



Reconstruction in 3-D

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Exercise Sheet 7

22 Analytic and Iterative Reconstruction

- (i) The X-ray transform models the process of how CT raw data is acquired. In 3-D, is there a difference between this transform and the Radon transform? Include the respective integrals in your answer.

Hint: How and in which geometry are the integrals computed?

- (ii) The algebraic reconstruction technique (ART) is a practical application of the Kaczmarc method. To get an idea how it works, assume that the following two equations describe two lines in the solution space of a synthetic CT projection:

$$\begin{aligned}3y - x &= 5, \\11x + 4y &= 19.\end{aligned}$$

Compute two iteration steps using ART to find an approximate solution $\mathbf{X}^{(2)} \in \mathbb{R}^2$. Initialize your algorithm with $\mathbf{X}^{(0)} = (0, 0)^T$.

How good is this estimate? Compute the exact solution of the linear system and compare. Comment on the convergence rate for this specific example.

- (iii) What is the main drawback of the elementary ART? Name and explain in a few words three different techniques of how we can tackle this problem.

2+2+2

23 Data Completeness

- (i) What is Orlov's condition? What is Tuy's condition? Show for each condition a trajectory that meets its criterion.

- (ii) Use Tuy's condition to explain under which condition the FDK algorithm performs an exact reconstruction.
- (iii) In the lecture, we have taken a closer look at one specific algorithm that can be used for a helical trajectory. Which reconstruction algorithm was that? This algorithm used a concept called π -lines. Explain them in your own words and discuss the treatment of redundant data.

2+1+1

Submit to your tutor until the last exercise session on 01/18/2017 or 01/19/2017.

Total: 10
