Forecasting - Part I

h-Step ahead prediction for **AR** processes

STAT 464 / 864 | Fall 2024

Discrete Time Series Analysis

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Properties of P(· | W) | Review from Week 8 Video Lecture

$$\boxed{P(Y|W) = \mu_Y + a^T(W - \mu_W)}$$

P1) Linearity $P(c_0 + c_1U + c_2V|W) = c_0 + c_1P(U|W) + c_2P(V|W)$

P2) If Y is uncorrelated with the predictors {**W**i}, $P(Y|W) = \mathrm{E}[Y]$

P3) Predictors uncorr. with residuals $\mathrm{Cov}\big(W_i, Y - P(Y|W)\big) = 0,$

P4) Best LP of any predictor Wi is itself $P(W_i|W) = W_i$

Forecasting

$$P_N X_{N+h} = P(X_{N+h} | (X_N, \ldots, X_1)^T)$$

Correlation matrix of **W**

1-step ahead prediction | AR(1)

$$X_t = \phi X_{t-1} + Z_t$$

$$Z_t \sim wn(0, \sigma^2)$$

$W =$

$Y =$

$\mu_Y =$

$\rho_X(h) =$

$$\begin{bmatrix} \phi^0 & \phi^1 & \phi^2 & \cdots & \phi^{N-1} \\ \phi^1 & & & \cdots & \vdots \\ \phi^2 & & & \ddots & \phi^2 \\ \vdots & \vdots & \ddots & \ddots & \phi^1 \\ \phi^{N-1} & \cdots & \phi^2 & \phi^1 & \phi^0 \end{bmatrix} \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}$$

ϕ times the first column of **R** is ρ → **Solution:**

1-step ahead prediction | AR(1)

$$X_t = \phi X_{t-1} + Z_t$$

$$Z_t \sim wn(0, \sigma^2)$$

$$a = (\phi, 0, \dots, 0)^T$$

$$|\phi| < 1$$

$$P(X_{N+1} | X_N, \dots, X_1) =$$

Another way to derive:

$$P_N X_{N+1} =$$

$$=$$

$$=$$

1-step ahead prediction | AR(1)

$$
\begin{aligned}
X_{N+h} &= \phi X_{N+h-1} + Z_{N+h} \\[1em]
P_N X_{N+h} &= \phi P_N X_{N+h-1} + P_N Z_{N+h} \\
&= \phi P_N X_{N+h-1} \\
&= \phi(\phi P_N X_{N+h-2}) \\
&= \qquad \vdots \\
&= \phi^h P_N X_{N+h-h} = \phi^h X_N
\end{aligned}
$$

$$X_t = \phi X_{t-1} + Z_t$$

$$Z_t \sim wn(0, \sigma^2)$$

$$a = (\phi, 0, \ldots, 0)^T$$

$$|\phi| < 1$$

$$a^T W = \phi X_N$$

1-step ahead prediction | AR(p)

$$X_{N+1} = Z_{N+1} + \sum_{i=1}^{p} \phi_i X_{N+1-i}$$

AR(p) process

$$1 - \phi_1 z - \phi_2 z^2 - \cdots - \phi_p z^p$$

$$P_N X_{N+1} =$$

$$=$$

h-step ahead prediction | AR(p)

$$P_N X_{N+h} =$$

MMSE | AR(1) – 1-step ahead

$$MMSE =$$

$\gamma_X(0)$

$\gamma_X(1)$

MMSE | AR(1) – h-step ahead

$$MMSE = \mathrm{Var}(Y) - a^T\gamma$$

$$\mathrm{Var}(Y) =$$

$$a =$$

$$a^T\gamma =$$

$$MMSE =$$

MMSE | AR(p) – 1-step ahead

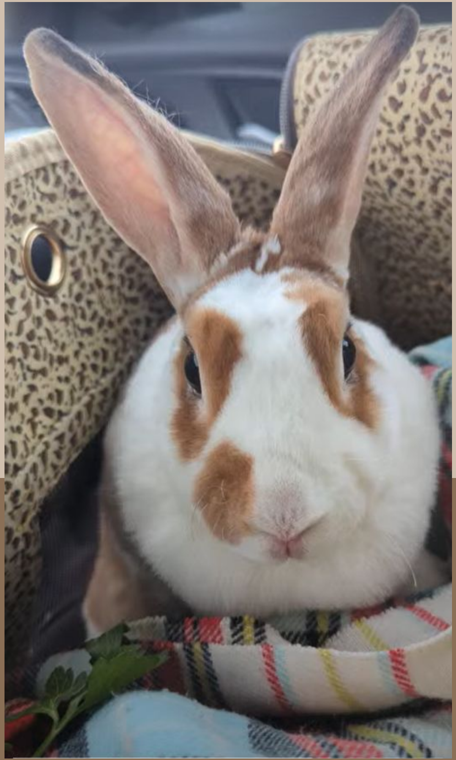
$$MMSE = \mathrm{Var}(Y) - a^T\gamma$$

$$X_{N+1} = \boxed{Z_{N+1}} + \sum_{i=1}^{p} \phi_i X_{N+1-i}$$

$$X_{N+1} - P_N X_{N+1} =$$

$$MMSE = \mathrm{E}\big[(X_{N+1} - P_N X_{N+1})^2\big] =$$

We learned something today, in Time Series 🕛



What do we tell quin?