Medical Image Processing for Diagnostic Applications

3-D Data – Line-Integrals

Online Course – Unit 44

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Topics

From 2-D to 3-D

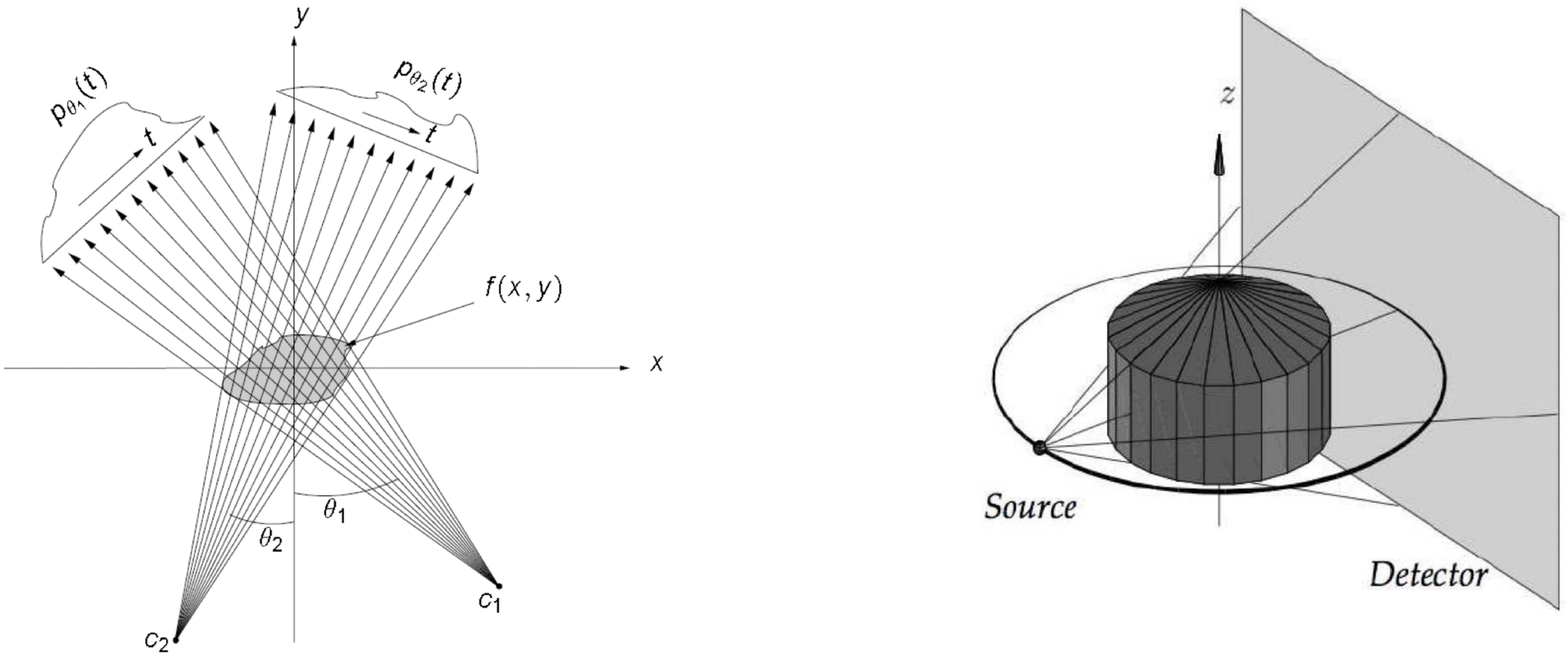

Figure 1: The fan beam geometry is a 2-D pattern (left), its corresponding 3-D structure is the cone beam (right).

Topics

Parallel Line-Integral Data

- **Problem:** Line-integrals may be along rays that are not perpendicular to the axis of rotation.
- How does this affect our reconstruction?

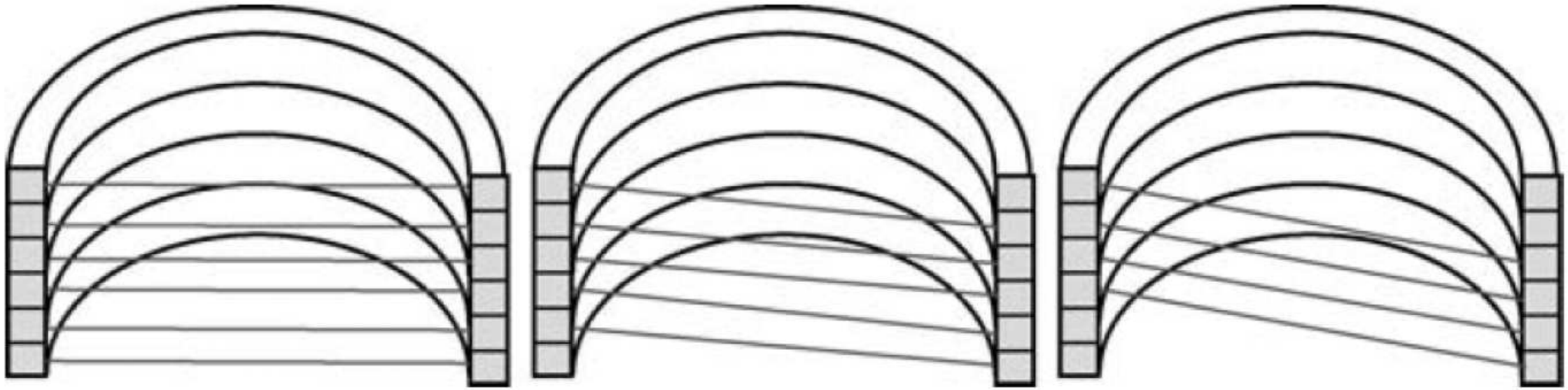

Figure 2: Several parallel ray paths from source to detector (Zeng, 2009)

Central Slice Theorem for Line-Integral Data

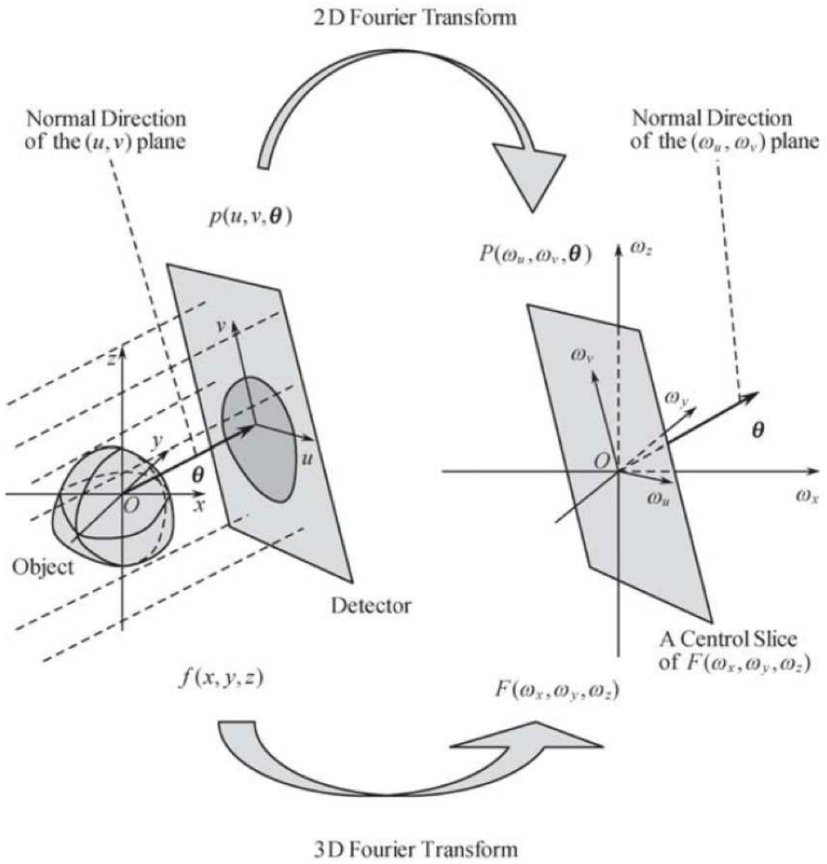

Figure 3: The 2-D Fourier transform of a projection corresponds to a slice through the origin in the 3-D Fourier transform of the object (Zeng, 2009).

Parallel Line-Integral Data

Parallel projections on rays that are perpendicular to the rotation axis can fill the complete Fourier space:

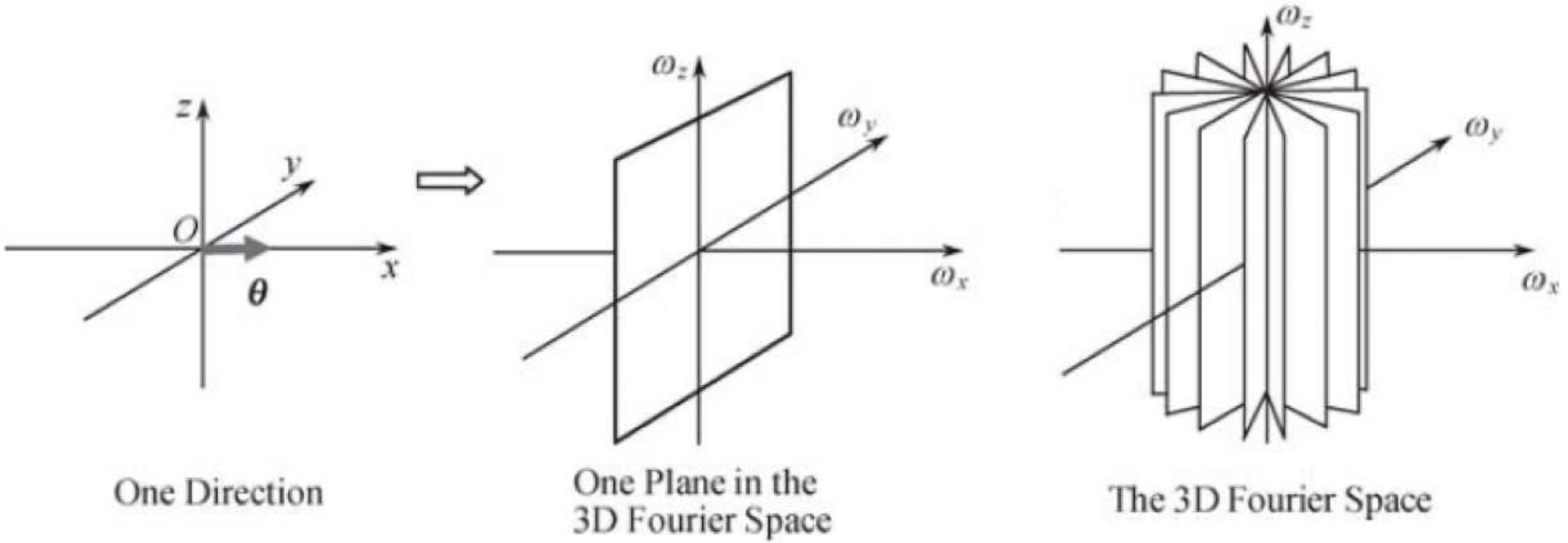

Figure 4: Sampling the Fourier space with parallel line-integrals (Zeng, 2009)

Parallel Line-Integral Data

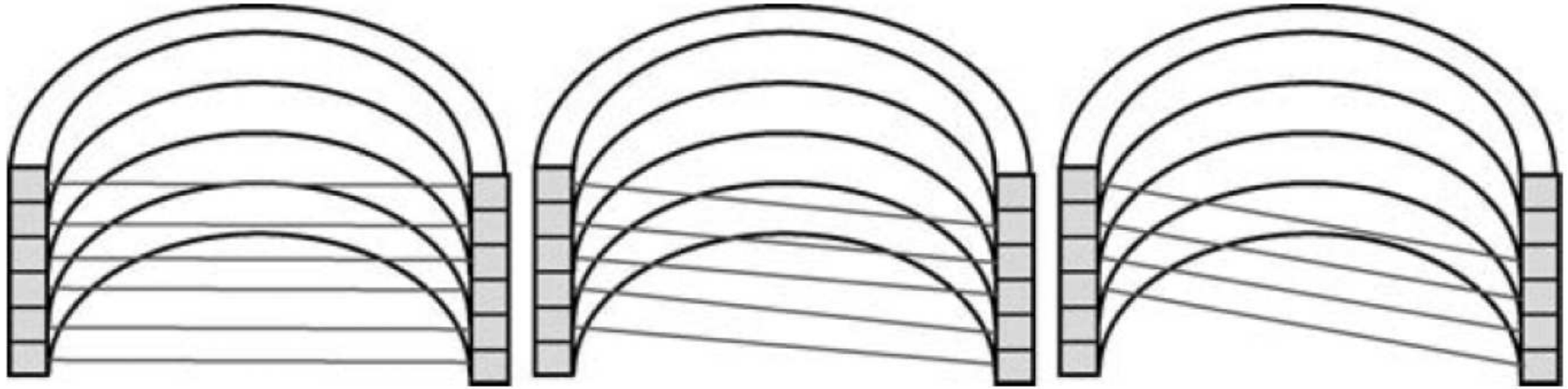

Figure 5: Several parallel ray paths from source to detector (Zeng, 2009)

Fourier Space:

Topics

Condition for Data Completeness

Orlov's condition: A complete data set can be obtained if every great circle intersects the trajectory.

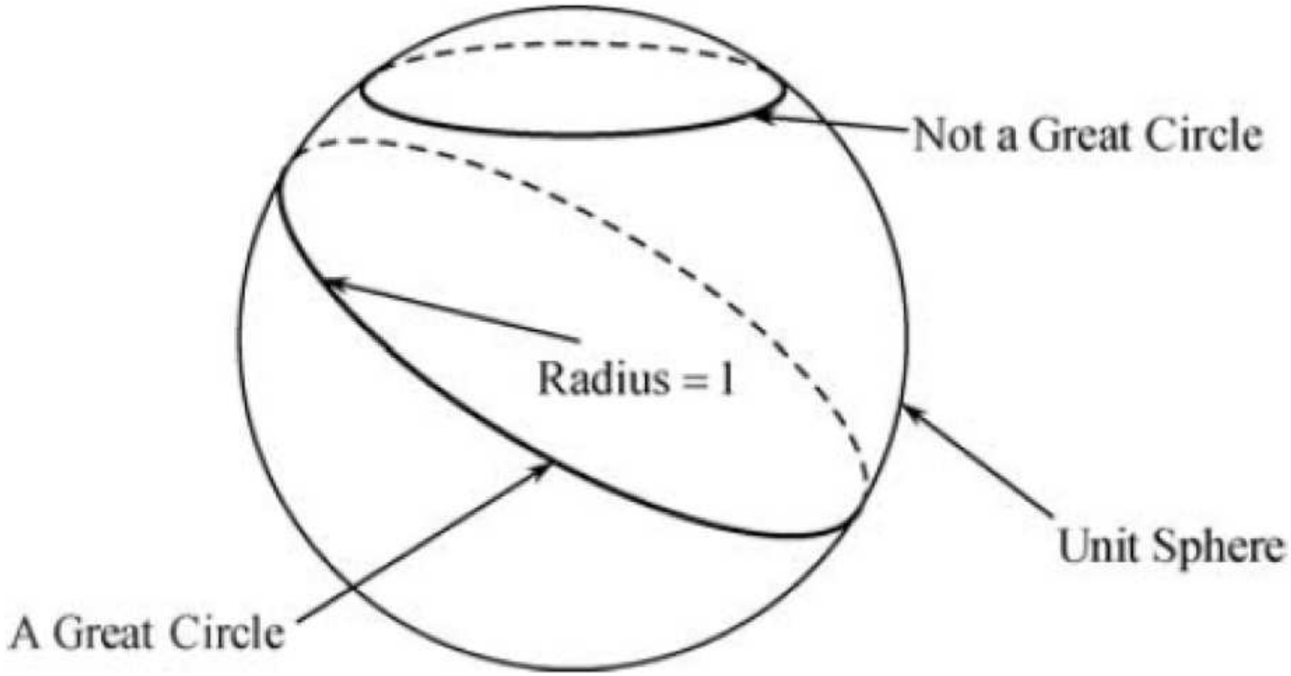

Figure 6: Diagram showing the definition of great circles (Zeng, 2009)

Examples of 3-D Trajectories

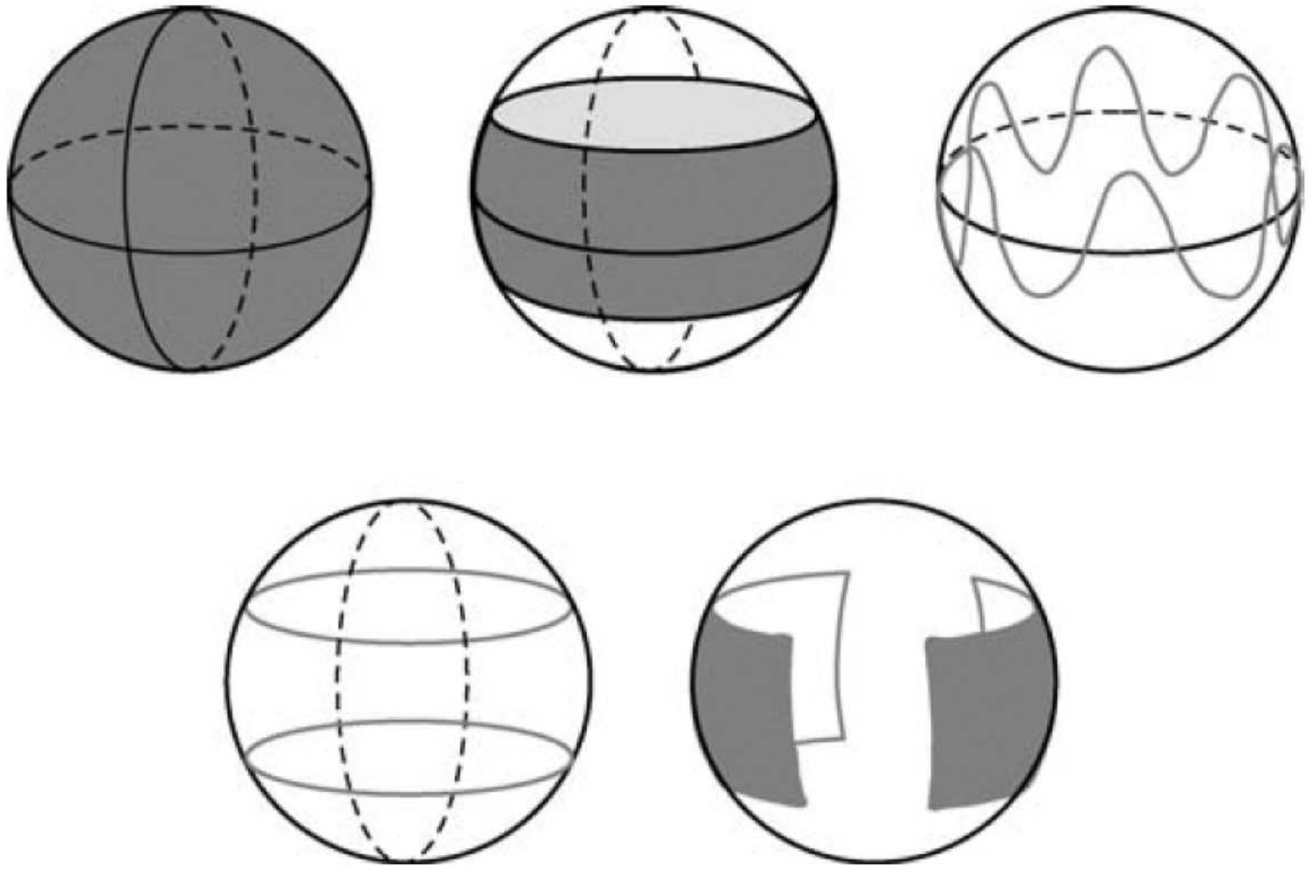

Figure 7: Several possible trajectories: which of these yield a complete data set (Zeng, 2009)?

Trajectories

Fourier Space:

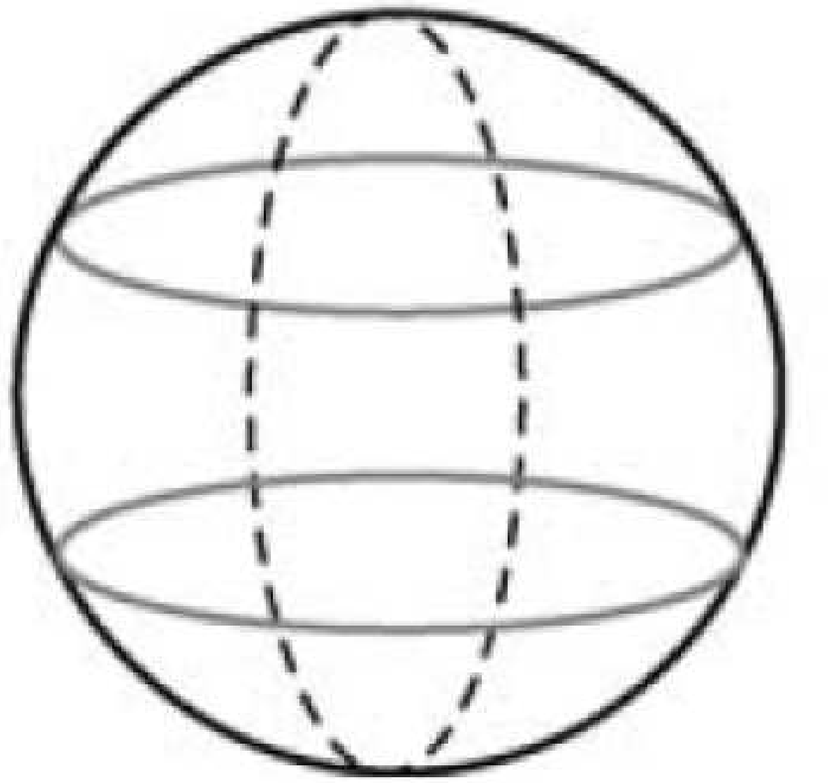

Figure 8: Trajectory consisting of two circles (Zeng, 2009)

Trajectories

Fourier Space:

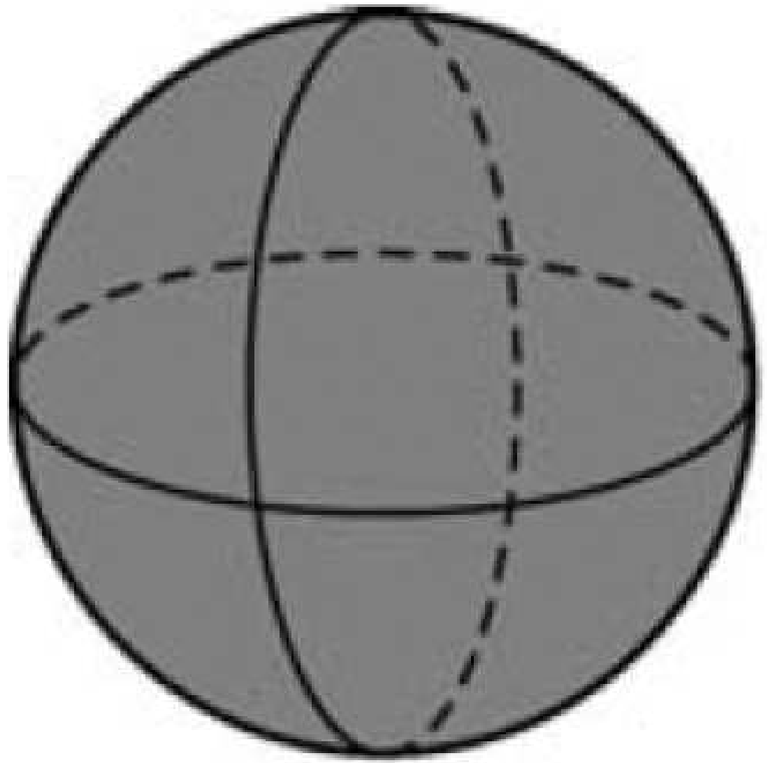

Figure 9: Trajectory containing every direction on the sphere (Zeng, 2009)

Trajectories

Fourier Space:

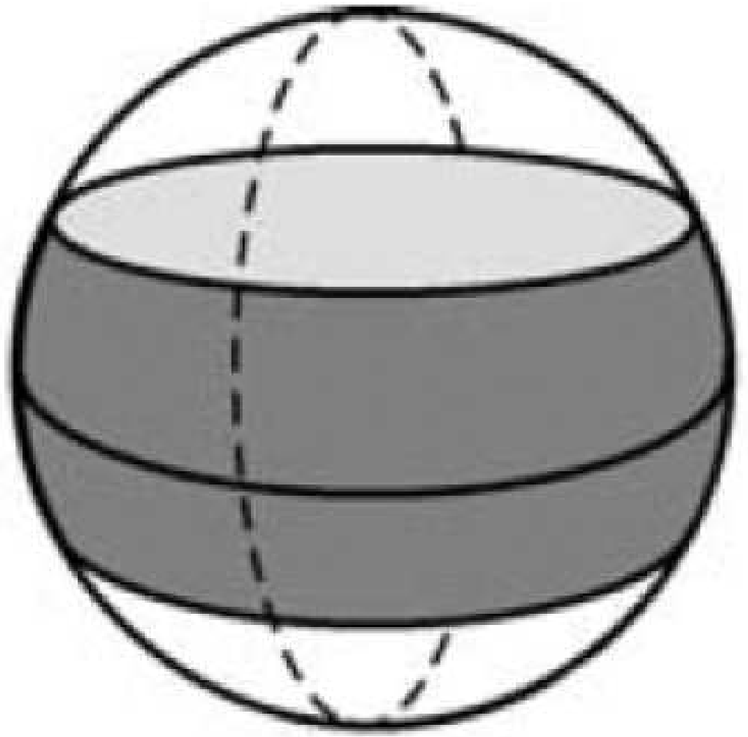

Figure 10: Trajectory consisting of a band segment of the unit circle (Zeng, 2009)

Topics

BPF Algorithm for Parallel Line-Integral Data

- For the unit sphere the PSF can be shown to be

$$h(x,y,z) = \frac{1}{x^2+y^2+z^2} = \frac{1}{r^2}.$$

- The back projection of the data is computed by

$$b = f ** * h,$$

$$B(\omega_x,\omega_y,\omega_z) = F(\omega_x,\omega_y,\omega_z) \cdot H(\omega_x,\omega_y,\omega_z),$$

where $***$ denotes the 3-D convolution, and

$$B = \mathrm{FT}(b),\ F = \mathrm{FT}(f),\ H = \mathrm{FT}(h).$$

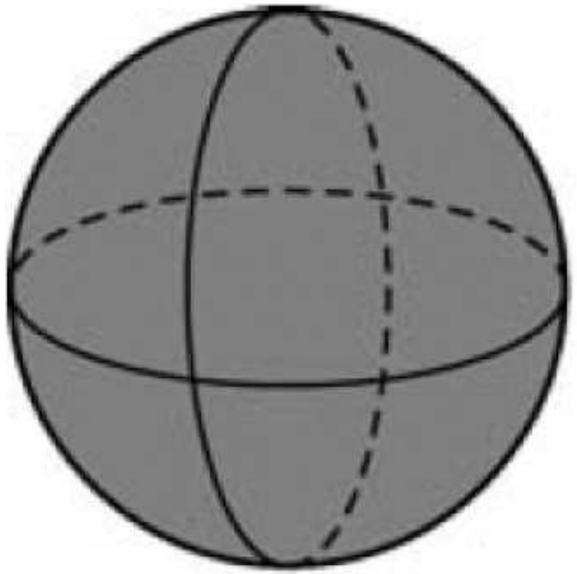

Figure 11: Trajectory containing every direction on the sphere (Zeng, 2009)

Topics

Take Home Messages

- There is a central slice theorem for parallel line-integral data in 3-D.
- Data completeness depends on the used trajectory $\rightarrow$ Orlov's condition.

Further Readings

The best way to augment your knowledge of the shown concepts is to read the companion book of the current chapter:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9