Medical Image Processing for Diagnostic Applications

Parallel Beam – Preparation

Online Course – Unit 30
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Topics

Important Methods

Central Slice Theorem

Summary

Take Home Messages

Further Readings

Fourier Transform

- Fourier transform in 1-D:

$$P(\omega) = \int_{-\infty}^{\infty} p(s) e^{-2\pi i s \omega} \mathrm{d}s$$

- Inverse Fourier transform in 1-D:

$$p(s) = \int_{-\infty}^{\infty} P(\omega) e^{2\pi i s \omega} \mathrm{d}\omega$$

$\longrightarrow$ Fourier pairs

Convolution

- Convolution:

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\,\mathrm{d}\tau = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)\,\mathrm{d}\tau$$

- Convolution theorem:

$$q(s) = f(s) * g(s) \qquad \Leftrightarrow \qquad Q(\omega) = F(\omega) \cdot G(\omega)$$

Hilbert Transform

- The spatial representation of the Hilbert transform:

$$(f * h)(t) = \text{p.v.} \int_{-\infty}^{\infty} f(t-\tau)h(\tau)\,\mathrm{d}\tau,$$

  where "p.v." denotes the principal value, has the transformation kernel

$$h(\tau) = \frac{1}{\pi\tau}.$$

- Its Fourier representation is:

$$H(\omega) = -i\,\mathrm{sgn}(\omega).$$

Topics

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Central Slice Theorem

$$P(\omega, \theta) = F(\omega \cos\theta, \omega \sin\theta)$$

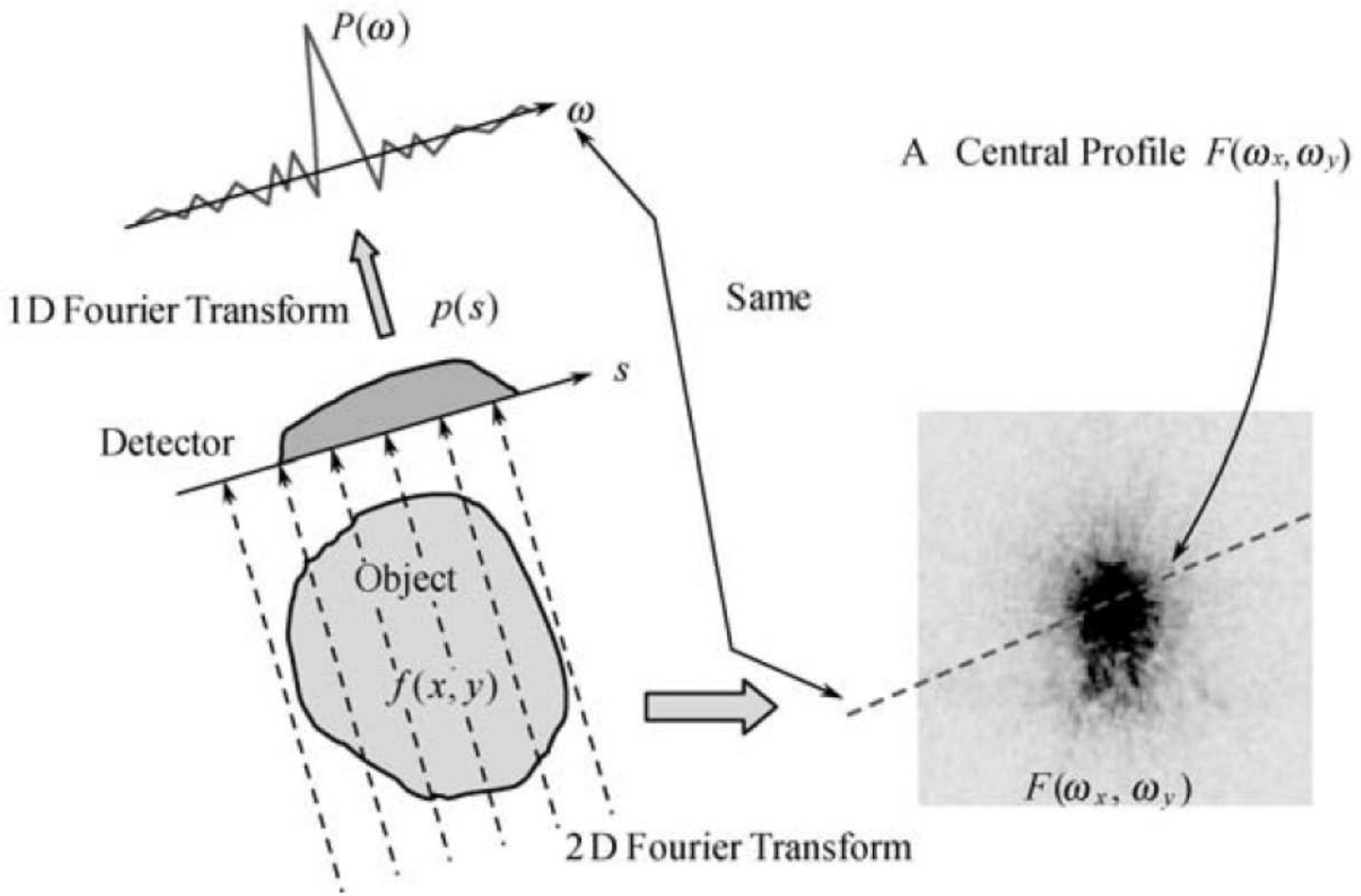

Figure 1: Central slice theorem (Zeng, 2009)

Topics

Important Methods

Central Slice Theorem

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- The Fourier transform, the convolution theorem, and the Hilbert transform are important tools for image reconstruction.
- The central slice theorem (also: Fourier slice theorem) is one of the most fundamental concepts in reconstruction theory!

Further Readings

The derivation of the Fourier slice theorem can also be found here (bibsource):

Joachim Hornegger, Andreas Maier, and Markus Kowarschik. “CT Image Reconstruction Basics”. In: *MR and CT Perfusion and Pharmacokinetic Imaging: Clinical Applications and Theoretical Principles*. Ed. by Roland Bammer. 1st ed. Alphen aan den Rijn, Netherlands: Wolters Kluwer, 2016, pp. 01–09

The concise reconstruction book from ‘Larry ’Zeng:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

If you want to learn more about applications of the Fourier transform:

Ronald N. Bracewell. *The Fourier Transform and Its Applications*. 3rd ed. Electrical Engineering Series. Boston: McGraw-Hill, 2000

Medical Image Processing for Diagnostic Applications

Parallel Beam – Filtered Backprojection

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Topics

Idea for Reconstruction

Filtered Backprojection

Summary

Take Home Messages

Further Readings

Idea for Reconstruction

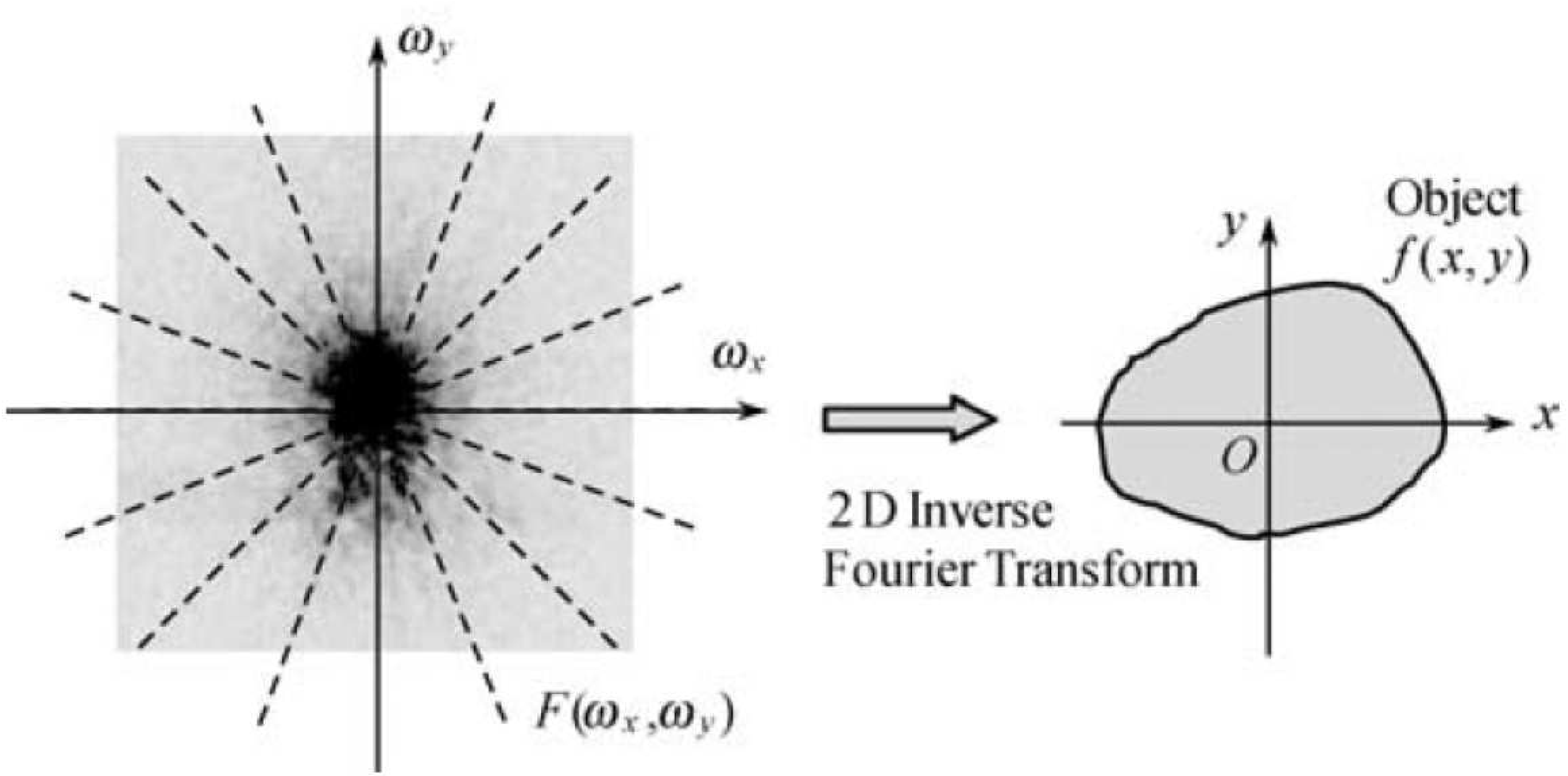

Figure 1: By projections the Fourier space is sampled, by inverse Fourier transform an image of the object can be reconstructed (Zeng, 2009).

Topics

Idea for Reconstruction

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Further Readings

Filtered Backprojection

The inverse Fourier transform of the 2-D Fourier measurement $F(u, v)$:

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{2\pi i(ux+vy)} \mathrm{d}u \mathrm{d}v$$

can be written in polar coordinates:

$$f(x, y) = \int_{0}^{\pi} \int_{-\infty}^{\infty} F_{\text{polar}}(\omega, \theta) |\omega| e^{2\pi i\omega(x\cos\theta + y\sin\theta)} \mathrm{d}\omega \mathrm{d}\theta.$$

According to the Fourier slice theorem $P(\omega, \theta) = F(\omega\cos\theta, \omega\sin\theta) = F_{\text{polar}}(\omega, \theta)$ this yields:

$$f(x, y) = \int_{0}^{\pi} \int_{-\infty}^{\infty} P(\omega, \theta) |\omega| e^{2\pi i\omega(x\cos\theta + y\sin\theta)} \mathrm{d}\omega \mathrm{d}\theta.$$

Filtered Backprojection

The inner integral in the last equation:

$$f(x,y) = \int_0^{\pi} \left(\int_{-\infty}^{\infty} P(\omega,\theta)|\omega| e^{2\pi i\omega(x\cos\theta + y\sin\theta)} \mathrm{d}\omega \right) \mathrm{d}\theta$$

represents the 1-D inverse Fourier transform of the product $P(\omega,\theta)|\omega|$.
According to the convolution theorem this corresponds to a convolution in spatial domain:

$$f(x,y) = \int_0^{\pi} p(s,\theta) * h(s)|_{s=x\cos\theta + y\sin\theta} \mathrm{d}\theta,$$

where $h(s)$ denotes the corresponding inverse Fourier transform of $|\omega|$.

Filtered Backprojection: Practical Algorithm

1. Apply filter on the detector row:

$$q(s,\theta) = p(s,\theta) * h(s).$$

2. Backproject $q(s,\theta)$:

$$f(x,y) = \int_0^{\pi} q(s,\theta)|_{s=x\cos\theta+y\sin\theta}\,\mathrm{d}\theta.$$

Topics

Idea for Reconstruction

Filtered Backprojection

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- The central slice theorem allows a very practical reconstruction algorithm for parallel beam geometry.
- The workflow includes filtering on the detector rows and successive backprojection.

Further Readings

The derivation of the filtered backprojection formula can also be found here (bibsource):

Joachim Hornegger, Andreas Maier, and Markus Kowarschik. "CT Image Reconstruction Basics". In: *MR and CT Perfusion and Pharmacokinetic Imaging: Clinical Applications and Theoretical Principles*. Ed. by Roland Bammer. 1st ed. Alphen aan den Rijn, Netherlands: Wolters Kluwer, 2016, pp. 01–09

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Medical Image Processing for Diagnostic Applications

Parallel Beam – Differentiated Backprojection

Online Course – Unit 32

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Topics

Differentiated Backprojection

Filtering Revisited

Differentiated Backprojection

Variety of Reconstruction Algorithms

Summary

Take Home Messages

Further Readings

Filtering Revisited

- Rewrite $|\omega|$:

$$|\omega| = (2\pi i\omega) \cdot \left(\frac{1}{2\pi} (-i \operatorname{sgn}(\omega)) \right).$$

- Note that multiplication in frequency space with $-i \operatorname{sgn}(\omega)$ is a **Hilbert transform**, i. e., equivalent to a convolution with $h(s) = \frac{1}{\pi s}$.
- Note that the inverse Fourier transform of $2\pi i\omega$ is the derivative operator:

$$\mathrm{FT}^{-1}(2\pi i\omega \cdot G(\omega)) = \frac{\mathrm{d}}{\mathrm{d}s} g(s).$$

Differentiation Hilbert Backprojection Algorithm

1. Compute first derivative of the detector row:

$$q_1(s,\theta) = \frac{\partial p(s,\theta)}{\partial s}.$$

2. Apply Hilbert transform:

$$q_2(s,\theta) = \frac{1}{2\pi^2 s} * q_1(s,\theta).$$

3. Backproject $q_2(s,\theta)$:

$$f(x,y) = \int_0^{\pi} q_2(s,\theta)|_{s=x\cos\theta+y\sin\theta}\,\mathrm{d}\theta.$$

Differentiated Backprojection

Definition of the backprojection:

$$b(x,y) = \int_0^{\pi} \mathbf{H}p(s,\theta)|_{s=x\cos\theta+y\sin\theta}\, \mathrm{d}\theta,$$

where **H** is the Hilbert transform with respect to s.

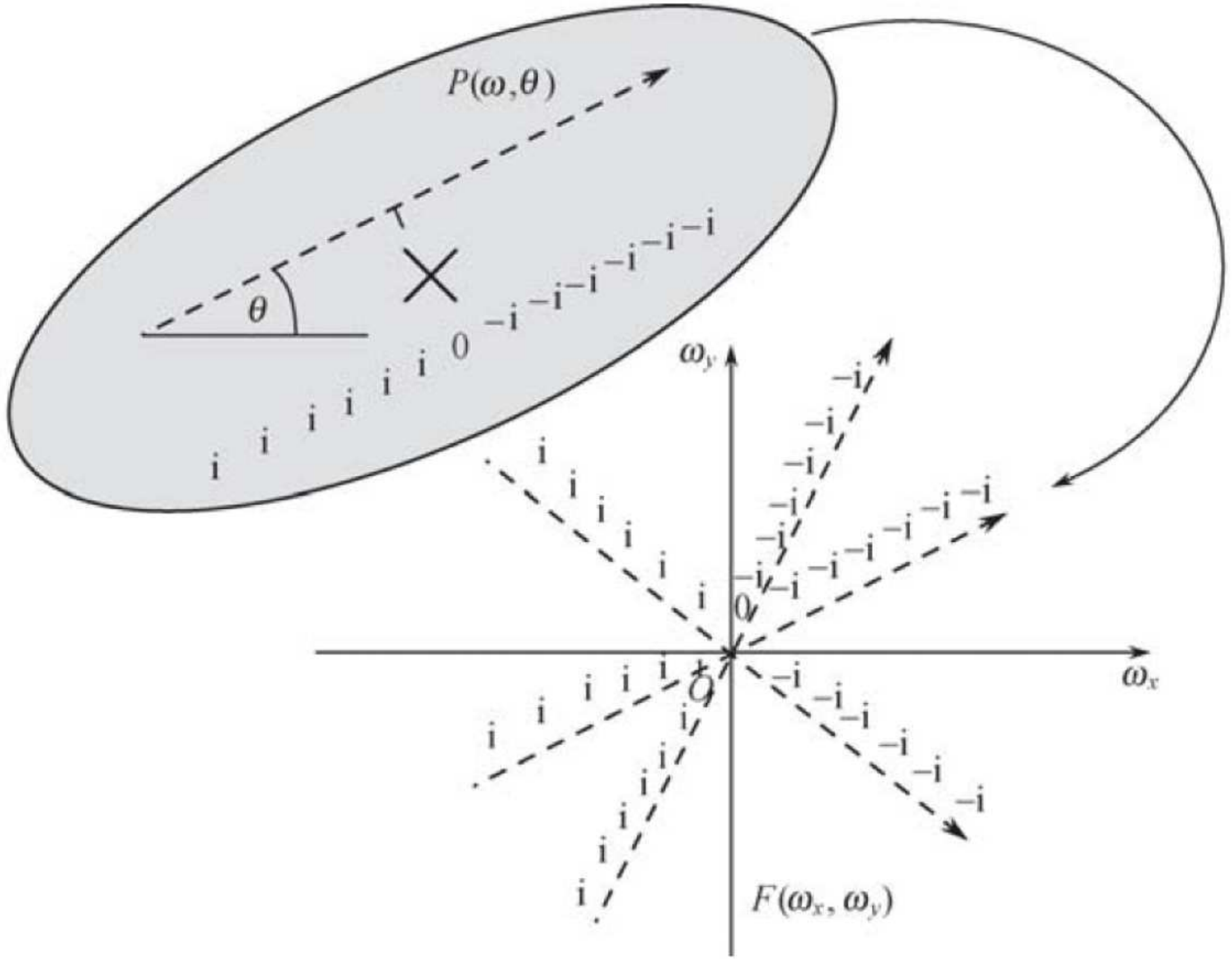

Figure 1: Computation scheme (Zeng, 2009)

Many reconstruction algorithms are possible ...

Step 1	*Step 2*	*Step 3*
1-D Ramp Filter with Fourier Transform	Backprojection	
1-D Ramp Filter with Convolution	Backprojection	
Backprojection	2-D Ramp Filter with Fourier Transform	
Backprojection	2-D Ramp Filter with 2-D Convolution	
Derivative	Hilbert Transform	Backprojection
Derivative	Backprojection	Hilbert Transform
Backprojection	Derivative	Hilbert Transform
Hilbert Transform	Derivative	Backprojection
Hilbert Transform	Backprojection	Derivative
Backprojection	Hilbert Transform	Derivative

Table 1: Valid combinations for analytical parallel-beam reconstruction algorithms (cf. Zeng, 2009)

Topics

Take Home Messages

- Reformulation of the ramp filter showed that the combination of Hilbert transform and the projection derivatives produce another analytical reconstruction algorithm.
- There is a multitude of valid algorithms that can be built using the tools: projection derivatives, Fourier and Hilbert transform, and backprojection.

Further Readings

The concise reconstruction book from ‘Larry ’Zeng:

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Medical Image Processing for Diagnostic Applications

Parallel Beam – Ram-Lak Filter

Online Course – Unit 33

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Topics

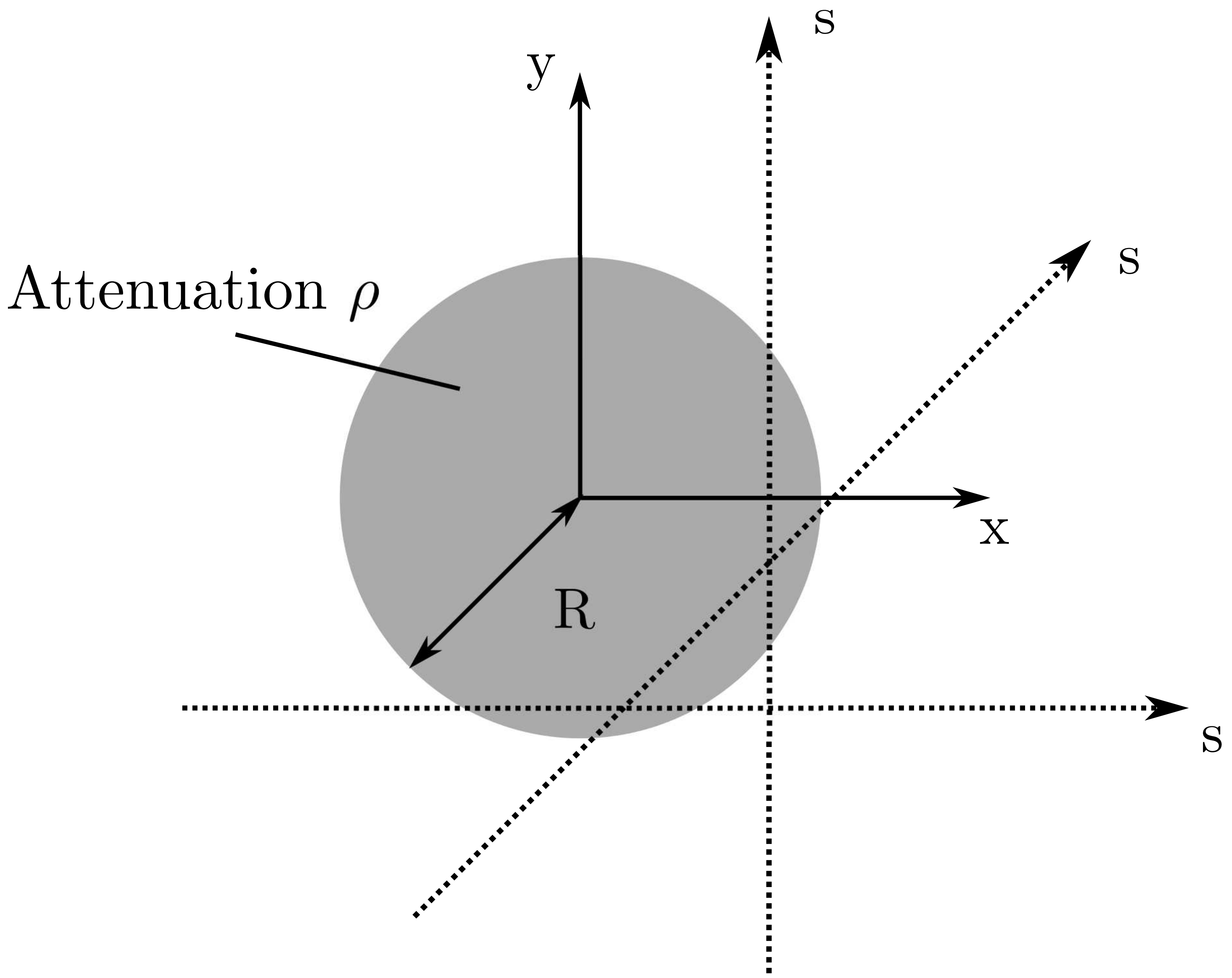
How to Implement a Parallel Beam Algorithm - Part 1

- Example Projection
- Implementation Scheme
- Discrete Spatial Form of the Ramp Filter

Summary

- Take Home Messages
- Further Readings

Example: Homogeneous Cylinder (My First Phantom)



Disc of radius R is in the coordinate center $\rightarrow$ projection is the same in all views:

$$p(s) = \begin{cases} 2\rho\sqrt{R^2 - s^2} & s \leq R, \\ 0 & s > R. \end{cases}$$

(The dotted lines indicate rays from different projection angles.)

Example: Homogeneous Cylinder

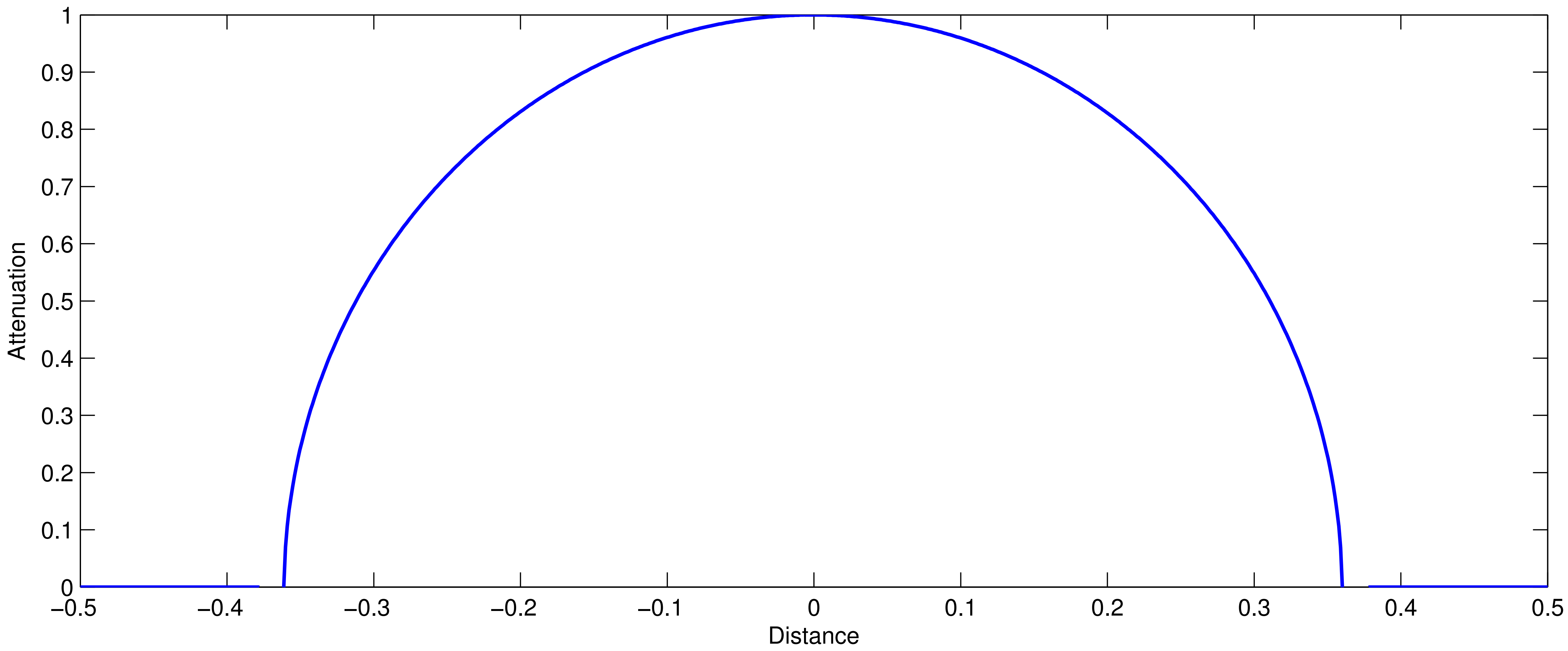


Figure 1: 1-D projection profile of the cylinder object in 2-D

Filtered Backprojection: Implementation Scheme

- Apply filter on the detector row:

$$q(s,\theta) = h(s) * p(s,\theta),$$

$$h(s) = \int_{-\infty}^{\infty} |\omega| e^{2\pi i \omega s} \mathrm{d}\omega.$$

- Backproject $q(s,\theta)$:

$$f(x,y) = \int_{0}^{\pi} q(s,\theta)|_{s=x\cos\theta+y\sin\theta} \mathrm{d}\theta.$$

Discrete Spatial Form of the Ramp Filter

Problem: Find the inverse Fourier transform of $|\omega|$.

Given a detector spacing τ, we know from the Nyquist-Shannon sampling theorem the maximum frequency that can be represented by the DFT:

$$2B = \frac{1}{\tau}.$$

Therefore, we set the cut-off frequency of the ramp filter at $\omega = B$.

So we want to determine

$$h(s) = \int_{-B}^{B} |\omega| e^{2\pi i \omega s} \mathrm{d}\omega = \int_{-\infty}^{\infty} |\omega| \,\mathrm{rect}\left(\frac{\omega}{2B}\right) e^{2\pi i \omega s} \mathrm{d}\omega,$$

where

$$\mathrm{rect}(t) = \begin{cases} 1, & \text{if } |t| < \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$

Discrete Spatial Form of the Ramp Filter

With the rect-function we can also rewrite $|\omega|$:

$$|\omega| = B - \text{rect}\left(\frac{\omega}{B}\right) * \text{rect}\left(\frac{\omega}{B}\right).$$

The convolution of both rect-functions yields a triangular shaped function with support on $[-B, B]$ and its maximum B at zero.

We now have:

$$\begin{aligned} h(s) &= \text{FT}^{-1}\left(\left(B - \text{rect}\left(\frac{\omega}{B}\right) * \text{rect}\left(\frac{\omega}{B}\right)\right) \text{rect}\left(\frac{\omega}{2B}\right)\right) \\ &= \text{FT}^{-1}\left(B\,\text{rect}\left(\frac{\omega}{2B}\right)\right) - \text{FT}^{-1}\left(\underbrace{\left(\text{rect}\left(\frac{\omega}{B}\right) * \text{rect}\left(\frac{\omega}{B}\right)\right)}_{\text{support on } [-B,B]} \underbrace{\text{rect}\left(\frac{\omega}{2B}\right)}_{=1 \text{ on } [-B,B]}\right) \\ &= \text{FT}^{-1}\left(B\,\text{rect}\left(\frac{\omega}{2B}\right)\right) - \text{FT}^{-1}\left(\text{rect}\left(\frac{\omega}{B}\right)\right) \cdot \text{FT}^{-1}\left(\text{rect}\left(\frac{\omega}{B}\right)\right). \end{aligned}$$

Discrete Spatial Form of the Ramp Filter

The Fourier transform of the rect-function is a sinc-function, and using the appropriate scaling properties of the Fourier transform, we get:

$$\begin{aligned} h(s) &= 2B^2 \,\mathrm{sinc}\left(2Bs\right) - B^2 \mathrm{sinc}^2\left(Bs\right) \\ &= \frac{1}{2\tau^2} \frac{\sin\left(\frac{\pi s}{\tau}\right)}{\frac{\pi s}{\tau}} - \frac{1}{4\tau^2} \left(\frac{\sin\left(\frac{\pi s}{2\tau}\right)}{\frac{\pi s}{2\tau}} \right)^2 . \end{aligned}$$

The detector is sampled by $s = n\tau$, $n \in \mathbb{Z}$, hence we find the discrete filter in the spatial domain:

$$h(n\tau) = \begin{cases} \frac{1}{4\tau^2} & n = 0, \\ 0 & n \text{ even}, \\ -\frac{1}{\pi^2 (n\tau)^2} & n \text{ odd}, \end{cases}$$

also known as the **“Ramachandran-Lakshminarayanan” convolver** or shortly the **“Ram-Lak” filter**.

Topics

How to Implement a Parallel Beam Algorithm - Part 1

- Example Projection
- Implementation Scheme
- Discrete Spatial Form of the Ramp Filter

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- In this unit we derived the discrete spatial filter version of the ramp filter. It is also called the Ram-Lak filter.
- By design, the Ram-Lak filter “fits” optimally on the detector grid which enhances the accuracy of the reconstruction algorithm (see next unit).

Further Readings

The original Ram-Lak article is:

G. N. Ramachandran and A. V. Lakshminarayanan. “Three-dimensional Reconstruction from Radiographs and Electron Micrographs: Application of Convolutions instead of Fourier Transforms”. In: *Proceedings of the National Academy of Sciences of the United States of America* 68.9 (Sept. 1971), pp. 2236–2240

The derivation shown in this unit is based on a document by Martin Berger.

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Medical Image Processing for Diagnostic Applications

Parallel Beam – Reconstruction Steps

Online Course – Unit 34

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Topics

How to Implement a Parallel Beam Algorithm - Part 2

- Discrete Spatial vs. Continuous Frequency Version
- Practical Algorithm
- Backprojection Example

Summary

- Take Home Messages
- Further Readings

Ram-Lak: Discrete Spatial Form of the Ramp Filter

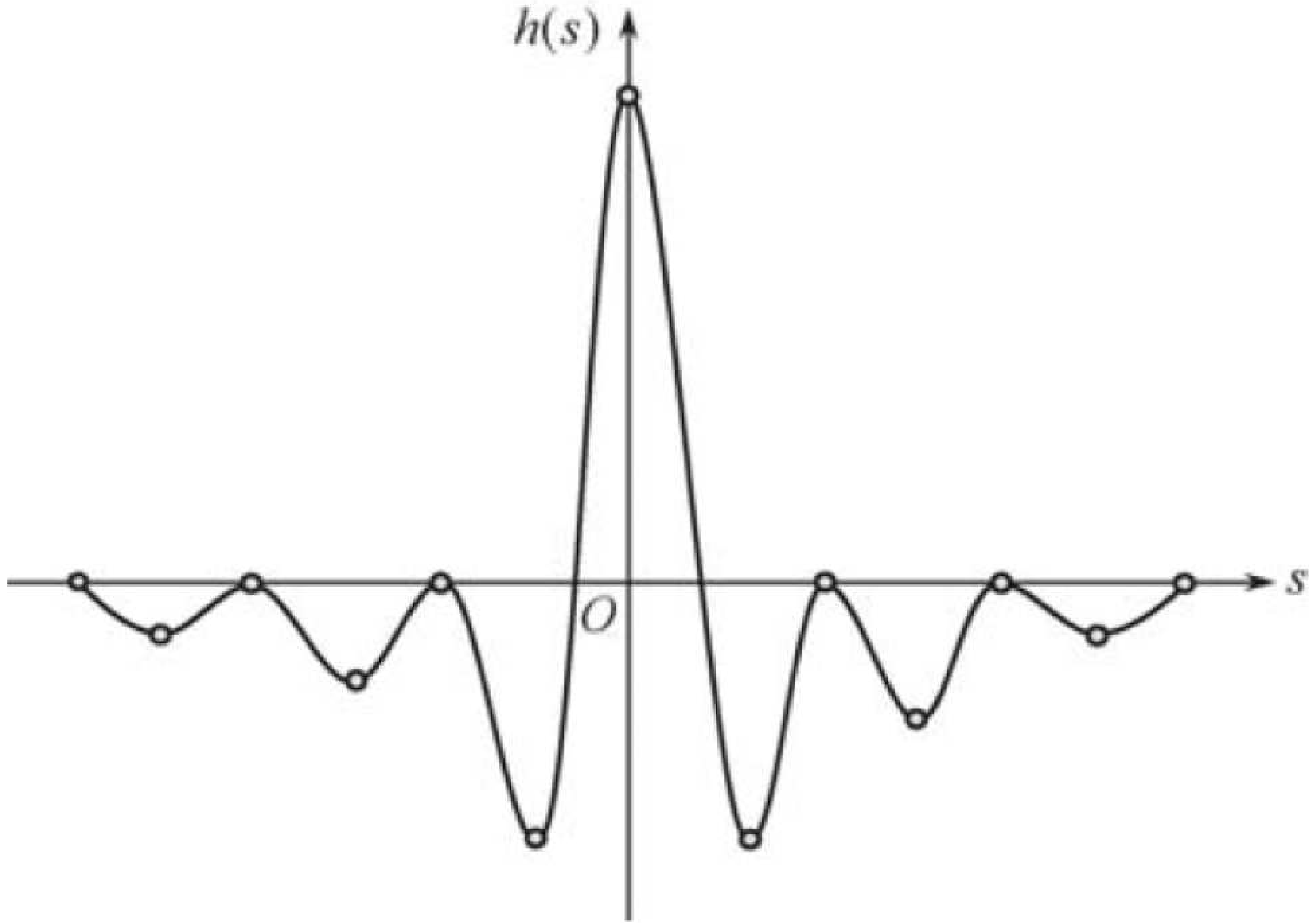

Figure 1: Continous and discrete graph of the Ram-Lak filter (Zeng, 2009)

Discrete Spatial vs. Continuous Frequency Version

- Continuous frequency representation of the ramp filter:

$$H(\omega) = |\omega|$$

- Discrete spatial form:

$$h(n\tau) = \begin{cases} \frac{1}{4\tau^2} & n = 0, \\ 0 & n \text{ even}, \\ -\frac{1}{\pi^2 (n\tau)^2} & n \text{ odd} \end{cases}$$

Discrete Spatial vs. Continuous Frequency Version

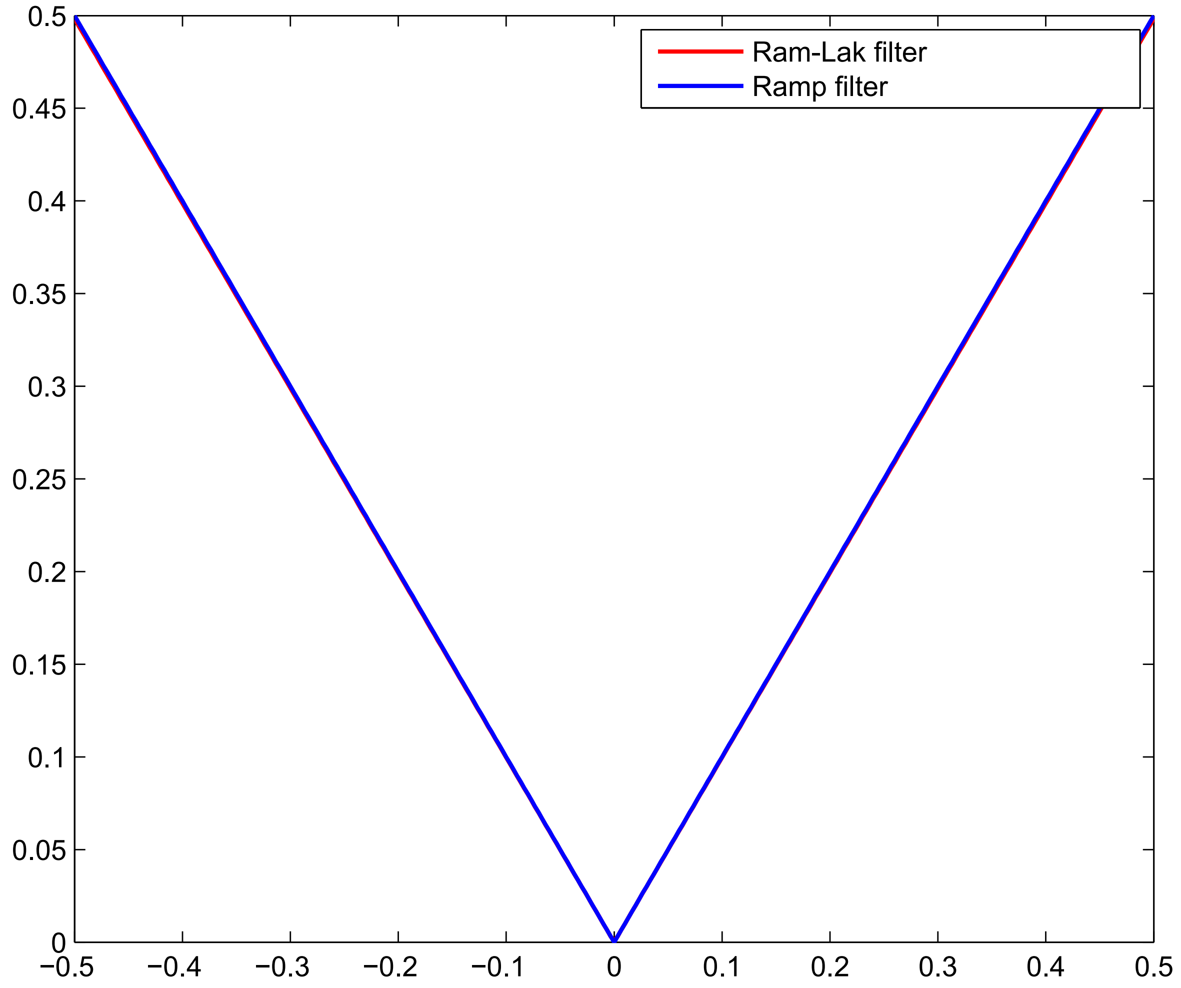

Figure 2: Plot of the ramp filter, whole scale

Discrete Spatial vs. Continuous Frequency Version

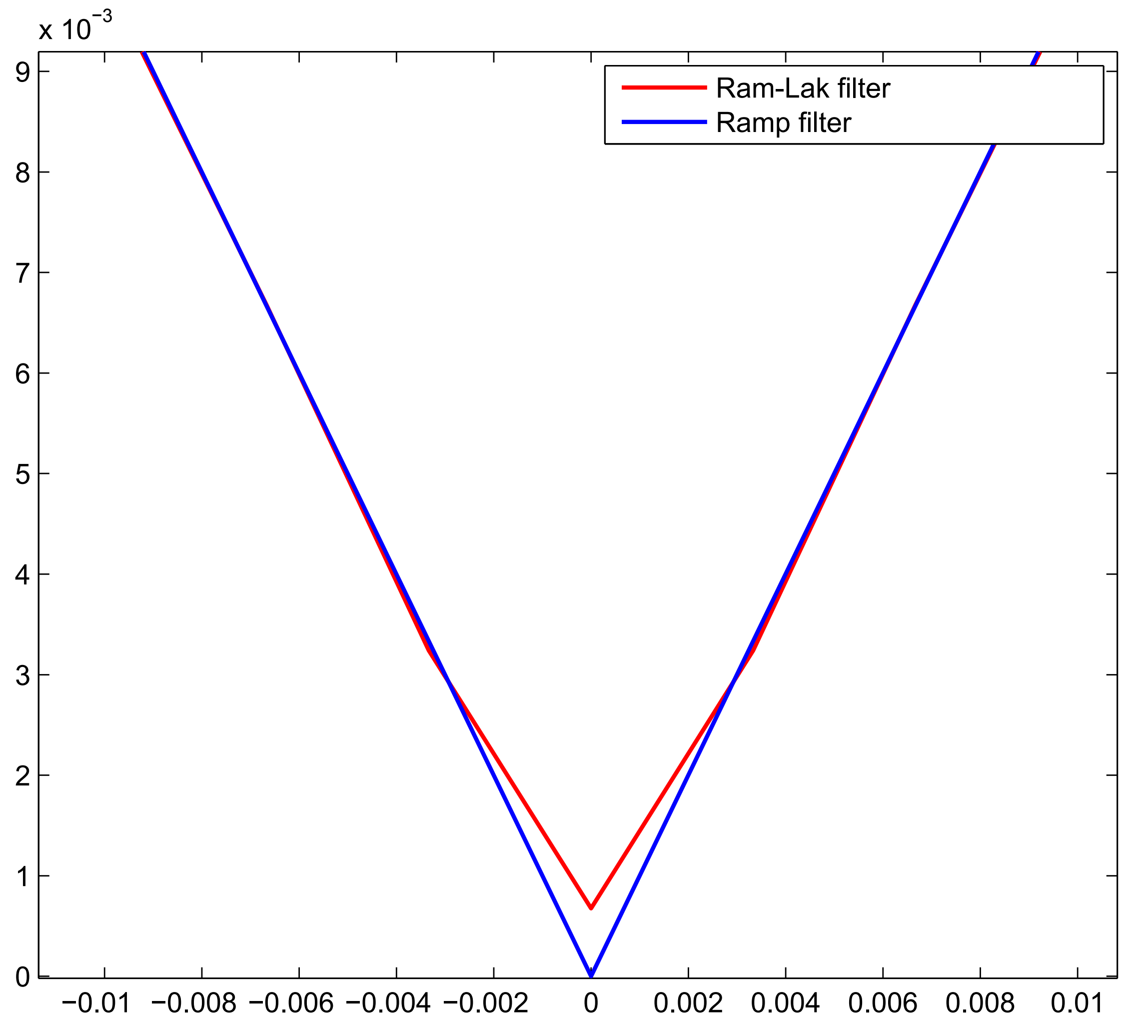

Figure 3: Comparison of the ramp and the Ram-Lak filter, zoomed in at zero

Example: Homogeneous Cylinder after Filter

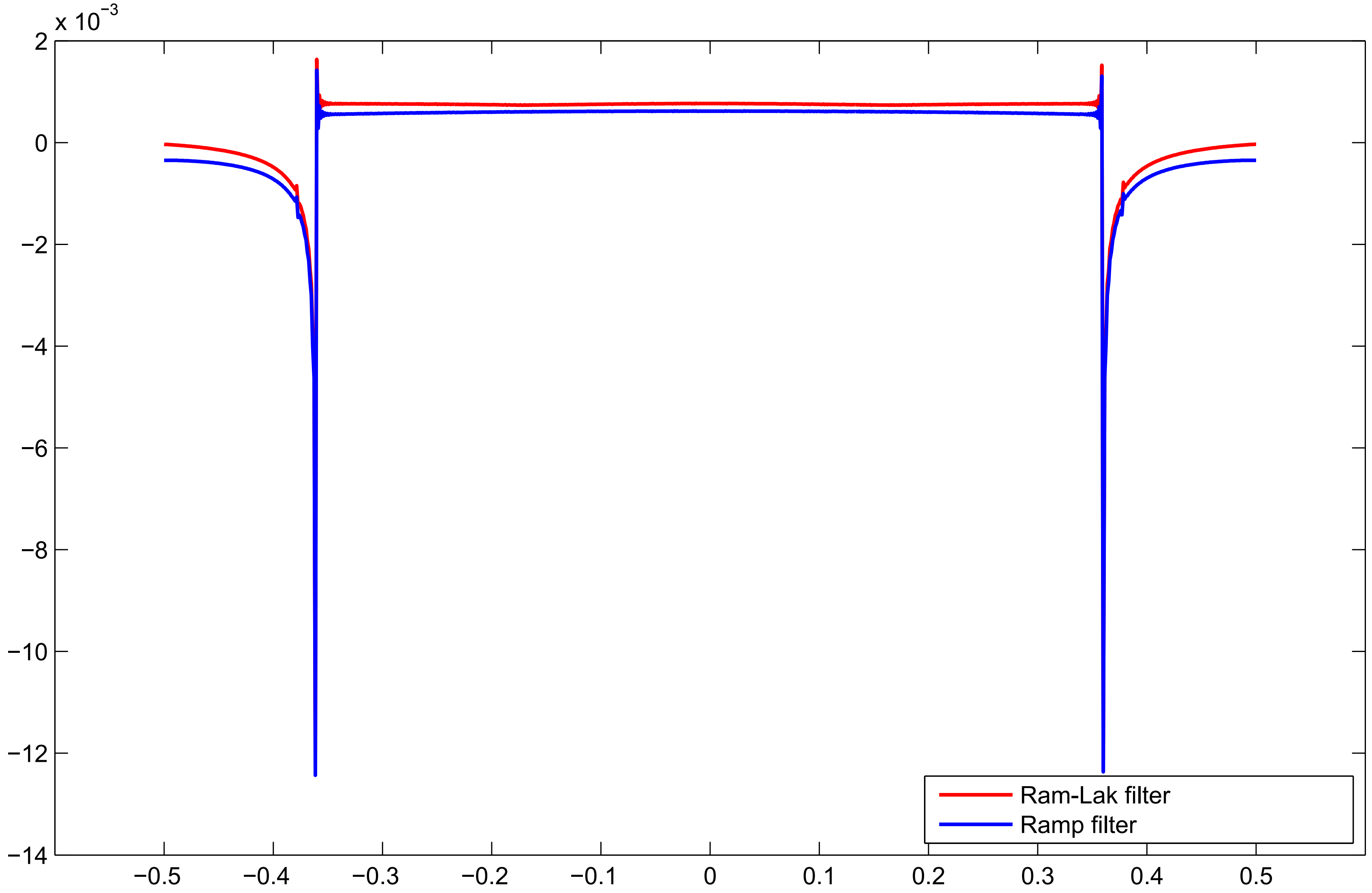

Figure 4: Filtered projection profile of the cylinder phantom

Practical Algorithm - Filtering

1. Precompute filter h(s) in spatial domain $O(N)$
2. Transform filter to frequency domain $H(\omega)$ via FFT $O(N \log N)$
3. For each of $\#P$ projections:
   - Compute FFT of $p(s,\theta)$ $O(N \log N)$
   - Apply filter $P(\omega,\theta) \cdot H(\omega)$ $O(N)$
   - Compute filtered projection $q(s)$ via iFFT $O(N \log N)$

Total complexity: $O(N + N \log N + \#P(N + 2N \log N)) = O(\#P\ N \log N)$

Practical Algorithm - Backprojection

1. Initialize $f(x,y) = 0$ $\quad O(N^2)$
2. For each of $N \times N$ pixels:

   For each of $\#P$ projections:

   - Compute $s = x\cos\theta + y\sin\theta$ $\quad O(1)$
   - Update $f(x,y) += q(s,\theta)$ $\quad O(1)$

Total complexity: $\quad O(N^2 + N^2\#P(1+1)) = O(N^2\#P)$

Practical Algorithm: Overall Complexity

- Apply filter on the detector row:

$$O(\#P\ N \log N)$$

- Backproject:

$$O(\#P\ N^2)$$

Backprojection Example

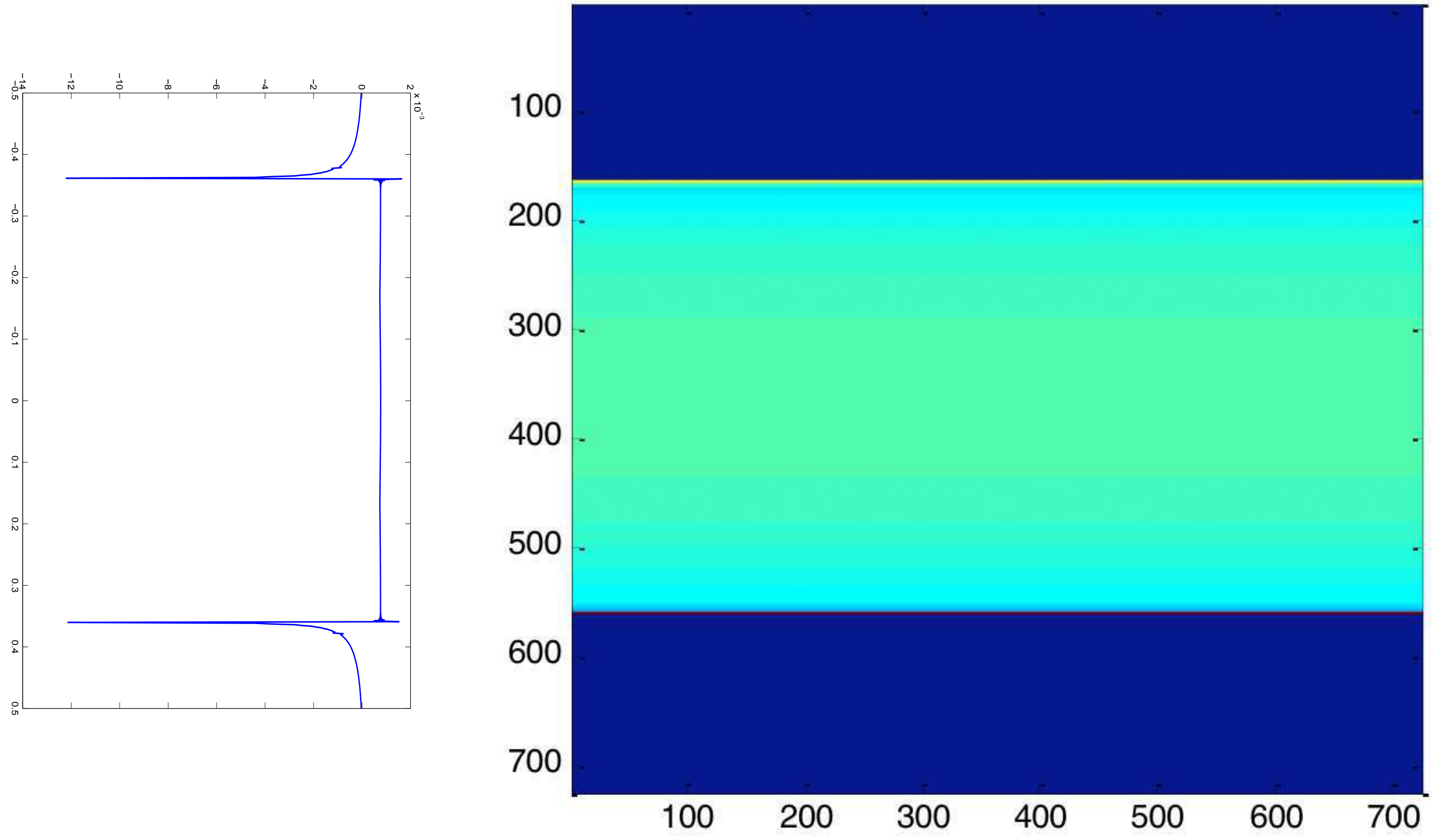

Figure 5: Backprojection of a single projection

Backprojection and Fourier Slice Theorem

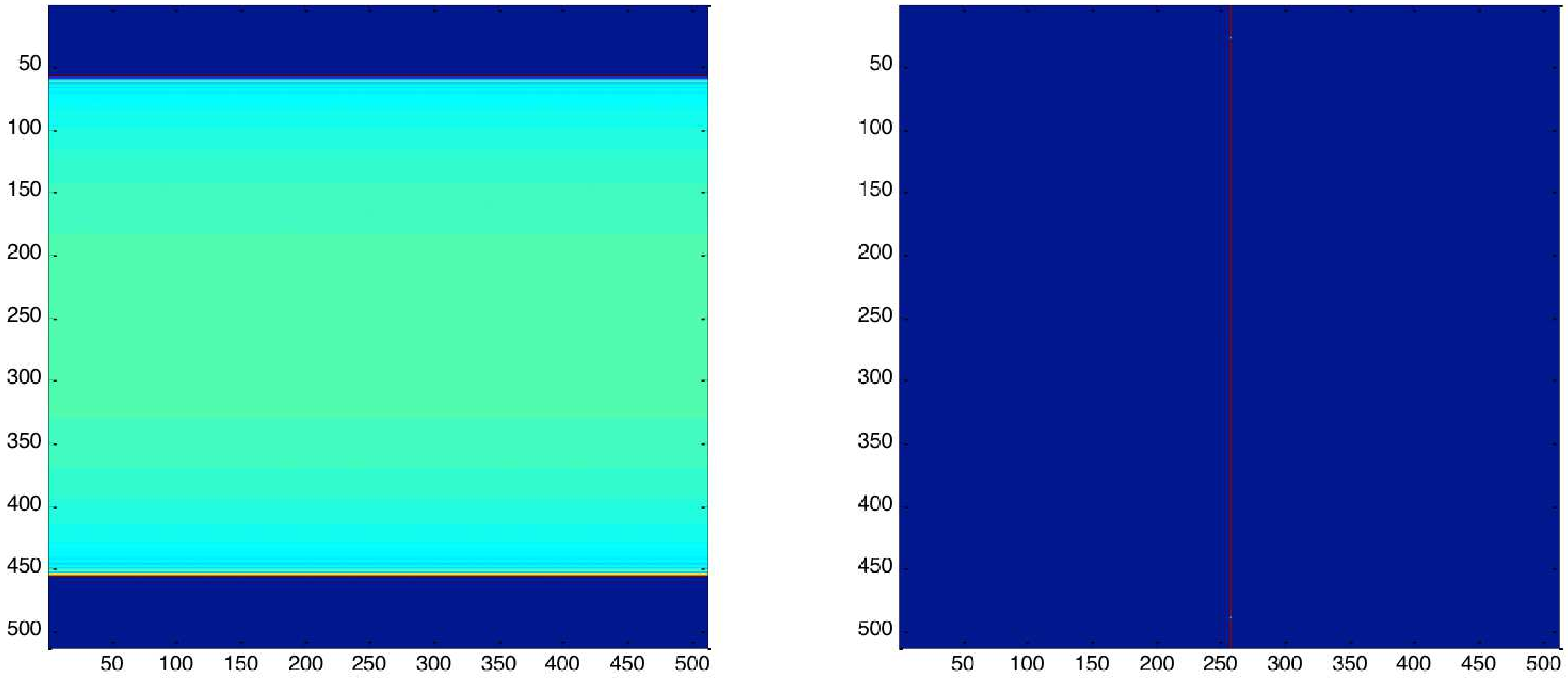

Figure 6: Backprojection (left) of a single line in the Fourier space (right)

Backprojection Example

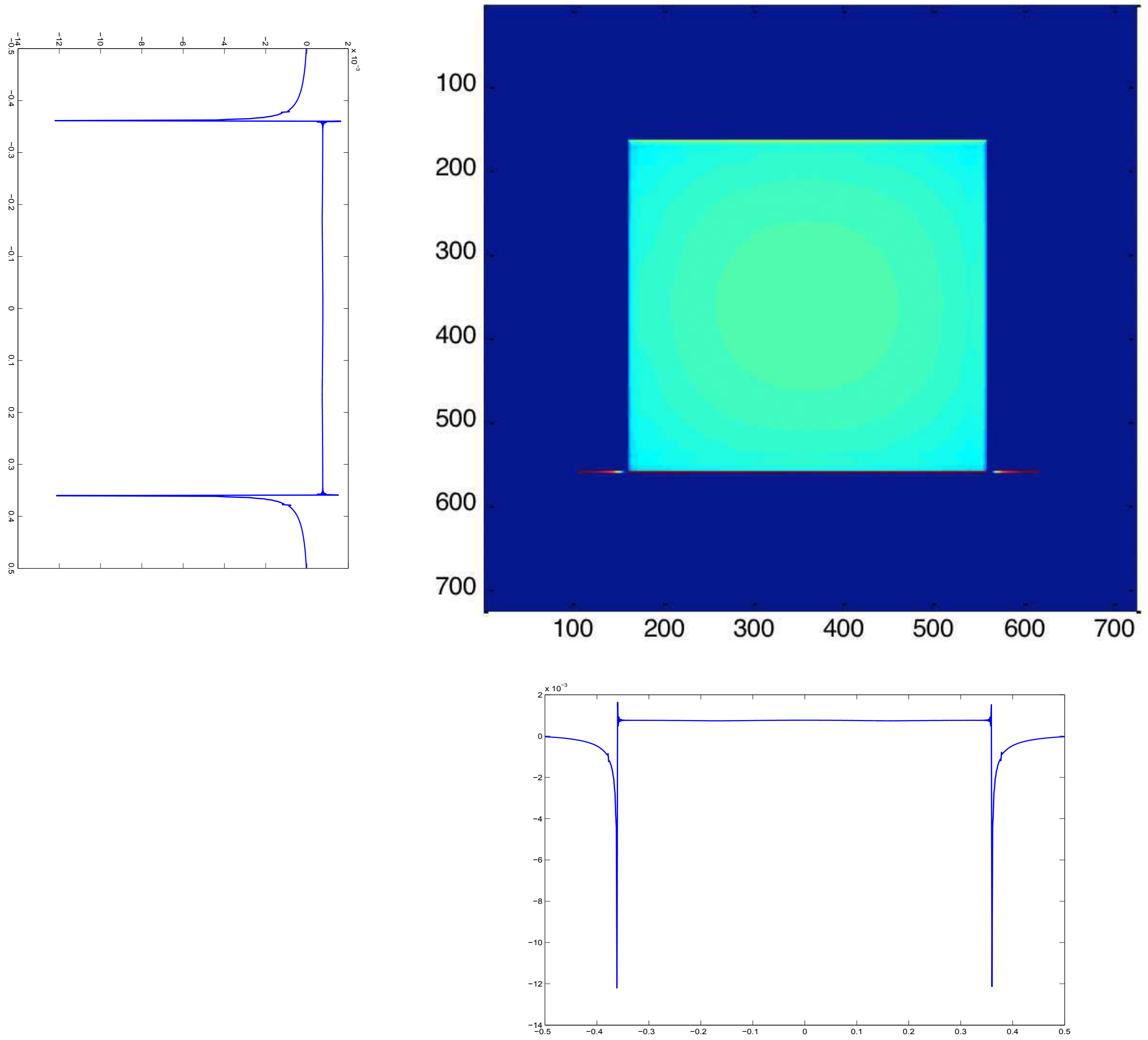

Figure 7: Backprojection of two projections (0°, 90°)

Backprojection Example

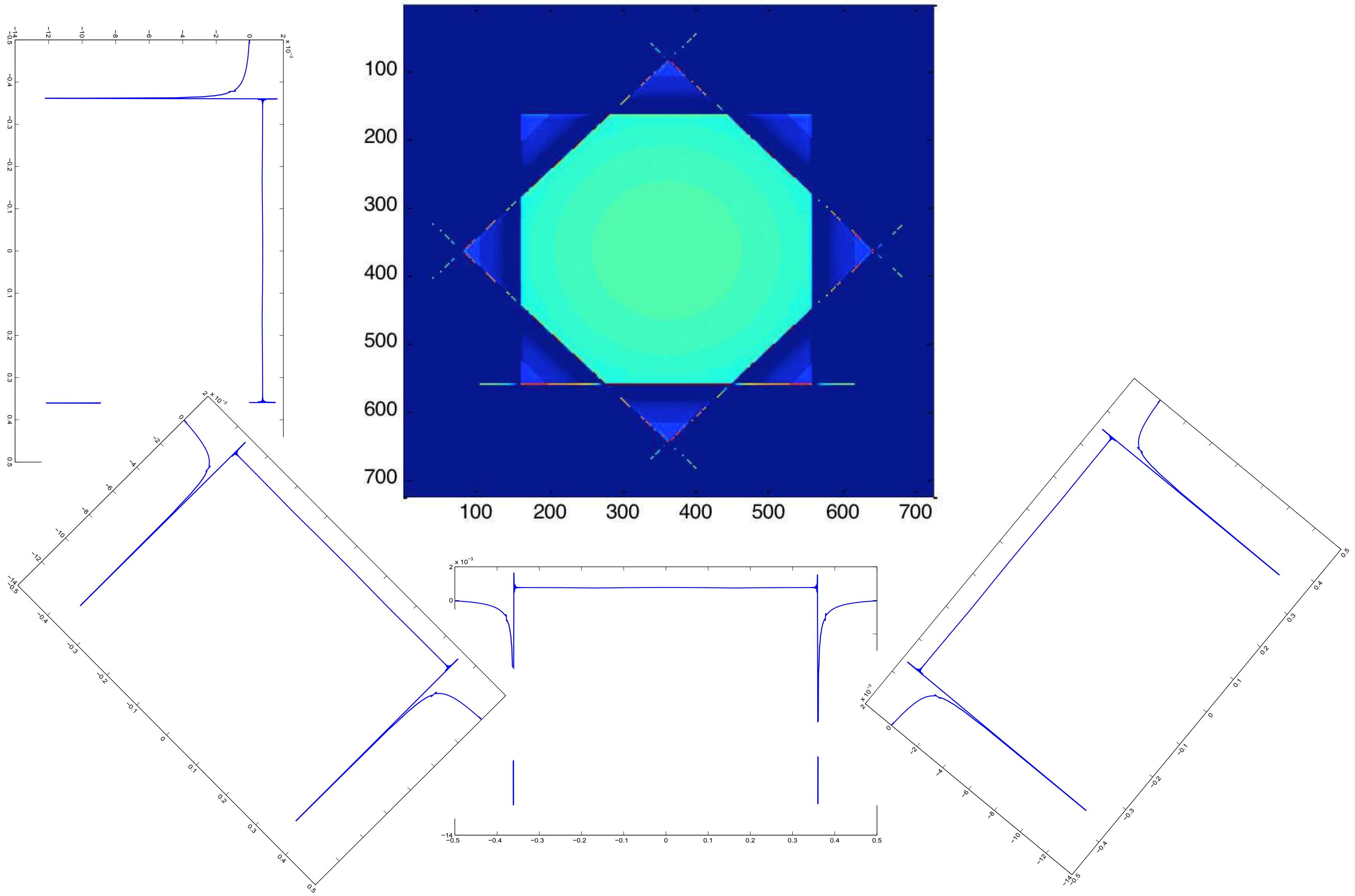

Figure 8: Backprojection of four projections (0°, 45°, 90°, 135°)

Backprojection Example

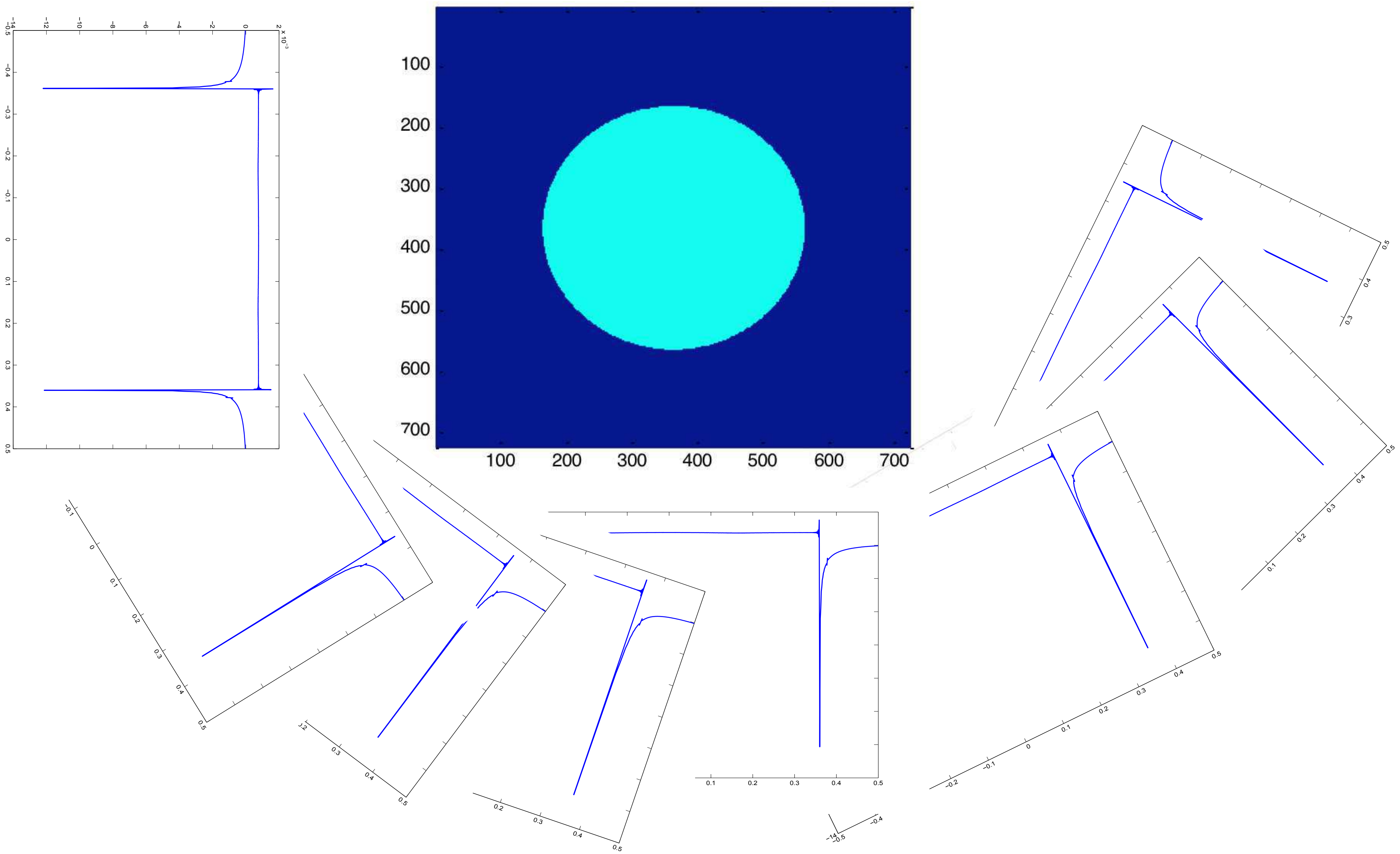

Figure 9: Backprojection of multiple projections (0°-180°)

Topics

How to Implement a Parallel Beam Algorithm - Part 2

- Discrete Spatial vs. Continuous Frequency Version
- Practical Algorithm
- Backprojection Example

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- Although the original ramp filter converges to zero at zero frequency, the Ram-Lak filter takes low frequencies into account. This enables discrete computations to be more accurate.
- The filtering has a complexity of $O(N \log N)$, and the backprojection a complexity of $O(N^2)$ per projection.
- Increasing the number of projections improves the reconstruction result.

Further Readings

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Medical Image Processing for Diagnostic Applications

Parallel Beam – On Noise, Filtering and Window Functions

Online Course – Unit 35

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Topics

Effect of Noise on Filtering

Window Functions
- General Idea
- Common Examples
- The According Filters
- Filter Results

Summary
- Take Home Messages
- Further Readings

Additive Noise (+2%)

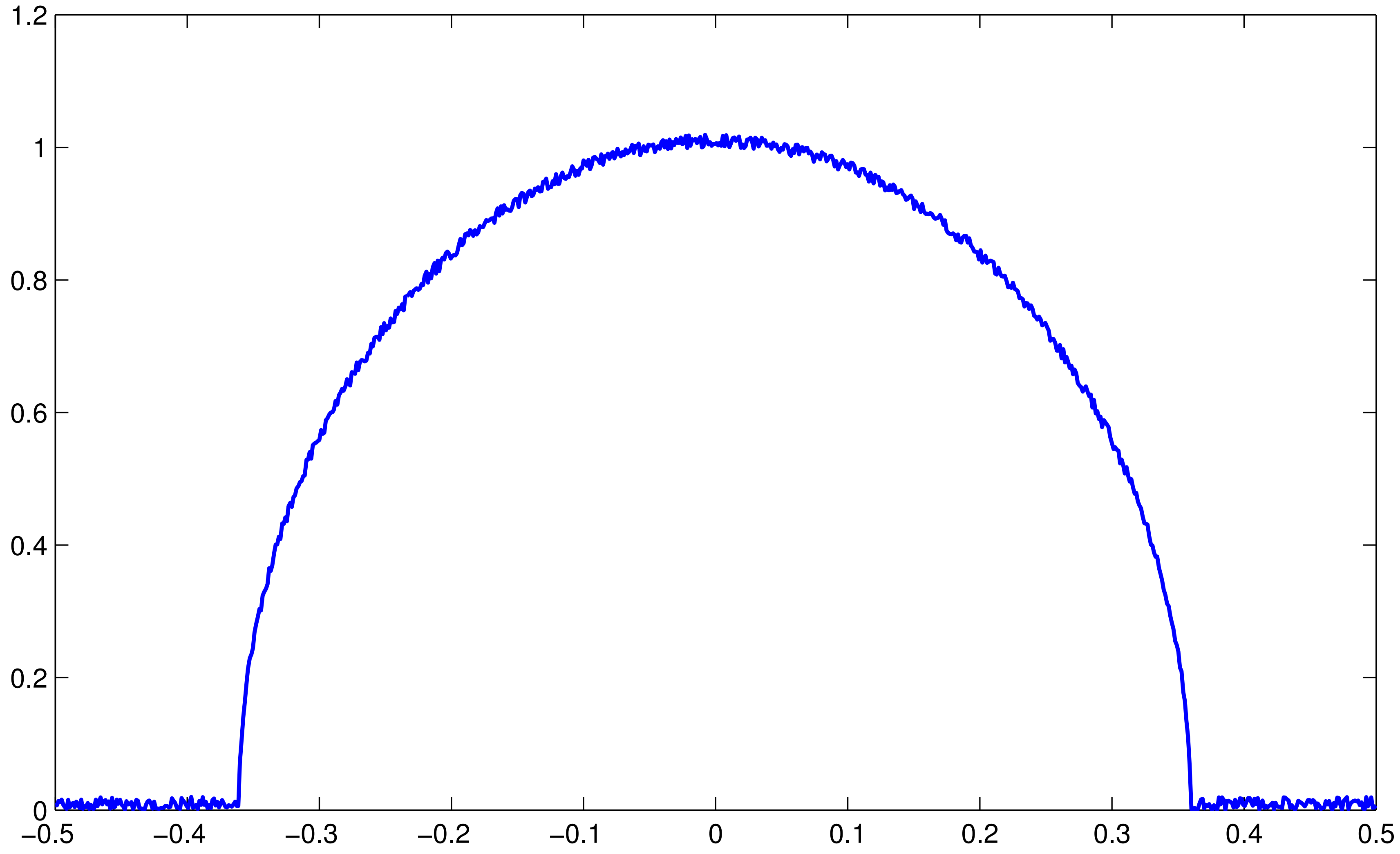


Figure 1: Projection of the cylinder phantom with 2% noise added

Additive Noise (+2%)

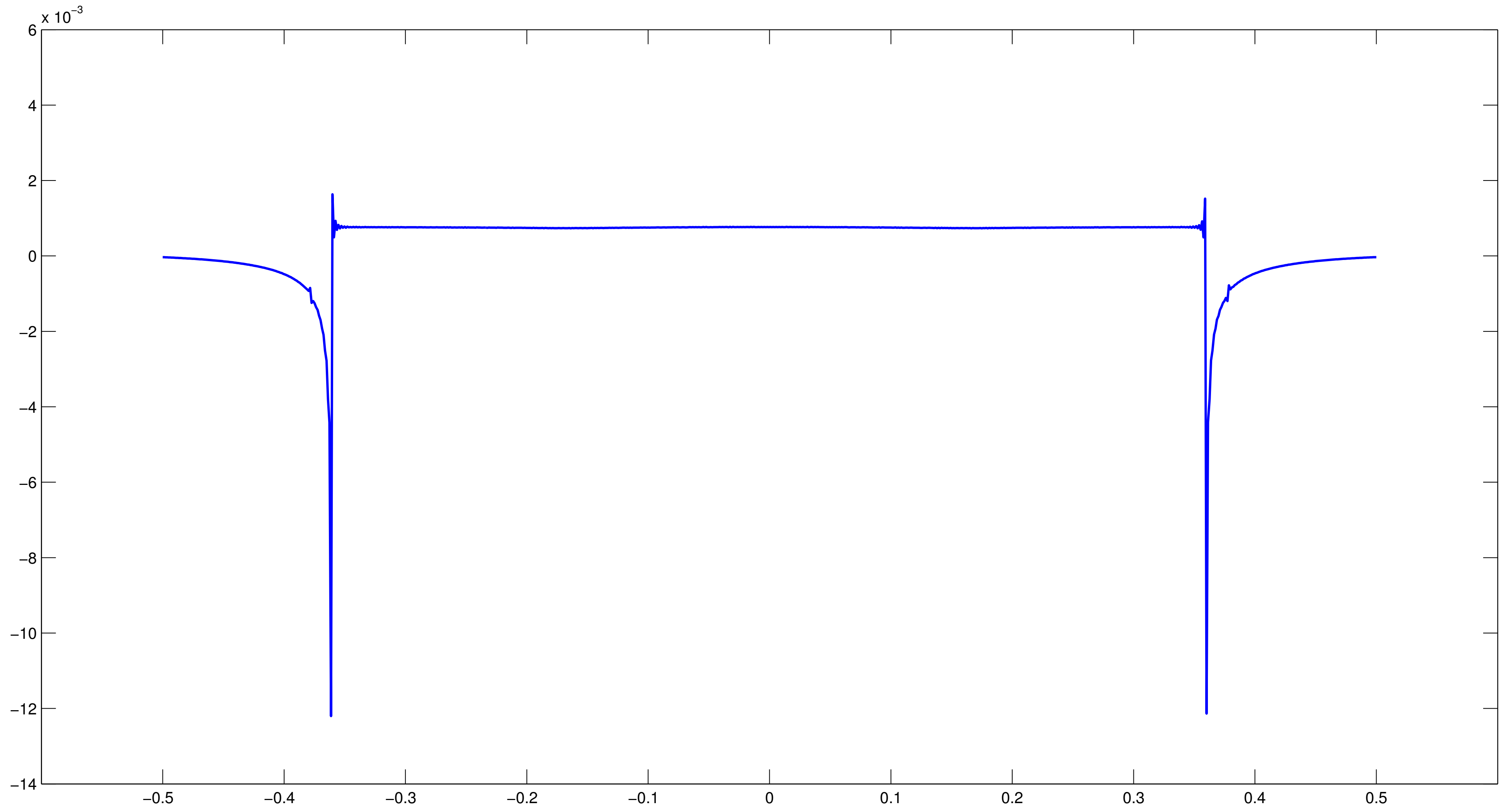


Figure 2: Filtered result of the noiseless projection

Additive Noise (+2%): After Filtering

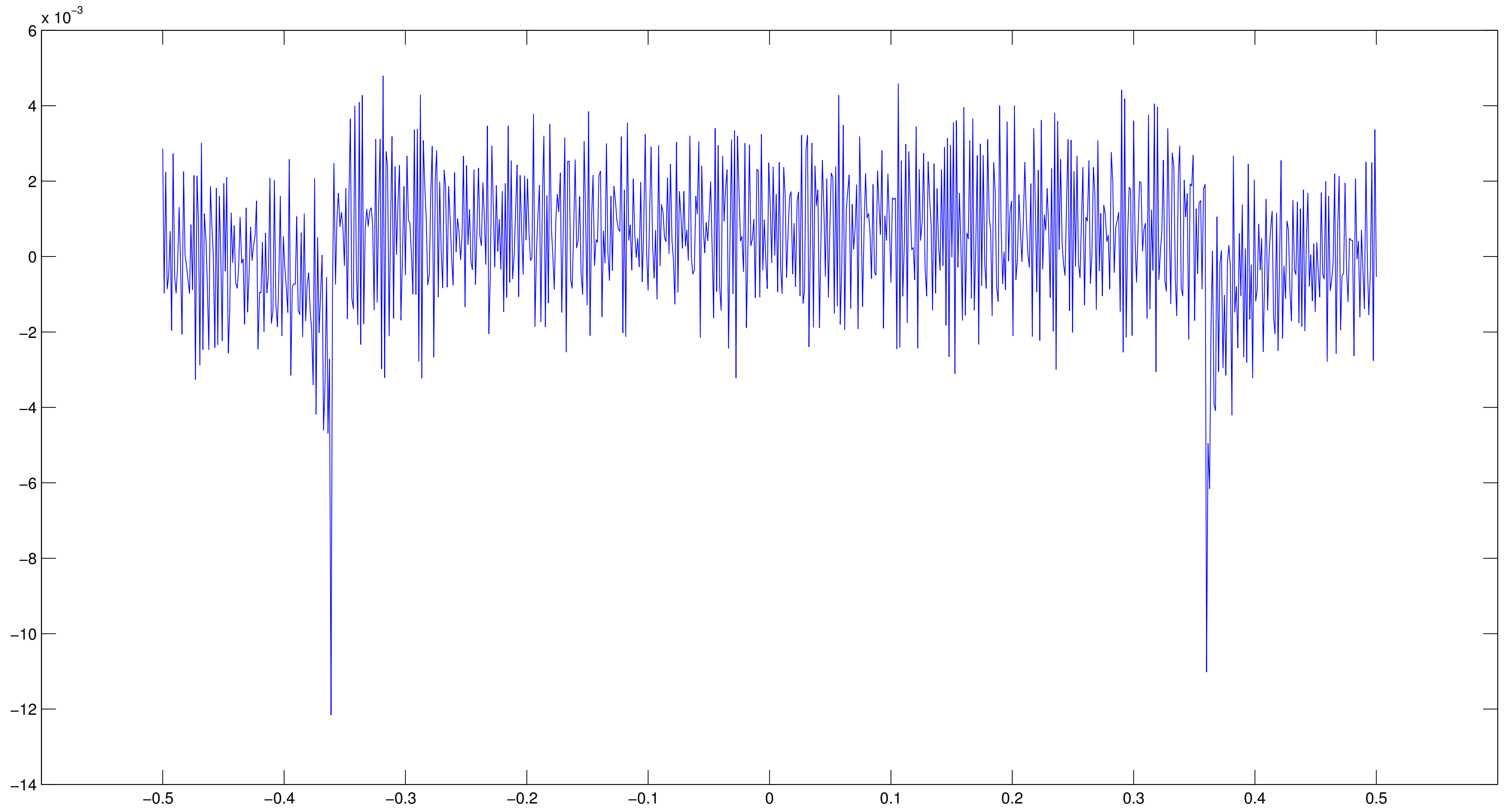


Figure 3: Filtered result of the noisy projection

Noise ...

- ... is amplified when filtering with the ramp filter.
- ... has to be taken care of in an appropriate manner.
- ... is indirectly proportional to the applied dose.
- ... affects different reconstruction methods differently.

Topics

Window Functions

Window functions are used to improve signals as high frequencies are reduced or even eliminated:

$\rightarrow$ Noise reduction

$\rightarrow$ Reduces high frequencies caused by cutting

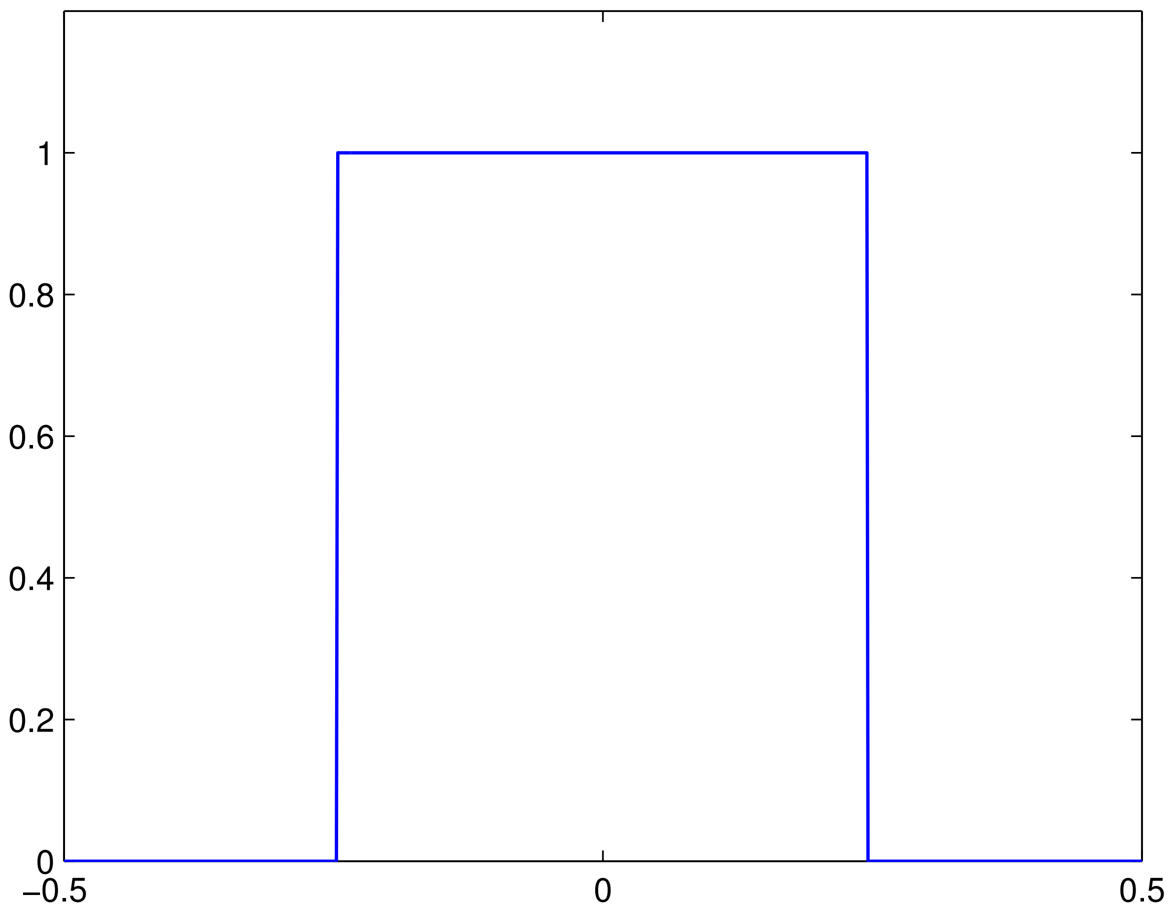
Many window functions are known:

- Cosine window
- Shepp-Logan window
- ...

Window Functions: Filter Adaptation

1. Apply the window function W in frequency domain:
$$P'(\omega,\theta) = W(\omega)\cdot P(\omega,\theta).$$
2. Then apply the filter H:
$$Q'(\omega,\theta) = H(\omega)\cdot P'(\omega,\theta) = H(\omega)\cdot W(\omega)\cdot P(\omega,\theta).$$
3. Rewrite the filtering equation to an adjusted filter H':
$$Q'(\omega,\theta) = H'(\omega)\cdot P(\omega,\theta)$$
$$\Rightarrow \qquad H'(\omega) = H(\omega)\cdot W(\omega).$$

Rectangular Window (Frequency Cut-off)



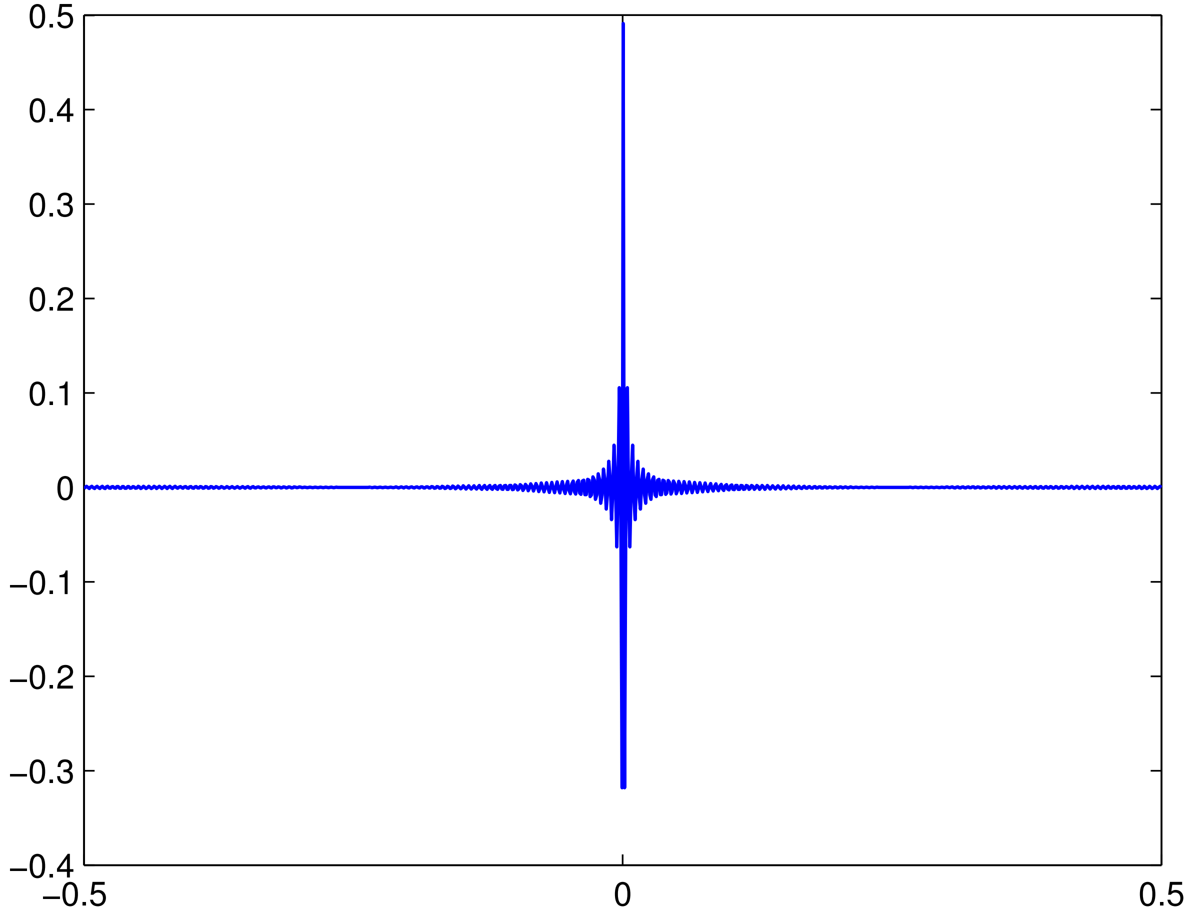


Figure 4: Rectangular window function in frequency domain (left) and its counterpart in spatial domain (right)

Cosine Window: $\cos(\pi \cdot x)$

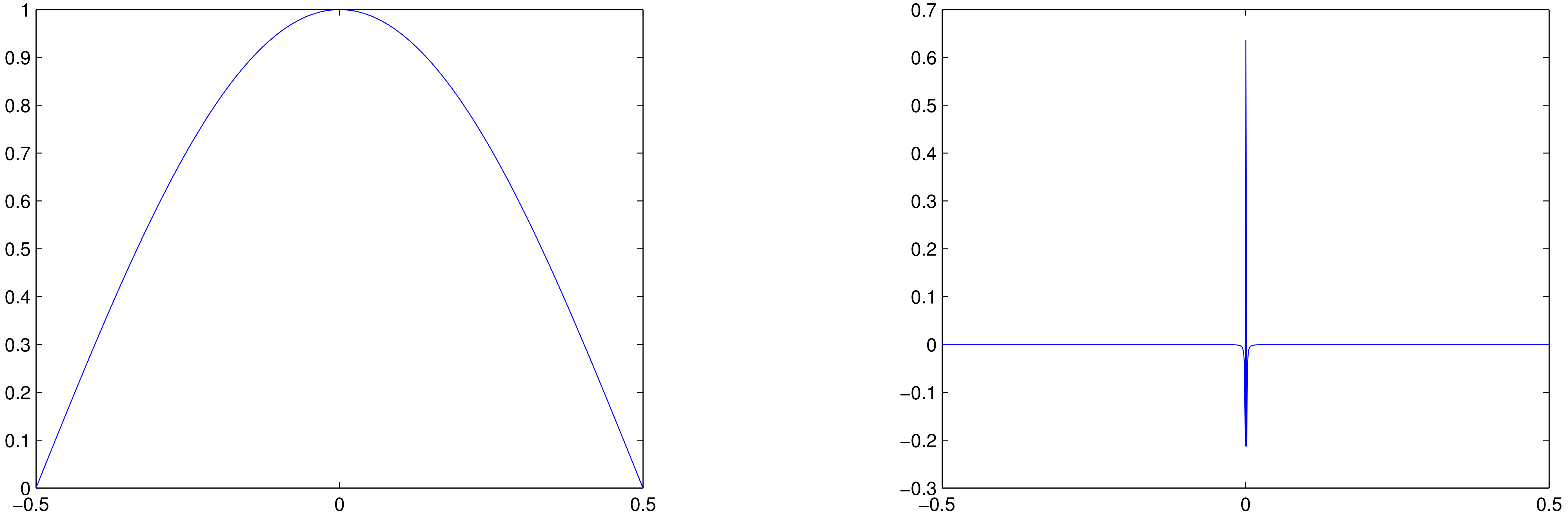
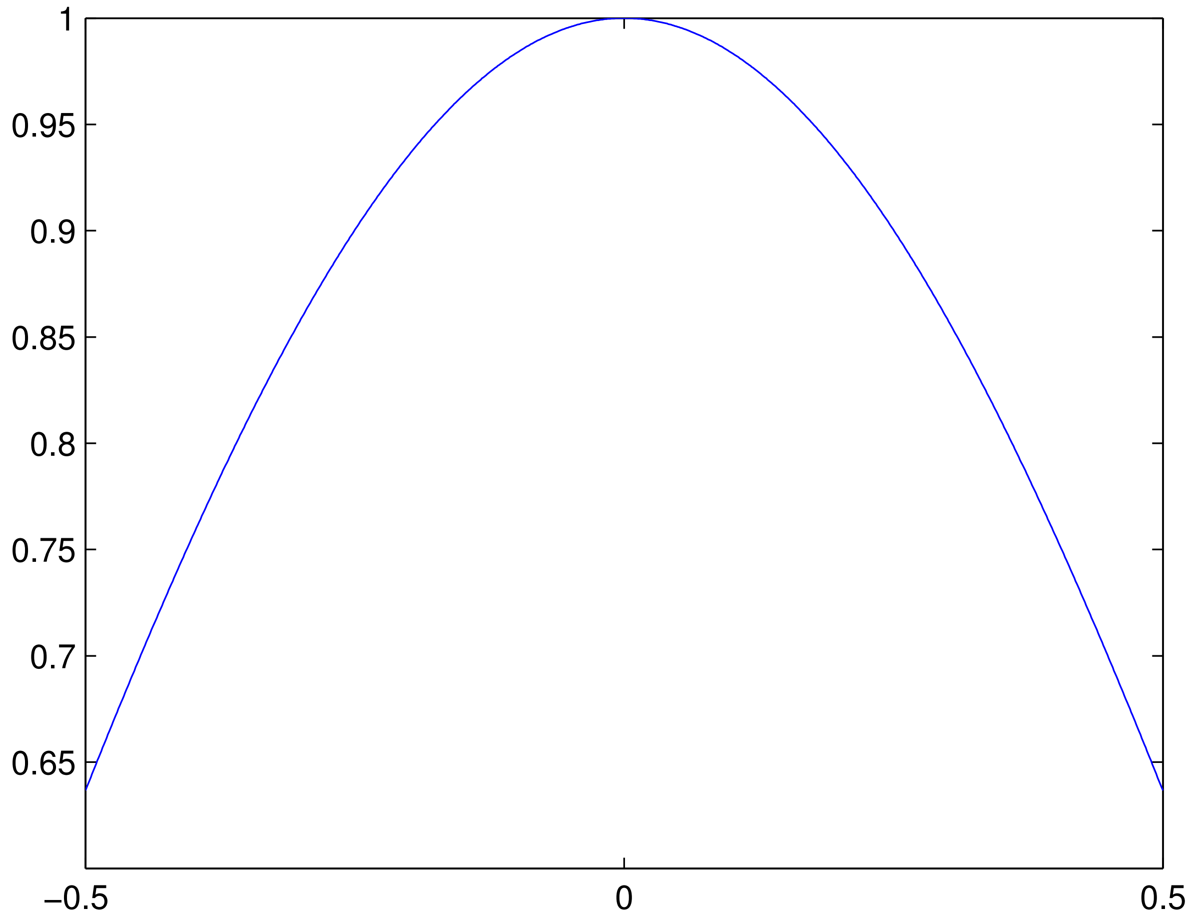


Figure 5: Cosine window function in frequency domain (left) and its counterpart in spatial domain (right)

Shepp-Logan Window: $\frac{\sin(\pi \cdot x)}{(\pi \cdot x)}$



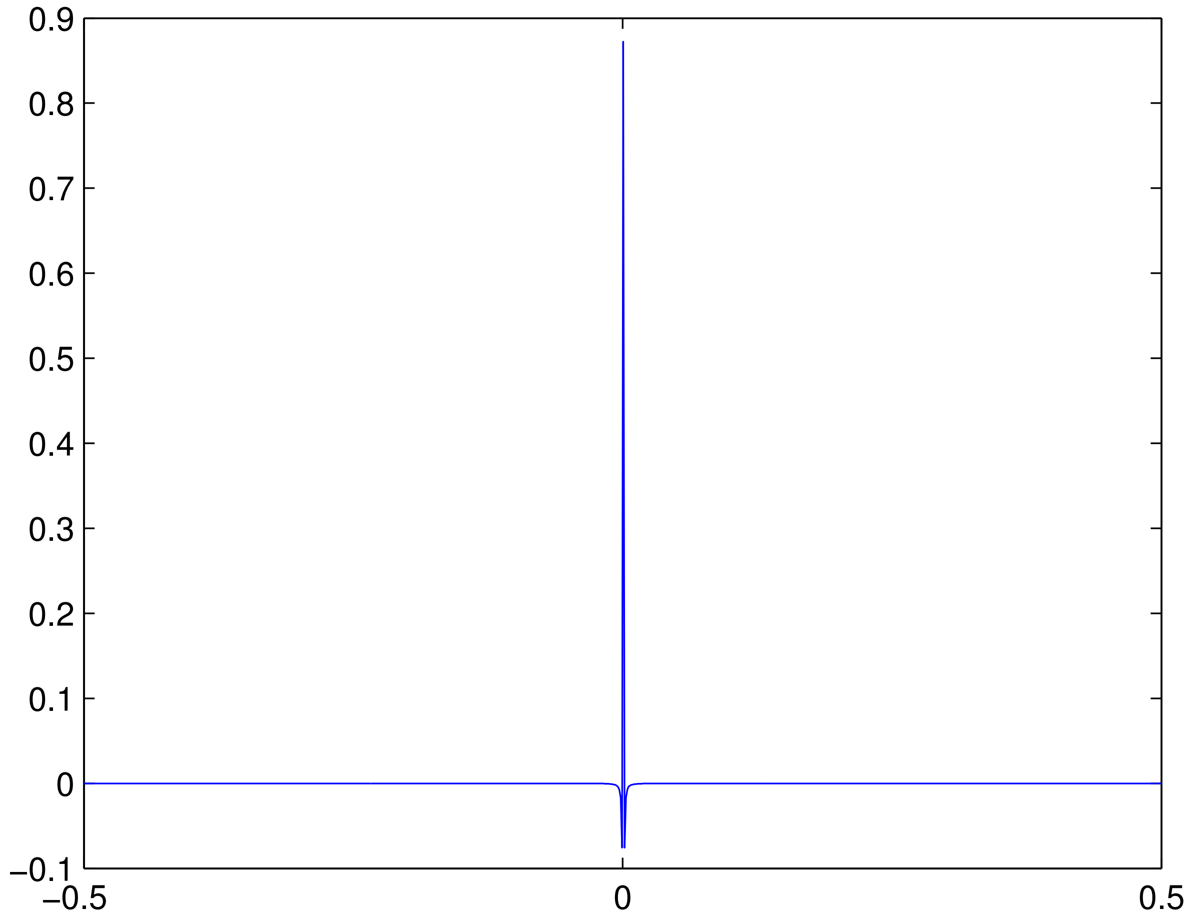


Figure 6: Shepp-Logan window function in frequency domain (left) and its counterpart in spatial domain (right)

Rectangular Filter

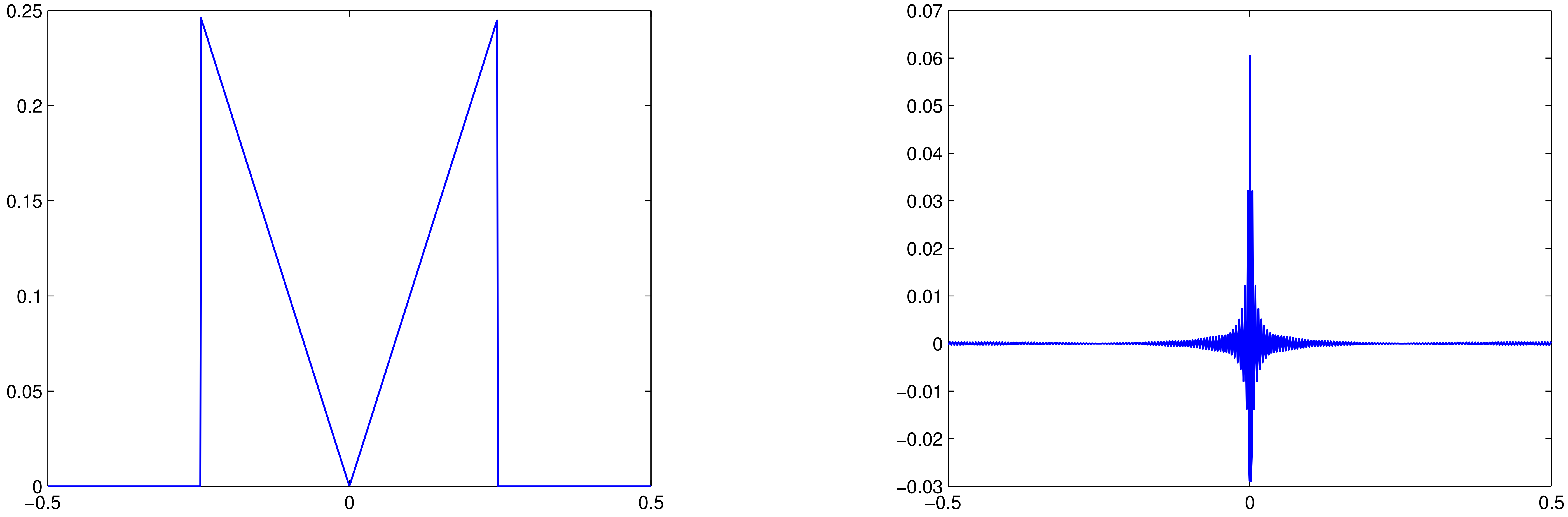


Figure 7: Rectangular filter in frequency domain (left) and its counterpart in spatial domain (right)

Cosine Filter

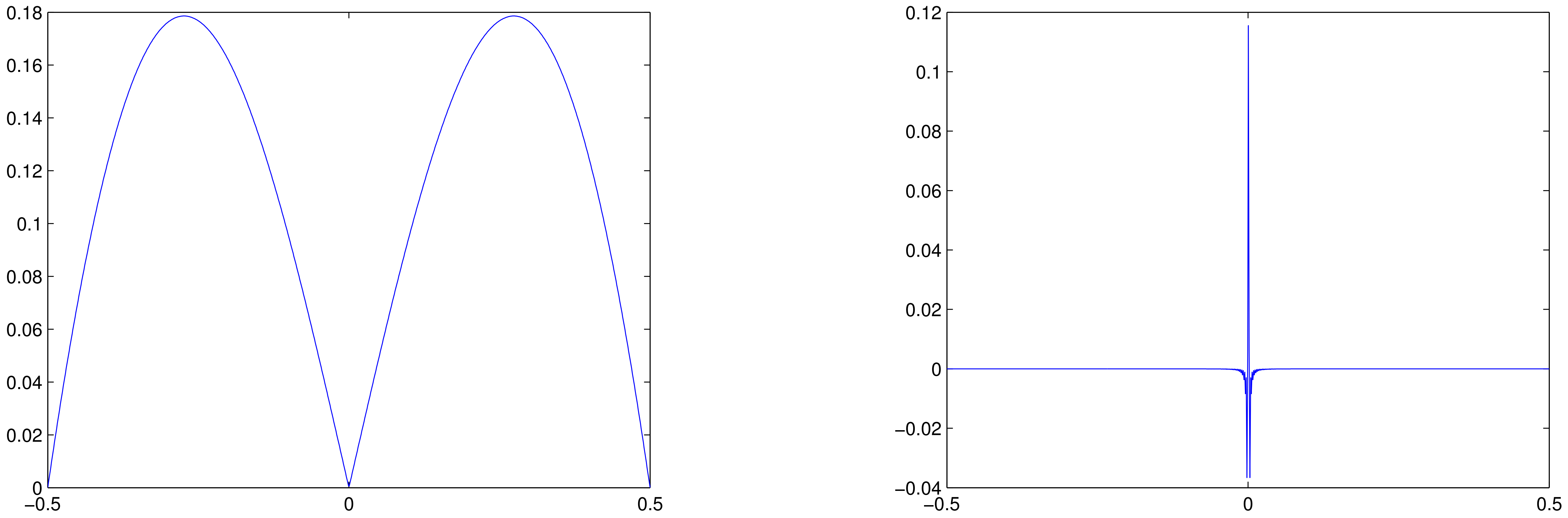
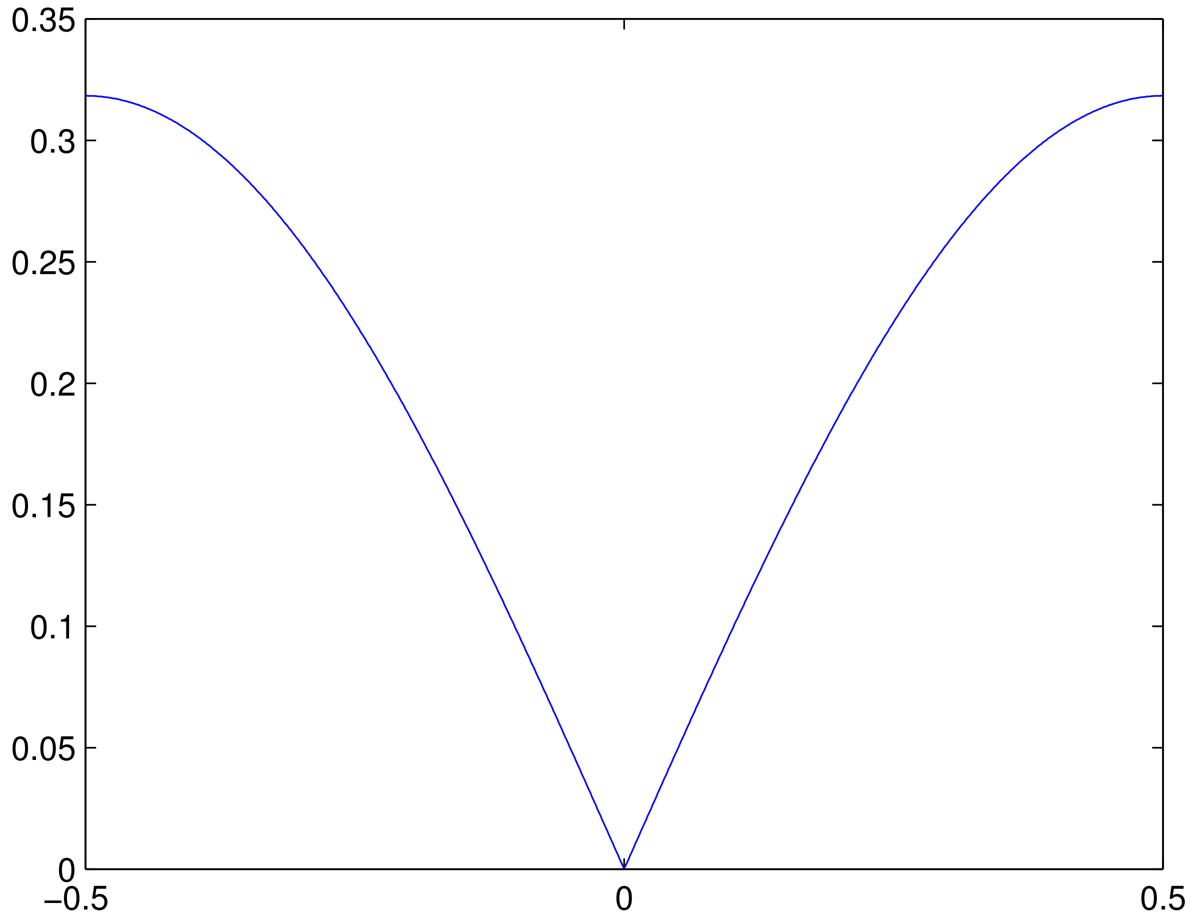


Figure 8: Cosine filter in frequency domain (left) and its counterpart in spatial domain (right)

Shepp-Logan Filter



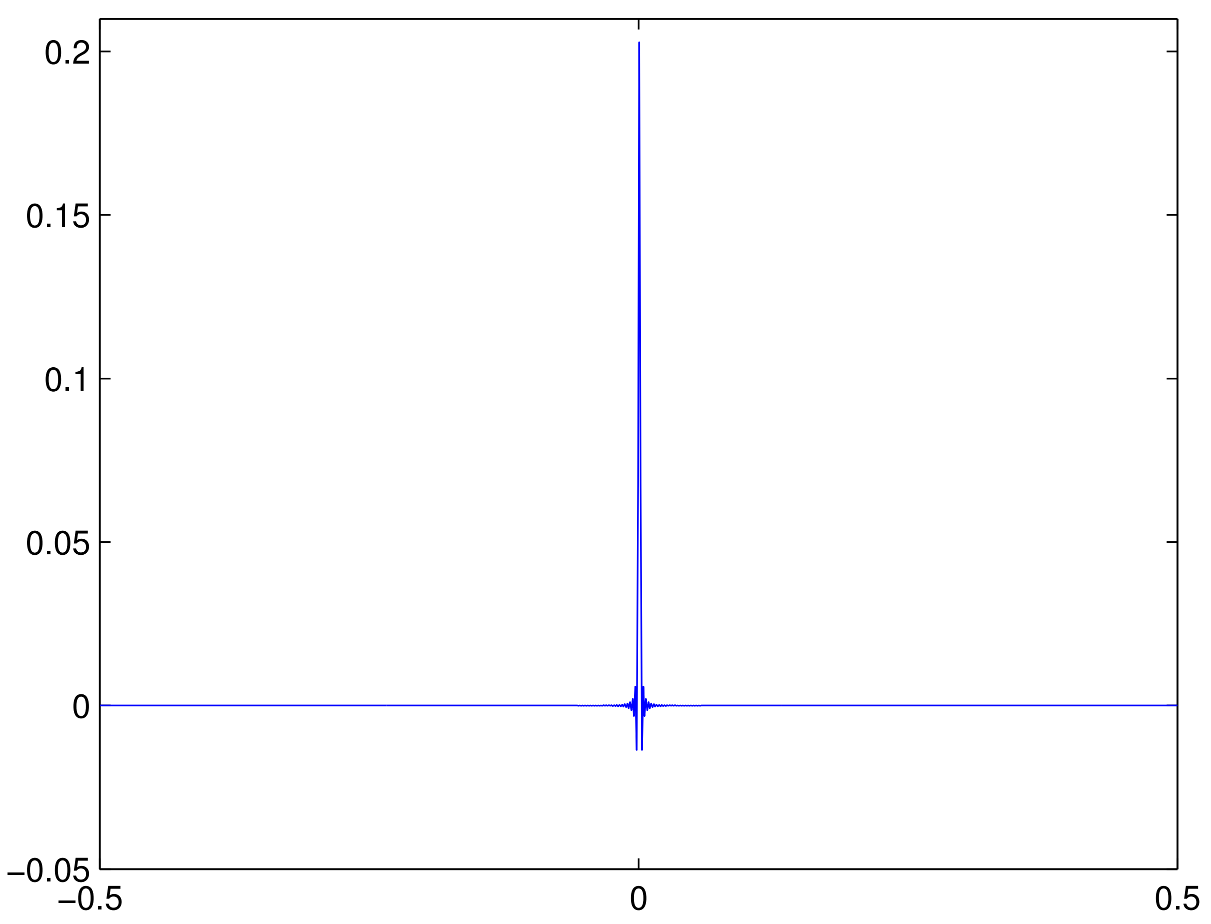


Figure 9: Shepp-Logan filter in frequency domain (left) and its counterpart in spatial domain (right)

Ramp Filter Result

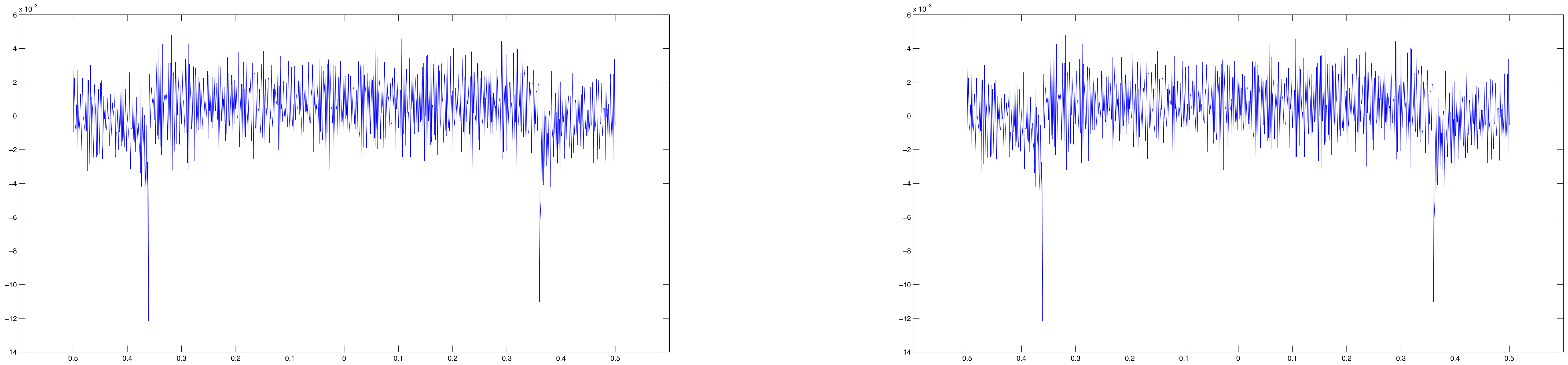


Figure 10: Filtered noisy projection using the rectangular window (left) vs. itself, the ramp filter (right)

Cosine Filter Result

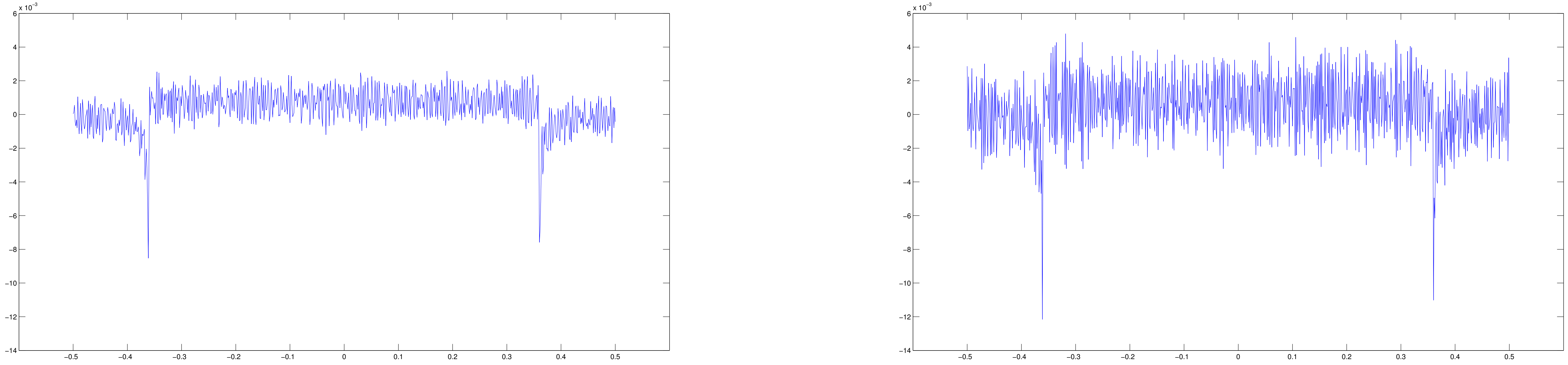


Figure 11: Filtered noisy projection using the cosine window (left) vs. the ramp filtered result (right)

Shepp-Logan Filter Result

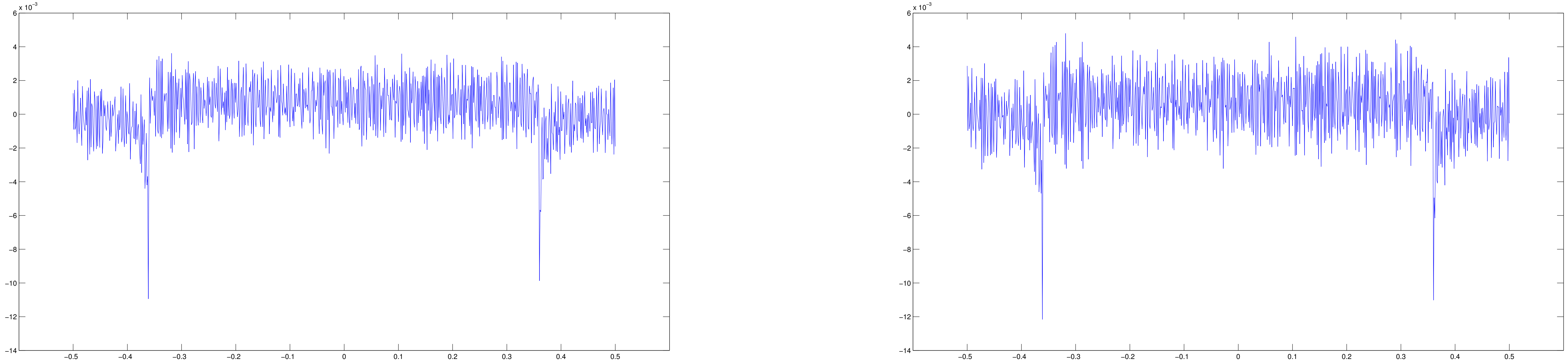


Figure 12: Filtered noisy projection using the Shepp-Logan window (left) vs. the ramp filtered result (right)

Topics

Effect of Noise on Filtering

Window Functions

- General Idea
- Common Examples
- The According Filters
- Filter Results

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- Noise has a severe effect on the filtering result.
- Window functions can be used to reduce this effect.
- We have learned about the frequency cut-off, the cosine window and the Shepp-Logan window.

Further Readings

The original Ram-Lak article is:

G. N. Ramachandran and A. V. Lakshminarayanan. "Three-dimensional Reconstruction from Radiographs and Electron Micrographs: Application of Convolutions instead of Fourier Transforms". In: *Proceedings of the National Academy of Sciences of the United States of America* 68.9 (Sept. 1971), pp. 2236–2240

The concise reconstruction book from 'Larry 'Zeng:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Another mathematical examination of filtered backprojection can be found in

Thorsten Buzug. *Computed Tomography: From Photon Statistics to Modern Cone-Beam CT*. Springer Berlin Heidelberg, 2008. DOI: 10.1007/978-3-540-39408-2