Medical Image Processing for Interventional Applications

Random Walker – Algorithm

Online Course – Unit 41
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TECHNISCHE FAKULTÄT

Topics

Random Walks for Image Segmentation

Algorithm

- Dirichlet Integral
- Decomposition
- Solution

Summary

- Take Home Messages
- Further Readings

Problem Statement

K-way image segmentation

- User-defined **seeds**
- Indicating regions of the image belonging to K objects

Random walk

- Labeling an **unseeded** pixel by resolving the question:
  What is the probability of a random walker starting at this pixel that it first reaches seed point k?
- Selecting the label of the most probable seed destination for each pixel
- Biasing the random walker to avoid crossing sharp intensity gradients

Problem Statement

Image as discrete object

- **Graph** with a fixed number of vertices and edges
- Each node represents one pixel in the image.
- Edges connect neighboring pixels: e. g., 4-connectivity (2-D), 6-connectivity (3-D), 8-connectivity (2-D).
- A real-valued **weight** is assigned to each edge representing the likelihood that a random walker will cross this edge.
  $\rightarrow$ Weight of zero: the random walker may not move along that edge.
- Purely combinatorial operators:
  - No discretization
  - No discretization errors or ambiguities

Problem Statement

Edge weights for adjacent pixels *i* and *j*

Gaussian weighting function:

$$w_{ij} = \exp\left(-\beta(g_i - g_j)^2\right)$$

where

- g_i: image intensity at pixel i
- β: only free parameter!

- Useful operation: prior normalization of the square gradients:

$$\forall e_{ij} \in E: \ (g_i - g_j)^2 \in [0,1]$$

- Modification to handle color or general vector-valued data: $(g_i - g_j)^2 \longrightarrow ||\boldsymbol{g}_i - \boldsymbol{g}_j||^2$

Problem Statement

Four mathematically equivalent ways (Grady, 2006)

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3. *“Assign the pixel to the label for which its seeds have the largest effective conductance (i. e., smallest effective resistance) with the pixel.”*

Problem Statement

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1. *“If a random walker leaving the pixel is most likely to first reach a seed bearing label s, assign the pixel to label s.”*
2. *“If the seeds are alternately replaced by grounds/unit voltage sources, assign the pixel to the label for which its seeds being ‘on’ produces the greatest electrical potential.”*
3. *“Assign the pixel to the label for which its seeds have the largest effective conductance (i. e., smallest effective resistance) with the pixel.”*
4. *“If a 2-tree is drawn randomly from the graph (with probability given by the product of weights in the 2-tree), assign the pixel to the label for which the pixel is most likely to remain connected to.”*

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Algorithm

Combinatorial Laplacian matrix L

$$L_{ij} = \begin{cases} d_i & \text{if } i = j, \\ -w_{ij} & \text{if } v_i \text{ and } v_j \text{ are adjacent nodes,} \\ 0 & \text{otherwise,} \end{cases}$$

where

- L_{ij} is indexed by vertices v_i and v_j,
- $d_i = \sum w(e_{ij})$ for all edges e_{ij} incident on node v_i.

Algorithm

Example: Pixels of a 4×4 *image*

	1	2	3	4
1	v_1	v_2	v_3	v_4
2	v_5	v_6	v_7	v_8
3	v_9	v_{10}	v_{11}	v_{12}
4	v_{13}	v_{14}	v_{15}	v_{16}

Algorithm

Example: Pixels of a 4×4 *image* and the according *combinatorial Laplacian matrix* L

	1	2	3	4
1	v_1	v_2	v_3	v_4
2	v_5	v_6	v_7	v_8
3	v_9	v_{10}	v_{11}	v_{12}
4	v_{13}	v_{14}	v_{15}	v_{16}

$$L = \begin{bmatrix}
d_1 & -w_{1,2} & 0 & 0 & -w_{1,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-w_{1,2} & d_2 & -w_{2,3} & 0 & 0 & -w_{2,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -w_{2,3} & d_3 & -w_{3,4} & 0 & 0 & -w_{3,7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -w_{3,4} & d_4 & 0 & 0 & 0 & -w_{4,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-w_{1,5} & 0 & 0 & 0 & d_5 & -w_{5,6} & 0 & 0 & -w_{5,9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -w_{2,6} & 0 & 0 & -w_{5,6} & d_6 & -w_{6,7} & 0 & 0 & -w_{6,10} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -w_{3,7} & 0 & 0 & -w_{6,7} & d_7 & -w_{7,8} & 0 & 0 & -w_{7,11} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -w_{4,8} & 0 & 0 & -w_{7,8} & d_8 & 0 & 0 & 0 & -w_{8,12} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -w_{5,9} & 0 & 0 & 0 & d_9 & -w_{9,10} & 0 & 0 & -w_{9,13} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -w_{6,10} & 0 & 0 & -w_{9,10} & d_{10} & -w_{10,11} & 0 & 0 & -w_{10,14} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -w_{7,11} & 0 & 0 & -w_{10,11} & d_{11} & -w_{11,12} & 0 & 0 & -w_{11,15} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -w_{8,12} & 0 & 0 & -w_{11,12} & d_{12} & 0 & 0 & 0 & -w_{12,16} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -w_{9,13} & 0 & 0 & 0 & d_{13} & -w_{13,14} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -w_{10,14} & 0 & 0 & -w_{13,14} & d_{14} & -w_{14,15} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -w_{11,15} & 0 & 0 & -w_{14,15} & d_{15} & -w_{15,16} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -w_{12,16} & 0 & 0 & -w_{15,16} & d_{16}
\end{bmatrix}$$

Algorithm

Combinatorial formulation of the Dirichlet integral

$$D(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T\boldsymbol{L}\boldsymbol{x} = \frac{1}{2}\sum_{e_{ij}\in E} w_{ij}(x_i - x_j)^2$$

Partitioning the vertices into two sets:

- marked/seed nodes V_M,
- unseeded nodes V_U,

such that $V_M \cup V_U = V$ and $V_M \cap V_U = \varnothing$.

Without loss of generality: The nodes in $\boldsymbol{L}$ and $\boldsymbol{x}$ are **ordered**, i. e., seed nodes are first, unseeded nodes are second.

Algorithm

Decomposition

$$D[\boldsymbol{x}_U] = \frac{1}{2}\left(\boldsymbol{x}_M{}^{\mathsf{T}}\ \boldsymbol{x}_U{}^{\mathsf{T}}\right)\begin{bmatrix}\boldsymbol{L}_M & \boldsymbol{B}\\ \boldsymbol{B}^{\mathsf{T}} & \boldsymbol{L}_U\end{bmatrix}\begin{pmatrix}\boldsymbol{x}_M\\ \boldsymbol{x}_U\end{pmatrix} = \frac{1}{2}\left(\boldsymbol{x}_M{}^{\mathsf{T}}\boldsymbol{L}_M\boldsymbol{x}_M + 2\boldsymbol{x}_U{}^{\mathsf{T}}B^{\mathsf{T}}\boldsymbol{x}_M + \boldsymbol{x}_U{}^{\mathsf{T}}\boldsymbol{L}_U\boldsymbol{x}_U\right)$$

$\boldsymbol{L}$ is positive semi-definite, i. e., the only critical points of $D[\boldsymbol{x}]$ will be minima.

Algorithm

Differentiating w. r. t. $\boldsymbol{x}_U$ and finding the critical points:

$$\boldsymbol{L}_U \boldsymbol{x}_U = -\boldsymbol{B}^\top \boldsymbol{x}_M$$

- System of linear equations with $|V_U|$ unknowns
- Equation will be non-singular
  - if the graph is connected, or
  - if every connected component contains a seed.

Algorithm

Solution to the combinatorial Dirichlet problem for label s

- x_i^s: probability (potential) assumed at node v_i for label s
- Set of labels: $\forall v_j \in V_M: \; Q(v_j) = s, s \in \mathbb{Z}, 0 < s \leq K$
- $V_M \times 1$ vector $\boldsymbol{m}^s$:

$$m_j^s = \begin{cases} 1 \text{ if } Q(v_j) = s, \\ 0 \text{ if } Q(v_j) \neq s \end{cases}$$

Algorithm

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Solution for one label:

$$\boldsymbol{L}_U \boldsymbol{x}^s = -\boldsymbol{B}^\top \boldsymbol{m}^s$$

Algorithm

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Solution for one label:

$$\boldsymbol{L}_U \boldsymbol{x}^s = -\boldsymbol{B}^\top \boldsymbol{m}^s$$

Solution for all labels:

$$\boldsymbol{L}_U \boldsymbol{X} = -\boldsymbol{B}^\top \boldsymbol{M}$$

where $\boldsymbol{X}, \boldsymbol{M}$ are matrices with K columns taken by each $\boldsymbol{x}^s$ and $\boldsymbol{m}^s$, respectively.

Algorithm

Note:

- At any node the probabilities x_i^s will sum to unity:

$$\forall v_i \in V : \sum_s x_i^s = 1 \, .$$

- Hence, only $K - 1$ sparse linear systems must be solved.

Topics

Random Walks for Image Segmentation

Algorithm

- Dirichlet Integral
- Decomposition
- Solution

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- For the segmentation using a random walker, we describe pixel transitions from one pixel to a neighboring pixel in form of graph edges.
- In the algorithm the combinatorial Dirichlet problem has to be solved to find a segmentation result.

Further Readings

These slides are based on the following publication:

L. Grady. "Random Walks for Image Segmentation". In: *IEEE Transactions on Pattern Analysis and Machine Intelligence* 28.11 (Nov. 2006), pp. 1768–1783. DOI: 10.1109/TPAMI.2006.233

His implementations in Matlab can be downloaded here:

- Graph Analysis Toolbox
- Random Walker

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Random Walker – Properties

Online Course – Unit 42

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Topics

Properties and Effects

- Neutral Segmentation
- Weak Boundaries
- Noise Robustness
- Ambiguous Unseeded Regions

Summary

- Take Home Messages
- Further Readings

Neutral Segmentation

→ Corresponds roughly to Voronoi cells

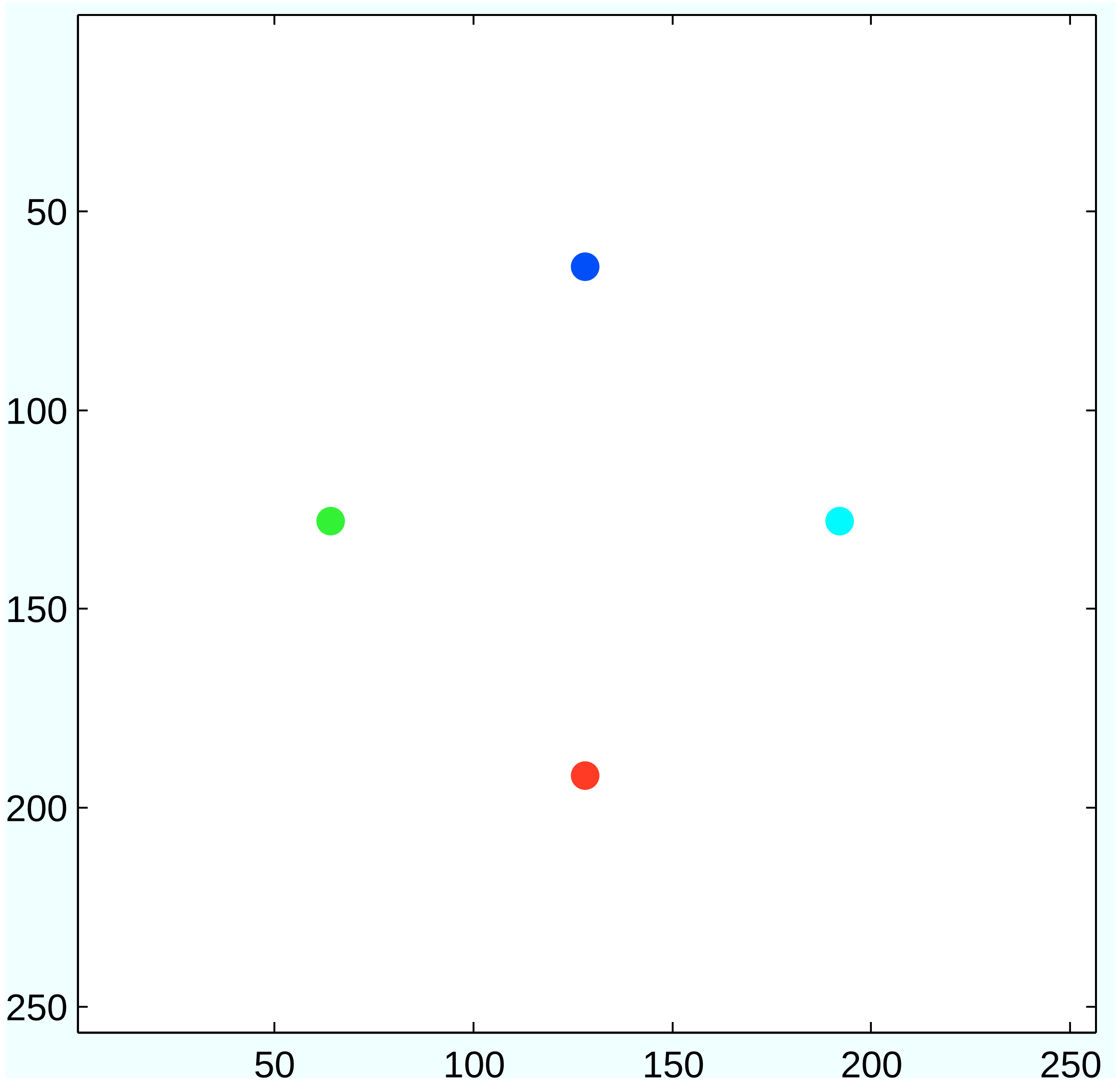

Figure 1: Original image with seed points

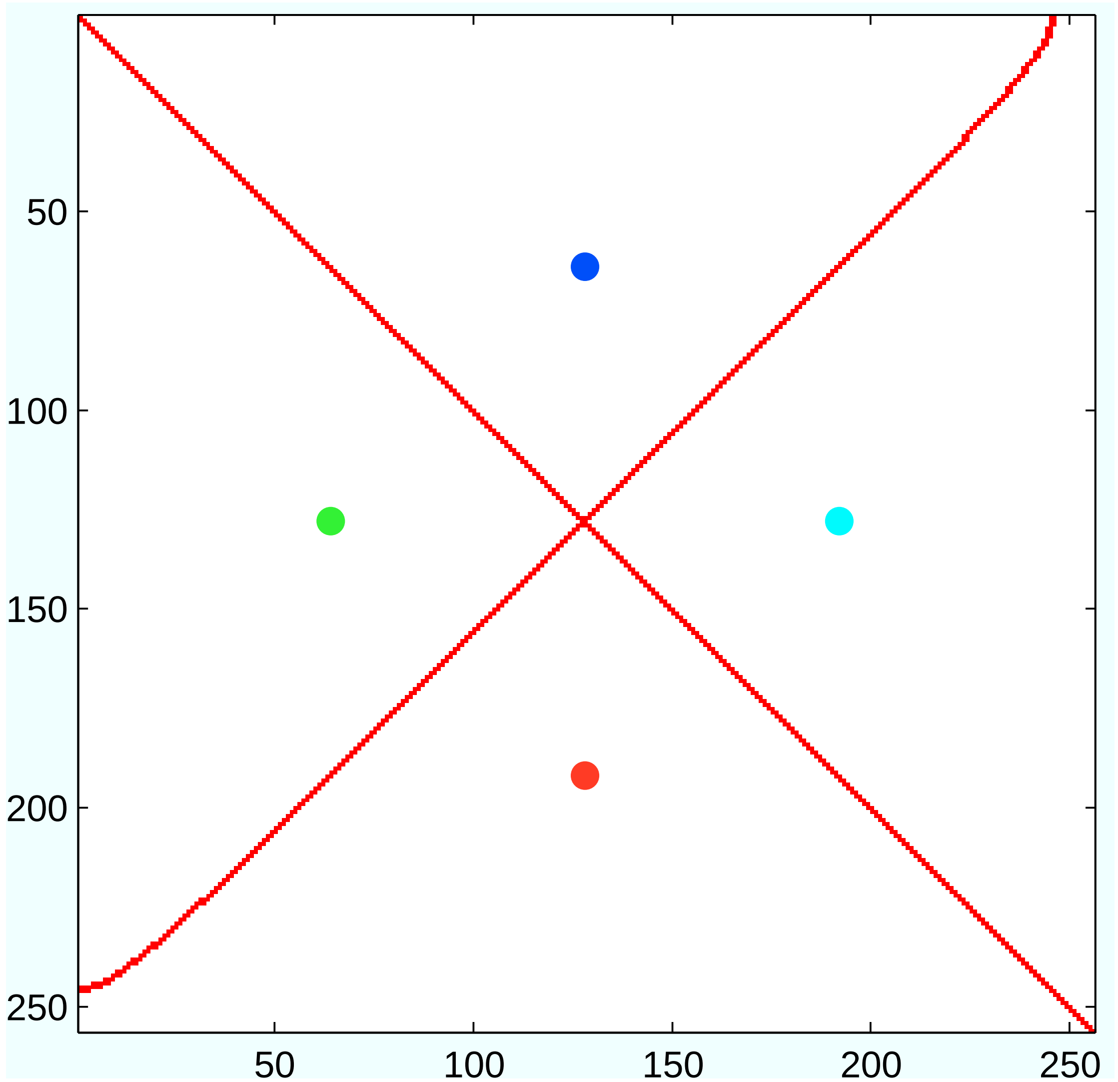

Figure 2: Outlined mask

Neutral Segmentation

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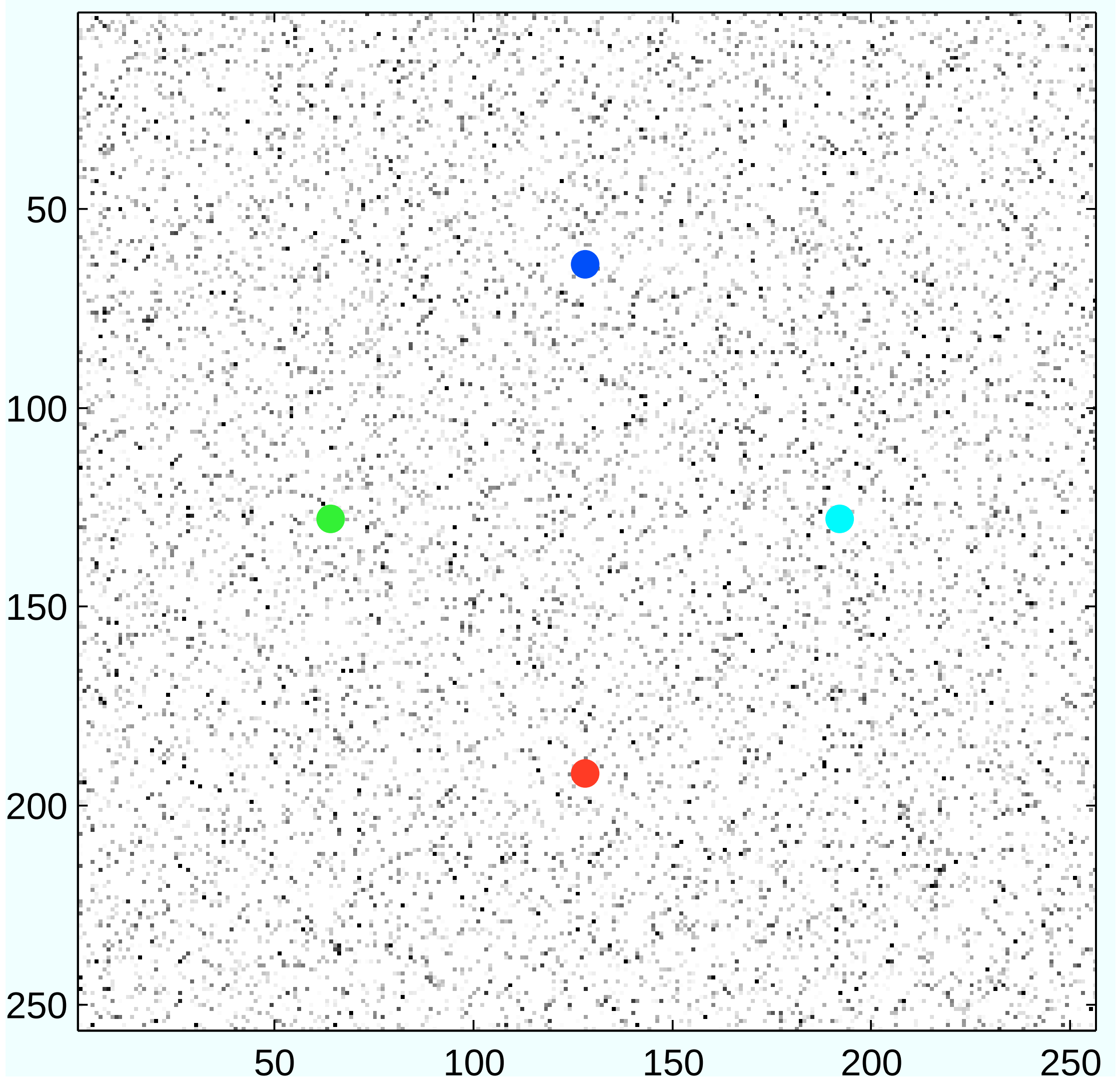

Figure 3: Original image with seed points

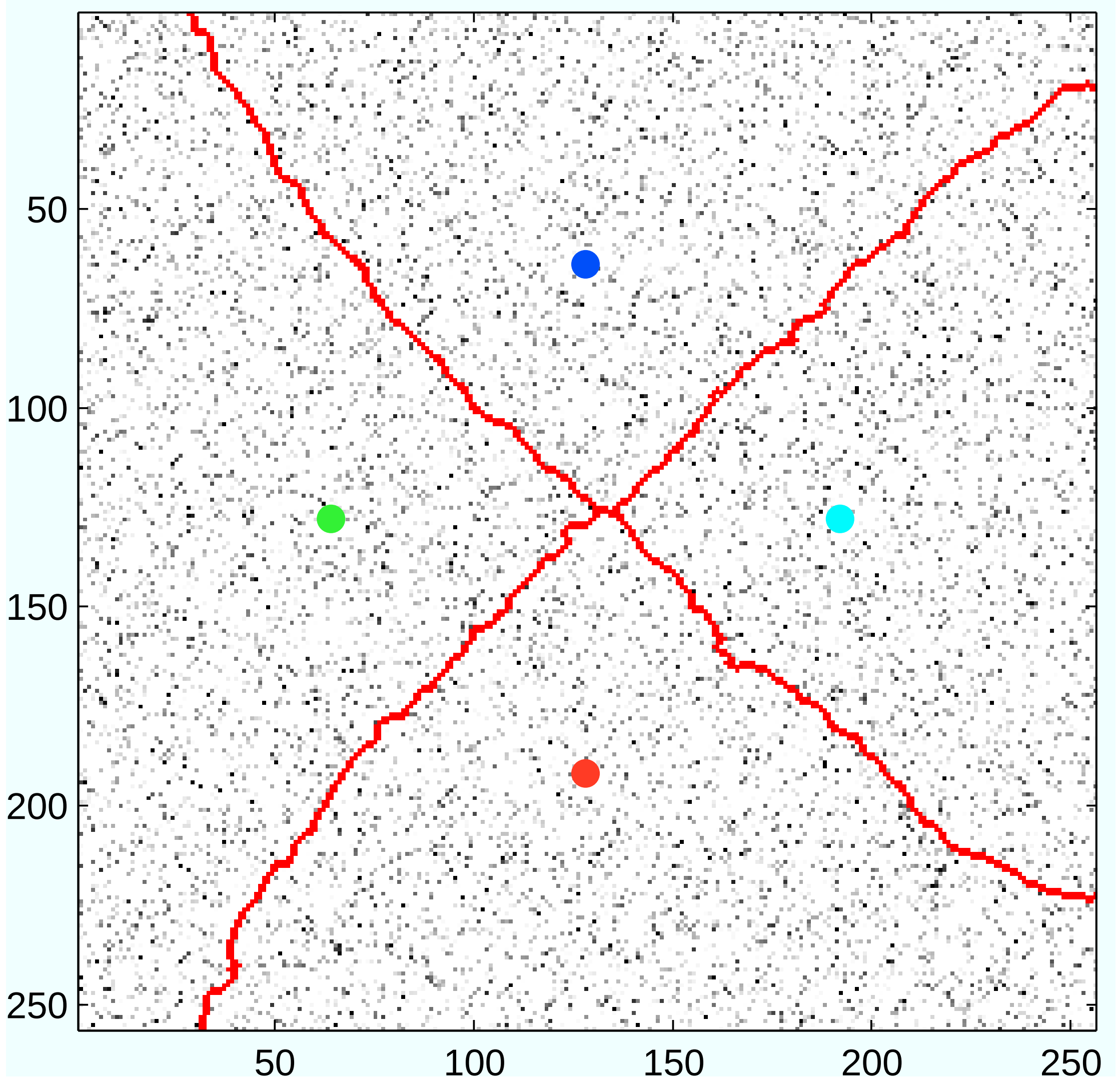

Figure 4: Outlined mask

Weak Boundaries

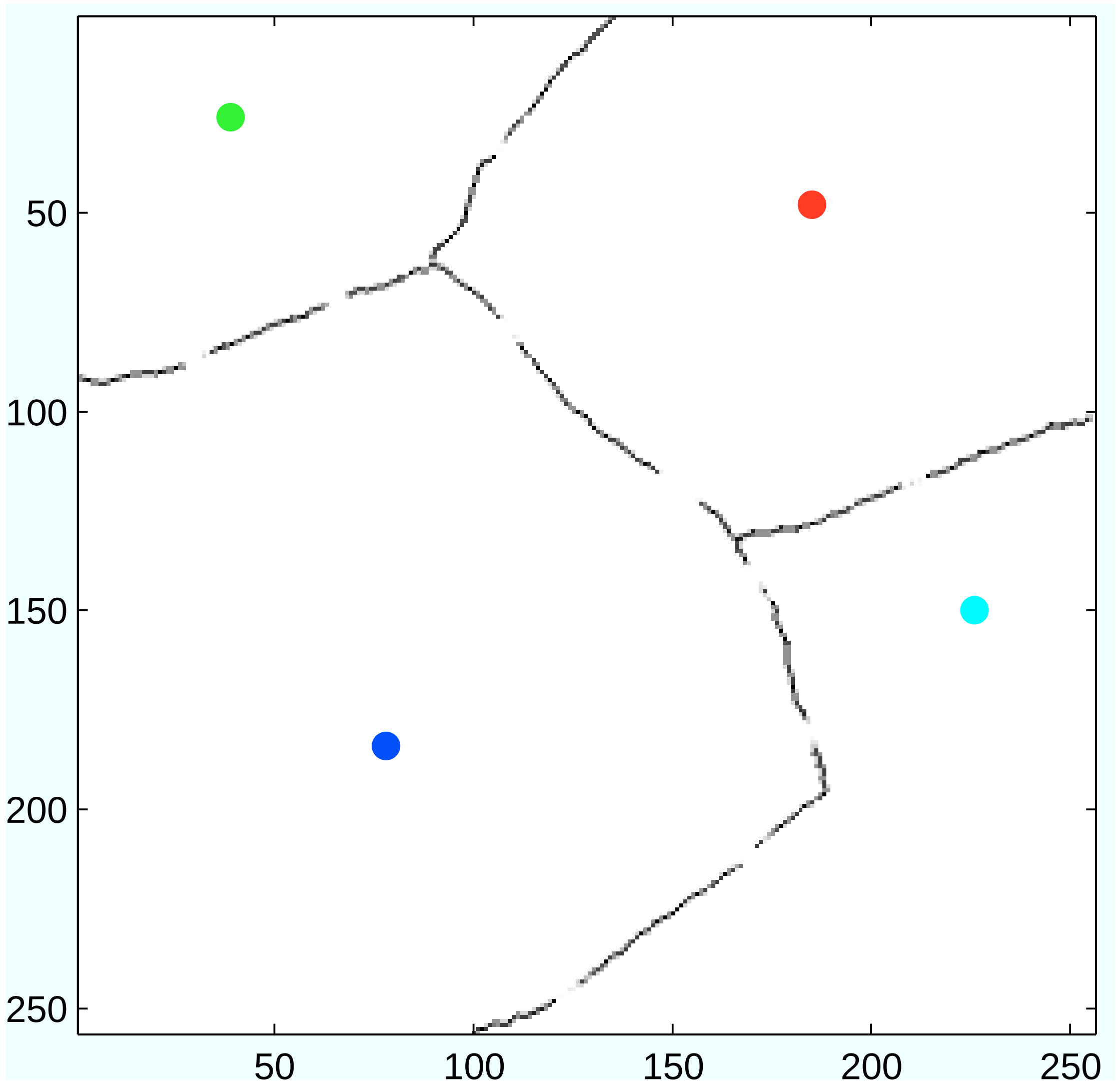

Figure 5: Original image with seed points

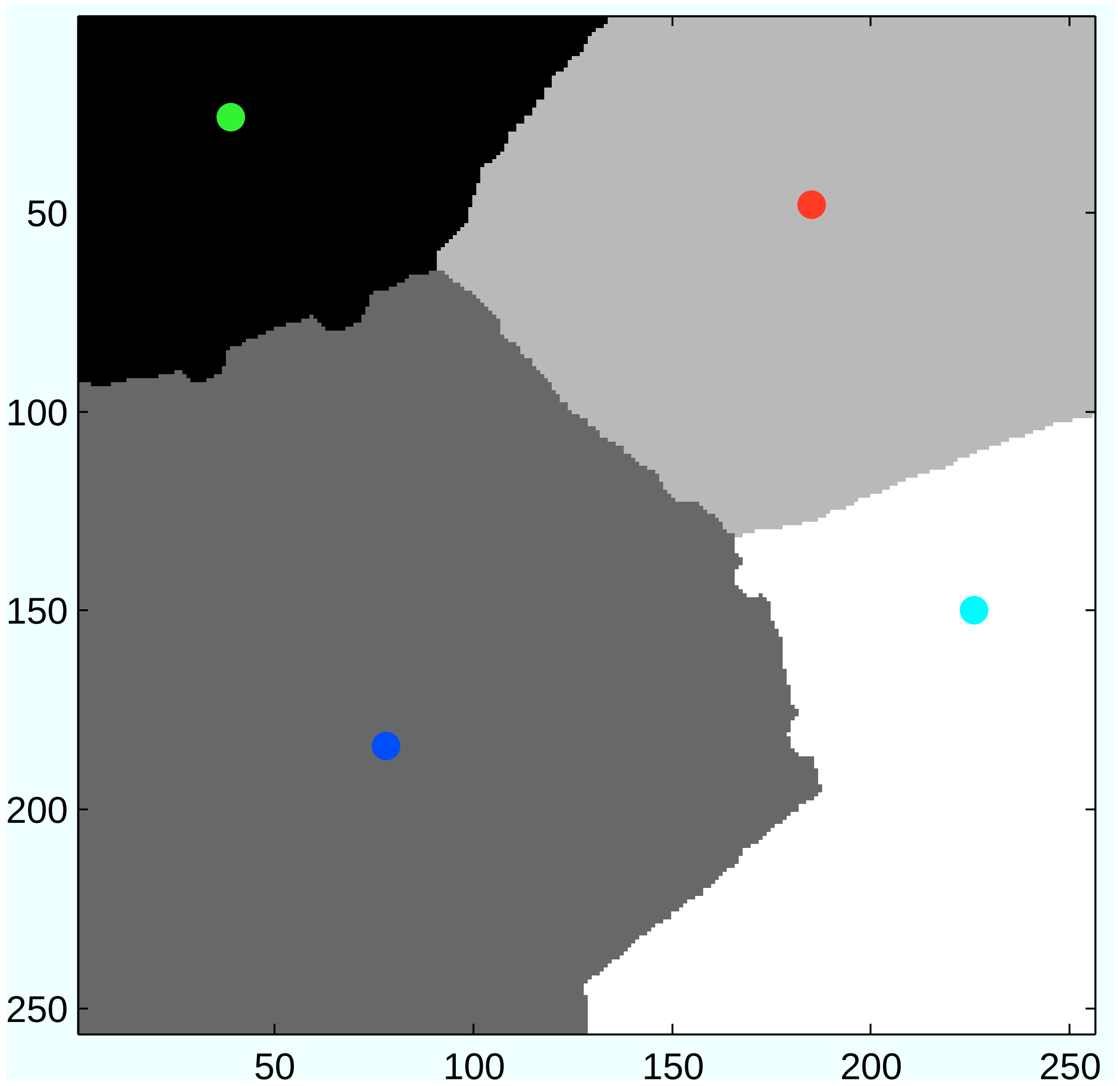

Figure 6: Output mask

Weak Boundaries

- On its initial step, the current pixel has 3 out of 4 chances to enter into the region that is likely to be labeled as belonging to the black circle.
- On the other side of the weak boundary, the same holds for the white circle.
- Due to the sharp drop in the probabilities, the segmentation will respect the weak boundary.

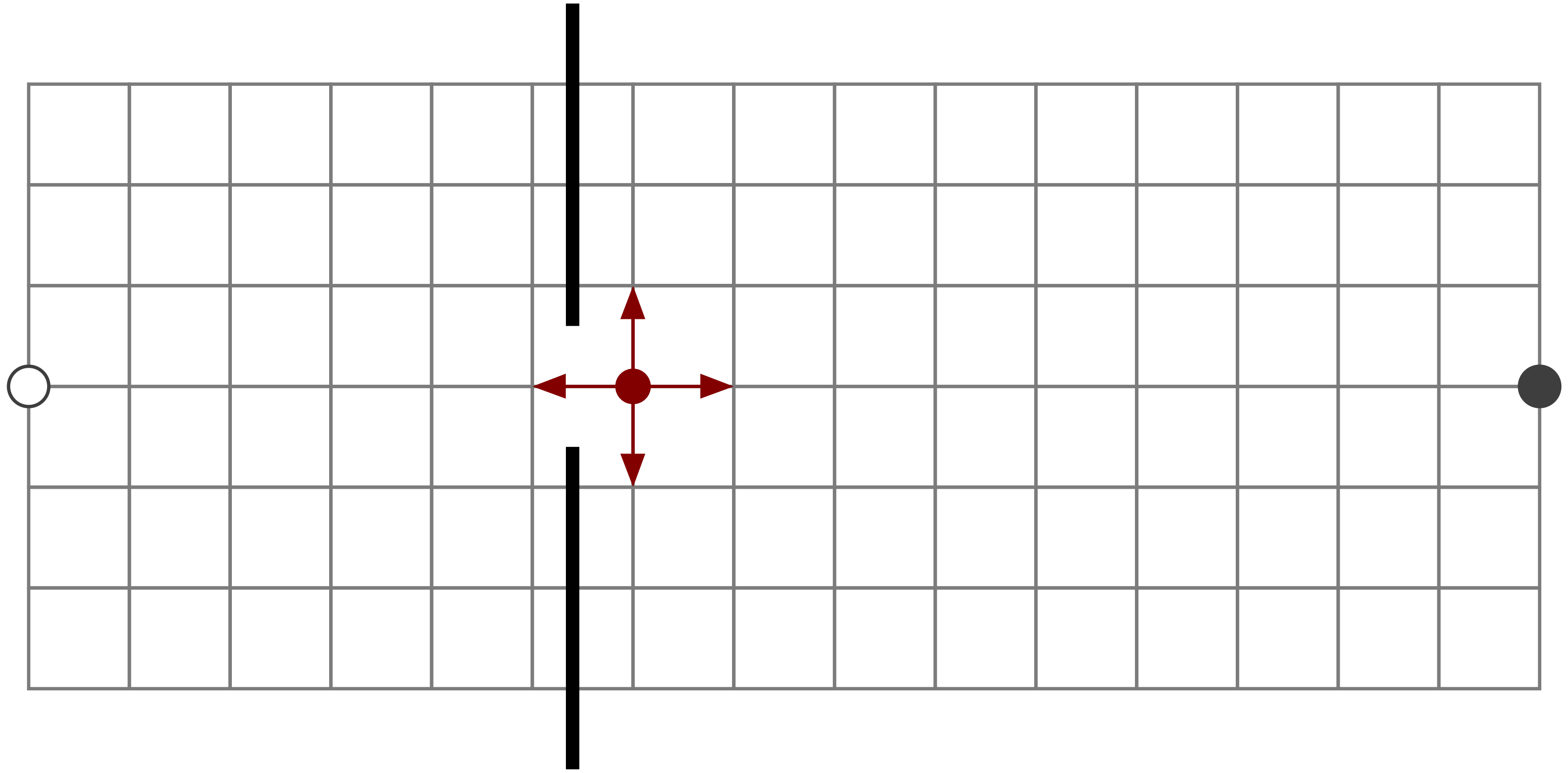

Figure 7: Random walk at a region boundary

Weak Boundaries

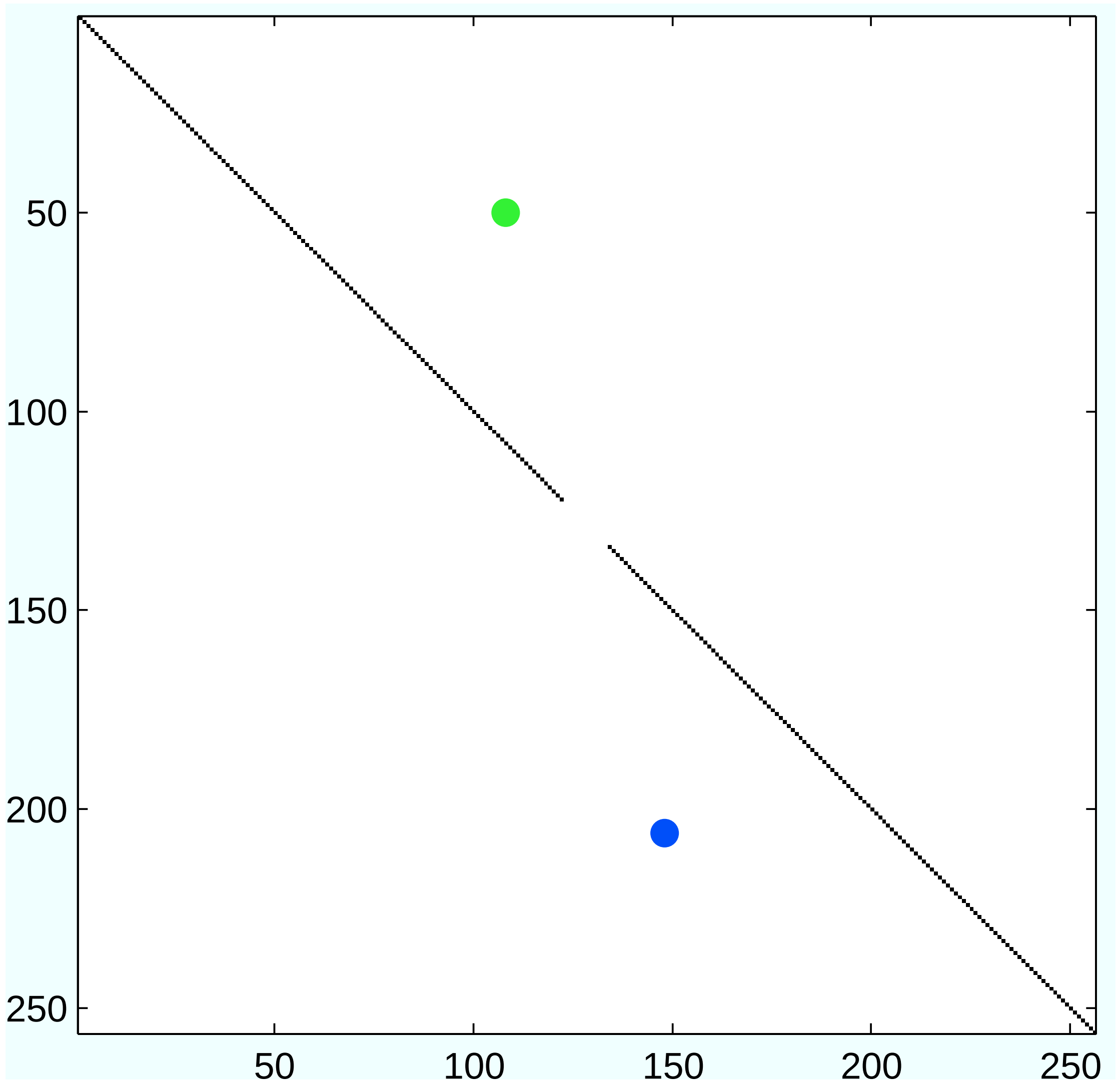

Figure 8: Original image with seed points

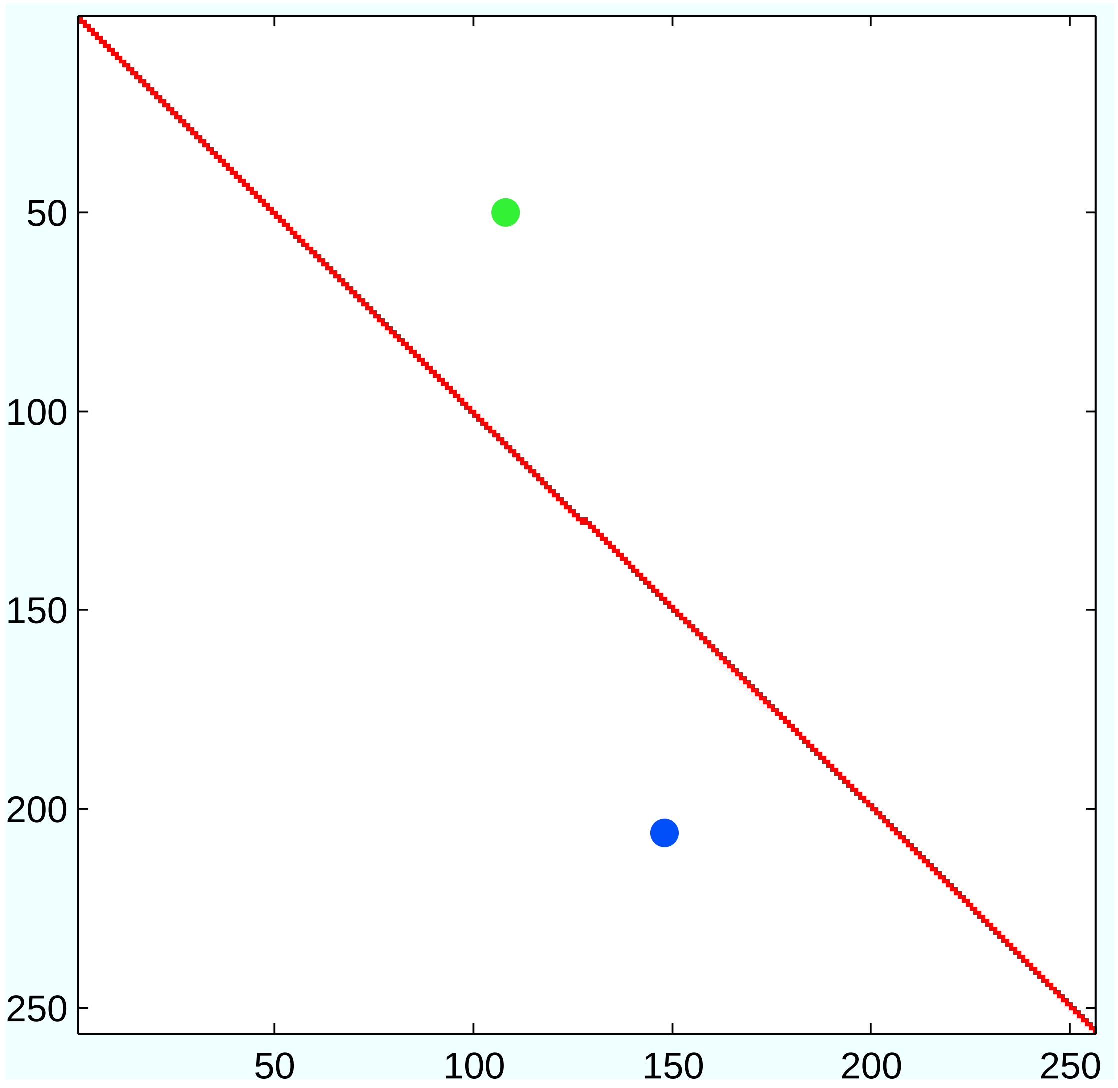

Figure 9: Outlined mask

Weak Boundaries

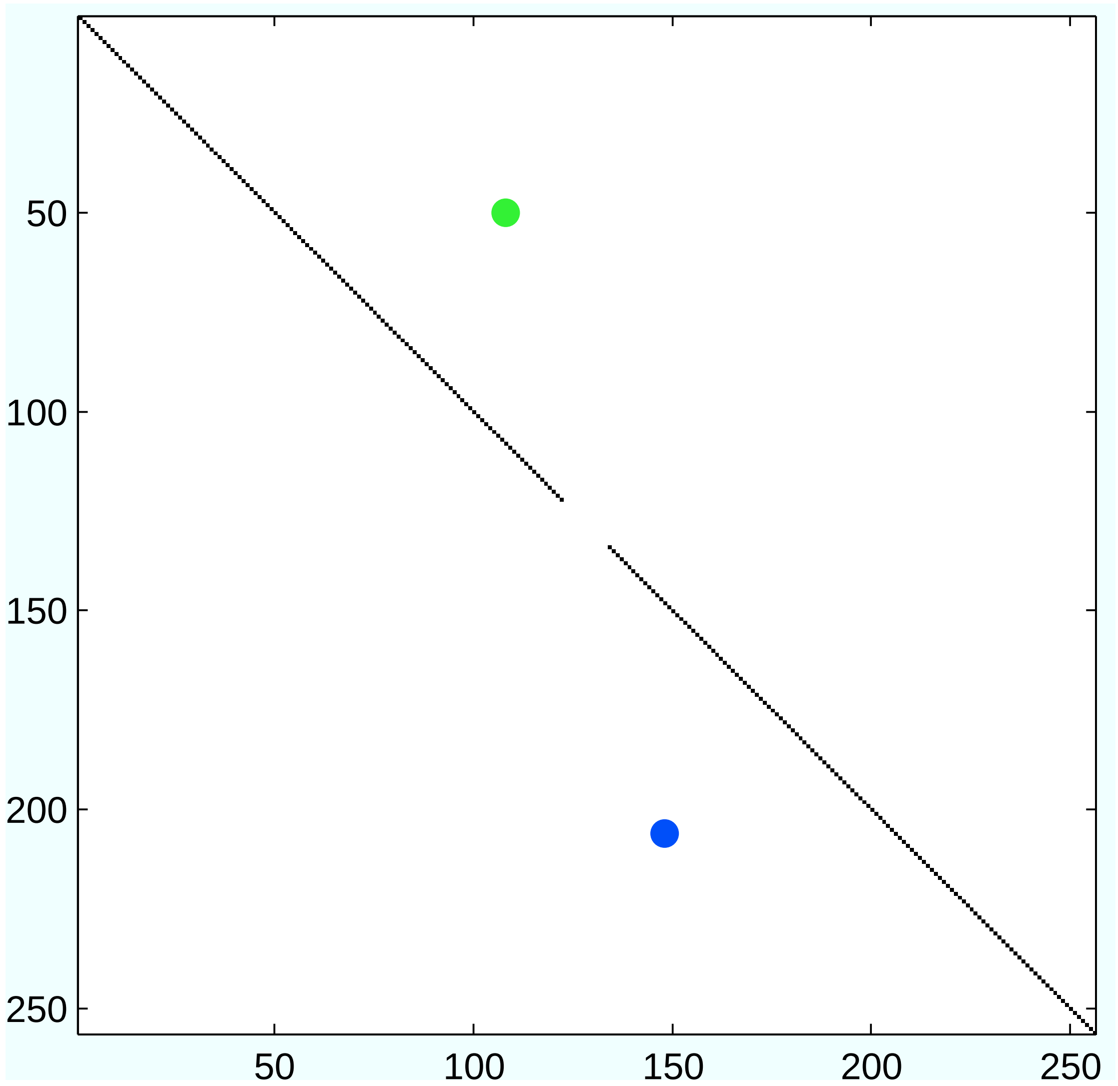

Figure 8: Original image with seed points

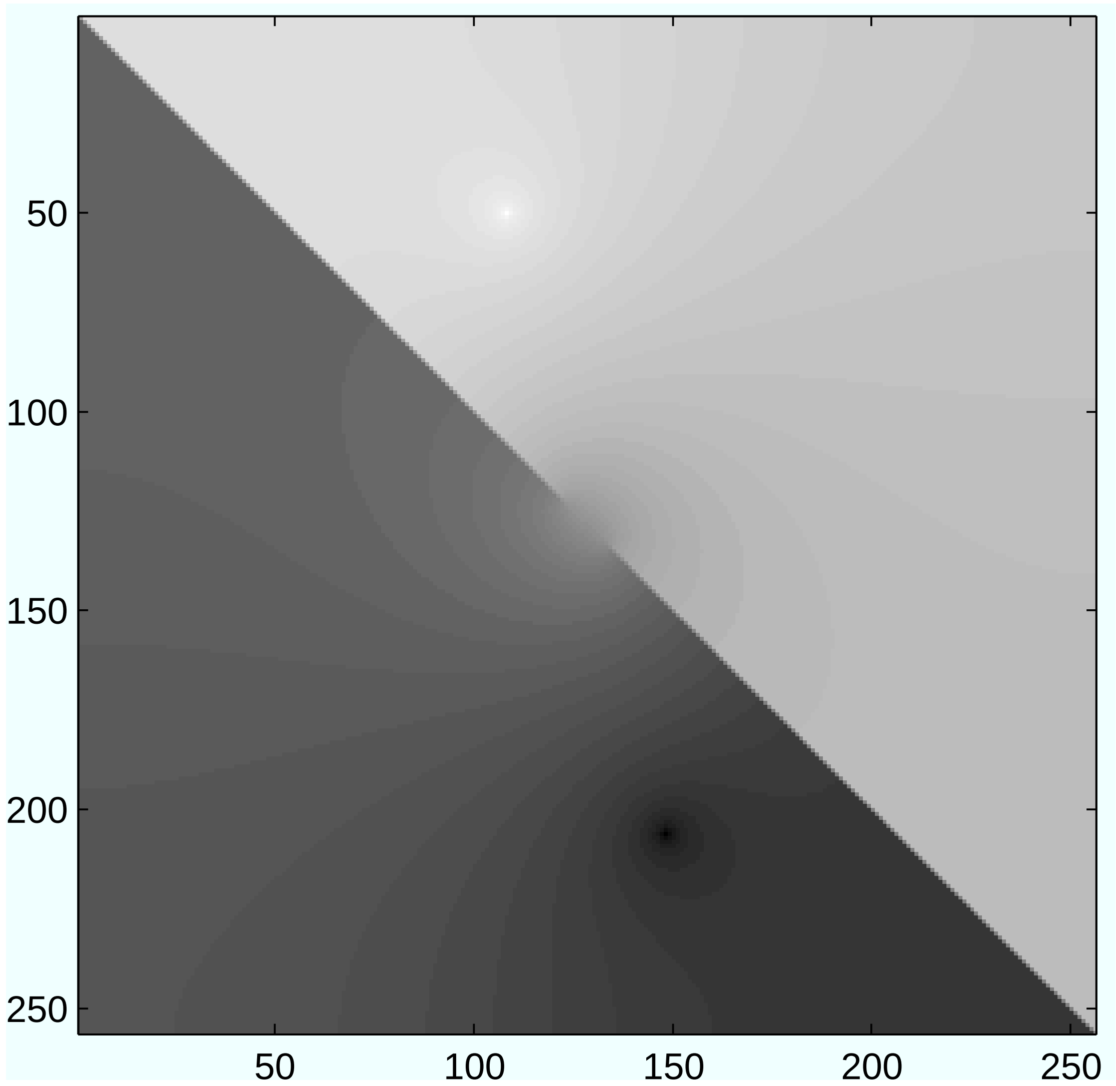

Figure 10: Probabilities for reaching seed 1

Weak Boundaries

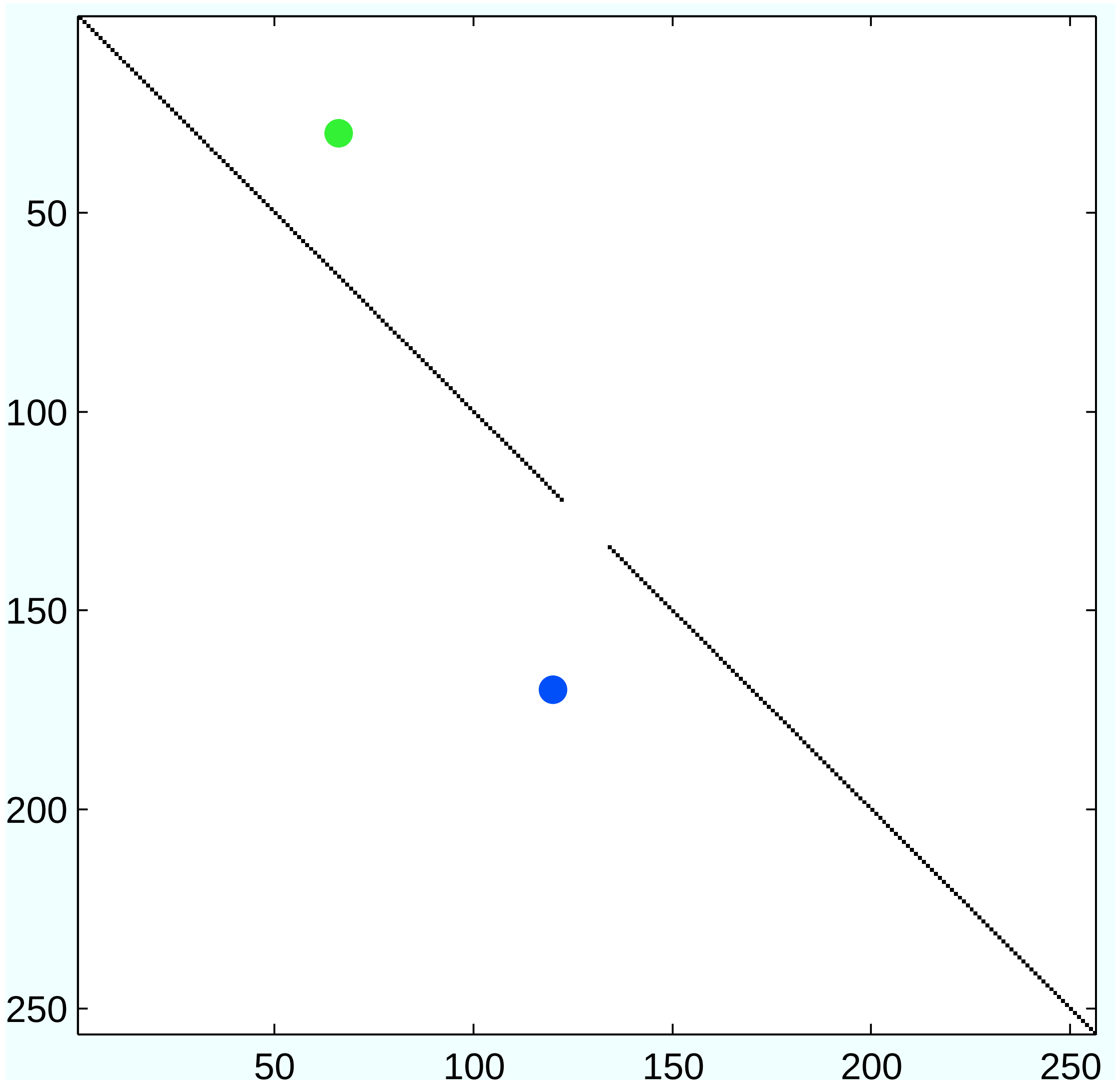

Figure 11: Original image with seed points

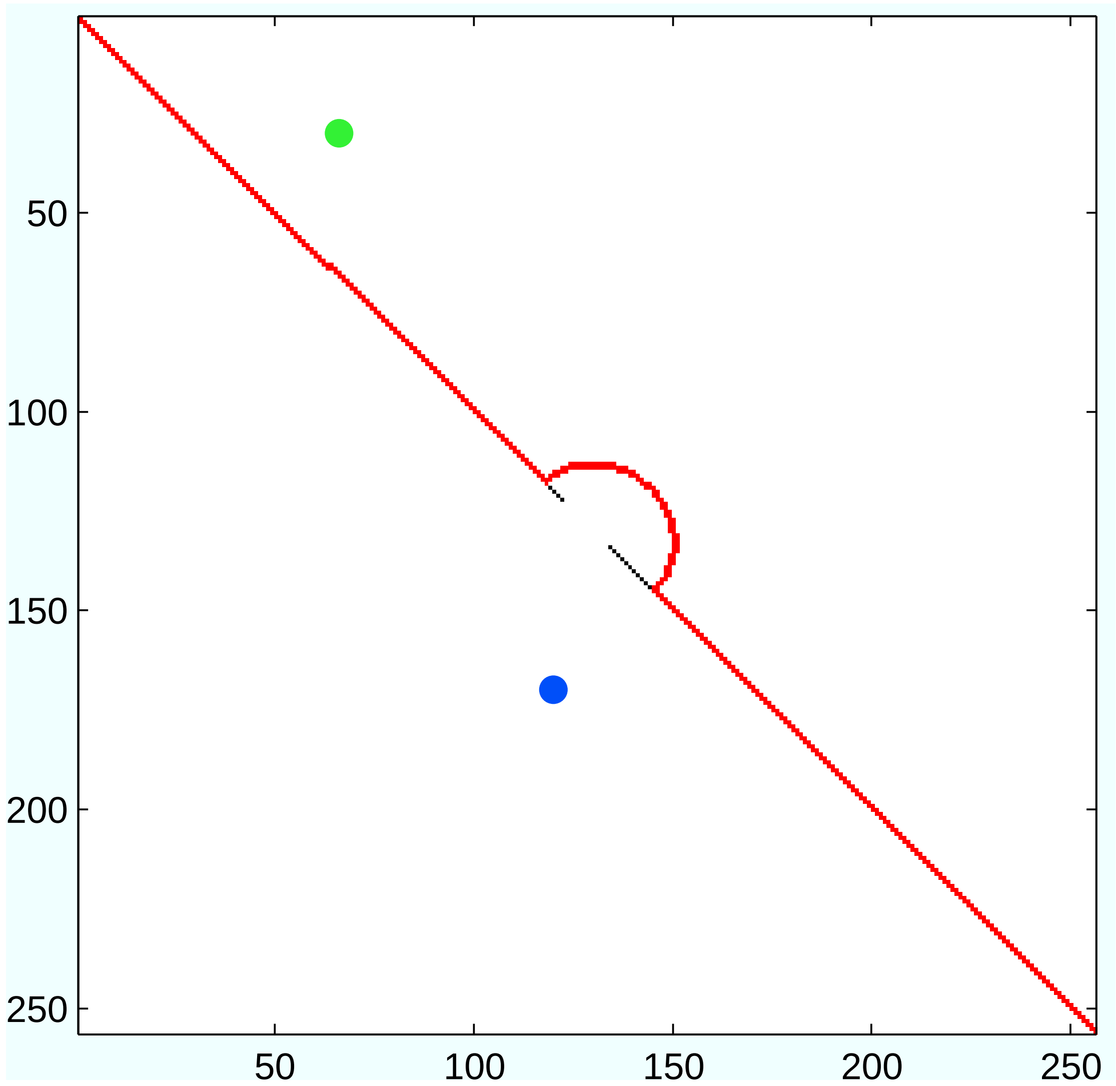

Figure 12: Outlined mask

Weak Boundaries

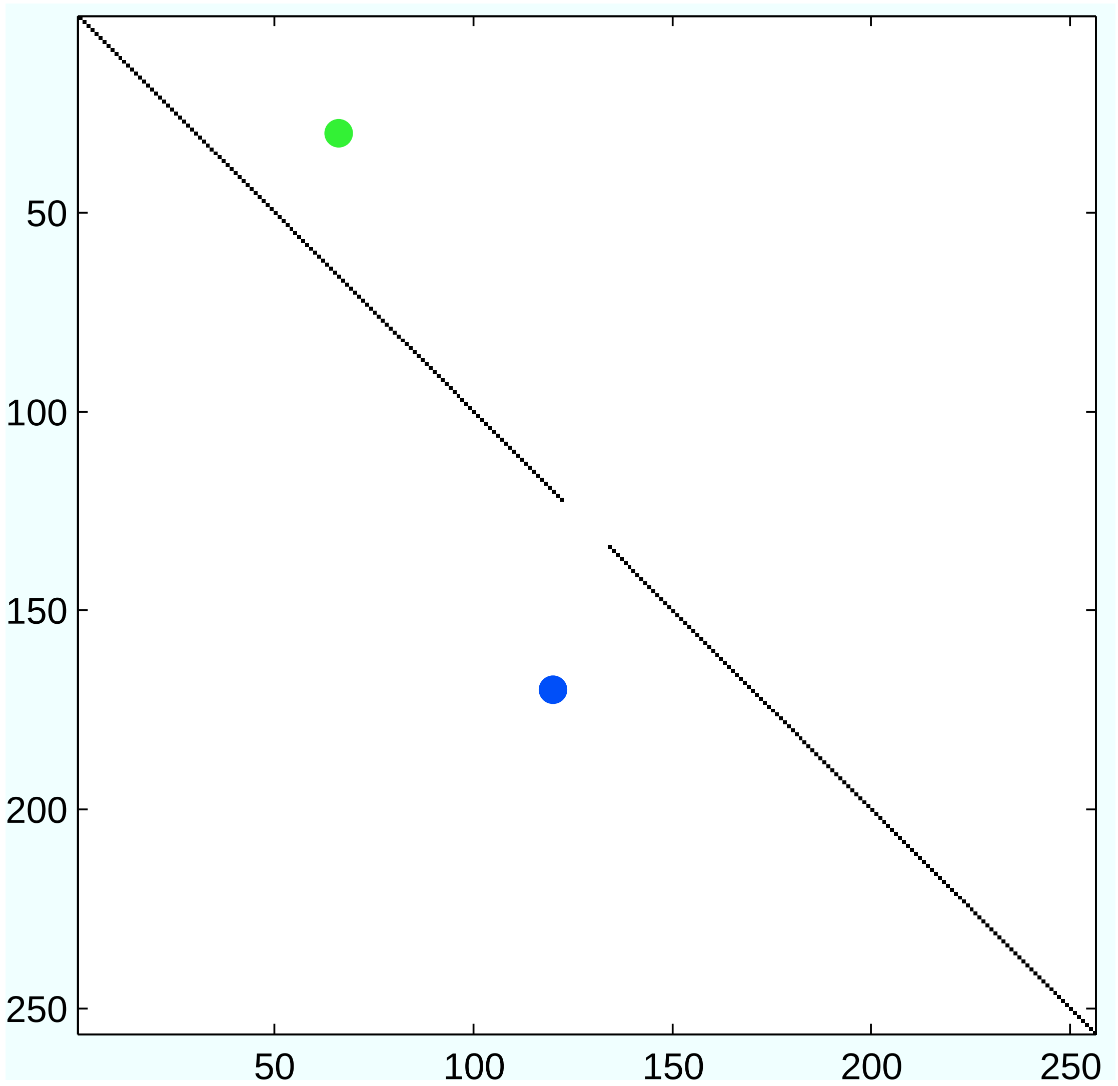

Figure 11: Original image with seed points

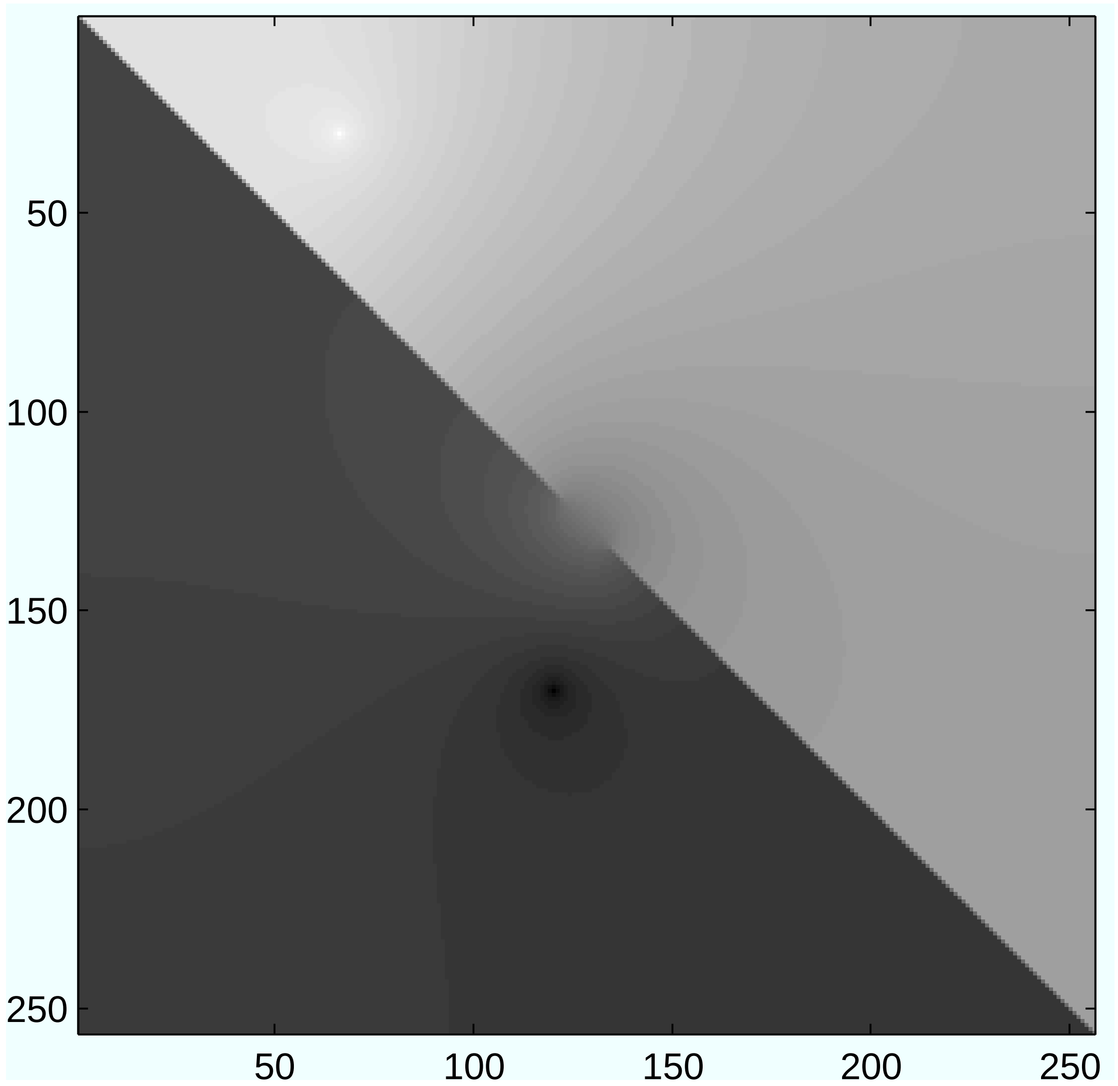

Figure 13: Probabilities for reaching seed 1

Noise Robustness

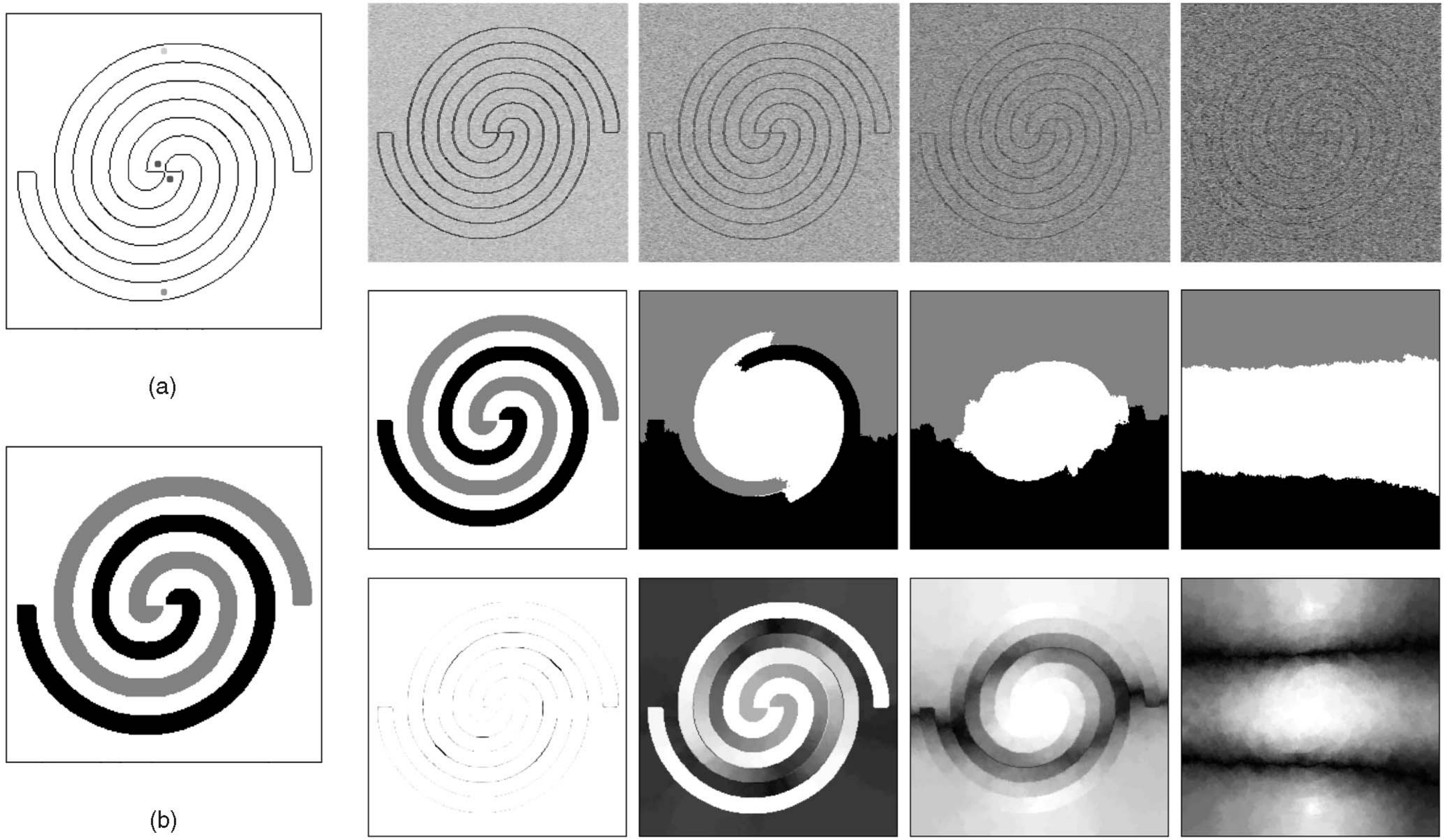

Figure 14: Edge mask (a) and ground truth segmentation (b) for a 3-class spiral segmentation task and segmentation results: edge masks with increasing noise level (right part, top row), random walker segmentation (right part, middle row), difference to ground truth (right part, bottom row)

Ambiguous Unseeded Regions

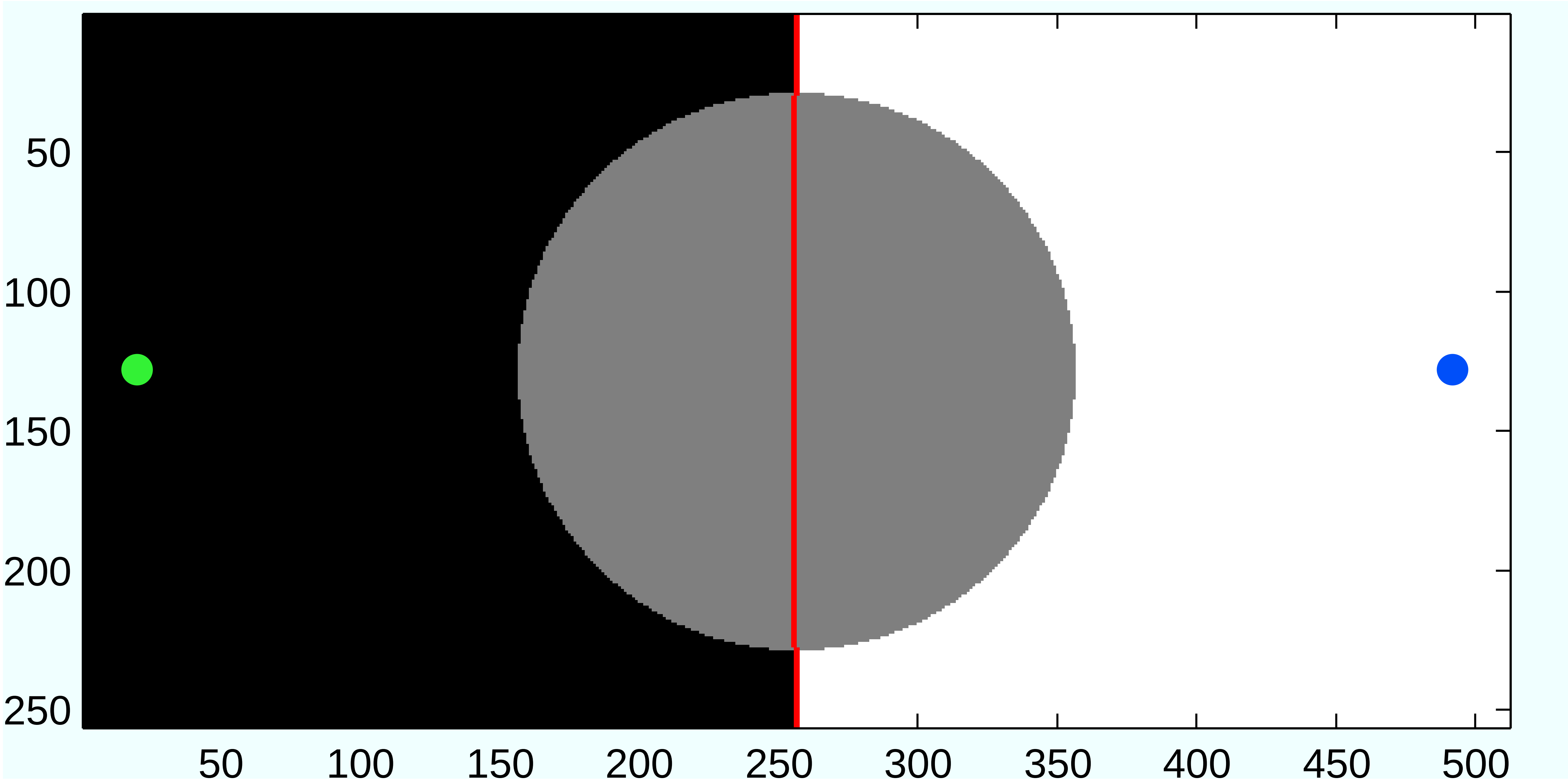

Figure 15: **Centered precisely** with respect to surface area and intensity

Ambiguous Unseeded Regions

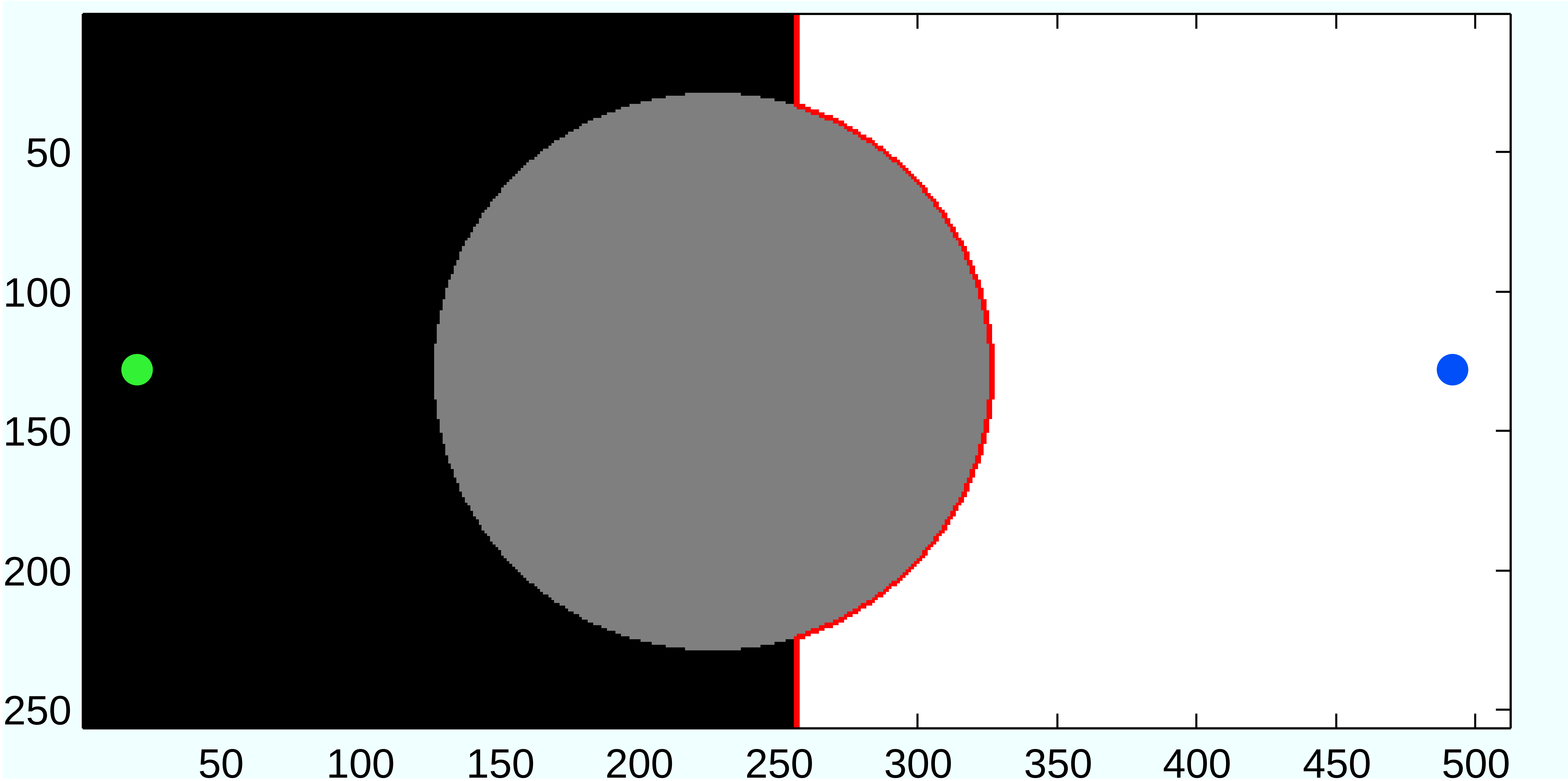

Figure 16: **Sharing more** surface area with **black region**

Ambiguous Unseeded Regions

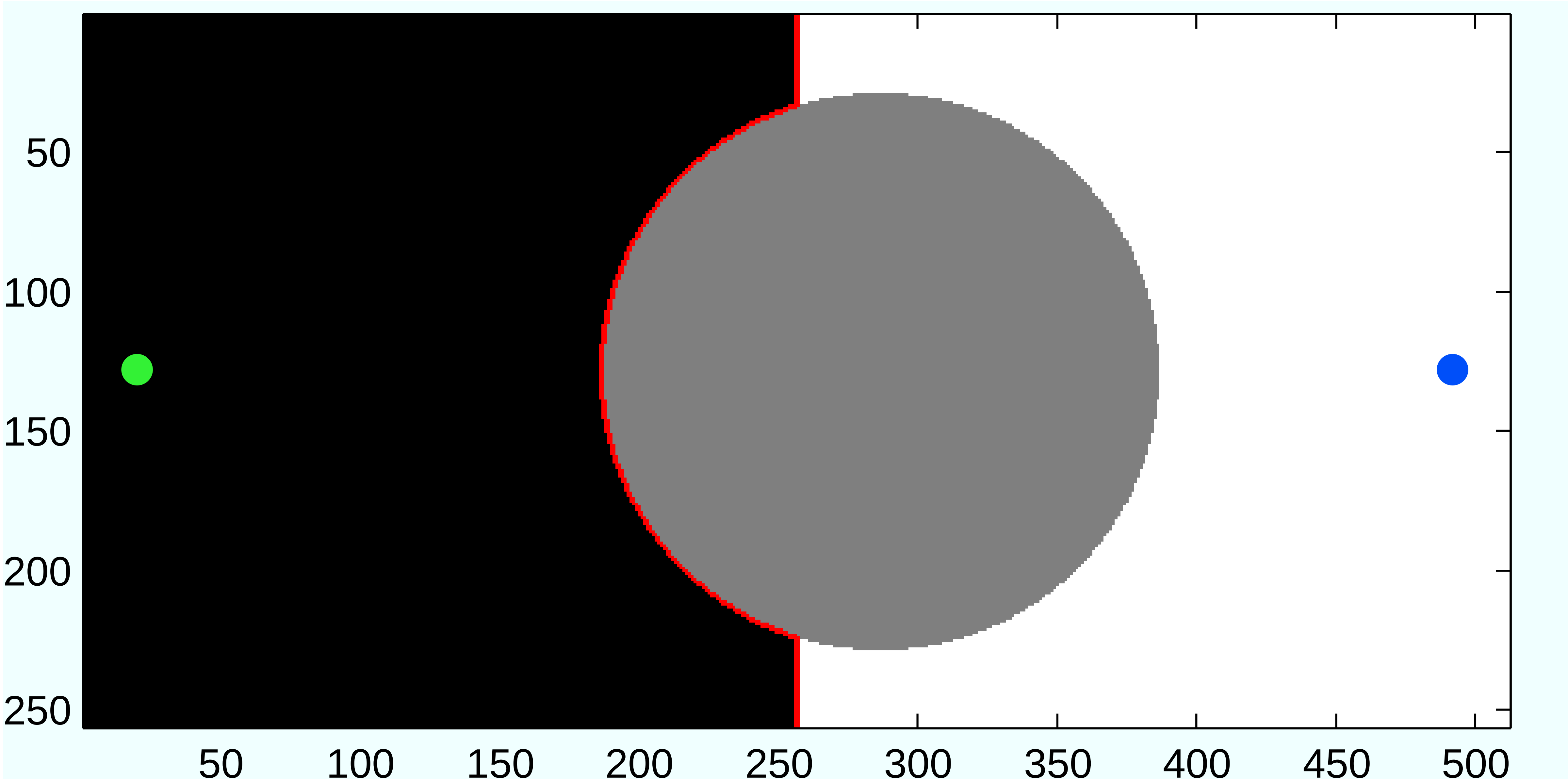

Figure 17: **Sharing more** surface area with **white region**

Ambiguous Unseeded Regions

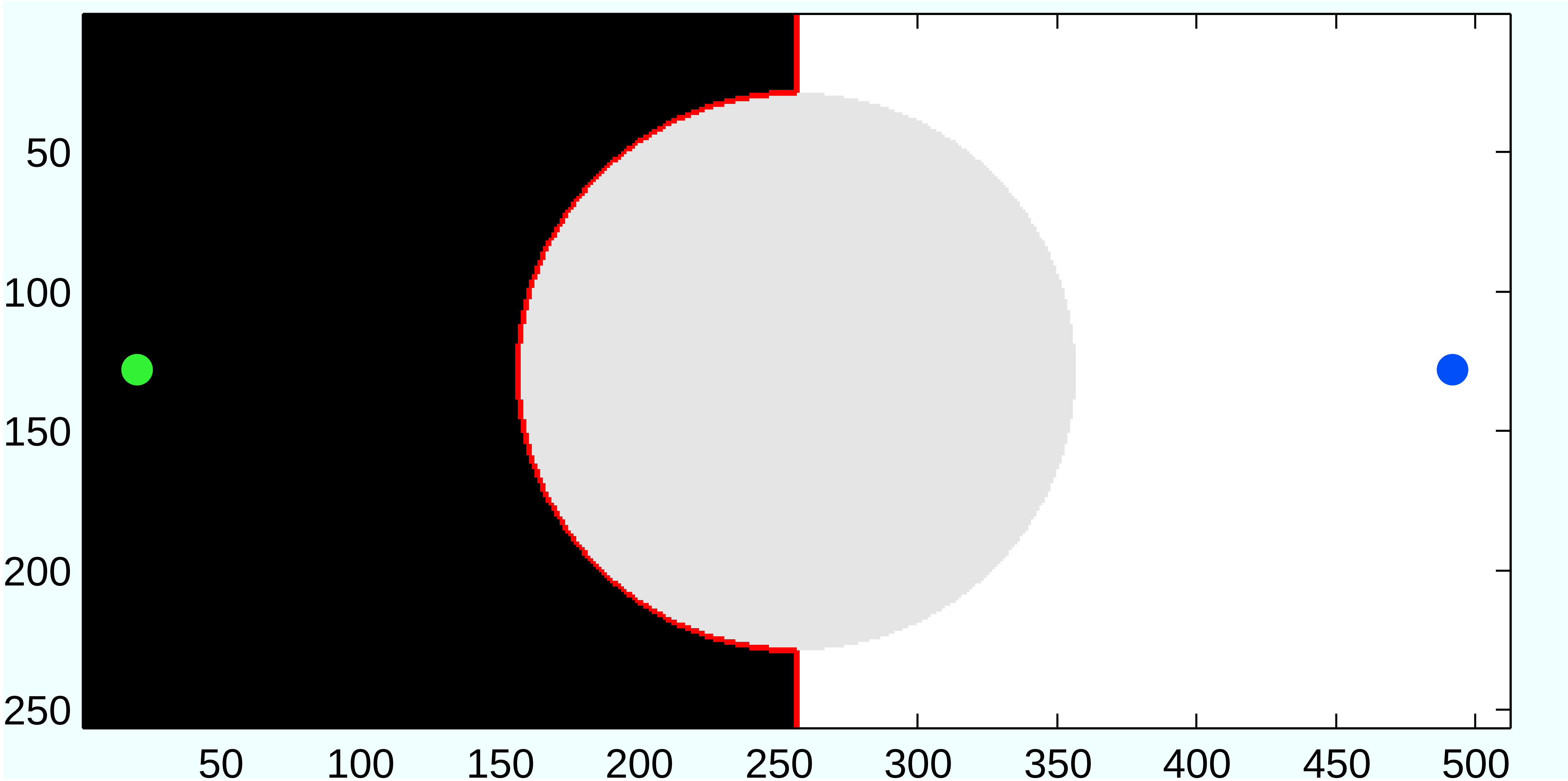

Figure 18: **Closer in intensity** to the **white** region (gray value **0.9**)

Ambiguous Unseeded Regions

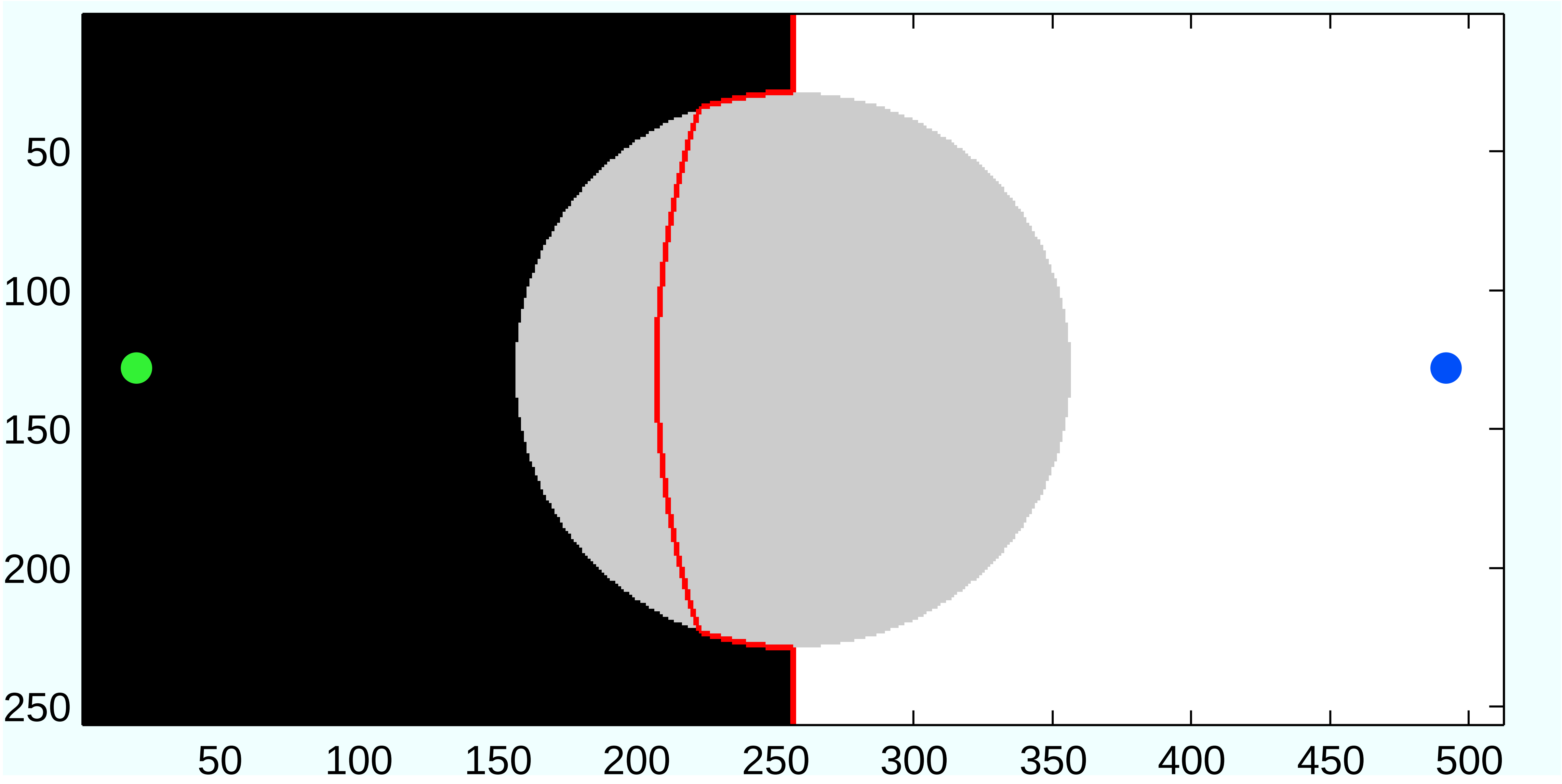

Figure 19: **Closer in intensity** to the **white** region (gray value **0.8**)

Ambiguous Unseeded Regions

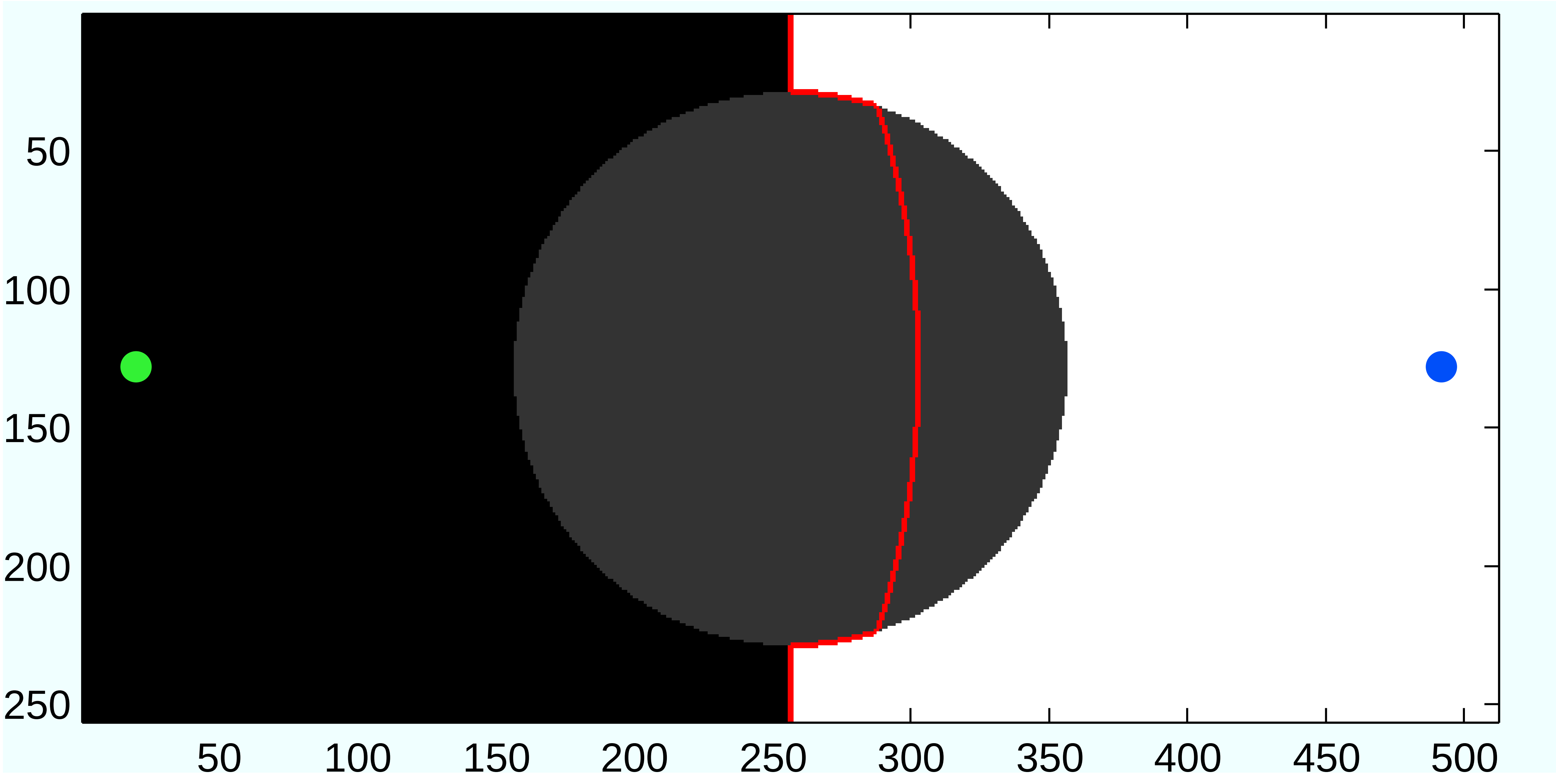

Figure 20: **Closer in intensity** to the **black** region (gray value **0.2**)

Ambiguous Unseeded Regions

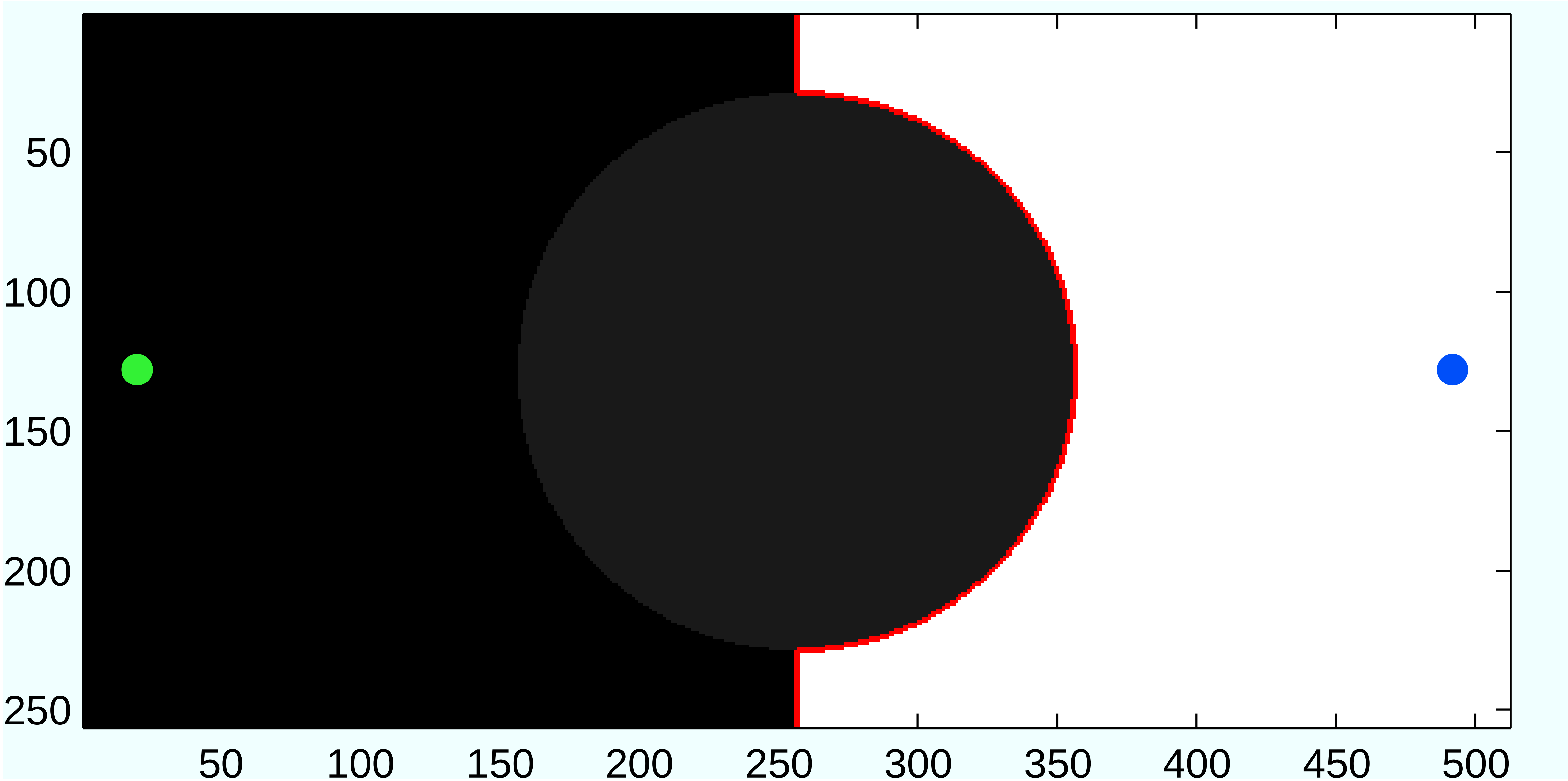

Figure 21: **Closer in intensity** to the **black** region (gray value **0.1**)

Topics

Properties and Effects
- Neutral Segmentation
- Weak Boundaries
- Noise Robustness
- Ambiguous Unseeded Regions

Summary
- Take Home Messages
- Further Readings

Take Home Messages

- The segmentation with random walker works quite well with weak boundaries.
- The algorithm is robust against noise to some degree.
- Unseeded regions are also handled by the algorithm in a useful way.

Further Readings

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Random Walker – Application Example

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Topics

Implementation

Example

Summary

Take Home Messages

Further Readings

Implementation

MATLAB Implementation

- Graph Analysis Toolbox available for MATLAB to:
  - easily build weighted image graphs,
  - solve the combinatorial Dirichlet problem.
- With specialty code using the toolbox above, the random walker segmentation can be performed:
  - recommended for research purposes,
  - sufficient for 512×512 images,
  - more industrial use requires C++ implementation of conjugate gradients or multigrid code.

Topics

Example: Axial CT Slice

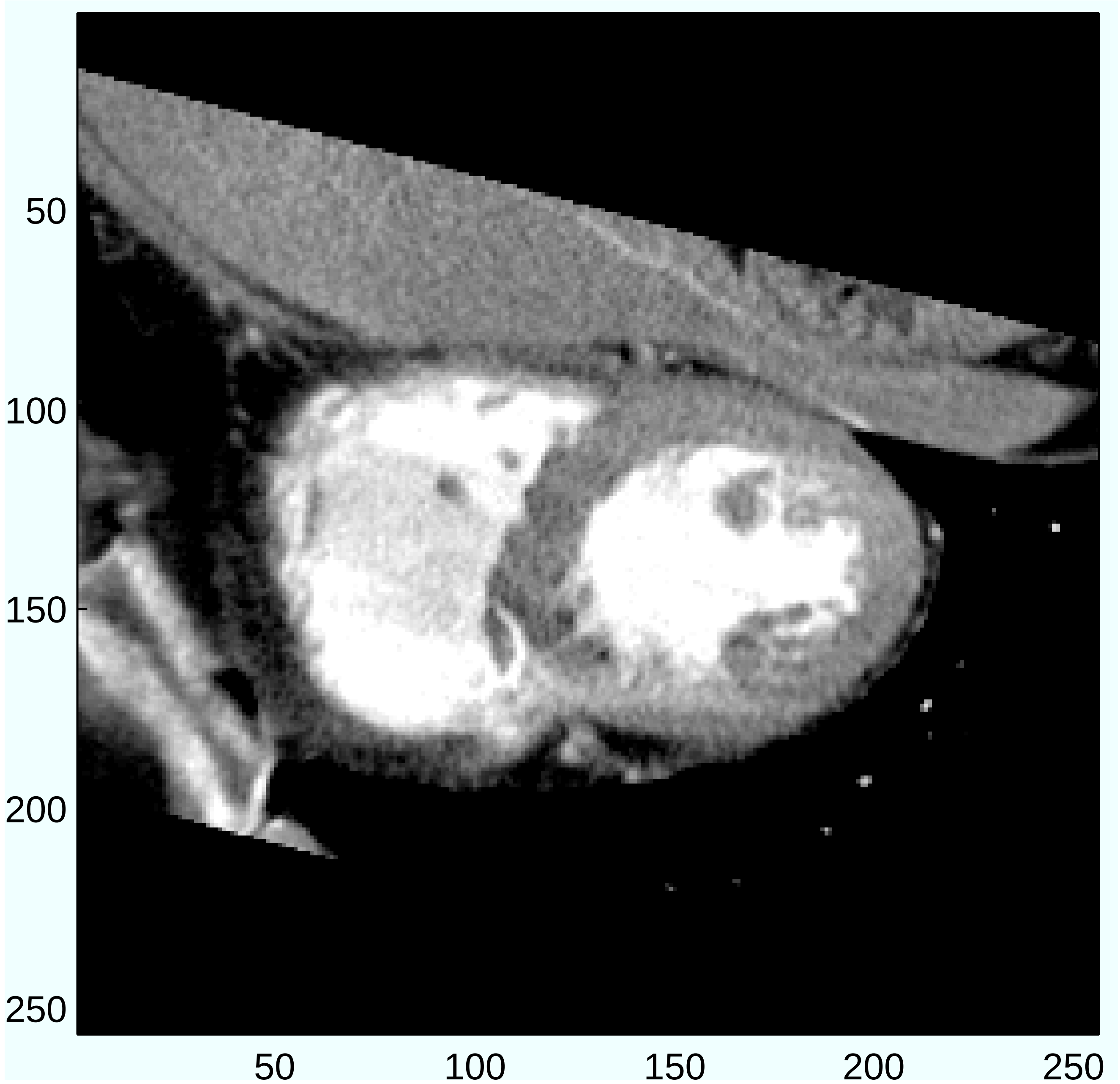
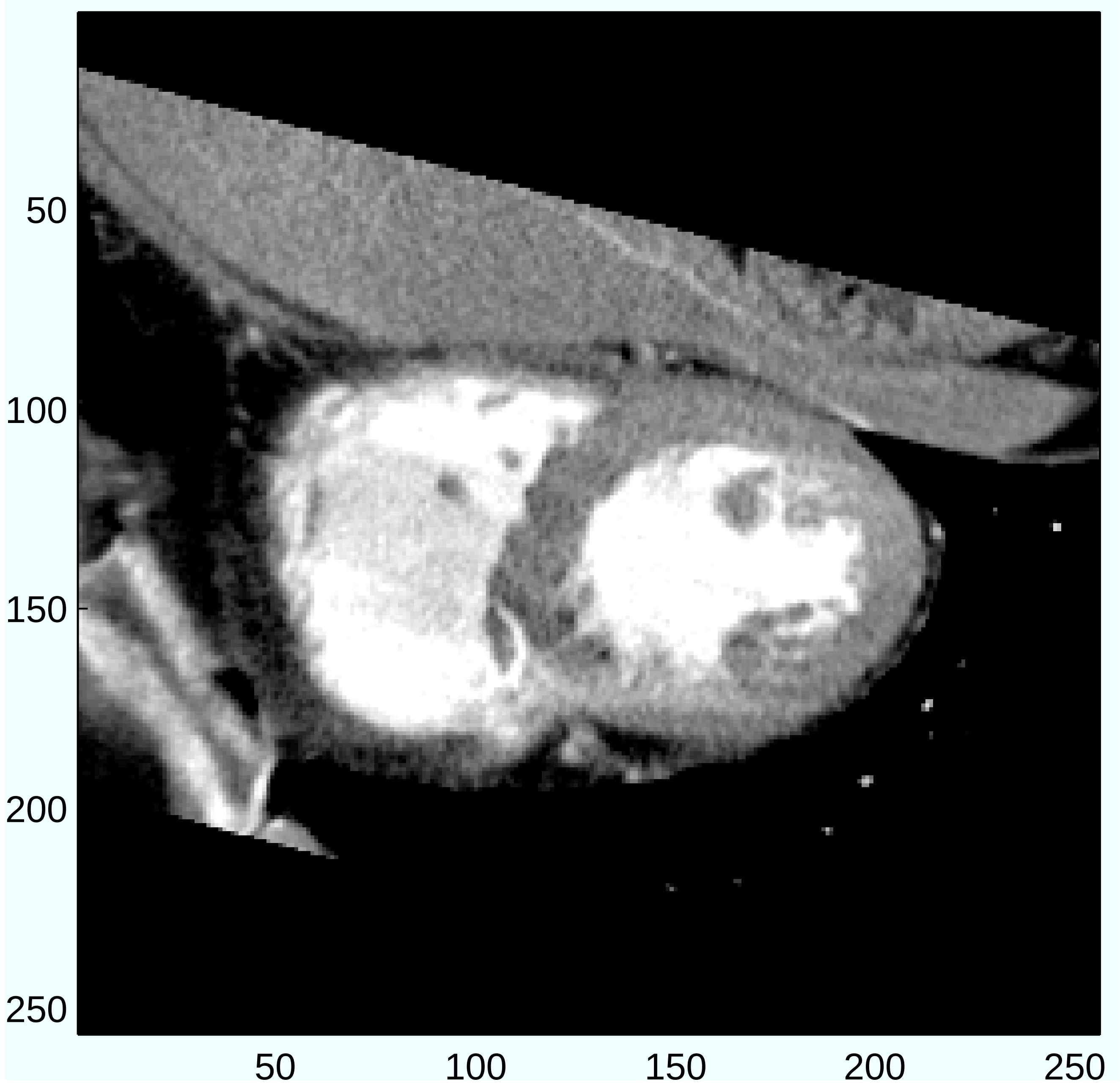


Figure 1: Original image

Example: Axial CT Slice



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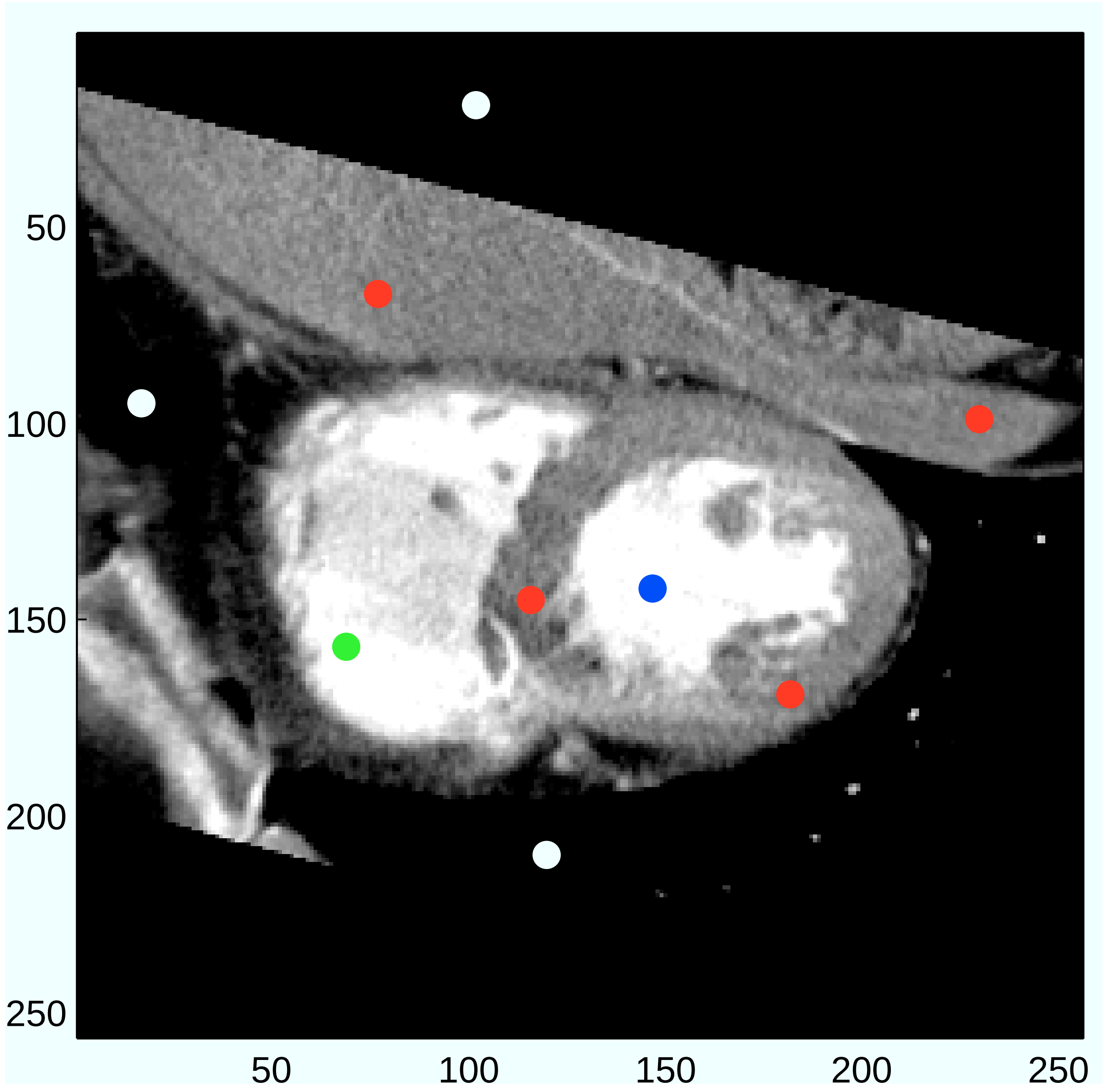

Figure 2: Original image with seed points

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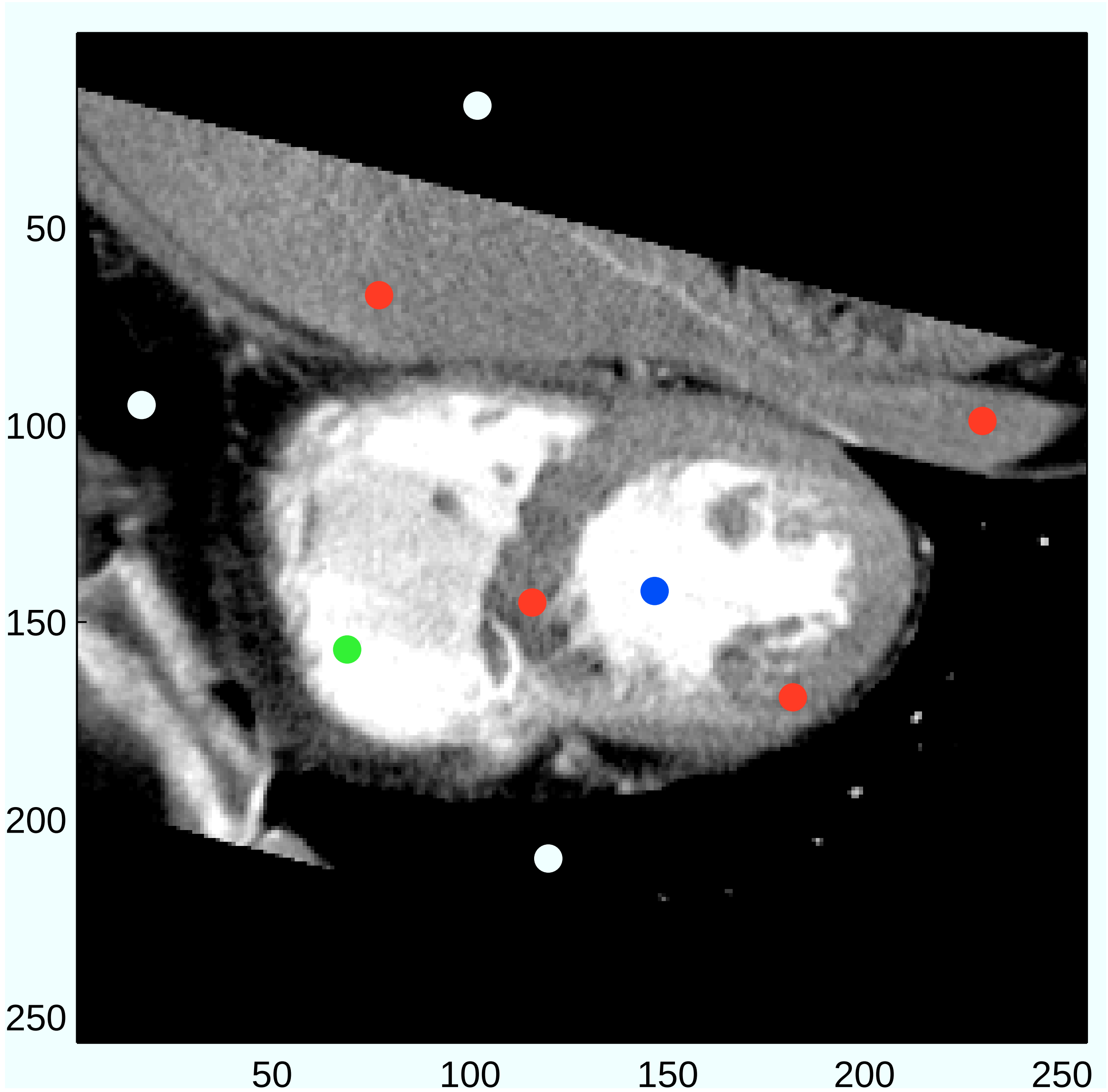

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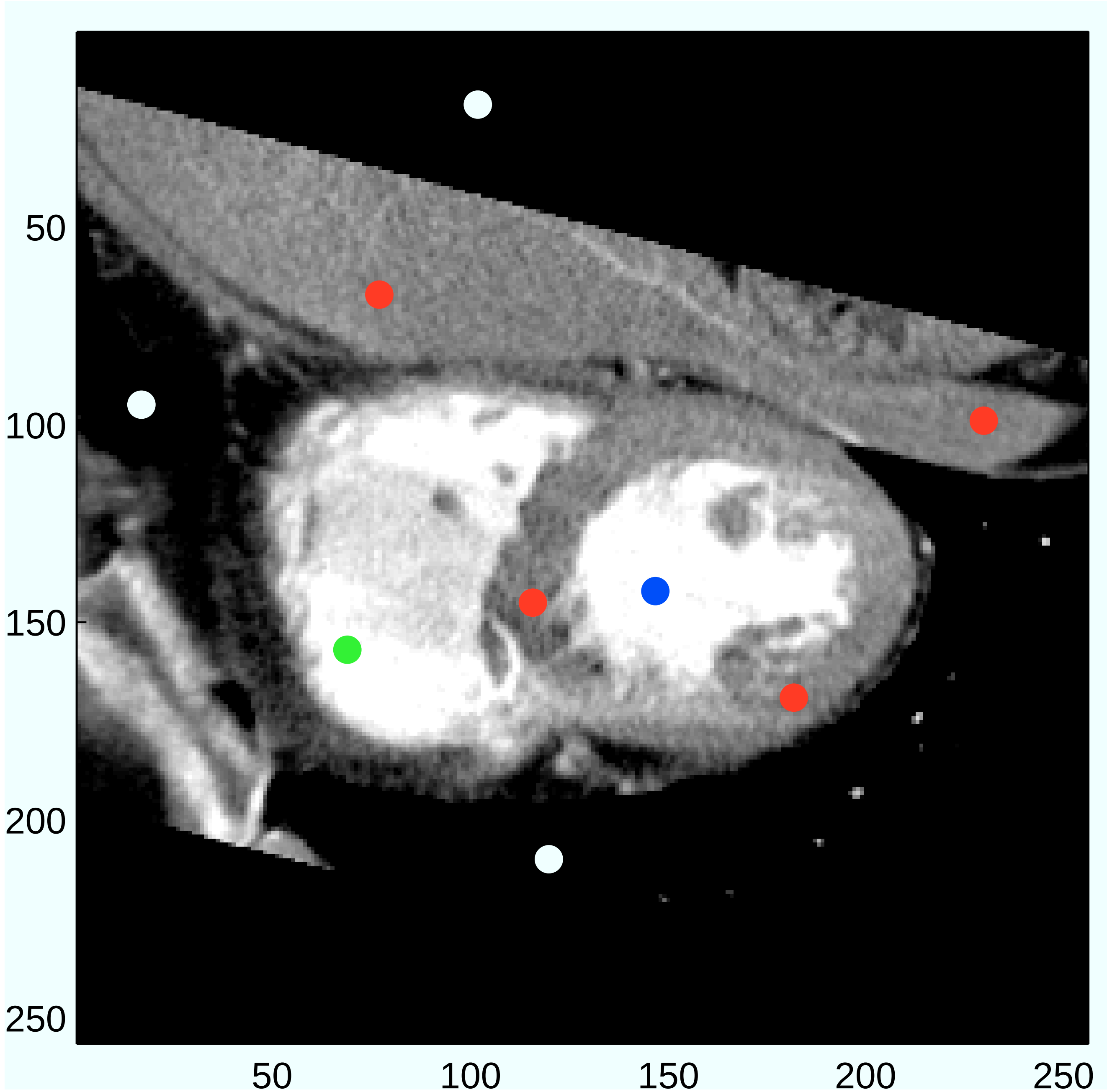

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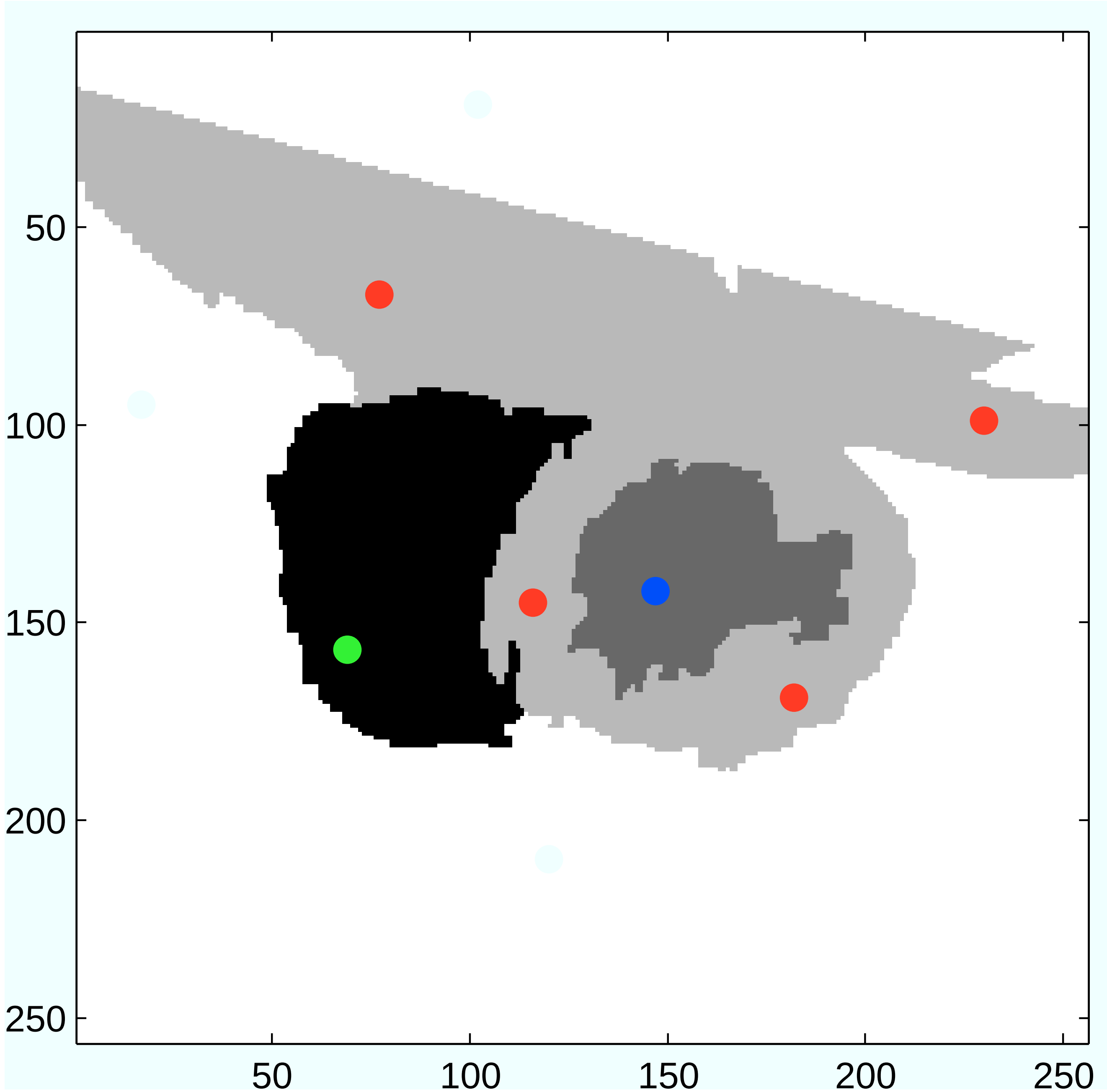

Figure 3: Output mask

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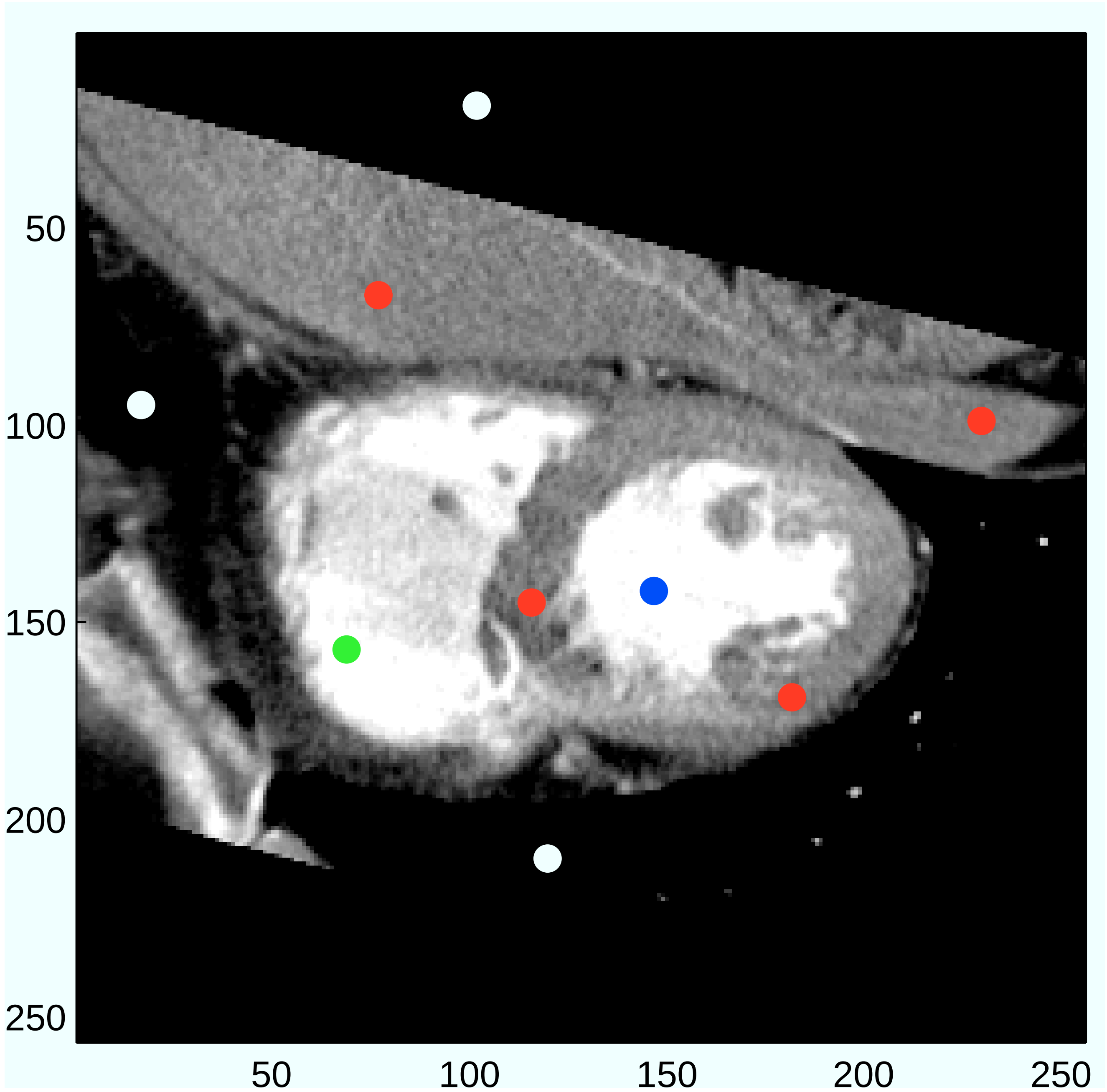

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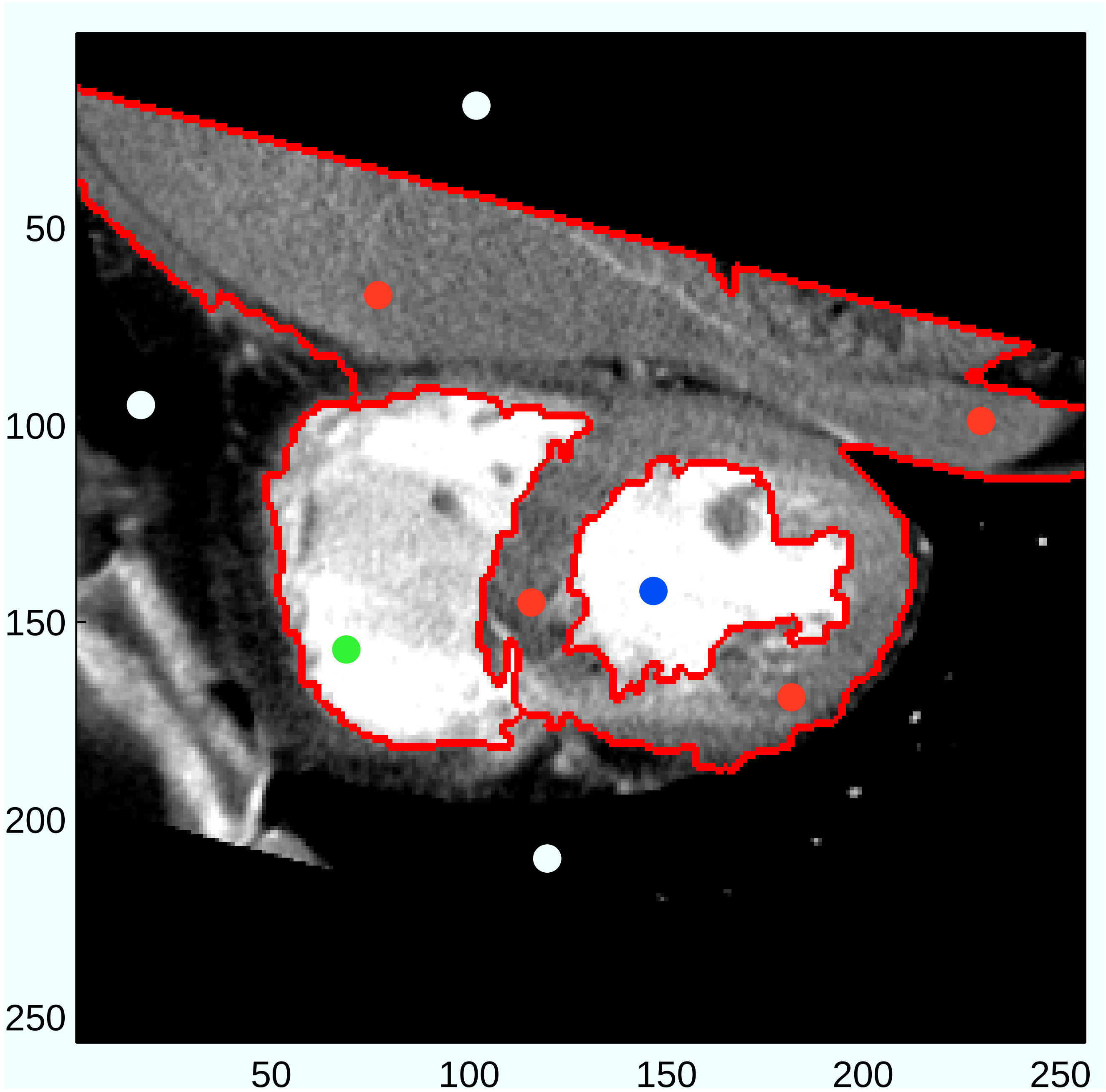

Figure 4: Outlined mask

Example: Axial CT Slice

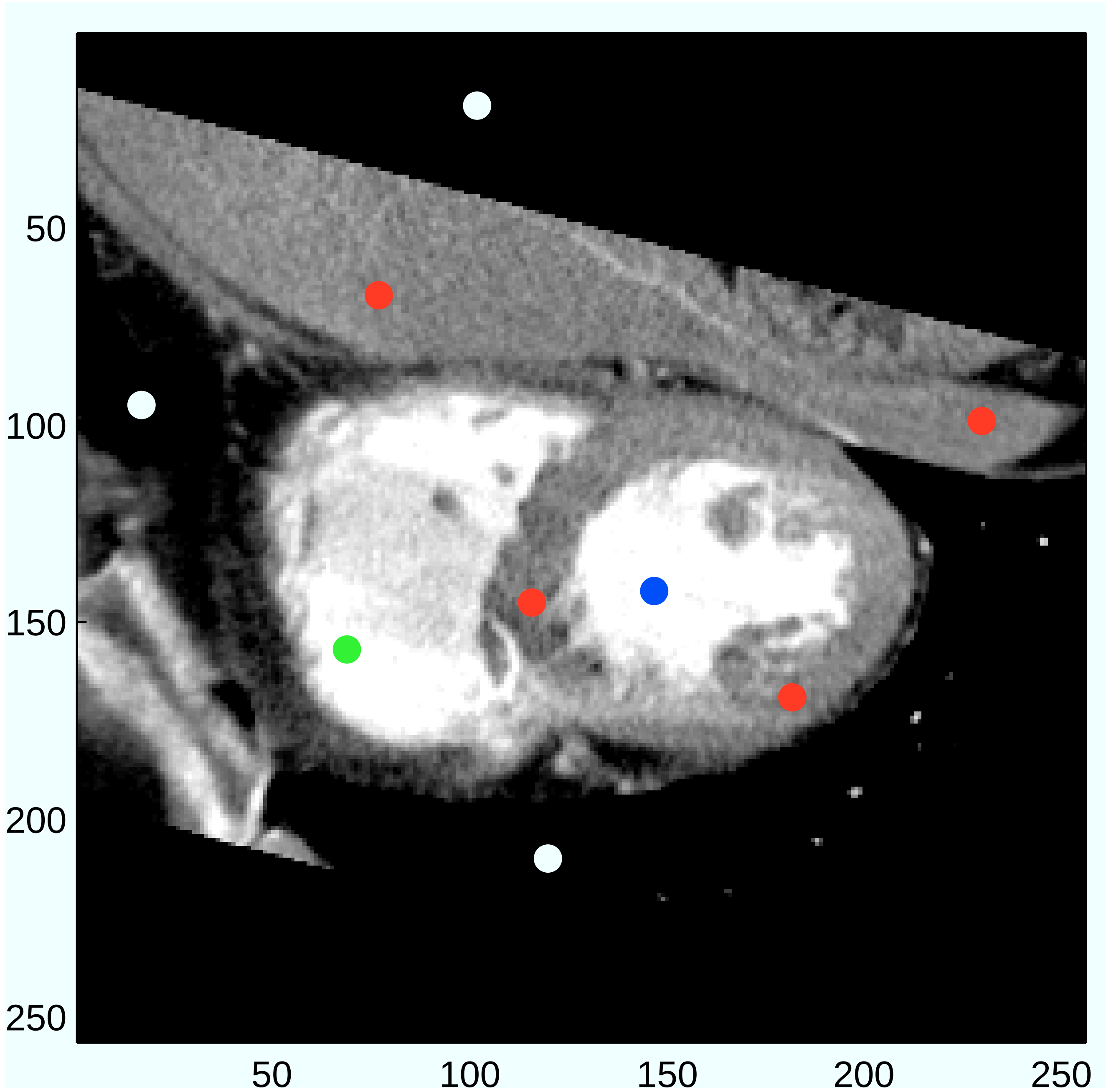

Figure 2: Original image with seed points

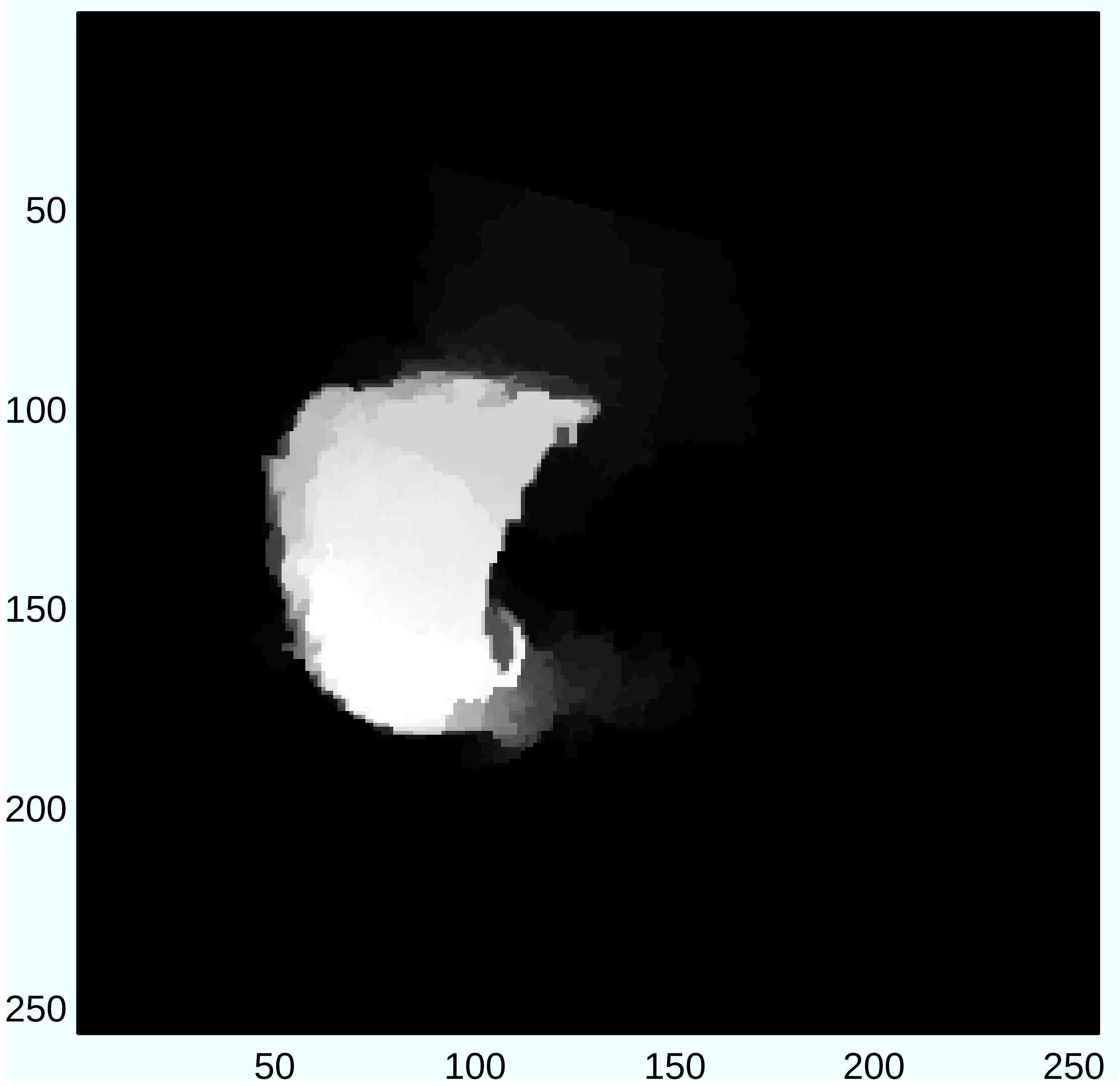

Figure 5: Probabilities for reaching seed 1

Example: Axial CT Slice

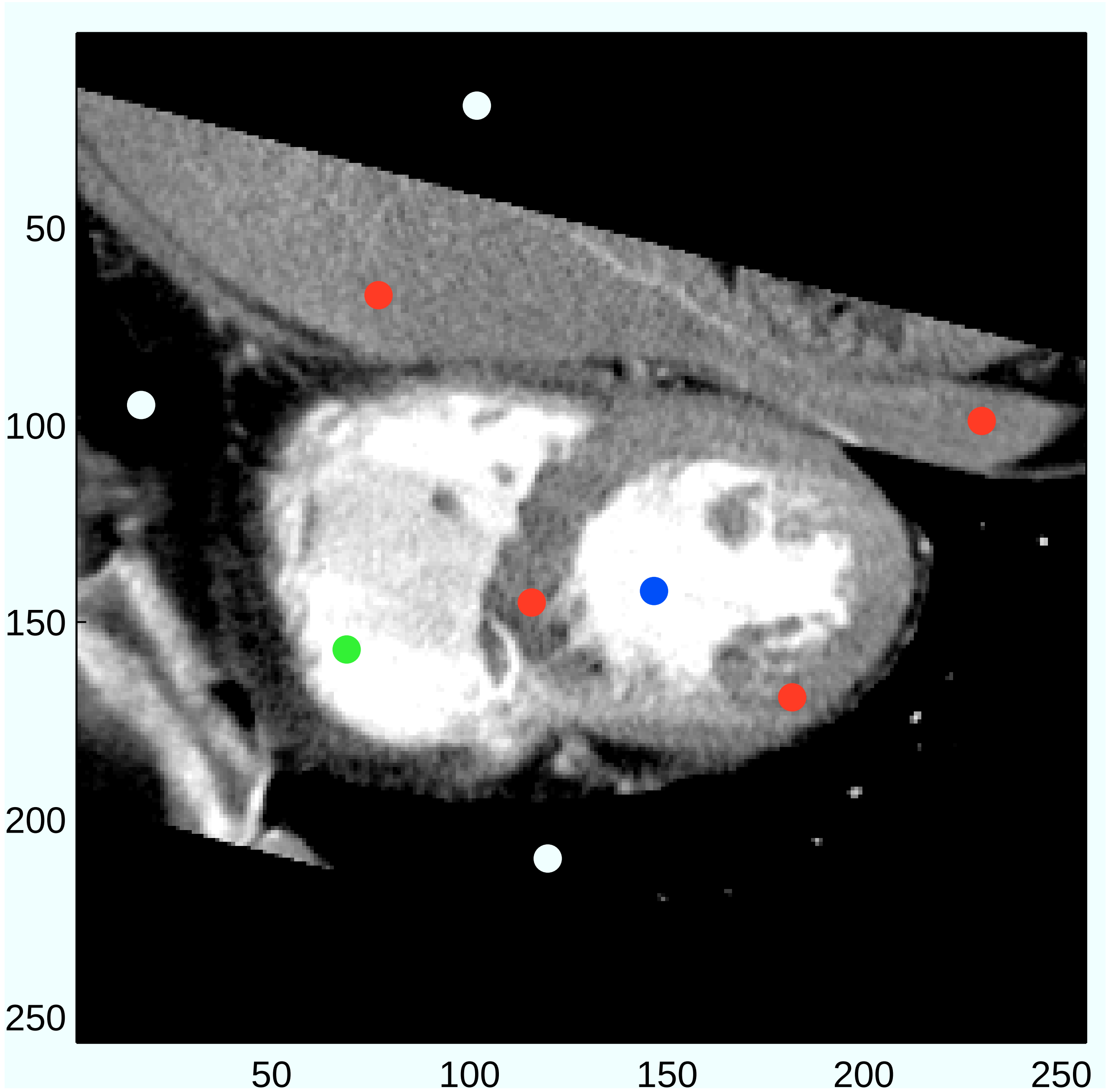

Figure 2: Original image with seed points

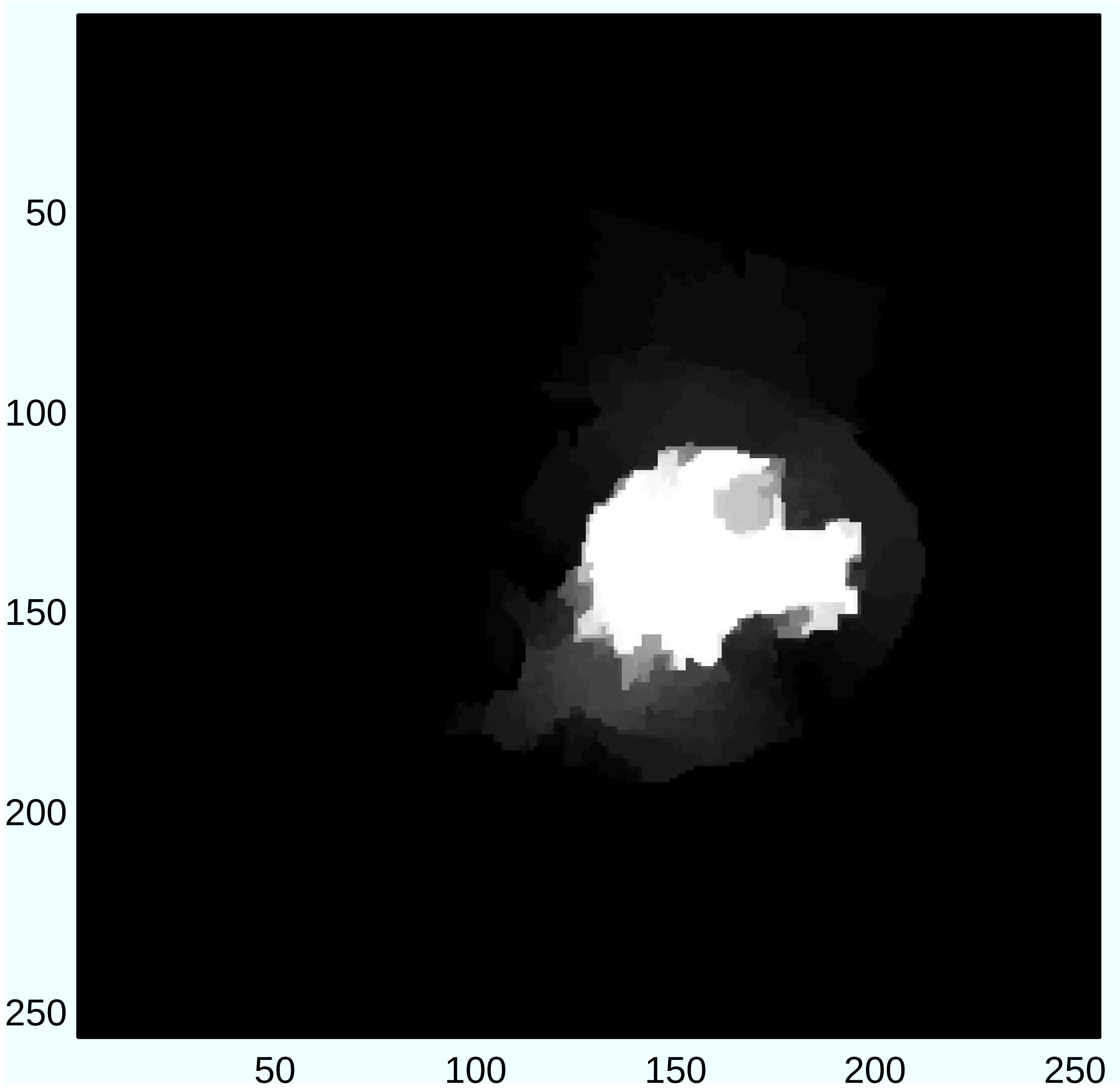

Figure 6: Probabilities for reaching seed 2

Example: Axial CT Slice

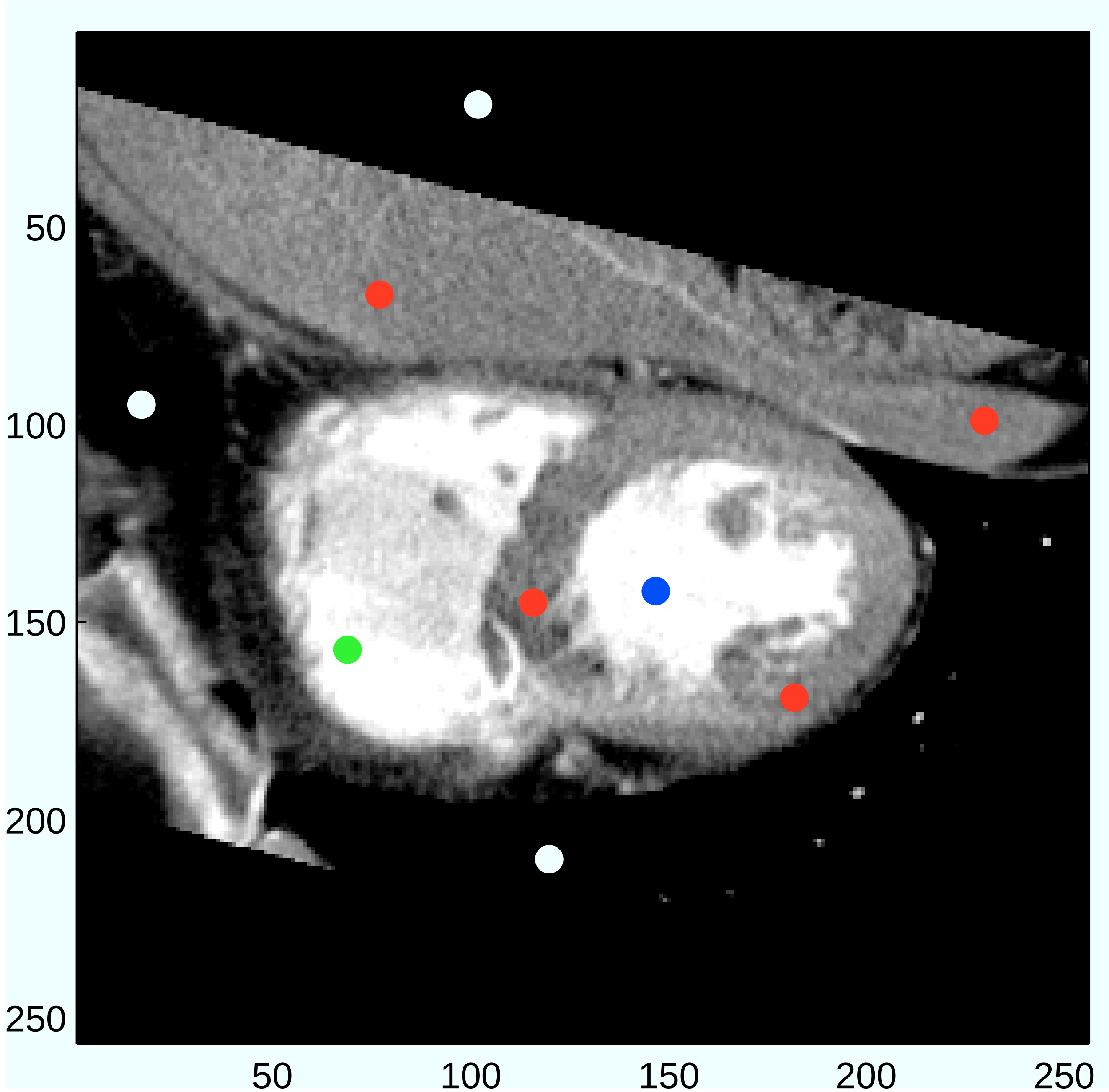

Figure 2: Original image with seed points

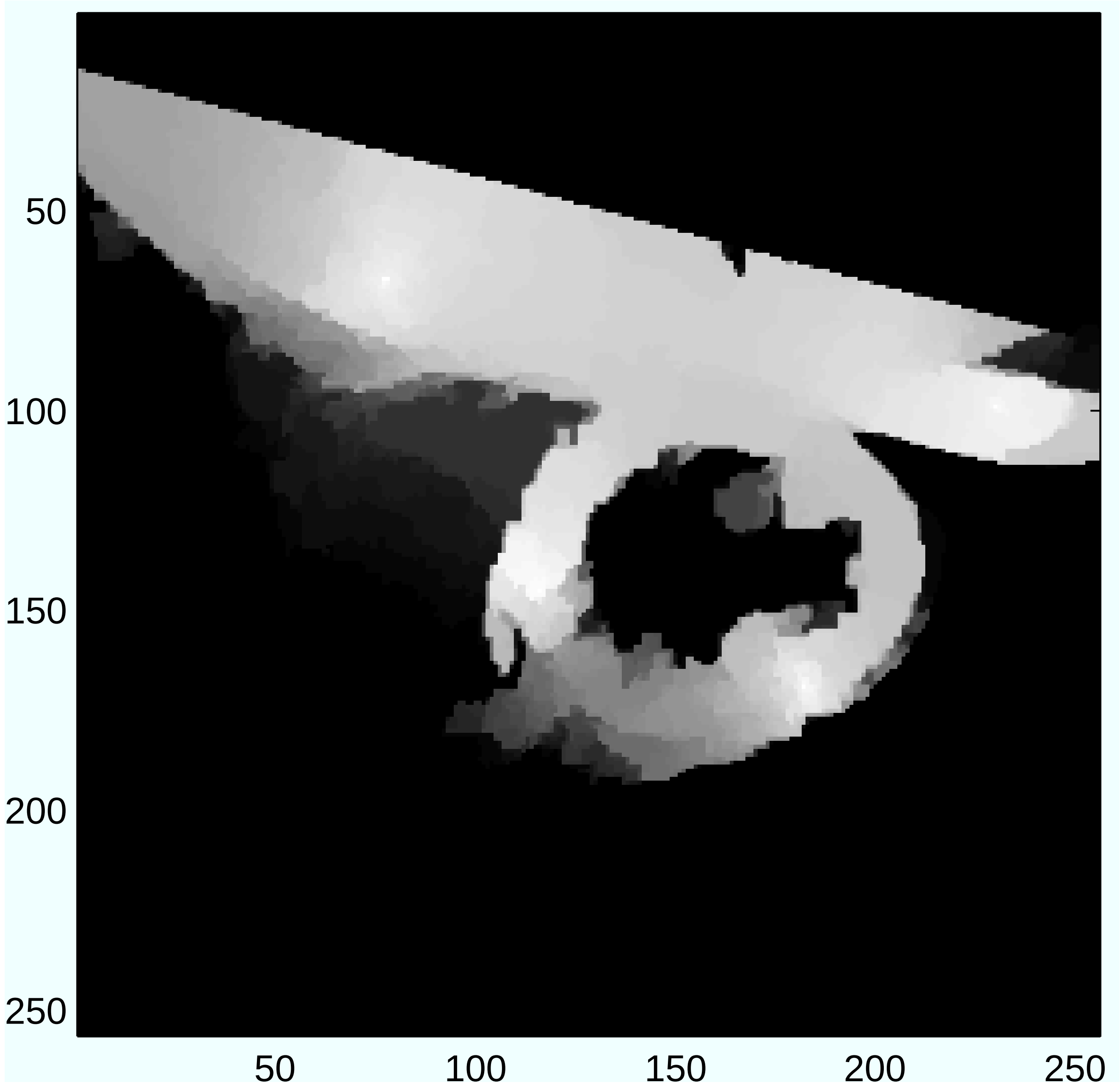

Figure 7: Probabilities for reaching seed 3

Example: Axial CT Slice

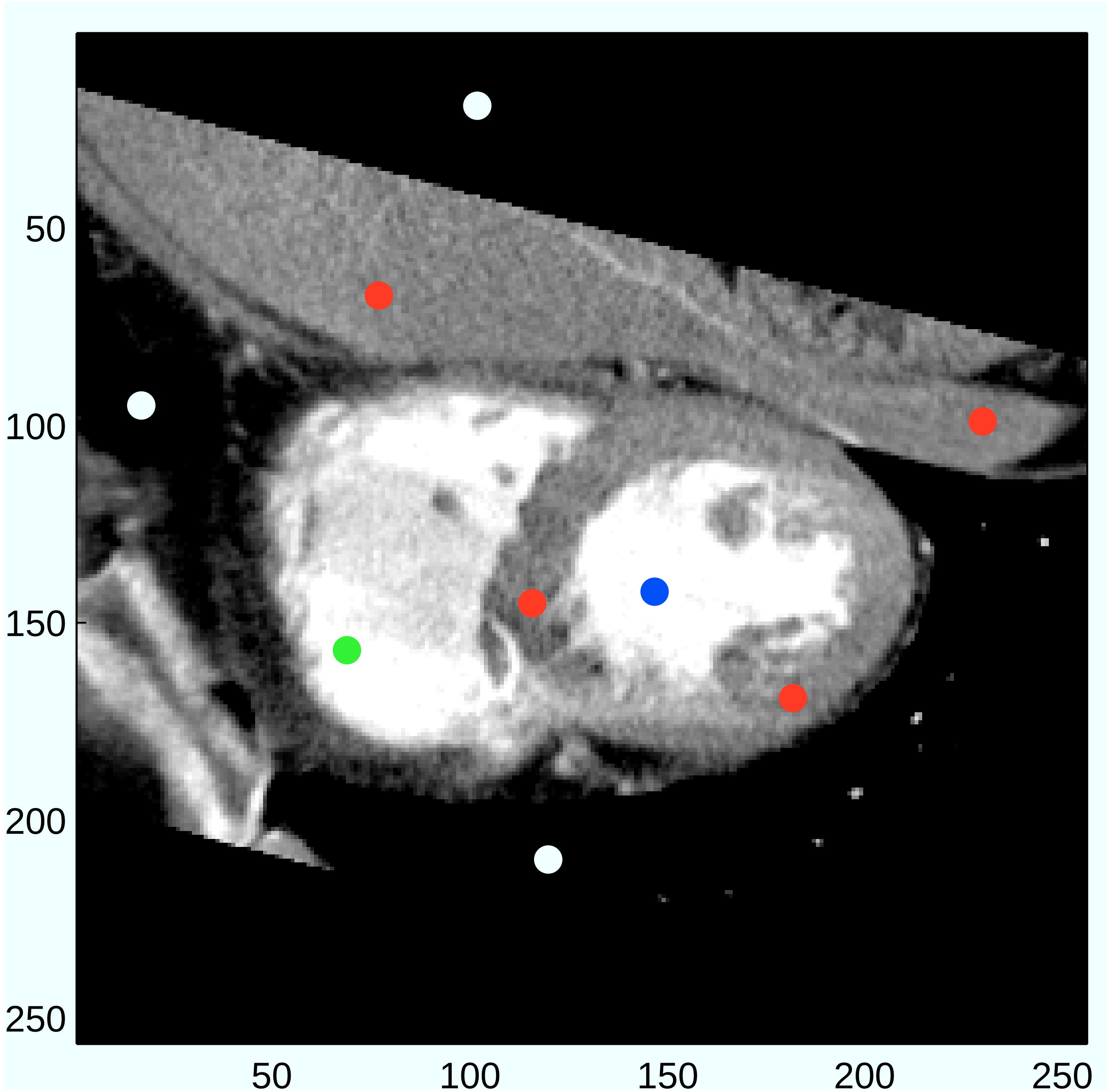

Figure 2: Original image with seed points

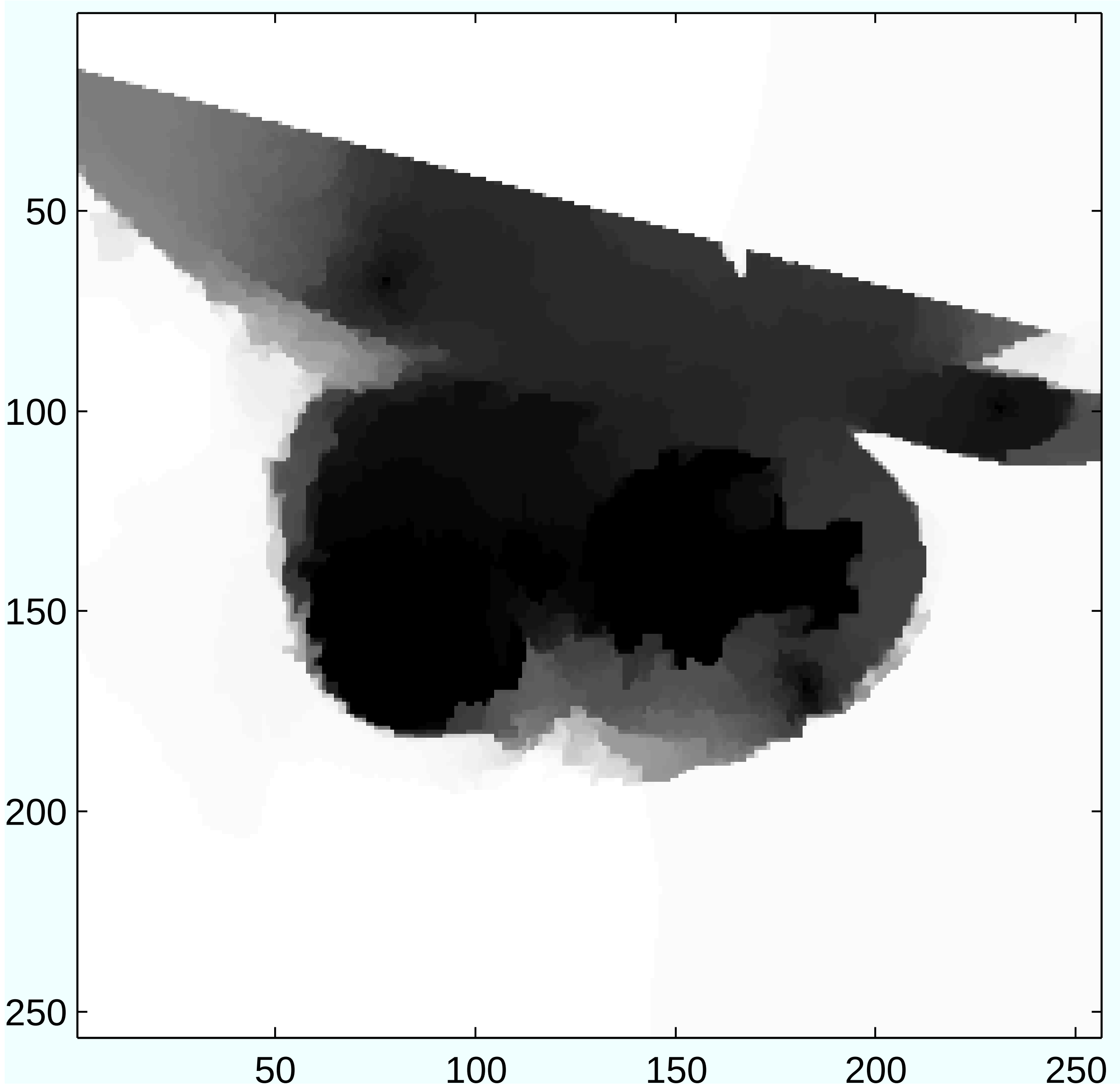

Figure 8: Probabilities for reaching seed 4

Topics

Implementation

Example

Summary

Take Home Messages

Further Readings

Take Home Messages

- The random walker algorithm has a straightforward implementation with only one free parameter.
- It is computed by solving a sparse, symmetric, positive-definite system of equations.
- This method has a lot of useful properties:
  - segments are guaranteed to be connected,
  - noise robustness,
  - no discretization errors,
  - efficient performance.
- Interactive editing is a possible extension, where a previous solution is used as an initial solution for an iterative matrix solver.

Further Readings

These slides are based on the following publication:

L. Grady. "Random Walks for Image Segmentation". In: *IEEE Transactions on Pattern Analysis and Machine Intelligence* 28.11 (Nov. 2006), pp. 1768–1783. DOI: 10.1109/TPAMI.2006.233

His implementations in Matlab can be downloaded here:

- Graph Analysis Toolbox
- Random Walker

Medical Image Processing for Interventional Applications

Statistical Shape Models – Part 1

Online Course – Unit 44

Andreas Maier, Frank Schebesch
Pattern Recognition Lab (CS 5)

virtuelle hochschule bayern

FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG
TECHNISCHE FAKULTÄT

Topics

Statistical Shape Models

- Introduction
- Overview of Methods
- Shape Representation

Summary

- Take Home Messages
- Further Readings

Introduction

Segmentation is difficult in presence of noise and artifacts (e.g. streaks).

$\rightarrow$ Prior knowledge about the general shape of the organ is helpful.

Problem: Human anatomy is very different between different subjects.

$\rightarrow$ Description needs to be able to describe these variations.

Model-based methods

- Applied since 1990s successfully in 2-D
- Expansion to 3-D available and applied successfully

- Model describes expected shape and appearance
- Applied in a top-down approach in contrast to low-level methods
- Increased stability compared to low-level methods

- Single template models only useful in industrial applications (e. g., no variations expected)
- In medical applications $\rightarrow$ Variation needs to be modeled!

Other Approaches

Freely deformable models:

- Seminal snakes (smoothness + image fit)
- Deformable simplex mesh (template shape + image fit)

→ Models can be adopted to specific shapes but do not allow to model variations.

Other Approaches

Freely deformable models:

- Seminal snakes (smoothness + image fit)
- Deformable simplex mesh (template shape + image fit)

$\rightarrow$ Models can be adopted to specific shapes but do not allow to model variations.

Level-sets (see also Cremers, Rousson, Deriche, 2007):

- Implicit shape representation with regional or edge-based features
- Learning-based methods can be integrated in the energy function.
- Distance maps do not form a linear space:

  $\rightarrow$ They can lead to invalid shapes (in cases with too much variations in the training data).

  $\rightarrow$ Transformation into a different space is beneficial.

General Structure

1. Shape representation
2. Extraction of principal modes
3. Finding correspondences between shapes

Shape Representation

Input:

- Segmented volumetric images
- Surface meshes
- Any other shape representation that can be converted into one of the above (B-Spline, NURBS)

$\rightarrow$ Representation is crucial for the resulting **statistical shape model**.

Landmarks and Meshes

- Simple, but very generic
- Fixed number K of points as a long vector:

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ \vdots \\ x_K \\ y_K \\ z_K \end{pmatrix}$$

- Points are referred to as ***landmarks***.
  (They do not need to be actual anatomical landmarks $\rightarrow$ ***semi-landmarks***.)
- If connectivity information available $\rightarrow$ mesh
- Landmarks only as a basis for a statistical shape model yield a **point distribution model** (PDM).

Other Models

- Medial models:
  - Use center lines and radii $\rightarrow$ centerline points and vectors pointing to boundary
  - Called ***m-rep*** in 3-D
- Fourier surfaces: Composition of a certain topology from base shapes
- Spherical Harmonics: Composition of spheres to describe a closed surface
- NURBS: **N**on-**u**niform **r**ational **B-s**plines
- Wavelets, etc. ...

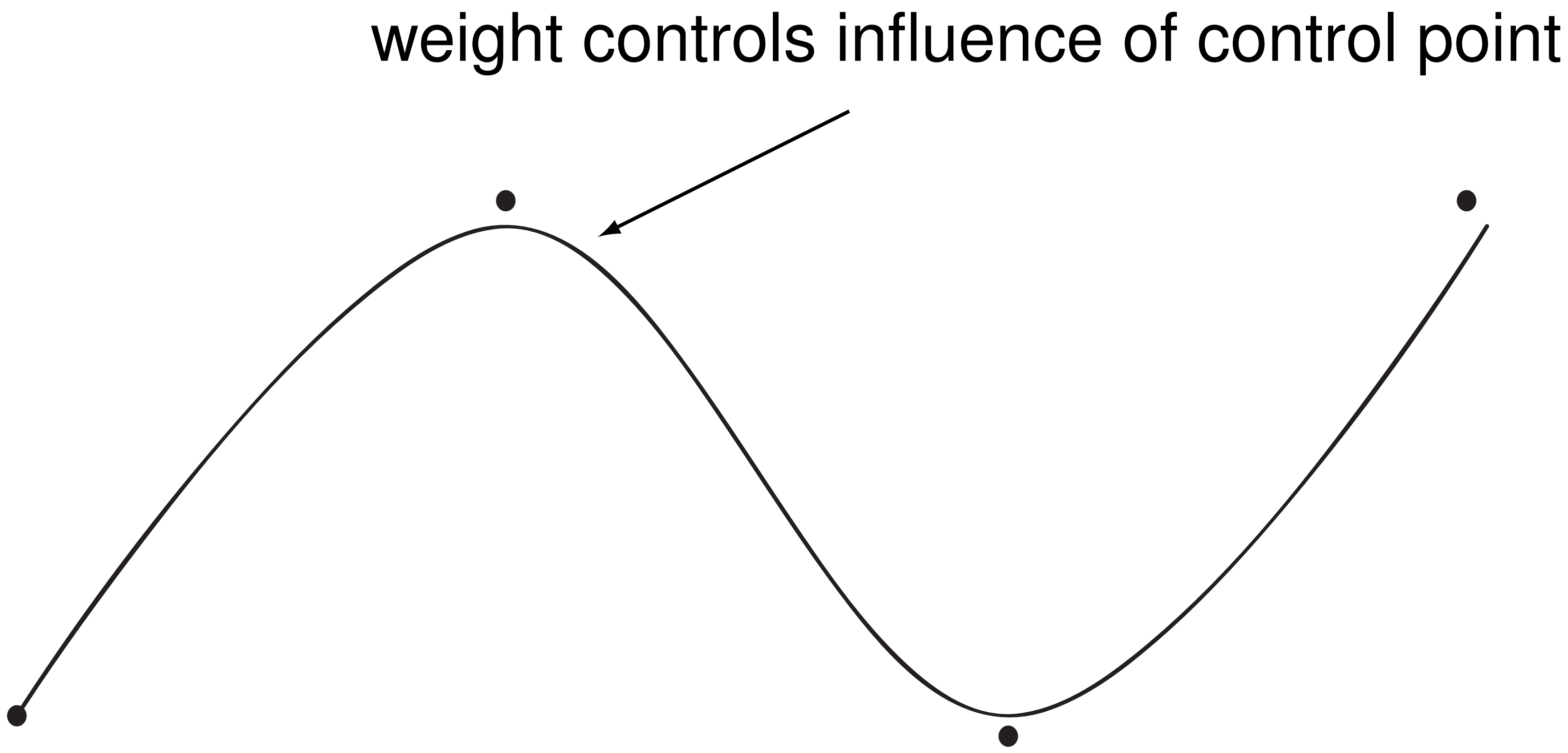

Figure 1: NURBS example

Topics

Statistical Shape Models
- Introduction
- Overview of Methods
- Shape Representation

Summary
- Take Home Messages
- Further Readings

Take Home Messages

- Statistical shape models are used in a series of approaches to incorporate prior data into a segmentation algorithm.
- A large variety of methods has been researched which can be studied starting with the review of Heimann and Meinzer, 2009.
- Shape can be represented simply by so-called landmarks, for example, but there exist other elaborate models as well.

Further Readings

These review papers are a good start for learning more about the methods described in this unit:

- Tobias Heimann and Hans-Peter Meinzer. "Statistical Shape Models for 3D Medical Image Segmentation: A Review". In: *Medical Image Analysis* 13.4 (Aug. 2009), pp. 543–563. DOI: 10.1016/j.media.2009.05.004
- Daniel Cremers, Mikael Rousson, and Rachid Deriche. "A Review of Statistical Approaches to Level Set Segmentation: Integrating Color, Texture, Motion and Shape". In: *International Journal of Computer Vision* 72.2 (Apr. 2007), pp. 195–215. DOI: 10.1007/s11263-006-8711-1

They also contain lots of references for further reading.

Medical Image Processing for Interventional Applications

Statistical Shape Models – Part 2

Online Course – Unit 45
Andreas Maier, Frank Schebesch
Pattern Recognition Lab (CS 5)

Topics

Shape Model Construction

- Extract mean shape
- Extract a number of modes of variation

$\rightarrow$ Varies with the shape representation, **here**: PDMs with known correspondences

Alignment

- Shape is independent of translation, rotation and scaling.
- Normalization is required.

Generalized Procrustes alignment (GPA):

- Procrustes alignment: Minimize mean square distance between two shapes (1 base shape $+$ 1 reference shape $\Rightarrow$ 1 transformation)
- GPA: Iterate over all shapes to minimize distance to mean shape $\bar{\boldsymbol{x}}$
- Other norms than L_2 are used (e. g., $L_1, L_\infty, \ldots$).
- Tangent space is useful for improved estimation of the variation modes.
- Misalignment yields incorrect modes of variation.

Dimensionality Reduction

$\rightarrow$ Find modes of variation

Principal component analysis (PCA):

- Mean shape:

$$\bar{\boldsymbol{x}} = \frac{1}{s} \sum_{i=1}^{s} \boldsymbol{x}_i$$

- Covariance matrix:

$$S = \frac{1}{s-1} \sum_{i=1}^{s} (\boldsymbol{x}_i - \bar{\boldsymbol{x}})(\boldsymbol{x}_i - \bar{\boldsymbol{x}})^{\mathrm{T}}$$

Dimensionality Reduction

- Eigenvectors $\boldsymbol{\phi}_m$ and eigenvalues λ_m are used to identify the c-modes of variations (e. g., scree test).
- SVD of the aligned landmark matrix L delivers more stable results:

$$L = ((\boldsymbol{x}_1 - \bar{\boldsymbol{x}}) \ldots (\boldsymbol{x}_s - \bar{\boldsymbol{x}})) .$$

- Maximum number of modes is $\max\{s-1, 3K\}$.
- Robust PCA can help with outliers.

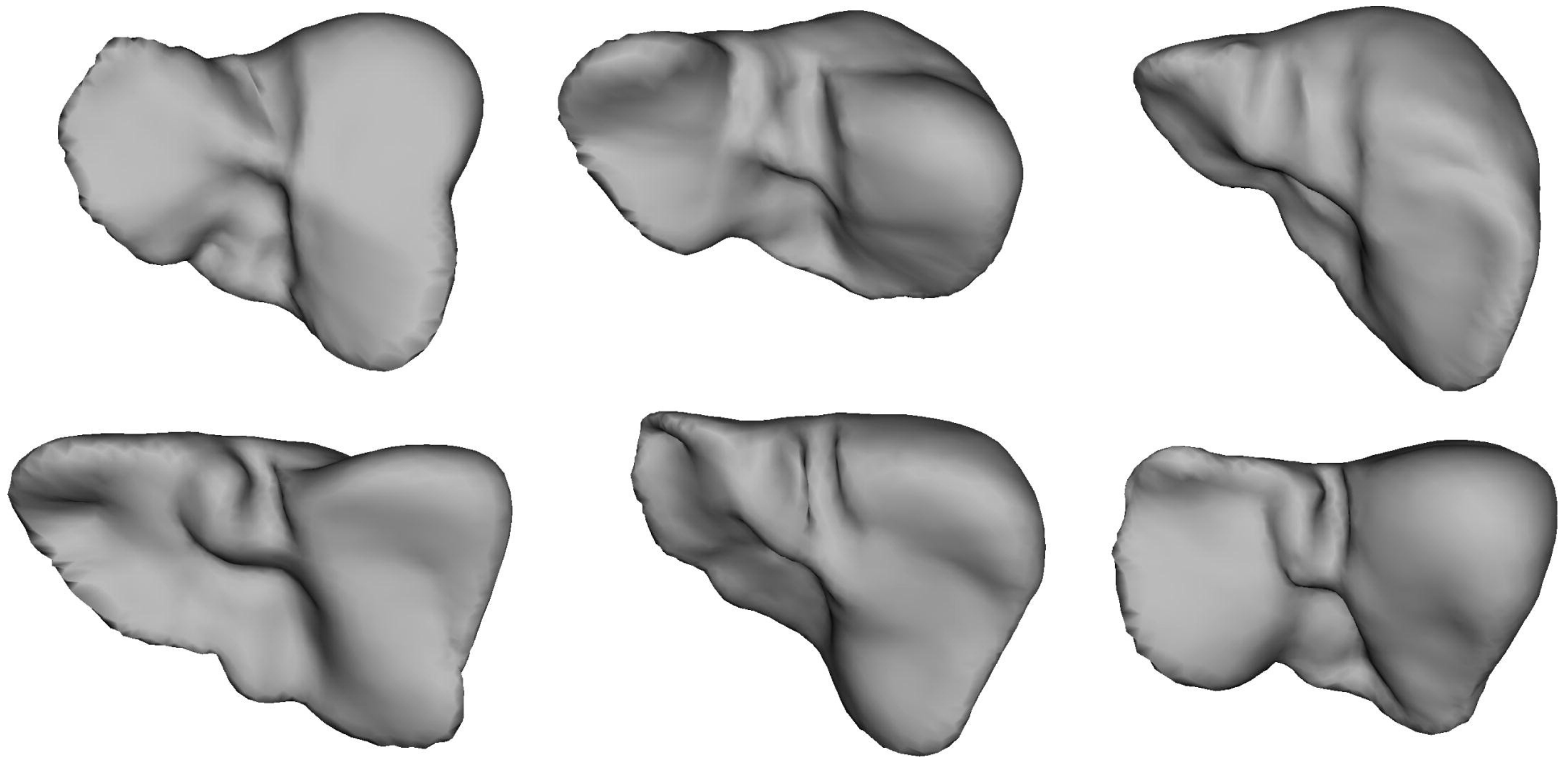

Figure 1: Variations of a shape model

Dimensionality Reduction

- Valid shapes are now expressed as linear combinations of the mean shape and its modes $\boldsymbol{\phi}_{\boldsymbol{m}}$:

$$\boldsymbol{x} = \bar{\boldsymbol{x}} + \sum_{m=1}^{c} b_m \boldsymbol{\phi}_m, \quad b_m \in [-3\lambda_m, 3\lambda_m].$$

- Independent component analysis (ICA) can also be used, but problems such as the ordering of the components arise.

Finding Correspondences: Mesh to Mesh

- Manual landmarks → high effort/infeasible
- Automatic registration

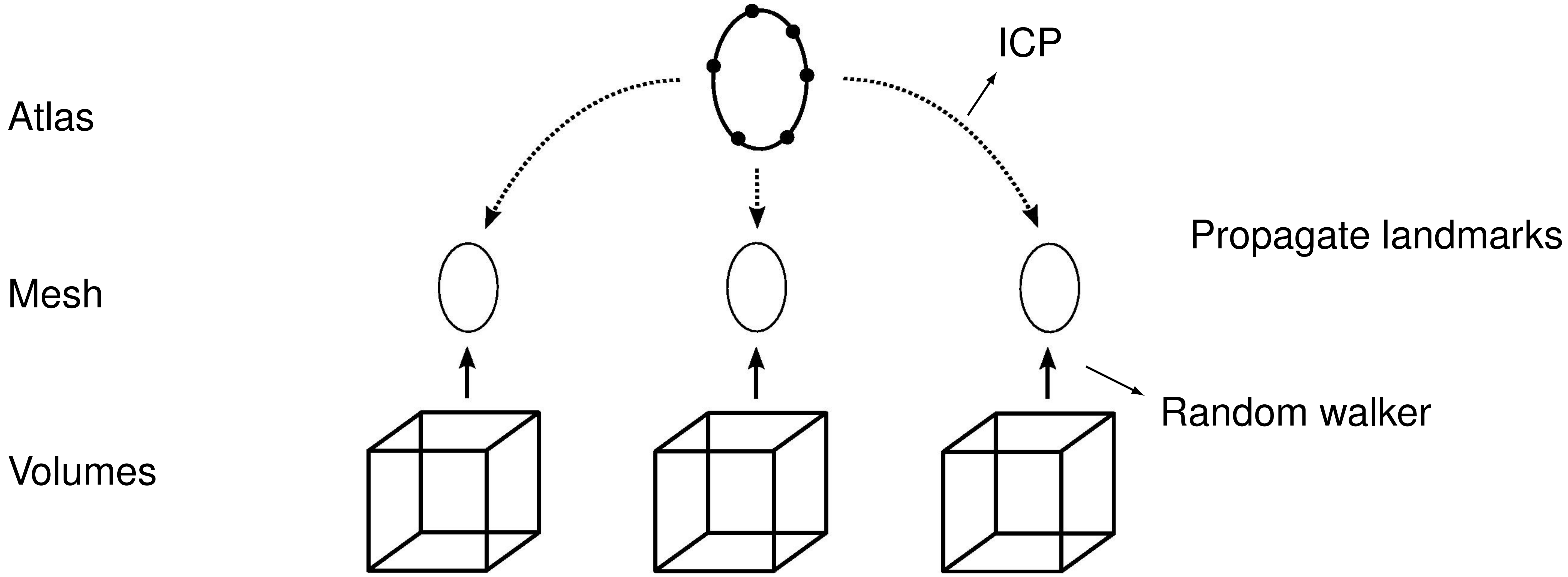

Figure 2: Mesh to mesh

Finding Correspondences: Mesh to Volume

→ Reference shape may introduce bias.

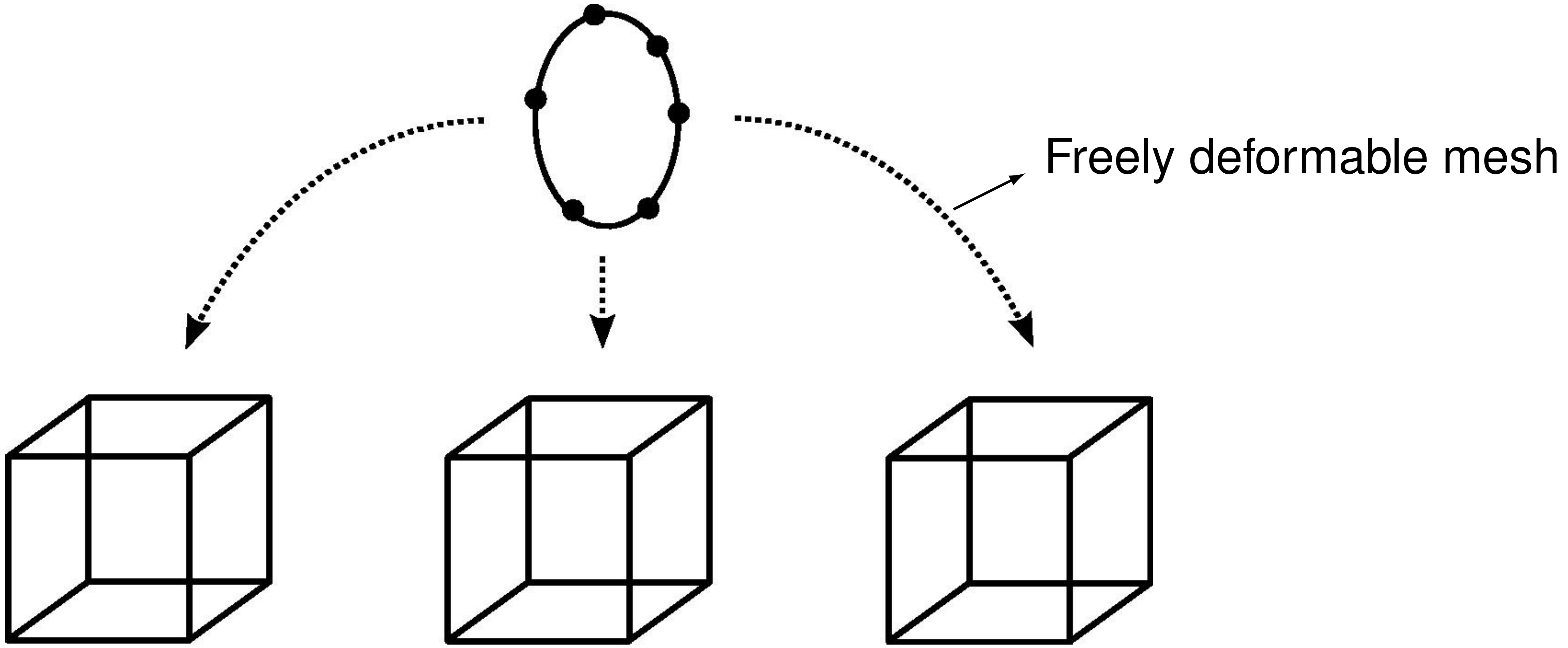

Figure 3: Mesh to volume

Finding Correspondences: Volume to Volume

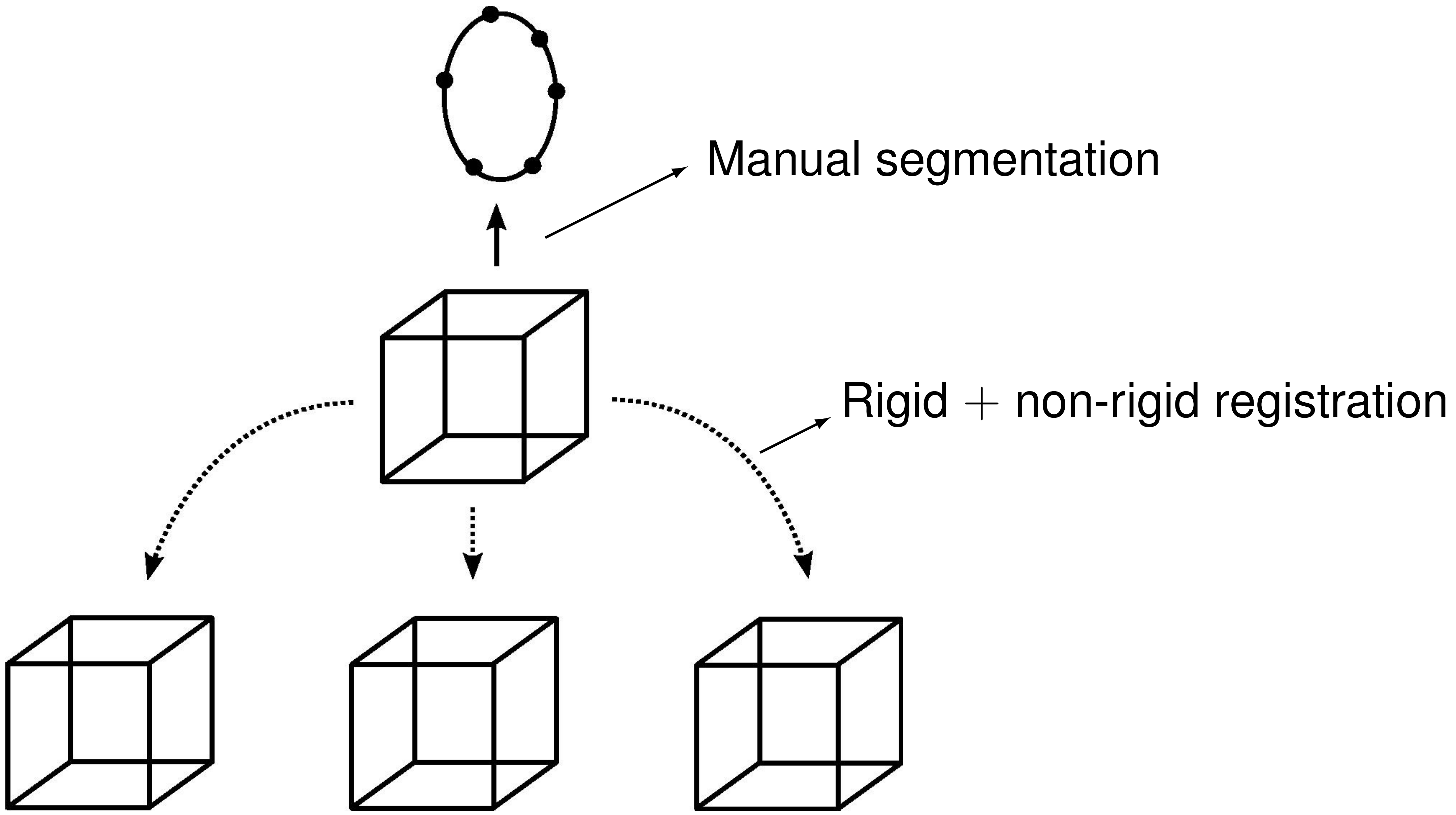

Figure 4: Volume to volume

Appearance Models

Models how the mode looks in the image:

- Boundary-based features (intensity + derivatives)
- Gabor wavelets
- Distances
- SIFT, MOG
- Classifiers, boosting
- Texture information

→ Delivers a fit / goodness for every landmark

Search Algorithms

Initialization:

- User / interaction
- Histogram
- Generalized Hough transform
- Particle filter
- Bounding box detection

Search Algorithms

Active shape models:

Search strategy based on a point distribution mode

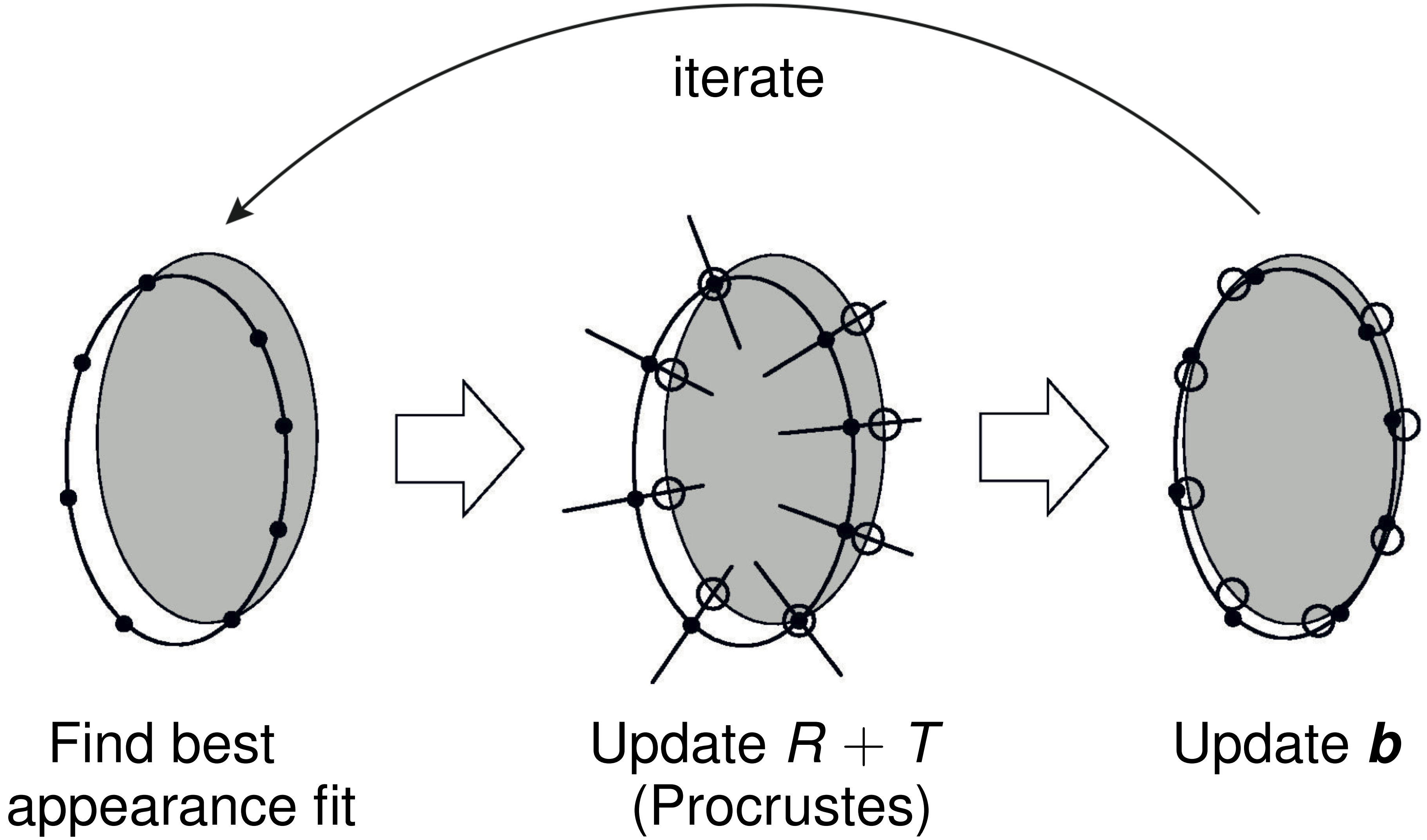

Figure 5: Scheme for active shape model

Search Algorithms

Active shape models:

Search strategy based on a point distribution mode

Active appearance models:

- Combine shape and appearance into a single model
- Use linear system to update parameters

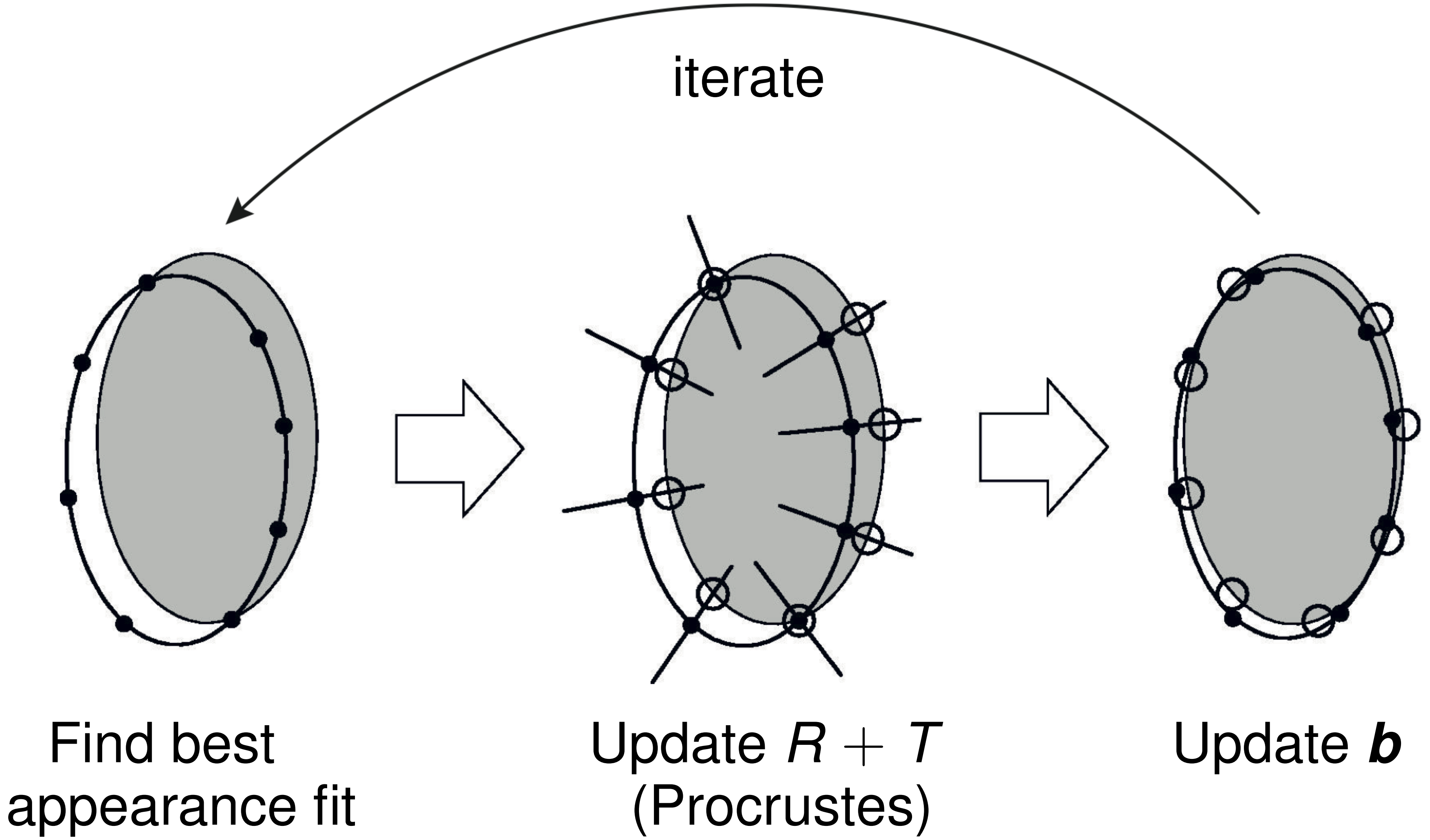

Figure 5: Scheme for active shape model

Applications

- Segmentation
  - Organs (heart, liver, etc.)
  - Bones (vertebrae, knee, femur, etc.)
- Interpretation of shapes
  - Hippocampus
    - Schizophrenia
    - Attention deficit disorder
    - Alzheimer's disease
  - Analysis of aging
- Shape extrapolation
  - 3-D+t heart shape from 2-D+t MRI / US data
  - Complete bone shapes from sparse 3-D data
- And many more applications

Topics

Statistical Shape Models

Shape Model Construction

Finding Correspondences

Appearance Models

Search Algorithms

Applications

Summary

Take Home Messages

Further Readings

Take Home Messages

- Concepts like GPA and PCA are used to align and analyze the shape for a given point set.
- New data is registered to the corresponding model which can be performed using mesh to mesh, mesh to volume, or volume to volume registration.
- Active shape models and active appearance models are popular concepts for statistical shape models.

Further Readings

These review papers are a good start for learning more about the methods described in this unit:

- Tobias Heimann and Hans-Peter Meinzer. "Statistical Shape Models for 3D Medical Image Segmentation: A Review". In: *Medical Image Analysis* 13.4 (Aug. 2009), pp. 543–563. DOI: 10.1016/j.media.2009.05.004
- Daniel Cremers, Mikael Rousson, and Rachid Deriche. "A Review of Statistical Approaches to Level Set Segmentation: Integrating Color, Texture, Motion and Shape". In: *International Journal of Computer Vision* 72.2 (Apr. 2007), pp. 195–215. DOI: 10.1007/s11263-006-8711-1

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