Medical Image Processing for Diagnostic Applications

Cone Beam Reconstruction – FDK

Online Course – Unit 47

Andreas Maier, Joachim Hornegger, Markus Kowarschik, Frank Schebesch
Pattern Recognition Lab (CS 5)

Topics

Feldkamp's Algorithm

Summary

Take Home Messages

Further Readings

Feldkamp's Algorithm ...

- ... is also known as the Feldkamp, Davis, Kress (FDK) algorithm named after the authors of the original publication.
- ... is designed for circular trajectories and thus is approximate.
- ... is a commonly used cone beam reconstruction algorithm because it is fast and robust.
- ... is based on a fan beam reconstruction algorithm with appropriate cosine weights.
- ... is exact for objects that do not vary in z-direction.

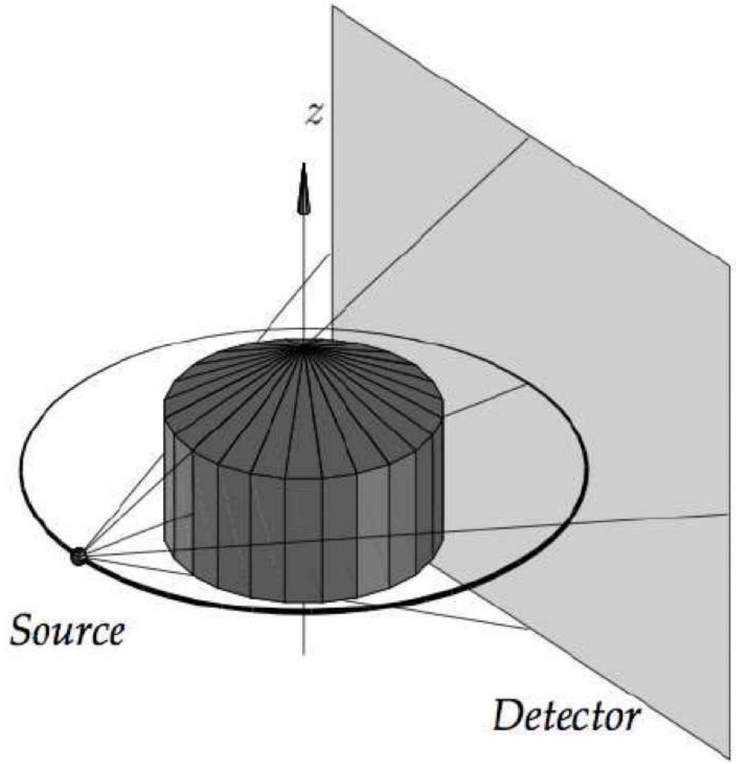

Figure 1: Cone beam scheme

Feldkamp's Algorithm: Geometry

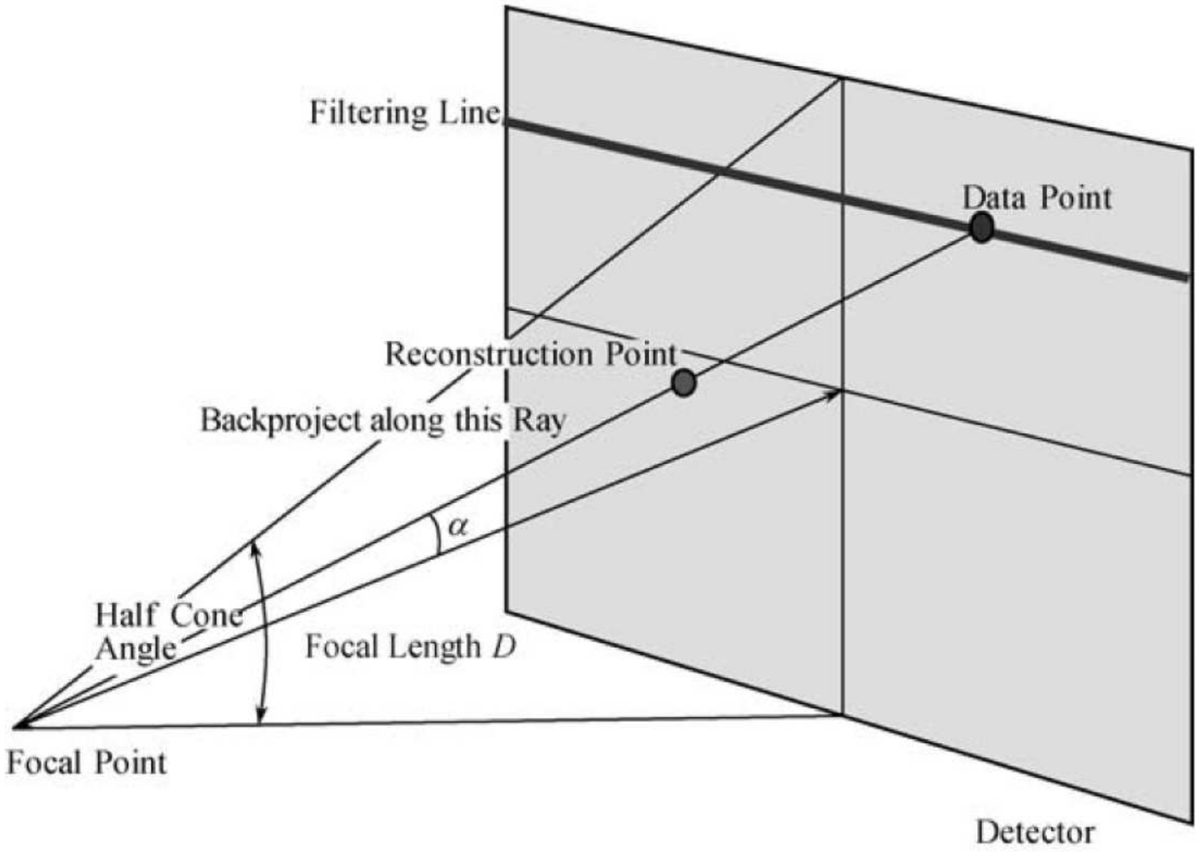

Figure 2: Geometry scheme for a point source cone beam acquisition on a flat panel detector (Zeng, 2009)

Fan Beam vs. Cone Beam

- FBP for fan beam data:

$$f(r,\varphi) = \frac{1}{2}\int_0^{2\pi} \frac{1}{U^2} \int_{-\infty}^{\infty} \frac{D}{\sqrt{D^2+t^2}} g(t,\beta) h(t'-t)\,\mathrm{d}t\,\mathrm{d}\beta,$$

where

$$U = \frac{D + r\sin(\beta-\varphi)}{D}, \quad t' = \frac{Dr\cos(\beta-\varphi)}{D + r\sin(\beta-\varphi)}.$$

- Feldkamp's algorithm for cone beam data:

$$f(r,\varphi,z) = \frac{1}{2}\int_0^{2\pi} \frac{1}{U^2} \int_{-\infty}^{\infty} \frac{D}{\sqrt{D^2+t^2+u^2}} g(t,u,\beta) h(t'-t)\,\mathrm{d}t\,\mathrm{d}\beta,$$

where u denotes the second detector coordinate for the projected point (r,φ,z) and is determined by:

$$u = z\frac{2D}{r}\frac{D + r\sin(\beta-\varphi)}{r\sin\left(2(\beta-\varphi)\right)}.$$

Feldkamp's Algorithm

1. Perform adjusted cosine weighting:

$$g_1(t,u,\beta) = g(t,u,\beta)\frac{D}{\sqrt{D^2+t^2+u^2}}.$$

2. Apply ramp filter for each detector row:

$$g_2(t,u,\beta) = g_1(t,u,\beta) * h(t).$$

3. Backproject with distance weight:

$$f(r,\varphi,z) = \frac{1}{2}\int_0^{2\pi} \frac{1}{U^2} g_2(t',u,\beta)\,\mathrm{d}\beta.$$

Feldkamp's Algorithm: Remarks

- Filtering is row-wise, hence its complexity is $O(N^2 \log N)$.
- Backprojection is in 3-D $\Rightarrow O(N^3)$.
- In C-arm CT, projection is performed using projection matrices.
- This processing speed can be improved if Horner's scheme is applied.
- Backprojection is often implemented on special hardware.

Topics

Feldkamp's Algorithm

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- The FDK algorithm is a 3-D cone beam reconstruction algorithm.
- It is based on the fan beam reconstruction algorithm and only row-wise filtering is applied.

Further Readings

The original work of Feldkamp, Davis, and Kress can be found here:

L. A. Feldkamp, L. C. Davis, and J. W. Kress. “Practical Cone-Beam Algorithm”. In: *Journal of the Optical Society of America A: Optics, Image Science, and Vision* 1.6 (June 1984), pp. 612–619

The best way to augment your knowledge of the shown concepts is to read the companion book of the current chapter:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9