# Medical Image Processing for Diagnostic Applications

## Cone Beam Reconstruction – Katsevich's Algorithm

Online Course – Unit 49

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# Topics

Katsevich's Algorithm

Summary

Take Home Messages

Further Readings

# Katsevich's Algorithm ...

- ... was first developed for helical trajectories,
- ... was later expanded to more general orbits.
- ... is in the form FBP,
- ... involves filtering that can be made shift-invariant, i. e., independent of the reconstruction location.

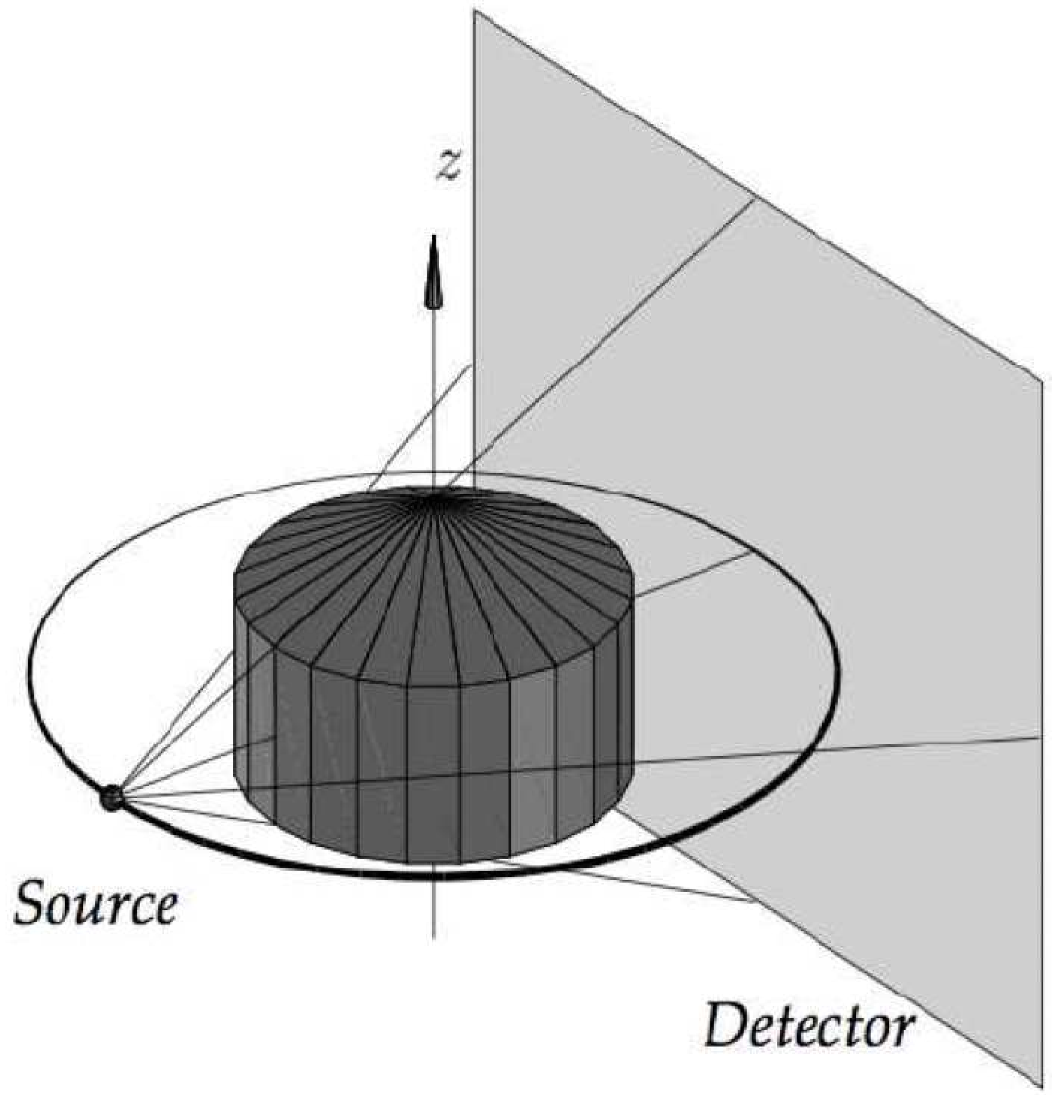


Figure 1: Cone beam scheme

# Katsevich's Algorithm: Geometry

- For all points inside the helix there is one line that passes the point and hits the helix at two points that are separated by less than one pitch.
- This line is called a $\pi$-line or $\pi$-segment.
- If redundant data occurs, it occurs three times.
- Redundancy is solved by assigning the weights $1, -1, 1$.

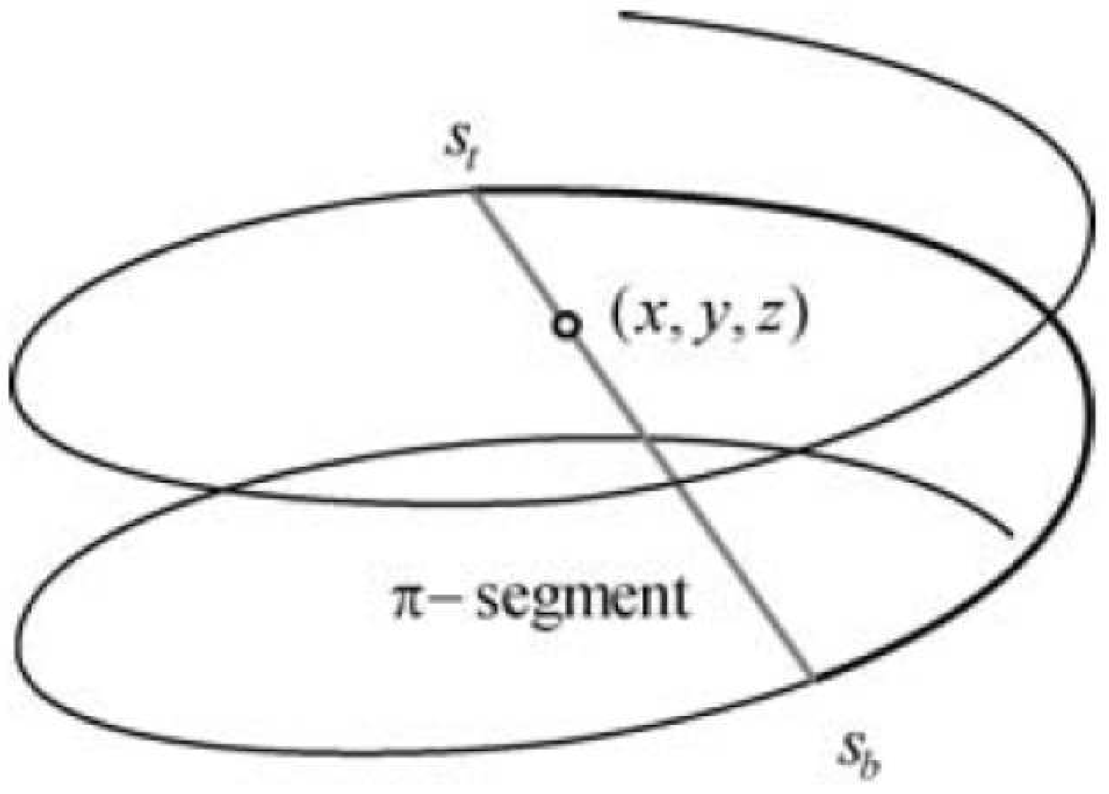


Figure 2: A $\pi$-segment intersects the helix at least twice (Zeng, 2009).

# Katsevich's Algorithm: Concept

Compute the derivative along the trajectory $\boldsymbol{a}(s)$ and filter along direction $\beta$:

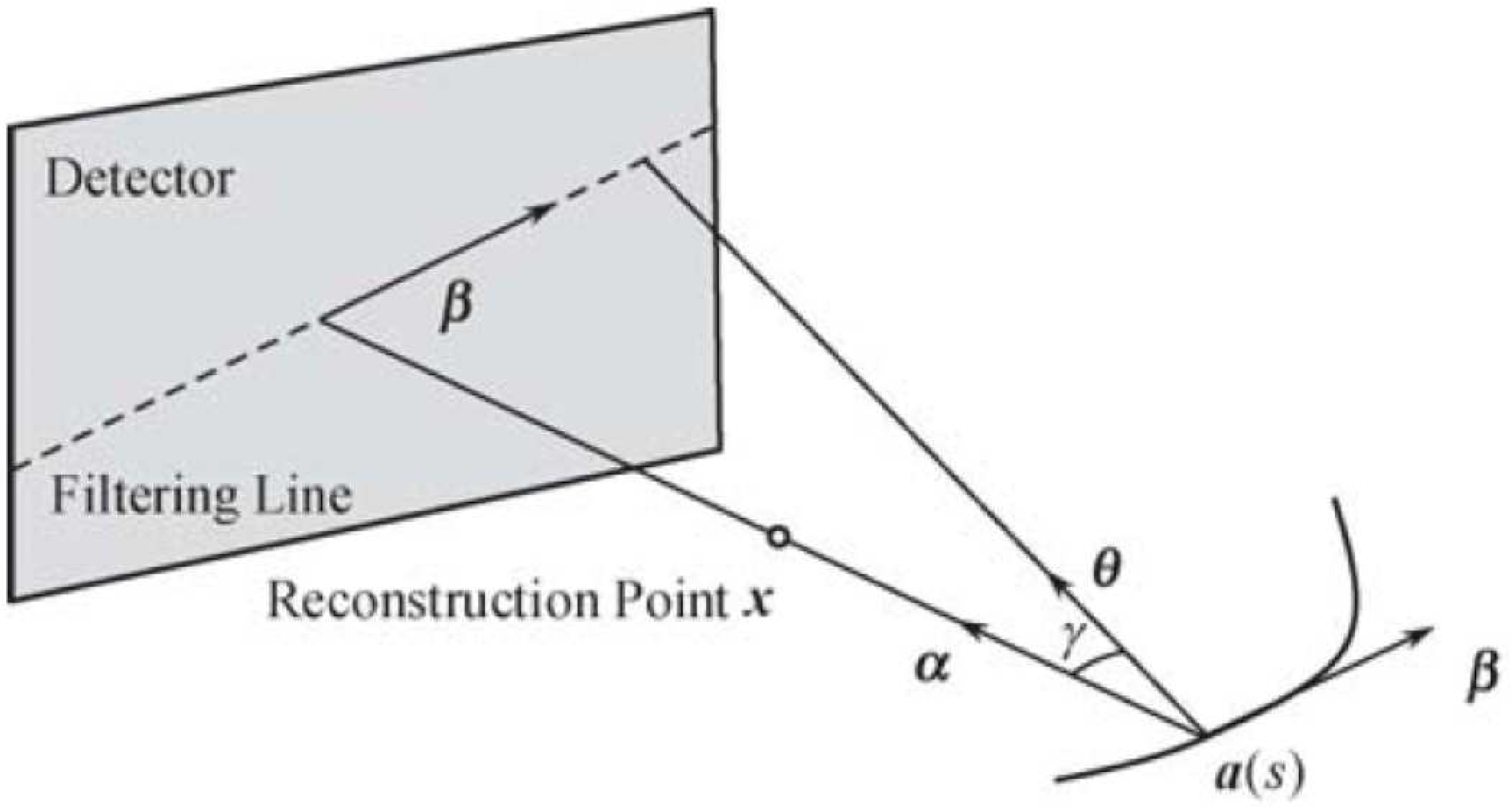


Figure 3: Direction of filtering (Zeng, 2009)

# Katsevich's Algorithm: Derivative

Compute the derivative along the trajectory as discrete difference between two neighboring projections:

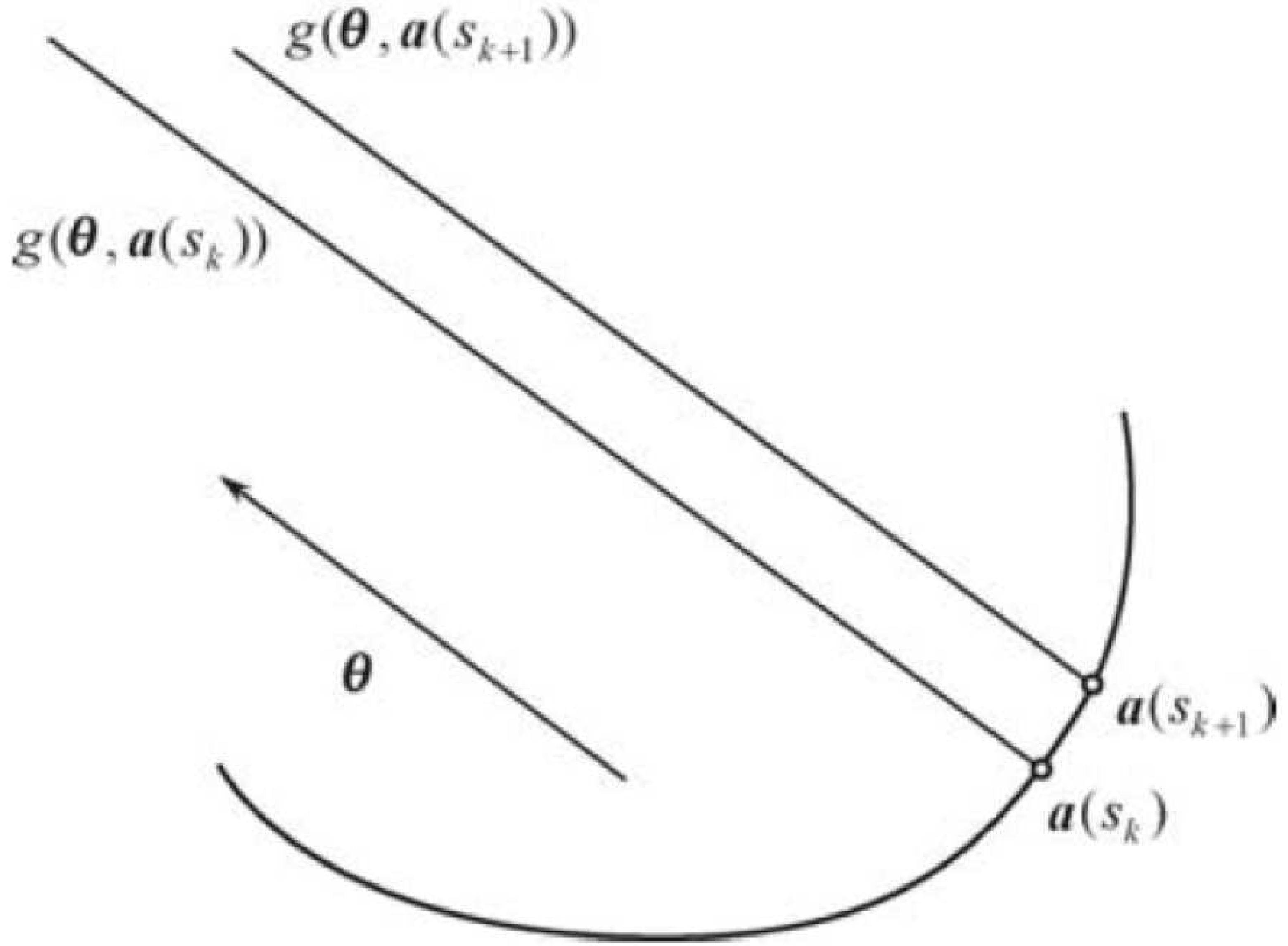


Figure 4: Computation of the directional derivative (Zeng, 2009)

# Katsevich's Algorithm: Weighting

Perform weighting of the projection data with

$$\frac{D}{\sqrt{D^2 + w^2}}$$

where $D$ is the source detector distance, and $w$ the axis of the detector that points along the rotation axis.

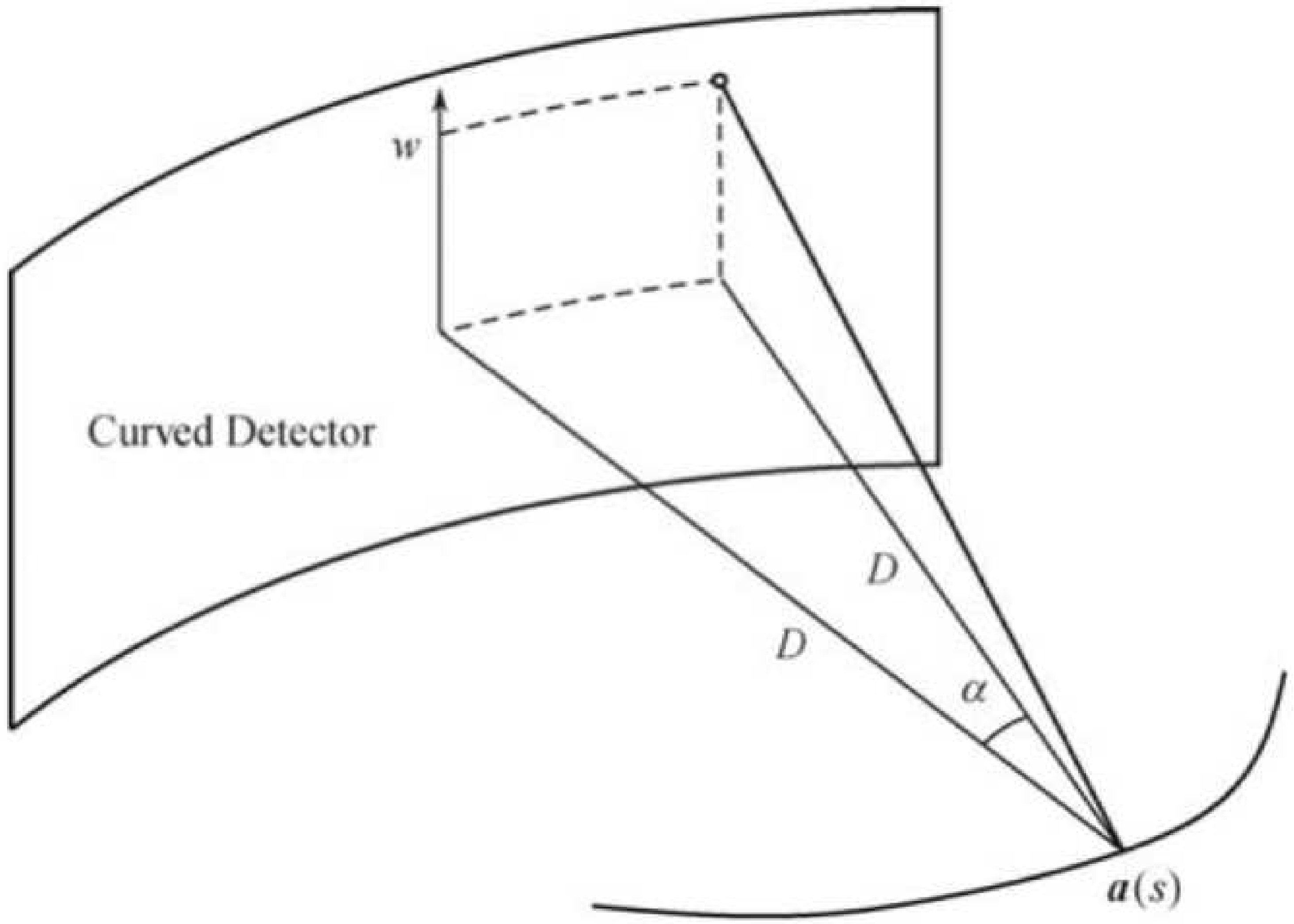


Figure 5: Weighting scheme (Zeng, 2009)

# Katsevich's Algorithm: Choosing $\beta$

- Compute derivative along the trajectory $\boldsymbol{a}(s)$.
- Choose $\beta$ as the angle of a plane $\kappa$ that contains the points $\boldsymbol{x}$, $\boldsymbol{a}(s)$, $\boldsymbol{a}(s+\psi)$, and $\boldsymbol{a}(s+2\psi)$.
- This is not unique. Hence, choose $|\psi|$ to be minimal.

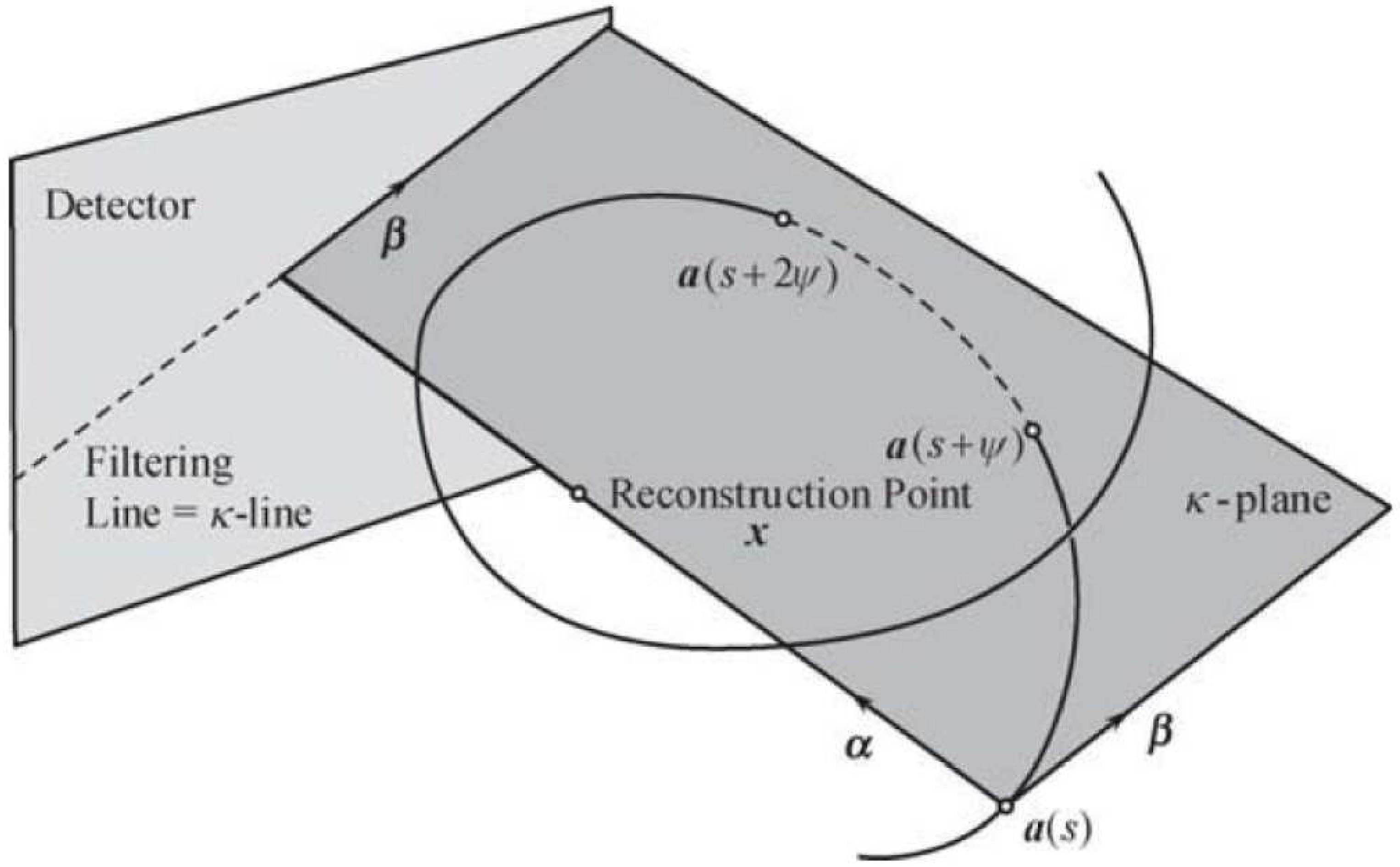


Figure 6: Weighting scheme (Zeng, 2009)

# $\kappa$-lines

- The projection of a $\kappa$-plane onto the detector is called a $\kappa$-line.
- The orientation changes with the reconstruction point.

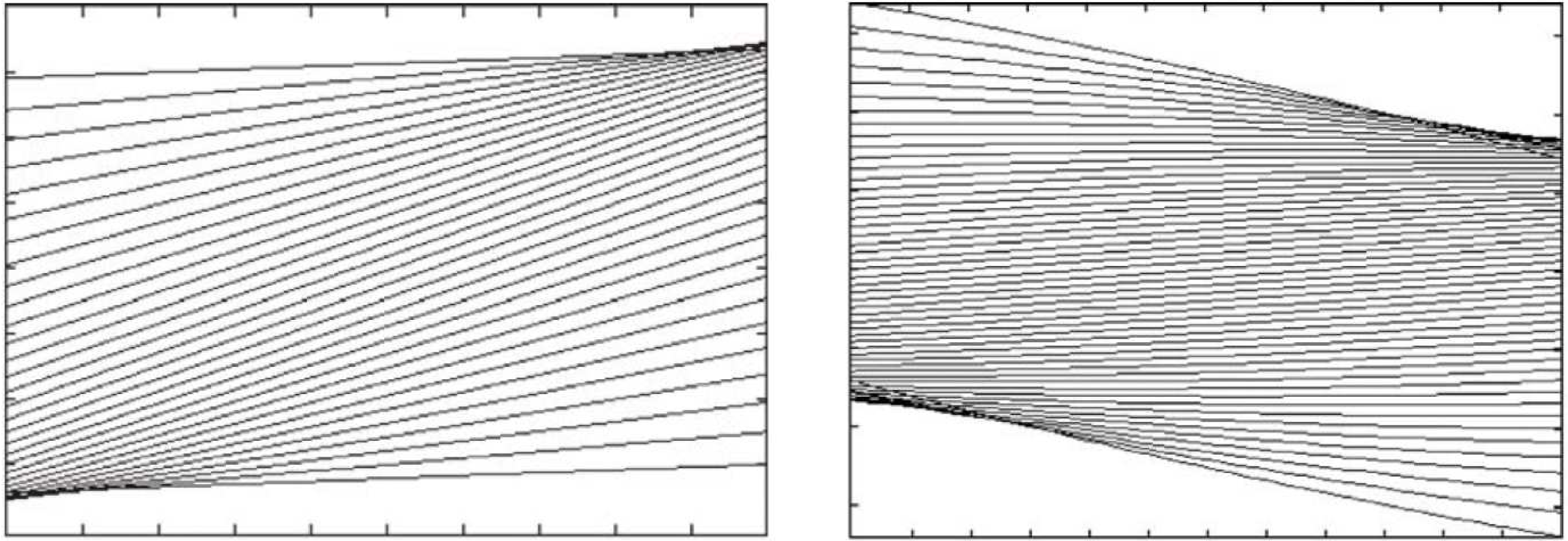

Figure 7: $\kappa$-lines of a flat detector (left), and a curved detector (right) (Zeng, 2009)

# Katsevich's Algorithm: Rebinning and Filtering

- Rebin the data to parallel lines to prepare for the Hilbert transform.
- Perform the 1-D Hilbert transform along each line.

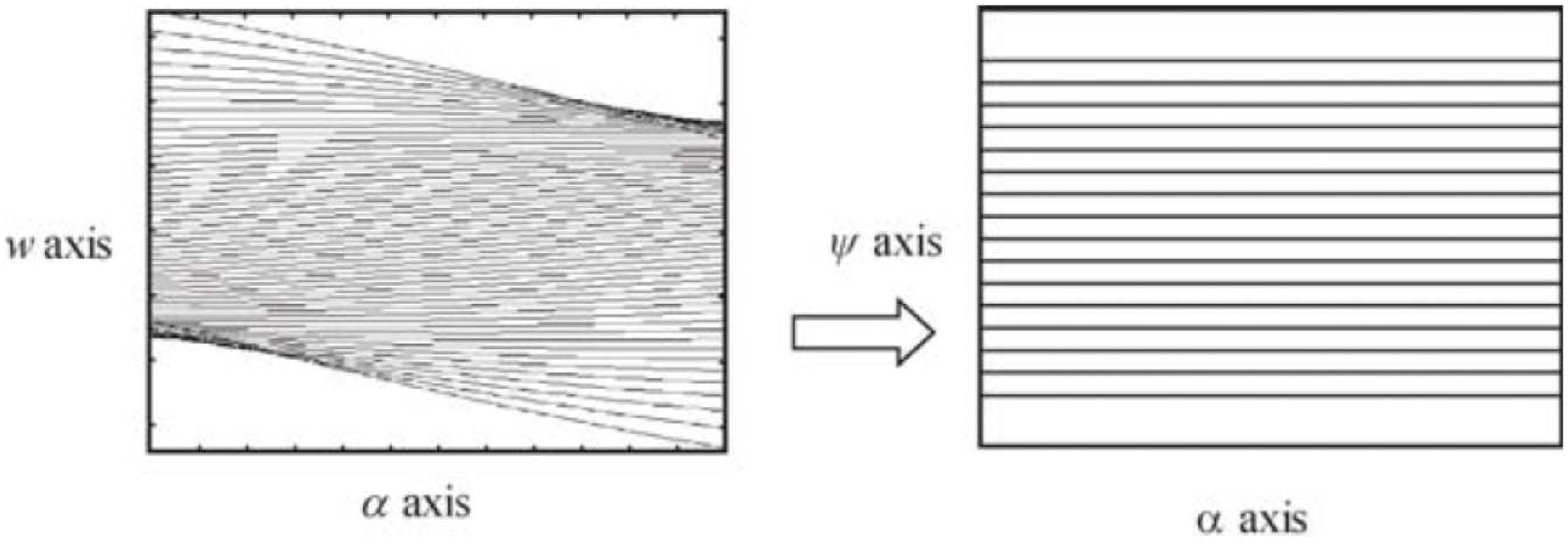


Figure 8: Rebinning of the $\kappa$-lines (Zeng, 2009)

# Katsevich's Algorithm: Final Steps

- Invert the rebinned data to go back to detector coordinates.
- Perform remaining weighting with $\cos\alpha$.
- Perform cone beam backprojection with distance weighting for all voxels $\boldsymbol{x}$.

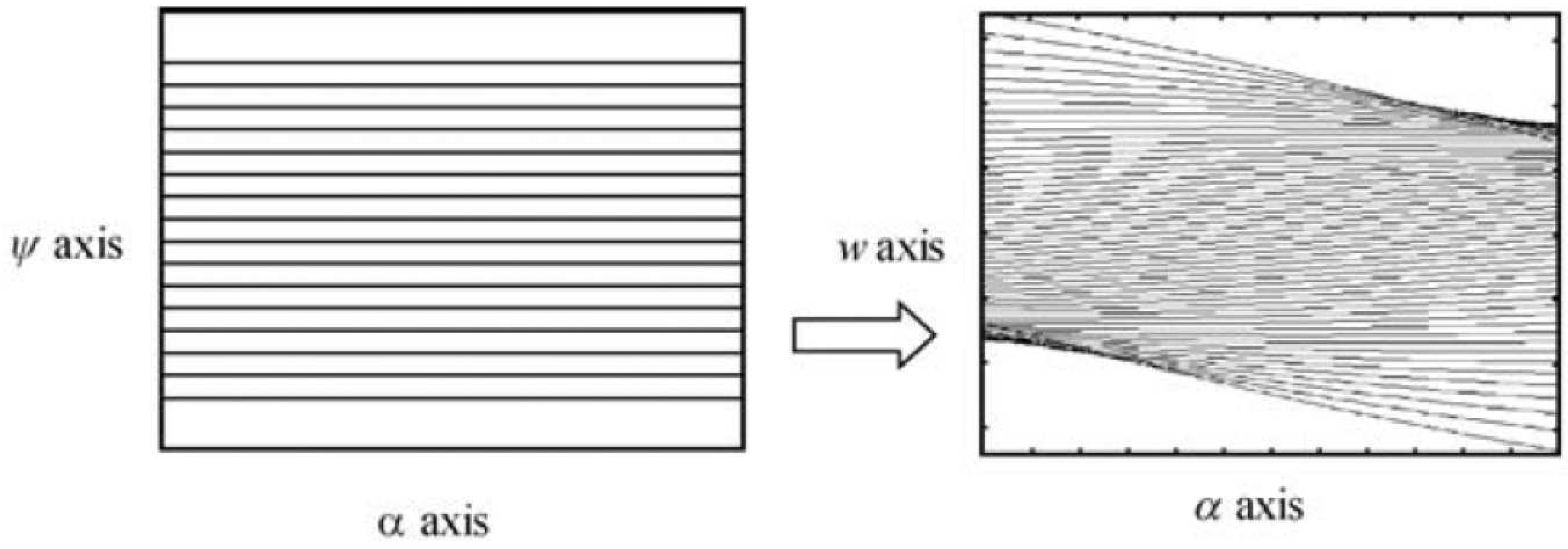


Figure 9: Invert the rebinning after Hilbert transform has been applied (Zeng, 2009)

# Katsevich's Algorithm: Remarks

- Katsevich's algorithm performs an exact reconstruction.
- The algorithms causes some additional computational overhead compared to other algorithms as the FDK algorithm.
- Katsevich's algorithm does not use all available data.
- Katsevich's algorithm deletes redundant data $\rightarrow$ $(1, -1, 1)$-weighting.

# Topics

Katsevich's Algorithm

Summary

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Further Readings

# Take Home Messages

- Katsevich's algorithm is a 3-D cone beam reconstruction algorithm.
- It is applicable to helical CT, and provides a theoretically exact algorithm.
- Katsevich's algorithm is a filtered backprojection algorithm.

# Further Readings

The original works of Katsevich can be found here:

Alexander Katsevich. "Theoretically Exact Filtered Backprojection-Type Inversion Algorithm for Spiral CT". In: *SIAM Journal on Applied Mathematics* 62.6 (2002), pp. 2012–2026. DOI: 10.1137/S0036139901387186

Alexander Katsevich. "Analysis of an Exact Inversion Algorithm for Spiral Cone-beam CT". In: *Physics in Medicine and Biology* 47.15 (July 2002), pp. 2583–2597

Alexander Katsevich. "An Improved Exact Filtered Backprojection Algorithm for Spiral Computed Tomography". In: *Advances in Applied Mathematics* 32.4 (May 2004), pp. 681–697. DOI: doi:10.1016/S0196-8858(03)00099-X

The best way to augment your knowledge of the shown concepts is to read the companion book of the current chapter:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9