Medical Image Processing for Diagnostic Applications

Histogram-based Bias Field Correction

Online Course – Unit 24

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Topics

Entropy and KL Divergence Minimization

Summary

Take Home Messages

Further Readings

Refresher: What are histograms?

- A ***histogram*** essentially is a frequency distribution of the pixel values in an image.
- Several values in a certain range are counted into so-called ***bins*** and then frequencies are plotted against bins.
- Normalization by the total sum makes a histogram a probability distribution.

Entropy and KL Divergence Minimization

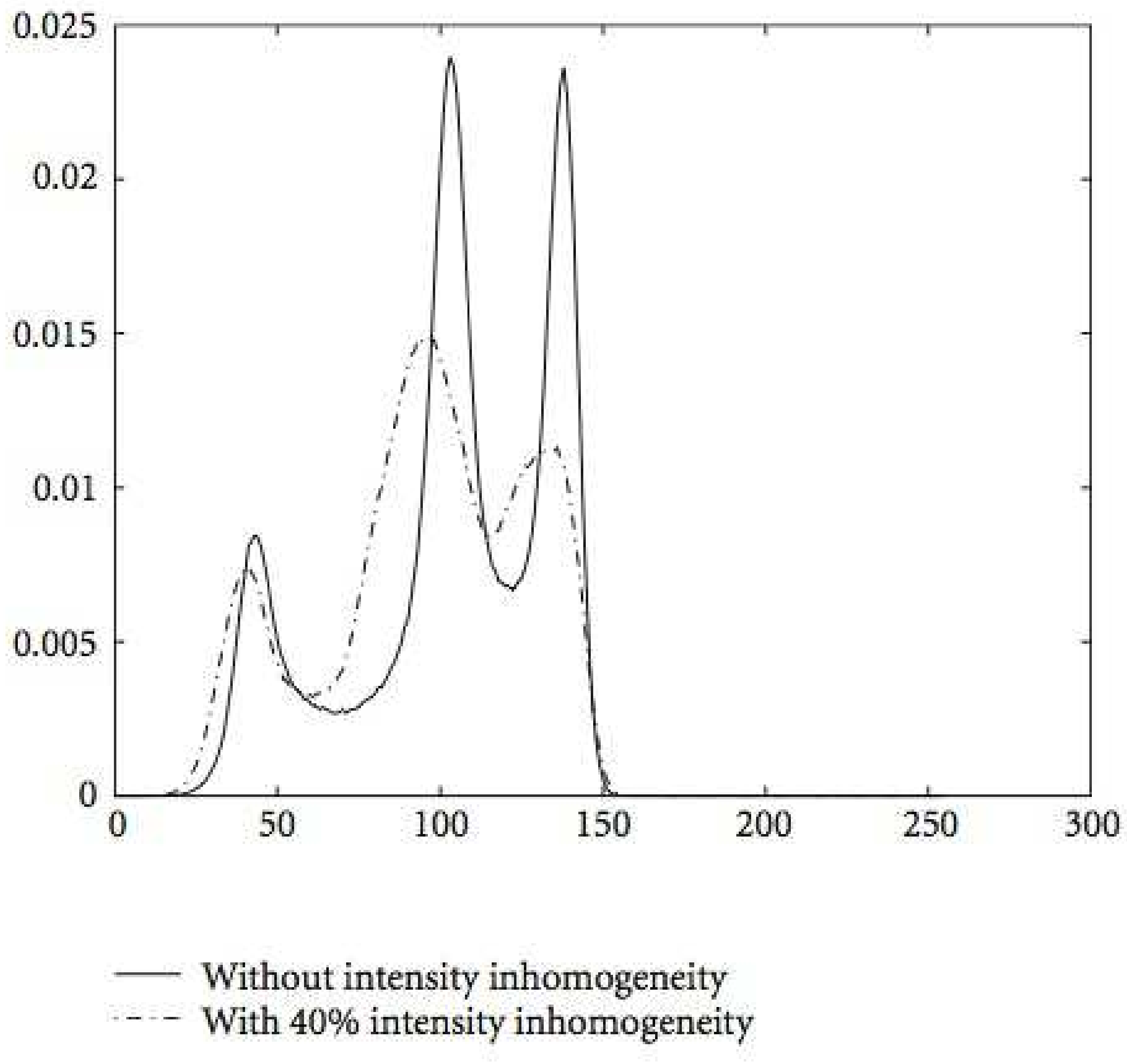

Figure 1: Histograms of brain tissue with IIH (dashed dotted line) and without bias (solid line)

Entropy and KL Divergence Minimization

Definition

The ***entropy*** H of a discrete random variable X is defined by

$$H(X) = -\sum_{i=1}^{n} p(x_i) \log p(x_i),$$

where the random measurements $x_1, x_2, \ldots, x_n$ underly the discrete probability density function

$$p(x_i) = p(X = x_i).$$

- The entropy is maximal, if the intensities are uniformly distributed.
- The entropy measures the amount of **disorder** in the image, and it is minimal for intensities with the least disorder.

Entropy and KL Divergence Minimization

Compute a bias field that minimizes the disorder in intensities.

Entropy and KL Divergence Minimization

The basic idea: Instead of fitting a parametric function with low frequencies by minimization of the squared distances between the function and the sampling values (i. e., intensities), we now apply a statistical method.

Entropy and KL Divergence Minimization

- It is assumed that the probability density function of the intensities of the original unbiased image is multimodal.
- IIH causes intensity overlap, and thus a “smearing” of the original probability density function. The peaks are no longer as sharp as they should be.
- Flattening brings the probability density function closer to a uniform density (increase of entropy).
- In terms of the statistical measures known from probability and information theory this means:
  - the bias field increases the entropy of the image,
  - the bias field decreases the Kullback-Leibler divergence between the probability density function of the image and the uniform density.

Entropy and KL Divergence Minimization

Definition

The ***Kullback-Leibler divergence*** (KL divergence) between two discrete probability density functions p and q is defined as:

$$\mathrm{KL}(p,q) = \sum_{i=1}^{n} p(x_i) \log \frac{p(x_i)}{q(x_i)}.$$

Entropy and KL Divergence Minimization

Using this definition and the functional equation of the logarithm, we find:

$$\begin{aligned}
\mathrm{KL}(p,q) &= \sum_{i=1}^{n} p(x) \log \frac{p(x)}{q(x)} \\
&= \underbrace{\sum_{i=1}^{n} p(x) \log p(x)}_{H(p)=-\sum_{i=1}^{n} p(x) \log p(x)} \quad - \quad \underbrace{\sum_{i=1}^{n} p(x) \log q(x)}_{H(p,q)=-\sum_{i=1}^{n} p(x) \log q(x)} \\
&= H(p,q) - H(p),
\end{aligned}$$

where $H(p)$ denotes entropy and $H(p,q)$ denotes cross entropy.

Entropy and KL Divergence Minimization

The KL divergence is a common similarity measure for probability density functions.

A few properties of the KL divergence:

$$\begin{aligned} \mathrm{KL}(p,q) &\neq \mathrm{KL}(q,p), && \text{(asymmetry)} \\ \mathrm{KL}(p,q) &\geq 0, && \text{(non-negativity)} \\ \mathrm{KL}(p,q) = 0 \quad &\Leftrightarrow \quad p = q, && \text{(definiteness)} \\ \mathrm{KL}(p,q) \to 0 \quad &\Rightarrow \quad p \to q. && \text{(continuity)} \end{aligned}$$

These properties show that the KL divergence is not a full metric, but resembles one, which is why it is used for the current purpose.

Entropy and KL Divergence Minimization

The algorithm for the estimation of the IIH:

- requires a parametric surface function that approximates the bias field (e. g., a bivariate polynomial), and
- the parameters are estimated such that
  - either the entropy of the resulting IIH corrected image is minimized, or
  - the Kullback-Leibler divergence w. r. t. to the uniform density is maximized.

Entropy and KL Divergence Minimization

Pros

- This approach can deal with rather complex and steep bias fields.
- It uses only a few parameters for the parametric representation of the bias field.

Cons

- Entropy minimization needs restrictions to avoid finding an all-white corrected image. This inhibits general applicability.
- The optimization task is difficult.

Entropy and KL Divergence Minimization

Note: In the Kullback-Leibler divergence based IIH correction, a **reference probability density function other than the uniform density** can be used to estimate the parameters of the bias field. In this case maximization is replaced by minimization.
This allows the incorporation of prior knowledge to IIH correction.

Reference probability density functions can be generated from:

- reference data, for instance, images of anatomic atlas data,
- ideal data that are supposed to be unbiased, for instance, manually corrected intensities,
- reference data that are acquired by costly high end MR scanners that provide superior image quality.

Topics

Entropy and KL Divergence Minimization

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- You learned about histograms, entropy and the Kullback-Leibler divergence.
- As a rough summary, for the IIH correction entropy is minimized, KL divergence maximized.

Further Readings

The webpage of the National High Magnetic Field Laboratory can be one starting point for more detailed information regarding MRI. For an initial overview of the technology, the following article is worth reading:

MRI: A Guided Tour by Kristen Coyne.

If you want to know more about segmentation of MR images, e. g., consult

the Google Scholar record of ‘Sandy’ Wells’ publications.

Another article worth reading is this survey paper on algorithms for intensity correction methods:

Zujun Hou. “A Review on MR Image Intensity Inhomogeneity Correction”. In: *International Journal of Biomedical Imaging* 2006.Article ID 49515 (Feb. 2006), pp. 1–11. DOI: 10.1155/IJBI/2006/49515