Diagnostic Medical Image Processing
Prof. Dr.-Ing. Andreas Maier
Exercises (DMIP-E)
WS 2016/17

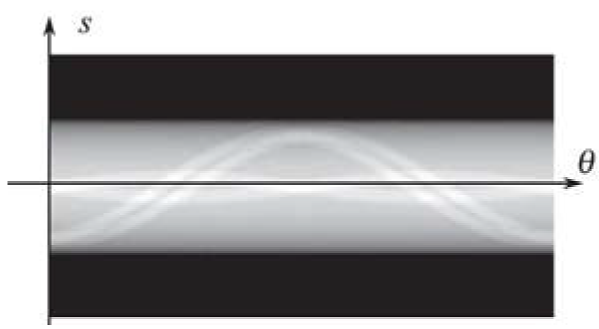
Parallel Beam Reconstruction

Frank Schebesch, Tobias Würfl, Matthias Utzschneider,
Yixing Huang, Asmaa Khdeir, Houman Mirzaalian

Exercise Sheet 5

16 Parallel Beam Projection & Backprojection

(i) Look at the following figure:



What is this type of image called? Explain what information the image contains and how it can be obtained.

(ii) Consider the following 2×2 image:

$$\begin{bmatrix} w & x \\ y & z \end{bmatrix}.$$

Given four projections:

$$
\begin{aligned}
p_0 &: x + z = 9, w + y = 11, \\
p_{\frac{\pi}{4}} &: w + z = 7, \\
p_{\frac{\pi}{2}} &: w + x = 12, y + z = 8, \\
p_{\frac{3\pi}{4}} &: y + x = 13,
\end{aligned}
$$

build the system matrix and compute the 2×2 image values. Check your result.

(iii) Now we acquire four projections of the image:

$$
\begin{bmatrix} 1 & 3 & 2 \\ 6 & 1 & 2 \\ 0 & 5 & 3 \end{bmatrix}
$$

at angles 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$. Obtain the sinogram. When computing the diagonal projections, assume, for simplicity, that the image intensities contribute to equal parts to the values of the detector pixels $1, 2, 3$ according to the following scheme (analoguously for the other diagonal angle):

2	1	1
3	2	1
3	3	2

(iv) In the same fashion, backproject the data of the sinogram. Is this a perfect reconstruction, and why?

(v) Which angular range is necessary for a complete parallel beam backprojection: $[0, \pi]$ or $[0, \pi)$? Explain graphically.

1+1+1+1+1

17 Filtered Backprojection

In this exercise we want to show the filtered backprojection algorithm in the Fourier domain. From the lecture, we know that simple backprojection of parallel beam data is not sufficient to obtain the original object. Filtering has to be applied which is usually implemented as a multiplication in Fourier domain. Here we derive the necessary steps:

(i) Start with the inverse Fourier transform of $f(x, y)$:

$$
f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{2\pi i (xu + yv)} \, \mathrm{d}u \, \mathrm{d}v
$$

and transform the integration variables from cartesian (detector) coordinates (u, v) to polar coordinates (ω, θ). The transformation of $F(u, v)$ is denoted by $F_{\text{polar}}(\omega, \theta)$.

(ii) Since we do not know F_{polar} directly, we want to connect this formula with something we can compute, for instance the 1-D Fourier transform of the sinogram $p(s, \theta)$:

$$P(\omega, \theta) = \int\limits_{-\infty}^{\infty} p(s, \theta) e^{-2\pi i \omega s} \, \mathrm{d}s.$$

Show that $F_{\text{polar}}(\omega, \theta) = P(\omega, \theta)$ for all $\omega \geq 0$, $\theta \in [0, 2\pi)$.

(iii) Finally, use what you have shown so far to state the filtered backprojection algorithm in Fourier domain. What steps do you have to compute?

(iv) What is the name of the filter you have found? Why do we need it in the algorithm? Please describe the reason from the aspect of sampling.

1.5+1.5+1+1

18 Ram-Lak Filter – Programming Exercise

In this exercise we have a look at the filtered backprojection of a Shepp-Logan phantom. Your task is to implement the appropriate filter methods. Therefore, please fill in the missing parts in `exercise5.java`. You can set the type of filtering in the `main()`-method.

(i) First, try backprojecting without filtering. Then implement the Ram-Lak filter and apply it in Fourier domain.

(ii) Compare your result with another filter. For that purpose implement the Shepp-Logan filter as well.

4.5+1.5

Latest submission date: 12/16/2016

Total: 16