Medical Image Processing for Diagnostic Applications

Fan Beam – Reconstruction Algorithm

Online Course – Unit 38

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Topics

Reprise

Fan Beam Reconstruction Algorithm

- Equiangular Case
- Backprojection and Fourier Slice Theorem
- Equally-spaced Case

Summary

- Take Home Messages
- Further Readings

Concept for finding a reconstruction algorithm

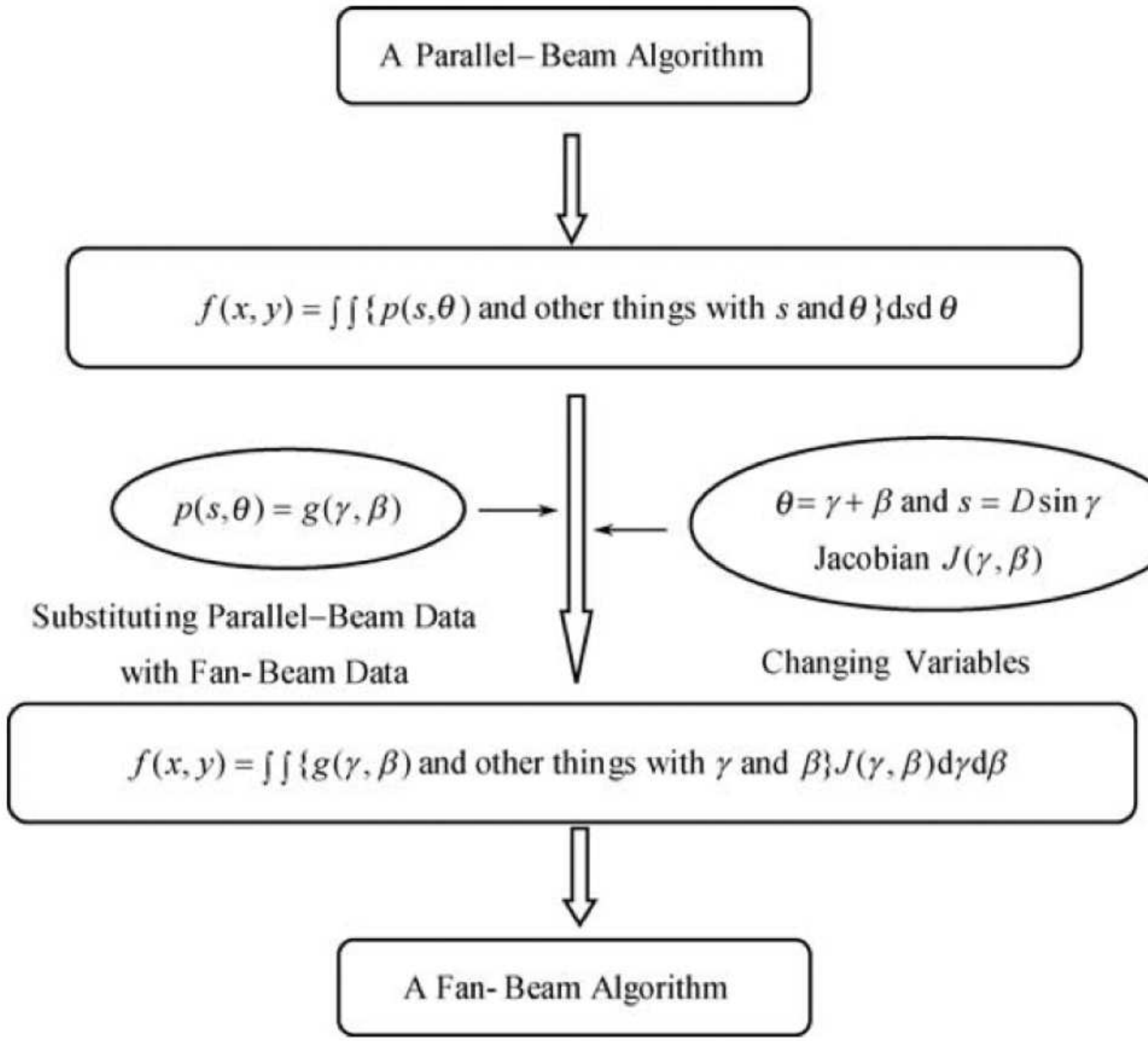

Figure 1: Flow chart showing the steps needed to develop a fan beam reconstruction algorithm (Zeng, 2009).

Equally-spaced and Equiangular Detectors

- Sampling is different in both detector geometries.
- Hence, different reconstruction formulas are obtained.

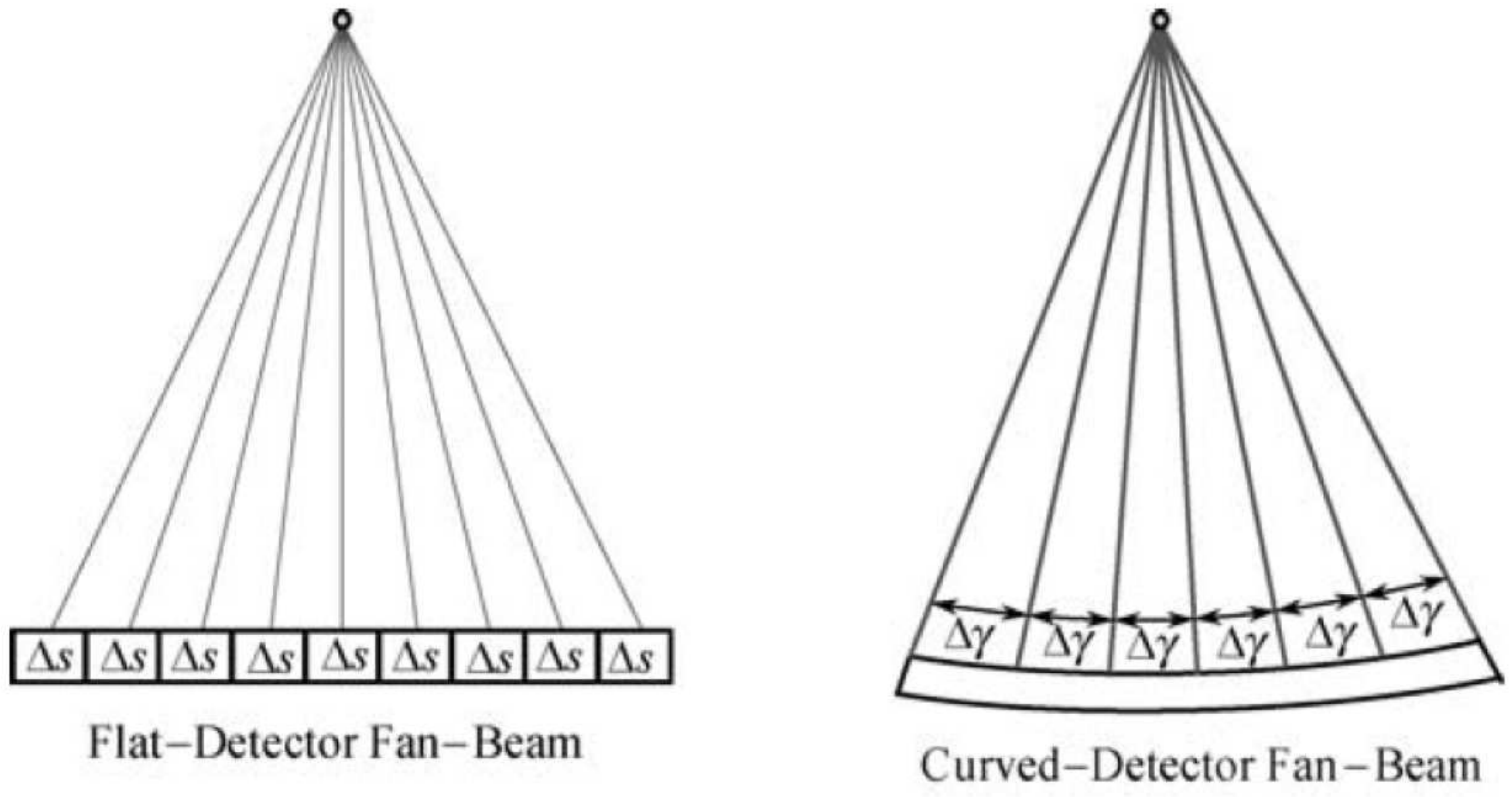

Figure 2: Schematics of flat and curved detector panels for fan beam (Zeng, 2009).

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FBP for the Equiangular Case

We start with a parallel beam backprojection:

$$f(x,y) = \frac{1}{2}\int_{0}^{2\pi}\int_{-\infty}^{\infty} p(s,\theta)h(x\cos\theta + y\sin\theta - s)\,\mathrm{d}s\,\mathrm{d}\theta,$$

and transform the reconstruction point into polar coordinates (r,φ), i. e., $x = r\cos\varphi$, $y = r\sin\varphi$:

$$f(r,\varphi) = \frac{1}{2}\int_{0}^{2\pi}\int_{-\infty}^{\infty} p(s,\theta)h(r\cos(\theta-\varphi) - s)\,\mathrm{d}s\,\mathrm{d}\theta.$$

FBP for the Equiangular Case

Using the described change of variables from (s,θ) to (γ,β) with the Jacobian $\left|\frac{\partial(s,\theta)}{\partial(\gamma,\beta)}\right| = D\cos\gamma$, and using distance D' and angle γ' of the reconstruction point with respect to the source, we get:

$$f(r,\varphi) = \frac{1}{2}\int_{0}^{2\pi}\int_{-\pi/2}^{\pi/2} g(\gamma,\beta)h(D'\sin(\gamma'-\gamma))\,D\cos\gamma\,\mathrm{d}\gamma\,\mathrm{d}\beta.$$

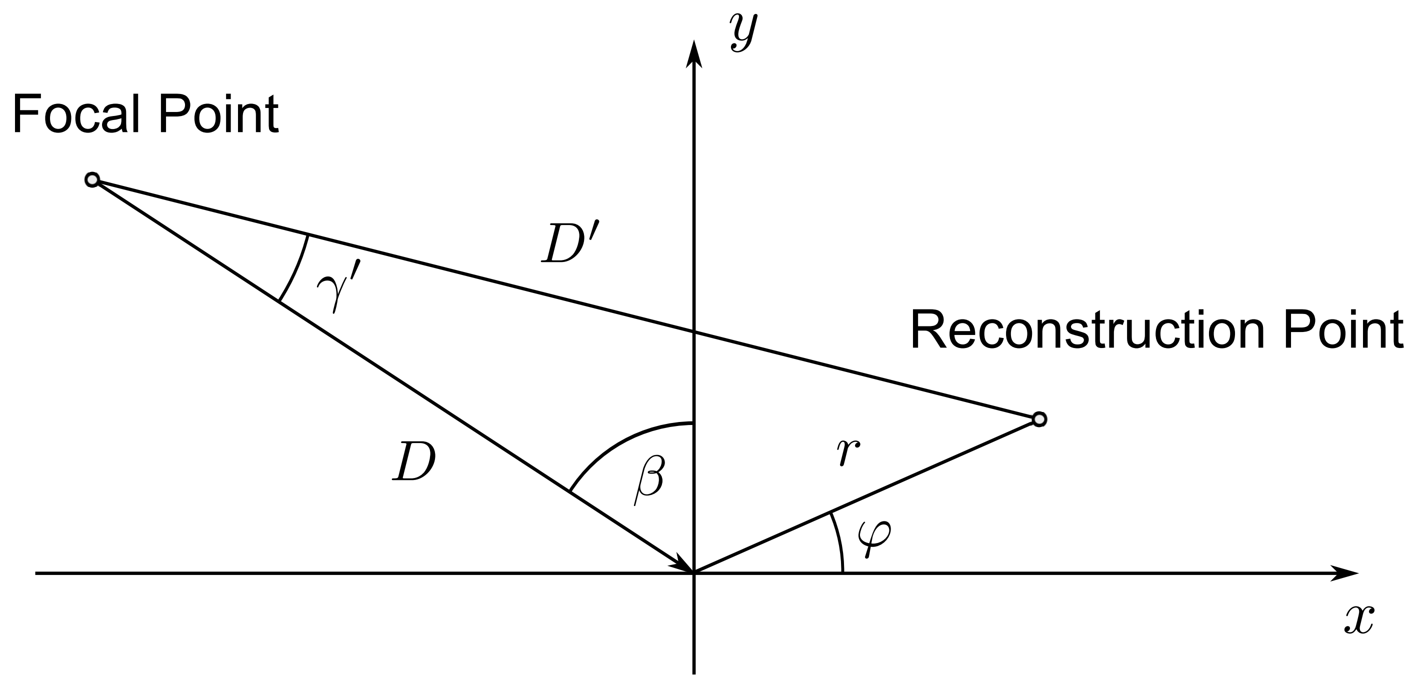

Figure 3: Definition of D' and γ'

FBP for the Equiangular Case

The ramp filter has the following property:

$$h(D' \sin\gamma) = \left(\frac{\gamma}{D' \sin\gamma}\right)^2 h(\gamma),$$

and defining

$$h_{\text{fan}}(\gamma) = \frac{D}{2}\left(\frac{\gamma}{\sin\gamma}\right)^2 h(\gamma),$$

we finally obtain the backprojection algorithm:

$$f(r,\varphi) = \int_0^{2\pi} \frac{1}{D'^2} \int_{-\pi/2}^{\pi/2} \cos\gamma\, g(\gamma,\beta)\, h_{\text{fan}}(\gamma' - \gamma)\, \mathrm{d}\gamma\, \mathrm{d}\beta.$$

FBP for the Equiangular Case: Algorithm

1. Perform cosine weighting

$$g_1(\gamma, \beta) = g(\gamma, \beta) \cos \gamma.$$

2. Apply fan beam filter:

$$g_2(\gamma', \beta) = (g_1 * h_{\text{fan}})(\gamma', \beta).$$

3. Backproject with distance weight:

$$f(r, \varphi) = \int_0^{2\pi} \frac{1}{D'^2} g_2(\gamma', \beta) \, \mathrm{d}\beta.$$

Backprojection and Fourier Slice Theorem

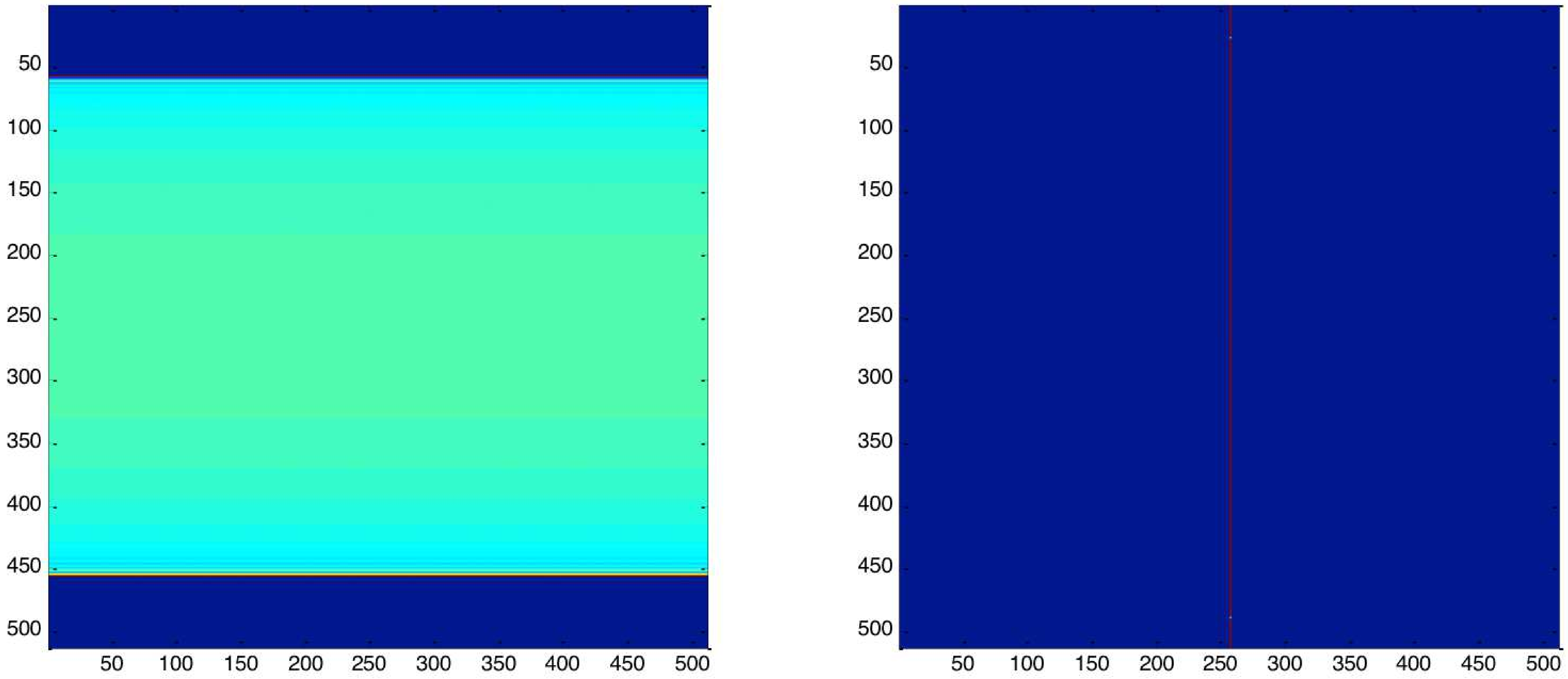

Figure 4: Backprojection of a single view (left) and its Fourier transform (right)

Backprojection and Fourier Slice Theorem

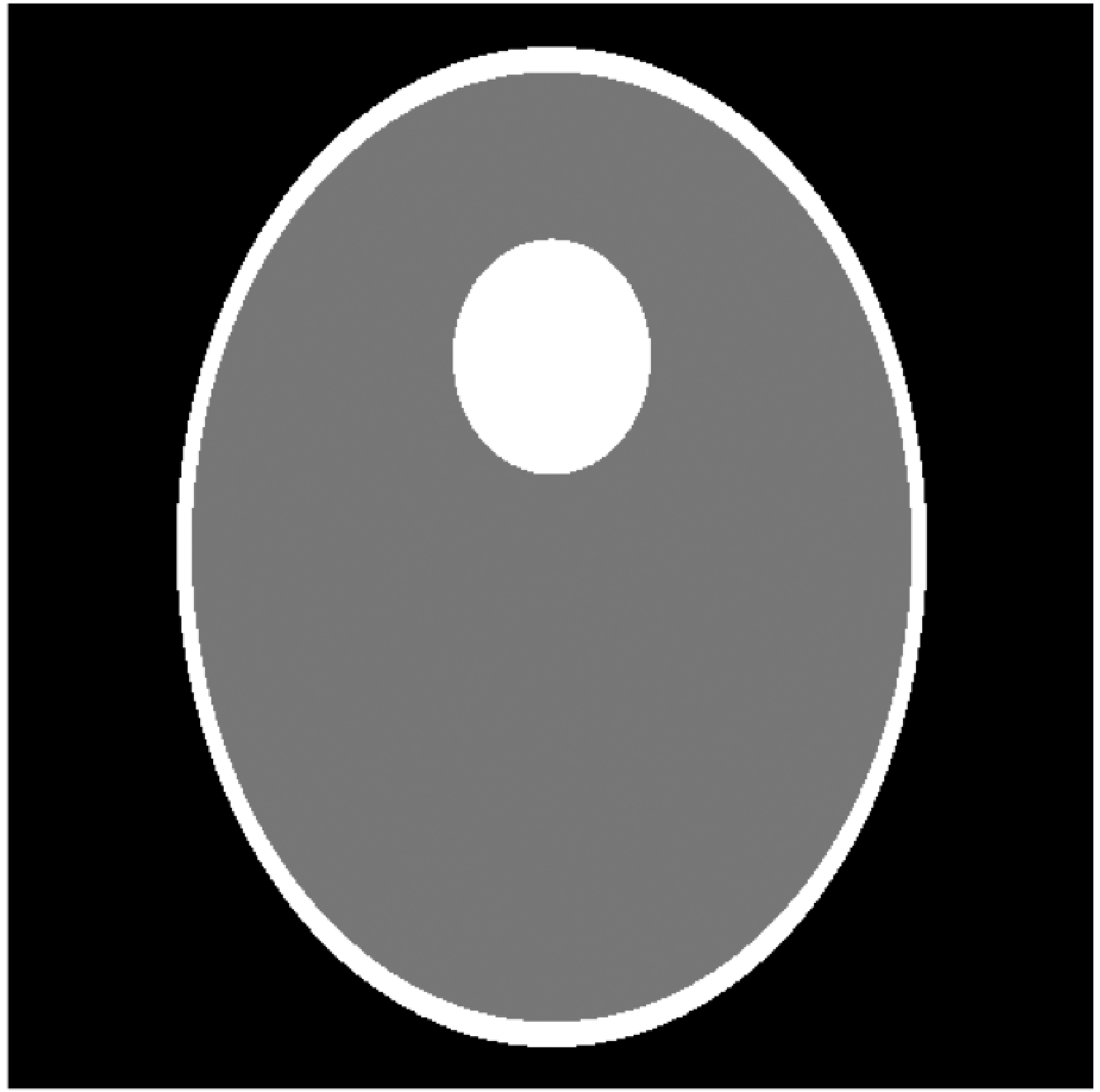

Figure 5: Slice view

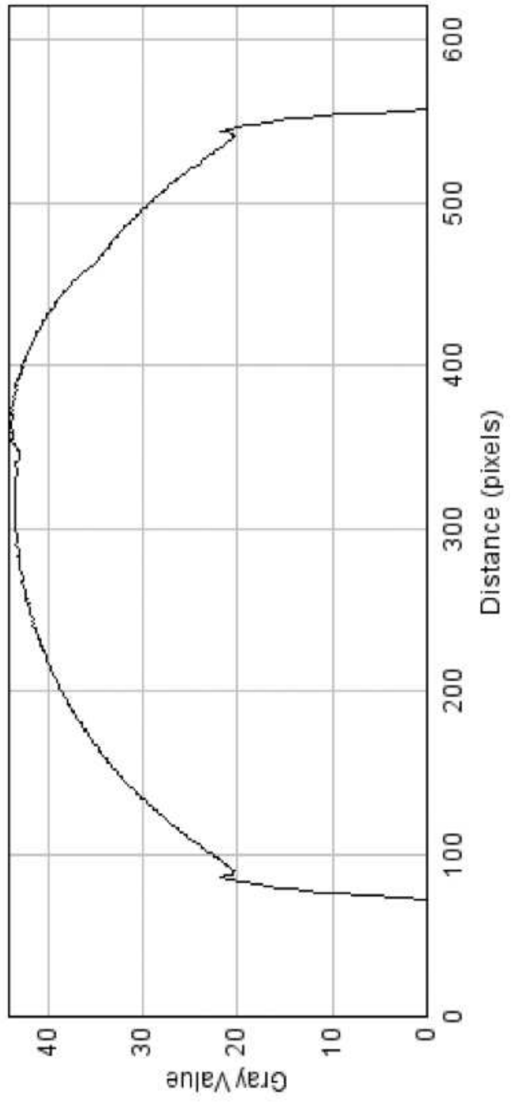

Figure 6: Projection profile

Backprojection and Fourier Slice Theorem

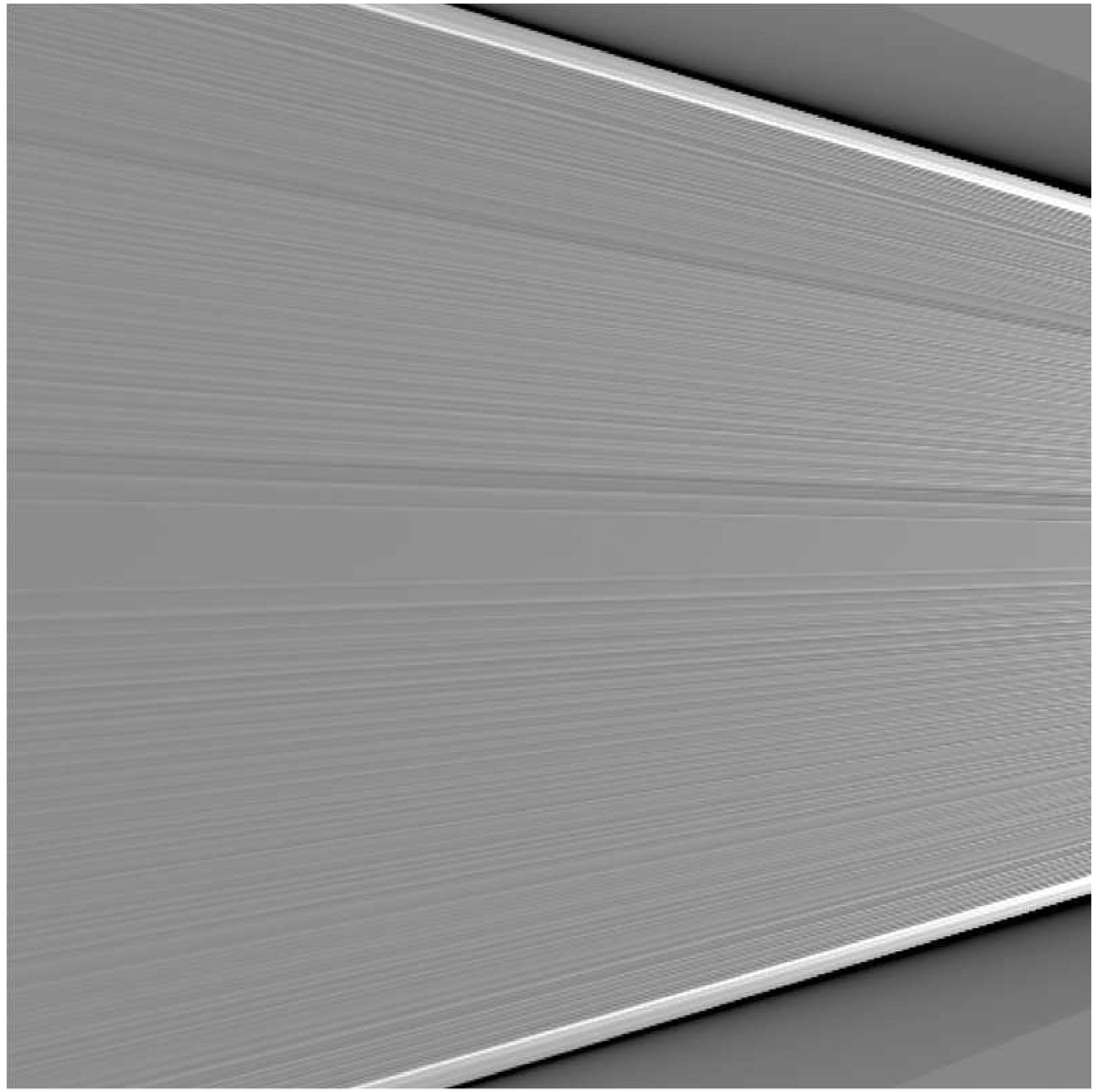

Figure 7: Fan beam backprojection

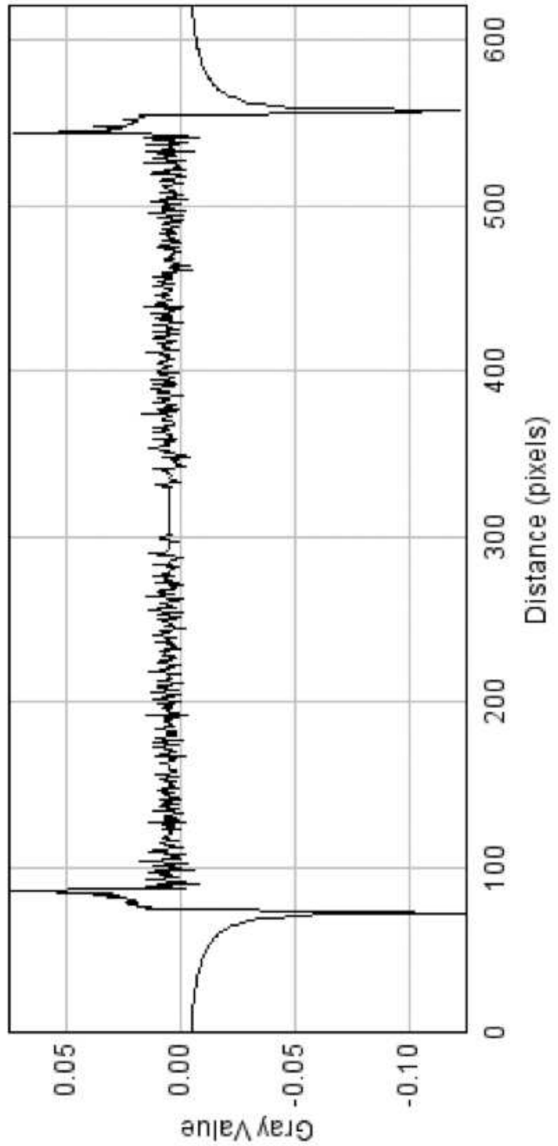

Figure 8: Filtered projection

Backprojection and Fourier Slice Theorem

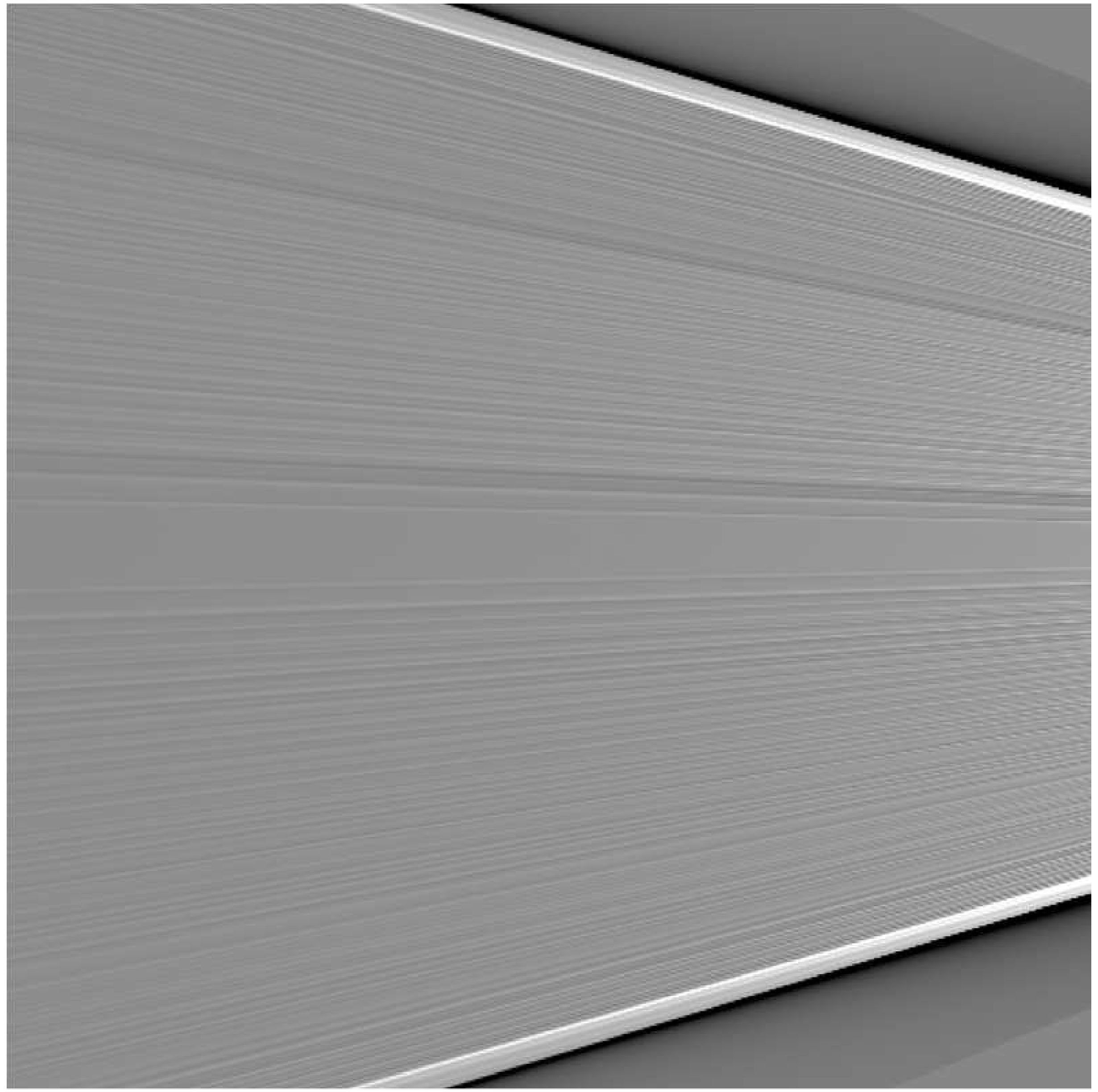

Figure 7: Fan beam backprojection

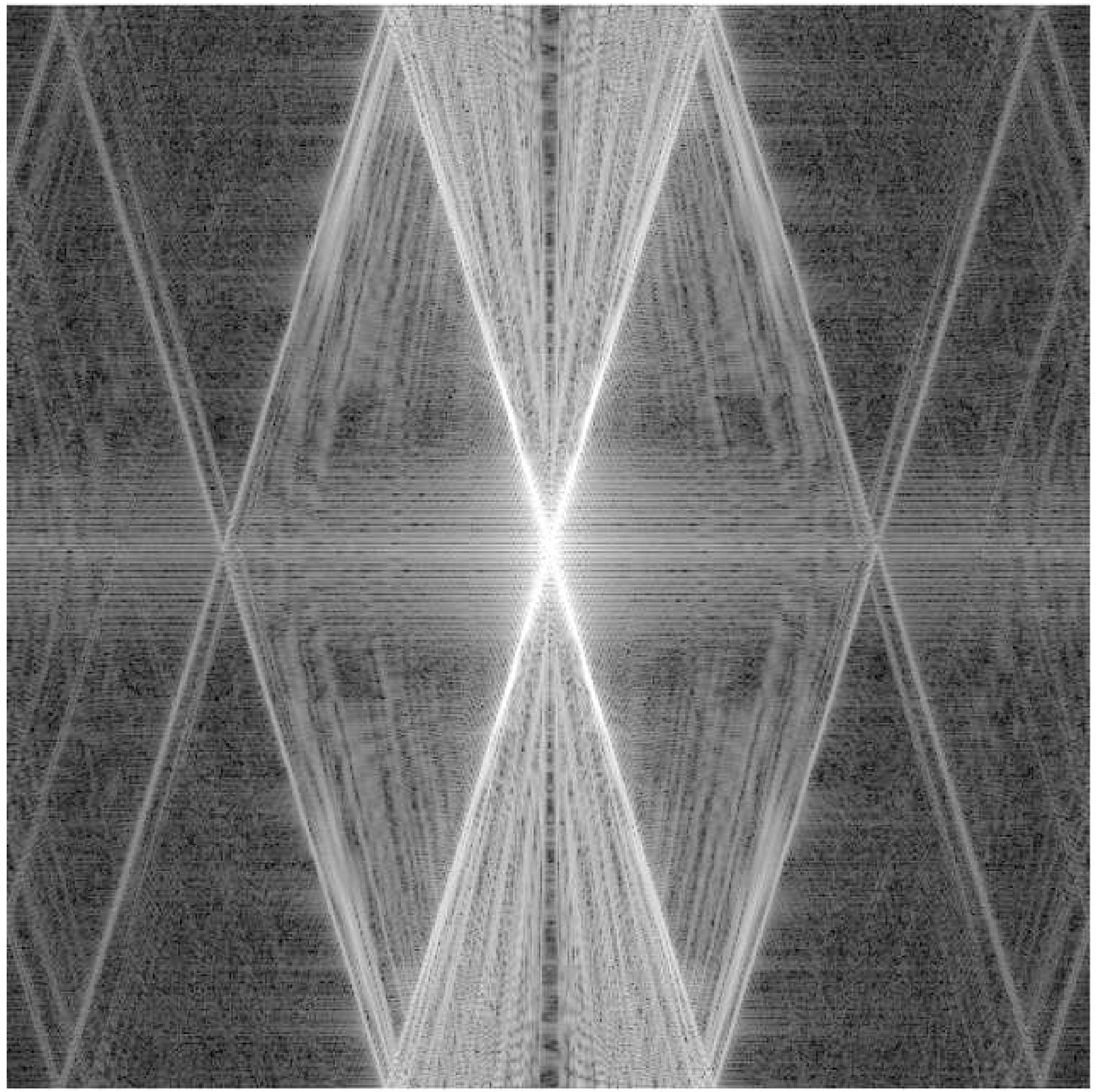

Figure 9: 2-D Fourier transform

Backprojection and Fourier Slice Theorem

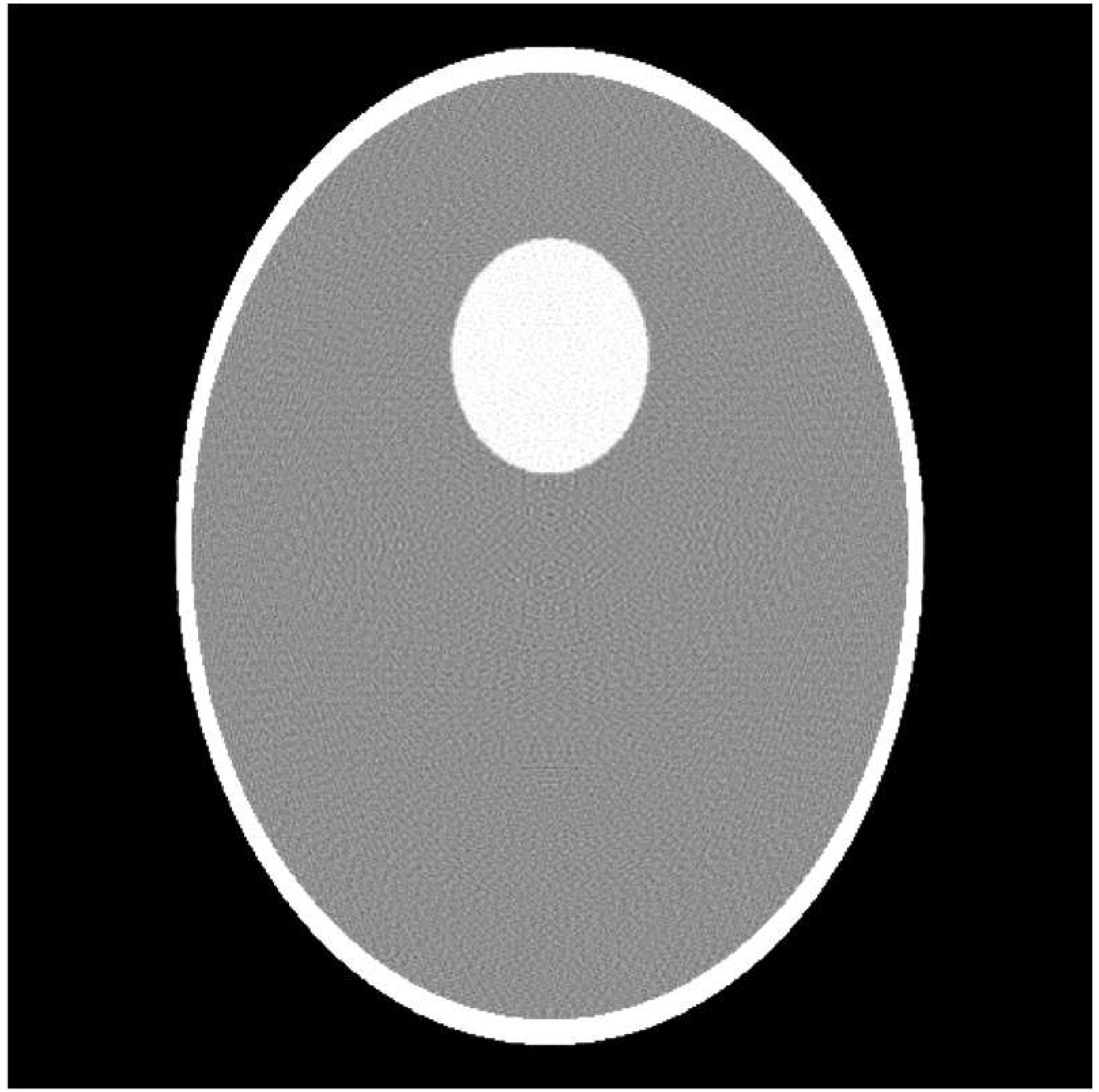

Figure 10: Reconstruction from fan beam data

FBP for the Equally-spaced Case

Like for the equiangular case we start with the parallel beam backprojection in polar coordinates:

$$f(r,\varphi) = \frac{1}{2}\int_{0}^{2\pi}\int_{-\infty}^{\infty} p(s,\theta)h(r\cos(\theta-\varphi)-s)\,\mathrm{d}s\,\mathrm{d}\theta.$$

Analogous considerations and transformations can be used to show:

$$f(r,\varphi) = \frac{1}{2}\int_{0}^{2\pi}\frac{1}{U^2}\int_{-\infty}^{\infty}\frac{D}{\sqrt{D^2+t^2}}g(t,\beta)h(t'-t)\,\mathrm{d}t\,\mathrm{d}\beta,$$

where

$$U = \frac{D+r\sin(\beta-\varphi)}{D}, \quad t' = \frac{Dr\cos(\beta-\varphi)}{D+r\sin(\beta-\varphi)}.$$

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Take Home Messages

- We derived two fan beam reconstruction algorithms based on the parallel beam to fan beam conversion.
- The detector geometry has to be considered.

Further Readings

More details on the derivations in this unit can be found in 'Larry's' book

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9