Diagnostic Medical Image Processing
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Exercises (DMIP-E)
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LEHRSTUHL FÜR MUSTER-ERKENNUNG

# Preprocessing/Image Undistortion

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Exercise Sheet 2

## 4 Preprocessing

(i) Shortly explain the term “preprocessing”.

(ii) Artifacts are often encountered in medical imaging, and might obscure or simulate pathologies. There are many types of image artifacts, name at least three types of image artifacts **and** their origins.

(iii) Explain interpolation and regression and the difference of both in your own words.

(iv) An undistorted image $\mathbf{I}_1$ has to be generated from a distorted image $\mathbf{I}_0$. Describe the interpolation problems associated with the mapping from the distorted image to the undistorted image. What are the disadvantages of using the points of $\mathbf{I}_0$ for sampling to generate $\mathbf{I}_1$? Outline a good solution and a solution fixing the associated problems directly.

$\boxed{\mathbf{1+1.5+1.5+2}}$

(i) See lecture

(ii) Possible examples of artifacts: noise, beam hardening, heel effect, scattering, pseudoenhancement, motion, cone beam, helical, ring, and metal artifacts, etc.

(iii) Definitions see lecture. The explanation should contain that in interpolation each sampling point lies on the estimated function, in regression they rarely do.

(iv) Parts of a solution:

- Blind spots in the output space are possible, where no points of $\mathbf{I}_0$ map to.

- One has to use some scheme to distribute a mapped value to the adjacent pixels. The sampling points of $\mathbf{I}_0$ have to be extrapolated in the output space, which leads to inaccuracies.
- The best and obvious solution is to map from output to input $\rightarrow$ *rule of thumb*: *Always* sample in the output space.
- However, one could also fix the problems very crude by distributing the values for example nearest neighbor like to all adjacent pixels and fix the holes by applying a median.

# 5 Polynomial Mappings

A parametric mapping for the distortion correction of a distorted image can be estimated by bivariate polynomials, i.e. $X(x', y')$ for the mapped $x$-coordinate of a point, in which $(x', y')$ is a point in the undistorted image:

$$x = X(x', y') = \sum_{i=0}^{d} \sum_{j=0}^{d-i} u_{i,j} x'^i y'^j.$$

(i) How many multiplications does a naive evaluation of this polynomial take?

(ii) How can the number of multiplications for the evaluation be efficiently reduced and how many multiplications are necessary to evaluate the polynomial after the reduction?

(iii) Use the Horner scheme to evaluate the polynomial for $d = 2$.

(iv) Given six calibration points from the undistorted image, i.e., $(x'_1, y'_1)$, $(x'_2, y'_2)$, $(x'_3, y'_3)$, $(x'_4, y'_4)$, $(x'_5, y'_5)$, $(x'_6, y'_6)$, and $d = 2$, write the measurement matrix $\mathbf{A}$. How would you call this matrix in 1D?

(v) Have a look at the diagrams. What is the value at $p = (3, 5)$ when the corner nodes of the rectangle in the left diagram are $(2, 4)$, $(6, 4)$, $(6, 7)$, $(2, 7)$, and $a = 2$, $b = 5$,$c = 3$,$d = -1$?

(vi) Looking at the process of how you computed the last value, can you also compute the value at $p = (3, 2.464)$ in the right diagram with the corner nodes $(1, 1)$, $(5, 1)$, $(3, 4.464)$, and $a = 100$, $b = 250$, and $c = 250$.

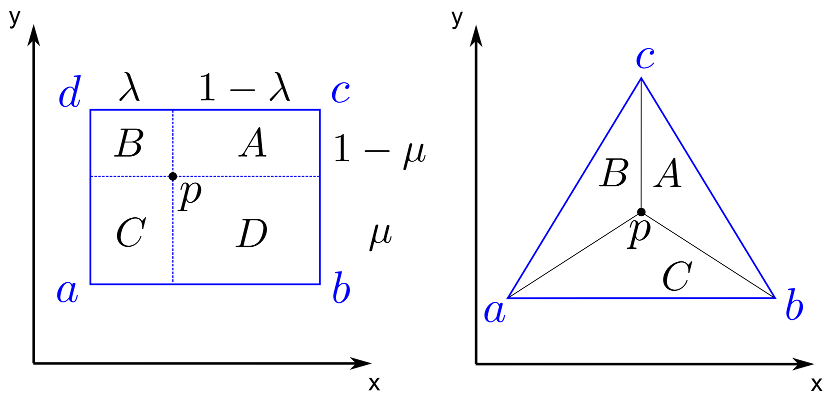
$\boxed{\textbf{1+1+1+1+1+1}}$

(i) The naive evaluation of this polynomial will take $\frac{d(d+1)(2d+1)}{6} + \frac{d(d+1)}{2}$ multiplications.

(ii) Using the *Horner scheme* the necessary multiplications can be reduced to $d + [(d^2 + d) \cdot 0.5]$.

(iii) First, we can evaluate the coefficients $\xi_i$, $i = 0, 1, 2$, for $x'$:

$$x = \sum_{i=0}^{2} \underbrace{\left( \sum_{j=0}^{2-i} u_{i,j} y'^j \right)}_{\xi_i} x'^i,$$




that is to use the Horner scheme for evaluation in $y'$. We have

$$\begin{aligned} i = 0: &\quad \xi_0 = (u_{0,2}y' + u_{0,1})y' + u_{0,0} \\ i = 1: &\quad \xi_1 = u_{1,1}y' + u_{1,0} \\ i = 2: &\quad \xi_2 = u_{2,0} \end{aligned}$$

Then, we need to fill in:

$$x = (\xi_2 x' + \xi_1)x' + \xi_0 = u_{2,0}x'^2 + u_{1,1}x'y' + u_{1,0}x' + u_{0,2}y'^2 + u_{0,1}y' + u_{0,0}.$$

(iv) From the last task we see:

$$x = \left(1, y', y'^2, x', x'y', x'^2\right) \begin{pmatrix} u_{0,0} \\ u_{0,1} \\ u_{0,2} \\ u_{1,0} \\ u_{1,1} \\ u_{2,0} \end{pmatrix}.$$

The measurement matrix consists of the row-vector above evaluated at the calibration points. In 1D we call this type of matrix Vandermonde matrix.

(v) Compare lecture diagram, $2\frac{3}{4}\frac{2}{3} + (-1)\frac{3}{4}\frac{1}{3} + 5\frac{1}{4}\frac{2}{3} + 3\frac{1}{4}\frac{1}{3} = \frac{11}{6}$

(vi) The ratios of the areas adjacent to the respective points to the total area enclosed by the convex hull of the nodes define the correct weights for a linear interpolation. In other symbols, we need to compute:

$$G := A + B + C = \frac{4(4.464 - 1)}{2} = 6.928,\ C = \frac{4(2.464 - 1)}{2} = 2.928,$$

which by means of

$$A = B = \frac{G - C}{2} = 2.000$$

yields

$$a\frac{A}{G} + b\frac{B}{G} + c\frac{C}{G} = 185.6.$$

## 6 Image Undistortion – Programming Exercise

Have a look at the code file *exercise2.java*. First, search for the main method and see what is happening there. Go through the code of `generateDistortedImage()` to understand how the distorted test images for this exercise are artificially created.
Your task is to undo this distortion by implementing the method `doImageUndistortion()` using the polynomial undistortion approach which you have seen in the lecture.

1. In real world we would know the relation between the undistorted and the distorted image by point correspondences of a calibration pattern. However, in this example we use the artificial distortion field.
   Therefore, we assume lattice points distributed over the whole image domain, e.g. a $8 \times 8$ lattice. For these 64 positions we know the relation between the undistorted (Xu,Yu) and the distorted image (Xd,Yd).

   Set the number of lattice points and sample the undistorted and distorted image at those positions.
   **Attention:** We want to **sample the distorted image** to get the corrected image. When creating the distortion field the ideal image was sampled to get the distorted one. Check the direction of the distortion regarding the undistorted and distorted image!
2. The distortion correction is described in the slide set for unit 11. $d$ is the polynomial's degree. What is the maximum number for $d$ w.r.t. the given number of sampling points?
3. Build the matrix $\mathbf{A}$ where the number of rows is the number of correspondences, and the number of columns is the number of coefficients (see "Gaußsche Summenformel").
4. Compute the pseudoinverse $\mathbf{A}^{\dagger}$ and use it to estimate the coefficients $u_{i,j}$ and $v_{i,j}$.
5. Compute the sampling grid for an undistorted image and resample the distorted image to obtain the undistorted image using bilinear interpolation. Show the result.

**2+1+0.5+1.5+1**

(i) code implementation

(ii) code implementation

(iii) code implementation

(iv) code implementation

(v) code implementation

**Total: 18**

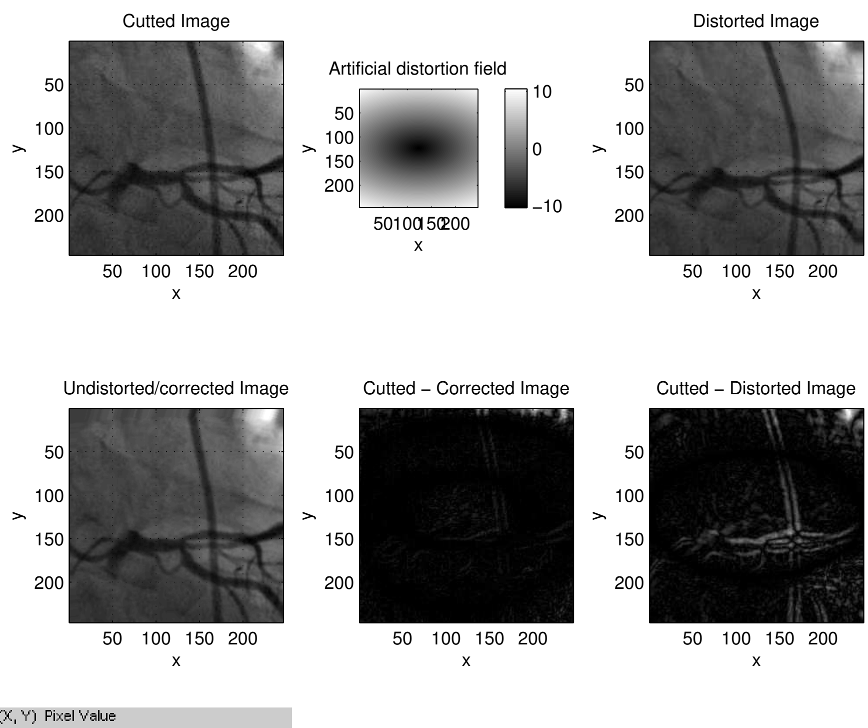



Figure 1: Original image, artificial distortion field and image undistortion result.