Medical Image Processing for Diagnostic Applications

Parallel Beam – Filtered Backprojection

Online Course – Unit 31

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Topics

Idea for Reconstruction

Filtered Backprojection

Summary

- Take Home Messages
- Further Readings

Idea for Reconstruction

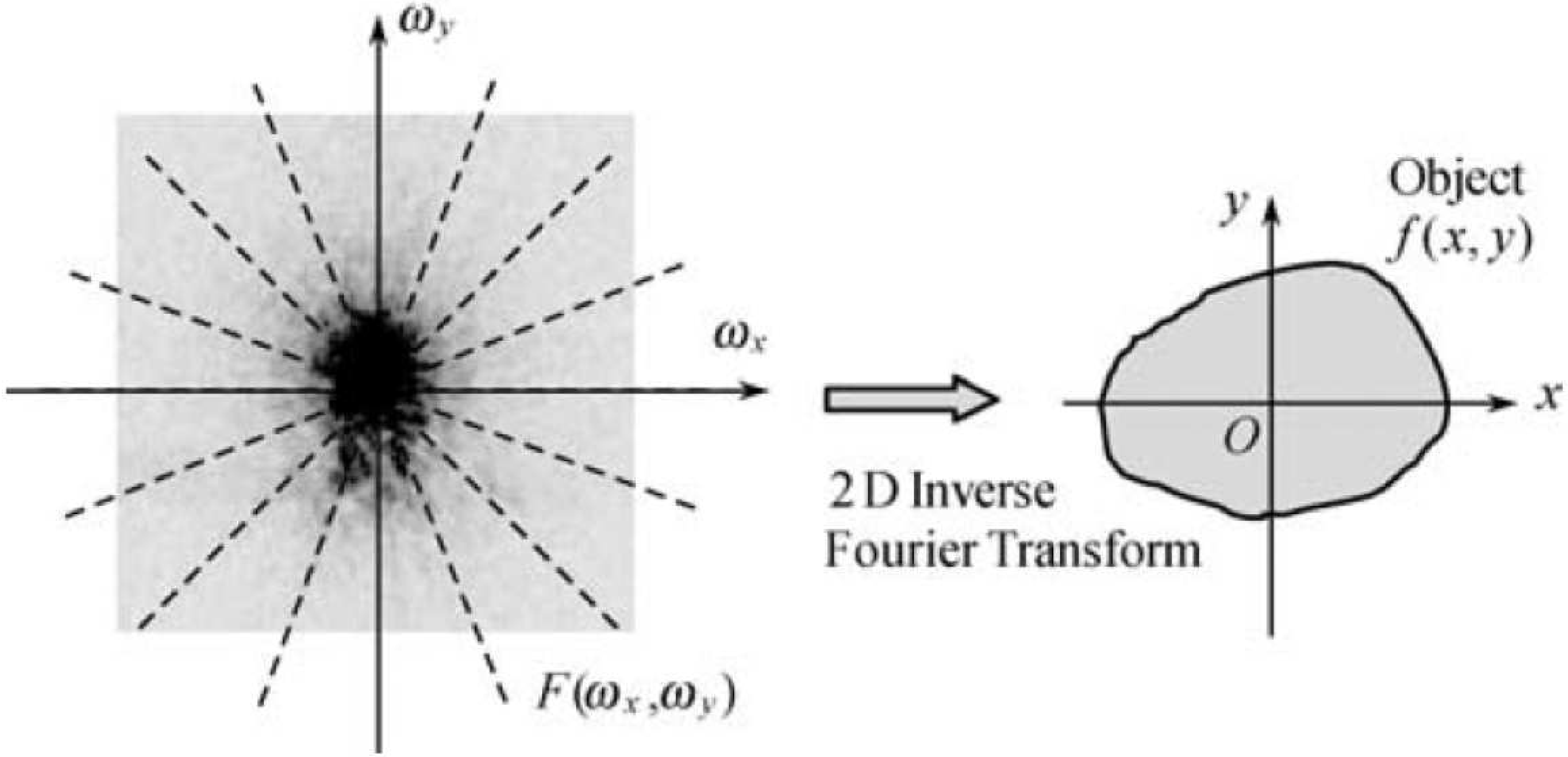

Figure 1: By projections the Fourier space is sampled, by inverse Fourier transform an image of the object can be reconstructed (Zeng, 2009).

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The inverse Fourier transform of the 2-D Fourier measurement $F(u, v)$:

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{2\pi i(ux+vy)} \mathrm{d}u \mathrm{d}v$$

can be written in polar coordinates:

$$f(x, y) = \int_{0}^{\pi} \int_{-\infty}^{\infty} F_{\text{polar}}(\omega, \theta) |\omega| e^{2\pi i\omega(x\cos\theta + y\sin\theta)} \mathrm{d}\omega \mathrm{d}\theta.$$

According to the Fourier slice theorem $P(\omega, \theta) = F(\omega\cos\theta, \omega\sin\theta) = F_{\text{polar}}(\omega, \theta)$ this yields:

$$f(x, y) = \int_{0}^{\pi} \int_{-\infty}^{\infty} P(\omega, \theta) |\omega| e^{2\pi i\omega(x\cos\theta + y\sin\theta)} \mathrm{d}\omega \mathrm{d}\theta.$$

Filtered Backprojection

The inner integral in the last equation:

$$f(x,y) = \int_0^{\pi} \left(\int_{-\infty}^{\infty} P(\omega,\theta)|\omega| e^{2\pi i\omega(x\cos\theta + y\sin\theta)} \mathrm{d}\omega \right) \mathrm{d}\theta$$

represents the 1-D inverse Fourier transform of the product $P(\omega,\theta)|\omega|$.
According to the convolution theorem this corresponds to a convolution in spatial domain:

$$f(x,y) = \int_0^{\pi} p(s,\theta) * h(s)|_{s=x\cos\theta + y\sin\theta} \mathrm{d}\theta,$$

where $h(s)$ denotes the corresponding inverse Fourier transform of $|\omega|$.

Filtered Backprojection: Practical Algorithm

1. Apply filter on the detector row:

$$q(s,\theta) = p(s,\theta) * h(s).$$

2. Backproject $q(s,\theta)$:

$$f(x,y) = \int_0^\pi q(s,\theta)\big|_{s=x\cos\theta+y\sin\theta}\,\mathrm{d}\theta.$$

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Take Home Messages

- The central slice theorem allows a very practical reconstruction algorithm for parallel beam geometry.
- The workflow includes filtering on the detector rows and successive backprojection.

Further Readings

The derivation of the filtered backprojection formula can also be found here (bibsource):

Joachim Hornegger, Andreas Maier, and Markus Kowarschik. “CT Image Reconstruction Basics”. In: *MR and CT Perfusion and Pharmacokinetic Imaging: Clinical Applications and Theoretical Principles*. Ed. by Roland Bammer. 1st ed. Alphen aan den Rijn, Netherlands: Wolters Kluwer, 2016, pp. 01–09

The concise reconstruction book from ‘Larry ’Zeng:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

If you want to learn more about applications of the Fourier transform:

Ronald N. Bracewell. *The Fourier Transform and Its Applications*. 3rd ed. Electrical Engineering Series. Boston: McGraw-Hill, 2000