Exercise 1

$$P(B) = 0.01$$
$$P(\neg B) = 1 - P(B) = 1 - 0.01 = 0.99$$

$$P(Test = positive|B) = 0.95$$
$$P(Test = negative|B) = 1 - P(Test = positive|B) = 0.05$$
$$P(Test = positive|\neg B) = 1 - P(Test = negative|\neg B) = 0.1$$
$$P(Test = negative|\neg B) = 0.9$$

Solution A

$$\begin{aligned}
P(B|Test = positive) &\overset{Bayes}{=} \frac{P(Test = positive|B) * P(B)}{P(Test = positive)} \\
&\overset{T.P.F}{=} \frac{P(Test = positive|B)P(B)}{P(Test = positive)P(B) + P(Test = positive|\neg B)P(\neg B)} \\
&= \frac{0.95 * 0.01}{0.95 * 0.01 + 0.10 * 0.99} \\
&= \frac{0.0095}{0.0095 + 0.0990} \\
&\approx 0.0876
\end{aligned}$$

Solution B

$$P(B|Test = positive) = 0.5$$

$$
\begin{aligned}
P(B|Test = positive) &\overset{Bayes}{=} \frac{P(Test = positive|B) * P(B)}{P(Test = positive)} \\
&\overset{T.P.F}{=} \frac{P(Test = positive|B)P(B)}{P(Test = positive)P(B) + P(Test = positive|\neg B)P(\neg B)}
\end{aligned}
$$

$$
\begin{aligned}
&[P(Test = positive)P(B)+ \\
&P(Test = positive|\neg B)P(\neg B)] \\
&P(B|Test = positive) = P(Test = positive|B)P(B) \\
&P(Test = positive)P(B)+ \\
&P(Test = positive|\neg B)P(\neg B) = \frac{P(Test = positive|B)P(B)}{P(B|Test = positive)} \\
&P(Test = positive|\neg B)P(\neg B) = \frac{P(Test = positive|B)P(B)}{P(B|Test = positive)} - P(Test = positive)P(B) \\
&P(Test = positive|\neg B) = \frac{P(Test = positive|B)P(B)}{P(\neg B)}(\frac{1}{P(B|Test = positive)} - 1) \\
&1 - P(Test = negative|\neg B) = \frac{P(Test = positive|B)P(B)}{P(\neg B)}(\frac{1}{P(B|Test = positive)} - 1) \\
&P(Test = negative|\neg B) = 1 - \frac{P(Test = positive|B)P(B)}{P(\neg B)}(\frac{1}{P(B|Test = positive)} - 1) \\
&P(Test = negative|\neg B) = 1 - \frac{0.95 * 0.01}{0.99}(\frac{1}{0.5} - 1) \\
&= 1 - \frac{0.95 * 0.01}{0.99} \\
&= 1 - \frac{0.0095}{0.99} \\
&\approx 1 - 0.0095 \\
&= 0.9905
\end{aligned}
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