

1.

1. FC $Y = W^T X + b$

$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial W} = X^T \frac{\partial L}{\partial Y}$$

$$\frac{\partial L}{\partial b} = \frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial b} = \frac{\partial L}{\partial Y}$$

$$\frac{\partial L}{\partial X} = \frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial X} = \frac{\partial L}{\partial Y} \cdot W^T$$

2. ReLU

~~$$\frac{\partial L}{\partial X} = \frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial X} = \frac{\partial L}{\partial Y}$$~~

$$\frac{\partial L}{\partial X_i} = \left(\frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial X} \right)_i = \begin{cases} \frac{\partial L}{\partial Y_i} & \text{if } x_i > 0 \\ 0 & \text{if } x_i \leq 0 \end{cases}$$

3. Dropout

$$\frac{\partial L}{\partial X} = \frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial X} = \frac{\partial L}{\partial Y} \cdot M$$

4. BN

~~$$\frac{\partial \mu}{\partial X_i} = \frac{1}{n} \quad \frac{\partial \sigma}{\partial X_i} = \frac{1}{2} \left(\frac{1}{n} \sum_j (x_j - \mu)^2 + \epsilon \right)^{-1/2} \cdot \frac{2}{n} (X_i - \mu)$$~~

$$\frac{\partial \sigma}{\partial X_i} = \frac{1}{2\sigma} \cdot \frac{1}{n} \cdot \left(\sum_j (x_j - \mu)^2 \right)^{-1/2}$$

$$= \frac{1}{2\sigma n} \left(\sum_j 2(x_j - \mu)(x_j - \mu)' \right)$$

$$= \frac{1}{\sigma n^2} \left(-\sum_j (x_j - \mu) + (n-1) \cdot (X_i - \mu) \right)$$

$$\begin{aligned}
 \frac{\partial L}{\partial x_i} &= \frac{\partial L}{\partial Y_i} \cdot \frac{\partial Y_i}{\partial x_i} \\
 &= \frac{\partial L}{\partial Y} \left(Y - \frac{(x_i - \mu) \frac{\partial \sigma}{\partial x_i}}{\sigma^2} \right) \\
 &= \frac{\partial L}{\partial Y} \cdot Y \cdot \frac{\frac{\partial(x_i - \mu)}{\partial x_i} \sigma - (x_i - \mu) \frac{\partial \sigma}{\partial x_i}}{\sigma^2} \\
 &= \frac{\partial L}{\partial Y} \cdot Y \cdot \frac{1}{\sigma^2} \left(\sigma \cdot \left(1 - \frac{1}{n}\right) - (x_i - \mu) \cdot \frac{1}{\sigma n^2} \cdot \left(- \sum_{j \neq i} (x_j - \mu) + (n-1)(x_i - \mu) \right) \right)
 \end{aligned}$$

5. Conv

$$\begin{aligned}
 H - H' + 1 + 2(H' - 1) &= H + H' - 1 \\
 W - W' + 1 + 2(W' - 1) &= W + W' - 1
 \end{aligned}$$

$$\begin{aligned}
 Y_{n,f} &= \sum_c X_{n,c} \cdot \text{valid } W_{f,c} \\
 (Y_{n,f})_{ij} &= \sum_{m,n'} X_{n,m} \cdot \bar{W}_{i-m+H', j-n'+W'}
 \end{aligned}$$

$$\left(\frac{\partial L}{\partial X_{n,c}} \right)_{ij} = \left(\frac{\partial L}{\partial Y_{n,f}} \cdot \frac{\partial Y_{n,f}}{\partial X_{n,c}} \right)_{ij} = \sum_{m,n'} \bar{W}_{i-m+H', j-n'+W'} \left(\frac{\partial L}{\partial Y_{n,f}} \right)_{i-m+H', j-n'+W'}$$

$$\begin{aligned}
 \frac{\partial L}{\partial X_{n,c}} &= \sum_f W_{f,c} \cdot \frac{\partial L}{\partial Y_{n,f}} \\
 \left(\frac{\partial L}{\partial W_{f,c}} \right)_{ij} &= \left(\frac{\partial L}{\partial Y_{n,f}} \cdot \frac{\partial Y_{n,f}}{\partial W_{f,c}} \right)_{ij} = \sum_n \sum_{m,n'} X_{n,m} \cdot \frac{\partial L}{\partial Y_{n,f}} \cdot \left(\frac{\partial Y_{n,f}}{\partial W_{f,c}} \right)_{m,n'}
 \end{aligned}$$

$$\frac{\partial L}{\partial W_{f,c}} = \sum_n X_{n,c} \cdot \frac{\partial L}{\partial Y_{n,f}}$$

EECS 498/598 Deep Learning HW1

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1. Logistic regression and beyond: Binary classification with a sigmoid

output layer

- a. No hidden units:
 - i. Performance: train acc: 0.964000; val acc: 0.952000
 - ii. Model: input_dim=20, hidden_dim=None, weight_scale=1e-3, reg=0.2
 - iii. Solver: update rule: sgd_momentum, learning_rate=1, lr_decay=0.95, num_epochs=20, batch_size = 50
- b. With hidden units:
 - i. Performance: train acc: 0.962000; val_acc: 0.952000
 - ii. Model: input_dim=20, hidden_dim=32, weight_scale=1e-3, reg=0.3
 - iii. Solver: update rule: sgd_momentum, learning_rate=8e-2, lr_decay=0.9, num_epochs=20, batch_size = 50

2. SVM and beyond: binary classification with a hinge-loss output

layer

- a. No hidden units:
 - i. Performance: train acc: 0.964000; val_acc: 0.944000
 - ii. Model: input_dim=20, hidden_dim=None, weight_scale=1e-3, reg=0.1
 - iii. Solver: update rule: sgd_momentum, learning_rate=1, lr_decay=0.95, num_epochs=50, batch_size = 50
- b. With hidden units:
 - i. Performance: train acc: 0.968000; val_acc: 0.928000
 - ii. Model: input_dim=20, hidden_dim=32, weight_scale=1e-3, reg=0.3
 - iii. Solver: update rule: sgd_momentum, learning_rate=1e-1, lr_decay=0.95, num_epochs=50, batch_size = 50

3. Softmax regression and beyond: multi-class classification with a

softmax output layer

- a. No hidden units:
 - i. Performance: train acc: 0.953000; val_acc: 0.935556
 - ii. Model: input_dim=784, hidden_dim=None, weight_scale=1e-3,

- reg=0.01
 - iii. Solver: update rule: rmsprop, learning_rate=5e-5, lr_decay=0.9, num_epochs=50, batch_size = 100
- b. With hidden units:
 - i. Performance: train acc: 0.993000; val_acc: 0.977037
 - ii. Model: input_dim=784, hidden_dim=64, weight_scale=1e-3, reg=0.01
 - iii. Solver: update rule: rmsprop, learning_rate=1e-4, lr_decay=0.9, num_epochs=50, batch_size = 100

4. Convolutional Neural Network for multi-class classification

- a. No batch normalization and drop out:
 - i. Performance: train acc: 0.984000; val_acc: 0.976200
 - ii. Model: input_dim=(1,28,28), num_filters=5, filter_size =7, hidden_dim=16, num_classes=10, weight_scale=1e-3, reg=0
 - iii. Solver: update rule: sgd_momentum, learning_rate=1e-3, lr_decay=0.9, num_epochs=50, batch_size = 100
- b. With batch normalization and drop out:
 - i. Performance: train acc: 0.914000; val_acc: 0.901200
 - ii. Model: input_dim=(1,28,28), num_filters=8, filter_size =7, hidden_dim=16, num_classes=10, weight_scale=1e-3, reg=0
 - iii. Solver: update rule: sgd_momentum, learning_rate=1e-2, lr_decay=0.9, num_epochs=50, batch_size = 100

5. Convolutional Neural Networks for CIFAR10 image classification.

- a. Performance: accuracy: 74%
- b. Architecture Improvement:
 - i. Double the filter_num of first two conv layers
 - ii. Add batch normalization and relu after each conv layers
 - iii. Add drop out after first fc layer
- c. Solver:
 - i. Learning_rate = 0.001
 - ii. Batch_size = 100
 - iii. Criterion = crossentropy loss
 - iv. Optimizer = adam

6. Short answer questions

- a. Sigmoid projects large inputs to 0 or 1, causing its dsigmoid = sigmoid * (1 - sigmoid) to vanish. In DNN, the gradient vanishing is even more severe due to chain rule, resulting in difficulty in training.

- b. When perform W in testing, multiply x by p to account for missing activations in training.
- c. I will reduce learning rate.