Computational MRI

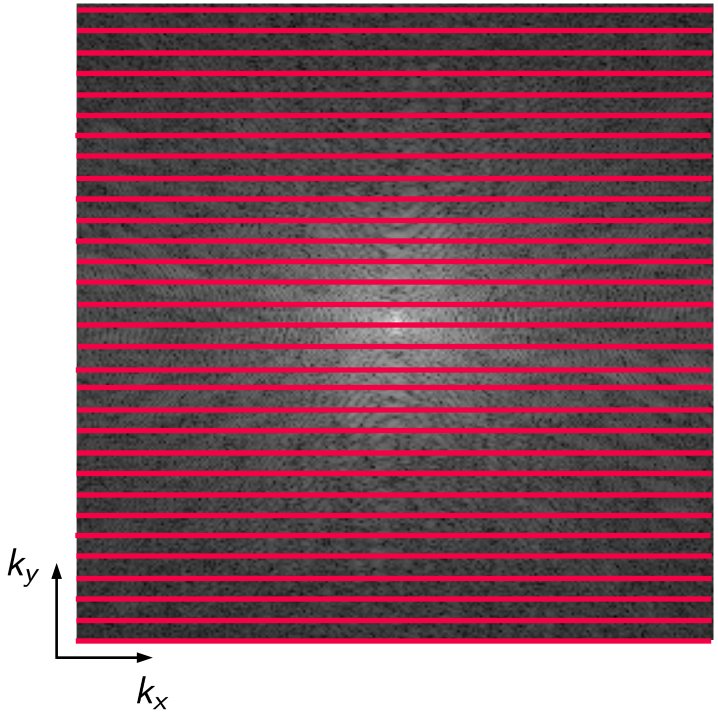
Partial Fourier Imaging

(and constrained reconstruction)

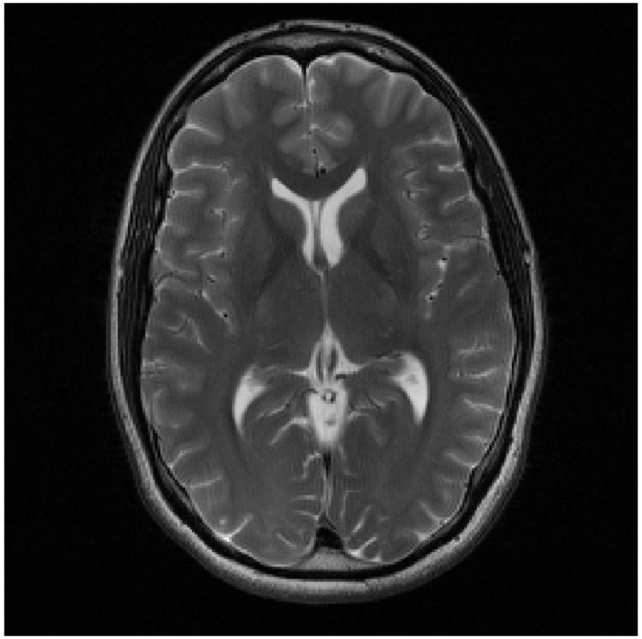
FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG

K-space

10min scantime (assumed)

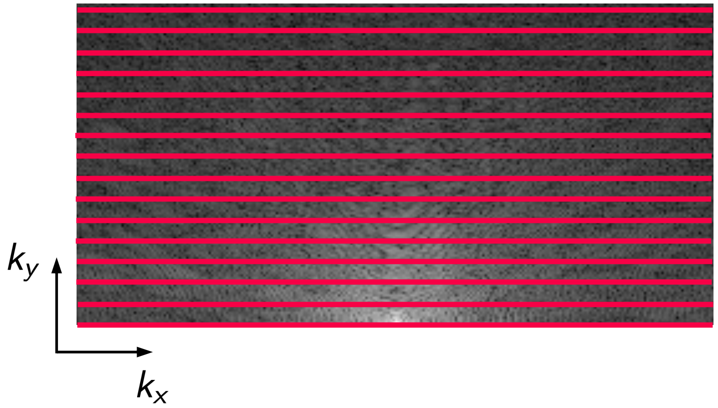


$$\xrightarrow{\mathcal{F}\mathcal{T}^{-1}}$$

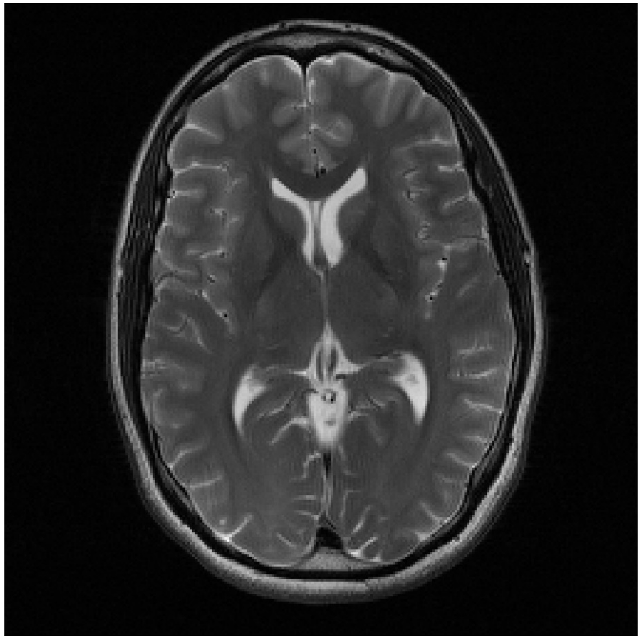


K-space

5min scantime (assumed)



$$\xrightarrow{\mathcal{FT}^{-1}}$$



Faster MR Imaging Methods

Paul Margosian, Franz Schmitt

MR Development Group, STEM 51, Siemens AG UB Med
Henkestraße 127, 8520 Erlangen West Germany

Abstract

Two different types of methods for acquiring MR data more quickly have been explored here. The first set of methods uses half the normal number of measurements and is capable of shortening acquisition times by a factor of two. The second type of method uses measurements from multiple echoes to synthesize a 'conventional' spin-warp raw data set. This second method is capable of shortening data acquisition times by factors of four to eight.

Three types of 'half the data' methods have been tested. The first uses the central half of the phase encoding steps, and reconstructs on the usual 256 x 256 matrix using a sinc interpolation; with this first method, signal to noise ratio is improved, but spatial resolution in the phase encoding direction is reduced. The second method uses all the upper (or lower) half of the normal phase encoding steps, places them in an otherwise empty 256 x 256 matrix, reconstructs, makes a phase correction, takes the real part. This 'half Fourier' method uses the phase information to retain full resolution with only half the data (really 53%) but at the cost of a reduction of signal to noise ratio. for the third method, every second phase encoding measurement is taken, along with eight extras in the center of the data space, empty data lines are filled by cubic interpolation, and a FFT recontruction is done. For this method, spatial resolution and signal to noise ratio are maintained, but two 'ghost' images (low amplitude-occur because of the inherent undersampling of this method.

The 'multiple echo' method for faster data acquisition makes use of a sequence that acquires different phase encoding steps in the data from different echoes. the best strategy is to take the central (low frequency) measurements from the lowest numbered echoes, and the higher frequency measurements from the later echoes. The practical details of this method include correcting position, phase, and amplitude of measurements from different echoes. It is also important to use data acquisition sequences that are very well adjusted refered to stimulated echoes and eddy currents. The result is that data can be acquired four to eight times faster than normal for spin-warp imaging, almost full resolution is retained, and signal to noise ratio is reduced somewhat.

Half Data Methods

The simplest way to reduce MR imaging time is to use methods that require only half (or approximately half) the normal data acquisition. Three methods have been tested here:

(1) Use the central 128 (half) of the data, reconstruct. This is the classic 'sinc interpolation' method.

(2) Use only the upper half (or lower half) of the normal phase encoding lines, make a phase correction, reconstruct; the real part is the desired (full resolution) image.

(3) Use every other data line of the normal phase encoding set; do special processing to minimize the 'ghost' images that come from the inherent undersampling of this method.

These three data acquisition strategies are illustrated by the following sketches (vertical direction is phase encoding direction, horizontal direction is readout direction, shaded areas indicate which measurements are actually taken, all else is set to zero):

Fourier transform properties

- Linearity: $F\{as_1(r)+bs_2(r)\}=aS_1(k)+bS_2(k)$

- Shifting: $F\{s(r-r_0)\}= e^{-i2\pi kr_0}S(k)$

- Modulation: $F\{e^{i2\pi k_0 r}s(r)\}=S(k-k_0)$

- Conjugate symmetry: $s(r)\,real \Rightarrow S(-k)=S^*(k)$

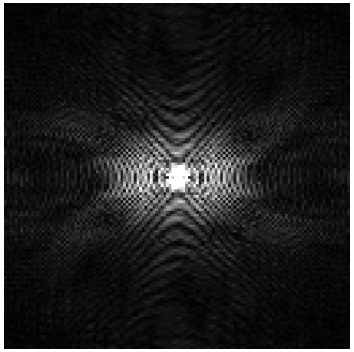
  时域只有real,不是复数

- Scaling: $F\{s(ar)\}=\frac{1}{|a|}S\left(\frac{k}{a}\right)$

Hermitian symmetry

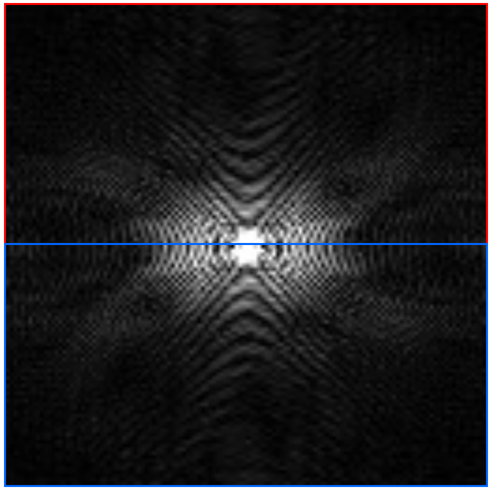
前提：数据收集过程中没有出现相位误差，即image只有实部。
相位误差来源于磁场不均匀，运动等。（见底下推断）

- Real-valued objects $\Rightarrow$ $S(-k)=S^*(k)$
  共轭复数



- k-space is redundant

- In theory: imaging with half the acquisition time

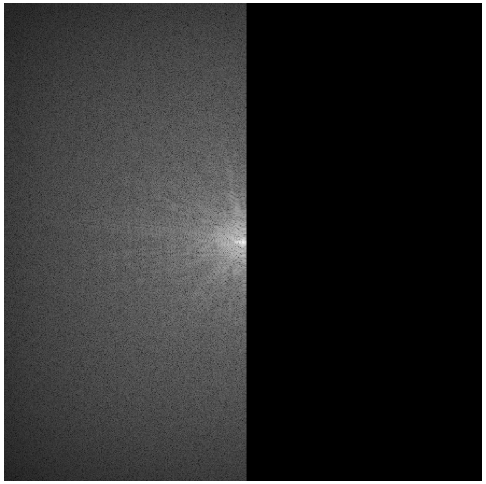


Acquired k-space

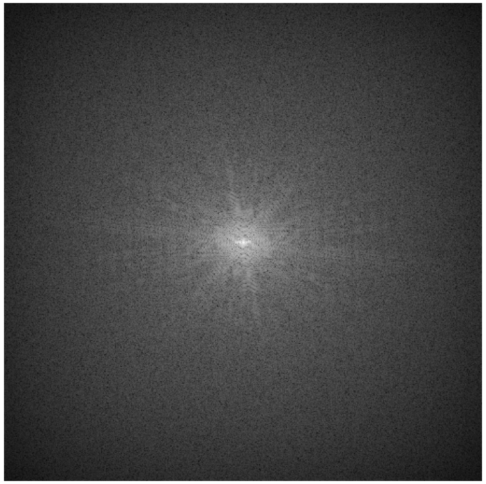
Synthesized k-space

The precession frequency becomes $\omega_0 \rightarrow \omega_0 + \Delta\omega(\boldsymbol{r})$ with
$\Delta\omega(\boldsymbol{r}) = \gamma \Delta B_z(\boldsymbol{r})$
(as for the gradients: only the z-component matters)
The associated phase is $\Delta\varphi(\boldsymbol{r}, t) = \Delta\omega(\boldsymbol{r}) \cdot t.$

Hermitian symmetry reconstruction

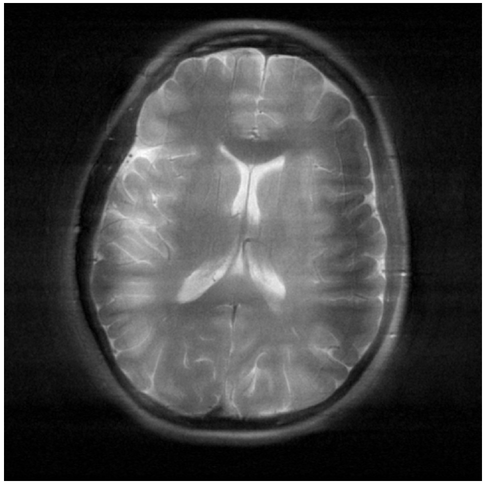


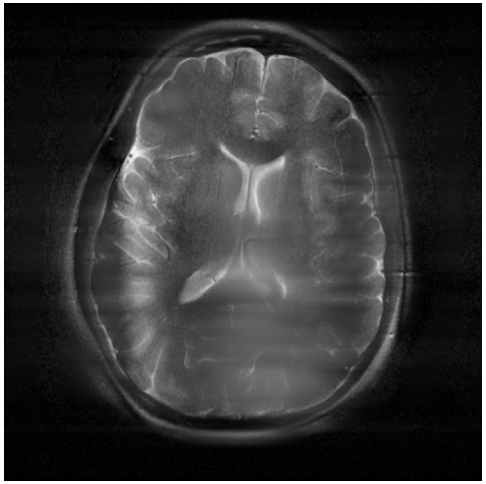
$$S(-k) = S^*(k)$$



$\mathcal{F}^{-1}$

$\mathcal{F}^{-1}$



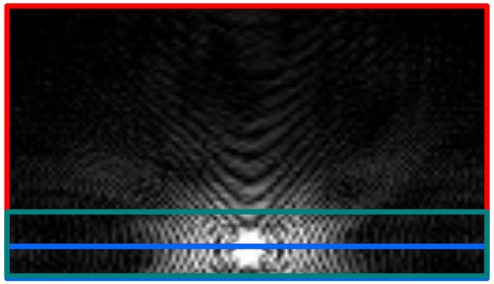


Practical considerations

- In practice, objects are not truly real-valued (phase)
  - Magnetic field inhomogeneities
  - Object motion

- Solution: measure more k-space samples, estimate the phase and reconstruct the magnitude image
  - Phase estimation
    - Low resolution image
    - Small symmetric region at the k-space center

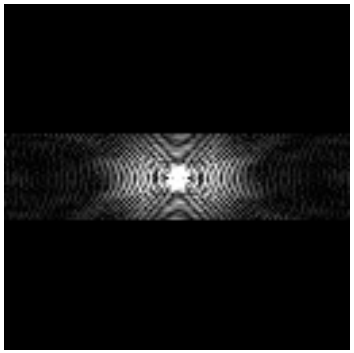
Asymmetric region in k-space



Partial Fourier Reconstruction

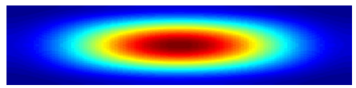
- Phase estimation

Zero-filled
phase reference



×

Hamming
window



$\xrightarrow{F^{-1}}$ $\varphi(r)$

- Phase-constrained reconstruction
  - One-step phase correction

    Margosian method (Margosian P et al. 1984)

    Homodyne method (Doll DC et al. 1991)

  - Iterative estimation of missing k-space data

    POCS method (Cuppen and Van Est 1987)

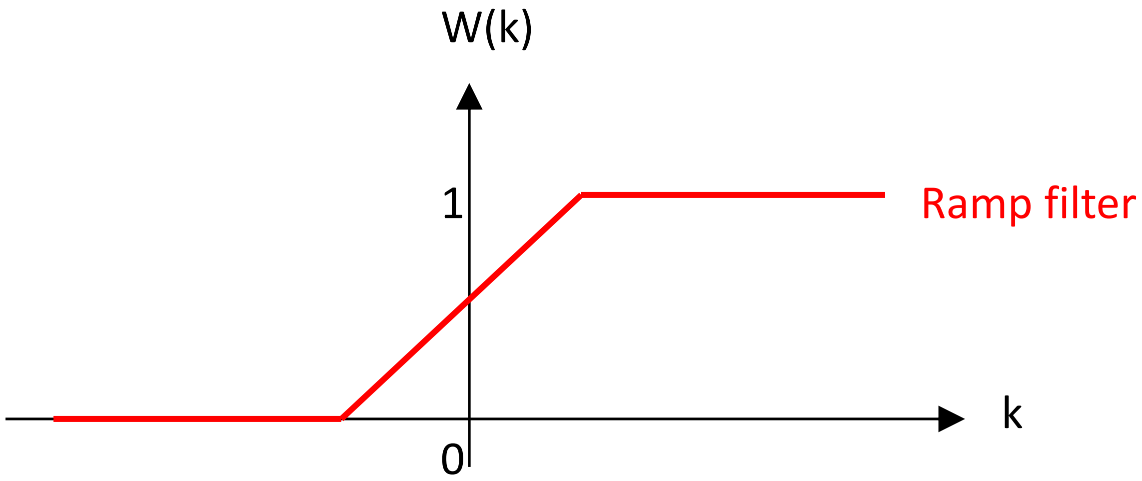
Margosian method

- Phase-corrected image

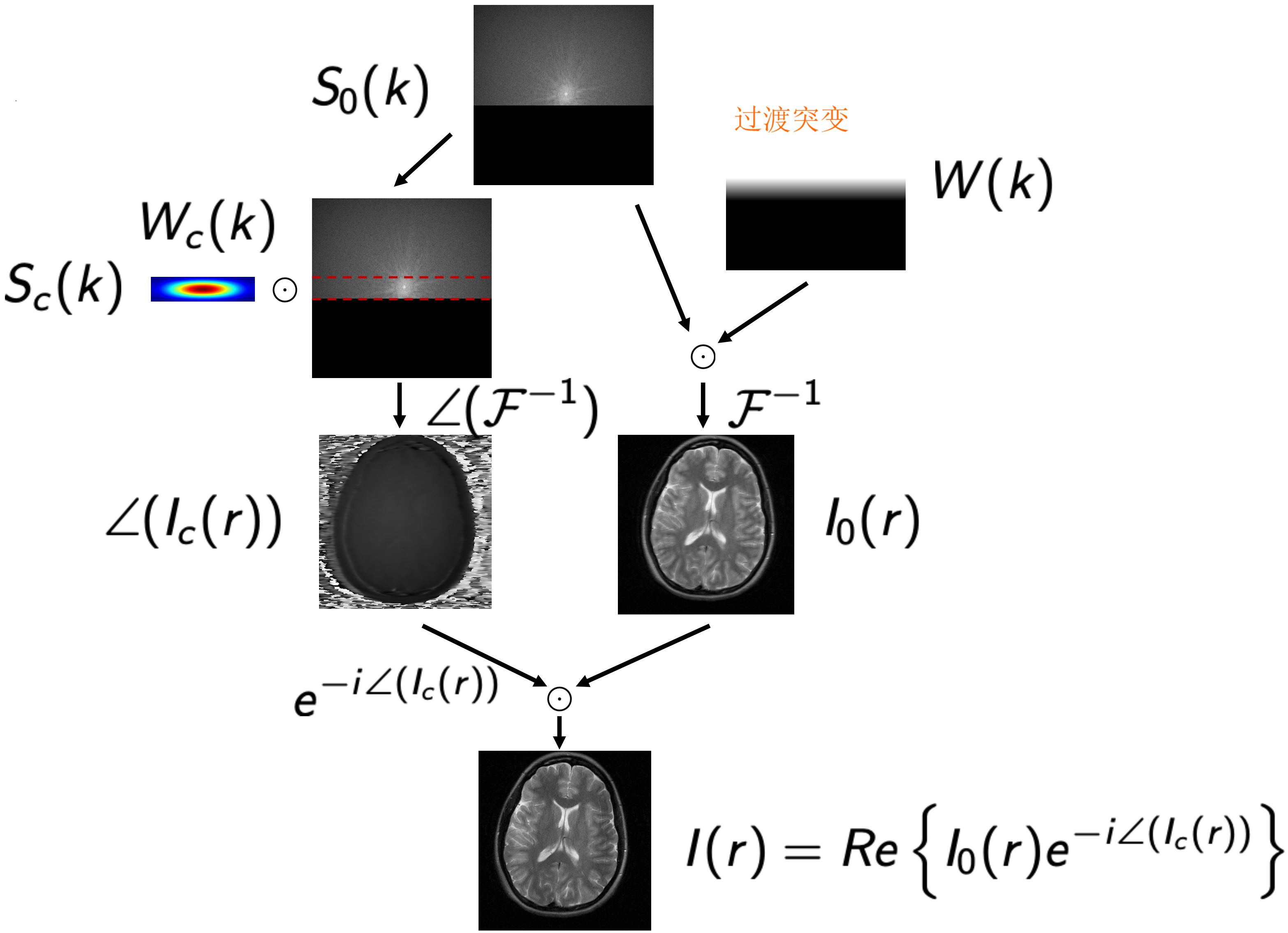
$$I(r) = Re\left\{I_0(r)e^{-i\angle(I_c(r))}\right\}$$

- $I_0(r)$: zero-filled FT reconstruction of the filtered k-space data

$$I_0(r) = \mathcal{F}^{-1}\left\{S_0(k)W(k)\right\}$$



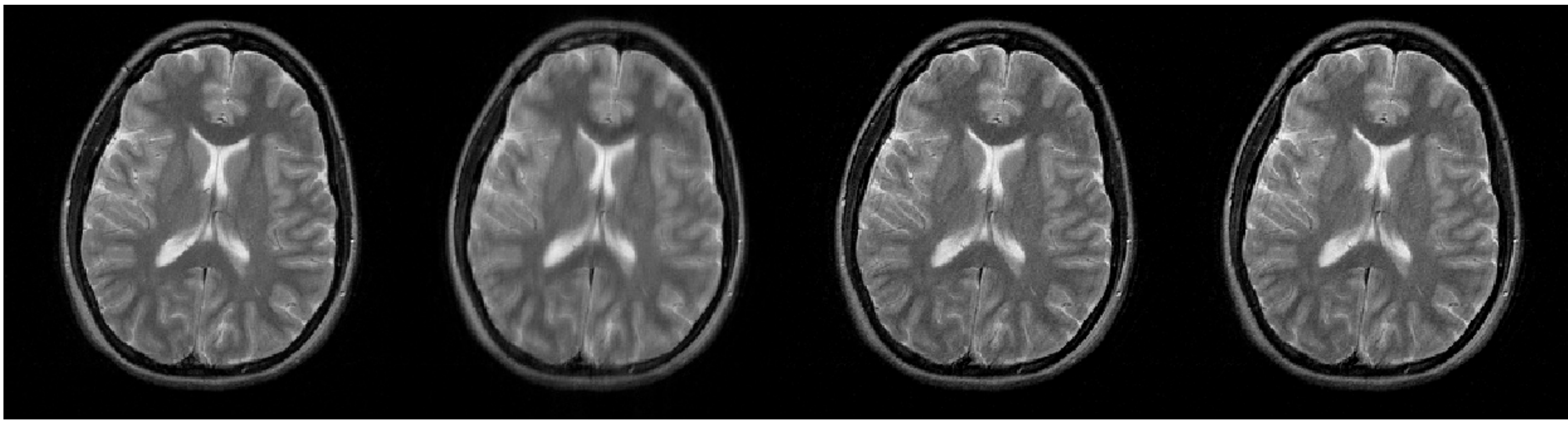
Algorithm 1: Margosian (homodyne) method

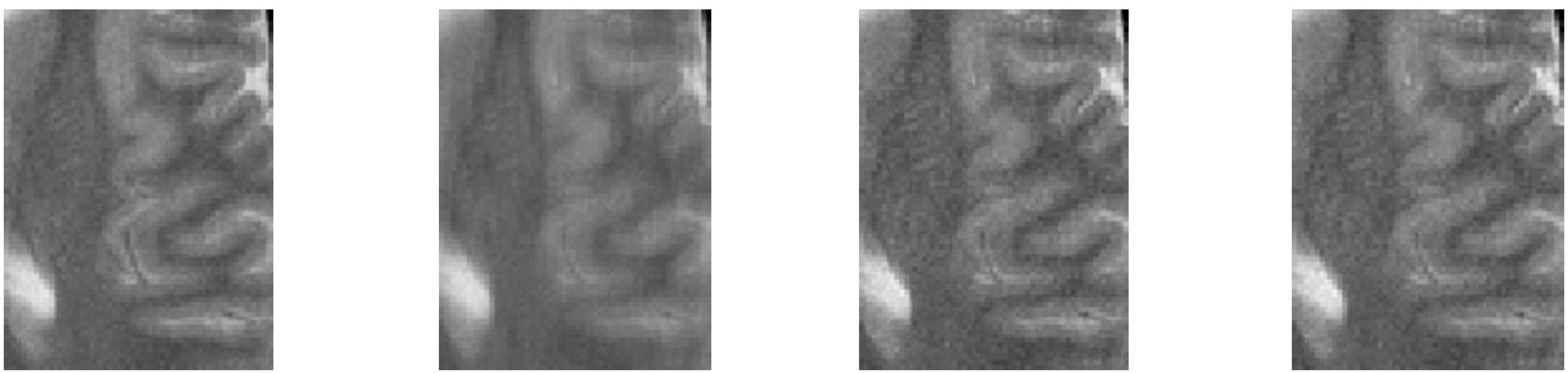


Margosian method example (PF=9/16)

- 512x512 image, 288 PE lines

Original | Zero-filled | Margosian (ramp) | Margosian (Hamming)

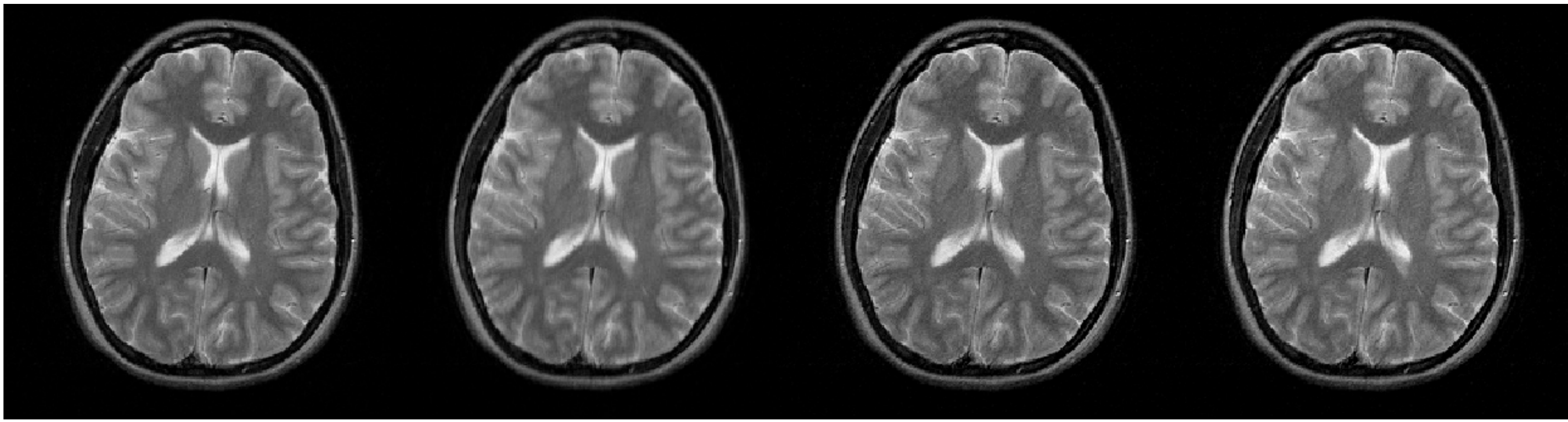


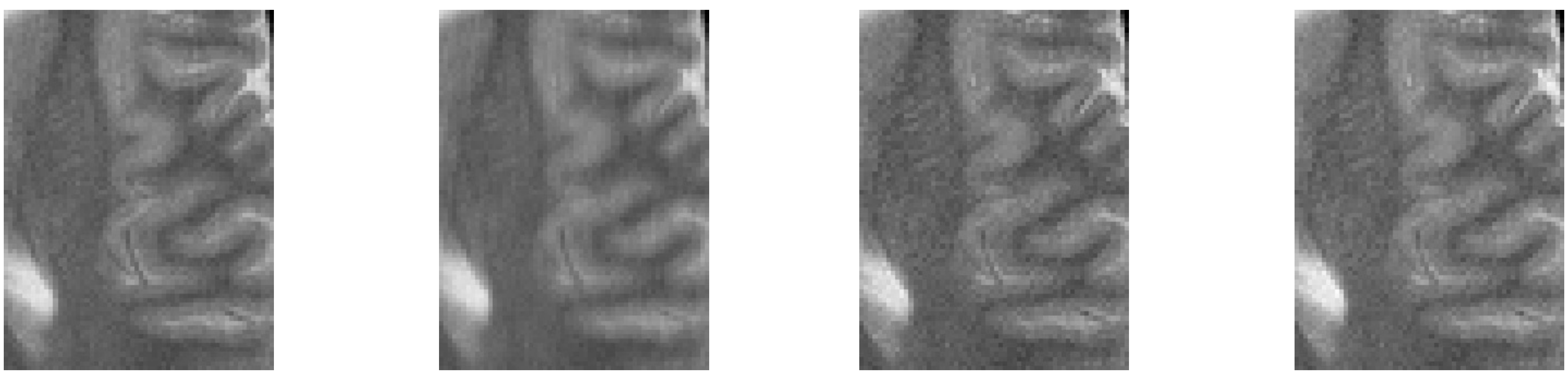


Margosian method example (PF=5/8)

- 512x512 image, 320 PE lines

Original	Zero-filled	Margosian (ramp)	Margosian (Hamming)





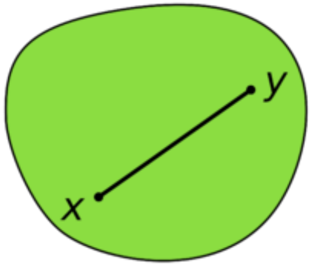
POCS method

- Partial Fourier reconstruction formulated as a constrained optimization problem

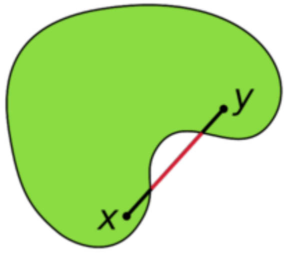
- Projection onto convex sets (POCS)
  - General tool to solve constrained optimization problems
  - Each constraint is defined as a convex set

$$\Omega \text{ is convex if for } x_1 \in \Omega \text{ and } x_2 \in \Omega \quad \Rightarrow \quad \lambda x_1 + (1-\lambda) x_2 \in \Omega$$

Convex:



Non-convex:



POCS method

- Alternating projection algorithm
  - A point in the intersection of two convex sets can be found iteratively by alternating projections on each set

    Ω_1 and Ω_2 are convex sets

    Goal is to find $x \in \Omega_1 \cap \Omega_2$

    - Start with any x_0
    - Alternately project onto Ω_1 and Ω_2

      $$y_l = P_{\Omega_2}(x_l)$$

      $$x_{l+1} = P_{\Omega_1}(y_l)$$

  - If constraints are well defined, the algorithm converges in a few iterations

POCS method for partial Fourier reconstruction

- Constraints
  - Phase: $\Omega_1 : \varphi(r) = \angle(I_c(r))$
  - Data consistency: $\Omega_2 : \mathcal{F}(I(r)) = S_0(k)$

- Algorithm

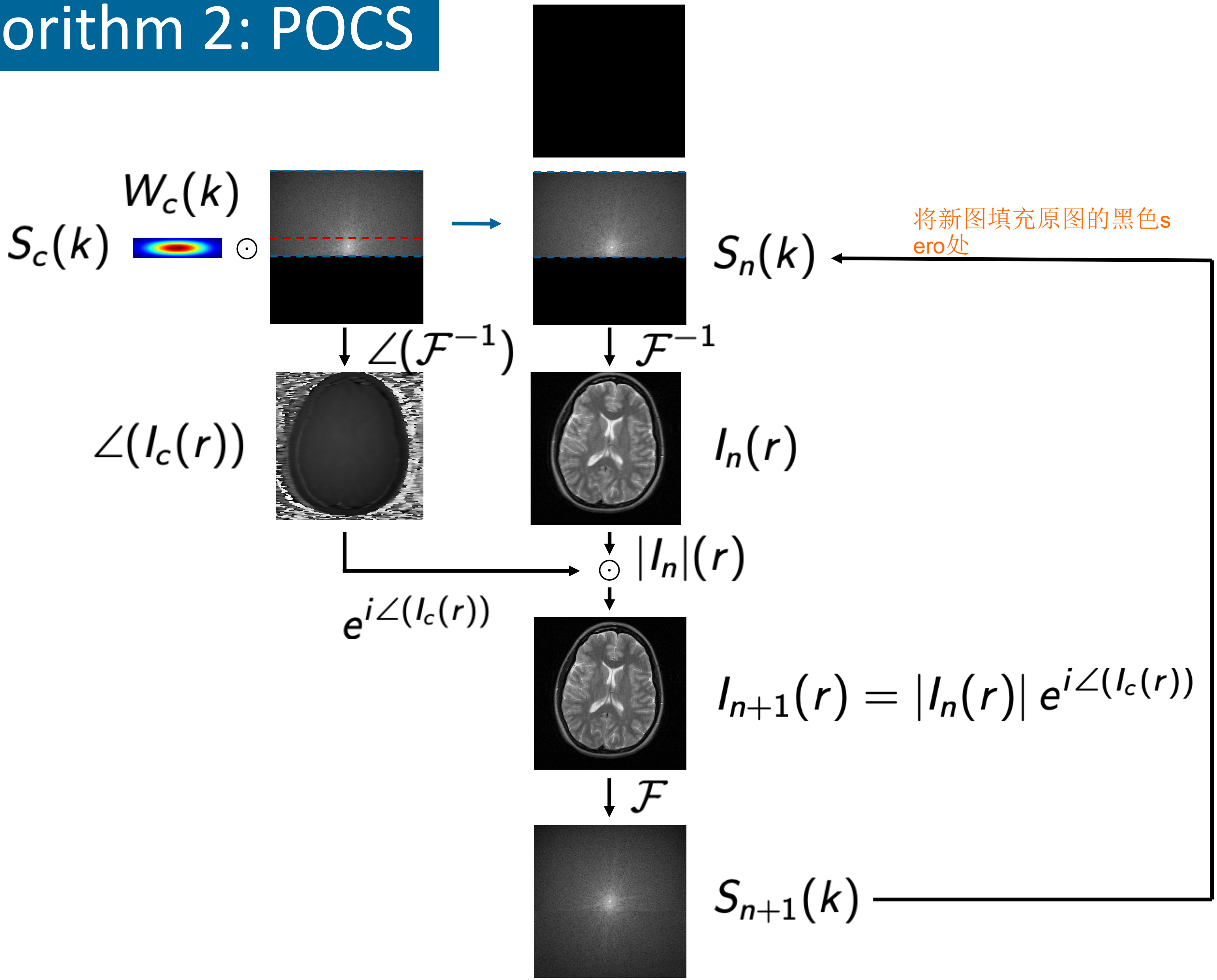
  Initialization $I_0(r) = \mathcal{F}^{-1}(S_0(k))$

  Iteration

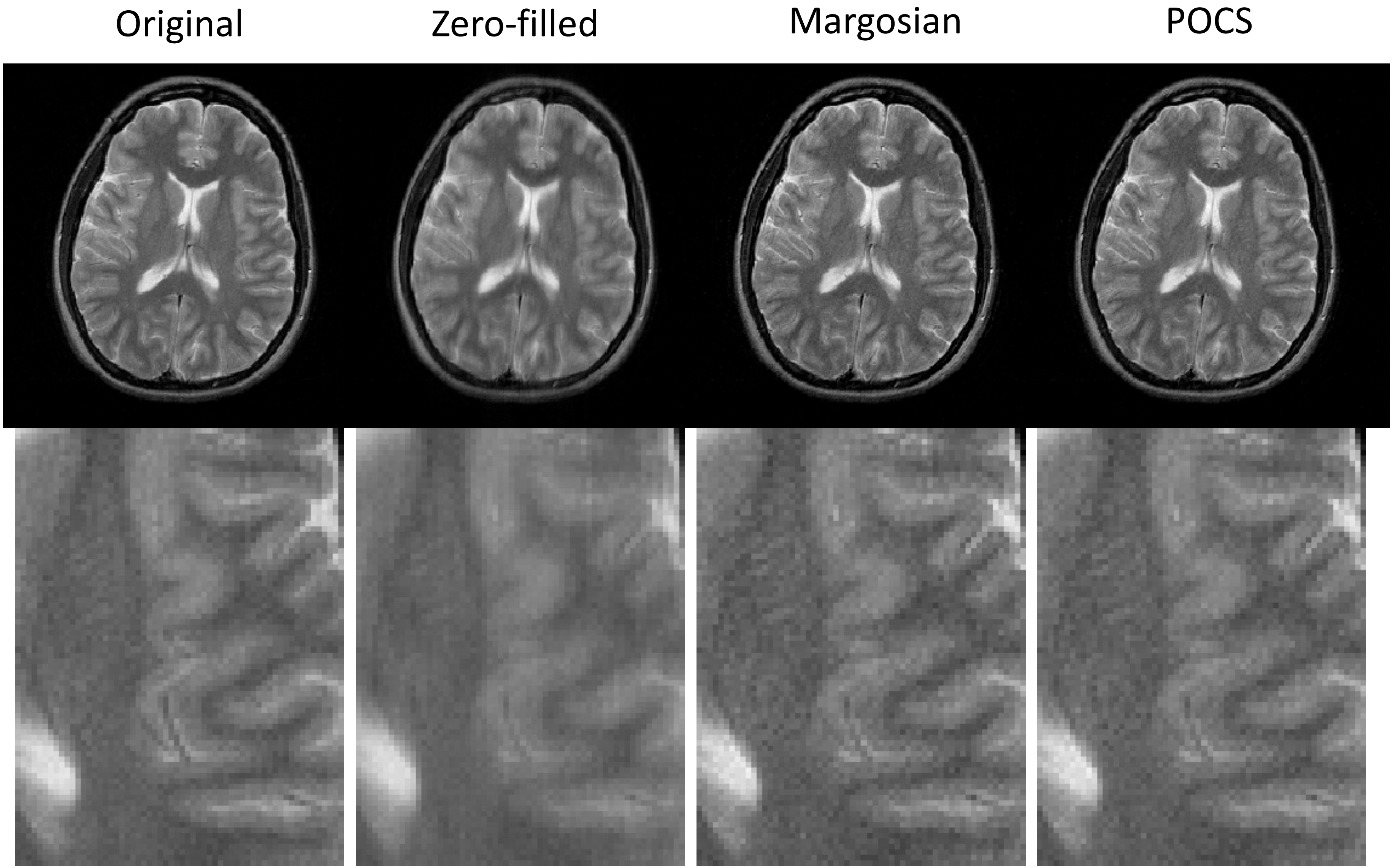
  Phase: $I_{n+1}(r) = |I_n(r)|\, e^{i\angle(I_c(r))}$

  Data consistency: $\begin{cases} S_{n+1}(k) = \mathcal{F}(I_{n+1}(r)) \\ S_{n+1}(k) = S_0(k) \quad \text{if k was sampled} \end{cases}$

Algorithm 2: POCS



Partial Fourier MRI (PF=9/16)



SNR for partial Fourier MRI

- SNR loss due to reduced number of k-space samples

$$SNR \propto \frac{1}{\sqrt{R}}, \text{ where R is the reduction factor}$$

- For partial Fourier MRI

$$R = \frac{N}{N_h}$$

$$\text{N : reconstructed image size}$$

$$\text{N}_\text{h} \text{ : number of k} - \text{space samples}$$

Summary

- Partial Fourier MRI
  - Hermitian symmetry
  - Reduce the number of samples to almost a half
- Practical methods
  - Phase-correction
  - Margosian method (one-step)
  - POCS methods (iterative)
- SNR loss close to square root of 2
- Applications
  - Reduce the number of phase-encoding steps
  - Reduce the readout duration (fractional/asymmetric readout)