Lab Session 11

MA-571 : Numerical Linear Algebra Lab 2022 R. Alam

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1. Consider the matrix $A := \begin{bmatrix} 1 & f \\ f & 1 \end{bmatrix}$, where $f = \sqrt{\texttt{eps}}$ and `eps` is matlab's machine epsilon.
   Let T be the result of 10 basic QR steps (without shift) performed on A. Determine T. Now compute eigenvalues of A using MATLAB command `eig`. Can the diagonals of T be considered as eigenvalues of A? Justify your answer. Now compute the eigenvalues of A from its characteristic polynomial. Which method is better?

2. Write a MATLAB function that implements explicit QR algorithm with Wilkinson's shift for symmetric tridiagonal matrices. Your task is to compute only eigenvalues and so no need to accumulate the Q matrices:
   ```
   function D = myqr(A, m)
   % A is a tridiagonal matrix, m is the number of iteration to be performed,
   % and D is result of m QR steps performed on A
   ```
   Use only `diag` command to generate an $n \times n$ symmetric tridiagonal matrix A such that $A(j,j) = -2$ for $j = 1 : n$ and $A(j+1, j) = 1$ for $j = 1 : n-1$. **2 marks**

   For $n = 100$, compute eigenvalues of A using the command `eig`. Now, perform 50 steps of QR algorithm with Wilkinson shift on A, that is, compute `D = myqr(A,50)`. Check the subdiagonal and diagonal entries of D and determine how many eigenvalues A have been found after 50 QR steps. Write down these eigenvalues and compare them with those computed by `eig(A)` and comment on their accuracy.

   If A_m denotes the result of m QR steps, then we know that $A_m(n, n-1) \to 0$ as $m \to \infty$ and the convergence is cubic. Prepare a table for $A_m(n, n-1)$ for as many values of m as you need to validate cubic convergence of QR algorithm. Do you observe cubic convergence?

3. Numerically, an eigenvalue is real if the imaginary part is negligible. Set `tol =` 10^{-6} and consider MATLAB code
   `A = randn(8); sum( abs( imag ( eig(A) ) ) < tol).`
   What output does this code return? Justify your answer.

   Now generate 1000 random (normally distributed) matrices of size 10. Run the code fragment on these matrices and prepare a histogram. Type `help hist` for information about histogram. What information do you get from the histogram? Can you guess the probability of the outcomes and their expectations from the histogram?

**********End**********