Medical Image Processing for Diagnostic Applications

Parallel Beam – Reconstruction Steps

Online Course – Unit 34

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Topics

How to Implement a Parallel Beam Algorithm - Part 2

- Discrete Spatial vs. Continuous Frequency Version
- Practical Algorithm
- Backprojection Example

Summary

- Take Home Messages
- Further Readings

Ram-Lak: Discrete Spatial Form of the Ramp Filter

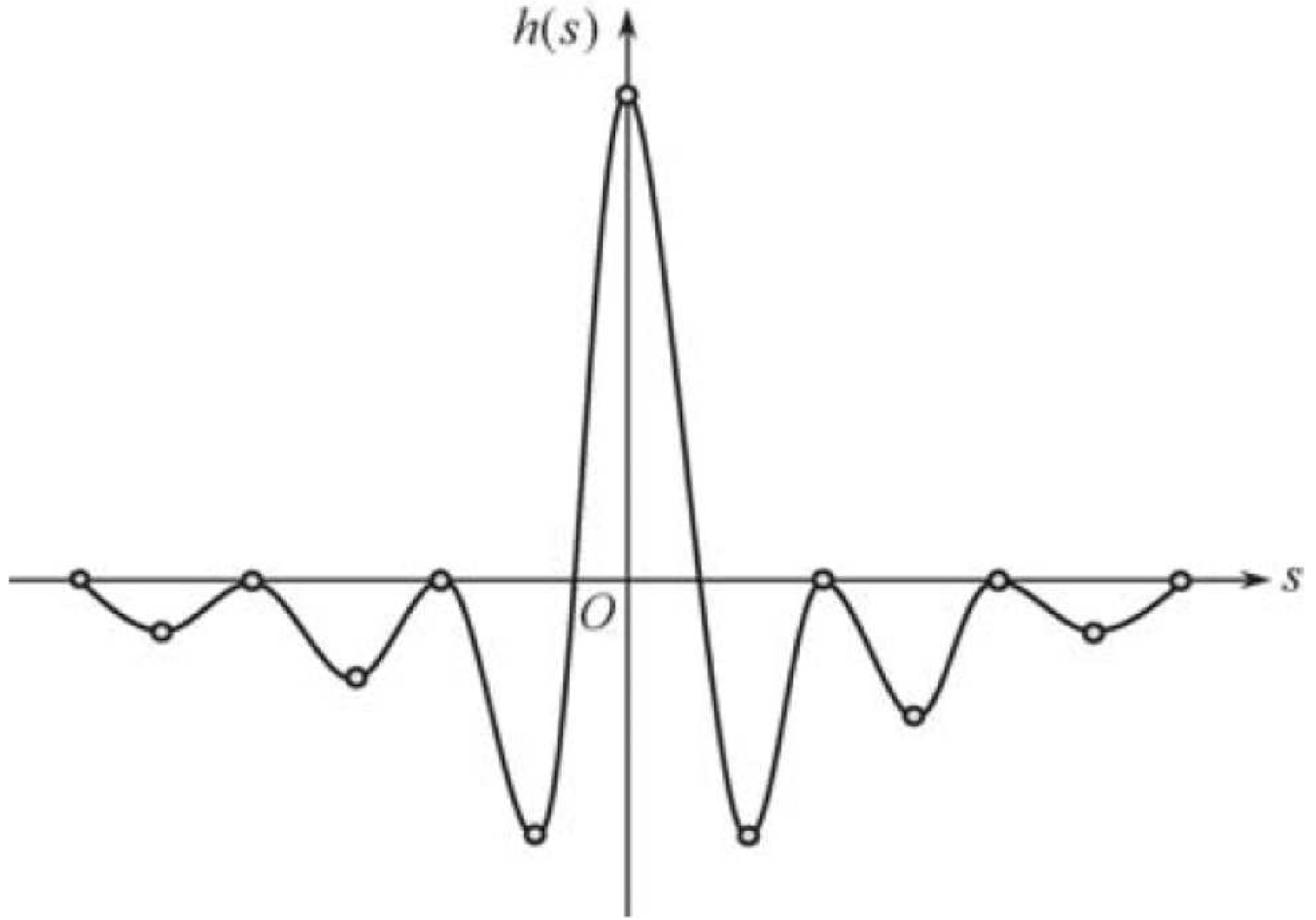

Figure 1: Continous and discrete graph of the Ram-Lak filter (Zeng, 2009)

Discrete Spatial vs. Continuous Frequency Version

- Continuous frequency representation of the ramp filter:

$$H(\omega) = |\omega|$$

- Discrete spatial form:

$$h(n\tau) = \begin{cases} \frac{1}{4\tau^2} & n = 0, \\ 0 & n \text{ even}, \\ -\frac{1}{\pi^2 (n\tau)^2} & n \text{ odd} \end{cases}$$

Discrete Spatial vs. Continuous Frequency Version

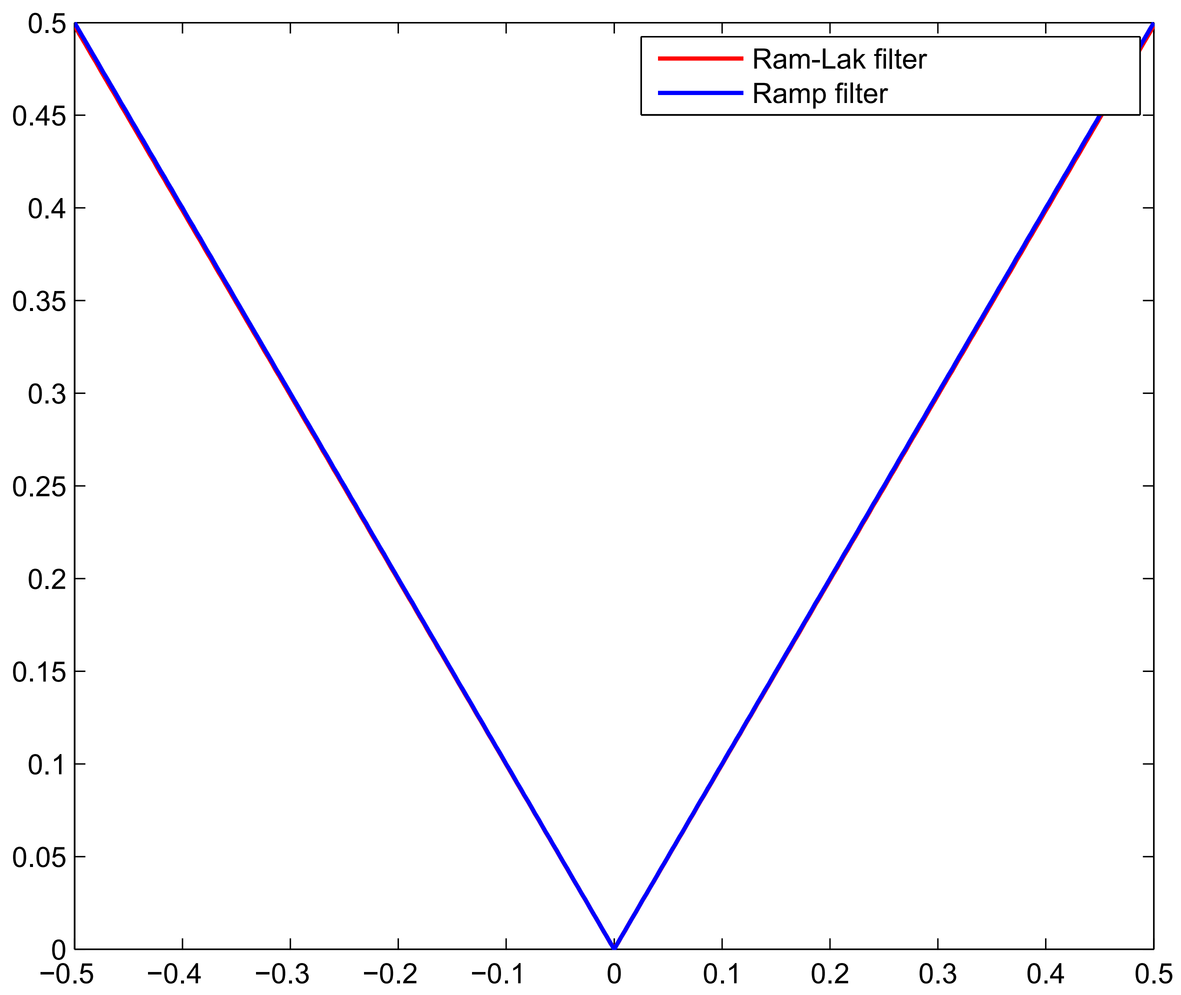

Figure 2: Plot of the ramp filter, whole scale

Discrete Spatial vs. Continuous Frequency Version

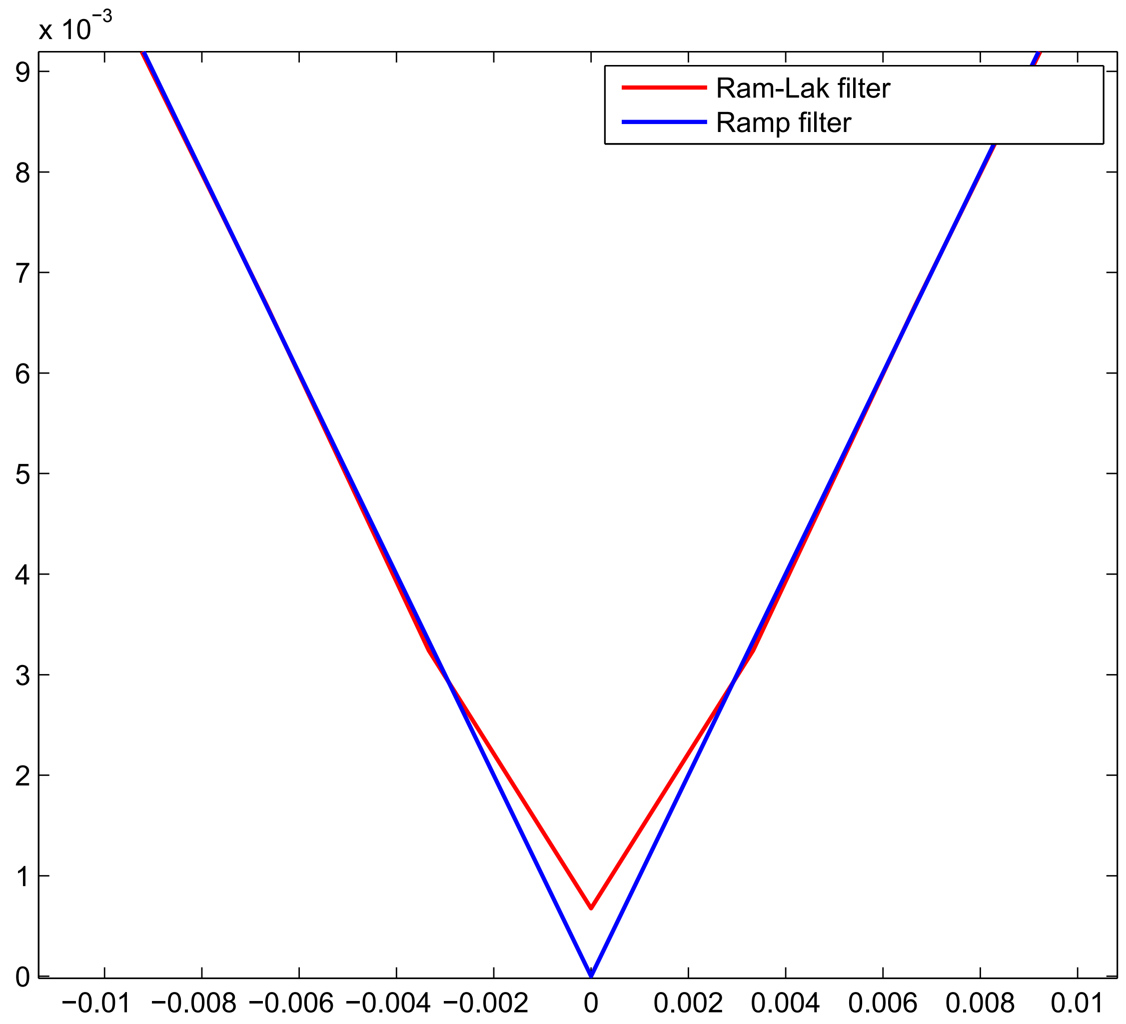

Figure 3: Comparison of the ramp and the Ram-Lak filter, zoomed in at zero

Example: Homogeneous Cylinder after Filter

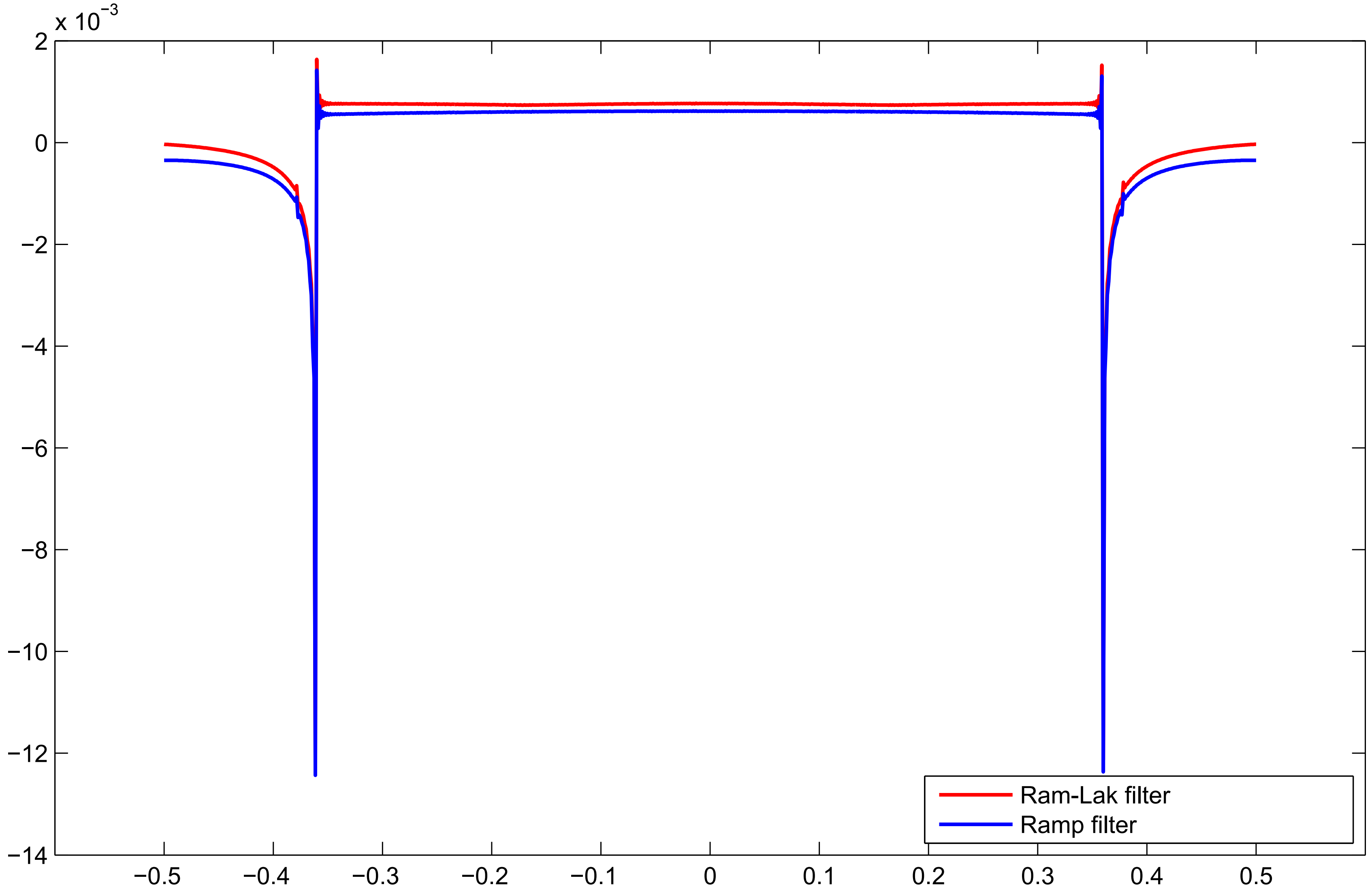

Figure 4: Filtered projection profile of the cylinder phantom

Practical Algorithm - Filtering

1. Precompute filter h(s) in spatial domain $O(N)$
2. Transform filter to frequency domain $H(\omega)$ via FFT $O(N\log N)$
3. For each of $\#P$ projections:
   - Compute FFT of $p(s,\theta)$ $O(N\log N)$
   - Apply filter $P(\omega,\theta)\cdot H(\omega)$ $O(N)$
   - Compute filtered projection $q(s)$ via iFFT $O(N\log N)$

Total complexity: $O(N+N\log N+\#P(N+2N\log N)) = O(\#P\ N\log N)$

Practical Algorithm - Backprojection

1. Initialize $f(x,y) = 0$ $\quad O(N^2)$
2. For each of $N \times N$ pixels:

   For each of $\#P$ projections:

   - Compute $s = x\cos\theta + y\sin\theta$ $\quad O(1)$
   - Update $f(x,y) += q(s,\theta)$ $\quad O(1)$

Total complexity: $O(N^2 + N^2\#P(1+1)) = O(N^2\#P)$

Practical Algorithm: Overall Complexity

- Apply filter on the detector row:

$$O(\#P \; N \log N)$$

- Backproject:

$$O(\#P \; N^2)$$

Backprojection Example

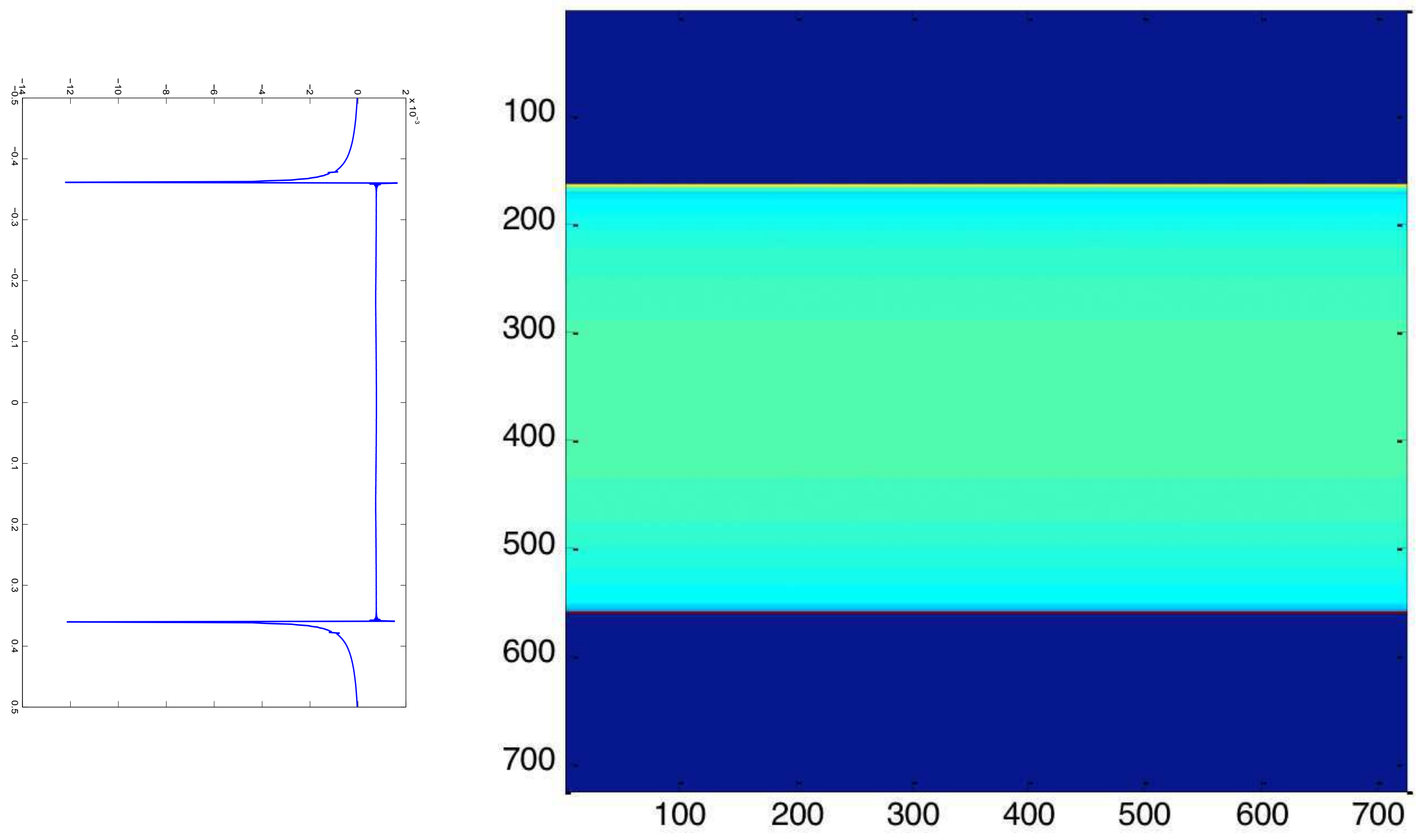

Figure 5: Backprojection of a single projection

Backprojection and Fourier Slice Theorem

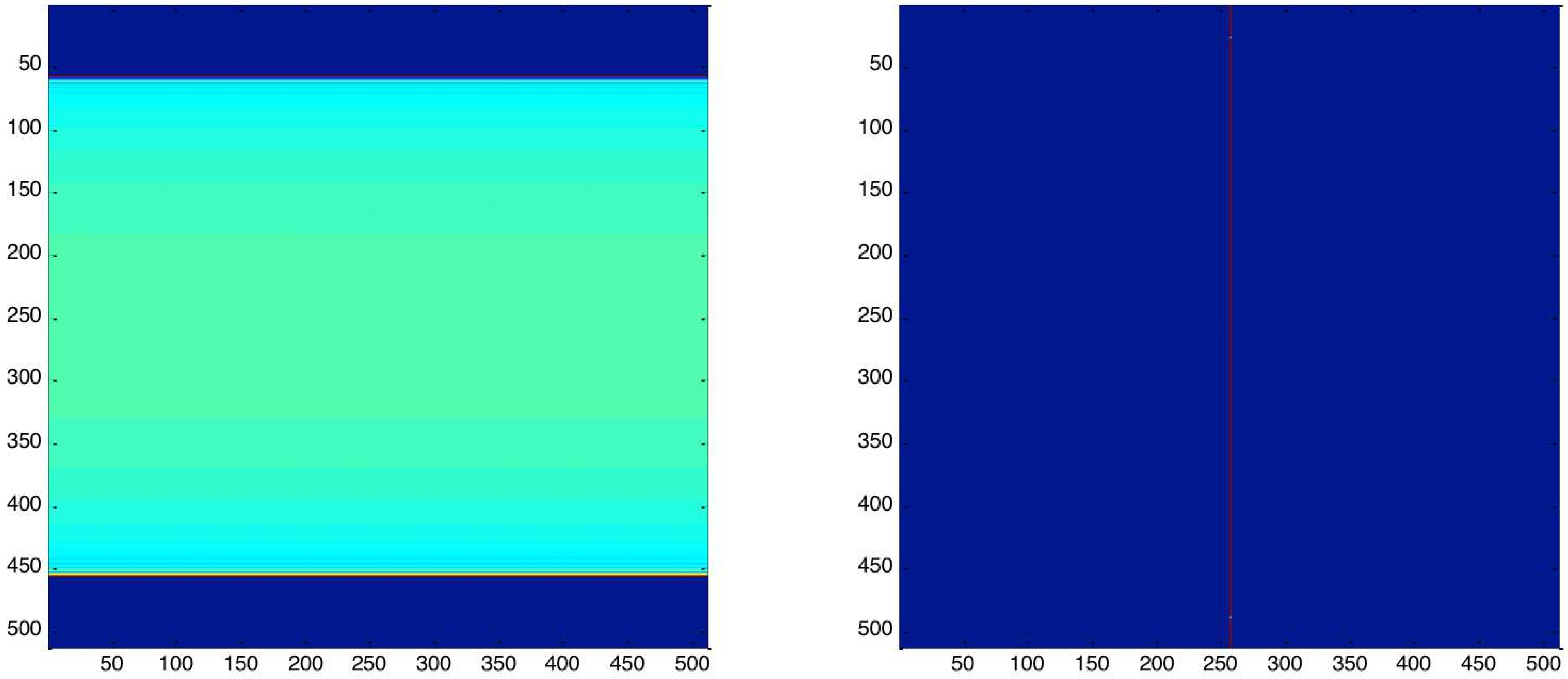

Figure 6: Backprojection (left) of a single line in the Fourier space (right)

Backprojection Example

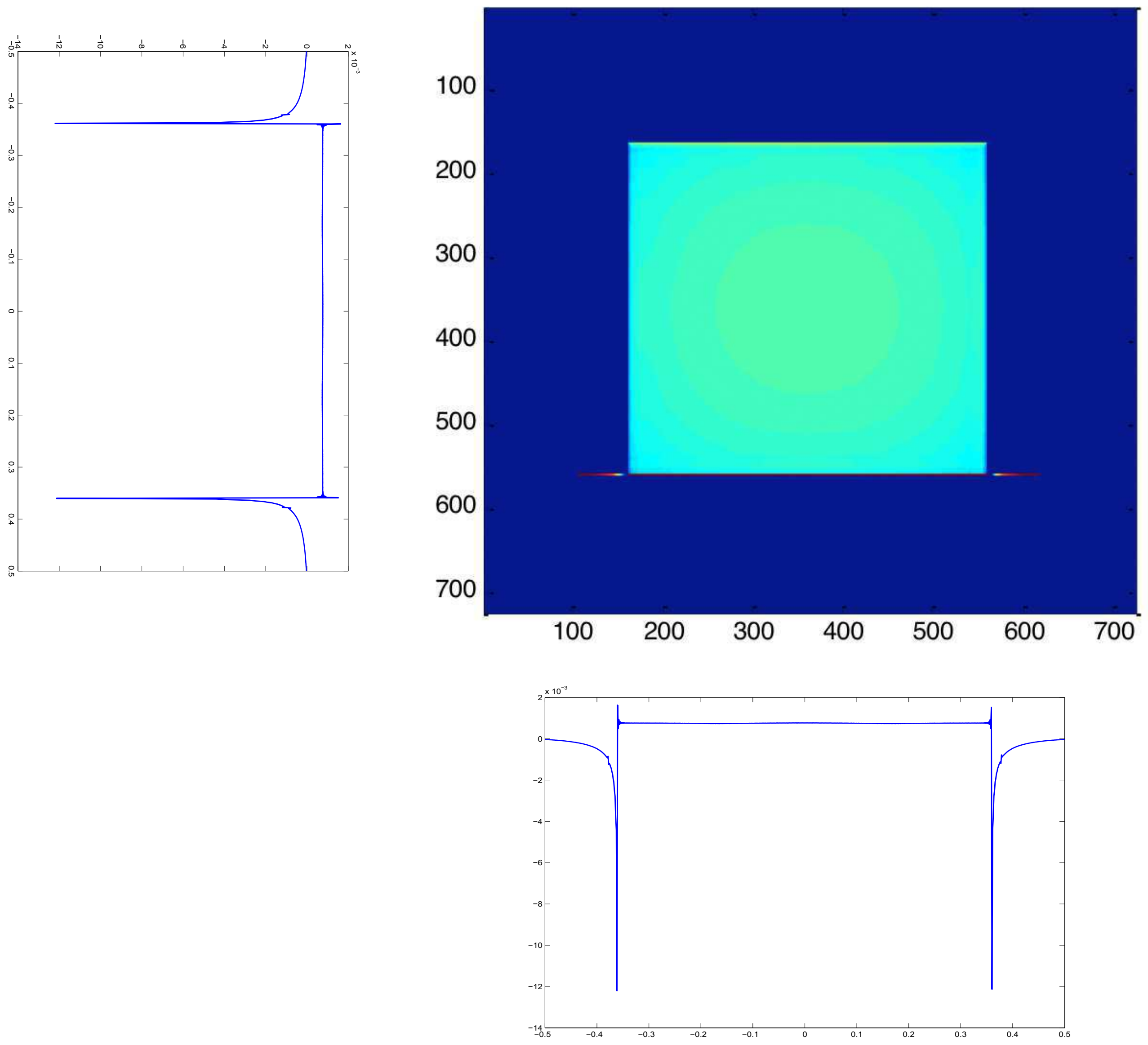

Figure 7: Backprojection of two projections (0°, 90°)

Backprojection Example

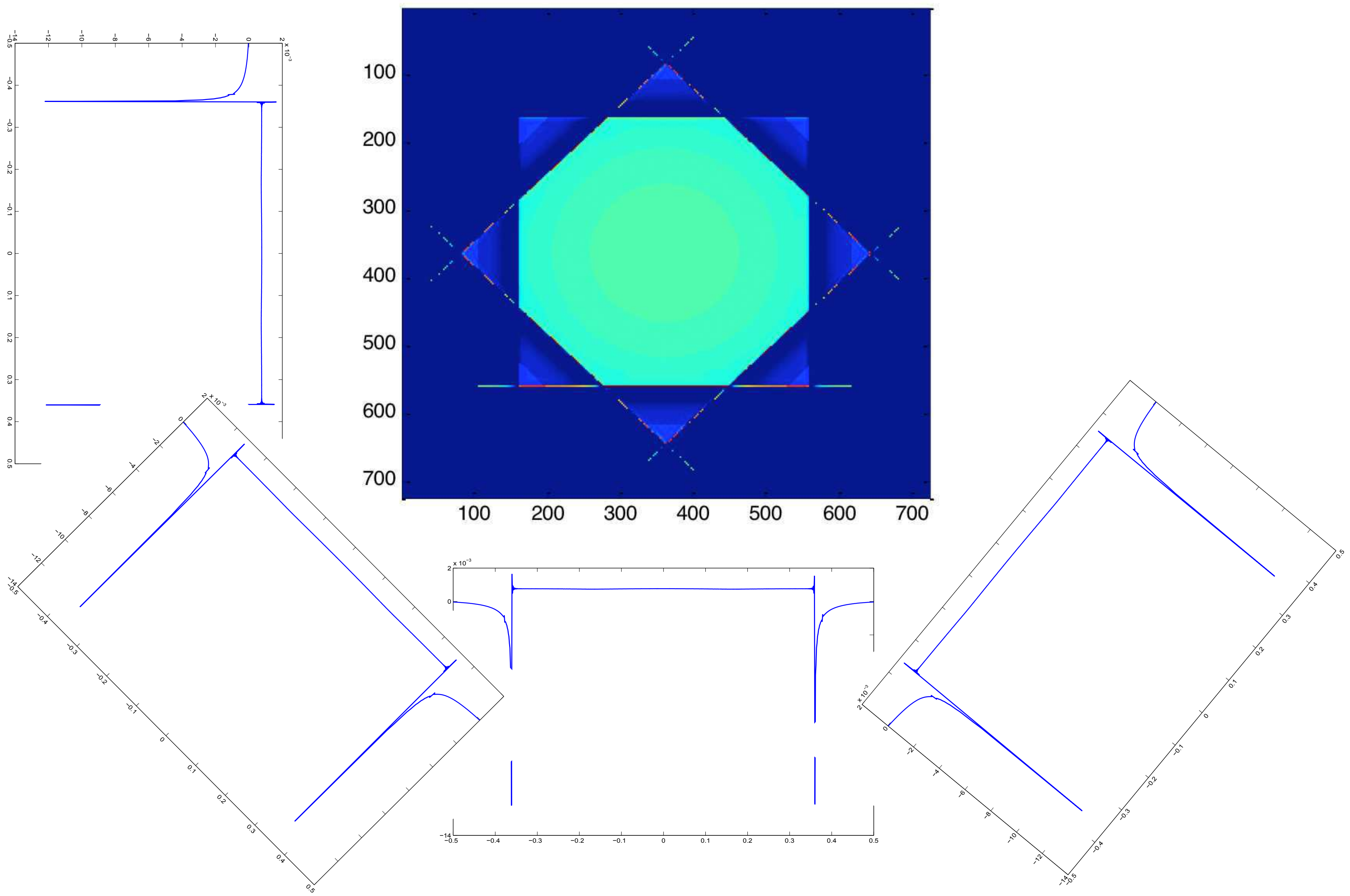

Figure 8: Backprojection of four projections (0°, 45°, 90°, 135°)

Backprojection Example

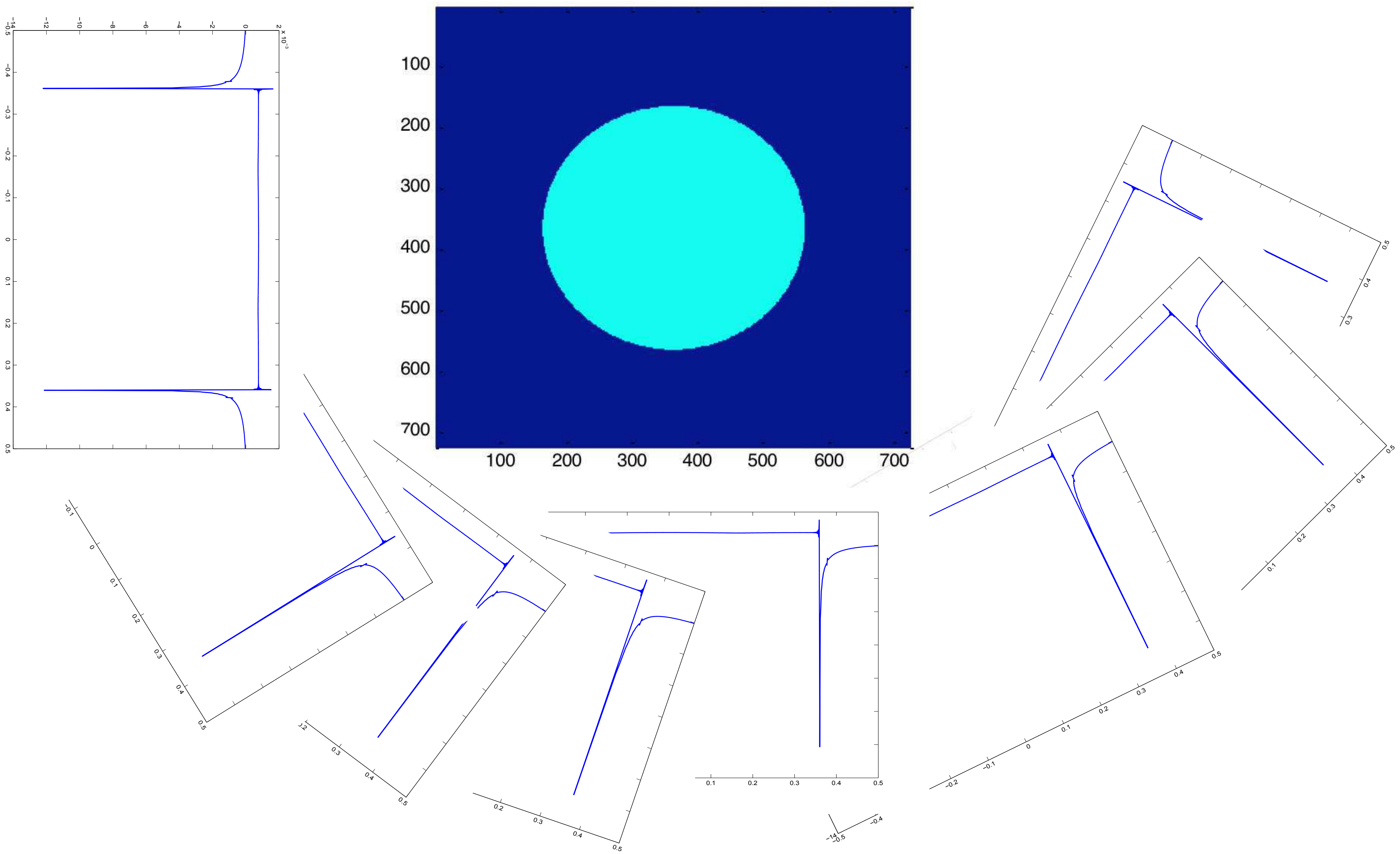

Figure 9: Backprojection of multiple projections (0°-180°)

Topics

Take Home Messages

- Although the original ramp filter converges to zero at zero frequency, the Ram-Lak filter takes low frequencies into account. This enables discrete computations to be more accurate.
- The filtering has a complexity of $O(N \log N)$, and the backprojection a complexity of $O(N^2)$ per projection.
- Increasing the number of projections improves the reconstruction result.

Further Readings

The original Ram-Lak article is:

G. N. Ramachandran and A. V. Lakshminarayanan. "Three-dimensional Reconstruction from Radiographs and Electron Micrographs: Application of Convolutions instead of Fourier Transforms". In: *Proceedings of the National Academy of Sciences of the United States of America* 68.9 (Sept. 1971), pp. 2236–2240

The derivation shown in this unit is based on a document by Martin Berger.

The concise reconstruction book from 'Larry 'Zeng:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Another mathematical examination of filtered backprojection can be found in

Thorsten Buzug. *Computed Tomography: From Photon Statistics to Modern Cone-Beam CT*. Springer Berlin Heidelberg, 2008. DOI: 10.1007/978-3-540-39408-2