Medical Image Processing for Diagnostic Applications

Parallel Beam – On Noise, Filtering and Window Functions

Online Course – Unit 35

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Pattern Recognition Lab (CS 5)

Topics

Effect of Noise on Filtering

Window Functions
- General Idea
- Common Examples
- The According Filters
- Filter Results

Summary
- Take Home Messages
- Further Readings

Additive Noise (+2%)

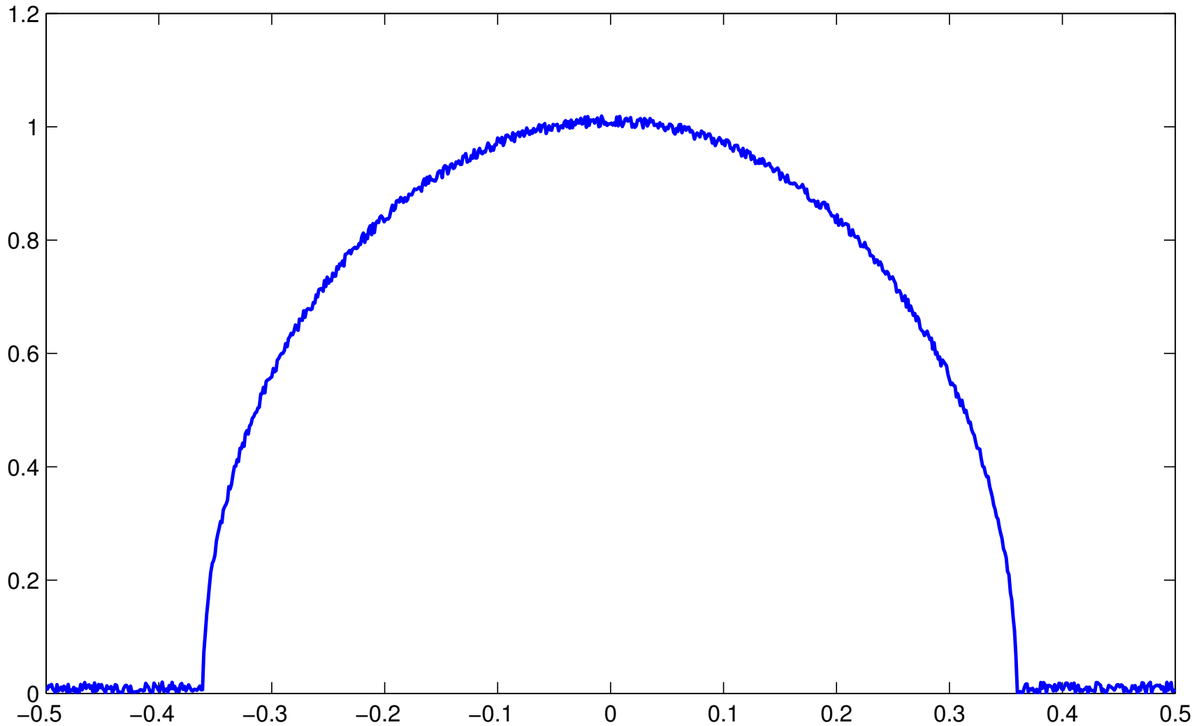


Figure 1: Projection of the cylinder phantom with 2% noise added

Additive Noise (+2%)

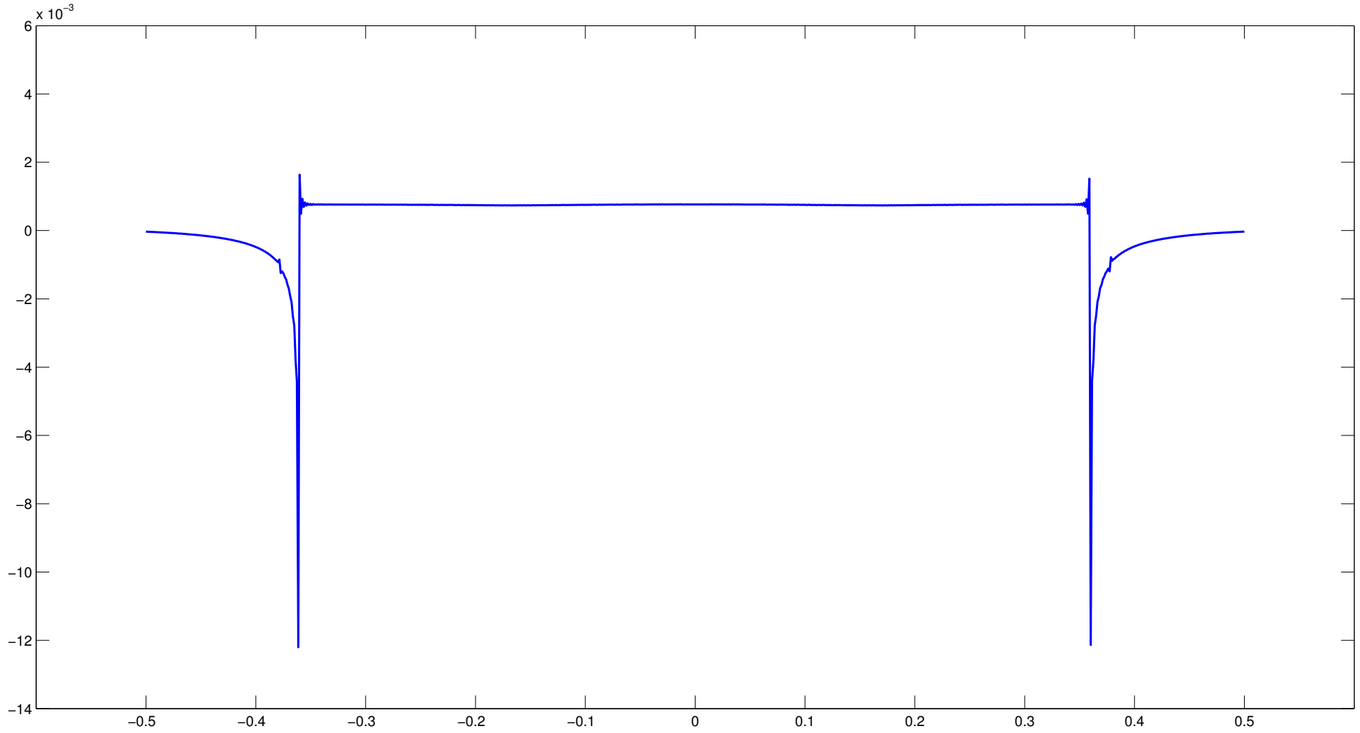


Figure 2: Filtered result of the noiseless projection

Additive Noise (+2%): After Filtering

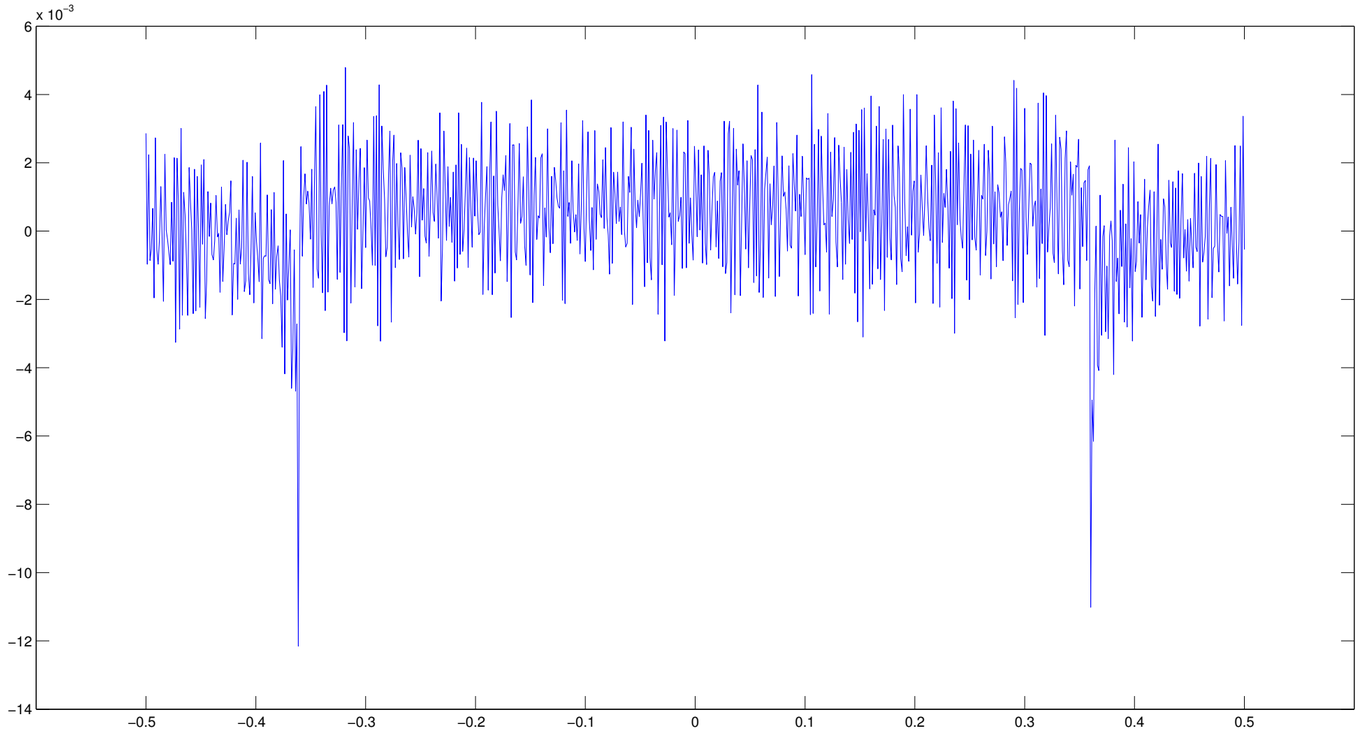


Figure 3: Filtered result of the noisy projection

Noise ...

- ... is amplified when filtering with the ramp filter.
- ... has to be taken care of in an appropriate manner.
- ... is indirectly proportional to the applied dose.
- ... affects different reconstruction methods differently.

Topics

Window Functions

Window functions are used to improve signals as high frequencies are reduced or even eliminated:

→ Noise reduction

→ Reduces high frequencies caused by cutting

Many window functions are known:

- Cosine window
- Shepp-Logan window
- ...

Window Functions: Filter Adaptation

1. Apply the window function W in frequency domain:

$$P'(\omega,\theta) = W(\omega) \cdot P(\omega,\theta).$$

2. Then apply the filter H:

$$Q'(\omega,\theta) = H(\omega) \cdot P'(\omega,\theta) = H(\omega) \cdot W(\omega) \cdot P(\omega,\theta).$$

3. Rewrite the filtering equation to an adjusted filter H':

$$Q'(\omega,\theta) = H'(\omega) \cdot P(\omega,\theta)$$

$$\Rightarrow \qquad H'(\omega) = H(\omega) \cdot W(\omega).$$

Rectangular Window (Frequency Cut-off)

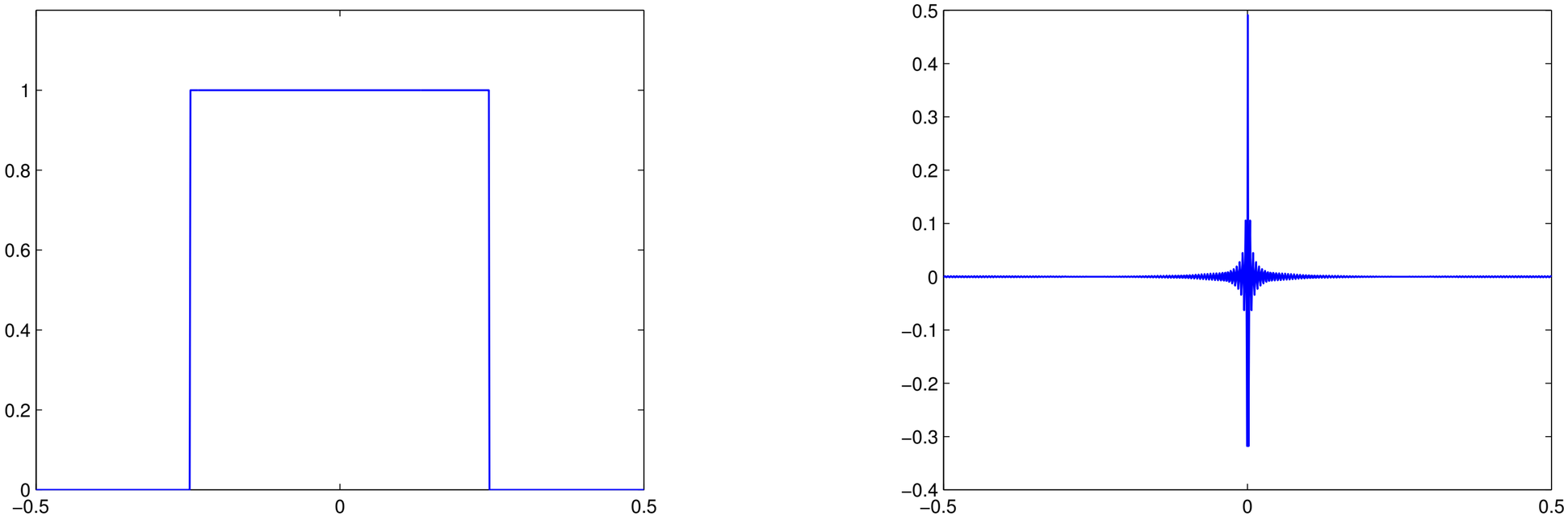
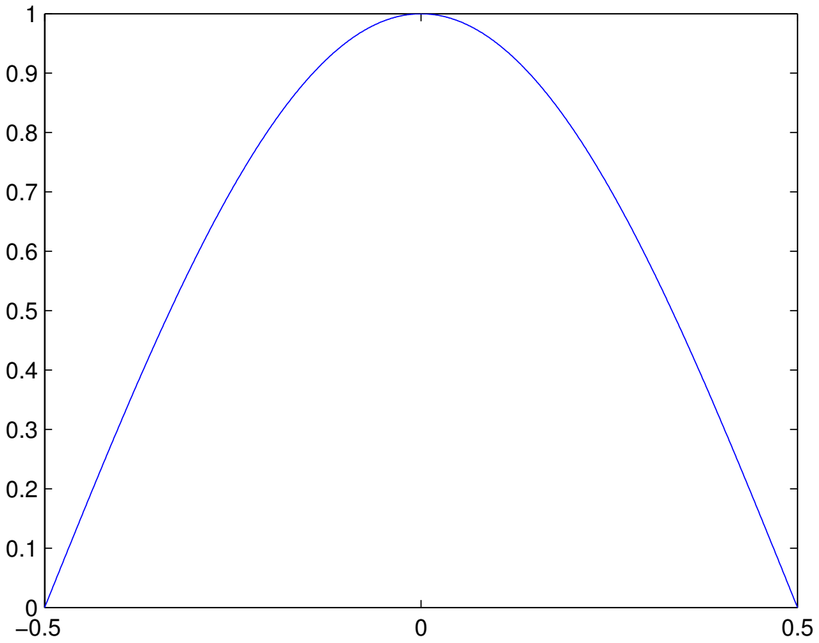


Figure 4: Rectangular window function in frequency domain (left) and its counterpart in spatial domain (right)

Cosine Window: $\cos(\pi \cdot x)$



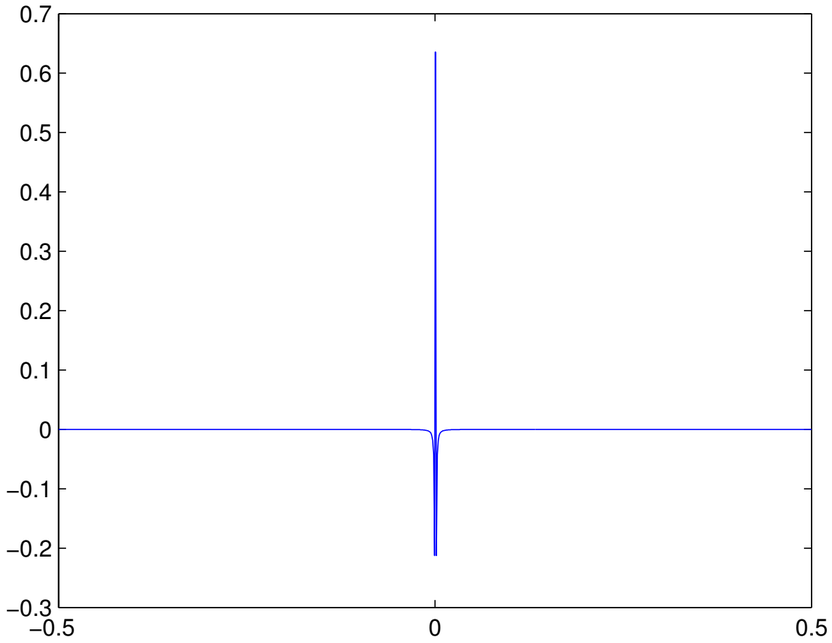
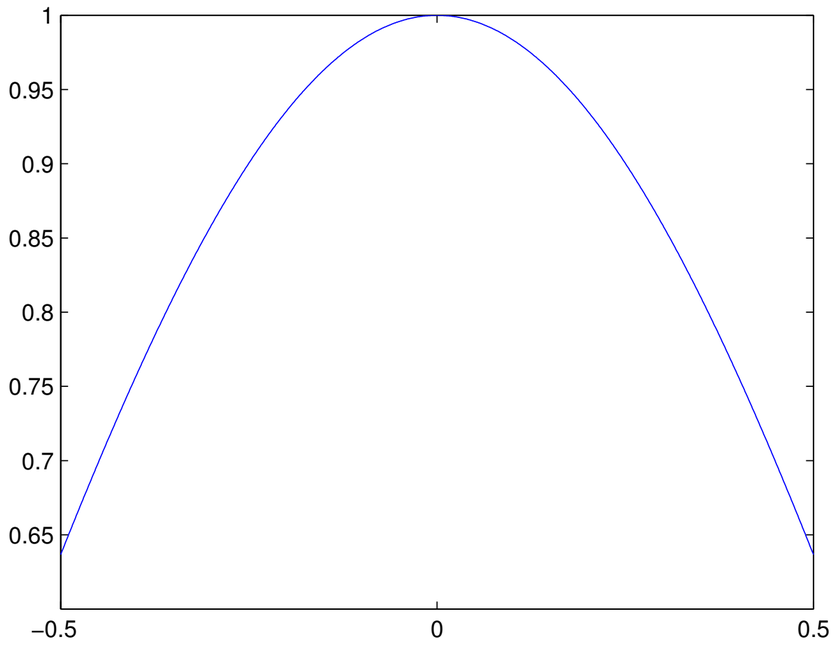


Figure 5: Cosine window function in frequency domain (left) and its counterpart in spatial domain (right)

Shepp-Logan Window: $\frac{\sin(\pi \cdot x)}{(\pi \cdot x)}$



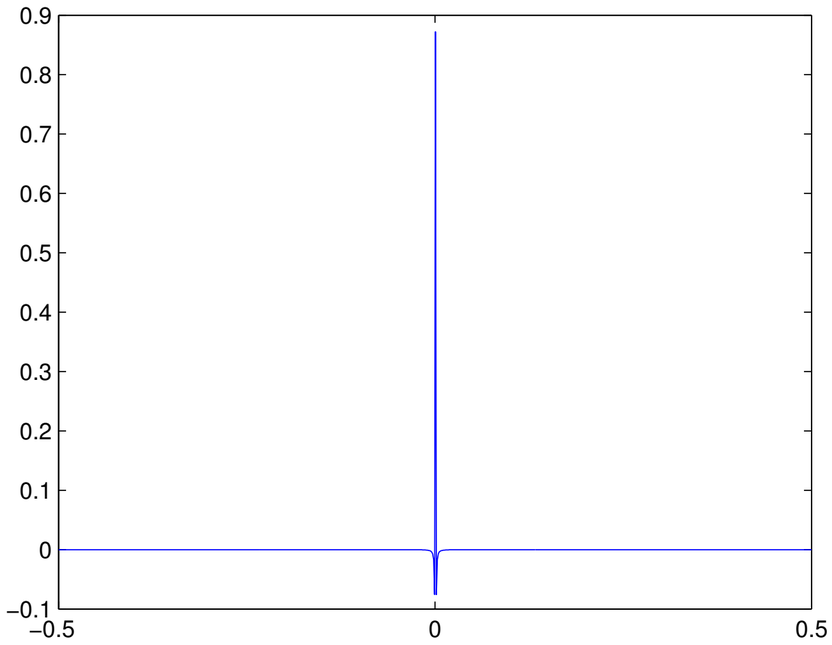


Figure 6: Shepp-Logan window function in frequency domain (left) and its counterpart in spatial domain (right)

Rectangular Filter

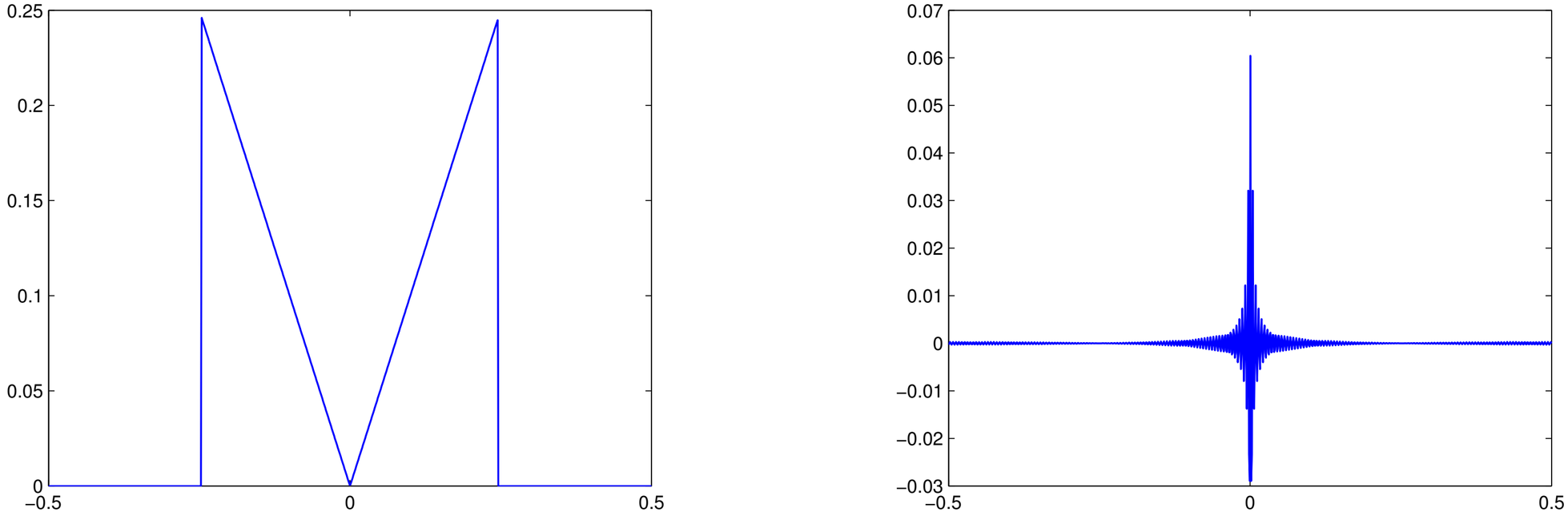


Figure 7: Rectangular filter in frequency domain (left) and its counterpart in spatial domain (right)

Cosine Filter

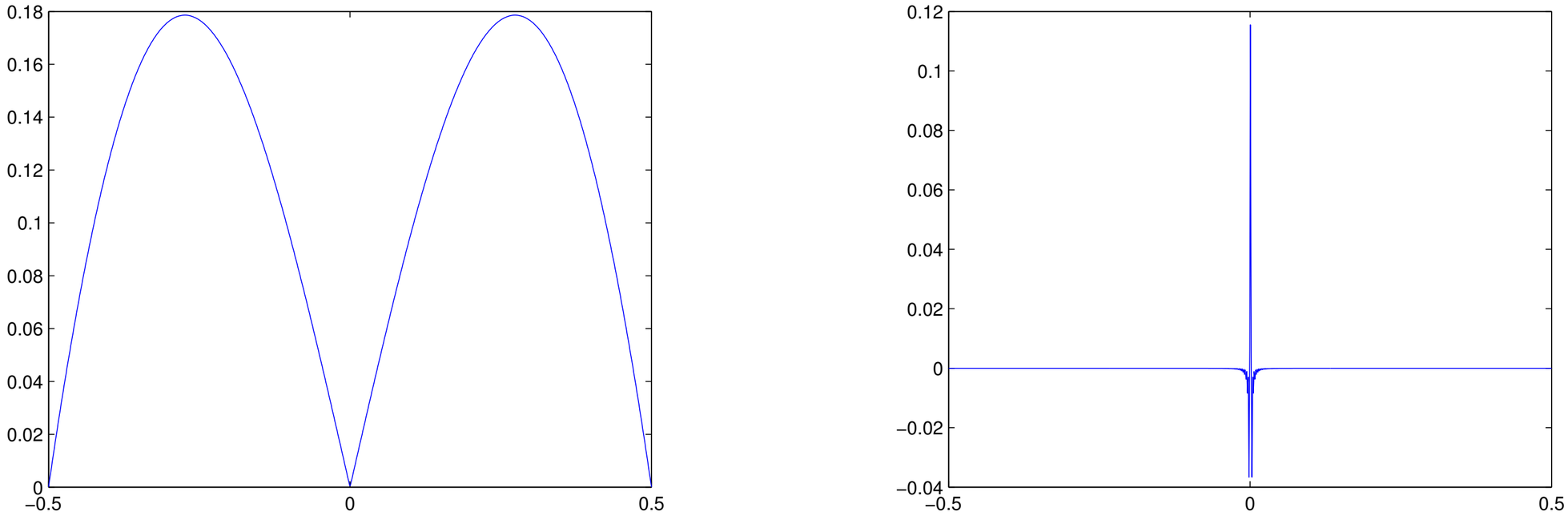


Figure 8: Cosine filter in frequency domain (left) and its counterpart in spatial domain (right)

Shepp-Logan Filter

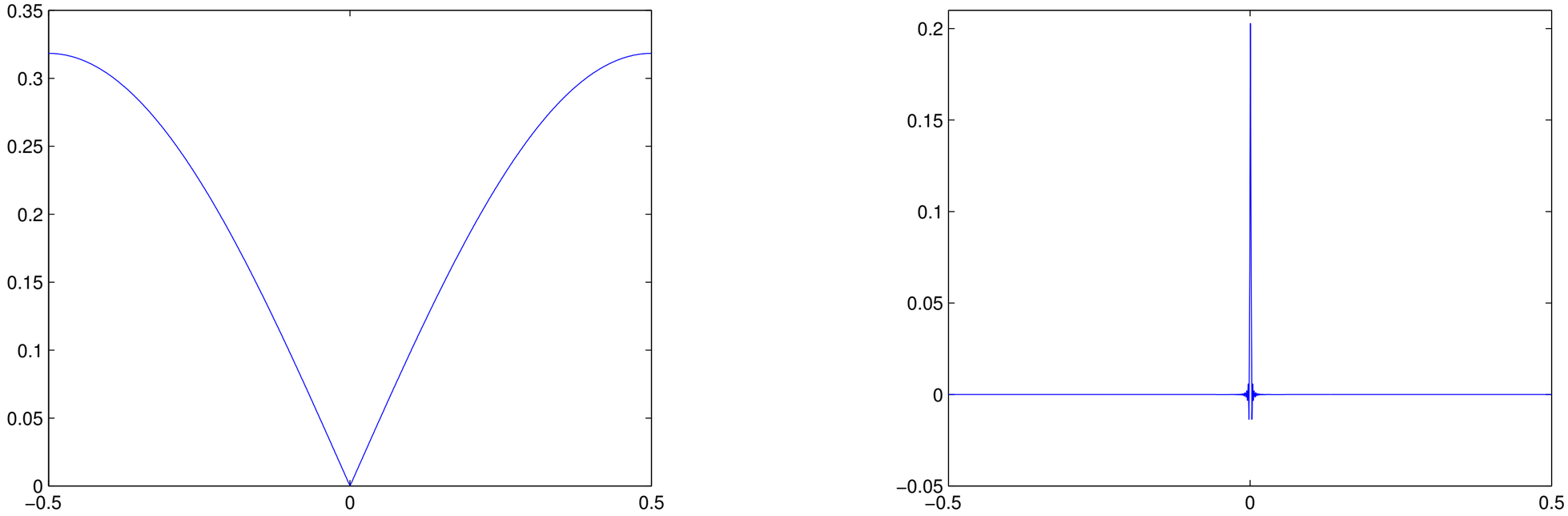


Figure 9: Shepp-Logan filter in frequency domain (left) and its counterpart in spatial domain (right)

Ramp Filter Result

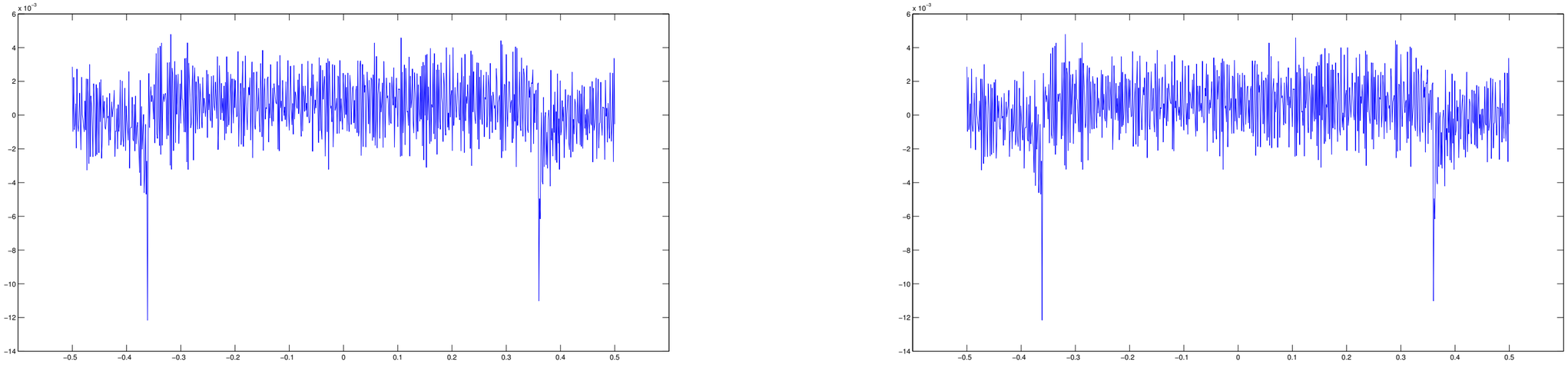


Figure 10: Filtered noisy projection using the rectangular window (left) vs. itself, the ramp filter (right)

Cosine Filter Result

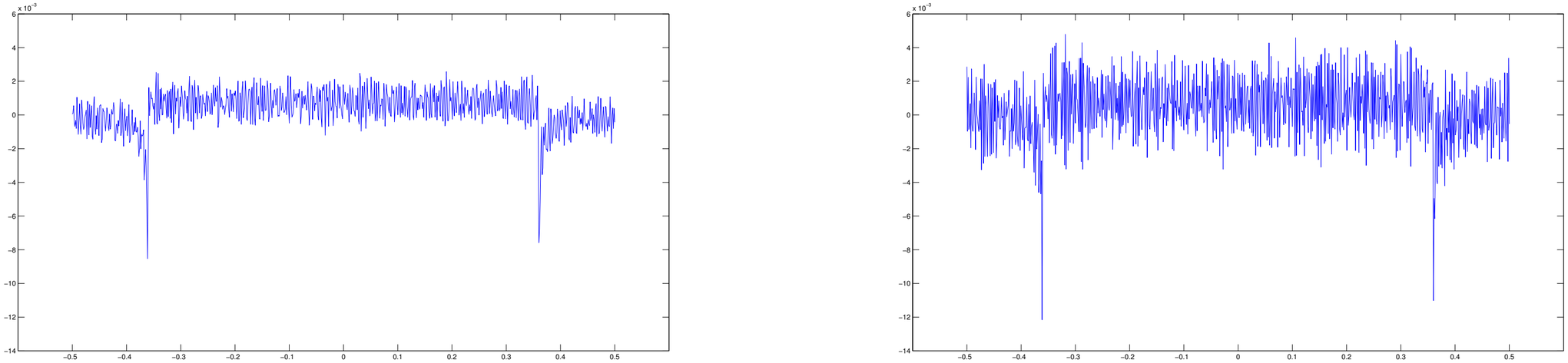


Figure 11: Filtered noisy projection using the cosine window (left) vs. the ramp filtered result (right)

Shepp-Logan Filter Result

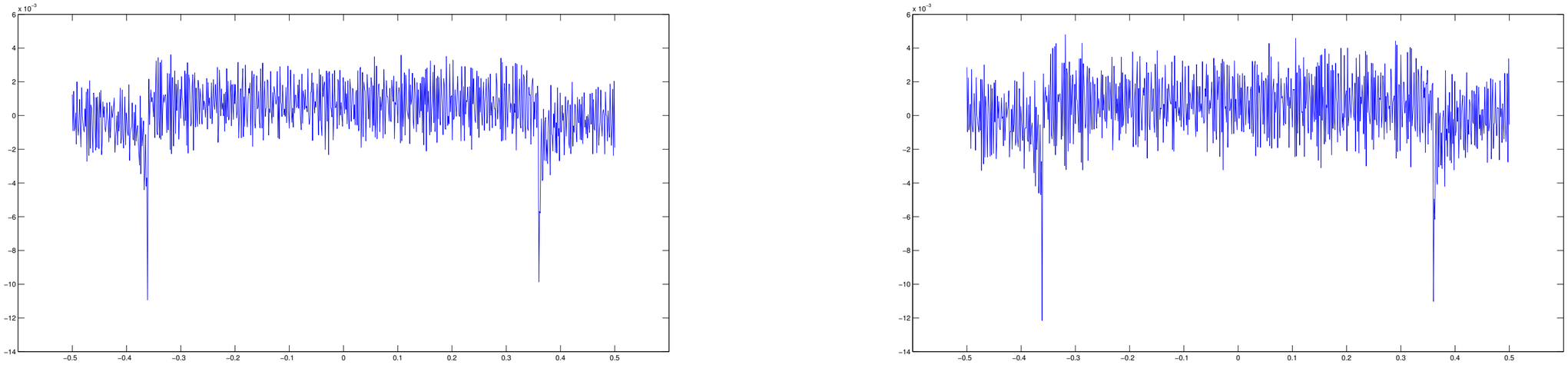


Figure 12: Filtered noisy projection using the Shepp-Logan window (left) vs. the ramp filtered result (right)

Topics

Take Home Messages

- Noise has a severe effect on the filtering result.
- Window functions can be used to reduce this effect.
- We have learned about the frequency cut-off, the cosine window and the Shepp-Logan window.

Further Readings

The original Ram-Lak article is:

G. N. Ramachandran and A. V. Lakshminarayanan. “Three-dimensional Reconstruction from Radiographs and Electron Micrographs: Application of Convolutions instead of Fourier Transforms”. In: *Proceedings of the National Academy of Sciences of the United States of America* 68.9 (Sept. 1971), pp. 2236–2240

The concise reconstruction book from ‘Larry ’Zeng:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Another mathematical examination of filtered backprojection can be found in

Thorsten Buzug. *Computed Tomography: From Photon Statistics to Modern Cone-Beam CT*. Springer Berlin Heidelberg, 2008. DOI: 10.1007/978-3-540-39408-2