Medical Image Processing for Diagnostic Applications

3-D Data – Plane-Integrals

Online Course – Unit 45

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Topics

3-D Radon Transform

The 3-D Radon transform is not equal to the X-ray transform!

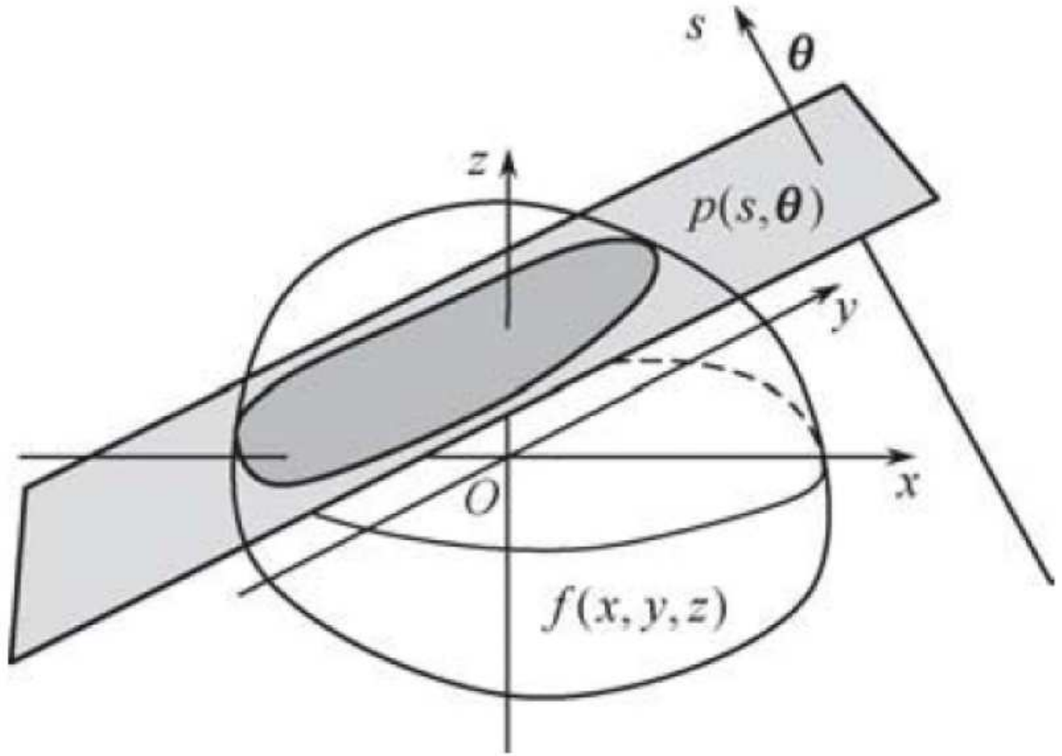

Figure 1: Integral over a plane through the object $f(x, y, z)$ at distance s from the origin and normal $\boldsymbol{\theta}$ (Zeng, 2009)

3-D Radon transform

- We do not have direct detectors for this transform.
- We can compute the value of such a “detector” as a line-integral on a 2-D parallel beam detector.
- There is a central slice theorem for this transform.
- Backprojection is different as a point has to be backprojected as a plane into 3-D.

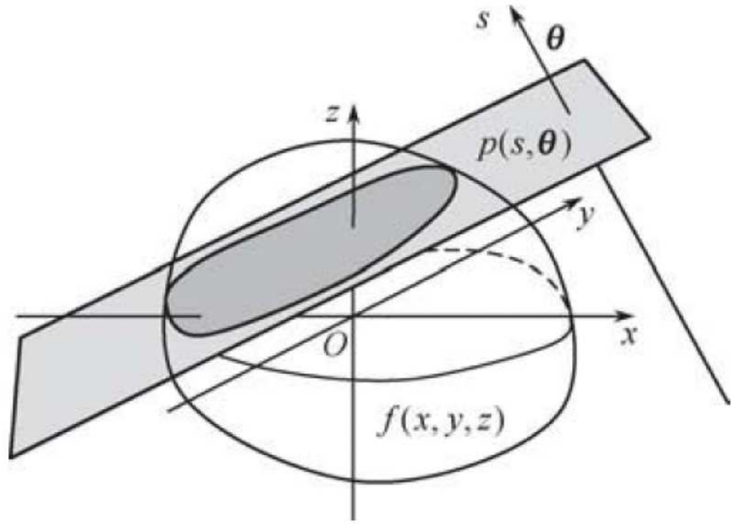

Figure 2: Scheme of the Radon transform (Zeng, 2009)

Central Slice Theorem for Plane-Integral Data

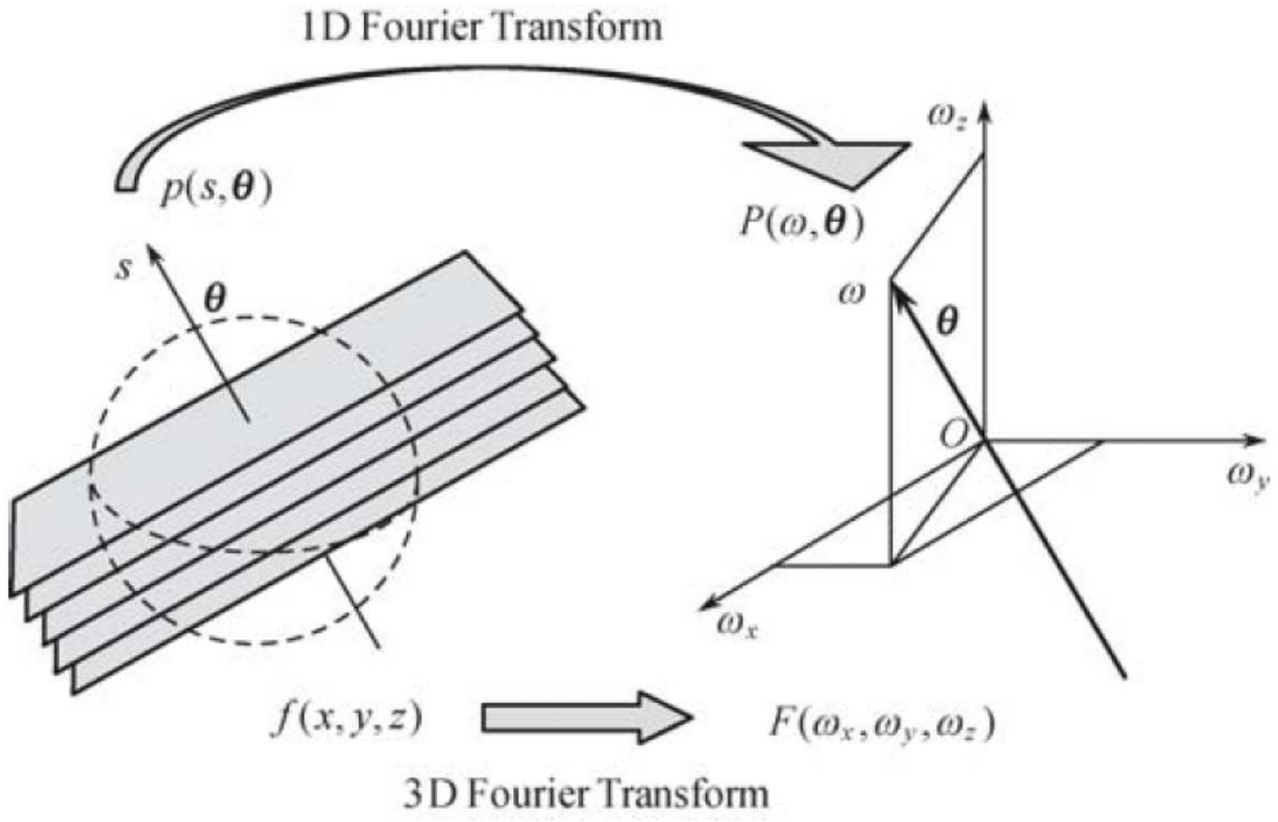

Figure 3: This central slice theorem connects the 3-D Fourier transform of the object with the 1-D Fourier transform of the plane-integrals (Zeng, 2009).

Parallel Plane-Integral Data: Backprojection

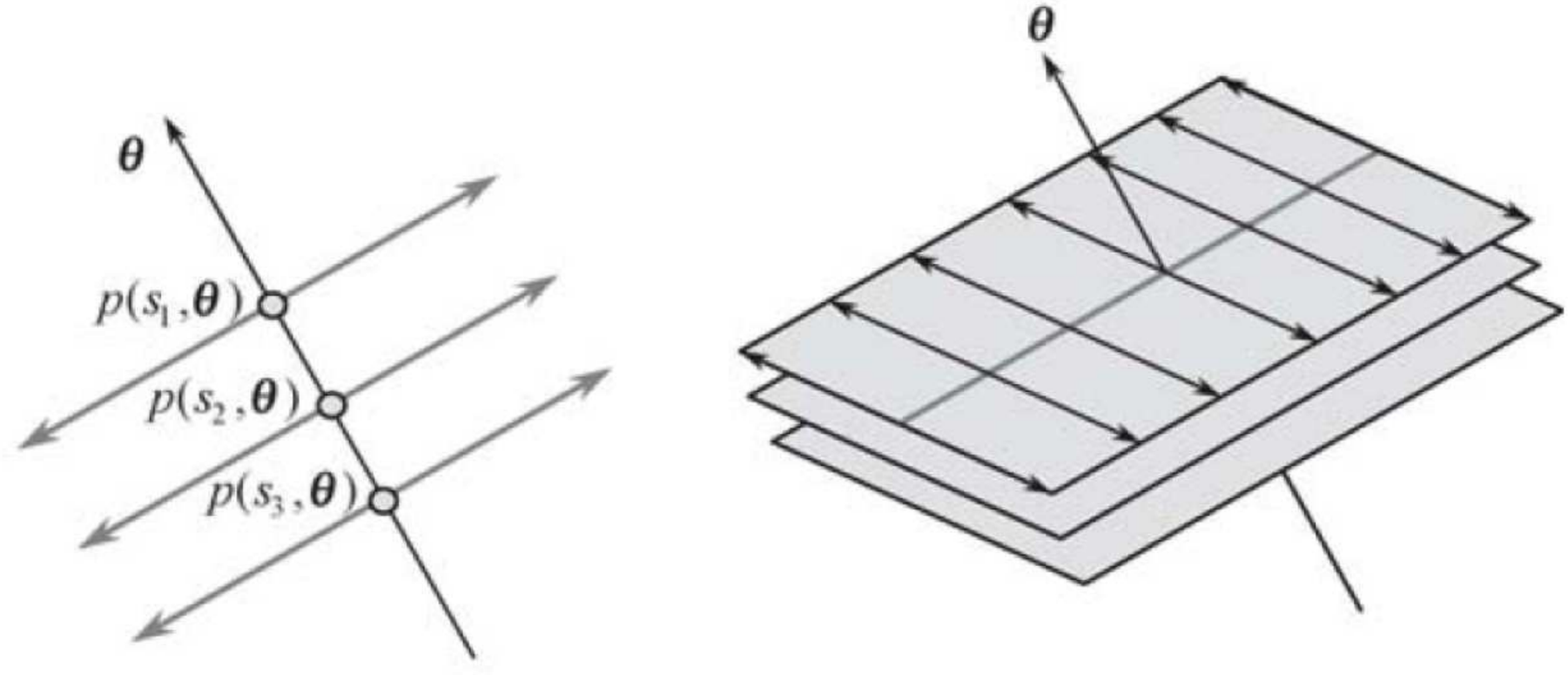

Figure 4: Scheme of how the plane-integral data can be backprojected (Zeng, 2009)

3-D Radon Inversion Formula

$$f(x,y,z) = -\frac{1}{8\pi^2} \iint\limits_{[0,2\pi]^2} \frac{\partial^2 p(s,\boldsymbol{\theta})}{\partial s^2}\Big|_{s=\boldsymbol{x}\cdot\boldsymbol{\theta}} \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi,$$

where

$$\boldsymbol{\theta} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

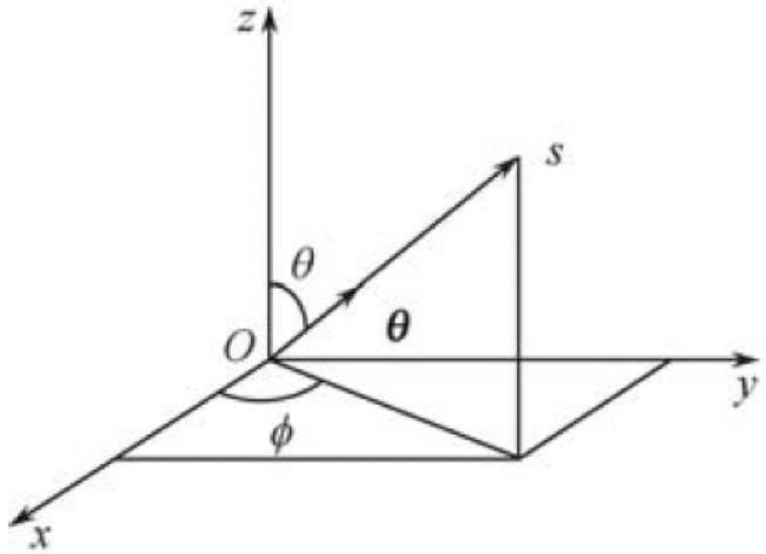

Figure 5: Diagram shows the different angles and other variables of the inversion formula (Zeng, 2009)

Topics

Take Home Messages

- There is a central slice theorem for parallel plane-integral data in 3-D.
- The 3-D Radon transform cannot be directly translated into detector hardware, but it can be inverted and therefore be considered for reconstruction algorithms.

Further Readings

The best way to augment your knowledge of the shown concepts is to read the companion book of the current chapter:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9