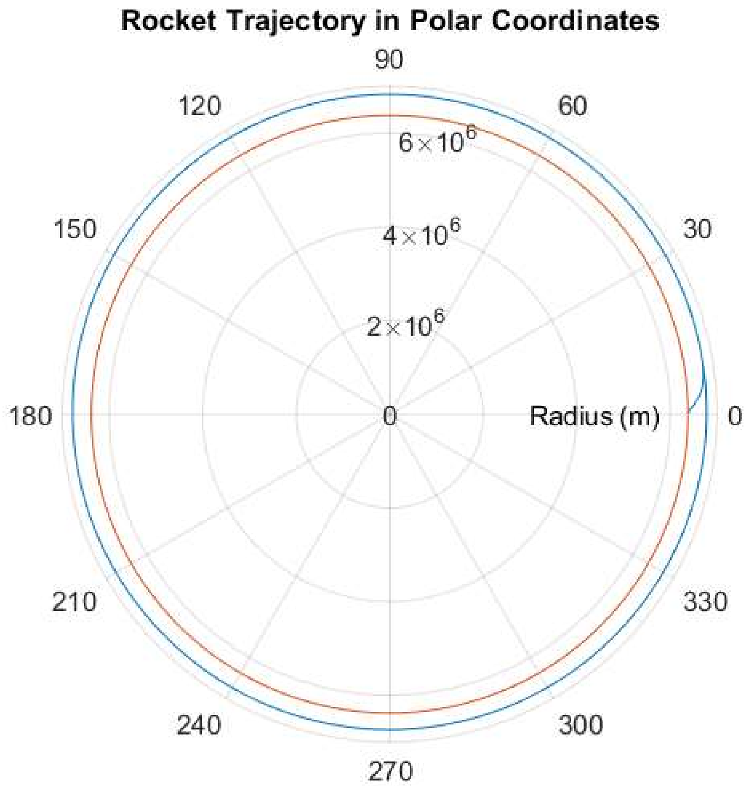

Figure 1: The position of the rocket with respect to the radius from the center of the Earth (m) and the and angle theta (degrees). The inner circle is the radius of the Earth and the outer circle is the rocket's trajectory to rendezvous with the International Space Station.

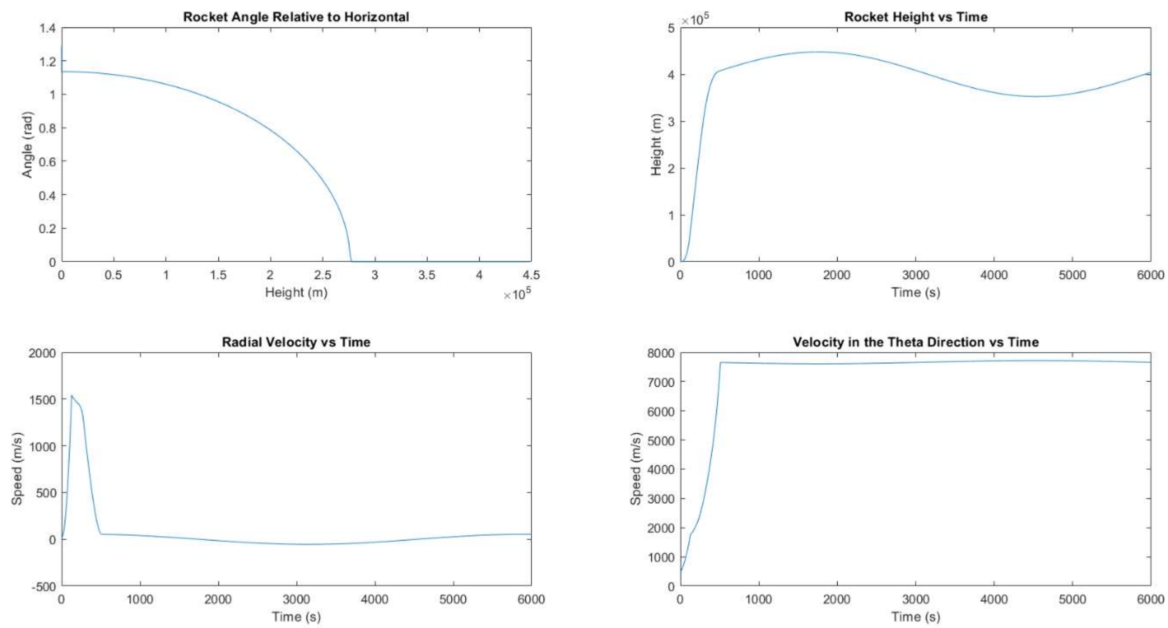

Figure 2: A (Top Left) is the angle of the rocket with respect the Earth's surface versus height. B (Top Right) is the height of the shuttle versus time. C (Bottom Left) is the radial velocity of the shuttle with respect to time. D (Bottom Right)is the velocity parallel to the earth's surface with respect to time.

In completing the launch code, several changes were made to account for different launch scenarios:

1. Changing the launch angle resulted in the shuttle either not achieving orbit and flying off into space or crashing back into the Earth, as shown in *Figure 3*. To account for a different initial launch angle, thrust times and magnitudes would require adjustments.
2. Reducing the thrust of the engines to a percentage of the maximum would cause the rocket to spend less time in the atmosphere, therefore decreasing the drag and consequently the required fuel, and is what real launches instigate.
3. Changing the mass of the shuttle required the thrust and burn time to increase to achieve orbit. This mostly affected the early stages of the launch when the shuttle and components had to overcome Earth's gravity.

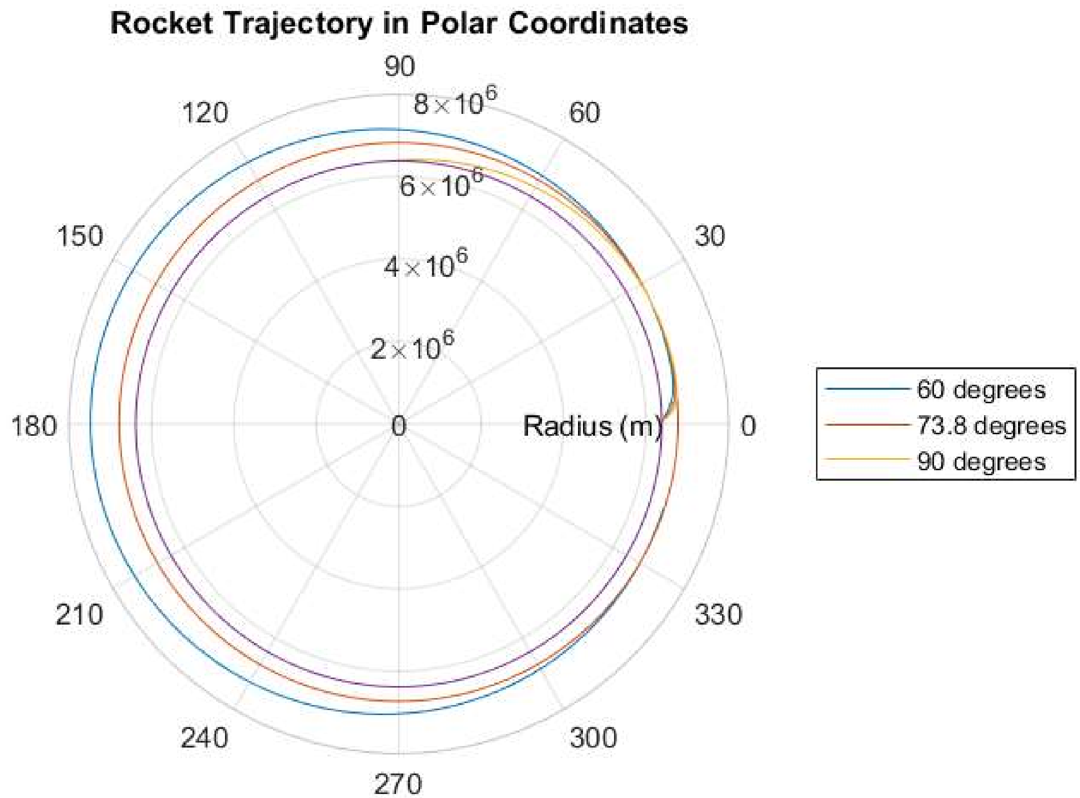

Figure 3: The trajectories of the rocket based on starting angles of 60, 73.8, and 90 degrees

Recommendations for Future Models and Conclusion

For future models, a more realistic scenario could be produced by taking into account such variables as changing air temperatures, pressures, and weather patterns. Additionally, introducing a user input prompt specifying the size of the shuttle payload could improve the usefulness of the model.

This model was greatly simplified in how it dealt with changing atmospheric conditions, flight paths, and drag. However, it provided an intriguing and informative start to understanding how to apply numerical analyses to such complicated scenarios, as well as a basis from which to build more complex versions of the shuttle launch.