# Medical Image Processing for Diagnostic Applications

Defect Pixel Interpolation – Utilizing Symmetry

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## **Topics**

#### Defect Pixel Interpolation using Symmetry Properties







#### **Notation**

For simplicity and without loss of generality we limit the following discussion to 1-D signals of length N, and use the following notation:

- discrete ideal signal: f(n),
- binary mask image: w(n),
- observed signal: g(n).

The respective Fourier transforms are denoted  $F(\xi)$ ,  $W(\xi)$ ,  $G(\xi)$ .







#### Observations:

- We consider real valued signals f(n), g(n), w(n).
- They satisfy the following relationship:

$$g(n) = f(n) \cdot w(n) \Leftrightarrow G(\xi) = F(\xi) * W(\xi).$$

 For the ideal, the mask, and the defect image, the Fourier transform satisfies the symmetry property:

$$F(\xi) = \overline{F}(N - \xi),$$

$$G(\xi) = \overline{G}(N - \xi),$$

$$W(\xi) = \overline{W}(N - \xi).$$

where the bar symbol in  $\overline{z}$  denotes the complex conjugate of z.







Now we make explicit use of the symmetry property of the Fourier transform to derive an interpolation algorithm:

- Select a pair G(s) and G(N-s) of the Fourier transform of the corrupted image showing pixels defects.
- Select a pair F(s) and F(N-s) of the Fourier transform of the ideal image.







- Let us assume that the Fourier transform of the ideal image  $F(\xi)$ consists only of two lines at s and N-s, where  $s \neq 0$ .
- We can then rewrite the Fourier transform of f(n) using Dirac's  $\delta$ -function:

$$F(\xi) = \widehat{F}(s)\delta(\xi - s) + \widehat{F}(N - s)\delta(\xi - N + s),$$

where  $\hat{F}$  denotes an estimate of F, and the  $\delta$ -function is defined by

$$\delta(k) = \begin{cases} 1, & \text{if } n = 0, \\ 0, & \text{otherwise.} \end{cases}$$







The convolution of F and the Fourier transform W of the given mask image leads to the Fourier transform of the observed corrupted image:

$$G(s) = \frac{1}{N} \left( \widehat{F}(s) W(0) + \overline{\widehat{F}}(s) W(2s) \right).$$

This can be shown as follows:

$$G(s) = F(s) * W(s) = \frac{1}{N} \sum_{k=0}^{N-1} F(k) * W(s-k).$$

Due to our assumption, we know that  $F \neq 0$  only at k = s or k = N - s, hence:

$$G(s) = \frac{1}{N} \left( \widehat{F}(s) W(0) + \widehat{F}(N-s) W(s-N+s) \right)$$
$$= \frac{1}{N} \left( \widehat{F}(s) W(0) + \overline{\widehat{F}}(s) W(2s) \right).$$







 For the conjugate complex Fourier transform of the observed image we get:

$$\overline{G}(s) = \frac{1}{N} \left( \overline{\widehat{F}}(s) \overline{W}(0) + \widehat{F}(s) \overline{W}(2s) \right).$$

- Since W is known, we get two equations linear in  $\widehat{F}(s)$  and  $\widehat{F}(s)$ .
- Hence, the final estimator for the Fourier transform of the ideal image is:

$$\widehat{F}(s) = N \frac{G(s)\overline{W}(0) - \overline{G}(s)W(2s)}{|W(0)|^2 - |W(2s)|^2},$$
 (FT-EST)

where |.| denotes the absolute value of the complex number.







## Error Spectrum

 An objective function to measure the quality of the interpolated image results from the least square error:

$$\Delta_{\varepsilon} = \frac{1}{N} \sum_{n=0}^{N-1} \left( g(n) - w(n) \widehat{f}(n) \right)^{2}.$$

• The spectrum of the error in the *i*-th iteration step is given by:

$$G^{(i)}(\xi) = G^{(i-1)}(\xi) - \frac{1}{N} \left( \widehat{F}^{(i)}(s) \delta(\xi - s) + \overline{\widehat{F}}^{(i)}(s) \delta(\xi - N + s) \right) * W(\xi).$$







#### **Topics**

#### Interpolation Algorithm







#### Interpolation Algorithm

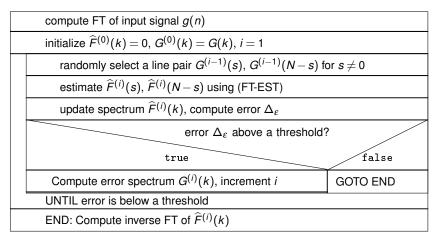


Figure 1: Interpolation algorithm according to Aach and Metzler







## **Topics**

Summary Take Home Messages **Further Readings** 







#### Take Home Messages

- Assuming the spectrum of a signal/function consists of two non-zero lines, then we find an estimate for the Fourier transform.
- The symmetry property of the Fourier transform w.r.t. real valued functions can be used to build a defect interpolation algorithm.







#### **Further Readings**

 The method presented for defect pixel interpolation in the frequency domain was published by Til Aach and Volker Metzler in 2001:

> Til Aach and Volker Metzler. "Defect Interpolation in Digital Radiography: How Object-Oriented Transform Coding Helps". In: Proc. SPIE 4322. Medical Imaging 2001: Image Processing. Vol. 4322. San Diego, CA: SPIE, Feb. 2001, pp. 824-835. DOI: 10.1117/12.431161

 A recent article about defect pixel interpolation with respect to image quality issues can be found here:

Jan Kuttig et al. "Effects of Defect Pixel Correction Algorithms for X-ray Detectors on Image Quality in Planar Projection and Volumetric CT Data Sets". In: Measurement Science and Technology 26.9 (Aug. 2015). 095406 (14pp). DOI: 10.1088/0957-0233/26/9/095406