Medical Image Processing for Diagnostic Applications

Parallel Beam – Differentiated Backprojection

Online Course – Unit 32
Andreas Maier, Joachim Hornegger, Markus Kowarschik, Frank Schebesch
Pattern Recognition Lab (CS 5)

virtuelle hochschule bayern

FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG
TECHNISCHE FAKULTÄT

Topics

Differentiated Backprojection

- Filtering Revisited
- Differentiated Backprojection
- Variety of Reconstruction Algorithms

Summary

- Take Home Messages
- Further Readings

Filtering Revisited

- Rewrite $|\omega|$:

$$|\omega| = (2\pi i\omega) \cdot \left(\frac{1}{2\pi} (-i \operatorname{sgn}(\omega)) \right).$$

- Note that multiplication in frequency space with $-i \operatorname{sgn}(\omega)$ is a **Hilbert transform**, i. e., equivalent to a convolution with $h(s) = \frac{1}{\pi s}$.
- Note that the inverse Fourier transform of $2\pi i\omega$ is the derivative operator:

$$\mathrm{FT}^{-1}(2\pi i\omega \cdot G(\omega)) = \frac{\mathrm{d}}{\mathrm{d}s} g(s).$$

Differentiation Hilbert Backprojection Algorithm

1. Compute first derivative of the detector row:

$$q_1(s,\theta) = \frac{\partial p(s,\theta)}{\partial s}.$$

2. Apply Hilbert transform:

$$q_2(s,\theta) = \frac{1}{2\pi^2 s} * q_1(s,\theta).$$

3. Backproject $q_2(s,\theta)$:

$$f(x,y) = \int_0^{\pi} q_2(s,\theta)|_{s=x\cos\theta+y\sin\theta}\,\mathrm{d}\theta.$$

Differentiated Backprojection

Definition of the backprojection:

$$b(x,y) = \int_0^{\pi} \mathbf{H}p(s,\theta)\big|_{s=x\cos\theta+y\sin\theta}\,\mathrm{d}\theta,$$

where **H** is the Hilbert transform with respect to s.

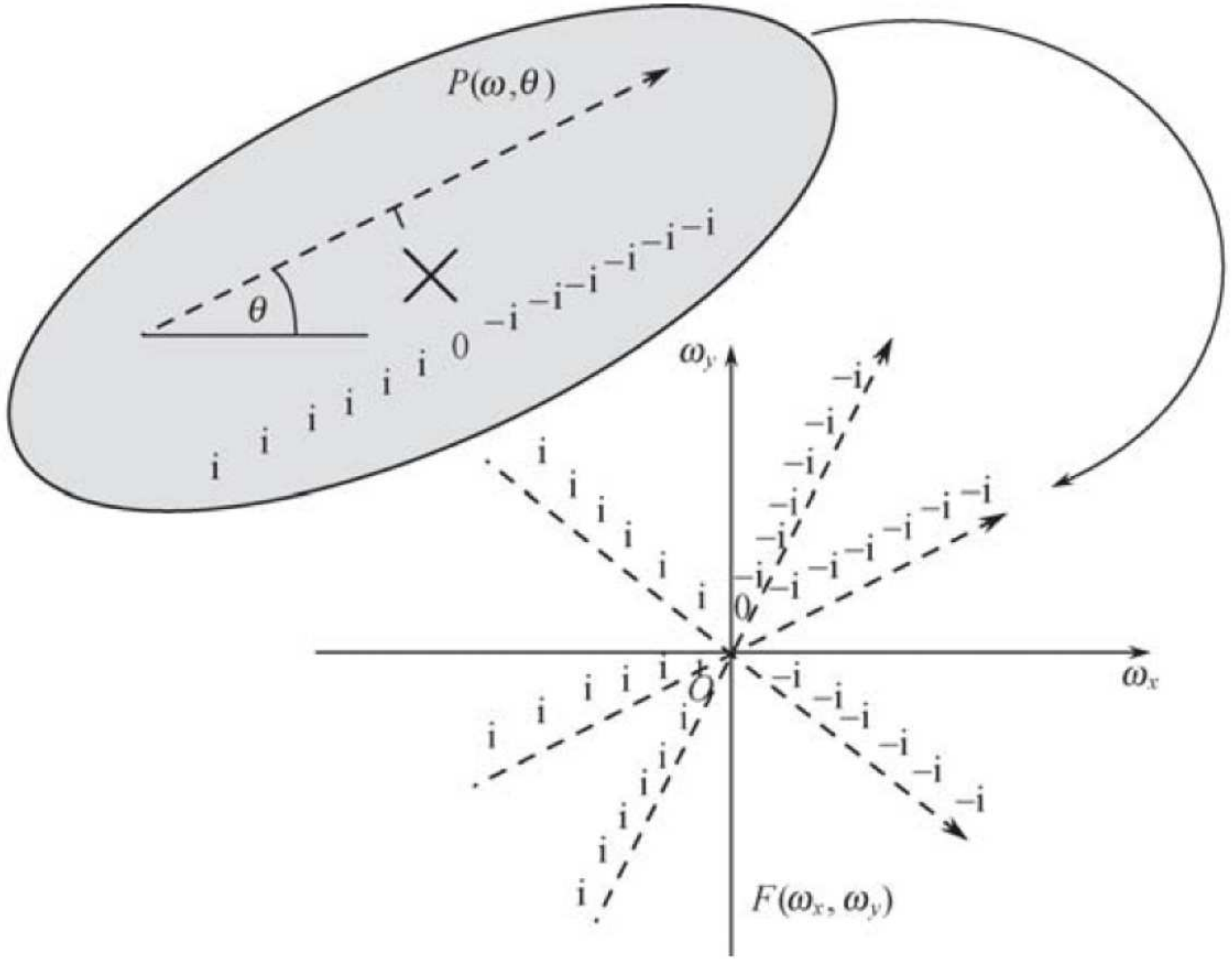

Figure 1: Computation scheme (Zeng, 2009)

Many reconstruction algorithms are possible ...

Step 1	*Step 2*	*Step 3*
1-D Ramp Filter with Fourier Transform	Backprojection	
1-D Ramp Filter with Convolution	Backprojection	
Backprojection	2-D Ramp Filter with Fourier Transform	
Backprojection	2-D Ramp Filter with 2-D Convolution	
Derivative	Hilbert Transform	Backprojection
Derivative	Backprojection	Hilbert Transform
Backprojection	Derivative	Hilbert Transform
Hilbert Transform	Derivative	Backprojection
Hilbert Transform	Backprojection	Derivative
Backprojection	Hilbert Transform	Derivative

Table 1: Valid combinations for analytical parallel-beam reconstruction algorithms (cf. Zeng, 2009)

Topics

Differentiated Backprojection

Filtering Revisited

Differentiated Backprojection

Variety of Reconstruction Algorithms

Summary

Take Home Messages

Further Readings

Take Home Messages

- Reformulation of the ramp filter showed that the combination of Hilbert transform and the projection derivatives produce another analytical reconstruction algorithm.
- There is a multitude of valid algorithms that can be built using the tools: projection derivatives, Fourier and Hilbert transform, and backprojection.

Further Readings

The concise reconstruction book from ‘Larry ’Zeng:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

If you want to learn more about applications of the Fourier transform:

Ronald N. Bracewell. *The Fourier Transform and Its Applications*. 3rd ed. Electrical Engineering Series. Boston: McGraw-Hill, 2000