Diagnostic Medical Image Processing
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Exercises (DMIP-E)
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Projection Models

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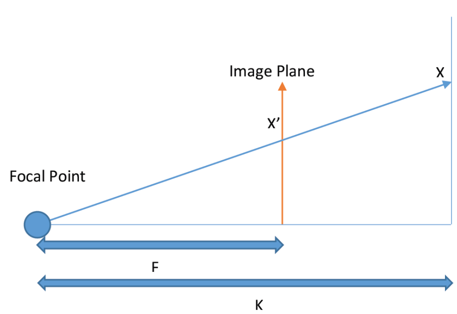
Exercise Sheet 3

7 Projections

(i) We want to find the projection of the point X on the image plane as shown in the figure.

- What kind of projection is that and how is X mapped to its projection point X' on the image plane?
- Which difficulty is connected with this mapping?
- Which kind of mapping could we use instead if we wanted to approximate the projection from above?
- For both projection models, write down the mapping for 3-D to 2-D *cartesian* coordinates.



(ii) Of the following projection models, for which ones do all projected points pass through the origin of the camera coordinate system?

☐ respective projection

■ weak-perspective projection

■ perspective projection

☐ orthographic projection

■ paraperspective projection

☐ no-perspective projection

2+1

(i) The shown projection is perspective:

$$\frac{X'}{F} = \frac{X}{K} \Rightarrow X' = F\frac{X}{K}.$$

It is not a linear mapping which makes it more difficult. Linear or affine mappings are easier to handle in general, so as an approximation one could use orthogonal, weak-perspective, or para-perspective projections. Either of these count as an answer. The perspective mapping in "normal" coordinates is:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} f \cdot x/z \\ f \cdot y/z \end{pmatrix}, \qquad \text{(perspective)}$$

$$\begin{pmatrix} k \cdot x \\ k \cdot y \end{pmatrix} = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad \text{(weak-perspective)}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad \text{(orthogonal)}$$

[perspective for a single point, needs elaboration, see lecture]. (para-perspective)

(ii) See marks

8 Homogeneous Coordinates

(i) Find the intersection point of the following two lines:

$$l_1 : 3x + 4y = 6,$$
$$l_2 : \quad x + y = 2.$$

(ii) Compute the intersection of the parallel 2-D lines [a,b,c] and [a,b,c'] (notation from the lecture).
(*Hint*: Check out which 2-D point you get if you calculate the cross product of the coordinate axes.)

(iii) Can you find an inhomogeneous 2-D representation of the intersection point? Otherwise describe why you cannot.

(iv) Do these results in 2-D match the intuition that parallel lines "meet at infinity"?

1+1+1+1

(i) Solve by Gaussian elimination or using homogeneous coordinates. For the latter, it would look like this:

$$\begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} \cong \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \quad \Rightarrow \quad \text{intersection point } (2,0)^T$$

(ii) The hint shows that by using the cross product the origin is found when looking for the intersection of the coordinate axes. The cross product for the given lines yields

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c' \end{pmatrix} = (c' - c) \begin{pmatrix} b \\ -a \\ 0 \end{pmatrix},$$

so this is an ideal point which lies on the line at infinity $\mathbf{l}_\infty = (0,0,1)^T$, thus it matches the intuition of the lines meeting infinitely far away.

(iii) No, you cannot. The point would be

$$\lim_{\epsilon \to 0} \begin{pmatrix} b/\epsilon \\ -a/\epsilon \end{pmatrix} \notin \mathbb{R}^2.$$

(iv) see (ii)

9 Camera Parameters

(i) What are extrinsic camera parameters and intrinsic camera parameters?

(ii) With your new digital camera (focal length f=0.3 cm, zero pinhole offset, perfect square pixels, i.e., $k_x = k_y$, camera skew is exactly 90°) you want to study the effects of perspective distortion. Therefore you take a picture of the rails at an inoperative side track near your local train station. The coordinates of the camera's optical center with respect to the world coordinate system are $C = (0, 0, h)^T$, where $h = 60$ cm. The camera's principal axis is parallel to the rails. Write the expression for the camera's full projection matrix P.

(iii) An object is observed and its center is located at $\mathbf{x}_0 = (1.2, 3.6, 2.0)^T \in \mathbb{R}^3$. The object is rotated around the x-axis by $\theta_x = 30°$ and by $\theta_y = 90°$ around the y-axis. Furthermore the camera is translated by $\mathbf{t} = (1.3, 2.2, 2.0)^T$. Finally the object is projected perspectively to the image plane with focal length $f = 4$.

- Does it make a difference which rotation to perform first? Why?
- State the translation, rotation and projection mapping in a single transformation matrix T and apply T appropriately to find the homogeneous point $\mathbf{x}_1' \in \mathbb{P}^3$.
- Calculate the projected center $\mathbf{x}_1 \in \mathbb{R}^3$.

$\boxed{\mathbf{0.5+1+1.5}}$

(i) See lecture

(ii) (x-axis should go along the rails, and z-axis from the ground upwards, i.e., $k_y = k_z$.) Intrinsic parameters are ideal and known up to the pixel size, parallel to the rails means parallel in 3-D, thus it is only translated:

$$\mathbf{K}\mathbf{P}_{\text{perspective}}\mathbf{D} = \begin{pmatrix} k_y & 0 & 0 \\ 0 & k_z & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 0.3k_y & 0 & 0 & 0 \\ 0 & 0.3k_z & 0 & 0 \\ 0 & 0 & 1 & 60 \end{pmatrix} [cm].$$

(iii) Rotation is not commutative, so its successive order is important.

$$\mathbf{R}_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} \\ 0 & \sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{pmatrix}, \quad \mathbf{R}_y = \begin{pmatrix} \cos\frac{\pi}{3} & 0 & \sin\frac{\pi}{3} \\ 0 & 1 & 0 \\ -\sin\frac{\pi}{3} & 0 & \cos\frac{\pi}{3} \end{pmatrix}$$

so

$$\mathbf{T} = \begin{pmatrix} \mathbf{R}_y\mathbf{R}_x & \mathbf{t} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{4} & \frac{3}{4} & 1.3 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} & 2.2 \\ -\frac{\sqrt{3}}{2} & \frac{1}{4} & \frac{\sqrt{3}}{4} & 2.0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

further

$$\mathbf{T}\mathbf{x}_0 \approx \begin{pmatrix} 0.50 & 0.43 & 0.75 & 1.30 \\ 0.00 & 0.87 & 0.0 & 2.20 \\ -0.87 & 0.25 & 0.43 & 2.00 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{pmatrix} \begin{pmatrix} 1.2 \\ 3.6 \\ 2.0 \\ 1 \end{pmatrix} \approx \begin{pmatrix} 4.96 \\ 6.32 \\ 2.73 \\ 1.00 \end{pmatrix}$$

Projection:

$$\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 4.96 \\ 6.32 \\ 2.73 \\ 1.00 \end{pmatrix} \approx \begin{pmatrix} 7.27 \\ 9.27 \\ 1.00 \end{pmatrix}$$

Total: 10