Prediction intervals

$(1 - \alpha) \times 100\%$

STAT 464 / 864 | Fall 2024

Discrete Time Series Analysis

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Distribution of the innovations

1) The process {**Xt**} is **Gaussian** (identically distributed **normal** RVs)
   (True for ARMA models if their noise process {**Zt**} is Gaussian)

2) ACVF of {**Xt**} is known

For h-step ahead prediction, the error $X_{N+h} - P_N X_{N+h}$

Is normally distributed with mean = 0, and variance

Prediction Intervals

Derivation

$(1 - \alpha/2)$ - quantile of std. Normal distribution

Implications of knowing the ACVF

Suppose the ACVF $\gamma_X(h)$ is known → we know exactly:

- 🕑 covariance matrix Γ of **X := (X$_N$, ..., X$_1$)**
- 🕑 Vector γ of covariances **Cov(X$_{N+h}$, X)**
- 🕑 The prediction $P_N X_{N+h} = a_1 X_N + \cdots + a_N X_1$

  where vector **a** satisfies $\Gamma a = \gamma$
- 🕑 $MSE = \gamma_X(0) - a^T \gamma$

Remarks

- 🕑 In practice, assumptions 1 & 2 (Gaussian, known ACVF) aren't very accurate.

  Can do checks / transformations → assumptions are approx. correct

- 🕑 Assumption 2: we almost always need to estimate the ACVF.

  Get sample ACVF → input into innovations algorithm

  → get h-step ahead prediction, minimum MSE