

The Innovations Algorithm

Introduction

STAT 464 / 864 | Fall 2024

Discrete Time Series Analysis

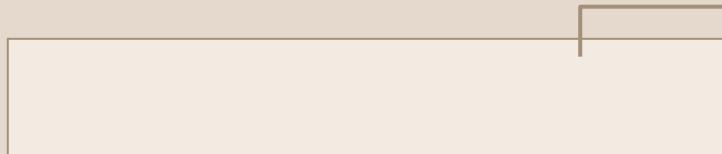
Skyepaphora Griffith, Queen's University

Innovations | RVs representing noise in a time series model

- 🕒 The Y_t in our classical decomp.
- 🕒 The Z_t in our AR(1) models
- 🕒 Prediction residuals, in the innovations algorithm's context
- 🕒 The j^{th} innovation is $(X_j - \hat{X}_j)$

Innovations Algorithm

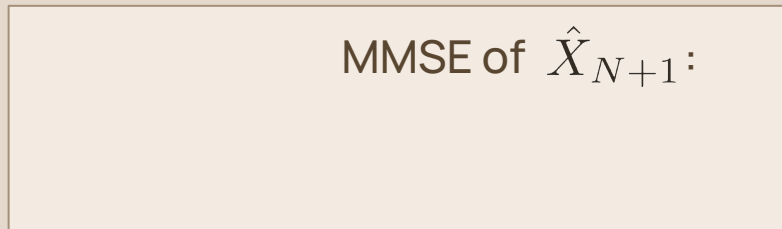
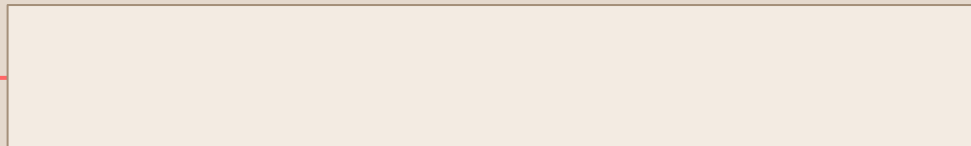
recursively computes:



\hat{X}_{N+1}

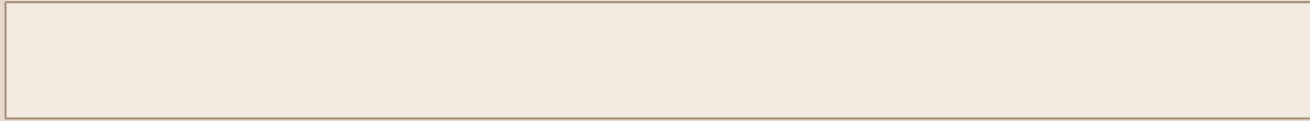
+ corresponding MMSEs $\{\nu_i\}_{i=0}^N$

MMSE of \hat{X}_{N+1} :



Innovations | RVs representing noise in a time series model

Algorithm expresses \hat{X}_{N+1} as a linear combo of innovations 1 to N



Note:

we can write \hat{X}_{N+1} in this form because \hat{X}_j , for $j \leq N$, is some linear combo of X_N, \dots, X_1

Innovations Algorithm

recursively computes:

$$P_0 X_1, P_1 X_2, \dots, P_N X_{N+1}$$

$$\hat{X}_{N+1}$$

+ corresponding MMSEs $\{\nu_i\}_{i=0}^N$

MMSE of \hat{X}_{N+1} :

$$\nu_N \stackrel{\text{def}}{=} E[(X_{N+1} - \hat{X}_{N+1})^2]$$

$$\nu_0, \hat{X}_1, \nu_1, \hat{X}_1, \nu_1, \dots, \nu_N, \hat{X}_{N+1}$$

THE ALGORITHM

$$\hat{X}_{N+1} = \theta_{N,1}(X_N - \hat{X}_N) + \cdots + \theta_{N,N}(X_1 - \hat{X}_1)$$

Step 0: Initialize ν_0



THE ALGORITHM

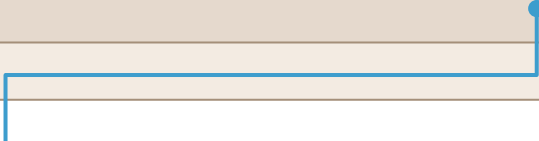
Step 0: Initialize $\nu_0 =$

Step i:

For i in $(1, \dots, N)$:

1)



$$\begin{bmatrix} \theta_{1,1} & & & \\ \theta_{2,1} & \theta_{2,2} & & \\ \theta_{3,1} & \theta_{3,2} & \theta_{3,3} & \\ \vdots & \ddots & \ddots & \ddots \\ \theta_{N,1} & \dots & \theta_{N,N-1} & \theta_{N,N} \end{bmatrix} \quad \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \vdots \\ \nu_N \end{bmatrix}$$


2)

Psst... What does this look like?

Remark 1 | Why do we initialize with $\text{Var}(X_1)$?

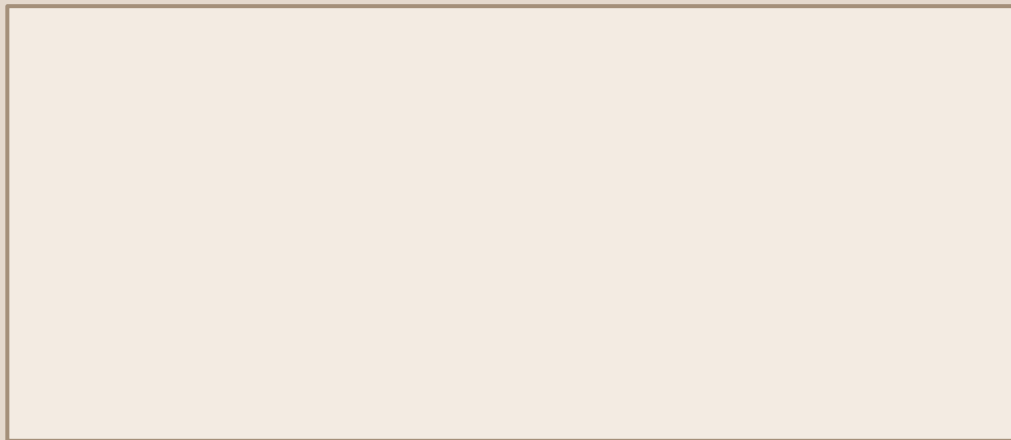
$$\nu_0 = \text{Var}(X_1) = \gamma_X(0)$$

In step 0, the best LP of X_1 based on no predictors – i.e.) $P_0 X_1$

is the constant a which minimizes

$$E[(X_1 - a)^2]$$

Differentiate w.r.t. a , set equal to 0



Remark 2 | Wait, we don't input the actual series, $\{X_t\}$?

Only input to the algorithm: ACVF at lags $\{h = 0, \dots, N-1\}$

Sample ACVF is unreliable at lags near N



Not good as input for this algorithm : (



Limited to prediction for time series where a **parametric model** is assumed





ACVF either **assumed known** at all lags,
or reliably estimated through **estimation of the model parameters**

Remark 3 | So the algorithm only works for stationary series?

We've been working in a **stationary** framework, this whole time.

There is a more general version, applicable to non-stationary series...

-  Required input: covariance function (2 variables: t_1 and t_2)
-  The text gives the version of this algorithm in section 2.5.4

This version is limited, however:

It's often infeasible to estimate $\gamma_X(t_1, t_2)$ without strict assumptions.