Week 2

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Day 2

Counting Techniques

(The mn Rule [Fundamental Counting Principle])

- First stage = m & Second stage = n
- mn ways to accomplish an experiment

(Extended mn Rule)

- k stages with n_1 ways for the first stage, n_2 ways for the second stage, and n_k ways to for the k^{th} stage
- $\prod_{x=1}^{k} n_x$

Permutation

Arrangement of Objects

(Distinct Permutation)

- Permutation of n objects is $n!$
- Permutation of n objects r at a time

$$\begin{aligned} P(n,r) &= \frac{n!}{(n-1)!} \\ &= nPr \\ &= P_r^n \end{aligned}$$

- Note that $P(n,n) = n!$

(Repeating Permutation)

- Permutation of n objects with k types

$$\frac{n!}{n_1!n_2!\cdots n_k!}$$

(Circular Permutation)

- Permutation in a circle with one fixed object is $(n-1)!$

Combination

- Counting without arrangement

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

Probability

(Uniform Probability Model)

$$\begin{aligned} P(A) &= \frac{\text{no. of simple events in A}}{\text{no. of simple events in S}} \\ &= \frac{|A|}{|S|} \end{aligned}$$

- In cases where out comes are <u>not equally likely to occur</u>:

$$P(A) = P(A_1) + P(A_2) + \cdots + P(A_n)$$

- Note that:
  1. $0 \le P(A) \le 1$
  2. $P(\emptyset) = 0$
  3. $P(S) = 1$
- Union and Intersection

$$P(A \cup B) = \frac{|A \cup B|}{|S|}$$
$$= P(A) + P(B)$$

$$P(A \cap B) = \frac{|A \cap B|}{|S|}$$

- Note the **Addition Rule**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Compliment $\longrightarrow P(A^C) = 1 - P(A)$

Conditional Probability

(Dependent Events)

- An event occurring affects the probability of the following event
- Probability of A given event B has occurred

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- Note the **Multiplication Rule**

$$\begin{aligned} P(A \cap B) &= P(A|B) \cdot P(B) \\ &= P(B|A) \cdot P(A) \end{aligned}$$

(Independent Events)

- Probability of one does not affect the other
- Independent if

$$P(A|B) = P(A)$$
$$\text{or}$$
$$P(B|A) = P(B)$$

$$\text{or}$$
$$P(A \cap B) = P(A) \cdot P(B)$$

- Mutual Independence
  - Events $A_1, A_2, \cdots, A_n$ are mutually independent if each pair of events A_i and A_j are independent.

$$P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1) \cdot P(A_2) \cdot \cdots \cdot P(A_n)$$

Day 3

Probability Distribution

- A formula, table, or graph that gives all the possible values k of the discrete random variable X, and the probability $p_X(k) = P(X = k)$ associated with each value
- $p_X(k) \geq 0$
- $\sum_{\text{all } k} p_x(k) = 1$

(Example)
Toss two fair coins and let X be the number of heads observed. Find the probability distribution for X.

Simple Event	Coin 1	Coin 2	Probability of Simple Event	Number of Heads Observed
	H	H		2
	H	T		1
	T	H		1
	T	T		0

Probability Distribution **Function** (pdf)

$$p_X(k) = \begin{cases} \frac{1}{4} \text{ if } k = 0 \\ \frac{1}{2} \text{ if } k = 1 \\ \frac{1}{4} \text{ if } k = 2 \end{cases}$$

Probability Distribution Table

0	
1	
2	

Cumulative Distribution

- formula, table or graph that gives all the possible values k and $F_X(k) = P(X \leq k)$, the probability that X is at most k

0	

1	
2	

Mean or Expected Value

- The average value of X in the population

$$\mu = E(X) = \sum_{\text{all } k} k \cdot p_X(k)$$

Standard Deviation and Variance

(Standard Deviation)

- Measures the spread or variability of the random variable

$$\begin{aligned} \sigma &= \sqrt{E((X-\mu)^2)} \\ &= \sqrt{\sum_{\text{all } k} (k-\mu)^2 \cdot p_X(k)} \end{aligned}$$

(Variance)

$$\begin{aligned} \sigma^2 &= E((X-\mu)^2) \\ &= \sum_{\text{all } k} (k-\mu)^2 \cdot p_X(k) \end{aligned}$$

Binomial Distribution

- Experiment consists of n identical trials
- Each trial results in one of two outcomes
- The probability of success on a single trial is equal to p and remains from trial to trial. Failure, $q = 1 - p$
- Trials are independent
- Each trial is called a Bernoulli Trial

(Example)
3 distinguishable biased coins $\longrightarrow$ 0.60 heads.

Coin 1	Coin 2	Coin 3	Number of Heads	Probability
H	H	H	3	`math 0.60^3 `
H	H	T	2	
H	T	H	2	

Coin 1	Coin 2	Coin 3	Number of Heads	Probability
H	T	T	1	
T	H	H	2	
T	H	T	1	
T	T	H	1	
T	T	T	0	

0.6^3

0.216