Week 2

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Day 2

Counting Techniques

(The mn Rule [Fundamental Counting Principle])

- First stage = m & Second stage = n
- mn ways to accomplish an experiment

(Extended mn Rule)

- k stages with n_1 ways for the first stage, n_2 ways for the second stage, and n_k ways to for the k^{th} stage
- $\prod_{x=1}^{k} n_x$

Permutation

Arrangement of Objects

(Distinct Permutation)

- Permutation of n objects is n!
- Permutation of n objects r at a time

$$P(n,r) = \frac{n!}{(n-1)!}$$

$$= nPr$$

$$= P_r^n$$

• Note that P(n,n) = n!

(Repeating Permutation)

Permutation of n objects with k types

$$\frac{n!}{n_1!n_2!\cdots n_k!}$$

(Circular Permutation)

• Permutation in a circle with one fixed object is (n-1)!

Combination

• Counting without arrangement

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

Probability

(Uniform Probability Model)

$$P(A) = \frac{\text{no. of simple events in A}}{\text{no. of simple events in S}}$$
$$= \frac{|A|}{|S|}$$

• In cases where out comes are <u>not equally likely to occur</u>:

$$P(A) = P(A_1) + P(A_2) + \cdots + P(A_n)$$

Note that:

1.
$$0 \le P(A) \le 1$$

2.
$$P(\emptyset) = 0$$

$$3. P(S) = 1$$

Union and Intersection

$$P(A \cup B) = \frac{|A \cup B|}{|S|}$$

$$= P(A) + P(B)$$

$$P(A \cap B) = \frac{|A \cap B|}{|S|}$$

Note the Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

• Compliment $\longrightarrow P(A^C) = 1 - P(A)$

Conditional Probability

(Dependent Events)

- An event occurring affects the probability of the following event
- Probability of A given event B has occurred

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

• Note the Multiplication Rule

$$P(A \cap B) = P(A|B) \cdot P(B)$$
$$= P(B|A) \cdot P(A)$$

(Independent Events)

- Probability of one does not affect the other
- Independent if

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

or
$$P(A \cap B) = P(A) \cdot P(B)$$

- Mutual Independence
 - Events A_1,A_2,\cdots,A_n are mutually independent if each pair of events A_i and A_j are independent.

$$P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1) \cdot P(A_2) \cdot \cdots \cdot P(A_n)$$

Probability Distribution

- A formula, table, or graph that gives all the possible values k of the discrete random variable X , and the probability $p_X(k)=\,P\,(X\,=\,k) \mbox{ associated with each value}$
- $p_X(k) \geq 0$
- $\bullet \quad \sum_{\text{all } k} p_x(k) = 1$

(Example)

Toss two fair coins and let \mathbf{X} be the number of heads observed. Find the probability distribution for \mathbf{X} .

Simple Event	Coin 1	Coin 2	Probability of Simple Event	Number of Heads Observed
	Н	Н		2
	Н	Т		1
	Т	Н		1
	Т	Т		0

Probability Distribution Function (pdf)

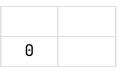
$$p_X(k) = \begin{cases} \frac{1}{4} & \text{if } k = 0 \\ \frac{1}{2} & \text{if } k = 1 \\ \frac{1}{4} & \text{if } k = 2 \end{cases}$$

Probability Distribution Table

0	
1	
2	

Cumulative Distribution

• formula, table or graph that gives all the possible values k and $F_X(k) = P(X \le k)$, the probability that X is at most k



1	
2	

Mean or Expected Value

• The average value of X in the population

$$\mu = \, E \, (X \,) = \, \textstyle \sum_{\text{all } k} k \cdot p_X(k)$$

Standard Deviation and Variance

(Standard Deviation)

• Measures the spread or variability of the random variable

$$\sigma = \sqrt[4]{\frac{E((X - \mu)^2)}{\sum_{\text{all } k} (k - \mu)^2 \cdot p_X(k)}}$$

(Variance)

$$\sigma^2 = E((X - \mu)^2)$$
$$= \sum_{\text{all } k} (k - \mu)^2 \cdot p_X(k)$$

Binomial Distribution

- Experiment consists of n identical trials
- Each trial results in one of two outcomes
- The probability of success on a single trial is equal to p and remains from trial to trial. Failure, $q=\,1-\,p$
- Trials are independent
- Each trial is called a Bernoulli Trial

(Example)

3 distinguishable biased coins \longrightarrow 0.60 heads.

Coin 1	Coin 2	Coin 3	Number of Heads	Probability
Н	Н	Н	3	math 0.60^3
Н	Н	Т	2	
Н	T	Н	2	

Coin 1	Coin 2	Coin 3	Number of Heads	Probability
Н	Т	Т	1	
Т	Н	Н	2	
Т	Н	Т	1	
Т	Т	Н	1	
Т	Т	Т	0	

 0.6^{3} 0.216