Medical Image Processing for Diagnostic Applications

Image Registration in 2-D

Online Course – Unit 64

Andreas Maier, Joachim Hornegger, Markus Kowarschik, Frank Schebesch
Pattern Recognition Lab (CS 5)

virtuelle hochschule bayern

FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG
TECHNISCHE FAKULTÄT

Topics

Image Registration

- Motivation
- Point Based 2-D/2-D Rigid Registration
- Complex Numbers and Rotations

Summary

- Take Home Messages
- Further Readings

Terminology

Definition

Image registration is the process of transforming two or more different images into one common coordinate system. The registration of volumes is also subsumed by the term image registration.

Dependent on the properties of the transform we have two major classes for image registration:

- The term ***rigid registration*** subsumes the process of computing a rigid transform for registration.
- The term ***non-rigid registration*** includes all the methods of deforming the different images such that they can be represented in one common coordinate system.

Fiducial Markers for Image Registration

Especially in therapeutic radiology the precise mapping of all available image information is required for the therapy of tumors.

Rigid registration methods are mostly applied to images of the skull. In this particular application physicians quite often make use of *fiducial markers* that are fixed to the patient.

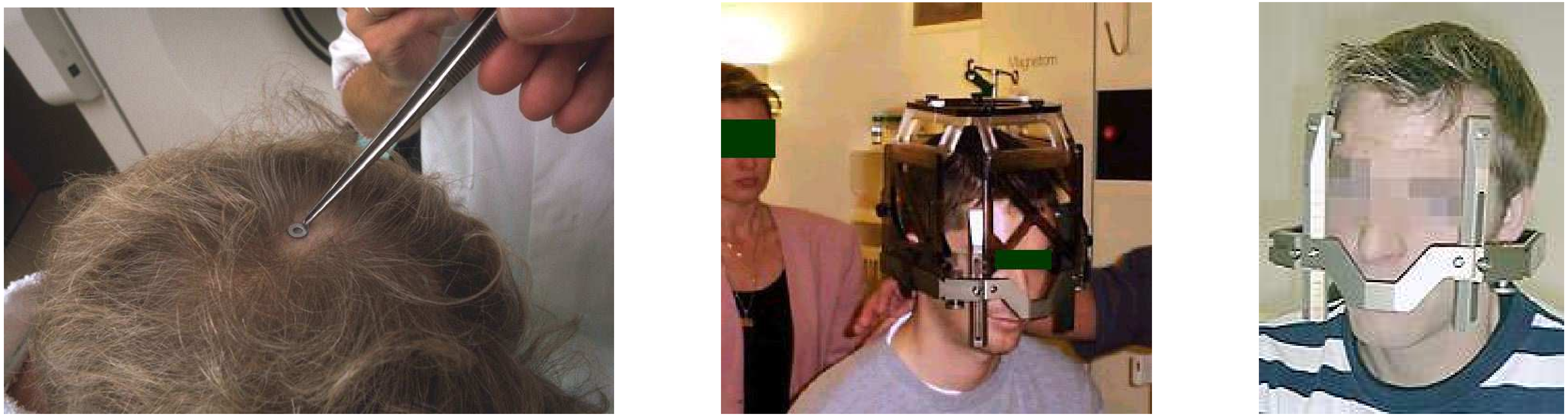

Figure 1: Fiducial markers used for Gamma Knife® treatment

Navigation System for Surgery

Further applications of markers in medical imaging:

Figure 2: Brainlab system for brain surgery

Point Based 2-D/2-D Registration

Assume a set of corresponding 2-D points in two images:

$$C = \{(\boldsymbol{p}_k, \boldsymbol{q}_k) \mid k = 1, 2, \ldots, N\},$$

where $\boldsymbol{p}_k, \boldsymbol{q}_k \in \mathbb{R}^2$ represent the k-th pair of corresponding image points.

Problem: Compute the transform that maps the $\boldsymbol{q}_k$'s to the $\boldsymbol{p}_k$'s.

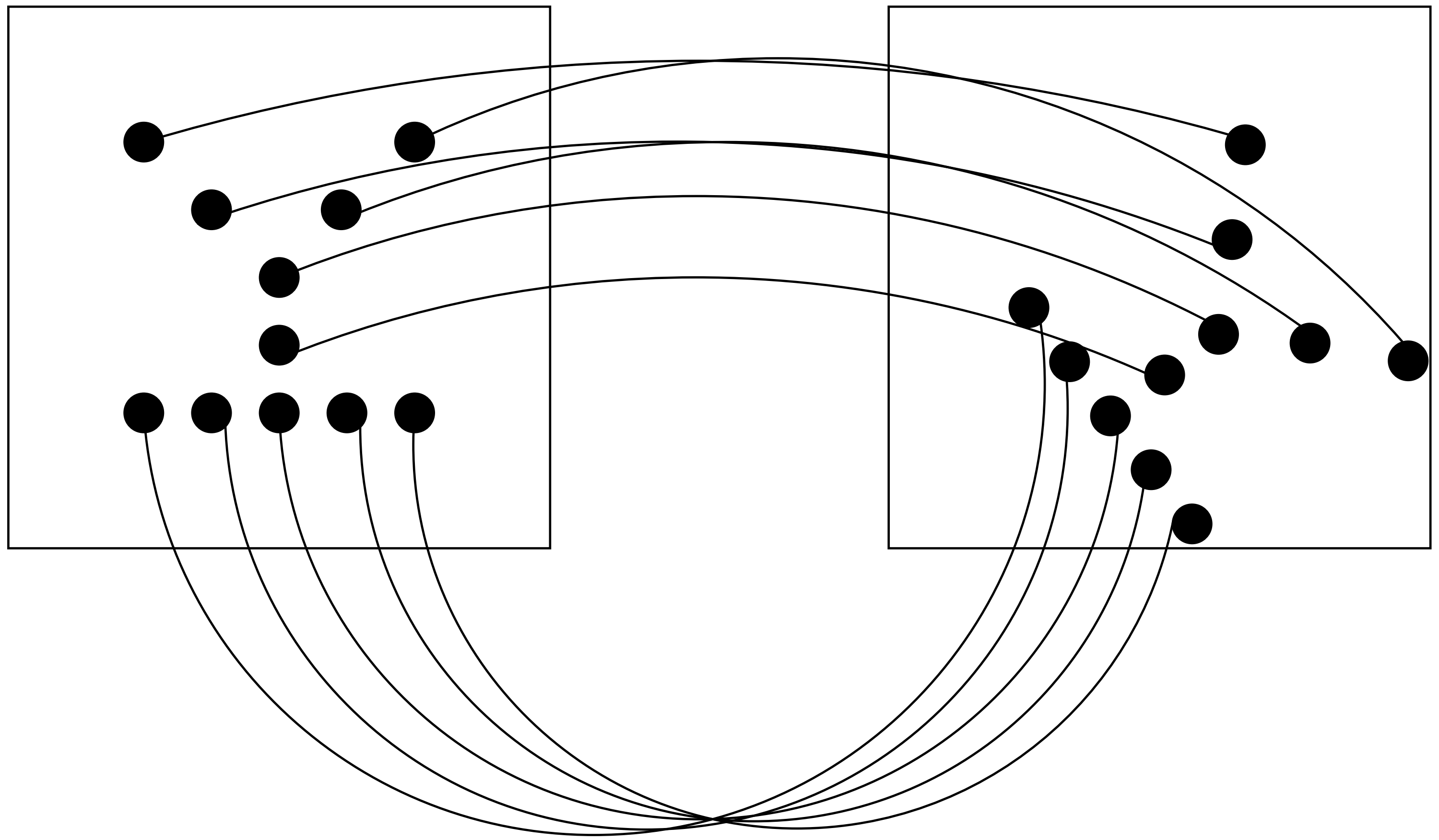

Figure 3: Corresponding 2-D point features

Optimization Problem

Using a rigid transform defined by rotation

$$\boldsymbol{R} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}, \quad \varphi \in [0, 2\pi),$$

and translation $\boldsymbol{t} = (t_1, t_2)^\mathsf{T}$, $t_1, t_2 \in \mathbb{R}$, we want:

$$\boldsymbol{p}_k = \boldsymbol{R}\boldsymbol{q}_k + \boldsymbol{t}.$$

This leads to the optimization problem for 2-D rigid registration:

$$\underset{\varphi, t_1, t_2}{\arg\min} \sum_{k=1}^{N} \|\boldsymbol{p}_k - \boldsymbol{R}\boldsymbol{q}_k - \boldsymbol{t}\|^2.$$

Conclusion: Rigid image registration turns out to be a nonlinear optimization problem.

Properties of Rotations

Some important properties of rotation matrices $\boldsymbol{R} \in \mathbb{R}^{n \times n}$:

- The columns of a rotation matrix are images of the base vectors of the original coordinate system (valid for all linear mappings!).
- Every rotation matrix is orthogonal:

  $$\boldsymbol{R}^\mathsf{T} = \boldsymbol{R}^{-1},$$

  and thus we have:

  $$\boldsymbol{R}^\mathsf{T}\boldsymbol{R} = \boldsymbol{R}\boldsymbol{R}^\mathsf{T} = \boldsymbol{I}_n.$$

- Rotation preserves the orientation (left/right-handedness) of the coordinate system.
- We have $\det \boldsymbol{R} = 1$.
- In 3-D, the eigenvector corresponding to the eigenvalue $\lambda_1 = 1$ defines the rotation axis.

Complex Numbers and Rotations

Complex numbers define a point in 2-D:

$$z = a + ib$$

where a is the real part and b the imaginary part of the complex number z.

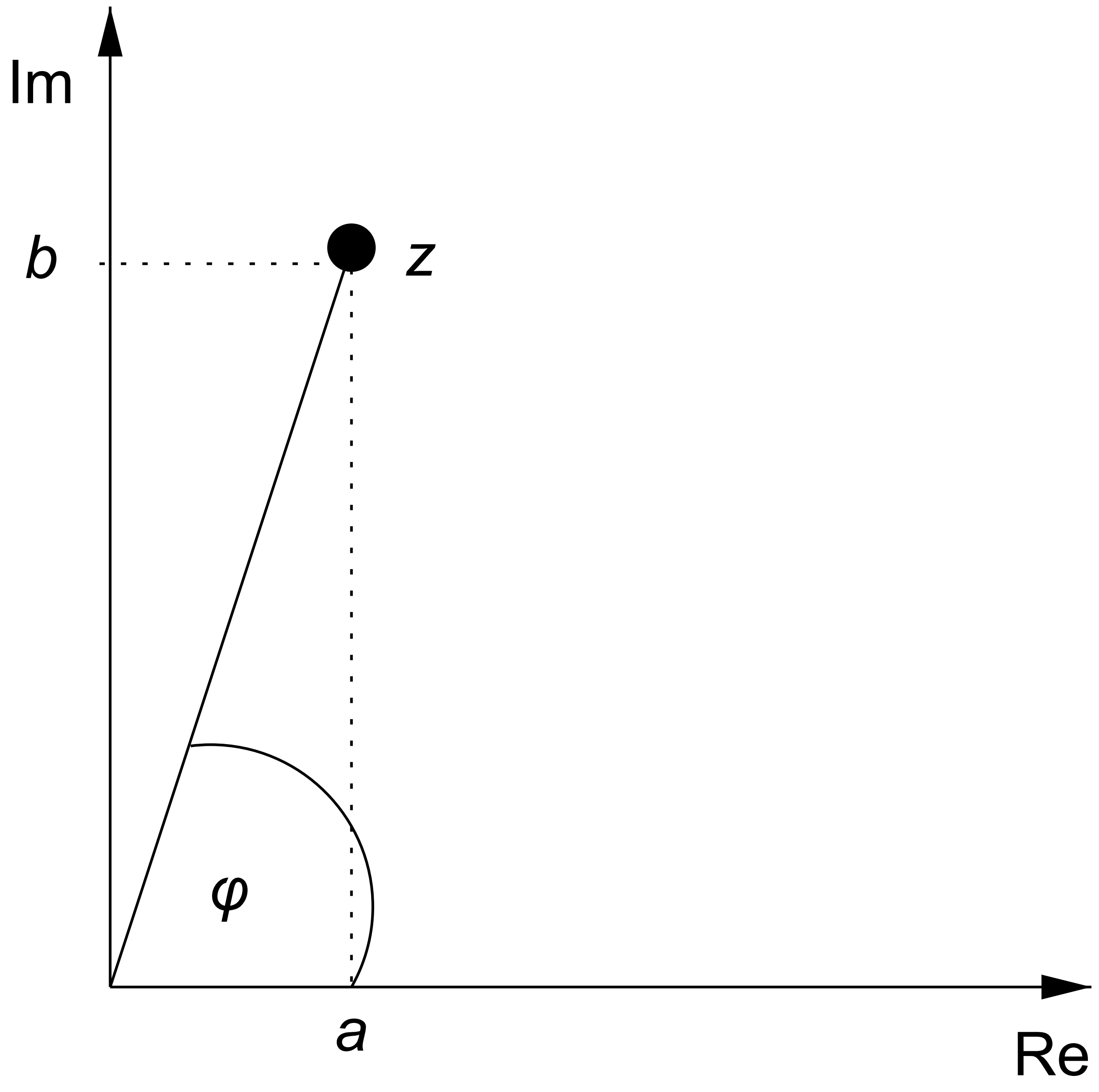

Figure 4: Geometric representation of complex numbers

Complex Numbers and Rotations

Proposition: Multiplication of complex numbers defines a 2-D scaling and rotation.

Multiplication of complex numbers is defined by:

$$z = z_1 \cdot z_2 = (a_1 + ib_1)(a_2 + ib_2) = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_2 b_1).$$

Complex numbers can be represented using Euler's notation:

$$z = |z| e^{i\varphi},$$

where

- $|z| = \sqrt{a^2 + b^2}$ is the length of the complex number, and
- $\varphi = \mathtt{atan2}(b, a)$ the angle w. r. t. the real axis.

Complex Numbers and Rotations

Multiplication of complex numbers using Euler notation:

$$z_1 \cdot z_2 = |z_1|e^{i\varphi_1} \cdot |z_2|e^{i\varphi_2} = |z_1||z_2|e^{i(\varphi_1+\varphi_2)}$$

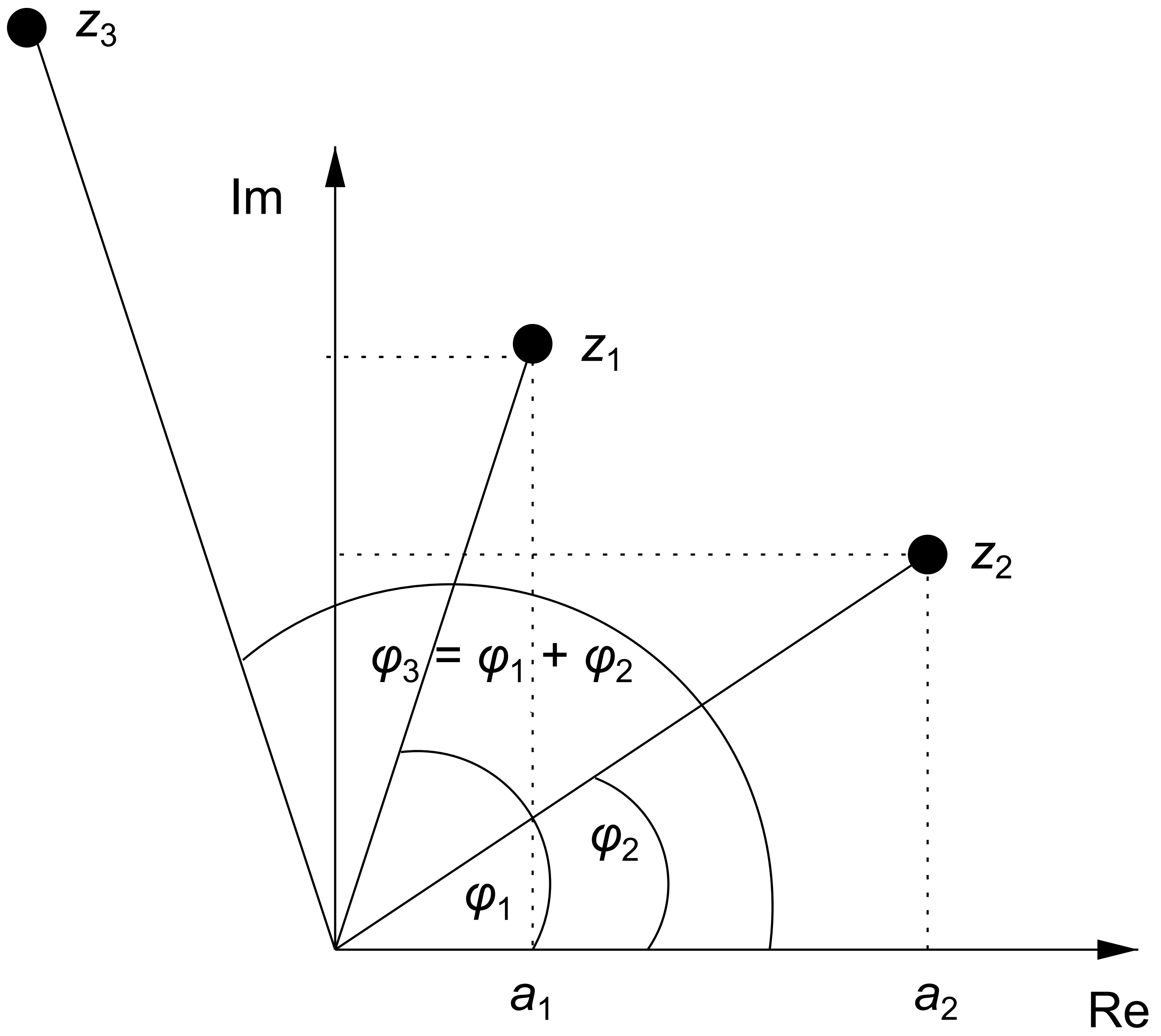

Figure 5: Geometric representation of complex numbers

Complex Numbers and Rotations

Conclusion: In terms of complex numbers we get the equations:

$$p_{k,1} + i p_{k,2} = (r_1 + i r_2)(q_{k,1} + i q_{k,2}) + t_1 + i t_2 \quad \text{for } k = 1, 2, \ldots, N.$$

This equation can be rewritten in two equations both linear in the components of the complex numbers corresponding to $\boldsymbol{R}$ and $\boldsymbol{t}$:

- for the real part we get the equation

$$p_{k,1} = r_1 q_{k,1} - r_2 q_{k,2} + t_1 = (q_{k,1}, -q_{k,2}, 1, 0) \begin{pmatrix} r_1 \\ r_2 \\ t_1 \\ t_2 \end{pmatrix},$$

- the imaginary parts results in the equation

$$p_{k,2} = r_1 q_{k,2} + r_2 q_{k,1} + t_2 = (q_{k,2}, q_{k,1}, 0, 1) \begin{pmatrix} r_1 \\ r_2 \\ t_1 \\ t_2 \end{pmatrix}.$$

Complex Numbers and Rotations

The final system of linear equations is:

$$\boldsymbol{A}\boldsymbol{x} = \begin{pmatrix} q_{1,1} & -q_{1,2} & 1 & 0 \\ q_{2,1} & -q_{2,2} & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ q_{N,1} & -q_{N,2} & 1 & 0 \\ q_{1,2} & q_{1,1} & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ q_{N,2} & q_{N,1} & 0 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ t_1 \\ t_2 \end{pmatrix} = \begin{pmatrix} p_{1,1} \\ p_{2,1} \\ \vdots \\ p_{N,1} \\ p_{1,2} \\ \vdots \\ p_{N,2} \end{pmatrix} = \boldsymbol{b}.$$

Complex Numbers and Rotations

Remarks:

- In this algorithm we compute rotation and translation simultaneously.
- Using SVD we compute the pseudoinverse of $\boldsymbol{A}$ and so get both rotation and translation.
- 2-D/2-D image registration using point correspondences results in a linear problem.
- Rotation matrices imply the constraint that $r_1^2 + r_2^2 = 1$. This can be enforced by a proper scaling of the solution of $\boldsymbol{Ax} = \boldsymbol{b}$.

Question: Can we *lift* the complex numbers to characterize 3-D rotations?

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Take Home Messages

- Point based image registration describes the attempt to determine a transform that maps corresponding image points.
- In rigid registration the transform model consists of rotation and translation.
- For 2-D/2-D image registration we found a linear method to compute both rotation and translation.

Further Readings – Part 1

Survey papers on medical image registration:

- Derek L. G. Hill et al. "Medical Image Registration". In: *Physics in Medicine and Biology* 46.3 (2001), R1–R45
- J. B.Antoine Maintz and Max A. Viergever. "A Survey of Medical Image Registration". In: *Medical Image Analysis* 2.1 (1998), pp. 1–36. DOI: 10.1016/S1361-8415(01)80026-8
- L. G. Brown. "A Survey of Image Registration Techniques". In: *ACM Computing Surveys* 24.4 (Dec. 1992), pp. 325–376. DOI: 10.1145/146370.146374
- Josien P. W. Pluim, J. B. Antoine Maintz, and Max A. Viergever. "Mutual-Information-Based Registration of Medical Images: A Survey". In: *IEEE Transactions on Medical Imaging* 22.8 (Aug. 2003), pp. 986–1004. DOI: 10.1109/TMI.2003.815867

A paper that inspired all the sections on complex numbers, quaternions, and dual quaternions:

Konstantinos Daniilidis. "Hand-Eye Calibration Using Dual Quaternions". In: *The International Journal of Robotics Research* 18.3 (Mar. 1999), pp. 286–298. DOI: 10.1177/02783649922066213

Further Readings – Part 2

Non-parametric mappings for image registration:

- Nonlinear registration methods applied to DSA can be found in Erik Meijering's papers.
- Jan Modersitzki. *Numerical Methods for Image Registration*. Numerical Mathematics and Scientific Computations. Oxford Scholarship Online, 2007. Oxford: Oxford University Press, 2003. DOI: 10.1093/acprof:oso/9780198528418.001.0001
- Many of Jan Modersitzki's and Bernd Fischer's papers on image registration can be found in the publication list of the Institute of Mathematics and Image Computing (Lübeck).
- The group of Martin Rumpf also published on non-parametric image registration. Details on their work can be found on the institute's webpage.