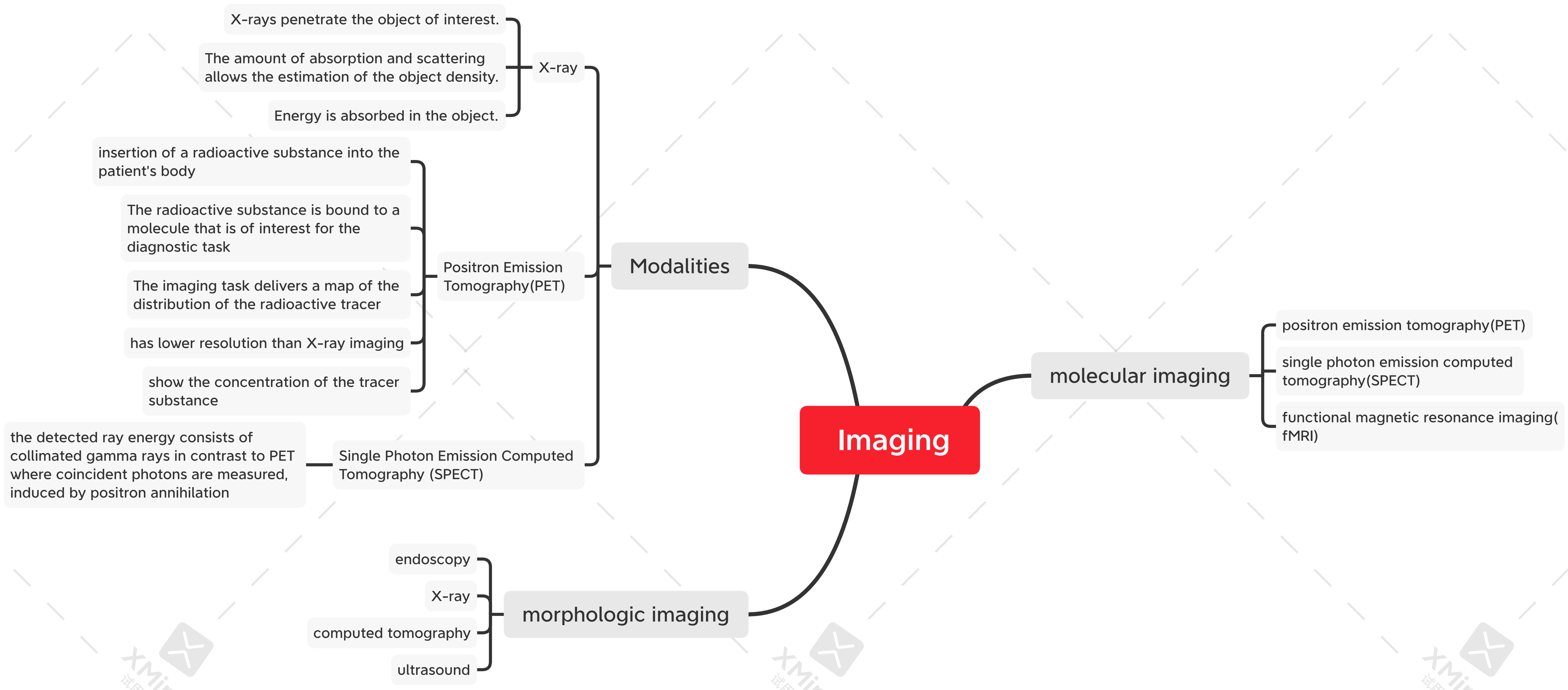
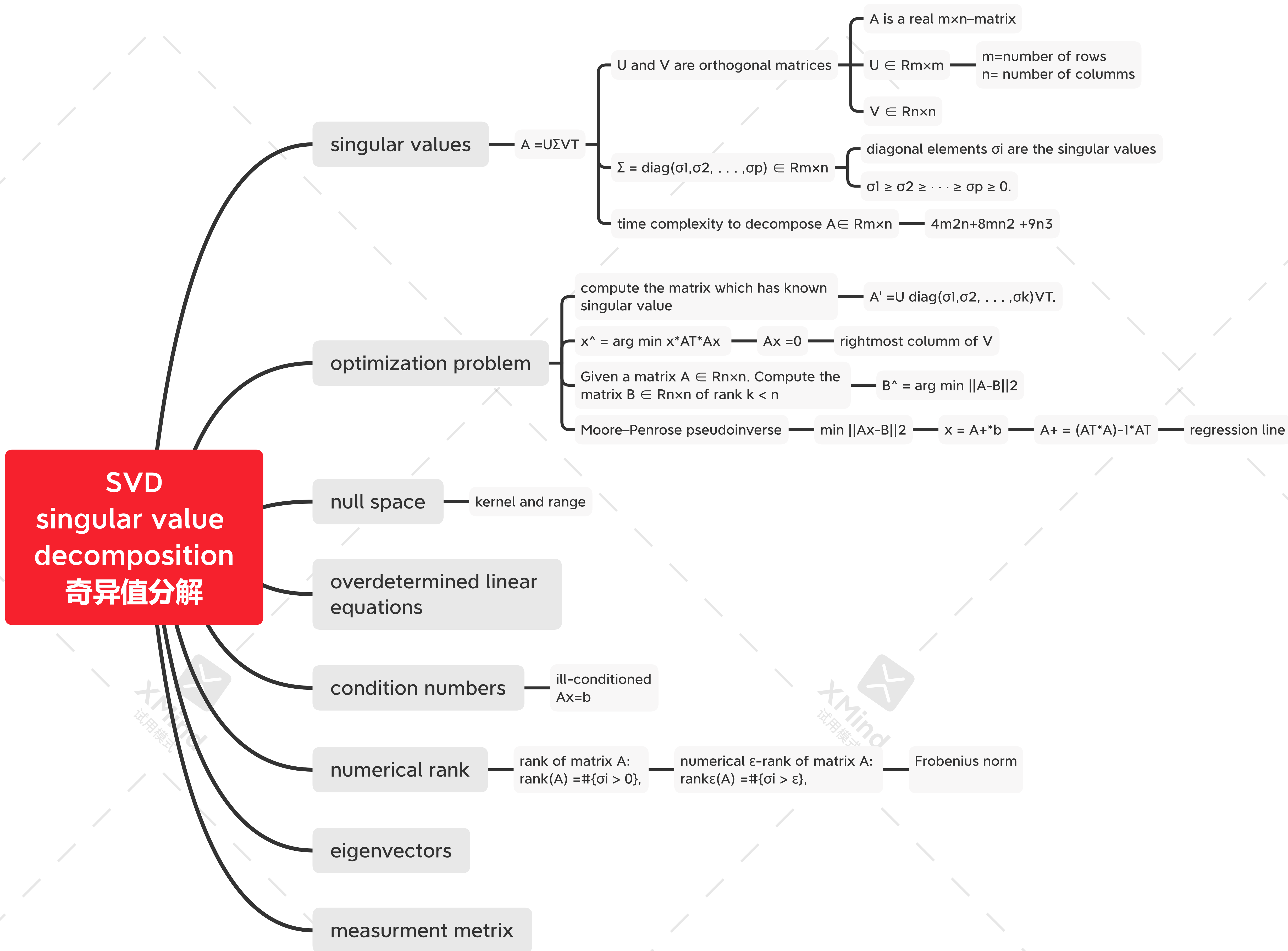
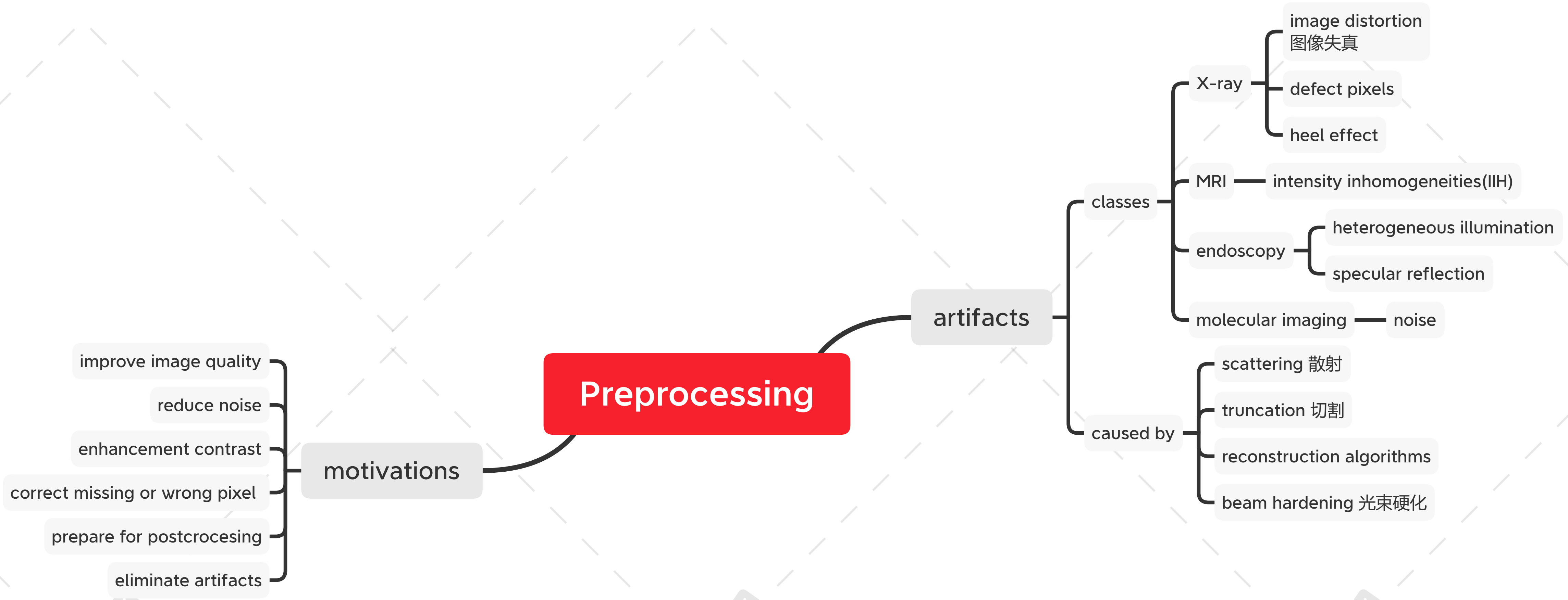

Imaging
Modalities
X-ray
X-rays penetrate the object of interest.
The amount of absorption and scattering allows the estimation of the object density.
Energy is absorbed in the object.
Positron Emission Tomography(PET)
insertion of a radioactive substance into the patient's body
The radioactive substance is bound to a molecule that is of interest for the diagnostic task
The imaging task delivers a map of the distribution of the radioactive tracer
has lower resolution than X-ray imaging
show the concentration of the tracer substance
Single Photon Emission Computed Tomography (SPECT)
the detected ray energy consists of collimated gamma rays in contrast to PET where coincident photons are measured, induced by positron annihilation
molecular imaging
positron emission tomography(PET)
single photon emission computed tomography(SPECT)
functional magnetic resonance imaging(fMRI)
morphologic imaging
endoscopy
X-ray
computed tomography
ultrasound


SVD
singular value
decomposition
奇异值分解
singular values
A =UΣVT
U and V are orthogonal matrices
A is a real m×n–matrix
U ∈ Rm×m
m=number of rows
n= number of columms
V ∈ Rn×n
Σ = diag(σ1,σ2, . . . ,σp) ∈ Rm×n
diagonal elements σi are the singular values
σ1 ≥ σ2 ≥ · · · ≥ σp ≥ 0.
time complexity to decompose A∈ Rm×n
4m2n+8mn2 +9n3
optimization problem
compute the matrix which has known singular value
A' =U diag(σ1,σ2, . . . ,σk)VT.
x^ = arg min x*AT*Ax
Ax =0
rightmost columm of V
Given a matrix A ∈ Rn×n. Compute the matrix B ∈ Rn×n of rank k < n
B^ = arg min ||A-B||2
Moore–Penrose pseudoinverse
min ||Ax-B||2
x = A+*b
A+ = (AT*A)-1*AT
regression line
null space
kernel and range
overdetermined linear equations
condition numbers
ill-conditioned
Ax=b
numerical rank
rank of matrix A:
rank(A) =#{σi > 0},
numerical ε-rank of matrix A:
rankε(A) =#{σi > ε},
Frobenius norm
eigenvectors
measurment metrix


Preprocessing
motivations
improve image quality
reduce noise
enhancement contrast
correct missing or wrong pixel
prepare for postcrocesing
eliminate artifacts
artifacts
classes
X-ray
image distortion 图像失真
defect pixels
heel effect
MRI
intensity inhomogeneities(IIH)
endoscopy
heterogeneous illumination
specular reflection
molecular imaging
noise
caused by
scattering 散射
truncation 切割
reconstruction algorithms
beam hardening 光束硬化

X-ray preprocessing

- Distortion Correction Mapping
  - Non-Parametric Mapping
    look-up table or displacement vector field
    - to describe the correct position of every pixel
  - Parametric Mapping
    - x =X(x',y') = ∑d∑d-i ui,j*bj(y')bi(x')
      y =Y(x',y') = ∑d∑d-i vi,j*bj(y')bi(x')
      - base function bK
        - commen :
          standard polynomials
          bi(x) = xi
          - use Horner's scheme to evaluation
      - image point on Undistorted image(x',y')
      - image point on distorted image(x,y)
      - mapping coefficients u,v
        - unknown
      - measurement matrix M
        - x= M*u
    - calibration parameter estimation
      校准参数估计（求u和v）
      - Least square estimation
        最小二乘
        - calibration by minimization of the sum of squared distances of data points from the curve
          - ∑(X(x'n,y'n)-xn)^2 →min
            ∑(Y(x'n,y'n)-yn)^2 →min
      - SVD
        - u=M+*x
      - the system can be solved only when M is full rank
    - interpolation,regression, overfitting
      - Interpolation defines the estimation of unknown data between observed data. In addition, we require the interpolation curve to fit all the training data.(perfect fitting)
      - regression defines a technique to discover a mathematical relationship between multiple variables using a set of data points, i. e., training data. In regression it is not required that the regression curve fits the training data perfectly.(not perfect)
      - Overfitting is defined as training a model, (e. g., a parametric model), so that it well fits the training data, but fails to predict well in between and outside the data.(dont fit general data)
    - proper scaling of input data
      - crucial for quality of the output data
        - minimization of condition number to minimize sensibility
    - interpolation of intensities
      - nearest neighbor interpolation
        - crispy and noisy, but fast
      - bilinear interpolation
  - Algorithm
    - compute coordinate x =X(x',y')
    - compute coordinate y =Y(x',y')
    - compute intensity f(x,y) = interpol(g;x,y)
- Defect Pixel Interpolation
  - type
    - in spatial domin
      - suitable for small defect areas,
      - non-linear
    - in frequency domin
      - bandlimitation by bandpass filtering
      - deconvolution of defect and ideal image,
      - binary defect image is computed in a calibration step
      - ideal image is multiplied with the binary defect image
  - mathematical model
    - the multiplication of the original image with a defect mask
      fi,j *wi,j = gi,j
      - fi,j denote the intensity value at grid point (i, j) of the ideal image f
      - wi,j denote the indicator value at (i, j) where w is the mask image
      - gi,j denote the intensity value at (i, j) of the observed image g
  - bandlimitation
    - Defect pixels cause high differences in intensities of neighboring pixels
    - Replace defect pixels iteratively by enforcing bandlimitation.
  - Frequency Domain
    Defect Pixel Interpolation
    - symmetry property
      - F(ξ) = F-(N−ξ),
        G(ξ) =G-(N−ξ),
        W(ξ) =W-(N−ξ),
        - z- denotes the complex conjugate of z
    - steps
      - compute FT of input signal g(n)
      - (initialize ^F(0)(k) = 0, G(0)(k) =G(k), i = 1)
        randomly select a line pair G(i−1)(s), G(i−1)(N−s) for s /= 0
        - cause by convalution
          G(s) = F(s) ∗W(s)
          =1/N ∑(k=0--N-1)F(k) ∗W(s−k).
          =1/N(F^(s)W(0)+ F^(N−s)W(s−N+s))
          =1/N(F^(s)W(0)+F^-(s)W(2s))
        - G-(s)
          =1/N(F^-(s)W(0)+F^(s)W(2s))
      - estimate ^F(i)(s), ^F(i)(N−s) using (FT-EST)
        - F^(s) =
          N*(G(s)W(0)−G(s)W(2s))/(|W(0)|^2 −|W(2s)|^2)
      - update spectrum ^F(0)(k), compute error delta e
        compute error spectrum G(i)(k)
        and iterate until deltae is lower than threshold
        - use least square error
      - compute inverse FT of F^(i)(k)
- heel effect
  (in flat panels)
  - anode heel
    阳极倾斜
    - heel effect is an example of intensity distortion
- Image distortion
  (in image intensifier)
  - geometric distortion
    - modifies the geometry of the mapped object
    - core
      - definition of a parametric or non-parametric mapping between undistorted and distorted image
      - interpolation of intensities of neighboring pixels
      - a robust and reliable estimation of parameters of the mapping
        - calibration
          定标
      - efficient and robust algorithms
  - intensity distortion
    - changes in intensities
    - heel effect is an example of intensity distortion
- devices
  - flat panel detectors(more commen)
    - +
      - Simple assembly and readout
      - Higher contrast resolution (high dynamic range)
      - Not sensitive to magnetic fields (no magnetic distortion)
      - More robust with respect to under- and overexposure
      - Reduced space requirements
      - Optimization of the clinical workflow
      - Mechanically rugged
    - -
      - Relatively slow readout
      - Still an expensive technology
      - High rejection rate in production
      - Elimination of defects with digital image processing
    - artifacts
      - Large detectors composed of four detectors → butting cross
      - Offset in intensities
        - correct and gain offset
      - Inactive pixels(defect pixel)
        - defect interpolation
  - image intensifier(out of style)

MRI preprocessing

- IIH
  - If there are slow and nonanatomic intensity variations present in the image of one and the same tissue class
  - resons
    - non-uniform radio-frequency,
    - inhomogeneity of the static main field
    - patient motion
- mathematical models
  - classify
    - Low-frequency model
      - IIH is caused by low-frequency components
        - The IIH map can be recovered using low-pass filtering
    - Hypersurface model
      - IIH map is represented by a smooth (low-frequency) parametric function
        - recovered by least-square-fitting (regression)
    - Statistical model
      - IIH map is represented by a stochastic process
        - can be recovered by parametric or non-parametric statistical estimation
  - statistical model
    - gain field (multiplicatiove)
      - b = [bi,j]
        - gi,j = fi,j*bi,j +ni,j
          - gi,j denotes the observed intensity at grid point (i, j), and ni,j is additive Gaussian noise
      - correction
        - fi,j =gi,j/bi,j
    - bias field (log additive)
      - logb = [logbi,j]
        - loggi,j = log fi,j +logbi,j,
      - correction
        - log fi,j = loggi,j −logbi,j
- frequency domain filters
  - high-pass filter
    - Design a high-pass filter that eliminates the low frequency bias field.
  - Homomorphic Filtering
    - Subtract the low-pass filtered image and normalize the mean.
- spatial filter
  - homomorphic unsharp masking
    - Apply mean normalization
  - polynomial surface fitting 多表面拟合
    - Approximate the low frequency bias field in the additive model by a multivariate regression polynomial and eliminate the bias by subtraction
    - Fit a parametric, smooth surface to the logarithm of the intensity values.
    - Estimate the parameters by minimizing the sum of squared differences of the surface points and the logarithmized image intensities
      - or by minimized entropy
      - or maximized KL divergence
    - The resulting surface is then subtracted from the logarithmic image
  - entropy and KL divergence minimization
    - basic idea
      - Compute a bias field that minimizes the disorder in intensities
      - the bias field increases the entropy of the image,
      - the bias field decreases the Kullback-Leibler divergence between the probability density function of the image and the uniform density.
    - entropy
      - H(X) =−∑p(xi)logp(xi)
    - KL-divergence
      - between two discrete probability density functions p and q
      - KL(p,q) = ∑p(xi)*log(p(xi)/q(xi))
      - KL(p,q) =H(p,q)−H(p)
        - H(p,q) denotes cross entropy H(p,q)=−∑p(x)*log(q(x))
      - propertiys
        - KL(p,q) □= KL(q,p),
        - KL(p,q) ≥ 0
        - KL(p,q) = 0 ⇔ p = q
        - KL(p,q) →0 ⇒ p→q
  - use (regularized) fuzzy c-means clustering to compute an estimator of the bias field

2D reconstruction

Phantoms

- Numeric/simulated phantoms
  - originate from computer simulations
  - are known exactly
  - have only limited realism
  - eg
    - Shepp-Logan phantom — a series of additive ellipsoids
    - FORBILD phantoms — A series of 3-D phantoms that mimic anatomic details
    - XCAT phantoms
      - Based on the Visible Human Project
      - Analytic description using splines
      - Comes with motion models for heart and torso
- Real phantoms
  - are designed with desired properties
  - are manufactured at a high accuracy
  - may be difficult to use
  - may still have a limited manufacturing accuracy
  - eg
    - Catphan — mimics a water cylinder
    - ART phantom — mimics a human body
    - Calibration phantoms — encodes information that can be used for calibration

Truncation 切断

- happens when the imaged object extends the field of view
- can be modeled as a multiplication with a rectangular window function in spatial domain
- introduces artificial frequencies into the reconstruction
- causes a typical artifact at the end of the field of view
- solution
  - 1: Defect pixel extrapolation
  - 2: Heuristic extrapolation — In order to enforce a limited size of the object, a cosine-like weighting is added
  - 3: Water cylinder assumption — Assume that the imaged object consists of water
  - 4: Use of prior knowledge
  - 5: Use of a semi-transparent filter — • Locate filter boundary. • Amplify filtered signal to original amplitude. • Reduce noise in the amplified signal. → This yields perfect truncation correction.
  - 6: ATRACT filtering

short scan

- use parker weights
- reduce the redundancy

sinogram

- visualize all projections in one image
- contains all information to reconstruct one slice
- is also called "fanogram" in fan beam geometry
- popular method for visualization with narrow detectors

Parallel Beam Reconstruction

- basic theorem
  - Backprojection
    - projection process P = A*X
    - Back projection B = AT*P
    - formular
  - Hibert Tranform
    - (f ∗h)(t) = p.v.*积分f(t−τ)h(τ)dτ
    - h(τ) =1/πτ
    - H(ω) =−i sgn(ω).
  - central slice theorem
    - P(ω,θ) = F(ωcosθ, ωsinθ) = F(ωx, ωy)
    - The 1-D Fourier transform of a projection corresponds to a line through the origin in the 2D Fourier transform of the object
- Filtered Backprojection
  - steps
    - 1. Apply filter on the detector row q(s,θ) = p(s,θ) ∗h(s)
    - 2.Backproject q(s,θ) f(x,y) =积分(0--π) q(s,θ)|s=xcosθ+ysinθ dθ.
  - filter
    - Ramp filter — continue
      - h(s) =积分(-B--B) |ω|exp(2πiωs) dω =积分(∞)|ω|rect(w/2B)exp(2πiωs) dω
      - H(ω)=|ω|
    - Ram-Lak — discrete spacial
      - h(nτ)=
        - 1/4τ^2, n=0
        - 0, n even
        - -1/(π^2*(nτ)^2), n odd
      - 2B=1/τ
  - adaptation with window functions to improve signals by reduce high frequencies — H'(ω)= H(ω)*W(ω)
- Differentiation Hilbert Backprojection
  - 1.Compute first derivative of the detector row:
  - 2. Apply Hilbert transform:
  - 3.Backproject q2(s,θ):

Fan Beam Reconstruction

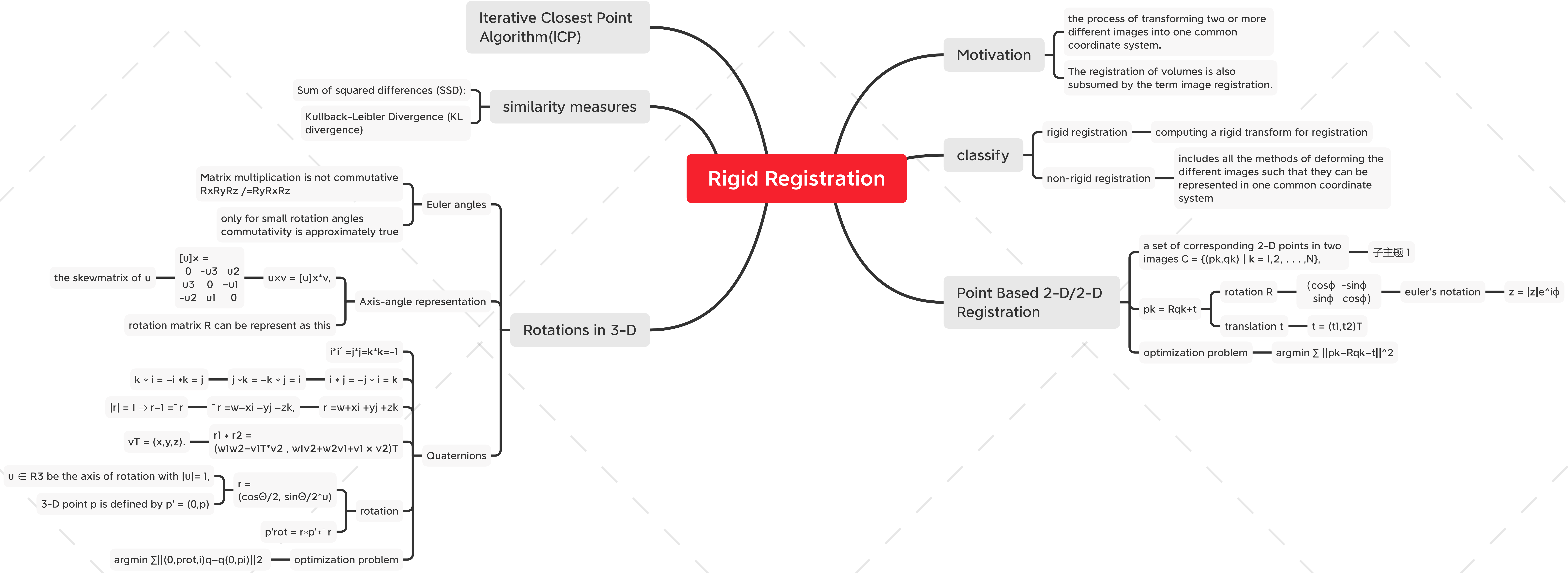
- no slice theorem anymore
- from parallel to fan a detected fan beam under the angle γ and source rotation β is equivalent to a parallel beam detected on a detector rotated by θ at a shifted position s.
  - θ = γ+β
    - γ fan展开脚的1/2
    - β fan的中心线相对于正对坐标轴旋转的角度
    - θ 等价的parallel对于正坐标旋转的角度
  - s =Dsinγ — s 等价的parallel平移的角度
  - rebinning
    - Dsi distance between x-ray source and isocenter
    - Ddi distance between isocenter and flat panel detector
    - tanγ = t/Dsi+Ddi — t half length of the flat panel
    - p(s,θ) = g(t,β)
- FBP for equiangular case
  - 1. perform cosine weighting for — g1(γ,β) = g(γ,β)cosγ.
  - 2.Apply fan beam filter — g2(γ',β) = (g1 ∗hfan)(γ',β) — hfan(γ) =D/2*(γ/sinγ)^2*h(γ)
  - 3.Backproject with distance weight
  - for equally-spaced case just change the h
- FBP for the Equiangular Case and Parker Weights
  - 1. Perform Parker weighting: — to do short scan
  - 2. Perform cosine weighting:
  - 3. Apply fan beam filter:
  - 4. Backproject with distance weight:

3D Reconstruction

- remark
  - · Cone beam geometry allows a much faster data acquisition.
  - · Approximate methods allow robust reconstruction.
  - · Exact reconstruction provides artifact-free reconstruction if the data is complete.
  - · Cone beam geometry suffers from physical effects (such as scatter) much more than fan beam geometries
- Exact vs. Approximate Reconstruction
  - · The higher the cone angle, the more exact methods benefit the reconstruction result.
  - · Helical CT scanners usually have rather small cone angles (due to image artifacts).
  - · Flat panel scanners usually have circular trajectories that allow only approximate reconstruction.
- 3D parallel beam or fan beam
  - 3D radon transform
    - not equal to the X-ray transform
    - do not have direct detectors for this transform
      - compute the value of such a "detector" as a line-integral on a 2-D parallel beam detector
    - central slice theorem for this transform
  - condition for data completenes
    - Orlov's condition
      - A complete data set can be obtained if every great circle intersects the trajectory.
  - Central Slice Theorem
    - for Line-integral data
      - The 2-D Fourier transform of a projection corresponds to a slice through the origin in the 3-D Fourier transform of the object
    - for plane integral data
      - This central slice theorem connects the 3-D Fourier transform of the object with the 1-D Fourier transform of the plane-integrals
- Cone Beam Reconstruction
  - geometry
    - Flat panel or multi-row detectors can detect considerably more data at a time.
    - Projection and backprojection can be described using projection matrices
    - do not have a central slice theorem
    - Data sufficiency conditions
      - Tuy's condition
        - Every plane that intersects the object of interest must contain a cone beam focal point
    - data redundancy
      - Redundant observations will occur on cutting planes that hit the helix more than once:
  - Feldkamp's Algorithm
    - designed for circular trajectories
    - fast and robust.
    - based on a fan beam reconstruction algorithm with appropriate cosine weights
    - based on a fan beam reconstruction algorithm with appropriate cosine weights
    - steps
      - 1. perform adjusted cosine weighting
      - 2.Apply fan beam filter
      - 3.Backproject with distance weight
    - speed can be improved if Horner's scheme is applied
  - Grangeat's Algorithm
    - converts the cone beam problem to a 3-D Radon inversion problem
    - can provide exact reconstructions, if Tuy's condition is met
    - convert cone beam ray sums to plane-integrals
    - · ... is not a filtered backprojection algorithm.
      · ... requires interpolation on non-uniformly sampled data.
      · ... needs to handle redundancy correctly → divide by the number of redundant observations.
      · ... is not commonly used for reconstruction.
      · ... is very useful to analyze reconstruction problems.
  - Katsevich's Algorithm
    - first developed for helical trajectories
      - later expanded to more general orbits
    - in the form FBP
    - involves filtering that can be made shift-invariant, i. e., independent of the reconstruction location
    - · Katsevich's algorithm performs an exact reconstruction.
      · The algorithms causes some additional computational overhead compared to other algorithms as the FDK algorithm.
      · Katsevich's algorithm does not use all available data.
      · Katsevich's algorithm deletes redundant data → (1,−1,1)-weighting.

Iterative Reconstruction

- **Kaczmarz Method**
  - Each pixel can be interpreted as a linear equation
  - This equation forms a line (2-D) or a hyperplane (higher dimensions) in the solution space
  - The point of intersection forms the correct solution
  - Projection onto the respective hyperplane forms a solution that fulfills the respective equation
  - Repetition yields an improved solution
- **Algebraic Reconstruction Technique (ART)**
  - steps
    - 1. For each pixel pi and each row Ai of A perform the Kaczmarz Method
    - 2. Repeat until convergence
  - drawback
    - Slow convergence
  - extensions
    - Simultaneous ART (SART)
      - multiple updates at the same time and combination of the result
        - 1. Estimate an initial solution of the system of linear equations. 2. Compute the orthogonal projections of the current estimate to all hyperplanes. 3. Compute the centroid of all projected points. 4. Use this centroid for the next iteration.
    - Simultaneous Iterative Reconstruction Technique (SIRT)
      - compute update once per iteration
        - All voxels of the volume are updated simultaneously using the same projection Pj in the following update rule
  - Ordered Subsets
    - clever technique to speed up convergence
    - employs knowledge on the acquisition sequence
    - selects subsets of equations (projections) that are most likely orthogonal to each other
    - For all subsets
      - Update using all items in the subset
- **Gradient Descent Algorithms**
  - Formulate the reconstruction problem as an optimization problem.
    - fast descent using conjugate gradients
    - regularization
  - Gradient descent algorithms are iterative methods for which the iteration update is dependent on the gradient of the objective function.
- **resolution and noise**
  - resolution
    - do not necessarily have a uniform image resolution
    - Resolution increases with the number of iterations
    - This makes image quality assessment difficult
  - noise
    - Image noise is dependent on the number of iterations
    - more iterations, the more noise is in the image
    - A common means to control noise is to adjust the number of iterations.
    - Bilateral filtering
      - a method for edge preserving noise reduction
      - Instead of simply averaging image values in dependence on their geometric closeness, also the photometric similarity of nearby pixels is considered
- **Regularization**
  - Introduction of additional information into the reconstruction process helps to enforce a certain solution.
  - especially advantageous if the problem is underdetermined
  - Additional weighting terms are also used to suppress noise or artifacts
  - Lp-Norms
    - |x|p =(∑xi^p)^1/p
      - L2 — 根号下平方和
      - L1 — 绝对值和
      - L0 — 绝对值的0次方和
      - L∞ — 绝对值的最大值
  - TV-norm
- **Maximum-Likelihood Expectation-Maximization Methods(ML-EM)**
  - Formulate the objective function as a likelihood function
  - Find the optimum of the likelihood function, i. e., the most likely solution
  - Commonly solved with the "Expectation Maximization Algorithm" (EM).



Rigid Registration
Iterative Closest Point Algorithm(ICP)
similarity measures
Sum of squared differences (SSD):
Kullback-Leibler Divergence (KL divergence)
Rotations in 3-D
Euler angles
Matrix multiplication is not commutative RxRyRz /=RyRxRz
only for small rotation angles commutativity is approximately true
Axis-angle representation
u×v = [u]x*v,
[u]× = 0 −u3 u2 u3 0 −u1 −u2 u1 0
the skewmatrix of u
rotation matrix R can be represent as this
Quaternions
i*i´ =j*j=k*k=-1
i ∗ j = −j ∗ i = k
j ∗k = −k ∗ j = i
k ∗ i = −i ∗k = j
r =w+xi +yj +zk
¯ r =w−xi −yj −zk,
|r| = 1 ⇒ r−1 =¯ r
r1 ∗ r2 = (w1w2−v1T*v2 , w1v2+w2v1+v1 × v2)T
vT = (x,y,z).
rotation
r = (cosΘ/2, sinΘ/2*u)
u ∈ R3 be the axis of rotation with |u|= 1,
3-D point p is defined by p' = (0,p)
p'rot = r∗p'∗¯ r
optimization problem
argmin ∑||(0,prot,i)q−q(0,pi)||2
Motivation
the process of transforming two or more different images into one common coordinate system.
The registration of volumes is also subsumed by the term image registration.
classify
rigid registration
computing a rigid transform for registration
non-rigid registration
includes all the methods of deforming the different images such that they can be represented in one common coordinate system
Point Based 2-D/2-D Registration
a set of corresponding 2-D points in two images C = {(pk,qk) | k = 1,2, . . . ,N},
子主题 1
pk = Rqk+t
rotation R
(cosϕ -sinϕ sinϕ cosϕ)
euler's notation
z = |z|e^iϕ
translation t
t = (t1,t2)T
optimization problem
argmin ∑ ||pk−Rqk−t||^2