Medical Image Processing for Diagnostic Applications

Parallel Beam – Preparation

Online Course – Unit 30

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Topics

Important Methods

Central Slice Theorem

Summary
- Take Home Messages
- Further Readings

Fourier Transform

- Fourier transform in 1-D:

$$P(\omega) = \int_{-\infty}^{\infty} p(s) e^{-2\pi i s \omega} \, \mathrm{d}s$$

- Inverse Fourier transform in 1-D:

$$p(s) = \int_{-\infty}^{\infty} P(\omega) e^{2\pi i s \omega} \, \mathrm{d}\omega$$

$\longrightarrow$ Fourier pairs

Convolution

- Convolution:

$$(f*g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\,\mathrm{d}\tau = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)\,\mathrm{d}\tau$$

- Convolution theorem:

$$q(s) = f(s) * g(s) \qquad \Leftrightarrow \qquad Q(\omega) = F(\omega) \cdot G(\omega)$$

Hilbert Transform

- The spatial representation of the Hilbert transform:

$$(f * h)(t) = \text{p.v.} \int_{-\infty}^{\infty} f(t-\tau)h(\tau)\,\mathrm{d}\tau,$$

where "p.v." denotes the principal value, has the transformation kernel

$$h(\tau) = \frac{1}{\pi\tau}.$$

- Its Fourier representation is:

$$H(\omega) = -i\,\mathrm{sgn}(\omega).$$

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$$P(\omega, \theta) = F(\omega \cos\theta, \omega \sin\theta)$$

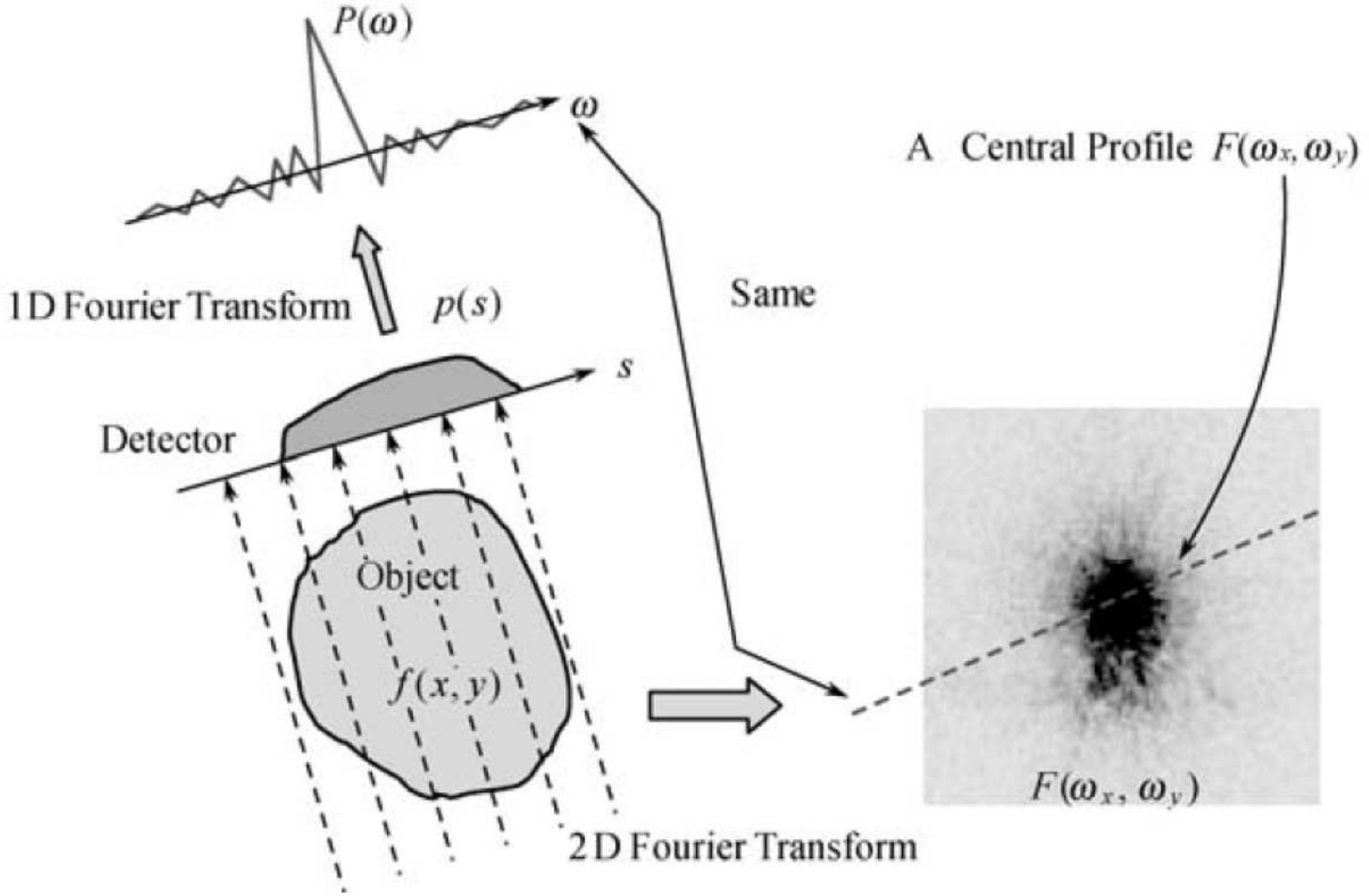

Figure 1: Central slice theorem (Zeng, 2009)

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Take Home Messages

- The Fourier transform, the convolution theorem, and the Hilbert transform are important tools for image reconstruction.
- The central slice theorem (also: Fourier slice theorem) is one of the most fundamental concepts in reconstruction theory!

Further Readings

The derivation of the Fourier slice theorem can also be found here (bibsource):

Joachim Hornegger, Andreas Maier, and Markus Kowarschik. “CT Image Reconstruction Basics”. In: *MR and CT Perfusion and Pharmacokinetic Imaging: Clinical Applications and Theoretical Principles*. Ed. by Roland Bammer. 1st ed. Alphen aan den Rijn, Netherlands: Wolters Kluwer, 2016, pp. 01–09

The concise reconstruction book from ‘Larry ’Zeng:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9

If you want to learn more about applications of the Fourier transform:

Ronald N. Bracewell. *The Fourier Transform and Its Applications*. 3rd ed. Electrical Engineering Series. Boston: McGraw-Hill, 2000