Exercise 1

What we know:
B: Is a disease
probability to get that said dissease is:

$$P(B) = 0.01$$

therefore:

$$P(\neg B) = 1 - P(B) = 0.99$$

our marginal probabilities:

$$P(Test = positive|B) = 0.95$$
$$P(Test = negative|B) = 1 - P(Test = positive|B) = 0.05$$
$$P(Test = positive|\neg B) = 1 - P(Test = negative|\neg B) = 0.1$$
$$P(Test = negative|\neg B) = 0.9$$

Solution A

We must compute $P(B|Test = positive)$

$$P(B|Test = positive) = \frac{P(Test = positive|B) * P(B)}{P(Test = positive)}$$

$$= \frac{P(Test = positive|B)P(B)}{P(Test = positive)P(B) + P(Test = positive|\neg B)P(\neg B)}$$

$$= \frac{0.95 * 0.01}{0.95 * 0.01 + 0.10 * 0.99} = \frac{0.0095}{0.0095 + 0.0990} \approx 0.0876$$

Solution B

$min(P(Test = negative|\neg B))$ such that $P(B|Test = positive) = 0.5$
From Bayes theory:

$$P(B|Test = positive) = \frac{P(Test = positive|B)P(B)}{P(Test = positive|B)P(B) + P(Test = positive|\neg B)P(\neg B)}$$

We get:

$$P(Test = positive|B)P(B) + P(Test = positive|\neg B)P(\neg B) = \frac{P(Test = positive|B)P(B)}{P(B|Test = positive)}$$

$$P(Test = positive|B)P(B) + P(Test = positive|\neg B)P(\neg B) = \frac{0.95 * 0.01}{0.5}$$

$$P(Test = positive|B)P(B) + P(Test = positive|\neg B)P(\neg B) = \frac{0.95 * 0.01}{0.5}$$
$$P(Test = positive|B)P(B) + P(Test = positive|\neg B)P(\neg B) = 0.019$$

$$P(Test = positive|\neg B)P(\neg B) = 0.019 - P(Test = positive|B)P(B)$$
$$P(Test = positive|\neg B)P(\neg B) = 0.019 - 0.95 * 0.01$$
$$P(Test = positive|\neg B)P(\neg B) = 0.0095$$
$$P(Test = positive|\neg B) = \frac{0.0095}{P(\neg B)}$$
$$P(Test = positive|\neg B) = \frac{0.0095}{0.99}$$
$$P(Test = positive|\neg B) \approx 0.0095$$

$$1 - P(Test = negative|\neg B) = 0.0095$$

$$P(Test = negative|\neg B) = 1 - 0.0095 = 0.9905$$

In conclusion, for the probability $P(B|Test = positive) = 0.5$ the minimum specificity is 0.99