Medical Image Processing for Interventional Applications

Introduction

Compute condition number

Determine the value of the condition number for the following matrices.

The matrix $B \in R^{20\times 20}$ is defined as $B = Udiag(a1,\ldots,a20)V^{\backslash\mathrm{T}}$ with elements $a_n = \frac{1}{(n-5)^2+4}, n = 1,\ldots,20$. (The matrix is defined like that, it is not necessarily a proper SVD according to the definition in the lecture.) Computing the condition number of this matrix we get the value $\kappa(B) = 57.25$

Test your knowledge about SVD

In this unit, you will learn about certain properties of the SVD, such as rank determination, the relationship to eigenvectors of $A^\top A$ and $AA^\top$ as well as the condition of matrices.

By **decomposing** a matrix with SVD, we can easily determine the **rank** of a matrix. It is equal to the number of **singular** values **greater than zero** . The eigenvalues of $A^T A$ are ** σ^2 ** . A high condition number is **bad** for computations, because the **ratio** of largest and smallest singular value is **large** .

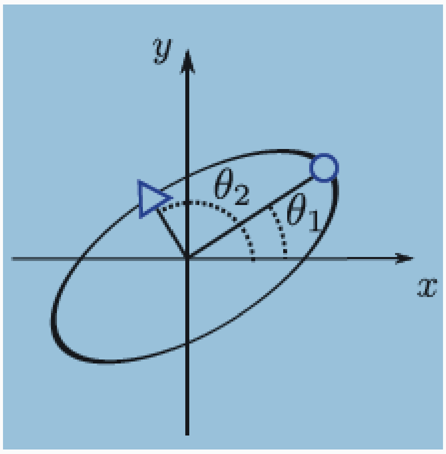
Applications of SVD

The singular value decomposition is an important theoretical and also practical tool for a lot of applications. Every linear system can be written as a matrix-vector equation $Ax = b$.
In medical imaging we often have to deal with this type of algebraic equation and knowing about the SVD is essential in this context. In this unit we find out, how this normal form of matrices works.

- [x] Computation of condition numbers
- [ ] Ranking matrices
- [x] Low-rank approximations of images
- [x] Solving linear systems
- [ ] Computation of nonsingular values
- [x] Computation of the null space

Understanding SVD

Take a look at the ellipse on the figure. Let $\theta_1 = 30^o$; $\theta_2 = 120^o$, the coordinates of the circle and triangle in the shown axes are $(3\sqrt{3}, 3)$and $(-1, \sqrt{3})$



We consider a mapping that transforms the ellipse into a unit circle.

Because this mapping is linear it can be represented by a 2x2 matrix.
Consider the effect of the components of the SVD of this matrix. Can you complete the following statement for such a mapping?

This mapping **is not** unique, because **any orthogonal transformation of ∑V⊤ maps the ellipse to the unit sphere**

Image rank k - approximations

Given the definition from the third optimization problem, how many different rank k-approximations can you compute for an image of size 32x64 (at most)?

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Inversion of a linear system

In this unit, we will learn more about optimization using SVD. In the first problem of this unit, we discuss rank k-approximations of matrices. The last problem of this chapter shows how the pseudoinverse optimally approximates the inverse for a overdetermined linear system (in a least square sense). This is applied to compute a regression line for a given set of points.

A linear system of type $Ax = b$ can always be solved using the **pseudoinverse oder Moore-Penrose pseudoinverse** . If we need to compute it, we could use **SVD oder singular value decomposition oder svd**

Features

Vesselness (Example)

```
Introduction:
```

Consider **two regions** of an image that can **locally** be **approximated** by the functions

$f(x, y) = -\frac{(x-y)^2}{2}$ (Region 1)

and

$g(x, y) = -\frac{x^2+y^2}{2}$ (Region 2).

We want to **analyze the structure** of these regions by using the **vesselness measure**.

First, **compute the Hessians** of $f(x, y)$ and $g(x, y)$:

$H_f =$

-1	1
1	-1

`-1 1` `1 -1` wurde wie folgt interpretiert: $\begin{matrix} -1 & 1 \\ 1 & -1 \end{matrix}$

$H_g =$

-1	0
0	-1

`-1 0` `0 -1` wurde wie folgt interpretiert: $\begin{matrix} -1 & 0 \\ 0 & -1 \end{matrix}$

Determine the eigenvalues of both matrices (sort them by descending absolute value):

$H_f : \lambda_1 =$ -2

`-2` wurde wie folgt interpretiert: -2

and $\lambda_2 =$ 0

`0` wurde wie folgt interpretiert: 0

$H_g : \lambda_1 =$ -1

`-1` wurde wie folgt interpretiert: -1

and $\lambda_2 =$ -1

`-1` wurde wie folgt interpretiert: -1

Assume $\beta = 0.5$ and $c = 1$. Compute the values of $R_B and S$ for both regions, use them to calculate the vesselness, and insert your results here:

(%e^2-1)/%e^2

`(%e^2-1)/%e^2` wurde wie folgt interpretiert: $\frac{e^2-1}{e^2}$

(%e-1)/%e^3

`(%e-1)/%e^3` wurde wie folgt interpretiert: $\frac{e-1}{e^3}$

Comparing V_f and V_g, which of the regions indicate a vessel-like structure?
(*Hint*: Type *Region 1* or *Region 2*.)

Region 1

`Region 1` wurde wie folgt interpretiert: Region 1

Structure Tensor

Consider a structure tensor J which has eigenvalues $\lambda 1$ and $\lambda 2$.
Select the correct relationships for $\lambda 1$and $\lambda 2$ for a straight edge, a flat area, or a corner.

Remark: The eigenvalues are sorted as we introduced it for our courses. The lowest index indicates the largest eigenvalue (and then in descending order).

Straight edge:

λ_2 ≪ λ_1

λ_2 = 0

Flat area:

λ_1 = 0

λ_2 = λ_1

Corner:

0 ≪ λ_2

λ_1 ≥ λ_2

Harris Corner Response

Select the qualitative response of the Harris corner detector depending on the type of local structure.

Corner: H is significantly greater than zero

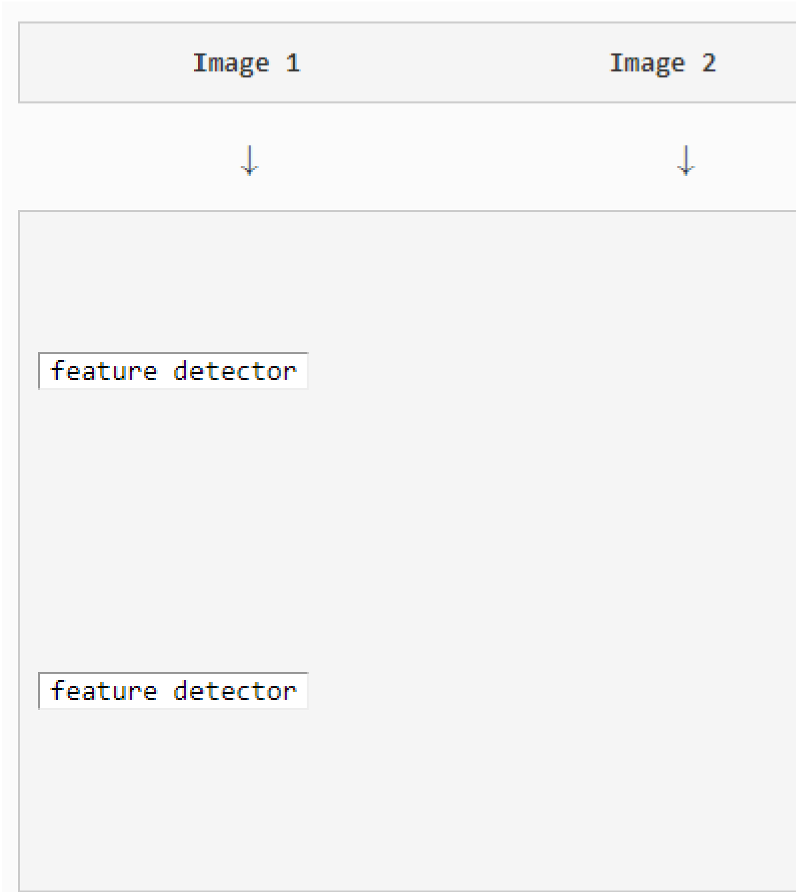
Flat area: H is small in absolute value

Edge: H is significantly smaller than zero

Feature Mapping

For a lot of applications we need to *match features* from one image to another.

Fill in the blanks with the appropriate steps for each image.



↓ ↓

`feature descriptor`

`feature descriptor`

↘ ↙

`Matching / Application`

Edges

Complete the following statements.

- The gradient points into the direction of the highest change in intensities.
- An edge is supposed to be orthogonal to the gradient.
- Derivatives are highly sensitive to noise.

Vesselness

Given a Hessian matrix H and its eigenvalues $\lambda_1,\ \lambda_2,$, write down the **parameters** for the vesselness measure first:

$R_B =$ l2/l1

`l2/l1` wurde wie folgt interpretiert: $\frac{l_2}{l_1}$

$S =$ sqrt(l2^2+l1^2)

`sqrt(l2^2+l1^2)` wurde wie folgt interpretiert: $\sqrt{l_2^2 + l_1^2}$

Now, state the term for the **vesselness measure** where it is **non-zero** (and at least one eigenvalue is non-zero):

%e^-(R^2/(2*b^2))*(1-%e^-(S^2/(2*c^2)))

`%e^-(R^2/(2*b^2))*(1-%e^-(S^2/(2*c^2)))` wurde wie folgt interpretiert: $e^{-\frac{R^2}{2 \cdot b^2}} \cdot \left(1 - e^{-\frac{S^2}{2 \cdot c^2}}\right)$

Use the syntax that is used for computer algebra systems, i.e.

- use brackets () to define the order of operations,
- use the slash / as division operator,
- use the star * as multiplication symbol,
- use the hat ^ to indicate power of a variable,
- use either %e or exp(...) for the exponential function.

Additionally write β simply as "b", λ_1 as "l1", and R_B as R, and so on.

Harris Corner Detector

Please **select** the correct option showing the formula for the Harris corner detector $H(x, y)$.

Note that $\lambda 1$ and $\lambda 2$ denote the two *eigenvalues* of the structure tensor $G(x, y)$ and vv denotes an adjustable *parameter*.

◉ $\lambda_1 * \lambda_2 - v * (\lambda_1+\lambda_2)^2$

Feature Mapping (Statements)

Select all true statements from the following:

- ☐ A feature is suitable for matching if it varies with rigid transformations.
- ☑ Feature matching requires robust and replicable features.
- ☑ The matching cascade is usually started with computation of low-level features to identify potential points of interest.
- ☐ High-level descriptors are computed right away in a matching algorithm.
- ☐ The cost of feature extraction is always maximized.
- ☑ Reduction of the search space helps to accelerate feature matching methods.
- ☑ Nearest neighbor matching makes use of certain distance metrics.
- ☐ The random square cover is an acceleration method for feature matching.

Gaussian Cascades

```
Introduction:
```

For the computation of the vesselness feature or the structure tensor, we filter the original image by Gaussians successively, depending on the number of different scale levels that we want to analyze.

Due to commutativity of successive convolutions we have:

$g_{\mu_2,\sigma 2} * (g_{\mu_1,\sigma 1} * I) = (g_{\mu_2,\sigma 2} * g_{\mu_1,\sigma 1}) * I = g_{\mu',\sigma'} * I,$

which means that **successive filtering with Gaussians** g_{μ_i,σ_i} **is the same as filtering with only one Gaussian** $g_{\mu',\sigma'}$ determined from the parameters $\{(\mu_i, \sigma_i)\}_i$ of the single filters $(i = 1, 2, \dots)$. We want to check that for some important cases.

```
Prerequisites:
```

For smoothing of images, the Gaussian blur is applied centrally for each pixel which means that we can assume $\mu_i = 0\ (i = 1, 2)$ in the following. In order to compute the cascaded Gaussian $g_{\mu',\sigma'}$ above, we make use of the **convolution theorem** (see course MIPDA for reference) that allows us to compute the **convolution in image domain as a multiplication of the Fourier transforms**:

$g_{\mu_1=0,\sigma 1} * g_{\mu_2=0,\sigma 2} = \mathcal{F}^{-1}(G_{\mu_1=0,\sigma 1} G_{\mu_2=0,\sigma 2})$

where $G_{\mu_i,\sigma_i} = \mathcal{F}(g_{\mu_i,\sigma_i})$.

```
Task:
```

Now it is your turn. Compute the cascaded Gaussian $g_{\mu',\sigma'}$ in **two steps**.

1. Compute the **Fourier transform** of $g_{0,\sigma 1}$.
   - Use the symbol *k* for the frequency argument and *x* for the pixel location in image domain. The imaginary unit is denoted as *i*. For the parameters use *s1* and *s2*, otherwise your answer will not get accepted.
   - Write the exponential function either as *%e^(...)* or *exp(...)*.
   - Use brackets *()* to define the order of operations.
   - Use the slash / as division operator, the star * for multiplication, and the hat ^ to indicate power of a variable.
   - **The integral has to be resolved completely.** Keep in mind that $\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{t^2}{2\sigma^2}} \mathrm{d}t = 1.$
   - *Hint:* You might need the following **trick** $t^2 + 2at = t^2 + 2at + a^2 - a^2 = (t + a)^2 - a^2.$

   $G_{0,\sigma 1}(k) = \mathcal{F}(g_{0,\sigma 1}(x)) =$ %e^-(2*%pi^2*k^2*s1^2)

   `%e^-(2*%pi^2*k^2*s1^2)` wurde wie folgt interpretiert: $e^{-2 \cdot \pi^2 \cdot k^2 \cdot s_1^2}$

2. With your result and the convolution theorem (see above in the task description), you can now compute the cascaded Gaussian $g_{\mu',\sigma'}$. Please **note its parameters** here, and use the **same syntax** from the first step.
   Hint: You will probably have to apply the result from the first step *several times*.

   $\mu' =$ 0

   `0` wurde wie folgt interpretiert: 0

   $\sigma' =$ sqrt(s2^2+s1^2)

   `sqrt(s2^2+s1^2)` wurde wie folgt interpretiert: $\sqrt{s_2^2 + s_1^2}$

Image Enhancement

Guided Filter

For the *guided filter*, we assume a **linear model** for each image pixel $f(\mathbf{x})$ of the enhanced image (locally for a neighborhood $\omega_{\mathbf{x}}$):

$f(\mathbf{x}') = a_{\mathbf{x}} i(\mathbf{x}') + b_{\mathbf{x}},\ \forall \mathbf{x}' \in \omega_{\mathbf{x}}.$

To find the parameters $a_{\mathbf{x}}$ and $b_{\mathbf{x}}$, we minimize the following *optimization goal*:

$\mathcal{J}(a_{\mathbf{x}}, b_{\mathbf{x}}) = \sum_{\mathbf{x}' \in \omega_{\mathbf{x}}} (f(\mathbf{x}') - g(\mathbf{x}'))^2 + \epsilon a_{\mathbf{x}}^2.$

Set $\nabla \mathcal{J} = \left(\frac{\partial \mathcal{J}}{\partial a_{\mathbf{x}}}, \frac{\partial \mathcal{J}}{\partial b_{\mathbf{x}}}\right)^\top \stackrel{!}{=} 0$ and **derive the formulas for both parameters** $a_{\mathbf{x}}$ and $b_{\mathbf{x}}$. We highly recommend to do this once by yourself.

Which **general result** do you get for the parameters? *(See below the question for the syntax to be used!)*

$a_{\mathbf{x}} =$ Cov(g(x),i(x))/(Var(i(x))+eps)

`Cov(g(x),i(x))/(Var(i(x))+eps)` wurde wie folgt interpretiert: $\mathrm{Cov}(g(x),i(x))/(\mathrm{Var}(i(x))+\mathrm{eps})$

$b_{\mathbf{x}} =$ Ewx(g(x))-ax*Ewx(i(x))

`Ewx(g(x))-ax*Ewx(i(x))` wurde wie folgt interpretiert: $\mathrm{Ewx}(g(x))\text{-}\mathrm{ax}*\mathrm{Ewx}(i(x))$

If we do not have a guidance image, we can try to replace $i(\mathbf{x})$ by the original image $g(\mathbf{x})$. The idea is to find a *smooth approximation* that does not differ from the original image "more than necessary".

Therefore let $i(\mathbf{x}) = g(\mathbf{x})$ from now on. What happens if $\epsilon = 0$?

$a_{\mathbf{x}}|_{\epsilon=0} =$ 1

`1` wurde wie folgt interpretiert: 1

$b_{\mathbf{x}}|_{\epsilon=0} =$ 0

`0` wurde wie folgt interpretiert: 0

Polychromatic Absorption

Select the correct formula for the computation of a single energy bin in the **polychromatic** case.

◉ $I_b = \int I_{0,b'} e^{-\int \mu(b',j) l_j \mathrm{d}j} \mathrm{d}b'$

Material Decomposition

Can you **decompose** the image into three different materials from the detector data?

Type *Y* or *N*: **Y**

What is the **maximum number** of materials that you can decompose?

3

Before you run your algorithm for material decomposition **you find something interesting** about your data.

For the bins measured by the detector, the attenuations of material 1 are *two-thirds* of the attenuations of material 2. Is that an issue?

Why/why not? **It is a problem because now the matrix M has an insufficient rank for three materials.**

Understanding Bilateral Filter

Select all **true** statements about the *bilateral filter*

- [x] The stronger the edges in an image, the less they are blurred by bilateral filtering.
- [x] The full kernel of the bilateral filter is not shift-invariant.
- [ ] Length measurements of a bone structure in CT are usually way more accurate when applying a 10x10 averaging filter than the appropriate bilateral filter.
- [x] The bilateral filter is a special type of normalized convolution.
- [x] The range similarity term allows preserving the edges in an image while the term for spatial closeness leads to denoising of the image.

Super-Resolution

Sampling Theory

Assume you **sampled a continuous signal** f and **measured discrete values** $f_{m,n}$ according to the scheme

$fm, n = f(m\Delta x, n\Delta y) \in R, fm, n = f(m\Delta x, n\Delta y) \in R,$

where
$\Delta x = 2mm, \Delta y = 3mm\Delta x = 2mm, \Delta y = 3mm, andm, n = 0, 1, 2, \ldots m, n = 0, 1, 2, \ldots.$

The highest frequency components that appear in the continous signal are $u_{max} = 0.5mm - 1$ along the x-axis and $v_{max} = 0.15mm - 1$ along the y-axis.

Is the signal **sampled without any loss**?

No, the sampling in x-direction is insufficient.

Now, to be more specific, can you **choose** the sampling distances **as above or** do you need to **change** them if you want **to sample all** of the signal?
Select the setting that you would choose **without making unnecessary adjustments** to the original setting:

Δx= **1** mm

Δy= **3** mm

Low Resolution Causes

Which of the following statements contain **(typical) reasons for non-ideal imaging** of a real scene?

- [x] The discrete representation of a continuous real world scene has finite resolution.
- [ ] Lateral sampling induces side effects.
- [x] Lighting and physical effects (blur/diffraction) lead to a non-ideal mapping of points and edges.
- [ ] Loud noise from the environment creates a tremendous disturbance in the camera sensor.
- [x] Small details of the scene get lost in the camera image due to blur and noise.

Super-Resolution Algorithmic Steps

Sort the steps to outline the principal workflow of a multi-frame super-resolution algorithm.

Motion Estimation

Motion Compensation

Interpolation

Restoration

Super-resolution Definition

Complete the definition of super-resolution.

Super-resolution is the **process** of **obtaining high-resolution images** from **observed low-resolution images** .

Single-Frame and Multi-Frame SR

Fill in the gaps in the following text properly.

Single-frame super-resolution

Super-resolution can be achieved on a single-frame bases by estimating a **high-resolution** image from one single low-resolution image by incorporation of **prior knowledge** .

For this, we have **learning-based** methods and **frequency interpolation** methods at our diposal.

Multi-frame super-resolution

On basis of motion estimation, multi-frame super- resolution is another approach to reconstruct **high-resolution** images from low-resolution frames . For multi-frame methods we need to capture **a sequence of warped and degraded images** of the ideal scene.

The multiple image acquisitions yield a more precise sampling due to **non-integer pixel shifts** caused by **moving cameras** and **object motion** .

Regularization

Tikhonov regularization:

$$p(\boldsymbol{x}) = \frac{1}{(2\pi)^{N/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}\mathbf{x}^T\boldsymbol{\Sigma}^{-1}\mathbf{x}\right)$$

General form of the prior distribution:

$$p(\boldsymbol{x}) = \frac{1}{Z}\exp(-\lambda R(\boldsymbol{x}))$$

Bilateral total variation:

$$p(\mathbf{x}) \propto \exp\left(-\lambda \sum_{u=-P}^{P}\sum_{v=-P}^{P} \alpha^{|u|+|v|} \left|\left|\mathbf{x} - \mathbf{S}_i^u\mathbf{S}_j^v\mathbf{x}\right|\right|_1\right)$$

Total variation: $p(\boldsymbol{x}) \propto \exp\left(-\lambda \left|\left|\boldsymbol{Q}\mathbf{x}\right|\right|_1\right)$

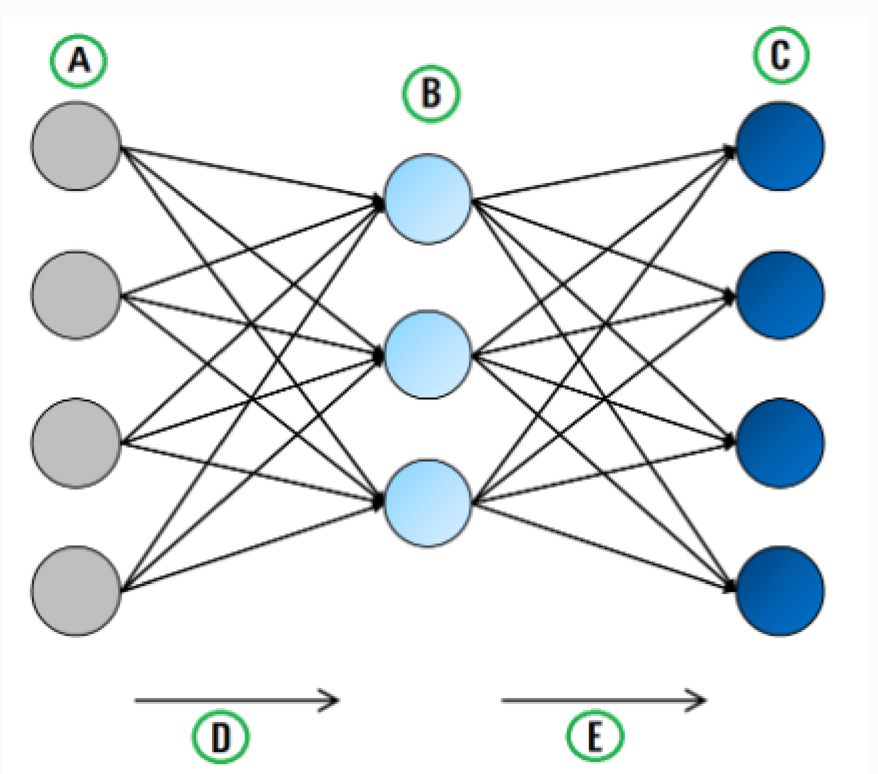
Huber prior:

$$p(\boldsymbol{x}) \propto \exp\left(-\lambda \sum_{i=1}^{N} h_\tau([\boldsymbol{Q}\mathbf{x}]_i)\right), \quad h_\tau(z) = \begin{cases} \frac{1}{2}z^2, & \text{if } |z| \leq \tau, \\ \tau\left(|z| - \frac{\tau}{2}\right), & \text{otherwise} \end{cases}$$

Deep Learning

Autoencoder

Have a look at the image and then **select the appropriate terms** for the parts or steps of the autoencoder scheme.



(A): Input Layer

(B): Hidden Layer(s)

(C): Output Layer

(D): Encoding Step

(E): Decoding Step

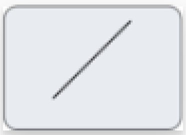
Fundamentals

Select all **completely correct** statements from the following list:

- [ ] Backpropagation is a concept similar to a gradient descent method, but it uses the second derivative instead of simple gradients.
- [x] Backpropagation can be used to associate the network's weighting parameters with the computed loss. Mathematically, this makes heavy use of the chain rule for derivatives.
- [ ] A fully connected network for an image input has always less parameters than a convolutional network.
- [ ] Deep learning can be used for denoising and inpainting, but not for segmentation.
- [x] The activation function is an important design part when creating a neural network.

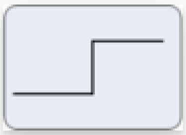
Activation Functions

Which of the diagrams belongs to which type of activation?



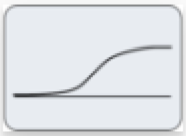
passt zu

Linear function



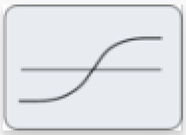
passt zu

Step function



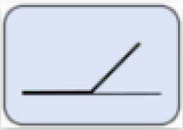
passt zu

Sigmoid function



passt zu

Hyperbolic tangent



passt zu

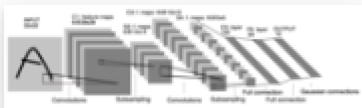
ReLU activation

Popular Architectures

Can you **associate** the names with the corresponding architecture?

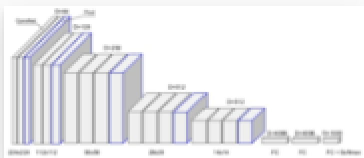
LeNet

passt zu



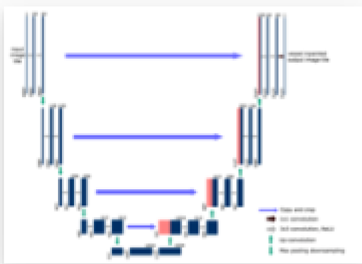
VGGNet

passt zu



U-Net

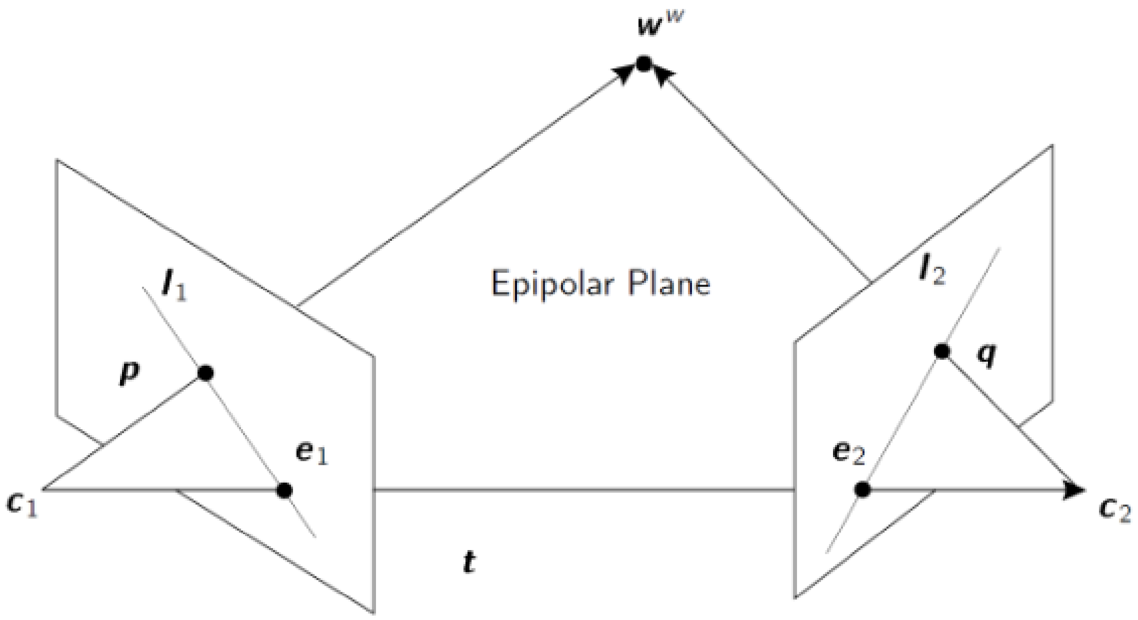
passt zu



Epipolar Geometry

Epipolar Geometry

Have a look at the diagram. Can you **name** all of the variables?



$\mathbf{c}_1 =$ optical center of camera 1

$\mathbf{c}_2 =$ optical center of camera 2

$\mathbf{w}^{\mathbf{w}} =$ 3-D world point

$\mathbf{t} =$ translation vector

$\mathbf{e}_1 =$ epipole on the image plane of camera 1

$\mathbf{e}_2 =$ epipole on the image plane of camera 2

$\mathbf{p} =$ projection of world point on the image plane of camera 1

$\mathbf{q} =$ projection of world point on the image plane of camera 2

$\mathbf{l}_1 =$ epipolar line corresponding to q

$\mathbf{l}_2 =$ epipolar line corresponding to p

Essential properties

- [ ] $\mathbf{E}$ has 8 degrees of freedom and can be computed by the five point algorithm.
- [x] The essential matrix has the same rank as the fundamental matrix.
- [x] $\mathbf{F}$ encodes intrinsic and extrinsic parameters.
- [ ] The essential matrix $\mathbf{E}$ maps an epipolar line $\mathbf{l}_2^\top$ to a point $\left(\tilde{\mathbf{p}}^i\right)^\top = \mathbf{l}_2\mathbf{E}$.
- [x] The fundamental matrix $\mathbf{F}$ maps a point $\tilde{\mathbf{p}}^p$ to its epipolar line $\mathbf{l}_2^\top = \mathbf{F}\tilde{\mathbf{p}}^p$.
- [x] All non-zero singular values of $\mathbf{E}$ are identical.
- [ ] In order to compute the nullspace of $\mathbf{E}$, you need both the rotation matrix and translation vector.
- [ ] The essential matrix $\mathbf{E}$ is the product of a skew matrix with rank 3 and of the matrix $\mathbf{R}$ which has rank 2.

Grangeat's Theorem

What does the following formula implicitly state? **Complete** the statement below accordingly.

$\frac{d}{dn}\rho f(E) \approx \frac{d}{dt}\rho I(l)$

The **derivative** of the **3-D Radon transform** in **normal direction** is approximately the derivative of the **2-D Radon transform** in intercept direction.

Essential Formulas

Assume you are using a system of *two cameras* for some experiments.

Your supervisor gave you two matrices $\mathbf{F}$ and $\mathbf{K}$ for this system and told you that you can compute the **essential matrix** $\mathbf{E}$ by

$\mathbf{E} = \mathbf{K}^\top \mathbf{F} \mathbf{K}$.

What did she give you?

$\mathbf{F}$ is the fundamental matrix

`fundamental matrix` wurde wie folgt interpretiert: **fundamental matrix**

$\mathbf{K}$ contains the intrinsic parameters

`intrinsic parameters` wurde wie folgt interpretiert: **intrinsic parameters**

Later, you acquire an image of a prepared scene with both cameras. Your supervisor marks one single point on the image from camera 2 and denotes it $\tilde{\mathbf{q}}^i$. She asks you if you can already tell something about the *corresponding point* $\tilde{\mathbf{p}}^i$ on the image from camera 1.

Of course, you answer, it has to **suffice the constraint**:

q^T*E*p=0

`q^T*E*p=0` wurde wie folgt interpretiert: $\mathbf{q}\hat{}\mathbf{T}*\mathbf{E}*\mathbf{p}=0$

Hints: Give the proper equation and use the following notation:

- For simplicity, use just $\mathbf{p}$ for $\tilde{\mathbf{p}}^i$, and $\mathbf{q}$ for $\tilde{\mathbf{q}}^i$.
- Use +, -, *, / as symbols for addition, subtraction, multiplication, and division.
- (!) For reasons of clarity, use * also for matrix-matrix and matrix-vector multiplication (normally no symbol needed).
- If you need to transpose a homogeneous vector $\mathbf{a}$, write $\mathbf{a}$^T.

Factorization

Measurement Matrix

Which of the following properties are **actual properties** of the measurement matrix for 3-D ultrasound as you have seen it in the course?

- [ ] The factorization of $\widehat{\mathbf{M}}$ is unique.
- [x] $\widehat{\mathbf{M}}$ has rank 3.
- [x] $\widehat{\mathbf{M}}$ can be factorized into three matrices.
- [x] $\widehat{\mathbf{M}}$ can be factorized into two matrices.
- [x] The measurements can be split into a matrix that contains the camera movement, and one that contains 3-D points.
- [ ] In practice the rank criterion for the matrix $\widehat{\mathbf{M}}$ does not need to be enforced.

Interpolation

Nearest

neighbor

interpolation:

- Run through **all pixels** .
- Fill the nearest voxel with the intensity value.
- Multiple contributions are **averaged** or use **maximum value** alternatively.
- Gap filling is done in a second step.

Distance

weighted

interpolation:

- Sampling is done **voxel by voxel** .
- Voxel values are assigned the averaged **sum** of nearby voxels.
- For example, consider a spherical **neighborhood** centered around each voxel.

Physical Phenomenons

Which **three physical wave phenomenons** have to be considered for ultrasound imaging?

Reflection

Refraction

Absorption

Ultrasound

`Introduction:`

In the course module you learned about the physical background of ultrasound imaging devices. It utilizes the reflection of sound waves at material boundaries.
Sound is propagated differently fast, depending on the transmission medium. Part of the signal is reflected, and the rest enters the new medium. We neglect absorption inside each medium and any other physical effects for this exercise (e.g. refraction angle).

If the acoustic impedances Z_1 for material 1 and Z_2 for material 2 are known, we know that the amplitude of the signal velocity of the reflected signal relatively to the incoming signal can be computed with the factor $\frac{1-\frac{Z_2}{Z_1}}{1+\frac{Z_2}{Z_1}}$.

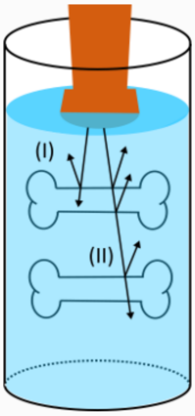
The signal intensity depends quadratically on the signal velocity, hence its reflection contains

$$r = \left(\frac{1-\frac{Z_2}{Z_1}}{1+\frac{Z_2}{Z_1}}\right)^2$$

of the original signal intensity. Of course, we do not want all of the signal reflected, otherwise we could not look "behind" the tissue boundary ($Z_2 \gg Z_1$, $Z_2 \ll Z_1$). Just as little as we want all of the signal passing through it ($Z_2 \approx Z_1$), because the reflected signal carries the information we can use for imaging.

`Task:`

In order to get some idea how much of the signal is reflected, you build a little experiment. At your university lab you acquired an ultrasound device and two bones from the biology department. Clever as you are, you put them into a bucket filled with water, dip the head of the ultrasound probe into the water, and start your measurements.



Later on you compare those with theoretical values. Assuming ideal conditions such that all of the reflected signal is measured, and neglecting all other effects as stated above, you compute two factors r_I and r_{II} for the two scenarios shown in the figure.

`Scenario I: Reflection at the transition from the water to the first bone`

$r_I =$ 0.372742382271

`0.372742382271` wurde wie folgt interpretiert: 0.372742382271

`Scenario II: Reflection at the transition from the water to the second bone after passing through the first bone`

The sound signal has to pass through the first bone and is therefore reflected several times (compare figure). You are only interested in the reflection fraction **directly** at the second bone **with respect to the original signal** emitted from the device. On its way back the signal will have to pass the first bone again, but this computation is **not** required for this exercise.

$r_{II} =$ 0.146656280145

`0.146656280145` wurde wie folgt interpretiert: 0.146656280145

Tomasi's Factorization Algorithm

Can you remember the workflow of *Tomasi's factorization algorithm*? **Sort** the following boxes accordingly.

Track points.

Compute SVD of $\widehat{\boldsymbol{M}}$:

$$\widehat{\boldsymbol{M}} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^{\mathsf{T}}.$$

Set all σ_k for $k \geq 4$ to zero, since $\operatorname{rank}\left(\widehat{\boldsymbol{M}}\right) = 3$.

Let $\boldsymbol{U}'$ be the $2N_{\mathrm{F}} \times 3$ submatrix of $\boldsymbol{U}$, and $\boldsymbol{V}'$ the $3 \times N_{\mathrm{p}}$ submatrix of $\boldsymbol{V}$ corresponding to σ_1, σ_2, and σ_3.
Let $\boldsymbol{\Sigma}' = \operatorname{diag}(\sigma_1, \sigma_2, \sigma_3)$, then compute:

$$\hat{\boldsymbol{R}} = \boldsymbol{U}'\boldsymbol{\Sigma}'^{\frac{1}{2}}, \qquad \hat{\boldsymbol{S}} = \boldsymbol{\Sigma}'^{\frac{1}{2}}\boldsymbol{V}'^{\mathsf{T}}.$$

Solve the following (nonlinear) equations for $\boldsymbol{Q}$:

$$\hat{\boldsymbol{u}}_i^\top \boldsymbol{Q}\boldsymbol{Q}^\top \hat{\boldsymbol{u}}_i = 1,$$
$$\hat{\boldsymbol{v}}_i^\top \boldsymbol{Q}\boldsymbol{Q}^\top \hat{\boldsymbol{v}}_i = 1,$$
$$\hat{\boldsymbol{u}}_i^\top \boldsymbol{Q}\boldsymbol{Q}^\top \hat{\boldsymbol{v}}_i = 0.$$

Compute the output:

$$\boldsymbol{R} = \hat{\boldsymbol{R}}\boldsymbol{Q}, \qquad \boldsymbol{S} = \boldsymbol{Q}^{-1}\hat{\boldsymbol{S}}.$$

Segmentation

Laplacian Matrix

We want to understand how the **Laplacian matrix** for the random walker is built. Therefore, let us have a look at a simple toy example which is the following 2×2 image:

[5123].[5213].

Further, instead of Gaussian weighting, we choose the following weights:

$wij = |gi - gj|$

where gi and gj indicate the pixel values at positions ii and j(linear indexing along rows first).

Complete the Laplacian matrix for this image:

7	-3	-4	0
-3	4	0	-1
-4	0	6	-2
0	-1	-2	3

ASM General Structure

Which of the following **choices** actually *describes* the *general workflow* for segmentation with active shape models?

First, the kind of shape representation has to be fixed (e.g. use landmarks). From this, several shape statistics (mean shape, modes of variation) are computed. For its application, correspondences between shapes have to be found (fit the model to an observation).

Random Walker

Select all **correct** statements:

- ☐ One drawback of the random walker algorithm is that it has a lot of parameters to set.
- ☑ Segments are guaranteed to be connected.
- ☑ The random walker algorithm is robust against noise and shows efficient performance.
- ☐ Discretization errors in the segmentation are a caused of inaccuracy depending on the initial seedpoints.
- ☐ The random walker segmentation is weak with boundaries.
- ☐ Unfortunately, there is no possible extension of the algorithm to interactively add seedpoints.
- ☐ The segmentation of the spiral problem with the random walker algorithm is almost perfect even with very strong noise.
- ☑ The random walker algorithm can result in segmentations with slight leakage at weak boundaries.

Variational Calculus

Partial and Total Derivative

Given is the functional $F(x, f, f') = x^2 + xf + (f')^2$ where $f = f(x)$ and $f' = \frac{\mathrm{d}f}{\mathrm{d}x}$.

Compute the partial derivatives $\frac{\partial F}{\partial f}$, $\frac{\partial F}{\partial f'}$, and the total derivative $\frac{\mathrm{d}F}{\mathrm{d}x}$.

$\frac{\partial F}{\partial f} =$ x

x wurde wie folgt interpretiert: x

$\frac{\partial F}{\partial f'} =$ 2*Df

2*Df wurde wie folgt interpretiert: $2 \cdot Df$

$\frac{\mathrm{d}F}{\mathrm{d}x} =$ Df*x+2*x+f+2*DDf*Df

Df*x+2*x+f+2*DDf*Df wurde wie folgt interpretiert: $Df \cdot x + 2 \cdot x + f + 2 \cdot DDf \cdot Df$

Hints: Give the proper equations by using the following notation:

- Use +, -, *, / as symbols for addition, subtraction, multiplication, and division.
- Use the hat ^ to indicate power of a variable.
- Use f, Df, DDf, and so on, to denote the function f and its derivatives f', f'', and so on.
- Use brackets () to organize the order of operations.
- Example: If you want to write $2f' + \frac{3}{f''+1}$, you are supposed to type it like this: 2*Df+3/(DDf+1).

Registration (Differentiation)

Complete the following statements such that they tell a correct differentiation between *both* types of image registration that you have learned about.

- The term **rigid** registration subsumes the process of computing a rigid **transform** for registration.
- The term **non-rigid** registration includes all the methods of **deforming** the different images such that they can be represented in **one common coordinate system** .

Why Non-Rigid Registration?

- [x] Long-term body changes like weight loss
- [x] Different position of patient's hands/arms in different image acquisitions
- [x] Patient repositioning
- [x] Breathing
- [x] Heart motion
- [ ] Bored patients
- [ ] Prestige
- [ ] Physician's hand in the scanner
- [ ] "Non-rigid" sounds more sophisticated

Euler-Lagrange Equation

```
Objective Function for Image Smoothing
```

Given an acquired image f_a, we want to find an enhanced image f by minimization of the objective function

$$I(f) = \frac{1}{2} \int_{\Omega} \left((f - f_a)^2 + \mu \parallel \nabla f \parallel_p^2 \right) \mathrm{d}\mathbf{x}$$

where $(f - f_a)^2$ is the similarity measure between both images and $\mu \parallel \nabla f \parallel_p^2$ enforces a certain regularity of the solution controlled by the parameter μ. In the lecture we learned, that calculus of variations can be used to compute f.

```
General Variational Problem
```

Such a problem as above can be written in a more general way as follows (for the sake of notational simplicity here only 1-D):

$$\text{Minimize} \quad I(f) = \int_{x1}^{x2} F(x, f(x), f'(x))\, \mathrm{d}x, \qquad f(x_1) = f_1, \quad f(x_2) = f_2,$$

with $x_1,\ x_2 \in \mathbb{R}^2$, and f' being the first derivative of f.

```
Euler-Lagrange Equation
```

If $f_0 : [x_1, x_2] \to \mathbb{R},\ x \mapsto f_0(x)$, is a function that minimizes $I(f)$, then the so-called Euler-Lagrange equation holds:

$$\frac{\partial}{\partial f} F(x, f_0(x), f_0'(x)) - \frac{\mathrm{d}}{\mathrm{d}x} \frac{\partial}{\partial f'} F(x, f_0(x), f_0'(x)) = 0.$$

```
Task
```

Assume that you are working on a problem with the following functional F:

$$F(x, f(x), f'(x)) = 3f(x)^2 + \left(-7x^3 + 6x^2 + 4x + -2\right) f'(x).$$

What is the function $f_0(x)$ that minimizes F ?

$f_0(x) =$ ((-21*x^2)+12*x+4)/6

((-21*x^2)+12*x+4)/6 wurde wie folgt interpretiert: $\frac{-21 \cdot x^2 + 12 \cdot x + 4}{6}$

Reconstruction

Gating Methods

Complete the following paragraph properly.

Motion is **a problem** in medical image reconstruction.

Gating is a technique that can be used to compensate for motion in the reconstruction results. Two possible basic gating methods were introduced in the module.

One is based on the patient's **ECG oder electrocardiogram** (signal) and image acquisitions are timed with the patient's heartbeat.

Another one is image-based gating where the motion is estimated from **variations in the sinogram .**

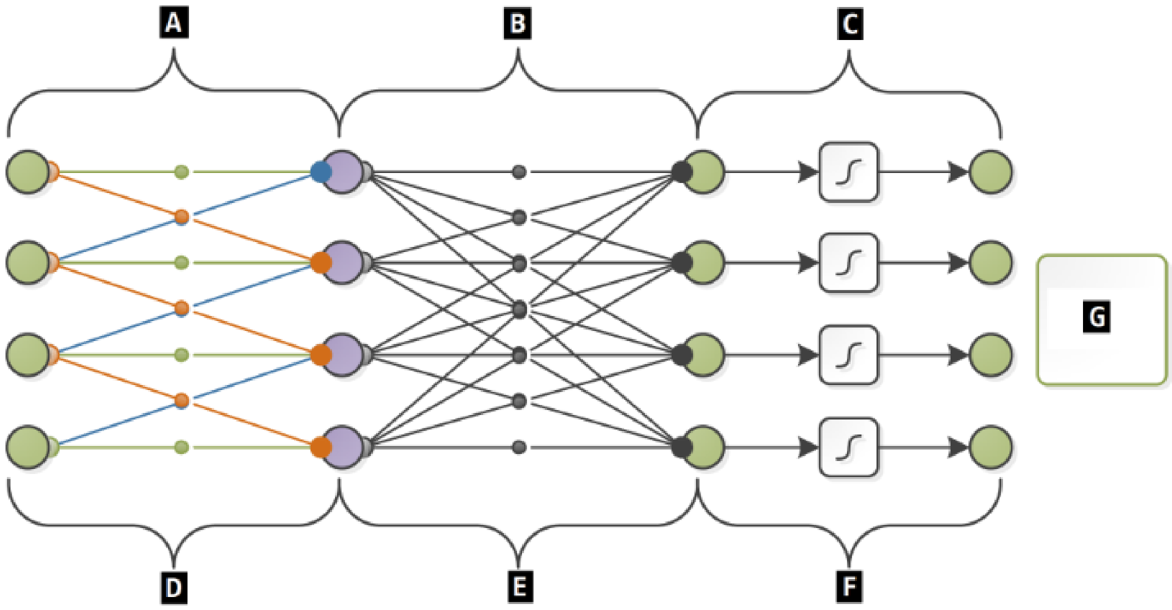
Usually, these methods are based on the assumption of motion **periodicity** . But certain diseases and handicaps of the patient (like arrhythmia or sedation) can cause a degraded image quality under this assumption.

Therefore, a less restrictive assumption is that the motion is **continuous** and **temporally** smooth which still helps to compensate for motion with certain methods.

DL Tomography

The following diagram shows an architecture that transfers parallel-beam FBP reconstruction into a neural network.

Match the letters shown in the diagram with the appropriate description.



A	passt zu	filtering
B	passt zu	back-projection
C	passt zu	non-negativity constraint
D	passt zu	convolution layer
E	passt zu	fully connected layer
F	passt zu	rectified linear unit
G	passt zu	loss function

Temporal Regularization

Which of the following are **methods to handle temporal inconsistencies** in cardiac imaging?

- [x] Temporal TV regularization in algebraic reconstruction
- [x] Motion estimation using 4-D B-Splines
- [ ] Adaptive Gaussian smoothing based on a road map
- [ ] Multilateral soothing in iterative motion confirmation
- [x] Bilateral smoothing in iterative motion estimation/compensation
- [ ] Temporal motion inhibition by blood flow gating

Motion Compensation

Bring these algorithmic steps into a **meaningful order** to create a method for motion compensated reconstruction.

Initial Image Reconstruction

Motion Estimation

Motion Compensation

Spatio-Temporal Regularization

Repeat Last 3 Steps

Output Final Reconstruction

Motion Patterns

Which three types of motion do we distinguish in medical imaging that are related to the patient?

Cardiac motion

Respiratory motion

Patient motion