

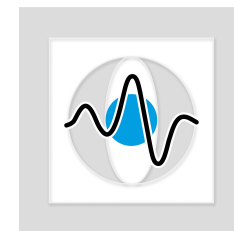
# Medical Image Processing for Diagnostic Applications

## Filtering in Frequency Domain

Online Course – Unit 22

Andreas Maier, Joachim Hornegger, Markus Kowarschik, Frank Schebesch

Pattern Recognition Lab (CS 5)



# Topics

Frequency Domain Filters

Homomorphic Filtering

Summary

Take Home Messages

Further Readings

# Frequency Domain Filters

**Design a high-pass filter that eliminates the low frequency bias field.**

# Frequency Domain Filters

Let us consider the idea of high-pass filtering first by designing a filter in frequency domain:

- First, the observed input image  $g = [g_{i,j}]$  is Fourier transformed:

$$G = \text{FT}([g_{i,j}]).$$

- Second, a high-pass filter is defined in the discrete frequency domain by:

$$H_{k,l} = 1 - \beta e^{-\frac{k^2+l^2}{2\sigma^2}},$$

where

- $\beta$  is a scaling factor that ensures that  $H_{k,l} \geq 0$  for all  $k, l = 0, 1, \dots, M-1$ ,
- and  $\sigma^2$  is closely related to the bandwidth of the filter-kernel.

# Frequency Domain Filters

The relation between low- and high-pass filters is:

$$\begin{aligned} f &= g * h_{\text{HP}} \\ &= \text{FT}^{-1}(\text{FT}(g * h_{\text{HP}})) \\ &= \text{FT}^{-1}(G \cdot H_{\text{HP}}) \\ &= \text{FT}^{-1}(G \cdot (1 - H_{\text{LP}})) \\ &= g * (1 - h_{\text{LP}}) \\ &= g - g * h_{\text{LP}}. \end{aligned}$$

# Frequency Domain Filters

Using the convolution theorem, high-pass filtering is simply a multiplication in the frequency domain:

$$F_{k,l} = G_{k,l} \cdot H_{k,l},$$

for all  $k, l = 0, 1, \dots, M - 1$ .

The final output image  $f$  is obtained by computing the inverse Fourier transform:

$$f = \text{FT}^{-1}([F_{k,l}]).$$

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# Homomorphic Filtering

These filtering approaches assume that IIH is

- an artifact with low frequencies, and
- anatomic structures contribute to the high frequencies of the image.

Elimination of image inhomogeneities can be done by low-pass filtering.



# Homomorphic Filtering

**Subtract the low-pass filtered image and normalize the mean.**

# Homomorphic Filtering

Homomorphic filtering is applied to log-transformed images:

- Make a low-pass filtering of the log-transformed image

$$[h_{i,j}] = \text{LPF}([\log g_{i,j}]),$$

where LPF denotes a low-pass filter (like averaging or a Gaussian filter).

- The IHH corrected, log-transformed image  $\log f$  results from the difference:

$$[\log f_{i,j}] = [\log g_{i,j}] - [h_{i,j}] + \mu,$$

where  $\mu$  ensures that the correction is mean preserving.

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## Take Home Messages

- The straightforward approach for IIH correction is low-pass filtering using the Fourier transform of the image.
- When using homomorphic filtering, a similar idea is applied on the log-transformed images including a mean preservation technique.

## Further Readings

The webpage of the [National High Magnetic Field Laboratory](#) can be one starting point for more detailed information regarding MRI. For an initial overview of the technology, the following article is worth reading:

[MRI: A Guided Tour](#) by Kristen Coyne.

If you want to know more about segmentation of MR images, e. g., consult the [Google Scholar record](#) of ‘Sandy’ Wells’ publications.

Another article worth reading is this survey paper on algorithms for intensity correction methods:

[Zujun Hou](#). “A Review on MR Image Intensity Inhomogeneity Correction”. In: *International Journal of Biomedical Imaging* 2006. Article ID 49515 (Feb. 2006), pp. 1–11. DOI: 10.1155/IJBI/2006/49515