1.

1. FC $\quad Y = W^T X + b$

$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial W} = X^T \frac{\partial L}{\partial Y}$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial b} = \cdot \frac{\partial L}{\partial Y}$$

$$\frac{\partial L}{\partial X} = \frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial X} = \frac{\partial L}{\partial Y} \cdot W^T$$

2. ReLU

$$\cancel{\frac{\partial L}{\partial X} = \frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial X} = \frac{\partial L}{\partial Y}}$$

$$\frac{\partial L}{\partial x_i} = \left(\frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial x}\right)_i = \begin{cases} \frac{\partial L}{\partial Y_i} & \text{if } x_i > 0 \\ 0 & \text{if } x_i \le 0 \end{cases}$$

3. Dropout

$$\frac{\partial L}{\partial X} = \frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial X} = \frac{\partial L}{\partial Y} \cdot M$$

4. BN

$$\frac{\partial \mu}{\partial x_i} = \frac{1}{n} \qquad \cancel{\frac{\partial \sigma}{\partial x_i} = \frac{1}{2}\left(\frac{1}{n}\sum_j (x_j-\mu)^2+\epsilon\right) \cdot \frac{2}{n}(x_i-\mu)}$$

$$\frac{\partial \sigma}{\partial x_i} = \frac{1}{2\sigma} \cdot \frac{1}{n} \cdot \left(\sum_j (x_j-\mu)^2\right)'$$

$$= \frac{1}{2\sigma n}\left(\sum_j 2(x_j-\mu)(x_j-\mu)'\right)$$

$$= \frac{1}{\sigma n^2} \cdot \left(-\sum_{j \neq i} (x_j-\mu) + (n-1)\cdot(x_i-\mu)\right)$$

$$\frac{\partial L}{\partial x_i} = \frac{\partial L}{\partial Y_i} \cdot \frac{\partial Y_i}{\partial x_i}$$

$$\cancel{= \frac{\partial L}{\partial Y}\left(\gamma - \frac{(x_i-\mu)\frac{d\sigma}{\partial x_i}}{\sigma^2}\right)}$$

$$= \frac{\partial L}{\partial Y}\gamma \cdot \frac{\frac{\partial(x_i-\mu)}{\partial x_i}\sigma - (x_i-\mu)\cdot\frac{d\sigma}{\partial x_i}}{\sigma^2}$$

$$= \frac{\partial L}{\partial Y}\cdot\gamma\frac{1}{\sigma^2}\left(\sigma\cdot\left(1-\frac{1}{n}\right) - (x_i-\mu)\cdot\frac{1}{\sigma n^2}\cdot\left(-\sum_{j\neq i}(x_j-\mu)+(n-1)(x_i-\mu)\right)\right)$$

5. Conv

$$H-H'+1+2(H'-1) = H+H'-1$$

$$W-W'+1+2(W'-1) = W+W'-1$$

$$Y_{n,f} = \sum_c X_{n,c} *_{\text{valid}} \overline{W_{f,c}}$$

$$(Y_{n,f})_{ij} = \sum_{m',n'} X_{m',n'} \cdot \overline{W}_{i-m'+H',\,j-n'+W'}$$

$$\left(\frac{\partial L}{\partial X_{n,c}}\right)_{ij} = \left(\frac{\partial L}{\partial Y_{n,f}}\cdot\frac{\partial Y_{n,f}}{\partial X_{n,c}}\right)_{ij} = \sum_f \sum_{m',n'} W_{m',n'}\left(\frac{\partial L}{\partial Y_{n,f}}\right)_{i-m'+1,\,j-n'+1}$$

$$\frac{\partial L}{\partial X_{n,c}} = \sum_f W_{f,c} *_{\text{full}} \frac{\partial L}{\partial Y_{n,f}}$$

$$\left(\frac{\partial L}{\partial W_{f,c}}\right)_{ij} = \left(\frac{\partial L}{\partial Y_{n,f}}\cdot\frac{\partial Y_{n,f}}{\partial W_{f,c}}\right)_{ij} = \sum_n \sum_{m',n'} X_{nc\,(i+m'-1,\,j+n'-1)}\left(\frac{\partial L}{\partial Y_{n,f}}\right)_{m'n'}$$

$$\frac{\partial L}{\partial W_{f,c}} = \sum_n X_{n,c} *_{\text{full}} \frac{\partial L}{\partial Y_{n,f}}$$