Medical Image Processing for Diagnostic Applications

Regularized Reconstruction Methods

Online Course – Unit 61

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Topics

Regularized Reconstruction Methods

- TV-Norm
- Gradient Methods
- ML Methods

Summary

- Take Home Messages
- Further Readings

Regularized Reconstruction: TV-Norm

- The **total variation norm (TV-norm)** is a popular combination of a sparsifying transform and the L_1-norm.
- The sparsifying transform is the gradient image:

$$\Psi(f(x,y)) = \sqrt{\left(\frac{\partial f}{\partial x}(x,y)\right)^2 + \left(\frac{\partial f}{\partial y}(x,y)\right)^2}.$$

- The norm is the L_1-norm:

$$|\boldsymbol{X}|_1 = \sum_{i,j} |x|.$$

- The TV-norm for the discretized image $\boldsymbol{f}$ then is computed by:

$$|\Psi(\boldsymbol{f})|_1 = \sum_{i,j} \left| \sqrt{\left(\frac{\partial f}{\partial x}(i,j)\right)^2 + \left(\frac{\partial f}{\partial y}(i,j)\right)^2} \right| = \sum_{i,j} \sqrt{\left(\frac{\partial f}{\partial x}(i,j)\right)^2 + \left(\frac{\partial f}{\partial y}(i,j)\right)^2}.$$

Regularized Reconstruction: TV-Norm

The TV-Norm ...

- ... promotes sparsity in the gradient domain.
- ... suppresses noise.
- ... preserves edges.
- ... promotes images with large uniform areas and sharp boundaries.

$\rightarrow$ These properties make it ideal for phantoms.

$\rightarrow$ Images are often described as “patchy”.

Regularized Reconstruction: TV-Norm

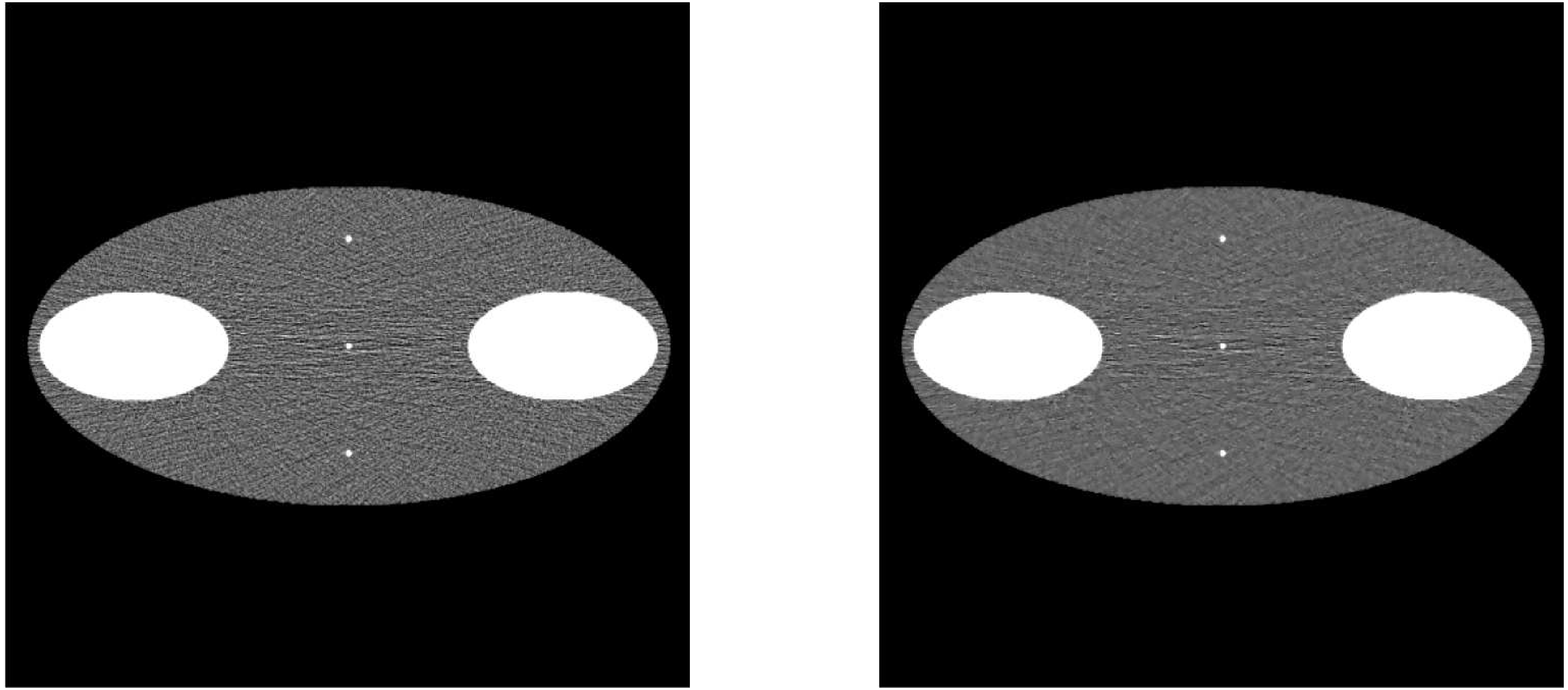

Figure 1: Streak artifacts in a phantom reconstruction without (left), and with TV regularization (right)

Regularized Reconstruction: TV-Norm

- Bruder et al. have shown that reconstruction with TV regularization is equivalent to FBP reconstruction with subsequent nonlinear edge preserving filtering.
- Non-linear filtering makes image quality assessment difficult.

$\rightarrow$ FBP reconstruction is much faster.

$\rightarrow$ The same “patch” image characteristic can be obtained if too much filtering is applied.

Regularized Reconstruction: Gradient Methods

Recall: The objective function is determined by:

$$\chi(\boldsymbol{X}) = |\boldsymbol{\Psi}\boldsymbol{X}|_p, \qquad \text{subject to } \boldsymbol{A}\boldsymbol{X} = \boldsymbol{P}.$$

We can formulate a single new objective function:

$$\chi(\boldsymbol{X}) = |\boldsymbol{\Psi}\boldsymbol{X}|_p + \boldsymbol{\Lambda}(\boldsymbol{A}\boldsymbol{X} - \boldsymbol{P})$$

with Lagrange multipliers

$$\boldsymbol{\Lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n).$$

$\rightarrow$ This method is often referred to as "**Compressed Sensing**".

Regularized Reconstruction: Gradient Methods

Similarly, we can add further constraints:

$$\chi(\boldsymbol{X}) = |\boldsymbol{\Psi}\boldsymbol{X}|_p + \boldsymbol{\Lambda}(\boldsymbol{A}\boldsymbol{X} - \boldsymbol{P}) + k(|\boldsymbol{X} - \boldsymbol{X}_{\text{prior}}|_p)$$

with weighting factor k and prior knowledge $\boldsymbol{X}_{\text{prior}}$.

$\rightarrow$ This method is known as "**Prior Image Constrained Compressed Sensing**".

Regularized Reconstruction: Gradient Methods

A solution is based on the following principles:

- compute the gradient & set to 0,
- follow the opposite gradient direction,
- perform projection onto convex sets:
  - enforce constraint A,
  - enforce constraint B,
  - repeat until convergence.

Regularized Reconstruction: ML Methods

- Introduction of new constraints is done using Bayes' rule:

$$P(X|p) = \frac{P(p|X)P(X)}{P(p)}.$$

- Taking the logarithm yields:

$$\ln P(X|p) = \ln P(p|X) + \ln P(X) - \ln P(p).$$

- The optimization is independent of $\ln P(p)$.
- This yields the following objective function:

$$L(X) = \ln P(p|X) + \ln P(X),$$

which could be interpreted as:

$$\text{Posterior Function} = \text{Likelihood Function} + \beta(\text{Prior Knowledge}).$$

Regularized Reconstruction: ML Methods

Recall: The unconstrained ML-EM update formula is:

$$x_j^{k+1} = \frac{x_j^k}{\sum_i a_{ij}} \sum_i a_{ij} \frac{p_i}{\sum_l a_{il} x_l^k}.$$

Adding the constraint $P(\boldsymbol{X})$ yields:

$$x_j^{k+1} = \frac{x_j^k}{\sum_i a_{ij} + \beta \frac{\partial \ln P(\boldsymbol{X}^k)}{\partial x_j^k}} \sum_i a_{ij} \frac{p_i}{\sum_l a_{il} x_l^k}.$$

$\rightarrow$ This method is also known as "Green's One-Step-Late Method".

$\rightarrow$ Methods using this Bayesian scheme are also known as **maximum-a-posteriori (MAP) methods**.

Regularized Reconstruction: Remarks

- Regularization leads to a new reconstruction problem that does not necessarily lead to the solution of the original problem.
- Sophisticated cost functions often also lead to increased computational effort.
- Regularization can lead to results that look beautiful.
- The result may not be the true image!

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Take Home Messages

- You have learned about several approaches to regularized reconstruction: TV-norm, gradient based and ML-EM methods.
- Regularization may change the image content towards, but also away from clinical realism, so these methods need to be tuned for the problem at hand.

Further Readings

References and related books for the discussed topics in iterative reconstruction:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

Stefan Kaczmarz. "Angenäherte Auflösung von Systemen linearer Gleichungen". In: *Bulletin International de l'Académie Polonaise des Sciences et des Lettres. Classe des Sciences Mathématiques et Naturelles. Série A, Sciences Mathématiques* 35 (1937), pp. 355–357 For this article you can find an English translation here (December 2016).

Avinash C. Kak and Malcolm Slaney. *Principles of Computerized Tomographic Imaging*. Classics in Applied Mathematics. Accessed: 21. November 2016. Society of Industrial and Applied Mathematics, 2001. DOI: 10.1137/1.9780898719277. URL: http://www.slaney.org/pct/

H. Bruder et al. "Adaptive Iterative Reconstruction". In: *Medical Imaging 2011: Physics of Medical Imaging*. Ed. by Norbert J. Pelc, Ehsan Samei, and Robert M. Nishikawa. Vol. 7961. Proc. SPIE 79610J. Feb. 2011, pp. 1–12. DOI: 10.1117/12.877953