Medical Image Processing for Diagnostic Applications

Basic Principles of Tomography

Online Course – Unit 27

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Topics

Basic Principles of Tomography

$\tau\acute{o}\mu o\varsigma$ ['tomos] $\rightarrow$ slice

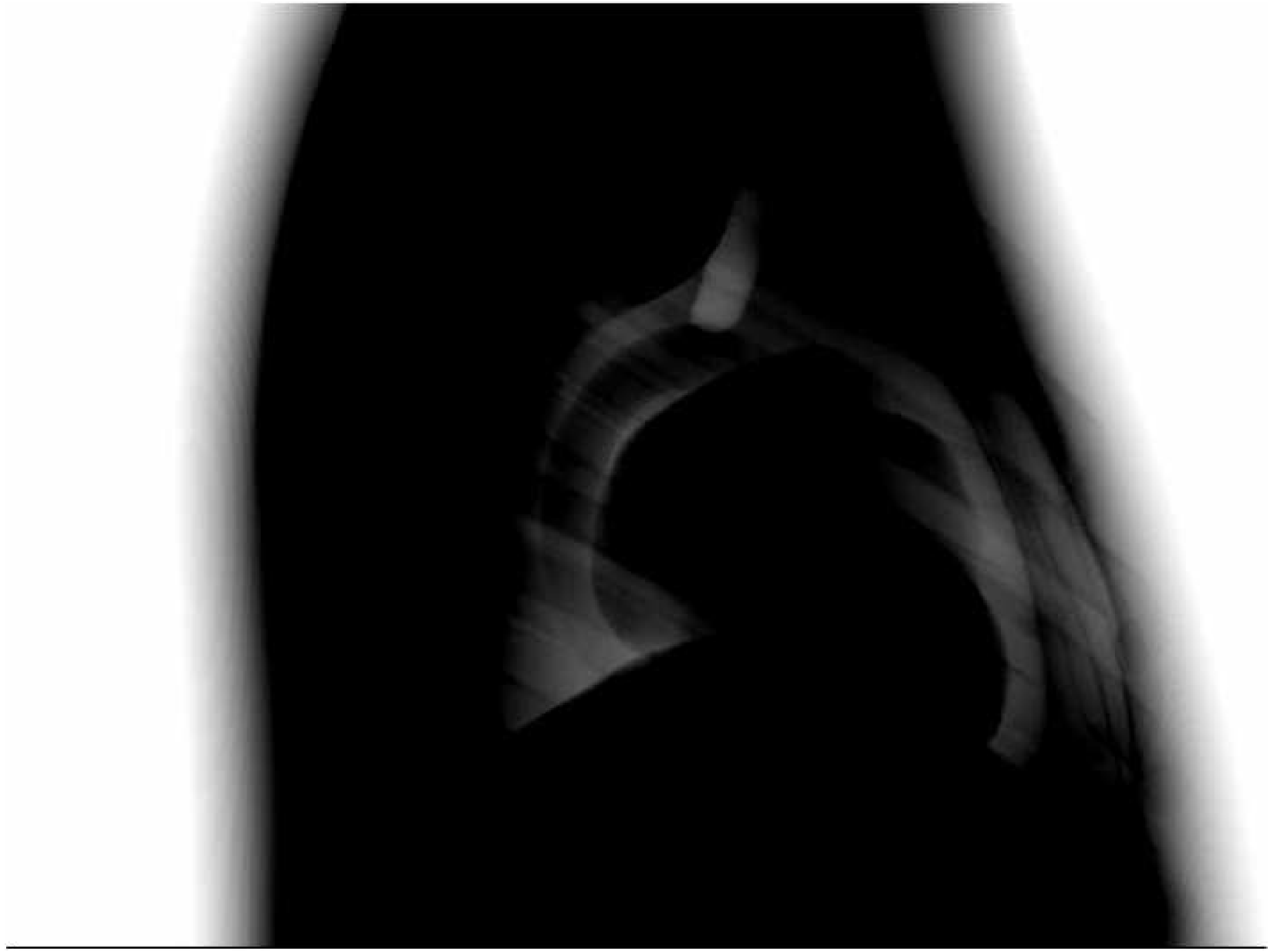


Figure 1: Single chest phantom projection

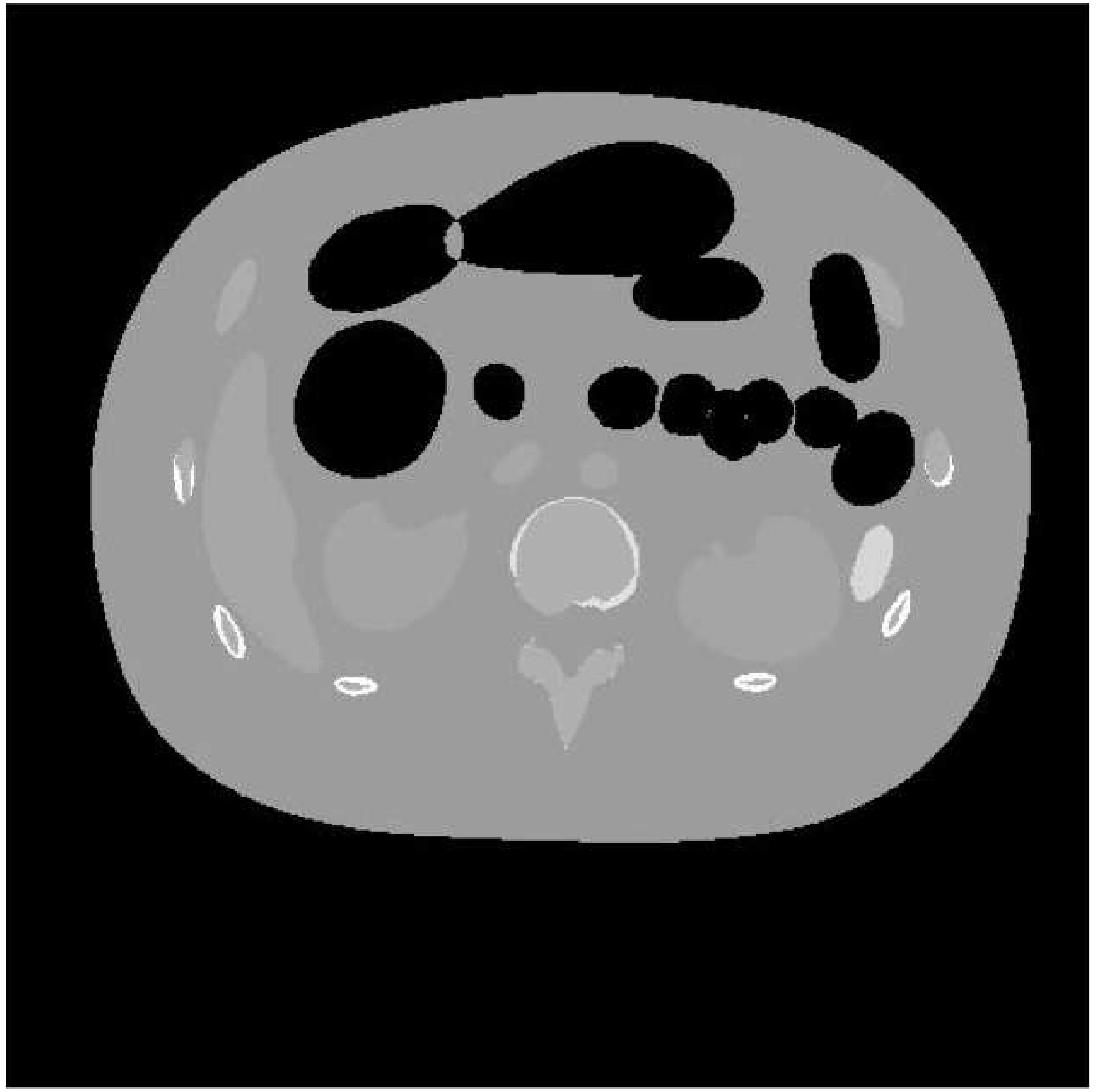


Figure 2: Slice view (click for video)

Basic Principles of Tomography

Idea: Observe the object of interest from multiple sides:

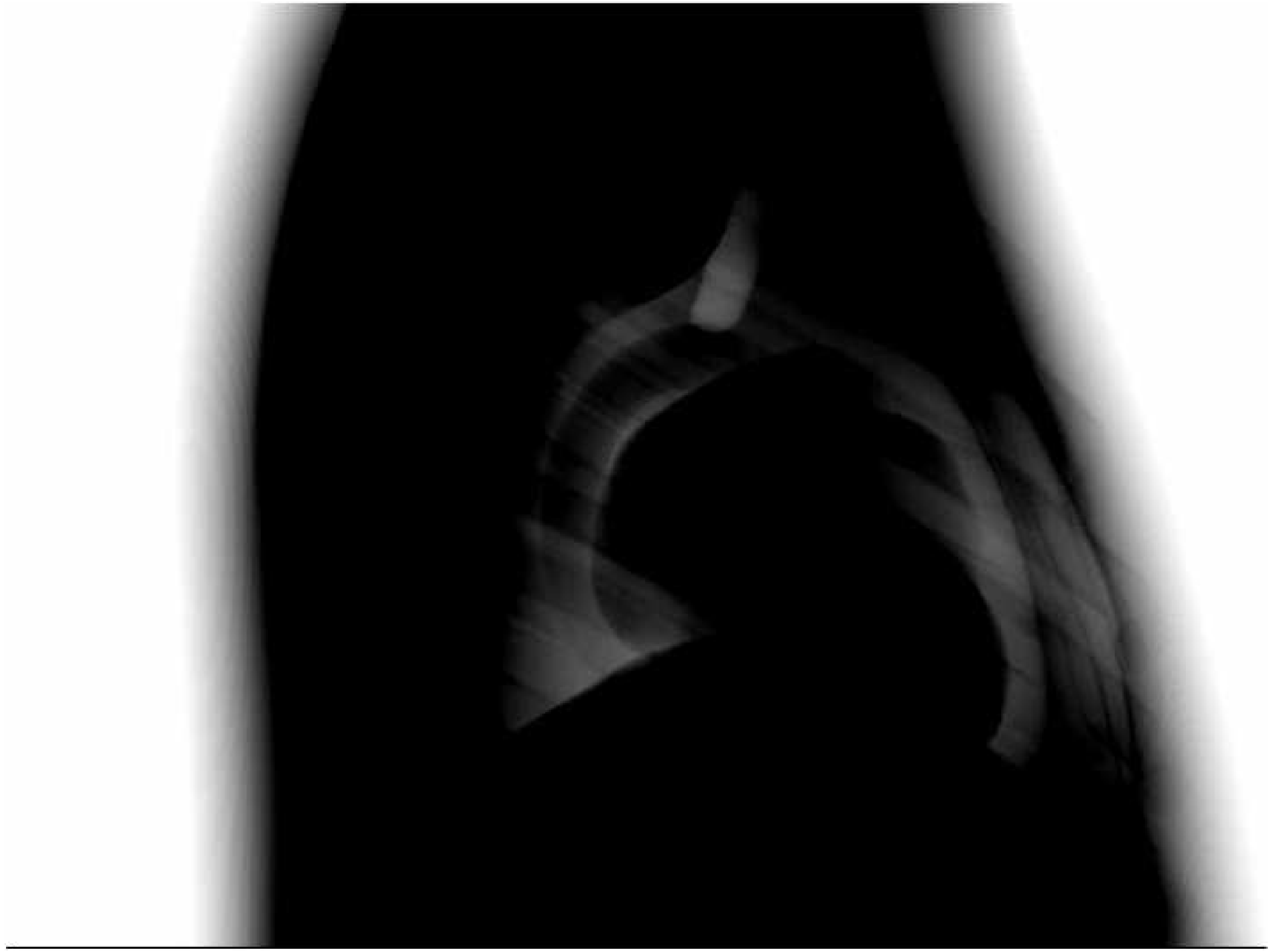
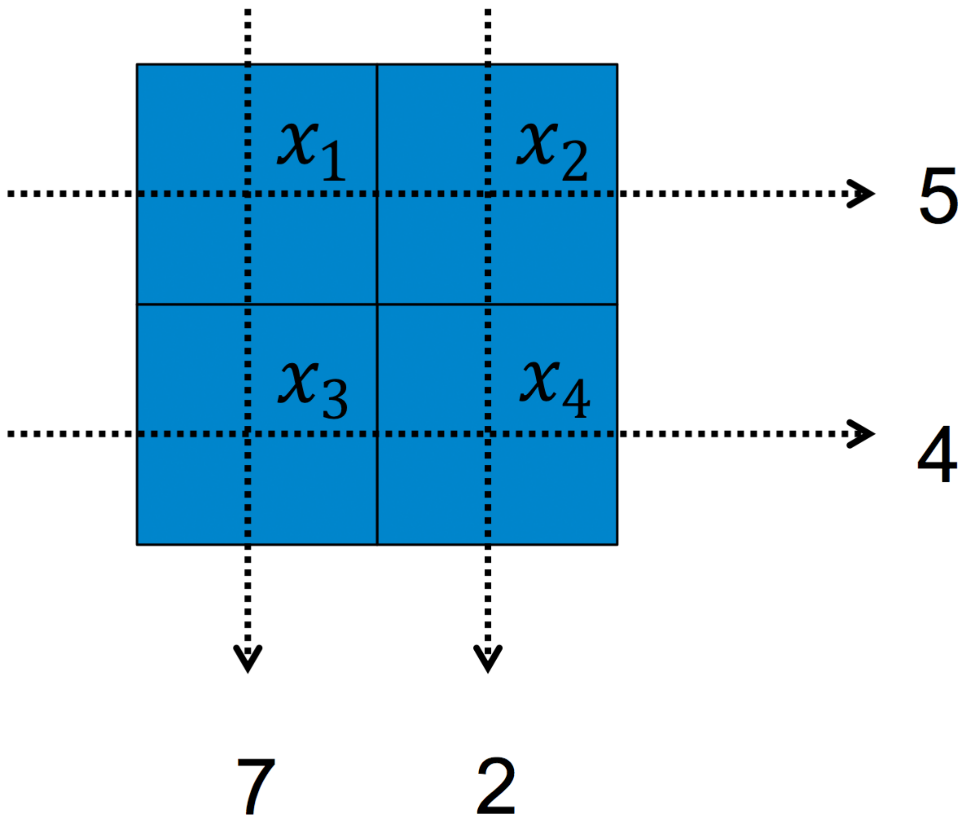


Figure 3: Multiple scan views (click for video)

Basic Principles of Tomography

Solve the puzzle:



$$
\begin{aligned}
x_1 + x_3 &= 7 \\
x_2 + x_4 &= 2 \\
x_1 + x_2 &= 5 \\
x_3 + x_4 &= 4
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
x_1 &= 3 \\
x_2 &= 2 \\
x_3 &= 4 \\
x_4 &= 0
\end{aligned}
$$

- Usually, the problem size in CT is $512 \times 512 \times 512 = 134\,217\,728$.
- **How can this problem be solved?**

Topics

Projection: X-ray Attenuation

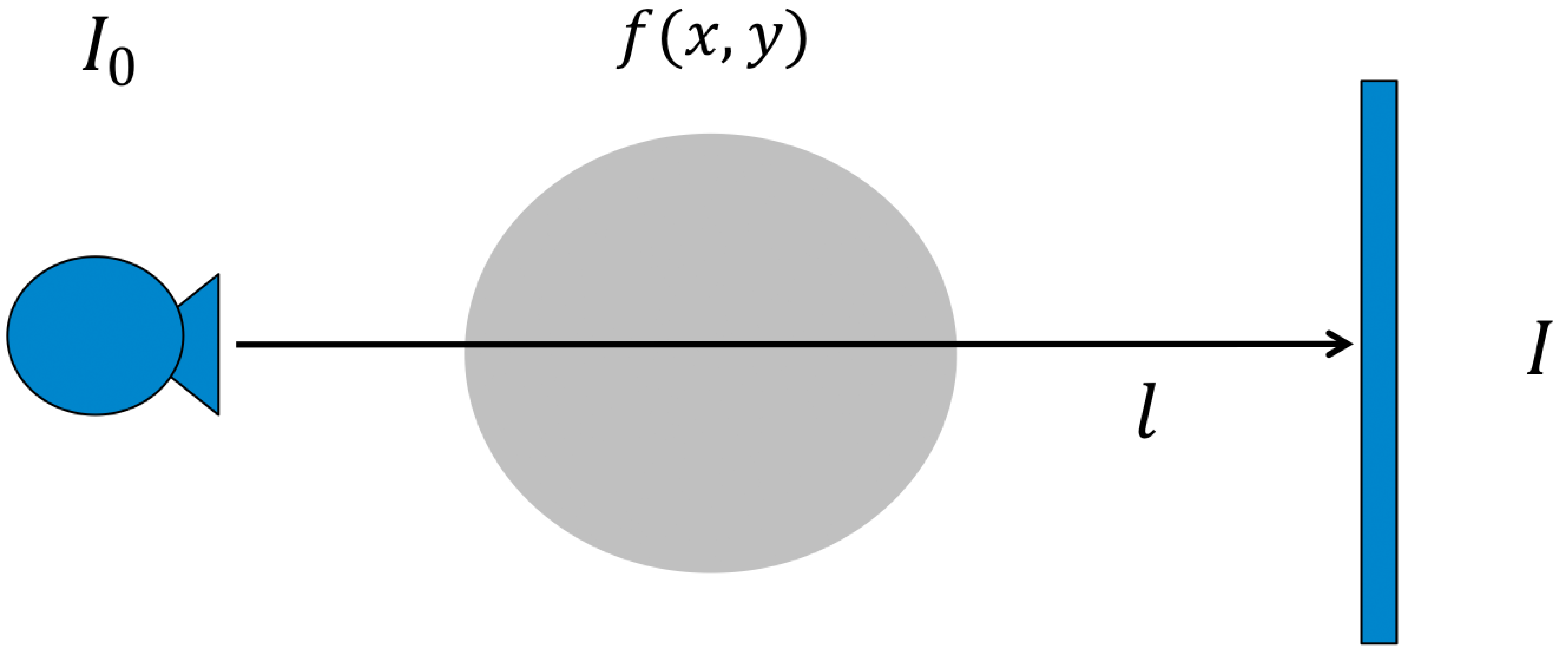


Figure 4: Beer–Lambert law: $I = I_0 e^{-\left(\int f(x,y) \mathrm{d}l\right)}$

Projection: Physical Observations

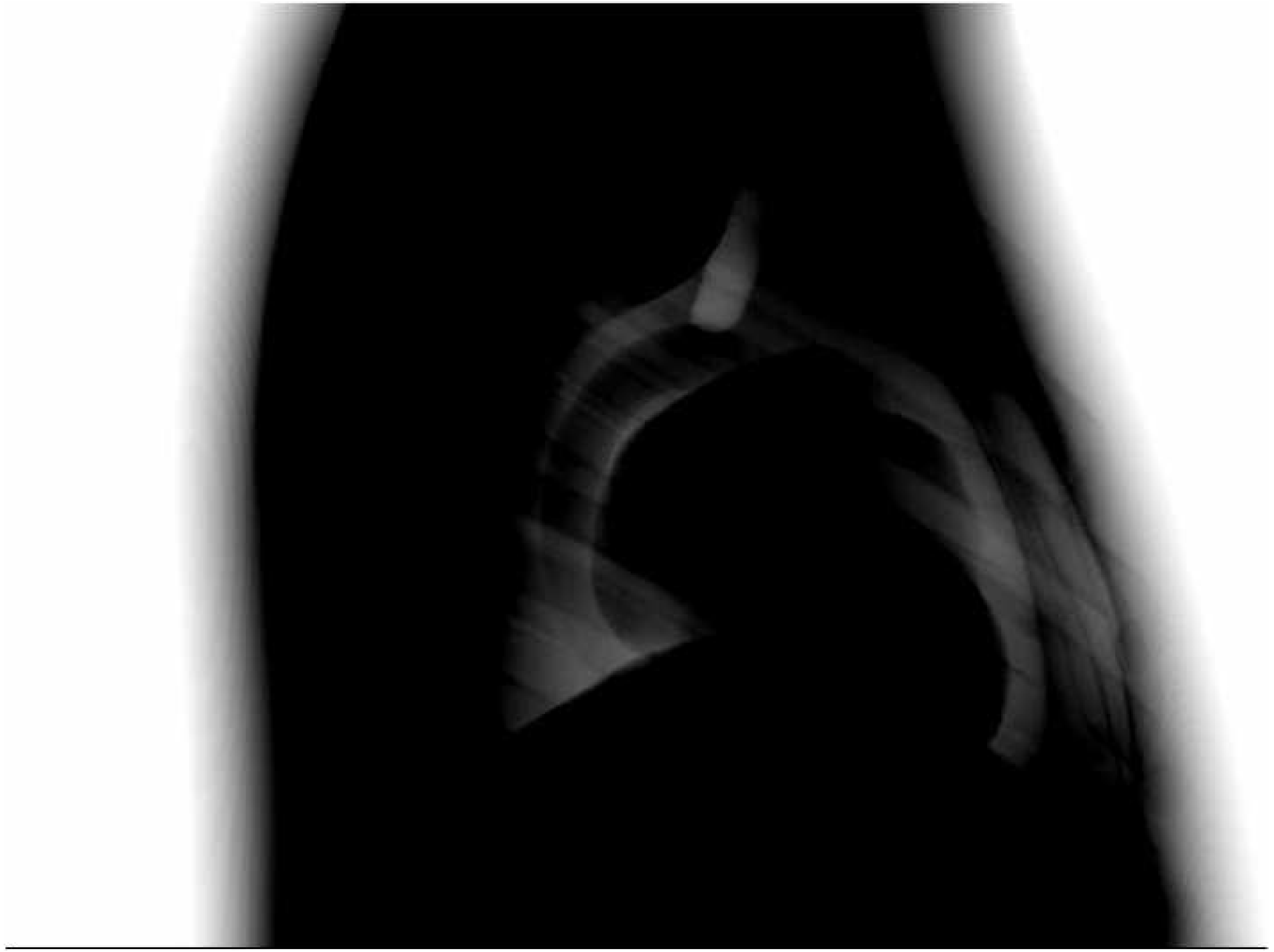


Figure 5: Observed projection signal (click for video)

Projection: Physical Observations

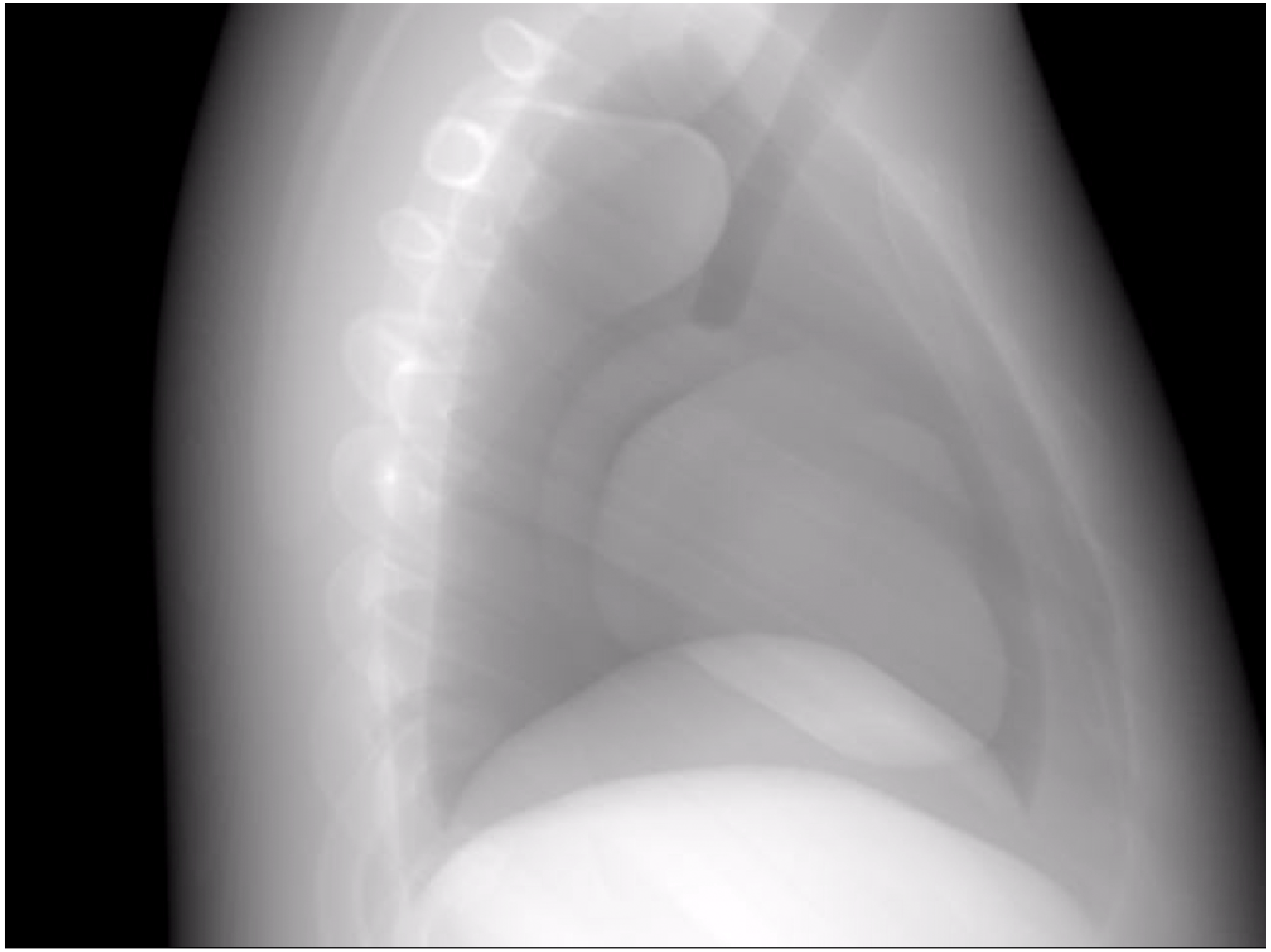


Figure 6: Line integral data (click for video)

Projection Formation

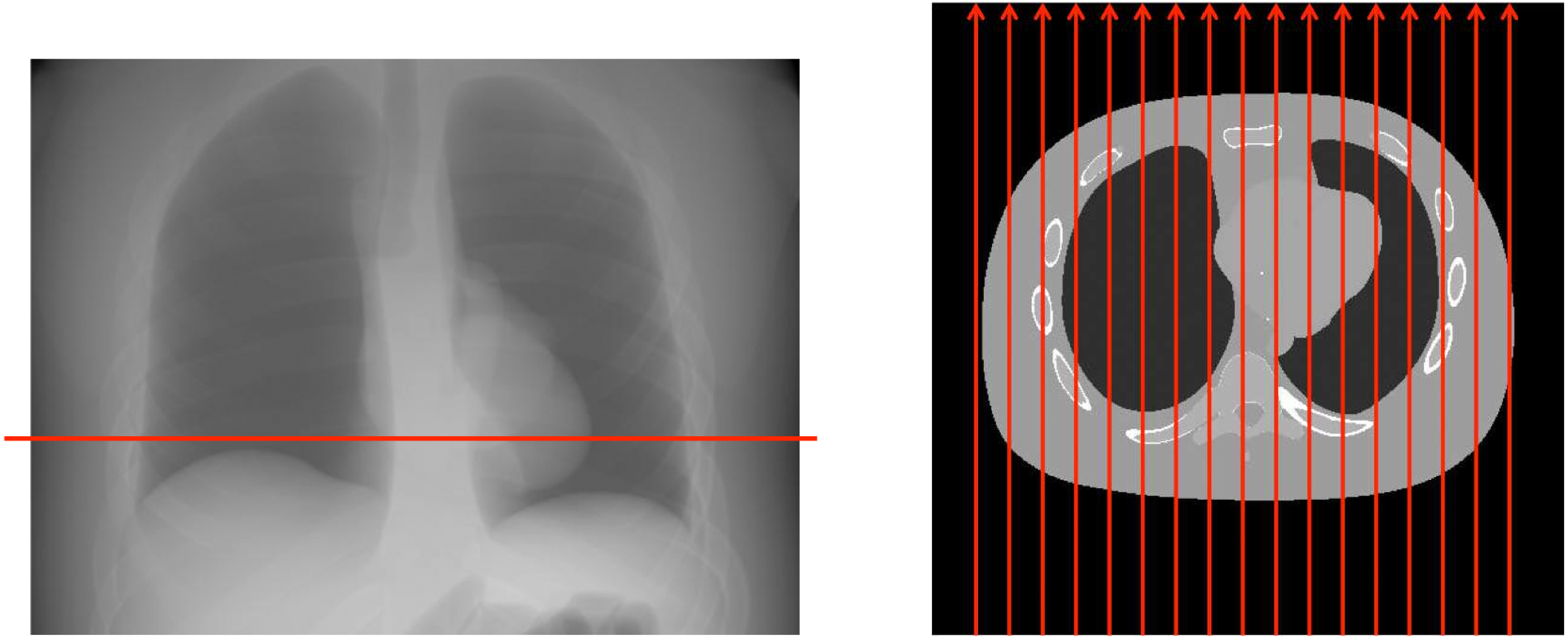


Figure 7: Slice by slice projection (left), projection ray scheme (right)

Projection: Mathematical Formulation

$$p(s,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x\cos\theta + y\sin\theta - s)\,\mathrm{d}x\,\mathrm{d}y$$

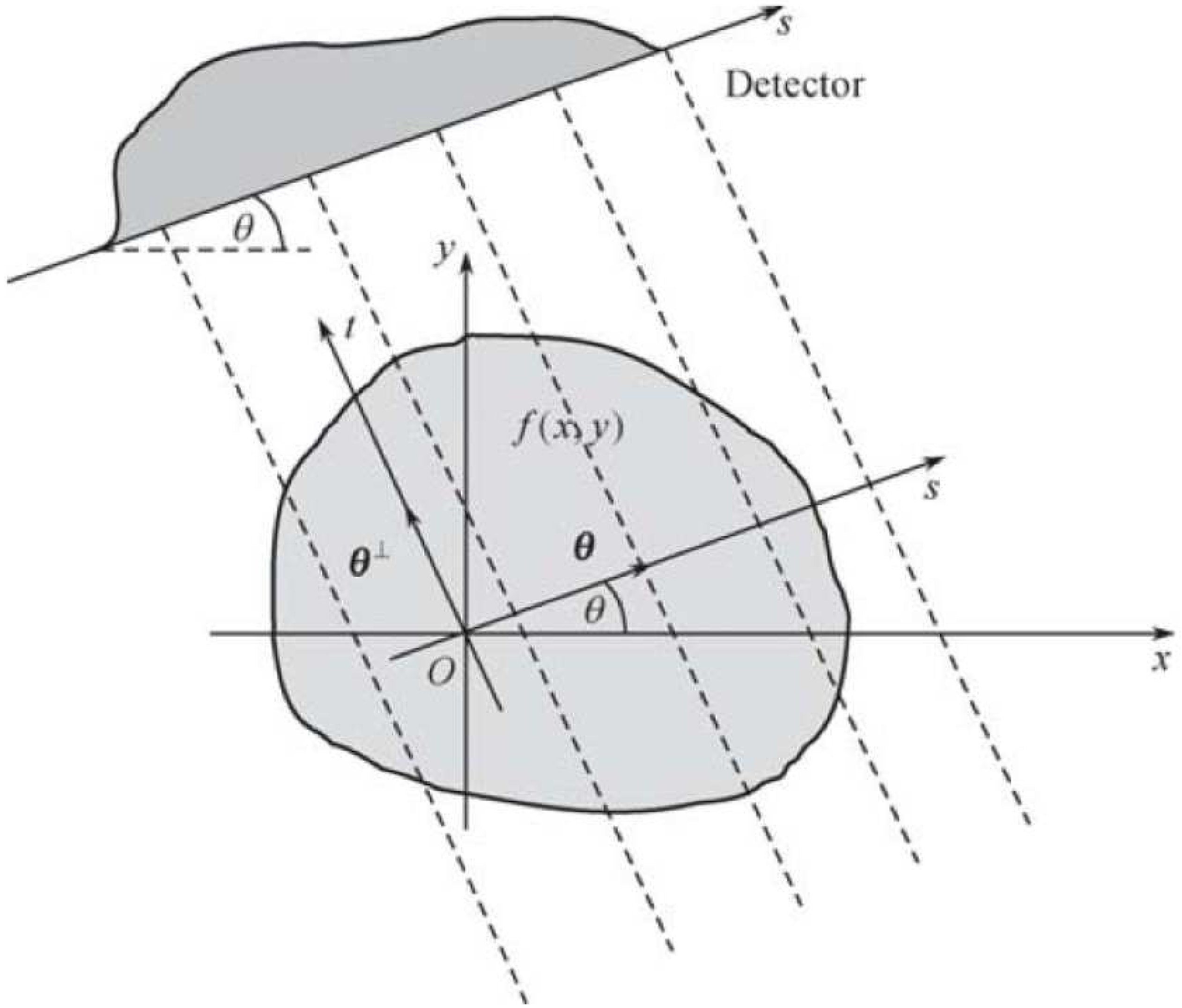


Figure 8: Parallel beam geometry (Zeng, 2009)

Projection: Example Point Object

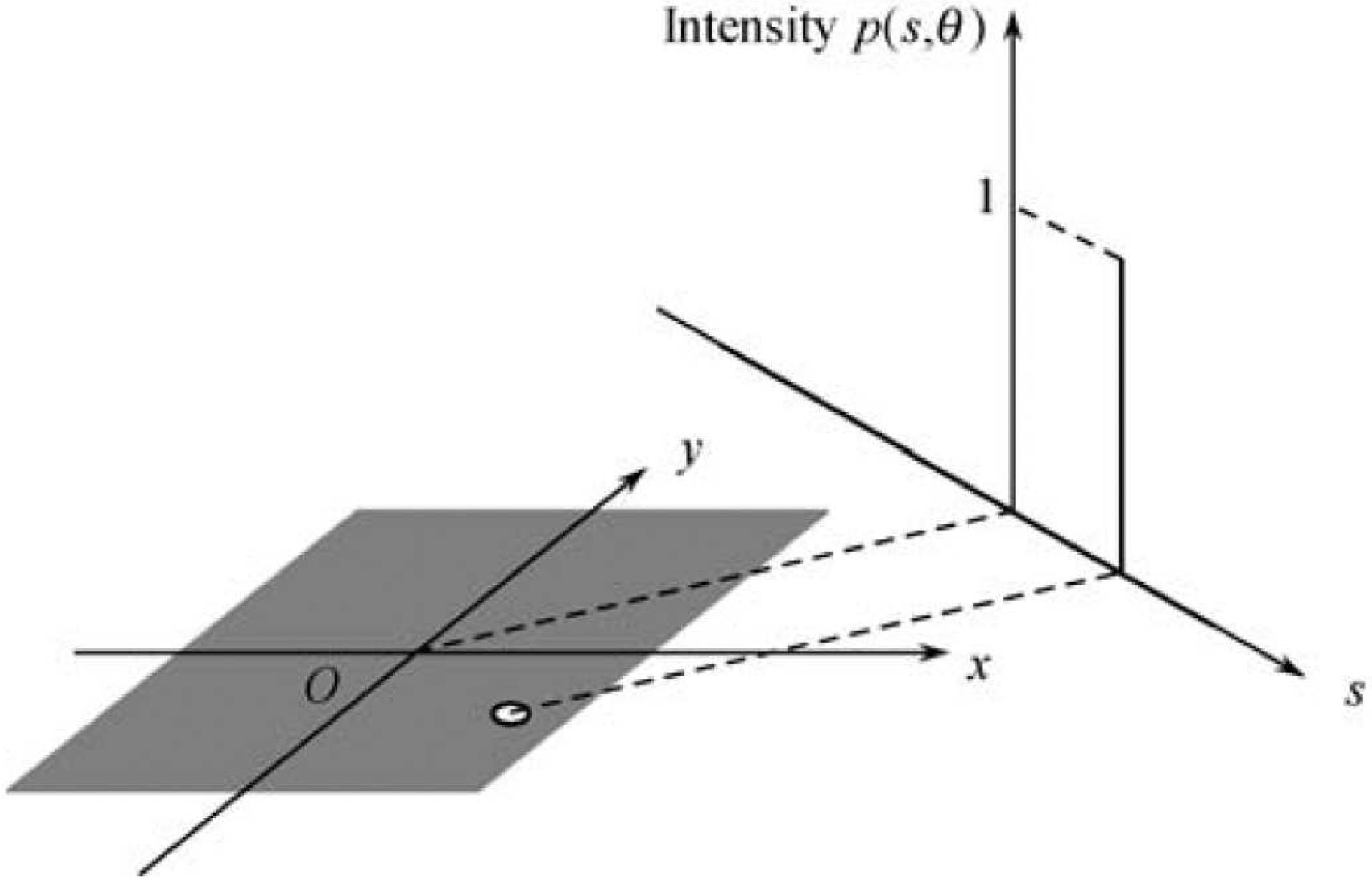


Figure 9: Intensity profile of a point object (Zeng, 2009)

Topics

Take Home Messages

- Tomography is looking through an object from several angles and recombining the projection views back to a 3-D volume of the object.
- 3-D data is usually represented as a stack of slices.
- The Beer-Lambert law describes the attenuation of X-rays on their path through an object.
- A single projection can be regarded as the integral of projections of every point inside the object.

Further Readings

Students learning about reconstruction should have a look at one of the following books:

- Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9
- Avinash C. Kak and Malcolm Slaney. *Principles of Computerized Tomographic Imaging*. Classics in Applied Mathematics. Accessed: 21. November 2016. Society of Industrial and Applied Mathematics, 2001. DOI: 10.1137/1.9780898719277. URL: http://www.slaney.org/pct/
- Thorsten Buzug. *Computed Tomography: From Photon Statistics to Modern Cone-Beam CT*. Springer Berlin Heidelberg, 2008. DOI: 10.1007/978-3-540-39408-2
- Willi A. Kalender. *Computed Tomography: Fundamentals, System Technology, Image Quality, Applications*. 3rd ed. Publicis Publishing, July 2011

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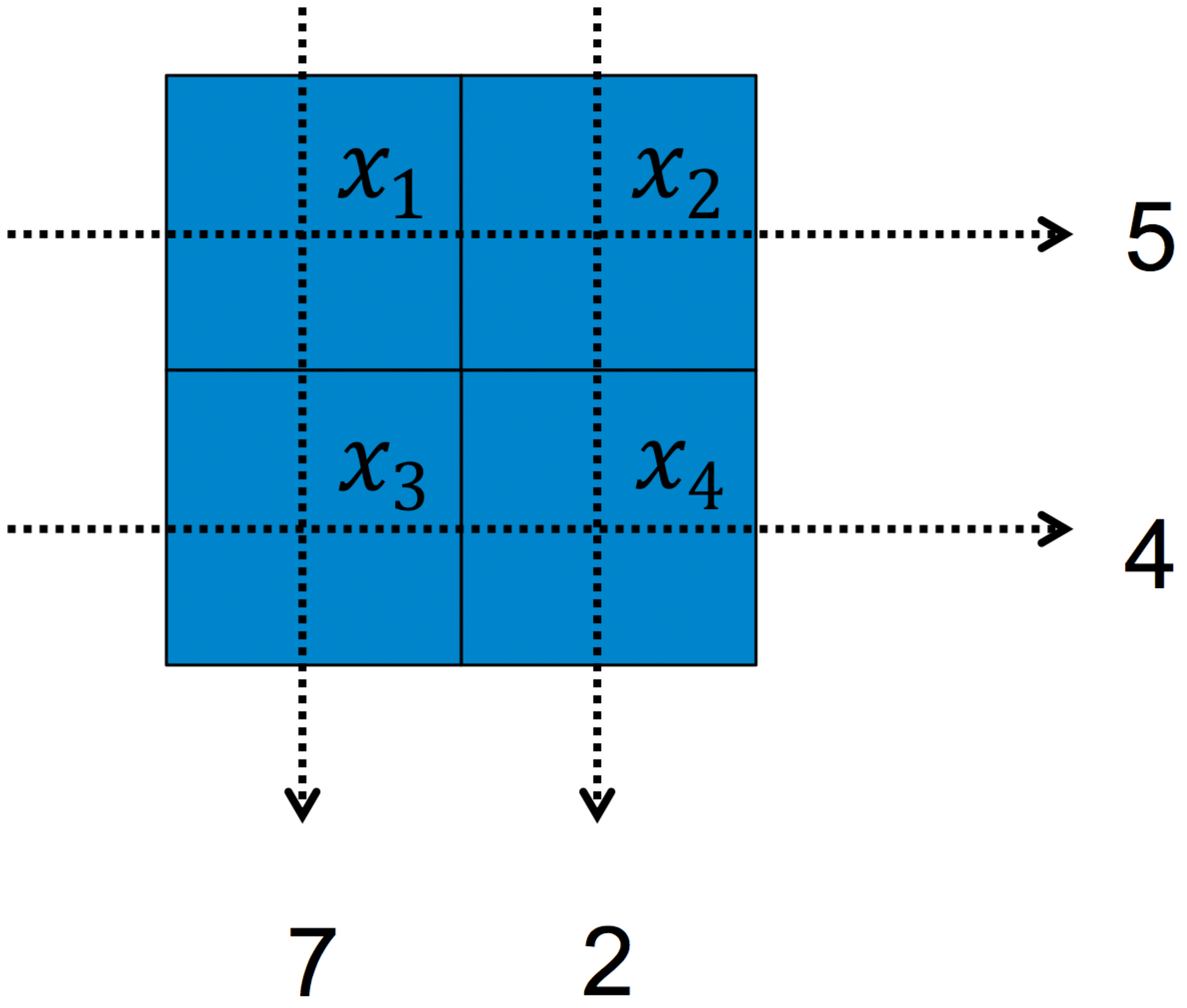
Topics

Reconstruction: Simple Example

Solve the puzzle:



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\end{aligned}
$$

$$
\Rightarrow \quad
\begin{aligned}
x_1 &= 3\\
x_2 &= 2\\
x_3 &= 4\\
x_4 &= 0
\end{aligned}
$$

Reconstruction: Simple Example

- The projection process can be formulated in matrix notation:

$$\boldsymbol{P} = \boldsymbol{A}\boldsymbol{X},$$

where

$$\boldsymbol{P} = \begin{pmatrix} 7 \\ 2 \\ 5 \\ 4 \end{pmatrix}, \quad \boldsymbol{A} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}, \quad \boldsymbol{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}.$$

- Can this be solved using the matrix inverse?

$$\boldsymbol{A}^{-1}\boldsymbol{P} = \boldsymbol{X}$$

- **Consider:** A common problem size is:

$$\boldsymbol{A} \in \mathbb{R}^{512^3 \times 512^2 \times 512},$$

which implicates

$$512^6 \cdot 4 \text{ Byte} = 2^{9 \cdot 6} \cdot 2^2 \text{ B} = 2^6 \cdot 2^{50} \text{ B} = 64 \text{ PB} = 65536 \text{ TB}.$$

Reconstruction Steps: Projection

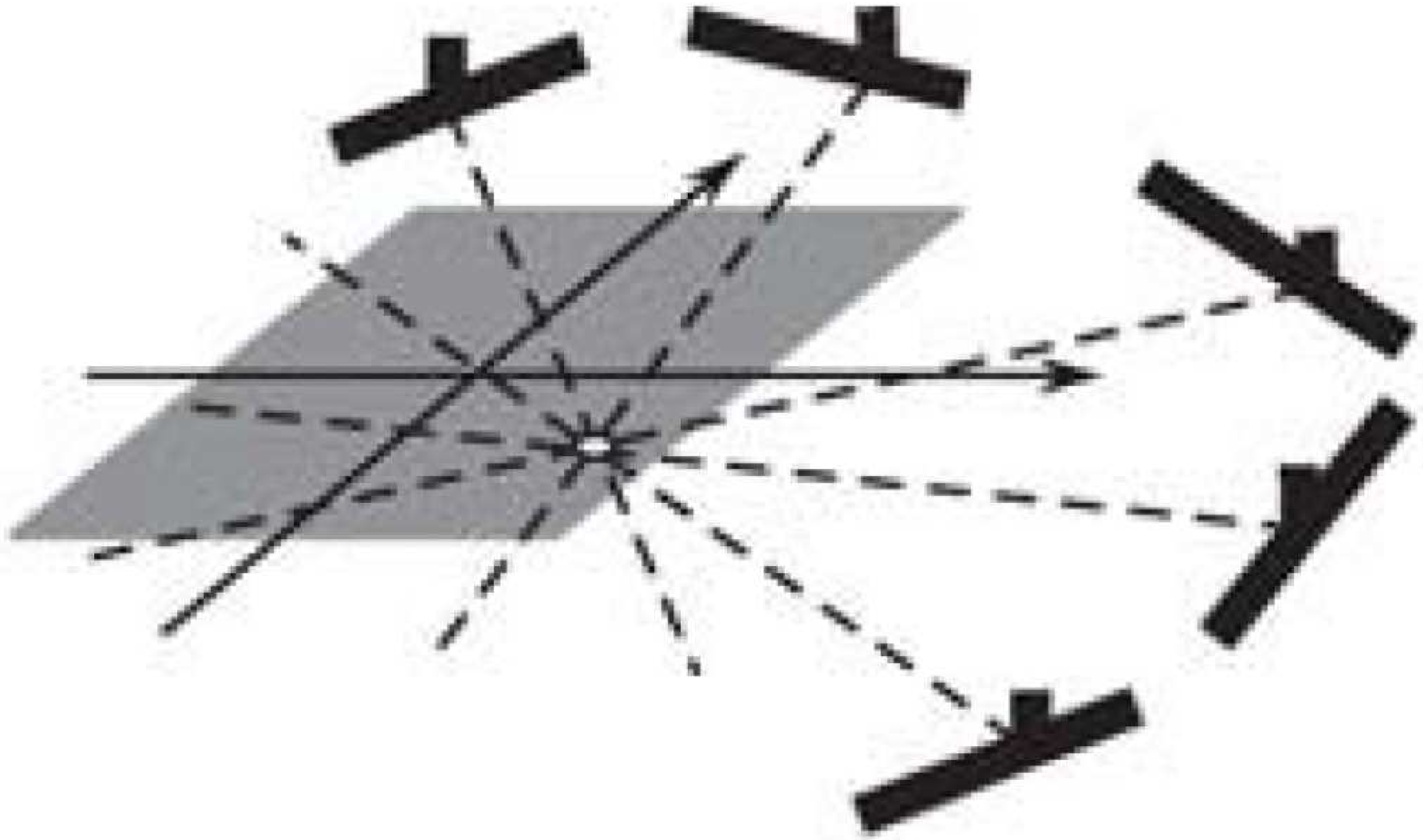


Figure 1: Schematic example for a set of projections (Zeng, 2009)

Reconstruction Steps: Backprojection (1)

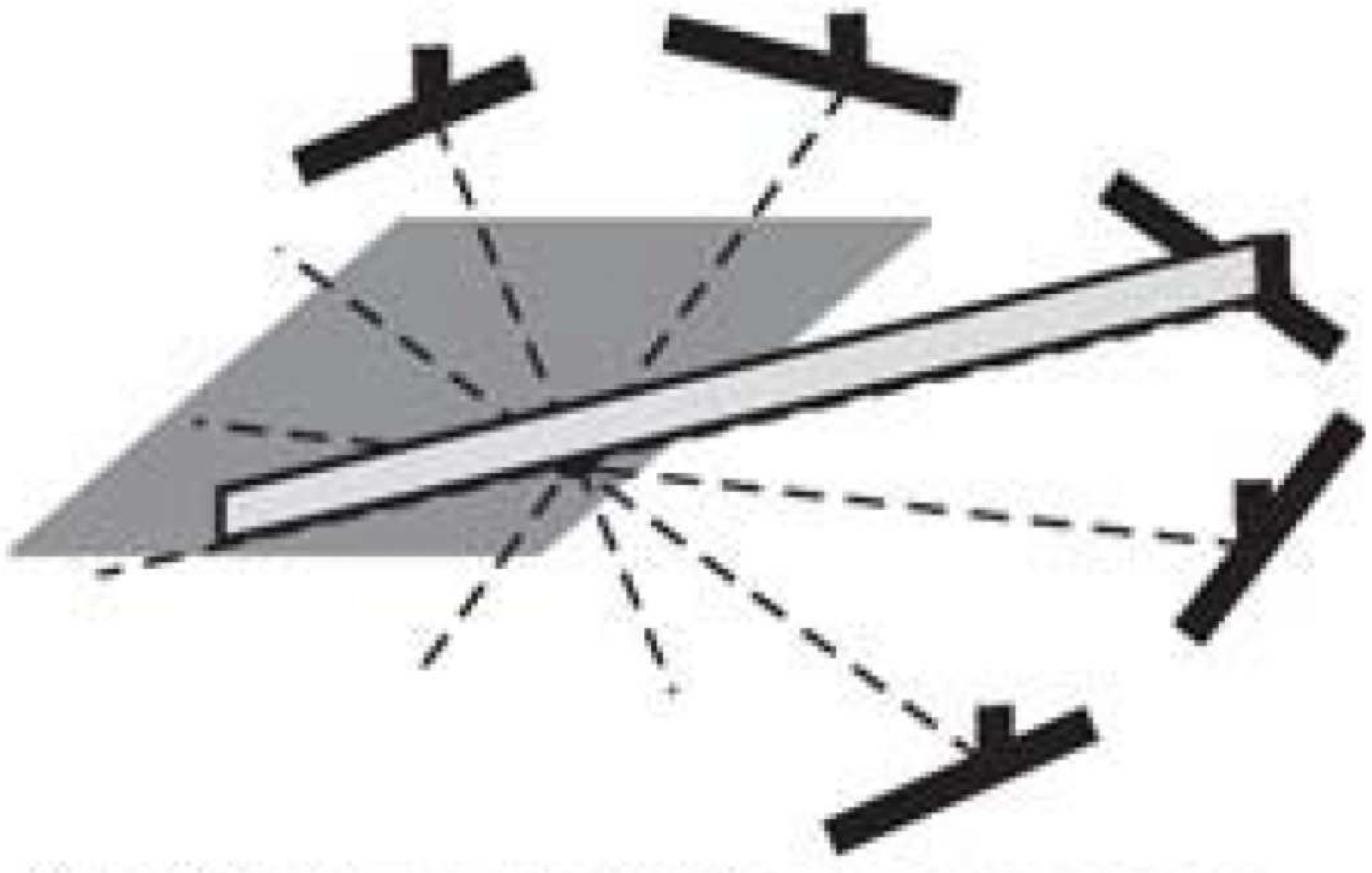


Figure 2: Schematic example for the backprojection process - one projection (Zeng, 2009)

Reconstruction Steps: Backprojection (2)

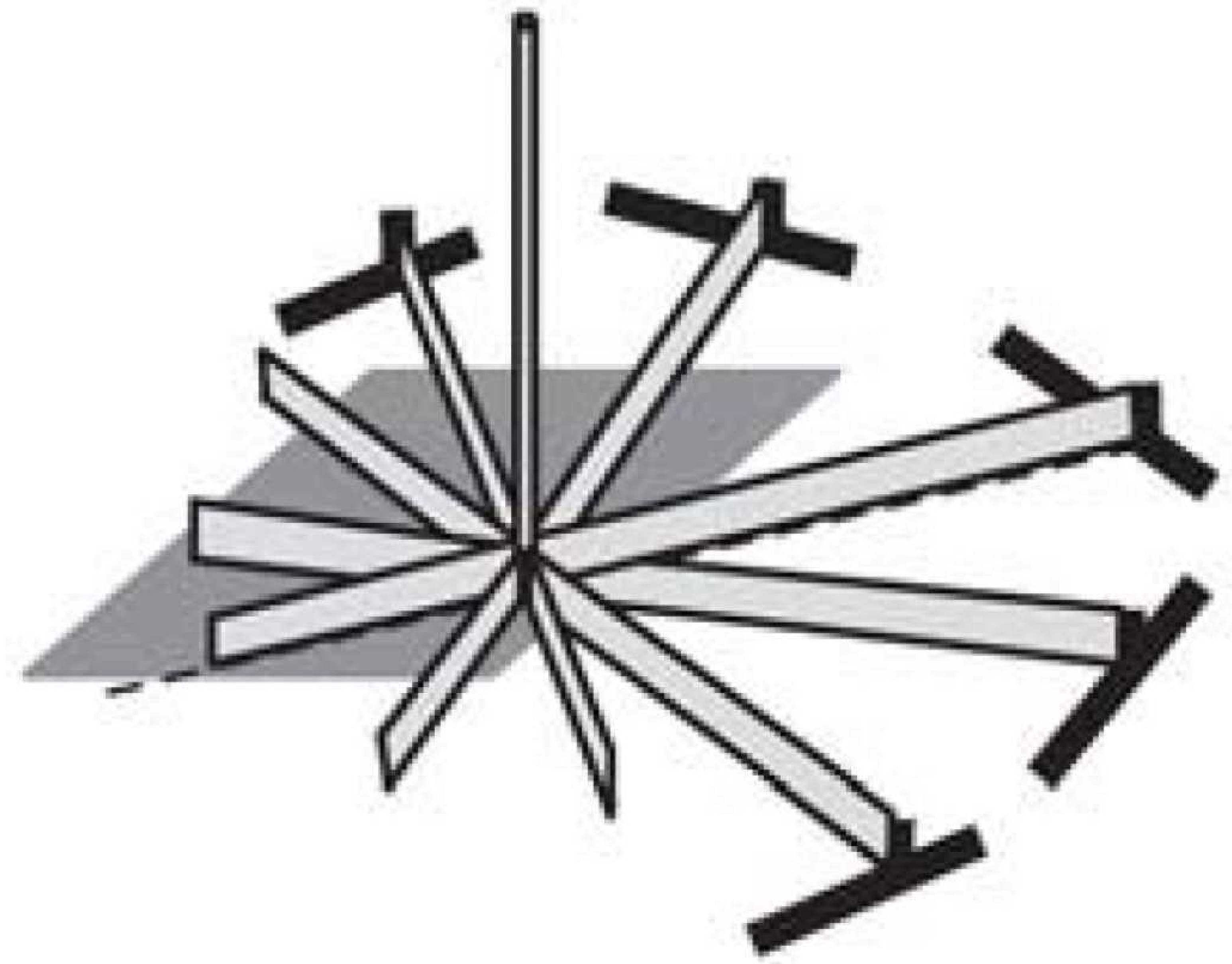


Figure 3: Schematic example for the backprojection process - multiple projections (Zeng, 2009)

Reconstruction Steps: Backprojection (3)

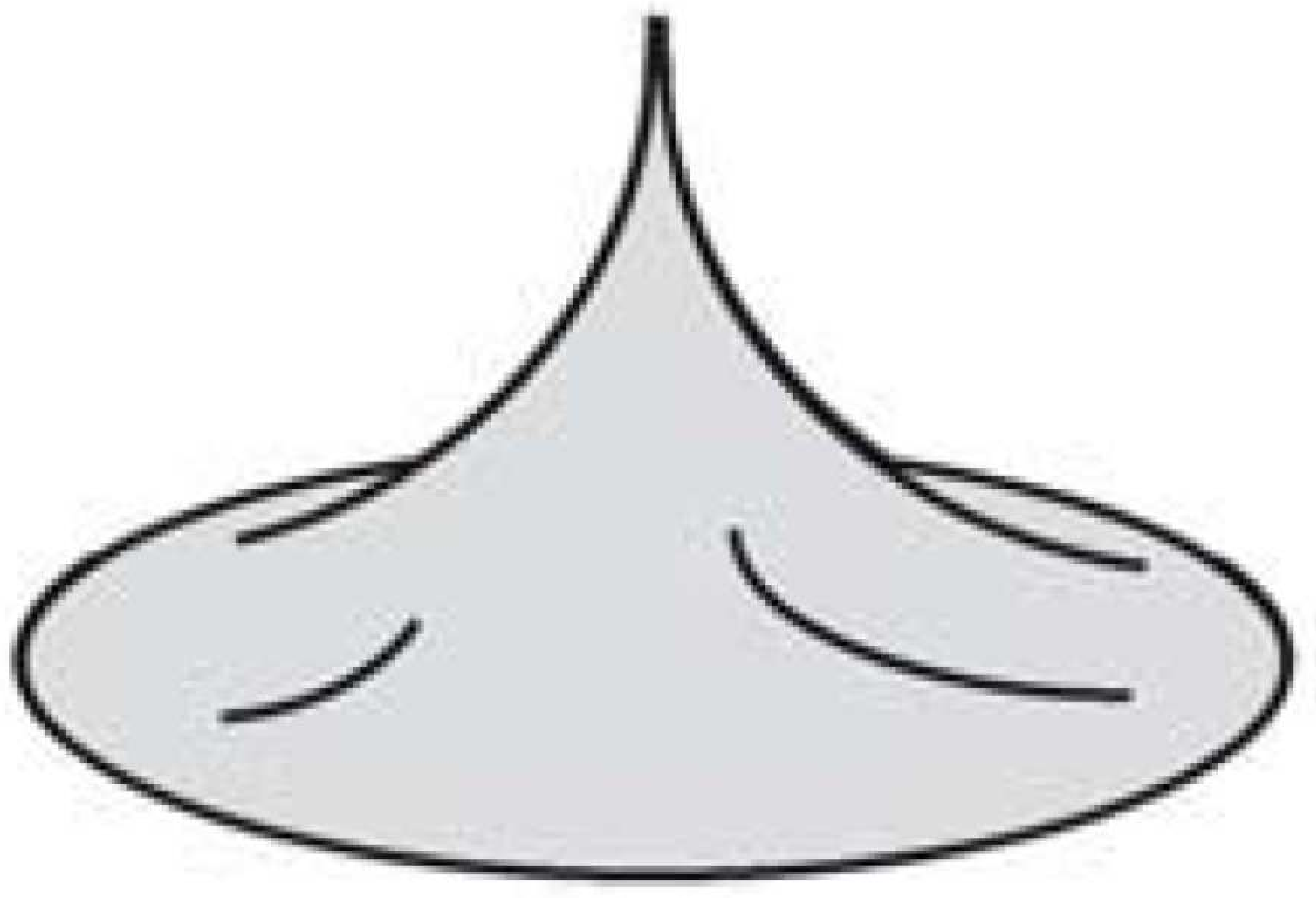


Figure 4: Schematic example for the backprojection process - infinitely many projections (Zeng, 2009)

Reconstruction Steps: “Negative Wings”

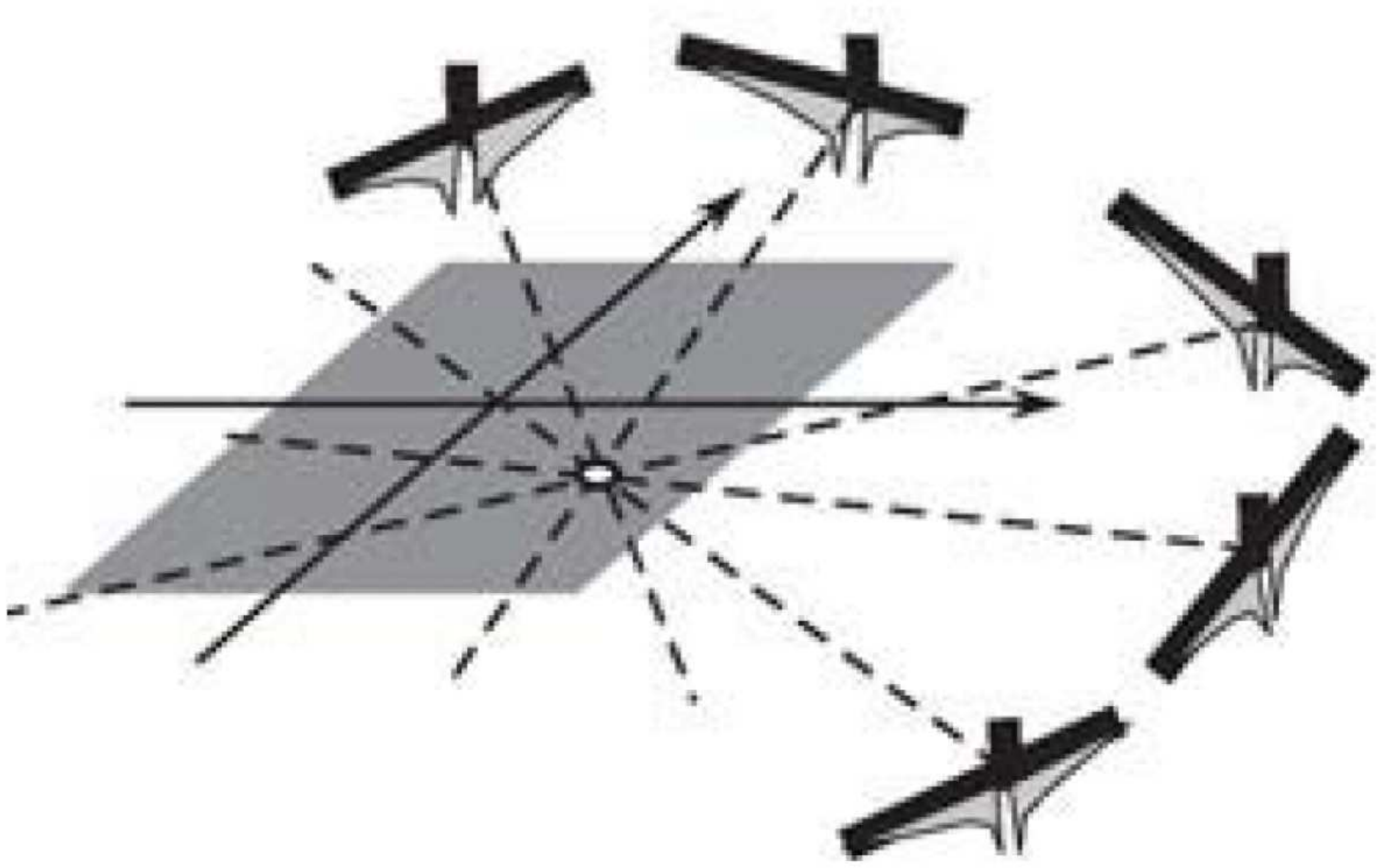


Figure 5: Schematic example for corrective filtering (Zeng, 2009)

Reconstruction Steps: Reconstruction Result

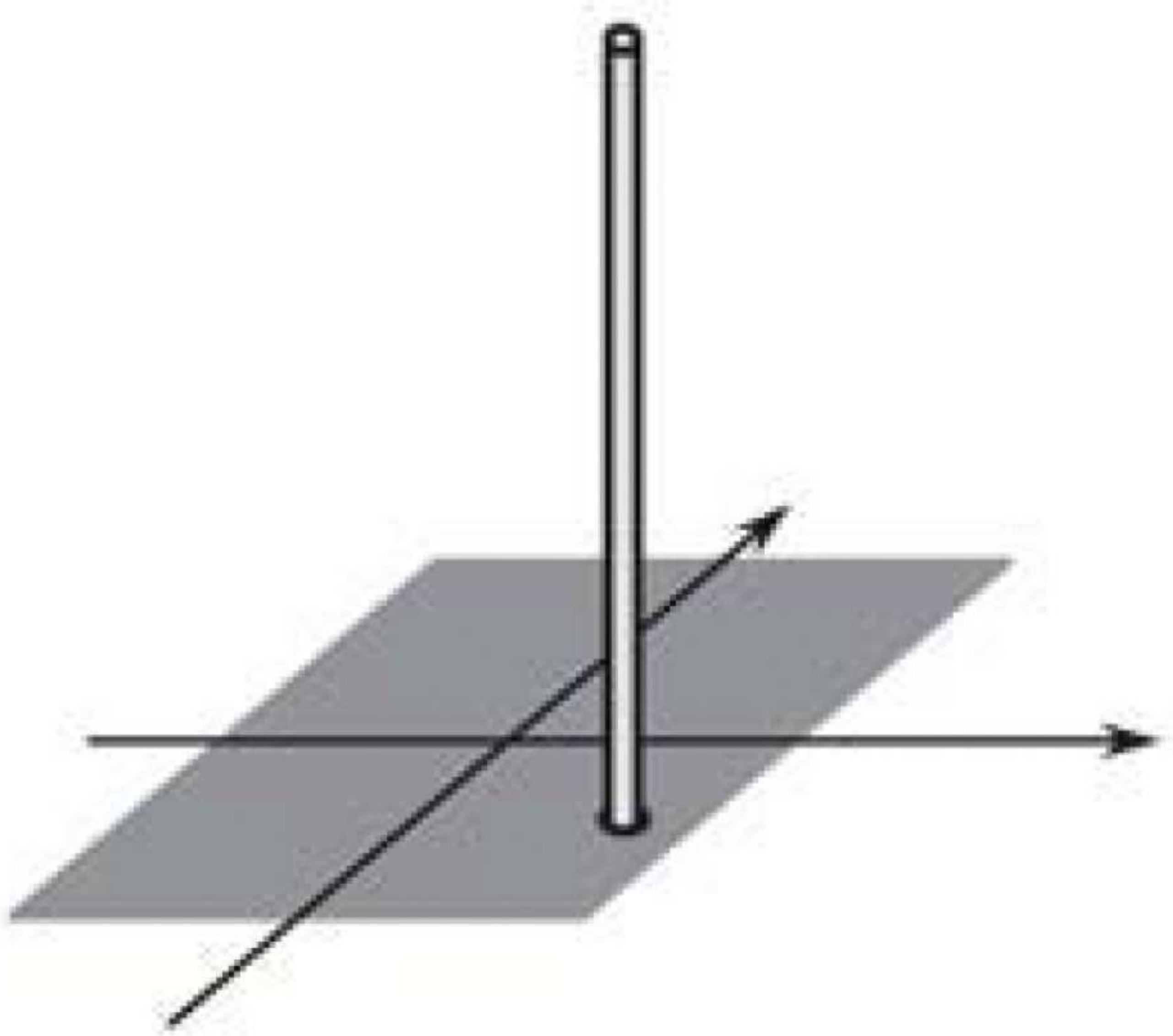
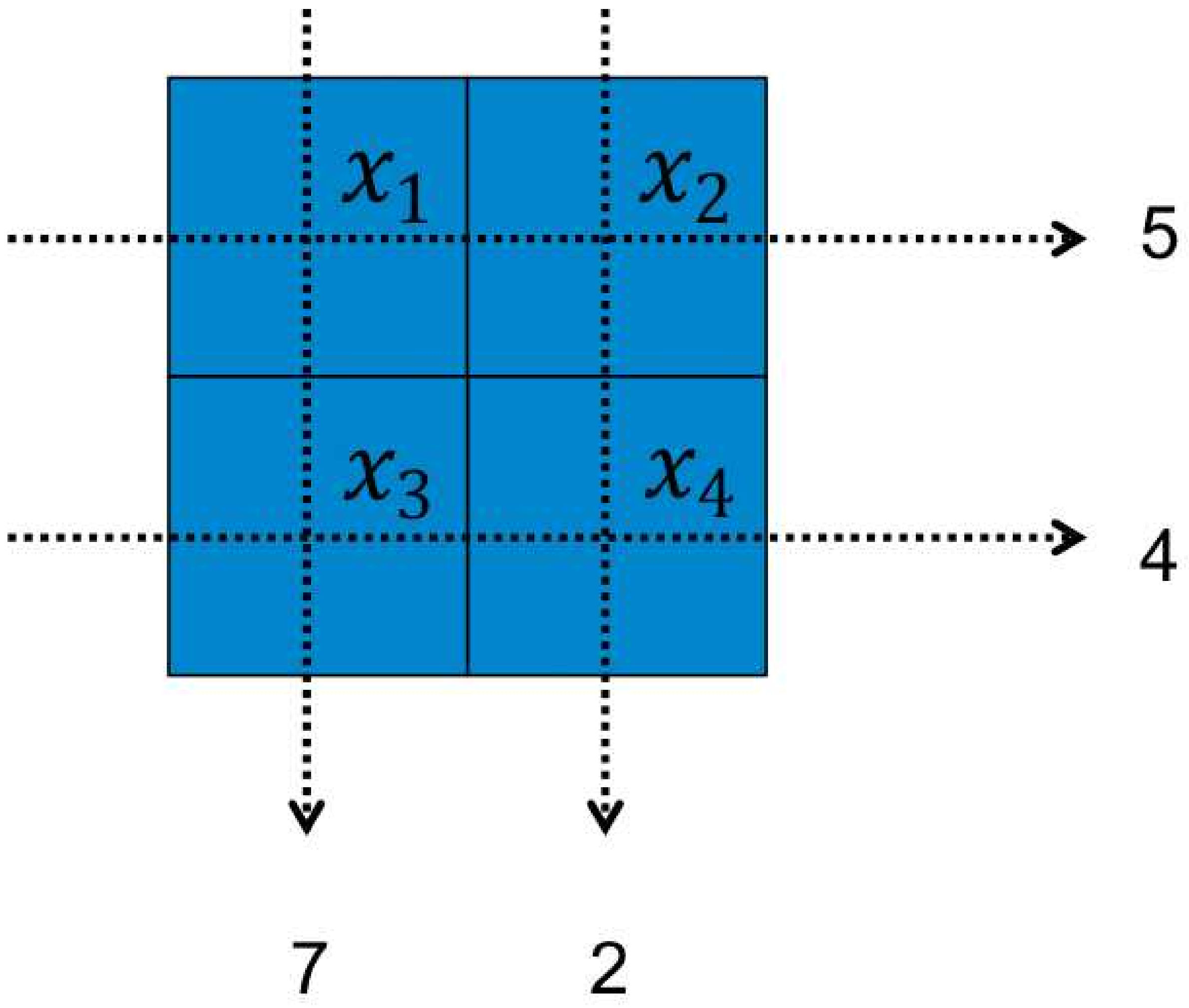


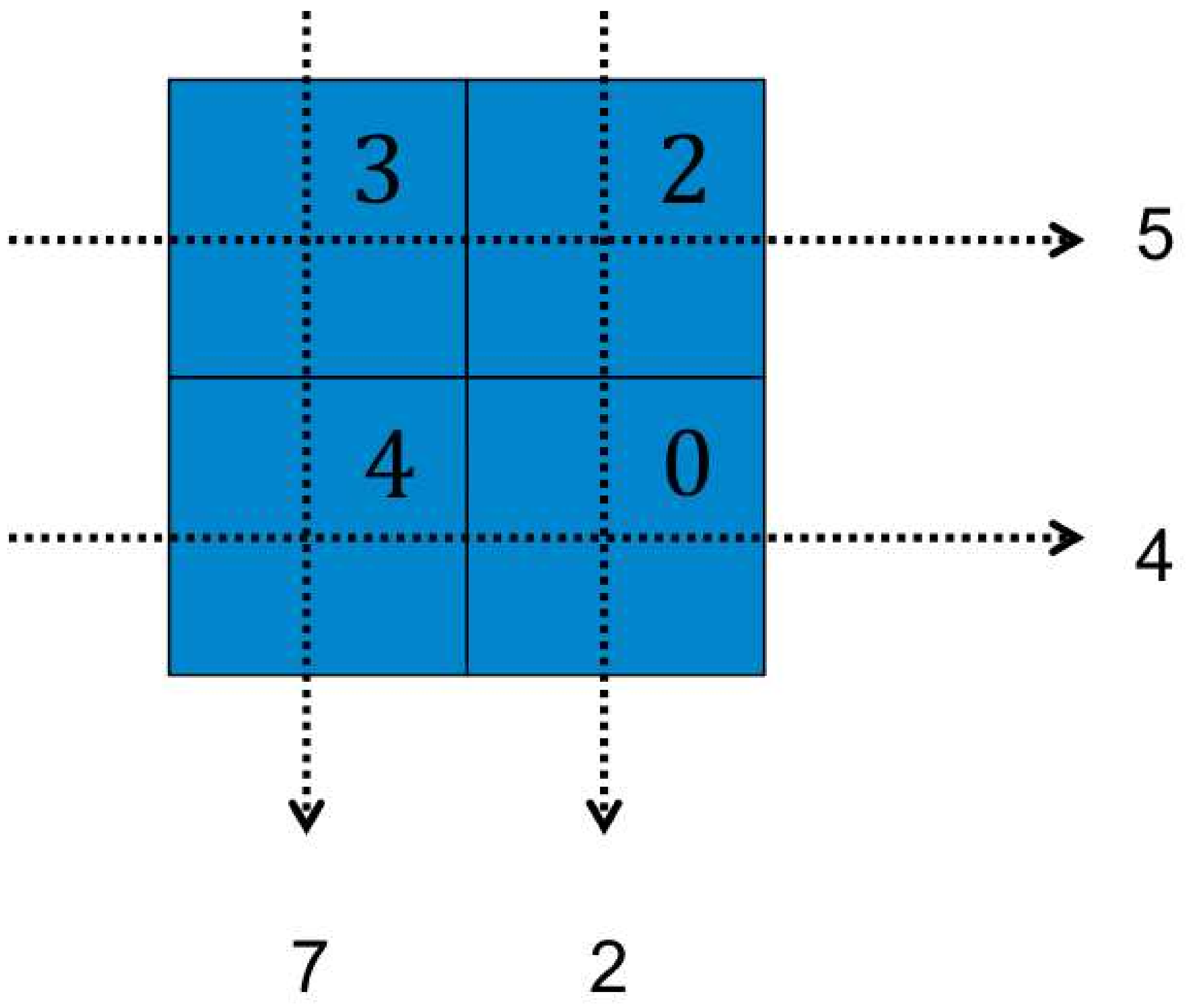
Figure 6: Schematic example for the reconstruction output (Zeng, 2009)

Topics

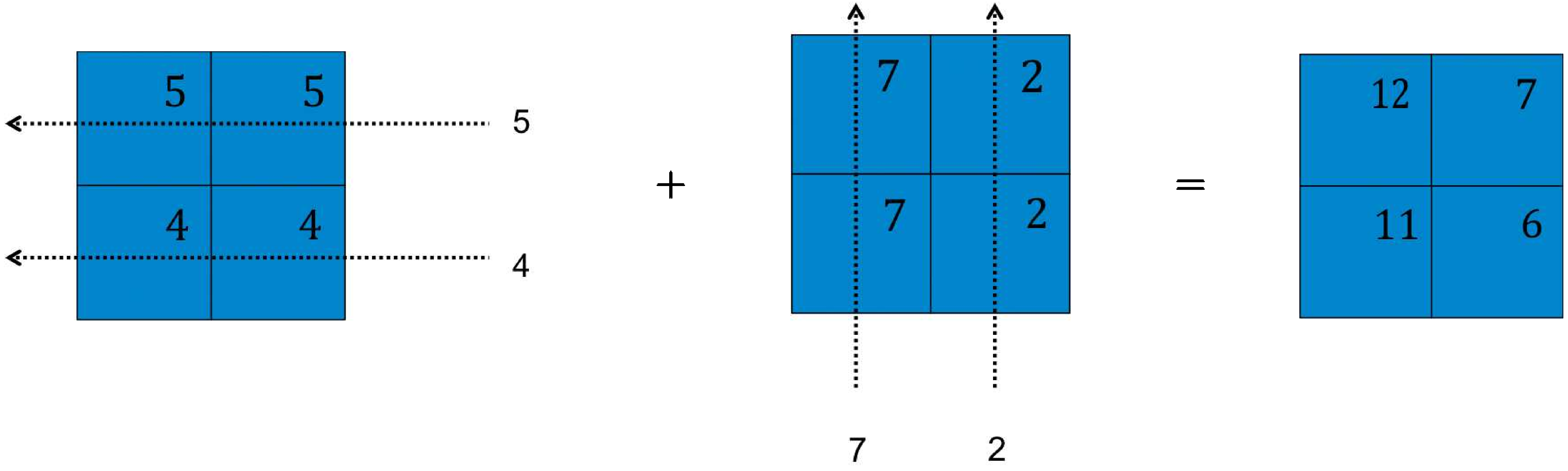
Backprojection: Simple Example



$\rightarrow$



Backprojection: Simple Example



Backprojection: Simple Example

- Backprojection is not the inverse of projection!
- In matrix notation, it is simply the matrix transpose:

$$\boldsymbol{B} = \boldsymbol{A}^T \boldsymbol{P},$$

where

$$\boldsymbol{A}^T = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \quad \boldsymbol{P} = \begin{pmatrix} 7 \\ 2 \\ 5 \\ 4 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} 12 \\ 7 \\ 11 \\ 6 \end{pmatrix}.$$

Backprojection: Mathematical Formulation

The following equivalent formulations are employed in literature:

$$b(x,y) = \int_0^{\pi} p(s,\theta)|_{s=x\cos\theta+y\sin\theta}\,\mathrm{d}\theta,$$

$$b(x,y) = \int_0^{\pi} p(s,\theta)|_{s=\boldsymbol{x}\cdot\boldsymbol{\theta}}\,\mathrm{d}\theta,$$

$$b(x,y) = \int_0^{\pi} p(\boldsymbol{x}\cdot\boldsymbol{\theta},\theta)\,\mathrm{d}\theta,$$

$$b(x,y) = \frac{1}{2}\int_0^{2\pi} p(x\cos\theta+y\sin\theta,\theta)\,\mathrm{d}\theta.$$

Topics

Take Home Messages

- Reconstruction involves several steps: projection, backprojection, and filtering.
- Backprojection is not the inverse of projection, but just the transpose.

Further Readings

Students learning about reconstruction should have a look at one of the following books:

- Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9
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About the History of CT

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Topics

Short History of CT

Development of the Geometry

Further developments

Summary

Take Home Messages

Further Readings

Parallel Beam Geometry

- Earliest acquisition geometry
- **Principle:** "Rotate & Translate"

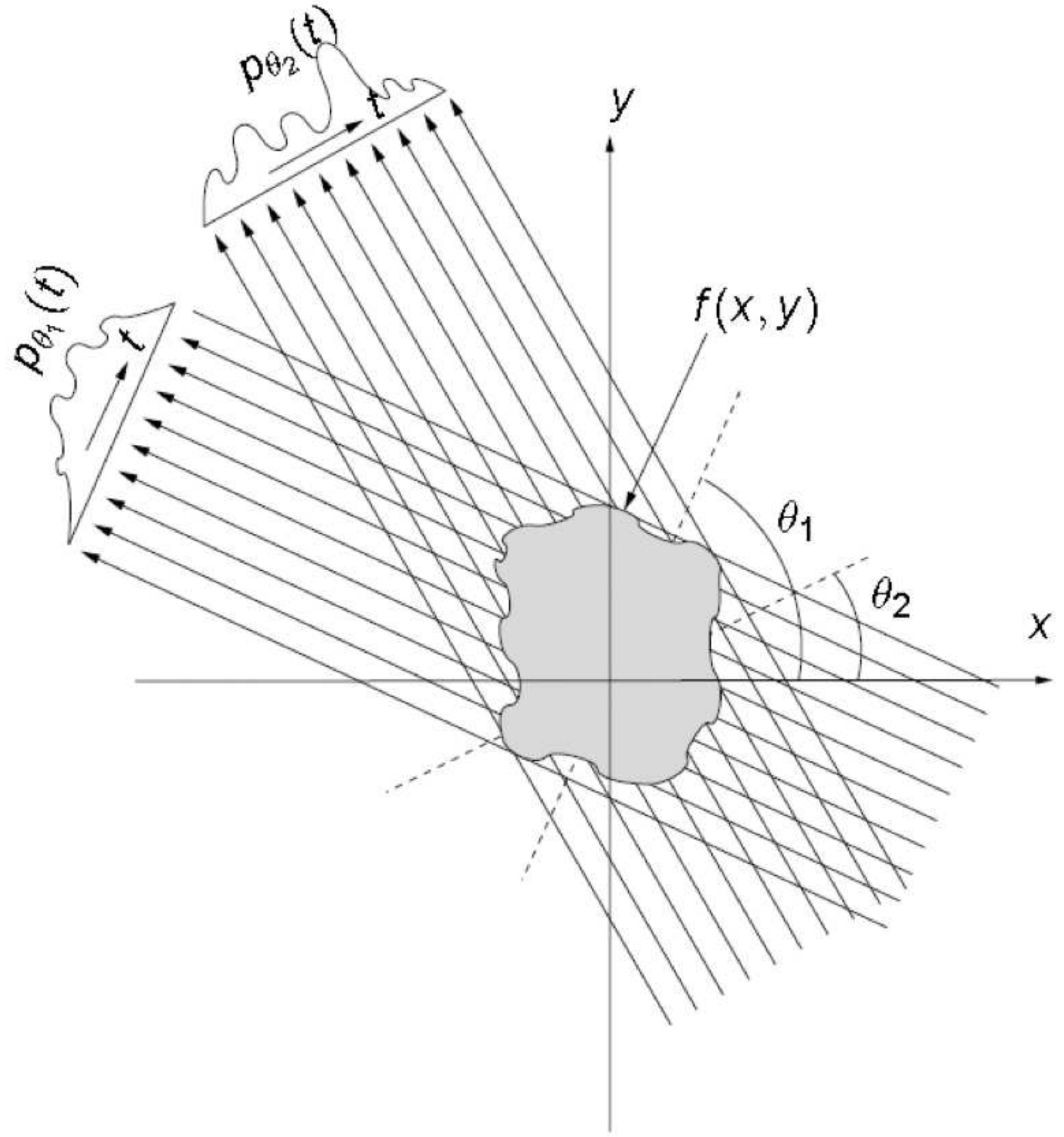


Figure 1: Parallel projection scheme with two different angles θ_1, θ_2 and the object $f(x, y)$

Parallel Beam Geometry

First CT scanner by EMI (1971)

- Acquisition took 5 minutes.
- Reconstruction took 30 minutes.
- Slice resolution was 80 $\times$ 80 pixels.

Figure 2: Image of the first commercial CT scanner model (Wikipedia)

Fan Beam Geometry

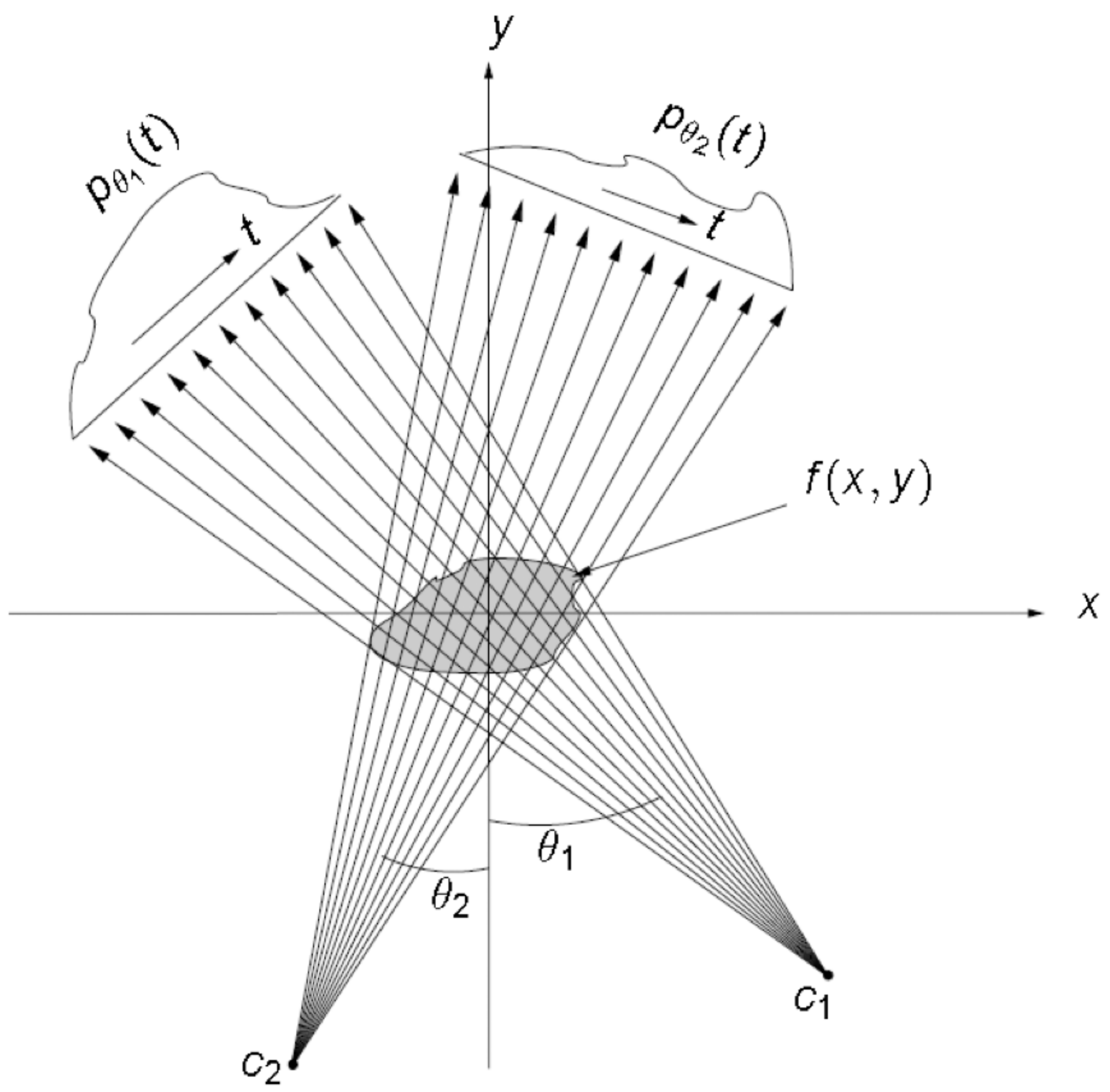
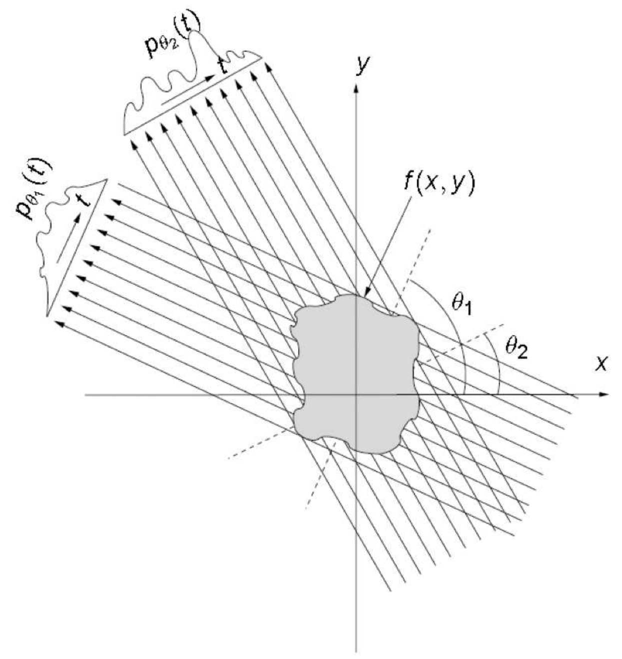


Figure 3: Fan beam projection scheme with two different angles θ_1, θ_2 and the object $f(x,y)$



Fan Beam Geometry

- Fan beam scanners became available in 1975 (20 s / slice).
- Fast rotations became possible 1987 with slip rings (300 ms / slice).

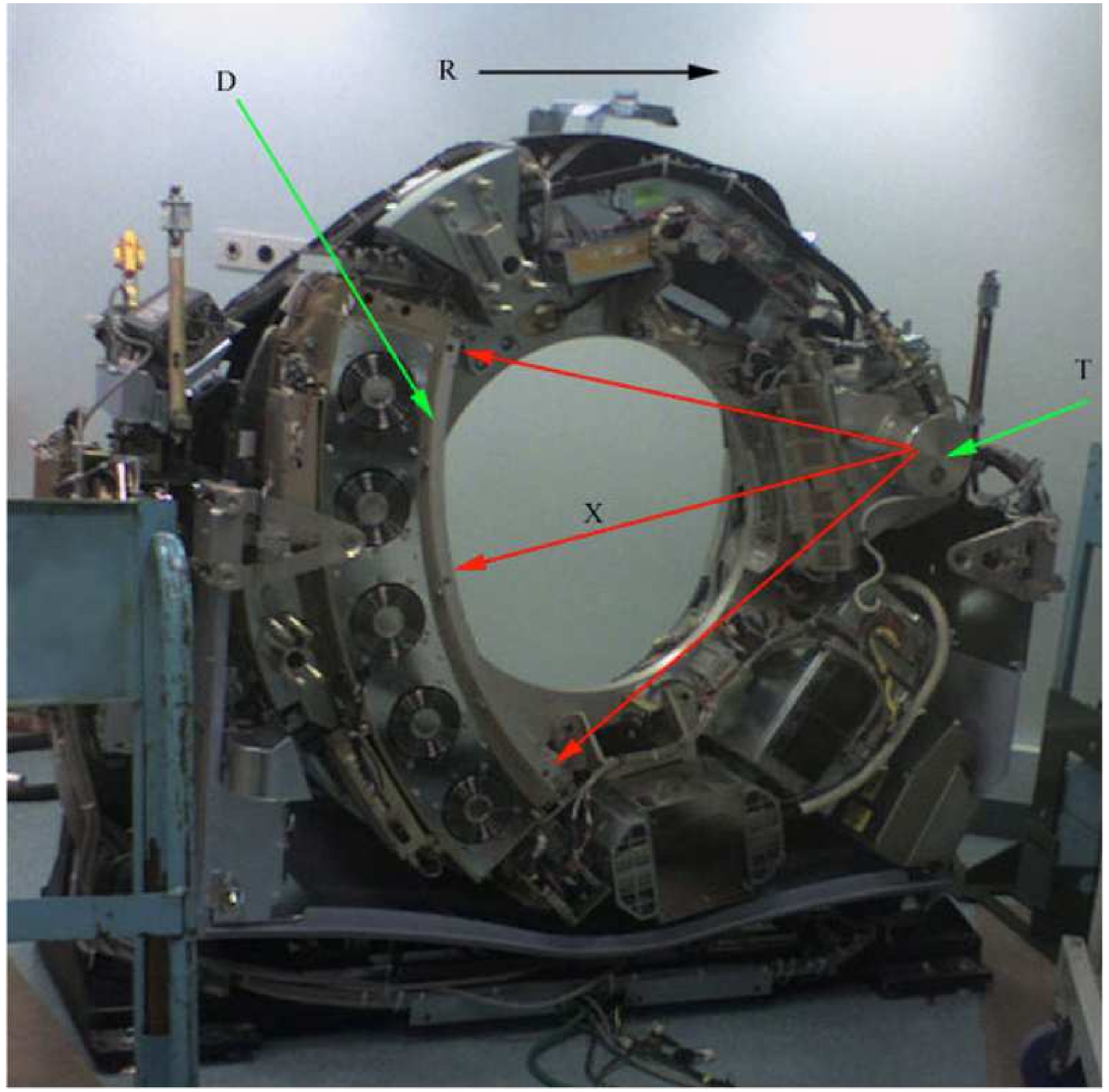
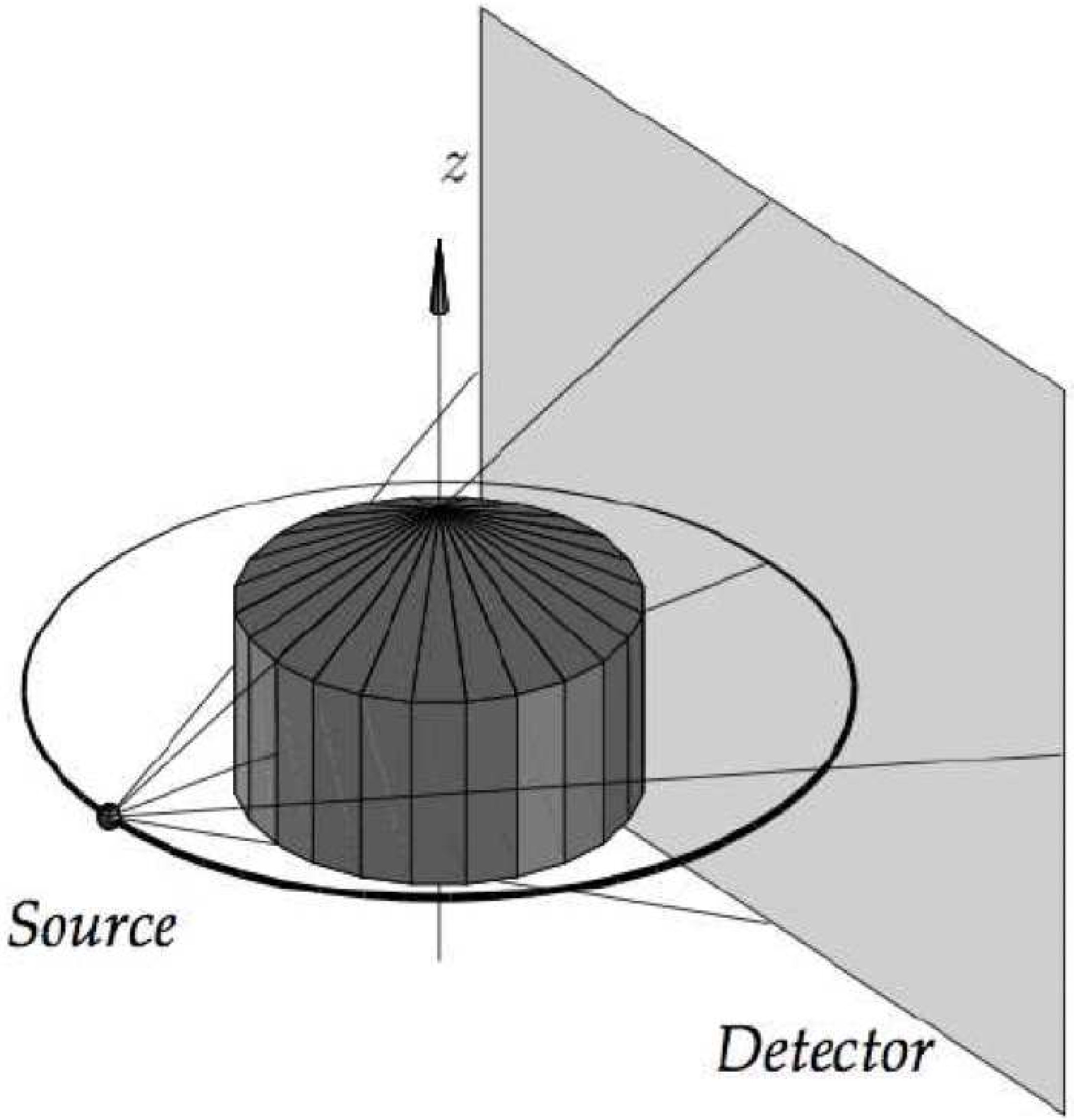


Figure 4: View inside a CT scanner (Wikipedia, GFDL)

Cone Beam Geometry



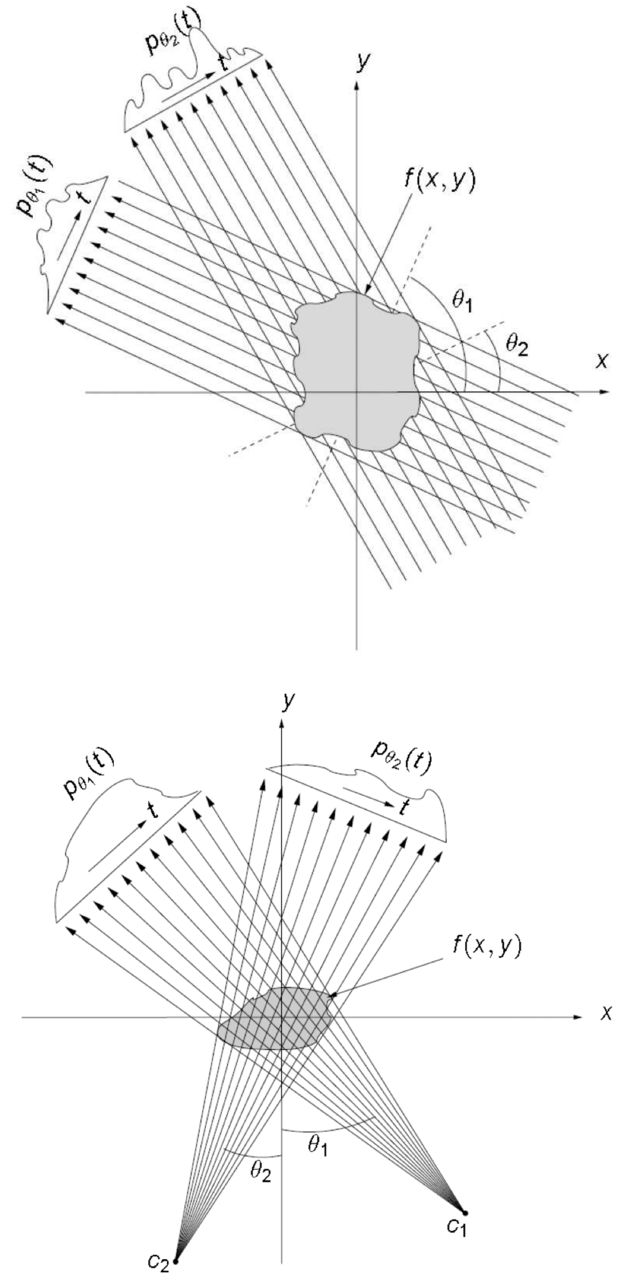


Figure 5: Cone beam projection scheme

Cone Beam Geometry

- Further increase in the number of rows did not take place so far.
- Physical effects such as scattered radiation currently limit the number of detector rows in CT.
- Flat panel detector technologies have even larger cone angles.

Figure 6: 320 Row Scanner by Toshiba (2007) (image courtesy of Toshiba)

3-D Reconstruction in Dual CT

- Dual source CT introduced 2005
- Fast scanning (75 ms)
- Material decomposition possible

Figure 7: Dual CT scanner (image courtesy of Siemens AG)

3-D Reconstruction in Dental Medicine

Figure 8: Introduced in October 2006 (image courtesy of Planmeca Oy)

3-D Reconstruction in the Angio Lab

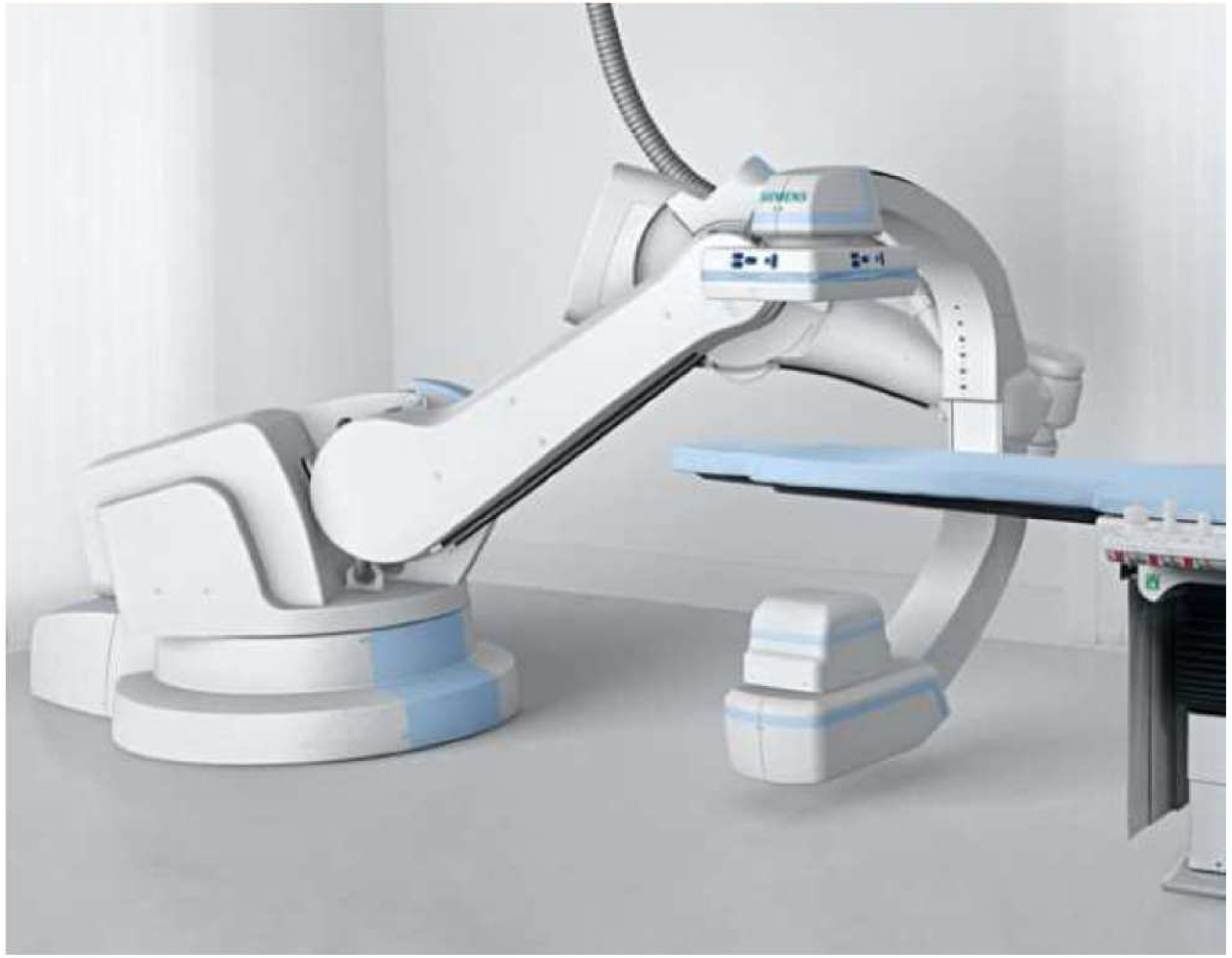


Figure 9: C-arm mounted on a robot system (November 2007) (image courtesy of Siemens AG)

3-D Reconstruction in the Neuro Lab

Figure 10: C-arm biplane device (image courtesy of Siemens AG)

Topics

Take Home Messages

- Over the years the geometries used for tomography developed from parallel beam and fan beam to cone beam geometries.
- Meanwhile CT scanners and thus 3-D reconstruction can be found in many different medical fields.

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