Medical Image Processing for Diagnostic Applications

Fan Beam – Short Scan

Online Course – Unit 39

Andreas Maier, Joachim Hornegger, Markus Kowarschik, Frank Schebesch
Pattern Recognition Lab (CS 5)

virtuelle hochschule bayern

FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG
TECHNISCHE FAKULTÄT

Topics

Short Scan

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Take Home Messages

Further Readings

Full Scan vs. Half Scan

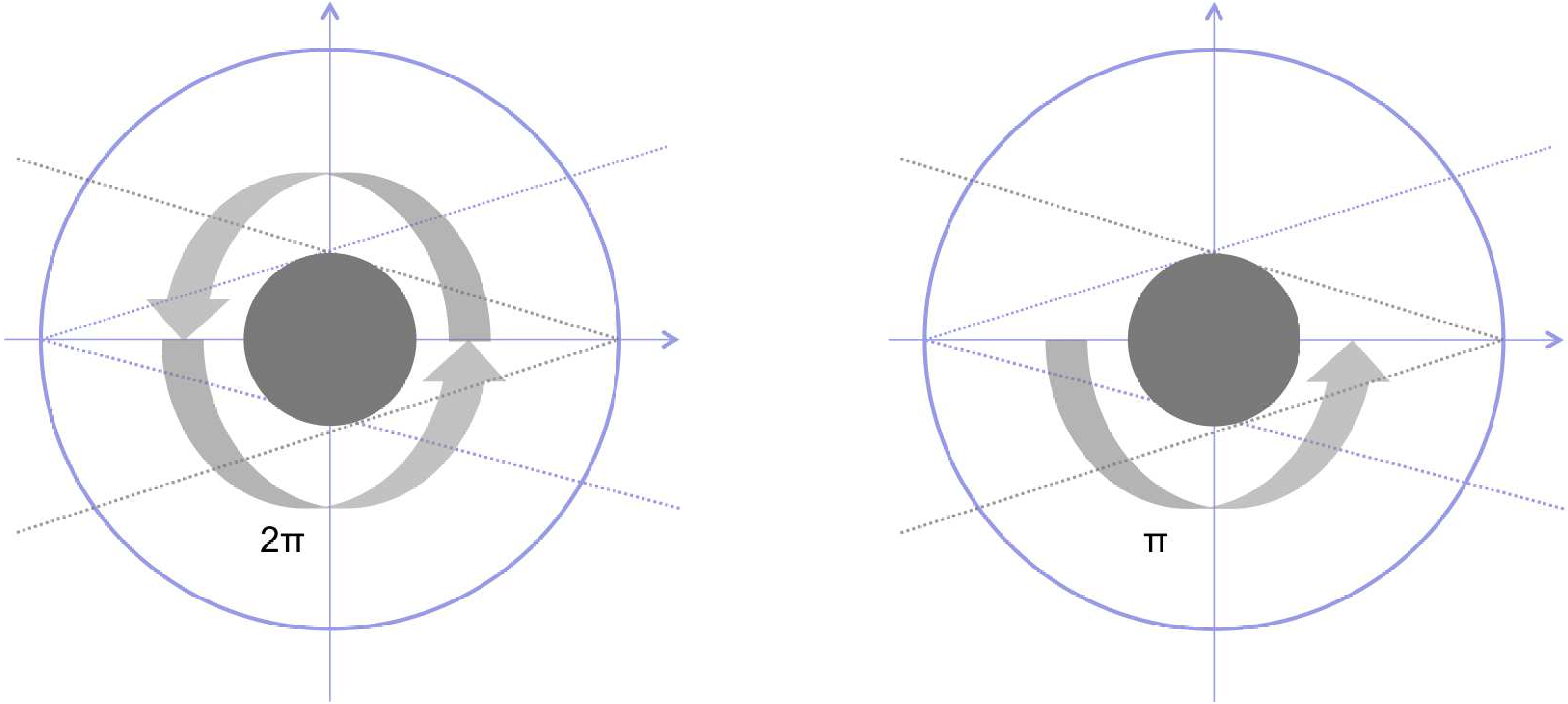


Figure 1: Full scan (left), half scan (right)

Redundant Areas: Sinogram

- Identical rays:

$$\gamma_1 = -\gamma_2,$$
$$\beta_2 = \beta_1 - 2\gamma_1 + \pi$$

- Upper triangle:

$$\pi + 2\gamma_1 \leq \beta_1 \leq \pi + 2\delta$$

- Lower triangle:

$$0 \leq \beta_2 \leq 2\gamma_2 + 2\delta$$

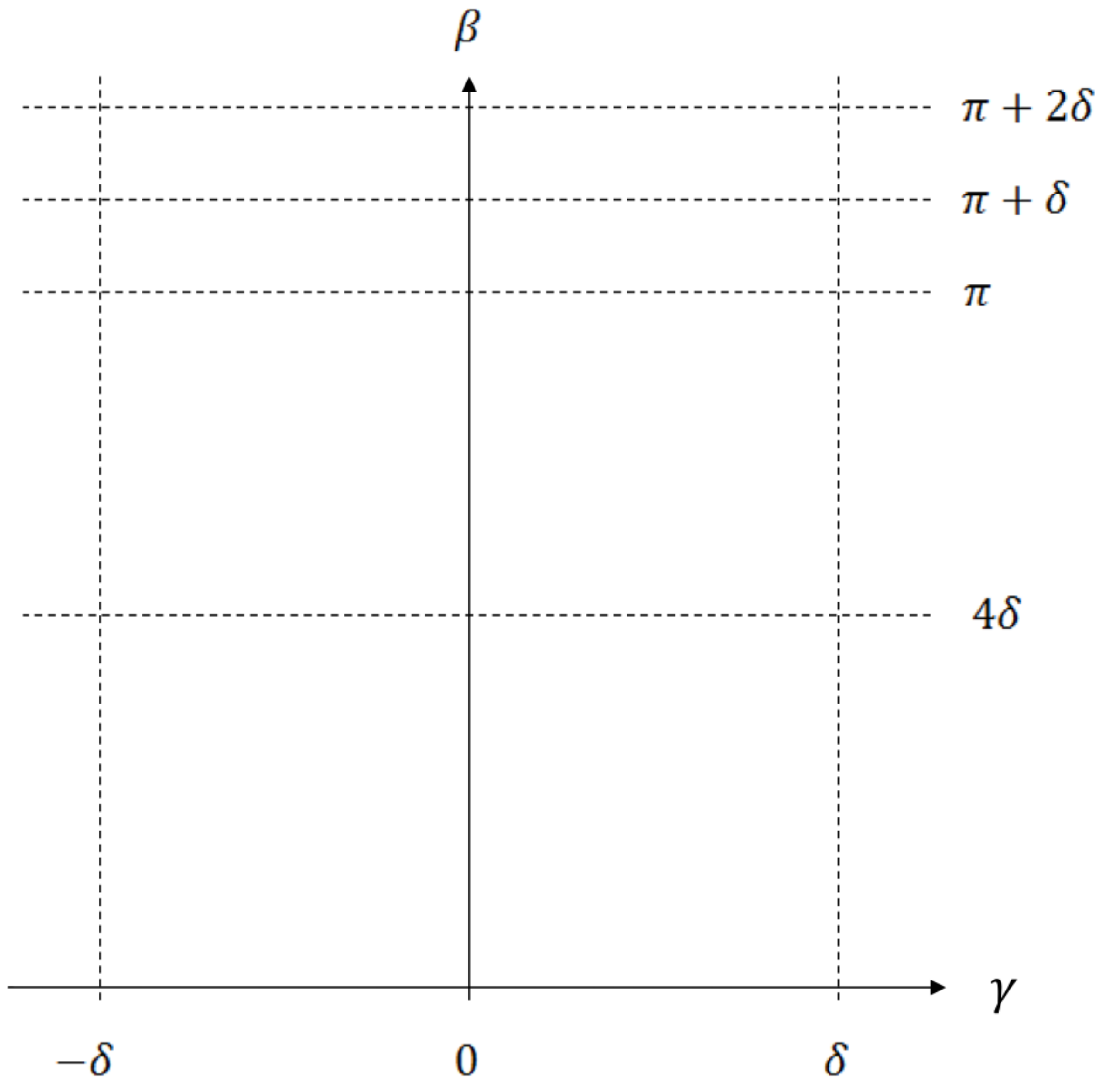


Figure 2: Sinogram range short scan

Parker Redundancy Weighting

Idea: Weight identical rays to reduce redundancy.

Constraints (upper triangle):

$$f_1(\pi + 2\delta) = 0, \quad (1')$$

$$f_1(\pi + 2\gamma) = 1 \quad (2')$$

Constraints (lower triangle):

$$f_2(0) = 0, \quad (1)$$

$$f_2(2\delta + 2\gamma) = 1 \quad (2)$$

Solve redundancy:

$$f_1(\beta_1) + f_2(\beta_2) = 1 \quad (3)$$

Parker Redundancy Weighting

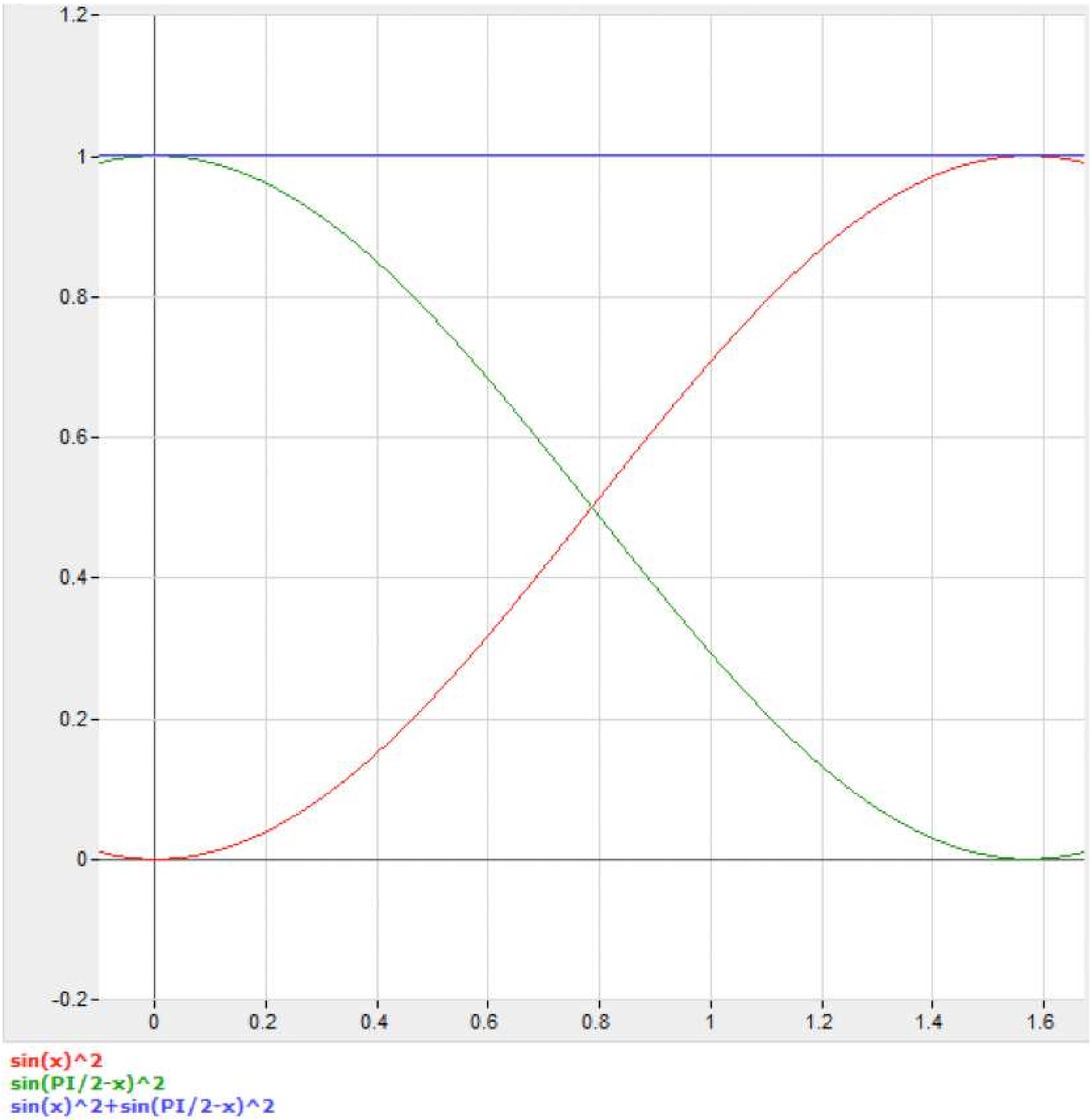


Figure 3: Plot of Parker weights

Parker Redundancy Weighting

Parker's trick:

$$\sin^2(\gamma) + \cos^2(\gamma) = 1,$$
$$\sin\left(\frac{\pi}{2} - \gamma\right) = \cos(\gamma)$$

New constraints:

$$f(x) = 0, \qquad (1 + 1')$$
$$f(x) = \frac{\pi}{2}, \qquad (2 + 2')$$
$$f_1(\beta_1) + f_2(\beta_2) = 1 \qquad (3)$$

Weighting functions:

$$f_1(\beta_1) = \frac{\pi}{2}\frac{\pi + 2\delta - \beta_1}{(\pi + 2\delta) - (\pi + 2\gamma)} = \frac{\pi}{4}\frac{\pi + 2\delta - \beta_1}{\delta - \gamma},$$
$$f_2(\beta_2) = \frac{\pi}{2}\frac{\beta_2}{2\delta + 2\gamma} = \frac{\pi}{4}\frac{\beta_2}{\delta + \gamma}$$

Polynomial Parker Weighting

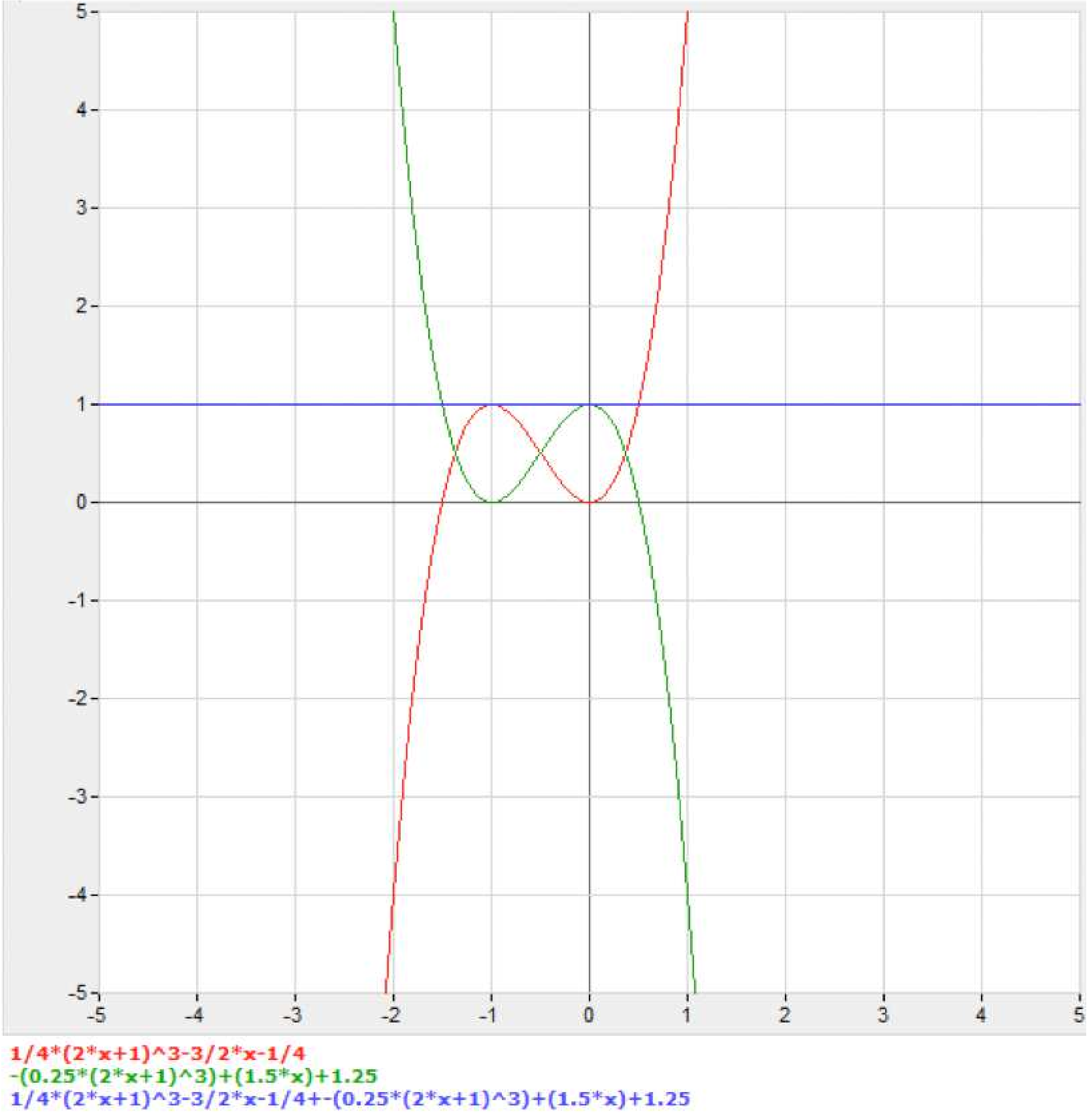


Figure 4: Plot of polynomial Parker weights

Parker Weighting: Example

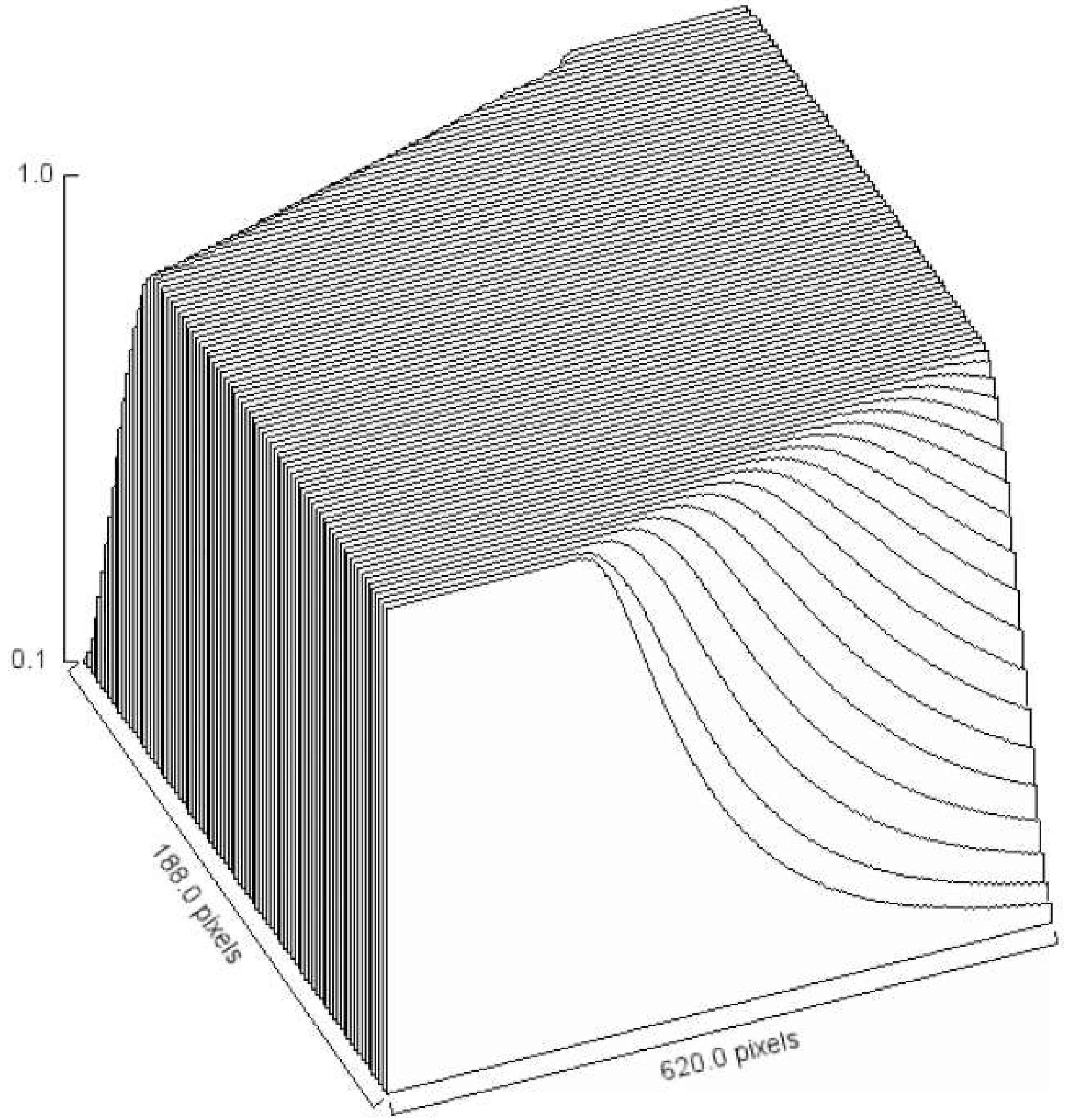
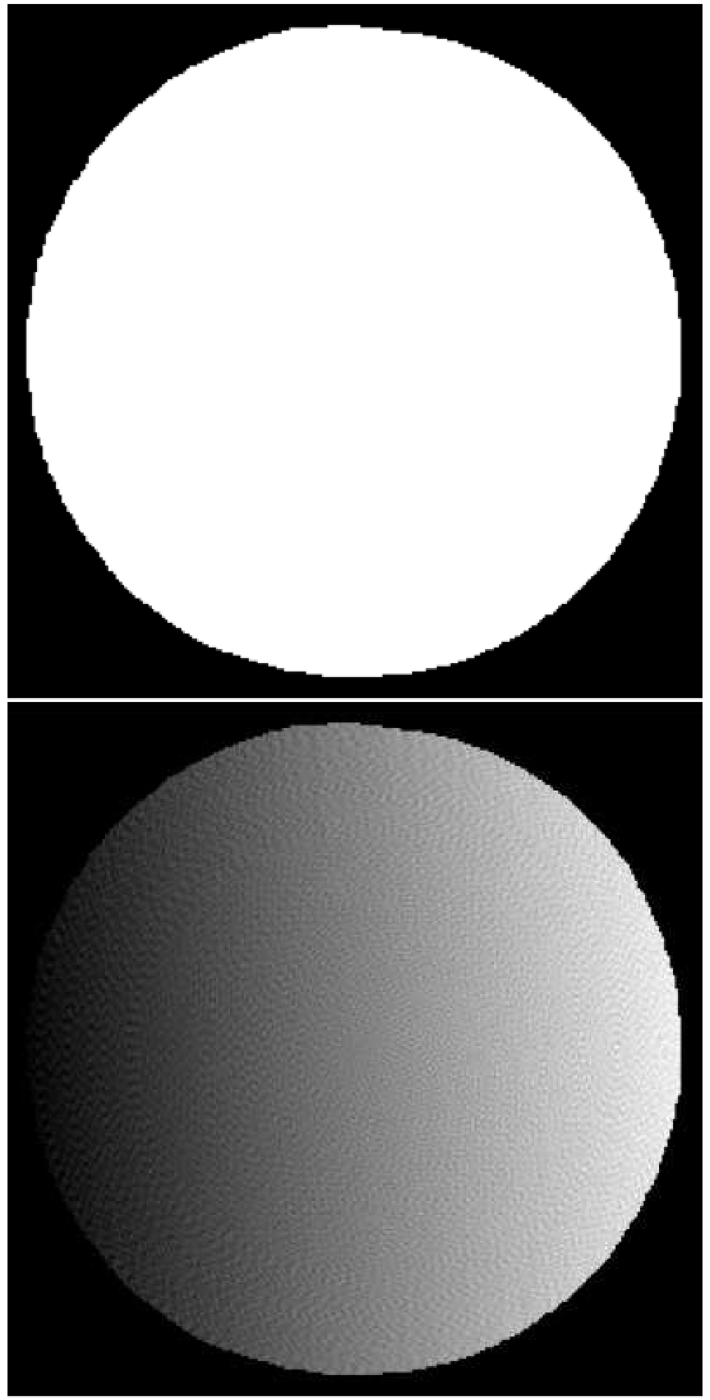


Figure 5: Parker weights for a short scan trajectory

FBP for the Equiangular Case and Parker Weights

1. Perform Parker weighting:
$$g_1(\gamma,\beta) = g(\gamma,\beta)\,w_{\text{Parker}}(\gamma,\beta).$$
2. Perform cosine weighting:
$$g_2(\gamma,\beta) = g_1(\gamma,\beta)\cos\gamma.$$
3. Apply fan beam filter:
$$g_3(\gamma',\beta) = (g_2 * h_{\text{fan}})(\gamma',\beta), \quad h_{\text{fan}}(\gamma) = \frac{D}{2}\left(\frac{\gamma}{\sin\gamma}\right)^2 h(\gamma).$$
4. Backproject with distance weight:
$$f(r,\varphi) = \int_0^{2\pi} \frac{1}{D'^2} g_3(\gamma',\beta)\,\mathrm{d}\beta.$$

No Redundancy Weights: Example



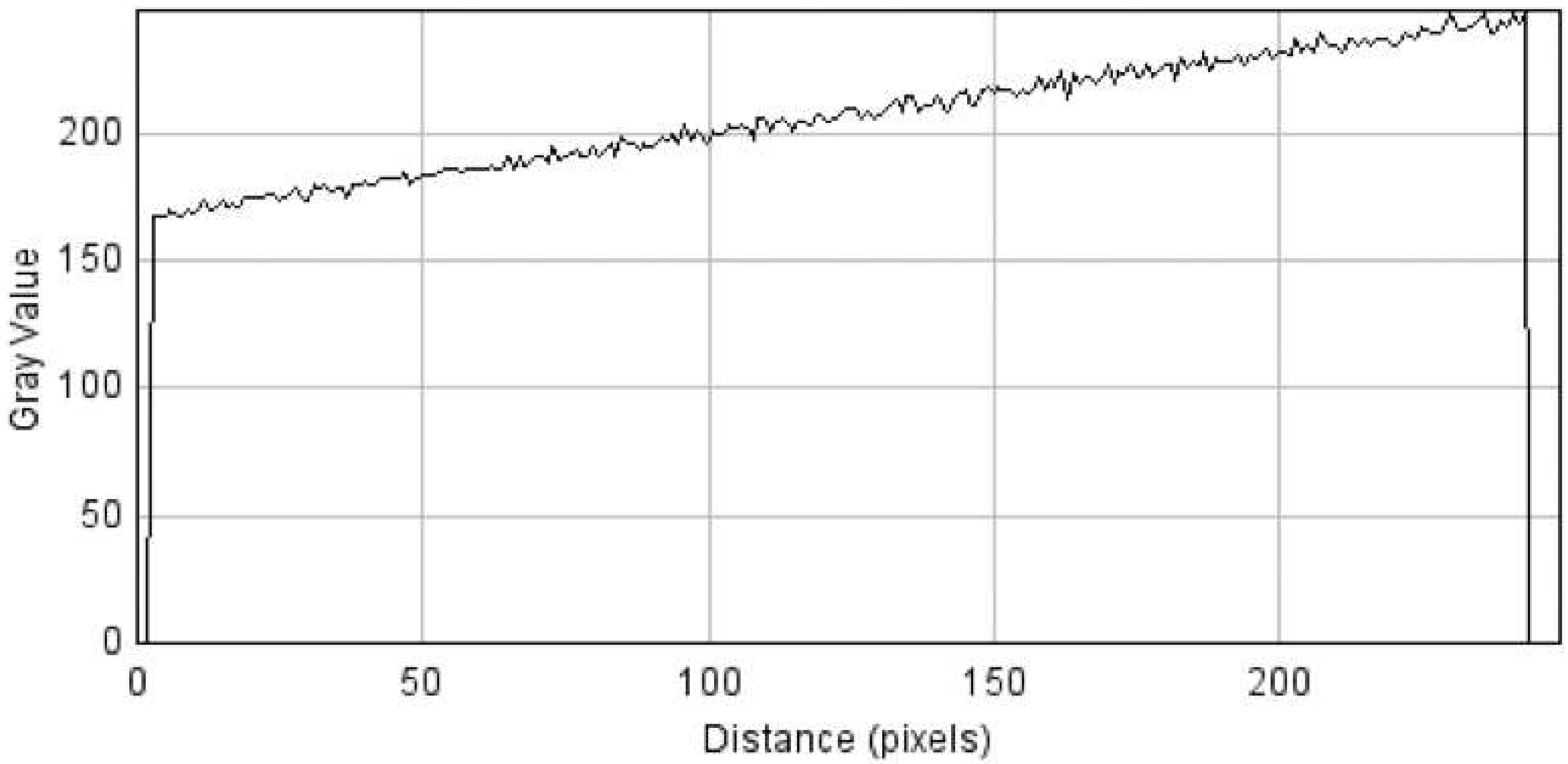


Figure 6: Gradient profile in the non-weighted image

Short Scan: Point Spread Function

- The point spread function is no longer uniform.
- Reconstruction resolution changes over the image.

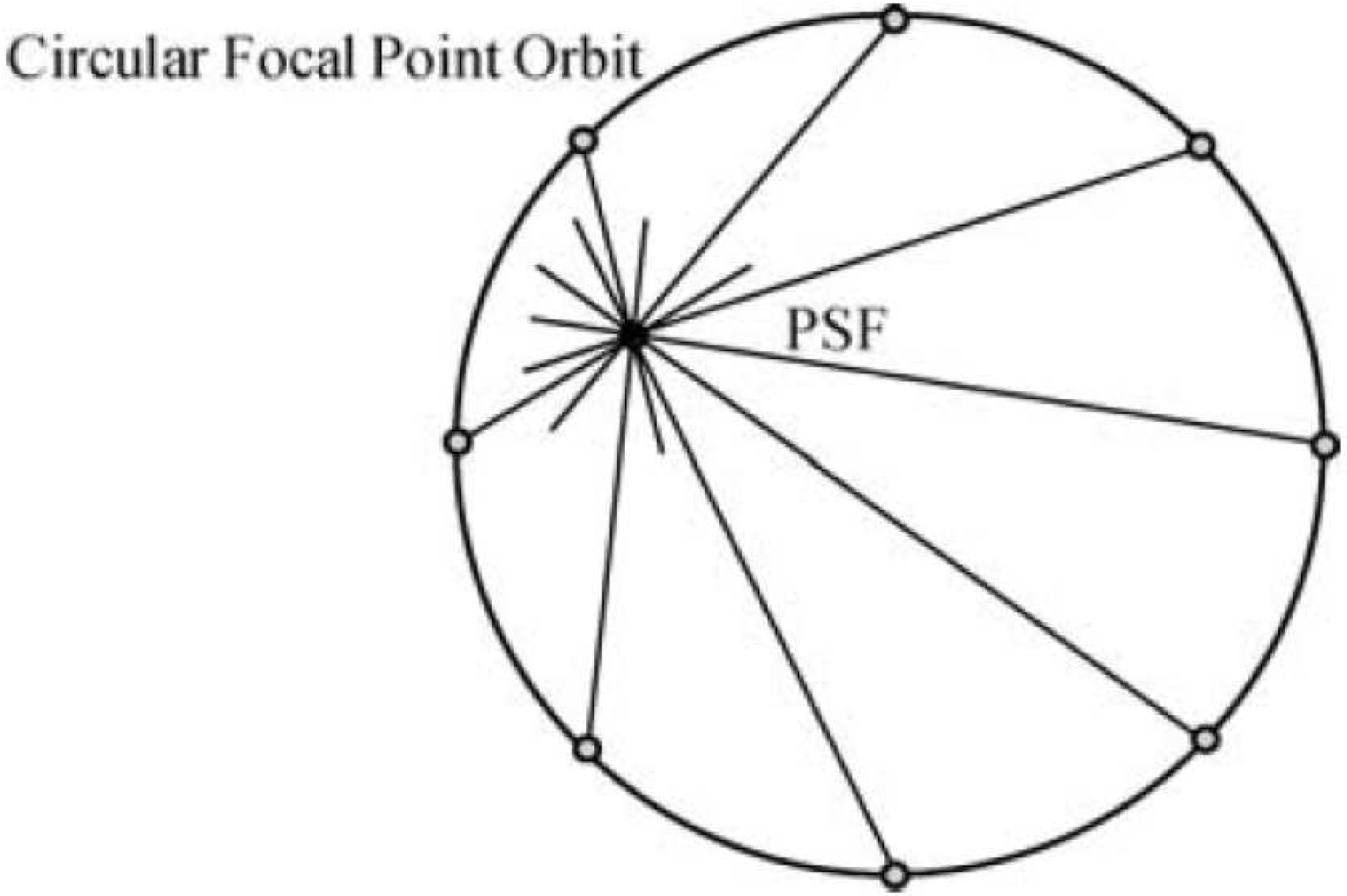


Figure 7: Scheme of the PSF (Zeng, 2009)

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Take Home Messages

- The short scan is important e. g., for C-arm systems.
- A complete fan beam dataset requires a rotation of 180° plus fan angle.
- Due to a data redundancy in the short scan we need to weight properly. The Parker weights allow a smooth weighting transition between redundant and singular data.

Further Readings

Helpful reads for the current unit:

Dennis L. Parker. “Optimal Short Scan Convolution Reconstruction for Fan Beam CT”. In: *Medical Physics* 9.2 (Mar. 1982), pp. 254–257. DOI: 10.1118/1.595078

Gengsheng Lawrence Zeng. *Medical Image Reconstruction – A Conceptual Tutorial*. Springer-Verlag Berlin Heidelberg, 2010. DOI: 10.1007/978-3-642-05368-9