Medical Image Processing for Diagnostic Applications

Iterative Reconstruction – ART Extensions

Online Course – Unit 57
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Topics

Extensions of ART

- SART
- SIRT
- Ordered Subsets
- Towards More Realism

Summary

- Take Home Messages
- Further Readings

Extensions of ART

- Slow convergence is the main drawback of ART.
  $\rightarrow$ Extensions aim at improving convergence speed.
- Compute update using more than one projected pixel:
  - Simultaneous ART (SART): multiple updates at the same time and combination of the result,
  - Simultaneous Iterative Reconstruction Technique (SIRT): compute update once per iteration.
- Use intelligent methods to select the order of the update equations (ordered subsets).
- Use more realistic models within the system matrix.

SART

1. Estimate an initial solution of the system of linear equations.
2. Compute the orthogonal projections of the current estimate to all hyperplanes.
3. Compute the centroid of all projected points.
4. Use this centroid for the next iteration.

SART

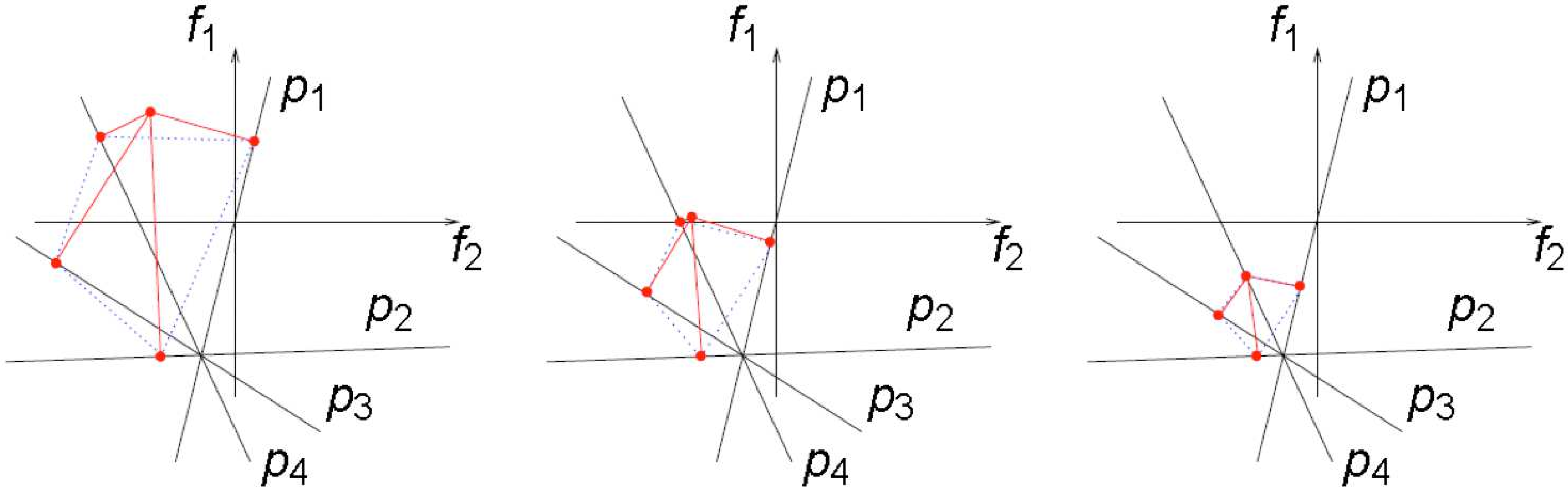

Figure 1: Scheme of the SART algorithm

SART

All pixels p_i are considered in the update simultaneously with the following update rule:

$$\boldsymbol{X}^{k+1} = \boldsymbol{X}^k + \lambda_k \sum_i u_{k,i} \frac{p_i - \boldsymbol{A_i}\boldsymbol{X}^k}{\boldsymbol{A_i}\boldsymbol{A_i}^\top} \boldsymbol{A_i}^\top,$$

where

$$\sum_i u_{k,i} = 1, \quad k = 0, 1, 2, \ldots,$$

and the parameter λ_k is used to control the step size in each iteration.

SIRT

- All voxels of the volume are updated simultaneously using the **same** projection $\boldsymbol{P_j}$ in the following update rule:

$$\boldsymbol{X}^{k+1} = \boldsymbol{X}^{k} + \frac{\boldsymbol{P_j} - \boldsymbol{A_j}\boldsymbol{X}^{k}}{||\boldsymbol{A_j}||}\boldsymbol{A_j}^{\top}$$

- $\boldsymbol{A_j}$ is the part of the system matrix that projects all the voxels of $\boldsymbol{X}$ onto the pixels of $\boldsymbol{P_j}$.
- $\boldsymbol{A_j}\boldsymbol{X}$ is a forward projection of $\boldsymbol{X}$.
- $\boldsymbol{P_j}\boldsymbol{A_j}^{\top}$ is a backprojection of $\boldsymbol{P_j}$.

Ordered Subsets ...

- ... is a clever technique to speed up convergence.
- ... employs knowledge on the acquisition sequence.
- ... selects subsets of equations (projections) that are most likely orthogonal to each other.

Iterations are performed in a nested way:

```
For all subsets
{
    Update using all items in the subset
}
```

Ordered Subsets

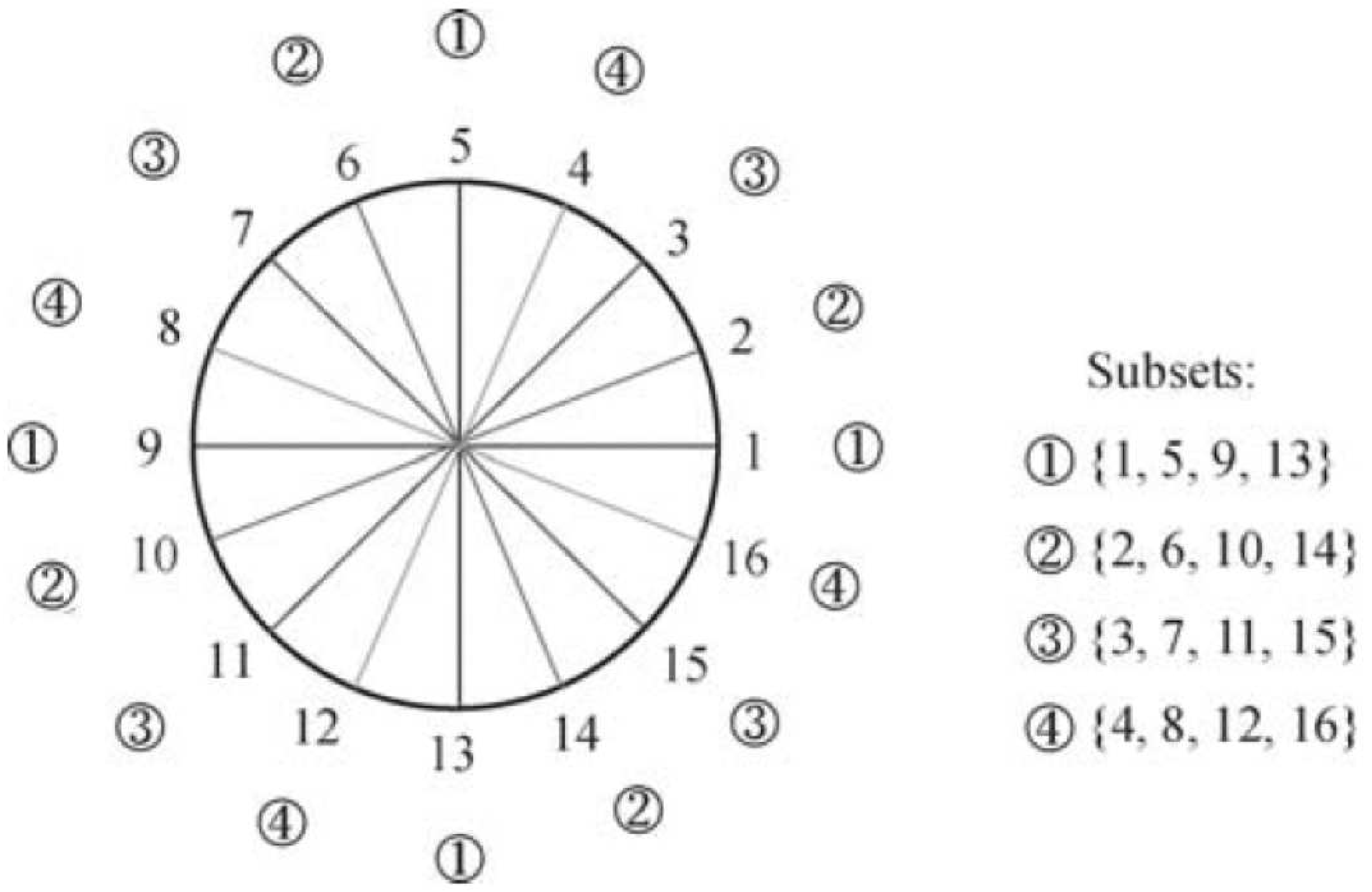

Figure 2: Illustration of the ordered subset scheme (Zeng, 2009)

Towards More Realism: Ray Intersection Model

The path length of the ray through a voxel is not the only way of modeling the imaging grid.

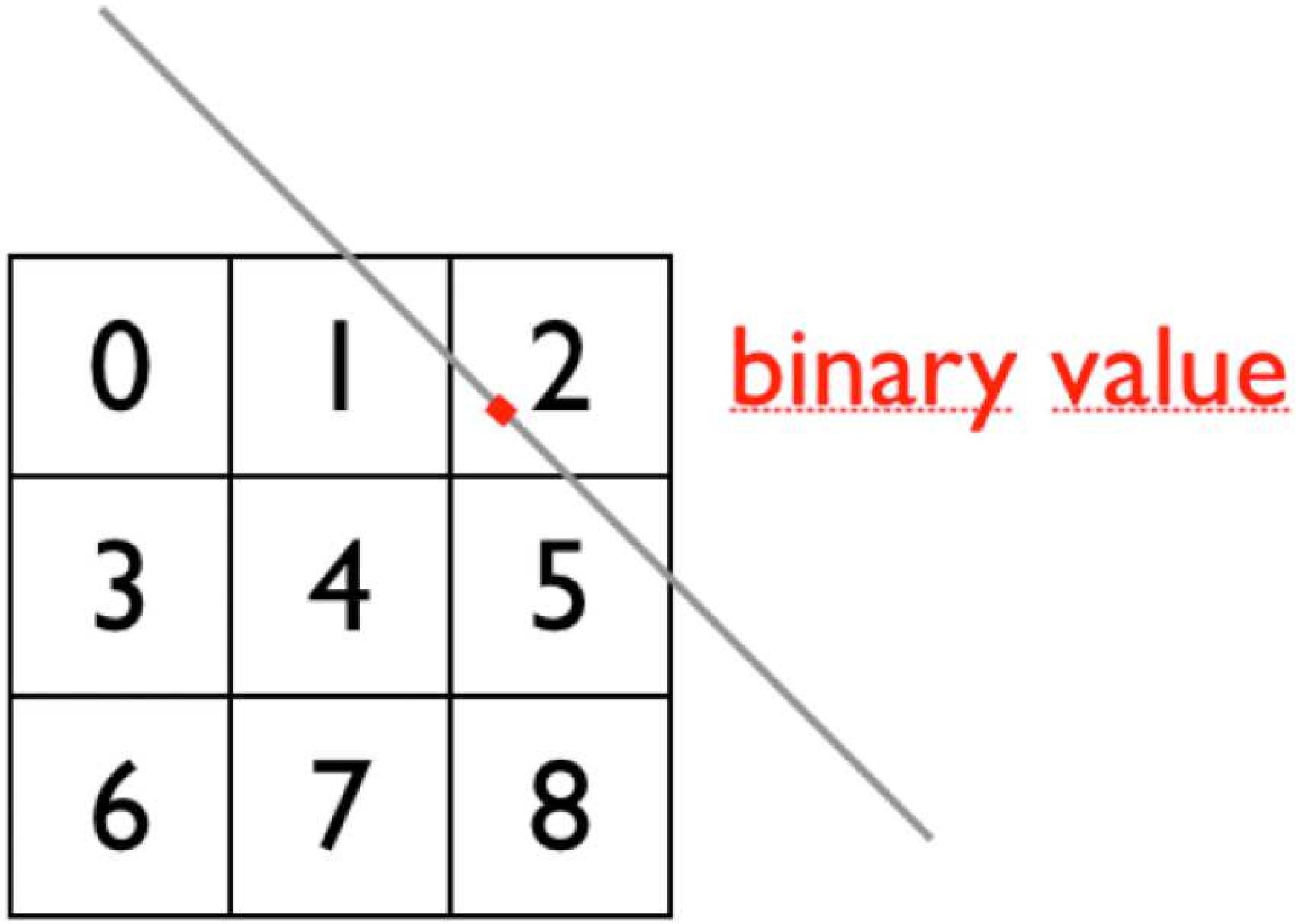

Figure 3: The projection value is evaluated in a single point.

Towards More Realism: Ray Intersection Model

The path length of the ray through a voxel is not the only way of modeling the imaging grid.

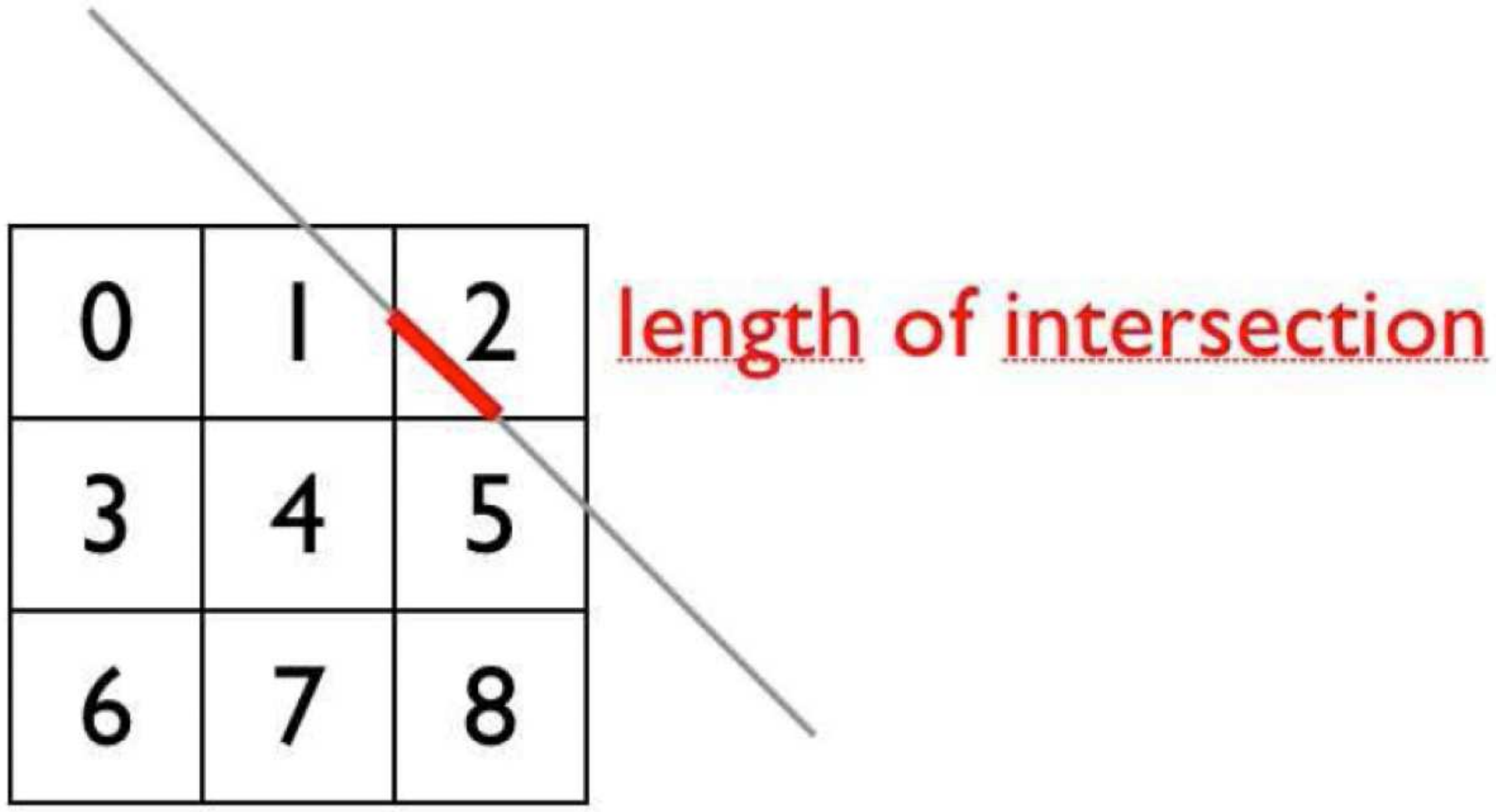

Figure 4: The length of a ray passing through a voxel is taken into account.

Towards More Realism: Ray Intersection Model

The path length of the ray through a voxel is not the only way of modeling the imaging grid.

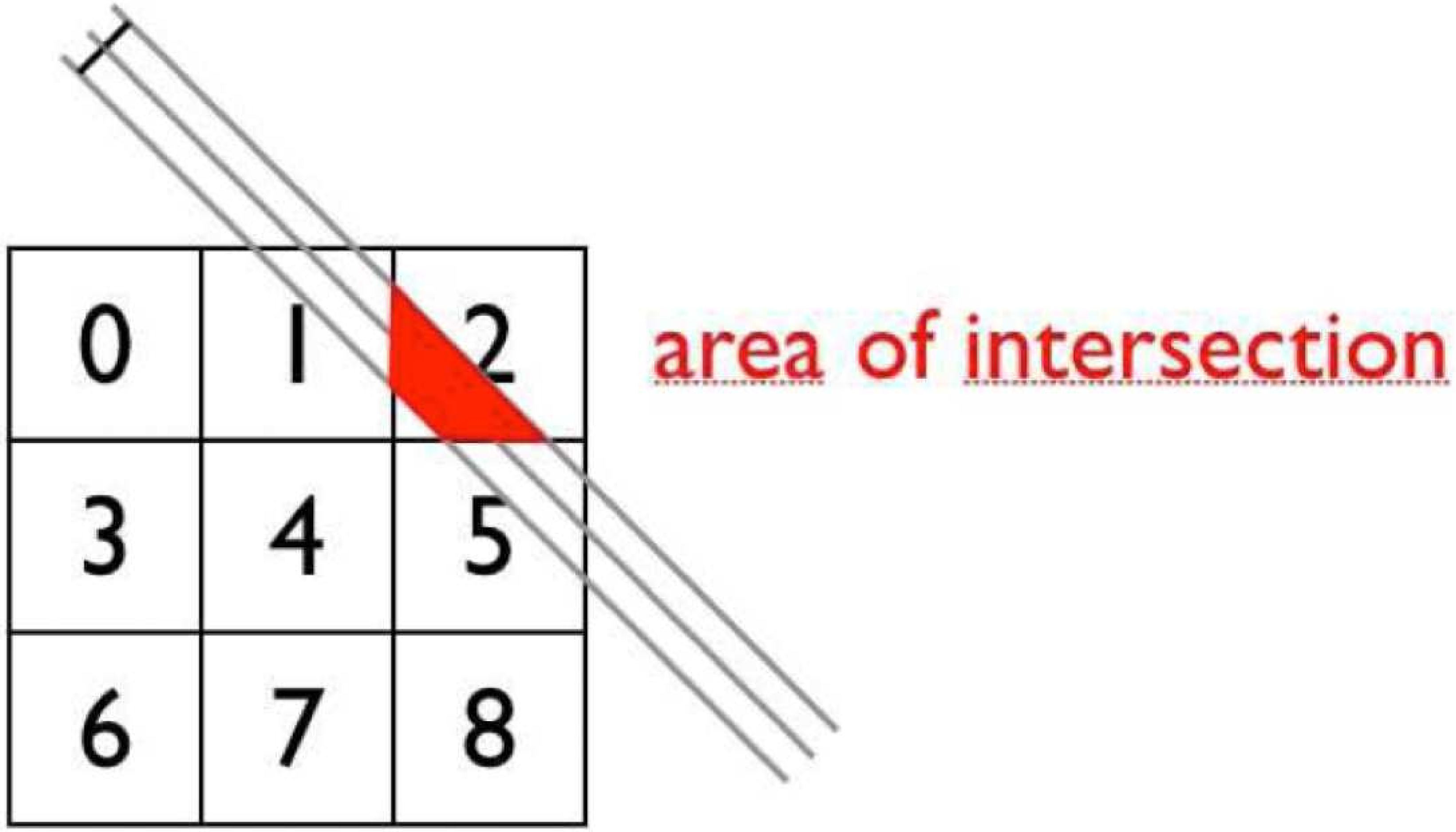

Figure 5: The ray model includes a second dimension such that the projection value scales with the area of intersection.

Towards more Realism: Alternative Voxels

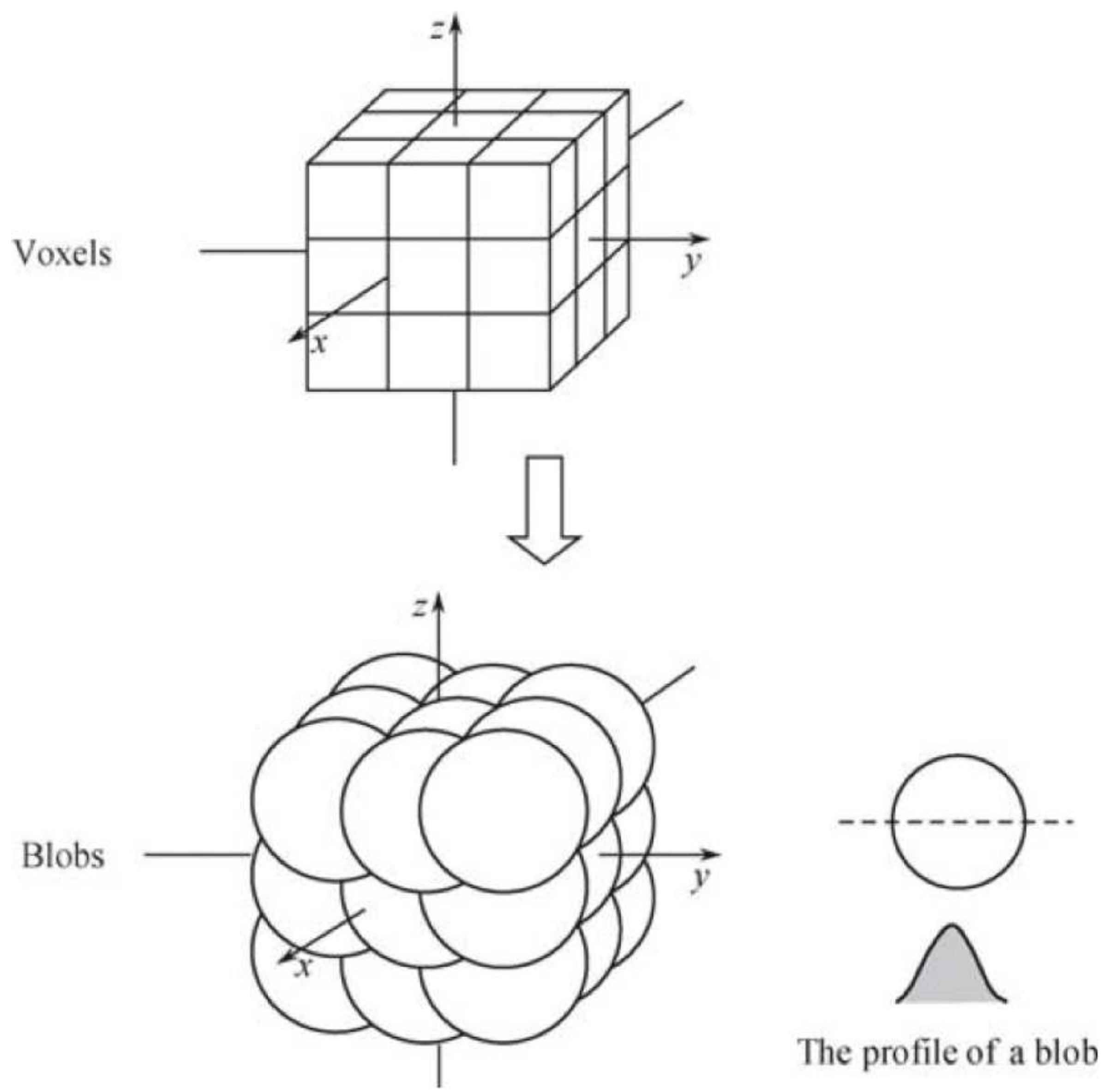

Figure 6: Use blobs as elements of the reconstruction volume (Zeng, 2009).

Towards more Realism: Alternative Voxels

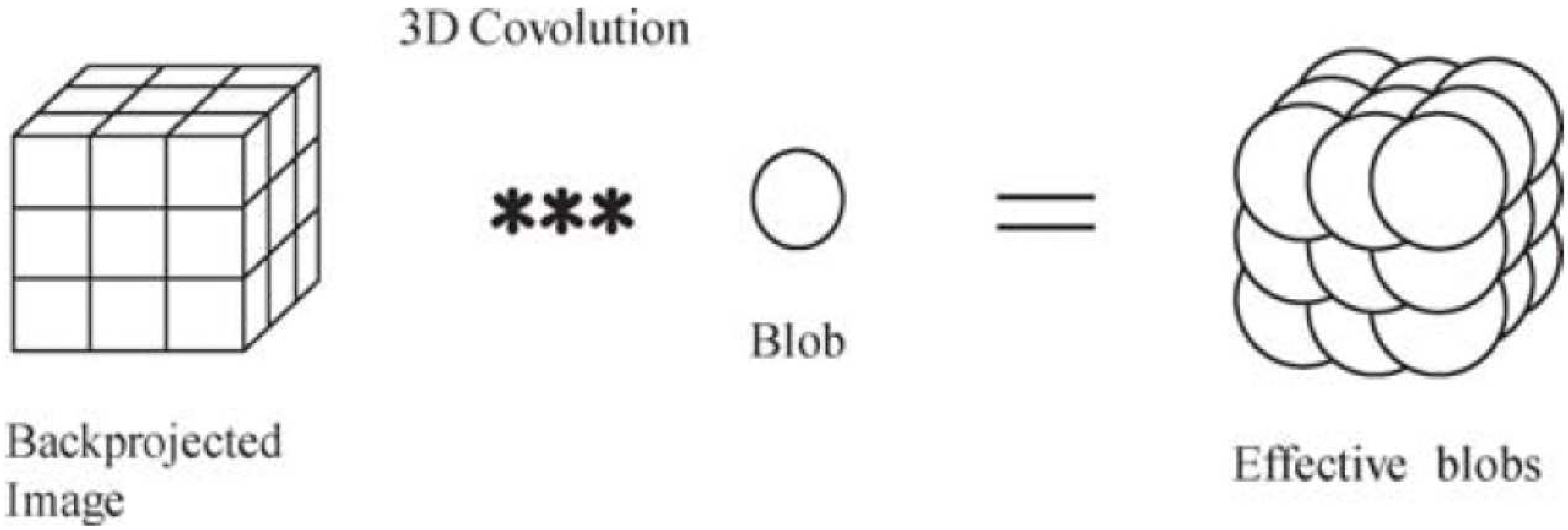

Figure 7: Illustration of the transfer from voxels to blobs by convolution (Zeng, 2009)

Towards more Realism: Final Remarks

Further extensions can be performed to include more realism:

- modeling of scattered photons,
- modeling of non-linear absorption effects,
- explicit noise modeling,
- modeling of the PSF of the imaging system.

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SIRT

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Further Readings

Take Home Messages

- Several extensions for ART are available (SART, SIRT, ...).
- Note that the projection and reconstruction model can be modified to achieve possibly better results (usually together with higher computational cost).

Further Readings

References and related books for the discussed topics in iterative reconstruction:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Stefan Kaczmarz. "Angenäherte Auflösung von Systemen linearer Gleichungen". In: *Bulletin International de l'Académie Polonaise des Sciences et des Lettres. Classe des Sciences Mathématiques et Naturelles. Série A, Sciences Mathématiques* 35 (1937), pp. 355–357 For this article you can find an English translation here (December 2016).

Avinash C. Kak and Malcolm Slaney. *Principles of Computerized Tomographic Imaging*. Classics in Applied Mathematics. Accessed: 21. November 2016. Society of Industrial and Applied Mathematics, 2001. DOI: 10.1137/1.9780898719277. URL: http://www.slaney.org/pct/

H. Bruder et al. "Adaptive Iterative Reconstruction". In: *Medical Imaging 2011: Physics of Medical Imaging*. Ed. by Norbert J. Pelc, Ehsan Samei, and Robert M. Nishikawa. Vol. 7961. Proc. SPIE 79610J. Feb. 2011, pp. 1–12. DOI: 10.1117/12.877953