Medical Image Processing for Diagnostic Applications

Fan Beam – Rebinning

Online Course – Unit 37

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Topics

Example: Homogeneous Cylinder

- Source is at position $\boldsymbol{a} = (a_x, a_y)^\mathsf{T}$.
- Detector detects rays:

$$g(\boldsymbol{a}, \gamma) = \int_{-\infty}^{\infty} f(a_x + t\cos\gamma, a_y + t\sin\gamma)\,\mathrm{d}t.$$

- Object is bounded by $\{(x, y)|R^2 = (x^2 + y^2)\}$.

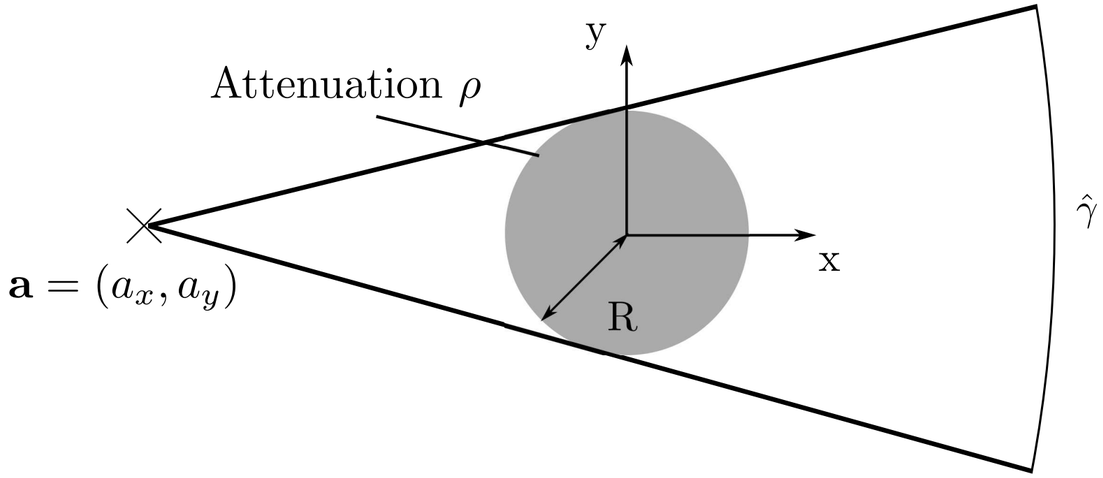
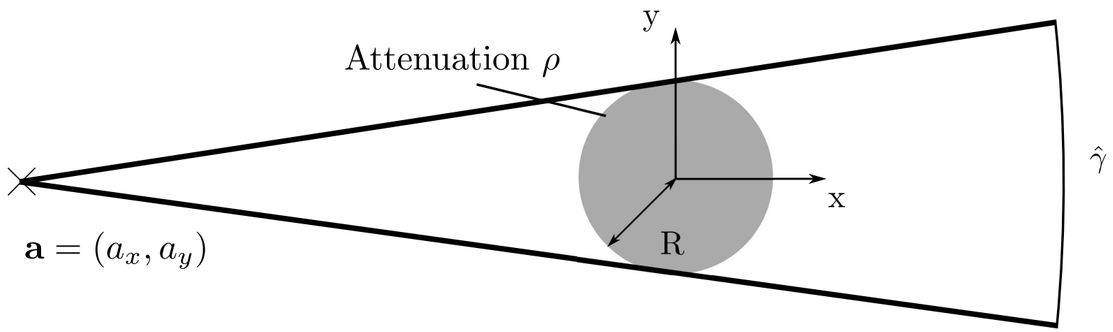


Figure 1: Cross section fan beam projection of a cylinder requires the opening angle $\hat{\gamma}$.

Homogeneous Cylinder: Far Source



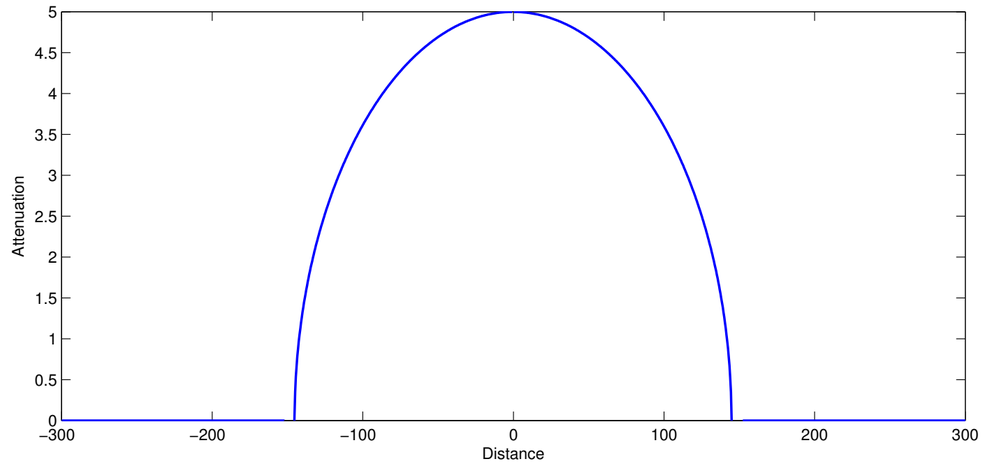


Figure 2: When the object is far from the source (left) its fan beam projection $g(\boldsymbol{a}, \gamma)$, $\gamma \in [-\frac{\hat{\gamma}}{2}, \frac{\hat{\gamma}}{2}]$, is condensed in a smaller part of the detector (right).

Homogeneous Cylinder: Close Source

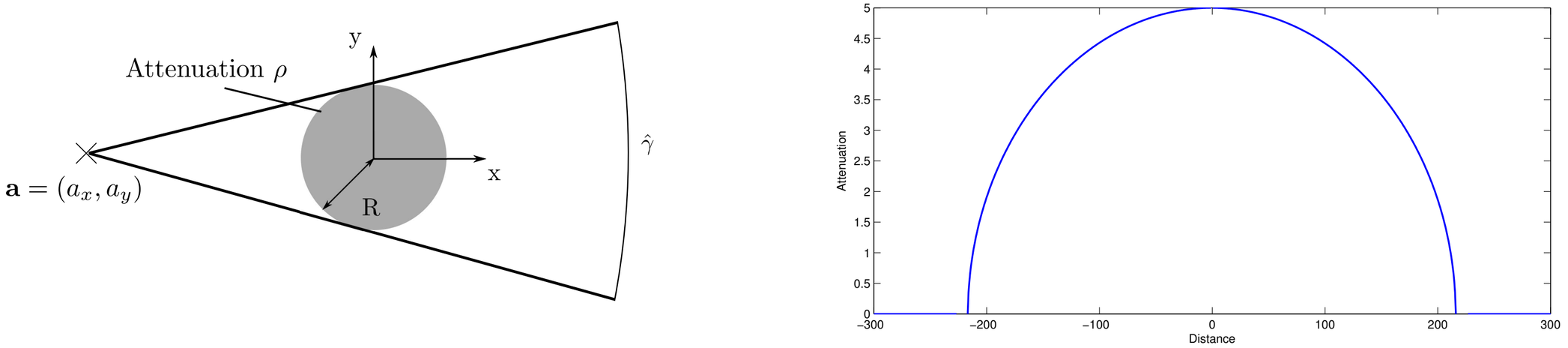
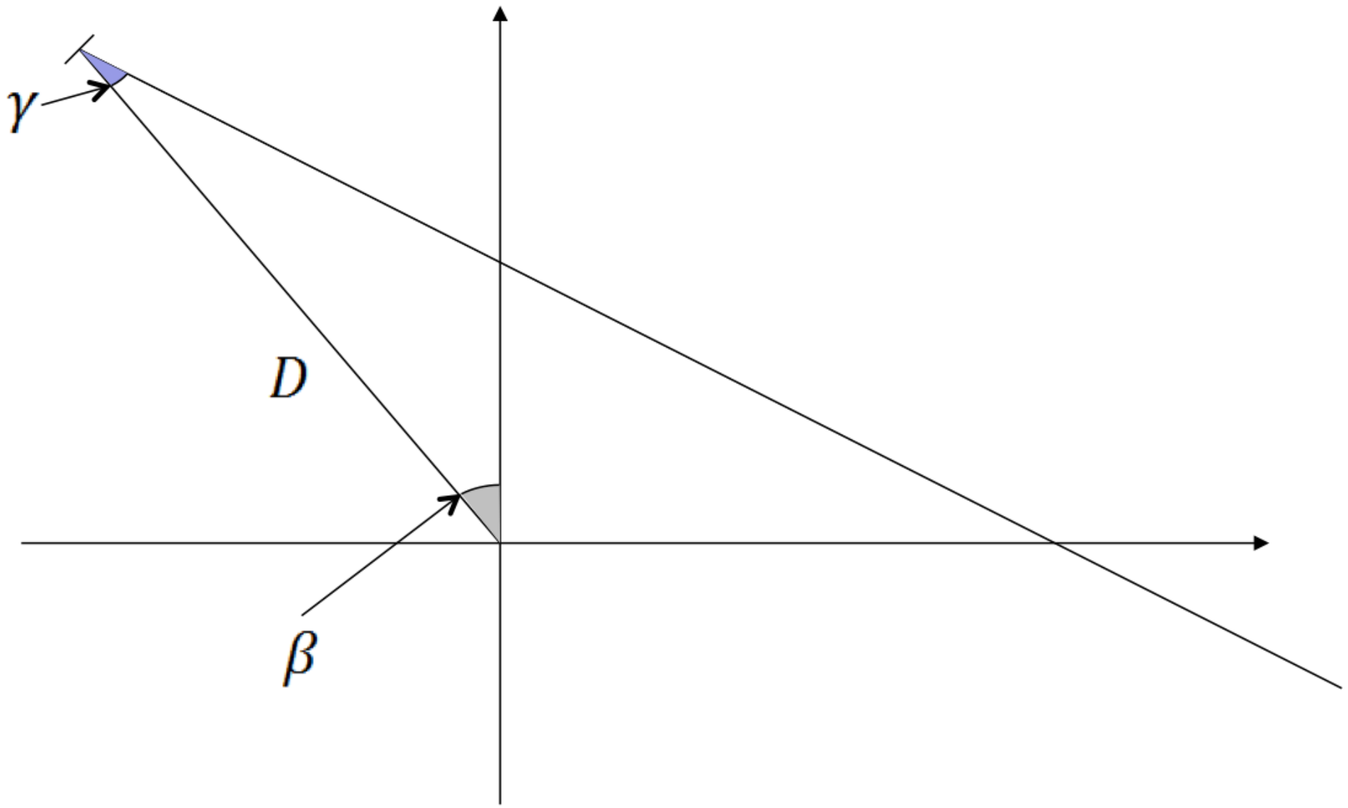


Figure 3: When the same object is closer to the source (left) its fan beam projection $g(\boldsymbol{a}, \gamma)$, $\gamma \in [-\frac{\hat{\gamma}}{2}, \frac{\hat{\gamma}}{2}]$, fills a larger part on the detector (right).

Basic Transform for Angle γ, Detector Center in Origin

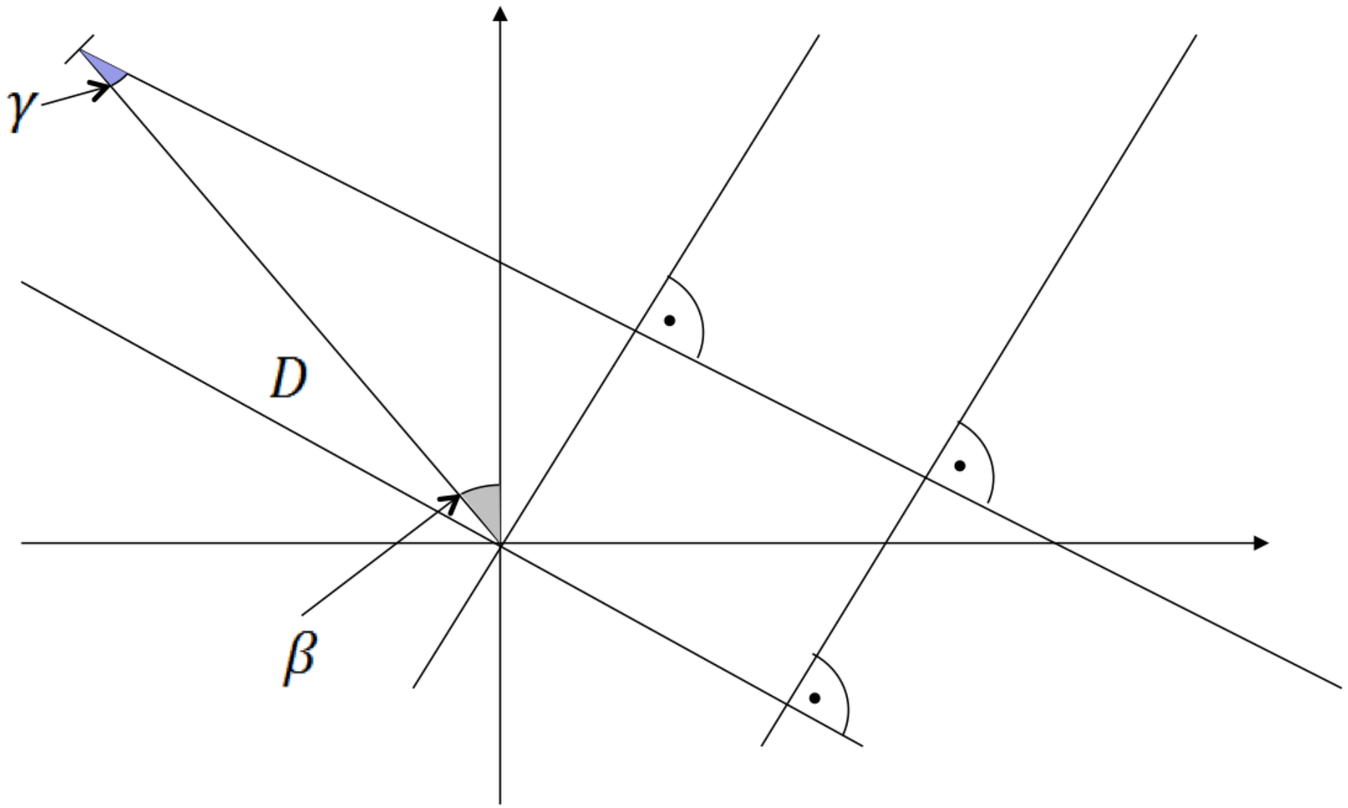


$$\theta = \gamma + \beta$$

$$s = D \sin \gamma$$

Figure 4: Graphical derivation for the general conversion from fan beam to parallel beam geometry. This diagram series shows how a detected fan beam under the angle γ and source rotation β is equivalent to a parallel beam detected on a detector rotated by θ at a shifted position s.

Basic Transform for Angle γ, Detector Center in Origin



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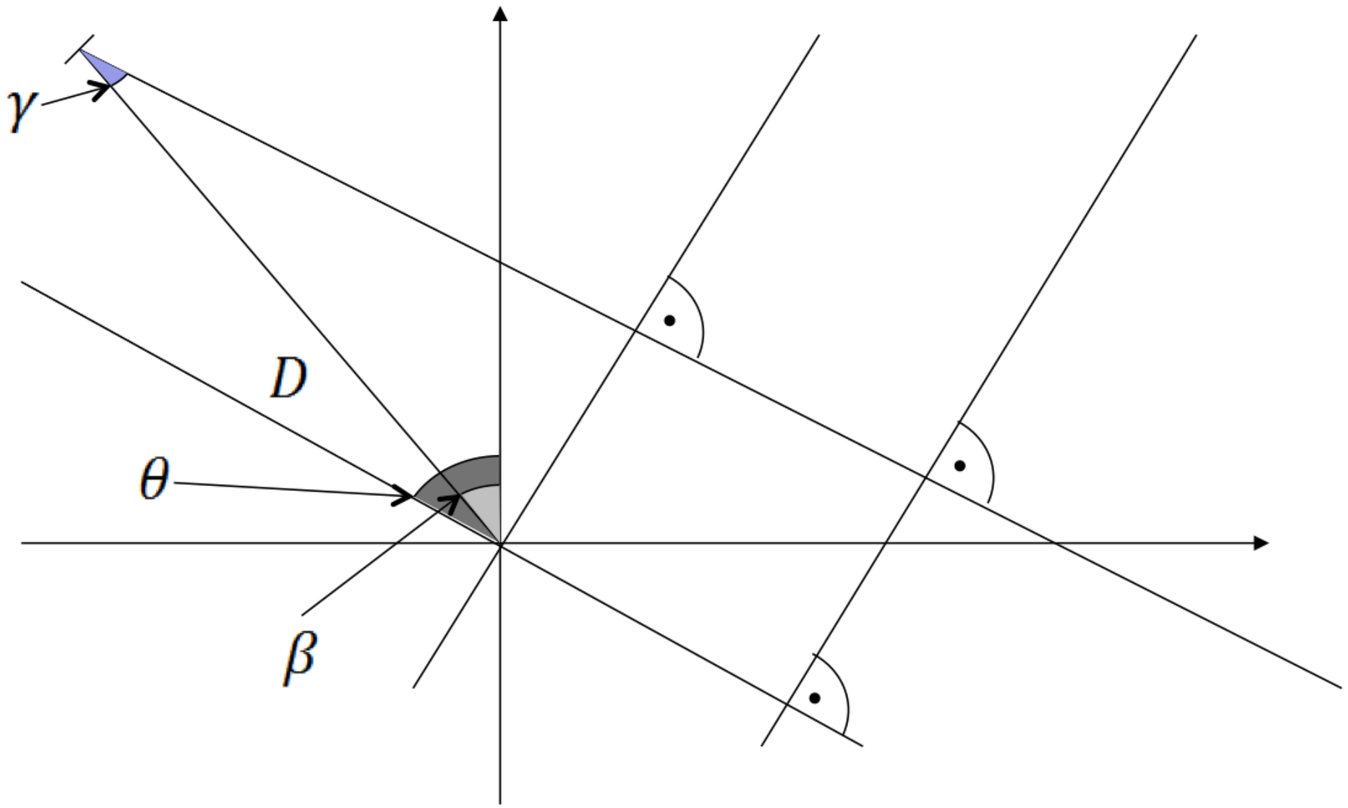

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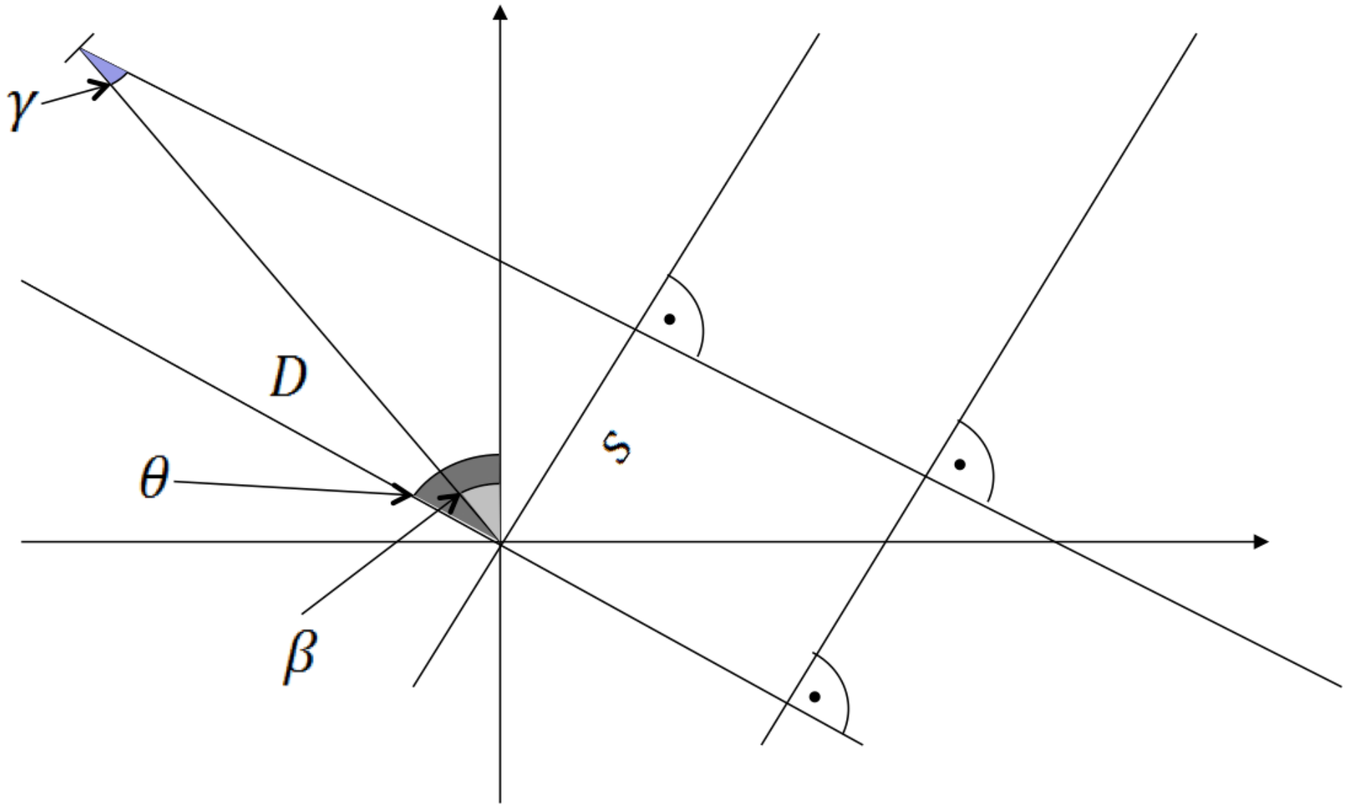

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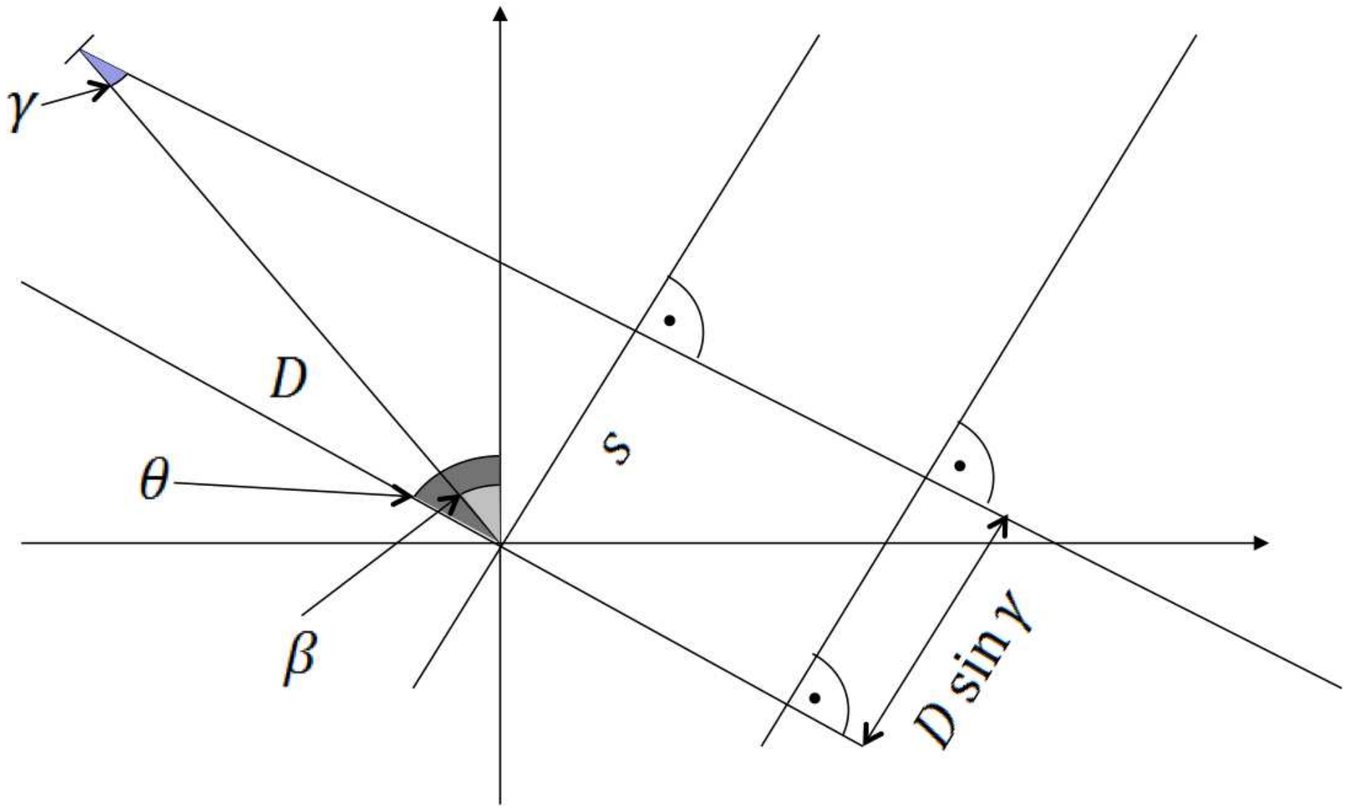

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Equally-spaced and Equiangular Detectors

Sampling is different in both detector geometries:

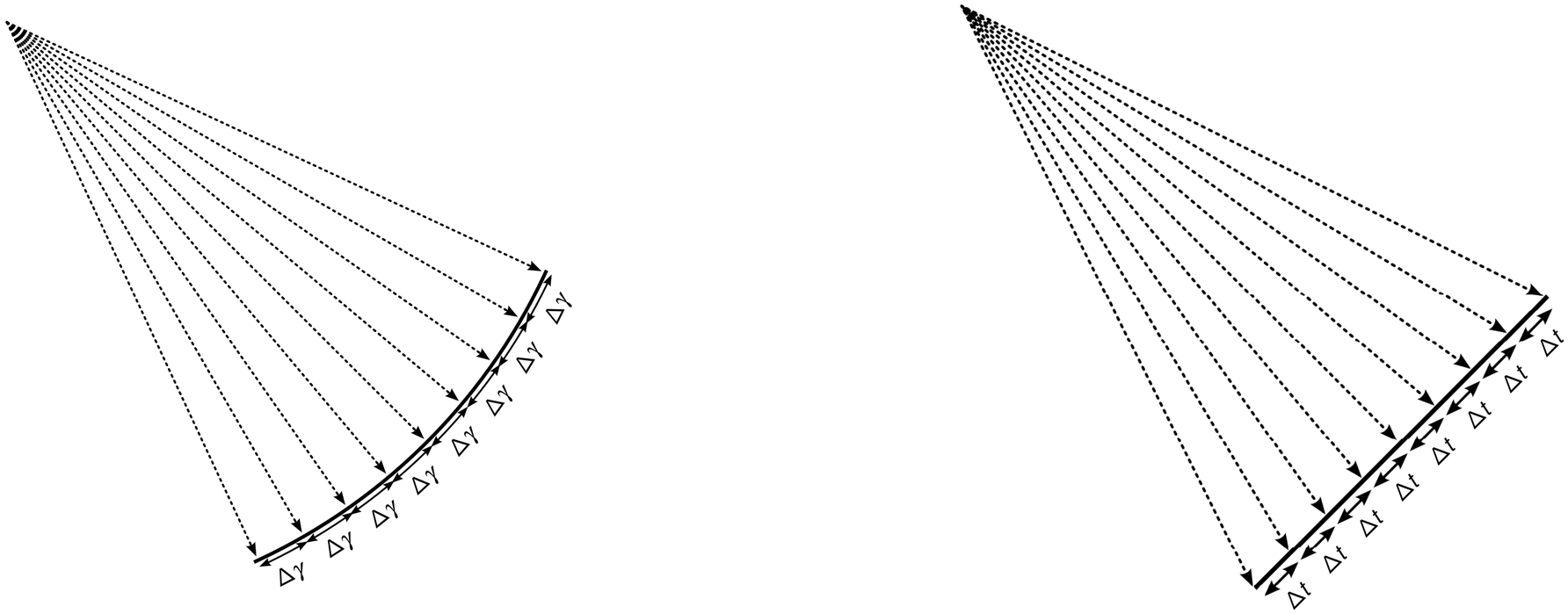

Figure 5: Flat detector with equiangular spacing $\Delta\gamma$ (left), curved detector fan beam with equal spacing Δt (right) (Magdalena Herbst)

Parallel Beam to Fan Beam Conversion: Flat-Panel

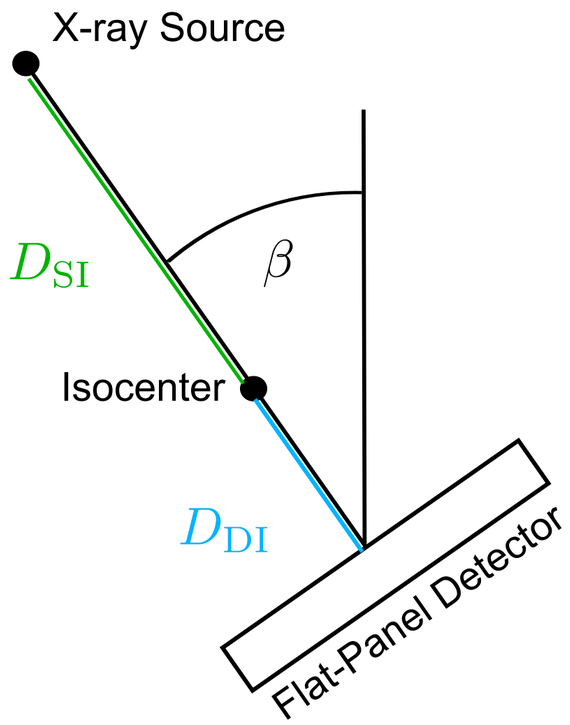

Figure 6: Graphical derivation of the conversion from fan beam to parallel beam geometry for a flat panel detector not centered in the origin. D_{DI} denotes the detector-isocenter distance and D_{SI} is the source-isocenter distance.

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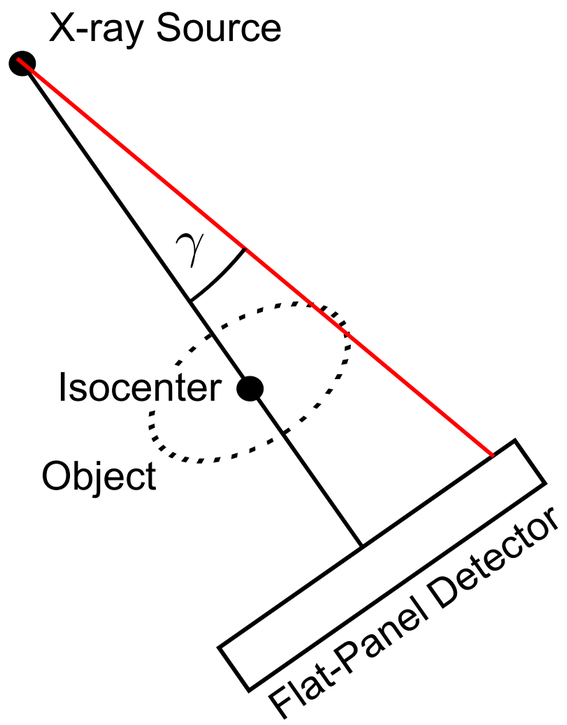

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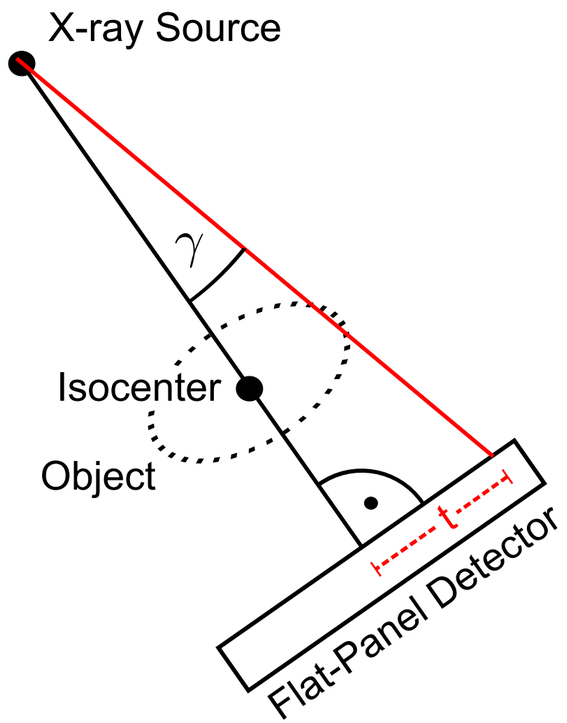

$$\tan\gamma = \frac{t}{D_{\mathrm{SI}}+D_{\mathrm{DI}}}$$

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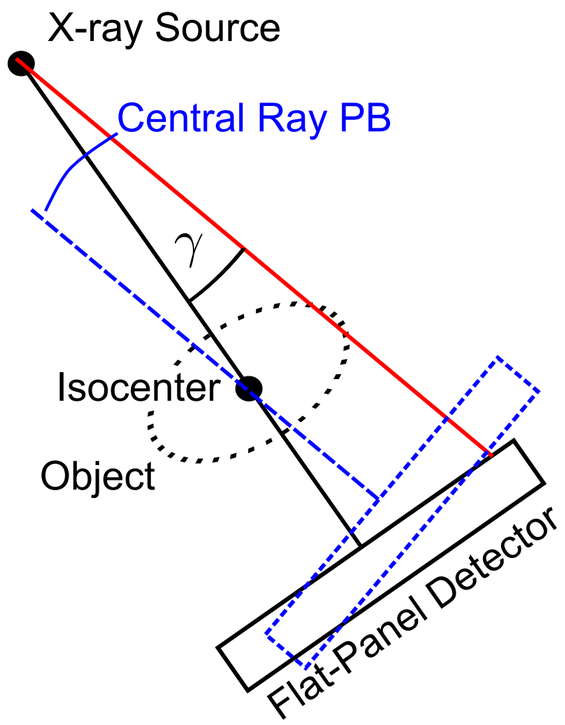

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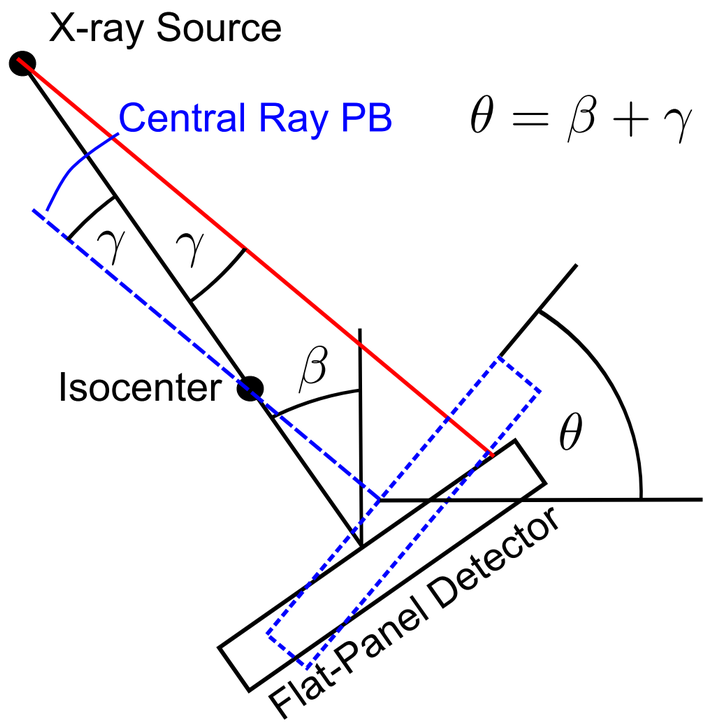

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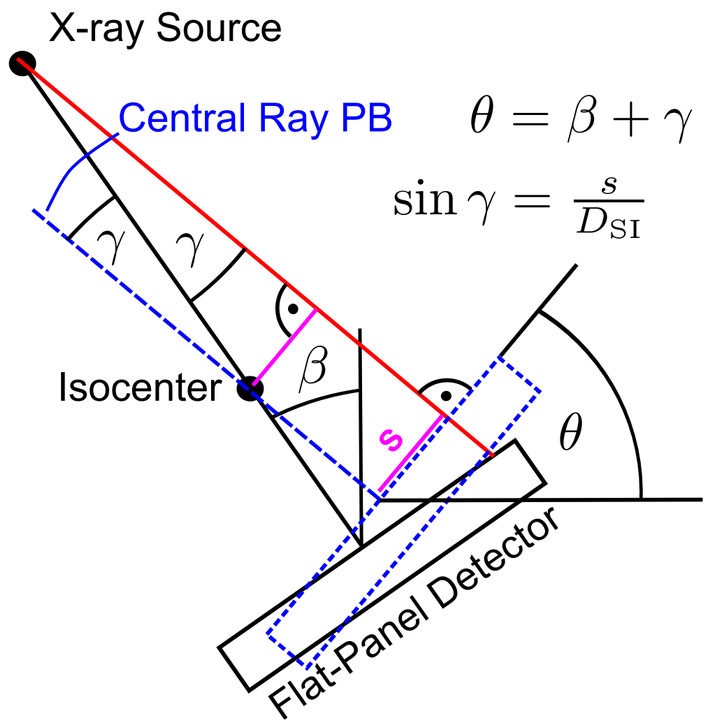

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Parallel Beam to Fan Beam Conversion

- **Idea:** Find equal rays in both geometries:

$$\theta = \gamma + \beta,$$
$$s = D \sin \gamma.$$

- Then set:

$$p(s, \theta) = g(\gamma, \beta).$$

- For flat panels these equations hold:

$$\theta = \beta + \arctan \frac{t}{D_{\mathrm{SI}} + D_{\mathrm{DI}}},$$
$$s = \frac{D_{\mathrm{SI}} t}{\sqrt{(D_{\mathrm{SI}} + D_{\mathrm{DI}})^2 + t^2}},$$
$$p(s, \theta) = g(t, \beta).$$

- This process is called ***rebinning***.

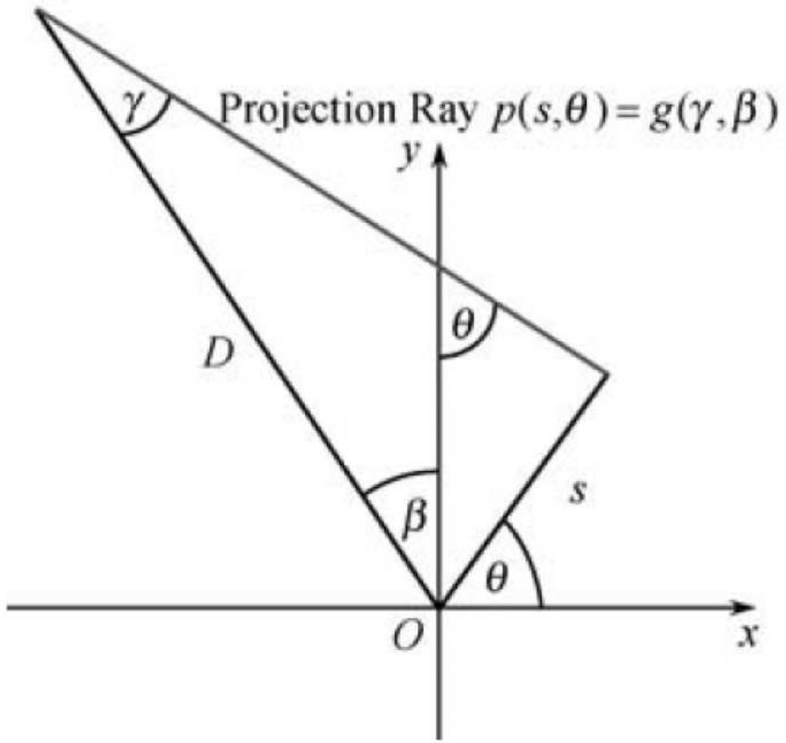

Figure 7: Corresponding rays yield the same projection value (Zeng, 2009).

Parallel Beam to Fan Beam Conversion

- Rebinning is a feasible solution.
- Change of coordinate systems requires interpolation which can introduce inaccuracies.
- Hence, rebinning might not be the method of choice.

$\Rightarrow$ Derive a reconstruction method for fan beam data by analytical conversion of the reconstruction algorithm.

Concept for finding a reconstruction algorithm

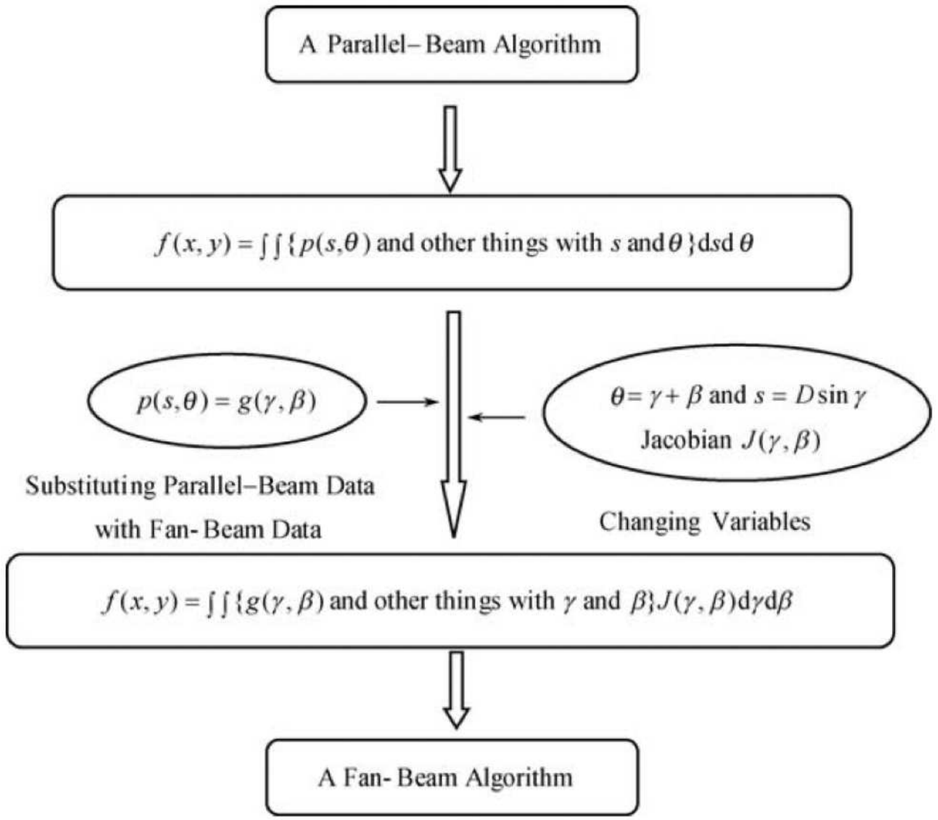

Figure 8: Flow chart showing the steps needed to develop a fan beam reconstruction algorithm (Zeng, 2009).

Topics

Take Home Messages

- Since the PSFs of fan beam and parallel beam geometry are equal, there is a process called rebinning which transforms one into the other.
- The rebinning concept mainly servers analytical purposes, because interpolation errors make reconstructions inaccurate.
- Fan beam reconstruction is dependent on the distances between source, isocenter and detector, in contrast to parallel beam.
- One has to distinguish flat panel detectors from curved detectors which both are in use.

Further Readings

Helpful reads for the current unit:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Ronald N. Bracewell. *The Fourier Transform and Its Applications*. 3rd ed. Electrical Engineering Series. Boston: McGraw-Hill, 2000