Medical Image Processing for Diagnostic Applications

Iterative Reconstruction – Algebraic Reconstruction Technique

Online Course – Unit 56

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Pattern Recognition Lab (CS 5)

Topics

Algebraic Reconstruction Technique

Summary

Take Home Messages

Further Readings

Algebraic Reconstruction Technique (ART)

Idea: Find an iterative solution of

$$\boldsymbol{A}\boldsymbol{X} = \boldsymbol{P},$$

using the Kaczmarz method:

1. For each pixel p_i and each row $\boldsymbol{A}_i$ of $\boldsymbol{A}$ perform the following update:

$$\boldsymbol{X}^{k+1} = \boldsymbol{X}^{k} + \frac{p_i - \boldsymbol{A}_i \boldsymbol{X}^{k}}{\boldsymbol{A}_i \boldsymbol{A}_i^{\mathsf{T}}} \boldsymbol{A}_i^{\mathsf{T}}.$$

2. Repeat until convergence.

Algebraic Reconstruction Technique: Remarks

- Tanabe has shown in 1971 that the iterative scheme converges to the solution if there exists a unique solution.
- The angle between hyperplanes influences the rate of convergence to the solution.
- If hyperplanes are orthogonal to each other, it is obvious that the method converges rapidly (consider the 2-D case for plausibility).
- Orthogonalization methods applied in advance to iterations will improve convergence.
  - **Cons:** This is computationally prohibitive, and orthogonalization amplifies noise in measurements.
  - An alternative to orthogonalization is careful selection of the sequence of projections.
- Overdetermined systems and noise often have no unique solution and suffer from oscillations.

Algebraic Reconstruction Technique: Case Studies

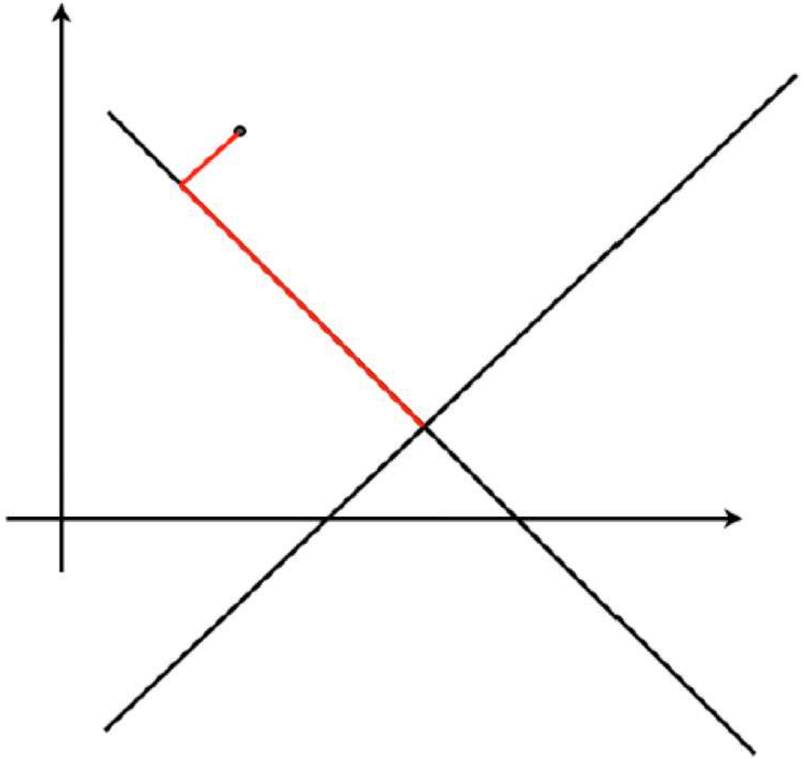

Figure 1: Two orthogonal projections

Algebraic Reconstruction Technique: Case Studies

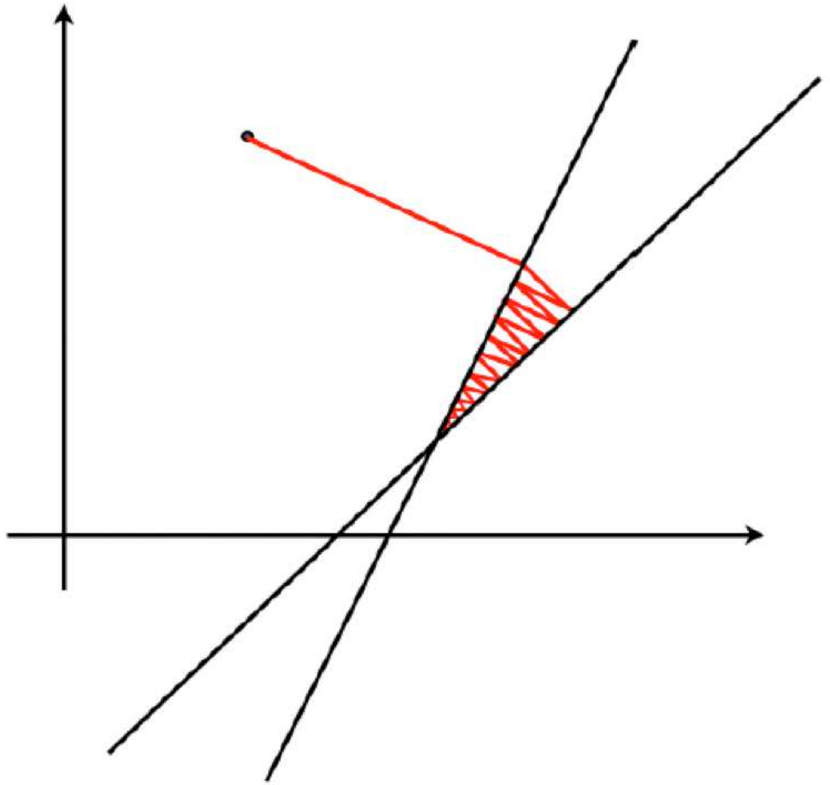

Figure 2: Oscillations

Algebraic Reconstruction Technique: Case Studies

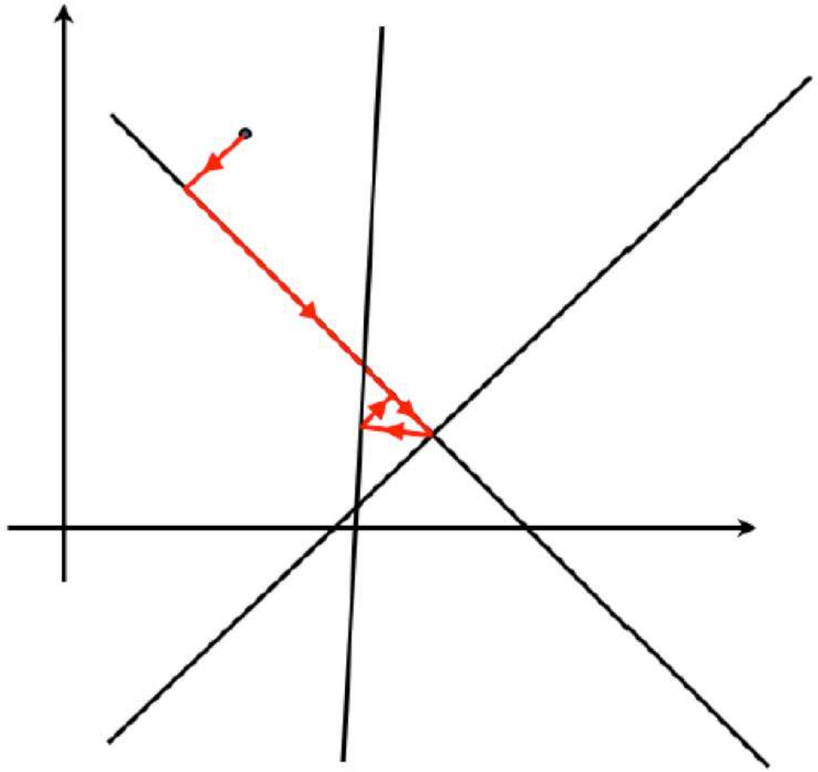

Figure 3: Four iterations

Algebraic Reconstruction Technique: Case Studies

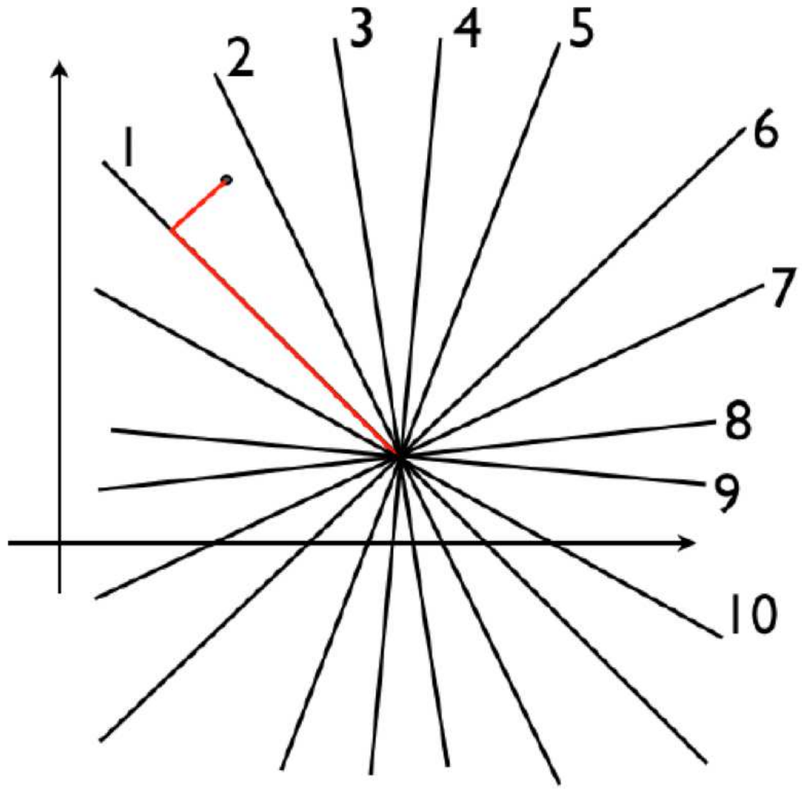

Figure 4: Consideration on convergence

Algebraic Reconstruction Technique: Case Studies

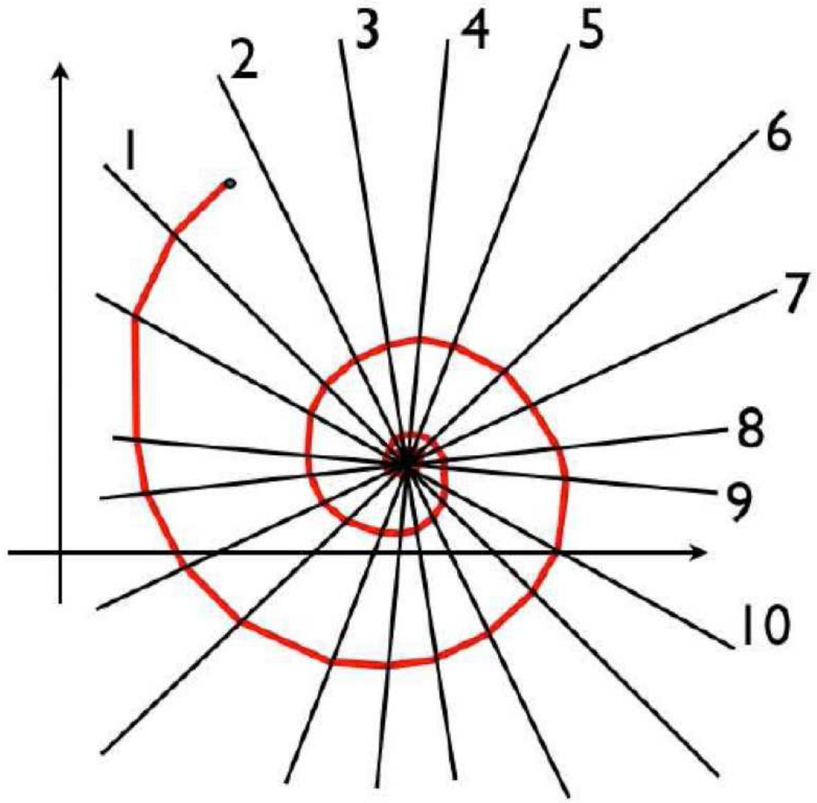

Figure 5: Consideration on convergence

Topics

Algebraic Reconstruction Technique

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- ART exploits the Kaczmarz method to iteratively compute a solution for a reconstruction problem.
- Iterative reconstruction usually demands significant higher computation times.
- Note that for this approach we do not have to make requirements on data completeness or acquisition geometry.
- The naive implementation has its problems, so we discuss some extensions in the next unit.

Further Readings

References and related books for the discussed topics in iterative reconstruction:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Stefan Kaczmarz. “Angenäherte Auflösung von Systemen linearer Gleichungen”. In: *Bulletin International de l’Académie Polonaise des Sciences et des Lettres. Classe des Sciences Mathématiques et Naturelles. Série A, Sciences Mathématiques* 35 (1937), pp. 355–357 For this article you can find an English translation here (December 2016).

Avinash C. Kak and Malcolm Slaney. *Principles of Computerized Tomographic Imaging*. Classics in Applied Mathematics. Accessed: 21. November 2016. Society of Industrial and Applied Mathematics, 2001. DOI: 10.1137/1.9780898719277. URL: http://www.slaney.org/pct/

H. Bruder et al. “Adaptive Iterative Reconstruction”. In: *Medical Imaging 2011: Physics of Medical Imaging*. Ed. by Norbert J. Pelc, Ehsan Samei, and Robert M. Nishikawa. Vol. 7961. Proc. SPIE 79610J. Feb. 2011, pp. 1–12. DOI: 10.1117/12.877953