Autoregressive Processes

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Discrete Time Series Analysis

Skyepaphora Griffith, Queen’s University

We learned something last time, in Time Series 🕛

Moving Average (MA) processes:

A little bit of noise from the present

A little bit of noise from the past

How far into the past? **p** steps → **Order p**

How much does each step contribute? k^{th} step → $\boldsymbol{\phi_k}$

MA processes are always zero mean

There's a formula for their ACVF

What did we tell quin?

Autoregressive Process of Order 1

An AR(1) process $\{X_t\}$ satisfies the autoregressive equation,

We assume a (stationary) solution $\{X_t\}$ exists

Autoregressive coefficient

Assume

Assume $Cov(Z_t, X_s) = 0$ for all $s < t$, and for all t.

AR(1) Processes | Expectation

(□) $X_t = \phi X_{t-1} + Z_t$

Take expectation of both sides of (□)

Stationarity → constant mean

Assume $\phi \neq 0$ (else $\{X_t\}$ is **wn**)

AR(1) Processes | Variance (□) $X_t = \phi X_{t-1} + Z_t$

Take variance of both sides of (□)

X_{t-1} and **Z_t** are uncorrelated

Final expression is time-independent

AR(1) Processes | Covariance

(□) $X_t = \underline{\phi X_{t-1}} + \underline{Z_t}$

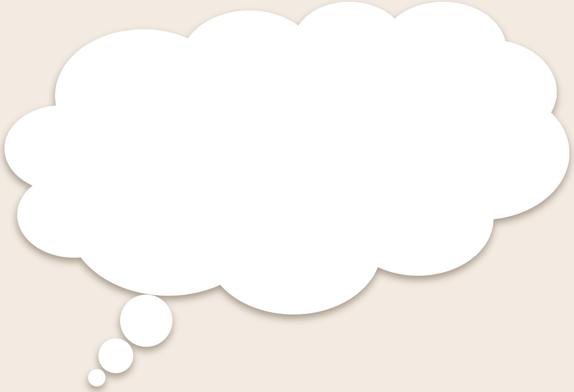
$$\begin{aligned}
\mathrm{Cov}(X_t, X_t + h) &= \mathrm{E}[X_t X_{t+h}] = \mathrm{E}[X_{t-h} X_t] \\
&= \mathrm{E}[X_{t-h} \underline{\phi X_{t-1}}] + \mathrm{E}[X_{t-h} \underline{Z_t}] \\
&= \phi \mathrm{E}[X_{t-h} X_{t-1}]
\end{aligned}$$

$$\boxed{\gamma_X(h) = \phi \gamma_X(h-1)}$$

AR(1) Processes | Covariance

(□) $X_t = \phi X_{t-1} + Z_t$

AR(1) Processes: ACVF / ACF (□) $X_t = \phi X_{t-1} + Z_t$



Autoregressive Process of Order p

Stationary $\{X_t\}$ satisfying:

AR polynomial:

(Complex valued z)

Assume roots exist **outside unit circle** in the complex plane. **$|z| > 1$**

AR(p): Expectation

(🌲) $X_t = \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p} + Z_t$

Stationarity → constant mean

Assume BWOC, μ is nonzero

Divide both sides by μ

z = 1 is a root of AR polynomial

But $|z| \leq 1$ not allowed! (unit circle) → μ must be zero

AR(p): Covariance

(🌲) $X_t = \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p} + Z_t$

Multiply both sides of (🌲) by X_{t-h}, where h > 0. Take expectation.

p + 1 linear equations, **p + 1** unknowns $\gamma_X(0), \ldots, \gamma_X(p)$
→ system of linear equations

For **h > p**, these equations form a recursion (easy to program)

We learned something today, in Time Series 🕛

What did we tell quin?

AR(1) Processes in R

ϕ can be estimated via linear regression:

Response: `y <- x_t[2:N]`

Predictor: `x <- x_t[1:(N-1)]`

Intercept: (None)

Model: `mod1 <- lm(y ~ x - 1)`

AR(p): Sample ACVFs in R

`Itsmr::aacvs()`
Computes ACVF for a given AR process… Or MA. (Or ARMA)