Medical Image Processing for Diagnostic Applications

Sinograms and Fan Beam Geometry

Online Course – Unit 36
Andreas Maier, Joachim Hornegger, Markus Kowarschik, Frank Schebesch
Pattern Recognition Lab (CS 5)

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Topics

Sinograms

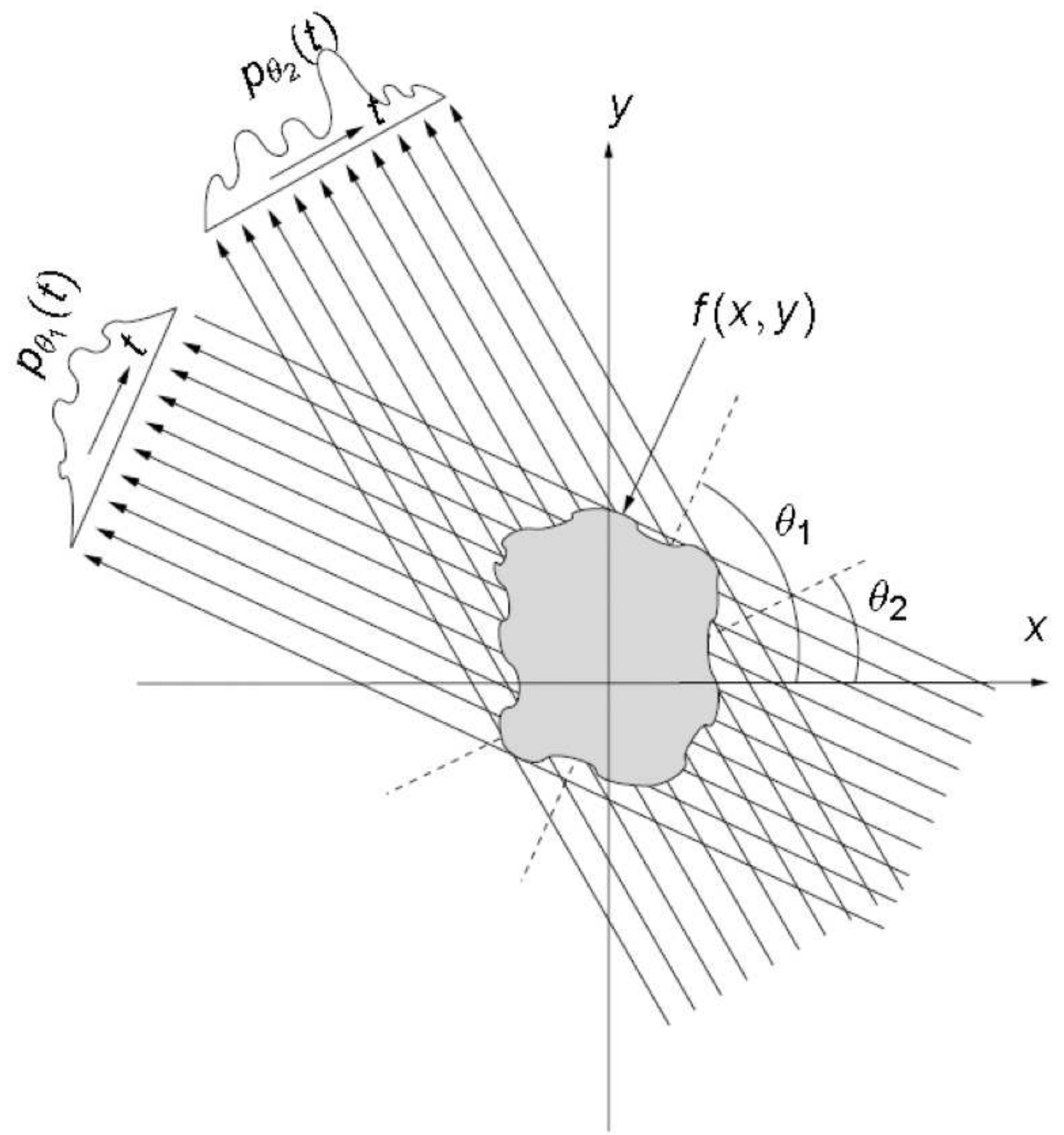
Fan Beam Geometry

Summary

Take Home Messages

Further Readings

Parallel Beam Geometry



(https://en.wikipedia.org/wiki/File:Emi1010.jpg)

Figure 1: Parallel projection scheme with two different angles θ_1, θ_2 and the object $f(x, y)$

A *sinogram* ...

- ... is a method to visualize all projections in one image.
- ... contains all information to reconstruct one slice.
- ... is also called *“fanogram”* in fan beam geometry.
- ... is a popular method for visualization with narrow detectors.

Parallel Beam Geometry: Sinogram

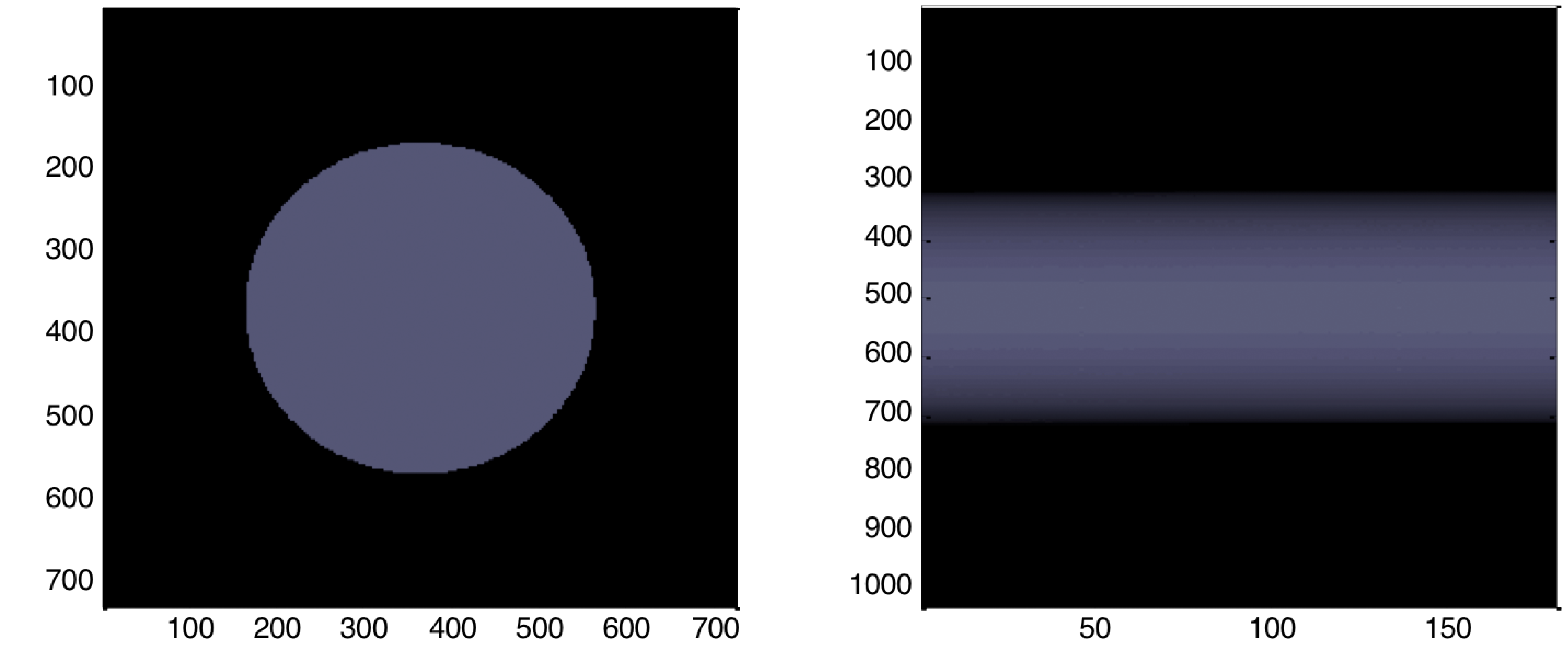
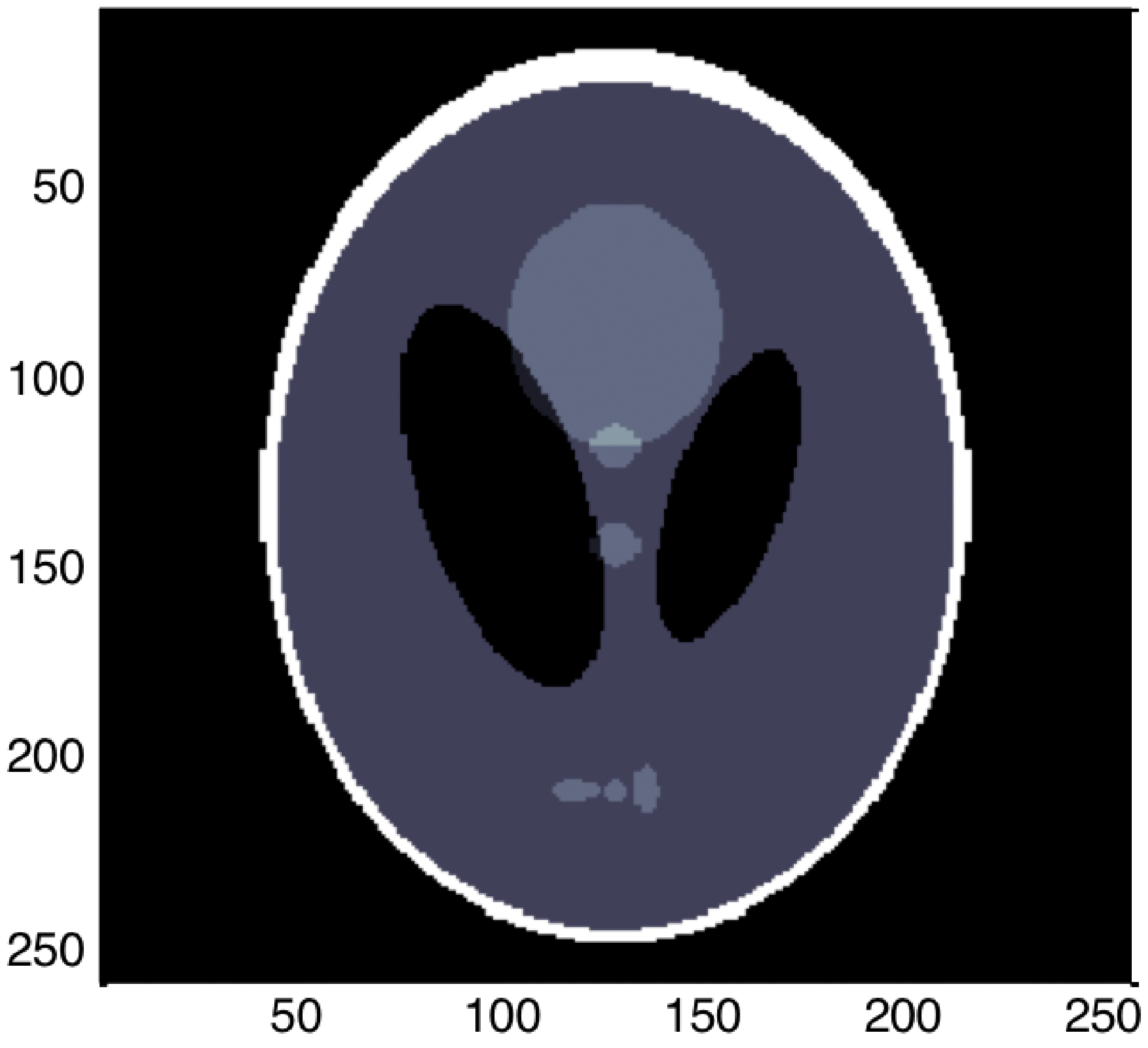


Figure 2: Circle phantom (left) and its sinogram (right)

Parallel Beam Geometry: Sinogram



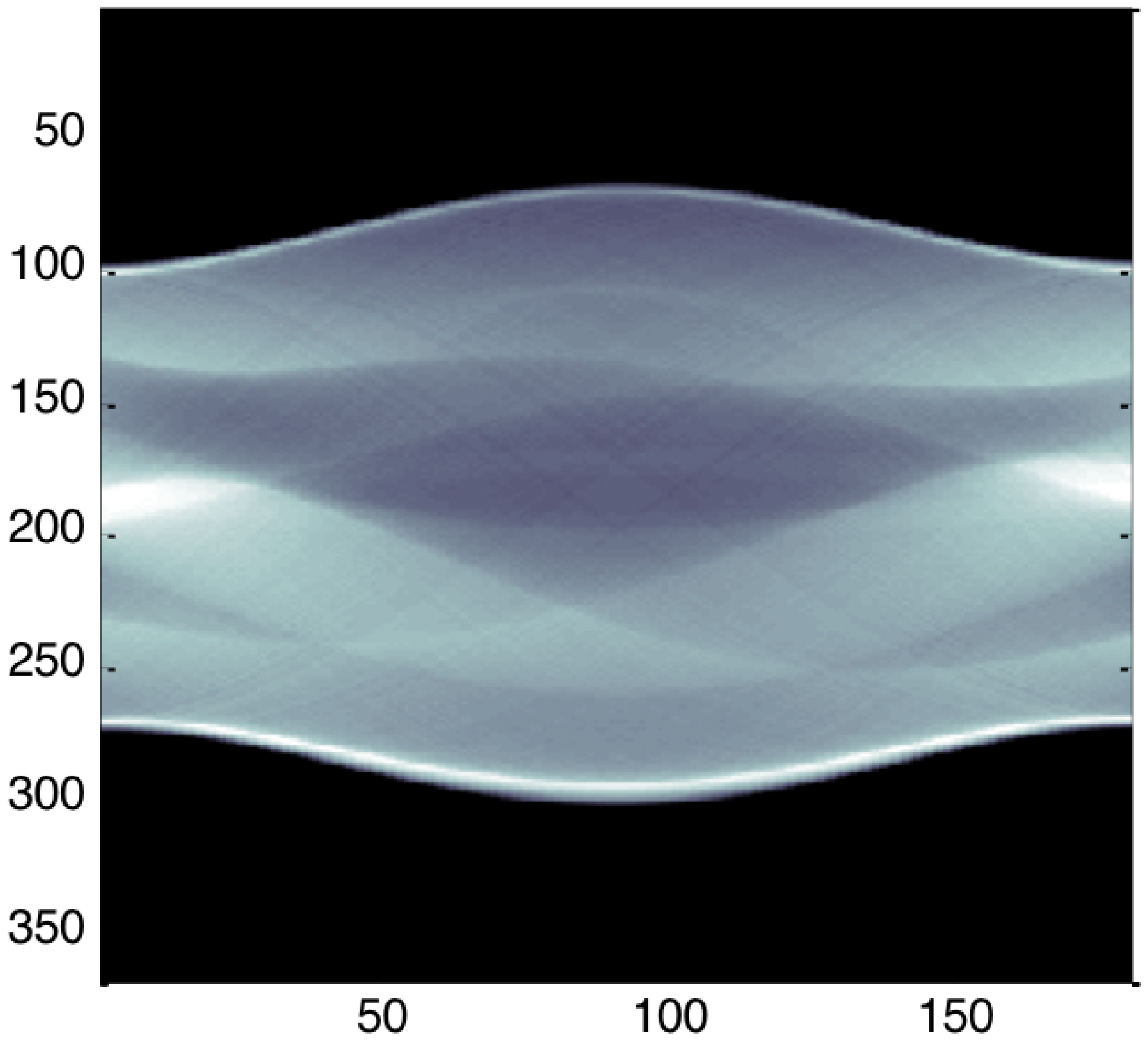


Figure 3: 2-D Shepp-Logan phantom (left) and the corresponding sinogram (right)

Parallel Beam Geometry: Sinogram

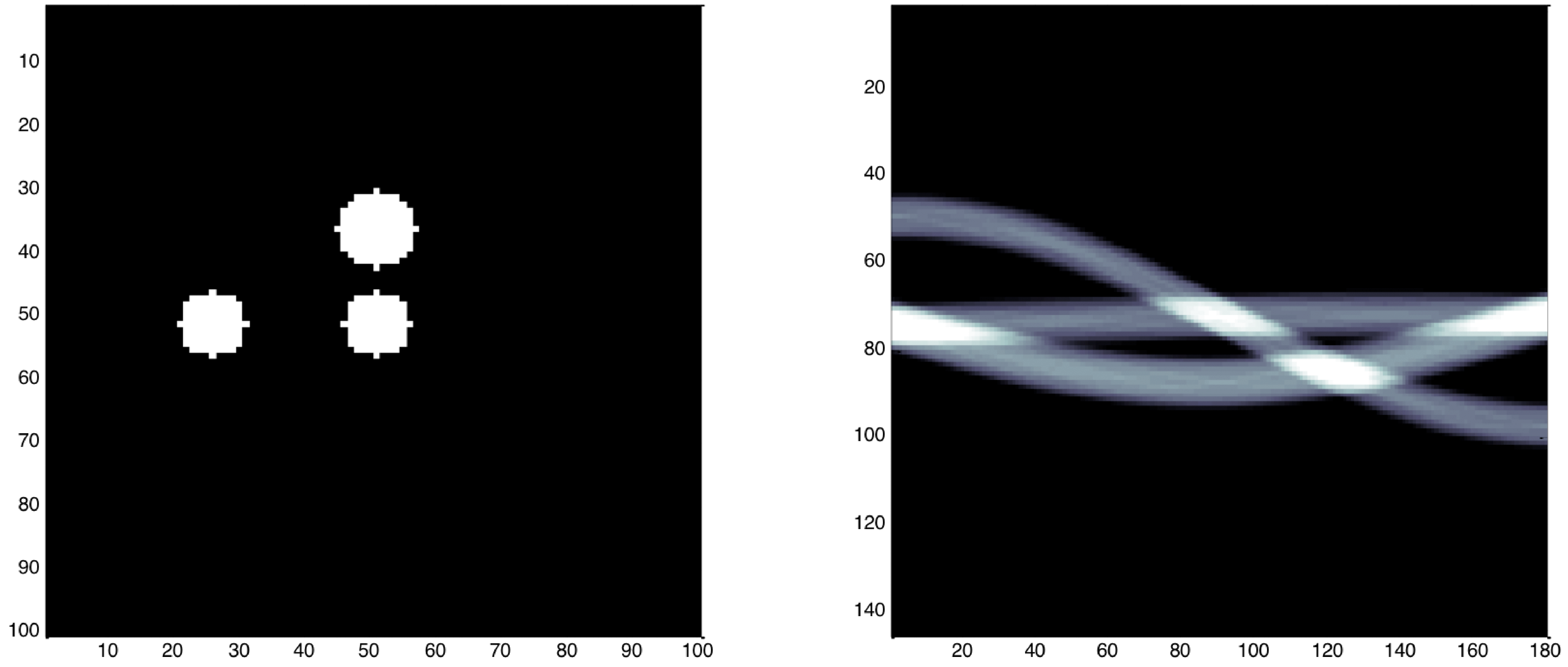


Figure 4: Several small objects (left) and their path in the sinogram (right)

Fan Beam Geometry

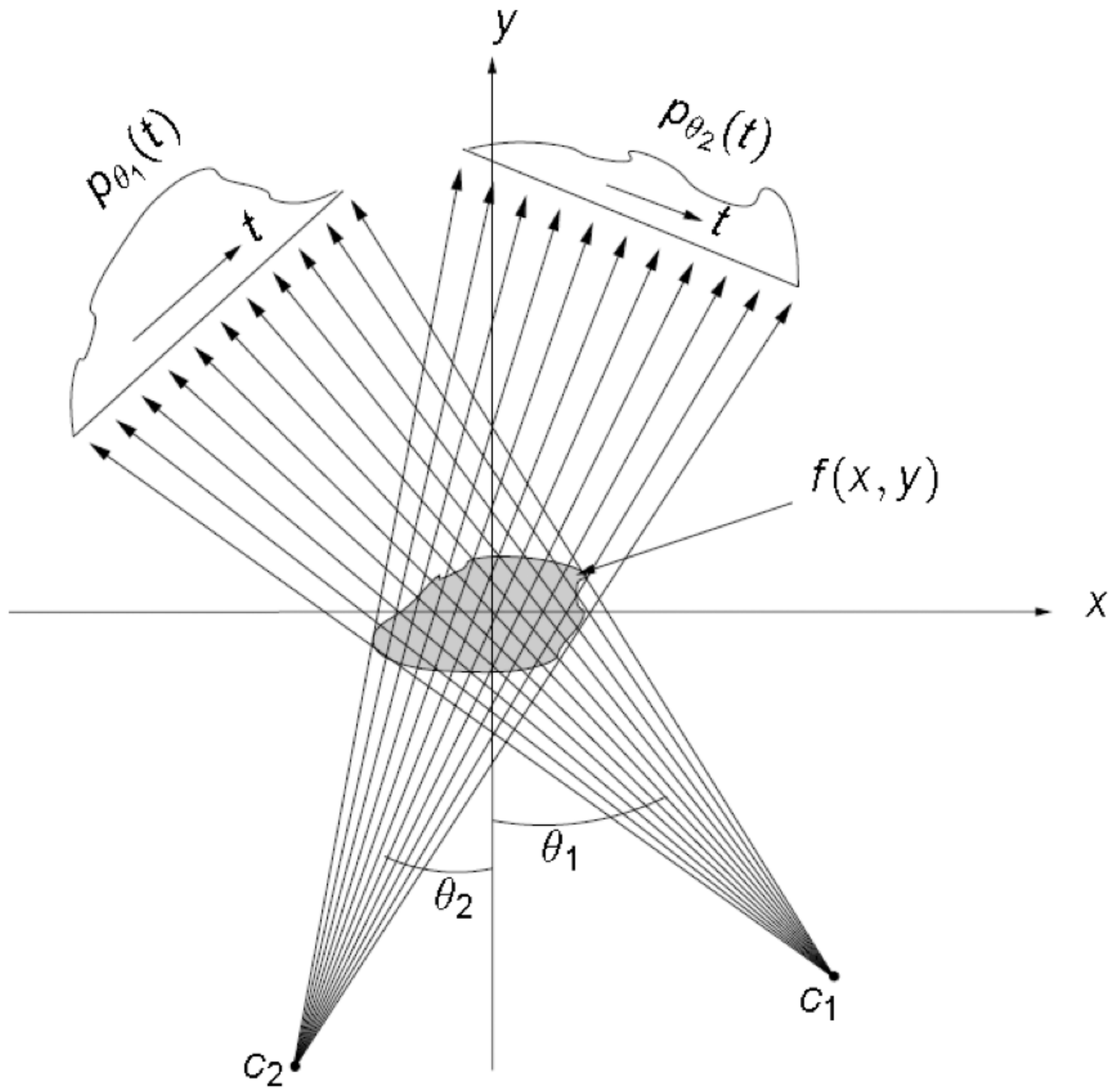
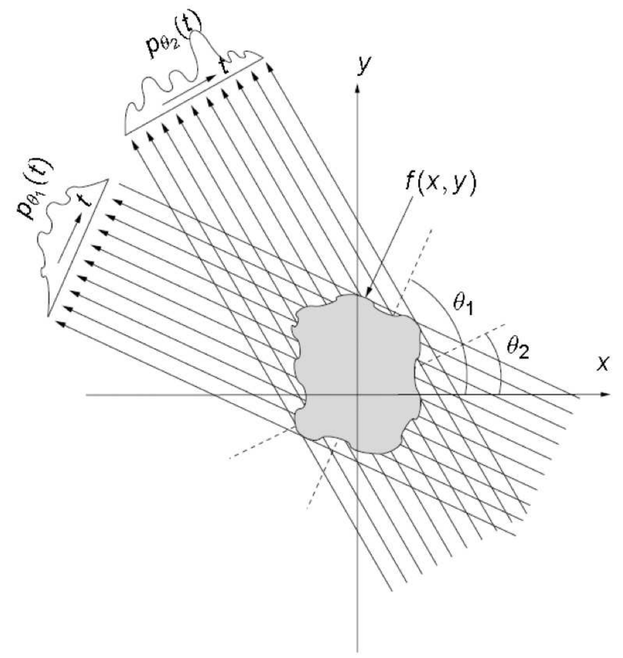
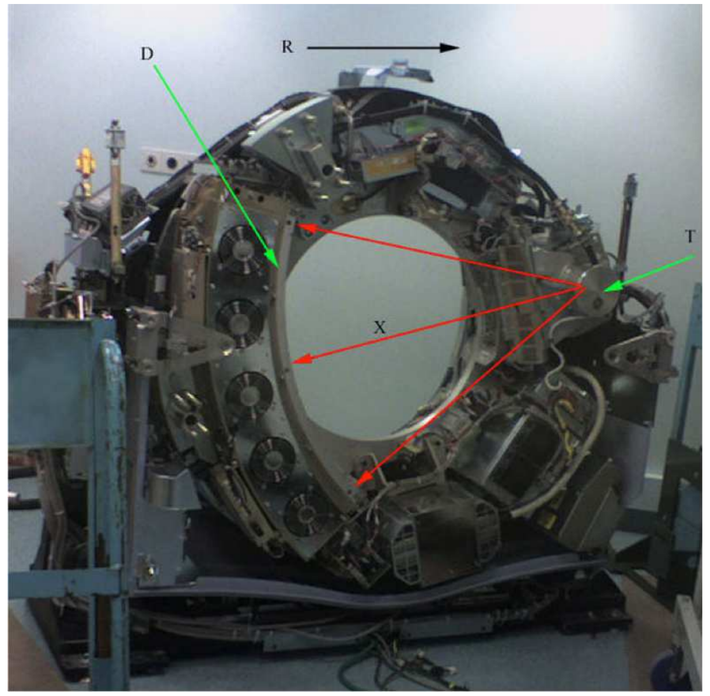


Figure 5: Fan beam projection scheme with two different angles θ_1, θ_2 and the object $f(x,y)$





(GFDL: https://commons.wikimedia.org/wiki/File:Ct-internals.jpg)

Topics

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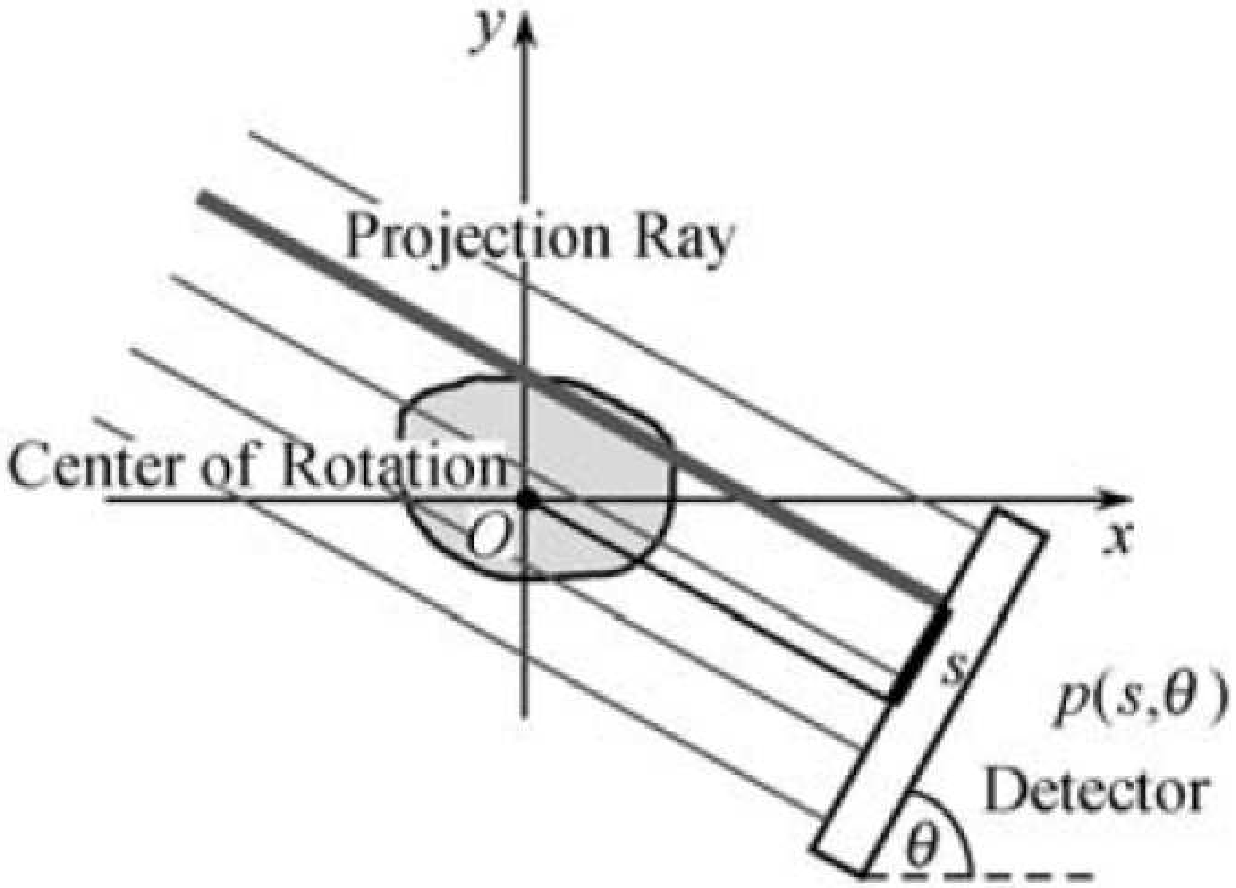
Summary

- Take Home Messages
- Further Readings

Fan Beam vs. Parallel Beam

- Parallel beam algorithms cannot be applied directly anymore.
- We do not have a central slice theorem anymore.



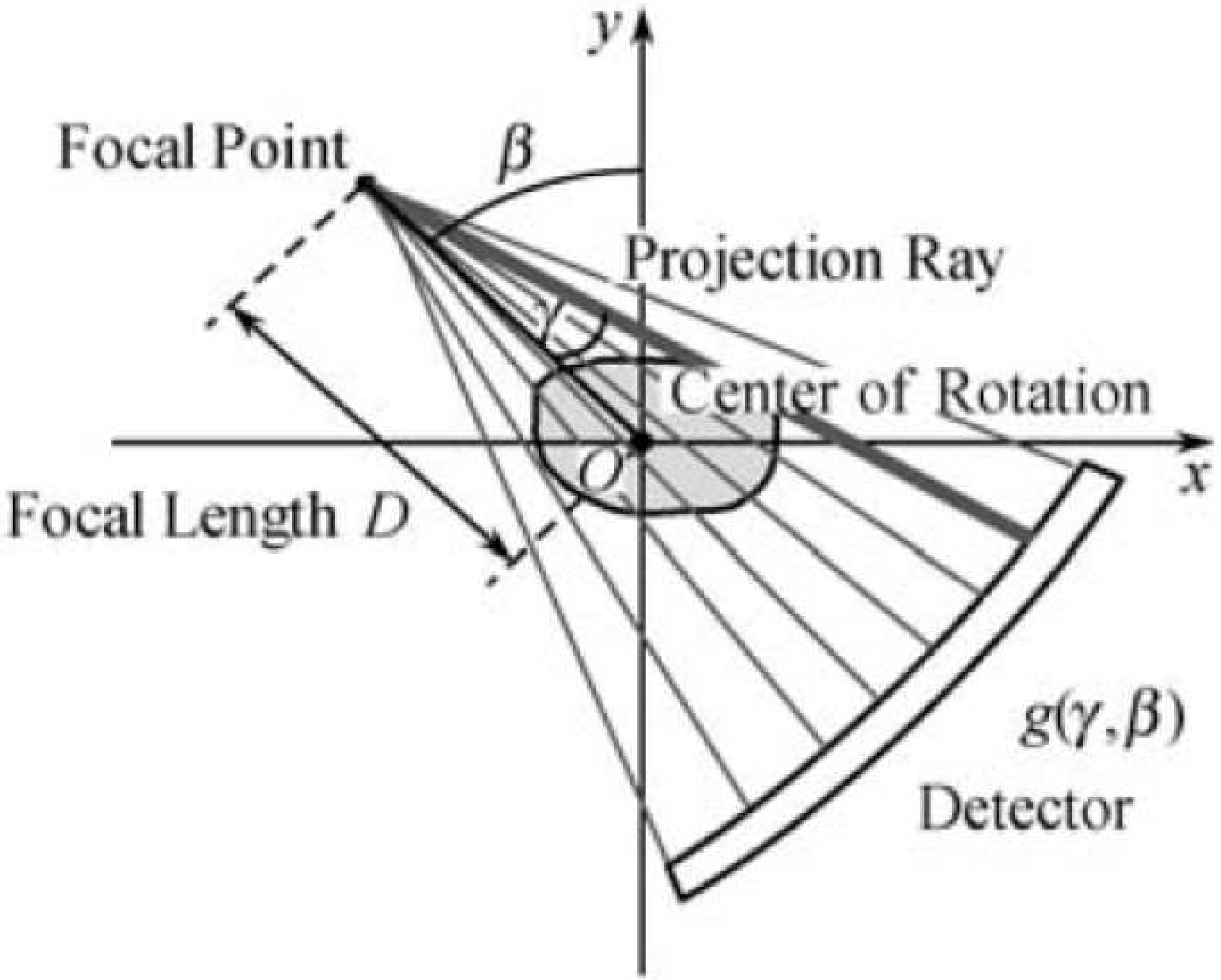


Figure 6: Parallel beam projection with a flat detector (left) and fan beam projection with a curved detector (right) (Zeng, 2009)

Point Spread Function (PSF): Parallel Beam

- Draw a line through the reconstructed point that is perpendicular to the detector.
- Repeat this for every detector position.
- In this case, the point spread function is shift-invariant, i. e., every reconstructed point shows the same pattern.

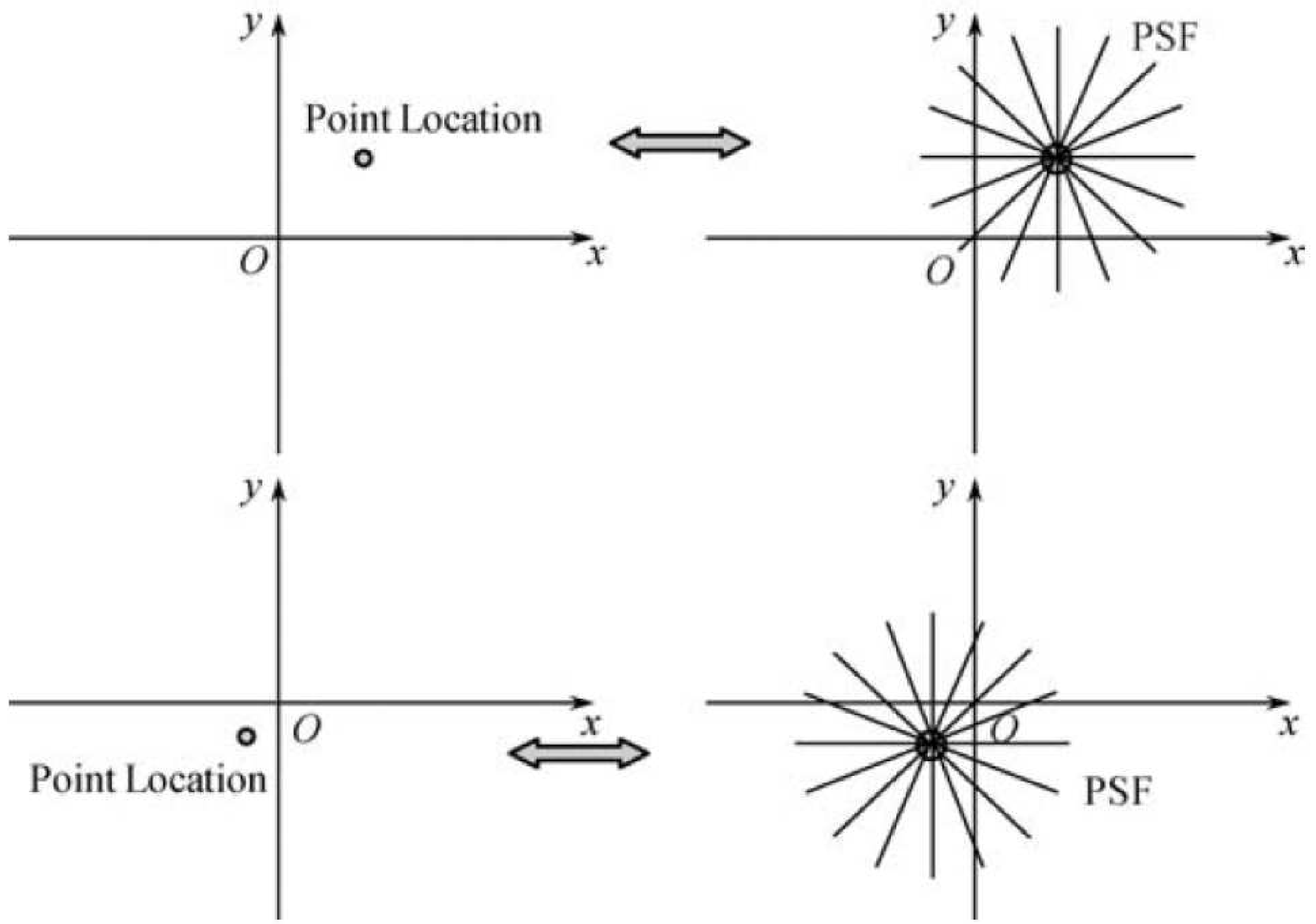


Figure 7: Relationship between point object and its PSF (Zeng, 2009)

Point Spread Function: Fan Beam

- Draw a line through the reconstructed point and the source position.
- Repeat this for every source position.
- For a complete circle, the pattern is also shift-invariant.
- It can be shown that the full circle PSF is equivalent to the parallel beam PSF!

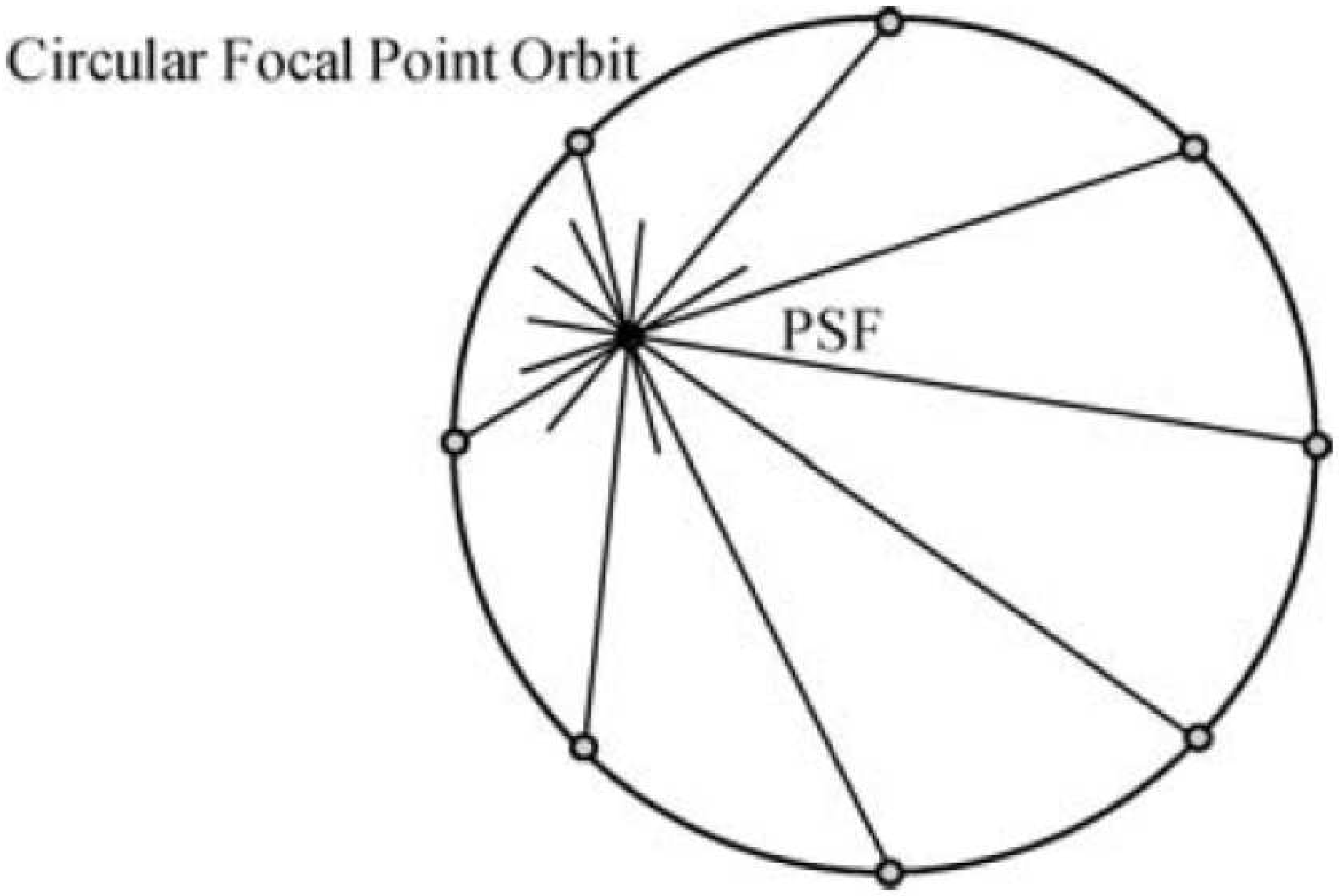


Figure 8: PSF in fan beam geometry (Zeng, 2009)

Topics

Sinograms

Fan Beam Geometry

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- Further Readings

Take Home Messages

- Since parallel beam scanners are slow, the next logical step are designs using fan beam geometry.
- We have learned what a sinogram is.
- The PSF is a useful tool to analyze the reconstruction output and relate it to the input.
- Our observation on the similarity of both parallel beam and fan beam PSF motivates the reconstruction algorithm in the following unit.

Further Readings

Helpful reads for the current unit:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Ronald N. Bracewell. *The Fourier Transform and Its Applications*. 3rd ed. Electrical Engineering Series. Boston: McGraw-Hill, 2000

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Fan Beam – Rebinning

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Topics

Parallel Beam to Fan Beam Conversion

- Distance Dependent Projections
- General Transform
- Transform for Equally-spaced Flat Panel Detectors
- Parallel Beam to Fan Beam Conversion – Summary

Summary

- Take Home Messages
- Further Readings

Example: Homogeneous Cylinder

- Source is at position $\boldsymbol{a} = (a_x, a_y)^\mathsf{T}$.
- Detector detects rays:

$$g(\boldsymbol{a}, \gamma) = \int_{-\infty}^{\infty} f(a_x + t\cos\gamma, a_y + t\sin\gamma)\,\mathrm{d}t.$$

- Object is bounded by $\{(x, y) | R^2 = (x^2 + y^2)\}$.

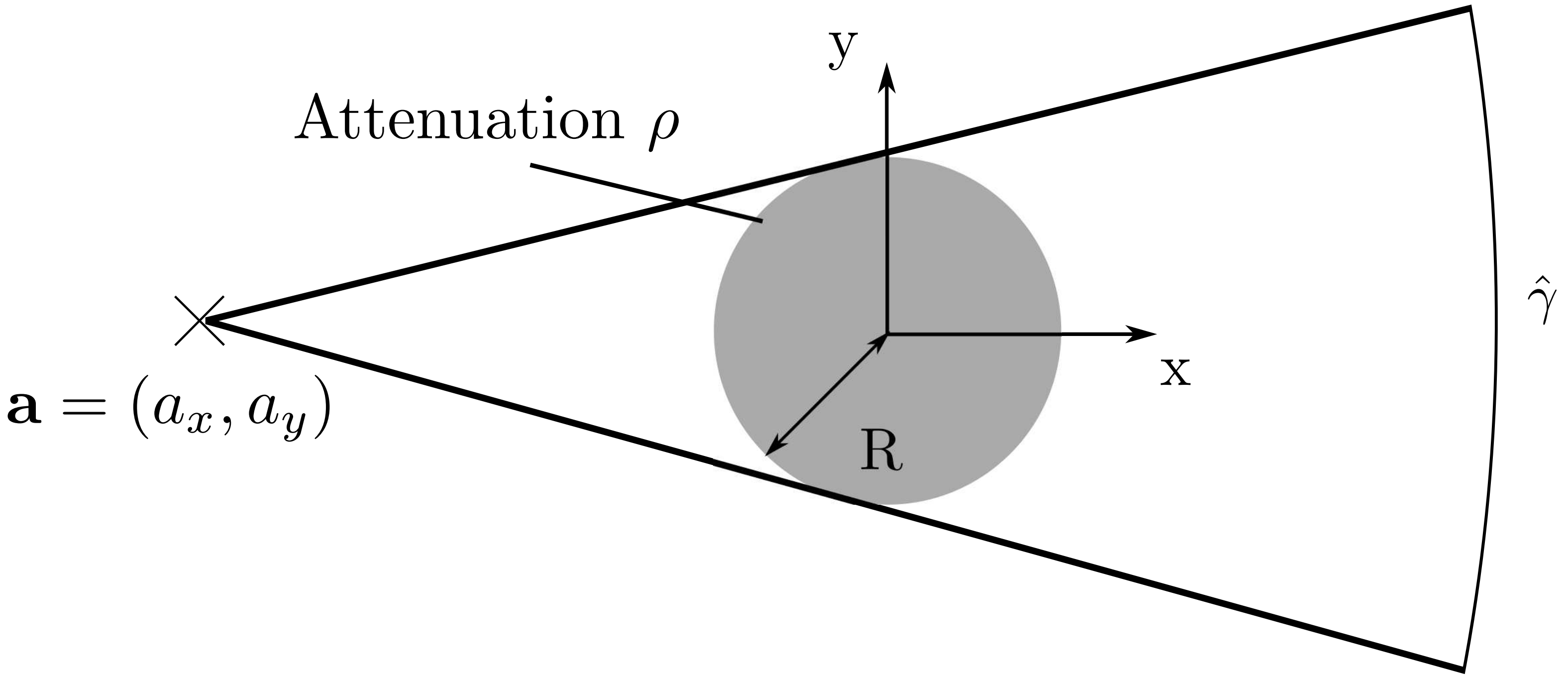
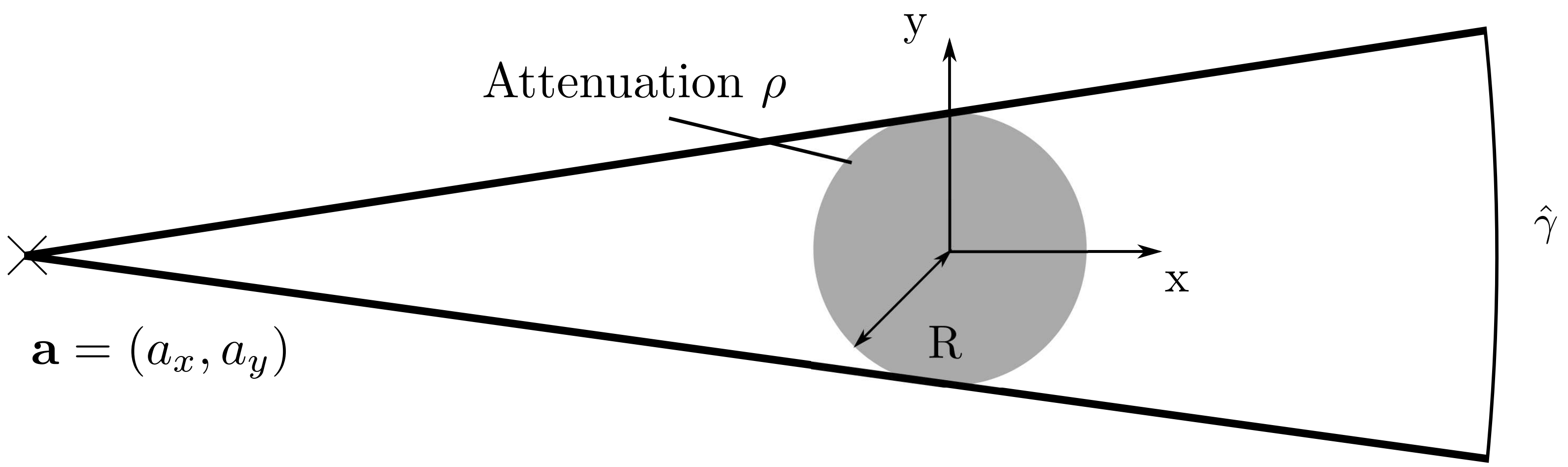


Figure 1: Cross section fan beam projection of a cylinder requires the opening angle $\hat{\gamma}$.

Homogeneous Cylinder: Far Source



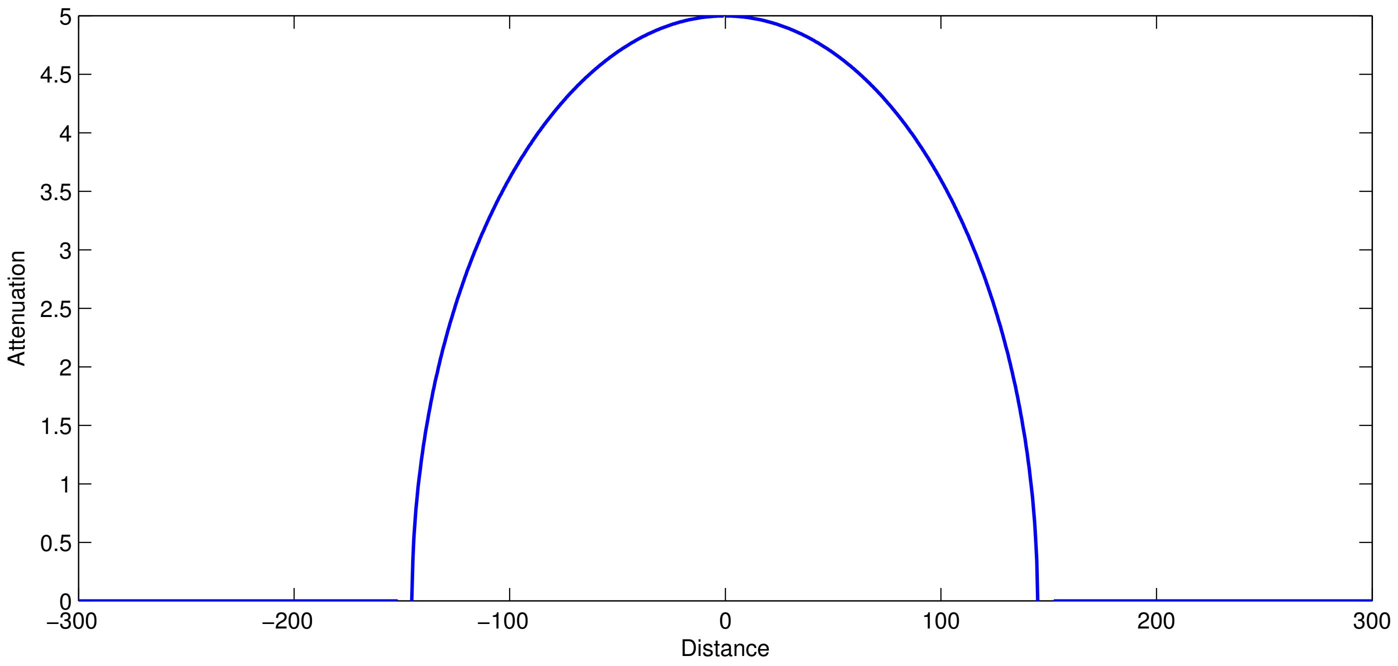
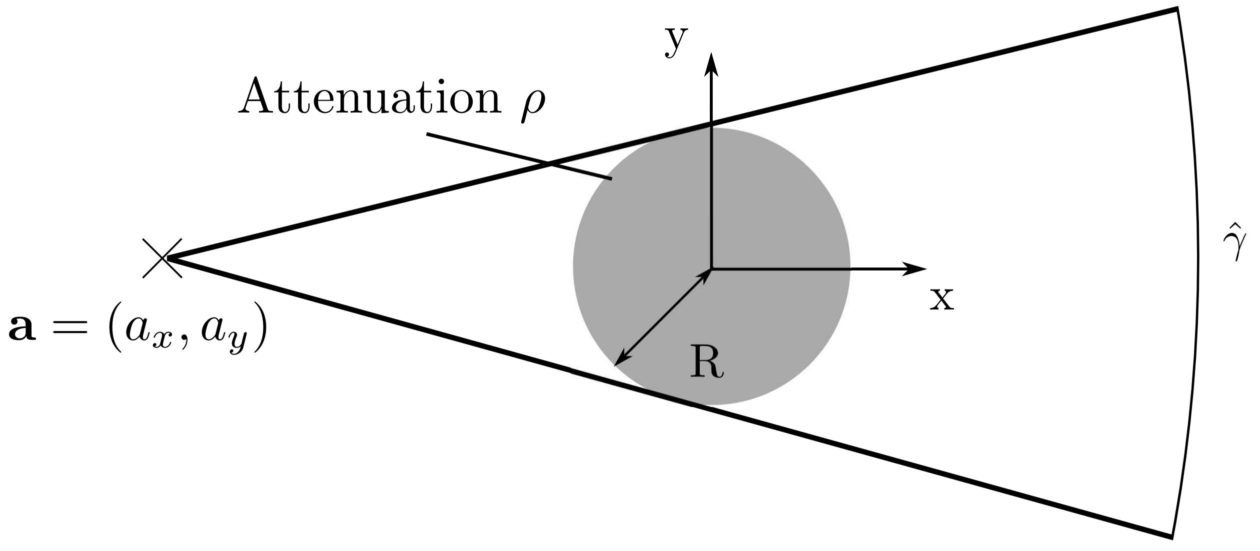


Figure 2: When the object is far from the source (left) its fan beam projection $g(\boldsymbol{a}, \gamma)$, $\gamma \in [-\frac{\hat{\gamma}}{2}, \frac{\hat{\gamma}}{2}]$, is condensed in a smaller part of the detector (right).

Homogeneous Cylinder: Close Source



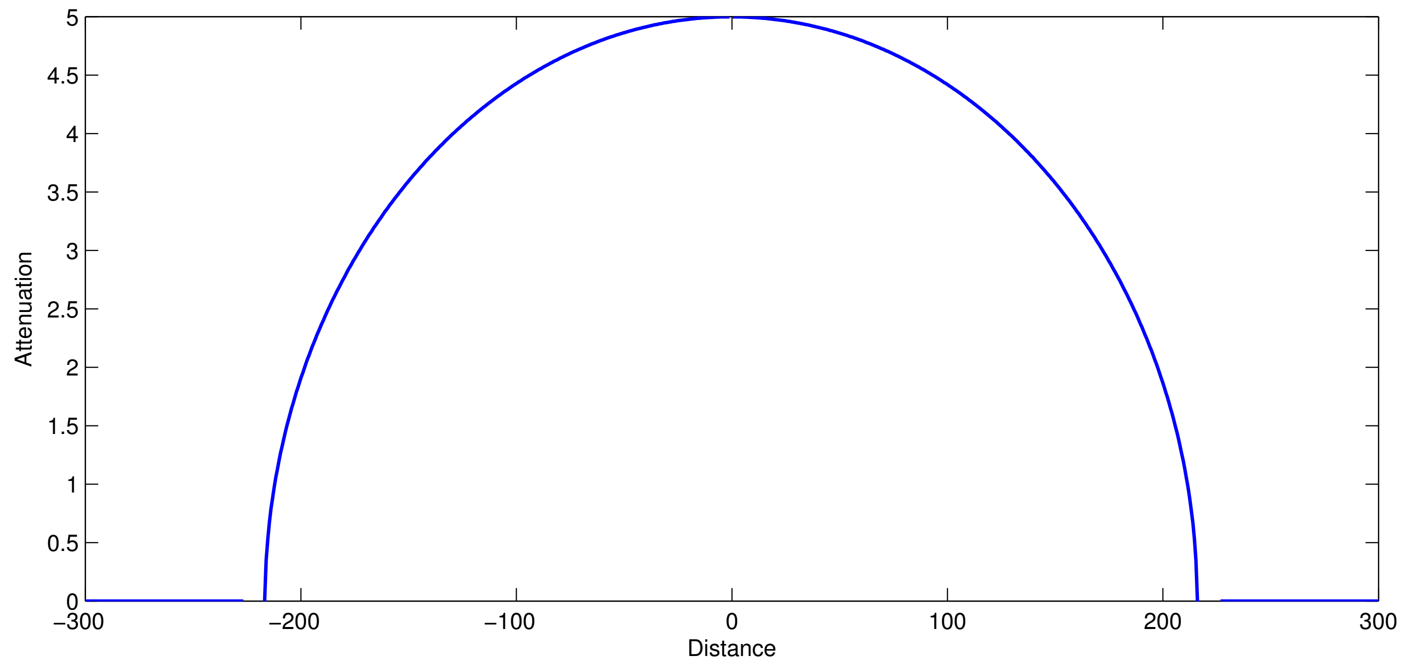
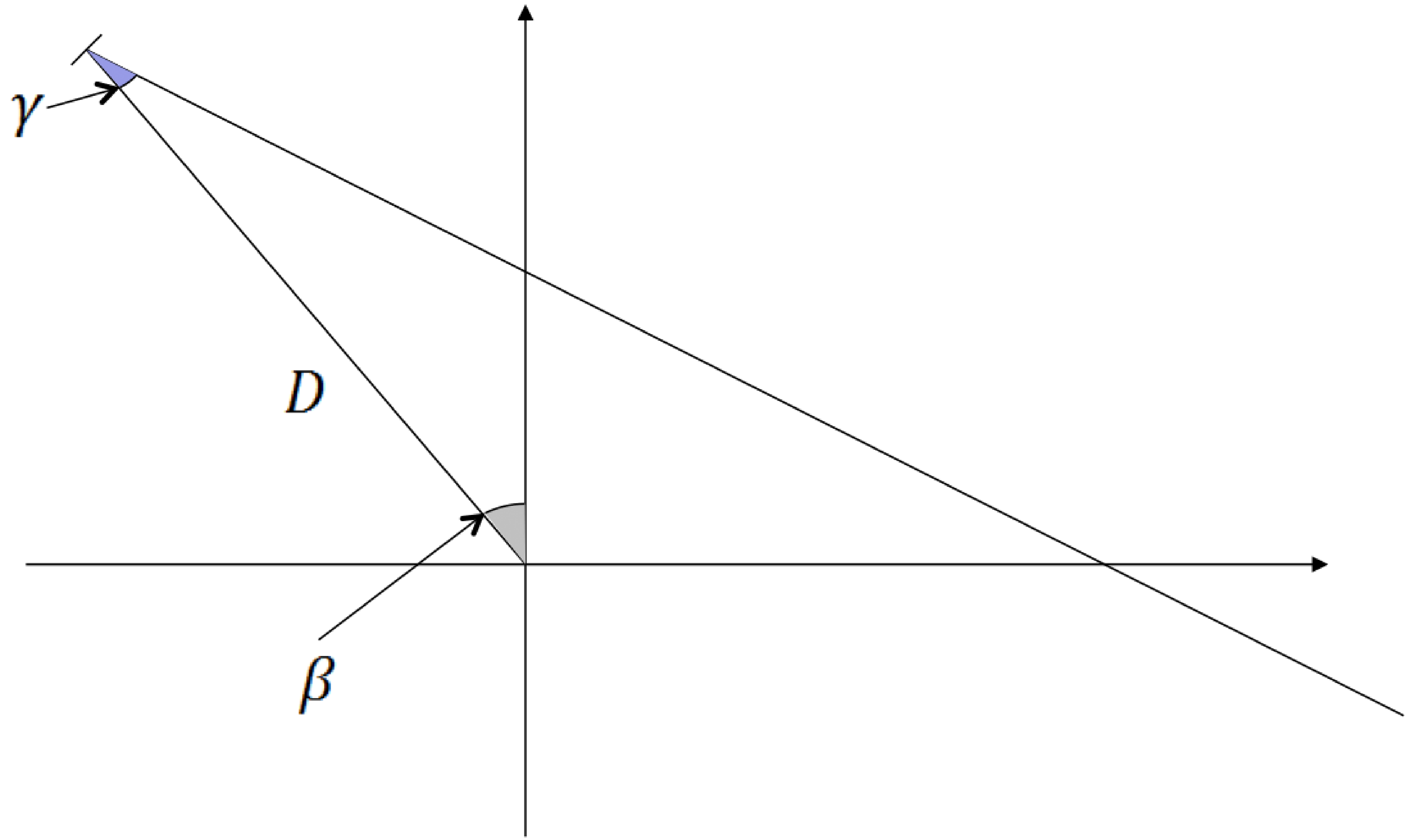


Figure 3: When the same object is closer to the source (left) its fan beam projection $g(\boldsymbol{a}, \gamma)$, $\gamma \in [-\frac{\hat{\gamma}}{2}, \frac{\hat{\gamma}}{2}]$, fills a larger part on the detector (right).

Basic Transform for Angle γ, Detector Center in Origin

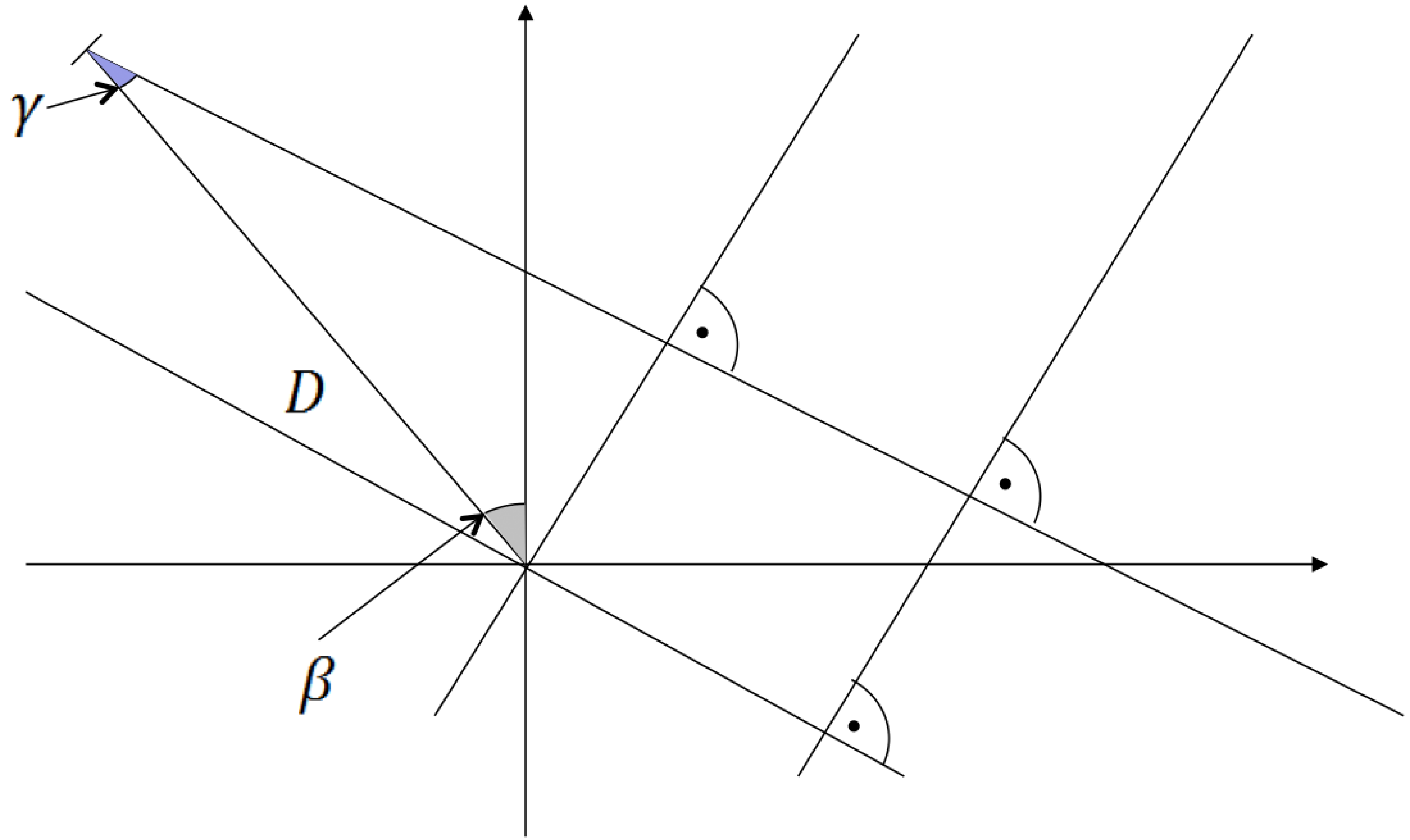


$$\theta = \gamma + \beta$$

$$s = D \sin \gamma$$

Figure 4: Graphical derivation for the general conversion from fan beam to parallel beam geometry. This diagram series shows how a detected fan beam under the angle γ and source rotation β is equivalent to a parallel beam detected on a detector rotated by θ at a shifted position s.

Basic Transform for Angle γ, Detector Center in Origin



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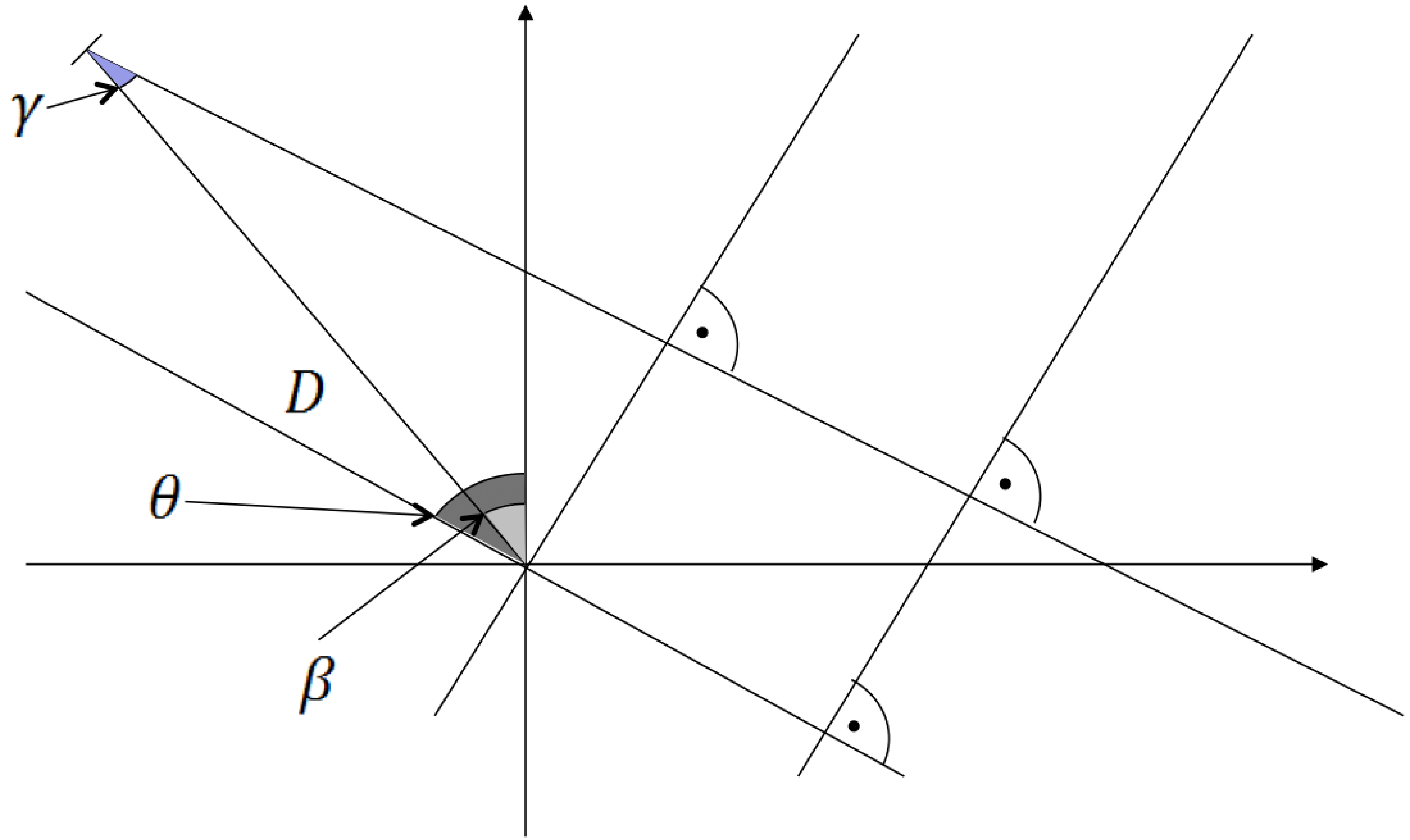

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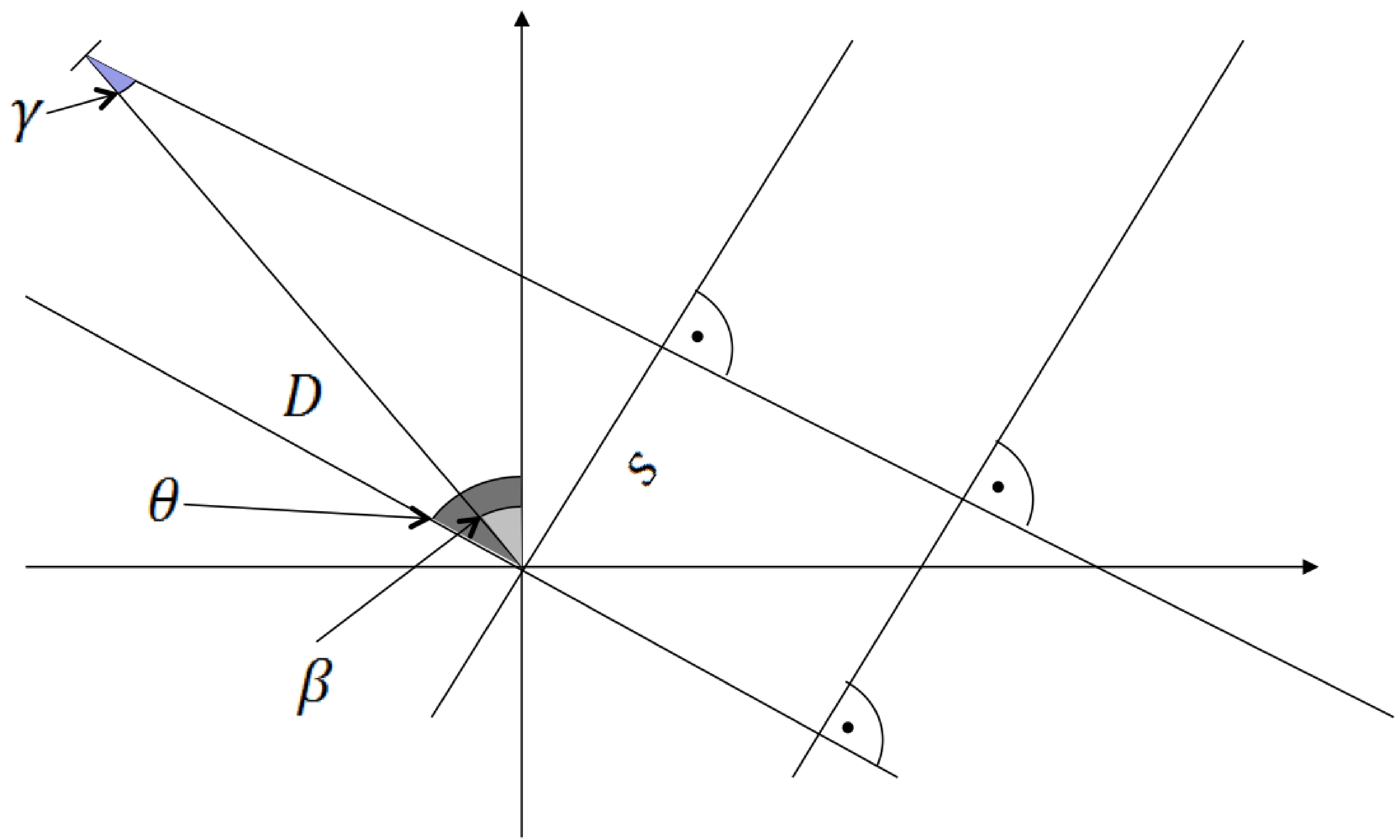

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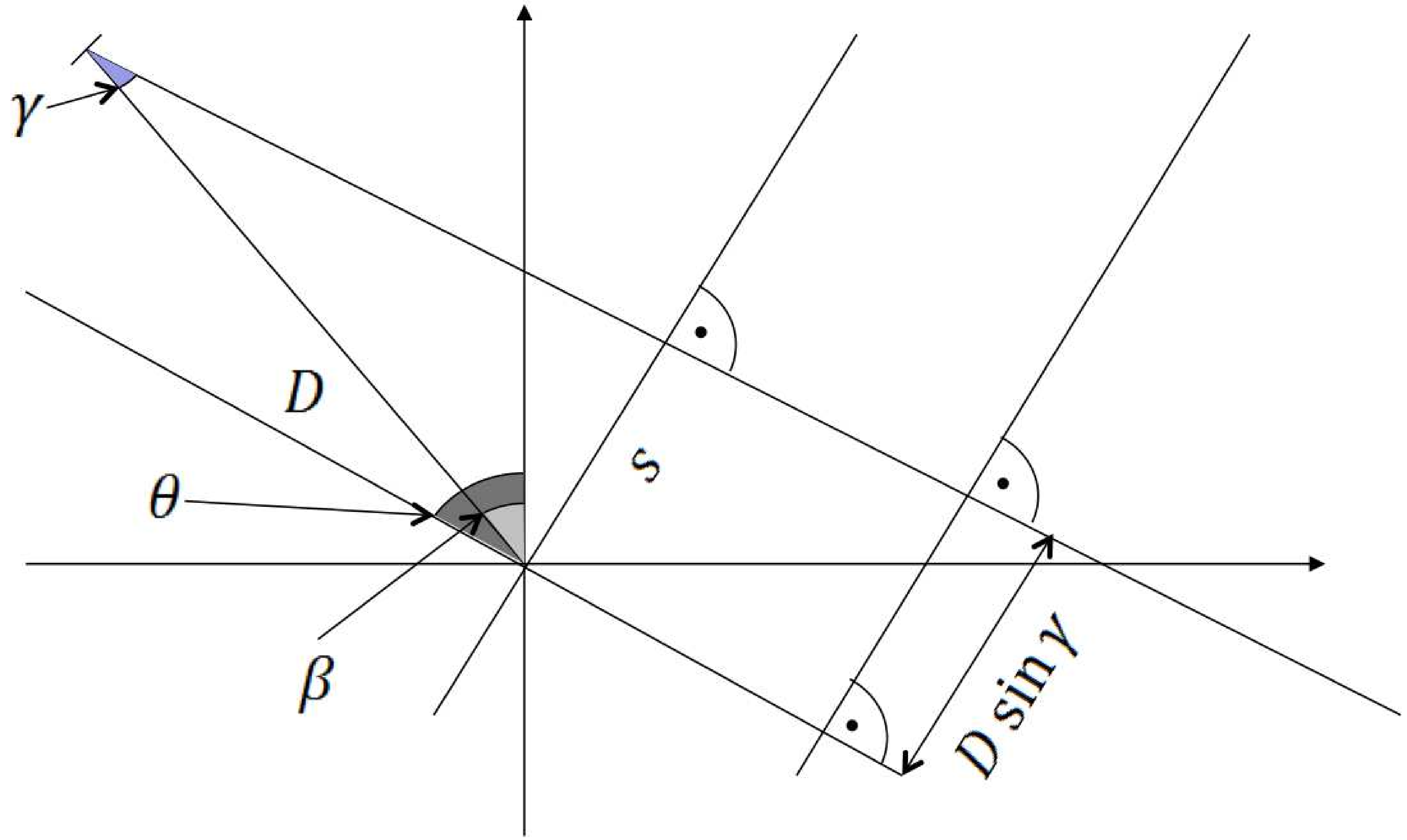

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Equally-spaced and Equiangular Detectors

Sampling is different in both detector geometries:

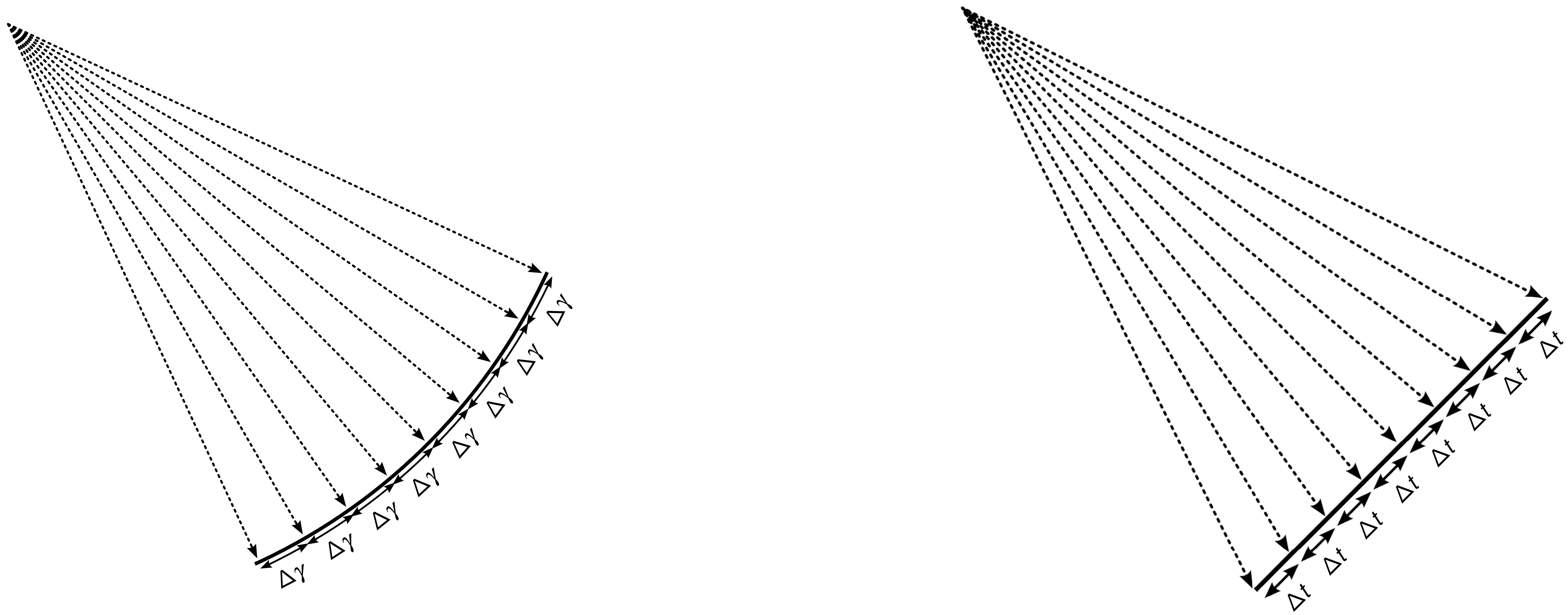


Figure 5: Flat detector with equiangular spacing $\Delta\gamma$ (left), curved detector fan beam with equal spacing Δt (right) (Magdalena Herbst)

Parallel Beam to Fan Beam Conversion: Flat-Panel

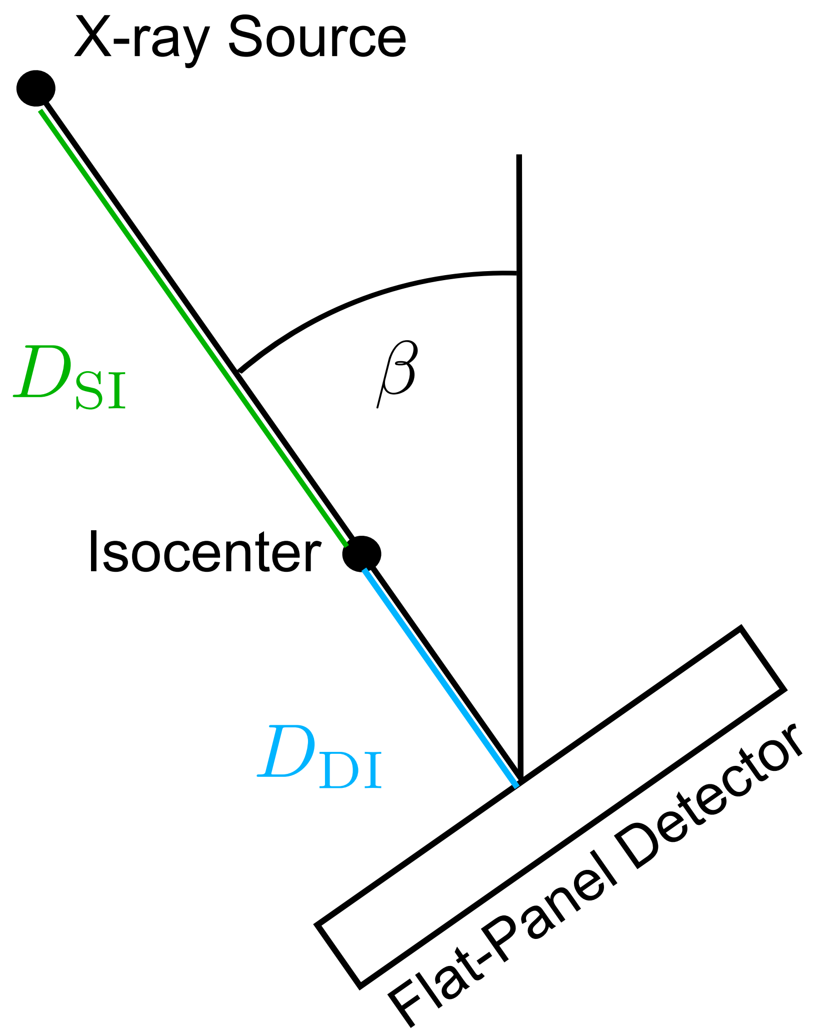
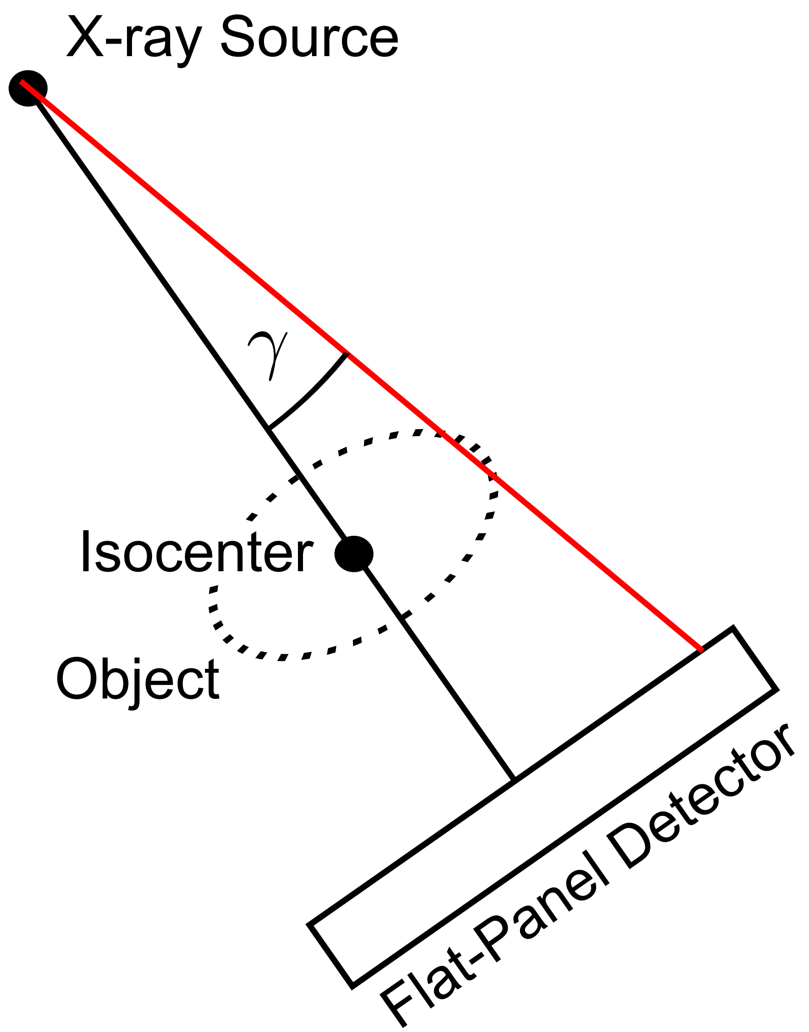


Figure 6: Graphical derivation of the conversion from fan beam to parallel beam geometry for a flat panel detector not centered in the origin. D_{DI} denotes the detector-isocenter distance and D_{SI} is the source-isocenter distance.

Parallel Beam to Fan Beam Conversion: Flat-Panel



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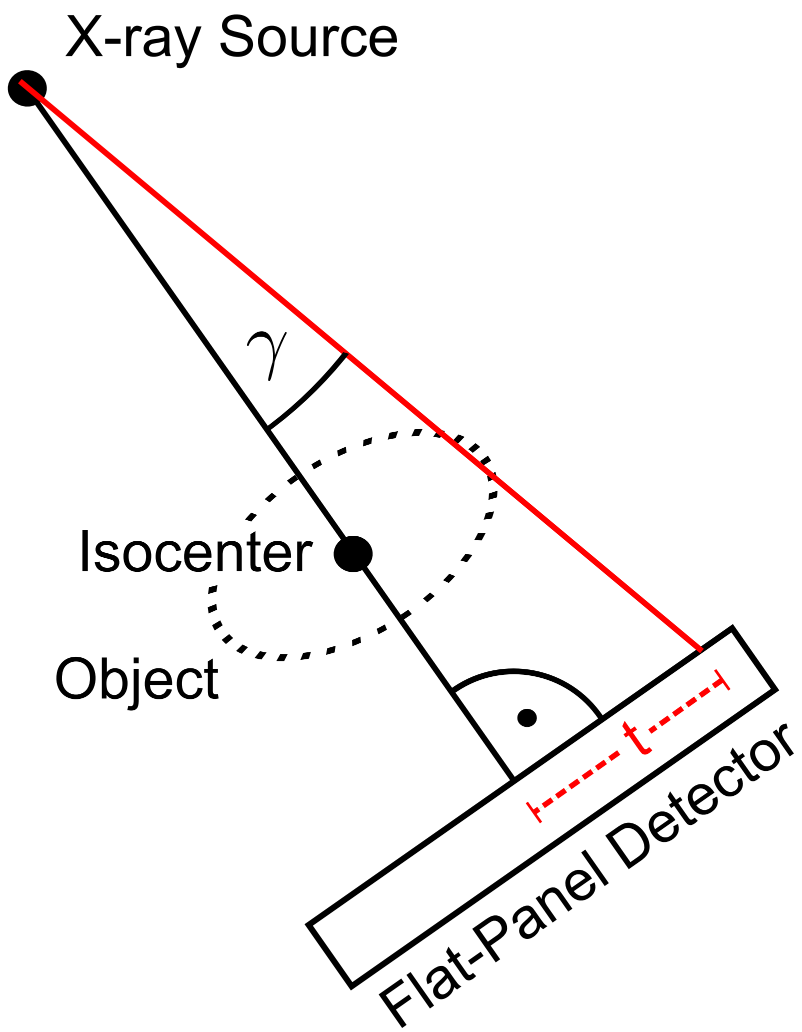

$$\tan\gamma = \frac{t}{D_{\mathrm{SI}}+D_{\mathrm{DI}}}$$

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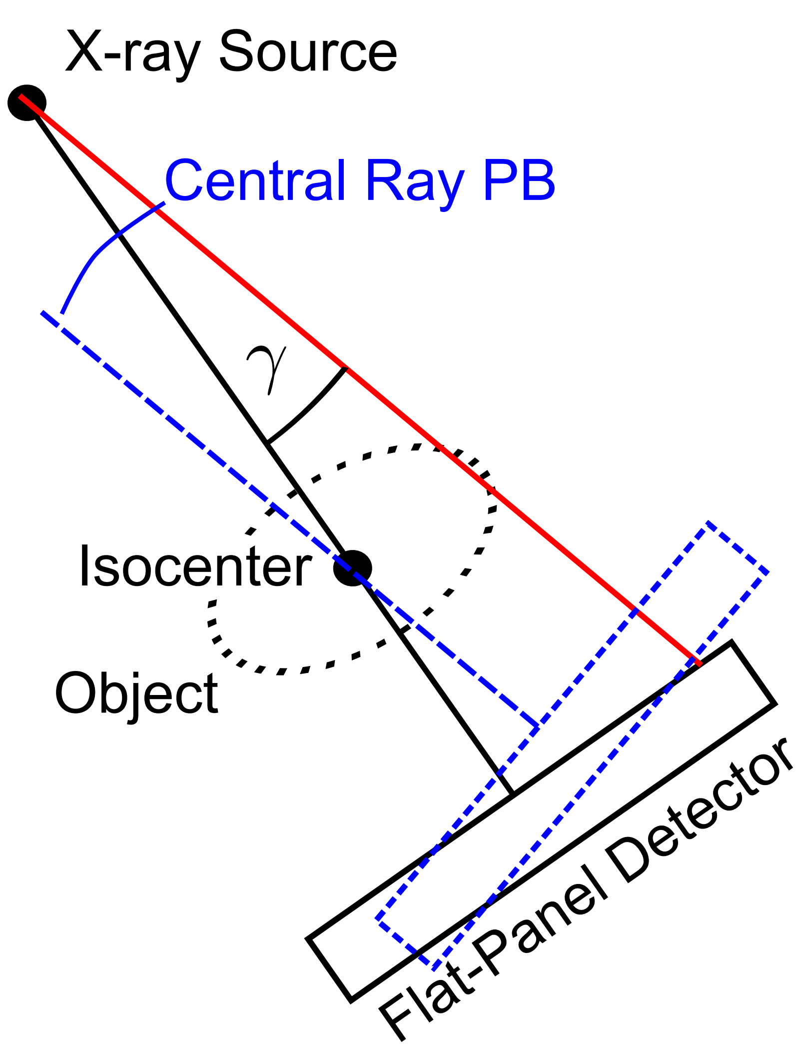

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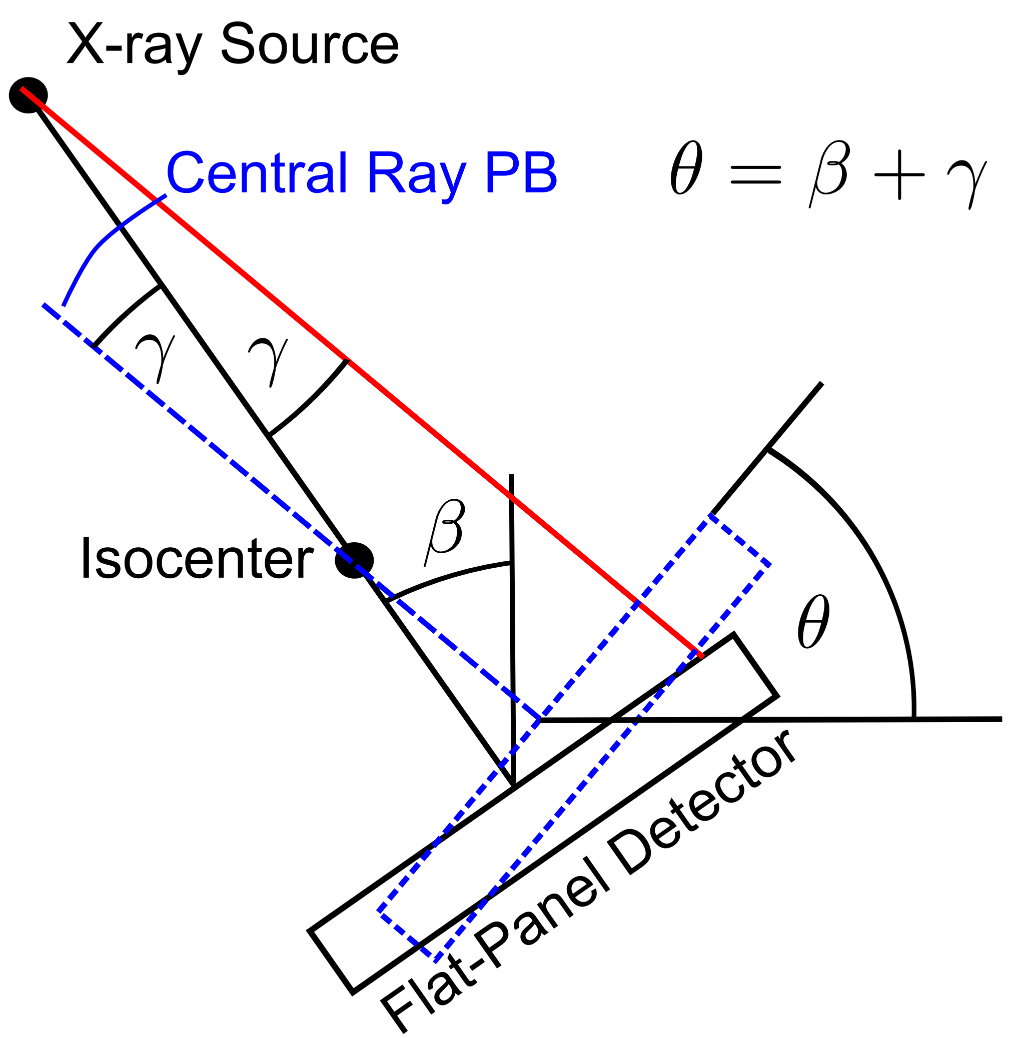

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Parallel Beam to Fan Beam Conversion: Flat-Panel

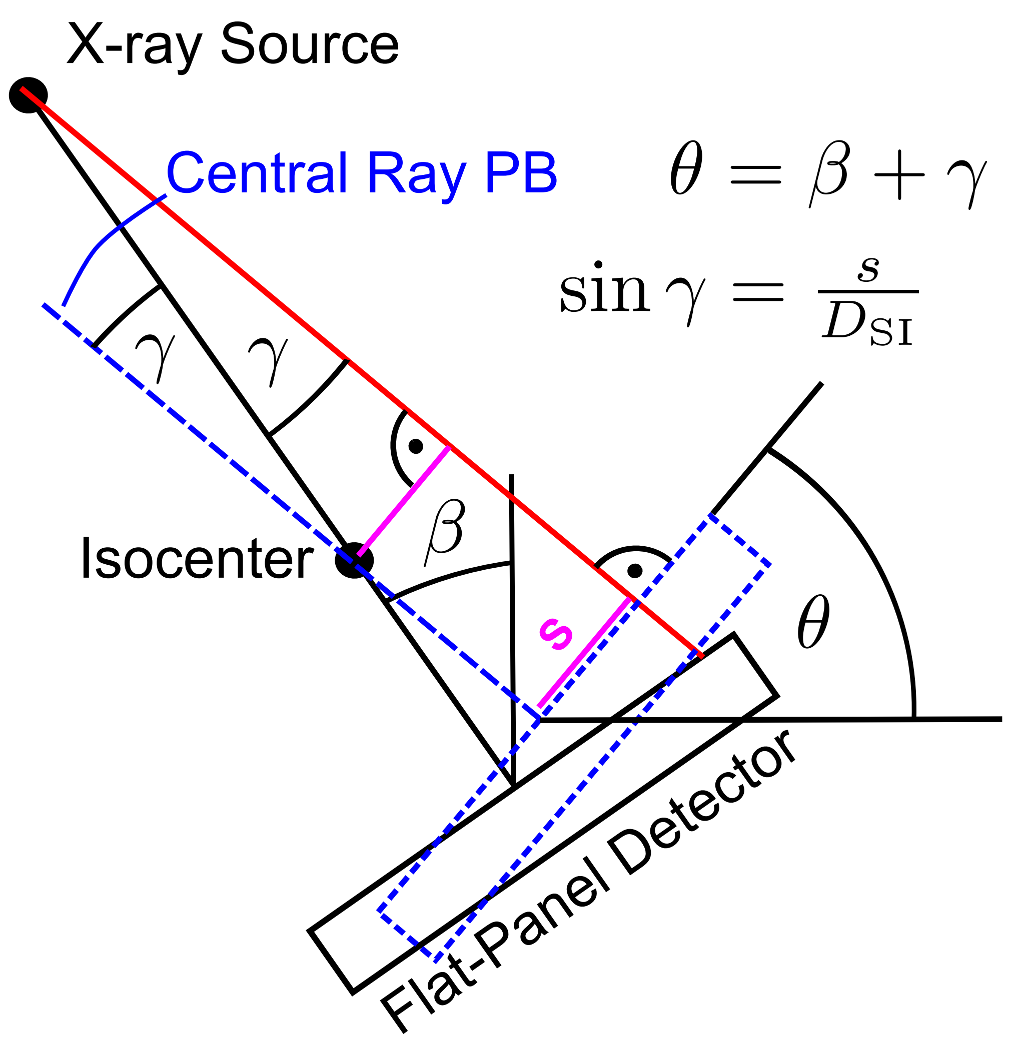

Figure 6: Graphical derivation of the conversion from fan beam to parallel beam geometry for a flat panel detector not centered in the origin. D_{DI} denotes the detector-isocenter distance and D_{SI} is the source-isocenter distance.

Parallel Beam to Fan Beam Conversion

- **Idea:** Find equal rays in both geometries:

$$\theta = \gamma + \beta,$$
$$s = D \sin \gamma.$$

- Then set:

$$p(s,\theta) = g(\gamma,\beta).$$

- For flat panels these equations hold:

$$\theta = \beta + \arctan \frac{t}{D_{\mathrm{SI}} + D_{\mathrm{DI}}},$$
$$s = \frac{D_{\mathrm{SI}} t}{\sqrt{(D_{\mathrm{SI}} + D_{\mathrm{DI}})^2 + t^2}},$$
$$p(s,\theta) = g(t,\beta).$$

- This process is called ***rebinning***.

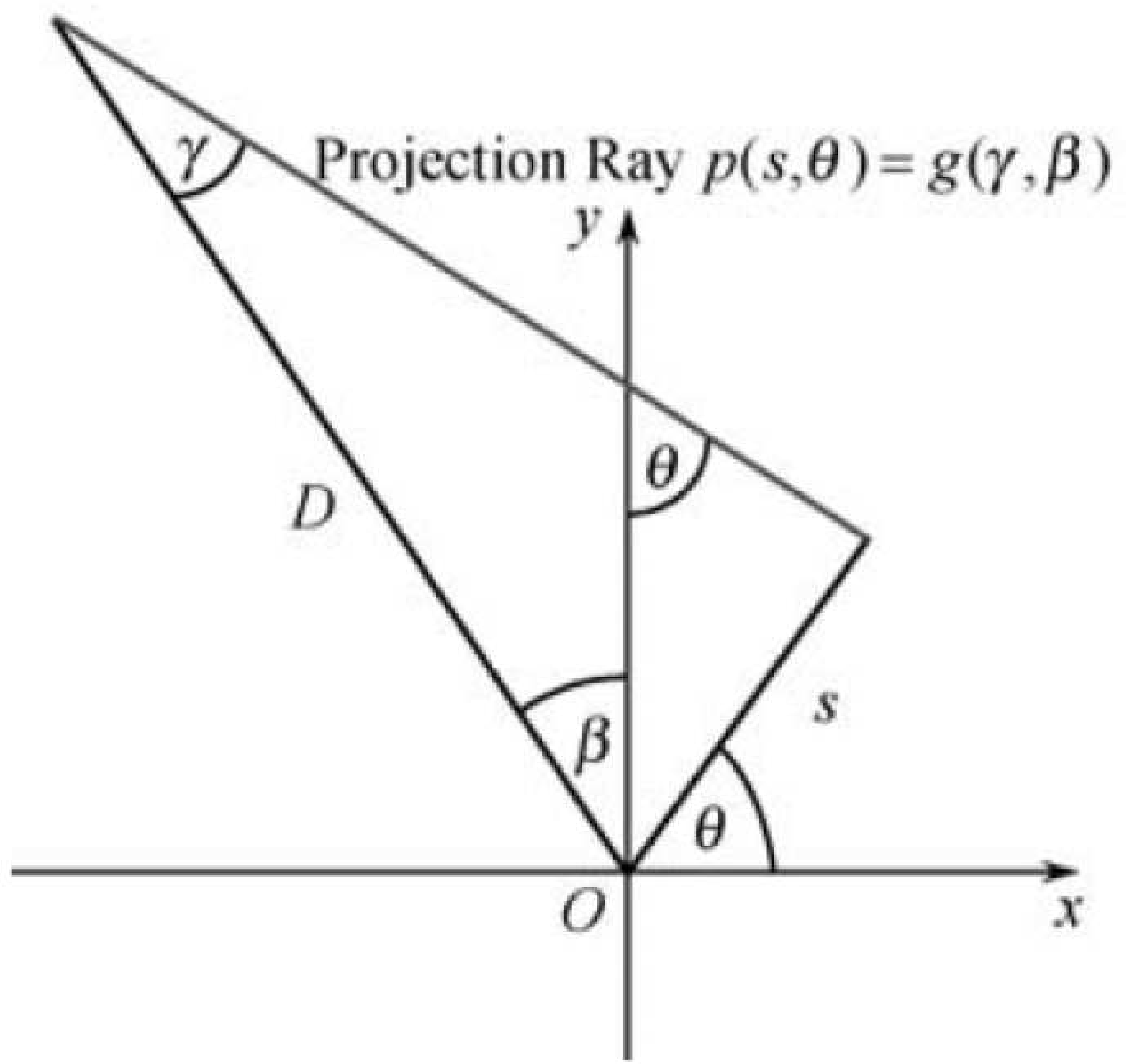

Figure 7: Corresponding rays yield the same projection value (Zeng, 2009).

Parallel Beam to Fan Beam Conversion

- Rebinning is a feasible solution.
- Change of coordinate systems requires interpolation which can introduce inaccuracies.
- Hence, rebinning might not be the method of choice.

$\Rightarrow$ Derive a reconstruction method for fan beam data by analytical conversion of the reconstruction algorithm.

Concept for finding a reconstruction algorithm

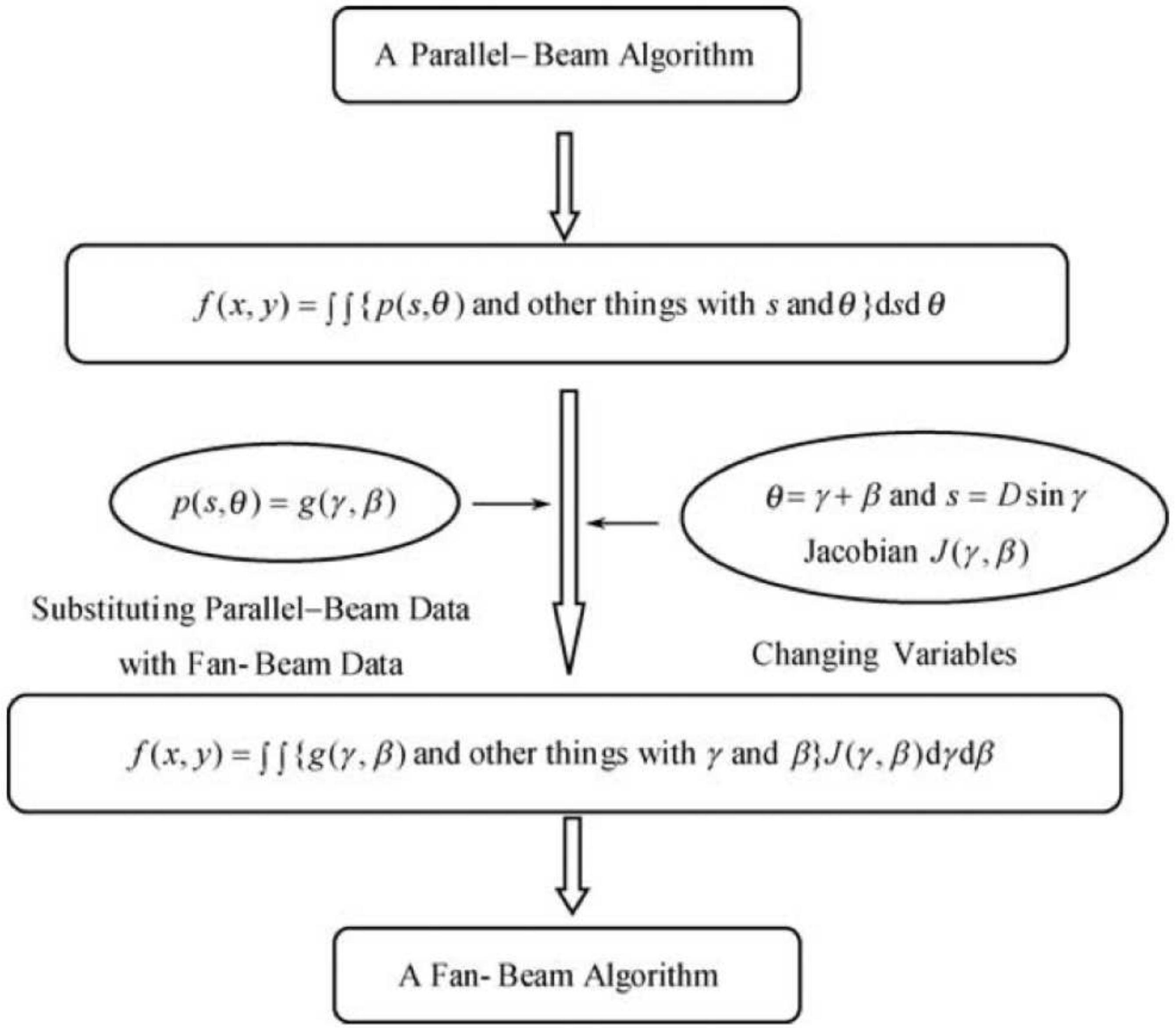

Figure 8: Flow chart showing the steps needed to develop a fan beam reconstruction algorithm (Zeng, 2009).

Topics

Take Home Messages

- Since the PSFs of fan beam and parallel beam geometry are equal, there is a process called rebinning which transforms one into the other.
- The rebinning concept mainly servers analytical purposes, because interpolation errors make reconstructions inaccurate.
- Fan beam reconstruction is dependent on the distances between source, isocenter and detector, in contrast to parallel beam.
- One has to distinguish flat panel detectors from curved detectors which both are in use.

Further Readings

Helpful reads for the current unit:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Ronald N. Bracewell. *The Fourier Transform and Its Applications*. 3rd ed. Electrical Engineering Series. Boston: McGraw-Hill, 2000

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Fan Beam – Reconstruction Algorithm

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Topics

Reprise

Concept for finding a reconstruction algorithm

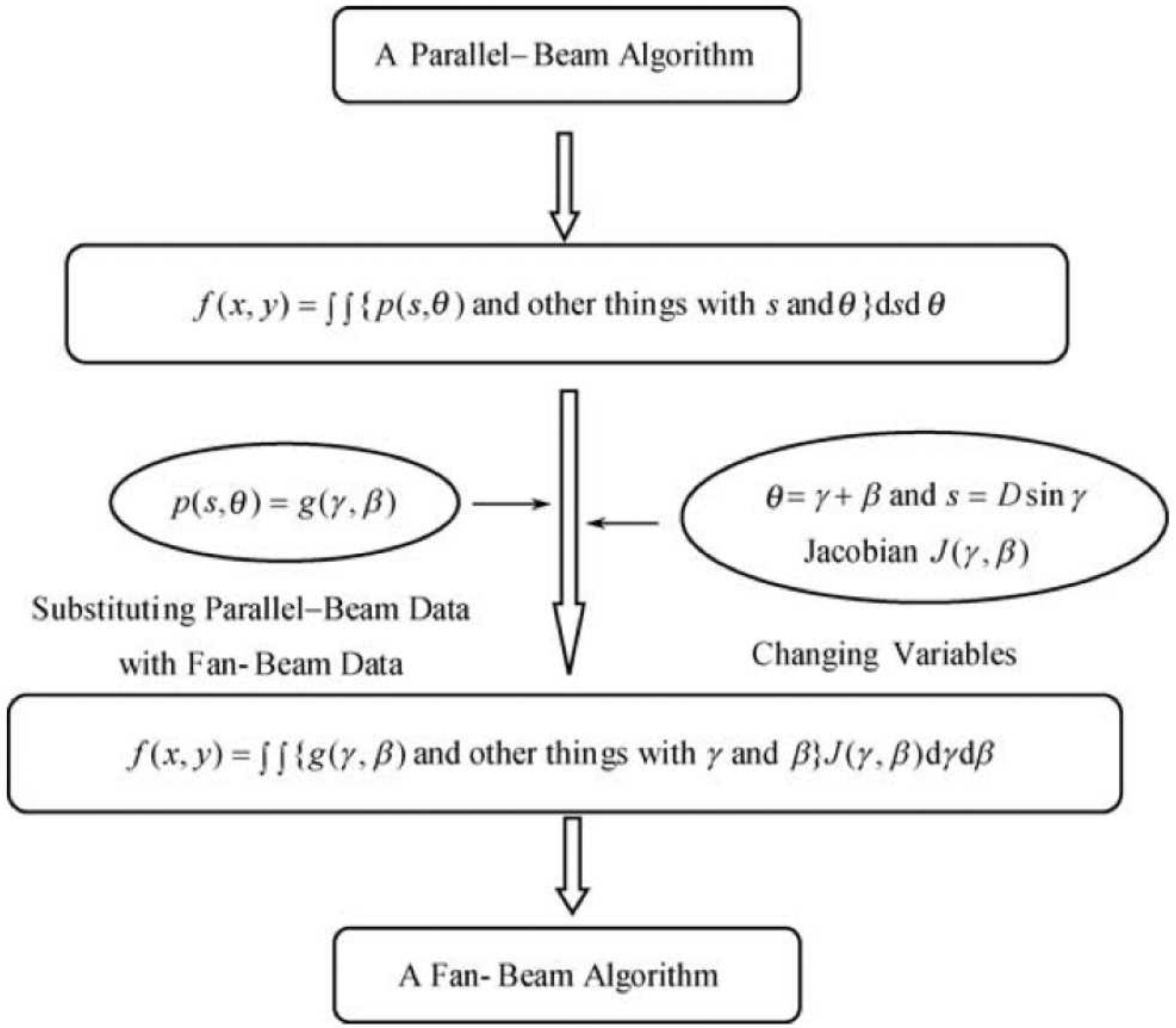


Figure 1: Flow chart showing the steps needed to develop a fan beam reconstruction algorithm (Zeng, 2009).

Equally-spaced and Equiangular Detectors

- Sampling is different in both detector geometries.
- Hence, different reconstruction formulas are obtained.

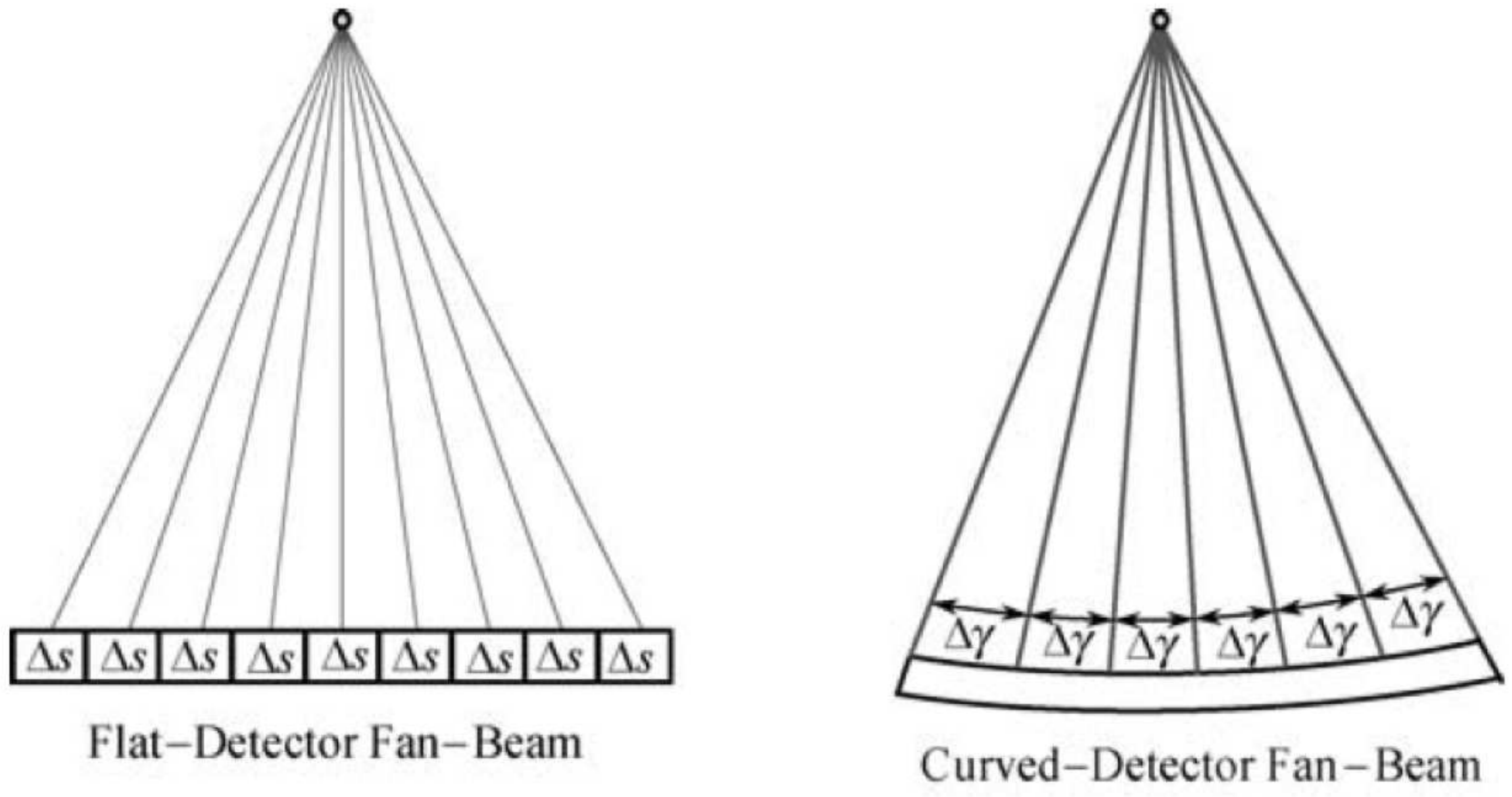


Figure 2: Schematics of flat and curved detector panels for fan beam (Zeng, 2009).

Topics

FBP for the Equiangular Case

We start with a parallel beam backprojection:

$$f(x,y) = \frac{1}{2}\int_0^{2\pi}\int_{-\infty}^{\infty} p(s,\theta)h(x\cos\theta + y\sin\theta - s)\,\mathrm{d}s\,\mathrm{d}\theta,$$

and transform the reconstruction point into polar coordinates (r,φ), i. e., $x = r\cos\varphi$, $y = r\sin\varphi$:

$$f(r,\varphi) = \frac{1}{2}\int_0^{2\pi}\int_{-\infty}^{\infty} p(s,\theta)h(r\cos(\theta-\varphi) - s)\,\mathrm{d}s\,\mathrm{d}\theta.$$

FBP for the Equiangular Case

Using the described change of variables from (s,θ) to (γ,β) with the Jacobian $\left|\frac{\partial(s,\theta)}{\partial(\gamma,\beta)}\right| = D\cos\gamma$, and using distance D' and angle γ' of the reconstruction point with respect to the source, we get:

$$f(r,\varphi) = \frac{1}{2}\int_{0}^{2\pi}\int_{-\pi/2}^{\pi/2} g(\gamma,\beta)h(D'\sin(\gamma'-\gamma))\,D\cos\gamma\,\mathrm{d}\gamma\,\mathrm{d}\beta.$$

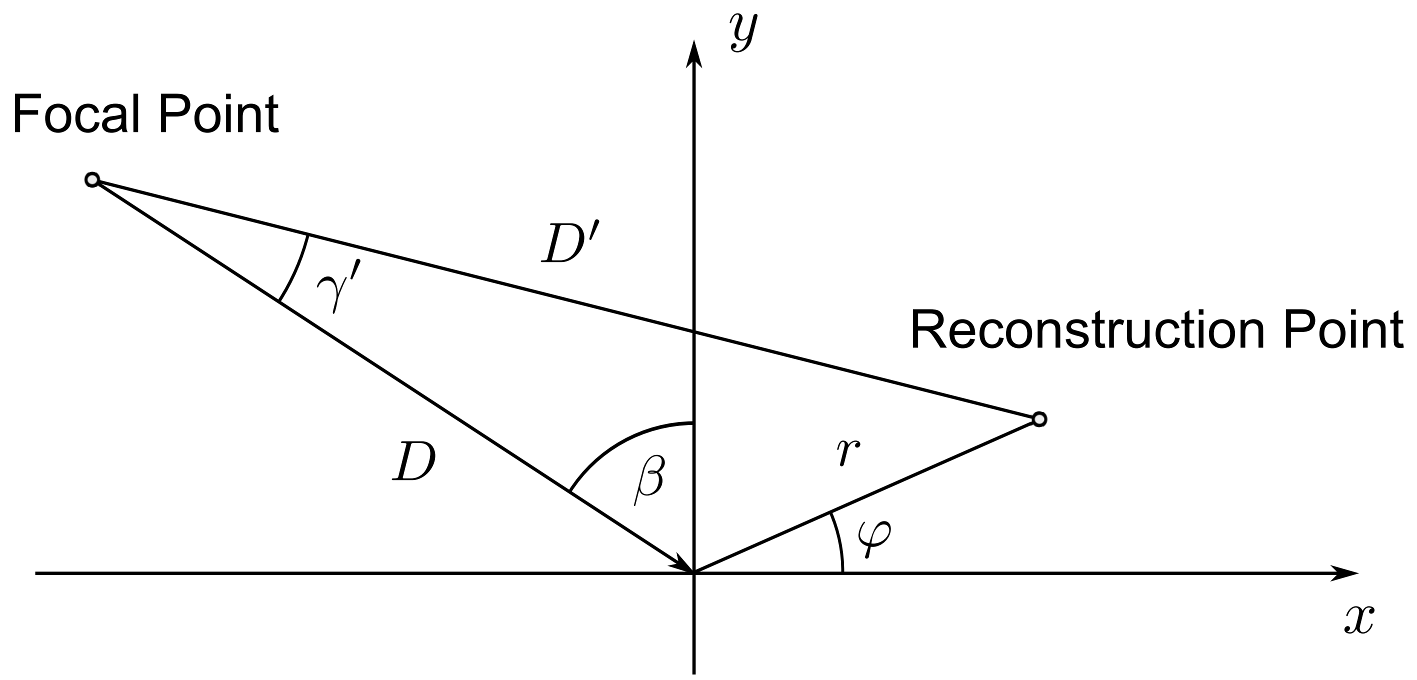


Figure 3: Definition of D' and γ'

FBP for the Equiangular Case

The ramp filter has the following property:

$$h(D' \sin \gamma) = \left(\frac{\gamma}{D' \sin \gamma} \right)^2 h(\gamma),$$

and defining

$$h_{\text{fan}}(\gamma) = \frac{D}{2} \left(\frac{\gamma}{\sin \gamma} \right)^2 h(\gamma),$$

we finally obtain the backprojection algorithm:

$$f(r, \varphi) = \int_0^{2\pi} \frac{1}{D'^2} \int_{-\pi/2}^{\pi/2} \cos \gamma \, g(\gamma, \beta) h_{\text{fan}}(\gamma' - \gamma) \, \mathrm{d}\gamma \mathrm{d}\beta.$$

FBP for the Equiangular Case: Algorithm

1. Perform cosine weighting

$$g_1(\gamma,\beta) = g(\gamma,\beta)\cos\gamma.$$

2. Apply fan beam filter:

$$g_2(\gamma',\beta) = (g_1 * h_{\text{fan}})(\gamma',\beta).$$

3. Backproject with distance weight:

$$f(r,\varphi) = \int_0^{2\pi} \frac{1}{D'^2} g_2(\gamma',\beta)\,\mathrm{d}\beta.$$

Backprojection and Fourier Slice Theorem

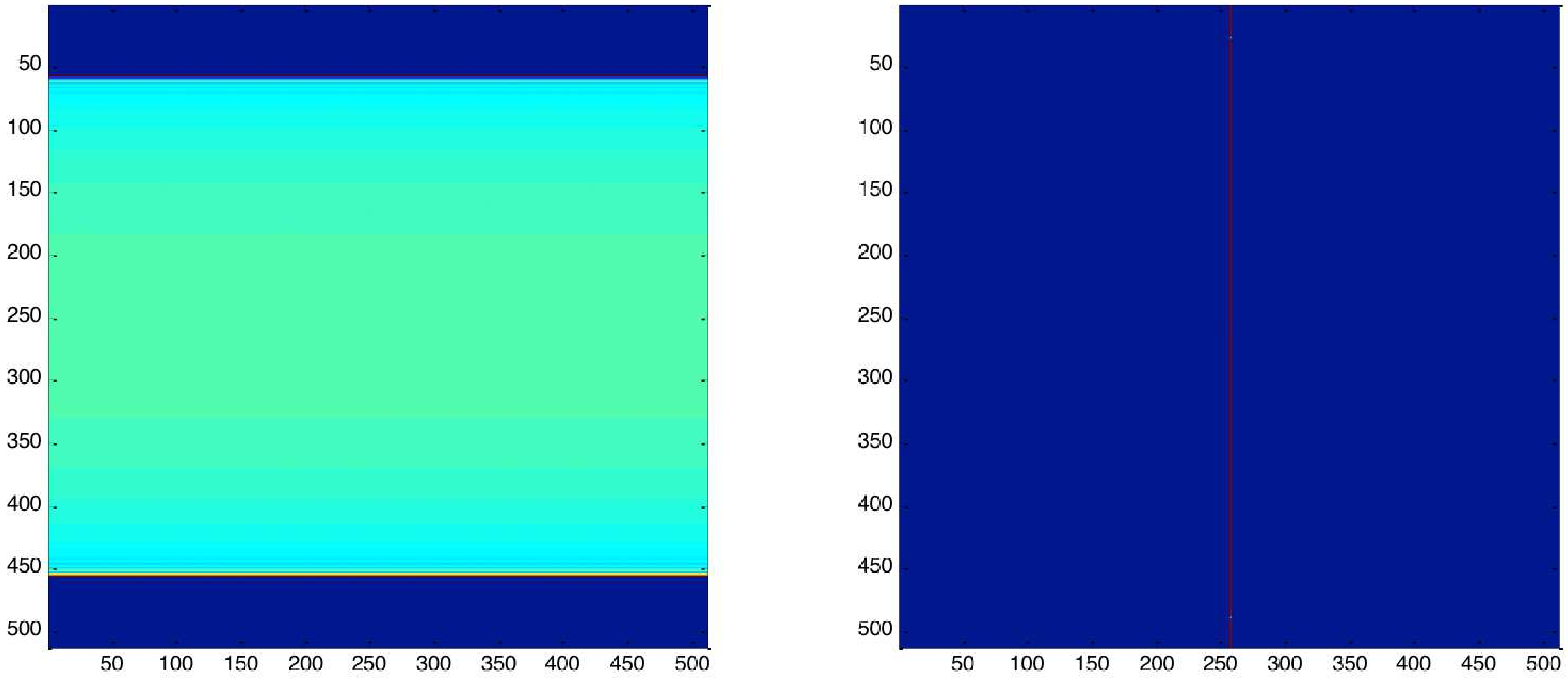


Figure 4: Backprojection of a single view (left) and its Fourier transform (right)

Backprojection and Fourier Slice Theorem

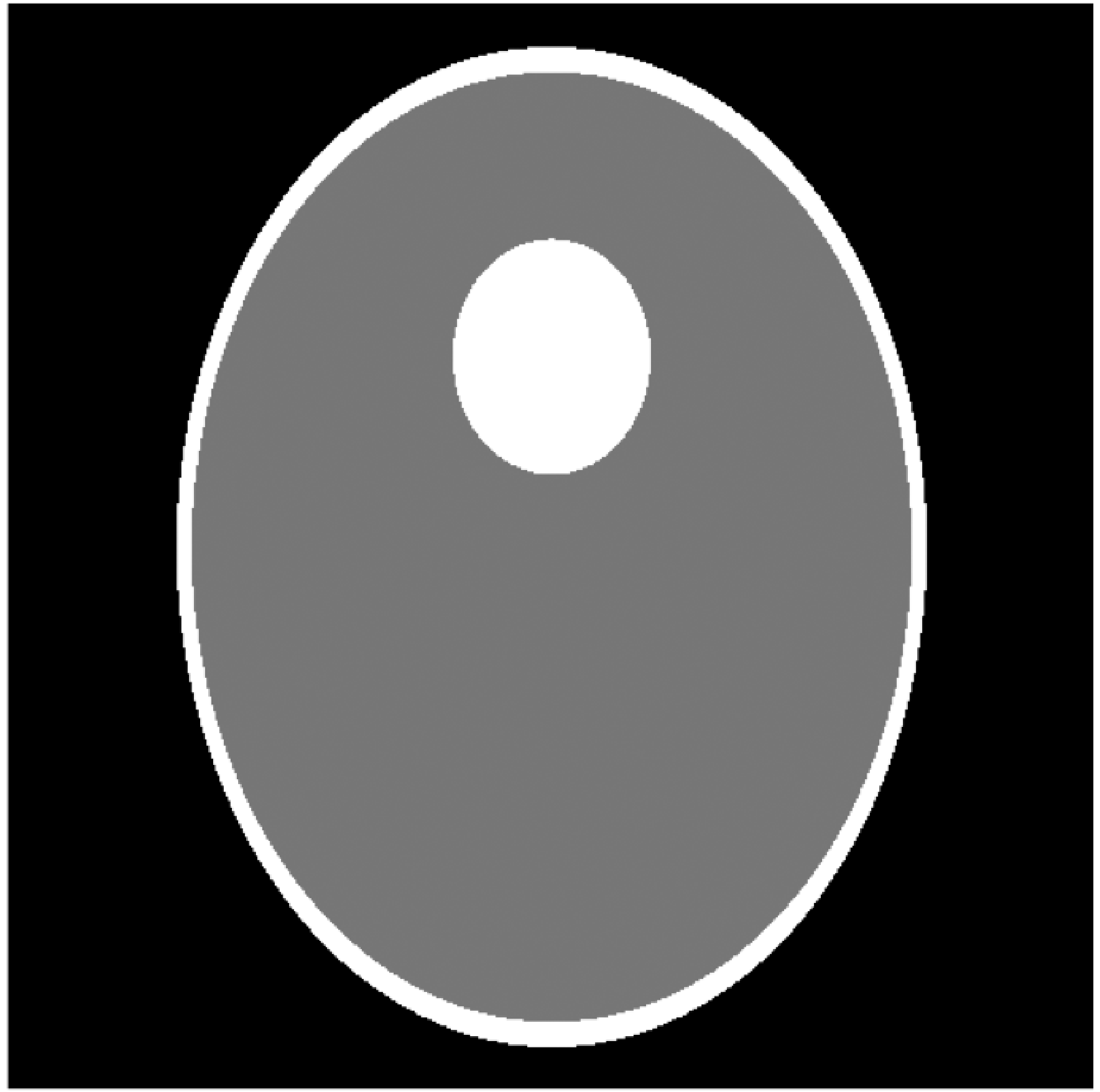


Figure 5: Slice view

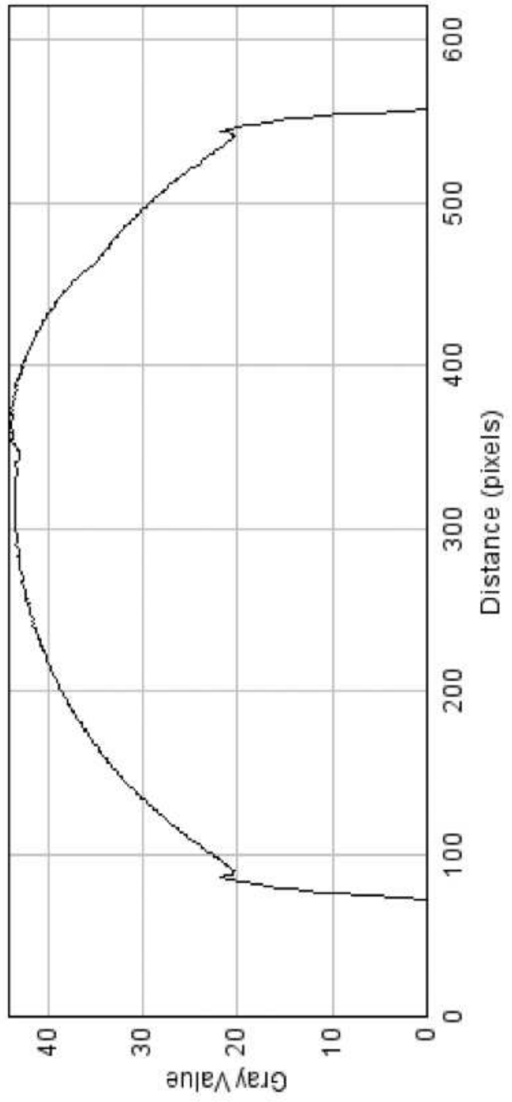


Figure 6: Projection profile

Backprojection and Fourier Slice Theorem

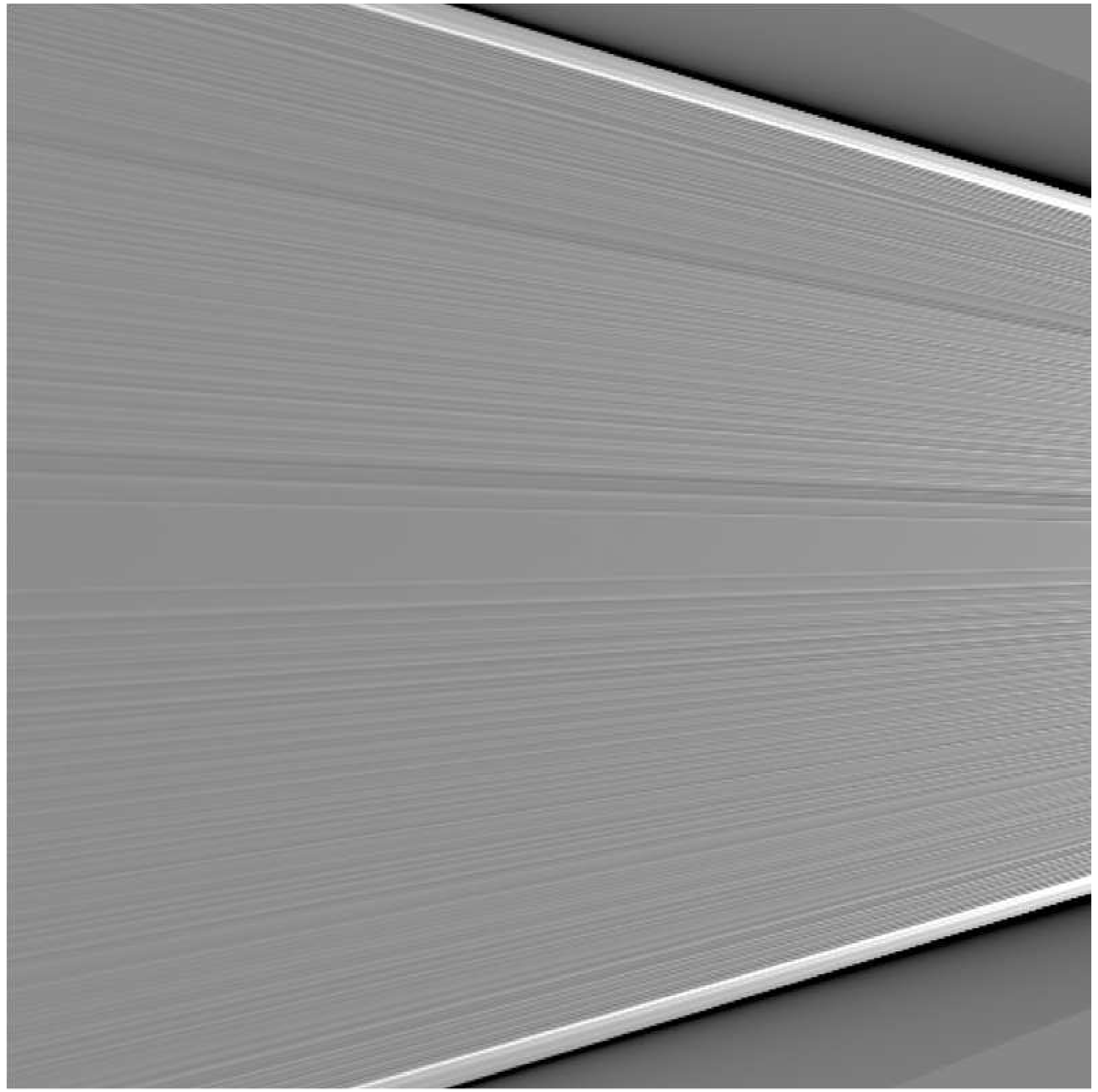


Figure 7: Fan beam backprojection

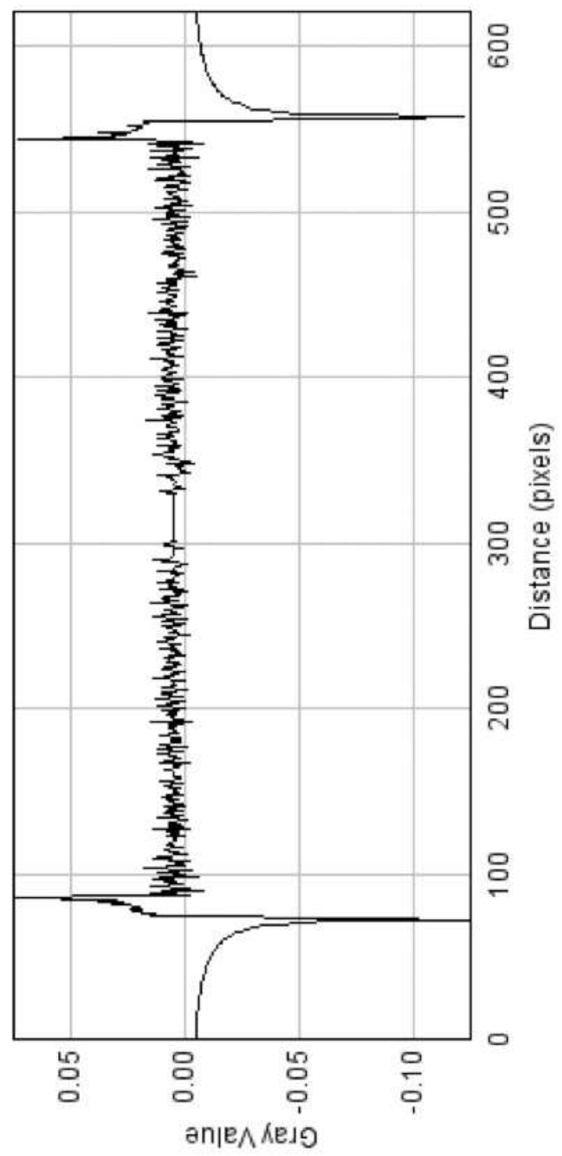
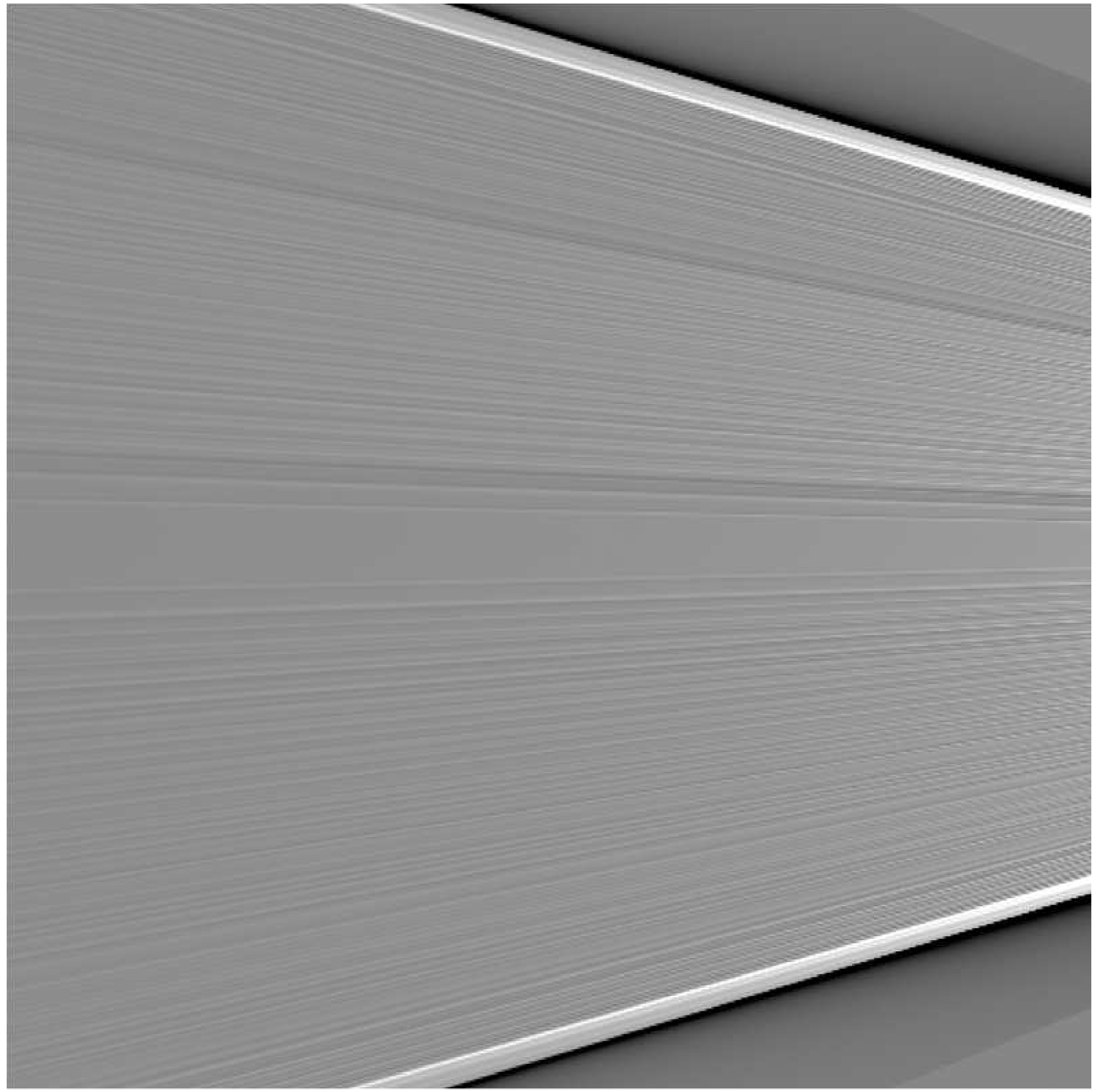


Figure 8: Filtered projection

Backprojection and Fourier Slice Theorem



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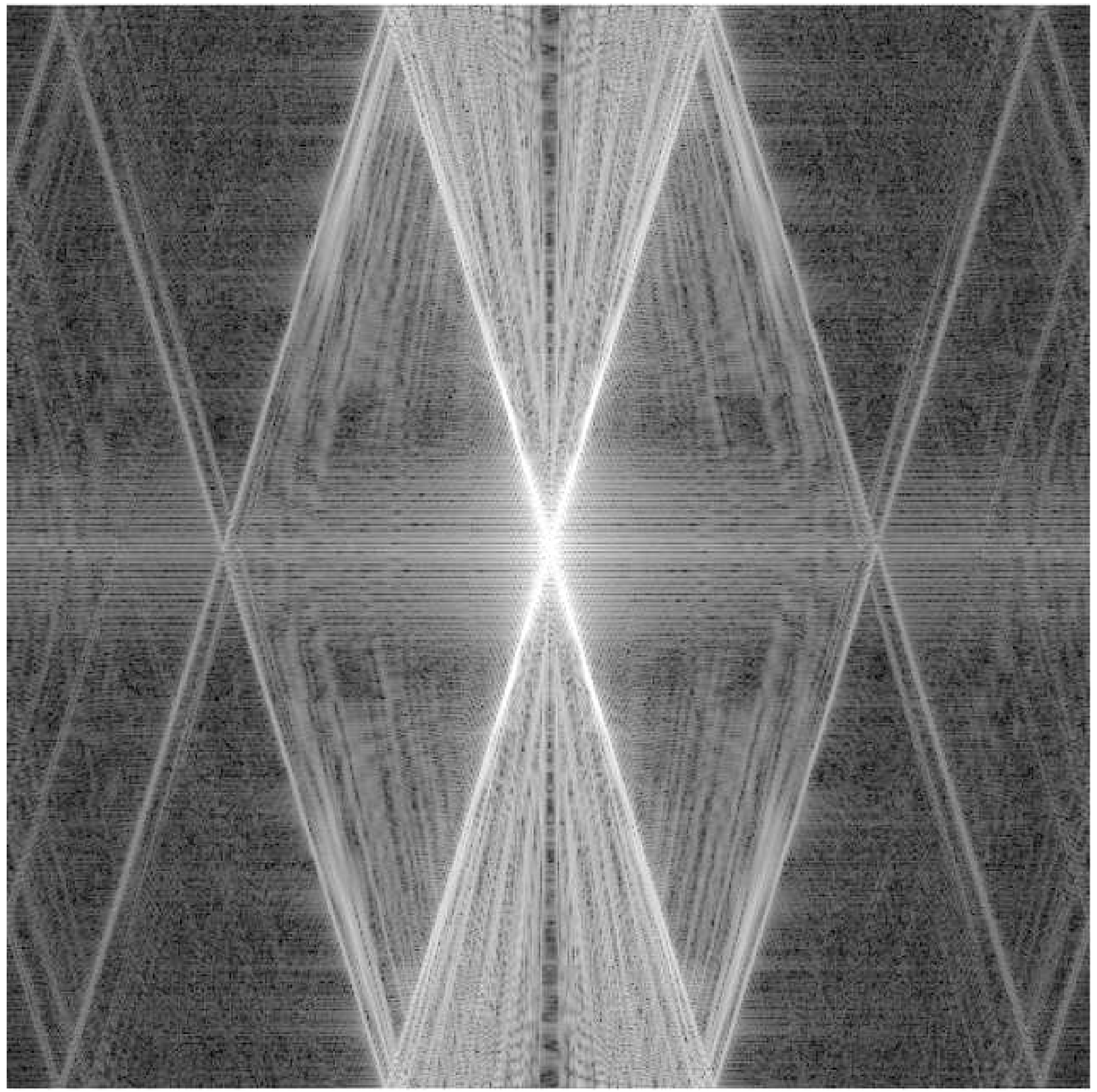

Figure 9: 2-D Fourier transform

Backprojection and Fourier Slice Theorem

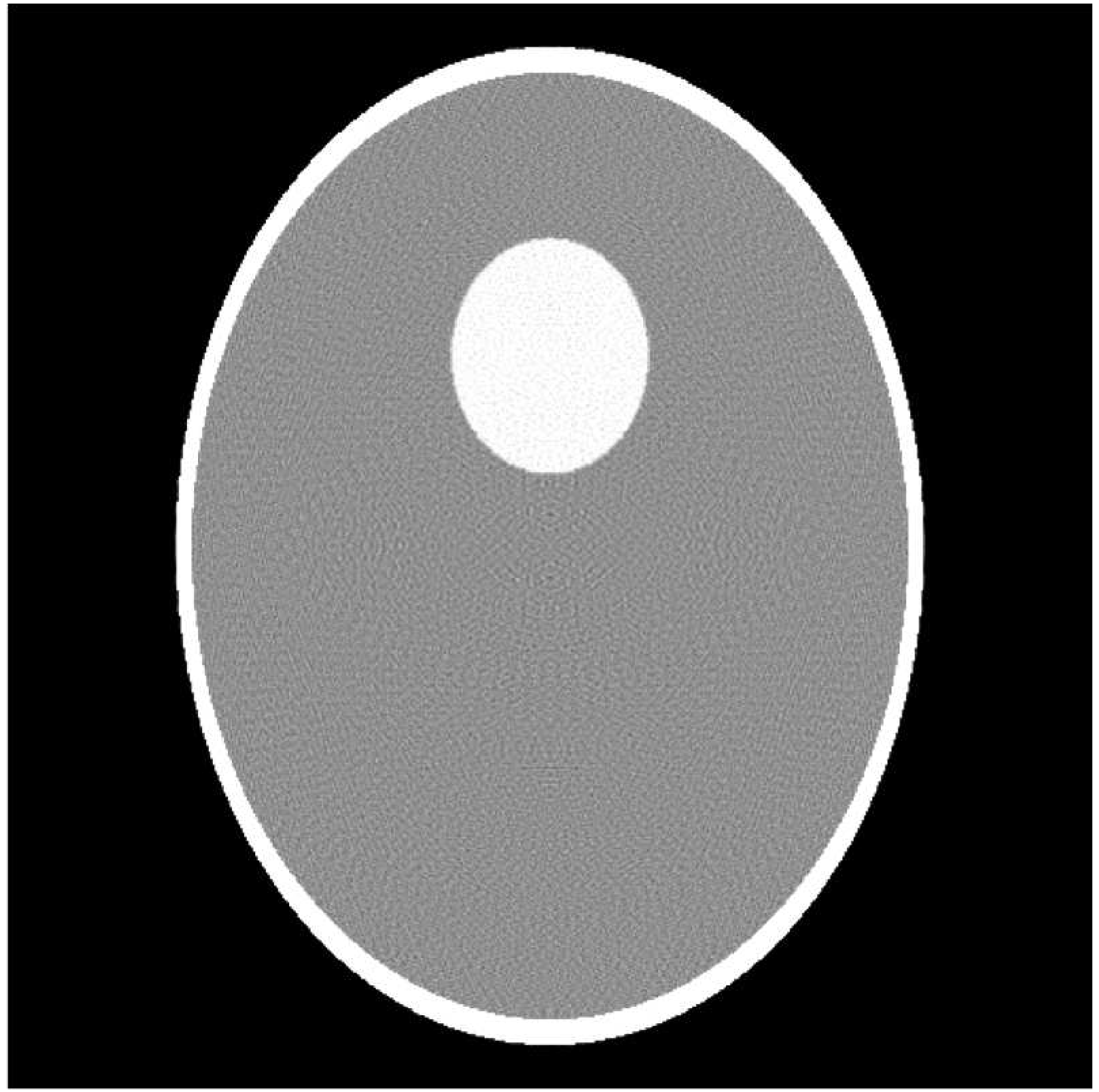

Figure 10: Reconstruction from fan beam data

FBP for the Equally-spaced Case

Like for the equiangular case we start with the parallel beam backprojection in polar coordinates:

$$f(r,\varphi) = \frac{1}{2}\int_0^{2\pi}\int_{-\infty}^{\infty} p(s,\theta)h(r\cos(\theta-\varphi)-s)\,\mathrm{d}s\,\mathrm{d}\theta.$$

Analogous considerations and transformations can be used to show:

$$f(r,\varphi) = \frac{1}{2}\int_0^{2\pi}\frac{1}{U^2}\int_{-\infty}^{\infty}\frac{D}{\sqrt{D^2+t^2}}g(t,\beta)h(t'-t)\,\mathrm{d}t\,\mathrm{d}\beta,$$

where

$$U = \frac{D+r\sin(\beta-\varphi)}{D}, \quad t' = \frac{Dr\cos(\beta-\varphi)}{D+r\sin(\beta-\varphi)}.$$

Topics

Reprise

Fan Beam Reconstruction Algorithm

- Equiangular Case
- Backprojection and Fourier Slice Theorem
- Equally-spaced Case

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- We derived two fan beam reconstruction algorithms based on the parallel beam to fan beam conversion.
- The detector geometry has to be considered.

Further Readings

More details on the derivations in this unit can be found in 'Larry's' book

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Medical Image Processing for Diagnostic Applications

Fan Beam – Short Scan

Online Course – Unit 39

Andreas Maier, Joachim Hornegger, Markus Kowarschik, Frank Schebesch
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Topics

Short Scan

Summary

Take Home Messages

Further Readings

Full Scan vs. Half Scan

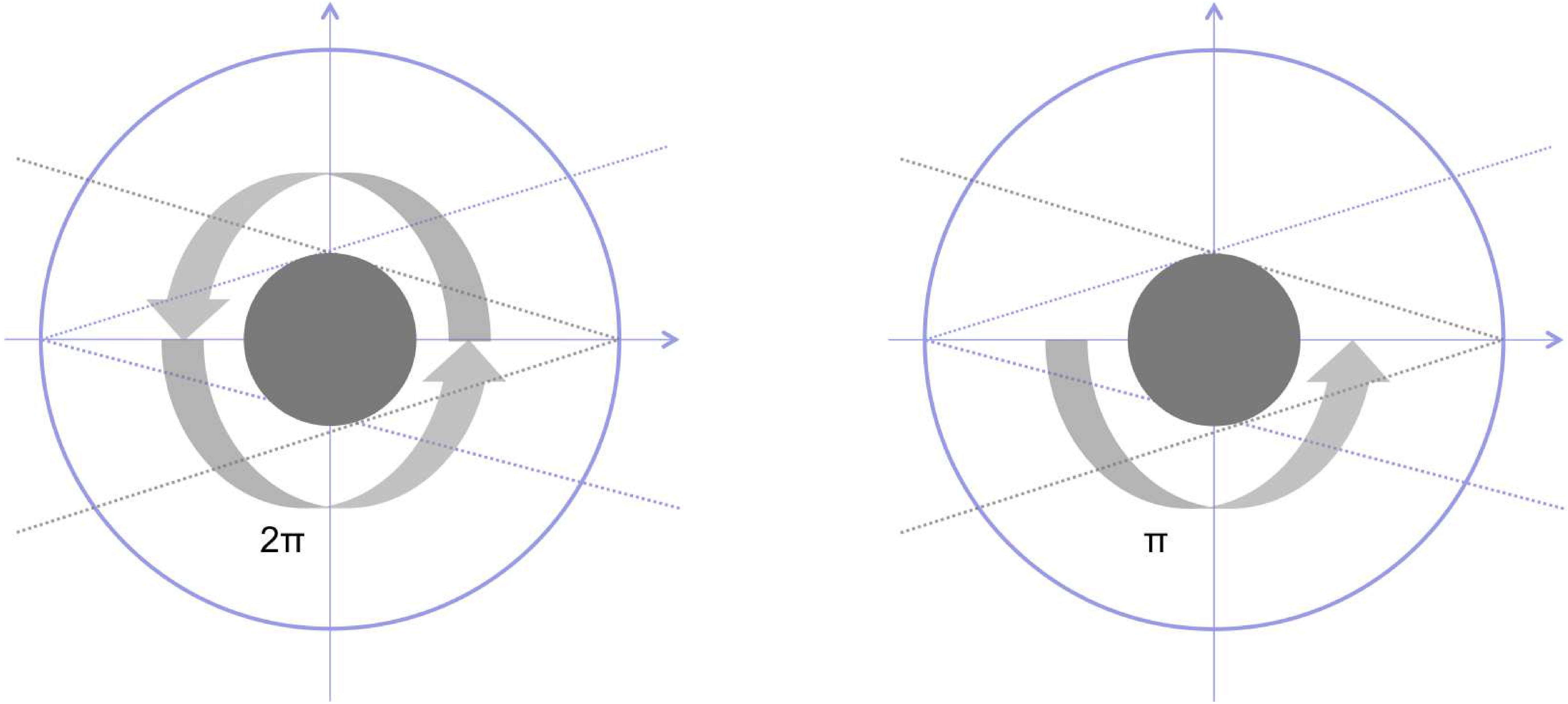


Figure 1: Full scan (left), half scan (right)

Redundant Areas: Sinogram

- Identical rays:

$$\gamma_1 = -\gamma_2,$$
$$\beta_2 = \beta_1 - 2\gamma_1 + \pi$$

- Upper triangle:

$$\pi + 2\gamma_1 \leq \beta_1 \leq \pi + 2\delta$$

- Lower triangle:

$$0 \leq \beta_2 \leq 2\gamma_2 + 2\delta$$

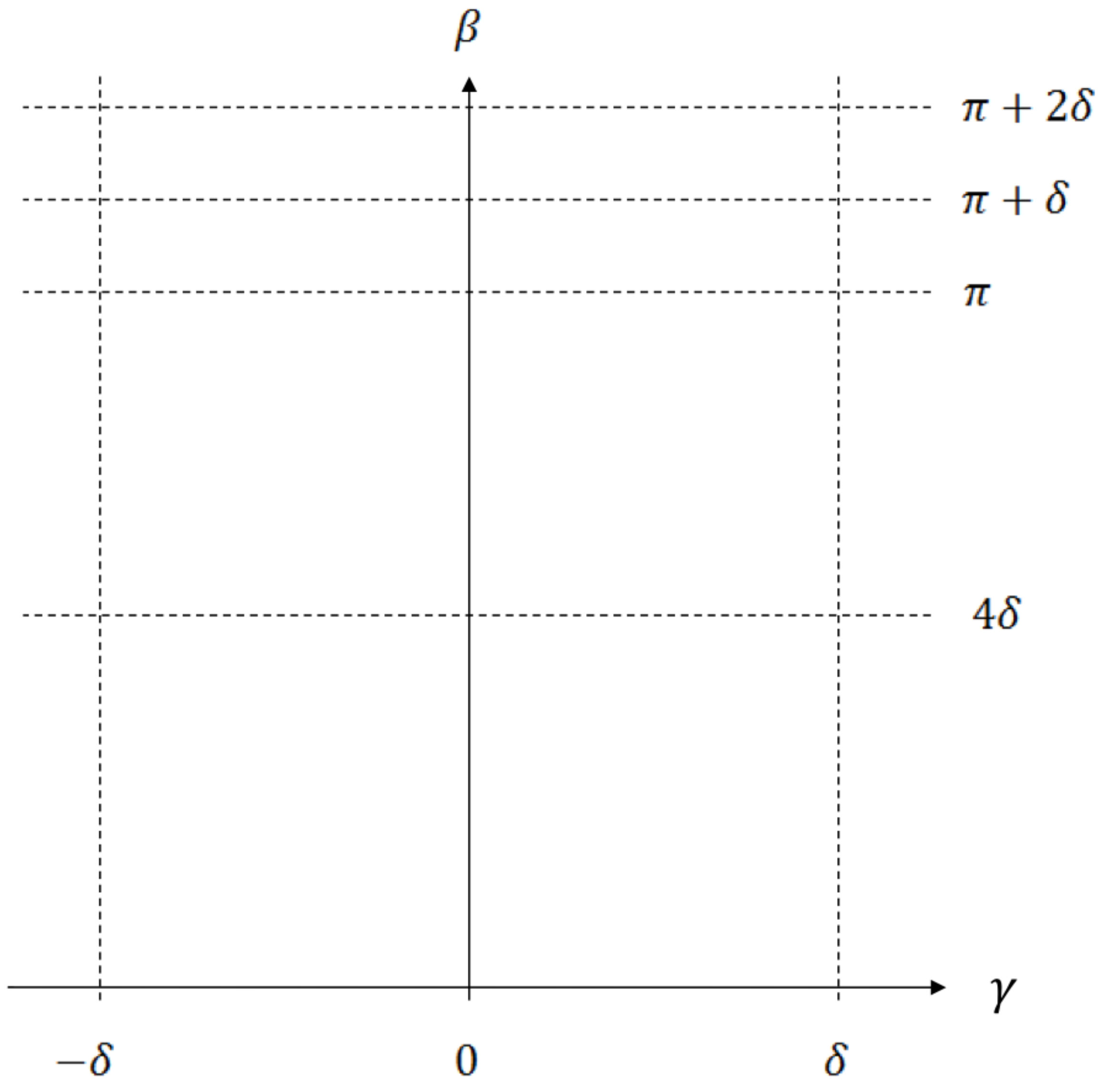


Figure 2: Sinogram range short scan

Parker Redundancy Weighting

Idea: Weight identical rays to reduce redundancy.

Constraints (upper triangle):

$$f_1(\pi + 2\delta) = 0, \tag{1'}$$
$$f_1(\pi + 2\gamma) = 1 \tag{2'}$$

Constraints (lower triangle):

$$f_2(0) = 0, \tag{1}$$
$$f_2(2\delta + 2\gamma) = 1 \tag{2}$$

Solve redundancy:

$$f_1(\beta_1) + f_2(\beta_2) = 1 \tag{3}$$

Parker Redundancy Weighting

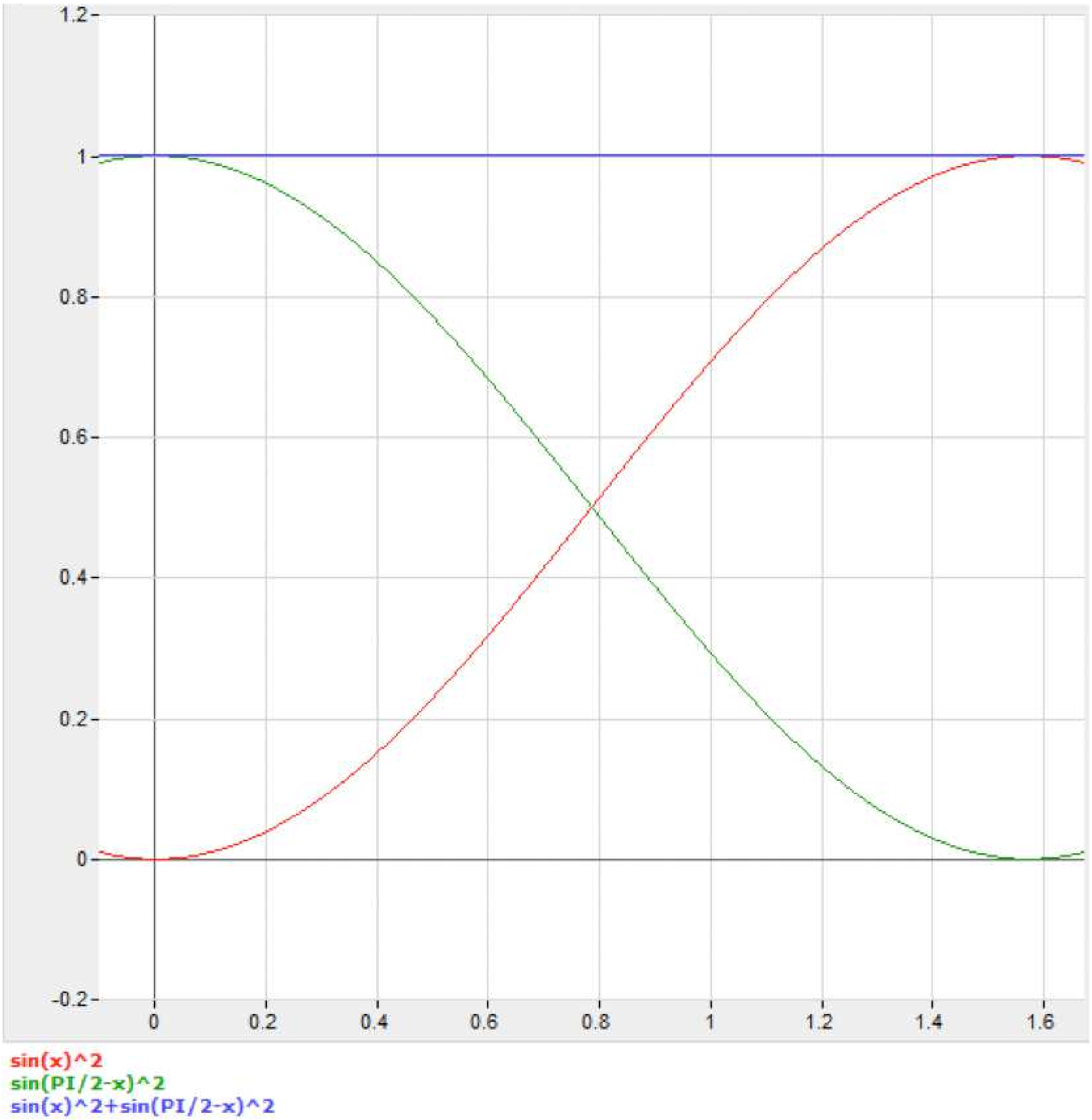


Figure 3: Plot of Parker weights

Parker Redundancy Weighting

Parker's trick:

$$\sin^2(\gamma) + \cos^2(\gamma) = 1,$$
$$\sin\left(\frac{\pi}{2} - \gamma\right) = \cos(\gamma)$$

New constraints:

$$f(x) = 0, \quad (1 + 1')$$
$$f(x) = \frac{\pi}{2}, \quad (2 + 2')$$
$$f_1(\beta_1) + f_2(\beta_2) = 1 \quad (3)$$

Weighting functions:

$$f_1(\beta_1) = \frac{\pi}{2} \frac{\pi + 2\delta - \beta_1}{(\pi + 2\delta) - (\pi + 2\gamma)} = \frac{\pi}{4} \frac{\pi + 2\delta - \beta_1}{\delta - \gamma},$$
$$f_2(\beta_2) = \frac{\pi}{2} \frac{\beta_2}{2\delta + 2\gamma} = \frac{\pi}{4} \frac{\beta_2}{\delta + \gamma}$$

Polynomial Parker Weighting

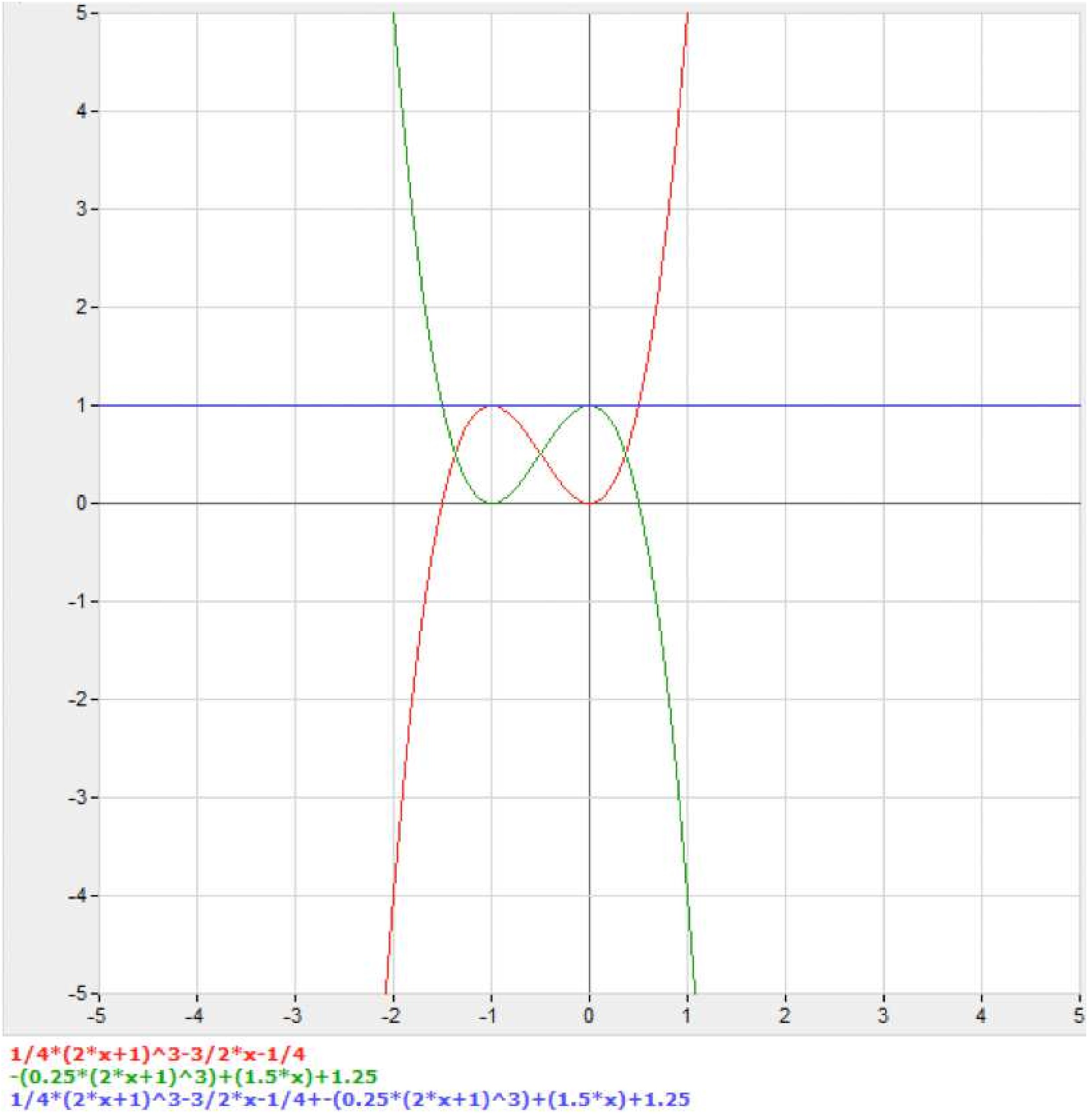


Figure 4: Plot of polynomial Parker weights

Parker Weighting: Example

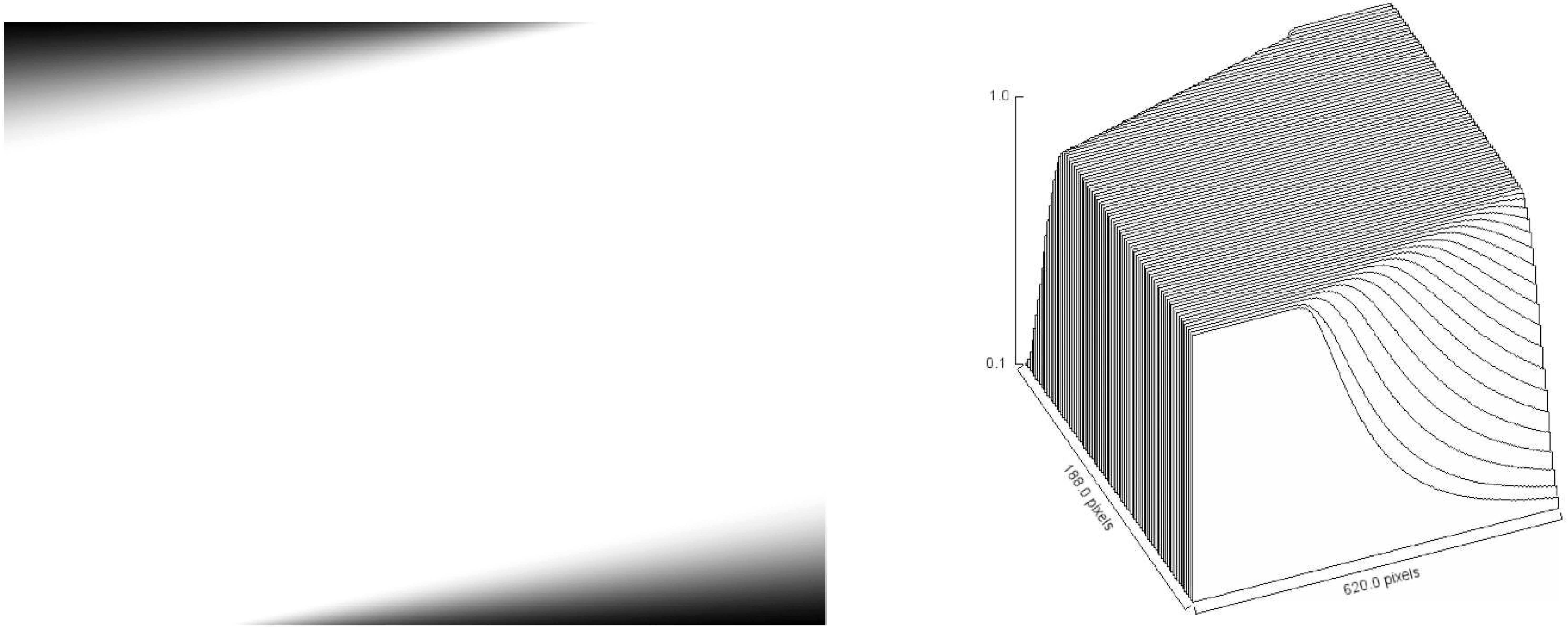


Figure 5: Parker weights for a short scan trajectory

FBP for the Equiangular Case and Parker Weights

1. Perform Parker weighting:

$$g_1(\gamma,\beta) = g(\gamma,\beta)\, w_{\text{Parker}}(\gamma,\beta).$$

2. Perform cosine weighting:

$$g_2(\gamma,\beta) = g_1(\gamma,\beta)\cos\gamma.$$

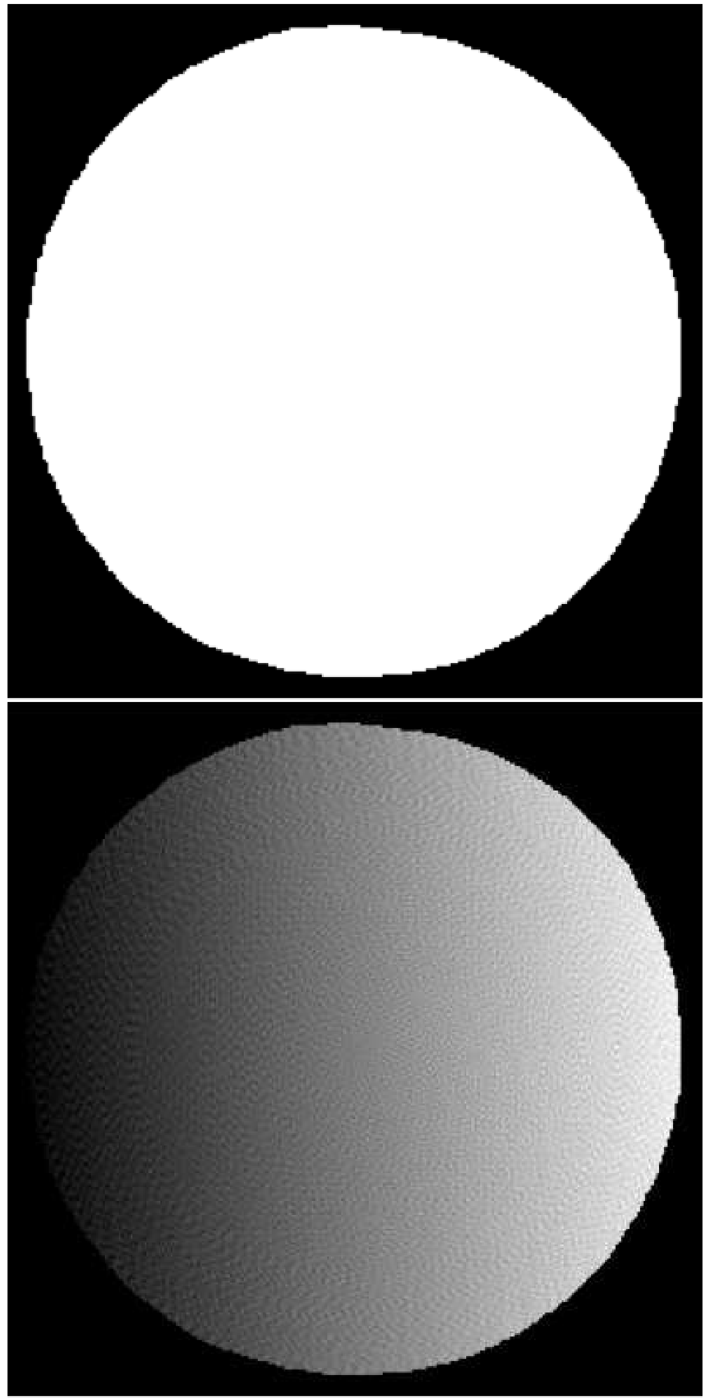
3. Apply fan beam filter:

$$g_3(\gamma',\beta) = (g_2 * h_{\text{fan}})(\gamma',\beta), \quad h_{\text{fan}}(\gamma) = \frac{D}{2}\left(\frac{\gamma}{\sin\gamma}\right)^2 h(\gamma).$$

4. Backproject with distance weight:

$$f(r,\varphi) = \int_0^{2\pi} \frac{1}{D'^2} g_3(\gamma',\beta)\,\mathrm{d}\beta.$$

No Redundancy Weights: Example



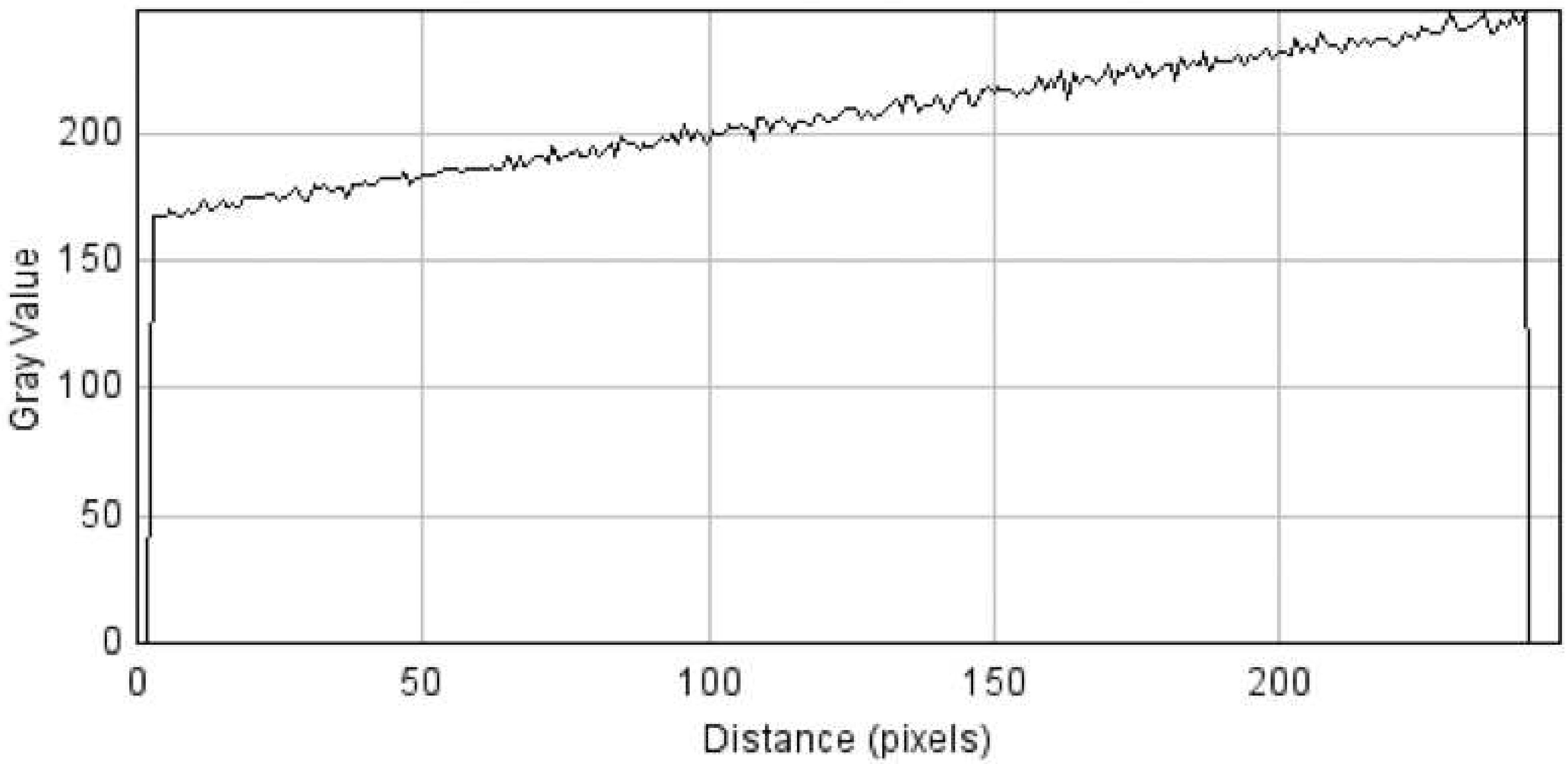


Figure 6: Gradient profile in the non-weighted image

Short Scan: Point Spread Function

- The point spread function is no longer uniform.
- Reconstruction resolution changes over the image.

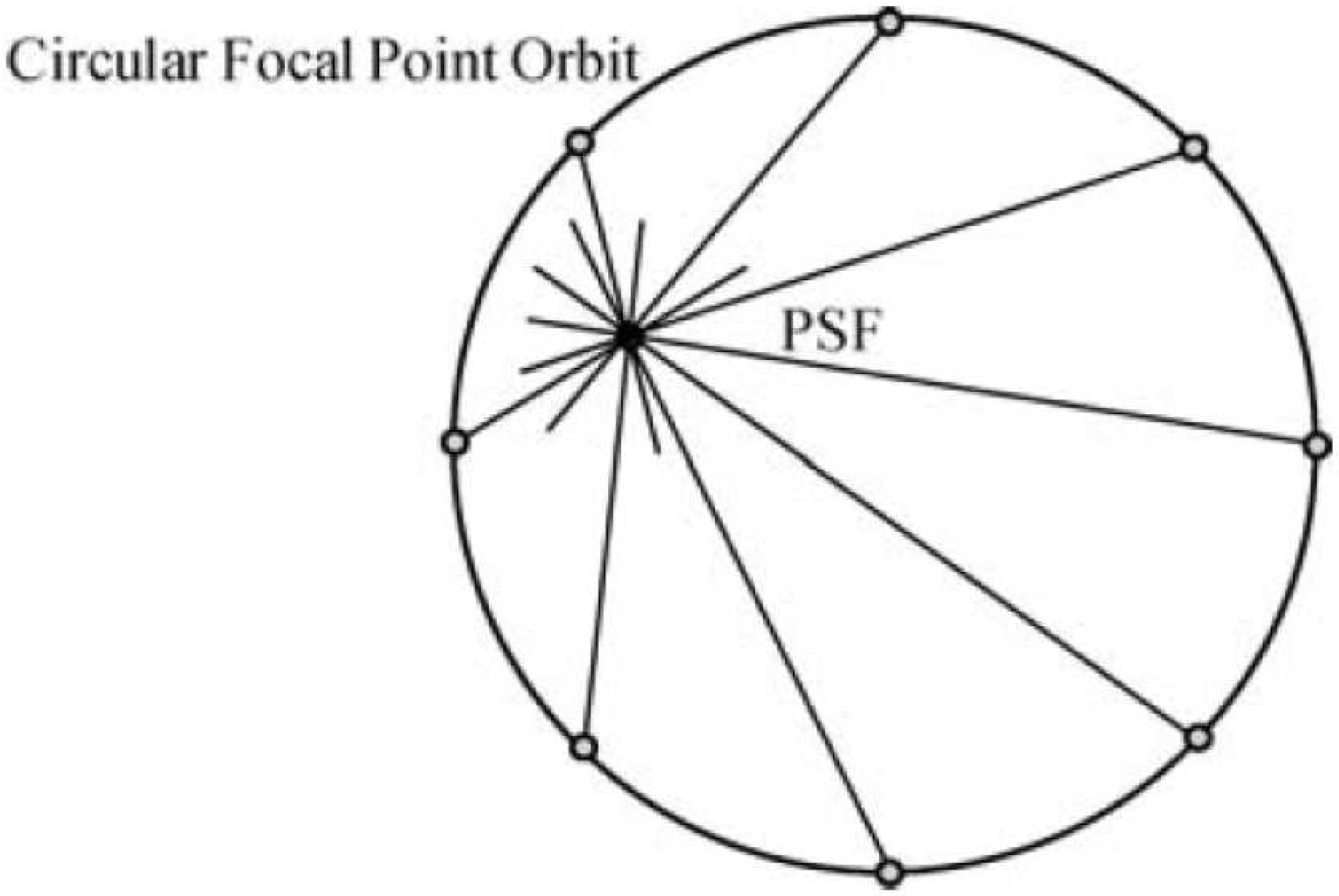


Figure 7: Scheme of the PSF (Zeng, 2009)

Topics

Short Scan

Summary

Take Home Messages

Further Readings

Take Home Messages

- The short scan is important e. g., for C-arm systems.
- A complete fan beam dataset requires a rotation of 180° plus fan angle.
- Due to a data redundancy in the short scan we need to weight properly. The Parker weights allow a smooth weighting transition between redundant and singular data.

Further Readings

Helpful reads for the current unit:

Dennis L. Parker. “Optimal Short Scan Convolution Reconstruction for Fan Beam CT”. In: *Medical Physics* 9.2 (Mar. 1982), pp. 254–257. DOI: 10.1118/1.595078

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Medical Image Processing for Diagnostic Applications

Fan Beam – Super Short Scan

Online Course – Unit 40

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Topics

Super Short Scan

Summary

Take Home Messages

Further Readings

Less than a short scan?

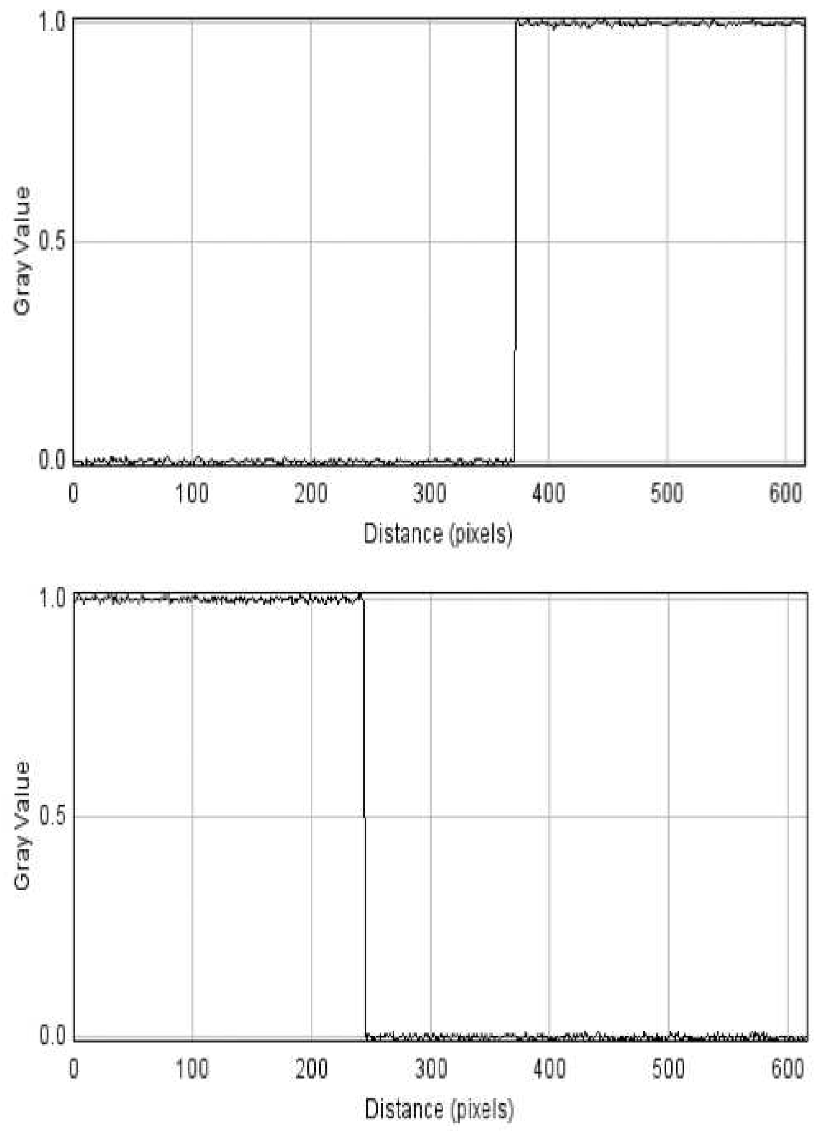


Figure 1: Weight profile

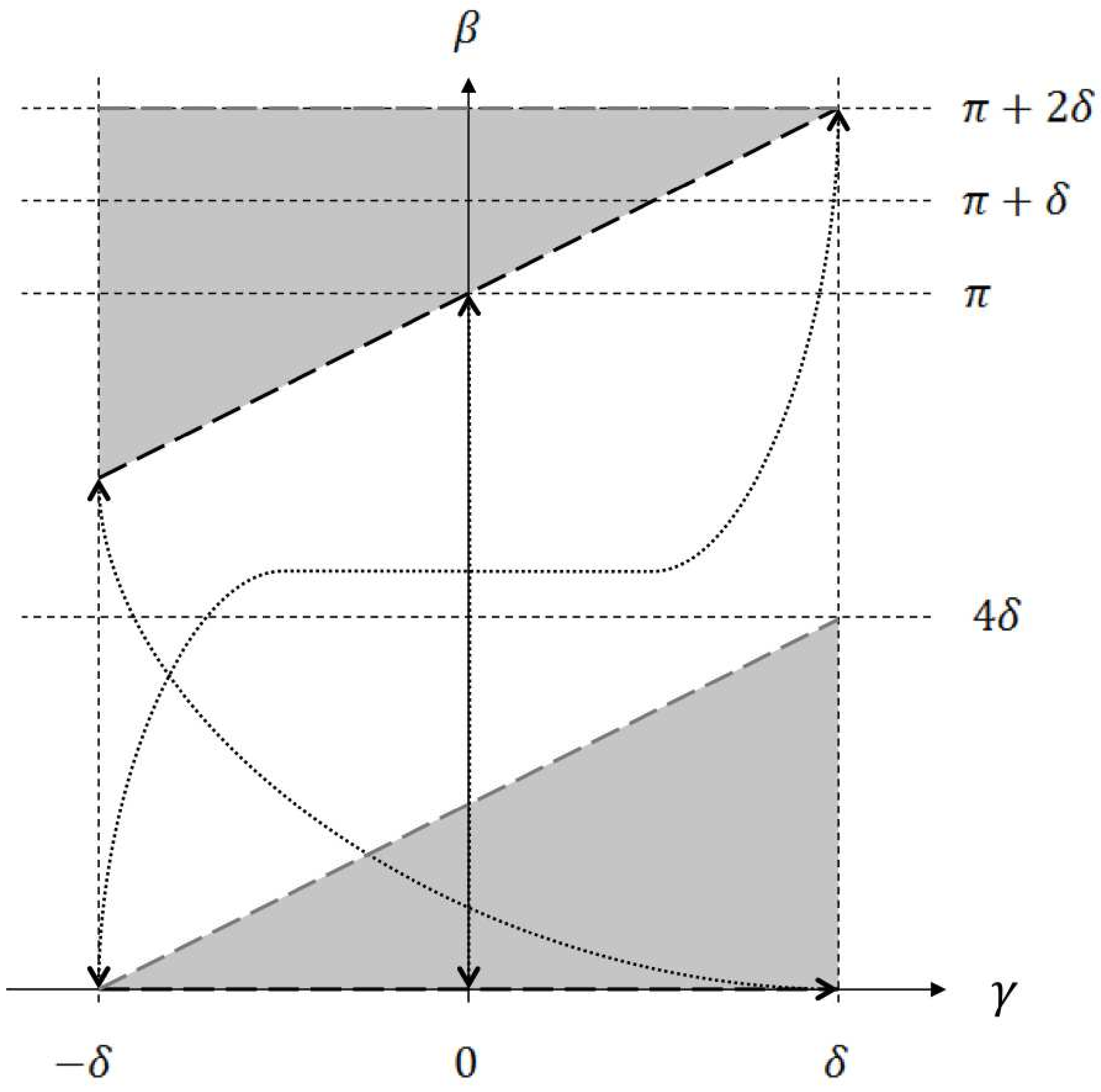


Figure 2: Redundant areas and ranges

Less than a short scan?

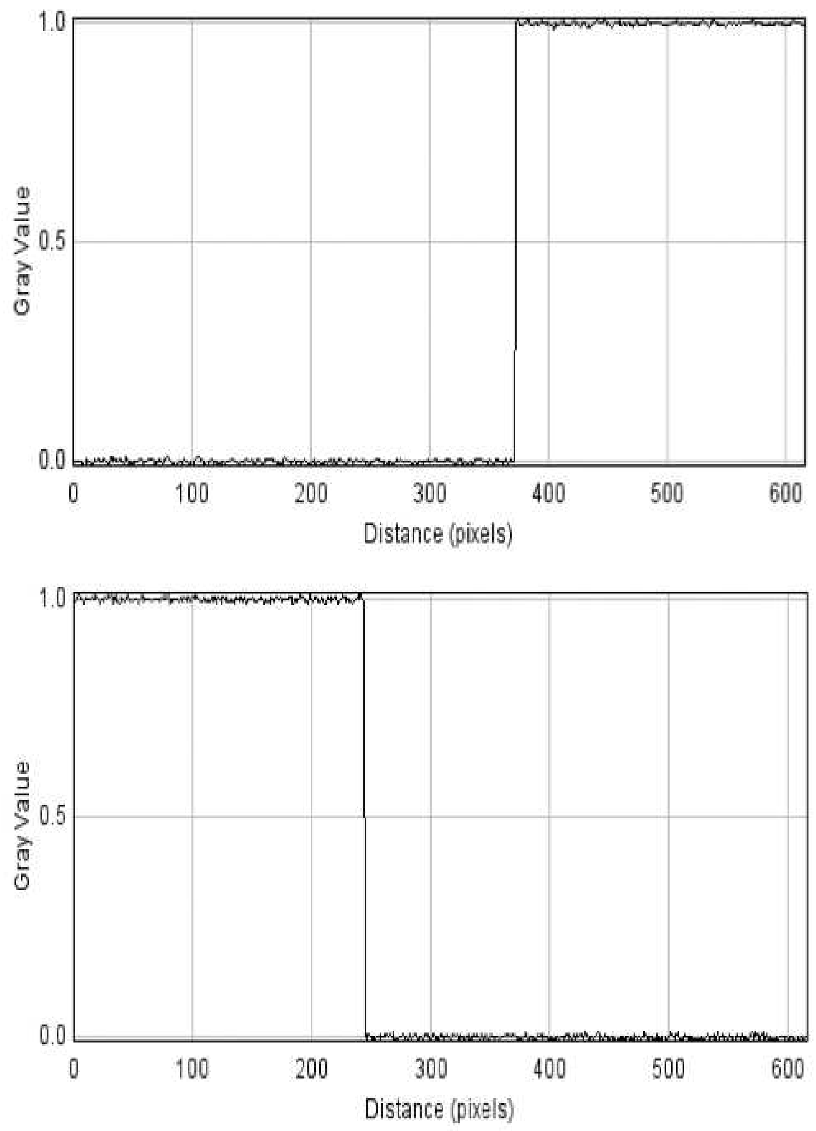


Figure 1: Weight profile

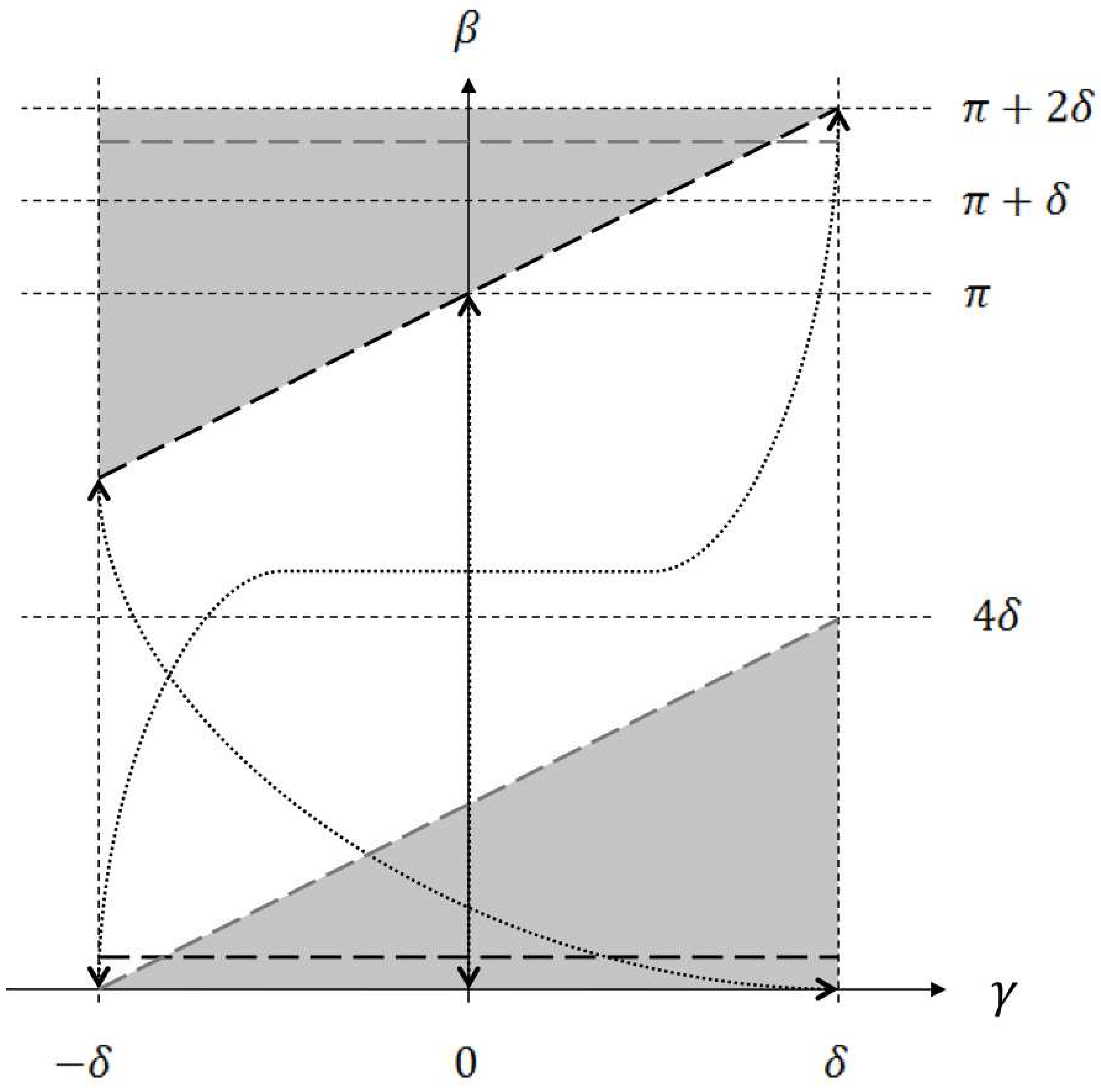


Figure 2: Redundant areas and ranges

Less than a short scan?

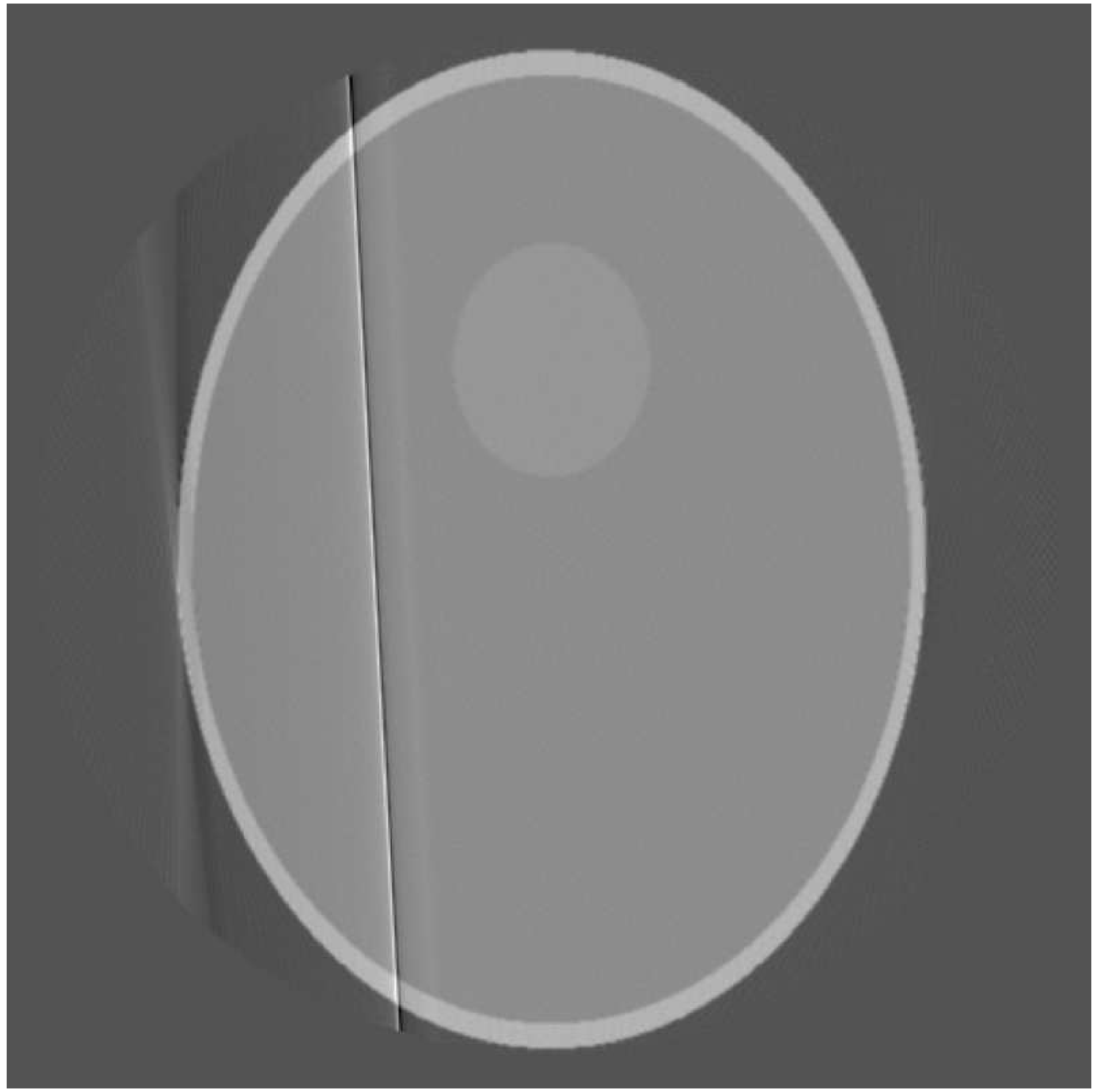


Figure 3: Super short scan reconstruction showing an artifact

Less than a short scan?

- Apply Parker weight $w_{\text{Parker}}(t,\beta)$ in reconstruction formula:

$$f(r,\varphi) = \frac{1}{2}\int_{0}^{2\pi} \frac{1}{U^2} \int_{-\infty}^{\infty} \frac{D}{\sqrt{D^2+t^2}} \left(w_{\text{Parker}}(t,\beta) g(t,\beta)\right) h(t'-t)\,\mathrm{d}t\,\mathrm{d}\beta,$$

  (cf. Unit 38).
- It is not possible to pull $w_{\text{Parker}}(t,\beta)$ after the convolution without introducing artifacts.
- There is a solution that can solve this problem, but it is not covered in this course.
- This reconstruction method is known as ***super short scan***.

Topics

Super Short Scan

Summary

Take Home Messages

Further Readings

Take Home Messages

- A super short scan with a lesser angular sampling range than required from theory can be feasible in practice.
- Parker weights are also important if a super short scan is acquired.

Further Readings

Helpful reads for the current unit:

Frédéric Noo et al. “Image Reconstruction from Fan-Beam Projections on Less Than a Short Scan”. In: *Physics in Medicine and Biology* 47.14 (July 2002), pp. 2525–2546. DOI: 10.1088/0031-9155/47/14/311

Dennis L. Parker. “Optimal Short Scan Convolution Reconstruction for Fan Beam CT”. In: *Medical Physics* 9.2 (Mar. 1982), pp. 254–257. DOI: 10.1118/1.595078

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Medical Image Processing for Diagnostic Applications

Fan Beam – Truncation

Online Course – Unit 41

Andreas Maier, Joachim Hornegger, Markus Kowarschik, Frank Schebesch
Pattern Recognition Lab (CS 5)

Topics

What is Truncation?

Summary

Take Home Messages

Further Readings

Example: Homogeneous Cylinder

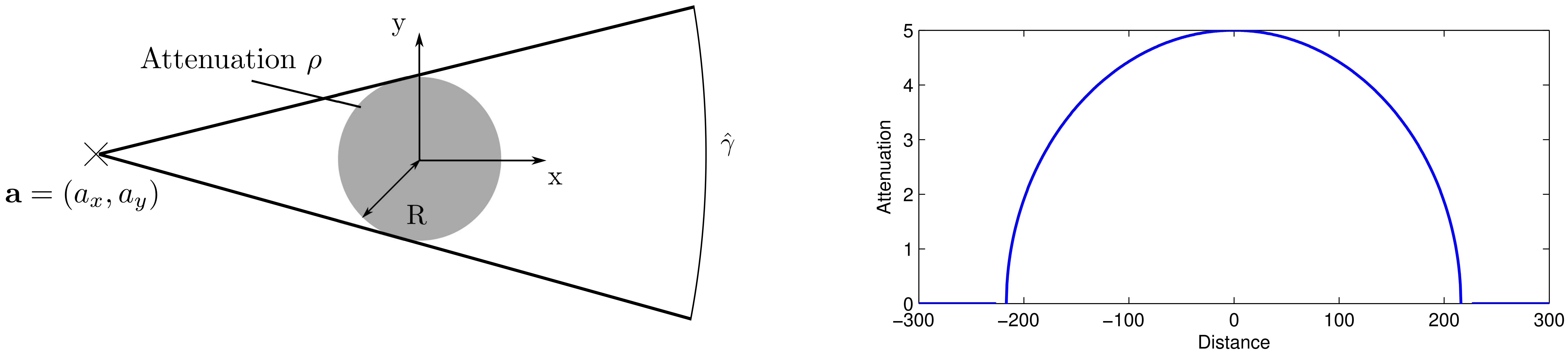
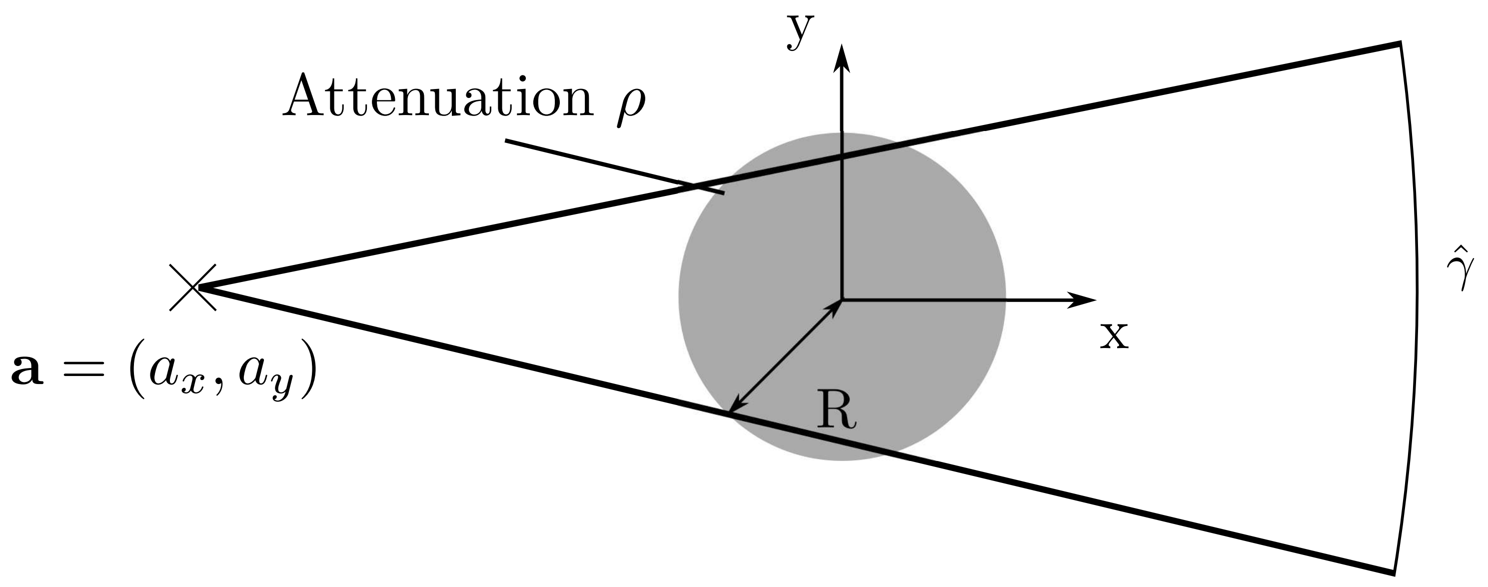


Figure 1: Complete object in field of view

Example: Homogeneous Cylinder



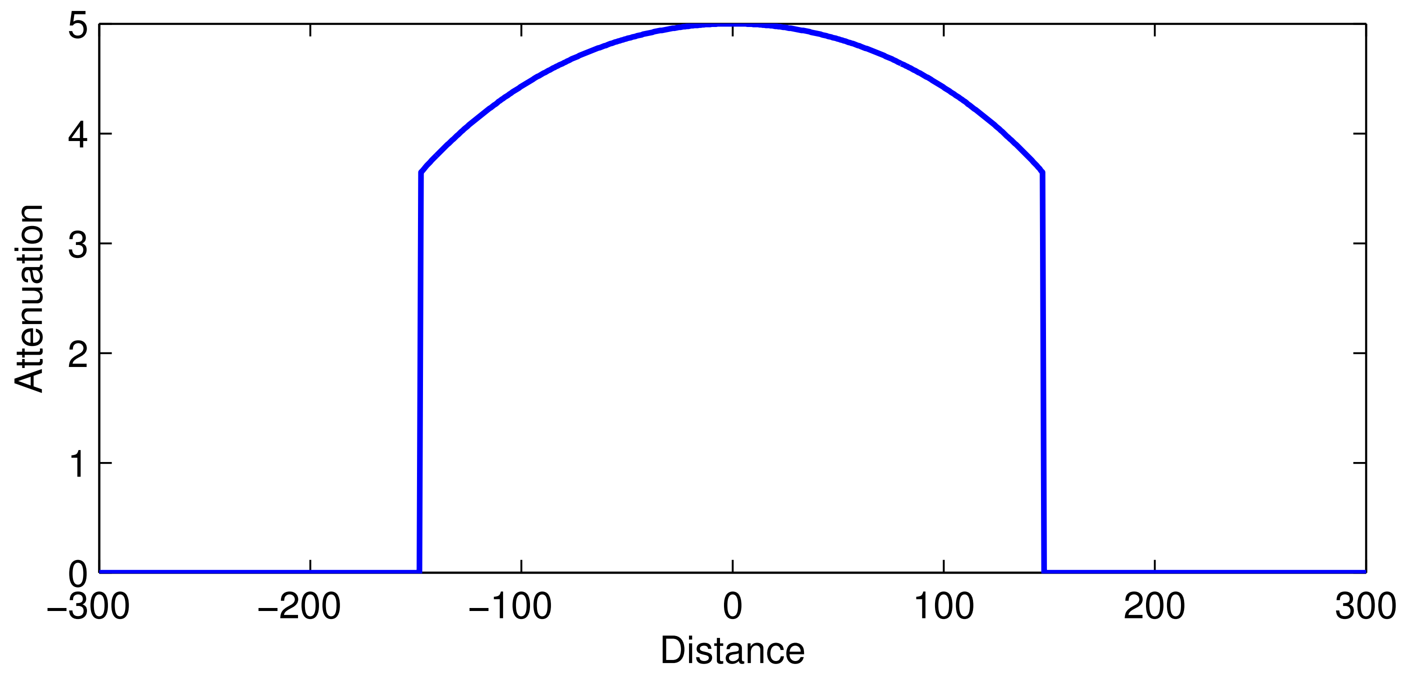


Figure 2: Truncated object projection

Example: Homogeneous Cylinder

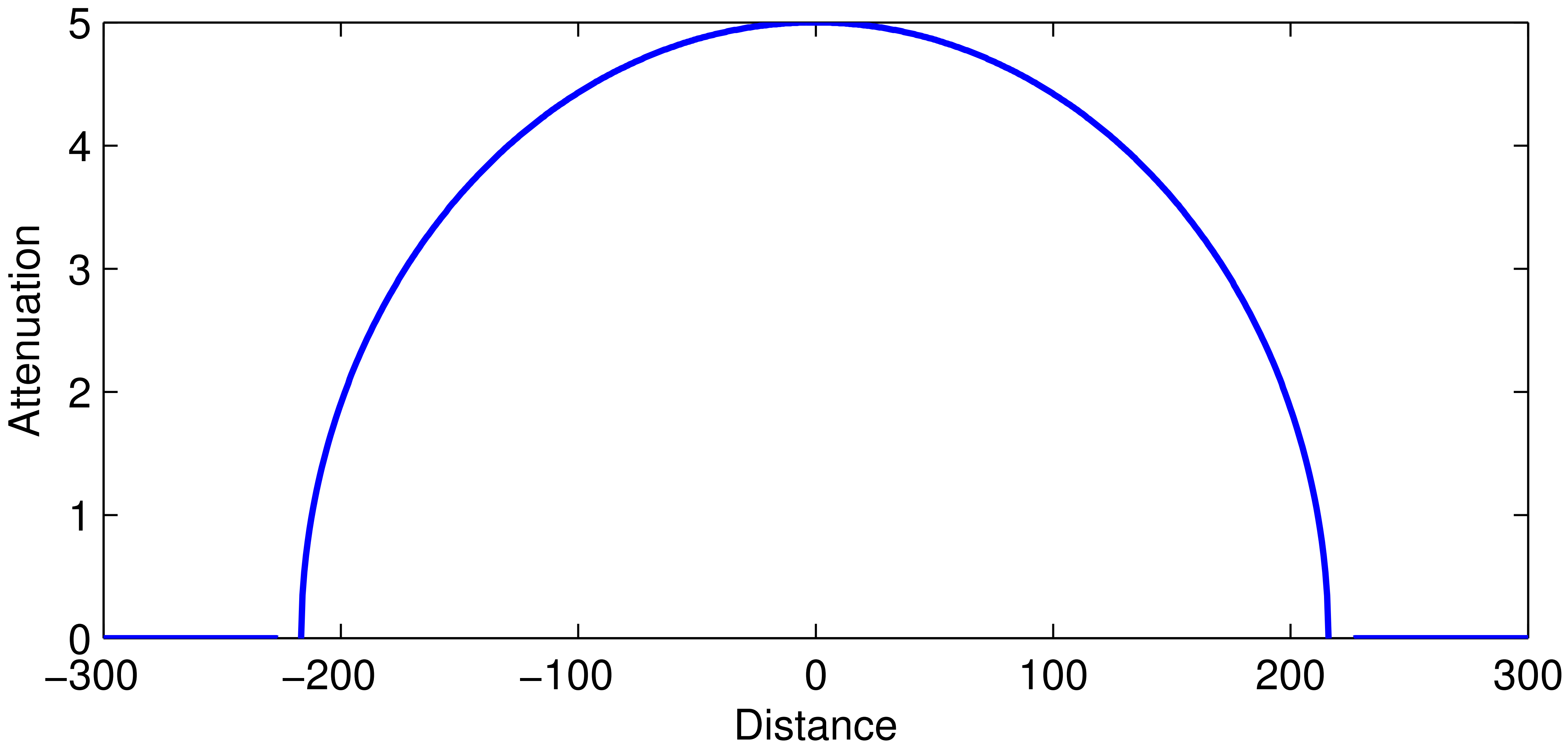


Figure 3: Full projection of the cylinder

Example: Homogeneous Cylinder

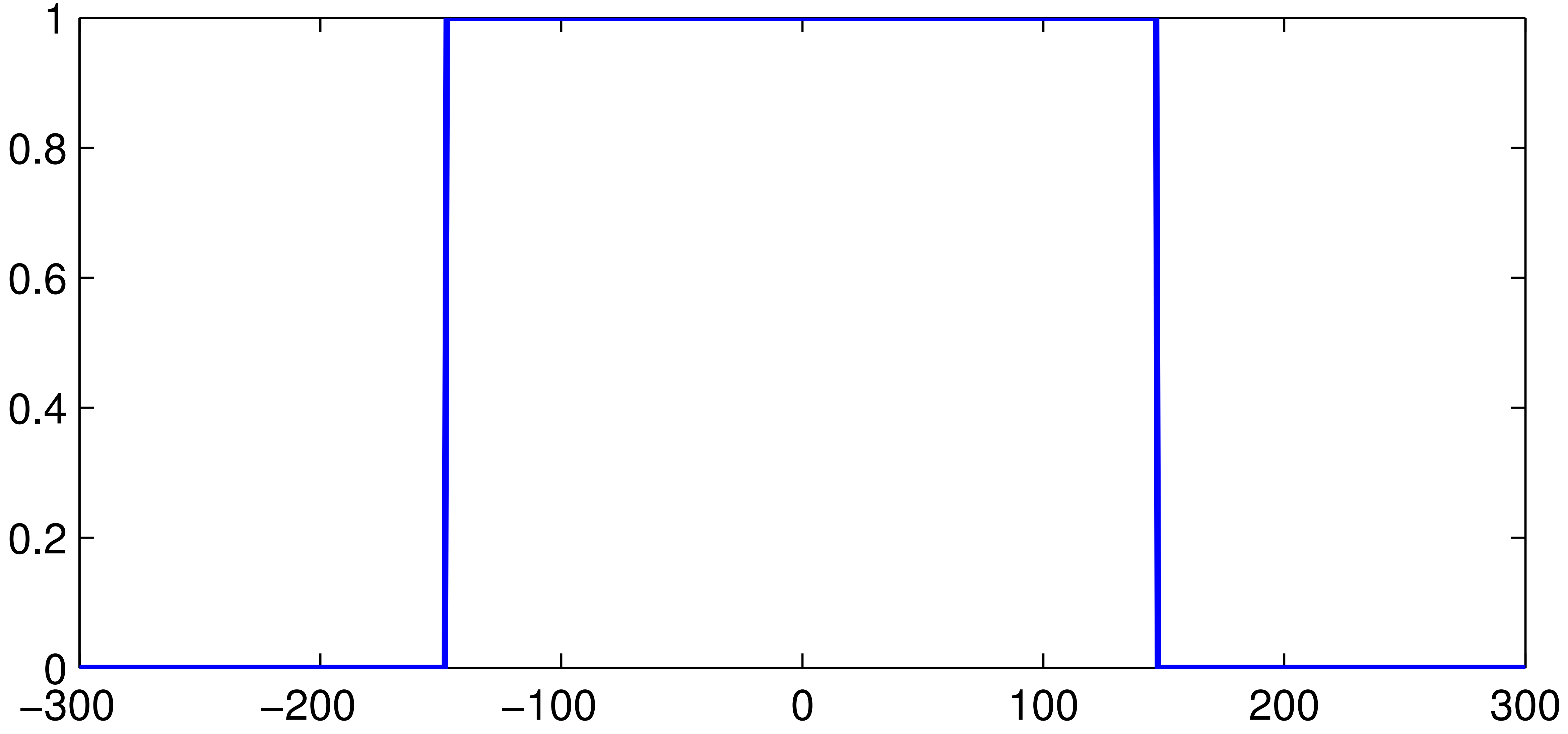


Figure 4: Truncation function

Example: Homogeneous Cylinder

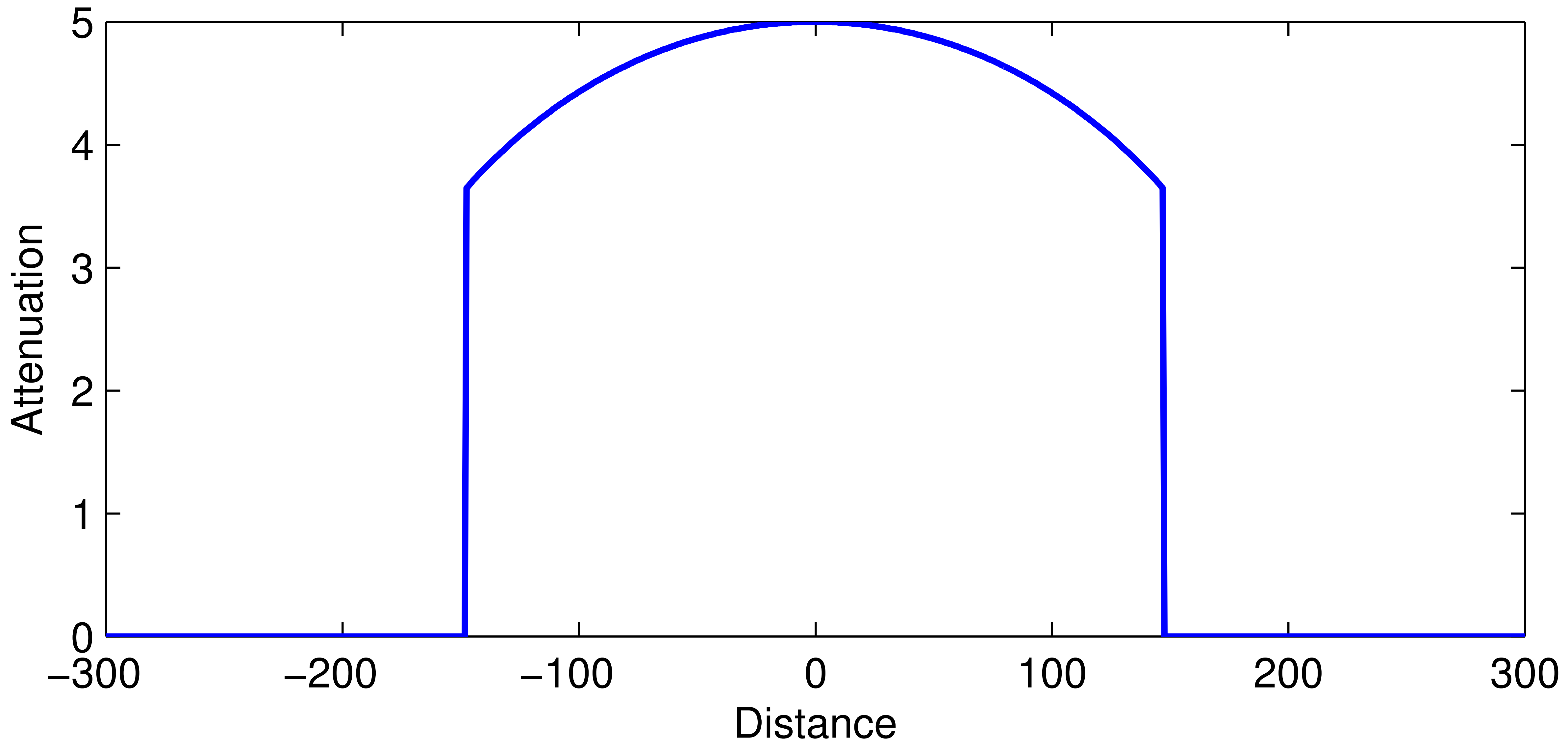
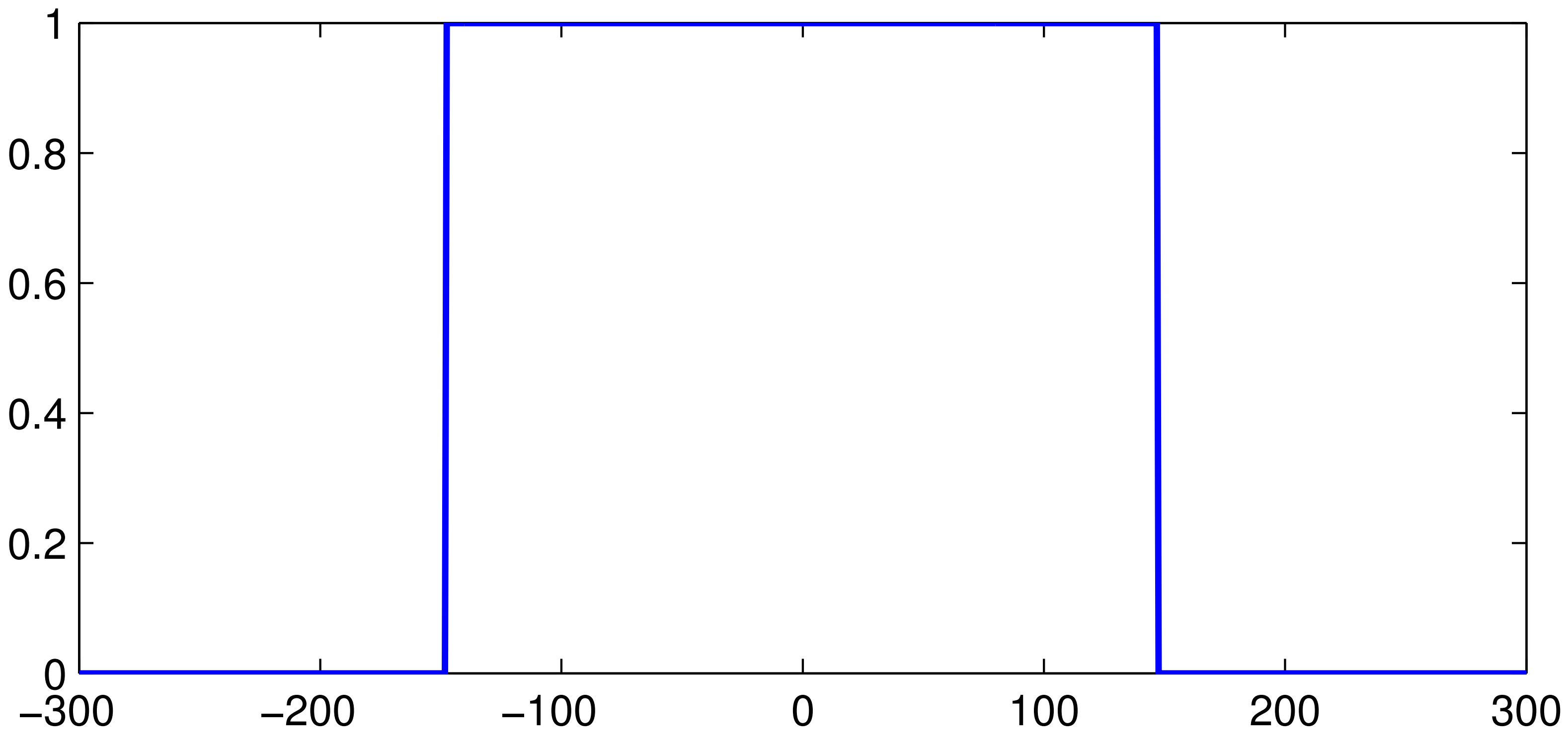


Figure 5: Truncated object in the projection

Example: Filter Results



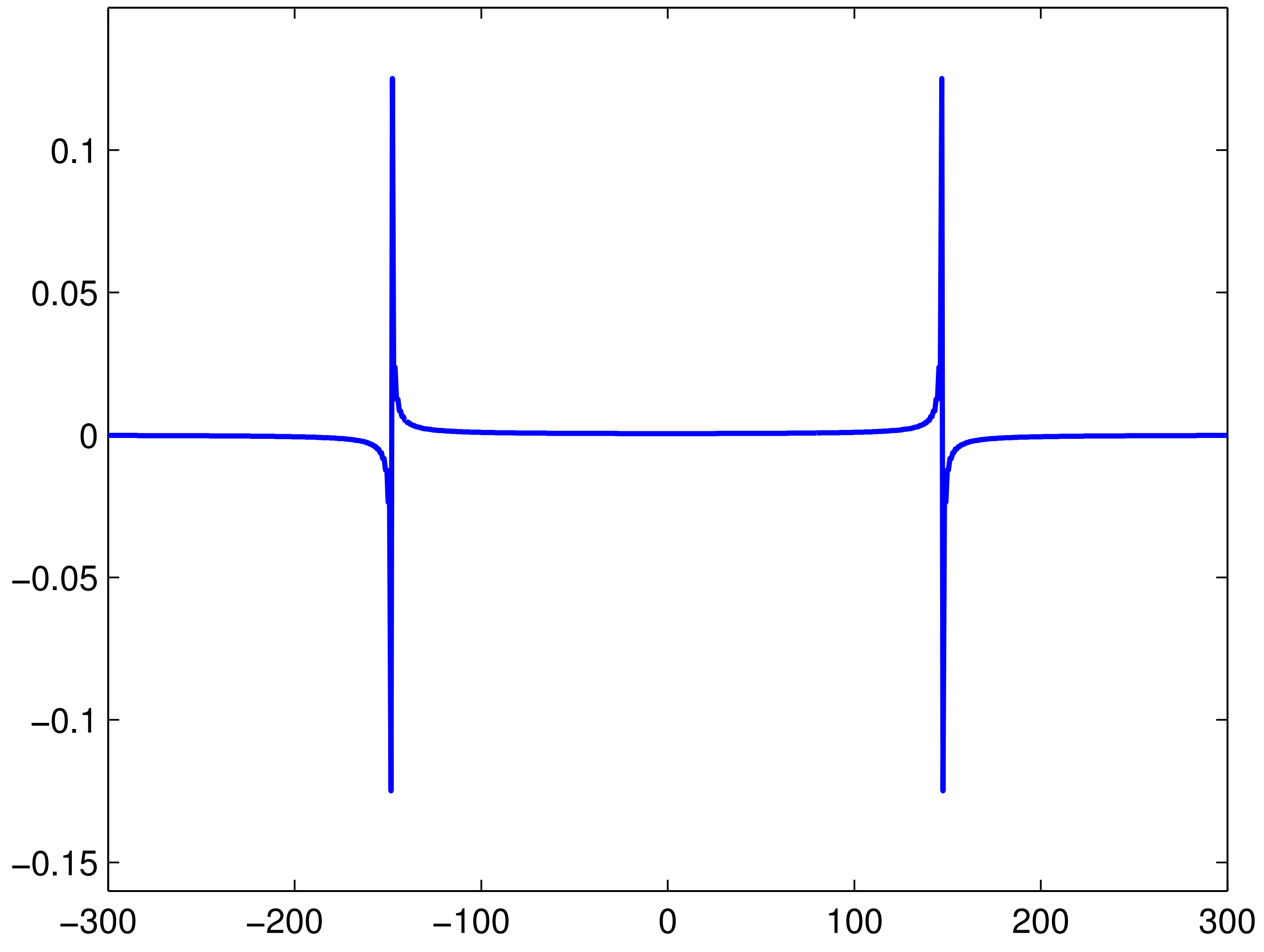


Figure 6: Filter result for the truncation function

Example: Filter Results

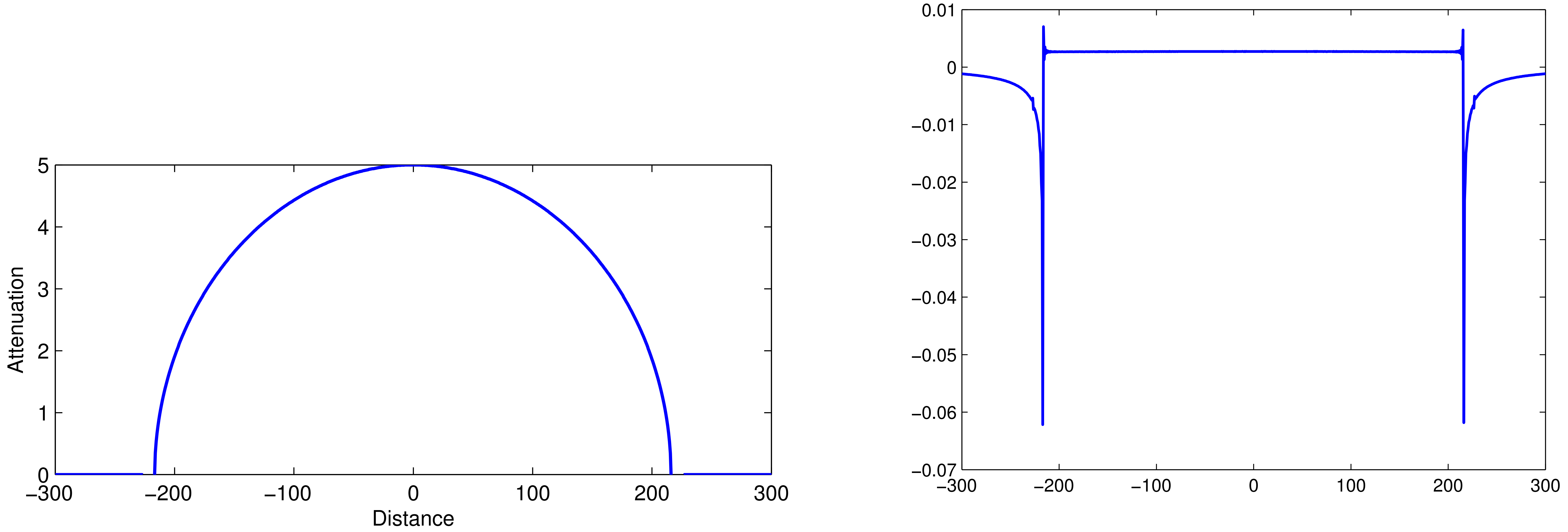
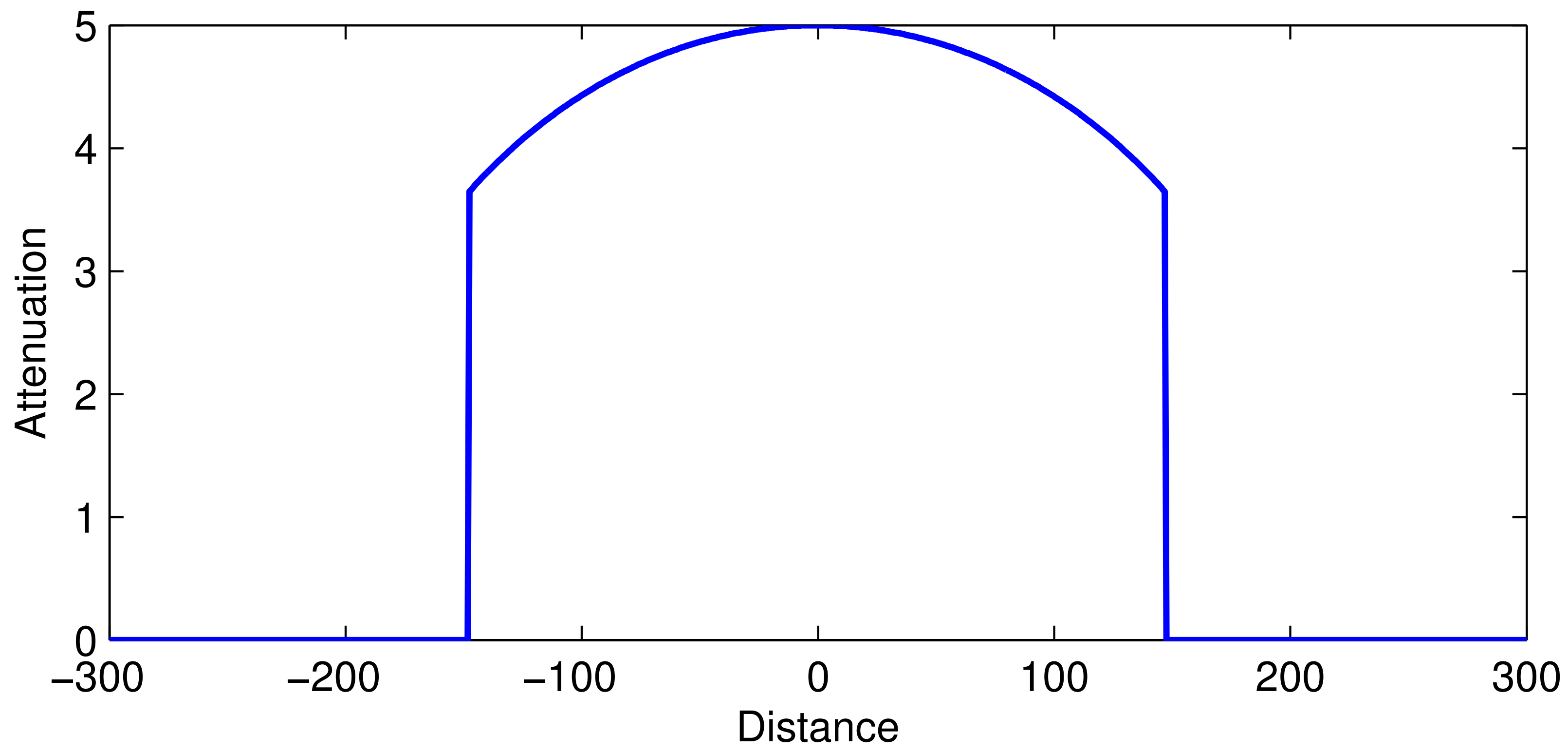


Figure 7: Filter result for the full projection

Example: Filter Results



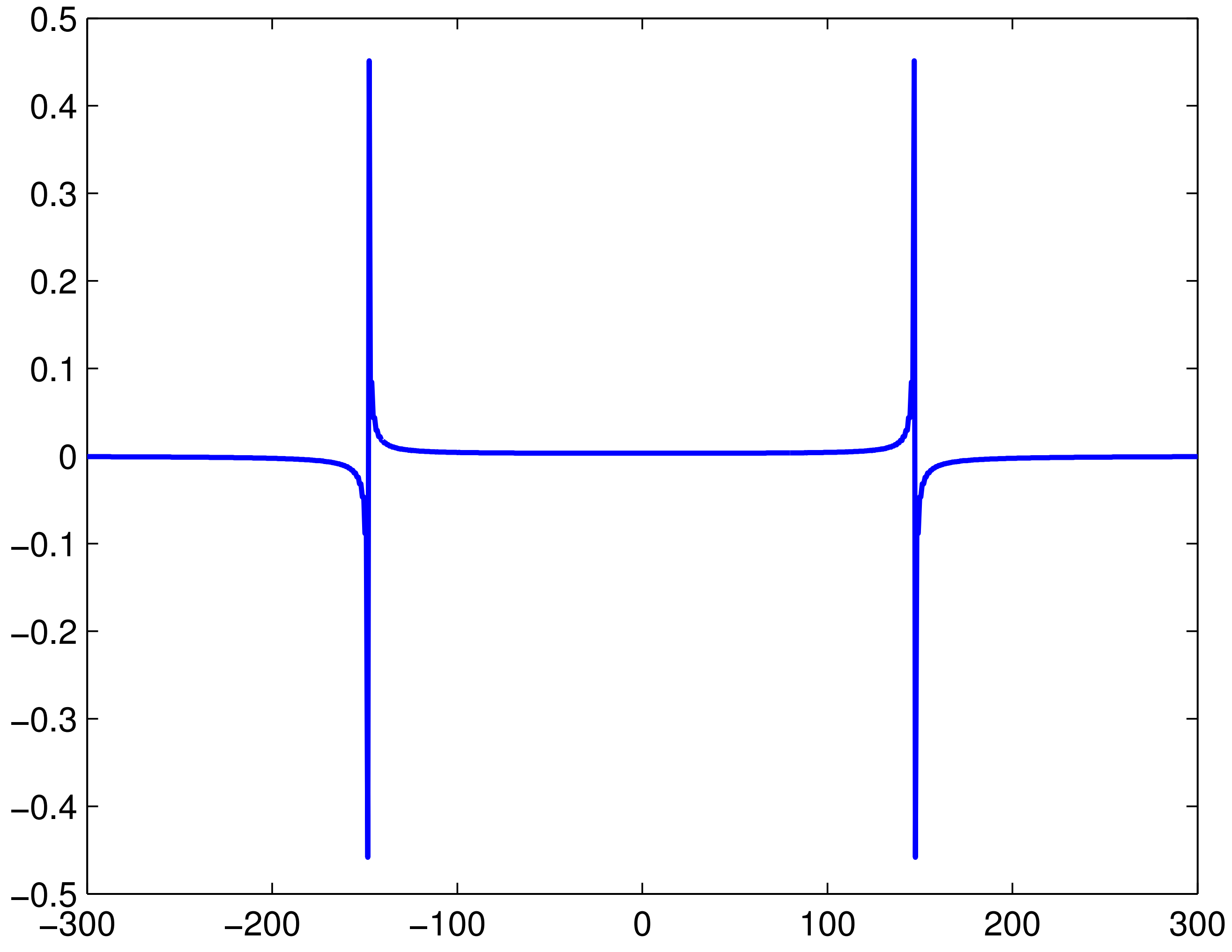


Figure 8: Filter result for the truncated projection

Example: Shepp-Logan Phantom

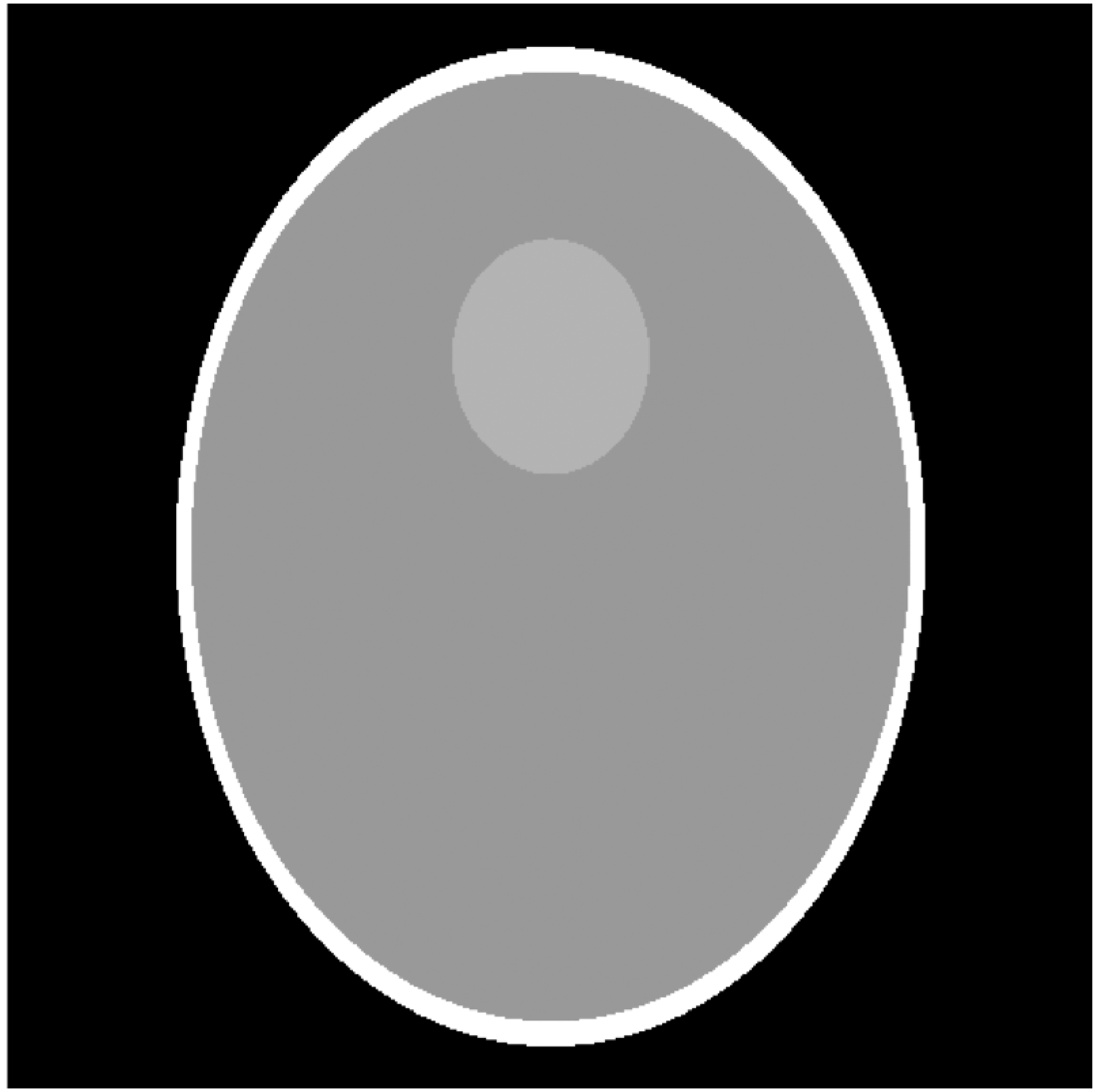


Figure 9: Original Shepp-Logan phantom

Example: Shepp-Logan Phantom

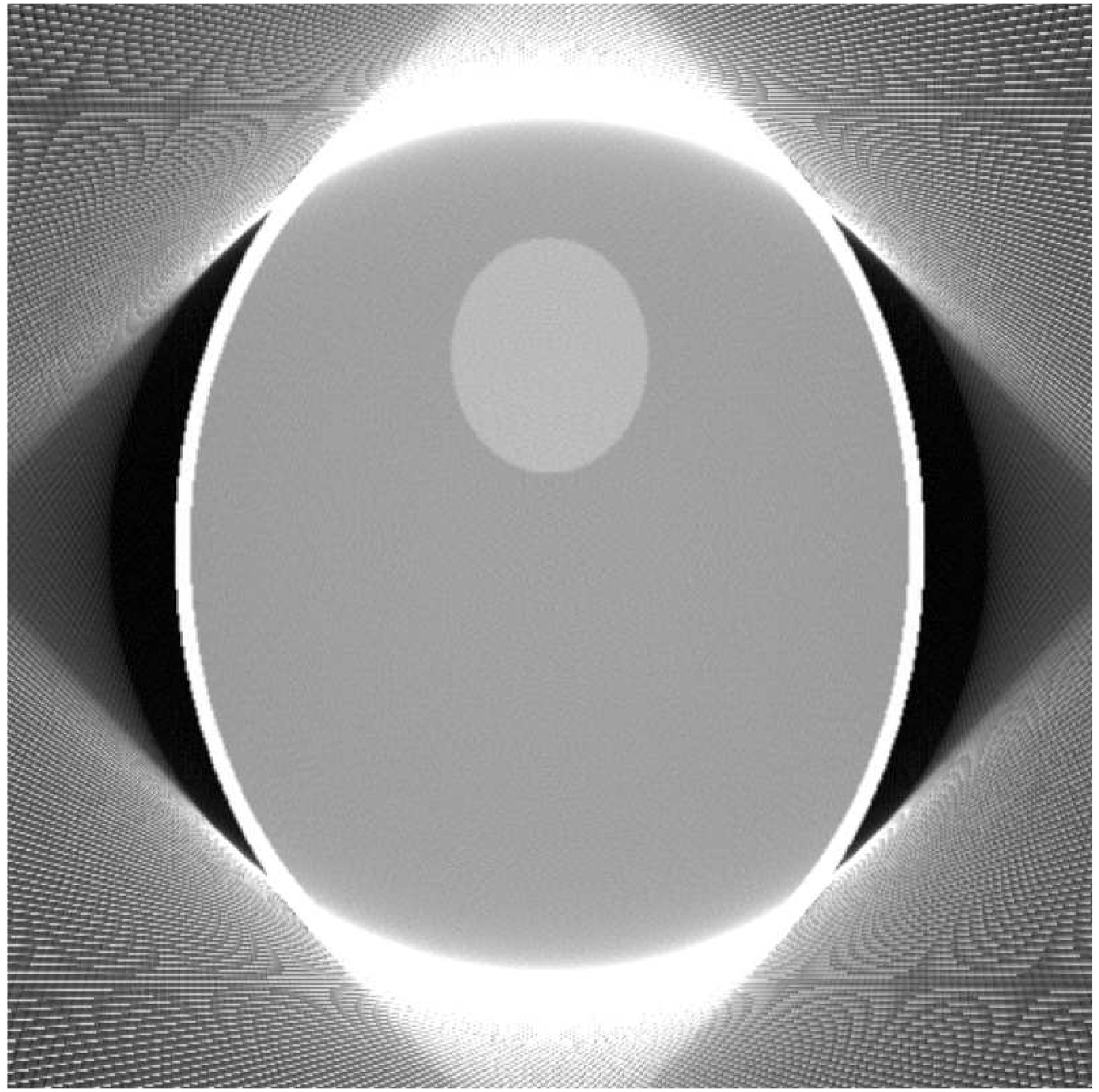


Figure 10: Shepp-Logan phantom with truncation artifact

Truncation ...

- ... happens when the imaged object extends the field of view.
- ... can be modeled as a multiplication with a rectangular window function in spatial domain.
- ... introduces artificial frequencies into the reconstruction.
- ... causes a typical artifact at the end of the field of view.

Topics

What is Truncation?

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- If the field of view is too small for the scanned object (or the source too closely positioned), truncation takes effect on the projections and therefore on the reconstruction result.
- Truncation artifacts can heavily degrade the image quality of a reconstruction output.

Further Readings

Helpful reads for the current unit:

B. Ohnesorge et al. "Efficient Correction for CT Image Artifacts Caused by Objects Extending Outside the Scan Field of View". In: *Medical Physics* 27.1 (Oct. 2000), pp. 39–46. DOI: 10.1118/1.598855

L. A. Shepp and Logan B. F. "The Fourier Reconstruction of a Head Section". In: *IEEE Transactions on Nuclear Science* 21.3 (June 1974), pp. 21–43. DOI: 10.1109/TNS.1974.6499235

W. P. Segars et al. "Realistic CT Simulation Using the 4D XCAT Phantom". In: *Medical Physics* 35.8 (Aug. 2008), pp. 3800–3808. DOI: 10.1118/1.2955743

Medical Image Processing for Diagnostic Applications

Fan Beam – Truncation Correction

Online Course – Unit 42

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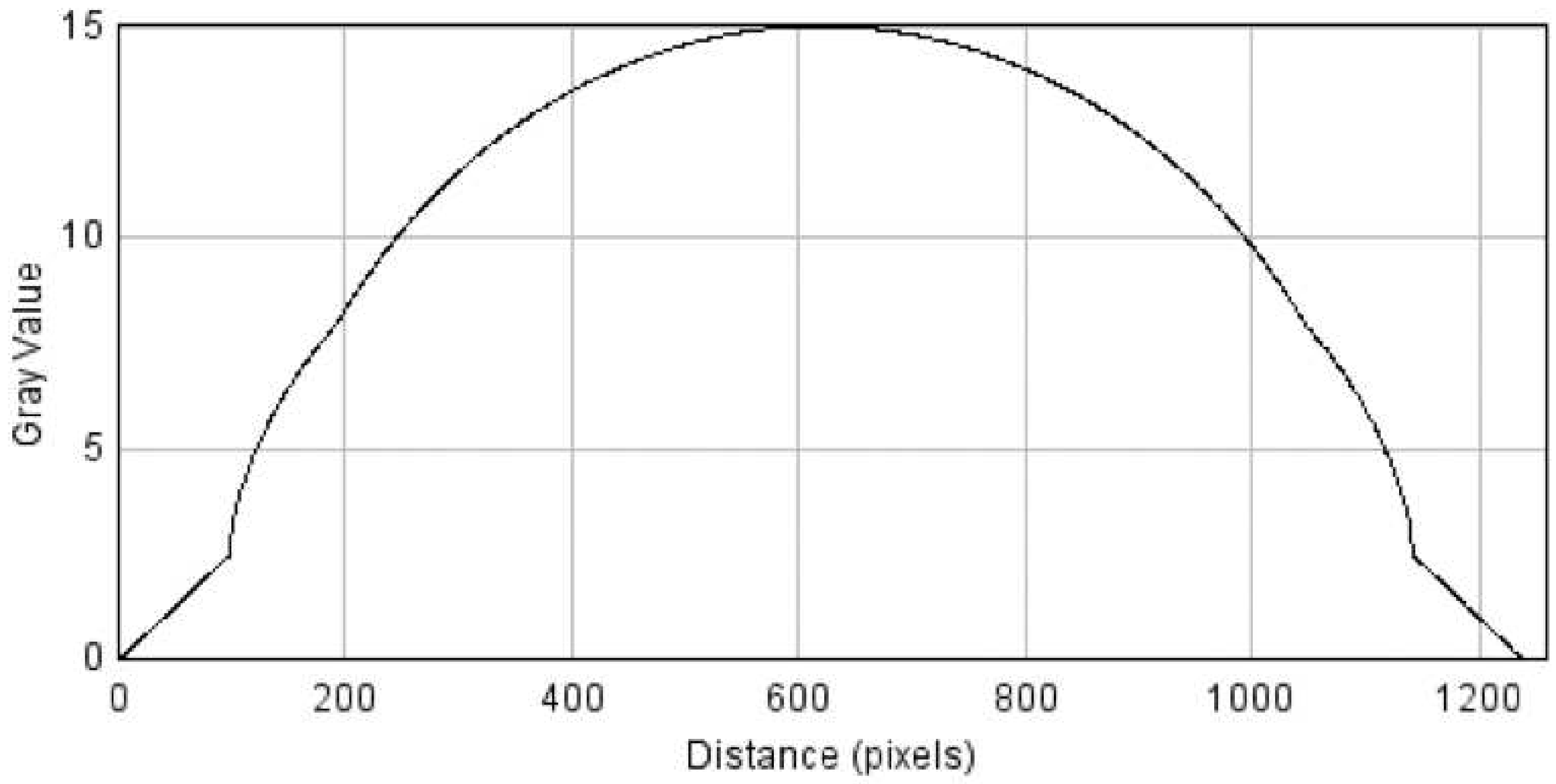
Topics

Truncation Correction via Extrapolation

- Solution 1: Defect pixel extrapolation
- Solution 2: Heuristic extrapolation
- Solution 3: Water cylinder assumption
- Solution 4: Use of prior knowledge
- Solution 5: Use of a semi-transparent filter
- Solution 6: ATRACT filtering

Defect Pixel Extrapolation

- Model extrapolation as deconvolution.
- Use a defect pixel interpolation algorithm.



$\rightarrow$ Unfortunately, the algorithm works not as well as expected.

Heuristic Extrapolation

- Use mirroring for extrapolation.
- In order to enforce a limited size of the object, a cosine-like weighting is added.

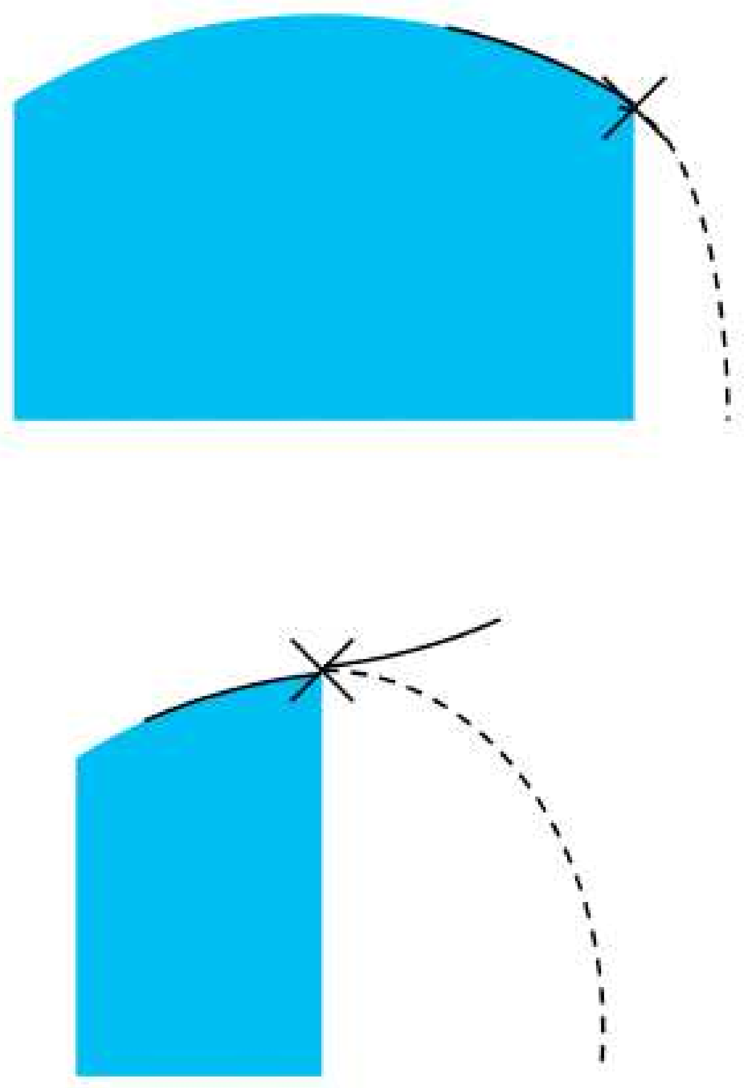


Figure 1: Heuristic extrapolation scheme

B. Ohnesorge et al. "Efficient Correction for CT Image Artifacts Caused by Objects Extending Outside the Scan Field of View". In: *Medical Physics* 27.1 (Oct. 2000), pp. 39–46. DOI: 10.1118/1.598855

Heuristic Extrapolation

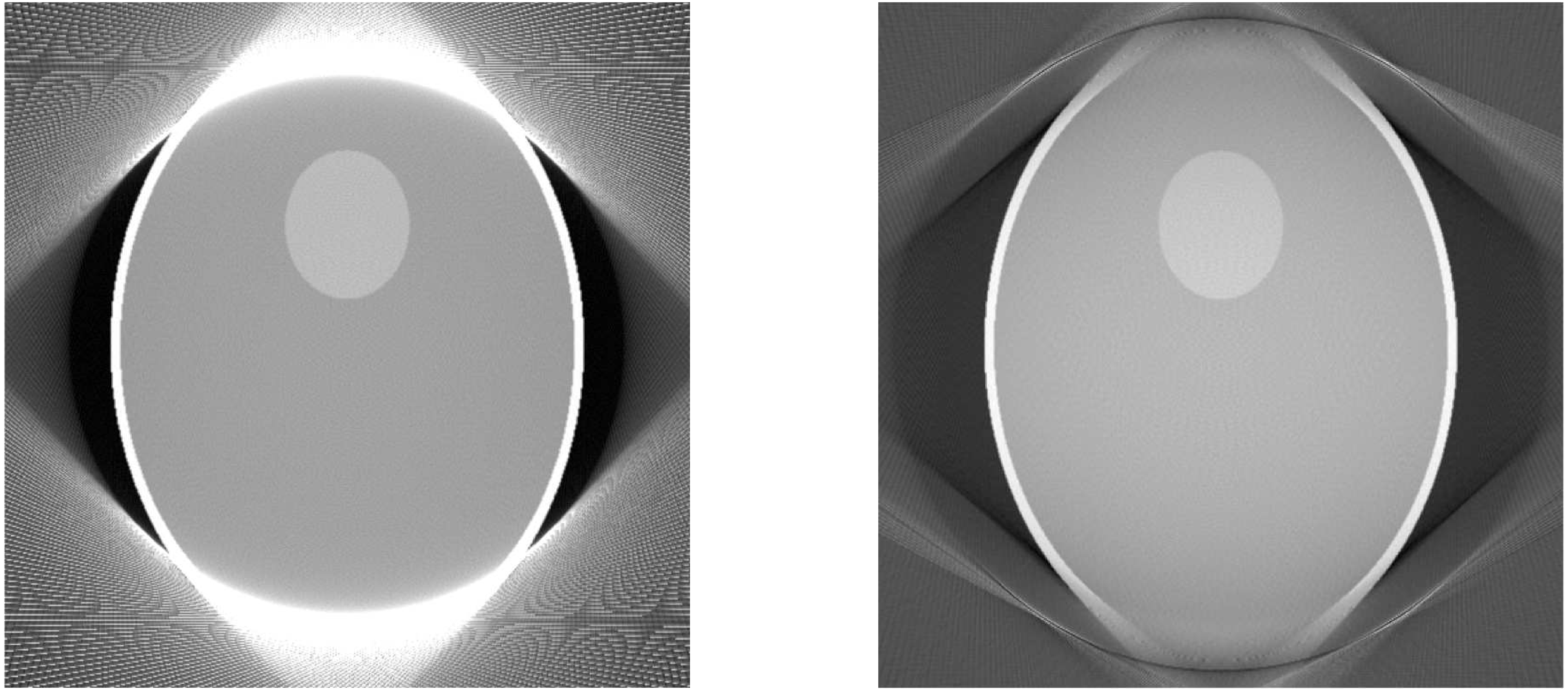


Figure 2: Heuristic extrapolation applied on a phantom

B. Ohnesorge et al. “Efficient Correction for CT Image Artifacts Caused by Objects Extending Outside the Scan Field of View”. In: *Medical Physics* 27.1 (Oct. 2000), pp. 39–46. DOI: 10.1118/1.598855

Water Cylinder Assumption

- Assume that the imaged object consists of water ($\rho = \rho_{H_2O}$).
- Fit water cylinder model to observed data

$$g(\gamma) = 2\rho_{H_2O}\sqrt{R^2 - D^2\sin^2\gamma}.$$

- Use model to extrapolate.

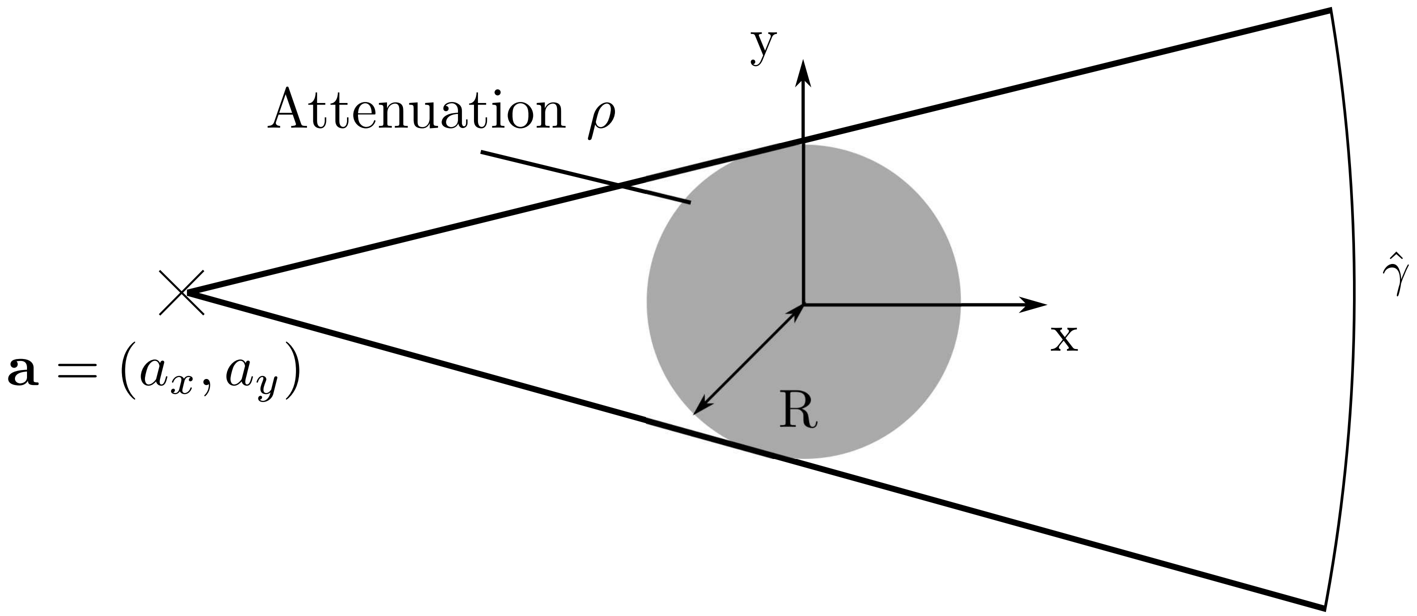


Figure 3: Assume the object to have a shape very similar to a water cylinder.

Water Cylinder Assumption

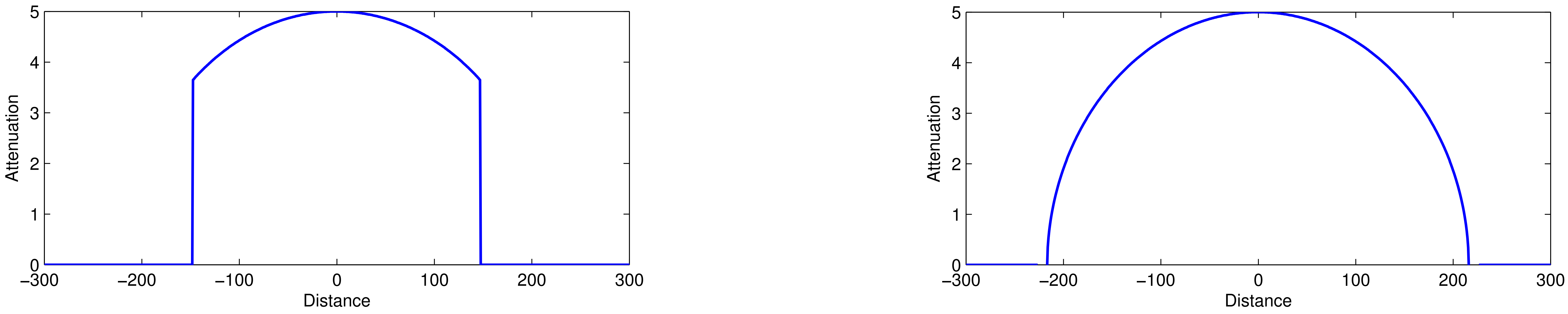


Figure 4: Extrapolate by assuming cylindric shape.

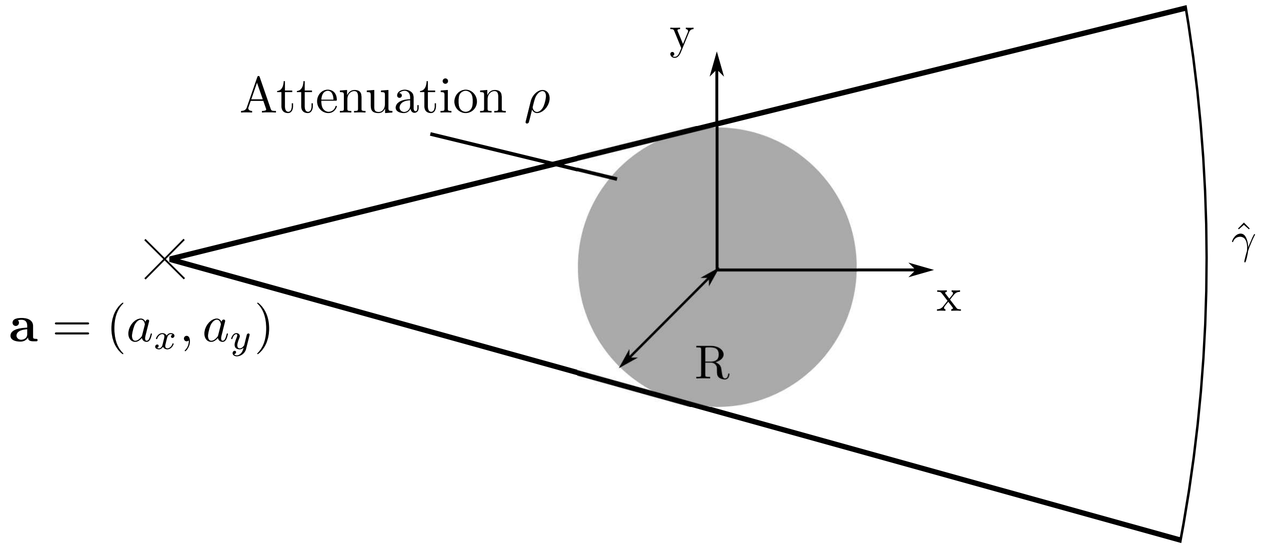
Water Cylinder Assumption

This approach ...

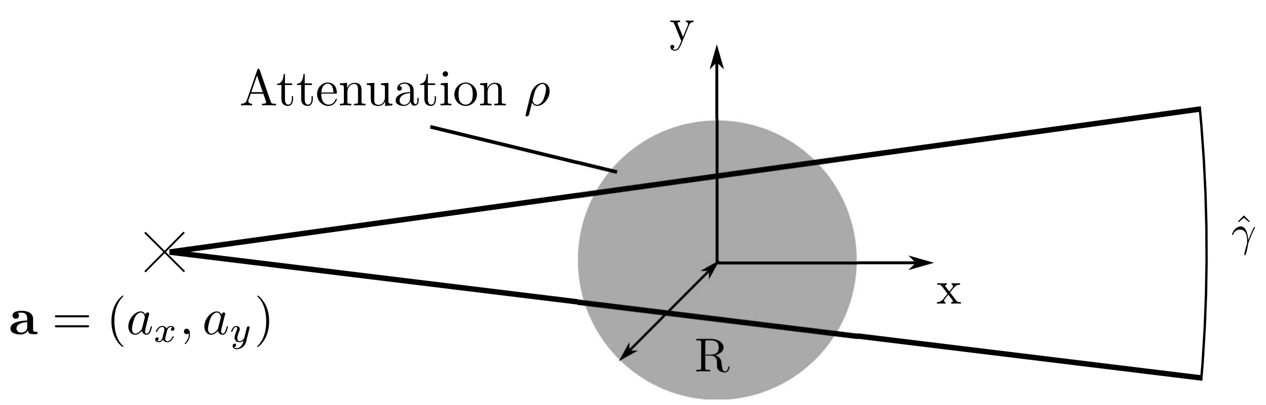
- ... will work perfectly if a water cylinder is imaged.
- ... yields good results for most objects (head, abdomen, etc...).
- ... will yield suboptimal results if the water cylinder assumption is violated (e. g., two cylinders).
- Different versions exist:
  - water ellipsoid assumption,
  - combination with cosine-like roll-off.

Use of Prior Knowledge

Prior scan (low dose)



Volume-of-interest scan (higher dose)



Use of Prior Knowledge

- Use data from a first scan to complete the data from a second scan.
- Correction will be perfect if the object did not change.
- One might also use a lower resolution prior scan.
- Movement and deformation of the object have to be compensated.
- This approach is only applicable if a prior scan exists.

Semi-transparent Filter

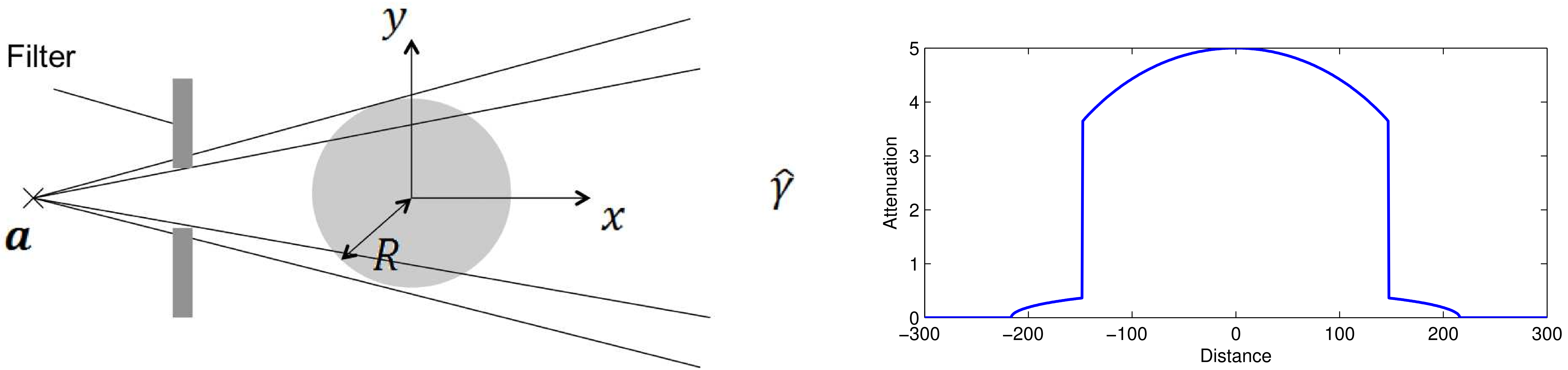


Figure 5: Scheme of a semi-transparent filter: setup (left) and example of a projection result (right)

Semi-transparent Filter

- Locate filter boundary.
- Amplify filtered signal to original amplitude.
- Reduce noise in the amplified signal.

$\rightarrow$ This yields perfect truncation correction.

Semi-transparent Filter

Challenges:

- Filter boundary must be located correctly (which may be influenced by the object).
- Correct amplification factor has to be estimated.
- Method has to be applied carefully in order not to introduce artificial high frequencies.
- Requires additional hardware in the scanner.

ATRACT

Idea:

$$|\omega| = 2\pi i\omega \cdot \left(-\frac{1}{2\pi} i\,\mathrm{sgn}(\omega)\right) = (2\pi i\omega)^2 \cdot \left(-\frac{1}{4\pi^2} i \frac{\mathrm{sgn}(\omega)}{\omega}\right)$$

The first term is the 2nd order derivative (local), and the right is called residual filter (global).
At the truncation boundaries the 2nd derivative produces a sparse signal. The resulting peaks are filled with zero and then the global filter is applied.

Remark: Without further considerations this does not preserve the mean value.

Topics

Truncation Correction Algorithms

- Defect Pixel Extrapolation
- Heuristic Extrapolation
- Water Cylinder Assumption
- Use of Prior Knowledge
- Use of a Semi-transparent Filter
- ATRACT Filtering

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- Truncation artifacts can be dealed with by extrapolating the projection data at the truncation boundaries.
- We have learned about six methods, which basically divide into
  - estimation of the truncated part,
  - incorporating prior knowledge from earlier scans, or
  - a special hardware setup.
  - filtering during reconstruction.

Further Readings

Helpful reads for the current unit:

B. Ohnesorge et al. "Efficient Correction for CT Image Artifacts Caused by Objects Extending Outside the Scan Field of View". In: *Medical Physics* 27.1 (Oct. 2000), pp. 39–46. DOI: 10.1118/1.598855

Frank Dennerlein and Andreas Maier. "Approximate Truncation Robust Computed Tomography–ATRACT". In: *Physics in Medicine and Biology* 58.17 (Aug. 2013), pp. 6133–6148. DOI: 10.1088/0031-9155/58/17/6133

Yan Xia et al. "Scaling Calibration in Region of Interest Reconstruction with the 1D and 2D ATRACT Algorithm". In: *International Journal for Computer Assisted Radiology and Surgery* 9.3 (May 2014), pp. 345–356. DOI: 10.1007/s11548-014-0978-z

L. A. Shepp and Logan B. F. "The Fourier Reconstruction of a Head Section". In: *IEEE Transactions on Nuclear Science* 21.3 (June 1974), pp. 21–43. DOI: 10.1109/TNS.1974.6499235

W. P. Segars et al. "Realistic CT Simulation Using the 4D XCAT Phantom". In: *Medical Physics* 35.8 (Aug. 2008), pp. 3800–3808. DOI: 10.1118/1.2955743

Medical Image Processing for Diagnostic Applications

Phantoms

Online Course – Unit 43

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Topics

Phantoms

- Introduction
- Numeric Phantoms
- Physical Phantoms

Summary

- Take Home Messages
- Further Readings

Phantoms: Purpose

- The performance of reconstruction algorithms has to be evaluated.
- X-rays are ionizing $\rightarrow$ we cannot use patients.
- We do not know the exact “geometry” of patients.
- We need an object that is precisely known.

Phantom Types

We distinguish two kinds of phantoms:

Numeric/simulated phantoms ...

- ... originate from computer simulations.
- ... are known exactly.
- ... have only limited realism.

Real phantoms ...

- ... are designed with desired properties.
- ... are manufactured at a high accuracy.
- ... may be difficult to use.
- ... may still have a limited manufacturing accuracy.

Examples for Numeric Phantoms

Commonly used numeric phantoms:

- Shepp-Logan phantom
- FORBILD phantoms
- XCAT phantoms
- Many more custom made phantoms

Shepp-Logan Phantom

- Described by a series of additive ellipsoids
- Only available in 2-D (extensions in 3-D exists, but are not standardized)

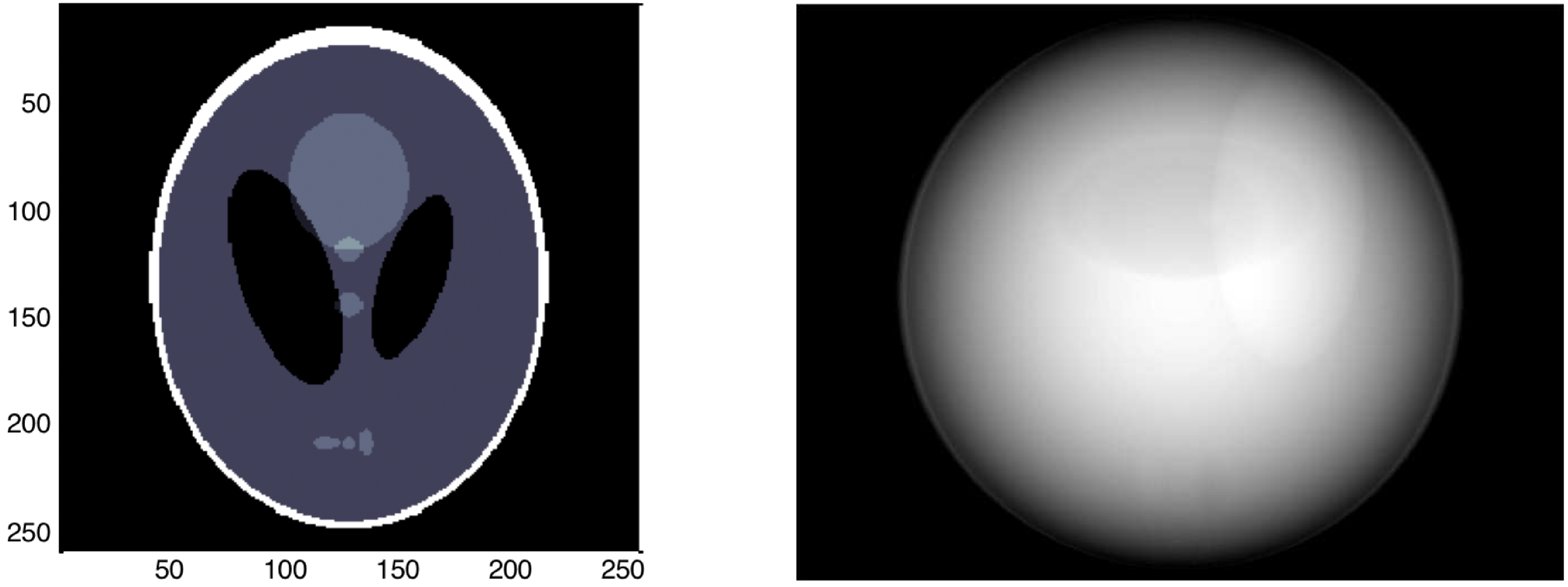


Figure 1: Shepp-Logan phantom in 2-D (left) and a 3-D version (right)

FORBILD Phantoms

- A series of 3-D phantoms that mimic anatomic details
- Descriptions based on simple geometric descriptors (cones, cubes, spheres, ...), and their intersections

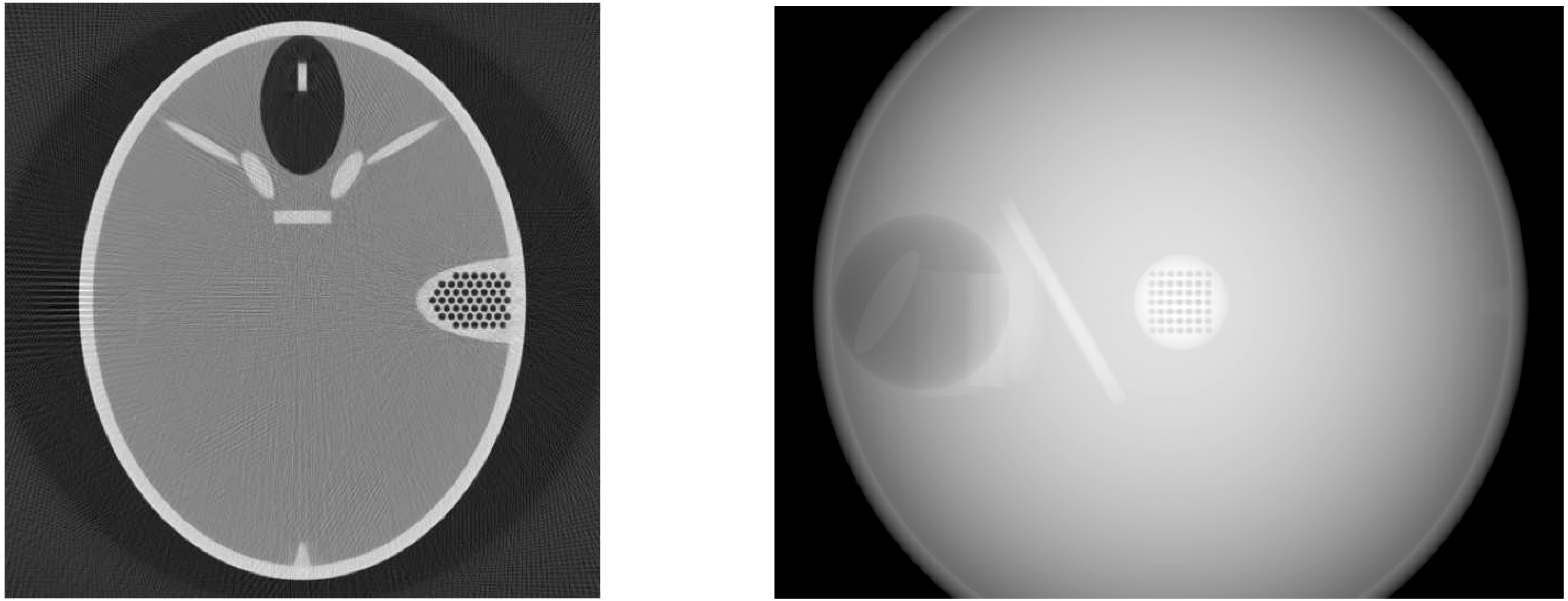


Figure 2: Examples for FORBILD phantoms

XCAT: Torso

- Based on the Visible Human Project®
- Analytic description using splines
- Comes with motion models for heart and torso

Figure 3: Beating heart in a breathing torso, link: https://www.youtube.com/v/Pbj0IFKh484

XCAT: Heart

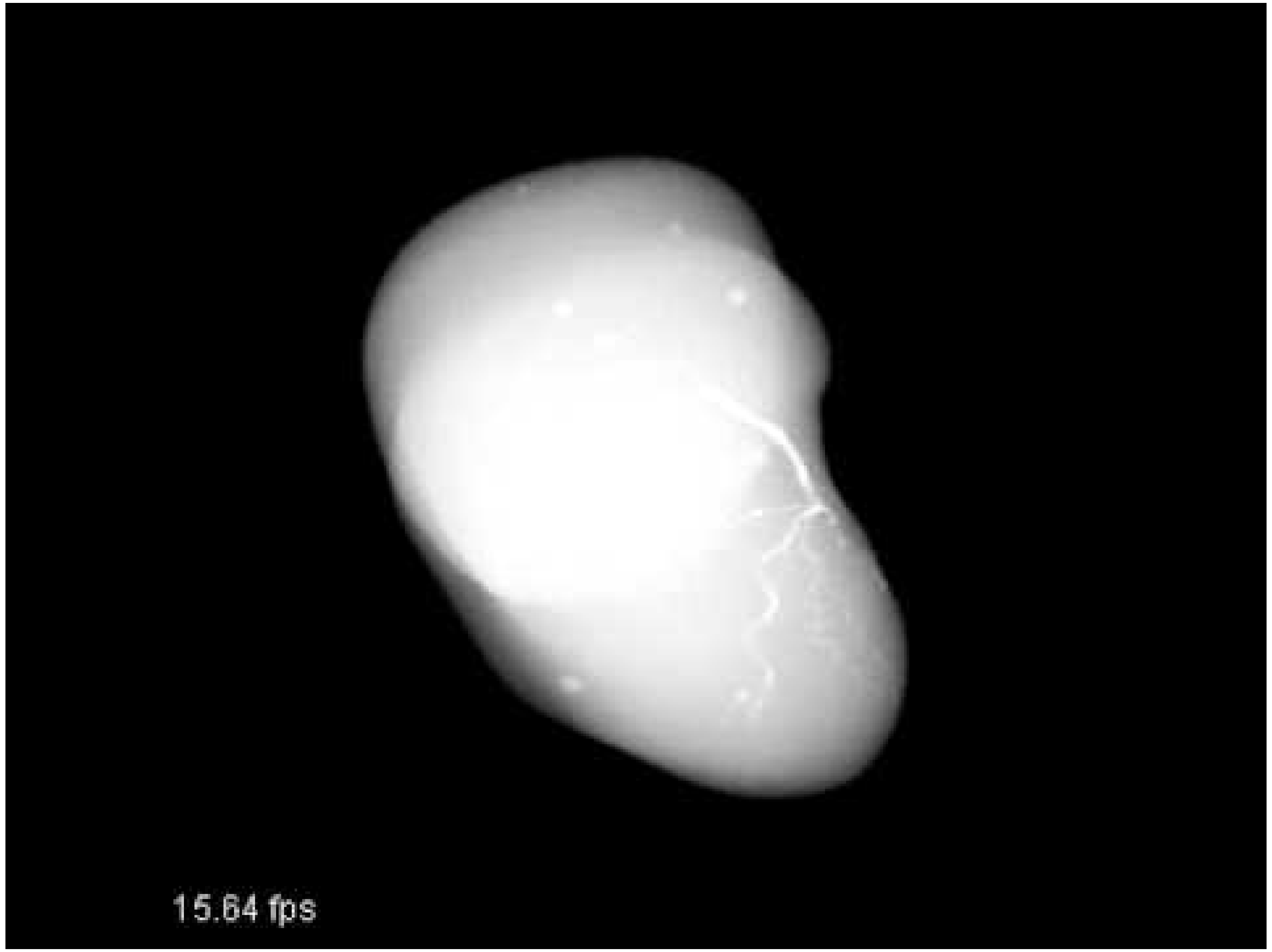


Figure 4: Beating heart with high contrast structures, link: https://www.youtube.com/v/aQCHZCTbXBU

XCAT: Legs

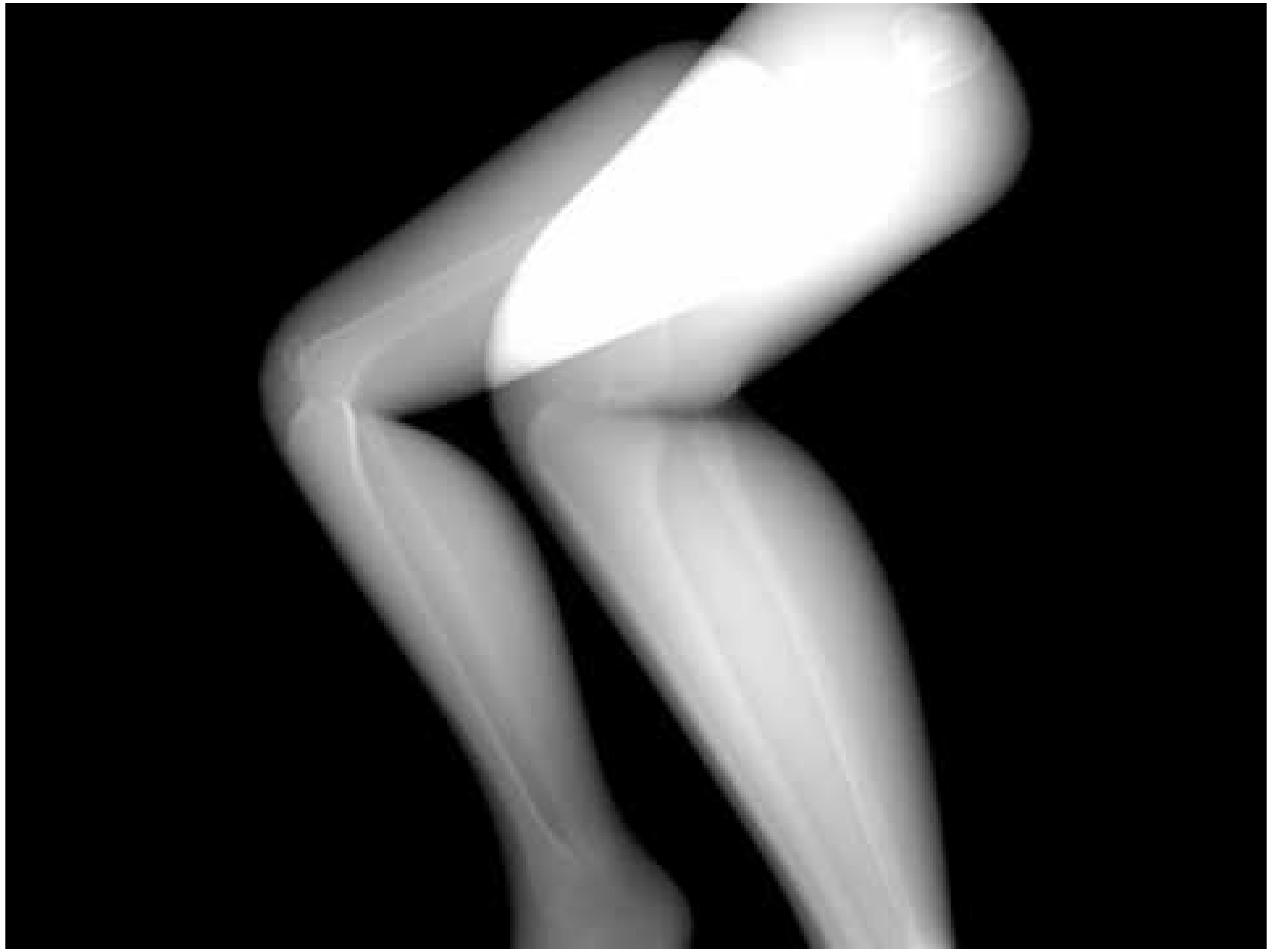


Figure 5: Squatting legs, link: https://www.youtube.com/v/Dp-s6TeEUwo

Examples for Physical Phantoms

Commonly used real phantoms:

- Catphan®
- ART phantom (earlier version: Alderson RANDO phantom)
- Calibration phantoms
- Many more custom made phantoms

Catphan®

- Phantom that mimics a water cylinder
- Contains exchangeable modules
- Manufactured at high accuracy

Figure 6: Catphan 600 (Image source: `https://www.phantomlab.com/`)

Alderson Radiation Therapy (ART) Phantom

- Phantom that mimics a human body
- Can be separated into slices
- Is also used to measure effective dose

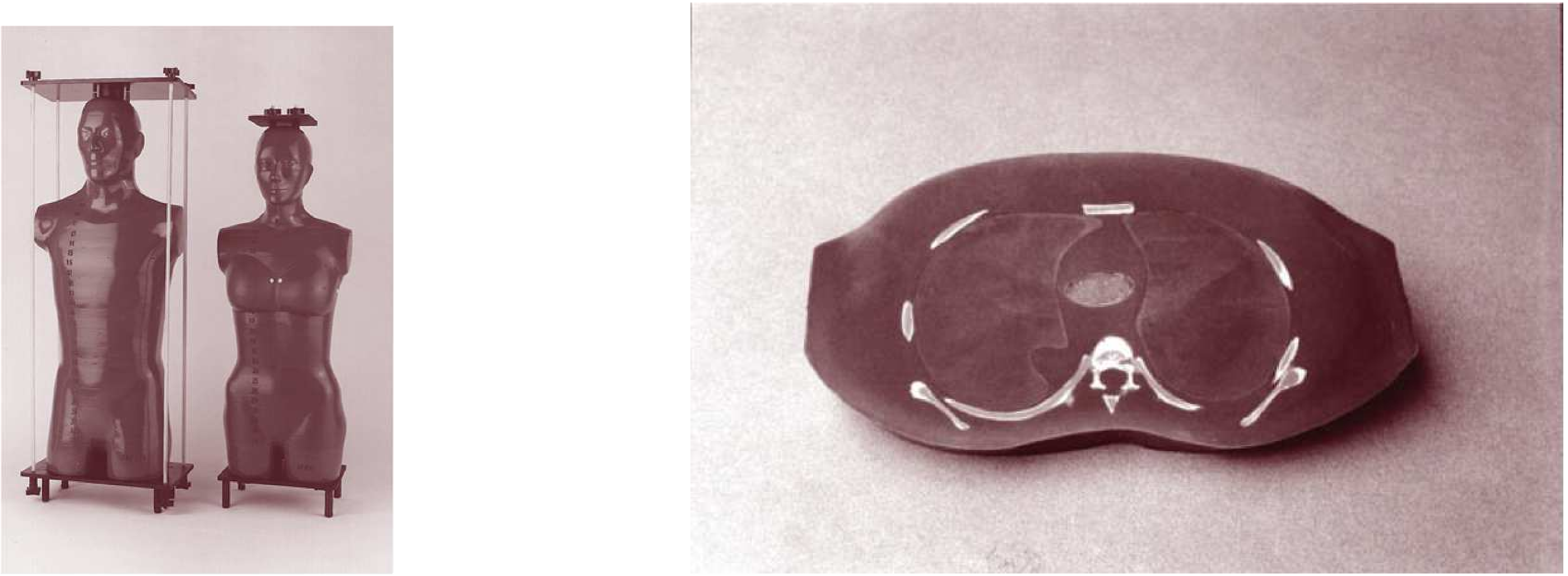
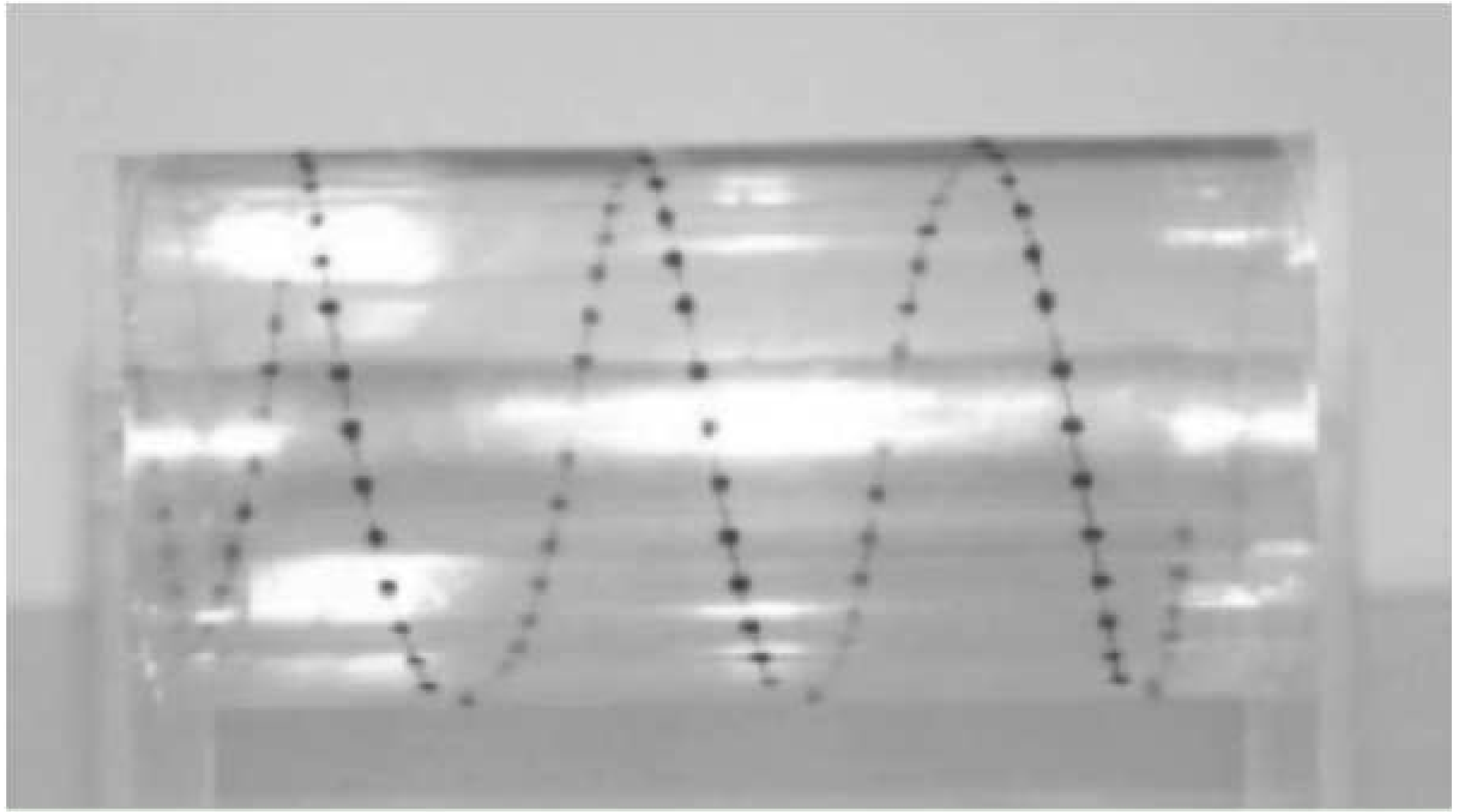


Figure 7: Male and female ART phantom (left) and a phantom slice (right) (Image source: `http://rsdphantoms.com/rt_art.htm`)

Calibration Phantoms

- Phantom that encodes information that can be used for calibration
- Enables to perform detailed accuracy analyses



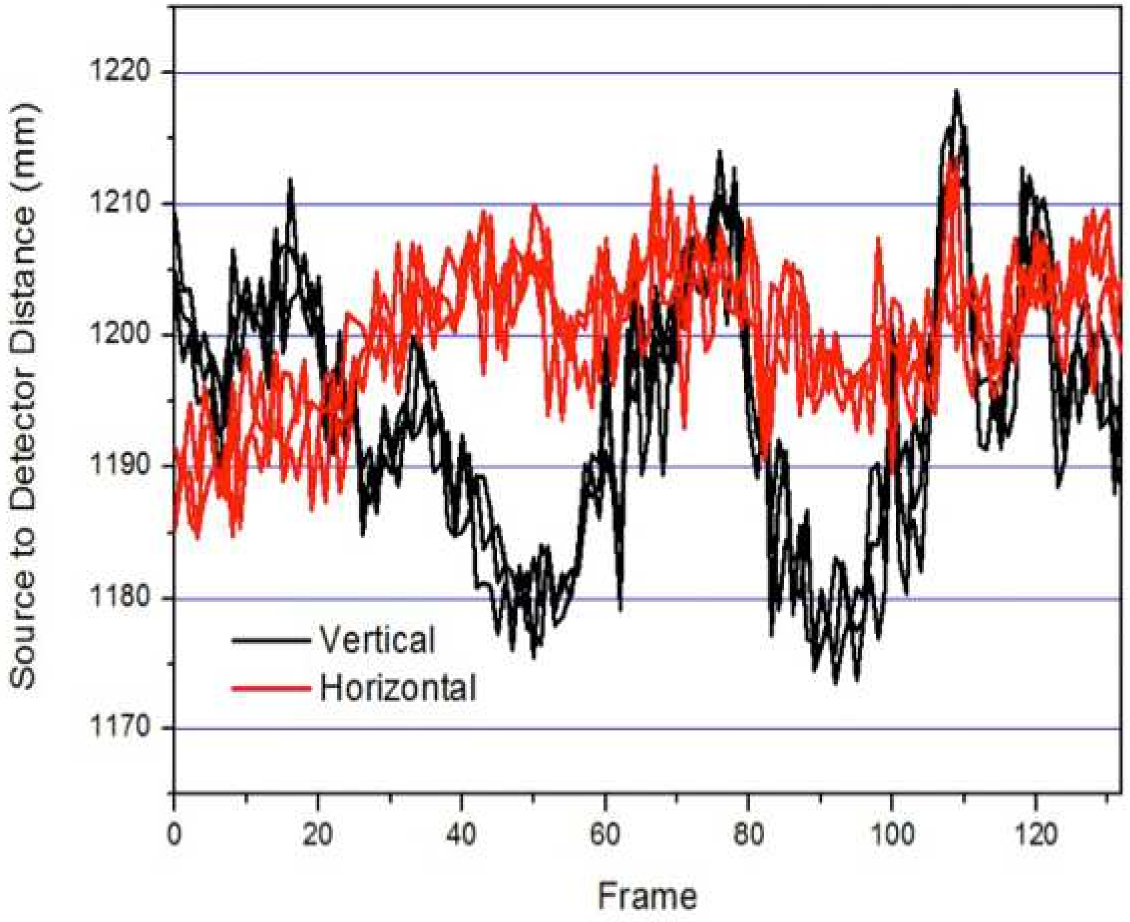


Figure 8: Phantom showing calibration pattern (left) and an example of an evaluation (right)

Topics

Take Home Messages

- Phantoms are one of the fundamental materials used in medical imaging (research).
- Both computational and physical phantoms are developed and used.
- The applications are manifold: algorithm testing, system calibration, dose measurements, image quality assessment, ...

Further Readings

Helpful reads for the current unit:

B. Ohnesorge et al. “Efficient Correction for CT Image Artifacts Caused by Objects Extending Outside the Scan Field of View”. In: *Medical Physics* 27.1 (Oct. 2000), pp. 39–46. DOI: 10.1118/1.598855

L. A. Shepp and Logan B. F. “The Fourier Reconstruction of a Head Section”. In: *IEEE Transactions on Nuclear Science* 21.3 (June 1974), pp. 21–43. DOI: 10.1109/TNS.1974.6499235

W. P. Segars et al. “Realistic CT Simulation Using the 4D XCAT Phantom”. In: *Medical Physics* 35.8 (Aug. 2008), pp. 3800–3808. DOI: 10.1118/1.2955743

A recent book on phantoms in medical imaging:

Larry A. DeWerd and Michael Kissick, eds. *The Phantoms of Medical and Health Physics: Devices for Research and Development*. Biological and Medical Physics, Biomedical Engineering. Springer New York, 2014. DOI: 10.1007/978-1-4614-8304-5