Medical Image Processing for Diagnostic Applications

Preprocessing Introduction

Online Course – Unit 8

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Topics

Motivation of Image Preprocessing
- Definition of Preprocessing
- Preprocessing Examples

Summary
- Take Home Messages
- Further Readings

Image Pre- and Postprocessing

Definition

Image preprocessing subsumes all image-to-image transforms that are done during image acquisition, i. e., in between the measurement at the detector (or other sensor) and the output on the monitor (or to hard disk).

Definition

All image-to-image transforms and image segmentation methods that are applied to images stored in the image data base are categorized to ***image postprocessing***.

Motivation of Image Preprocessing

There are **obvious reasons** for the need of image preprocessing:

- improvement of image quality to meet the requirements of the physician,
- noise reduction,
- contrast enhancement,
- correction of missing or wrong pixel (or voxel) values,
- optimal preparation of the data for post-processing,
- elimination of acquisition-specific artifacts.

Motivation of Image Preprocessing

Our task in the following lectures is to study:

- image acquisition procedures,
- their implications in terms of image artifacts, and
- the design of algorithms to eliminate image artifacts that are caused by certain image acquisition procedures.

Motivation of Image Preprocessing

The need of image preprocessing is illustrated by the image examples on the following slides.
We consider artifacts as they appear in:

- X-ray imaging (e. g., image distortion, defect pixels, heel effect),
- magnetic resonance imaging (e. g., elimination of intensity inhomogeneities in magnetic resonance imaging),
- endoscopy (e. g., heterogeneous illumination, specular reflection),
- molecular imaging (e. g., noise reduction).

Image Preprocessing in X-ray Imaging

Figure 1: Original image from X-ray device: colon filled with contrast agent (Stefan Böhm, Siemens Medical Solutions)

Image Preprocessing in X-ray Imaging

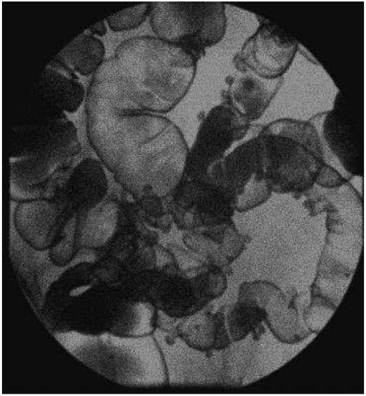

Figure 2: Image enhancement, step 1: corrupted image lines eliminated by interpolation (Stefan Böhm, Siemens Medical Solutions)

Image Preprocessing in X-ray Imaging

Figure 3: Image enhancement, step 2: contrast enhancement (Stefan Böhm, Siemens Medical Solutions)

Image Preprocessing in X-ray Imaging

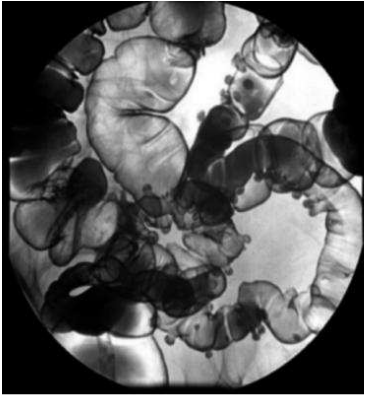

Figure 4: Image enhancement, step 3: image denoising (Stefan Böhm, Siemens Medical Solutions)

Image Preprocessing in X-ray Imaging

Figure 5: Image enhancement, step 4: edge enhancement (Stefan Böhm, Siemens Medical Solutions)

Image Preprocessing in MRI

Inhomogeneities in the magnetic field lead to images with intensity bias:

Figure 6: Image with inhomogeneities (left), and the intensity corrected preprocessing result (right) (Florian Jäger, Pattern Recognition Lab, FAU)

Image Preprocessing in MRI

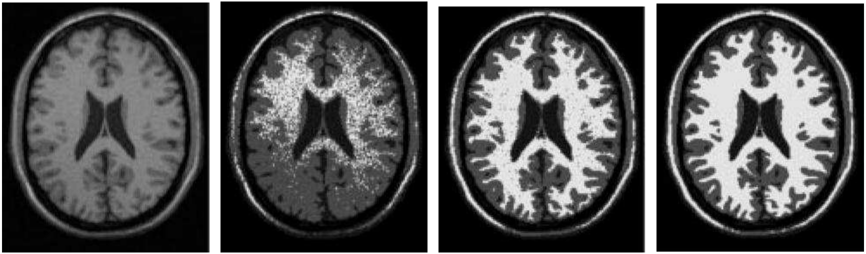

Figure 7: Image with bias field is corrected by different bias correction methods (Michael Balda, Pattern Recognition Lab, FAU).

Image Preprocessing in Endoscopy

One problem in imaging is the appearance of particles. By temporal in addition to spatial filtering images can be enhanced significantly.

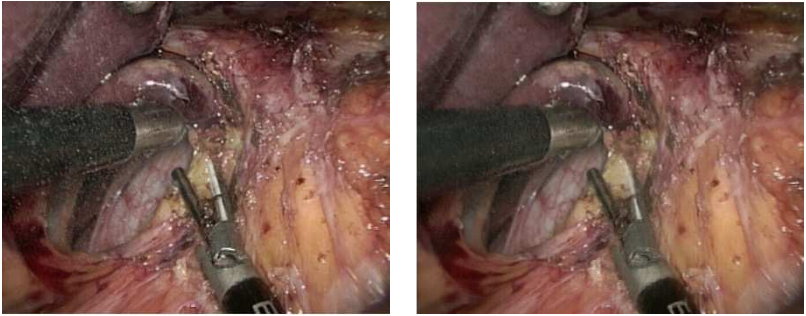

Figure 8: Images corrupted by flying particles (left), and the enhanced image (right) (Florian Vogt, Pattern Recognition Lab, FAU)

Artifacts

Common artifacts are caused by

- scattering,
- truncation,
- reconstruction algorithms, or
- beam hardening.

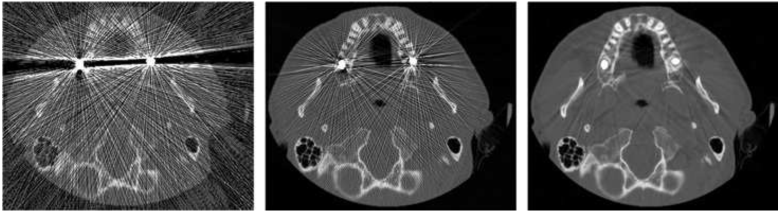

Figure 9: Reduction of streak artifacts (image courtesy of Stanford University)

Topics

Motivation of Image Preprocessing
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Summary
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Take Home Messages

Image preprocessing ...

- ... is done **before** the image appears on the monitor (“hidden algorithms”).
- ... is an art and an algorithmic challenge.
- ... requires the use of special hardware in most cases.
- ... is a trade-off (e. g., dose, run-time, hardware, ease of use, image quality).
- ... is driving business decisions (“to buy or not to buy”).
- ... is not an option, it is **mandatory**.

Further Readings

A book that covers many image preprocessing methods applied in medical imaging systems is:

> Jiří Jan. *Medical Image Processing, Reconstruction, and Restoration: Concepts and Methods*. Signal Processing and Communications. CRC Press, Taylor & Francis Group, Nov. 2005

This book is rather expensive. It is not required to buy this book to follow the lectures.

Medical Image Processing for Diagnostic Applications

X-ray Basics

Online Course – Unit 9

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Topics

X-ray Basics

- X-ray Devices – Concepts and Heel Effect
- Examples

Summary

- Take Home Messages
- Further Readings

X-ray Devices

X-ray

Two devices are typically used to convert X-rays into intensity images:

- image intensifiers (II), introduced $\sim$1940,
- flat panel detectors (FP or FD), introduced $\sim$2000.

Notes:

- Both technologies are still used in hospitals.
- Modern equipment is mostly shipped with flat panels.
- Research systems use flat panels, image intensifiers are obsolescent.

Image artifacts in X-ray imaging can have many sources. In the following units we consider artifacts that are due to the used detector technology.

X-ray Tube

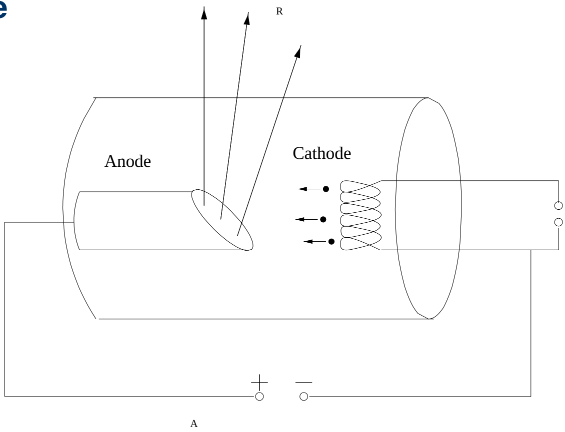

Figure 1: Concept of a traditional X-ray tube (R: ray emission, A: power source)

X-ray Tube: Heel Effect

The ***heel effect*** causes a gray level ramp in X-ray images:

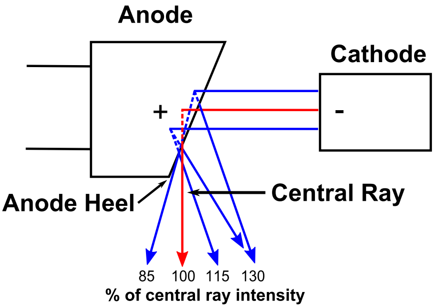

Figure 2: Scheme of the heel effect: rays with longer pathways through the anode are more likely absorbed.

The elimination of such inhomogeneities is discussed in upcoming units.

STRATON X-ray Tube

The **engineers from Erlangen** who developed this X-ray tube were under the final four for the “Deutschen Zukunftspreis” 2005! (*en.:* German future prize)

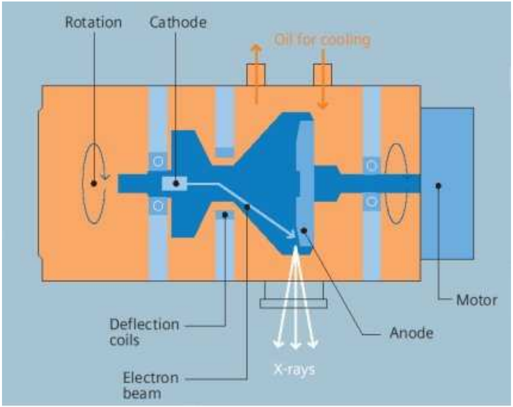

Figure 3: Concept of the STRATON X-ray tube (image courtesy of Siemens AG)

X-ray Tubes: Comparison

Figure 4: Traditional X-ray tube (left), Straton X-ray tube (right) (images courtesy of Siemens Healthcare)

X-Ray Detectors

Figure 5: Image intensifier (left), flat panel detector (right) (images courtesy of Siemens Healthcare)

X-Ray Detectors in Cardiology

Figure 6: C-arm device with image intensifier (left) and flat panel detector (right) (images courtesy of Siemens Healthcare)

Topics

X-ray Basics
- X-ray Devices – Concepts and Heel Effect
- Examples

Summary
- Take Home Messages
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Take Home Messages

- Over time different designs for X-ray emission and measurement devices have been developed.
- There are physical effects inherent to the system design that might degrade your images and therefore need preprocessing.

Further Readings

An excellent overview of different detectors used in X-ray equipment can be found in

> Heinz Morneburg, ed. *Bildgebende Systeme für die medizinische Diagnostik: Röntgendiagnostik und Angiographie, Computertomographie, Nuklearmedizin, Magnetresonanztomographie, Sonographie, integrierte Informationssysteme*. 3rd ed. Publicis MCD Verlag, June 1995 (in German).

Information on the distortion correction products can be found on the vendors' homepages. Try, for instance, www.healthcare.siemens.com.

Medical Image Processing for Diagnostic Applications

Image Undistortion for Image Intensifiers

Online Course – Unit 10

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Topics

Image Intensifier

Distortion Correction
- Image Distortion
- Distortion Correction – Design Considerations

Summary
- Take Home Messages
- Further Readings

Image Intensifier

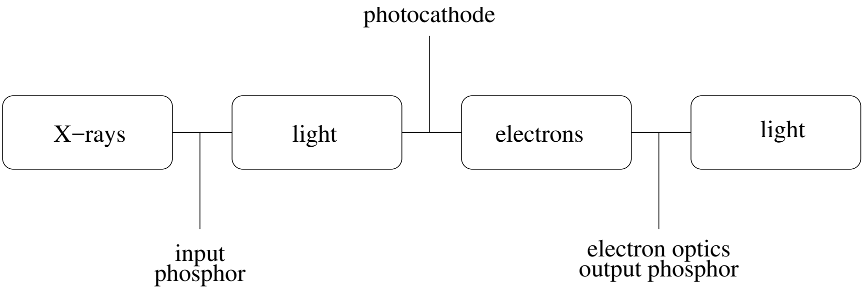

Figure 1: Basic principle of an image intensifier

Image Intensifier

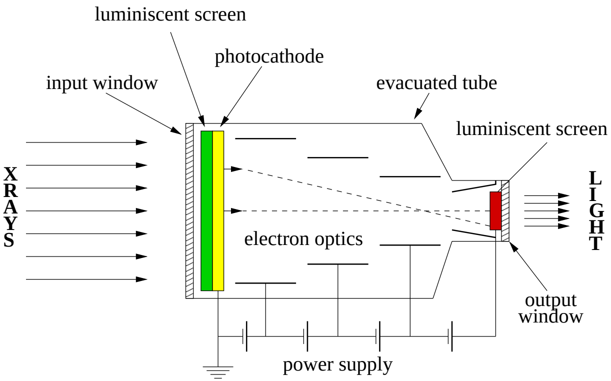

Figure 2: Internal structure of an image intensifier

X-Ray to Intensity Conversion

Materials used in image intensifiers:

input luminescent screen:	CsI:Na
photocathode:	$SbCs_3$
output luminescent screen:	ZnCdS:Ag

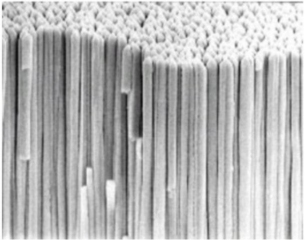

Figure 3: Due to its crystal structure, CsI minimizes lateral diffusion and scattering, i. e., it helps preserving spatial resolution.

Topics

Image Intensifier

Distortion Correction

- Image Distortion
- Distortion Correction – Design Considerations

Summary

- Take Home Messages
- Further Readings

Image Intensifier (II) and Image Distortion

Image distortion using II technology is caused by several phenomena:

- a magnetic field affects the accelerated electrons in the vacuum tube,
  - like the earth magnetic field, or
  - an artificial magnetic field (e. g., MR scanner, or Niobe system),
- scattering (veiling glare),
- convex entrance screen.

Image Distortion

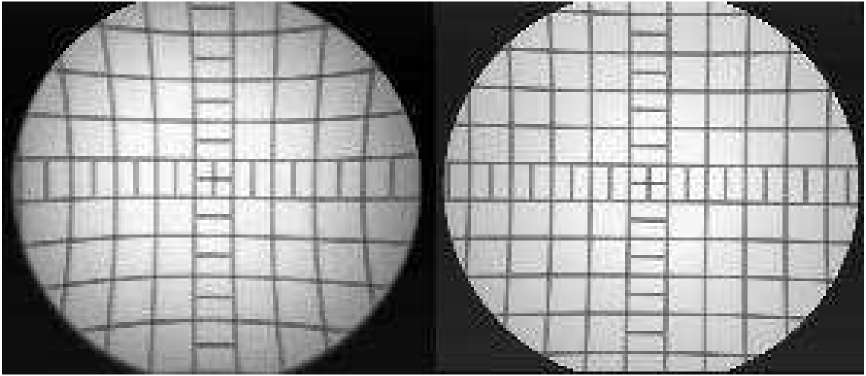

Figure 4: Example of a distorted (left) and an undistorted (right) image (image courtesy of RMIT University, Melbourne)

Image Distortion

We distinguish between two different types of image distortion:

- ***Geometric distortion:***
  - The acquisition device modifies the geometry of the mapped object.
  - In simple terms, we expect that in undistorted images straight lines in 3-D end up as straight lines in the 2-D image plane.
- ***Intensity distortion:***
  - The acquisition device induces changes in intensities.
  - In simple terms, we expect that in undistorted images identical tissue classes are mapped to identical intensities.
  - The heel effect is an example of intensity distortion.
  - Color normalization or homogenization of illumination can be used to tackle this type of distortion.

Distortion Correction

Definition

Image undistortion (or ***distortion correction***) is an image-to-image mapping that eliminates the distortions implied by the image acquisition device in the image plane.

Distortion Correction

How can we correct geometric image distortion?

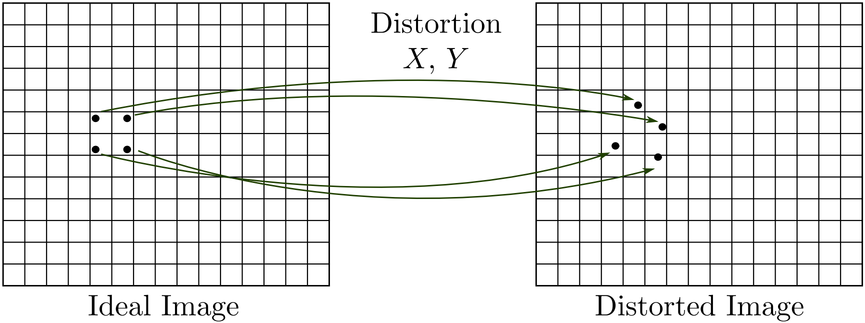

Figure 5: Image (un-)distortion $\leftrightarrow$ mapping of pixels or image points

Geometric Image Undistortion: Core Problems

These **problems** have to be solved for implementation of a geometric image undistortion algorithm:

- definition of a parametric or non-parametric **mapping** between undistorted and distorted image,
- **interpolation** of intensities of neighboring pixels, because lattice points of the undistorted image are not necessarily mapped to lattice points in the distorted image,
- a **robust and reliable estimation** of parameters or displacement vectors of the mapping,
- development of **efficient and robust algorithms** to run distortion correction (e. g., real-time image undistortion in cardiology with 30 frames per second).

Geometric Image Undistortion: Stages

Geometric image undistortion is a **three-stage process**:

- model design,
- estimation of model parameters (calibration),
- inference.

Geometric Image Undistortion: Model Design

Remarks on design issues:

- **Rule of thumb:** *always* sample in the space of your output!
- Consider parametric vs. non-parametric models.
- The dimension of the parameter space should be selected carefully (recall the curse of dimensionality!).
- Consider linear vs. non-linear estimators.
- Make optimal use of available hardware (e. g., manycore architectures, graphics card (GPU computing), cell processor, etc.).

Topics

Take Home Messages

- You have learned how an image intensifier works and that image distortions are a common problem that has to be dealt with.
- Geometric distortions and intensity distortions can occur during image acquisition.
- Several steps have to be considered to correct distortion in images. Hardware and algorithm design have an impact on the efficiency and usefulness of the distortion correction.

Further Readings

An excellent overview of different detectors used in X-ray equipment can be found in

> Heinz Morneburg, ed. *Bildgebende Systeme für die medizinische Diagnostik: Röntgendiagnostik und Angiographie, Computertomographie, Nuklearmedizin, Magnetresonanztomographie, Sonographie, integrierte Informationssysteme*. 3rd ed. Publicis MCD Verlag, June 1995 (in German).

Information on the distortion correction products can be found on the vendors' homepages. Try, for instance, www.healthcare.siemens.com.

Medical Image Processing for Diagnostic Applications

Polynomial Undistortion

Online Course – Unit 11

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Topics

Polynomial Undistortion
- Distortion Correction Mapping
- Parameter Estimation/Calibration

Summary
- Take Home Messages
- Further Readings

Distortion Correction Mapping

Non-Parametric Mapping: $\rightarrow$ look-up table or displacement vector field

Parametric Mapping: We assume separable base functions.

- $\rightarrow$ univariate base functions $b_k : \mathbb{R} \rightarrow \mathbb{R}$, where $k = 0, \ldots, d \in \mathbb{N}$
- image point of undistorted image: (x', y')
- image point of distorted image: (x, y)
- mapping coefficients $u_{i,j}, v_{i,j} \in \mathbb{R}, i = 0, \ldots, d, j = 0, \ldots, d$

$$x = X(x', y') = \sum_{i=0}^{d} \sum_{j=0}^{d-i} u_{i,j} b_j(y') b_i(x')$$

$$y = Y(x', y') = \sum_{i=0}^{d} \sum_{j=0}^{d-i} v_{i,j} b_j(y') b_i(x')$$

Distortion Correction Mapping

Example

We choose the **standard polynomials (monomials)**:

$$b_i(x) = x^i,$$

and get the following bi-variate polynomial of degree d for the x-coordinates:

$$x = X(x', y') = \sum_{i=0}^{d} \sum_{j=0}^{d-i} u_{i,j} y'^j x'^i.$$

Example: Solving Steps

Let $d = 2$. We can rewrite the mapping in matrix notation for corresponding image points $(x_1, y_1), \ldots, (x_n, y_n)$ and $(x'_1, y'_1), \ldots, (x'_n, y'_n)$:

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & x'_1 & (x'_1)^2 & y'_1 & x'_1 y'_1 & (y'_1)^2 \\ \vdots & & & \ddots & & \vdots \\ 1 & x'_n & (x'_n)^2 & y'_n & x'_n y'_n & (y'_n)^2 \end{pmatrix}}_{\text{measurement matrix } \boldsymbol{A}} \begin{pmatrix} u_{0,0} \\ u_{0,1} \\ u_{0,2} \\ u_{1,0} \\ u_{1,1} \\ u_{2,0} \end{pmatrix} .$$

Distortion Correction: Calibration of the Mapping

Definition

We call the estimation process of parameters that define the mapping of real-world objects into the camera image plane ***calibration***.

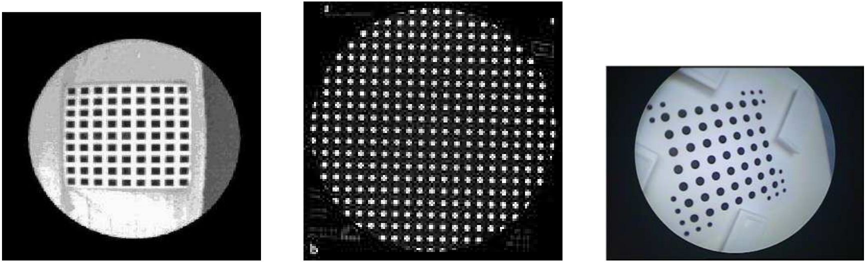

Figure 1: A few examples of calibration patterns with squares, dots, and circles (images courtesy of Siemens Healthcare)

Distortion Correction: Calibration of the Mapping

Calibration problem:

- N 2-D points on the planar calibration pattern are precisely known:

$$(x'_1, y'_1), (x'_2, y'_2), \ldots, (x'_N, y'_N).$$

- N points in the distorted image are observed:

$$(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N).$$

Problem: Estimate the parameters of the distortion function.

Distortion Correction: Calibration of the Mapping

Definition

Least square estimation is a numerical procedure that fits a parametric or non-parametric curve to data points by minimization of the sum of squared distances of data points from the curve.

For the calibration of the mapping, the least square estimation results in the following two optimization problems:

$$\sum_{n=1}^{N} \left(X(x'_n, y'_n) - x_n \right)^2 \rightarrow \min,$$

$$\sum_{n=1}^{N} \left(Y(x'_n, y'_n) - y_n \right)^2 \rightarrow \min.$$

Distortion Correction: Calibration of the Mapping

Using univariate base functions, we have the following $2N$ equations that are linear in $u_{i,j}$ and $v_{i,j}$:

$$x_n = \sum_{i=0}^{d} \sum_{j=0}^{d-i} u_{i,j} b_j(y'_n) b_i(x'_n), \quad n = 1, \ldots, N,$$

$$y_n = \sum_{i=0}^{d} \sum_{j=0}^{d-i} v_{i,j} b_j(y'_n) b_i(x'_n), \quad n = 1, \ldots, N.$$

Distortion Correction: Calibration of the Mapping

These equations can be rewritten in matrix notation (here shown for the x – correspondences, y – correspondences analogously):

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} u_{0,0} \\ u_{0,1} \\ \vdots \\ u_{d,0} \end{pmatrix} .$$

The matrix $\boldsymbol{A}$ is the measurement matrix for the particular problem of image undistortion.

Distortion Correction: Calibration Parameter Estimation

Estimates of the coefficients are computed by:

$$\begin{pmatrix} u_{0,0} \\ u_{0,1} \\ \vdots \\ u_{d,0} \end{pmatrix} = \boldsymbol{A}^{\dagger} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix},$$

where $\boldsymbol{A}^{\dagger}$ is the pseudoinverse of $\boldsymbol{A}$. As we already know, the pseudoinverse can be computed by using the singular value decomposition.

Topics

Polynomial Undistortion

- Distortion Correction Mapping
- Parameter Estimation/Calibration

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- A distortion correction method needs a mapping that is estimated from the correspondences of a calibration pattern and the measured data.
- Choosing univariate base functions is a good option. Using monomials, we find the Vandermonde matrix to be the measurement matrix.
- In the general case, we can use least square estimation to find the mapping parameters.

Further Readings

In case you need to learn more about polynomials and the efficient evaluation of polynomials, you have to read Volume 2 of Prof. Knuth's classic work on The Art of Computer Programming.

A book that covers many image preprocessing methods applied in medical imaging systems is:

Jiří Jan. *Medical Image Processing, Reconstruction, and Restoration: Concepts and Methods*. Signal Processing and Communications. CRC Press, Taylor & Francis Group, Nov. 2005

Medical Image Processing for Diagnostic Applications

Implementation Issues

Online Course – Unit 12

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Topics

Remarks on Parameterization

- Regress Carefully
- Interpolation, Regression, and Overfitting

Scaling of Input Data

Summary

- Take Home Messages
- Further Readings

Remarks on Parameterization

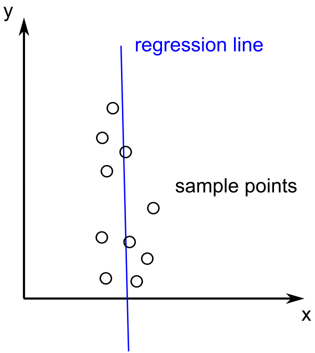

Figure 1: Regression line with infinite slope

Remarks on Parameterization

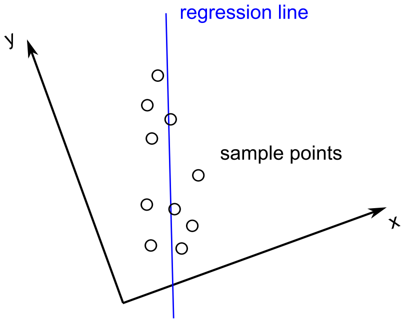

Figure 2: Regression line (rotated reference coordinate system)

Remarks on Parameterization

The parameterization of the straight line decides on the **sensitivity** of estimated parameters **to variations in input data**.

A **well-conditioned problem might appear ill-conditioned** if the parameterization of the problem is not done properly.

Remarks on Parameterization

For straight lines we observe:

- The line representation $y = mx + t$ has **singularities**: the more parallel the regression line to the y-axis, the larger m. For lines parallel to the y-axis, we observe the singularity $m = \infty$ (infinite slope).
- A **fair** representation of straight lines is

$$x \cos \alpha + y \sin \alpha = d,$$

  where $\alpha \in [0, 2\pi]$, $d \in \mathbb{R}$.

Conclusion: Select a parameterization that is independent from orthogonal transforms of the reference coordinate system.

Interpolation, Regression, and Overfitting

Definition

Interpolation defines the estimation of unknown data between observed data. In addition, we require the interpolation curve to fit all the training data.

Definition

Like interpolation, ***regression*** defines a technique to discover a mathematical relationship between multiple variables using a set of data points, i. e., training data. In regression it is not required that the regression curve fits the training data perfectly.

Note: The regression function is usually estimated using a least square approach. The transition from interpolation to regression is smoothly, and some authors do not differentiate between these two techniques explicitly.

Interpolation, Regression, and Overfitting

Definition

Overfitting is defined as training a model, (e. g., a parametric model), so that it well fits the training data, but fails to predict well in between and outside the data.

Overfitting can occur, if a complex model (e. g., a model with many parameters) is trained with a sparse set of data, i. e., too few training examples.

Interpolation, Regression, and Overfitting

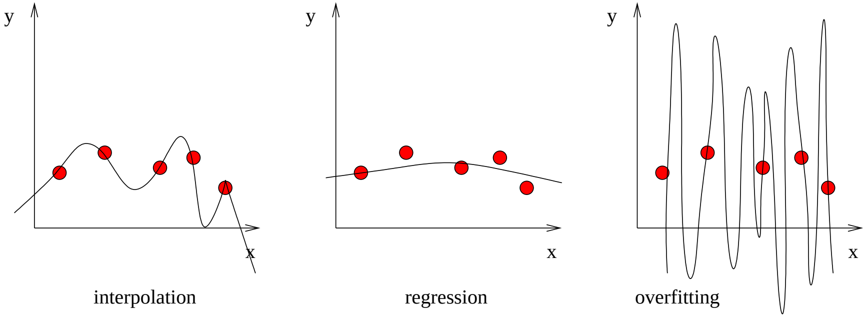

Figure 3: Interpolation vs. regression vs. overfitting

Remarks on Parameter Estimation

Problem: Compute the sensitivity of the estimated parameters from a set of N point correspondences.

- The parameters shall fit for all data that is processed by the used algorithm.
- How can we figure if the estimated parameters are sufficient for the observed data in practice? We might get different data as input for the algorithm.
- To compute the sensitivity (robustness) of the estimated parameters, we need many data samples.
- If we do not have many samples, we can try a bootstrapping approach, but we will not go into detail here.

Topics

Scaling of Input Data

- Proper scaling of data is **crucial** for the quality of the output, a fact that is often overseen.
- Limited numerical accuracy requires certain ranges.
- "*Data normalization must not be considered optional!*" (Richard Hartley)
- Select the optimal scaling by minimization of the condition number to **minimize sensibility** and to find a **proper data range**:

$$\kappa(\boldsymbol{A}^{\mathsf{T}}\boldsymbol{A}) \rightarrow \min .$$

Scaling of Input Data

Example

- Use a polynomial of total degree 5 to undistort images.
- The dimensions of the input images are 1024×1024 pixels.
- The $x-$ and $y-$coordinates are represented in pixels,
  i. e., $x, y \in \{1, 2, \dots 1024\}$.
- The monomials range from 1 to $1024^5 = 1125899906842624$.
- The result has to be between 0 and 1023!!!

$\rightarrow$ Think about it! Do you have a good feeling in doing this?

Minimization of κ

The Gramian matrix $\boldsymbol{A}^\mathsf{T}\boldsymbol{A}$ can be used to test for linear independence of functions. Any decrease of the condition number will be useful, even if it is not a global optimum!

Method to compute a proper scaling:

1. Select two constants k and l.
2. Scale all data points (x_i, y_i) to (kx_i, ly_i).
3. Rewrite the linear system for solving for the calibration coefficients from the last unit.
4. Compute the new measurement matrix $\boldsymbol{A}$.
5. Compute the condition number $\kappa(\boldsymbol{A}^\mathsf{T}\boldsymbol{A})$.
6. Minimize κ with respect to k and l (e. g., by gradient descent).
7. Finally, recover the original coefficients $u_{i,j}, v_{i,j}$ and invert the scaling process.

Topics

Take Home Messages

- A parameterization has to be chosen wisely.
- Know the differences between interpolation and regression.
- Also be aware of overfitting, i. e., that a model can adapt too much to its training data.
- Data normalization is mandatory.

Further Readings

A book that covers many image preprocessing methods applied in medical imaging systems is:

Jiří Jan. *Medical Image Processing, Reconstruction, and Restoration: Concepts and Methods*. Signal Processing and Communications. CRC Press, Taylor & Francis Group, Nov. 2005

For the original article about the bootstrapping method see

Bradley Efron. "Bootstrap Methods: Another Look at the Jackknife". In: *The Annals of Statistics* 7.1 (1979), pp. 1–26. DOI: doi:10.1214/aos/1176344552

Medical Image Processing for Diagnostic Applications

Efficient Implementation

Online Course – Unit 13

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Topics

Efficient Evaluation of Polynomials

Interpolation of Intensities

Distortion Algorithm: Summary and Further Applications

Summary
- Take Home Messages
- Further Readings

Efficient Evaluation of Polynomials

Problem: Each acquired image requires a distortion correction.

Observations:

- Bivariate polynomials can be understood as univariate polynomials where the coefficients are univariate polynomials (instead of a simple constant value):

$$x = \sum_{i=0}^{d} \left(\sum_{j=0}^{d-i} u_{i,j} y'^{j} \right) x'^{i}.$$

- Univariate polynomials are evaluated using Horner's scheme:

$$p(x) = \sum_{i=0}^{d} a_i x^i = (\ldots(a_d x + a_{d-1})x + \ldots)x + a_0.$$

Efficient Evaluation of Polynomials

Definition

The ***Horner scheme***, named after William George Horner, is an algorithm for the efficient evaluation of polynomials in monomial form. Horner's method describes a manual process by which one may approximate the roots of a polynomial equation. The Horner scheme can also be viewed as a fast algorithm for dividing a polynomial by a linear polynomial. (Wikipedia)

- For each line we get a univariate polynomial ($y' = \text{const}$).
- Row and column increments are constant.
- Arithmetic progression $\rightarrow$ reuse of former evaluations

Conclusion: After an initialization, for each pixel only sums have to be computed.

Example: Horner Scheme

Let us assume we have a matrix $\boldsymbol{M}$ and observations $(x', y')^\mathsf{T}$, and we want to find $(x, y)^\mathsf{T}$ such that:

$$\boldsymbol{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}.$$

Suppose we implement the necessary operations:

$$\begin{pmatrix} m_{1,1} & m_{1,2} \\ m_{2,1} & m_{2,2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} m_{1,1}x + m_{1,2}y \\ m_{2,1}x + m_{2,2}y \end{pmatrix}$$

for several pairs of x and y in two nested `for`-loops over x and y, we will do some computations more often than necessary.
Using the Horner scheme, we reuse earlier computations constant in the inner loop.

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Interpolation of Intensities

- **Nearest Neighbor Interpolation** is the simplest interpolation method. We assign to an image point in between pixels the intensity value of the closest pixel. Usually the Euclidean distance is applied.
- In **Bilinear Interpolation**, we compute for the new image point a weighted mean of neigboring intensities. Here the intuition is applied that pixels closer to the new image point have an higher impact on the final intensity value.

- Mostly the results of nearest neighbor interpolation are not well appreciated. The images appear crispy and noisy, though the interpolation method is extremely fast.
- In most practical applications where interpolation is required, bilinear interpolation is applied.

Interpolation of Intensities

Computation of the intensity *i*

- Nearest neighbor interpolation:

$$i = a$$

- Bilinear interpolation:

$$x = a(1-d') + dd',$$
$$y = b(1-d') + cd',$$
$$i = x(1-d'') + yd''$$

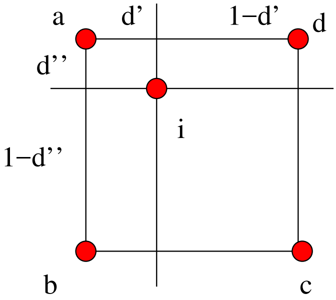

Figure 1: Interpolation: i is interpolated using the known intensities a, b, c, d.

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Accelerated Distortion Correction Algorithm

FOR $x :=$ first to last line DO
- FOR $y :=$ first to last column DO
  - compute coordinate $x = X(x', y')$
  - compute coordinate $y = Y(x', y')$
  - compute intensity $f(x, y) = \mathrm{interpol}(g; x, y)$

Figure 2: Image undistortion routine (g distorted image, f corrected output image)

Hardware Accelerated Image Warping

- Image is decomposed into squares.
- Map the vertices of the square.
- Undistortion can be implemented using texture mapping unit.
- Bilinear interpolation hardware is supported in GPUs.
- Texture mapping hardware is supported in GPUs.

Application of Image Undistortion: Endoscopy

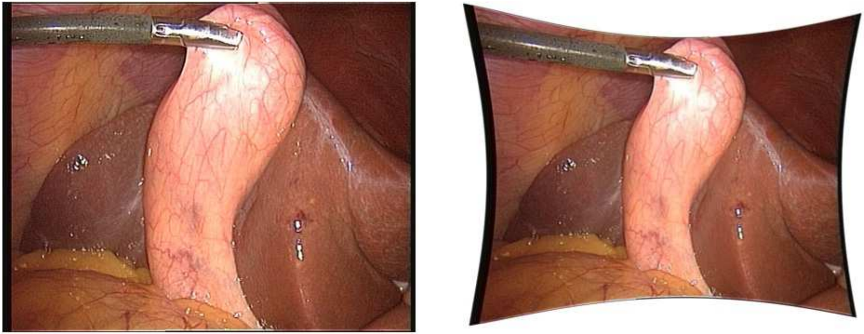

Figure 3: Original, distorted endoscope image (left), and the result of distortion correction (right)

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Take Home Messages

- The Horner scheme can be used for efficient evaluation of polynomials.
- Bilinear interpolation is sufficient for nearly all practical problems.

Further Readings

In case you need to learn more about polynomials and the efficient evaluation of polynomials, you have to read Volume 2 of Prof. Knuth's classic work on The Art of Computer Programming.

A book that covers many image preprocessing methods applied in medical imaging systems is:

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