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OR PRACTICE

Optimization of Vacation Timeshare Scheduling

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This paper reports on an application of network-flow integer programming to a vacation timeshare exchange problem. A typical timeshare owner has purchased yearly access to a specific week at a specific resort. The resulting lack of vacation variety is mitigated by systems that allow owners to exchange owned weeks for different weeks at different resorts according to their preferences, the assessed value of what they are exchanging, their contractual priority, and resort availability. The timeshare exchange problem is similar to other preference-based assignment problems such as labor scheduling, preferential bidding, and traditional timetabling, but different in the formulation of the objective function. This paper demonstrates how the effectiveness of timeshare exchange processes can be improved through mathematical optimization, as measured by increased satisfaction of participant preferences. Optimization also presents exchange managers with the opportunity to more precisely manage preference and priority trade-offs among various classes of participants. The trade-off decisions are aided by sensitivity analysis utilizing a minmax criterion.

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Introduction

This paper reports on a study of customer scheduling that exists in the hospitality portion of the service sector. Specifically, the situation involves the timeshare industry, where individuals “own” specific weeks of resort properties or own credits for annual access to weeks of resort properties. (These units of ownership are called *intervals*.) In the United States alone, there are more than 1,600 timeshare resorts with an estimated annual economic impact of $92 billion (American Resort Development Association 2006).

This research considers an assignment problem that occurs in a timeshare exchange system. Timeshare exchanges are extremely popular because they allow timeshare owners to have vacation variety from year to year (Withiam 1993). A 2002 survey of 1,062 timeshare owners revealed that 62% planned to participate in an exchange within the next 12 months, which emphasizes how important exchanges are to the timeshare industry (RCI 2002). The company studied in this paper is Owners’ Resorts and Exchange (ORE), a timeshare management company based in the United States, which manages resorts in North America. ORE offers a program called “Annual Scheduling” (AS), in which owners of various resort intervals can participate in an exchange process that occurs once per year. AS differs from traditional timeshare exchanges in that AS is an event, whereas traditional exchanges are ongoing.

AS operates as follows: Once each year all interval owners desiring to participate in AS submit their owned intervals to the AS exchange pool and also submit lists of choices for substitute intervals. The participants are given assignment priorities based on what they own, including subpriority levels that rotate values year to year (to give more people access to the best resort weeks). Priority rules are dictated by ownership contracts and company policies, yet are nonetheless subject to some degree of interpretation. All exchange choices must be submitted to ORE by May 31 of the specific year, with the choices being for resort assignments for the following calendar year. On a day in June, the AS participants’ choices are considered as a large-scale assignment problem.

Most interval owners who are allowed to participate in AS have the option of instead participating in a more traditional exchange, wherein they submit their intervals and look for substitute intervals at any time of the year. In the past, AS has been quite effective at satisfying the requests of most participants at a level that is believed to be superior to traditional exchanges. Nevertheless, ORE management would prefer to increase the effectiveness of AS as much as possible because more successful exchanges means more collected exchange fees and greater loyalty from the interval owners associations who contract with ORE.

The purpose of this paper is to describe the successful application of optimization to the AS problem. Three major objectives of this paper are: (1) to demonstrate how the complexities of AS were incorporated in a solvable optimization model, (2) to illustrate the dilemma that comes from having multiple objectives, and (3) to show how sensitivity analysis can be used to mitigate that dilemma.

The next section will discuss related literature. A mathematical formulation is then developed, including the complexities of interpreting the priority rules in forming candidate objective functions. A section discusses the solution methodology, data collection, and results. Managerial decision implications are then discussed. A letter from the company president, which attests to the value of this application, is included in the online appendix. An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

Summary of Related Literature

A search of literature revealed no prior published research concerning the application of optimization techniques to the timeshare industry. The closest published research involves fractional aircraft ownership scheduling, which Martin et al. (2003) says is similar to timeshare resort scheduling except "owners are guaranteed access to an aircraft whenever and wherever they need it with as little as four hours of notice" (p. 23). One key element of successful timeshare resort scheduling is advance planning, which is particularly important in applying mathematical optimization as we describe below.

The AS problem is a people-scheduling problem, and is related to other people-scheduling problems such as services staffing (Mabert 1986) and scheduling (Mason et al. 1998, Sarin and Aggarwal 2001). Some of the service labor scheduling research assumes the task assignments are homogenous (Easton and Rossin 1996, Mason et al. 1998, Thompson 1992), thus allowing preferences for time periods but not task assignments. However, other such research considers multiple tasks with distinct preferences for assigning given individuals to given tasks (Loucks and Jacobs 1991, Love and Hoey 1990, Ritzman et al. 1976). In some cases, the assignments are governed by complex rule systems, as is the case with AS (e.g., Loucks and Jacobs 1991, Rachamadugu 1991). Perhaps the most similar problem type is nurse scheduling (Jaumard et al. 1998).

A primary difference between traditional labor scheduling problems and the AS assignment problem is in objective function specification. A variety of objective functions have been used in labor scheduling (Bechtold et al. 1991). One common objective is maximizing the total labor hours scheduled (e.g., Mabert and Showalter 1990, Mason et al. 1998). Other objectives include minimizing the number of employees scheduled (Baker et al. 1979, Easton and Rossin 1991, Hung 1994), minimizing labor costs (Easton and Rossin 1996, Jaumard et al. 1998, Lauer et al. 1994), minimizing overstaffing (Loucks and Jacobs 1991, Love and Hoey 1990), and maximizing the utilization of employed labor (Jacobs and Bechtold 1993). These are all highly correlated with minimizing the total number of labor hours.

Granted, some instances of labor scheduling take into consideration employee preferences for assignments (Thompson 1999). This is particularly true of nurse scheduling (Miller et al. 1976, Warner 1976). Nevertheless, even in nurse scheduling, the primary objective (or constraint) is meeting the given task requirements, which is not the case for the AS problem.

Other related topics from the research literature include preferential bidding such as airline crew rostering (e.g., Cappanera and Gallo 2004), conference scheduling (e.g., Eglese and Rand 1987, Sampson and Weiss 1995), exam scheduling (e.g., Carter and Laporte 1996), class timetabling (e.g., Graves et al. 1993, Schaerf 1999), and even sports scheduling (e.g., Nemhauser and Trick 1998). As will be discussed, AS has a unique set of contextual issues that provide unique complexities. We will also describe how the AS problem is an interesting instance of multicriteria decision making (MCDM), with competing objectives to be satisfied (Keeney and Raiffa 1976).

AS Problem Formulation

The objective of AS is to assign participants to resort weeks according to their preferences. What makes AS complex is that there are different classes of participants with different assignment rights pertaining to different types of interval inventory (i.e., intervals available to be assigned to participants). When ORE took over the management of specific properties, they also had to assume the contracts and traditions that defined what each interval owner had coming to him or her each year. This often included some specification of rights and priorities that pertain to participation in AS. The following describes participant classes.

Classes of AS Participants

MROP Owners. The Multi Resort Ownership Plan (MROP) is an association that owns a pool of resort weeks that are not owned by any one particular owner. The MROP association actually owns the intervals (in 18 different resorts, as of this writing). MROP owners do not own a specific interval, but simply a credit. Their ownership is called *floating weeks* because the vacation week (and sometimes resort) typically changes from year to year. Some floating weeks are always "red," whereas others rotate colors year to year (colors are explained below).

Some MROP owners have one or more "home resorts" and thus have special priority for getting assignments into those resorts. Further, all MROP owners have some degree of priority for requested assignment into any of the base MROP inventory, even if it is not in one of their home resorts. MROP owners are subject to the standard color scheme, which categorizes intervals according to an estimation of popularity: "red" for high popularity, "white" for mid-popularity, and "blue" for off season (some resort destinations like Hawaii are red all year round). MROP participants can only request assignments in intervals that are the color they own.

Gold Owners. Gold owners typically own intervals in prime resorts. They participate in AS by submitting their intervals to the AS inventory pool and paying the AS participation fee. Gold owners can select from any available week—red, white, or blue. One condition particular to gold owners is that they are guaranteed to either get a week of their choice or to get their original submitted week back; that guarantee is not extended to nongold owners.

Other Owners. Owners of intervals at various other resorts can participate in AS by submitting their intervals to the AS inventory pool and paying the participation fee. These owners generally own weeks of resorts that are listed in the AS inventory catalog. Some own "rotating weeks" of a given resort, in which the week assignment automatically changes on a 20-year rotation. Unlike gold owners, these other owners are not guaranteed to get their original weeks back if they do not get an assignment from AS. Instead, they can be left to seek an interval from the regular ongoing exchange system. Like MROP owners, these other owners can only request intervals that are the same popularity color as what they submit to AS inventory.

Once participant preferences are gathered, the AS assignment process is to grant requests based on participant class and priority. Priority numbers between 1 and 5 are assigned to AS participants on a five-year rotation (1, 5, 2, 3, then 4), with different participants at different stages of the rotation. This means that once every five years an individual will get preferential (priority 1) treatment, followed by a year of less-preferential (priority 5) treatment, and so forth.

Preferences were collected by forms mailed out a few months before the May 31st AS deadline. The mailing also included a current list of resorts, showing which weeks are red, white, or blue for each property. The list shows how many units (condominiums) are part of the base AS inventory, which participants can use to estimate the likelihood of getting a week at a given resort. This also requires estimating demand for a given resort week, which is not known prior to AS. A resort catalog and company website also provide information about various resorts. Each participant returns a list of interval "choices" they would like to receive from the AS exchange, which is considered to be a ranked list.

Assignment Rules

The major source of complexity in developing an AS formulation comes from rules that define which participant requests for intervals have priority. The variations of distinctions of priority are summarized in the following rules (listed somewhat from highest priority to lowest):

RULE 1. Gold owners have priority for getting their own interval back if none of their other requests can be met.

RULE 2. MROP owners with home resorts have priority for requests in those resorts.

RULE 3. All MROP owners have some priority for requests in any MROP inventory.

RULE 4. Gold owners have priority over red, white, and blue owners, except for those who fall under Rules 2 or 3. (However, gold owners have priority over MROP owners for their original week if they cannot get something else.)

RULE 5. Within Rules 2, 3, or 4, owners with higher priority (i.e., lower priority numbers) have precedence over owners with lower priority (i.e., higher priority numbers).

RULE 6. The ranking of choices should be respected as much as possible. Obviously, participants prefer to get their first choices if possible. This is easier said than done because some resort intervals are highly sought after and others are not.

There are different ways to implement the rules in an IP formulation. One way is to devise a constraint set that strictly enforces the rules so that no participant of higher priority will be denied a choice for the sake of even multiple participants of lower priority. Another reasonable interpretation of "priority" is simply "a greater probability of granted requests," allowing some trade-offs between requests of different priority. For example, it may be preferable to grant two requests of secondary priority instead of only one request of higher priority. This can be implemented in a math program through an appropriately devised objective function, wherein rules might be considered goals with higher priority goals receiving greater weight. The important advantage of the objective function approach is that it allows us the opportunity to study trade-offs between requests that warrant priority according to different rules. In addition, we can select a set of objective function weighting coefficients that will synthesize the absolute priority interpretation for comparison purposes.

The prior system employed by ORE took a more rudimentary approach—it enforced rules via a construction heuristic that determined AS assignments request by request, according to a strict interpretation of priority rule ordering. In other words, MROP owners selecting their home properties were scheduled (person by person), followed by MROP owners selecting nonhome MROP properties, followed by gold owners, and so forth. This approach is "greedy" in that it simply gives each owner the first interval on his or her choice list that is available, with no attempt to backtrack assignments to allow more subsequent assignments. It fails to capitalize on the flexibility owners have in resort-week selection. We will use the prior solution as a baseline for comparing the optimized solutions.

Formulation Parameters and Decision Variables

The following are fundamental indices and sets of the AS problem:

$m \equiv$ AS participants (also called "members").
GOLD $\equiv$ set of members who are gold owners.

$c \equiv$ interval value color (red = 1, white = 2, blue = 3).
$r \equiv$ resort number.
$t \equiv$ week number (1 through 52).

The decision variable matrix is as follows:

- $A_{m,r,t} \equiv$ number of units at resort r during week t assigned to participant m.

Input data parameters are as follows:

- $\text{UNITS}_{r,t} \equiv$ number of units (condominiums) available for scheduling at resort r during week t. This parameter defines the available "inventory."
- $S_c \equiv$ set of intervals (i.e., resort, week tuples) designated as color c.
- $\text{OWN}_{m,c} \equiv$ number of week credits of color c owned by participant m.
- $\text{GOLD}_m \equiv$ number of gold credits owned by participant m.
- $\text{REQ}_{m,r,t} \equiv 1$ if participant m requests week t at resort r, otherwise zero.

The actual preferences that individual owners have for resort weeks is represented as follows:

$\text{PREF}_{m,r,t} \equiv$ preferential value for assigning owner m to week t at resort r.

These PREF parameters are the key element of the objective function, and their derivation will be described after the formulation.

AS Formulation

The AS formulation is as follows:

$$\text{Maximize:}\quad \text{SAT} = \sum_{m,r,t} \text{PREF}_{m,r,t}\, A_{m,r,t} \tag{1}$$

$$\text{subject to:}\quad A_{m,r,t} \leqslant \text{REQ}_{m,r,t} \quad \forall\, m, r, t, \tag{2}$$

$$\sum_{r,t \in S_c} A_{m,r,t} \leqslant \text{OWN}_{m,c} \quad \forall\, m \notin \text{GOLD}, c, \tag{3a}$$

$$\sum_{r,t} A_{m,r,t} = \text{GOLD}_m \quad \forall\, m \in \text{GOLD}, \tag{3b}$$

$$\sum_{m} A_{m,r,t} \leqslant \text{UNITS}_{r,t} \quad \forall\, r, t, \tag{4}$$

$$A_{m,r,t} \in \{0, 1\} \quad \forall\, m, r, t. \tag{5}$$

The objective function of Equation (1) seeks to maximize aggregate preference satisfaction. Equation (2) assures that owners are only given requests they request. Equation (4) prevents more than the number of units available from being scheduled. Equation (5) defines assignment decision variables as binary—no partial weeks are granted. It also assures that no owner is given the same week at the same resort. (The desirability of multiple weeks at a given resort is discussed in the final section.)

Equations (3a) and (3b) warrant a little more explanation. They assure that each person does not get more than owned credits of each given color priority. Equation (3a) allows individuals to get up to the number of credits they own of each color. Note that red owners cannot access white or blue weeks, which would leave white or blue owners with fewer available weeks without compensation. (Even though it would be mathematically simple to allow red owners to access white and blue weeks, it would not be politically simple.)

Gold owners have a different assignment constraint (3b). As mentioned, gold owners are assured they will either get a requested resort week or get their original week back. This formulation implies that we create a PREF value for each gold owner's original resort week as a low-ranked choice. These extra PREF values are to assure a feasible solution exists because (3b) is a binding constraint and there exists a possibility that none of a given gold owner's choices are available. Note that the extra PREF values are not necessary for gold owners who already included their original week in their personal choice sets, although it is not a problem if they are created anyway. The equality of Equation (3b) assures that gold owners will always get something, which "something" includes at least their submitted weeks.

Coding of PREF Elements

Five parameters influence PREF values: MROP "homeness," MROP "baseness," "goldness," priority number, and choice rank. Homeness, which corresponds to Rule 2 above, is a binary element which indicates whether a given interval choice represents a MROP participant requesting a MROP interval in their home resort. Homeness could potentially cause conflicting priorities if a person gave a higher preference rank to a nonhome resort and a lower preference rank to a home resort. To resolve this dilemma in the formulation, we only allow owners to have home-resort priority for their highest weighted preference selections. Therefore, homeness is represented as follows:

$h \equiv h(m, r, t) = 1$ for a given choice if it is in the participant's home resort *and* the owner has not chosen any resort weeks of higher rank outside of his/her home resort, otherwise zero.

Similarly, MROP baseness (which corresponds to Rule 3) is represented as follows:

$b \equiv b(m, r, t) = 1$ if it is an MROP owner selecting any MROP inventory *and* the owner has not chosen any resort weeks of higher rank outside of MROP, otherwise zero.

Note that if h equals one, then b also equals one because a home-MROP resort is certainly an MROP resort. For simplicity, we will assume that homeness is twice as important as baseness alone, allowing $h+b$ to act as a single parameter with the following interpretation:

$$h+b=\begin{cases}2 & \text{if choice is a top-ranked MROP interval that } \textit{is} \text{ in a home resort,}\\ 1 & \text{if choice is a top-ranked MROP interval that is } \textit{not} \text{ a in a home resort,}\\ 0 & \text{otherwise.}\end{cases}$$

Interval choices from gold owners (considered in Rules 1 and 4) are coded as follows:

$g \equiv g(m) = 1 \quad$ if $m \in$ GOLD, otherwise zero.

As specified in Rule 5, lower-priority numbers have precedence over higher-priority numbers, so we simply invert the scale:

$p \equiv p(\text{priority number}) = (6\text{-priority})$.

The priority value p is in the range 1 to 5, with 5 being most preferred.

AS participants can include dozens of intervals on their choice lists, but for simplicity, the choices were categorized into six rank levels (with all intervals in a given choice rank level being considered of equal rank). The choice rank parameter is as follows:

$k \equiv k(\text{choice rank}) =$ 7 for a choice of highest interest to the participant, 6 for a choice of next highest interest to the participant…and so forth for 5, 4, and 3 … 2 for a choice of lowest interest to the participant, 1 for gold owners getting their original week back.

As mentioned, gold owners are guaranteed to get their submitted interval back if none of their exchange choices can be granted. Therefore, the choice-rank value $k=1$ is attributed to gold owners' submitted weeks, which means that getting their submitted week back is considered to be their least-preferred choice. Of course, this is not the only coding of choice ranks that could be used and it does impose a linear utility interpretation of the ordinal values (Angilella 2004, p. 64). An alternative is to determine nonlinear estimates of the utility mapping (Liberatore and Stylianou 1995). However, that mapping could be precarious in that it would impose a uniform utility mapping across all participants. We also explored the potential for soliciting interval data directly from each AS participant, such as by having owners allocate preference points to desired resort weeks. However, focus groups with employees and customers showed that an ordinal specification of choices was highly preferred. Because this was an actual application, there was a business need to let customer concerns take priority over research interests to some degree, and the coding was certainly adequate for the present research study.

Deriving PREF Alternatives

We recognize that there are various PREF functional forms that could be argued as representing the prioritization rules. In this paper, we derive and present a few candidate functions that can be used as the basis for obtaining problem solutions and studying solution sensitivity. We will later mention other possible candidate functions, including a nonlinear option. For the present analysis, we will primarily consider PREF functions that are linear in problem parameters. The general linear PREF function is as follows:

$$\text{PREF}(g, h, b, p, k) = (h+b)W_h + gW_g + pW_p + kW_k,$$

where W values are objective function weight values. The weight values not only define the quality of solutions but also enforce some interpretation of the rule hierarchy.

The research literature shows it is extremely common to represent multiattribute models as a linear function of attributes, as we have done (Bolton and Drew 1991, Mateos et al. 2003, Sampson 1999). Nonlinear models described in the literature (e.g., Henig and Ritz 1986, Triantaphyllou and Sánchez 1997) could be used instead. However, even the linear function provides sufficient complexity for an interesting study. A subsequent section presents logic for considering nonlinear function forms.

The research literature also describes various methodologies for deriving objective function coefficients. Cunningham and Frances (1976) apply Bayesian decision theory to refine estimates of a coefficient. McKeown and Minch (1982) show a method for simultaneously analyzing multiple coefficients. More recent literature considers ranges of coefficient values across multiple coefficients, with a minmax regret goal (e.g., Mausser and Laguna 1999, Wendell 2004). Solving the minmax regret problem, where objective function values can take any value over a given range, has been shown to be NP-hard (Averbakh and Lebedev 2005).

For the AS problem, we also utilize a minmax regret goal but simplify matters by calculating discrete objective coefficients candidates. The primary factor driving the determination of objective function coefficients is the acceptability of the cost function by management and by AS participants. The ownership contracts imply prioritization of assignment requests, as embodied in the rules discussed previously. Therefore, we can narrow the coefficient set based on interpretation of the rules.

PREF_1**: Strict Rule Interpretation.** If we assume that the rule hierarchy is absolute, then we would have that $W_h \gg W_g \gg W_p \gg W_k$. Initially we will derive weights that represent this strict hierarchy interpretation. To start, we first set $W_k = 1$ because rank is the least important. W_p needs to be large enough such that larger p values always have precedence over smaller p values, even if the k value is increasing. To determine an appropriate W_p value, we define the following partial PREF function:

$$f(p, k) = pW_p + k.$$

The condition we seek is for any set of M requests of priority p to have a higher objective function measure than any M requests that are all of priority p except for one of priority $p-1$, even if the second set has higher k values. This condition is represented as

$$Mf(p, L_k) > (M-1)f(p, U_k) + f(p-1, U_k),$$

where $L_k = 1$ is the lower bound of k and $U_k = 7$ is the upper bound of k. That simplifies to

$$W_p > M(U_k - L_k).$$

We initially hypothesized that $M = 3$ would be sufficient to enforce the ordering of rules (with priority taking precedence over choice rank) because it provides that a one-step increase in priority for one person has more influence than any three individuals with potentially superior choice ranks. We tested this hypothesis empirically by calculating optimal AS solutions for W_p values of 30, 25, 20, 18, 15, and 10, and 5 (with W_g and W_h parameters as specified below). The solutions for $W_p \in \{30, 25, 20, 18\}$ were all identical (by any of the candidate objective measures), but those solutions differed from solutions in which $W_p < 18$. We conclude that all studied solutions with $W_p \geqslant 18$ are identical because that is the point of strict enforcement of Rules 5 and 6. In other words, $W_p \geqslant 18$ provides for no trade-off between owner priority and choice rank. Because $U_k - L_k = 6$, we have $W_p > M$ (6), which implies that our $M = 3$ assumption was supported. We therefore use a rounded $W_p = 20$ as an adequate constraining value.

Next, we determine the W_g coefficient, which is multiplied by a binary parameter g. Any set of M requests where one has $g = 1$ should have priority over any M requests that all have $g = 0$. This is represented as

$$W_g + f(L_p, L_k) + (M-1)f(L_p, L_k) > Mf(U_p, U_k) \quad \text{or}$$

$$W_g > Mf(U_p, U_k) + f(L_p, L_k),$$

where $L_p = 1$ is the lower bound of p and $U_p = 5$ is the upper bound of p. Using $M = 3$ results in $W_g > 3(107 - 21) = 258$. For convenience, we might specify $W_g = 300$. A similar procedure can be applied for determining that $W_h > 1{,}158$, allowing us to use $W_h = 1{,}200$. Therefore, we have a first preference function of

$$\text{PREF}_1(g, h, b, p, k) = 1200(h+b) + 300g + 20p + k.$$

This function reasonably assures that any home resort preference has precedence over any priority number advantage, which has precedence over any choice ranking.

PREF$_2$: Allow Priority/Rank Trade-Offs. An alternative form would allow trade-offs between rule sets. At issue is the interpretation of the p scheduling "priority numbers." If they indeed mean an absolute ordering of priority, then a PREF function like that above would be warranted. However, another possible interpretation is that the priority numbers simply give the person *probabilistic advantage* over people of lower priority. Such might be represented in the following PREF function:

$$\text{PREF}_2(g, h, b, p, k) = 1200(h+b) + 300g + 2p + k.$$

This preference function sustains the home request rule, and otherwise gives a slight advantage to people of higher priority values. The actual ranking of the choices is of lesser importance, but trade-offs are allowed. For example, a $p = 5$ person's first and second requests will take precedence over the first request of a $p = 4$ person, but the $p = 5$ person's third request will be an equal trade-off with the first request of $p = 4$. Although the priority number values still have meaning, they can be overridden by some high-ranked choices.

PREF$_3$: Emphasize High-Ranked Requests. One might ask if it is best to even have the choice-rank weight be of lower importance than the priority level. Even though Rule 6 is listed after Rule 5, it is not completely clear that Rule 6 should be subservient to Rule 5. The fact is, ORE prides itself in meeting high-priority requests. It is also valuable to be able to report that the AS process meets a high percent of AS participants' high-priority requests. If we believe that meeting highly-ranked choices should be a major objective, we might choose the following preference function:

$$\text{PREF}_3(g, h, b, p, k) = 1200(h+b) + 300g + p + W_k k.$$

Using the methodology above, it can be determined that $W_k > M(U_p - L_p) = M(5-1) = 3(4) = 12$. W_k of 20 should suffice.

PREF$_4$: Maximize High-Ranked Requests. To assure that top-ranked choices get precedence, we might specify an objective that favors some higher ranking over arbitrary combinations of lower rankings. We replace the "$W_k k$" term in the PREF function with an arbitrary function $f(k)$ that assures any request of rank k will have priority over any M requests of rank $k = 1$, which is represented by

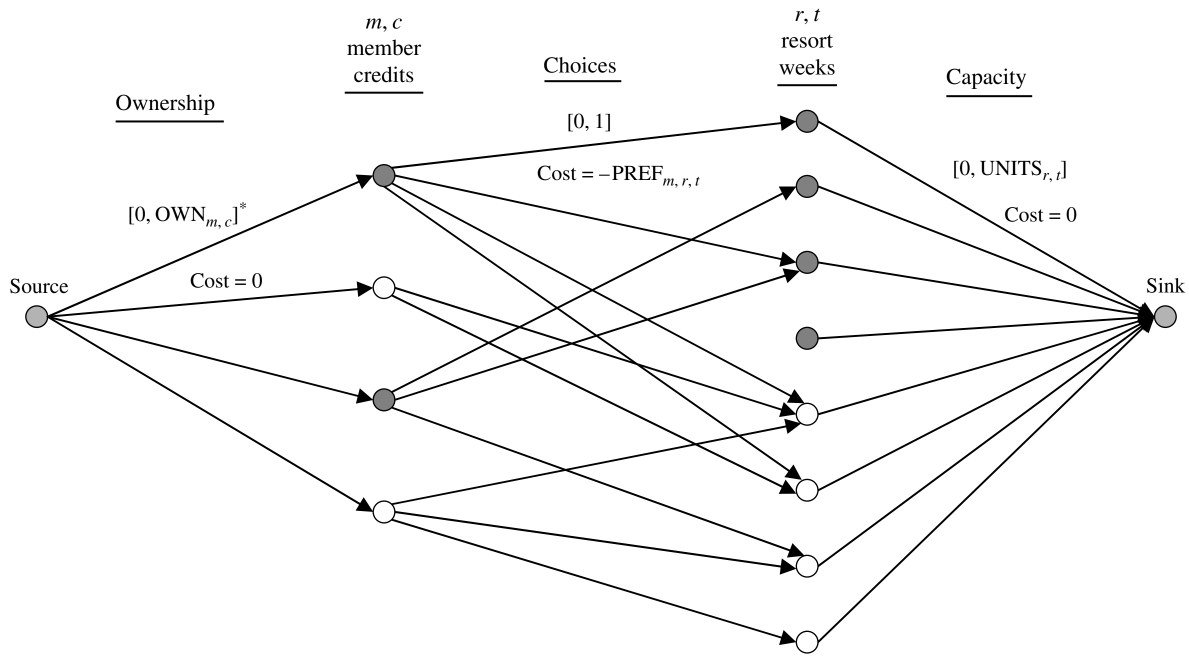
$$f(k) > Mf(k-1).$$

An obvious choice is $f(k) = n^k$, implying that

$$n^k > Mn^{k-1}.$$

Dividing both sides by n^{k-1} produces $n > M$. Again, using $M = 3$ produces an acceptable $n = 4$. It is more than sufficient to use the same W_k value used previously (actually,

Figure 1. Network-flow representation.



*Or $[\mathrm{OWN}_{m,c}, \mathrm{OWN}_{m,c}]$ for $m \in$ GOLD.

a lesser W_k would do because 4^k is not a linear function of k). This gives us the following nonlinear function:

$$\mathrm{PREF}_4(g,h,b,p,k) = 1200(2h+b)+300g+p+20(4^k).$$

This assures that meeting a high-ranked request is not overruled by any combination of M lower-ranked requests that are otherwise the same. We recognize that $4^7 = 16{,}384$, which is greater than the 1200 coefficient for the home-resort variables. This does not jeopardize the priority given to home-resort selections because we have defined home-resort selections as *only* being at the highest choice rankings (where k is largest).

One may argue that PREF_3 and PREF_4 violate the strict interpretation of the rule-set hierarchy. Still, they are worth considering as alternative interpretations of the rule set. They allow us to explore the impact of strictness of the rule hierarchy on the ability to satisfy high-ranked requests. Also, company management prides itself in granting high-ranked requests, and likes to publicize "XX percent of AS participants got their first choice." PREF_3 and PREF_4 will give management some idea of the potential for satisfying top-ranked requests under conditions of a relaxed rule hierarchy, which will facilitate a discussion of opportunities and trade-offs.

Solution, Data, and Results

We conducted an experiment based on actual AS data. The experiment allowed the AS process to proceed and obtain a solution according to the prior method, then generate additional AS solutions using the IP formulation and candidate objective functions. The questions of interest are how the IP solutions compare to the heuristic solutions and how the IP solutions compare to one another.

Solution Methodology

Although the AS formulation is an IP, the reader will observe that it can be transformed into a network-flow representation, which allows us to obtain integer solutions in polynomial time. Various network designs exist. We elected to use an open-loop, unidirectional flow, mini-cost representation, as depicted in Figure 1. The particular solver we used is one devised by Andrew Goldberg called "cs2," which has been proven to be extremely efficient ($O(n^2 m \log(nC))$) (Goldberg 1997, Stony Brook Algorithm Repository 2006). AS problems with $150{,}000+$ arcs and almost 7,000 nodes were solved to optimality in under 10 seconds on a 2 GHz Pentium 4 PC.

Problem Data

For the study AS cycle, 4,715 participant credits were submitted with resort-week preferences (some being individuals who participated with multiple intervals, each with distinct choice sets). Participants submitted an average of 18.13 intervals on their choice lists. Of the 4,715 participants, 197 submitted only one choice, and the largest choice list had 109 entries. Those who gave more choices would theoretically have a greater chance of having one of their choices satisfied.

Optimization Results

Table 1 shows how many of the 4,715 AS participants (participating intervals) received one of their choices from the

Table 1. Results summary.

Participants receiving	Prior method	Optimization with objective function...			
		$PREF_1$	$PREF_2$	$PREF_3$	$PREF_4$
Something	3,598	4,064	4,023	3,893	3,658
	*change (%):	+13.0	+11.8	+8.2	+1.7
Nothing	1,117	651	692	822	1,057
	*change (%):	**−41.7**	−38.0	−26.4	−5.4
First choice	1,588	1,599	1,782	1,998	2,302
	*change (%):	+0.7	+12.2	+25.8	**+45.0**

*Change is percent change relative to prior method.

prior construction heuristic and from optimization under each of the candidate PREF functions. The prior method resulted in 3,598 of the participants (76.3%) receiving some assignment, with 1,117 (23.7%) receiving no assignment. Under the prior method, 33.7% received their first choice, which is commendable but clearly not optimal.

In terms of satisfying the most participants, $PREF_1$ provided a superior solution by increasing the number of satisfied participants by 13%, which represents a full 41.7% drop in participants receiving nothing. On the other hand, $PREF_4$ had little impact on raw numbers of satisfied requests, but had a major impact on the number of first choices which were granted, increasing it by 45%! As you can see, the four candidate PREF specifications form an efficient frontier of the optimal solutions. This, of course, leads to the difficult issue of deciding which point on that frontier would be the most appropriate for actual implementation.

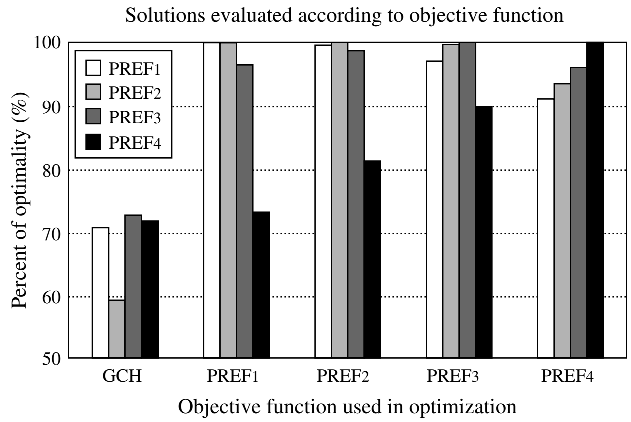
Selecting from Solution Alternatives

There are various criteria management might use to select the appropriate objective function. The solution using $PREF_4$ makes a lot of people highly satisfied without decreasing the overall number of people satisfied relative to the prior heuristic approach. Therefore, it seems participants could not argue against using that solution instead of the prior approach—unless, of course, low-priority owners become aware that the $PREF_1$ solution is possible!

The $PREF_1$ solution focuses the advantage on those of lower priority, who otherwise might have gotten nothing, but instead got one of their requests. As shown in Table 1, 466 more people were granted requests through the use of the $PREF_1$ optimization, which is significant considering the 466 more-satisfied owners came from *the exact same available inventory*.

Interestingly, when the solution results were presented to the ORE management team, they were divided over which interpretation of rules (i.e., PREF function solution) to use. One senior executive favored the $PREF_4$ solution, but others favored $PREF_1$. All of the four candidate solutions are on the efficient frontier of the solution space and are thus Pareto optimal (Keeney and Raiffa 1976, p. 70).

Figure 2. Comparison of solutions.



All were superior to the prior greedy solution. To help select from the four, we show how using any one of the objective functions would impact solution quality as measured by the other object function forms. This approach to resolving the trade-off dilemma is similar to the traditional minmax regret reduction strategy (Conde 2004, Fernandez et al. 2000). Figure 2 illustrates how much regret those favoring one PREF function would have if another of the PREF functions were used.

The leftmost set of bars (GCH) shows that the prior greedy construction heuristic does not do well under any of the objective functions. The set of bars labeled PREF1 at the bottom shows how the optimal solution using $PREF_1$ as the objective function evaluates according to each of the four objective functions. (The left bar of that set goes to 100% because optimizing with $PREF_1$ produces the optimal solution measured by $PREF_1$.) The second bar of that set is at 99.89%, meaning that optimizing with $PREF_1$ produces a solution which is 99.89% of the optimal solution when solved and measured by using $PREF_2$. That means that if management uses $PREF_1$ when they should have used $PREF_2$, almost nothing (0.11%) would be lost.

Most striking is that the fourth bar of the PREF1 set shows only 73.02%, meaning that optimizing with $PREF_1$ produces a solution that is 73.02% of the optimal solution when solved and measured by using $PREF_4$. That means that if management uses $PREF_1$ when they should have used $PREF_4$, a great deal (almost 27%) of the $PREF_4$ optimality would be lost. Thus, people who favor $PREF_4$ would not likely be satisfied using $PREF_1$ instead.

On the other hand, we see that if $PREF_4$ was used to solve the problem (shown in Figure 2 by the PREF4 cluster of bars), then the solution relative to $PREF_1$ would be 90.9% of what it would be if $PREF_1$ would have been used to solve the problem. Thus, individuals who would have liked $PREF_1$ to be used would lose 9.1% of their optimality if $PREF_4$ were used instead.

We conclude that if the decision of which objective function to use was split among the four PREF options (which it was), and management desired to minimize the dissatisfaction of the least-satisfied group (the minmax criteria), they should perhaps favor using PREF_4. However, PREF_4 is probably the hardest to justify in terms of complying with AS rules. If PREF_4 were thrown out of consideration, it appears that PREF_2 (shown by the PREF_2 bar set of Figure 2) would be the best choice, satisfying the other objective functions within 2% of optimality.

Outcome and Conclusion

One significant outcome of the success of this application of optimization was the potential for raising participant expectations for AS in future years. This is exacerbated by management desire to publicize key statistics to ORE owners, such as the great percentage of AS participants who receive a first choice. Those who previously were delighted to get a first choice may come to be surprised if they do not get a first choice.

Various methods have been implemented at ORE to further enhance the ability to make AS an effective exchange. Choice data collection has become fully online, allowing participants to provide even more flexibility in their choice sets. Even the formulation has evolved according to ongoing experience. Solution improvement was achieved by using a nonlinear PREF function form that scales the intervals between successive rank weights according to each individual's priority value.

Finally, we see that this application of optimization holds other interesting issues to study. For example, we have studied the potential for allowing individuals who own multiple intervals of similar color to provide joint requests for those intervals. Some owners may want two units at the same resort during the same week, or want the same unit at a given resort for two consecutive weeks. Other owners may want their two weeks at different times and at different resorts to provide variety. The formulation for this extension is more complicated and violates total unimodularity, precluding the use of a polynomial-time network-flow solution approach (Ahuja et al. 1993, Nemhauser and Wolsey 1988). However, we have developed a hybrid solution methodology that combines a flexible construction heuristic with an optimal network formulation (Sampson 2004). Results are very promising, with initial results granting 80% of the requests for joint assignments.

Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

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