Medical Image Processing for Diagnostic Applications

Bilateral Filtering

Online Course – Unit 63
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Topics

Edge Preserving Filtering
- Motivation
- Low-Pass Domain Filter
- Low-Pass Range Filter
- The Combination to the Bilateral Filter

Examples

Summary
- Take Home Messages
- Further Readings

Motivation

Problem: How can we prevent averaging across edges while still averaging within a smooth region?

Definition

Bilateral filtering is a method for edge preserving noise reduction. Instead of simply averaging image values in dependence on their geometric closeness, also the photometric similarity of nearby pixels is considered.

Low-Pass Domain Filter

Let us consider spatial domain filtering of continuous 2-D image functions defined as follows:

$$h_{\text{domain}}(x,y) = k_d^{-1}(x,y) \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\mu,\nu)c(x,y,\mu,\nu)\,\mathrm{d}\mu\,\mathrm{d}\nu,$$

where

- the filtering kernel is usually restricted to a local neighborhood,
- $f(x,y)$ denotes the observed image,
- $c(x,y,\mu,\nu)$ measures the *geometric closeness* between the image point (x,y) and the point (μ,ν), and
- the normalization function k_d is defined by

$$k_d(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} c(x,y,\mu,\nu)\,\mathrm{d}\mu\,\mathrm{d}\nu.$$

Low-Pass Domain Filter

We can make use of a standard definition from system theory:

Definition

The filter is called ***shift-invariant*** if $c(x, y, \mu, \nu)$ is bivariate in (μ, ν) and $k_d(x, y)$ is constant.

Low-Pass Range Filter

Now we apply a similar idea to the intensity values:

$$h_{\text{range}}(x,y) = k_r^{-1}(x,y) \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\mu,\nu)s(f(x,y),f(\mu,\nu))\,\mathrm{d}\mu\,\mathrm{d}\nu,$$

where

- again the filtering kernel is usually restricted to a local neighborhood,
- $f(x,y)$ denotes the observed image,
- $s(f(x,y),f(\mu,\nu))$ measures the *photometric similarity* between the intensity value at (x,y) and a neighboring position (μ,ν), and
- the normalization function k_r is defined by

$$k_r(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} s(f(x,y),f(\mu,\nu))\,\mathrm{d}\mu\,\mathrm{d}\nu.$$

Low-Pass Range Filter

Definition

The similarity function $s(f(x,y), f(\mu,v))$ is ***unbiased*** if it depends only on the difference $f(x,y) - f(\mu,v)$.

Bilateral Filter

Now we combine domain and range filtering in a proper manner:

$$h_{\text{bilateral}}(x,y) = k^{-1}(x,y) \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\mu,\nu)c(x,y,\mu,\nu)s(f(x,y),f(\mu,\nu))\,\mathrm{d}\mu\,\mathrm{d}\nu$$

where

- $f(x,y)$ denotes the observed image,
- $c(x,y,\mu,\nu)$ measures the *geometric closeness* between the image point (x,y) and the point (μ,ν),
- $s(f(x,y),f(\mu,\nu))$ measures the *photometric similarity* between the intensity value at (x,y) and a neighboring position (μ,ν), and
- the normalization function k is defined by

$$k(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} c(x,y,\mu,\nu)s(f(x,y),f(\mu,\nu))\,\mathrm{d}\mu\,\mathrm{d}\nu.$$

Geometric Closeness and Photometric Similarity: Examples

Example

Closeness function

$$c(x,y,\mu,\nu) = \exp\left(-\frac{1}{2}\frac{\left\|\begin{pmatrix}x\\y\end{pmatrix} - \begin{pmatrix}\mu\\\nu\end{pmatrix}\right\|^2}{\sigma_d^2}\right)$$

Example

Similarity function

$$s(f_1,f_2) = \exp\left(-\frac{1}{2}\frac{\|f_1 - f_2\|^2}{\sigma_r^2}\right)$$

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Edge Preserving Smoothing

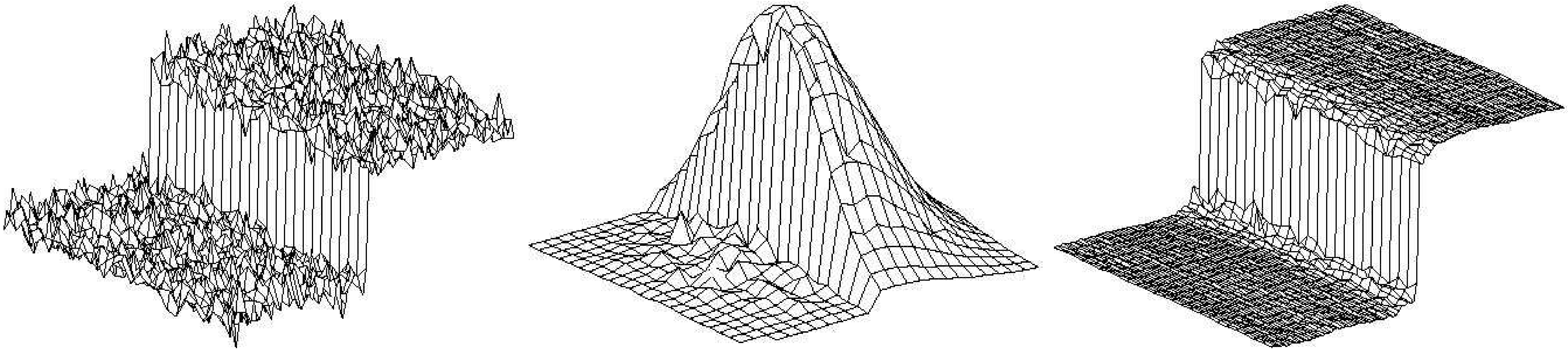

Figure 1: A 100-gray level step perturbed by Gaussian noise with $\sigma = 10$ gray levels (left). Combined similarity weights $c(x,y,\mu,\nu)s(f(x,y),f(\mu,\nu))$ for a 23×23 neighborhood centered two pixels to the right of the step (middle). Result of bilateral filtering with $\sigma_r = 50$ gray levels and $\sigma_d = 5$ pixels (right) (Carlo Tomasi)

Image Example: CT Slice

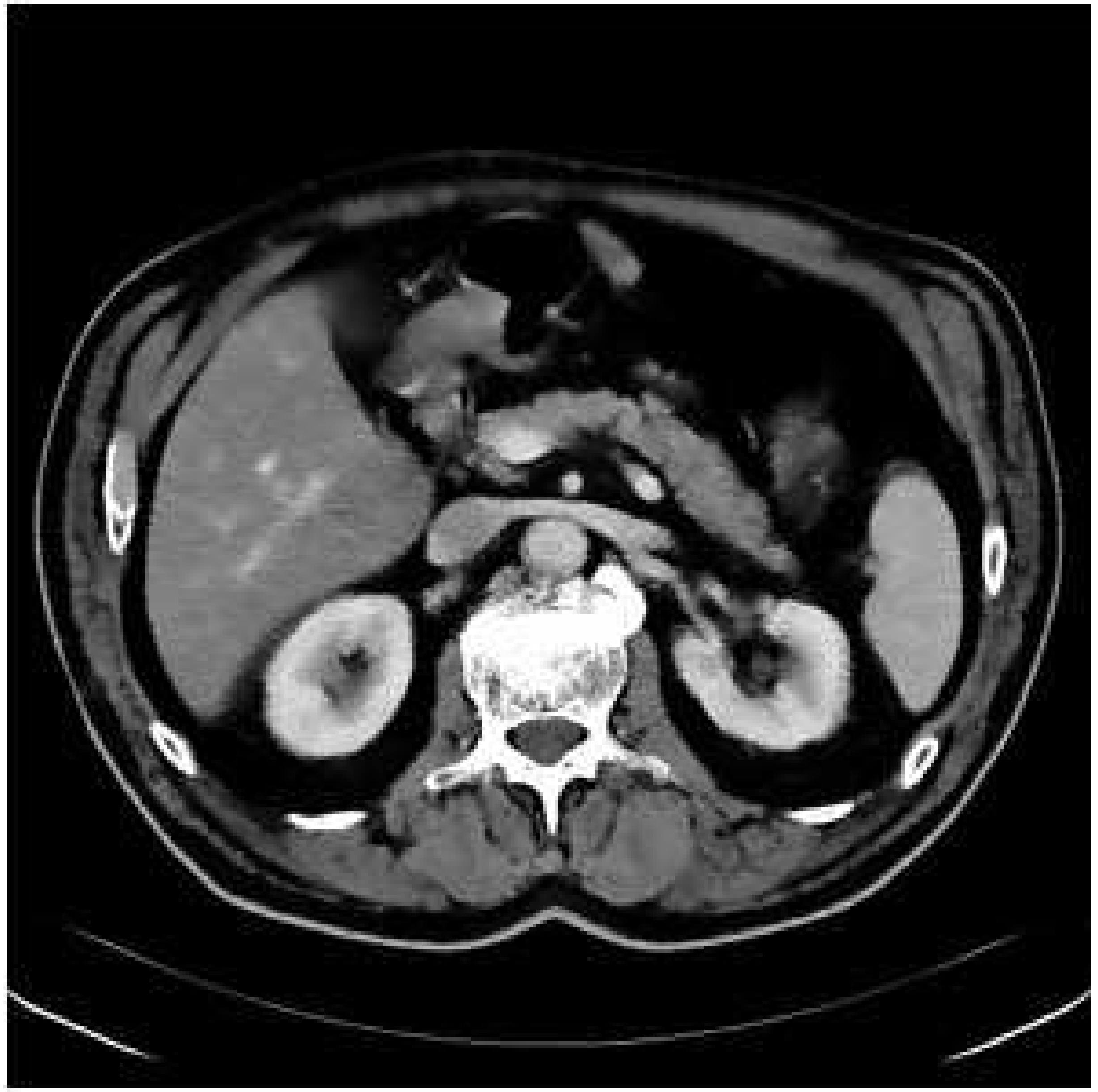

Figure 2: (Anja Borsdorf, Pattern Recognition Lab, FAU)

Example: Limited Angle Reconstruction (180 Degrees Only)

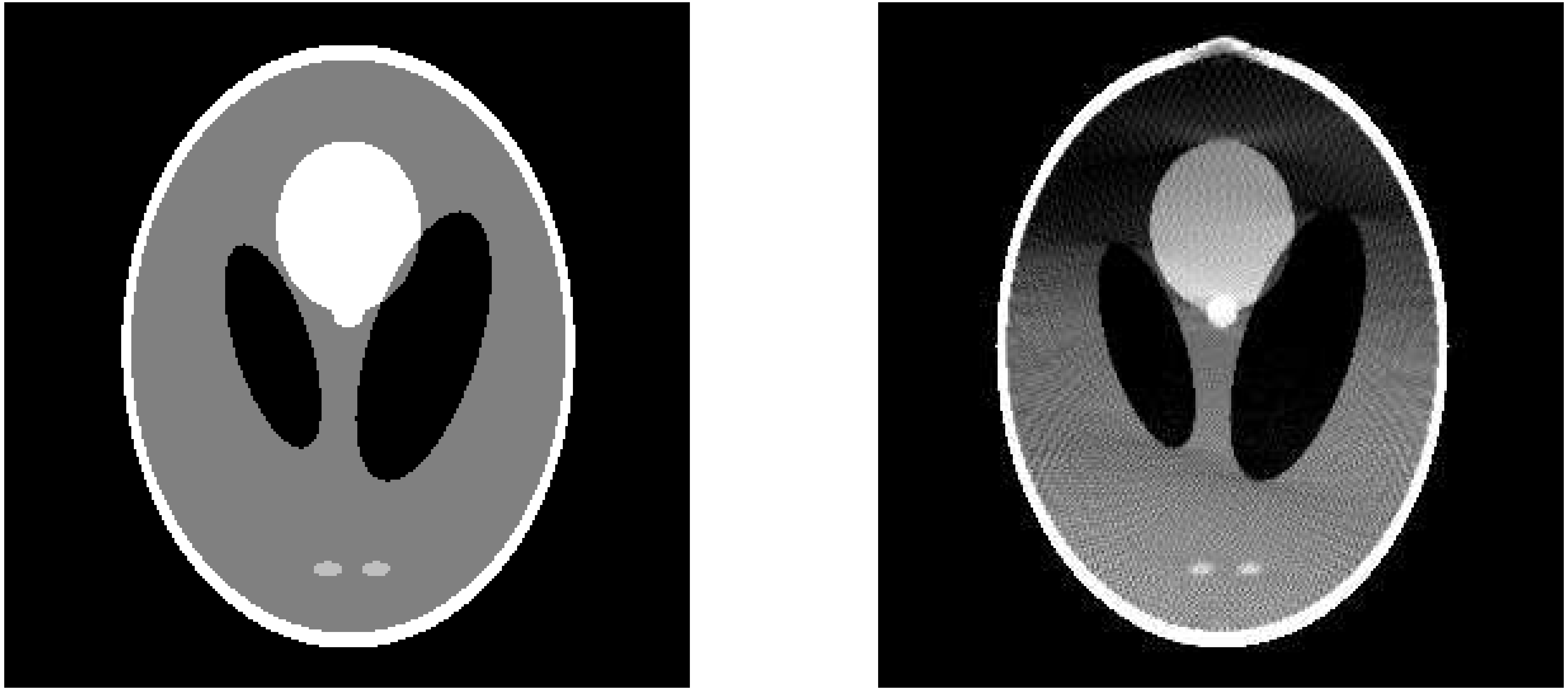

Figure 3: Shepp-Logan phantom (left), FBP with Parker weights (right)

Example: Limited Angle Reconstruction (Line Profiles)

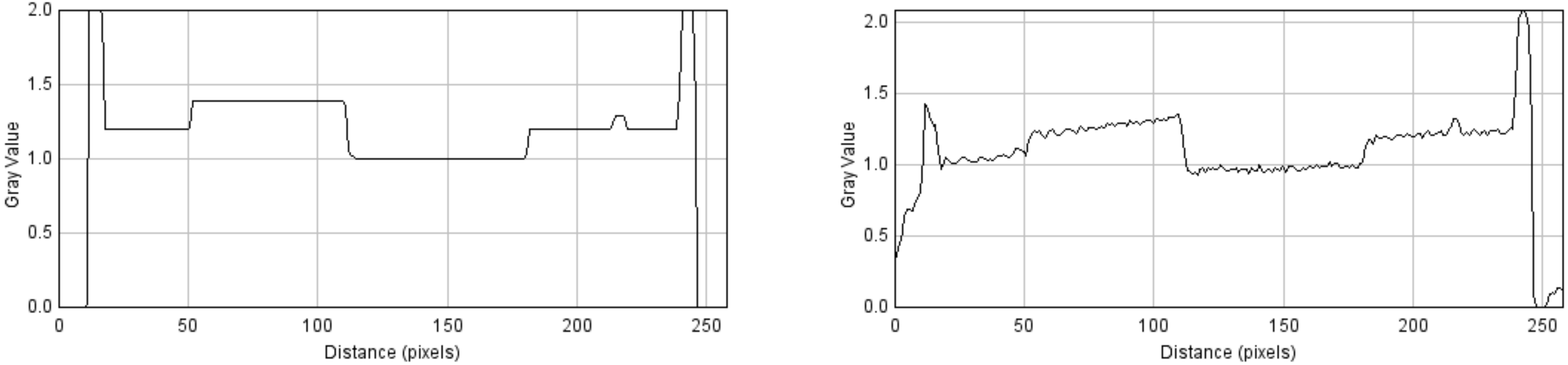

Figure 4: Shepp-Logan phantom (left), FBP with Parker weights (right)

Idea: Apply Multiplicative Weighting for Compensation

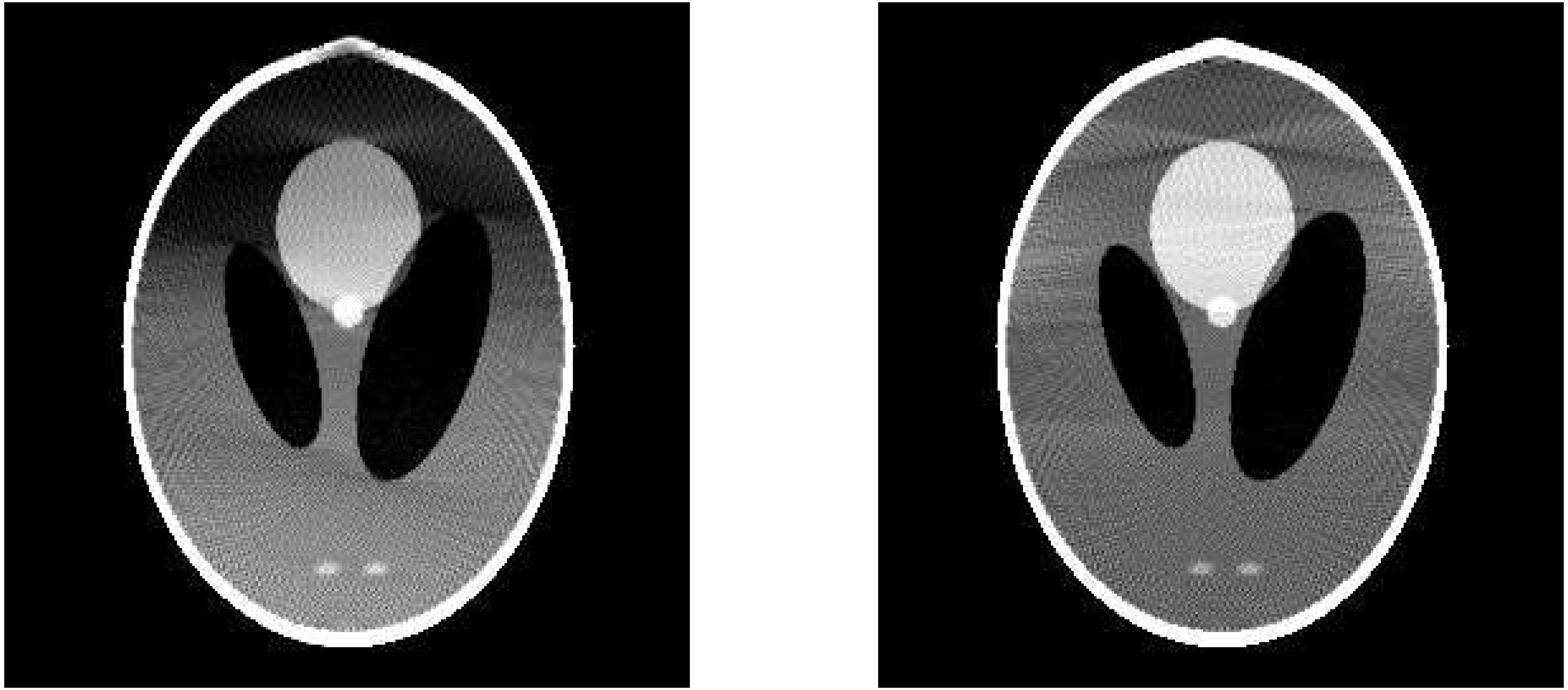

Figure 5: FBP with Parker weights (left), FBP with compensation (right)

Multiplicative Weighting (Line Profiles)

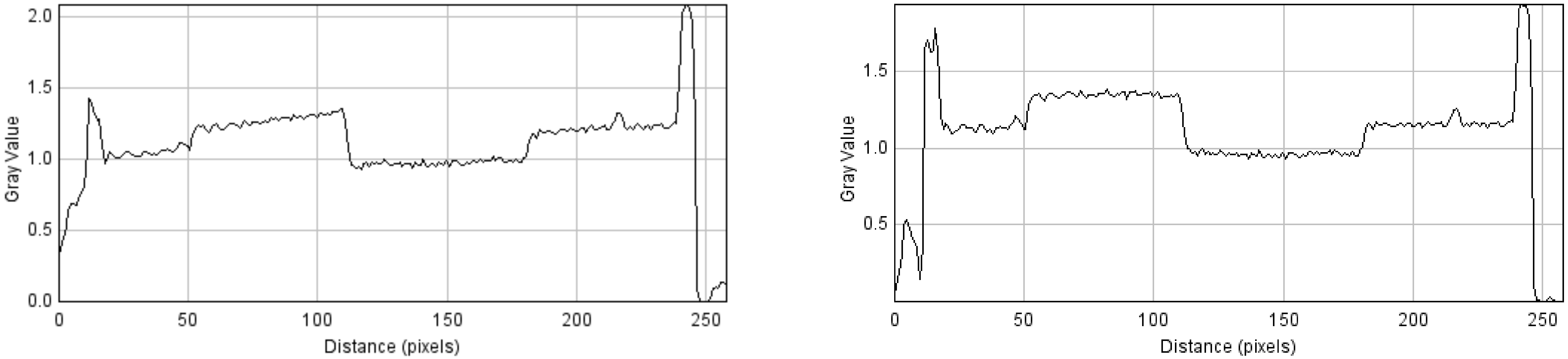

Figure 6: FBP with Parker weights (left), FBP with compensation (right)

Comparison: Regularized Iterative Reconstruction

Figure 7: Iterative reconstruction with TV (left), FBP with compensation (right)

TV vs. Multiplicative Weighting (Line Profiles)

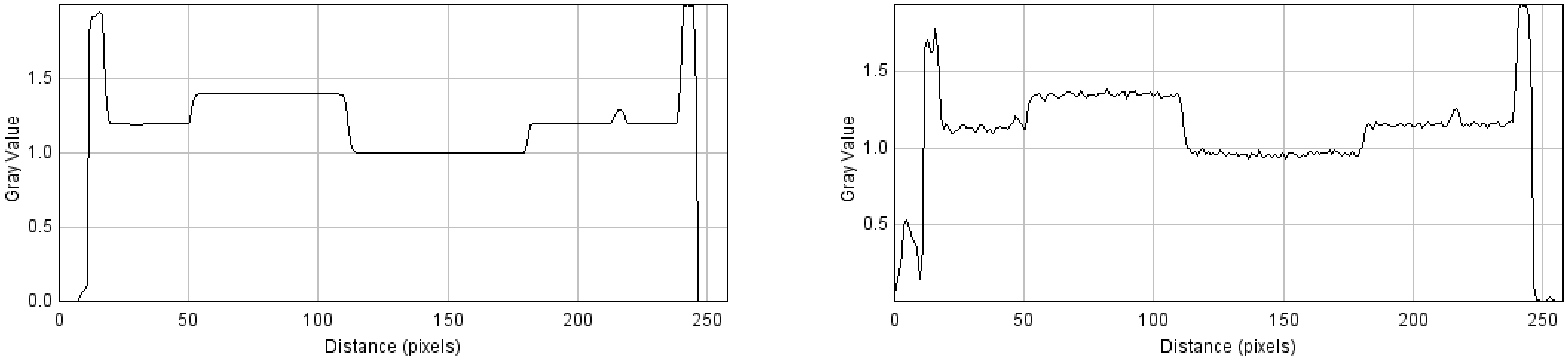

Figure 8: Iterative reconstruction with TV (left), FBP with compensation (right)

Image Space Iterative Reconstruction?

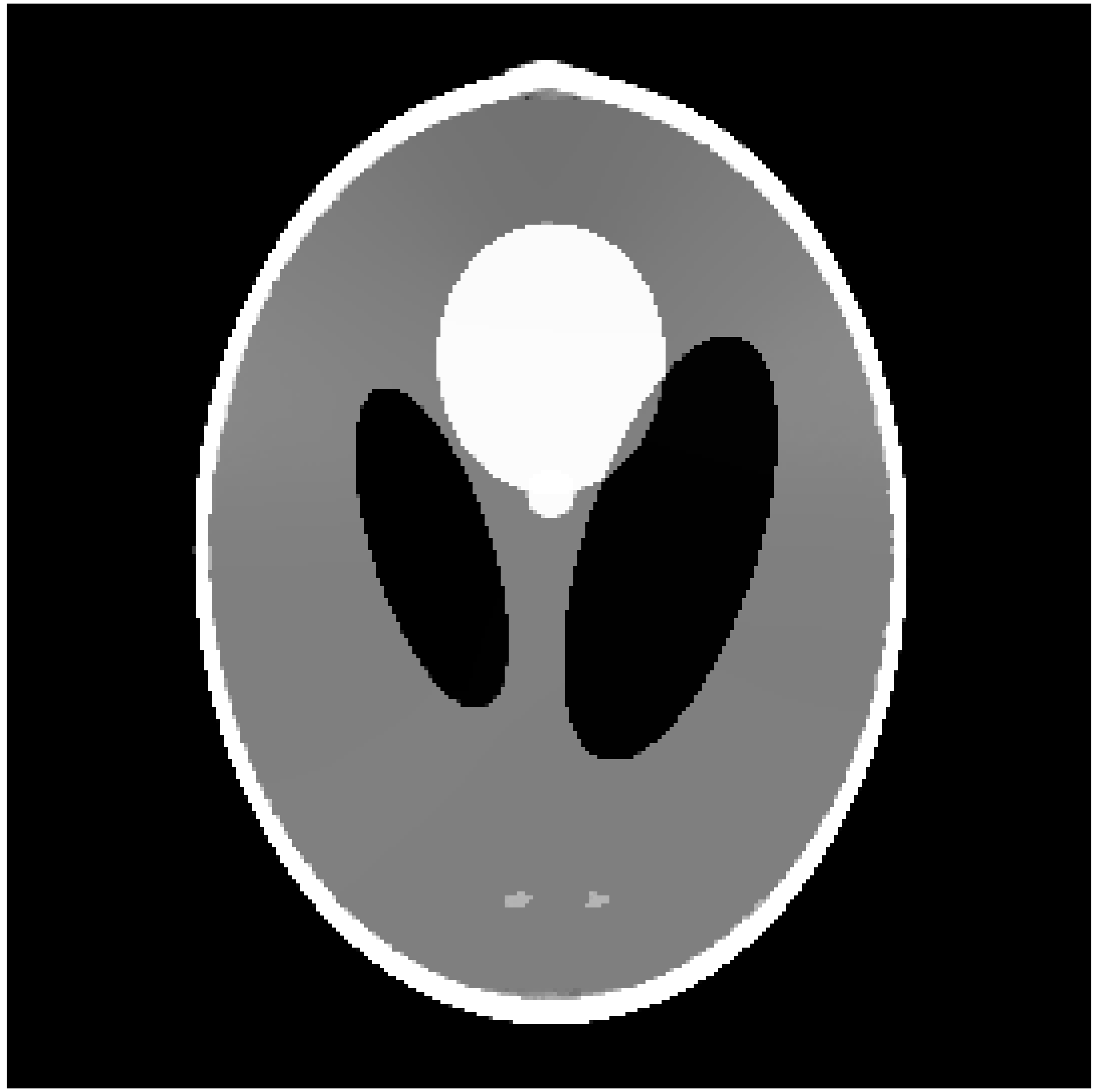

Figure 9: Iterative reconstruction with TV - 1000 iterations (left), FBP with compensation + BF, 1 iteration (right)

Line Plot Comparison

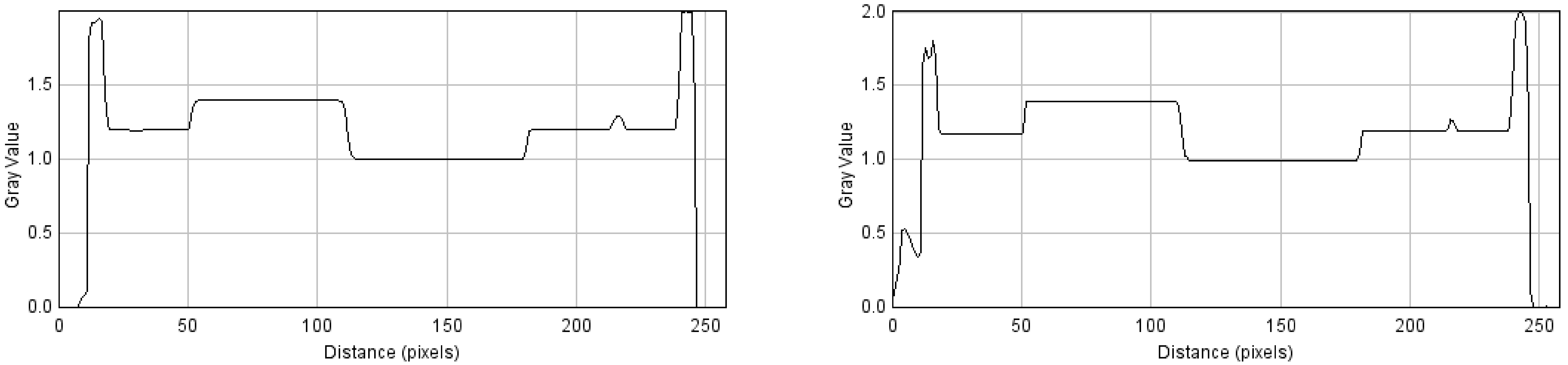

Figure 10: Iterative reconstruction with TV (left), FBP with compensation + BF (right)

TV or not TV? That is the question...

Figure 11: Iterative reconstruction with TV (left), FBP with Parker weights + BF (right)

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Take Home Messages

- Simple ideas are often the best ones!
- Bilateral filtering is computationally expensive and requires acceleration.
- Bilateral filtering is used in medical products and can be applied to any image (independent of the modality) where edge preserving smoothing is an issue.
- Edge preserving filtering can compensate for many disadvantages of expensive iterative reconstruction methods.

Further Readings

Read the original paper of the year 1998:

Carlo Tomasi and Roberto Manduchi. "Bilateral Filtering for Gray and Color Images". In: *Sixth International Conference on Computer Vision, 1998*. IEEE, Jan. 1998, pp. 839–846. DOI: 10.1109/ICCV.1998.710815

A very nice paper on ways to improve bilateral filtering can be found in:

Michael Elad. "On the Origin of the Bilateral Filter and Ways to Improve It". In: *IEEE Transactions on Image Processing* 11.10 (Oct. 2002), pp. 1141–1151. DOI: 10.1109/TIP.2002.801126

Developments of fast bilateral filtering are subject of the following webpage:

http://people.csail.mit.edu/sparis/bf/