Week 2

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Day 2

Counting Techniques

(The mn Rule [Fundamental Counting Principle])

- First stage = m & Second stage = n
- mn ways to accomplish an experiment

(Extended mn Rule)

- ullet stages with n_1 ways for the first stage, n_2 ways for the second stage, and n_k ways to for the k^{th} stage
- $\prod_{x=1}^k n_x$

Permutation

Arrangement of Objects

(Distinct Permutation)

- Permutation of n objects is n!
- ullet Permutation of n objects r at a time

$$P(n,r) = rac{n!}{(n-1)!} \ = nPr \ = P_r^n$$

• Note that P(n,n) = n!

(Repeating Permutation)

ullet Permutation of n objects with k types

$$\frac{n!}{n_1!n_2!\cdots n_k!}$$

(Circular Permutation)

• Permutation in a circle with one fixed object is (n-1)!

Combination

• Counting without arrangement

$$C(n,r)=rac{n!}{r!(n-r)!}$$

Probability

(Uniform Probability Model)

$$P(A) = rac{ ext{no. of simple events in } A}{ ext{no. of simple events in } S} = rac{|A|}{|S|}$$

• In cases where out comes are <u>not equally likely to occur</u>:

$$P(A) = P(A_1) + P(A_2) + \cdots + P(A_n)$$

- Note that:
 - 1. $0 \le P(A) \le 1$
 - 2. $P(\varnothing) = 0$
 - 3. P(S) = 1
- Union and Intersection

$$P(A \cup B) = rac{|A \cup B|}{|S|} = P(A) + P(B)$$

$$P(A\cap B)=rac{|A\cap B|}{|S|}$$

Note the Addition Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

• Compliment $\longrightarrow P(A^C) = 1 - P(A)$

Conditional Probability

(Dependent Events)

- An event occurring affects the probability of the following event
- ullet Probability of A given event B has occurred

$$P(A|B) = rac{P(A\cap B)}{P(B)}$$

• Note the Multiplication Rule

$$P(A \cap B) = P(A|B) \cdot P(B)$$
$$= P(B|A) \cdot P(A)$$

(Independent Events)

- Probability of one does not affect the other
- Independent if

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

or
$$P(A \cap B) = P(A) \cdot P(B)$$

- Mutual Independence
 - Events A_1,A_2,\cdots,A_n are mutually independent if each pair of events A_i and A_j are independent.

$$P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1) \cdot P(A_2) \cdot \cdots \cdot P(A_n)$$

Probability Distribution

- A formula, table, or graph that gives all the possible values k of the discrete random variable X, and the probability $p_X(k)=P(X=k)$ associated with each value
- $p_X(k) \ge 0$
- $ullet \sum_{ ext{all } k} p_x(k) = 1$

(Example)

Toss two fair coins and let X be the number of heads observed. Find the probability distribution for X.

Simple Event	Coin 1	Coin 2	Probability of Simple Event	Number of Heads Observed
	Н	Н		2
	Н	Т		1
	Т	Н		1
	Т	Т		0

Probability Distribution Function (pdf)

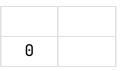
$$p_X(k) = egin{cases} rac{1}{4} ext{ if } k=0 \ rac{1}{2} ext{ if } k=1 \ rac{1}{4} ext{ if } k=2 \end{cases}$$

Probability Distribution Table

0	
1	
2	

Cumulative Distribution

• formula, table or graph that gives all the possible values k and $F_X(k)=P(X\leq k)$, the probability that X is at most k



1	
2	

Mean or Expected Value

$$\mu = E(X) = \sum_{ ext{all } k} k \cdot p_X(k)$$