Medical Image Processing for Diagnostic Applications

Efficient Implementation

Online Course – Unit 13

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Topics

Efficient Evaluation of Polynomials

Interpolation of Intensities

Distortion Algorithm: Summary and Further Applications

Summary
- Take Home Messages
- Further Readings

Efficient Evaluation of Polynomials

Problem: Each acquired image requires a distortion correction.

Observations:

- Bivariate polynomials can be understood as univariate polynomials where the coefficients are univariate polynomials (instead of a simple constant value):

$$x = \sum_{i=0}^{d} \left(\sum_{j=0}^{d-i} u_{i,j} y'^j \right) x'^i .$$

- Univariate polynomials are evaluated using Horner's scheme:

$$p(x) = \sum_{i=0}^{d} a_i x^i = (\ldots(a_d x + a_{d-1})x + \ldots)x + a_0 .$$

Efficient Evaluation of Polynomials

Definition

The ***Horner scheme***, named after William George Horner, is an algorithm for the efficient evaluation of polynomials in monomial form. Horner's method describes a manual process by which one may approximate the roots of a polynomial equation. The Horner scheme can also be viewed as a fast algorithm for dividing a polynomial by a linear polynomial. (Wikipedia)

- For each line we get a univariate polynomial ($y' = \text{const}$).
- Row and column increments are constant.
- Arithmetic progression $\rightarrow$ reuse of former evaluations

Conclusion: After an initialization, for each pixel only sums have to be computed.

Example: Horner Scheme

Let us assume we have a matrix $\boldsymbol{M}$ and observations $(x', y')^{\mathsf{T}}$, and we want to find $(x, y)^{\mathsf{T}}$ such that:

$$\boldsymbol{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}.$$

Suppose we implement the necessary operations:

$$\begin{pmatrix} m_{1,1} & m_{1,2} \\ m_{2,1} & m_{2,2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} m_{1,1}x + m_{1,2}y \\ m_{2,1}x + m_{2,2}y \end{pmatrix}$$

for several pairs of x and y in two nested `for`-loops over x and y, we will do some computations more often than necessary.
Using the Horner scheme, we reuse earlier computations constant in the inner loop.

Topics

Interpolation of Intensities

- **Nearest Neighbor Interpolation** is the simplest interpolation method. We assign to an image point in between pixels the intensity value of the closest pixel. Usually the Euclidean distance is applied.
- In **Bilinear Interpolation**, we compute for the new image point a weighted mean of neigboring intensities. Here the intuition is applied that pixels closer to the new image point have an higher impact on the final intensity value.

- Mostly the results of nearest neighbor interpolation are not well appreciated. The images appear crispy and noisy, though the interpolation method is extremely fast.
- In most practical applications where interpolation is required, bilinear interpolation is applied.

Interpolation of Intensities

Computation of the intensity i

- Nearest neighbor interpolation:

$$i = a$$

- Bilinear interpolation:

$$\begin{aligned} x &= a(1-d') + dd', \\ y &= b(1-d') + cd', \\ i &= x(1-d'') + yd'' \end{aligned}$$

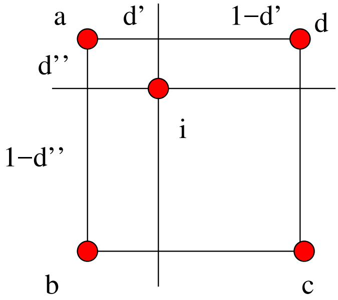

Figure 1: Interpolation: i is interpolated using the known intensities a, b, c, d.

Topics

Accelerated Distortion Correction Algorithm

FOR $x :=$ first to last line DO

FOR $y :=$ first to last column DO

compute coordinate $x = X(x', y')$

compute coordinate $y = Y(x', y')$

compute intensity $f(x, y) = \text{interpol}(g; x, y)$

Figure 2: Image undistortion routine (*g* distorted image, *f* corrected output image)

Hardware Accelerated Image Warping

- Image is decomposed into squares.
- Map the vertices of the square.
- Undistortion can be implemented using texture mapping unit.
- Bilinear interpolation hardware is supported in GPUs.
- Texture mapping hardware is supported in GPUs.

Application of Image Undistortion: Endoscopy

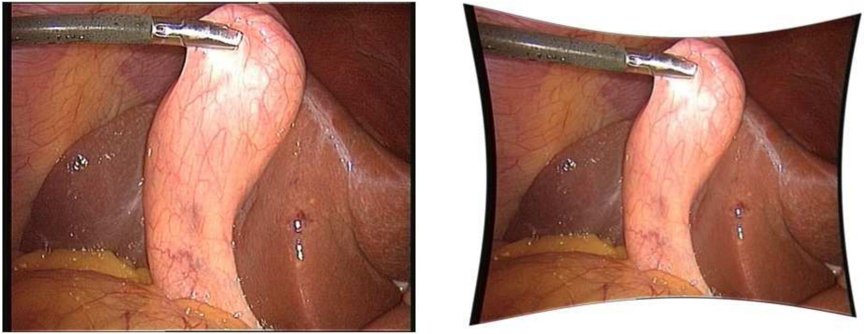

Figure 3: Original, distorted endoscope image (left), and the result of distortion correction (right)

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Take Home Messages

- The Horner scheme can be used for efficient evaluation of polynomials.
- Bilinear interpolation is sufficient for nearly all practical problems.

Further Readings

In case you need to learn more about polynomials and the efficient evaluation of polynomials, you have to read Volume 2 of Prof. Knuth's classic work on The Art of Computer Programming.

A book that covers many image preprocessing methods applied in medical imaging systems is:

Jiří Jan. *Medical Image Processing, Reconstruction, and Restoration: Concepts and Methods*. Signal Processing and Communications. CRC Press, Taylor & Francis Group, Nov. 2005