Diagnostic Medical Image Processing
Prof. Dr.-Ing. Andreas Maier
Exercises (DMIP-E)
WS 2016/17

LEHRSTUHL FÜR MUSTER-ERKENNUNG

Fan Beam Reconstruction

Frank Schebesch, Tobias Würfl, Matthias Utzschneider,
Yixing Huang, Asmaa Khdeir, Houman Mirzaalian

Exercise Sheet 6

19 Fan Beam Reconstruction

(i) Please explain graphically how the point spread function (PSF) for fan beam looks like and show its relation to the parallel beam PSF.
Using this insight, what trick have you learned in the lecture to derive an analytical reconstruction algorithm for fan beam reconstruction? State its name and a very short description of the general idea.

(ii) The parallel beam sinogram is denoted by $p(s, \theta)$ and the equal angle fan beam sinogram is denoted by $g(\gamma, \beta)$. The distance from the X-ray source to the isocenter is denoted by D. The filtered backprojection (FBP) algorithm for parallel beam reconstruction is given as follows:

$$f(x, y) = \frac{1}{2} \int_0^{2\pi} \int_{-\infty}^{\infty} p(s, \theta) h(x \cos\theta + y \sin\theta - s) \mathrm{d}s \mathrm{d}\theta.$$

The FBP algorithm for fan beam reconstruction is derived from the parallel beam reconstruction. Before we apply the fan beam filter, we apply the cosine weighting $\cos\gamma$ first. Why do we need the cosine weighting?
Hint: We used a transformation to polar coordinates for the derivation of the algorithm.

(iii) In order to write the final fan beam algorithm in form of a convolution, we used a specific property of the ramp filter h:

$$h(D' \sin\gamma) = \left(\frac{\gamma}{D' \sin\gamma}\right)^2 h(\gamma).$$

Show that this relation is correct.

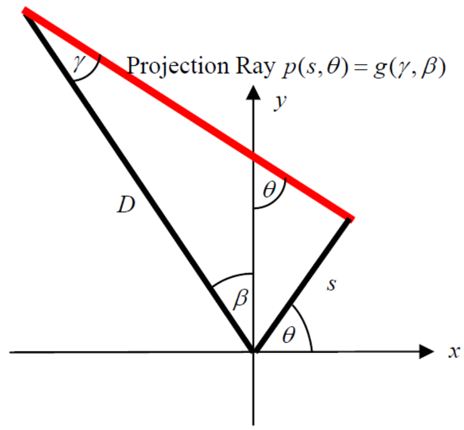

Figure 1: Geometry and notation for the fan beam system

(iv) Assume there is a point object at position (x_0, y_0) and we use the fan beam system as shown in **Fig.** 1. When the X-ray source and the curved detector rotate over 360°, the projection of the point will be a curve in the sinogram. Please give the function of the sinogram curve, i.e., the relation of γ and β.

$\boxed{4 \times 0.5\mathbf{+1+1+1}}$

20 Short Scan

(i) A CT acquisition of a Christmas tree (see **Fig.** 2) is reconstructed by a fan beam reconstruction with $\delta = 11°$. It is performed by a short-scan. Projections are taken with $\Delta\gamma = 2°$. How many projections have to be acquired to reconstruct the image?

(ii) In **Fig.** 3, you find a short-scan and three other trajectories. The object is assumed to occupy the circular shaded region of radius R_m, and the trajectory lies concentrically on a circle of radius R_0. Assume that the detector is always large enough to image the full object, thereby accommodating a maximum fan angle of $\gamma_m (= \delta)$. Explain for each figure if we can get a whole object from the reconstruction process?

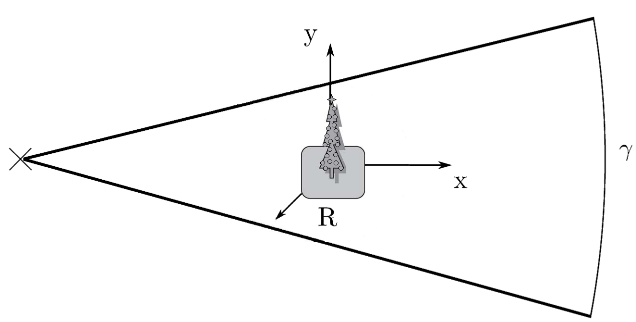

Figure 2: CT image for Christmas

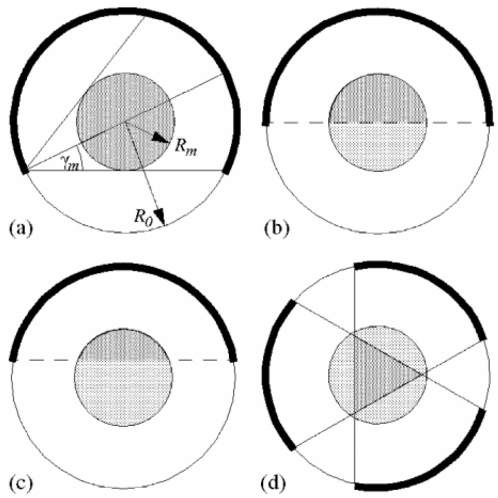

Figure 3: Different scan trajectories, how much of the object do they cover completely? (Source: `http://iopscience.iop.org/article/10.1088/0031-9155/47/14/311/`)

(iii) What problem arises with short scans and how can it be fixed?

$\boxed{\mathbf{1{+}2{+}1}}$

21 FBP for a Fan Beam Short Scan – Programming Exercise

The goal for this exercise is to reconstruct short scan data using the fan beam FBP algorithm with Parker weights.

Fan Beam FBP Algorithm

Given the fan beam opening angle δ and the flat panel 'fanogram' data $g\,(s, \beta)$, we use the following reconstruction steps from the lecture (see **Fig.** 4(a) for the notation):

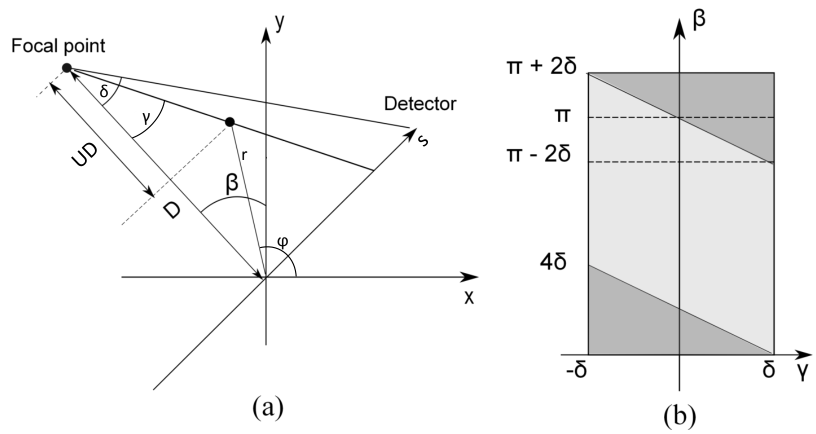

Figure 4: (a) Fan beam imaging geometry, (b) fan beam sinogram and data redundancy.

1. Cosine weighting of projection data to obtain $g_1(s, \beta)$:
$$g_1(s, \beta) = \frac{D}{\sqrt{D^2 + s^2}} g(s, \beta) \ ,$$

2. Perform fan beam filtering:
$$g_F(s, \beta) = \int_{-\infty}^{\infty} h_R\left(s - s'\right) g_1\left(s', \beta\right) ds' \ ,$$
where $h_R(s)$ is the filter kernel.

3. Backprojection with a weighting function of object to focal point distance U:
$$f(r, \varphi) = \int_0^{2\pi} \frac{1}{U^2} g_F(s, \beta)\, d\beta, \qquad U = \frac{D + r\sin(\beta - \varphi)}{D}$$

Parker Weights

A short scan measures some redundant rays at the beginning and at the end of data acquisition (see the two dark triangles in **Fig.** 4(b)). Corresponding redundant rays determined by the relation $g(\gamma, \beta) = g(-\gamma, \beta + \pi + 2\gamma)$ are commonly by the Parker weighting function:
$$\omega(\gamma, \beta) = \begin{cases} \sin^2\left(\frac{\pi}{4}\frac{\beta}{\delta - \gamma}\right), & 0 \leq \beta \leq 2\delta - 2\gamma \\ 1, & 2\delta - 2\gamma \leq \beta \leq \pi - 2\gamma \\ \sin^2\left(\frac{\pi}{4}\frac{\pi + 2\delta - \beta}{\delta + \gamma}\right), & \pi - 2\gamma \leq \beta \leq \pi + 2\delta \end{cases}$$
Projection rays measured twice are normalized to unity while guaranteeing smooth transitions between non-redundant and redundant data.

Implementation Tasks

Complete the gaps in the provided CONRAD class that are marked with "**TODO**"

(i) Initialize the parameters. Compute the fan angle and the short scan range.

(ii) Compute direction and position of the detector border at a rotation angle β.

(iii) Transform the pixel coordinates to world coordinates.

(iv) Compute the intersection point of a ray with the detector.

(v) Compute the distance weights for this point at the given rotation angle β.

(vi) Complete the cosine weights computation.

(vii) Complete the Parker weights implementation.

(viii) Test all available sinograms `Sinogram0.tif`, `Sinogram1.tif`, and `Sinogram2.tif`.

$\boxed{\mathbf{1+0.5+0.5+1+1+0.5+1+0.5}}$

Your tutors wish you merry Christmas and some relaxing holidays!

Latest submission date: 01/10/2017

$\boxed{\textbf{Total: 15}}$