Medical Image Processing for Diagnostic Applications

Image Registration in Practice – Part 2

Online Course – Unit 68

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Topics

Multimodal Registration

Historical remarks on the application of mutual information in computer vision and image processing:

- Maximization of mutual information is applied to solve parameter estimation problems, e. g., speech recognition (∼1980).
- Maximization of mutual information was first applied to image registration by P. Viola, W. Wells and by Collignon (1995).
- Maximization of mutual information was first applied to active object recognition by B. Schiele and J. Crowley (1997).

Kullback-Leibler Divergence

A proper similarity measure for density functions is the *Kullback-Leibler Divergence* (KL divergence). The KL divergence between two bivariate probability density functions $p(f,g)$ and $q(f,g)$ is defined as:

$$\mathrm{KL}(p,q) = \int\int p(f,g) \log \frac{p(f,g)}{q(f,g)} \mathrm{d}f \, \mathrm{d}g.$$

Idea for Image Registration: Images that are correctly registered imply the highest probabilistic dependency of aligned intensity values. The more the random variables depend on each other, the more the probability density functions $p(f,g)$ and $p(f) \cdot p(g)$ differ. This difference can be measured by the KL divergence:

$$\mathrm{KL}(p,q) = \int\int p(f,g) \log \frac{p(f,g)}{p(f)p(g)} \mathrm{d}f \, \mathrm{d}g.$$

Optimization Problem for Image Registration

The transform T that maps the gray level of the model image to the image point of the reference image can be estimated as follows:

$$\widehat{T} = \arg\max_{T} \sum_{i} p(f_i, g_{T(i)}) \log \frac{p(f_i, g_{T(i)})}{p(f_i)\ p(g_{T(i)})}.$$

In terms of information theory this is the maximization of **mutual information**:
We send an image $[f_i]_{i=1,\ldots,N}$ and receive the image $[g_i]_{i=1,\ldots,N}$, where the channel applies a transform to image coordinates and the intensity values change.

Registration of CT and SPECT Images

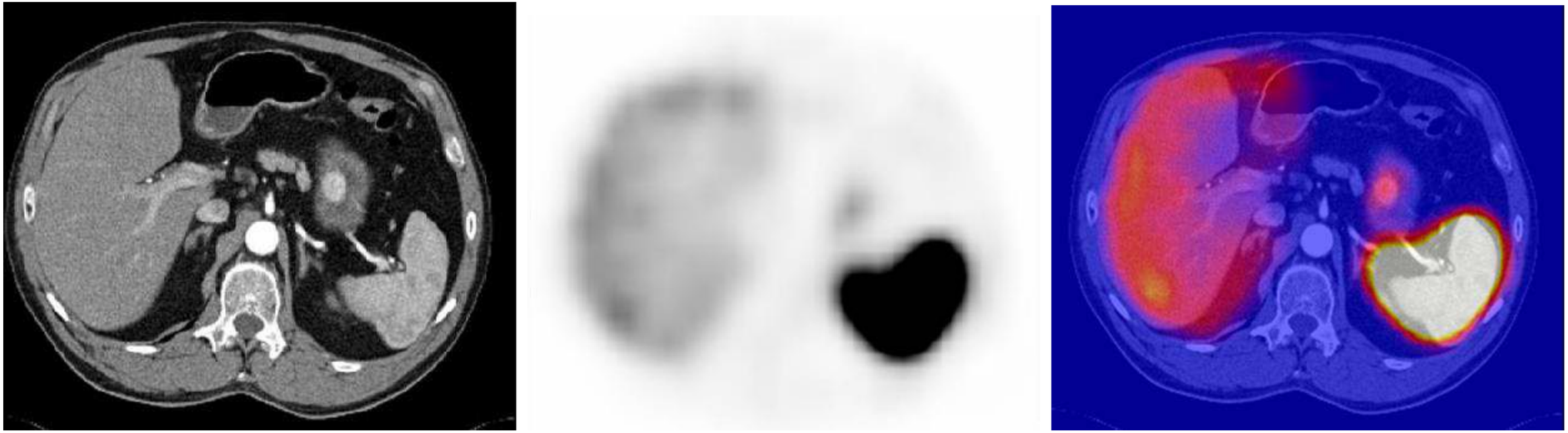

Figure 1: CT image (left), SPECT image (middle), result of image registration (right) (images courtesy of Dr. W. Römer, Nuclear Medicine, FAU)

Channel Model

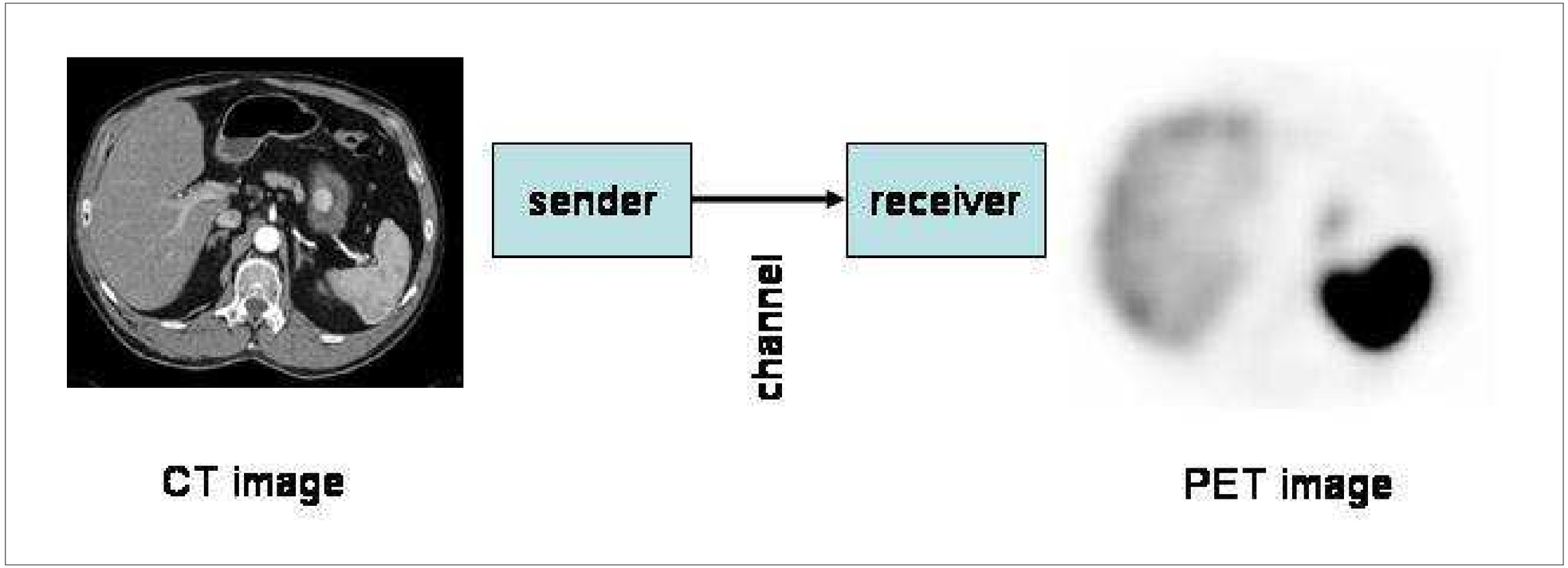

Figure 2: Channel model that defines the transition of CT to PET images (according to Wells et al., 1995).

Definition of Mutual Information

- Entropy is defined as:

$$H(F) = -\sum_{f} p(f) \log p(f).$$

- Entropy in the bivariate case is:

$$H(F,G) = -\sum_{f,g} p(f,g) \log p(f,g).$$

- Mutual information is defined as:

$$I(F,G) = H(F) + H(G) - H(F,G) = I(G,F).$$

Stochastic Maximization of Mutual Information

draw N_A samples for i
draw N_B samples for i
Set $\widehat{T} = \widehat{T} + \lambda \frac{\mathrm{d}I}{\mathrm{d}T}$
UNTIL convergence

Figure 3: Stochastic maximization

Topics

Take Home Messages

- Multimodal registration is often performed using measures of probability theory.
- KL divergence and mutual information are important similarity measures for registration.

Further Readings – Part 1

Survey papers on medical image registration:

- Derek L. G. Hill et al. “Medical Image Registration”. In: *Physics in Medicine and Biology* 46.3 (2001), R1–R45
- J. B.Antoine Maintz and Max A. Viergever. “A Survey of Medical Image Registration”. In: *Medical Image Analysis* 2.1 (1998), pp. 1–36. DOI: 10.1016/S1361-8415(01)80026-8
- L. G. Brown. “A Survey of Image Registration Techniques”. In: *ACM Computing Surveys* 24.4 (Dec. 1992), pp. 325–376. DOI: 10.1145/146370.146374
- Josien P. W. Pluim, J. B. Antoine Maintz, and Max A. Viergever. “Mutual-Information-Based Registration of Medical Images: A Survey”. In: *IEEE Transactions on Medical Imaging* 22.8 (Aug. 2003), pp. 986–1004. DOI: 10.1109/TMI.2003.815867

A paper that inspired all the sections on complex numbers, quaternions, and dual quaternions:

Konstantinos Daniilidis. “Hand-Eye Calibration Using Dual Quaternions”. In: *The International Journal of Robotics Research* 18.3 (Mar. 1999), pp. 286–298. DOI: 10.1177/02783649922066213

Further Readings – Part 2

Non-parametric mappings for image registration:

- Nonlinear registration methods applied to DSA can be found in Erik Meijering's papers.
- Jan Modersitzki. *Numerical Methods for Image Registration*. Numerical Mathematics and Scientific Computations. Oxford Scholarship Online, 2007. Oxford: Oxford University Press, 2003. DOI: 10.1093/acprof:oso/9780198528418.001.0001
- Many of Jan Modersitzki's and Bernd Fischer's papers on image registration can be found in the publication list of the Institute of Mathematics and Image Computing (Lübeck).
- The group of Martin Rumpf also published on non-parametric image registration. Details on their work can be found on the institute's webpage.