Medical Image Processing for Diagnostic Applications

3-D Data – Line-Integrals

Online Course – Unit 44
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Topics

From 2-D to 3-D

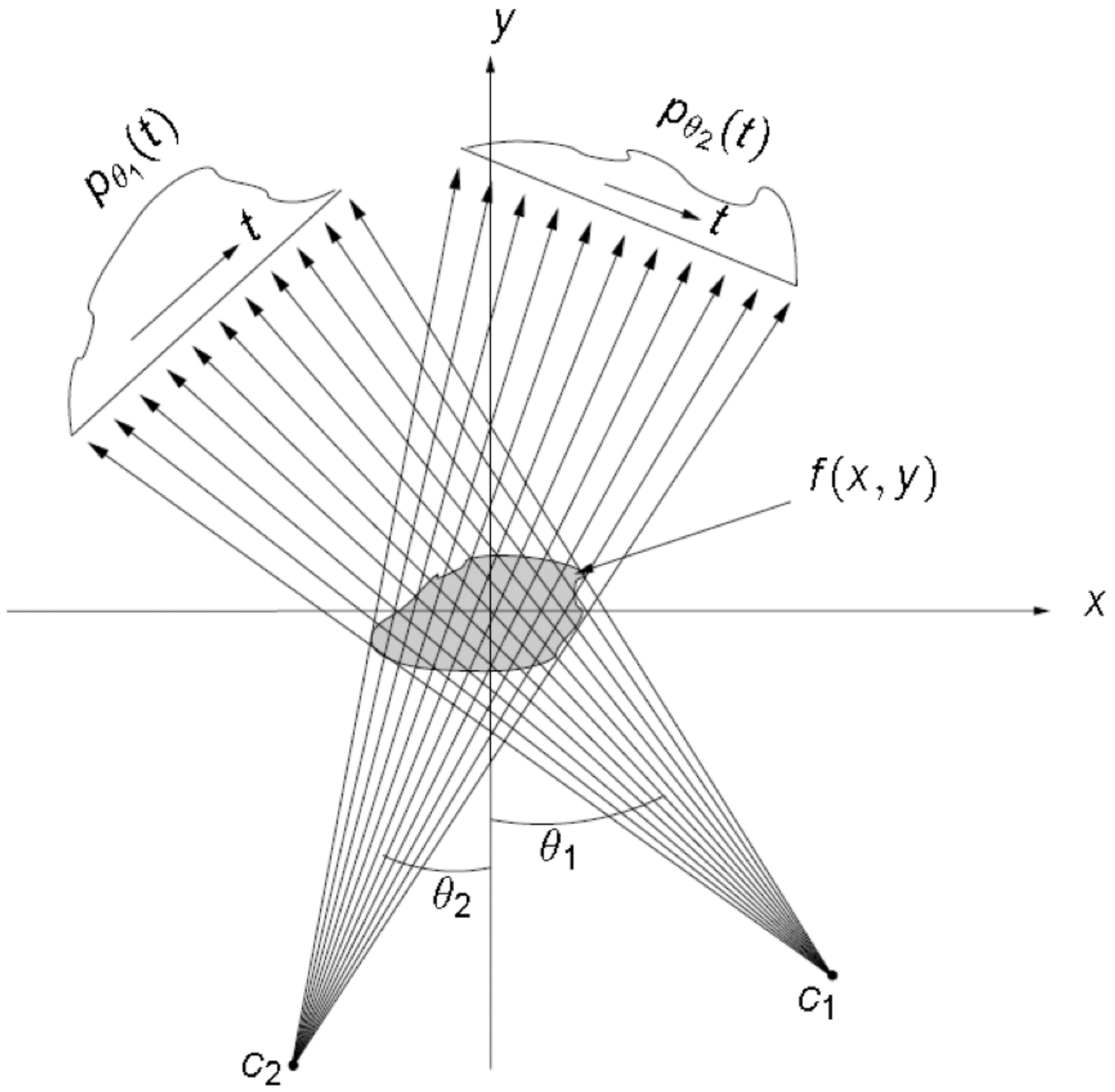
Parallel Line-Integral Data

Orlov's Condition

Outline: Backprojection-then Filtering (BPF) Algorithm

Summary
- Take Home Messages
- Further Readings

From 2-D to 3-D



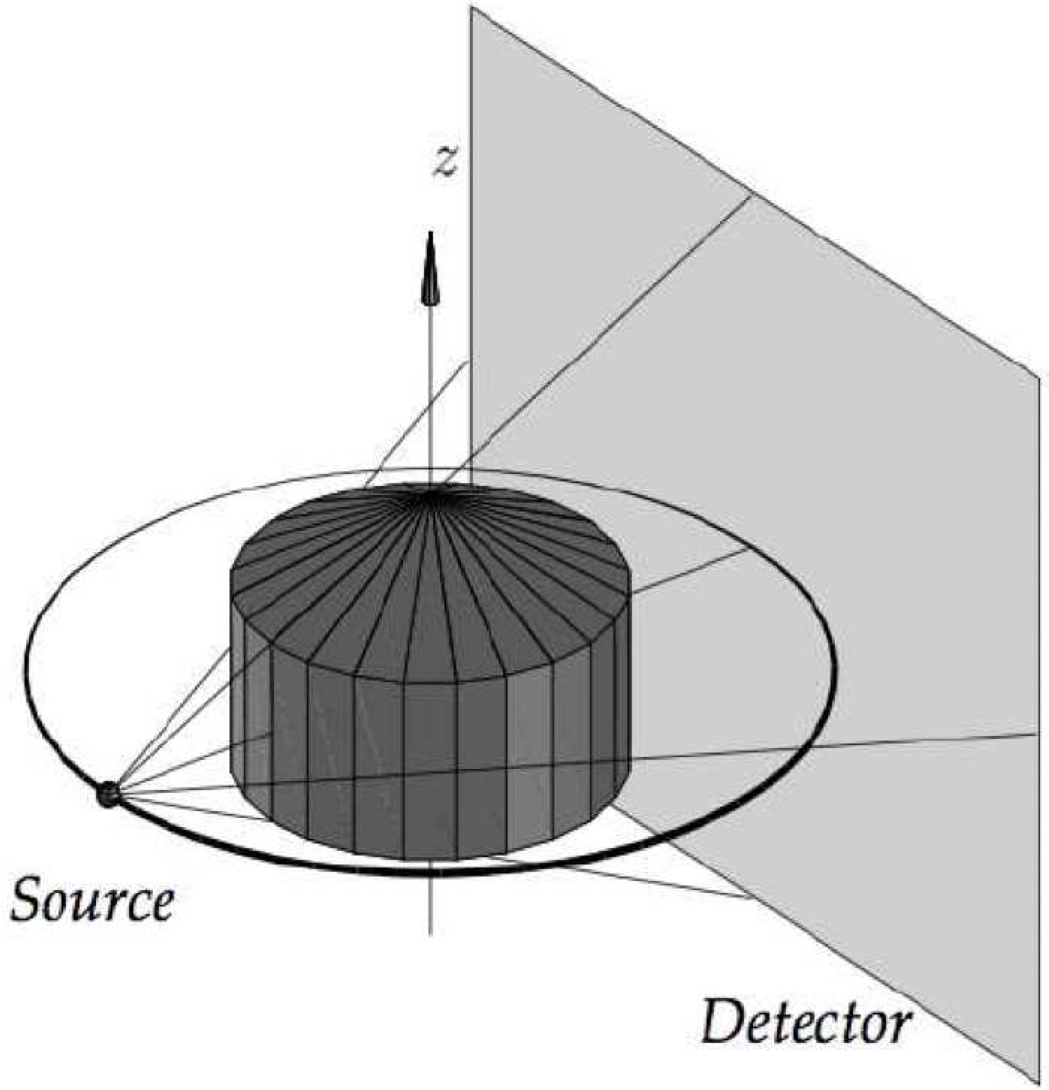


Figure 1: The fan beam geometry is a 2-D pattern (left), its corresponding 3-D structure is the cone beam (right).

Topics

From 2-D to 3-D

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Parallel Line-Integral Data

- **Problem:** Line-integrals may be along rays that are not perpendicular to the axis of rotation.
- How does this affect our reconstruction?

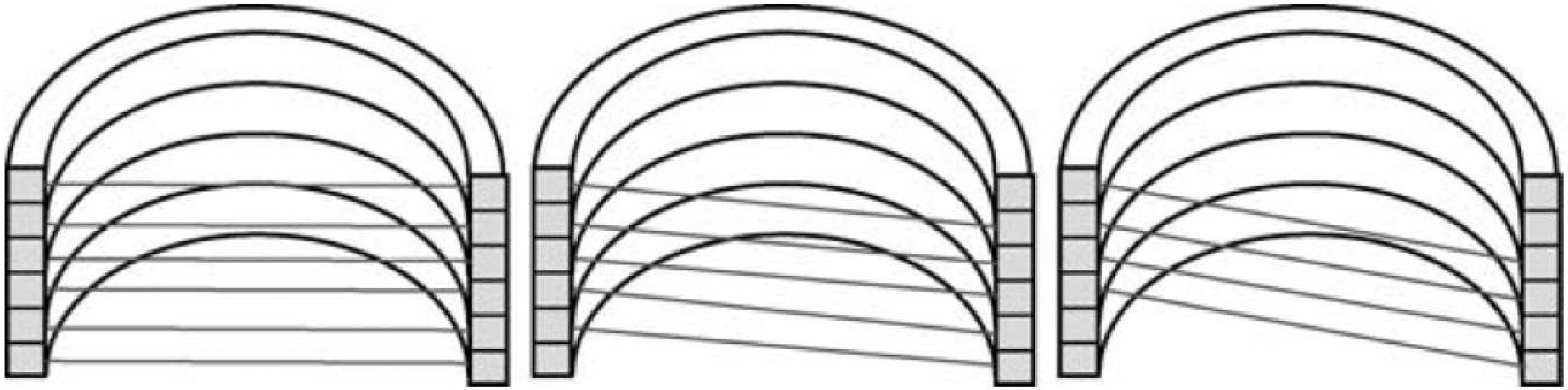

Figure 2: Several parallel ray paths from source to detector (Zeng, 2009)

Central Slice Theorem for Line-Integral Data

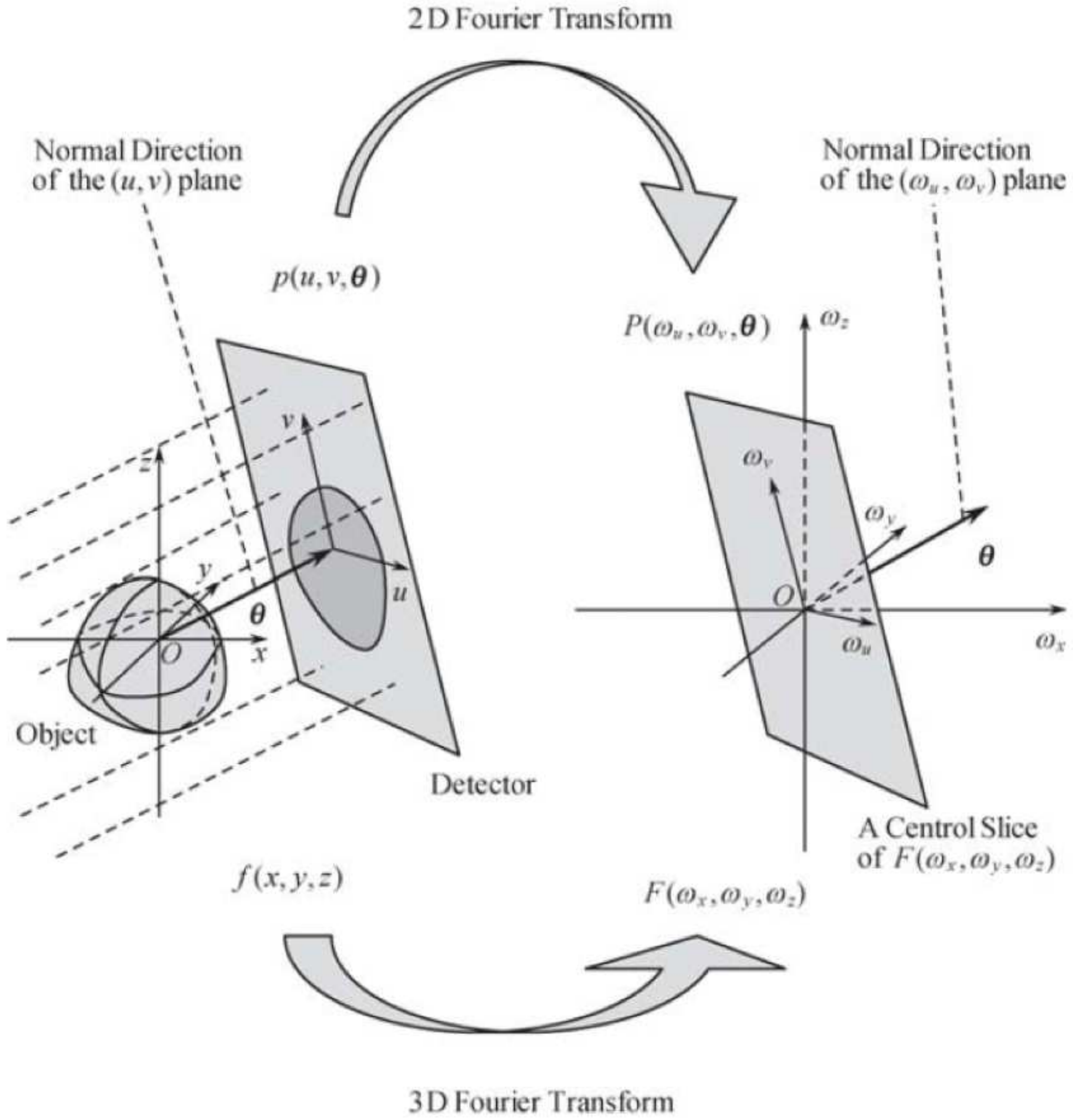

Figure 3: The 2-D Fourier transform of a projection corresponds to a slice through the origin in the 3-D Fourier transform of the object (Zeng, 2009).

Parallel Line-Integral Data

Parallel projections on rays that are perpendicular to the rotation axis can fill the complete Fourier space:

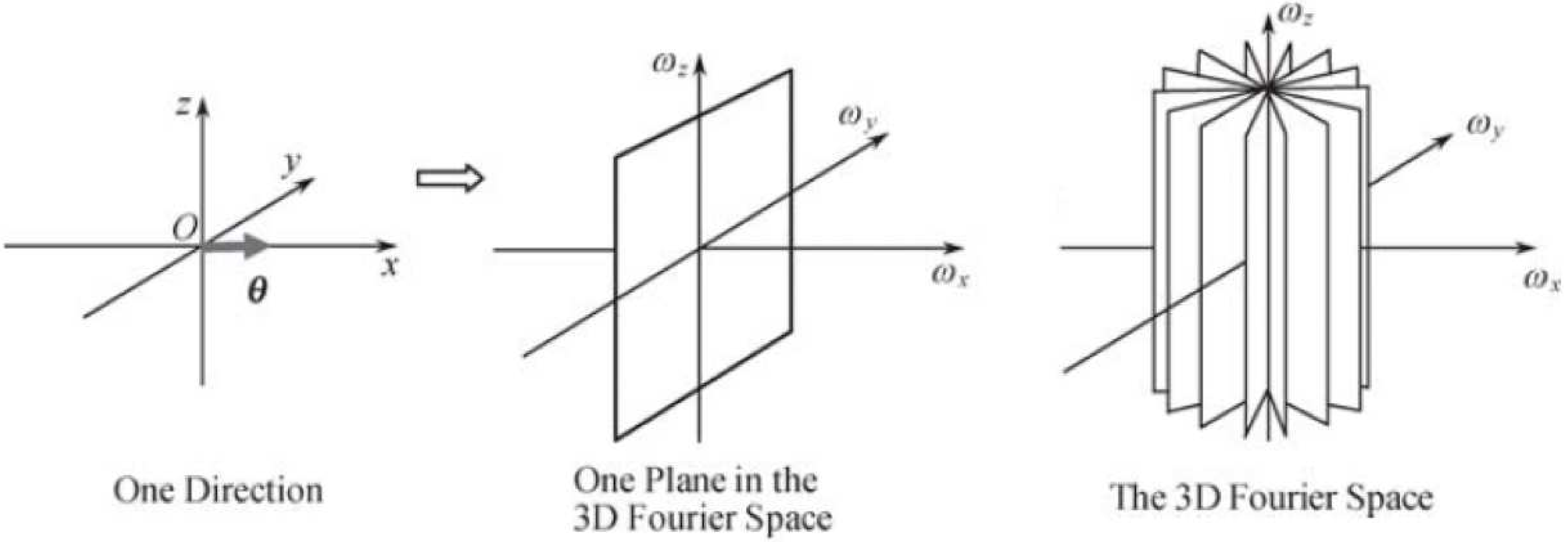

Figure 4: Sampling the Fourier space with parallel line-integrals (Zeng, 2009)

Parallel Line-Integral Data

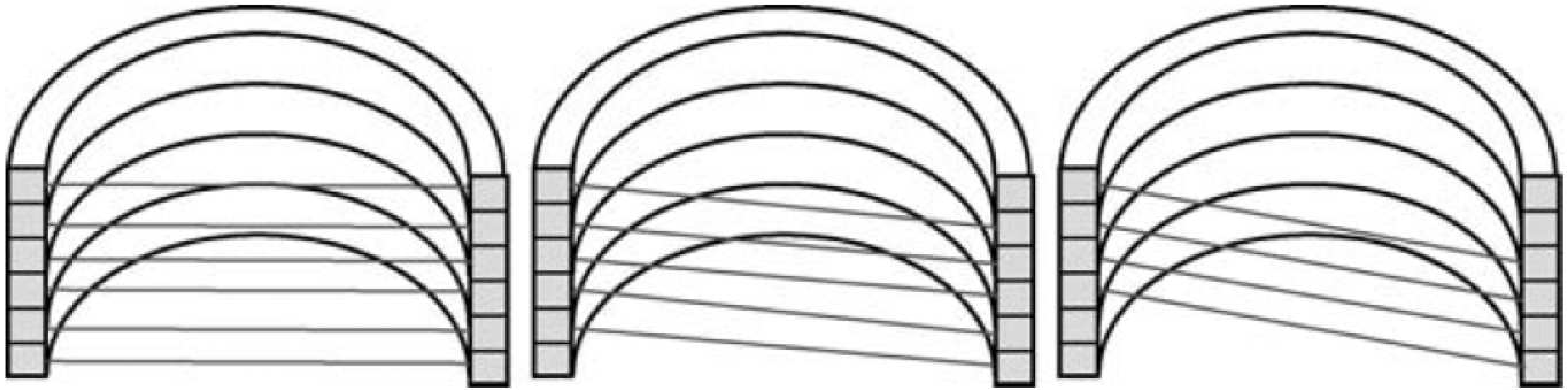

Figure 5: Several parallel ray paths from source to detector (Zeng, 2009)

Fourier Space:

Topics

From 2-D to 3-D

Parallel Line-Integral Data

Orlov's Condition

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Condition for Data Completeness

Orlov's condition: A complete data set can be obtained if every great circle intersects the trajectory.

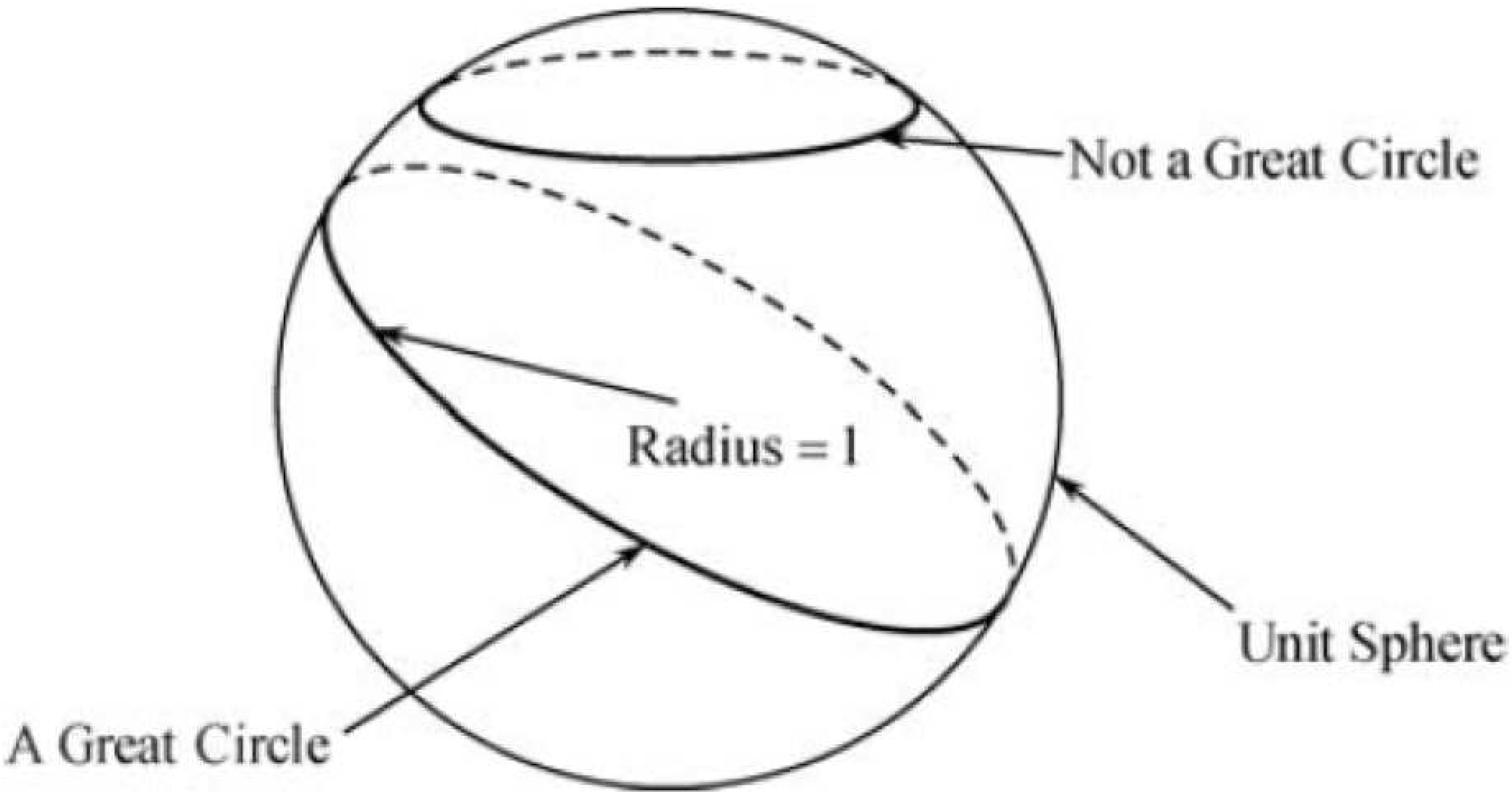

Figure 6: Diagram showing the definition of great circles (Zeng, 2009)

Examples of 3-D Trajectories

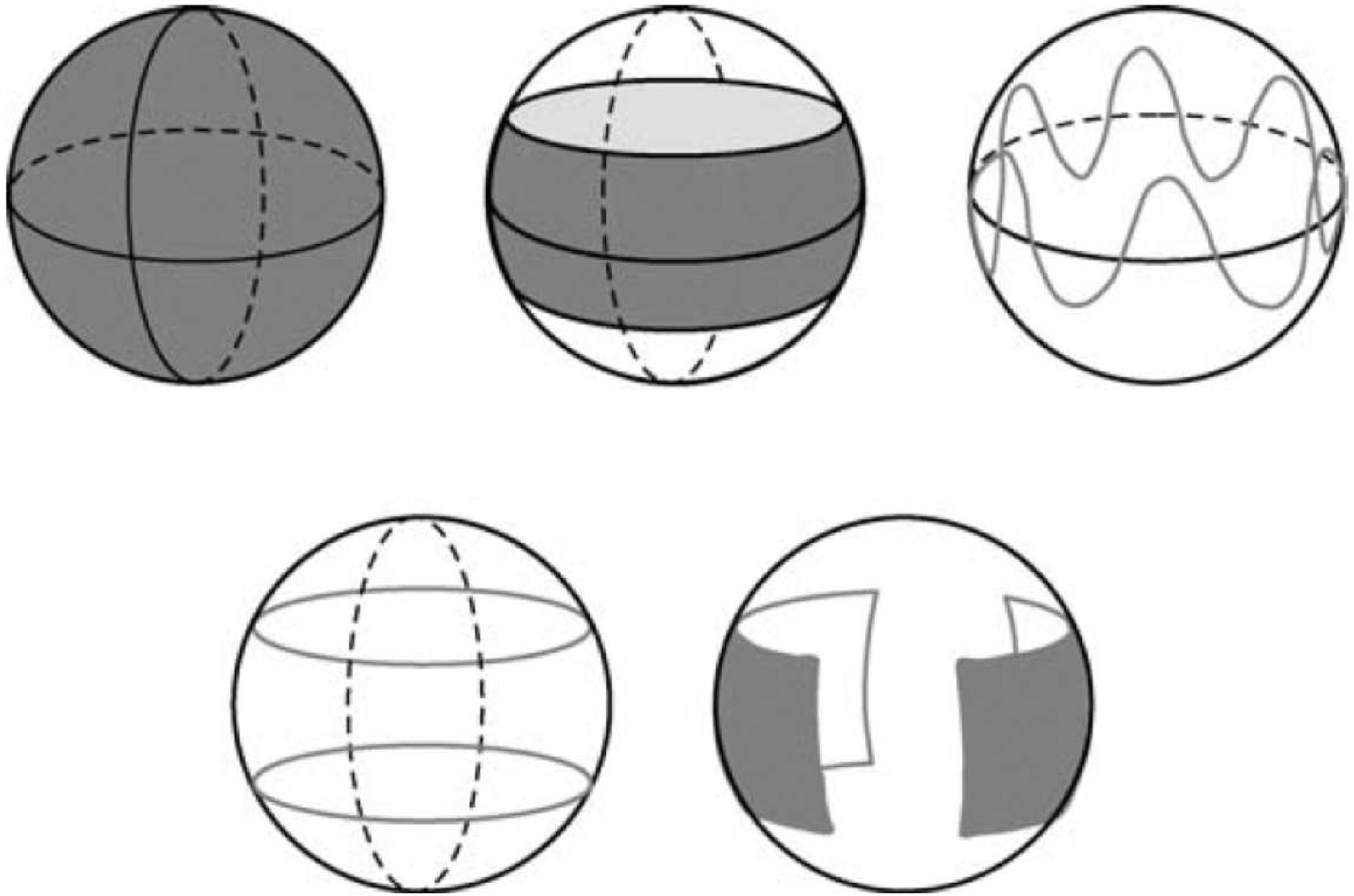

Figure 7: Several possible trajectories: which of these yield a complete data set (Zeng, 2009)?

Trajectories

Fourier Space:

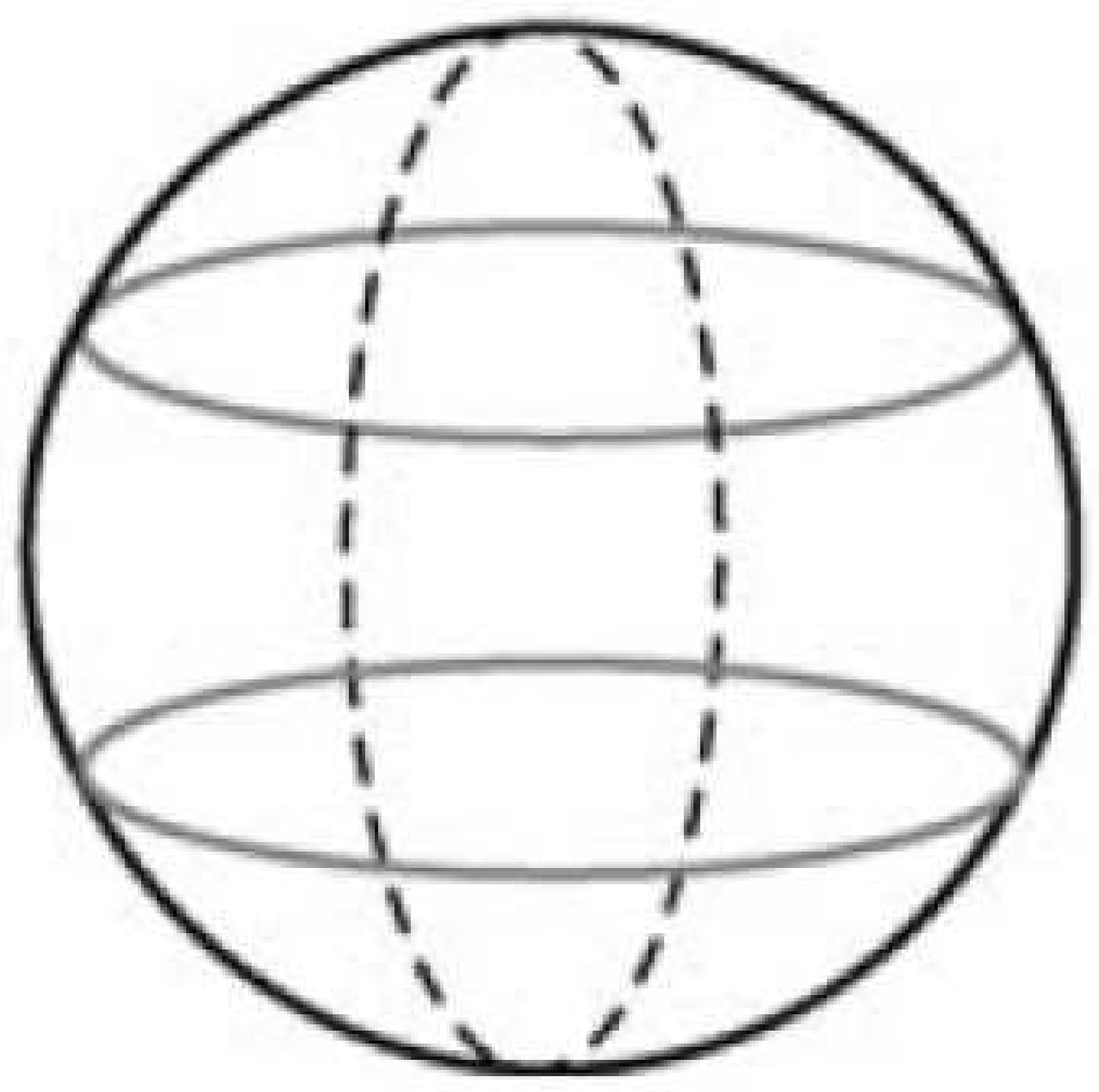

Figure 8: Trajectory consisting of two circles (Zeng, 2009)

Trajectories

Fourier Space:

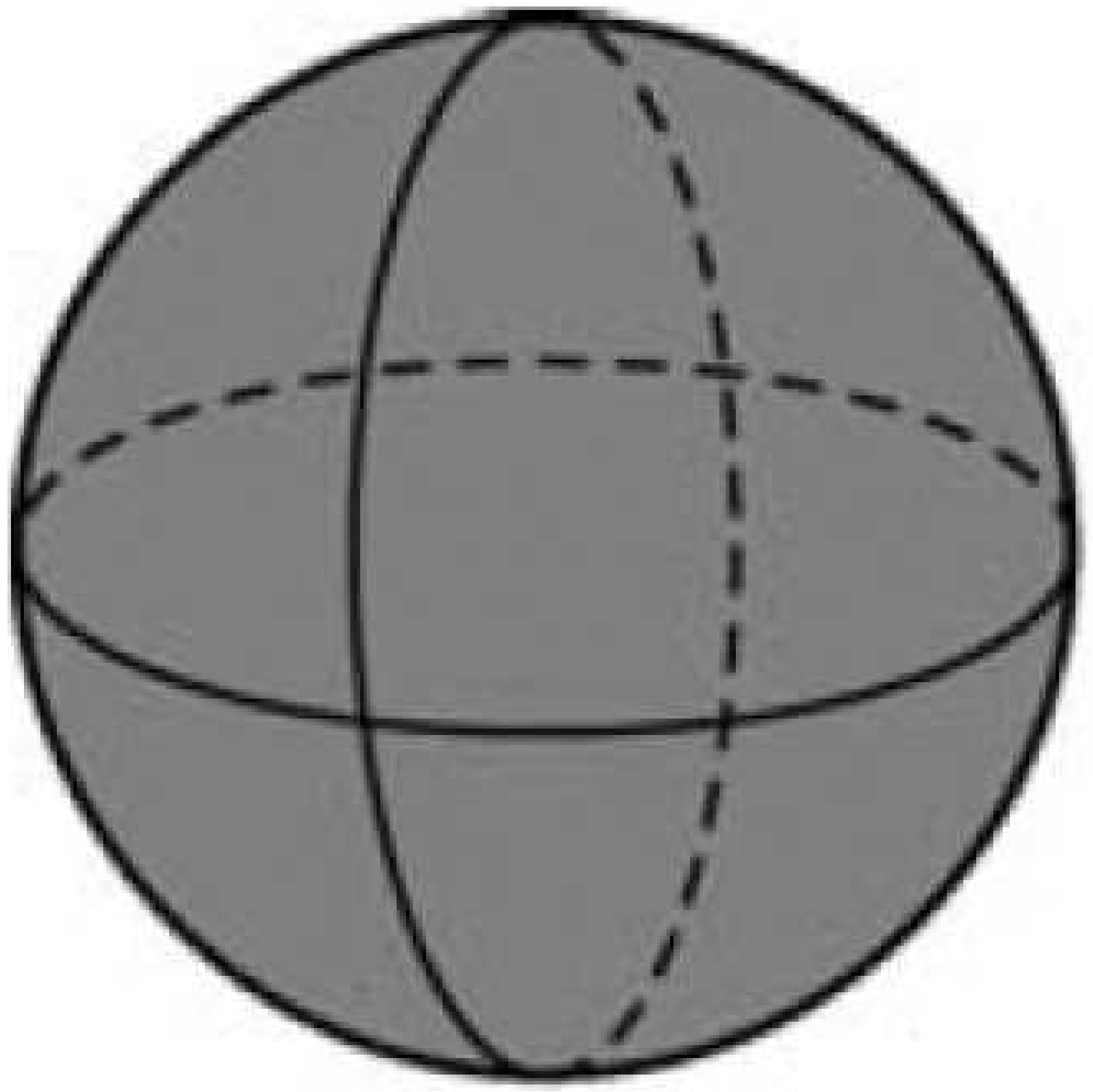

Figure 9: Trajectory containing every direction on the sphere (Zeng, 2009)

Trajectories

Fourier Space:

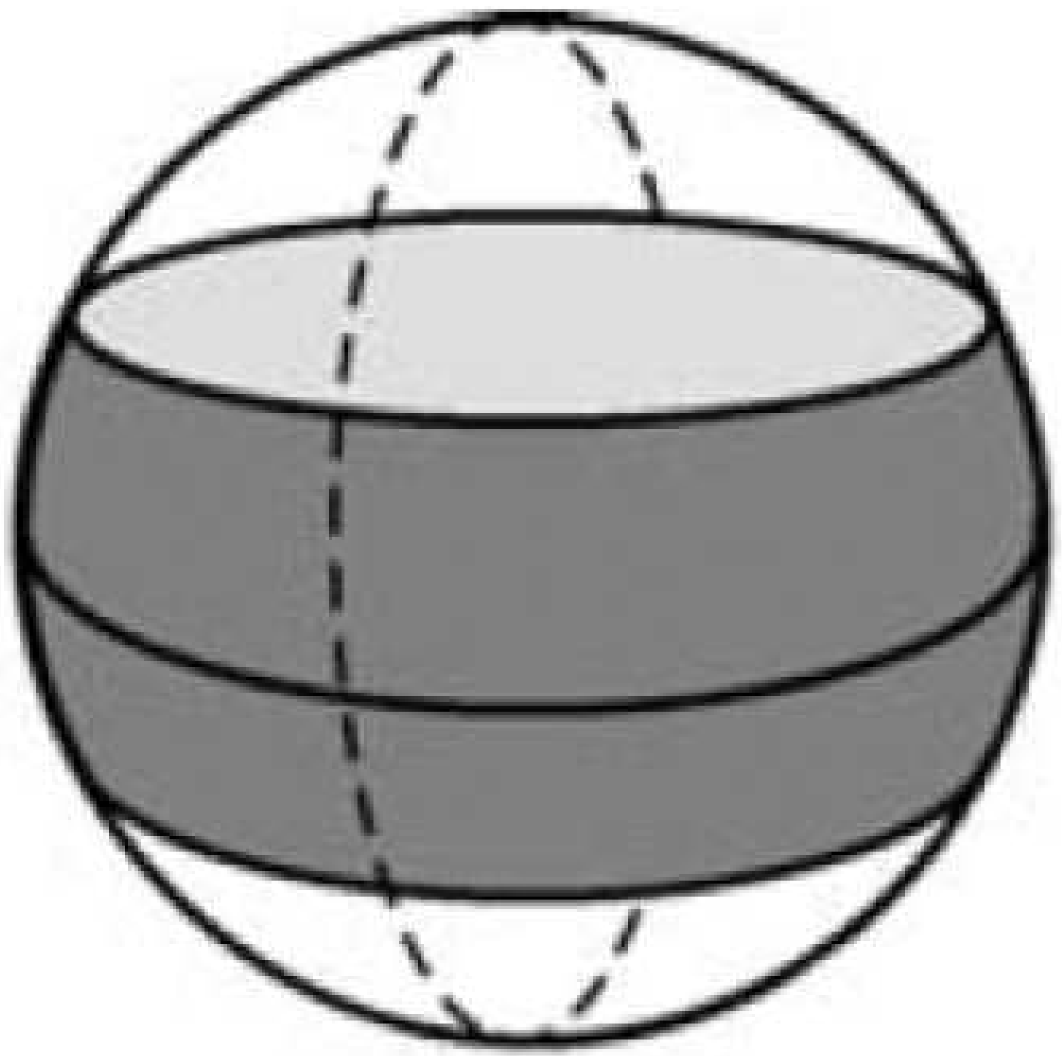

Figure 10: Trajectory consisting of a band segment of the unit circle (Zeng, 2009)

Topics

From 2-D to 3-D

Parallel Line-Integral Data

Orlov’s Condition

Outline: Backprojection-then Filtering (BPF) Algorithm

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BPF Algorithm for Parallel Line-Integral Data

- For the unit sphere the PSF can be shown to be

$$h(x,y,z) = \frac{1}{x^2+y^2+z^2} = \frac{1}{r^2}.$$

- The back projection of the data is computed by

$$b = f *** h,$$
$$B(\omega_x,\omega_y,\omega_z) = F(\omega_x,\omega_y,\omega_z) \cdot H(\omega_x,\omega_y,\omega_z),$$

where $***$ denotes the 3-D convolution, and

$$B = \text{FT}(b),\ F = \text{FT}(f),\ H = \text{FT}(h).$$

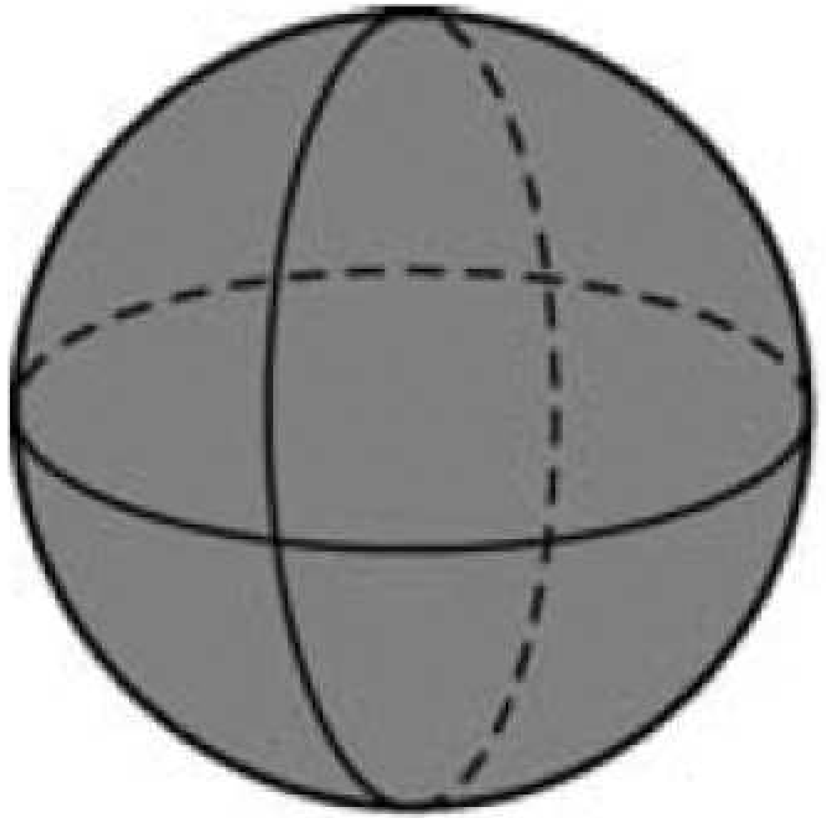

Figure 11: Trajectory containing every direction on the sphere (Zeng, 2009)

Topics

From 2-D to 3-D

Parallel Line-Integral Data

Orlov's Condition

Outline: Backprojection-then Filtering (BPF) Algorithm

Summary

Take Home Messages

Further Readings

Take Home Messages

- There is a central slice theorem for parallel line-integral data in 3-D.
- Data completeness depends on the used trajectory $\rightarrow$ Orlov's condition.

Further Readings

The best way to augment your knowledge of the shown concepts is to read the companion book of the current chapter:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Medical Image Processing for Diagnostic Applications

3-D Data – Plane-Integrals

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Topics

Parallel Plane-Integral Data

- 3-D Radon Transform
- Central Slice Theorem
- Backprojection

Summary

- Take Home Messages
- Further Readings

3-D Radon Transform

The 3-D Radon transform is not equal to the X-ray transform!

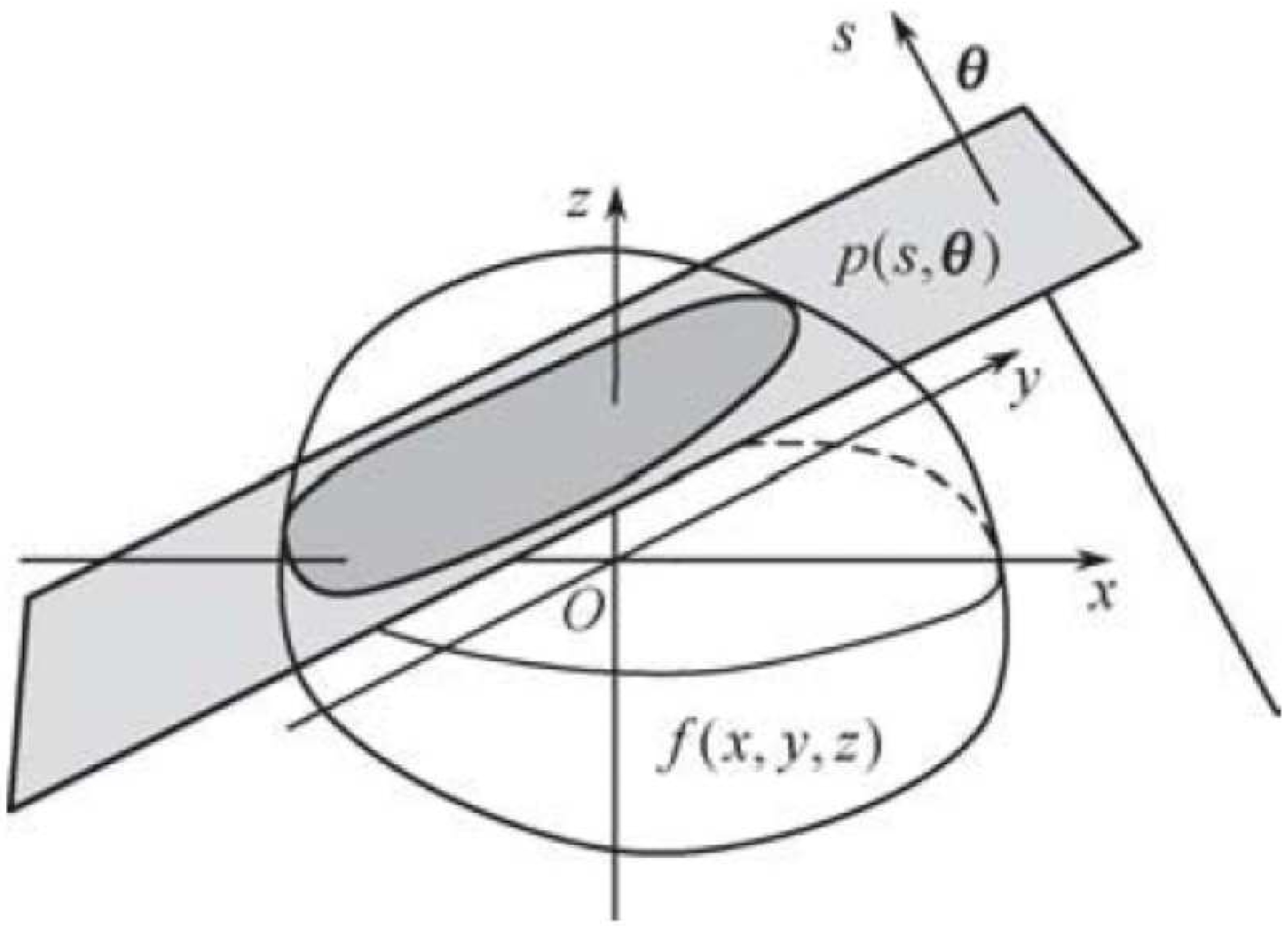

Figure 1: Integral over a plane through the object $f(x, y, z)$ at distance s from the origin and normal $\boldsymbol{\theta}$ (Zeng, 2009)

3-D Radon transform

- We do not have direct detectors for this transform.
- We can compute the value of such a “detector” as a line-integral on a 2-D parallel beam detector.
- There is a central slice theorem for this transform.
- Backprojection is different as a point has to be backprojected as a plane into 3-D.

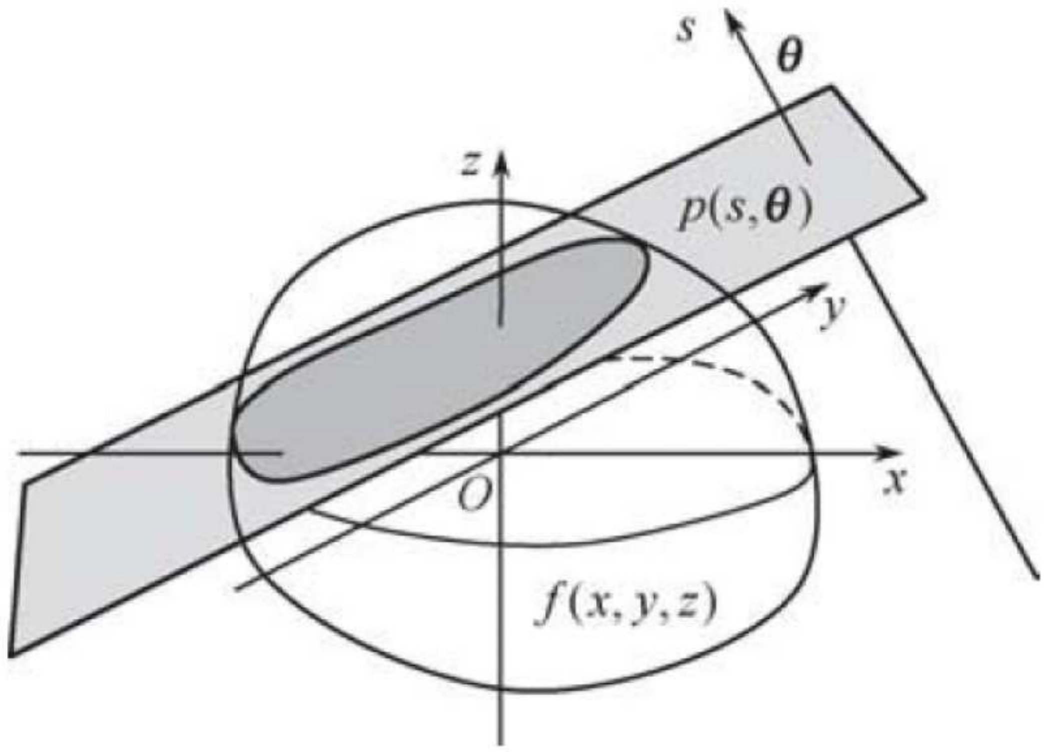

Figure 2: Scheme of the Radon transform (Zeng, 2009)

Central Slice Theorem for Plane-Integral Data

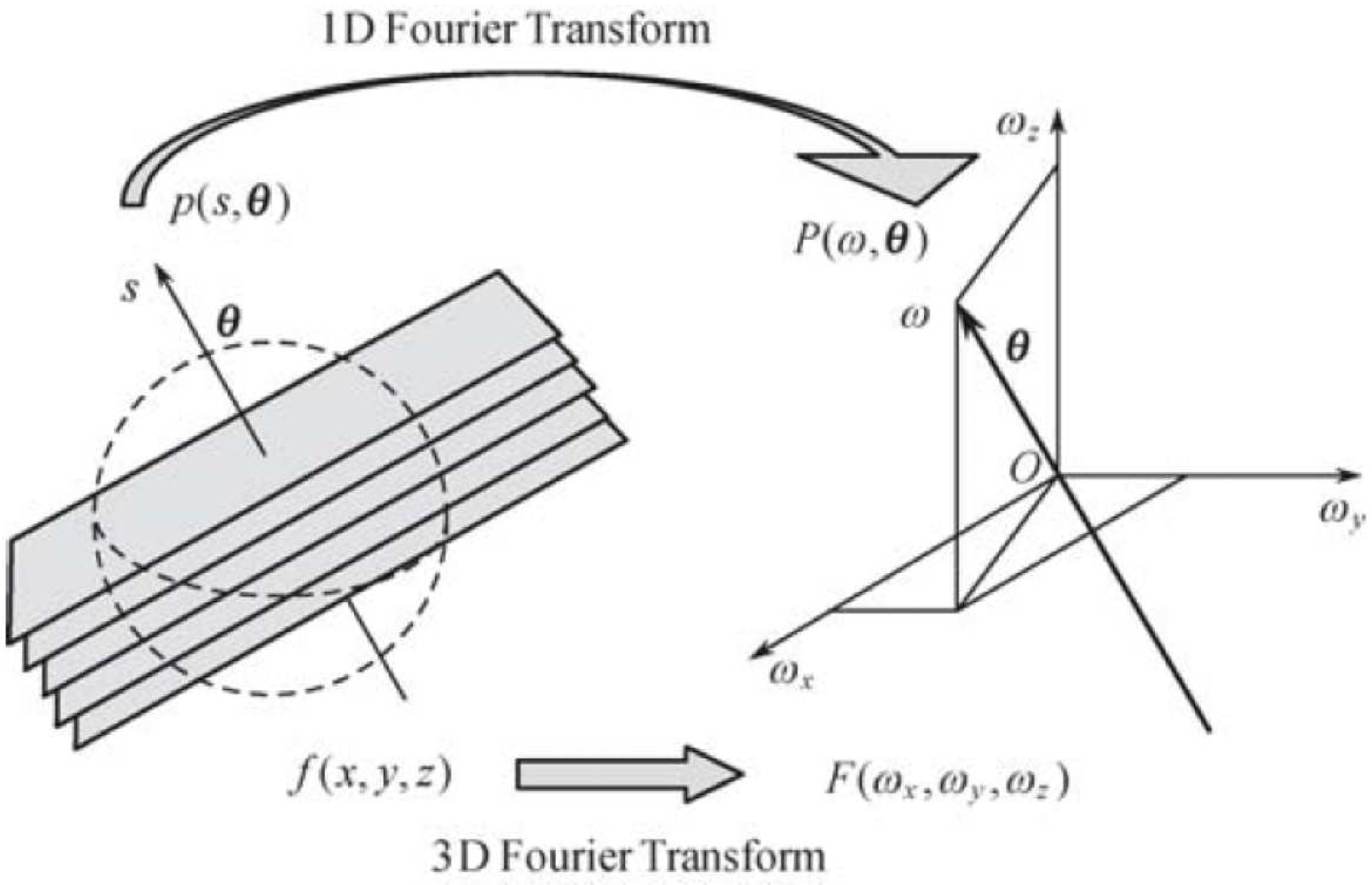

Figure 3: This central slice theorem connects the 3-D Fourier transform of the object with the 1-D Fourier transform of the plane-integrals (Zeng, 2009).

Parallel Plane-Integral Data: Backprojection

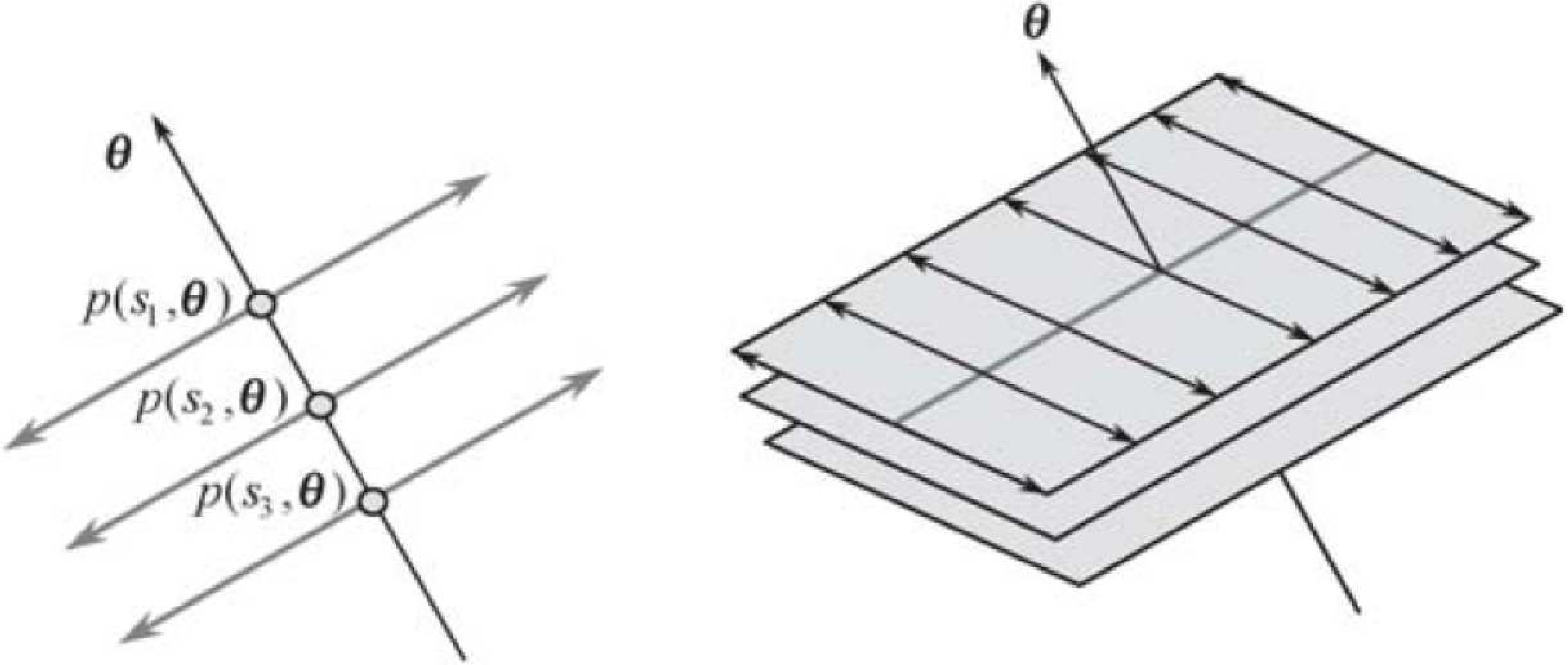

Figure 4: Scheme of how the plane-integral data can be backprojected (Zeng, 2009)

3-D Radon Inversion Formula

$$f(x,y,z) = -\frac{1}{8\pi^2} \iint\limits_{[0,2\pi]^2} \frac{\partial^2 p(s,\boldsymbol{\theta})}{\partial s^2}\Big|_{s=\boldsymbol{x}\cdot\boldsymbol{\theta}} \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi,$$

where

$$\boldsymbol{\theta} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

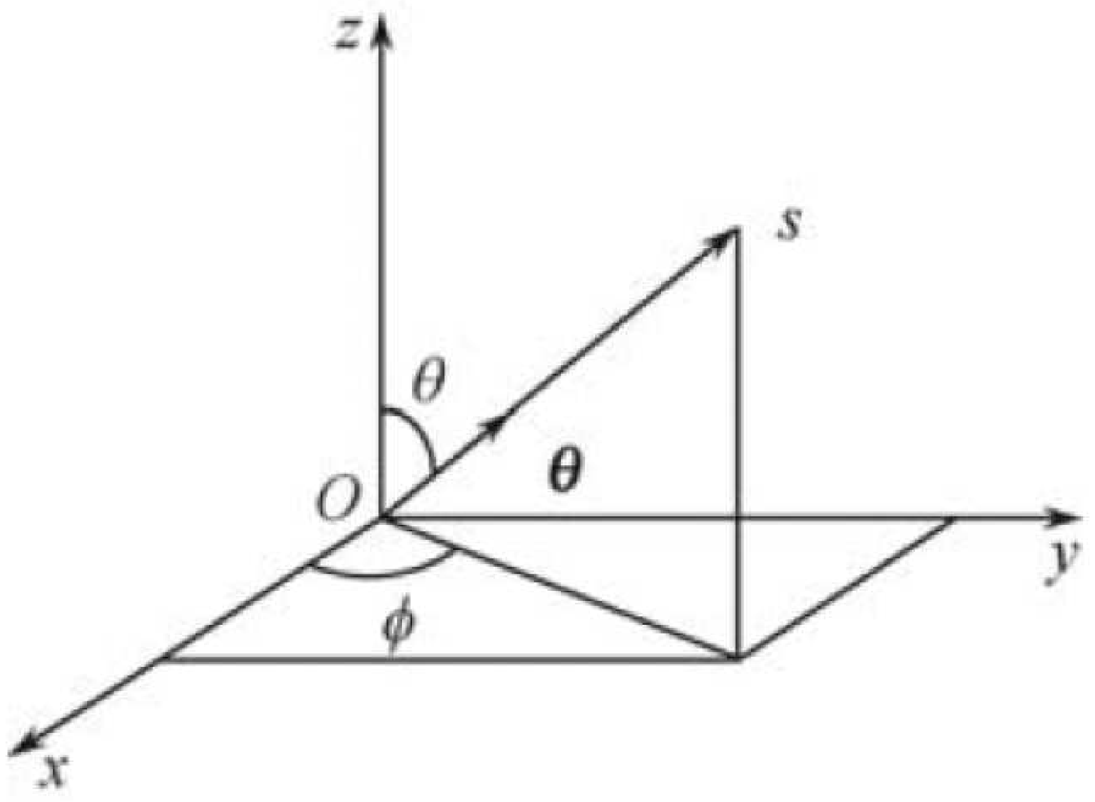

Figure 5: Diagram shows the different angles and other variables of the inversion formula (Zeng, 2009)

Topics

Parallel Plane-Integral Data
- 3-D Radon Transform
- Central Slice Theorem
- Backprojection

Summary
- Take Home Messages
- Further Readings

Take Home Messages

- There is a central slice theorem for parallel plane-integral data in 3-D.
- The 3-D Radon transform cannot be directly translated into detector hardware, but it can be inverted and therefore be considered for reconstruction algorithms.

Further Readings

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Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Medical Image Processing for Diagnostic Applications

3-D Data – Cone Beam Data

Online Course – Unit 46
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Topics

Cone Beam Data

- Cone Beam Geometry
- Data Sufficiency
- Data Redundancy

Summary

- Take Home Messages
- Further Readings

Cone Beam Geometry

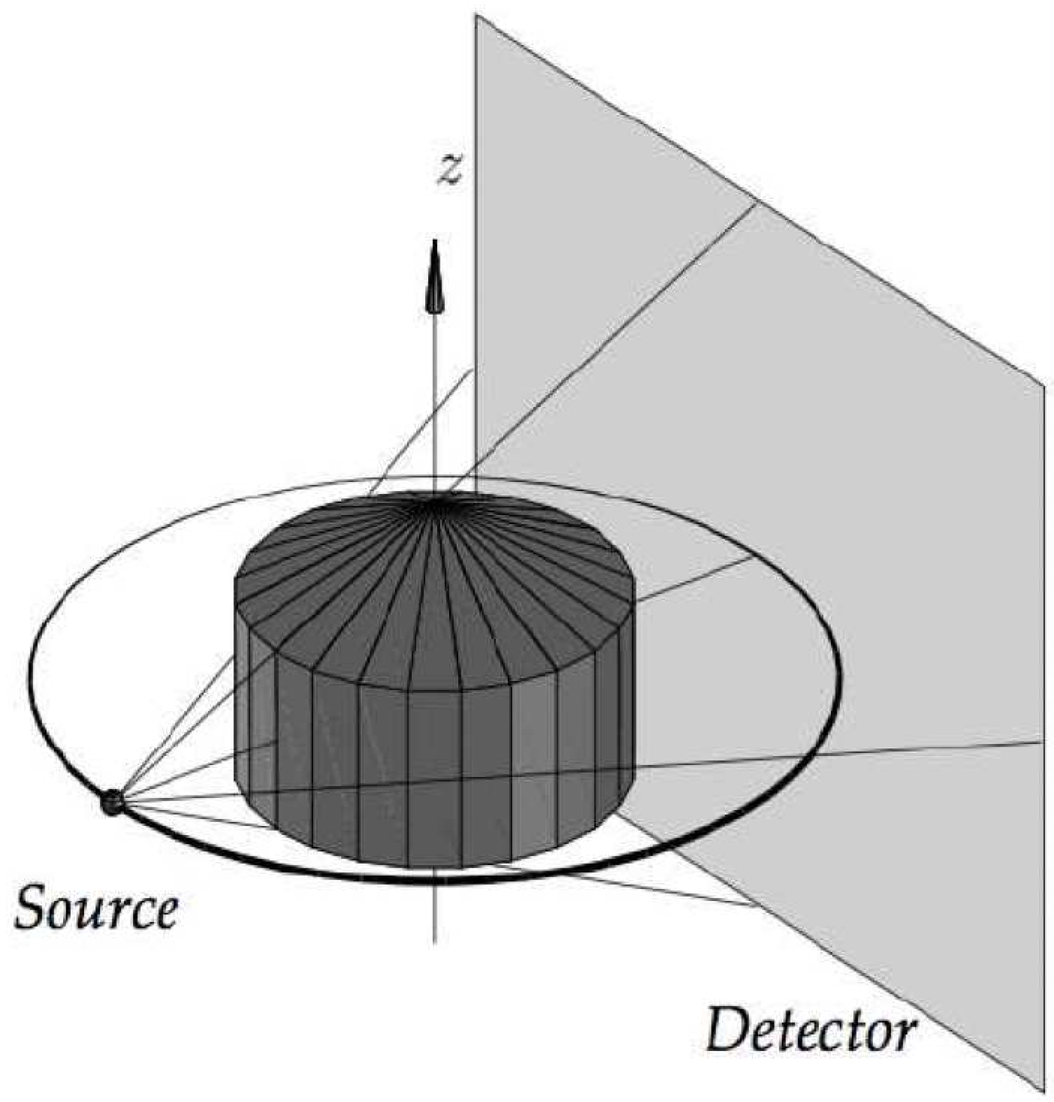

Figure 1: Schematic of a cone beam projection

Cone Beam Geometry

- Flat panel or multi-row detectors can detect considerably more data at a time.
- This geometry is very popular.
- Projection and backprojection can be described using projection matrices.
- Unlike in fan beam geometry, we do not have a central slice theorem.
- Data sufficiency conditions are different than in 3-D parallel beam geometry.

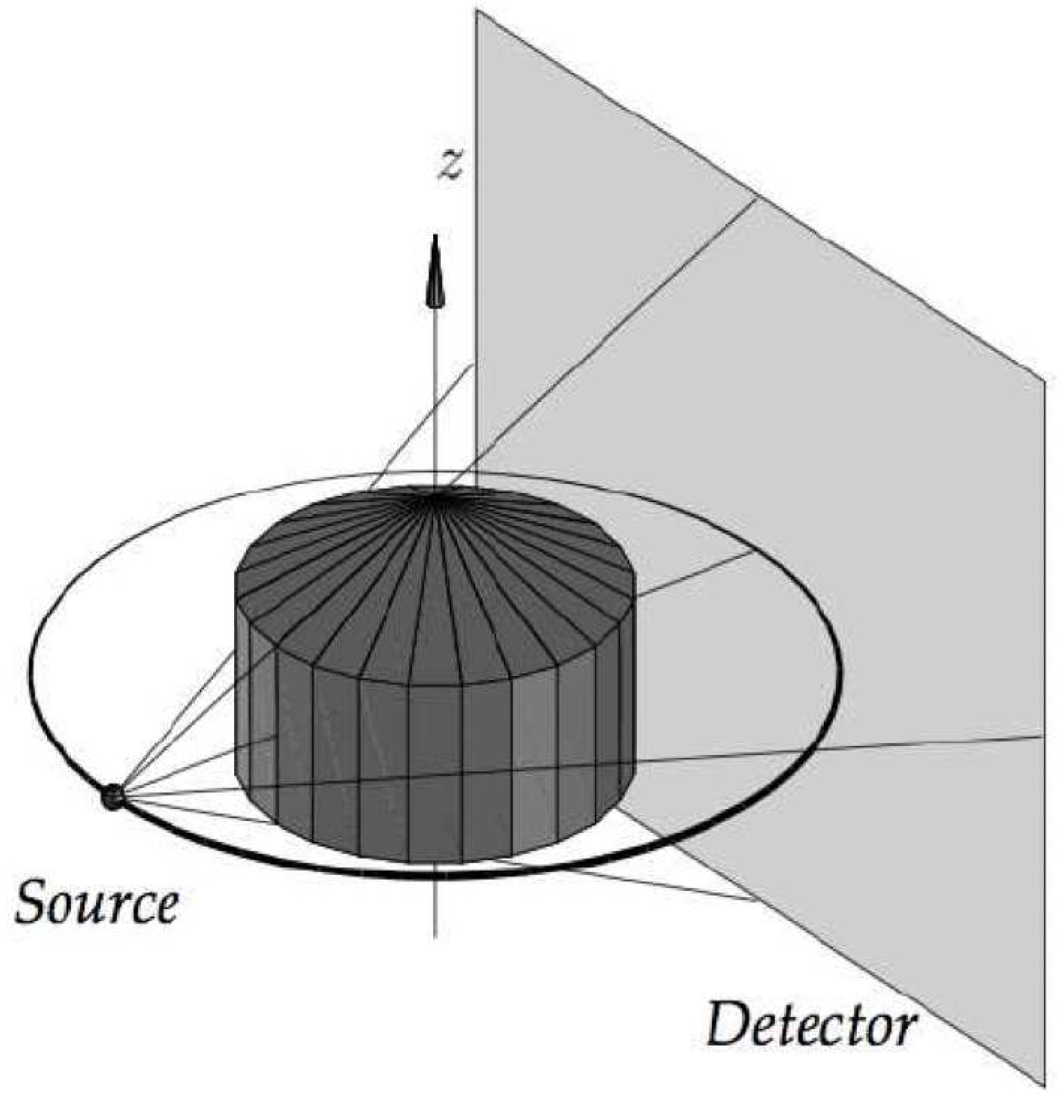

Figure 2: Schematic of a cone beam projection

Condition for Data Completeness

Tuy's condition: Every plane that intersects the object of interest must contain a cone beam focal point.

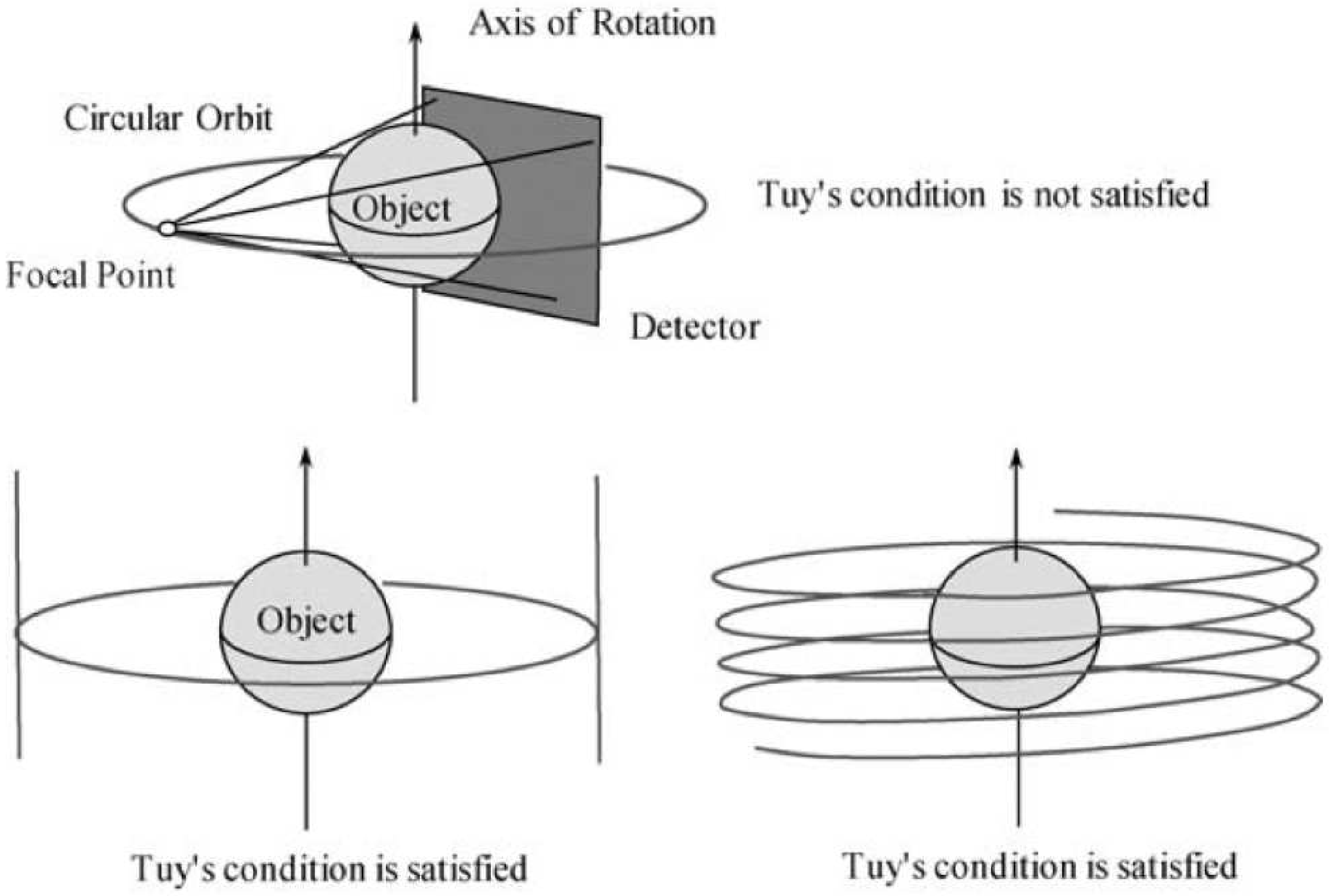

Figure 3: A circular trajectory not satisfying Tuy's condition (top) can be adjusted by moving up and down twice to satisfy it (bottom left). Following this idea, the helical trajectory (bottom right) is a much more practical version (Zeng, 2009).

Cone Beam Geometry: Data Sufficiency

- How to obtain a helical trajectory?

- How to make a circular scan complete?

Cone Beam Geometry: Data Sufficiency

- Consider plane-integrals along $\boldsymbol{\theta}$ for a reconstruction point in the plane of rotation.
- All angles are observed within the plane that is perpendicular to the viewing direction.

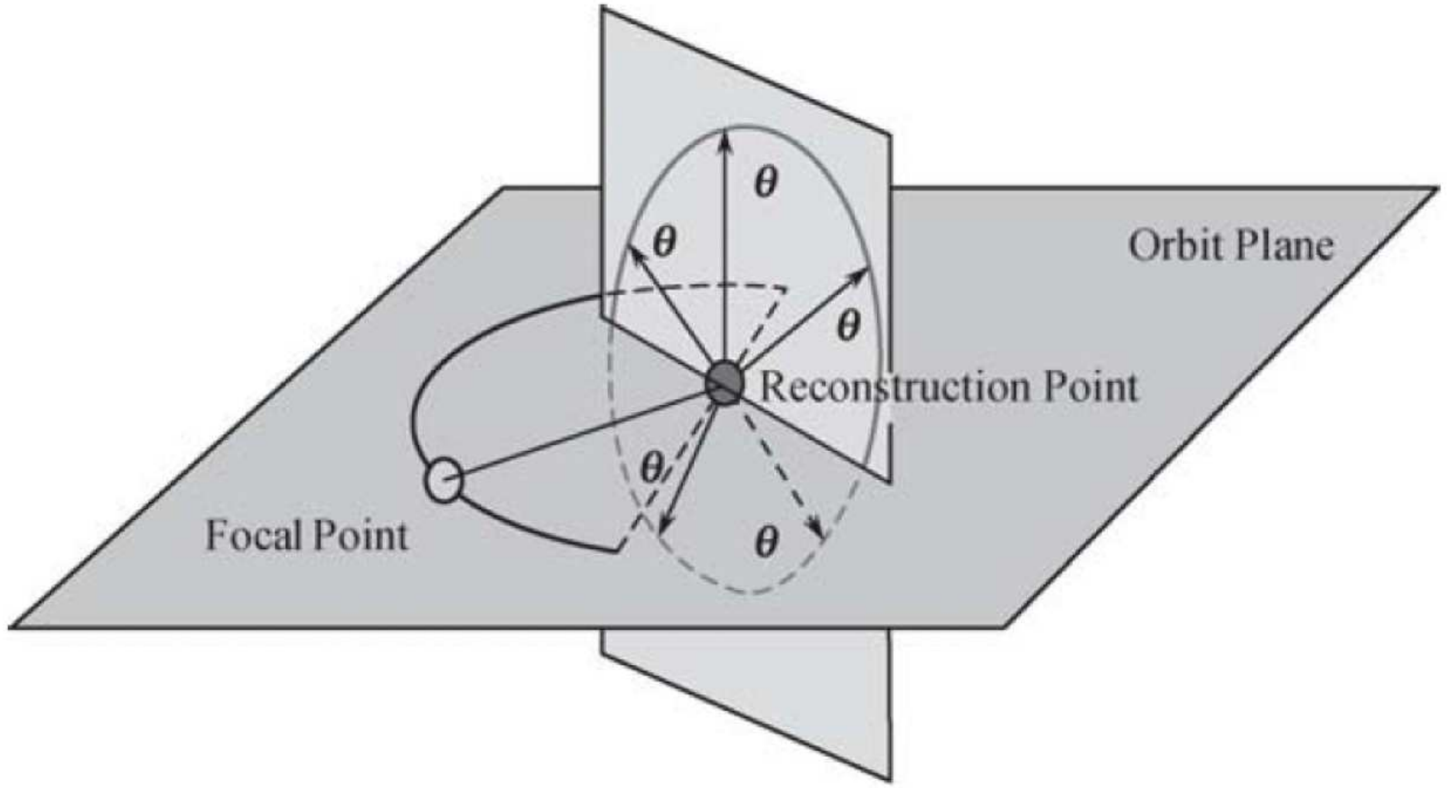

Figure 4: Reconstruction point lies in the plane of rotation (Zeng, 2009).

Cone Beam Geometry: Data Sufficiency

- If this is repeated for every point on the orbit, a full sphere will be sampled.
- Hence, data for this point is complete.
- We refer to this as a π-segment.

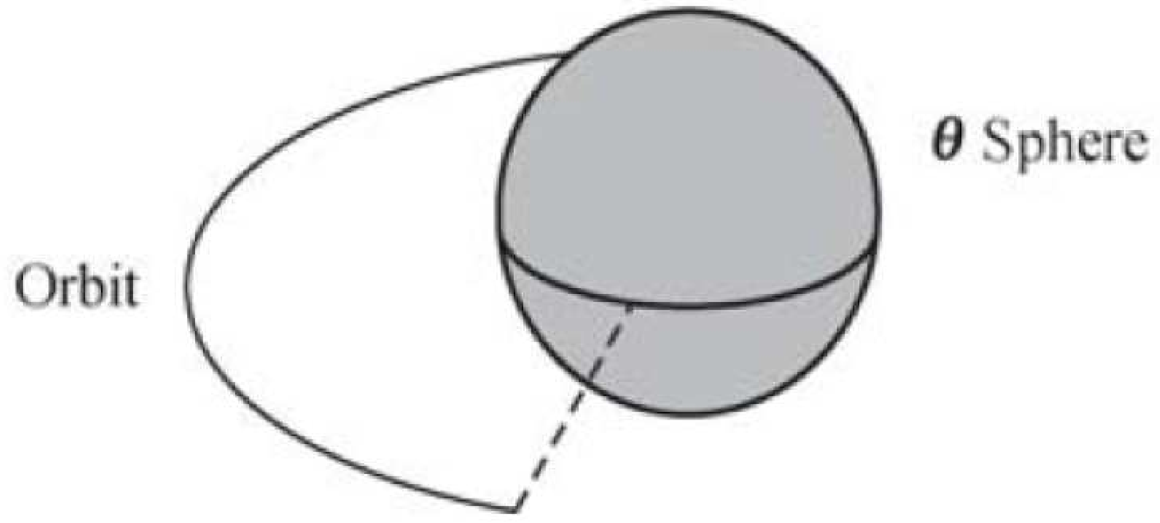

Figure 5: A π-segment (Zeng, 2009)

Cone Beam Geometry: Data Sufficiency

- On points above and below the orbit plane, there is missing data.
- The reconstruction will contain artifacts.
- The higher the angle to the reconstruction point, the stronger the cone beam artifact will appear.

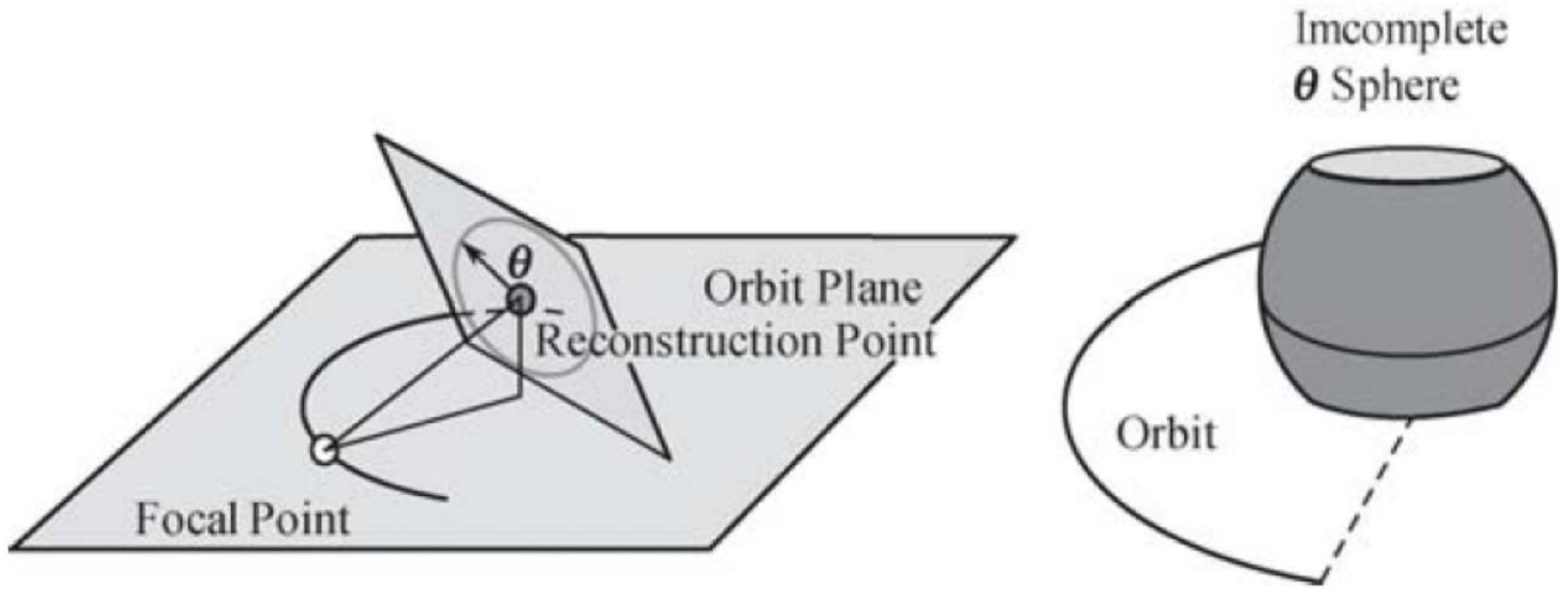

Figure 6: Reconstruction point lies outside the plane of rotation (Zeng, 2009).

Circular Trajectory

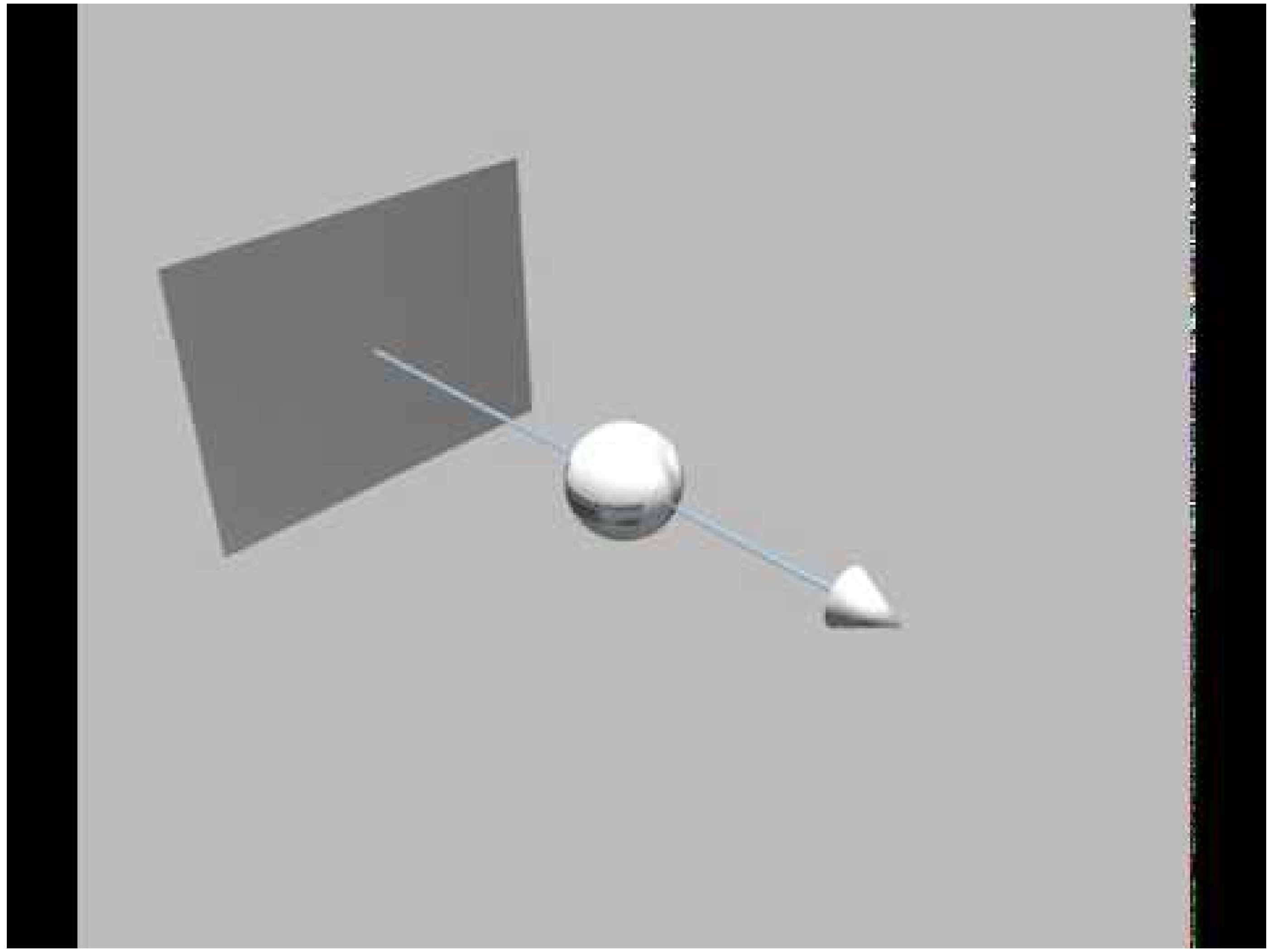

Figure 7: CT sampling circular trajectory in rotation plane, link: https://www.youtube.com/v/hFD9EZp2vok

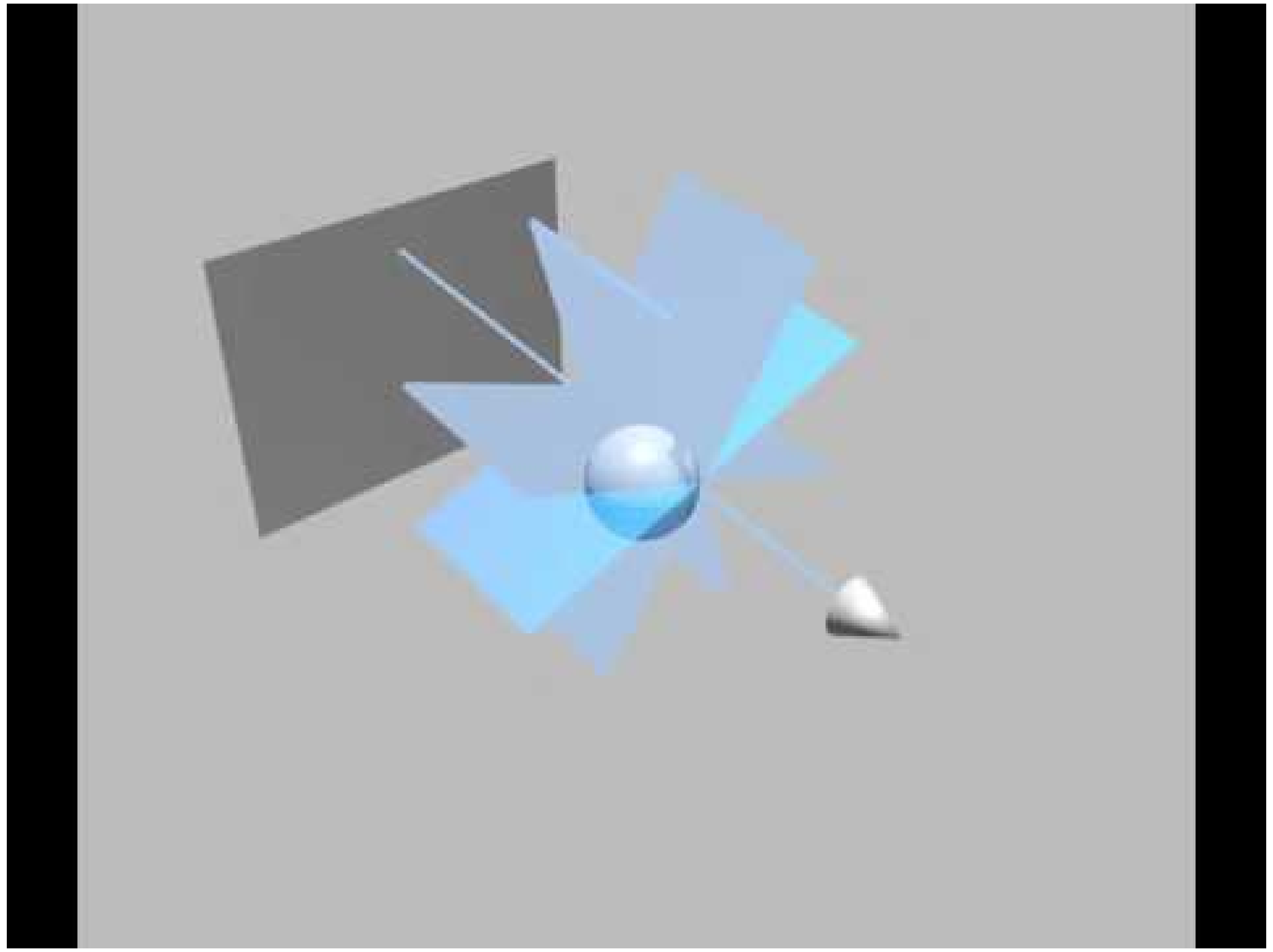

Figure 8: CT sampling circular trajectory out of rotation plane, link: https://www.youtube.com/v/iWIeScosduk

Cone Beam Geometry: Data Redundancy

A helical orbit will contain redundant observations:

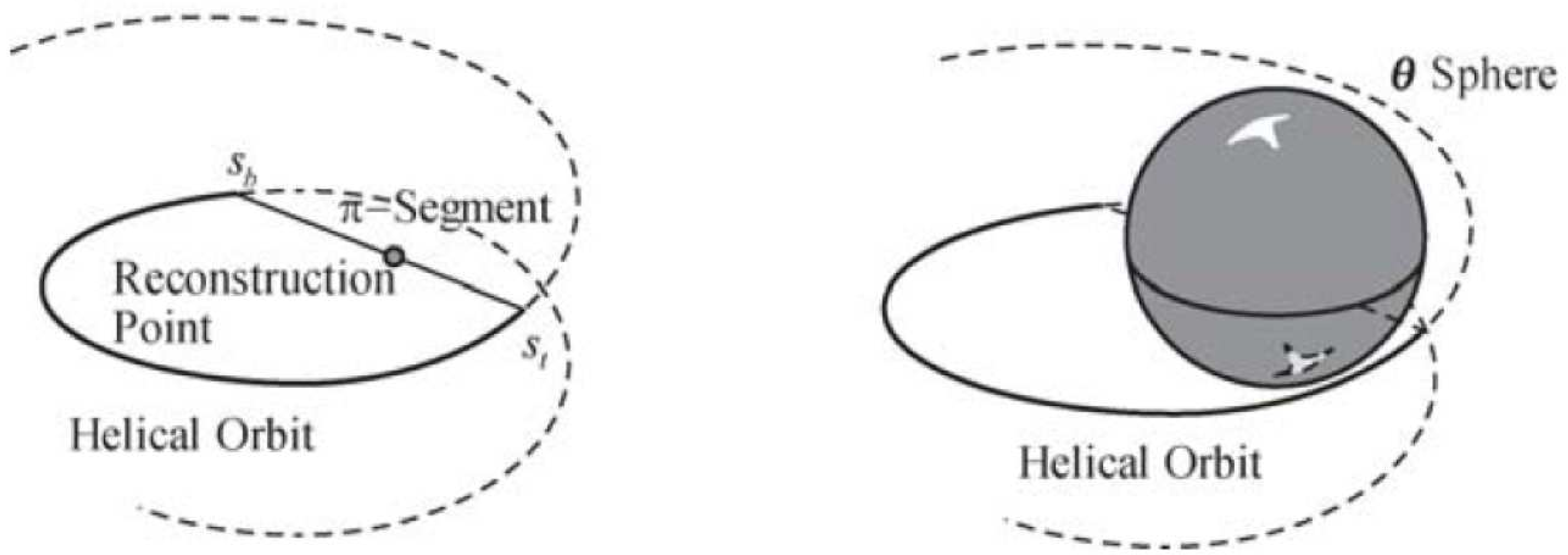

Figure 9: Some rays through the object hit the helical trajectory more than once (Zeng, 2009).

Cone Beam Geometry: Data Redundancy

Redundant observations will occur on cutting planes that hit the helix more than once:

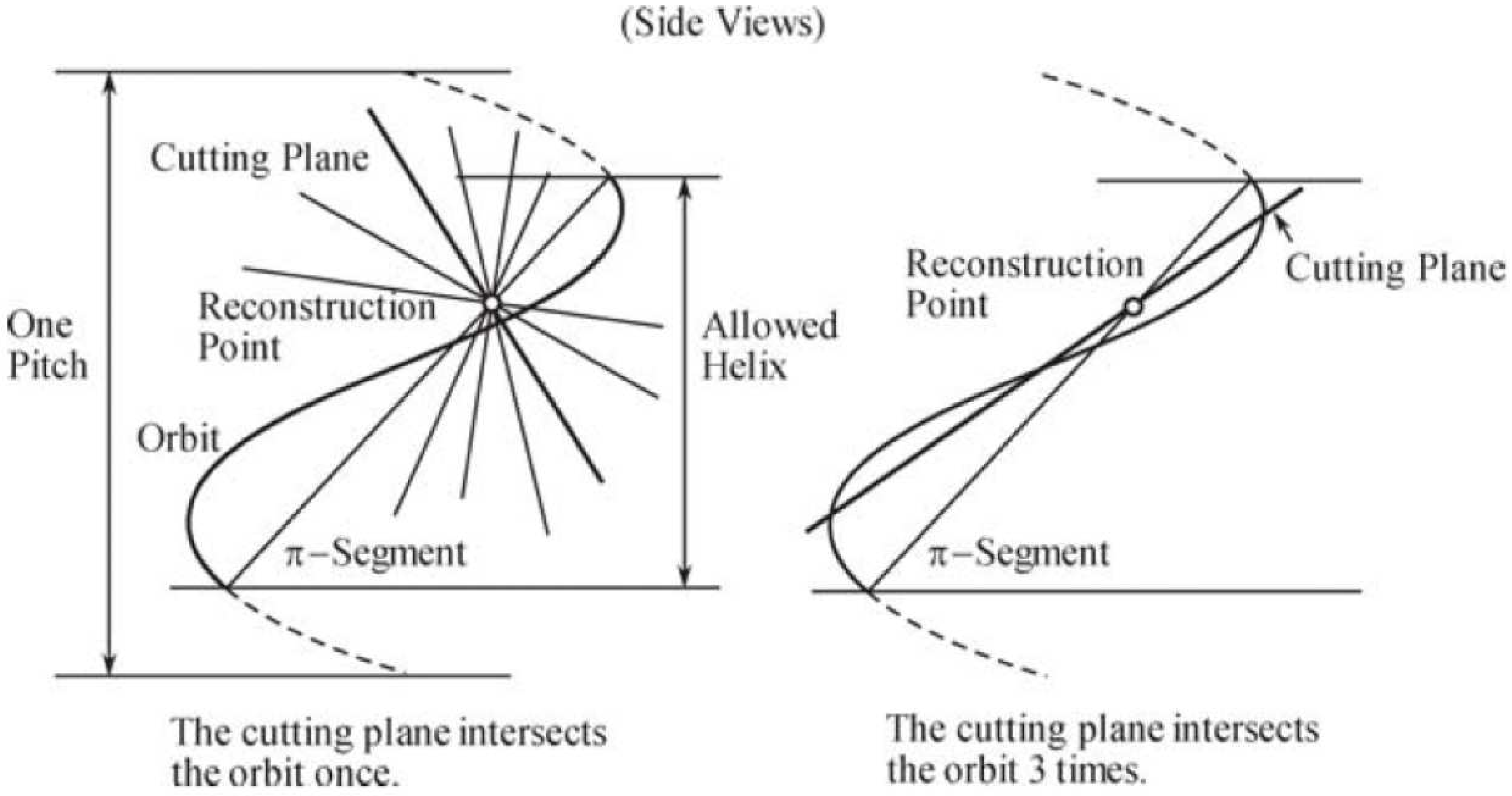

Figure 10: Some cutting planes intersect the orbit multiple times (Zeng, 2009).

Topics

Cone Beam Data
Cone Beam Geometry
Data Sufficiency
Data Redundancy

Summary
Take Home Messages
Further Readings

Take Home Messages

- There is no central slice theorem for cone beam data.
- If we want to check for data completeness, Tuy's condition is a helpful tool.
- In cone beam geometry, reconstruction areas out of plane are not completely sampled, such that artifacts appear.
- In a helical scan there is data redundancy.

Further Readings

The best way to augment your knowledge of the shown concepts is to read the companion book of the current chapter:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Medical Image Processing for Diagnostic Applications

Cone Beam Reconstruction – FDK

Online Course – Unit 47

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Topics

Feldkamp's Algorithm

Summary

Take Home Messages

Further Readings

Feldkamp's Algorithm ...

- ... is also known as the Feldkamp, Davis, Kress (FDK) algorithm named after the authors of the original publication.
- ... is designed for circular trajectories and thus is approximate.
- ... is a commonly used cone beam reconstruction algorithm because it is fast and robust.
- ... is based on a fan beam reconstruction algorithm with appropriate cosine weights.
- ... is exact for objects that do not vary in z-direction.

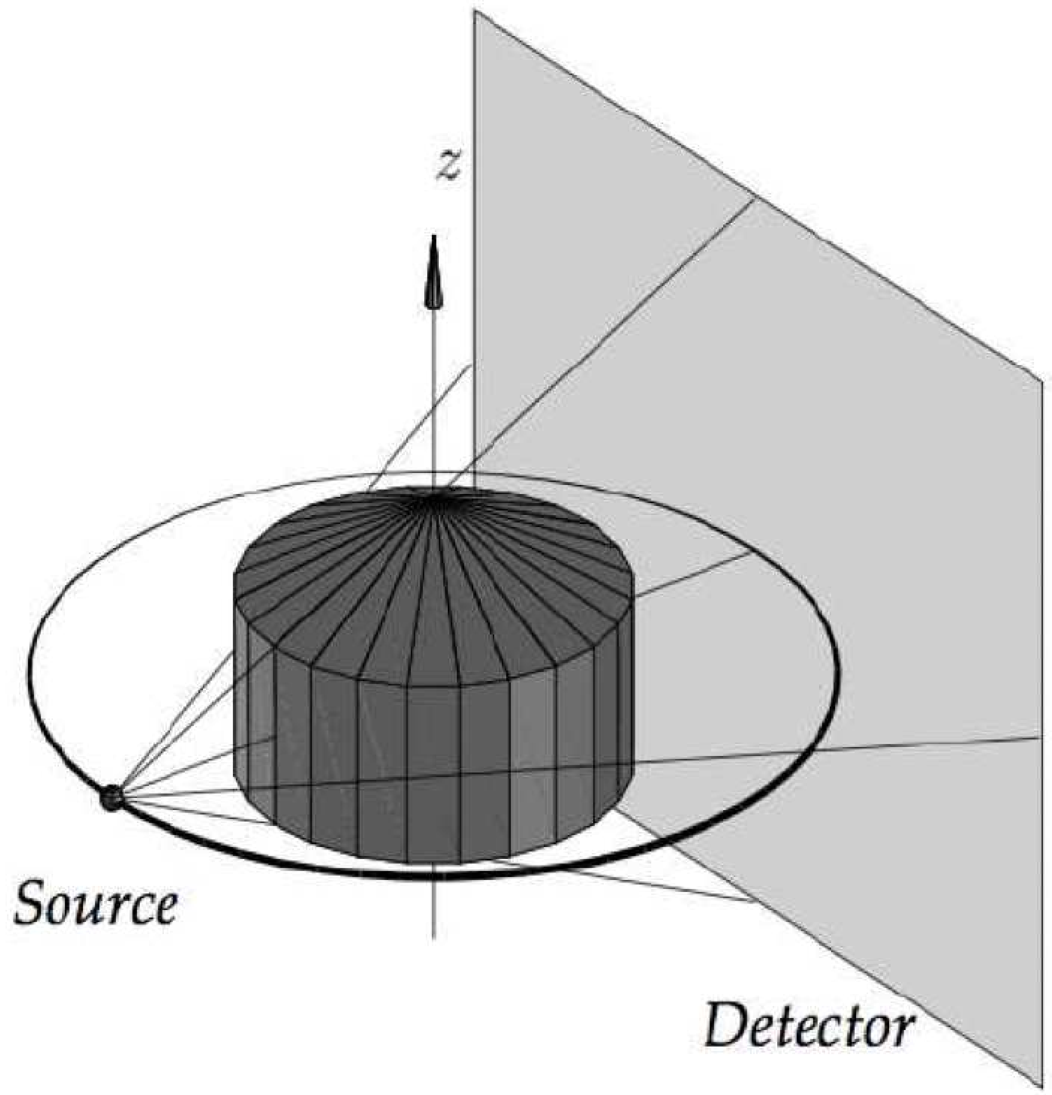

Figure 1: Cone beam scheme

Feldkamp’s Algorithm: Geometry

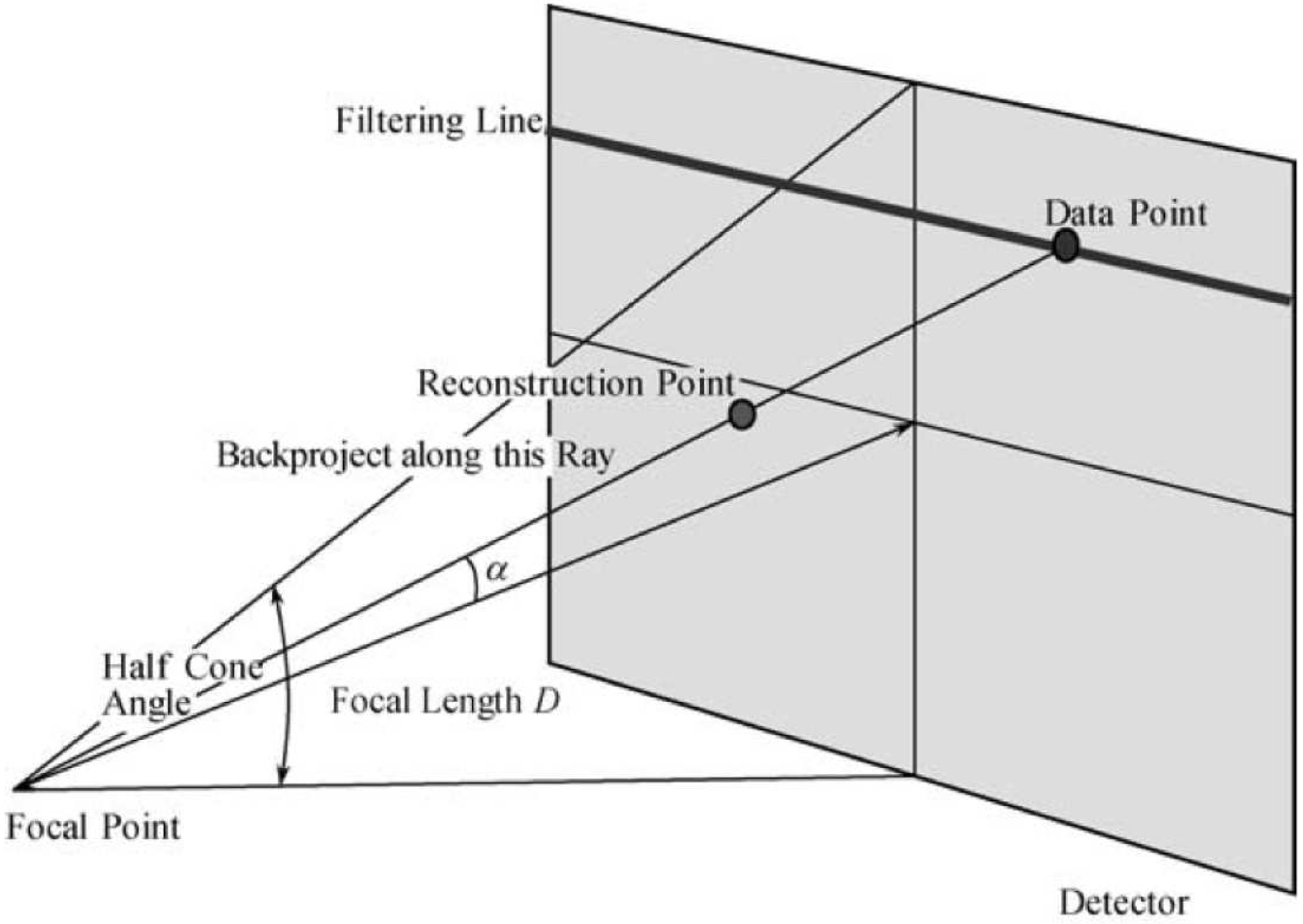

Figure 2: Geometry scheme for a point source cone beam acquisition on a flat panel detector (Zeng, 2009)

Fan Beam vs. Cone Beam

- FBP for fan beam data:

$$f(r,\varphi) = \frac{1}{2}\int_0^{2\pi} \frac{1}{U^2} \int_{-\infty}^{\infty} \frac{D}{\sqrt{D^2+t^2}} g(t,\beta) h(t'-t) \, \mathrm{d}t \, \mathrm{d}\beta,$$

where

$$U = \frac{D + r\sin(\beta-\varphi)}{D}, \quad t' = \frac{Dr\cos(\beta-\varphi)}{D + r\sin(\beta-\varphi)}.$$

- Feldkamp's algorithm for cone beam data:

$$f(r,\varphi,z) = \frac{1}{2}\int_0^{2\pi} \frac{1}{U^2} \int_{-\infty}^{\infty} \frac{D}{\sqrt{D^2+t^2+u^2}} g(t,u,\beta) h(t'-t) \, \mathrm{d}t \, \mathrm{d}\beta,$$

where u denotes the second detector coordinate for the projected point (r,φ,z) and is determined by:

$$u = z\frac{2D}{r}\frac{D + r\sin(\beta-\varphi)}{r\sin(2(\beta-\varphi))}.$$

Feldkamp's Algorithm

1. Perform adjusted cosine weighting:

$$g_1(t,u,\beta) = g(t,u,\beta)\frac{D}{\sqrt{D^2+t^2+u^2}}.$$

2. Apply ramp filter for each detector row:

$$g_2(t,u,\beta) = g_1(t,u,\beta) * h(t).$$

3. Backproject with distance weight:

$$f(r,\varphi,z) = \frac{1}{2}\int_0^{2\pi} \frac{1}{U^2} g_2(t',u,\beta)\,\mathrm{d}\beta.$$

Feldkamp's Algorithm: Remarks

- Filtering is row-wise, hence its complexity is $O(N^2 \log N)$.
- Backprojection is in 3-D $\Rightarrow O(N^3)$.
- In C-arm CT, projection is performed using projection matrices.
- This processing speed can be improved if Horner's scheme is applied.
- Backprojection is often implemented on special hardware.

Topics

Feldkamp's Algorithm

Summary

Take Home Messages

Further Readings

Take Home Messages

- The FDK algorithm is a 3-D cone beam reconstruction algorithm.
- It is based on the fan beam reconstruction algorithm and only row-wise filtering is applied.

Further Readings

The original work of Feldkamp, Davis, and Kress can be found here:

L. A. Feldkamp, L. C. Davis, and J. W. Kress. "Practical Cone-Beam Algorithm". In: *Journal of the Optical Society of America A: Optics, Image Science, and Vision* 1.6 (June 1984), pp. 612–619

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Medical Image Processing for Diagnostic Applications

Cone Beam Reconstruction – Grangeat's Algorithm

Online Course – Unit 48

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Topics

Grangeat's Algorithm

Summary

Take Home Messages

Further Readings

Grangeat's Algorithm ...

- ... converts the cone beam problem to a 3-D Radon inversion problem.
- ... can provide exact reconstructions, if Tuy's condition is met.
- ... uses the idea to convert cone beam ray sums to plane-integrals.

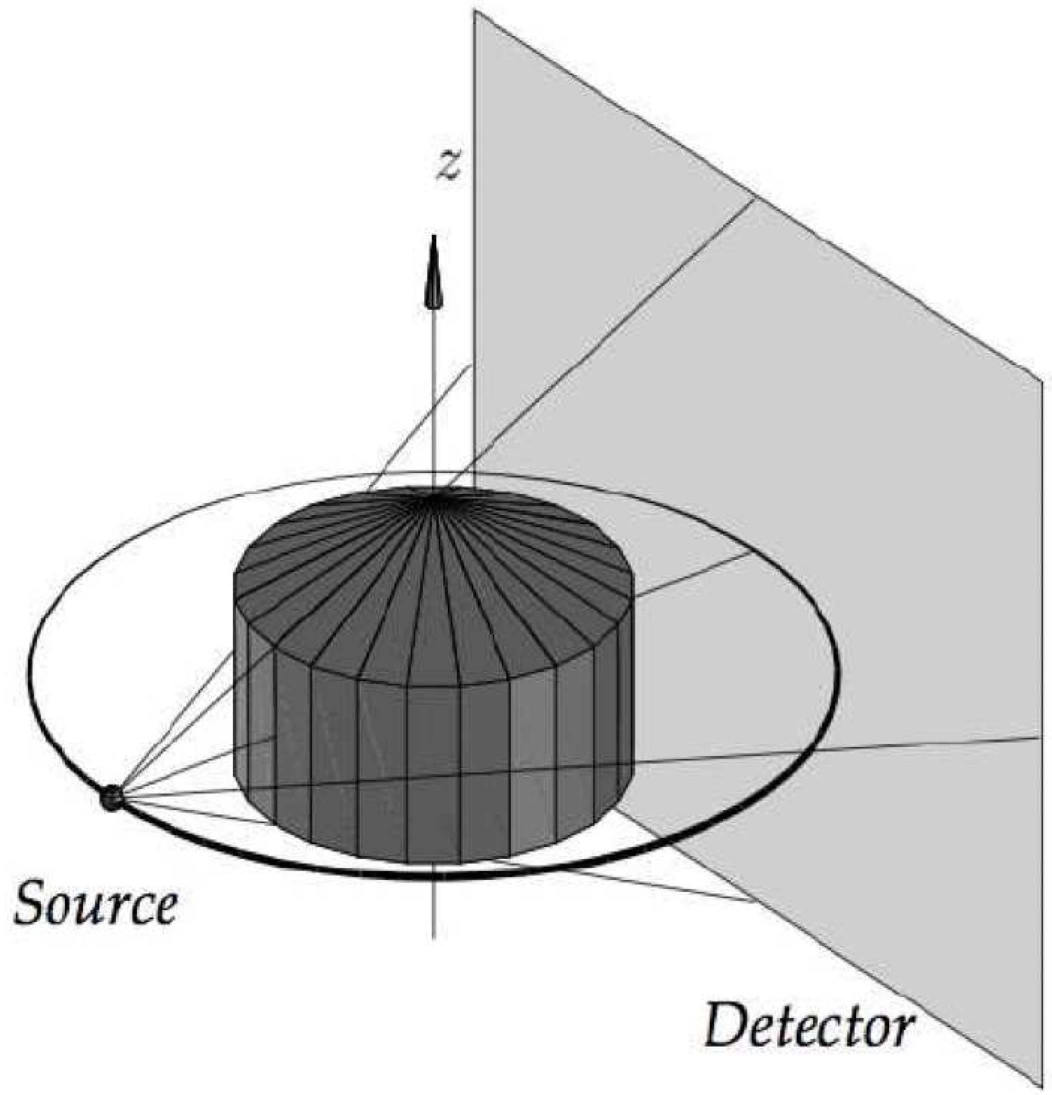

Figure 1: Cone beam scheme

Grangeat's Algorithm: Concept

- Line-integral on a cone beam detector is a weighted plane-integral.
- The line integral has to be weighted with $\frac{1}{r}$ to get the regular unweighted plane-integral.

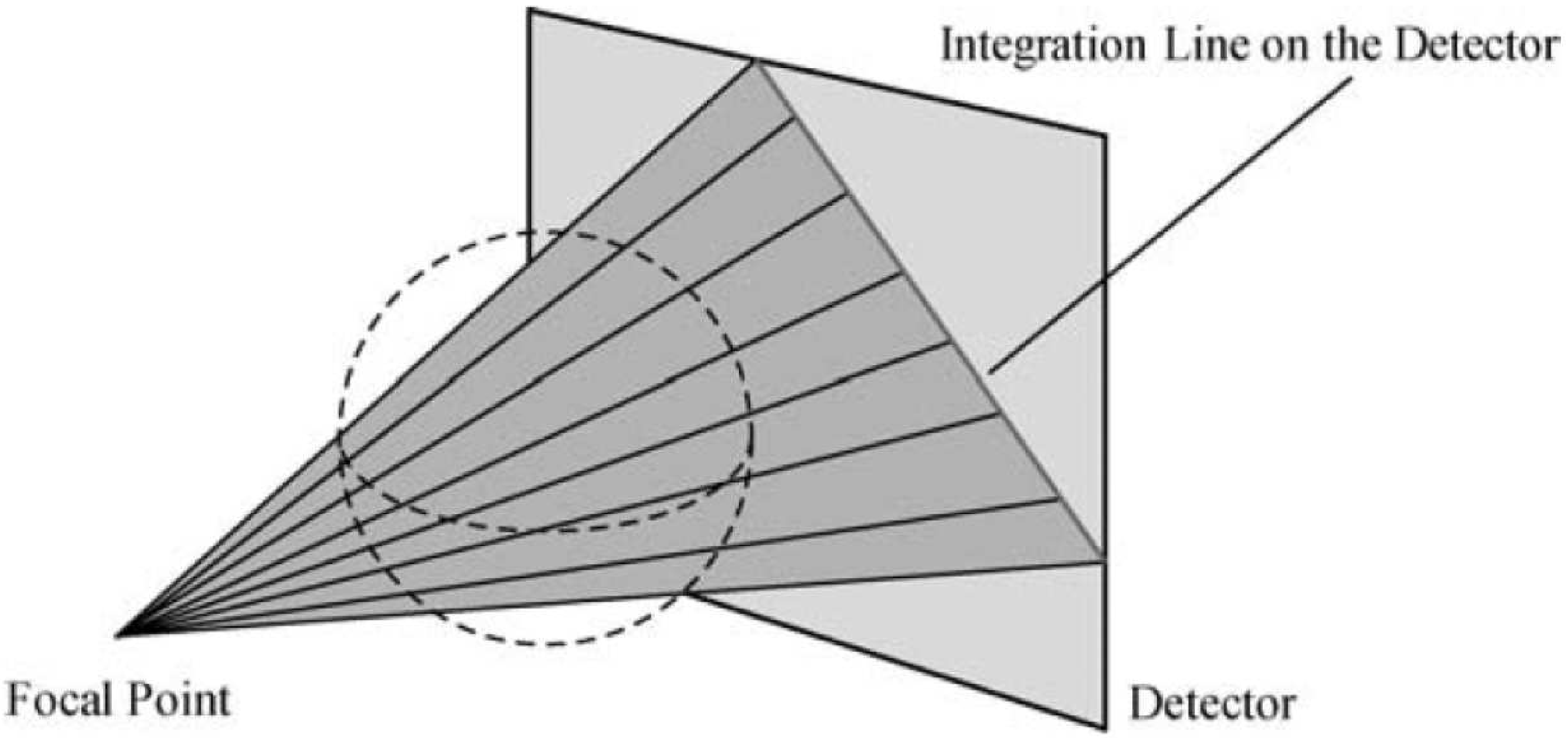

Figure 2: Weighting scheme (Zeng, 2009)

Grangeat's Algorithm: Concept

The derivative along the tangential direction dt is equal to the derivative of a $\frac{1}{r}$ weighted plane integral along dα:

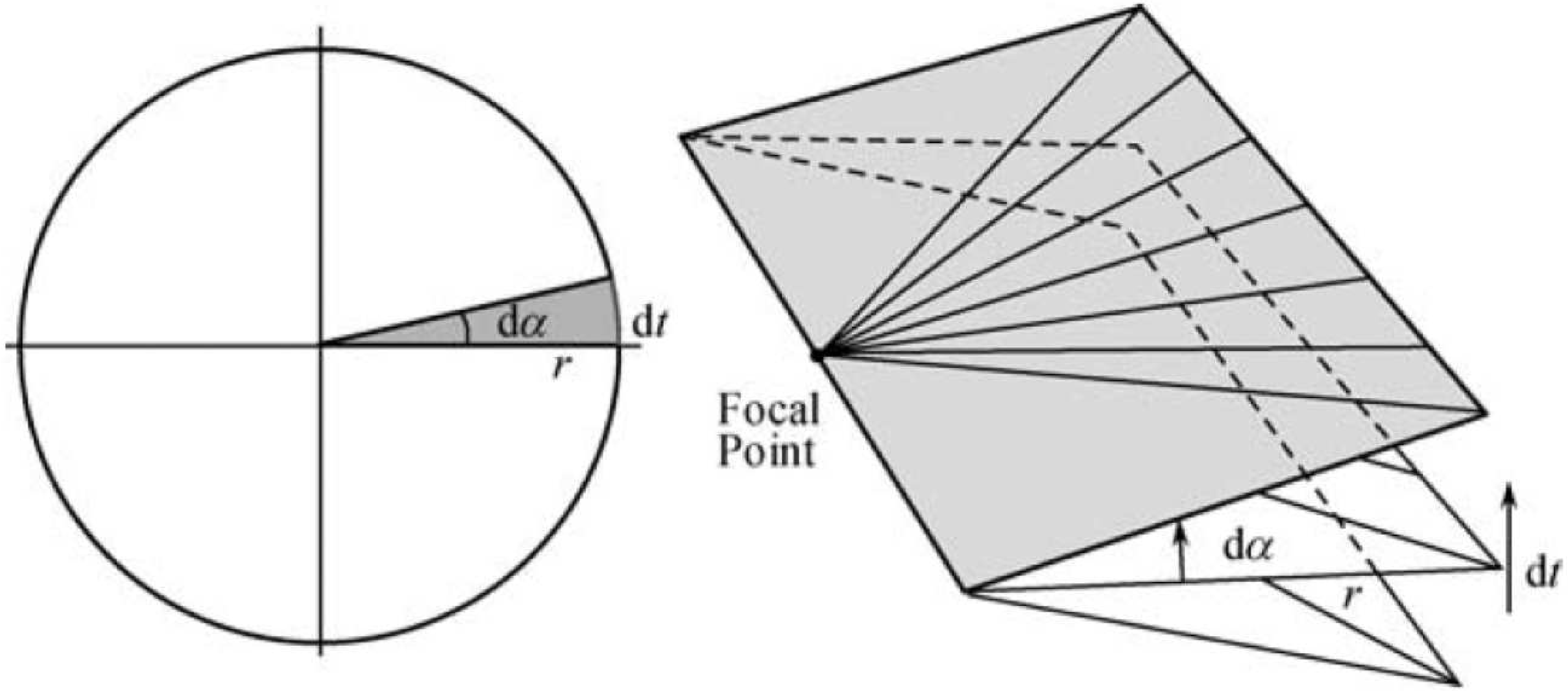

Figure 3: Tangential derivative (Zeng, 2009)

Grangeat's Algorithm

1. Form all possible line-integrals on each detector plane (all locations and orientations).
2. Compute the angular derivative.
3. Rebin the data to Radon space.
4. Take the derivative with respect to t.
5. Perform the 3-D Radon backprojection.

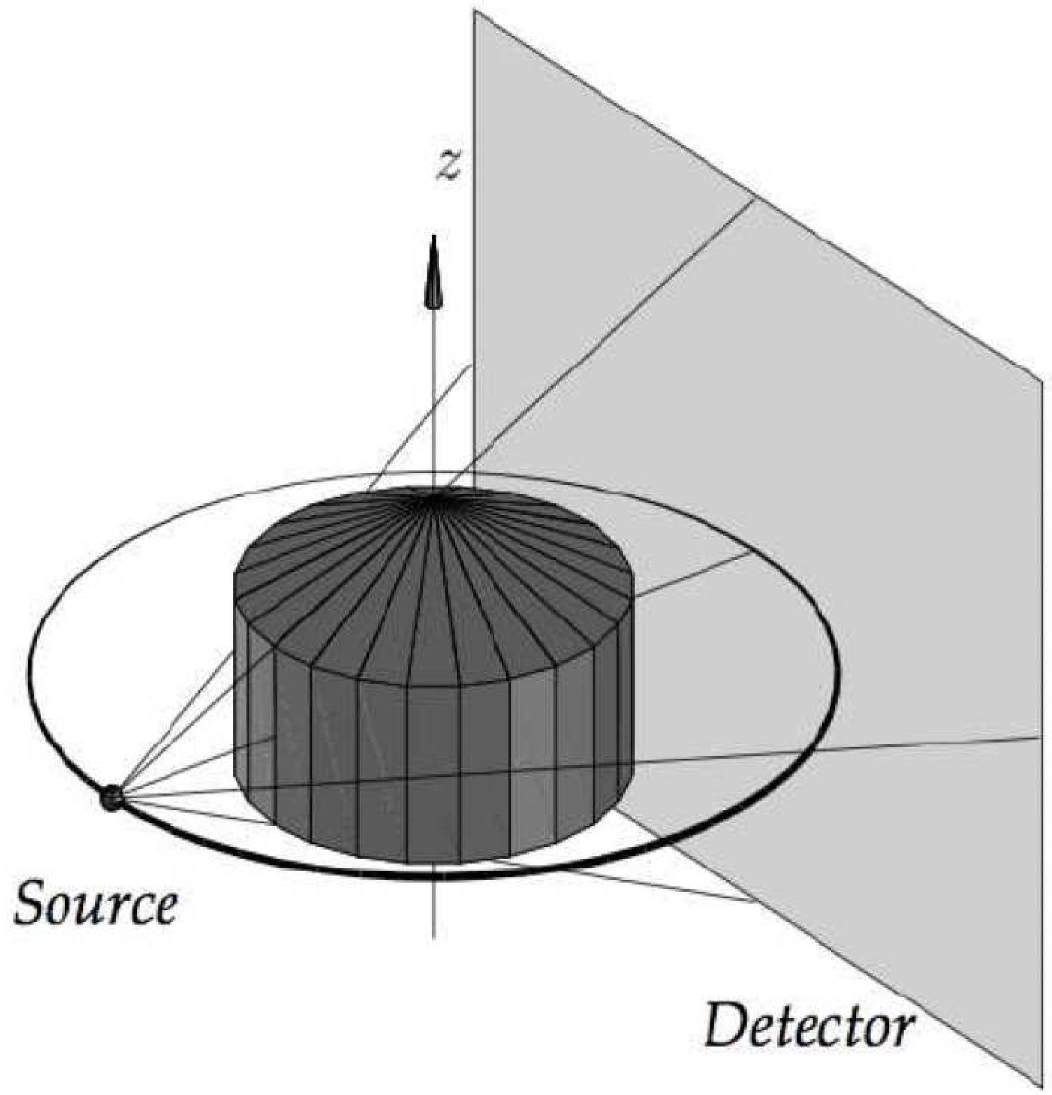

Figure 4: Cone beam scheme

Grangeat’s Algorithm: Remarks

Grangeat’s algorithm ...

- ... is not a filtered backprojection algorithm.
- ... requires interpolation on non-uniformly sampled data.
- ... needs to handle redundancy correctly $\rightarrow$ divide by the number of redundant observations.
- ... is not commonly used for reconstruction.
- ... is very useful to analyze reconstruction problems.

Topics

Grangeat's Algorithm

Summary
Take Home Messages
Further Readings

Take Home Messages

- Grangeat's algorithm is a 3-D cone beam reconstruction algorithm.
- It is useful for theoretical considerations, but rarely used in practice.

Further Readings

The original work of Grangeat can be found here:

Pierre Grangeat. “Mathematical Framework of Cone Beam 3D Reconstruction via the First Derivative of the Radon Transform”. In: *Mathematical Methods in Tomography*. Ed. by Gabor T. Herman, Alfred K. Louis, and Frank Natterer. Vol. 1497. Lecture Notes in Mathematics. Springer Berlin Heidelberg, 1991, pp. 66–97. DOI: 10.1007/BFb0084509

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Medical Image Processing for Diagnostic Applications

Cone Beam Reconstruction – Katsevich's Algorithm

Online Course – Unit 49
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Topics

Katsevich's Algorithm

Summary

Take Home Messages

Further Readings

Katsevich's Algorithm ...

- ... was first developed for helical trajectories,
- ... was later expanded to more general orbits.
- ... is in the form FBP,
- ... involves filtering that can be made shift-invariant, i. e., independent of the reconstruction location.

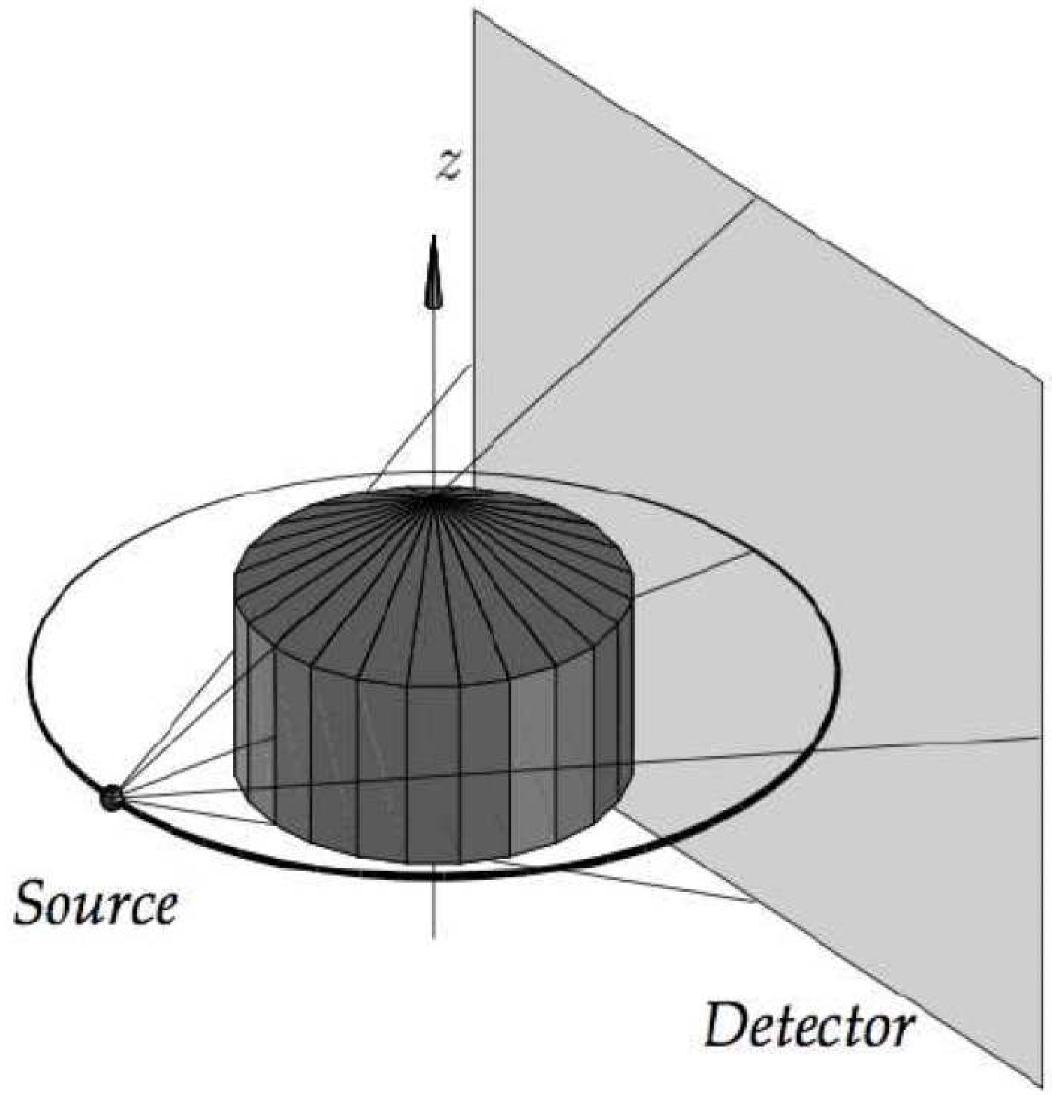

Figure 1: Cone beam scheme

Katsevich's Algorithm: Geometry

- For all points inside the helix there is one line that passes the point and hits the helix at two points that are separated by less than one pitch.
- This line is called a π-line or π-segment.
- If redundant data occurs, it occurs three times.
- Redundancy is solved by assigning the weights $1, -1, 1$.

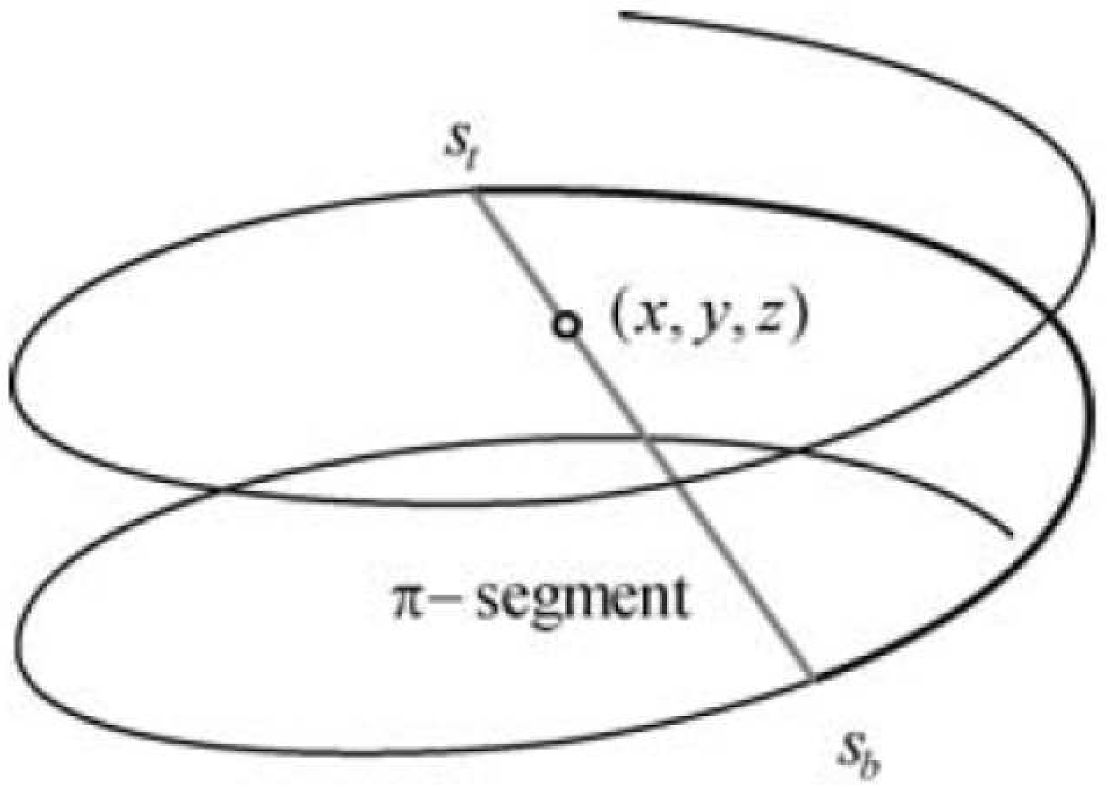

Figure 2: A π-segment intersects the helix at least twice (Zeng, 2009).

Katsevich's Algorithm: Concept

Compute the derivative along the trajectory $\boldsymbol{a}(s)$ and filter along direction β:

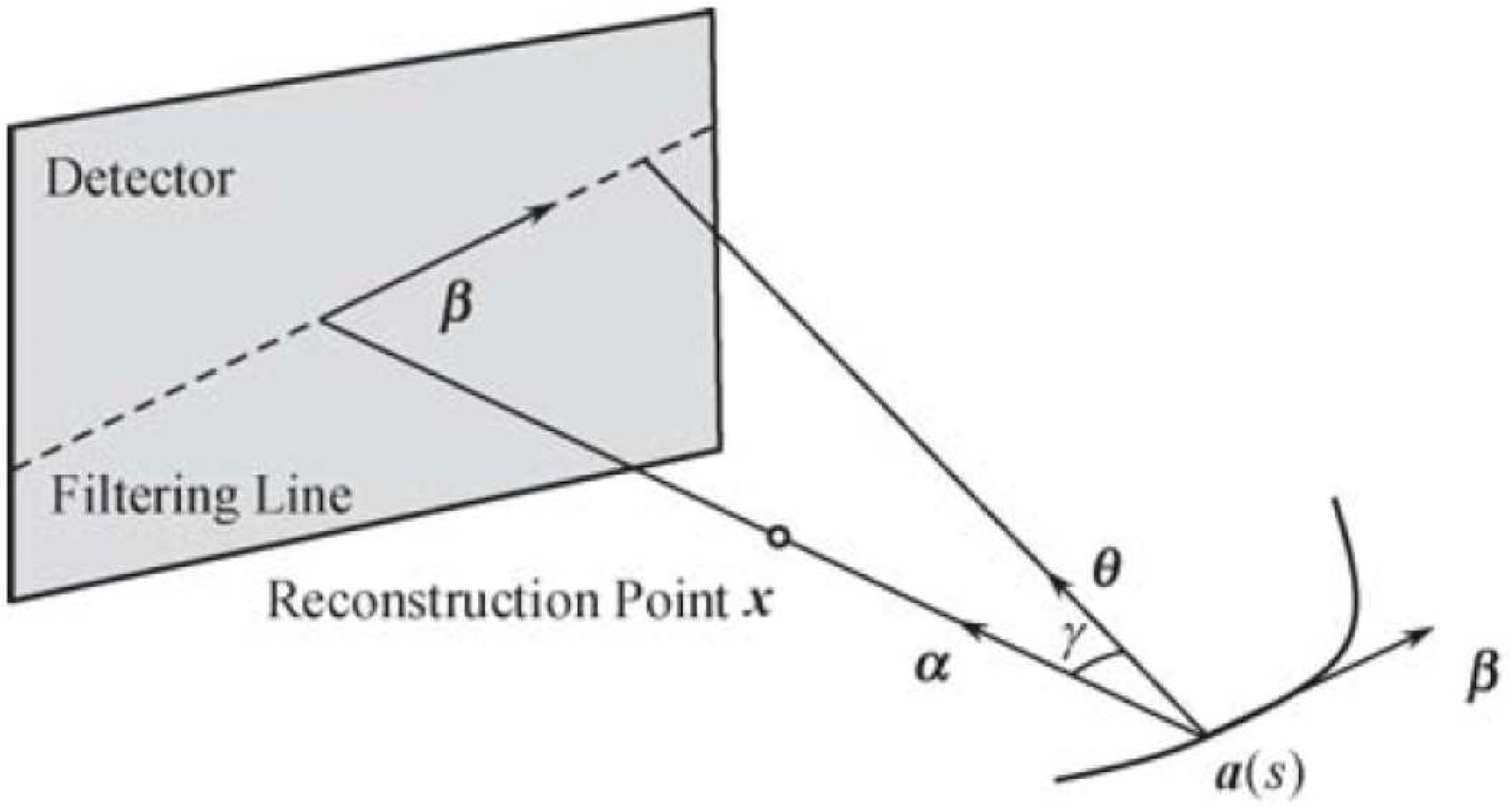

Figure 3: Direction of filtering (Zeng, 2009)

Katsevich's Algorithm: Derivative

Compute the derivative along the trajectory as discrete difference between two neighboring projections:

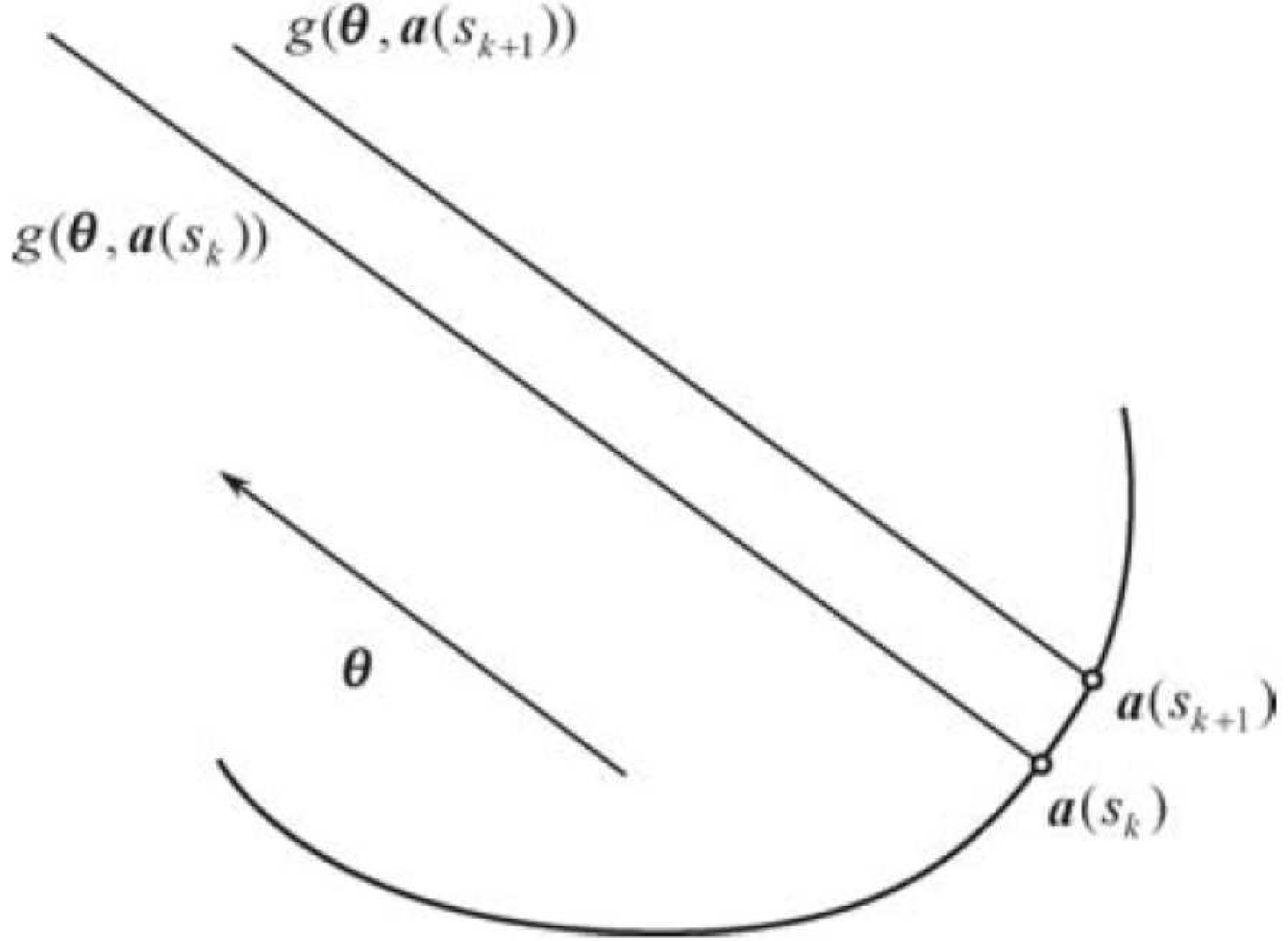

Figure 4: Computation of the directional derivative (Zeng, 2009)

Katsevich's Algorithm: Weighting

Perform weighting of the projection data with

$$\frac{D}{\sqrt{D^2 + w^2}}$$

where D is the source detector distance, and w the axis of the detector that points along the rotation axis.

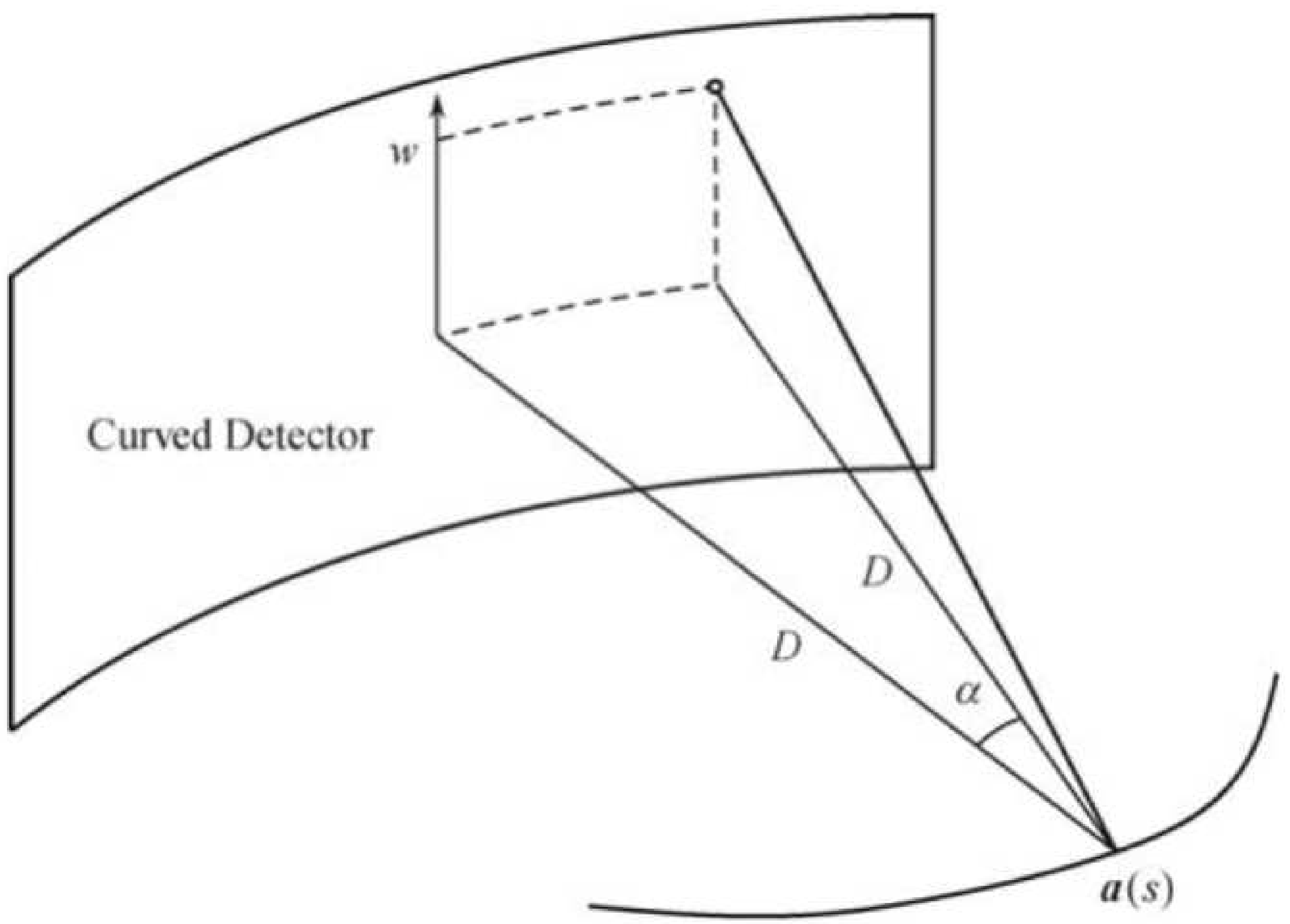

Figure 5: Weighting scheme (Zeng, 2009)

Katsevich's Algorithm: Choosing β

- Compute derivative along the trajectory $\boldsymbol{a}(s)$.
- Choose β as the angle of a plane κ that contains the points $\boldsymbol{x}$, $\boldsymbol{a}(s)$, $\boldsymbol{a}(s+\psi)$, and $\boldsymbol{a}(s+2\psi)$.
- This is not unique. Hence, choose $|\psi|$ to be minimal.

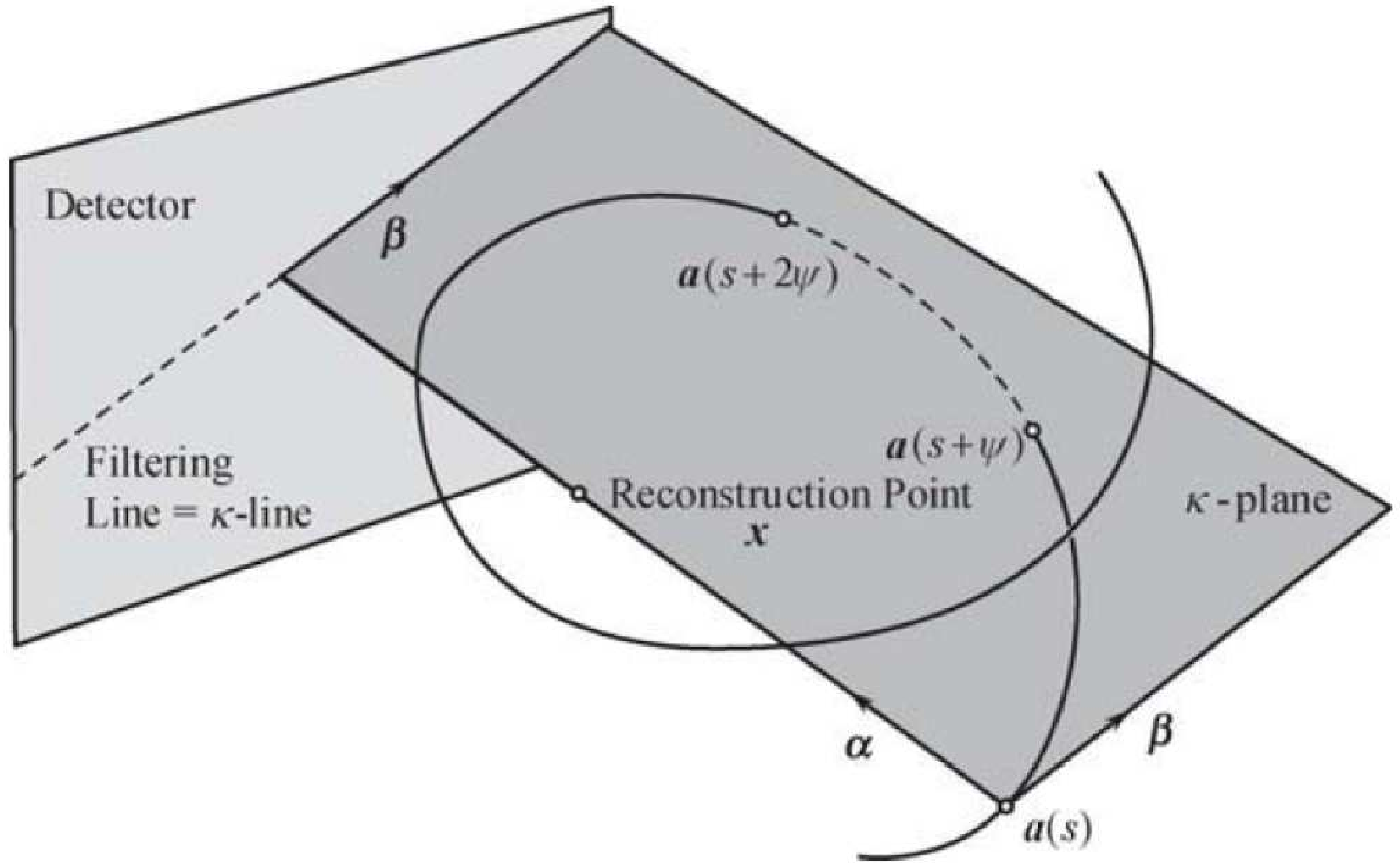

Figure 6: Weighting scheme (Zeng, 2009)

κ-lines

- The projection of a κ-plane onto the detector is called a κ-line.
- The orientation changes with the reconstruction point.

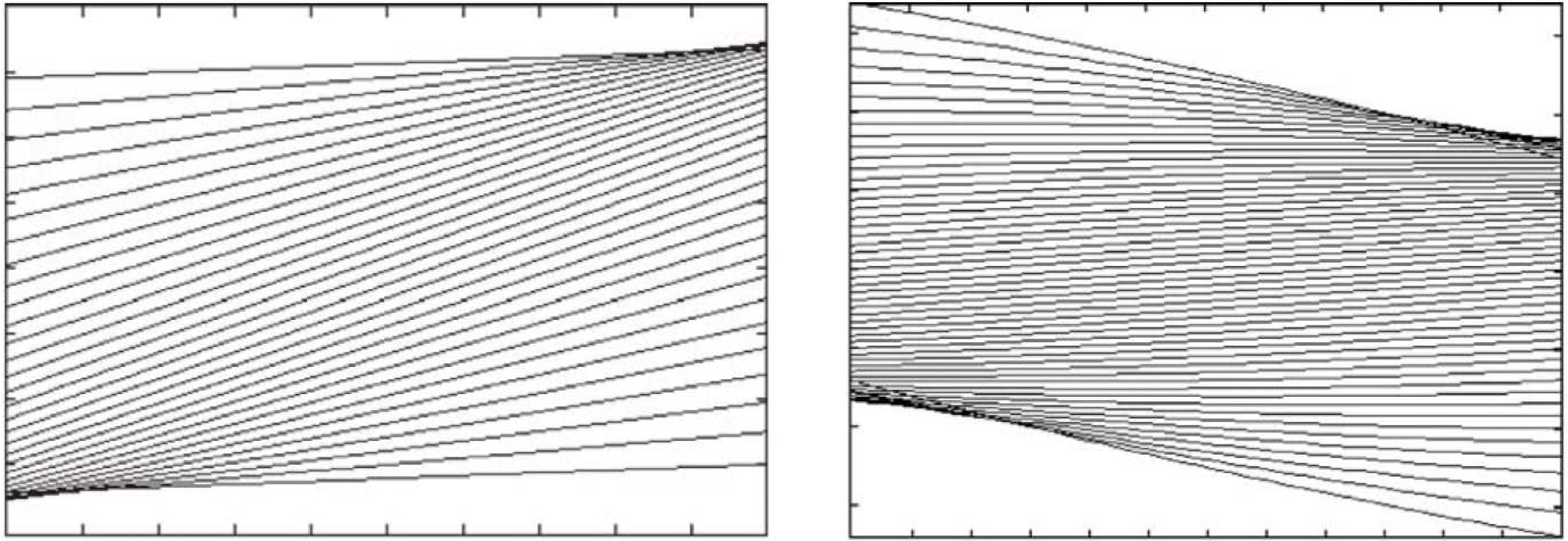

Figure 7: κ-lines of a flat detector (left), and a curved detector (right) (Zeng, 2009)

Katsevich's Algorithm: Rebinning and Filtering

- Rebin the data to parallel lines to prepare for the Hilbert transform.
- Perform the 1-D Hilbert transform along each line.

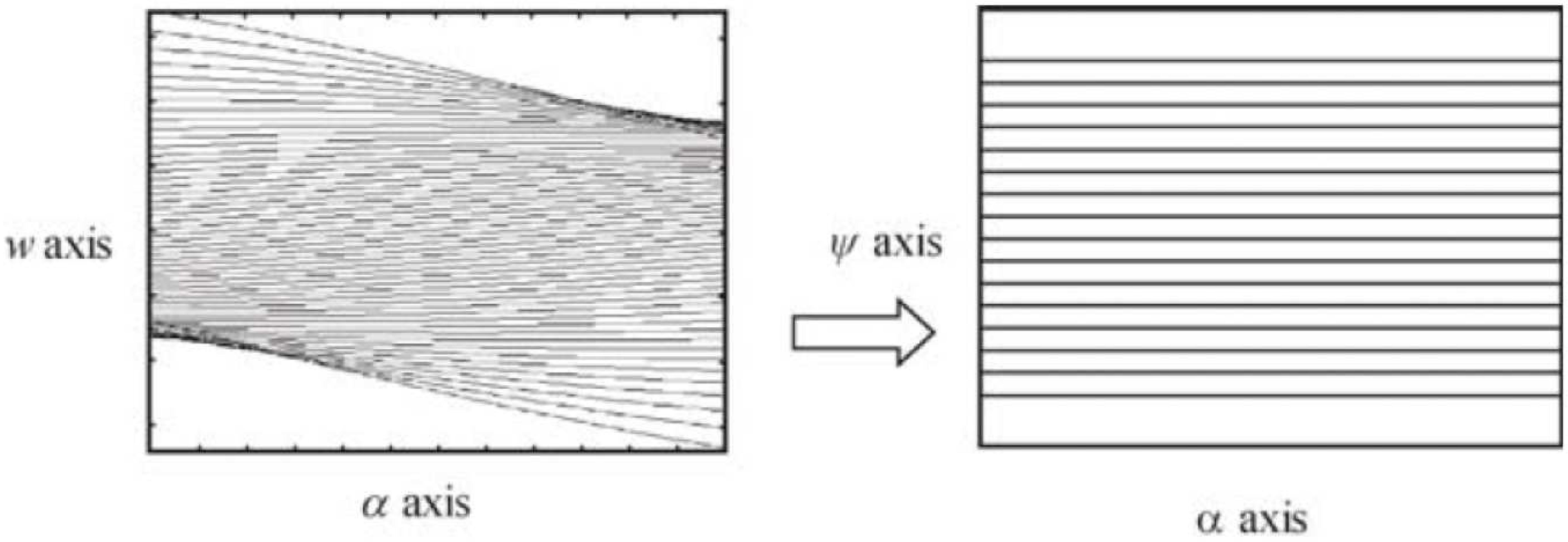

Figure 8: Rebinning of the κ-lines (Zeng, 2009)

Katsevich's Algorithm: Final Steps

- Invert the rebinned data to go back to detector coordinates.
- Perform remaining weighting with $\cos\alpha$.
- Perform cone beam backprojection with distance weighting for all voxels $\boldsymbol{x}$.

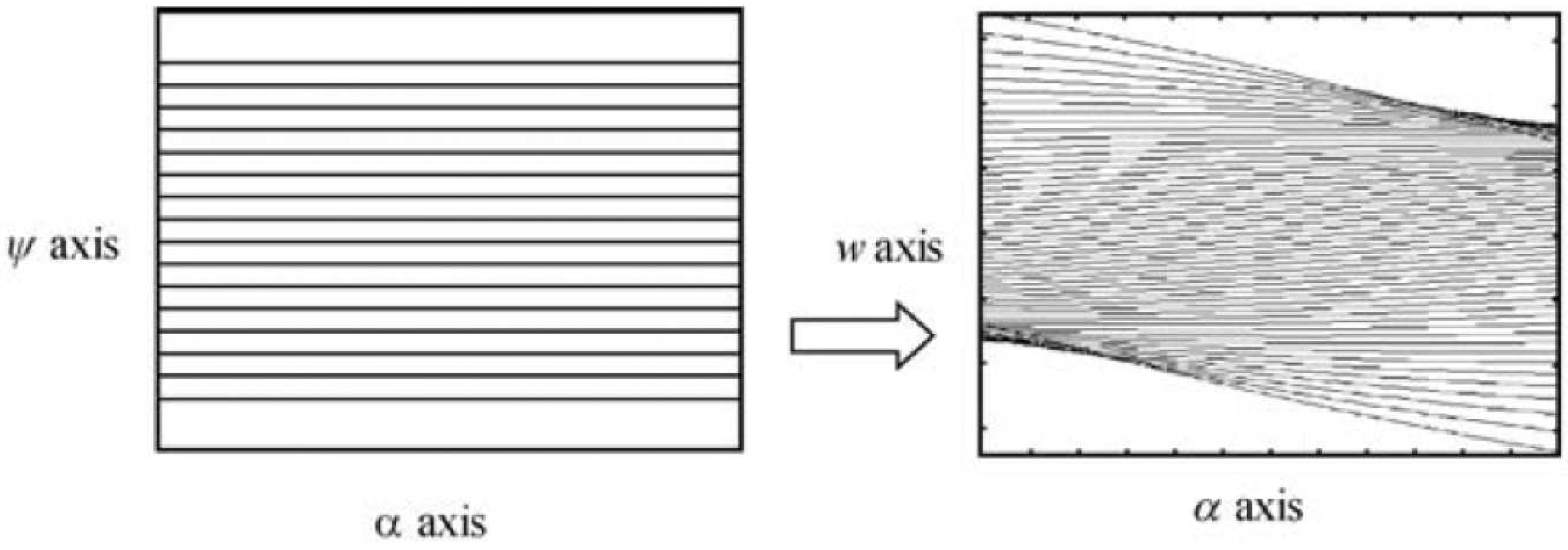

Figure 9: Invert the rebinning after Hilbert transform has been applied (Zeng, 2009)

Katsevich's Algorithm: Remarks

- Katsevich's algorithm performs an exact reconstruction.
- The algorithms causes some additional computational overhead compared to other algorithms as the FDK algorithm.
- Katsevich's algorithm does not use all available data.
- Katsevich's algorithm deletes redundant data $\rightarrow$ $(1, -1, 1)$-weighting.

Topics

Katsevich's Algorithm

Summary

Take Home Messages

Further Readings

Take Home Messages

- Katsevich's algorithm is a 3-D cone beam reconstruction algorithm.
- It is applicable to helical CT, and provides a theoretically exact algorithm.
- Katsevich's algorithm is a filtered backprojection algorithm.

Further Readings

The original works of Katsevich can be found here:

Alexander Katsevich. "Theoretically Exact Filtered Backprojection-Type Inversion Algorithm for Spiral CT". In: *SIAM Journal on Applied Mathematics* 62.6 (2002), pp. 2012–2026. DOI: 10.1137/S0036139901387186

Alexander Katsevich. "Analysis of an Exact Inversion Algorithm for Spiral Cone-beam CT". In: *Physics in Medicine and Biology* 47.15 (July 2002), pp. 2583–2597

Alexander Katsevich. "An Improved Exact Filtered Backprojection Algorithm for Spiral Computed Tomography". In: *Advances in Applied Mathematics* 32.4 (May 2004), pp. 681–697. DOI: doi:10.1016/S0196-8858(03)00099-X

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Medical Image Processing for Diagnostic Applications

Reconstruction in 3-D

Online Course – Unit 50

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Exact vs. Approximate Reconstruction

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Exact vs. Approximate Reconstruction

The Defrise phantom is often used to investigate the cone beam artifact:

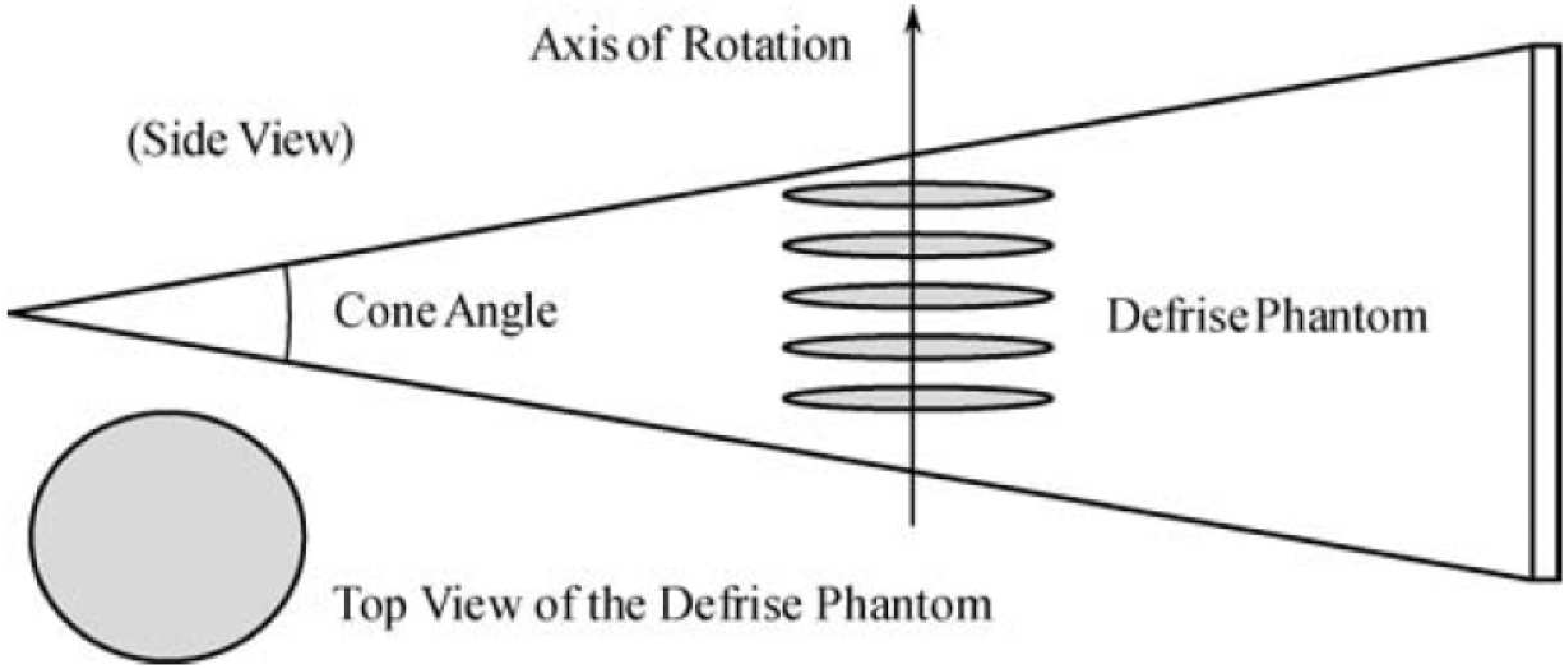

Figure 1: Scheme of studying the cone beam artifact with the Defrise phantom (Zeng, 2009)

Exact vs. Approximate Reconstruction

The higher the cone angle the stronger the cone beam artifact will appear:

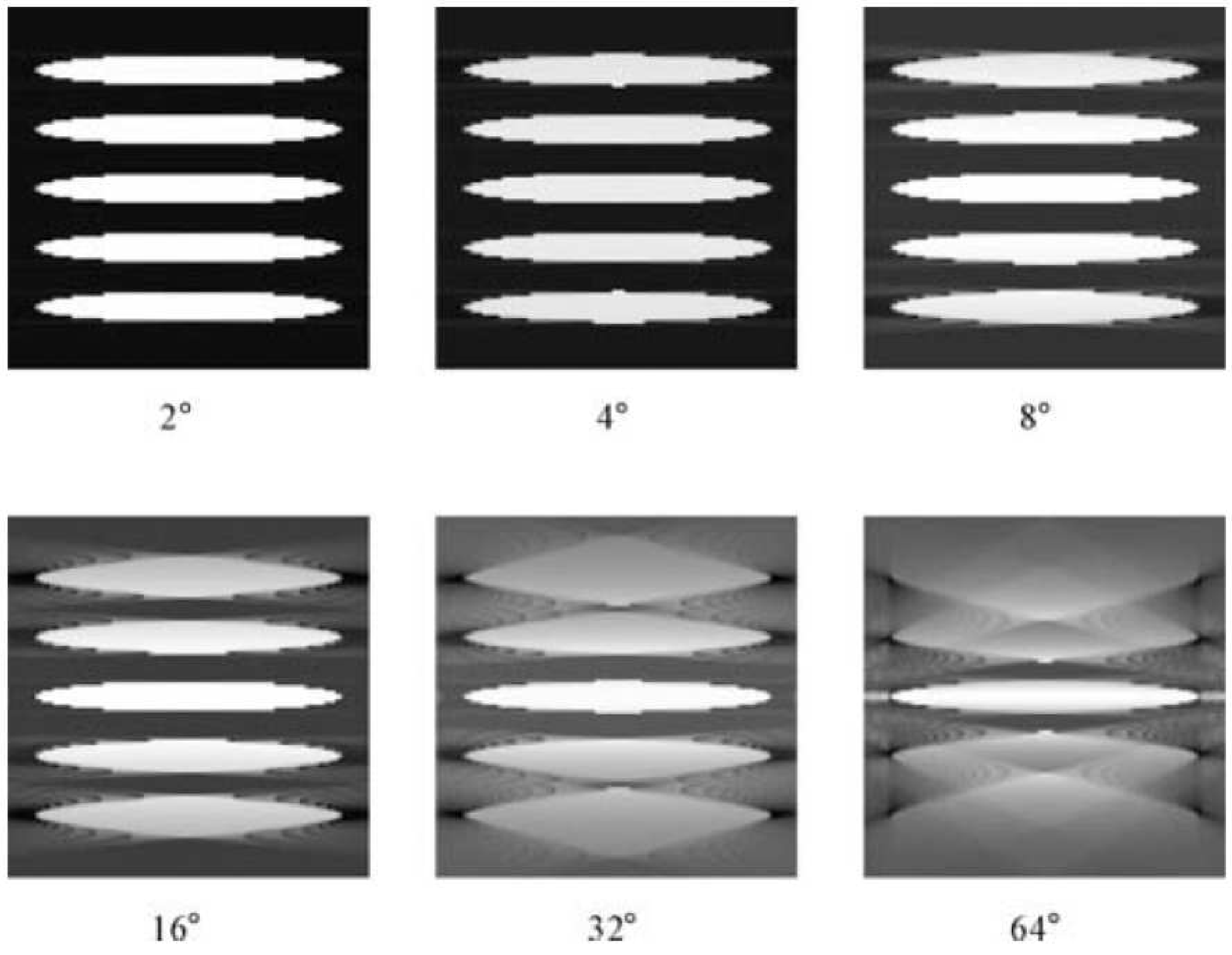

Figure 2: The artifact gets stronger for larger cone angles (Zeng, 2009).

Exact vs. Approximate Reconstruction

- The higher the cone angle, the more exact methods benefit the reconstruction result.
- Helical CT scanners usually have rather small cone angles (due to image artifacts).
- Flat panel scanners usually have circular trajectories that allow only approximate reconstruction.

$\Rightarrow$ Only few exact methods are used in clinical practice.

Topics

Exact vs. Approximate Reconstruction

Final Remarks

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Further Readings

3-D Reconstruction

- Cone beam geometry allows a much faster data acquisition.
- Approximate methods allow robust reconstruction.
- Exact reconstruction provides artifact-free reconstruction if the data is complete.
- Cone beam geometry suffers from physical effects (such as scatter) much more than fan beam geometries.

4-D Reconstruction?

- There are methods to model even more dimensions such as time.
- This makes the reconstruction problem even more difficult.
- Some approaches are:
  - fast scanning,
  - motion gating (regular motion), or
  - motion estimation.
- All these methods are ongoing research.

Topics

Exact vs. Approximate Reconstruction

Final Remarks

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- Cone beam artifacts are an issue when designing a reconstruction method in 3-D.
- Technical difficulties often discourage exact algorithms.
- There is ongoing research for 3-D and 4-D reconstruction methods and their applications.

Further Readings

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