

Puzzle Piece Galaxy — Example Star

Find the best LP of X_3 in terms of X_1 and X_2

$$P(X_3 | (X_2, X_1))$$

$$\Rightarrow \Gamma a = \gamma$$

$$\gamma_x(h) = \begin{cases} \sigma^2(1+\theta^2) & h=0 \\ \sigma^2\theta & h=\pm 1 \\ 0 & |h|>1 \end{cases}$$

$$\begin{aligned} & \hookrightarrow (\text{Cov}[X_3, X_2], \text{Cov}[X_3, X_1]) \\ & \hookrightarrow \text{Covariance matrix of } (X_2, X_1) = \begin{bmatrix} \text{Cov}[X_2, X_2] & \text{Cov}[X_2, X_1] \\ \text{Cov}[X_1, X_2] & \text{Cov}[X_1, X_1] \end{bmatrix} \end{aligned}$$

$\text{Var}[X_2]$

$$\Gamma = \begin{bmatrix} \sigma^2(1+\theta^2) & \sigma^2\theta \\ \sigma^2\theta & \sigma^2(1+\theta^2) \end{bmatrix}$$

$$\gamma_x(h) = \begin{cases} \sigma^2(1+\theta^2) & h=0 \\ \sigma^2\theta & |h|=1 \\ 0 & |h|>1 \end{cases}$$

\uparrow
 $\gamma_x(0)$

$$\gamma = (\gamma_x(1), \gamma_x(2)) = (\sigma^2\theta, 0)$$

$$\Gamma a = \gamma$$

\downarrow

$$(\text{Cov}[X_3, \underbrace{X_2}_{h=1}], \text{Cov}[X_3, X_1])$$

$$a = \left(\frac{\theta(1+\theta^2)}{1+\theta^2+\theta^4}, \frac{-\theta^2}{1+\theta^2+\theta^4} \right)^T$$

$$P(X_3 | (X_2, X_1)) = E[X_3] + a^T (X_2, X_1)$$

$$= \underset{\downarrow}{0} + \left(\frac{\theta(1+\theta^2)}{1+\theta^2+\theta^4} \right) X_2 - \left(\frac{\theta^2}{1+\theta^2+\theta^4} \right) X_1$$