Medical Image Processing for Interventional Applications

Super-Resolution: Introduction

Online Course – Unit 20

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Topics

What is Image Super-Resolution?

Cameras and Sampling

- The Sampling Theorem
- Sampling of Real Cameras
- Quantization and Image Noise

Summary

- Take Home Messages
- Further Readings

What is Image Super-Resolution?

Digital imaging systems perform a non-ideal mapping of a scene to the image plane of a camera:

- **(Down-)sampling**: continuous real world scene $\leftrightarrow$ discrete representation with finite resolution
- **Blur/diffraction**: non-ideal mapping of points and edges
- **Noise**: induced by camera sensors

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$\rightarrow$ Small details in the scene get lost in a 2-D image.

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Definition

Super-resolution is the process of obtaining high-resolution images from observed low-resolution images.

What is Image Super-Resolution?

Basic approaches to image super-resolution:

- Instrumental super-resolution (hardware-based):
  - Engineering of the characteristics of the imaging system
  - Widely used in microscopy (STED, RESOLFT)

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- *In this course*: computational super-resolution (software-based):
  - Retrospective approach to image super-resolution (reconstruction algorithms)
  - Aims at overcoming limitations related to digital sampling and/or diffraction
  - No modifications of the underlying camera hardware (sensor and optics) $\rightarrow$ low-cost solution

Super-Resolution Applications

Various applications for image super-resolution algorithms:

- Consumer electronics
- Surveillance cameras
- Remote sensing
- Medical imaging:
  - Ophthalmic imaging
  - Image-guided surgery
  - Radiology
- and more . . .

Figure 1: Super-resolving car license plates

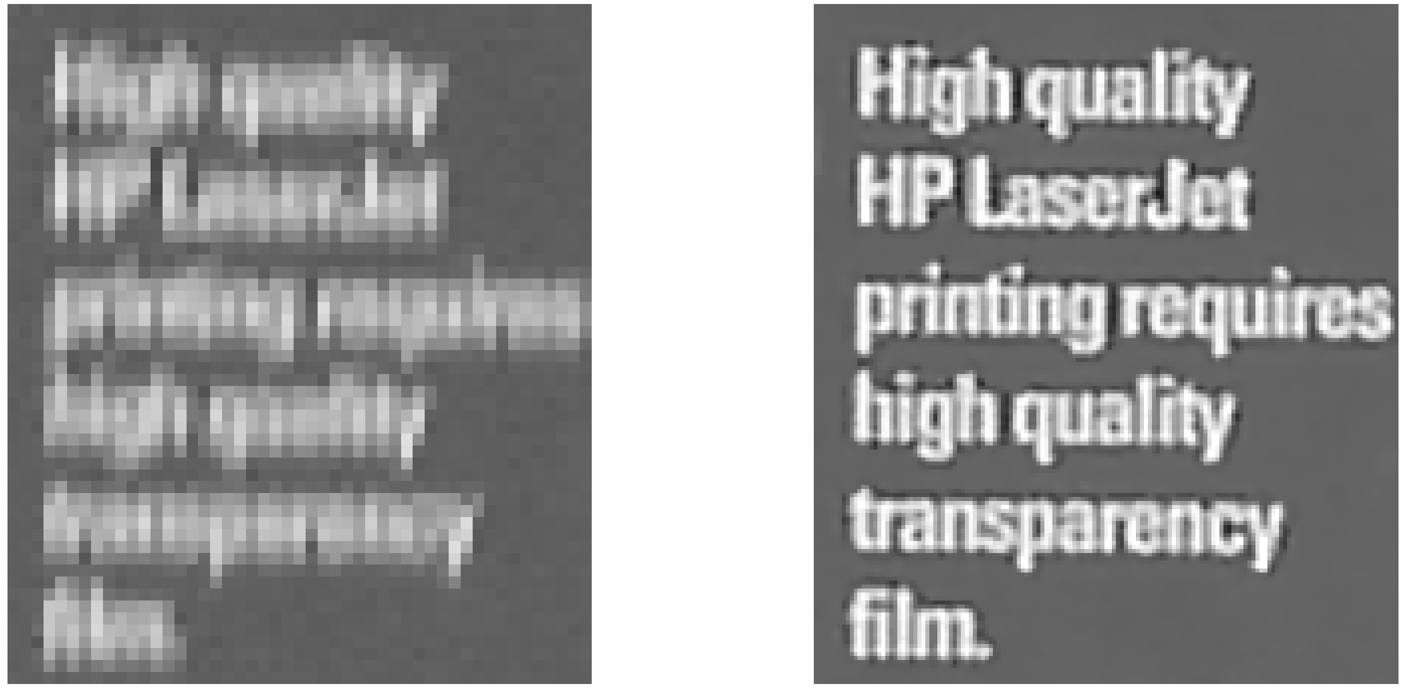

Figure 2: Super-resolving text document images

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Sampling in 2-D

Definitions:

- $f(x, y)$ is a continuous, real-valued signal.
- If $f(x, y)$ is sampled in x- and y-direction, it can be represented by discrete values $f_{m,n}$ where:
  - $f_{m,n} = f(m \cdot \Delta x, n \cdot \Delta y) \in \mathbb{R}$,
  - Δx and Δy denote the sample spacing (sample pitch) on a regular grid,
  - for a finite regular grid $f(x, y)$ is limited to a range $x_0 \leq x \leq x_1$ and $y_0 \leq y \leq y_1$.

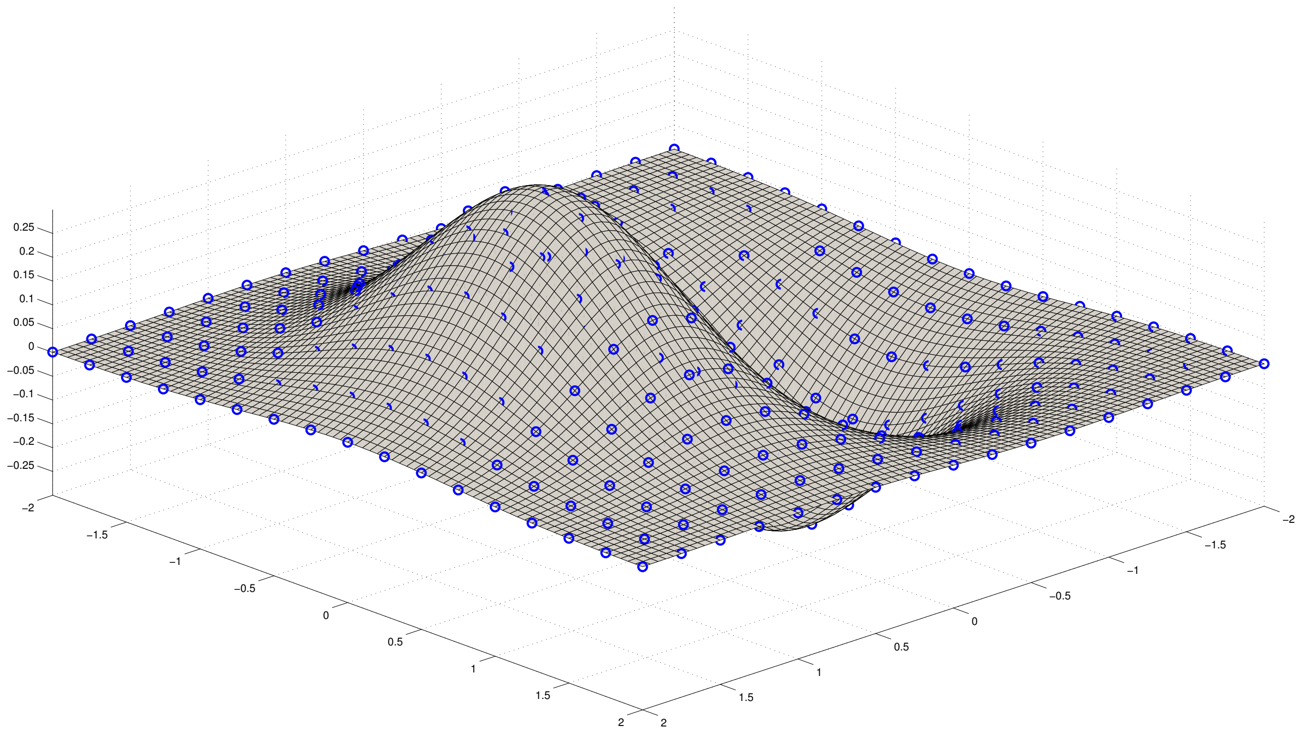

Figure 3: Example of a function sampling

Is it possible to reconstruct $f(x, y)$ from samples $f_{m,n}$ without loss of information? $\rightarrow$ Sampling theorem

Sampling in 2-D

Mathematical modeling of the sampling process:

- **Ideal sampling** is modeled by a sequence of Dirac delta functions:

$$\Delta(x,y) = \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(x - m\Delta x, y - n\Delta y),$$

where a single Dirac delta is given by

$$\delta(x,y) = \begin{cases} 1, & \text{if } x = 0 \text{ and } y = 0, \\ 0, & \text{otherwise.} \end{cases}$$

- The discrete samples are determined by

$$f_{m,n} = \Delta(x,y) f(x,y).$$

Sampling Theorem

Band-limited signals:

- Let $F(u,v)$ be the Fourier transform of the continuous signal $f(x,y)$:

$$F(u,v) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y) e^{-2\pi i(ux+vy)} \mathrm{d}x\,\mathrm{d}y$$

- For the formulation of the sampling theorem we consider **band-limited** signals $f(x,y)$:

$$F(u,v) = 0 \qquad \text{for } |u| > u_0 \text{ or } |v| > v_0.$$

Sampling Theorem

Sampling theorem according to Shannon and Nyquist (for low-pass signals):

The continuous signal $f(x,y)$ is completely determined by its discrete samples :

$$f_{m,n} = f(m \cdot \Delta x, n \cdot \Delta y) \in \mathbb{R} \qquad \text{for } m,n = 0,1,2,\dots$$

without loss of information **if and only if**

$$\Delta x \leq \frac{1}{2u_0} \quad \text{and} \quad \Delta y \leq \frac{1}{2v_0}.$$

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without loss of information **if and only if**

$$\Delta x \leq \frac{1}{2u_0} \quad \text{and} \quad \Delta y \leq \frac{1}{2v_0}.$$

In other words: We can reconstruct $f(x,y)$ from $f_{m,n}$ if the sampling rates $1/\Delta x$ and $1/\Delta y$ are high enough.

Aliasing

Violation of the sampling theorem:

- If the sampling theorem is not fulfilled, ***aliasing* is induced**.
- Aliasing: High frequencies in the original signal are mapped to low frequencies in the sampled signal.

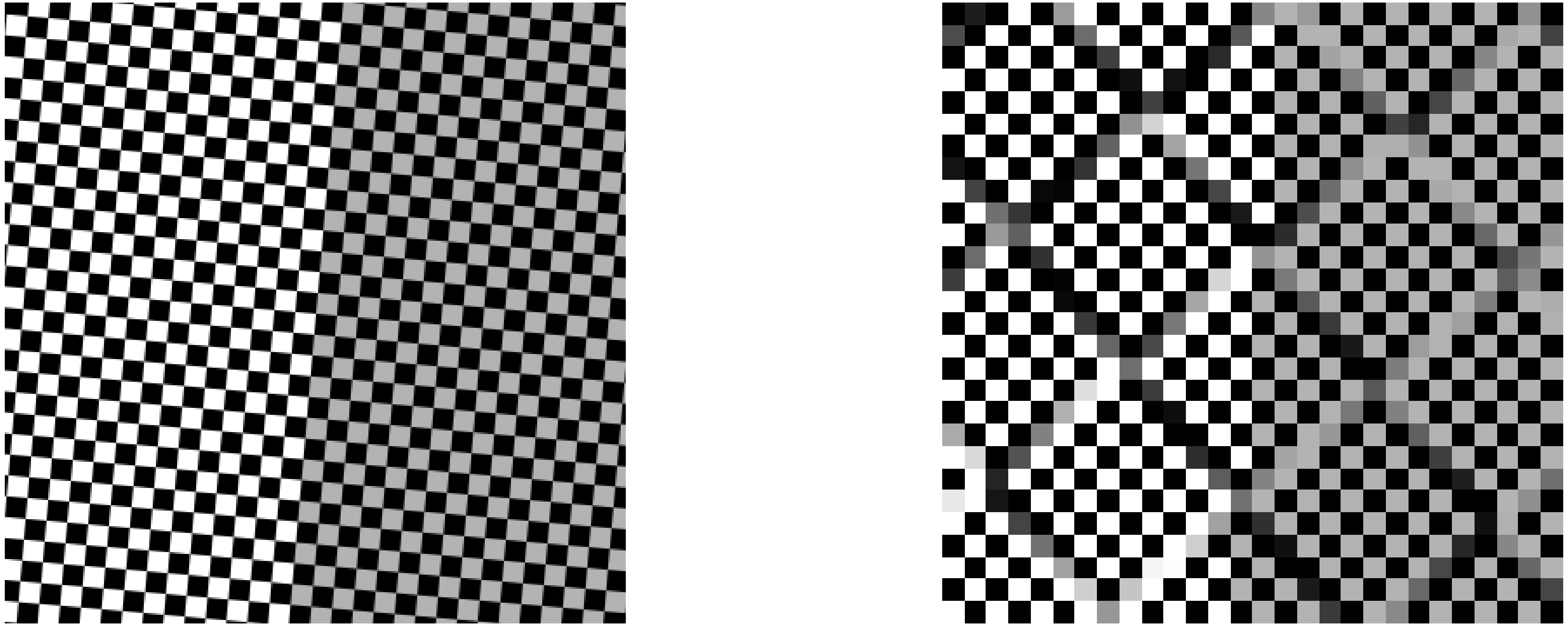


Figure 4: Original checkerboard pattern (left), resampled pattern with aliasing artifacts (right)

Sampling of Real Cameras

Generalization of the sampling process:

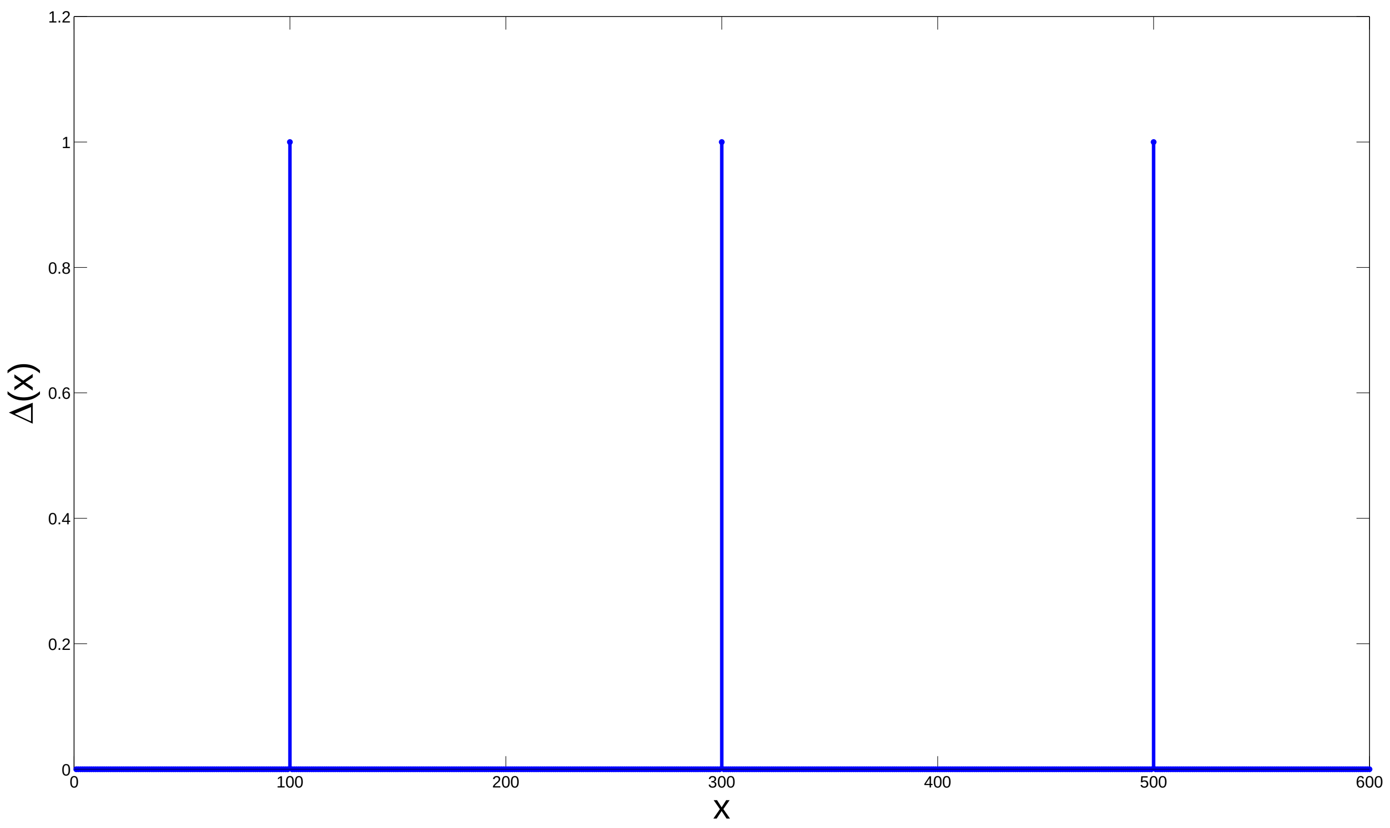
- A real camera cannot sample with ideal Dirac functions since the sensor array consists of pixels of finite size.
- The signal has to pass the ***point spread function*** (PSF) $h(x,y)$.
- For a space-invariant PSF, one discrete sample $f_{m,n}$ is given by

$$f_{m,n} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)h(x-m\Delta x, y-n\Delta y)\,\mathrm{d}x\,\mathrm{d}y,$$

i. e., $f_{m,n}$ is a weighted sum of the surrounding intensities $f(x,y)$ collected at the sensor array (convolution of $f(x,y)$ with the PSF).

Sampling of Real Cameras

Ideal sampling: If we could sample with an ideal Dirac sequence, ideal edges from the real world would be mapped onto ideal edges in an image.



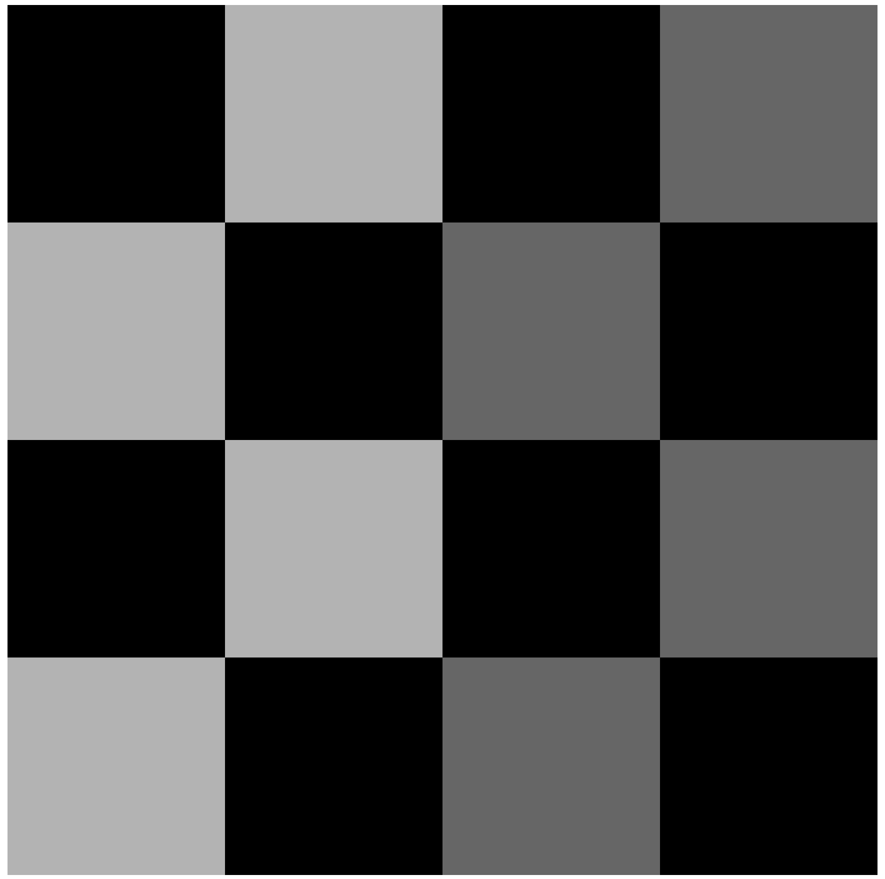
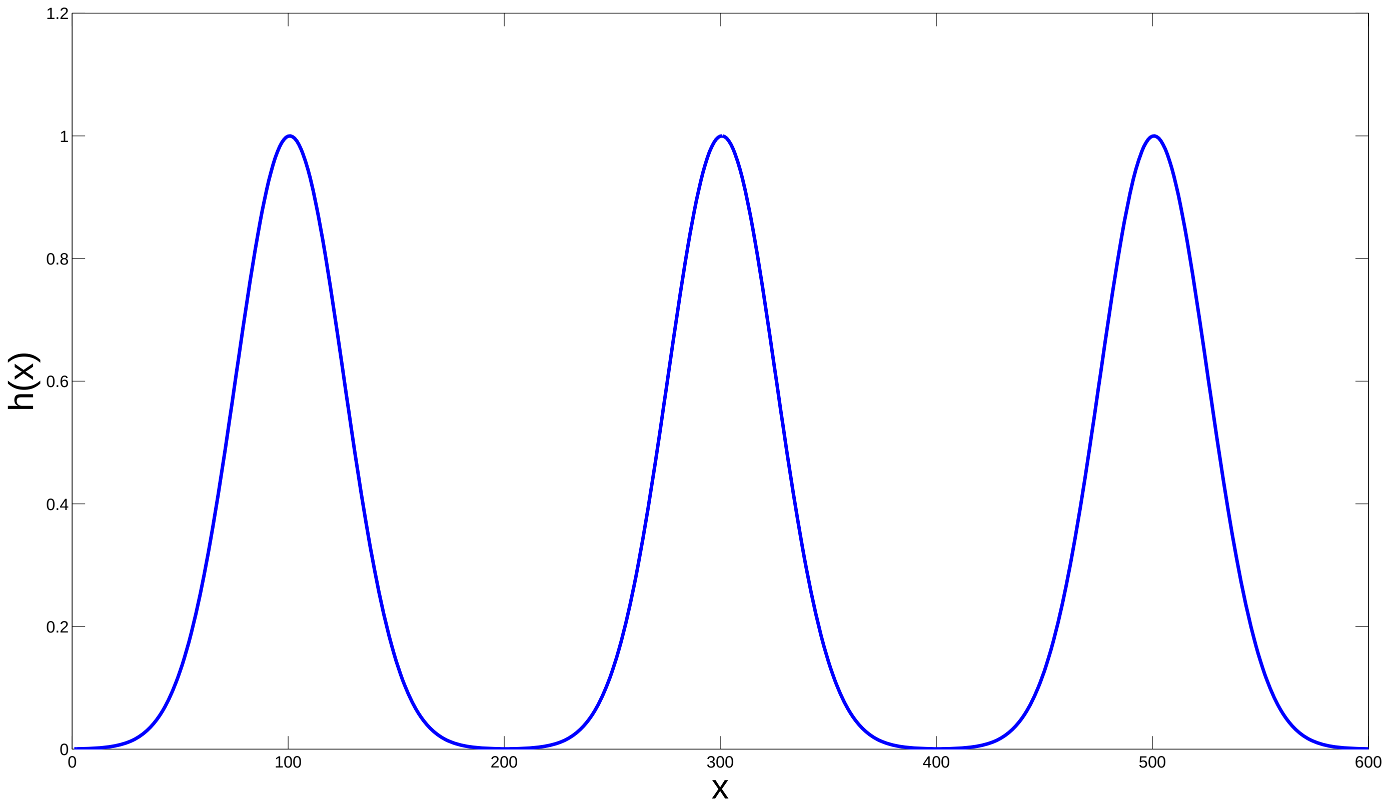


Figure 5: Ideal Dirac sequence in 1-D (left), ideal sampling (right)

Sampling of Real Cameras

Sampling under real-world conditions: If we sample with a real camera, ideal edges get blurred in the observed images.



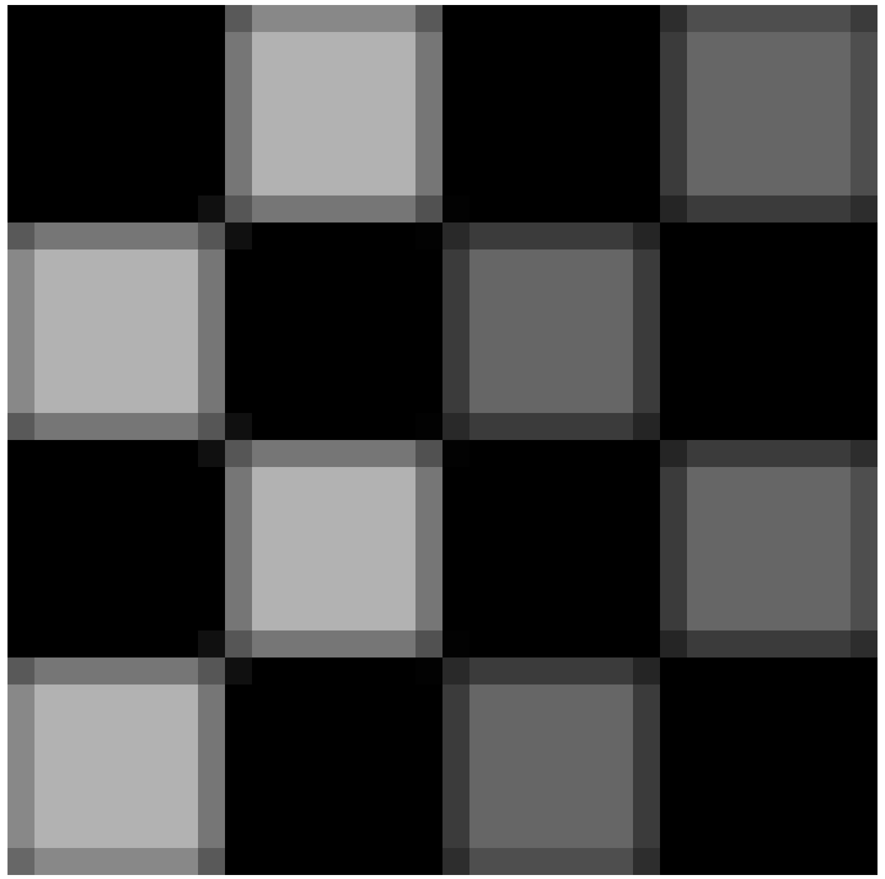


Figure 6: Gaussian PSF kernel in 1-D (left), real sampling (right)

Quantization and Image Noise

Consider noise in the image formation process:

- Discrete samples cannot be obtained and stored with infinite accuracy:
  - 8-bit quantization for grayscale images,
  - 24-bit quantization for RGB color images.
- Furthermore, noise is induced in the sensor array.

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Consider noise in the image formation process:

- Discrete samples cannot be obtained and stored with infinite accuracy:
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- Furthermore, noise is induced in the sensor array.

Total observation model: The sampled signal is disturbed by additive noise $\varepsilon_{m,n}$:

$$f_{m,n} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x,y) h(x - m\Delta x, y - n\Delta y) dx dy + \varepsilon_{m,n}.$$

We assume $\varepsilon_{m,n}$ to be the interference of different noise sources and therefore to be spatially invariant Gaussian noise ($\rightarrow$ central limit theorem).

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Take Home Messages

- Super-resolution algorithms enhance the resolution of an image which makes them highly interesting not only for medical applications.
- The sampling theorem allows us to determine a discrete sampling pattern which perfectly samples a given signal.
- A real camera has a limited sampling capability and we have to deal with noise as well.

Further Readings

Theory of image super-resolution (books and review articles):

- Hayit Greenspan. "Super-Resolution in Medical Imaging". In: *The Computer Journal* 52.1 (Feb. 2008), pp. 43–63. DOI: 10.1093/comjnl/bxm075
- Peyman Milanfar, ed. *Super-Resolution Imaging*. Digital Imaging and Computer Vision. CRC Press, 2011
- Sina Farsiu et al. "Advances and Challenges in Super-Resolution". In: *International Journal of Imaging Systems and Technology* 14.2 (Aug. 2004), pp. 47–57. DOI: 10.1002/ima.20007
- Sung Cheol Park, Min Kyu Park, and Moon Gi Kang. "Super-Resolution Image Reconstruction: A Technical Overview". In: *IEEE Signal Processing Magazine* 20.3 (May 2003), pp. 21–36. DOI: 10.1109/MSP.2003.1203207

Medical Image Processing for Interventional Applications

Super-Resolution: Computational Methods

Online Course – Unit 21

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Topics

Computational Methods for Image Super-Resolution

Summary

Take Home Messages

Further Readings

Single-Frame Super-Resolution

Principles of single-frame super-resolution:

- Single-frame methods estimate a high-resolution image from one single low-resolution image by incorporation of prior knowledge.
- Methods:
  - **Learning-based methods**: Estimate high-resolution image by learning the image degradation process using training data.
  - **Frequency interpolation methods**: Represent images in frequency domain (e. g., with wavelet coefficients) to estimate high-frequency information not present in low-resolution images.

Multi-Frame Super-Resolution

Principles of (motion-based) multi-frame super-resolution:

- Capture sequence of **warped and degraded** images of ideal scene
- More precise sampling due to **non-integer pixel** shifts caused by
  - moving cameras
  - object motion

Goal: Reconstruct high-resolution (ideal) image from low-resolution frames.

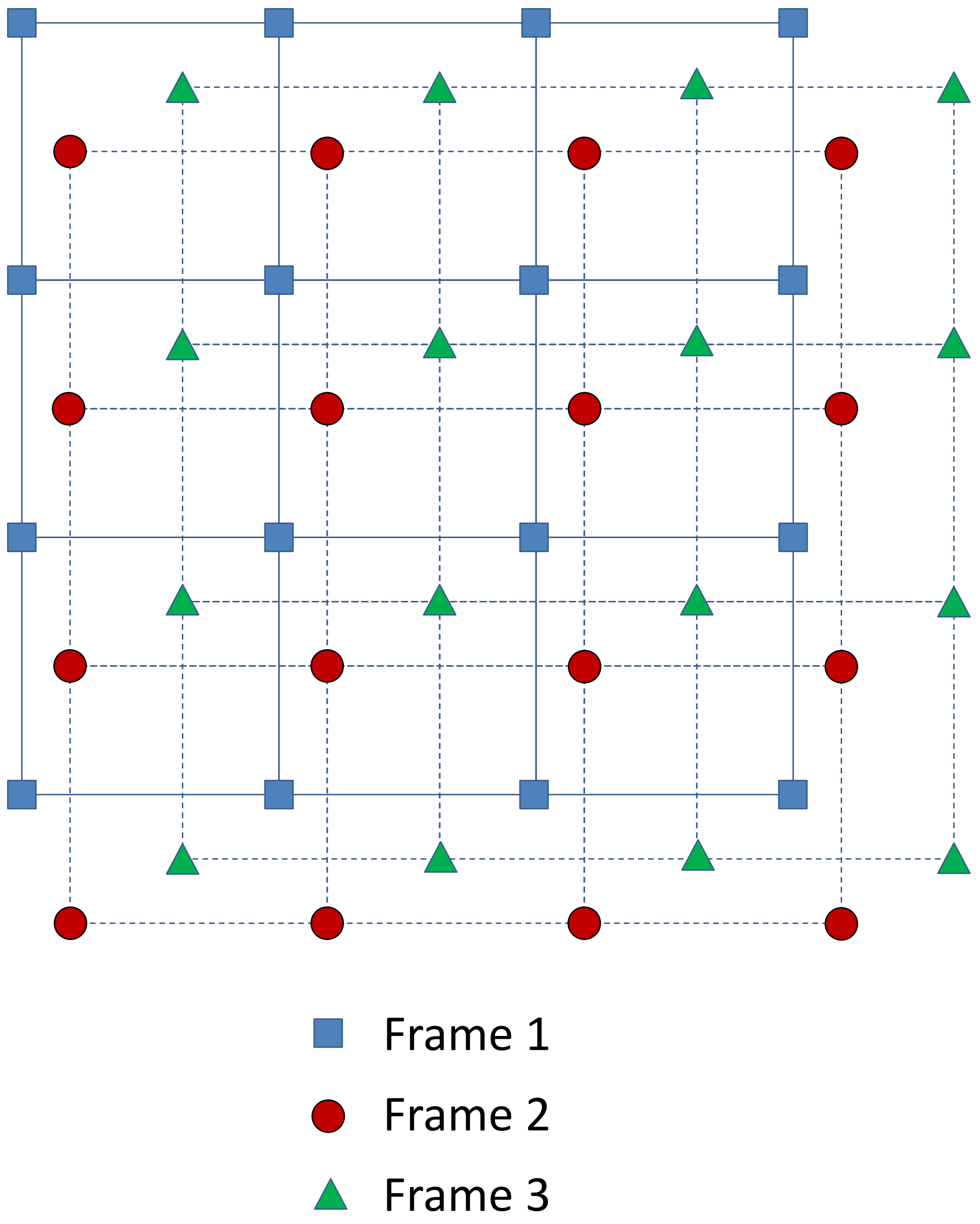

Figure 1: Relative motion shifts between frames

Super-Resolution via Non-Uniform Interpolation

Properties and assumptions:

- Direct approach (in contrast to a formulation as an inverse problem)
- Given: motion estimate for low-resolution frames

Algorithm: super-resolution performed in three stages

1. **Motion compensation**: Warp all low-resolution frames to the desired high-resolution grid according to the motion estimate.
2. **Interpolation**: Interpolate the high-resolution image from the warped low-resolution samples.
3. **Restoration** (optional): Deblur the interpolated image.

Super-Resolution via Non-Uniform Interpolation

Overview of the algorithm:

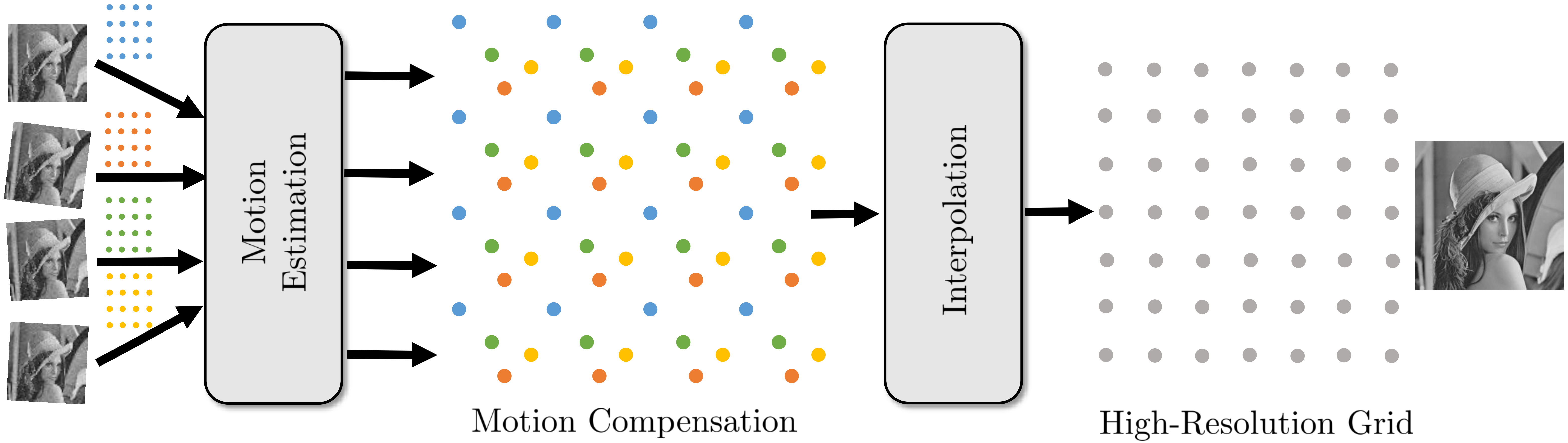

Figure 2: Schematic of the algorithm

Motion Models for Multi-Frame Super-Resolution

Description of image warping by motion model:

- **Parametric motion model** $\longrightarrow$ image-to-image homography $\boldsymbol{M}$ consisting of:
  - Rigid motion: rotation matrix $\boldsymbol{R}$ and translation $\boldsymbol{t}$ (3 degrees of freedom)

$$\boldsymbol{M} = \begin{pmatrix} \boldsymbol{R} & \boldsymbol{t} \\ \boldsymbol{0}^\top & 1 \end{pmatrix}$$

  - Affine motion: rotation, translation, scaling and shearing (6 degrees of freedom)

$$\boldsymbol{M} = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{t} \\ \boldsymbol{0}^\top & 1 \end{pmatrix}$$

- **Non-parametric model** $\longrightarrow$ displacement vector fields via optical flow

Non-Uniform Interpolation: Example

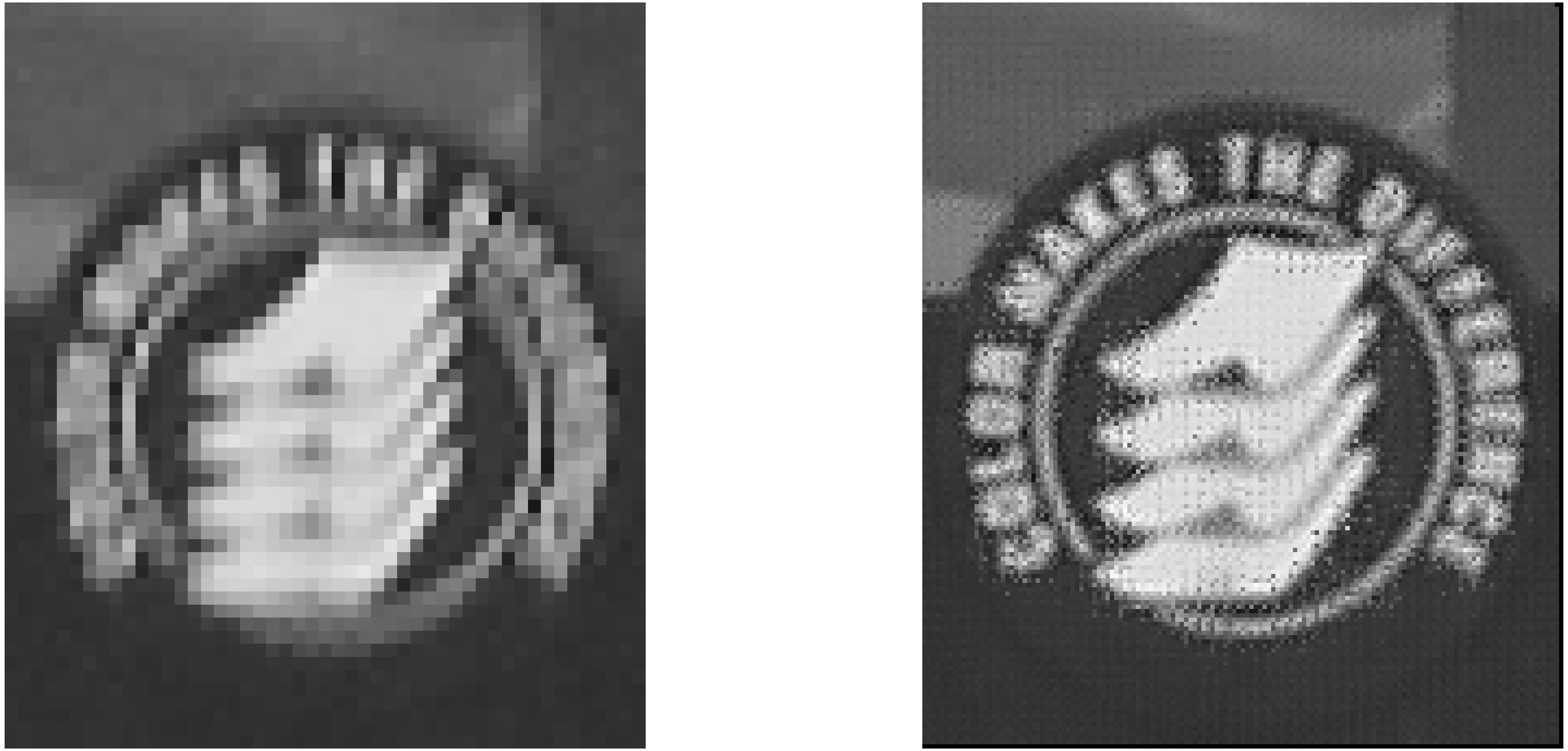

Figure 3: Single low-resolution frame (left) and result of non-uniform interpolation (right) using bicubic interpolation with $K = 26$ frames and $3\times$ magnification

Non-Uniform Interpolation: Discussion

Properties of the non-uniform interpolation approach:

- Easy to implement
- Computationally efficient (direct approach)
- Flexible in terms of motion models
- Prone to artifacts in case of misregistrations (error propagation)
- Difficult to model a priori knowledge regarding high-resolution images

Topics

Computational Methods for Image Super-Resolution

Summary

Take Home Messages

Further Readings

Take Home Messages

- Super-resolution algorithms can be subdivided into single and multiframe methods.
- The direct approach involves warping the frames to a high-resolution grid and subsequent interpolation.

Further Readings

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- Hayit Greenspan. "Super-Resolution in Medical Imaging". In: *The Computer Journal* 52.1 (Feb. 2008), pp. 43–63. DOI: 10.1093/comjnl/bxm075
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Medical Image Processing for Interventional Applications

Super-Resolution: ML Estimation

Online Course – Unit 22

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Topics

Super-Resolution as an Inverse Problem

Maximum Likelihood Estimation
- Bayesian Formulation
- Maximum Likelihood Estimation
- Numerical Optimization

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Problem Statement

Given: Set of low-resolution frames given as continuous functions (irradiance light fields)

$$y^{(1)}(\boldsymbol{u}), \ldots, y^{(K)}(\boldsymbol{u}),$$

where $\boldsymbol{u} \in \mathbb{R}^2$ (pixel grid)

We want to reconstruct a high-resolution image $x(\boldsymbol{u})$ that generated these frames according to:

$$y^{(k)}(\boldsymbol{u}) = \mathcal{W}^{(k)}\{x(\boldsymbol{u})\}, \qquad \text{for all } k = 1, \ldots, K,$$

where $\mathcal{W}^{(k)}\{\cdot\}$ is the (frame-wise) image formation model that:

- models characteristics of the camera optics,
- models spatial sampling on the sensor array.

$\rightarrow$ We investigate different approaches to model and solve this inverse problem.

Image Formation Model

Mathematical description of the image formation process:

Given an ideal image $x(\boldsymbol{u})$, $\boldsymbol{u} \in \mathbb{R}^2$, as continuous function, we can model the formation of a low-resolution image $y^{(k)}(\boldsymbol{u})$:

$$y^{(k)}(\boldsymbol{u}) = \mathcal{D}\left\{\mathcal{M}^{(k)}\{x(\boldsymbol{u})\} * h^{(k)}(\boldsymbol{u})\right\} + \varepsilon(\boldsymbol{u}).$$

$\mathcal{D}\{\cdot\}$ and $\mathcal{M}^{(k)}\{\cdot\}$: sampling and motion operators
$h^{(k)}(\boldsymbol{u})$: space invariant point spread function (PSF)
$\varepsilon(\boldsymbol{u})$: additive noise

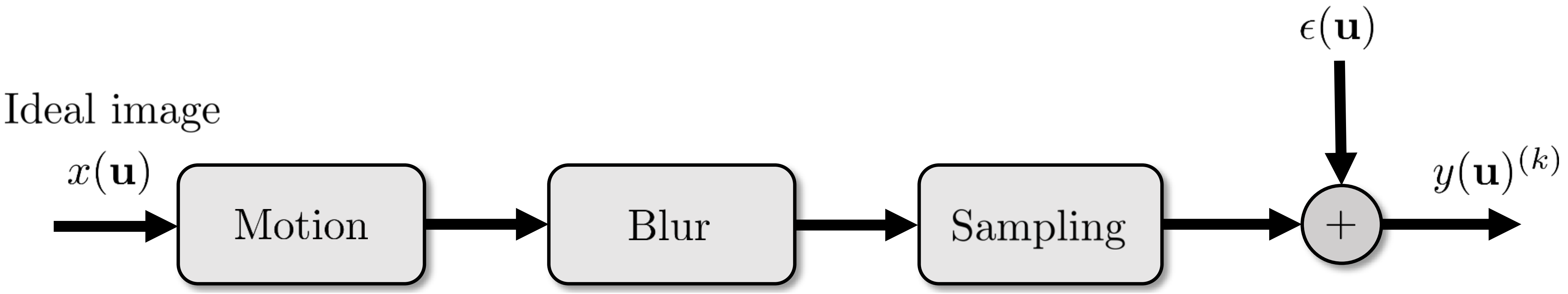

Figure 1: Steps of the image formation from an ideal image to a low-resolution output

Image Formation Model

Discretization of the continuous model:

- We need to discretize the image formation model to employ it for digital super-resolution algorithms:

$$\boldsymbol{y}^{(k)} = \boldsymbol{W}^{(k)}\boldsymbol{x} + \boldsymbol{\varepsilon}^{(k)}.$$

- Image formation modeled by matrix/vector operations:

$\boldsymbol{x}$: high-resolution image $\boldsymbol{x} \in \mathbb{R}^N$,
$\boldsymbol{y}^{(k)}$: k-th low-resolution frame $\boldsymbol{y}^{(k)} \in \mathbb{R}^M$ where $M < N$,
$\boldsymbol{W}^{(k)}$: system matrix of k-th frame to model motion, PSF and downsampling.

Anatomy of the System Matrix

Definition of the matrix: The system matrix models the mapping from $\mathbf{x}$ to $\mathbf{y}^{(k)}$:

$$W_{mn}^{(k)} = h(\mathbf{v}_n - \mathbf{u}'_m)\,,$$

where

$h(\mathbf{u})$: camera PSF as space and time invariant kernel,
$\mathbf{v}_n$: coordinates of n-th pixel in $\mathbf{x}$,
$\mathbf{u}'_m$: coordinates of m-th pixel in $\mathbf{y}$ warped to $\mathbf{x}$.

The elements are normalized according to:

$$\sum_n W_{mn}^{(k)} = 1\,.$$

Example: Isotropic Gaussian kernel of width σ_{PSF}

$$h(\mathbf{u}) = \exp\left(-\frac{||\mathbf{u}||_2^2}{2\sigma_{\text{PSF}}^2}\right)$$

Anatomy of the System Matrix

Properties and practical considerations:

- The system matrix $\mathbf{W}^{(k)}$ consists of:
  - N columns, where N denotes the number of high-resolution pixels,
  - M rows, where M is the number of low-resolution pixels.

  $\longrightarrow$ This is infeasible to store for larger instances (e. g., $N = 1024^2$ and $M = 512^2$).

- For a practical computation, we approximate $\mathbf{W}^{(k)}$ as **sparse matrix** by assuming a narrow kernel $h(\mathbf{u})$:

$$W_{mn}^{(k)} := 0 \quad \text{if } \left|\left|\mathbf{v}_n - \mathbf{u}'_m\right|\right|_2 > d_{max},$$

  e. g., $d_{max} = 3\sigma$ for isotropic Gaussian PSF.

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Bayesian Formulation of Multi-Frame Super-Resolution

Definitions and nomenclature:

Let us assign probability distributions to the quantities of the image formation model:

- We model a high-resolution image with a prior distribution $\mathbf{x} \sim p(\mathbf{x})$.
- Similarly, we model a low-resolution image as random variable $\mathbf{y}^{(k)} \sim p(\mathbf{y}^{(k)})$.

According to Bayes rule we obtain the posterior distribution:

$$p(\mathbf{x} \mid \mathbf{y}) = p(\mathbf{x} \mid \mathbf{y}^{(1)} \dots \mathbf{y}^{(K)}) = \frac{p(\mathbf{x}) \cdot p(\mathbf{y}^{(1)} \dots \mathbf{y}^{(K)} \mid \mathbf{x})}{p(\mathbf{y}^{(1)} \dots \mathbf{y}^{(K)})} = \frac{p(\mathbf{x}) \cdot p(\mathbf{y} \mid \mathbf{x})}{p(\mathbf{y})},$$

under the assumption of independent and identically distributed (i.i.d.) observations $\boldsymbol{y}$.

Maximum Likelihood Estimation

Derivation of the log-likelihood:

- For maximum likelihood (ML) estimation, $\boldsymbol{x}$ is assumed to be uniformly distributed (no prior available).
- The negative log-likelihood under this assumption is given by:

$$L(\mathbf{x}, \mathbf{y}^{(1)} \dots \mathbf{y}^{(K)}) = -\log p(\mathbf{y}^{(1)} \dots \mathbf{y}^{(K)} \,|\, \mathbf{x}).$$

  $p(\mathbf{y}^{(1)} \dots \mathbf{y}^{(K)} \,|\, \mathbf{x})$ is referred to as the Bayesian observation model.
- Reconstruct $\mathbf{x}$ that explains $\mathbf{y}$ best:

$$\hat{\mathbf{x}}_{ML} = \arg\max_{\mathbf{x}} p(\boldsymbol{y} \,|\, \boldsymbol{x}) = \arg\min_{\mathbf{x}} L(\mathbf{x}, \mathbf{y}^{(1)} \dots \mathbf{y}^{(K)}).$$

Maximum Likelihood Estimation

Definition of the observation model:

Let $\varepsilon \sim N(0, \sigma^2 \mathbf{I})$ be spatially uncorrelated, additive Gaussian noise:

$$p(\mathbf{y}^{(k)} \,|\, \mathbf{x}) = \left(\frac{1}{2\pi\sigma}\right)^{\frac{M}{2}} \exp\left(-\frac{\left|\left|\mathbf{y}^{(k)} - \mathbf{W}^{(k)}\mathbf{x}\right|\right|_2^2}{2\sigma^2}\right).$$

Using the observation model $p(\mathbf{y}^{(k)} \,|\, \mathbf{x})$, ML estimation is equivalent to the energy minimization:

$$\hat{\mathbf{x}}_{ML} = \arg\min_{\mathbf{x}} \sum_{k=1}^{K} \left|\left|\mathbf{y}^{(k)} - \mathbf{W}^{(k)}\mathbf{x}\right|\right|_2^2 = \arg\min_{\mathbf{x}} ||\mathbf{y} - \mathbf{W}\mathbf{x}||_2^2 .$$

Numerical Optimization

Optimization of the log-likelihood:

- **Closed-form solution**: Solve for $\hat{\mathbf{x}}_{ML}$ using the pseudoinverse $\mathbf{W}^{+}$:

$$\hat{\mathbf{x}}_{ML} = \mathbf{W}^{+}\mathbf{y}.$$

  For a large system $\mathbf{W}$, it is not feasible to compute $\mathbf{W}^{+}$ directly.

- **Iterative numerical optimization** to determine $\hat{\mathbf{x}}_{ML}$ from an initial guess $\boldsymbol{x}^0$:
  - Gradient descent iterations: $\boldsymbol{x}^{t+1} = \boldsymbol{x}^t + \alpha^t \cdot \boldsymbol{p}^t$
  - Calculation of the search direction $\boldsymbol{p}^t$ according to steepest descent:

$$\boldsymbol{p}^t = \nabla_{\boldsymbol{x}} ||\mathbf{y} - \mathbf{W}\mathbf{x}||_2^2 = -2\boldsymbol{W}^\top(\mathbf{y} - \mathbf{W}\mathbf{x})$$

  $\longrightarrow$ Different strategies available to compute $\boldsymbol{p}^t$
  - Calculation of α^t by line search or use of constant step size ($\alpha^t = \alpha$)

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- The multiframe super-resolution problem can be stated as an inverse problem and yields a system matrix after discretization of the image formation model.
- The system matrix is normally quite large, so that sparsity assumptions are made.
- One possibility to solve the inverse problem is maximum likelihood estimation where high- and low-resolution images are regarded as probability distributions.

Further Readings

Theory of image super-resolution (books and review articles):

- Hayit Greenspan. "Super-Resolution in Medical Imaging". In: *The Computer Journal* 52.1 (Feb. 2008), pp. 43–63. DOI: 10.1093/comjnl/bxm075
- Peyman Milanfar, ed. *Super-Resolution Imaging*. Digital Imaging and Computer Vision. CRC Press, 2011
- Sina Farsiu et al. "Advances and Challenges in Super-Resolution". In: *International Journal of Imaging Systems and Technology* 14.2 (Aug. 2004), pp. 47–57. DOI: 10.1002/ima.20007
- Sung Cheol Park, Min Kyu Park, and Moon Gi Kang. "Super-Resolution Image Reconstruction: A Technical Overview". In: *IEEE Signal Processing Magazine* 20.3 (May 2003), pp. 21–36. DOI: 10.1109/MSP.2003.1203207

ML/MAP super-resolution:

- Lyndsey C. Pickup. "Machine Learning in Multi-frame Image Super-resolution". PhD Thesis. Robotics Research Group, University of Oxford, 2007
- Michael Elad and Arie Feuer. "Restoration of a Single Superresolution Image from Several Blurred, Noisy, and Undersampled Measured Images". In: *IEEE Transactions on Image Processing* 6.12 (Dec. 1997), pp. 1646–1658. DOI: 10.1109/83.650118

Medical Image Processing for Interventional Applications

Super-Resolution: Regularization

Online Course – Unit 23

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Topics

Maximum A Posteriori Estimation

Image Priors

Summary

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Super-Resolution and Ill-posed Problems

ML estimation on a synthetic example:

- Super-resolution is an **ill-posed** problem
  $\rightarrow$ ML reconstruction might lead to unstable solutions with amplified noise.

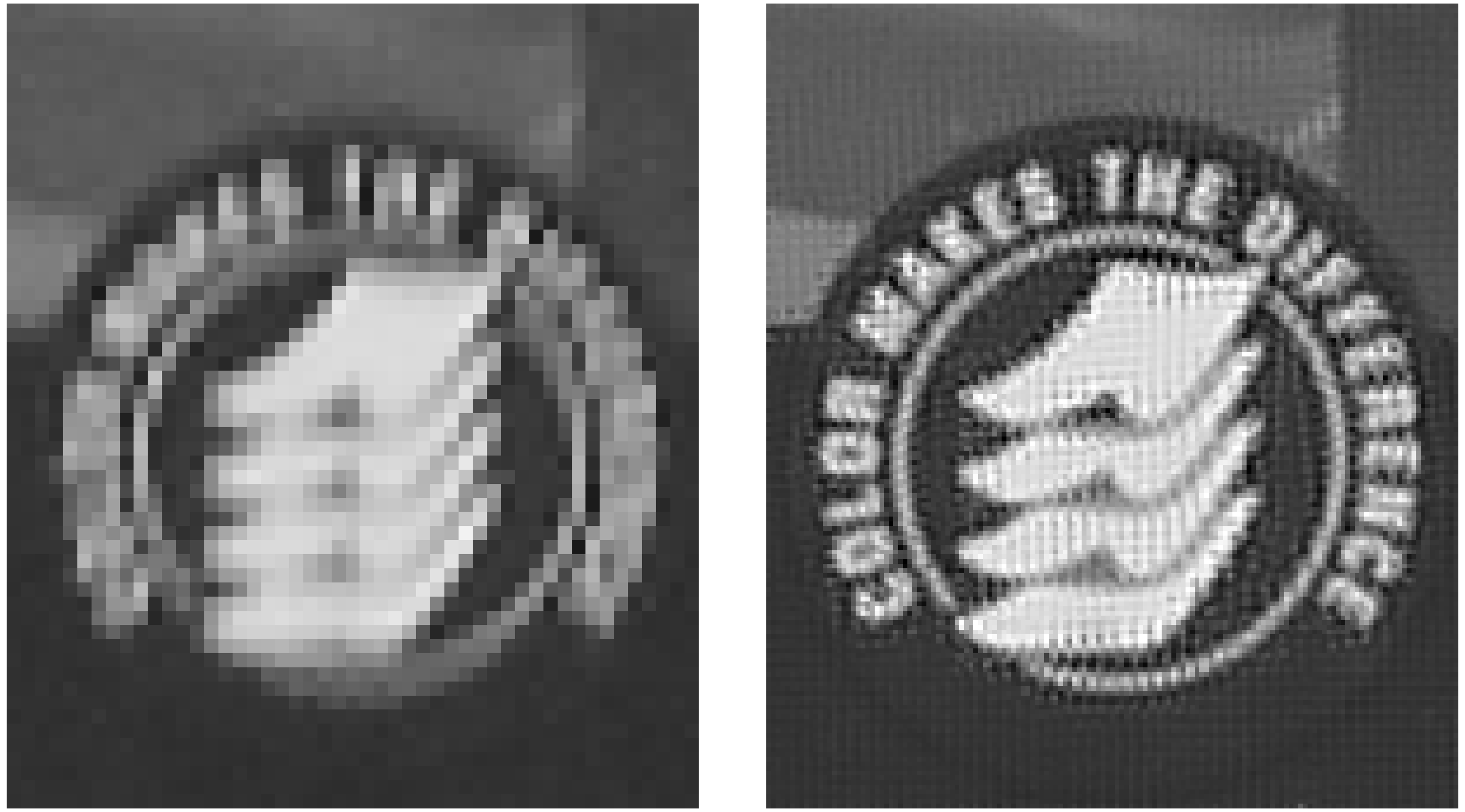

Figure 1: Example for insufficient result from ML estimation

Super-Resolution and Ill-posed Problems

ML estimation on a synthetic example:

- Super-resolution is an **ill-posed** problem $\rightarrow$ ML reconstruction might lead to unstable solutions with amplified noise.
- **Way out:** Incorporate prior knowledge into super-resolution algorithm.

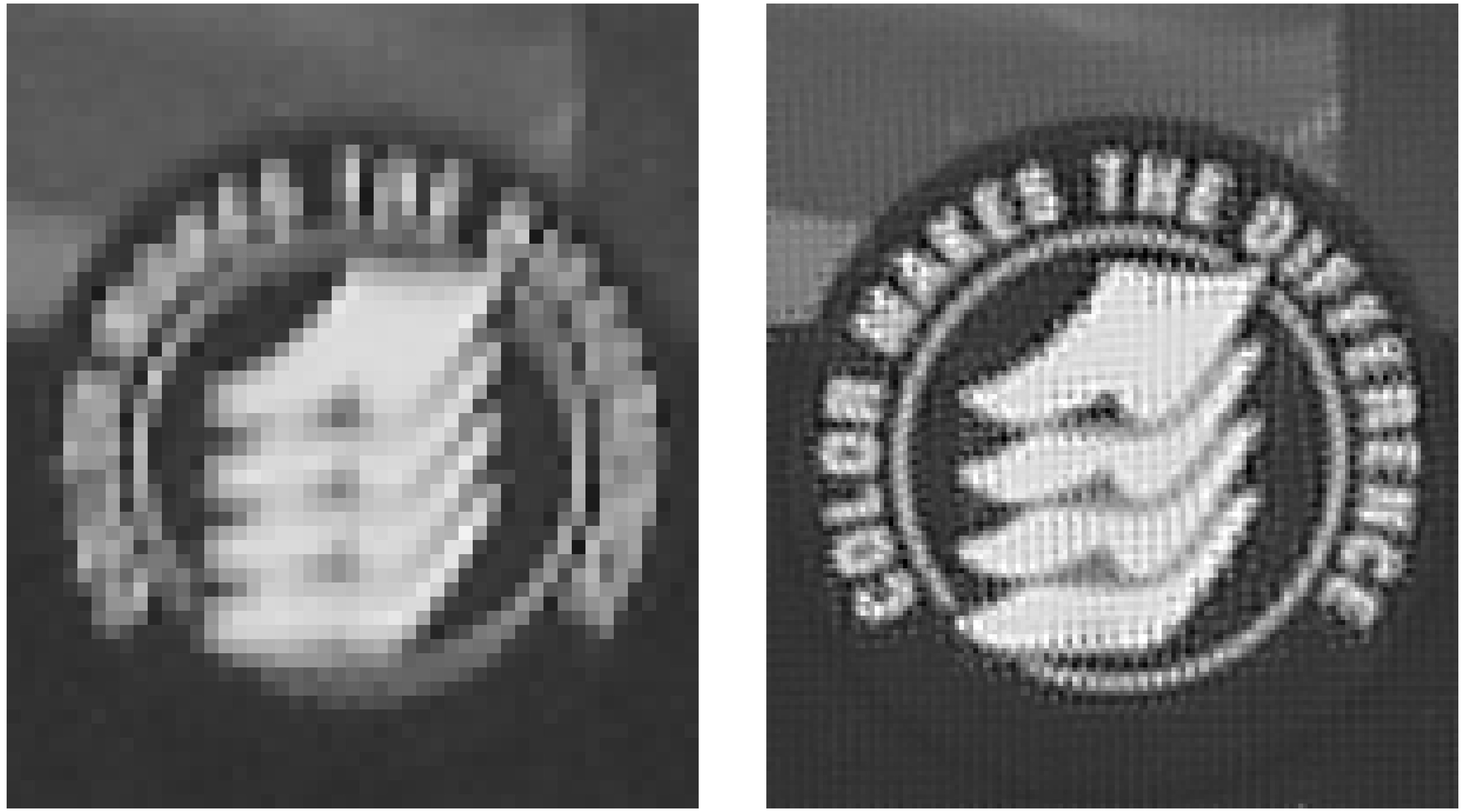

Figure 1: Example for insufficient result from ML estimation

Maximum A Posteriori Estimation

Extension of the ML estimation to MAP estimation:

- For maximum a posteriori (MAP) estimation, a suitable prior $p(\mathbf{x})$ is used.

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- For additive Gaussian noise, MAP estimation is equivalent to:

$$\hat{\mathbf{x}}_{MAP} = \arg\min_{\mathbf{x}} \sum_{k=1}^{K} \left|\left|\mathbf{y}^{(k)} - \mathbf{W}^{(k)}\mathbf{x}\right|\right|_2^2 - \log p(\mathbf{x}) = \arg\min_{\mathbf{x}} ||\mathbf{y} - \mathbf{W}\mathbf{x}||_2^2 - \log p(\mathbf{x}).$$

Topics

Maximum A Posteriori Estimation

Image Priors

Summary

Take Home Messages

Further Readings

Image Prior and Regularization Term

Mathematical framework for image priors in super-resolution:

- We define the prior distribution according to the exponential form:

$$p(\boldsymbol{x}) = \frac{1}{Z} \exp\left(-\lambda R(\boldsymbol{x})\right),$$

where Z is a normalization constant, and $R(\boldsymbol{x})$ denotes a regularization term with regularization weight $\lambda \geq 0$.

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- The regularizer $R(\boldsymbol{x})$ can be defined such that it penalizes large variations in $\boldsymbol{x}$.
- The regularization weight λ measures the impact of the prior:
  - $\lambda = 0$: simple ML estimation,
  - $\lambda \to \infty$: estimation dominated by the prior distribution.

Selected Priors for Super-Resolution

Gaussian prior (Tikhonov regularization):

- $p(\boldsymbol{x})$ is a normal distribution, i. e., $\boldsymbol{x} \sim N(0, \boldsymbol{\Sigma})$ and accordingly:

$$p(\boldsymbol{x}) = \frac{1}{(2\pi)^{N/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}\mathbf{x}^T\boldsymbol{\Sigma}^{-1}\mathbf{x}\right).$$

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- This regularization favors smooth solutions for $\boldsymbol{x}$.
- It facilitates a closed-form solution, but does not preserve discontinuities.

Selected Priors for Super-Resolution

Total variation (TV):

- $p(\boldsymbol{x})$ is modeled by a Laplacian distribution:

$$p(\boldsymbol{x}) \propto \exp\left(-\lambda \, ||\boldsymbol{Q}\mathbf{x}||_1\right).$$

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- Compare **x** to multiple shifted versions as generalization of the gradient in a $(2P+1) \times (2P+1)$ window:

$$p(\mathbf{x}) \propto \exp\left(-\lambda \sum_{u=-P}^{P} \sum_{v=-P}^{P} \alpha^{|u|+|v|} \left|\left|\mathbf{x} - \mathbf{S}_i^u \mathbf{S}_j^v \mathbf{x}\right|\right|_1\right).$$

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- It reduces "‘staircasing"’ artifacts which occur using the common TV prior.

Selected Priors for Super-Resolution

Huber prior:

- $p(\boldsymbol{x})$ is modeled by the Huber loss function:

$$p(\boldsymbol{x}) \propto \exp\left(-\lambda \sum_{i=1}^{N} h_\tau([\boldsymbol{Qx}]_i)\right),$$

$$h_\tau(z) = \begin{cases} \frac{1}{2}z^2, & \text{if } |z| \leq \tau, \\ \tau\left(|z| - \frac{\tau}{2}\right), & \text{otherwise.} \end{cases}$$

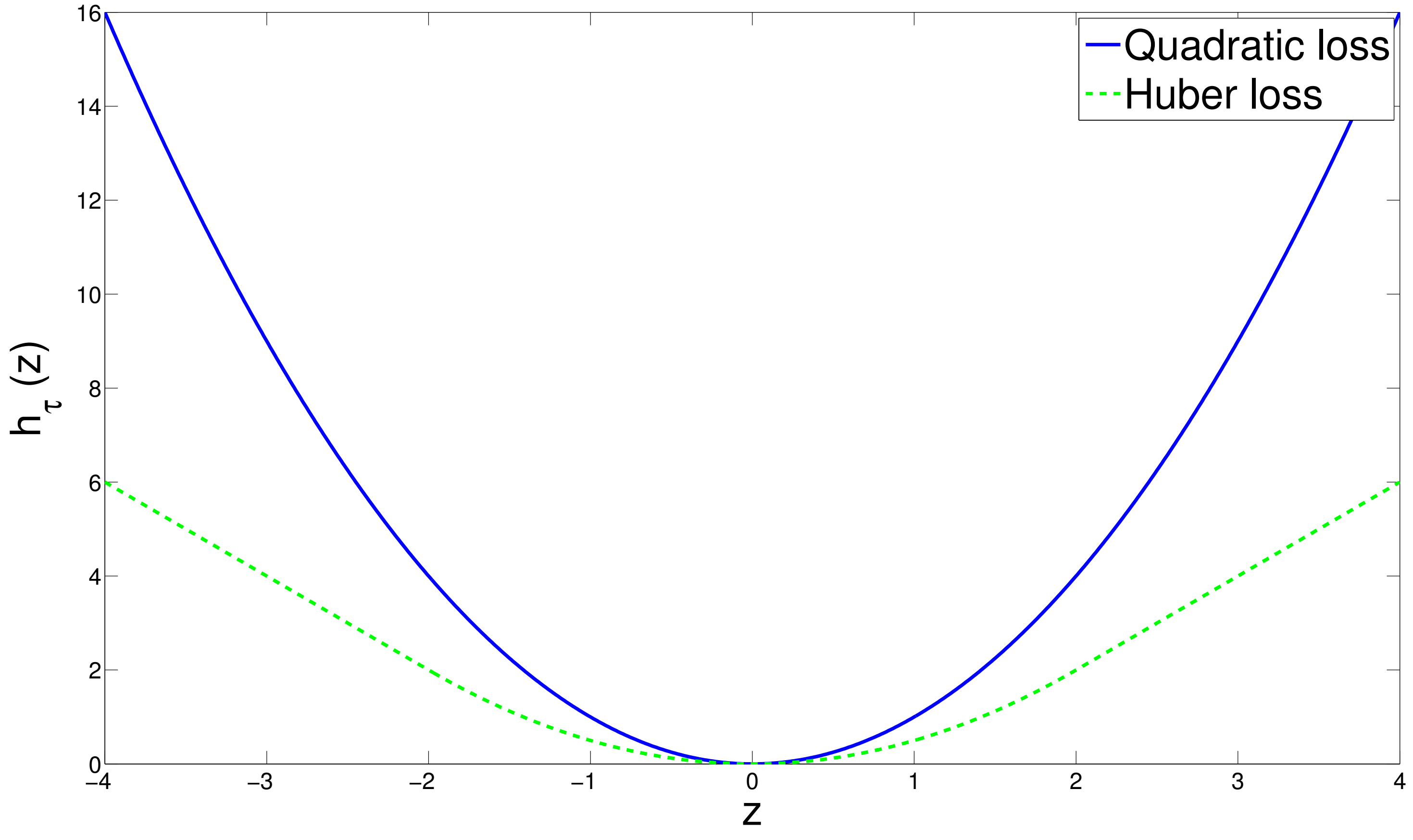

Figure 2: Comparison of quadratic loss and Huber loss

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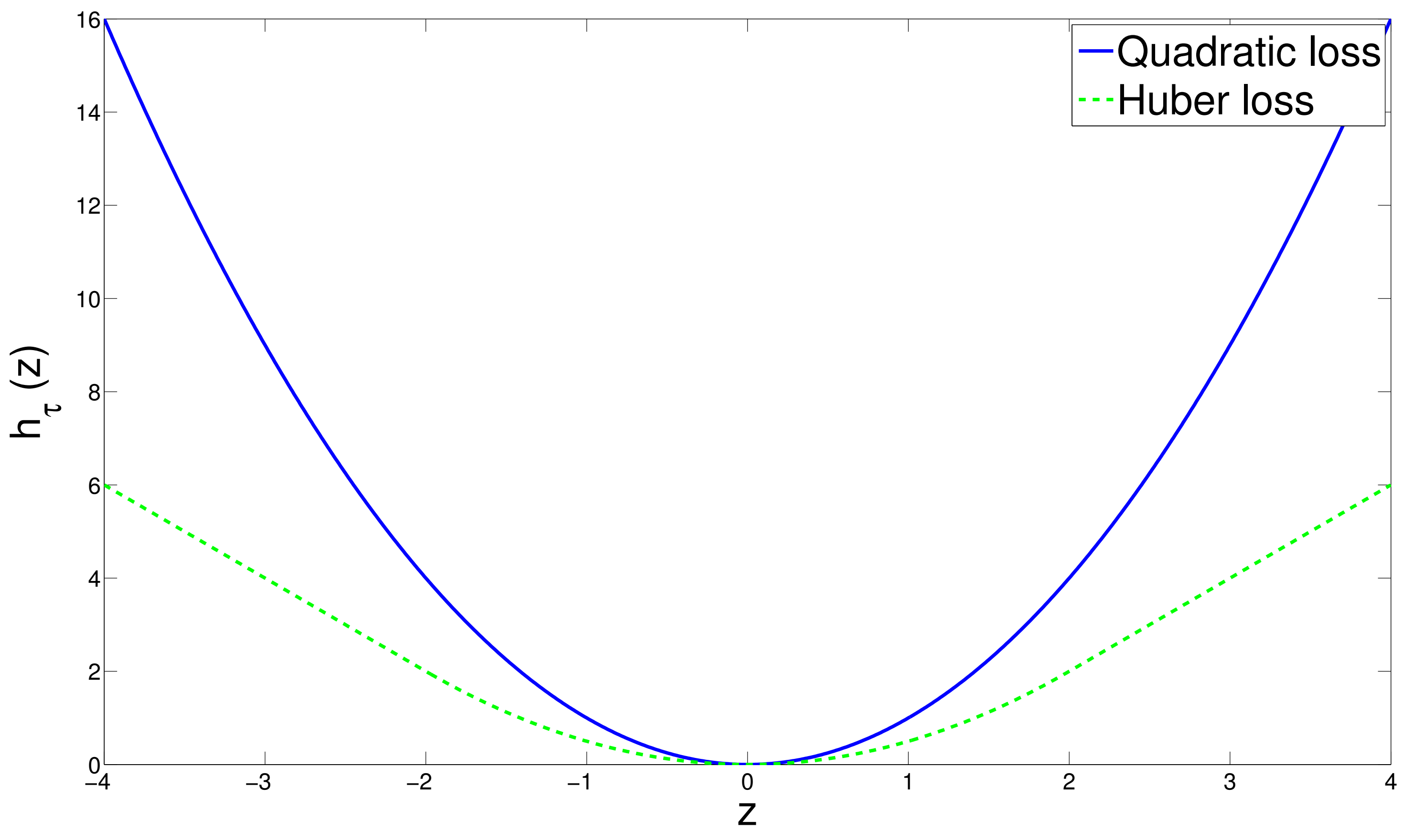

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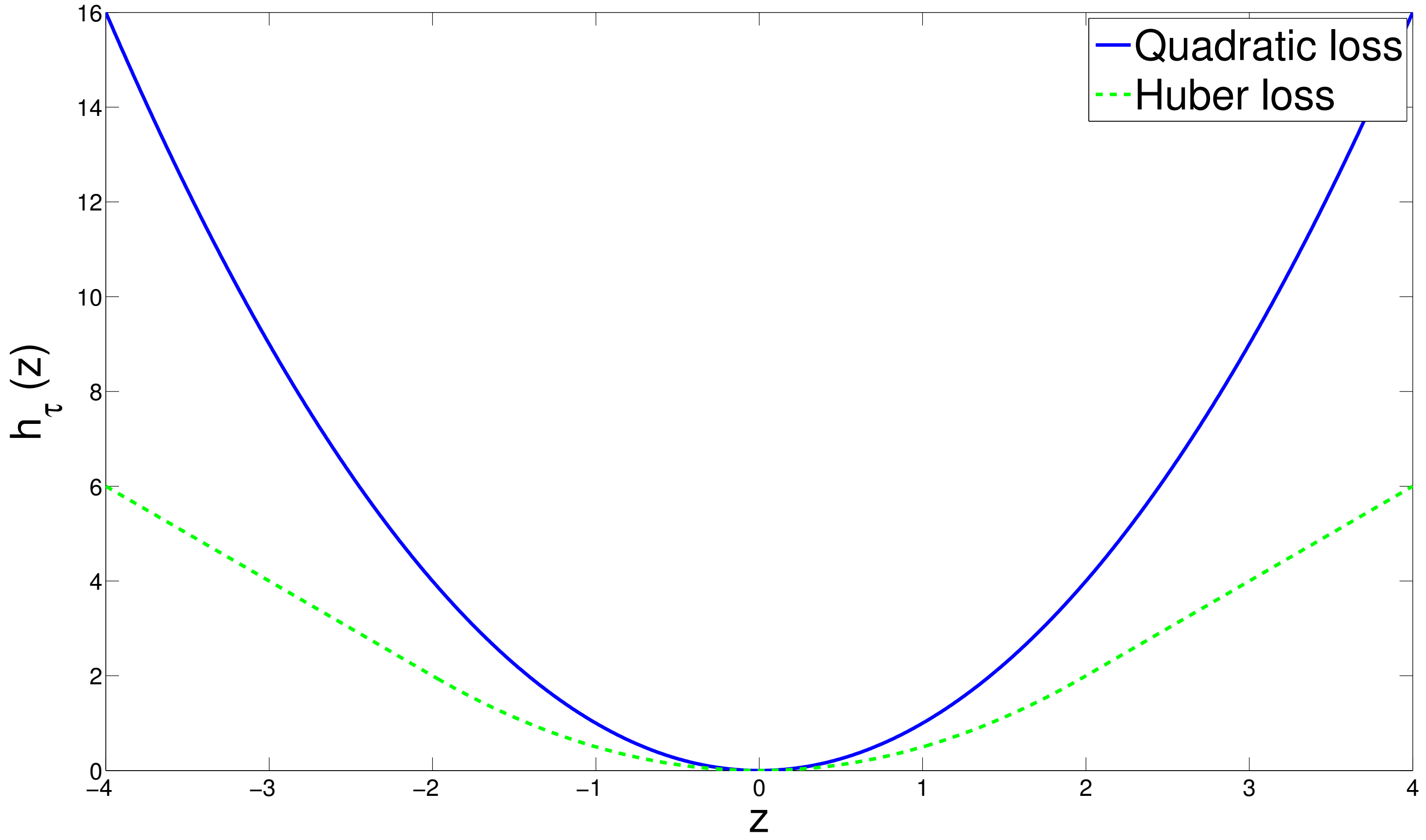

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ML and MAP Estimation: Discussion

Figure 3: Low-resolution frame (left), ML estimation (middle), MAP estimation with TV prior (right)

Topics

Maximum A Posteriori Estimation

Image Priors

Summary

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Further Readings

Take Home Messages

- Super-resolution is an ill-posed problem, so using regularization by utilizing prior knowledge is a good idea.
- We introduced several regularizers: Tikhonov, TV, BTV, and the edge-preserving approach using the Huber loss function.
- Regularization needs extra thought when implementing, but it is an important method to achieve adequate image quality.

Further Readings

Theory of image super-resolution (books and review articles):

- Hayit Greenspan. "Super-Resolution in Medical Imaging". In: *The Computer Journal* 52.1 (Feb. 2008), pp. 43–63. DOI: 10.1093/comjnl/bxm075
- Peyman Milanfar, ed. *Super-Resolution Imaging*. Digital Imaging and Computer Vision. CRC Press, 2011
- Sina Farsiu et al. "Advances and Challenges in Super-Resolution". In: *International Journal of Imaging Systems and Technology* 14.2 (Aug. 2004), pp. 47–57. DOI: 10.1002/ima.20007
- Sung Cheol Park, Min Kyu Park, and Moon Gi Kang. "Super-Resolution Image Reconstruction: A Technical Overview". In: *IEEE Signal Processing Magazine* 20.3 (May 2003), pp. 21–36. DOI: 10.1109/MSP.2003.1203207

ML/MAP super-resolution:

- Lyndsey C. Pickup. "Machine Learning in Multi-frame Image Super-resolution". PhD Thesis. Robotics Research Group, University of Oxford, 2007
- Michael Elad and Arie Feuer. "Restoration of a Single Superresolution Image from Several Blurred, Noisy, and Undersampled Measured Images". In: *IEEE Transactions on Image Processing* 6.12 (Dec. 1997), pp. 1646–1658. DOI: 10.1109/83.650118

Medical Image Processing for Interventional Applications

Super-Resolution: Applications

Online Course – Unit 24
Andreas Maier, Thomas Köhler, Frank Schebesch
Pattern Recognition Lab (CS 5)

virtuelle hochschule bayern
FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG
TECHNISCHE FAKULTÄT

Topics

Retinal Fundus Video Imaging

Hybrid 3-D Endoscopy

Summary

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Further Readings

Retinal Fundus Video Imaging

- Fundus imaging is a standard technique to acquire **photographs** of the human eye background.
- Fundus video imaging enables the observation of fast temporal changes on the retina.
- Video frames have a lower spatial resolution and signal-to-noise ratio (compared to common fundus cameras).

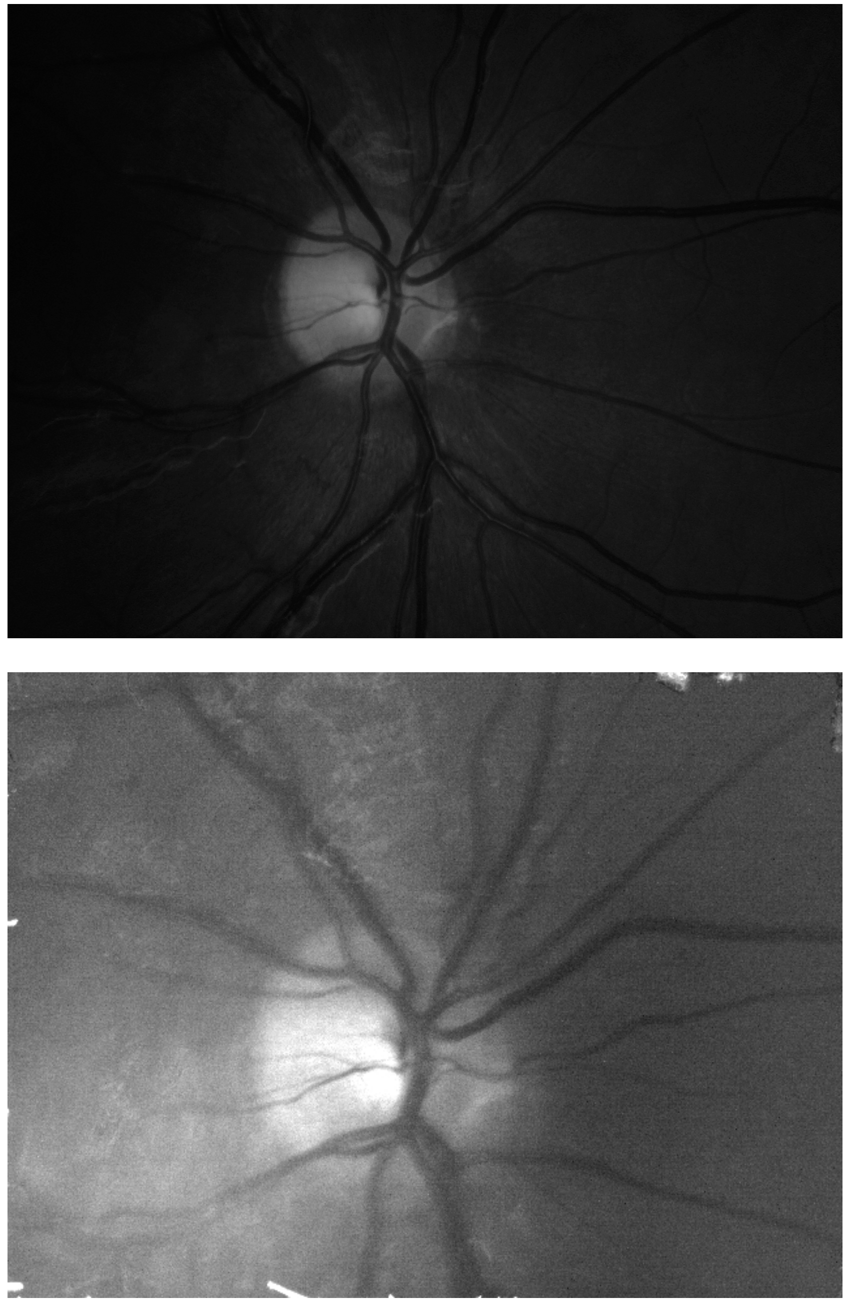
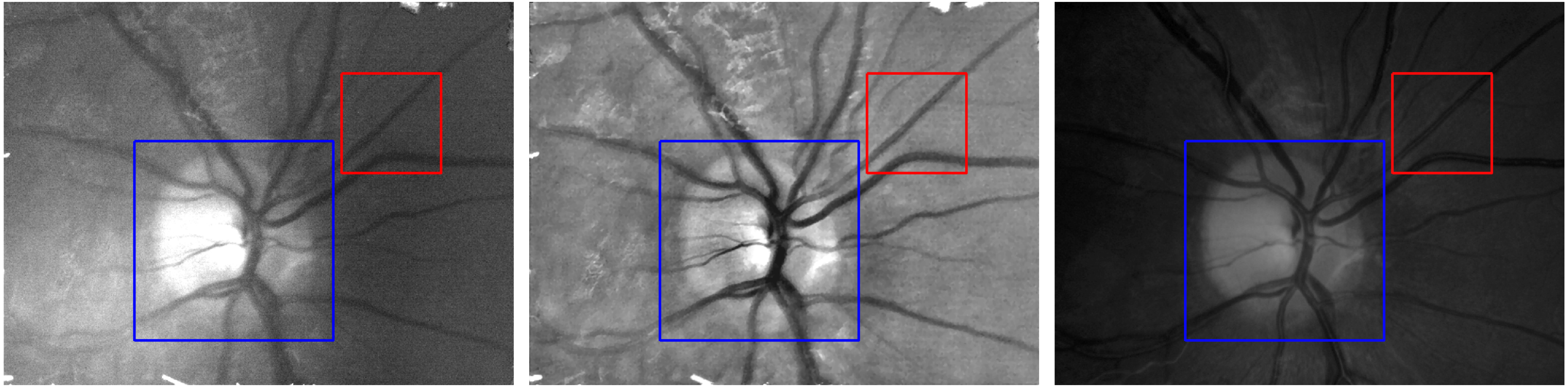


Figure 1: Fundus images

(Low-cost) Fundus Video vs. Kowa Nonmyd Camera

We utilize super-resolution to reconstruct high-resolution fundus images:



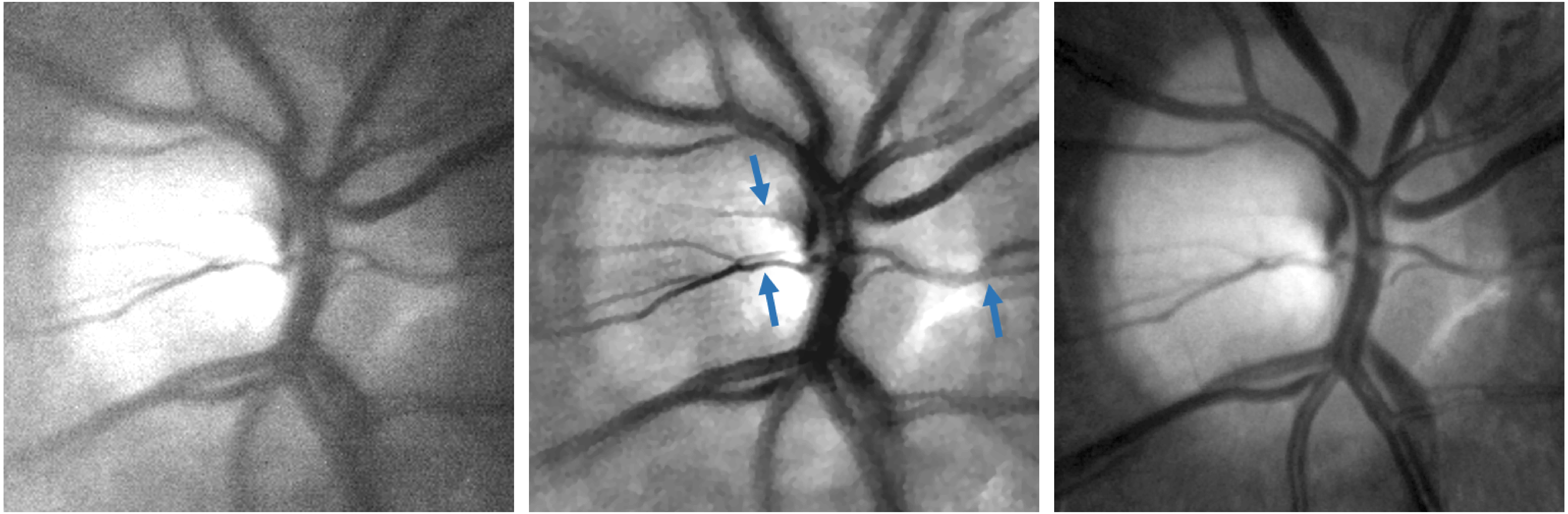
LR frame　　SR image　　Kowa nonmyd

Super-resolution approach (Köhler et al., 2014):

- Enhance spatial resolution and signal-to-noise ratio
- Correct inhomogeneous and temporally varying illumination

Analysis of the Optic Nerve Head

(Low-cost) video imaging vs. Kowa nonymd camera (Köhler et al., 2014):

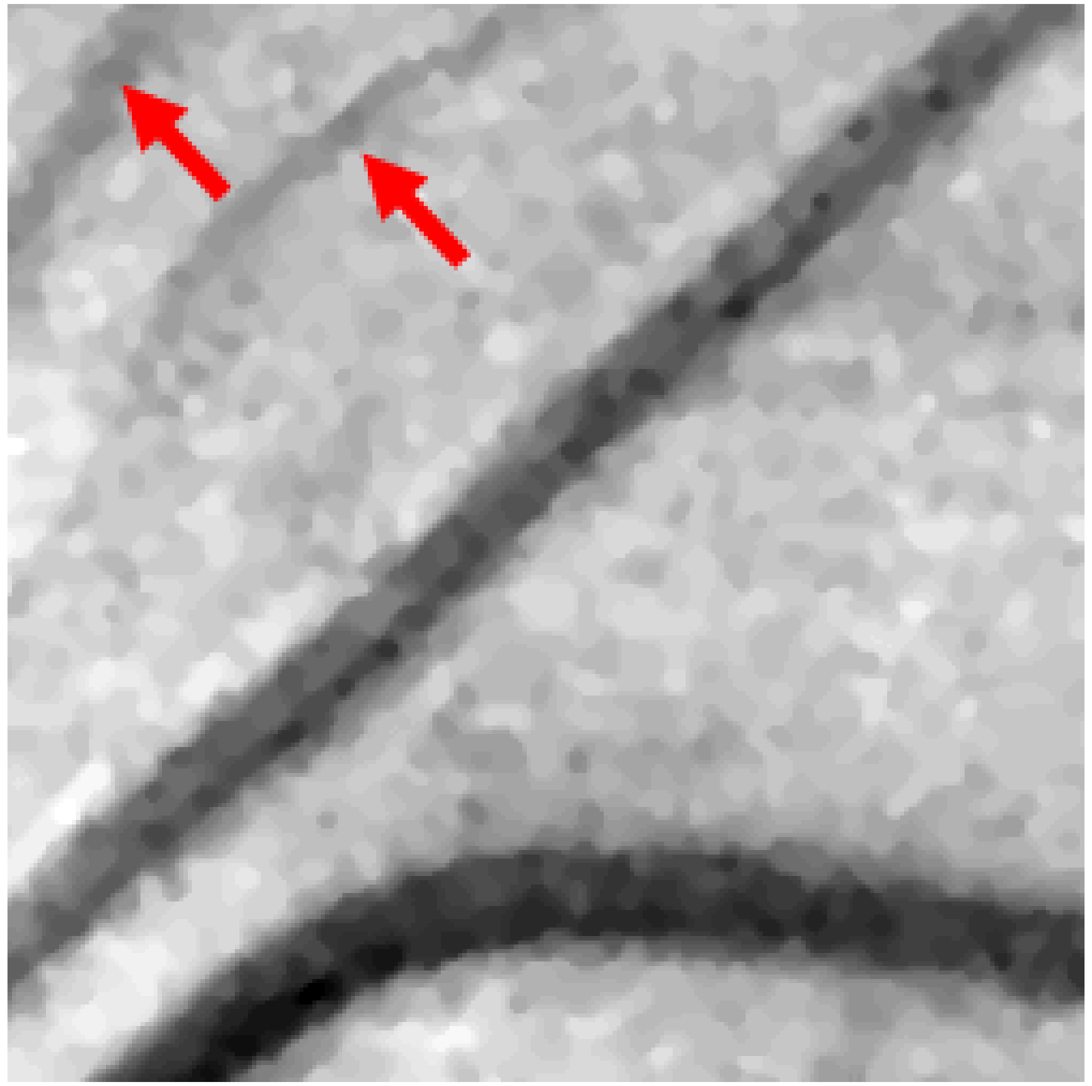


LR frame | SR image | Kowa nonmyd

Analysis of Retinal Blood Vessels

(Low-cost) video imaging vs. Kowa nonymd camera (Köhler et al., 2014):

LR frame



SR image

Kowa nonmyd

Topics

Retinal Fundus Video Imaging

Hybrid 3-D Endoscopy

Summary

- Take Home Messages
- Further Readings

Hybrid 3-D Endoscopy

- **Sensor fusion** of photometric (RGB) and 3-D range data (e. g., time-of-flight, structured light) in one endoscope (Haase et al., 2013)
- Exploit information of **complementary modalities**
- Restoration of low-resolution range data by means of super-resolution guided by photometric data
  $\rightarrow$ **Multi-sensor super-resolution**

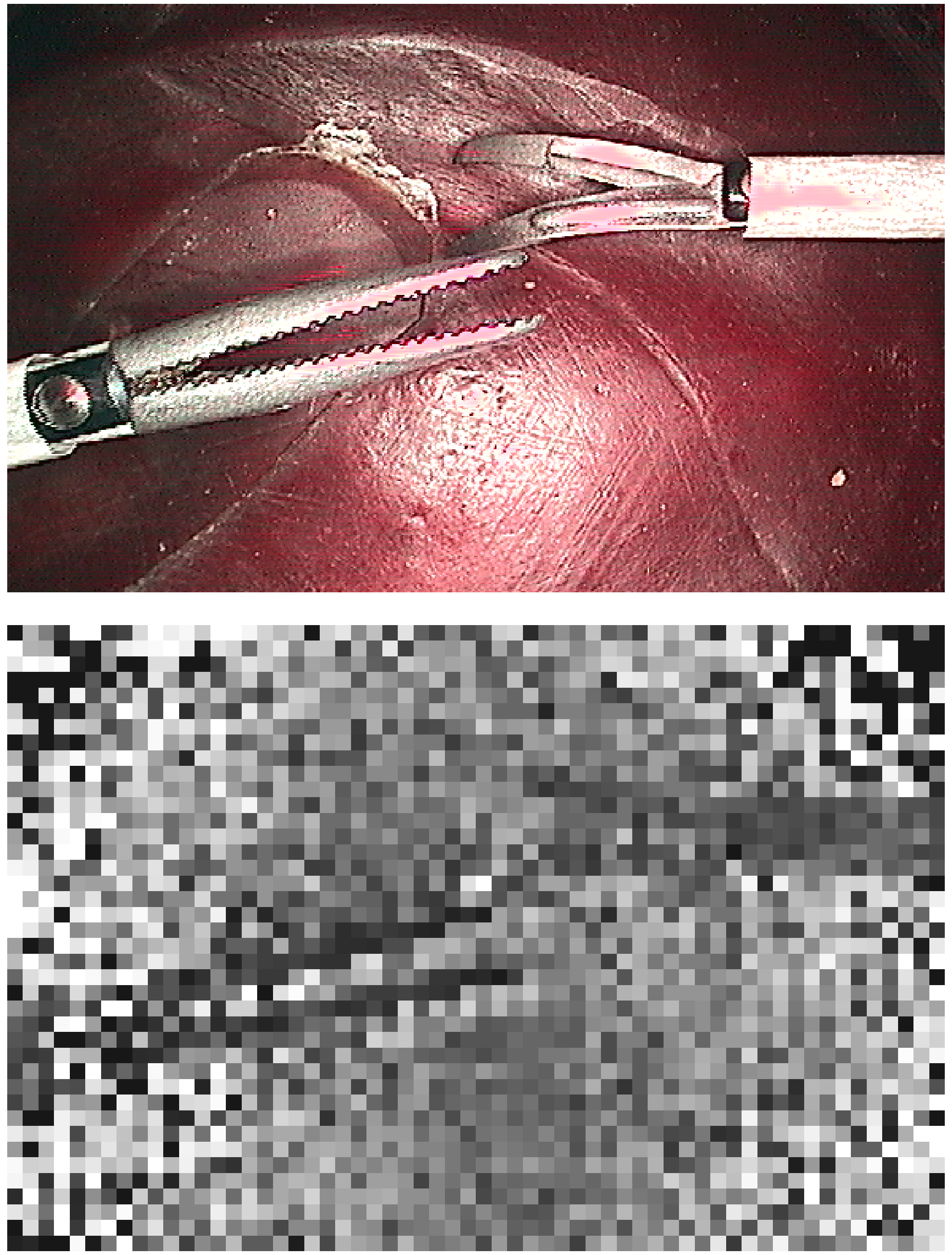


Figure 5: RGB (top) and time-of-flight (ToF) data (bottom)

Multi-Sensor Super-Resolution

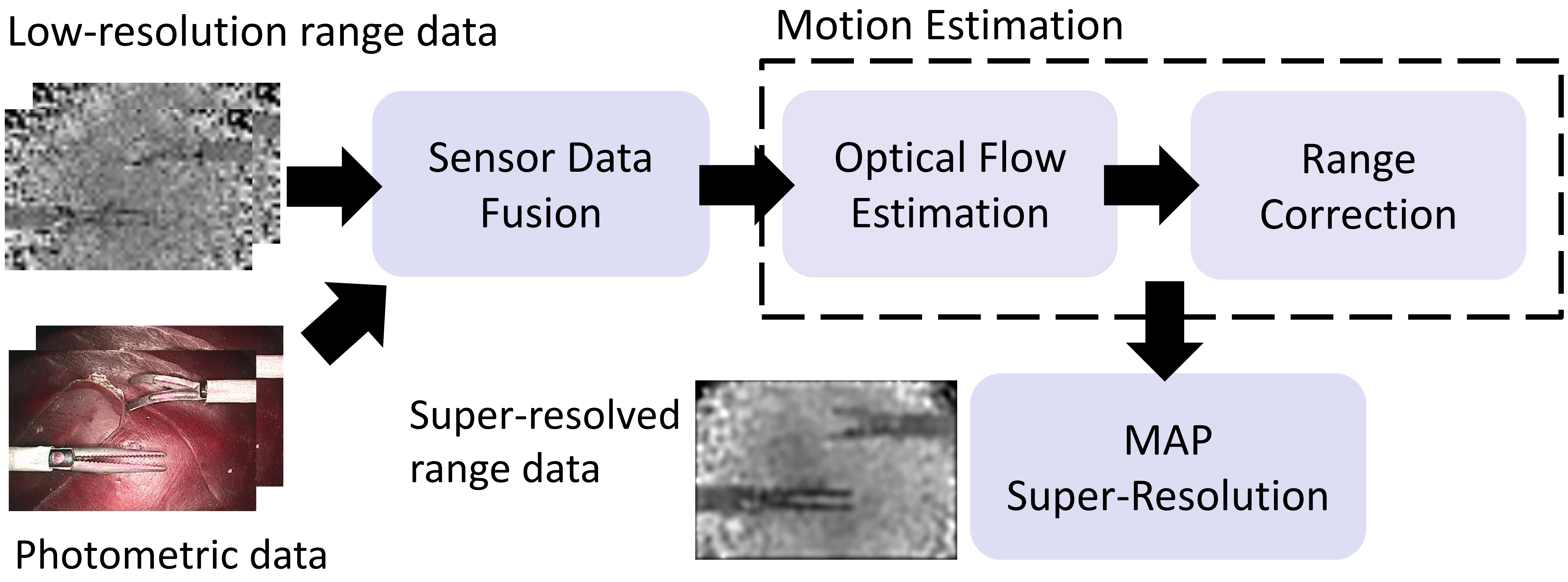
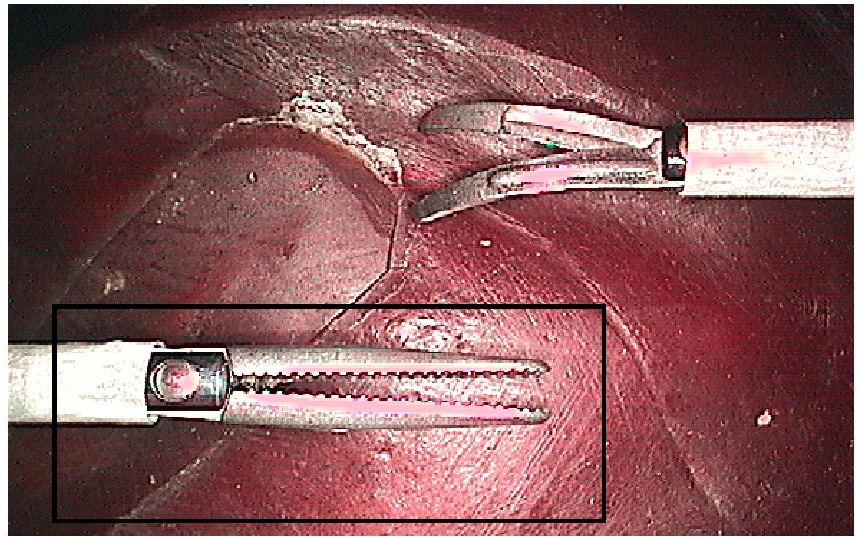


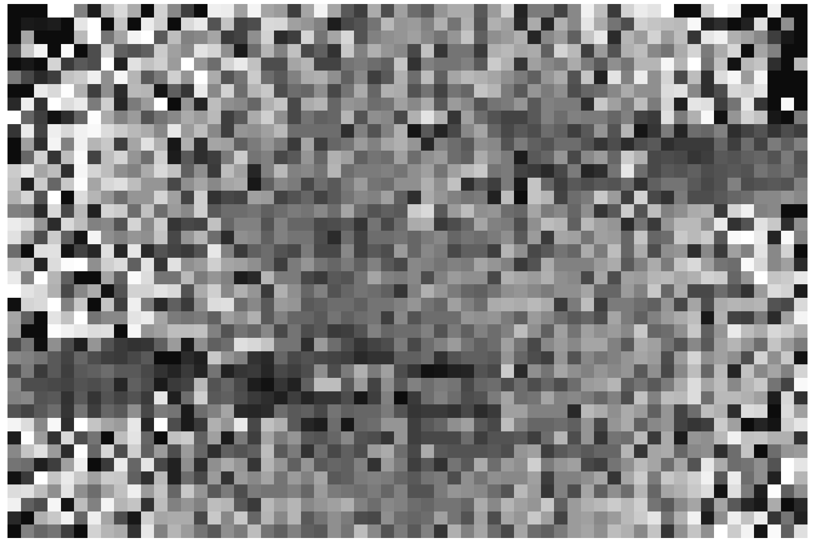
Figure 6: Flowchart for multi-sensor super-resolution (Köhler et al., 2013)

- Robust motion estimation (optical flow) on photometric data
- MAP super-resolution for range data reconstruction

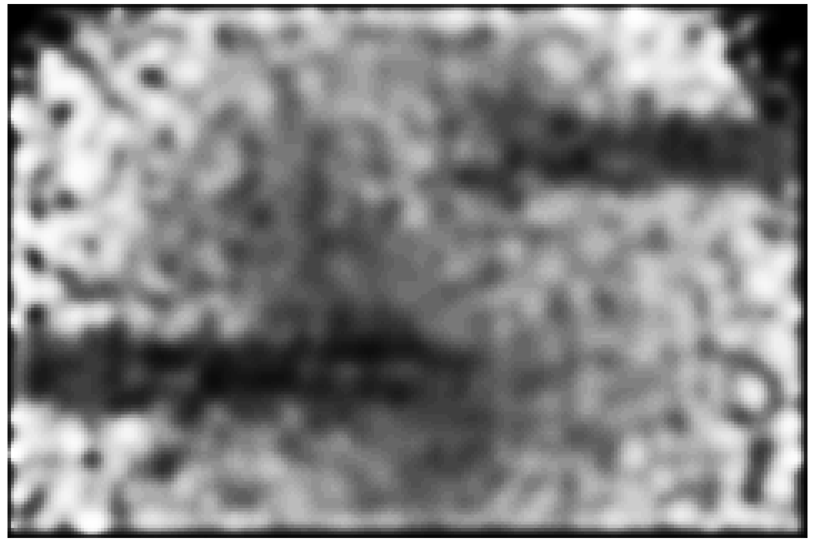
Single-sensor (SSR) vs. multi-sensor super-resolution (MSR)



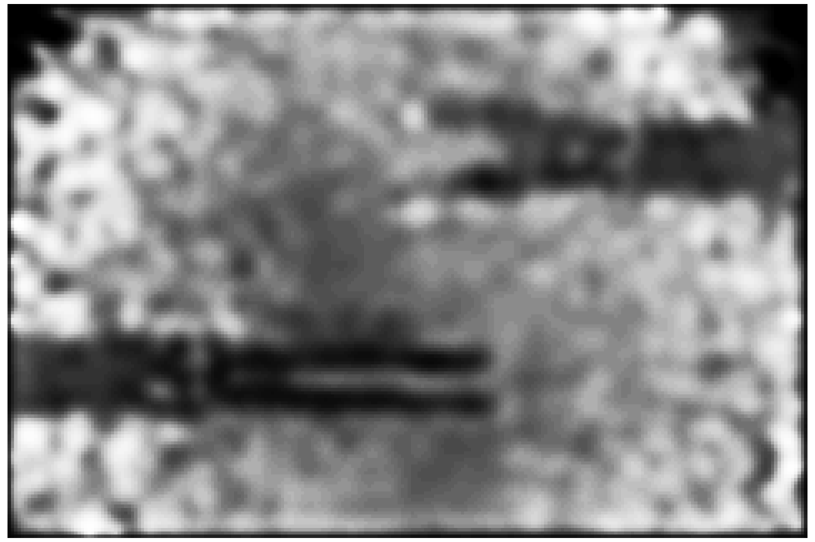
RGB image



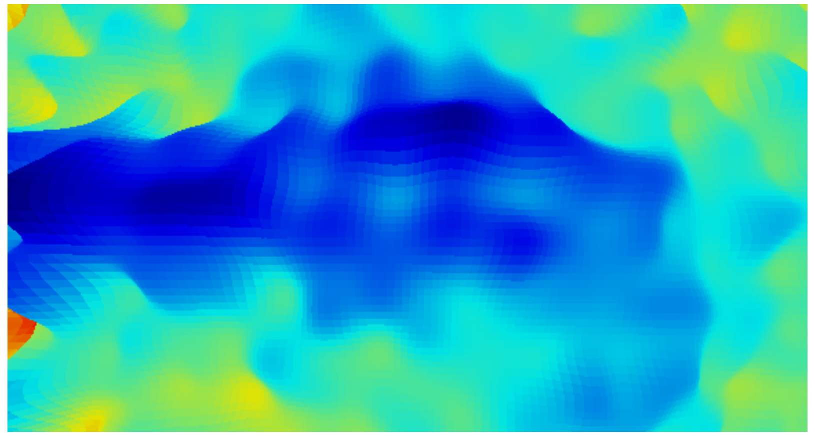
Range image



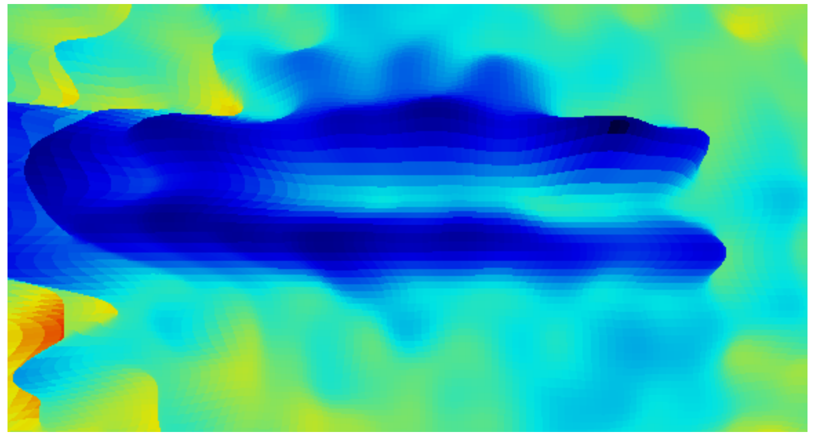
SSR



MSR



SSR (3-D mesh)



MSR (3-D mesh)

Adaptive MSR

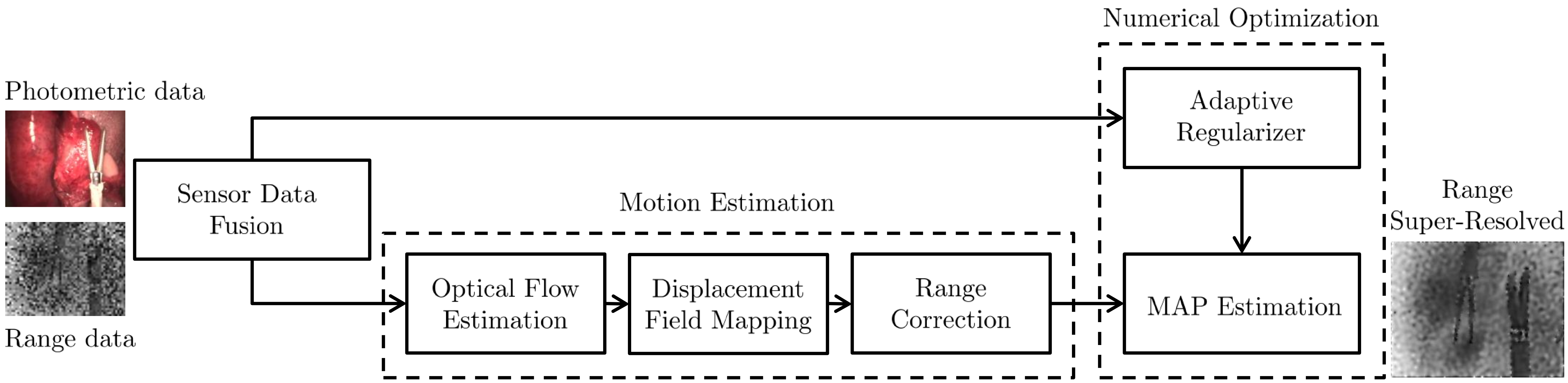
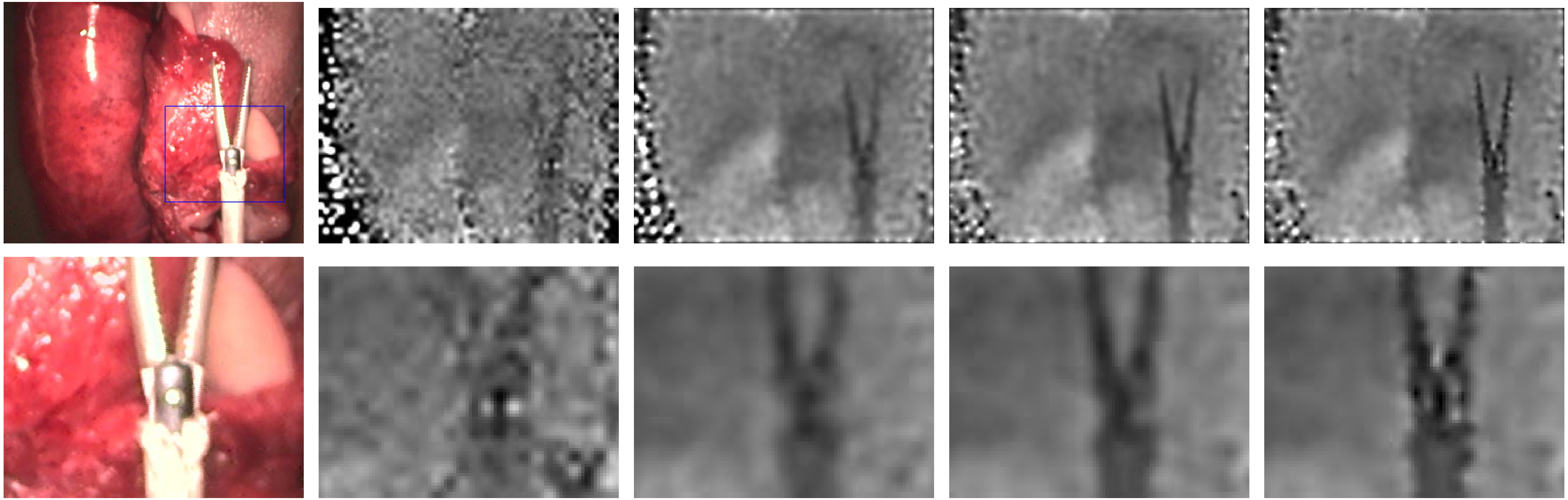


Figure 7: Extended flow chart including an adaptive scheme in the numerical optimizer (Köhler et al., 2015)

- Decrease the impact of the regularizer on edges
- Two-stage algorithm: first stage MSR, second stage spatially adaptive weighted regularization

Comparison of Methods



RGB image | Range image | SSR | MSR | AMSR

Topics

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Further Readings

Take Home Messages

- Super-resolution can improve image resolution in both applications from diagnostic as well as interventional imaging.
- Super-resolution algorithms are quite involved and there is a variety of approaches documented in literature.

Further Readings

The shown methods are described in detail in the following articles:

- Sven Haase et al. "ToF/RGB Sensor Fusion for 3-D Endoscopy". In: *Current Medical Imaging Reviews* 9.2 (2013), pp. 113–119. DOI: 10.2174/1573405611309020006
- Thomas Köhler et al. "ToF Meets RGB: Novel Multi-Sensor Super-Resolution for Hybrid 3-D Endoscopy". In: *Medical Image Computing and Computer-Assisted Intervention – MICCAI 2013: 16th International Conference, Nagoya, Japan, September 22-26, 2013, Proceedings, Part I*. ed. by Kensaku Mori et al. Vol. 8149. Lecture Notes in Computer Science. Berlin, Heidelberg: Springer, 2013, pp. 139–146. DOI: 10.1007/978-3-642-40811-3_18
- Thomas Köhler et al. "Multi-frame Super-resolution with Quality Self-assessment for Retinal Fundus Videos". In: *Medical Image Computing and Computer-Assisted Intervention – MICCAI 2014: 17th International Conference, Boston, MA, USA, September 14-18, 2014, Proceedings, Part I*. ed. by Polina Golland et al. Vol. 8673. Lecture Notes in Computer Science. Cham: Springer International Publishing, 2014, pp. 650–657. DOI: 10.1007/978-3-319-10404-1_81
- Thomas Köhler et al. "Multi-Sensor Super-Resolution for Hybrid Range Imaging with Application to 3-D Endoscopy and Open Surgery". In: *Medical Image Analysis* 24.1 (Aug. 2015), pp. 220–234. DOI: 10.1016/j.media.2015.06.011