

# **Autocovariance Functions (ACVFs)**

## & Autocorrelation Functions (ACFs)

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Discrete Time Series Analysis  
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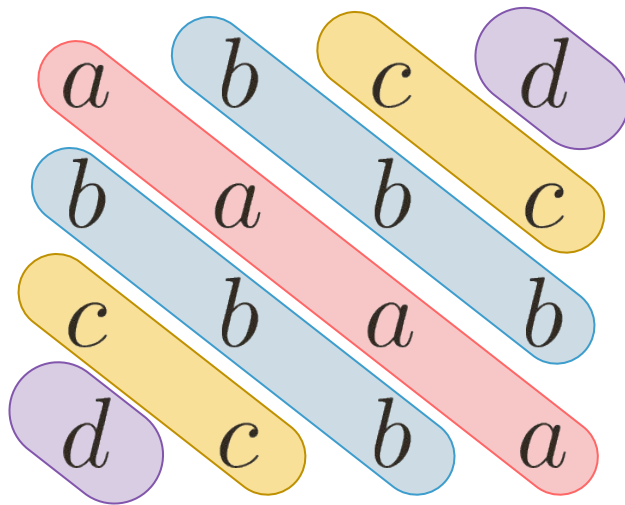
# Covariance Matrices of Stationary Processes

The covariance matrix  $\Gamma_N$   
of a **stationary** time series is **Toeplitz**

**Toeplitz:** Identical entries along diagonals

There are only **N** distinct values in  $\Gamma_N$

We can capture all info about  $\Gamma_N$   
in one **N**-length vector (or... function?)



# Autocovariance Functions | ACVFs

**Covariance Kernel:**  $\gamma_X(t, t+h) \stackrel{\text{def}}{=} \text{Cov}(X_t, X_{t+h}) \quad t \in T, h \in \mathbb{Z}$

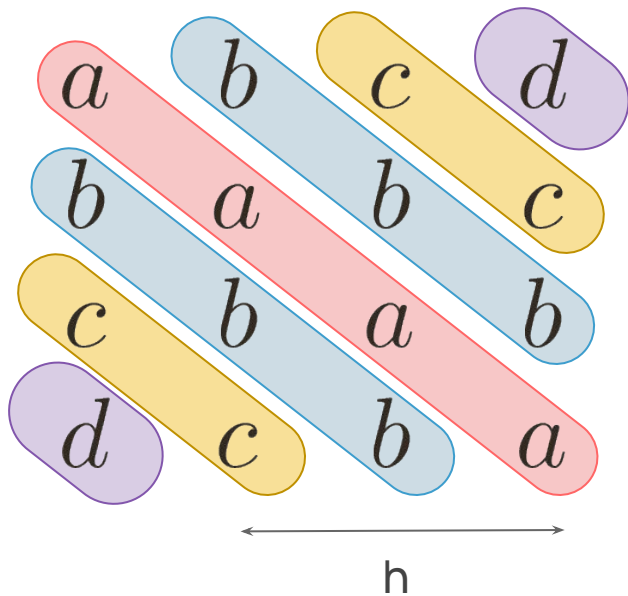
$$= \text{Cov}(X_s, X_{s+h}) \quad \forall s \in T$$

The time difference  $h$  is called “lag”

**Autocovariance Function:**

$$\gamma_X(h) \stackrel{\text{def}}{=} \text{Cov}(X_t, X_{t+h}) \quad t \in T, h \in \mathbb{Z}$$

$$\gamma_X(0, h)$$



# ACVFs and Stationarity

To show  $X_t$  is stationary:

- 1) Show that  $E[X_t]$  is time independent
- 2) Show that  $\text{Cov}(X_t, X_{t+h})$  is time independent

This will show up in your assignments and stuff. A lot.

If I ask you to find the ACVF of  $X_t$ , you can assume  $X_t$  is stationary

# Autocorrelation Functions (ACFs)

Denoted  $\rho_X(h)$ , or just  $\rho(h)$

ACVFs are easy to standardize!

$$\begin{aligned}\rho(h) = \rho(X_t, X_{t+h}) &= \frac{\text{Cov}(X_t, X_{t+h})}{\sqrt{\text{Var}(X_t)\text{Var}(X_{t+h})}} \\ &= \frac{\gamma(h)}{\sqrt{\gamma(0)\gamma(0)}} = \frac{\gamma(h)}{\gamma(0)}\end{aligned}$$

## Properties of ACVFs and ACFs

- 1)  $\gamma_X(0)$  is the variance  $\text{Var}(X_t) \quad \forall t \in T$  (independent of  $t$ )
- 2)  $\gamma_X$  is an even function. So is  $\rho_X$ , by extension.

$$\gamma_X(-h) = \text{Cov}(X_t, X_{t-h})$$

$$= \text{Cov}(X_{t-h}, X_t)$$

$$= \text{Cov}(X_t, X_{t+h}) = \gamma_X(h)$$



## Example: White Noise

### Definition:

A time series  $\{X_t\}$  is called **white noise** if

- 1) It's weakly stationary (assume mean = 0, WLOG)
- 2) All  $X_t$  are pairwise uncorrelated:  $\rho_X(h) = 0 \quad \forall h \neq 0$

If the mean is  $\mu$  and the variance is  $\sigma^2$ , we write  $X_t \sim \text{wn}(\mu, \sigma^2)$

i.i.d. Time Series  $\begin{matrix} \Rightarrow \\ \Leftarrow \end{matrix}$  White noise