Medical Image Processing for Diagnostic Applications

Cone Beam Reconstruction – Grangeat's Algorithm

Online Course – Unit 48

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Topics

Grangeat's Algorithm ...

- ... converts the cone beam problem to a 3-D Radon inversion problem.
- ... can provide exact reconstructions, if Tuy's condition is met.
- ... uses the idea to convert cone beam ray sums to plane-integrals.

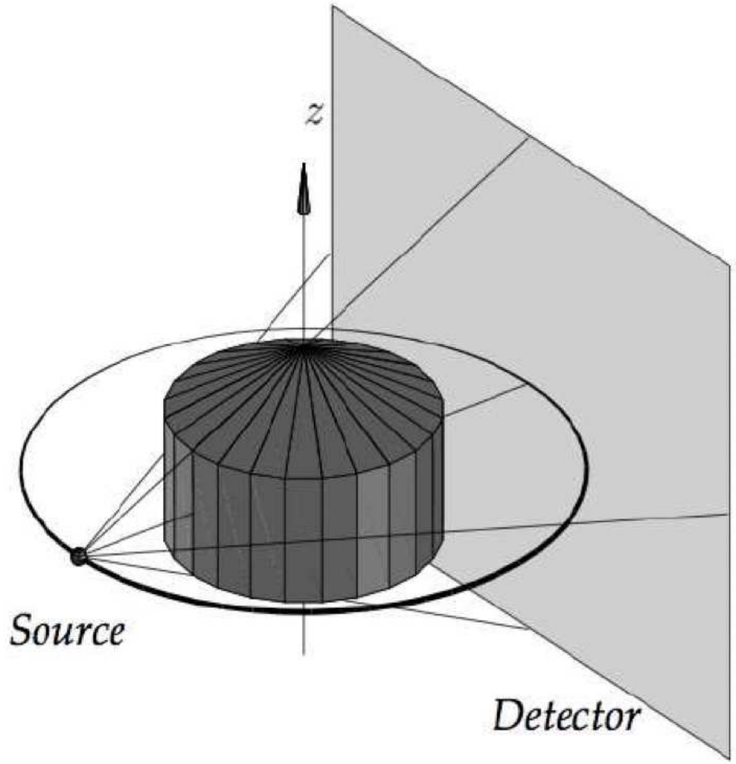

Figure 1: Cone beam scheme

Grangeat's Algorithm: Concept

- Line-integral on a cone beam detector is a weighted plane-integral.
- The line integral has to be weighted with $\frac{1}{r}$ to get the regular unweighted plane-integral.

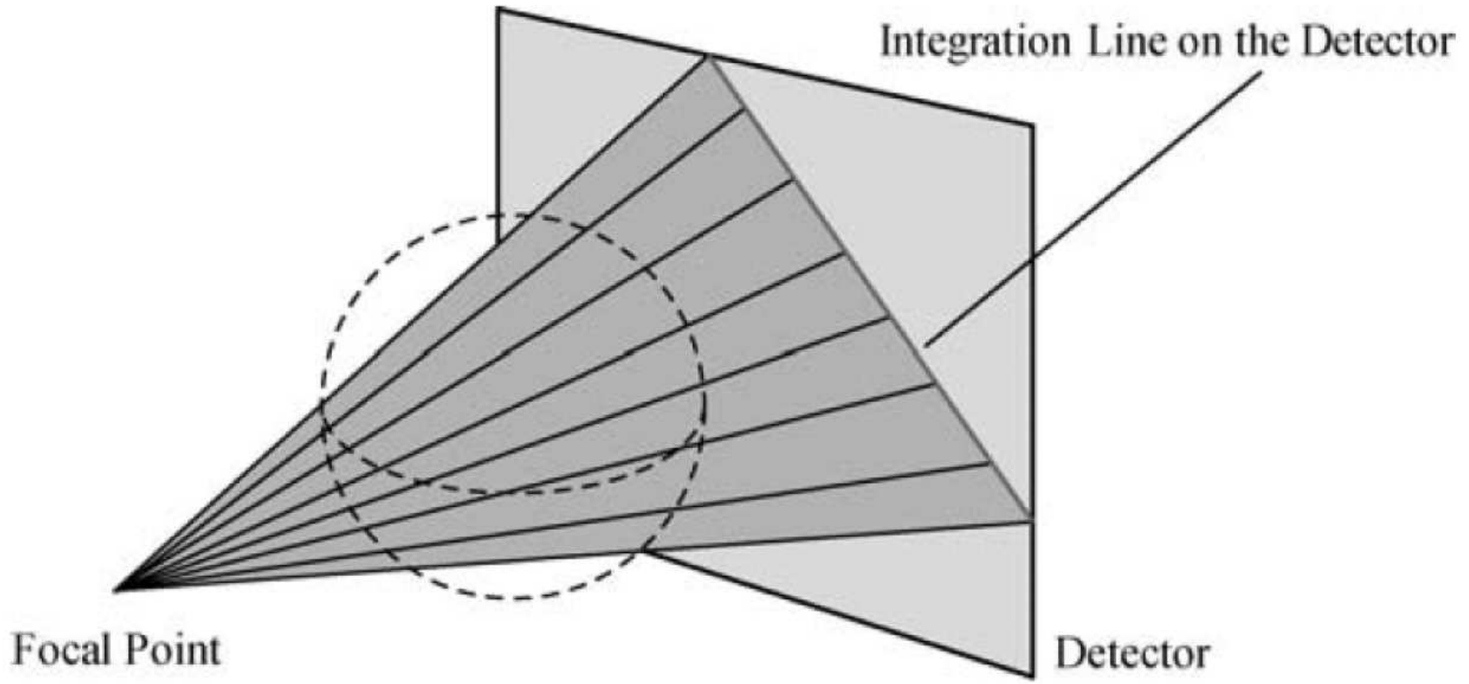

Figure 2: Weighting scheme (Zeng, 2009)

Grangeat's Algorithm: Concept

The derivative along the tangential direction dt is equal to the derivative of a $\frac{1}{r}$ weighted plane integral along dα:

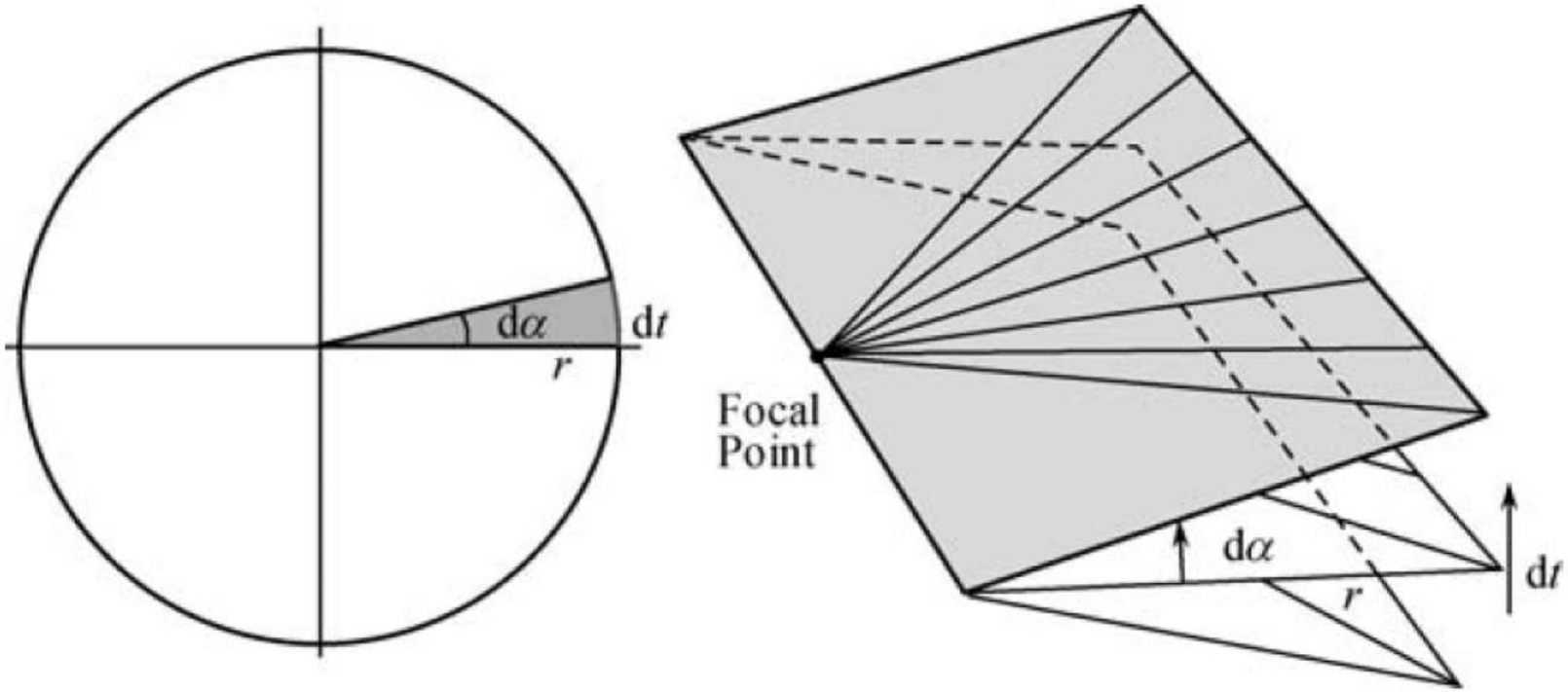

Figure 3: Tangential derivative (Zeng, 2009)

Grangeat's Algorithm

1. Form all possible line-integrals on each detector plane (all locations and orientations).
2. Compute the angular derivative.
3. Rebin the data to Radon space.
4. Take the derivative with respect to t.
5. Perform the 3-D Radon backprojection.

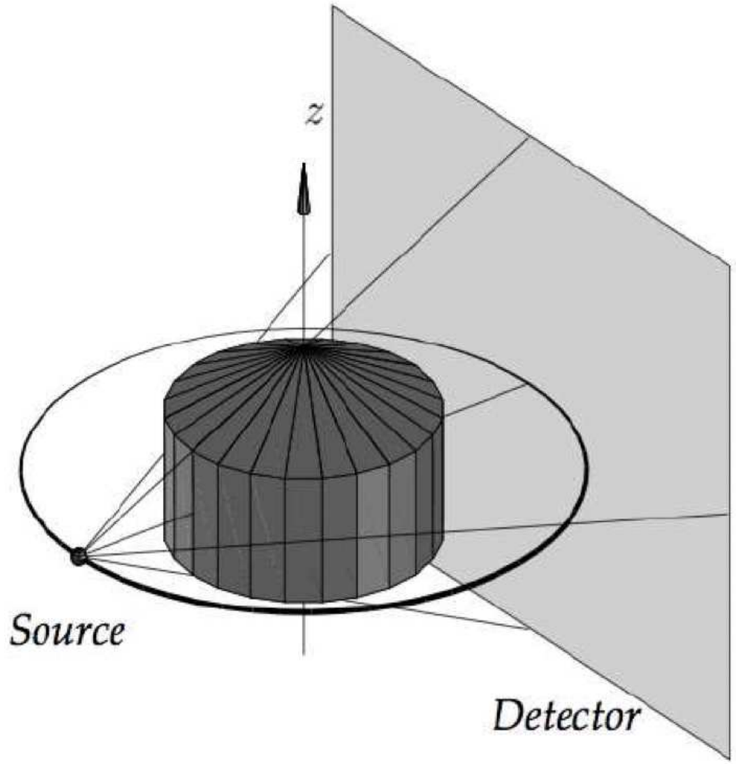

Figure 4: Cone beam scheme

Grangeat's Algorithm: Remarks

Grangeat's algorithm ...

- ... is not a filtered backprojection algorithm.
- ... requires interpolation on non-uniformly sampled data.
- ... needs to handle redundancy correctly $\rightarrow$ divide by the number of redundant observations.
- ... is not commonly used for reconstruction.
- ... is very useful to analyze reconstruction problems.

Topics

Grangeat's Algorithm

Summary

- Take Home Messages
- Further Readings

Take Home Messages

- Grangeat's algorithm is a 3-D cone beam reconstruction algorithm.
- It is useful for theoretical considerations, but rarely used in practice.

Further Readings

The original work of Grangeat can be found here:

Pierre Grangeat. “Mathematical Framework of Cone Beam 3D Reconstruction via the First Derivative of the Radon Transform”. In: *Mathematical Methods in Tomography*. Ed. by Gabor T. Herman, Alfred K. Louis, and Frank Natterer. Vol. 1497. Lecture Notes in Mathematics. Springer Berlin Heidelberg, 1991, pp. 66–97. DOI: 10.1007/BFb0084509

The best way to augment your knowledge of the shown concepts is to read the companion book of the current chapter:

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9