Computational MRI

Parallel imaging I: Image-domain methods

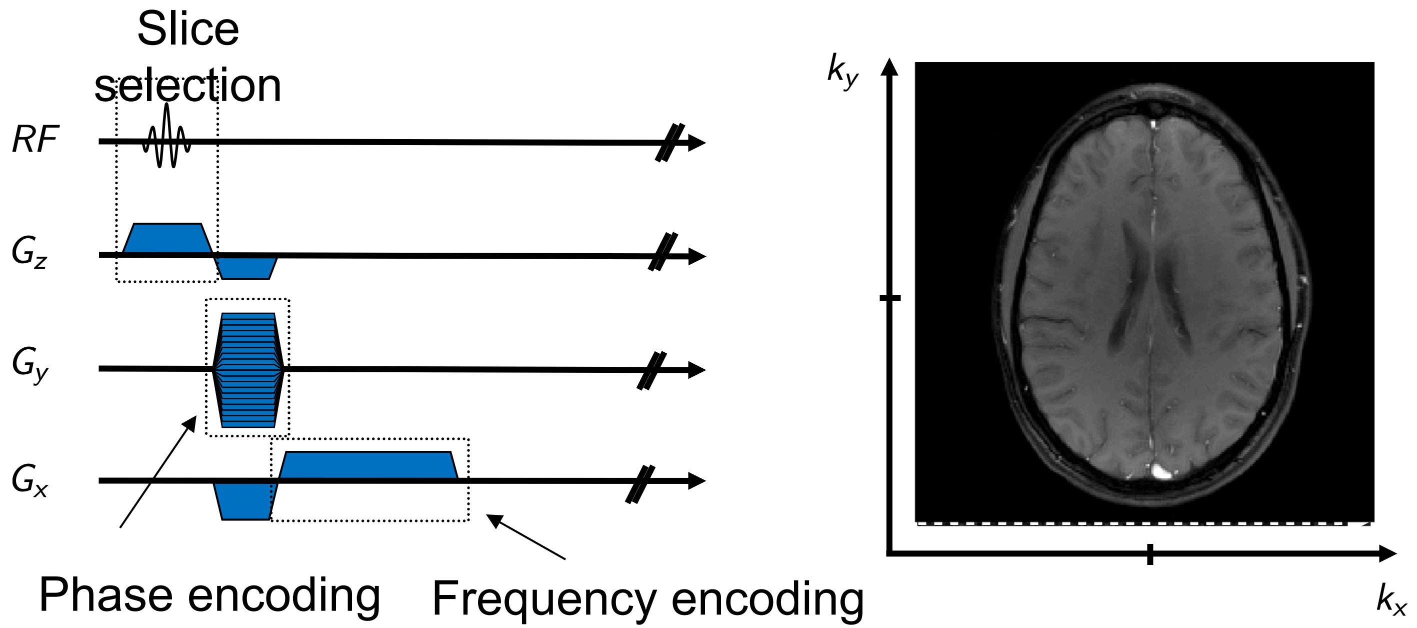
FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG

NYU

k-space undersampling

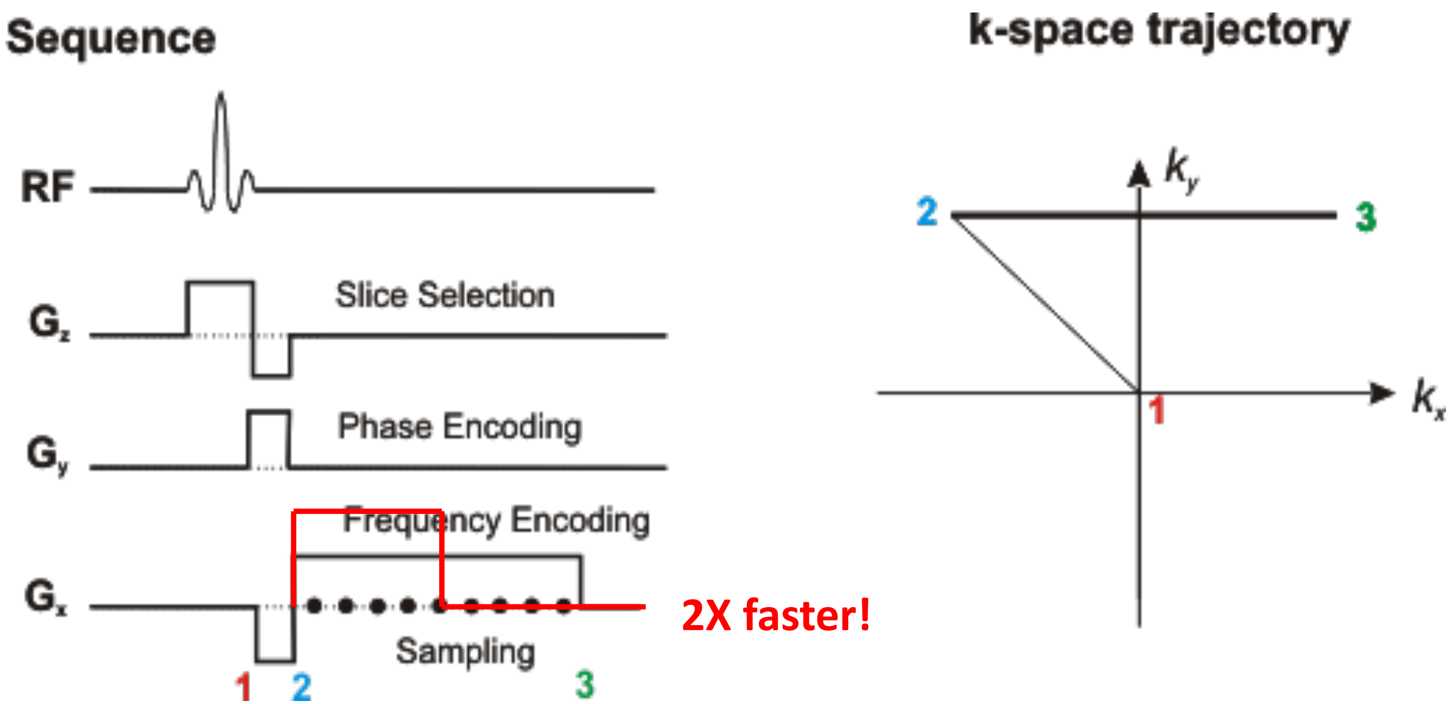
K-space encoding

- Speed of k-space traversal
- Switching rate and amplitude of magnetic field gradients

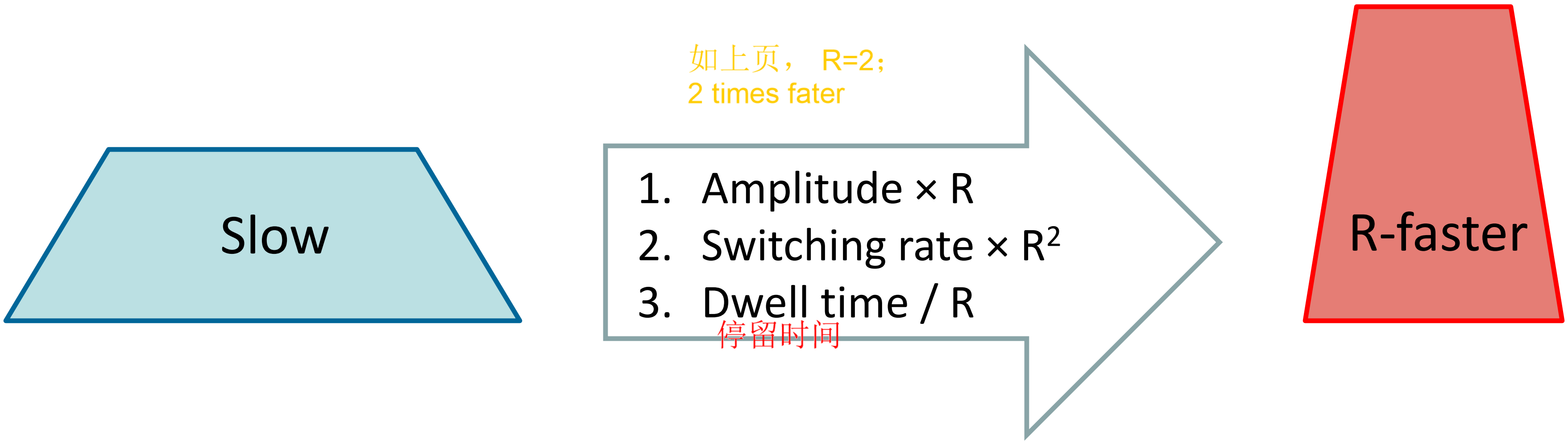


Imaging speed in MRI

- Speed of k-space traversal
- Switching rate and amplitude of magnetic field gradients

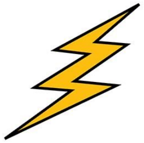


Speed of conventional MRI is limited

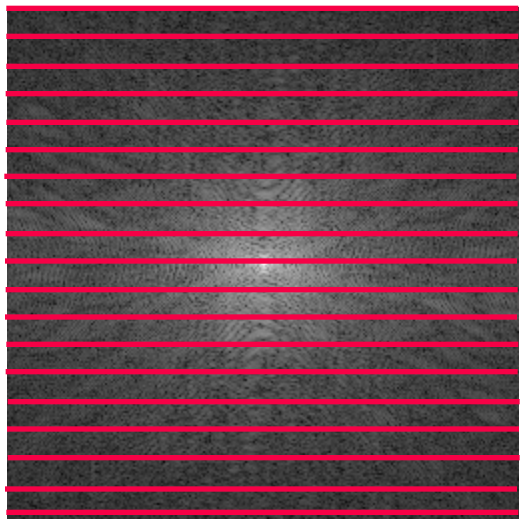


- What is the effect on the gradient amplifier?
  - Power increases with R^3

- What is the effect on SNR?
  - SNR decreases with $\sqrt{R}$
- It can also cause peripheral nerve stimulation



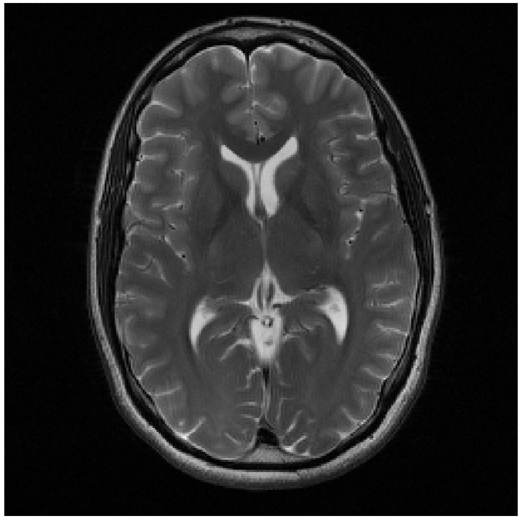
k-space undersampling



Δk_y

IFT

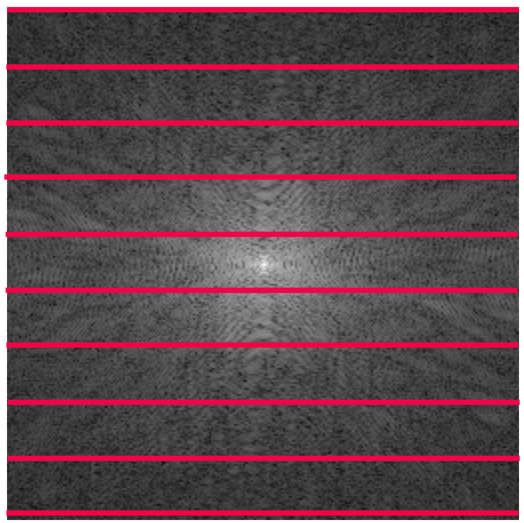
$\longrightarrow$



FOV

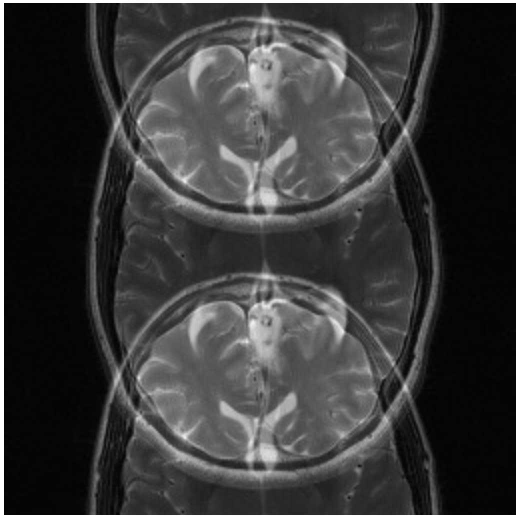
k-space undersampling

- Faster, no changes in gradient switching, but conventional Fourier reconstruction will result in aliasing artifacts



$2 \cdot \Delta k_y$

IFT $\longrightarrow$



$\frac{FOV}{2}$???

Question: Can we undersample in the readout dimension?

no need to do

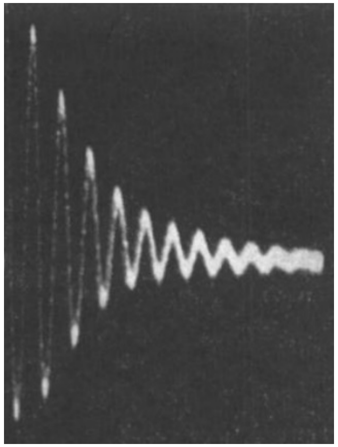
k-space undersampling

- Reconstruction
  - Exploit data redundancies!
- Parallel imaging
  - Multiple coils with different spatial sensitivities (real data redundancy)
- Compressed sensing, constrained reconstruction, machine learning,…
  - Image compressibility/sparsity
    (inherent redundancy)
    固有的

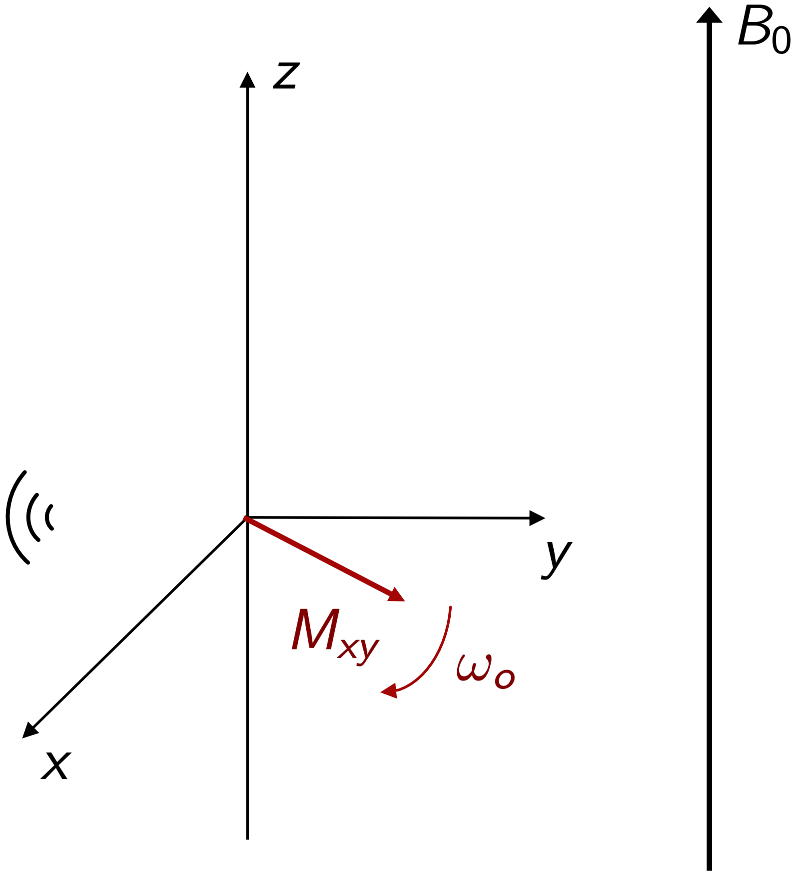
Multi-channel receive coils

Signal reception: MR receive coils

$$u_{ind} = -\frac{d\Phi}{dt}$$

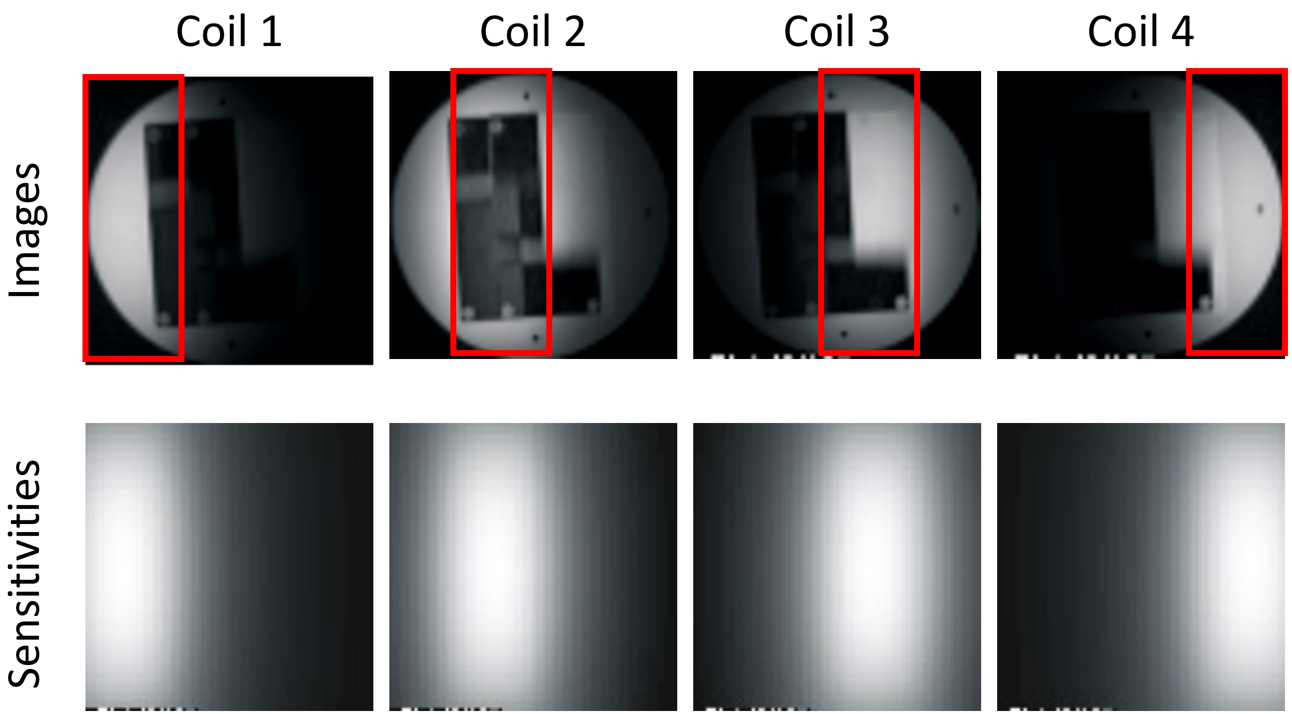


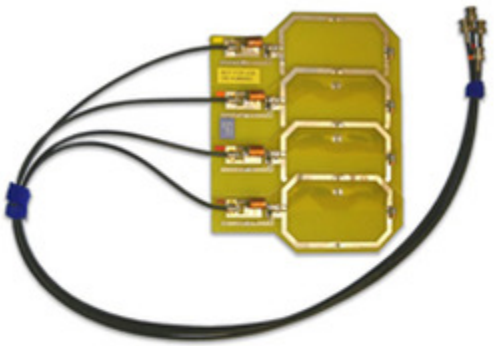
Hahn 1950



Multiple receiver coils

- Different spatial sensitivities



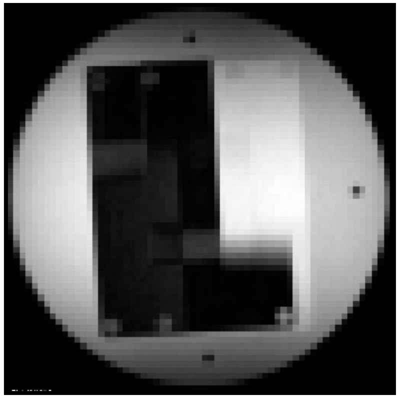


Multiple receiver coils

- Sensitivity-encoding equation

$$m_{\mathrm{i}}(r) = f(r) \times c_{\mathrm{i}}(r) + n_{\mathrm{i}}(r)$$

Coil image



Image

Coil sensitivity

Noise

Multiple receiver coils

变成好几个小的receiver coils， 重建的图像更好

- First used to improve SNR
  - What is the optimal coil combination?

- Matched-filter or least-squares combination

$$f(r) = \frac{\sum_{i=1}^{N_c} c_i^*(r) m_i(r)}{\sqrt{\sum_{l=1}^{N_c} |cl(r)|^2}}$$

$m_i(r)$: single coil images
$c_i(r)$: coil sensitivities

Roemer FB et al. Magn Reson Med. 1990; 16(2):192-225.

Multiple receiver coils

- Matched-filter or least-squares combination
  - In matrix form (for each pixel)

m = f×c

$$f = \left(\mathbf{C}^H \mathbf{C}\right)^{-1} \mathbf{C}^H \mathbf{m}$$

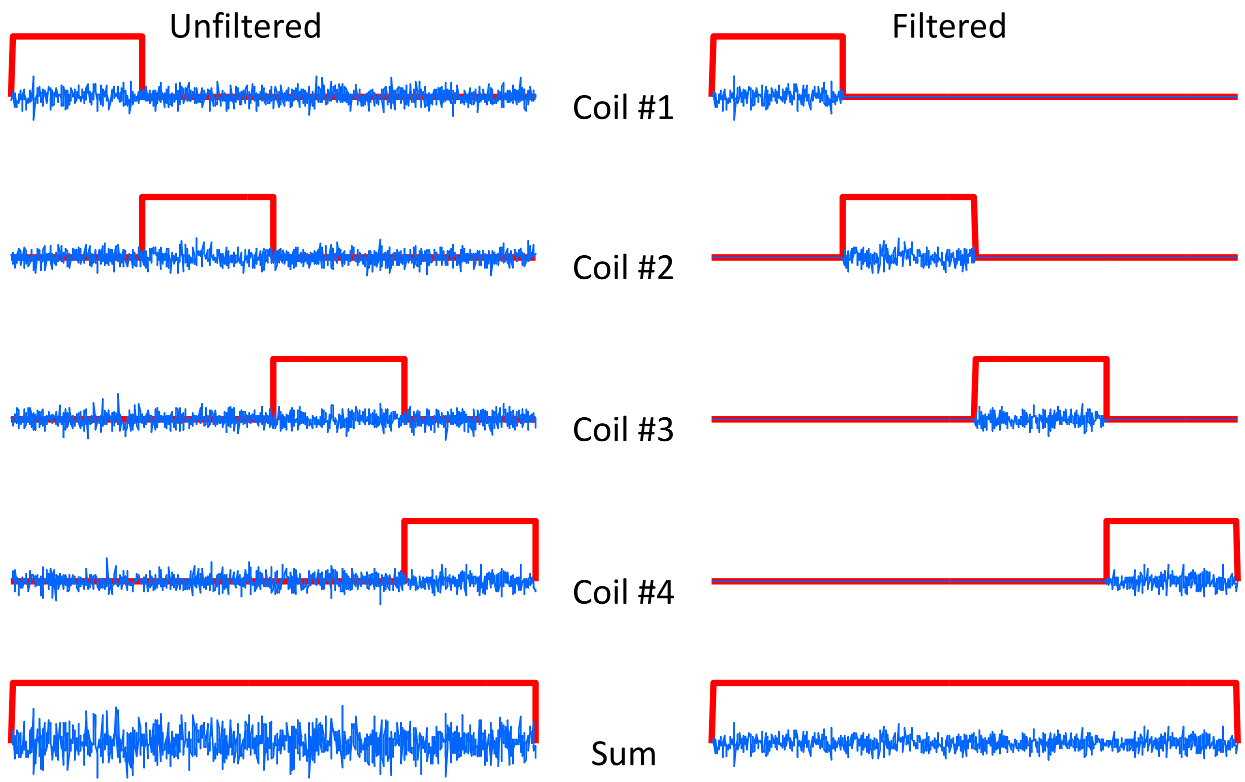
$$\mathbf{C} = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_N \end{pmatrix} \qquad \mathbf{m} = \begin{pmatrix} m_1 \\ m_2 \\ \vdots \\ m_N \end{pmatrix}$$

Cn: diagnose matrix, 256×256行， 256×256列. img value存放在对角。

Roemer FB et al. Magn Reson Med. 1990; 16(2):192-225.

Multiple receiver coils

- Matched-filter or sensitivity-weighted combination
  - Effects on noise



前提是noise均匀分布的

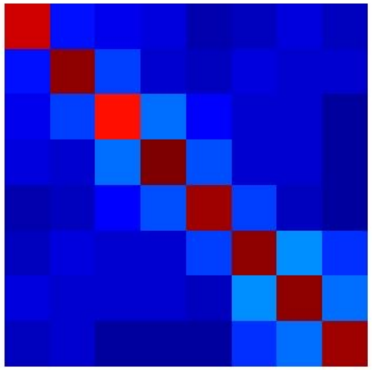
Multiple receiver coils

- Noise signals from different coils are correlated

Ψ: coil noise covariance matrix

noise不是均匀分布的，
有noise covariance matrix

8-coil array



- Least-squares combination using the covariance matrix

covariance matrix : 8×8

$$f = \left(\mathbf{C}^H \boldsymbol{\Psi}^{-1} \mathbf{C}\right)^{-1} \mathbf{C}^H \boldsymbol{\Psi}^{-1} \mathbf{m}$$

C： vector of 8
The element in the vector is a diagnal matrix with 256×256 rows and 256×256 columns

Multiple receiver coils

- Pre-whitening
  - Virtual coils with uncorrelated noise

$$\mathbf{m}_w = \mathbf{\Psi}^{-\frac{1}{2}}\mathbf{m}$$

$$\mathbf{C}_w = \mathbf{\Psi}^{-\frac{1}{2}}\mathbf{C}$$

- Solution

$$f = \left(\mathbf{C}_w{}^H \mathbf{C}_w\right)^{-1} \mathbf{C}_w^H \mathbf{m}_w$$

Multiple receiver coils

- Sum of squares
  - Approximation to the optimal combination
  - Images as coil sensitivities
  - SNR penalty of about 10%

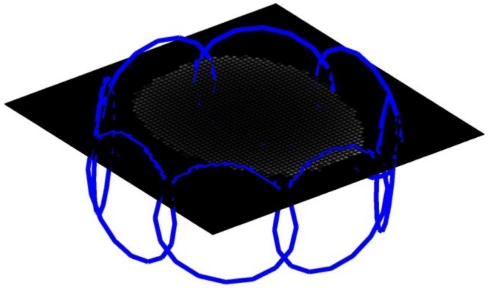
perfect received coil $$c_i(r) = m_i(r) \quad \Rightarrow \quad f(r) = \sqrt{\sum_{i=1}^{N_c} |m_i(r)|^2}$$

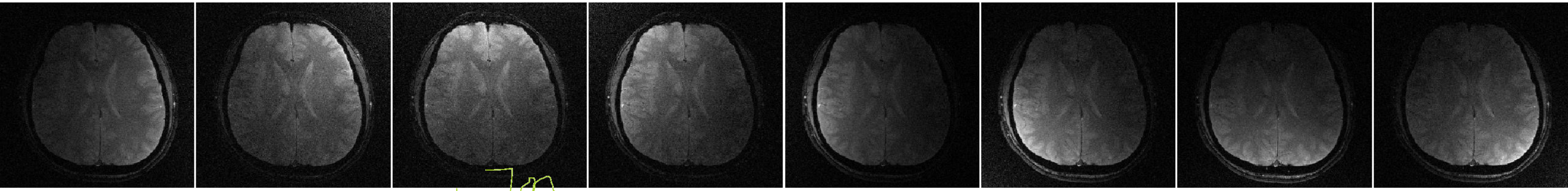
m:8-vextor

$$\mathbf{c} = \mathbf{m} \quad \Rightarrow \quad f = \sqrt{\mathbf{m}^H \mathbf{m}}$$

Signal combination example

- 8-coil circular array



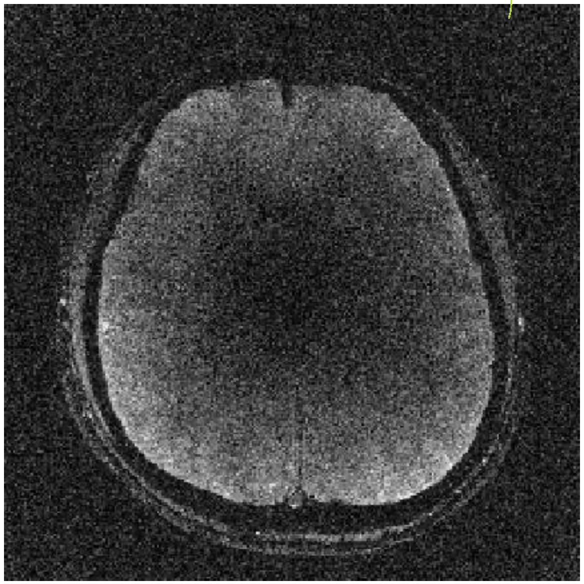
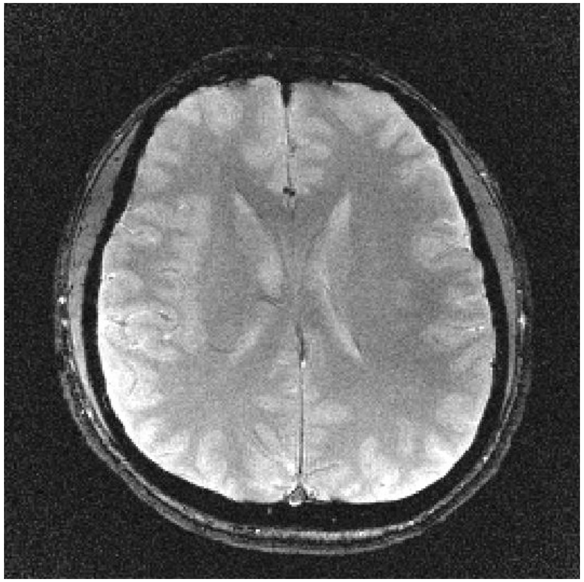
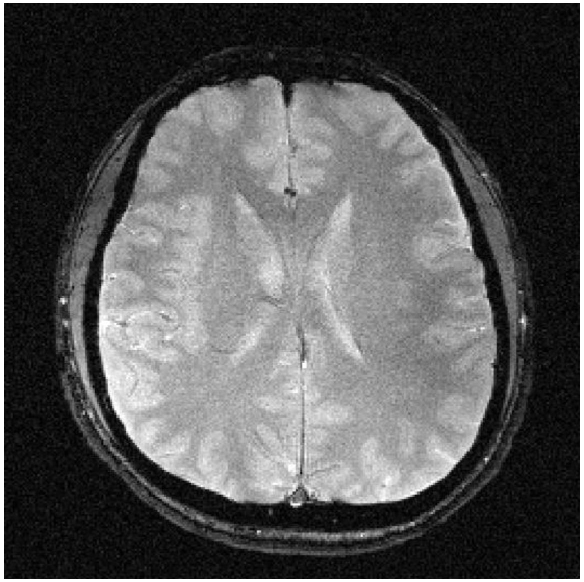


直接complex sum会导致phase cancellation

Im

Re

因此要用sum of squares

Complex sum	Sum of squares	Least-squares (matched-filter)
		

Signal combination example

- Matched-filter combination

sequence中不给RFpulse(没有img的信号), 就能得到noise covariance matrix
sequence中给RFpulse,得到的就是img signal和noise

Without Ψ

With Ψ

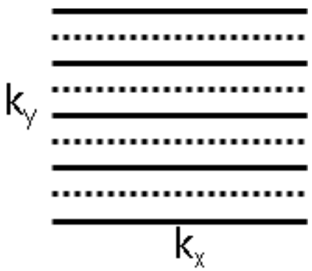
covariance matrix:8×8
表示不同channel之间的corelation

Parallel imaging

Parallel imaging

- Multiple coils enable acceleration of MRI data acquisition
  - Multiple coil data are redundant!

- Regular k-space undersampling

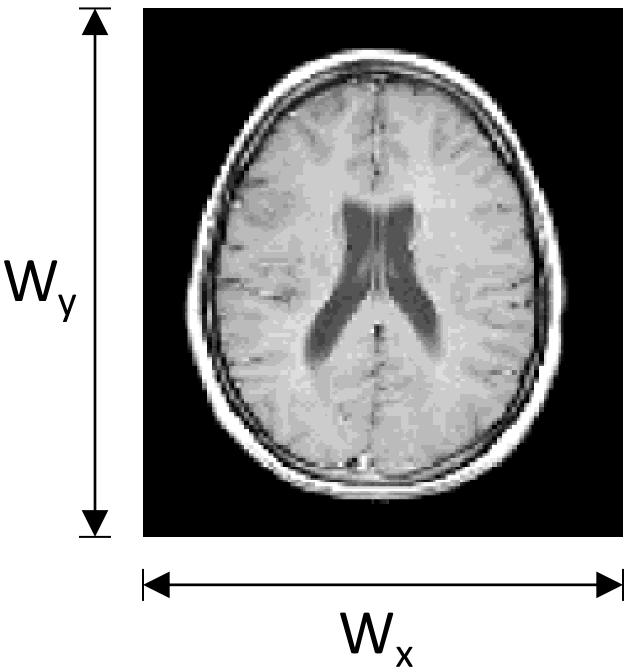


- Reconstruction using matrix inversion

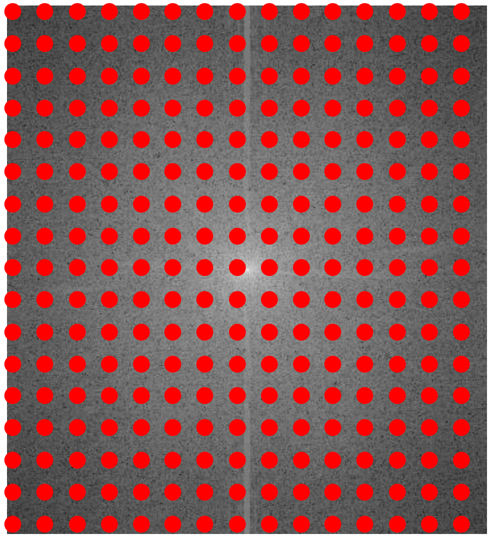
Sodickson DK, Manning WJ. Magn Reson Med. 1997; 38: 591-603
Pruessmann KP et al. Magn Reson Med 1999; 42: 952-962

Recap: k-space sampling density and image FOV

Image



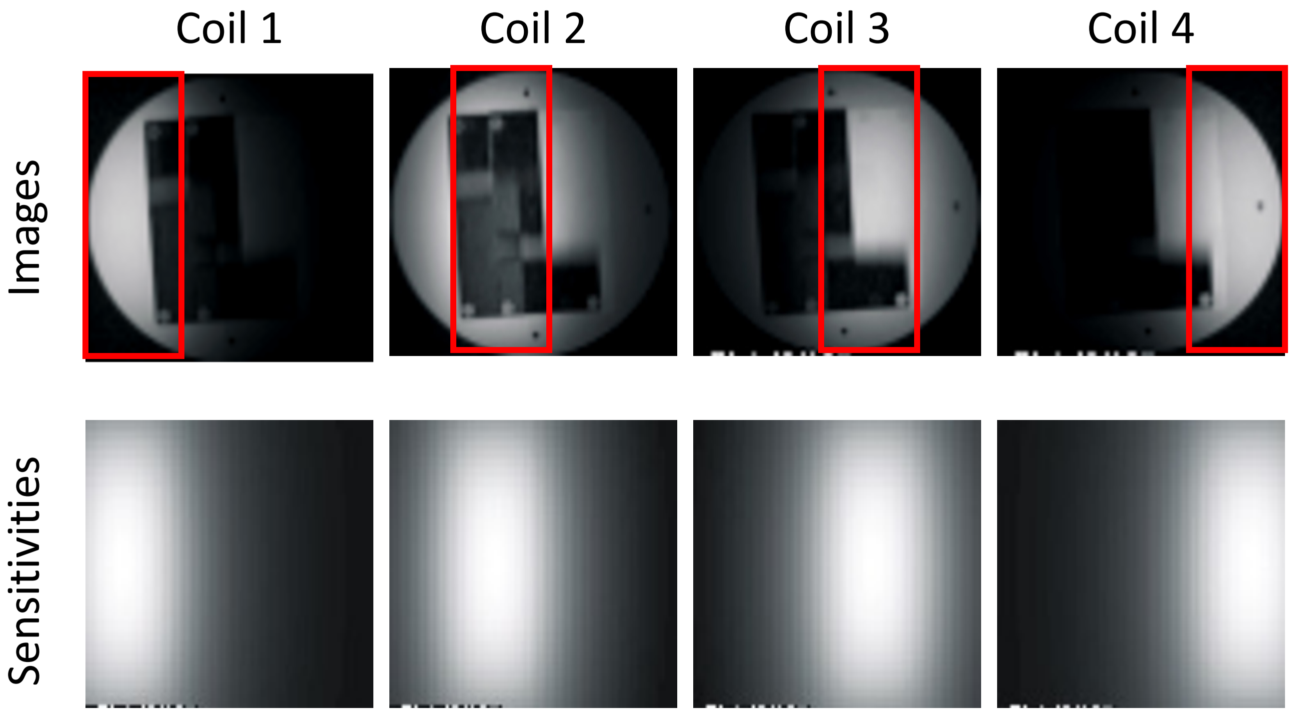
k-space

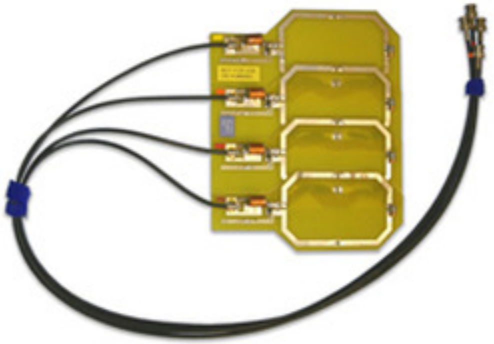


Nyquist rate:

$$\Delta k_x = \frac{1}{W_x}; \quad \Delta k_y = \frac{1}{W_y}$$

Spatial encoding of receive coils

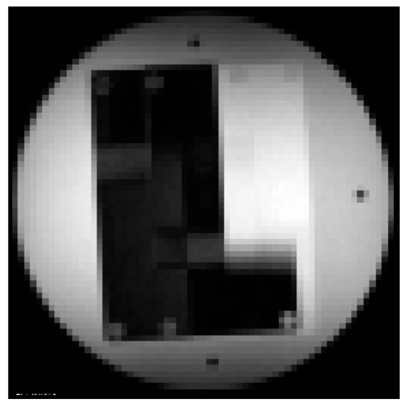




Coils also perform spatial encoding

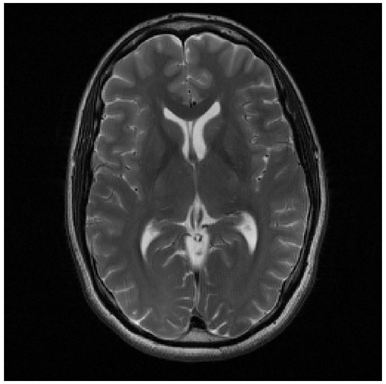
- Pixels close to the coil are bright
- Pixels far from the coil are dark

Back to sensitivity-encoding equation

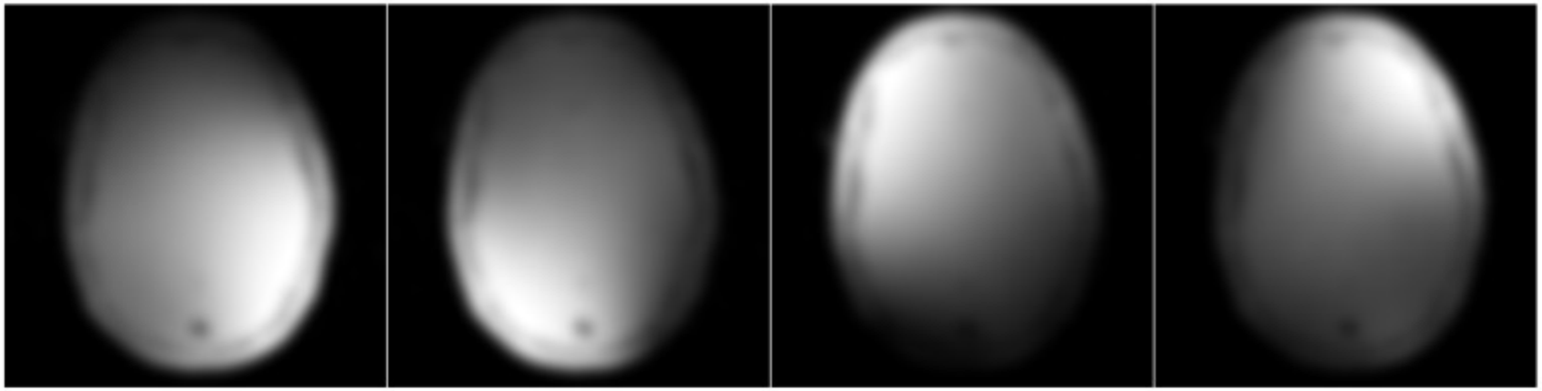
$m_i(r)$  = $f(r)$  x $c_i(r)$ 

Pruessmann et al., MRM 42: 952-962 (1999)

Cartesian SENSE



- T2 weighted brain scan
- 4-Channel receive coil
- Sensitivities are known



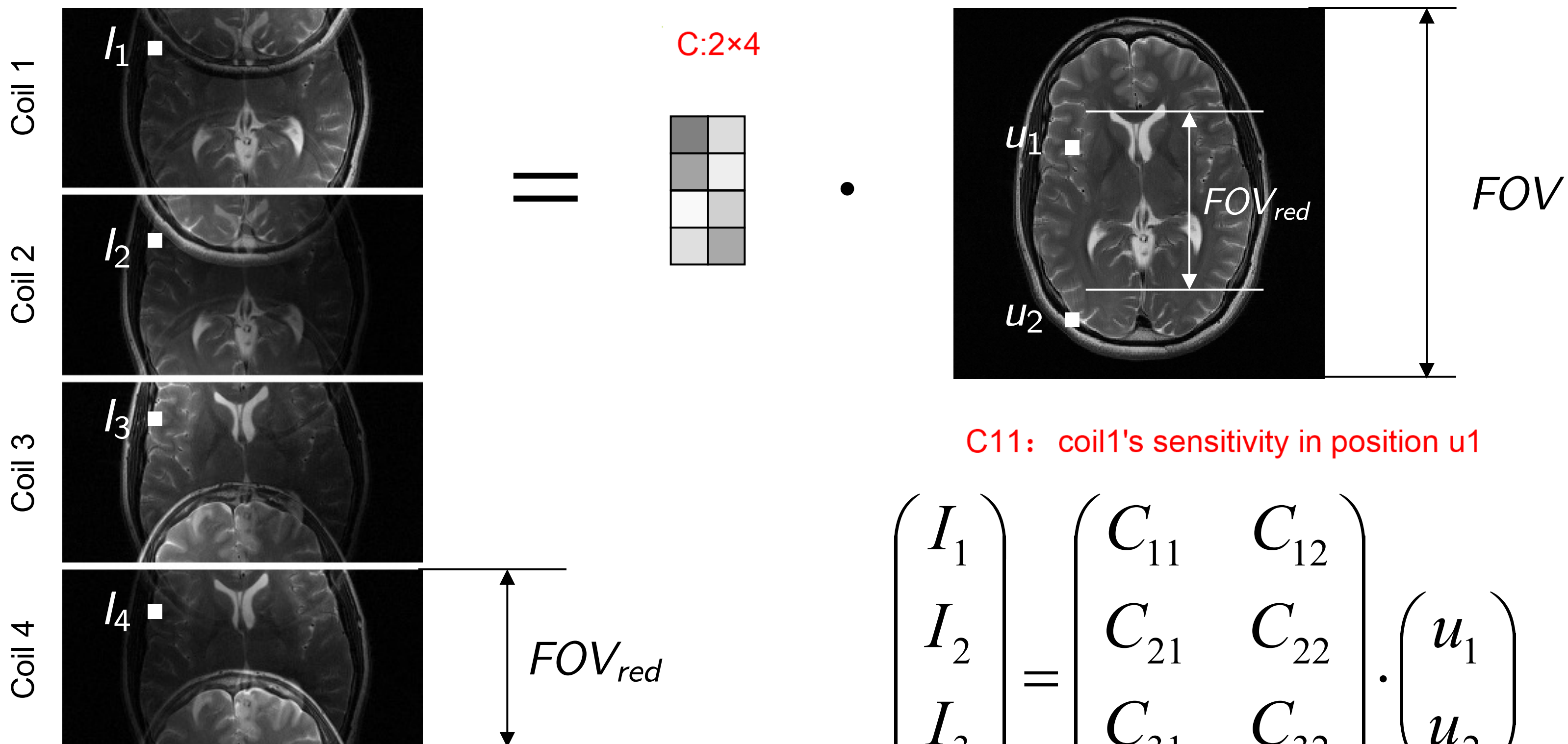
Pruessmann et al., MRM 42: 952-962 (1999)

Cartesian SENSE

wrap out artifact/alias

FOV read一定在中间

C:2×4



C11： coil1's sensitivity in position u1

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \\ C_{41} & C_{42} \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

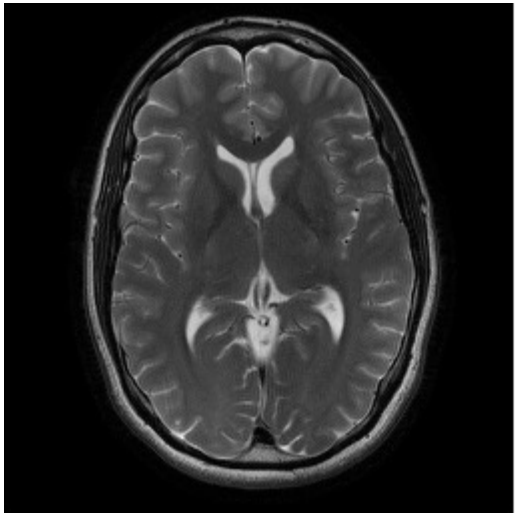
求u1,u2，这是重建的图

C needs rank of 2;
2 indenpendent eqaution
上式有4 independent equation

Parallel Imaging: SENSE

$$u = (C^H C)^{-1} C^H I$$

SENSE, R=2



SNR penalty in parallel imaging → SNR降低

$$SNR_{acc}=\frac{SNR_{no-acc}}{g\sqrt{R}}$$

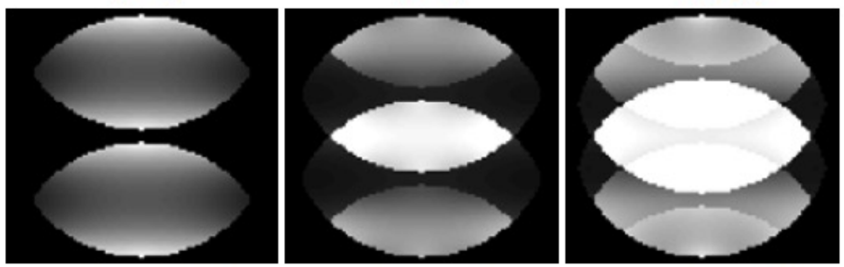
- g-factor: noise amplification due to ill-conditioning of the encoding matrix

rho:取对角元素(np.diagonal)

$$g(r)=\sqrt{\left(\mathbf{E}^{H}\boldsymbol{\Psi}^{-1}\mathbf{E}\right)^{-1}_{\text{rho}}\mathbf{E}^{H}\boldsymbol{\Psi}^{-1}\mathbf{E}_{(r)\,\text{rho}}}$$

g-factor

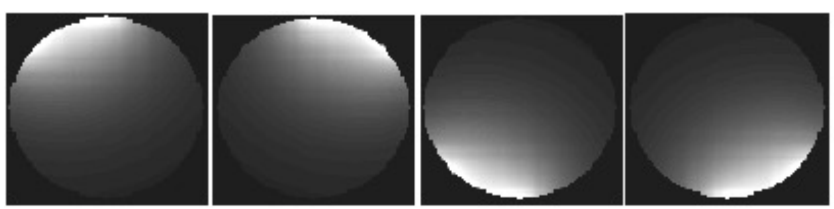
R=2 R=3 R=4



R=2	R=3	R=4
1<g<1.2	1<g<3	1<g<5
mean=1.07	mean=2.2	mean=5.4
max=1.2	max=3.1	max=8.6

R 增大， noise增大
因为R
增加（更多coil）,更容易有overlap,overlap会导致not independent equation

Coil sensitivities

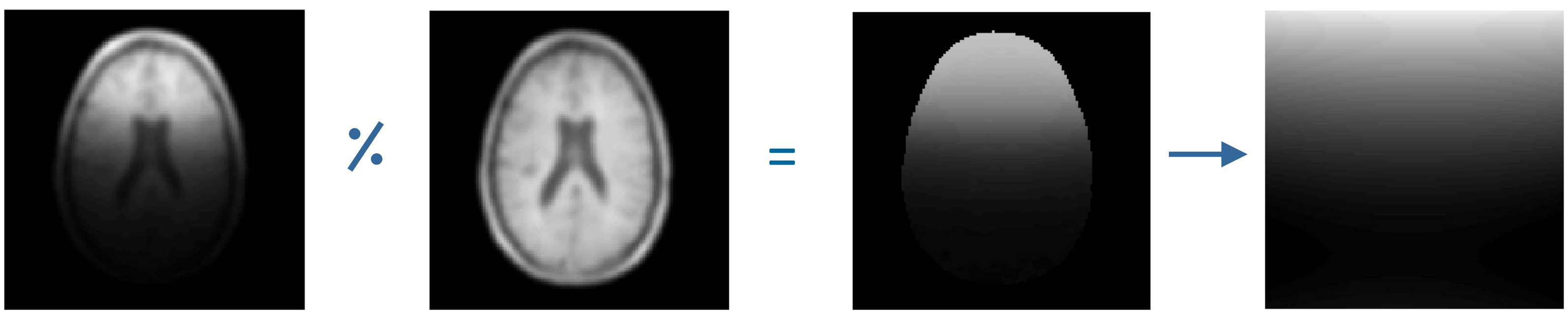


Coil sensitivity estimation for SENSE

- Estimation of pure coil sensitivities (Pruessmann et al. MRM 1999).
  - Separate low resolution image for each coil.

m = f×c　所以：c = m/c

REFERENCE ÷ COMBINATION = RAW SENSITIVITY → SMOOTHING EXTRAPOLATION



How to reduce noise amplification?

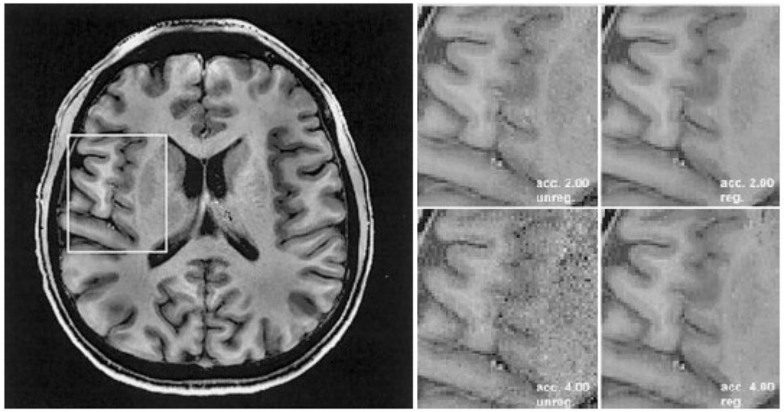
- Use more coils
- Improve coil array design
- Regularization of the inverse reconstruction if the coils are not strictly independent
- 2D acceleration instead of 1D acceleration (3D imaging)

Regularization of the inverse reconstruction

- Constrain the inverse problem to reduce noise amplification and control numerical instabilities

- Method 1: Tikhonov regularization
  - Constrain the power of the solution

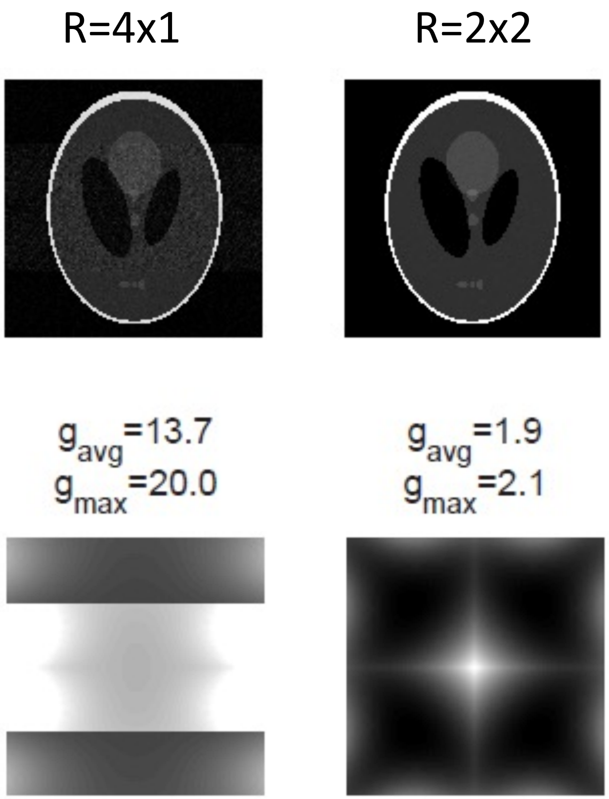
$$\hat{\mathbf{m}} = \min_{\mathbf{m}} \left\{ \left\| \mathbf{Em} - \mathbf{s} \right\|_2^2 + \lambda \left\| \mathbf{m} \right\|_2 \right\} = \left(\mathbf{E}^H \mathbf{E} + \lambda \mathbf{I} \right)^{-1} \mathbf{E}^H \mathbf{s}$$

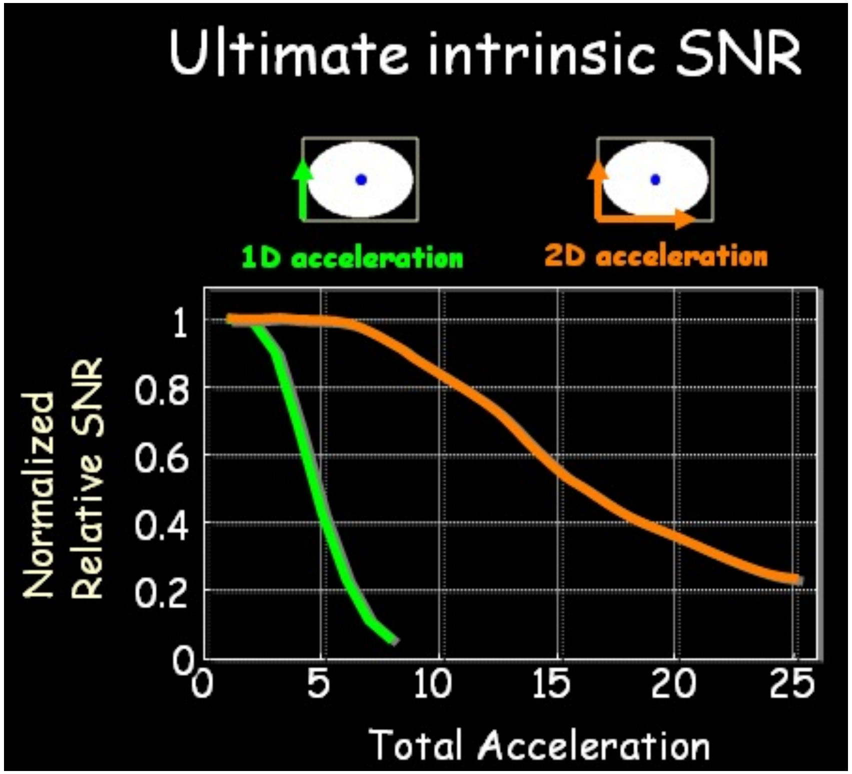


Lin FH et al, Magn Reson Med 2004; 51:559-567

2D acceleration Vs. 1D acceleration

- 2D acceleration reduces g-factor

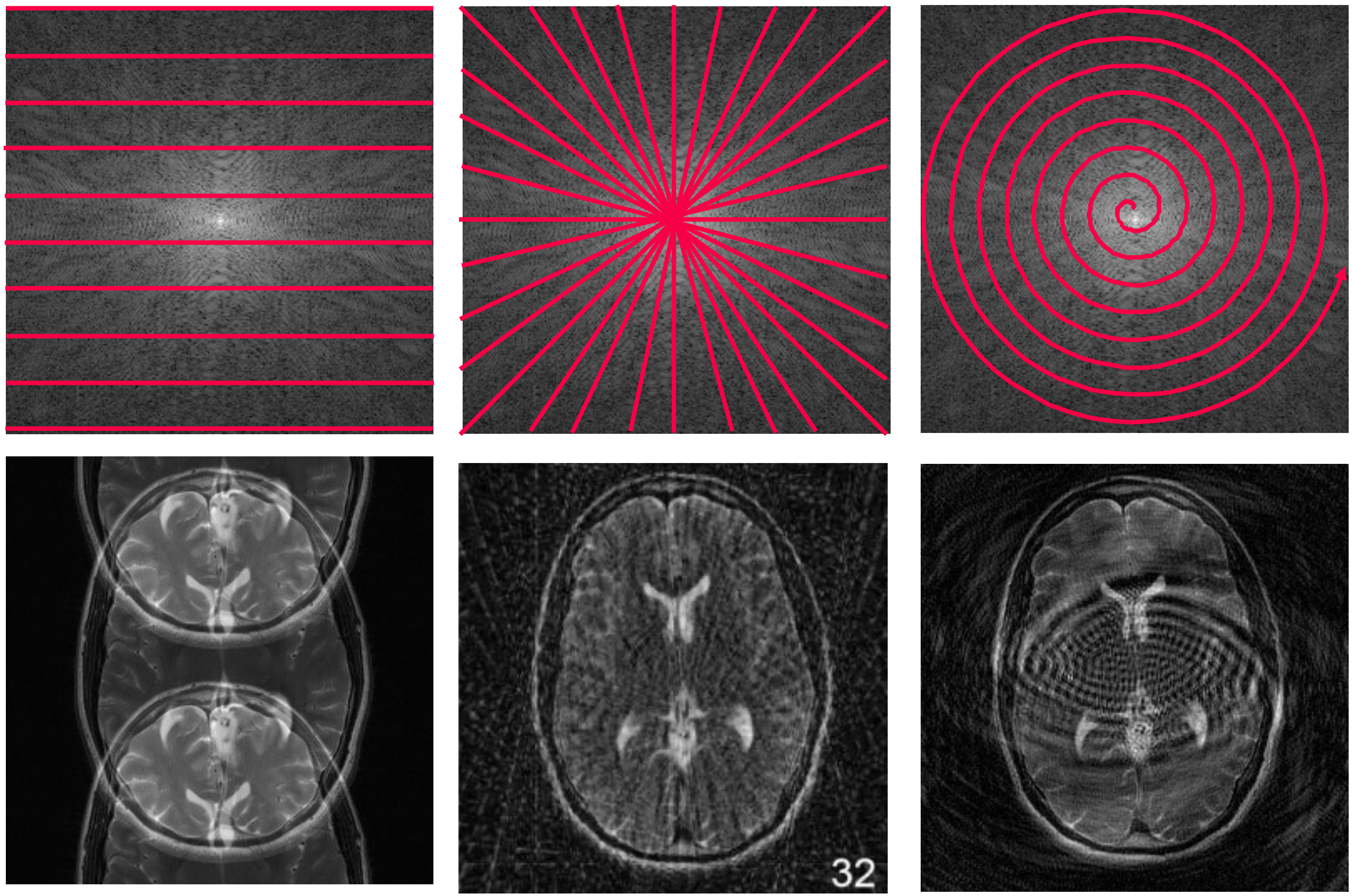




Ohliger MA et al. MRM 2003;50:1018-30

Non-Cartesian undersampling

Cartesian Radial Spiral



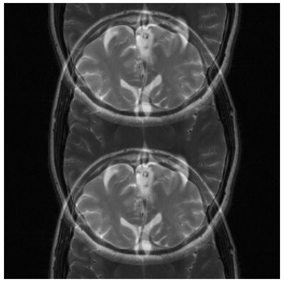
Non-Cartesian SENSE

- Decoupling is lost

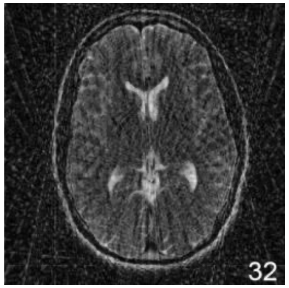
  解耦

- Each pixel is aliased with all other pixels
  - e.g. streaks in undersampled radial imaging
- Need to invert the full encoding equation
- Calls for an iterative algorithm
  - No explicit matrix inverse
  - Matrix-vector multiplications only

Cartesian



Radial



Summary

- Speed of gradient encoding is limited

  增大gradient amplitude来加速是不可行的

  - Physical and physiological constraints on the gradient amplitude and switching rate

- Fast, rapid or accelerated MRI

  用undersampling(multi-coild来加速)

  - k-space undersampling
  - Reconstruction is more challenging, but more fun
  - Exploit redundancies in the acquired data

Summary

- Parallel imaging
  - Exploit additional encoding provided by multiple receiver coils with different sensitivities
  - SNR penalty
  - SENSE (image-domain)
    - Unfolding images using coil sensitivities
    - Matrix inversion
  - SMASH, GRAPPA (k-space)
    - Next lecture

Outlook lab exercise

Computational MR imaging
Laboratory 5: Image space parallel imaging

Report is due on Wednesday the week after the lab session at 23:59. Send your report by email to Bruno Riemenschneider (bruno.riemenschneider@fau.de) and Florian Knoll (florian.knoll@fau.de).

Learning objectives

- Combine multicoil images
- Reconstruct undersampled multicoil data using SENSE algorithm
- Compute g-factor and SNR

Before the lab: Get familiar with the functions inv (matrix inverse) and pinv (matrix pseudo-inverse), and the operators ‘ (conjugate transpose) and * (matrix multiplication).

1. **Multicoil combination:** Load the file data_brain_8coils.mat. The variable d is the fully-sampled k-space data (256×256×8), the dimensions of the data are [PE,FE,channels], c is the coil sensitivity maps (256×256×8) and n is the noise-only scan (256×8). Combine the multicoil images using sum-of-squares and matched-filter (least-squares) algorithms. You might want to create a function for each combination, so that you can use it again. Comment of the effect of using the noise correlation matrix.

```
function [mc] = sos_comb(m,Psi)
% Input:
% m: multicoil images [nPE,nFE,nCh]
% Psi: noise correlation matrix [nCh, nCh]
% Output:
% mc: combined image [nPE,nFE,nCh]
```

```
function [mc] = ls_comb(m,c,Psi)
% Input:
% m: multicoil images [nPE,nFE,nCh]
% c: coil sensitivity maps [nPE,nFE,nCh]
% Psi: noise correlation matrix [nCh, nCh]
% Output:
% mc: combined image [nPE,nFE,nCh]
```

2. **Cartesian SENSE reconstruction and g-factor:** Write a function that reconstructs regularly undersampled data along the phase-encoding dimension using the SENSE method and computes the corresponding g-factor. The function will unfold multicoil aliased images using coil sensitivity maps in the image domain.

```
function [ir,g] = sense1d(ia,c,Psi,R)
% Input:
% ia: multicoil aliased images [Nx, Ny/R,Nc]
% c: coil sensitivity maps [Nx,Ny,Nc]
% Psi: noise correlation matrix [Nc,Nc]
% R: acceleration factor
% Output:
% ir: unaliased image [Nx,Ny]
% g: g-factor map [Nx,Ny]
```

Simulate acceleration factors of R= 2, 3, 4 along the phase-encoding dimension for the 8-coil data set from exercise 1. Reconstruct each undersampled data set using your SENSE implementation; compute the average g-factor and SNR loss (make sure to exclude the pixels outside the brain). Compute the RMSE with respect to the matched-filter combination of the fully-sampled data in exercise 1. Plot the reconstructed image, reconstruction error and g-factor map for each R.