Medical Image Processing for Diagnostic Applications

Filtering in Frequency Domain

Online Course – Unit 22

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Topics

Frequency Domain Filters

Homomorphic Filtering

Summary

- Take Home Messages
- Further Readings

Frequency Domain Filters

Design a high-pass filter that eliminates the low frequency bias field.

Frequency Domain Filters

Let us consider the idea of high-pass filtering first by designing a filter in frequency domain:

- First, the observed input image $g = [g_{i,j}]$ is Fourier transformed:

$$G = \mathrm{FT}\left([g_{i,j}]\right).$$

- Second, a high-pass filter is defined in the discrete frequency domain by:

$$H_{k,l} = 1 - \beta e^{-\frac{k^2+l^2}{2\sigma^2}},$$

  where
  - β is a scaling factor that ensures that $H_{k,l} \geq 0$ for all $k, l = 0, 1, \ldots, M-1$,
  - and σ^2 is closely related to the bandwidth of the filter-kernel.

Frequency Domain Filters

The relation between low- and high-pass filters is:

$$
\begin{aligned}
f &= g * h_{\mathrm{HP}} \\
&= \mathrm{FT}^{-1}\left(\mathrm{FT}(g * h_{\mathrm{HP}})\right) \\
&= \mathrm{FT}^{-1}\left(G \cdot H_{\mathrm{HP}}\right) \\
&= \mathrm{FT}^{-1}\left(G \cdot (1 - H_{\mathrm{LP}})\right) \\
&= g * \left(1 - h_{\mathrm{LP}}\right) \\
&= g - g * h_{\mathrm{LP}}.
\end{aligned}
$$

Frequency Domain Filters

Using the convolution theorem, high-pass filtering is simply a multiplication in the frequency domain:

$$F_{k,l} = G_{k,l} \cdot H_{k,l},$$

for all $k, l = 0, 1, \dots, M-1$.

The final output image f is obtained by computing the inverse Fourier transform:

$$f = \mathrm{FT}^{-1}\left([F_{k,l}]\right).$$

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Homomorphic Filtering

These filtering approaches assume that IIH is

- an artifact with low frequencies, and
- anatomic structures contribute to the high frequencies of the image.

Elimination of image inhomogeneities can be done by low-pass filtering.

Homomorphic Filtering

Subtract the low-pass filtered image and normalize the mean.

Homomorphic Filtering

Homomorphic filtering is applied to log-transformed images:

- Make a low-pass filtering of the log-transformed image
$$[h_{i,j}] = \mathrm{LPF}\left([\log g_{i,j}]\right),$$
where LPF denotes a low-pass filter (like averaging or a Gaussian filter).
- The IIH corrected, log-transformed image $\log f$ results from the difference:
$$[\log f_{i,j}] = [\log g_{i,j}] - [h_{i,j}] + \mu,$$
where μ ensures that the correction is mean preserving.

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Take Home Messages

- The straightforward approach for IIH correction is low-pass filtering using the Fourier transform of the image.
- When using homomorphic filtering, a similar idea is applied on the log-transformed images including a mean preservation technique.

Further Readings

The webpage of the National High Magnetic Field Laboratory can be one starting point for more detailed information regarding MRI. For an initial overview of the technology, the following article is worth reading:

MRI: A Guided Tour by Kristen Coyne.

If you want to know more about segmentation of MR images, e. g., consult

the Google Scholar record of ‘Sandy’ Wells’ publications.

Another article worth reading is this survey paper on algorithms for intensity correction methods:

Zujun Hou. “A Review on MR Image Intensity Inhomogeneity Correction”. In: *International Journal of Biomedical Imaging* 2006.Article ID 49515 (Feb. 2006), pp. 1–11. DOI: 10.1155/IJBI/2006/49515