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Test Exam

You have 60 minutes for the exam. It contains three sections with 20, 24, and 16 points.

Preprocessing

Question 1: MRI-Inhomogeneities

A common artifact in magnet-resonance imaging (MRI) are MRI-inhomogeneities. Name three possible reasons for these artifacts and one possible solution.

4 P.

Question 2: Defect Pixel Interpolation

Defect pixels on detectors can be compensated by defect pixel interpolation in frequency domain. In a 1-D case a signal g can be represented by:

$$g(t) = f(t) \cdot w(t), \tag{1}$$

$f(t)$ is the ideal signal, $g(t)$ is a measured signal with missing pixels, $w(t)$ is a binary mask describing the missing pixels. The corresponding Fourier transforms are $G(\xi)$, $W(\xi)$ and $F(\xi)$. For this task we consider the signal $F(\xi)$ only at the two frequencies s and $N - s$:

$$F(\xi) = \hat{F}(s)\delta(\xi - s) + \hat{F}(N - s)\delta(\xi - N + s). \tag{2}$$

$\hat{F}$ denotes an estimate of F and δ is the Dirac-delta function defined by:

$$\delta(t) = \begin{cases} 1 & \text{if } t = 0, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Find an estimator for $\hat{F}(s)$ to interpolate the corrupted signal g in frequency domain.

$\boxed{\textbf{14 P.}}$

Image Reconstruction

Question 3: Parallel-Beam Reconstruction

a) Describe shortly two possible alternative analytic parallel-beam reconstruction algorithms besides the "Filtered Backprojection".

2 P.

b) The CT reconstruction algorithm "Filtered Backprojection" consists of a ramp filter $h(s)$ and a backprojection. $h(s)$ is defined by

$$h(s) = \int_{-B}^{B} |\omega| e^{2\pi i \omega s} \, \mathrm{d}s, \tag{4}$$

with the bandwidth $B = \frac{1}{2\tau}$, the frequency ω, and τ the detector spacing. For this task we use a cut-off frequency $B = \frac{1}{2}$. In this case, the filter can be reformulated using the rectangular function $\mathrm{rect}(t)$:

$$h(s) = \int_{-\frac{1}{2}}^{\frac{1}{2}} |\omega| \, e^{2\pi i \omega s} \mathrm{d}\omega = \int_{-\infty}^{\infty} |\omega| \, \mathrm{rect}\left(\omega\right) e^{2\pi i \omega s} \mathrm{d}\omega \tag{5}$$

where $\mathrm{rect}(t)$ is defined by:

$$\mathrm{rect}(t) = \begin{cases} 1 & \text{if } |t| < \frac{1}{2}, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

For $B = \frac{1}{2}$, $|\omega|$ can be rewritten as $|\omega| = \frac{1}{2} - \text{rect}\,(2\omega) * \text{rect}\,(2\omega)$ on $[-\frac{1}{2}, \frac{1}{2}]$. Note that the inverse Fourier transform of $\text{rect}(t)$ is $\text{FT}^{-1}(\text{rect}(C\,t)) = \frac{1}{|C|}\,\text{sinc}(\frac{1}{C}\,t)$ with a constant C and $\text{sinc(t)} = \frac{\sin(\pi \text{t})}{\pi \text{t}}$.

Task: Derive the continuous form of the **Ramachandran-Lakshminarayanan** convolver by using the inverse Fourier transform.

$\boxed{\textbf{14 P.}}$

c) Use your result to derive the discrete form of the convolver.

[Alternatively, use the following substitutional result: $h(s) = 2B^2 \operatorname{sinc}(2Bs) - B^2 \operatorname{sinc}^2(Bs)$, where $B = \frac{1}{2\tau}$.]

$\boxed{\textbf{4 P.}}$

Question 4: 3-D Reconstruction

a) What does Orlov's condition state for trajectories for 3-D CT reconstruction?

$\boxed{\textbf{2 P.}}$

b) Draw two different trajectories around the unit sphere, which fulfill Orlov's condition.

$\boxed{\textbf{2 P.}}$

Rigid Registration

Question 5: Quaternions

Marker positions $\mathbf{q}_k \in \mathbb{R}^3, k = 1, 2, ..., N$, are observed on a 3-D CT image I_1, and on a 3-D PET image I_2 markers are observed at the positions $\mathbf{p}_k \in \mathbb{R}^3$. The rotation $\mathbf{R} \in \mathbb{R}^3$ of all markers occurring in the rigid transformation from I_1 to I_2 can be described by a quaternion $\mathbf{r} = w + xi + yj + zk$.

a) Show how an arbitrary point $\mathbf{p} \in \mathbb{R}^3$ is rotated by a quaternion $\mathbf{r}_\text{s}$.

$\boxed{\textbf{4 P.}}$

b) Show a way to estimate the rotation quaternion **r** such that the estimation is linear in the components of **r**.

$\boxed{\textbf{12 P.}}$