

## Markov Random Fields

The goal of this last programming exercise is to denoise an image with a Markov Random Field (MRF), as seen in lecture 03f, “MRF Example, Math, and Gibbs Sampling”. In this example, the MRF models the pixels of the noisy input image as observations, and the pixels of the denoised output image as hidden variables. Each hidden variable has 4 neighbors (pixels on the left, right, top, and bottom). The clique potentials are  $e^{-E(x_i, y_i)}$  with energy term  $E(x_i, y_i) = -\eta x_i y_i$ , and  $e^{-E(y_i, y_j)}$  with energy term  $E(y_i, y_j) = -\beta y_i y_j$ . For more details, please refer to lecture 3f.

**Exercise 1** Reproduce the MRF example from the lecture on the raccoon image. Shift the binary intensities to  $\{-1, 1\}$ . Add noise to the image by randomly flipping pixels.

The denoised image shall be found via Gibbs sampling. To this end, visit each pixel once in random order and calculate the locally optimal update to approximate the objective function

$$y_1^*, \dots, y_N^* = \operatorname{argmax}_{y_1, \dots, y_N} \exp(-\sum E(x_i, y_i) - \sum E(y_i, y_j)) . \quad (1)$$

Figure 1 shows from left to right the binarized raccoon, the raccoon with 10% added noise, and the denoised raccoon using  $\eta = 2.0$  and  $\beta = 1.5$ .

Try out different amounts of noise. Note that by tendency, you will have to run the Gibbs sampler longer if the noise level is higher. At which percentage of noise pixels do you find that the restoration does not work well anymore? What is the influence on the parameters  $\eta$  and  $\beta$ ?

Please post a figure of one of your results on the forum and add a short text with your observations.

*Comments:*

*We ask for only one figure per group. Please also state your group number. Bring your code to the joint meeting on August 2 or August 3 for a potential little extra experiment.*



Figure 1: From left to right: binarized, noisy, and denoised raccoons (for  $\eta = 2.0$  and  $\beta = 1.5$ ).