3.2 Theorem

Jeawon Na

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If φ is convex on (a, b) then φ is continuous on (a, b).

proof

Let $x, y \in (a, b)$. Then $\exists \alpha$ such that $y < \alpha < b$. Since φ is convex,

$$\frac{\varphi(y) - \varphi(x)}{y - x} \leq \frac{\varphi(b) - \varphi(\alpha)}{b - \alpha}. \tag{1}$$

Let $M = \frac{\varphi(b) - \varphi(\alpha)}{b - \alpha}$. Then

$$|\varphi(y) - \varphi(x)| \leq M|y - x| \tag{2}$$

For all $\epsilon > 0$, we can choose $\delta > 0$ less then ϵ/M, and $|y - x| < \delta < \frac{\epsilon}{M}$ implies $|\varphi(y) - \varphi(x)| < M\delta < \epsilon$. Note that M depends on x and y.