(a) Since:

$$a_1 = 1, a_2 = 3, a_n = a_{n-1} + a_{n-2} (n \geq 3)$$

For

$$x^{n+2} = x^{n+1} + x^n$$

We can have:

$$x^2 - x - 1 = 0$$

Thus,

$$x_1 = \frac{1+\sqrt{5}}{2}, x_2 = \frac{1-\sqrt{5}}{2}$$

Therefore, we let $L(n) = C \cdot (\frac{1+\sqrt{5}}{2})^n + D \cdot (\frac{1-\sqrt{5}}{2})^n$ We can have:

$$\begin{cases} C \cdot (\frac{1+\sqrt{5}}{2})^1 + D \cdot (\frac{1-\sqrt{5}}{2})^1 = 1, (n = 1) \\ C \cdot (\frac{1+\sqrt{5}}{2})^2 + D \cdot (\frac{1-\sqrt{5}}{2})^2 = 3, (n = 2) \end{cases}$$

By solving the equation, we can have $C = 1, D = 1$

Thus,

$$L(n) = (\frac{1+\sqrt{5}}{2})^n + (\frac{1-\sqrt{5}}{2})^n$$

Therefore,

$$\begin{aligned} \sum_{i=1}^{n} L(n) &= \sum_{i=1}^{n} [(\frac{1+\sqrt{5}}{2})^i + (\frac{1-\sqrt{5}}{2})^i] \\ &= \frac{\frac{1+\sqrt{5}}{2} \cdot [1 - (\frac{1+\sqrt{5}}{2})^n]}{1 - \frac{1+\sqrt{5}}{2}} + \frac{\frac{1-\sqrt{5}}{2} \cdot [1 - (\frac{1-\sqrt{5}}{2})^n]}{1 - \frac{1+\sqrt{5}}{2}} \\ &= \frac{1+\sqrt{5}}{1-\sqrt{5}} \cdot [1 - (\frac{1+\sqrt{5}}{2})^n)] + \frac{1-\sqrt{5}}{1+\sqrt{5}} \cdot [1 - (\frac{1-\sqrt{5}}{2})^n)] \end{aligned}$$

(b)

$$
\begin{aligned}
&\sum_{i=1}^{n} L(n)^2 = \sum_{i=1}^{n}[(\frac{1+\sqrt{5}}{2})^i + (\frac{1-\sqrt{5}}{2})^i]^2 \\
&= \sum_{i=1}^{n}(\frac{1-\sqrt{5}}{2})^{2i} + \sum_{i=1}^{n}(\frac{1+\sqrt{5}}{2})^{2i} + 2\cdot\sum_{i=1}^{n}(-1)^i \\
&= (\frac{1-\sqrt{5}}{2})^2 \cdot \frac{1-(\frac{1-\sqrt{5}}{2})^{2n}}{1-(\frac{1+\sqrt{5}}{2})^2} + \frac{1-(\frac{1+\sqrt{5}}{2})^{2n}}{1-(\frac{1+\sqrt{5}}{2})^2} + 2(-1)^n \\
&= \frac{1-\sqrt{5}}{2}[1-\frac{1-\sqrt{5}}{2}^{2n}] - \frac{1+\sqrt{5}}{2}[1-\frac{1+\sqrt{5}}{2}^{2n}] + (-1)^n - 1
\end{aligned}
$$