Forecasting - Part II b

h-Step ahead prediction for MA(q) processes

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Discrete Time Series Analysis

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We learned something last time, in Time Series 🕛

Predicting MA(1) using infinitely many past values

so it gets messy if we avoid approximation

approximation should be sufficient

What do we tell quin?

Invertibility

$$X_{N+1} = Z_{N+1} + \theta Z_N$$

$$\tilde{P}_N X_{N+1} = \tilde{P}_N Z_{N+1} + \theta \tilde{P}_N Z_N.$$

Z_N is uncorrelated with all predictors **X_N, X_{N-1}, . . .** → $\tilde{P}_N Z_{N+1} = 0$

Z_N is not uncorrelated with **X_N** → we can't say $\tilde{P}_N Z_N = 0$

Approach: treat **X** as an ARMA(1,1) process, solve for **Z** in terms of **X**

To do this, we must invert the operator ***θ*(B)**

Definition: An ARMA(1,1) process with MA coefficient θ is <u>invertible</u> if $|\theta| < 1$

Suppose {Xt} is an invertible ARMA(1,1) process:

$$\phi(B)X_t = \theta(B)Z_t$$

$$\theta(B) = (1 + \theta B)$$

$$|\theta| < 1$$

$$\phi(B) =$$

$$\theta(B)^{-1} =$$

$$Z_t =$$

$$=$$

$$=$$

$$= \Big(B^0$$

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$$=$$

Suppose {Xt} is an invertible ARMA(1,1) process:

$$\phi(B)X_t = \theta(B)Z_t$$

$$\theta(B) = (1 + \theta B)$$

$$|\theta| < 1$$

$$Z_{N+1} = X_{N+1} - (\theta + \phi)\sum_{i=1}^{\infty}(-\theta)^{j-1}X_{N+1-i}$$

$$\tilde{P}_N Z_{N+1} =$$

$$0 =$$

Filter

$$a_j =$$

$$\tilde{P}_N X_{N+1} =$$

Algorithms...

Solving these equations exactly for $a_1, \ldots, a_N$ is curiously difficult

1) Solve numerically
2) Approximation

Plot twist: there is a 3rd option
(Actually it's a modification of the 1st)

Text discusses 2 **algorithms** for exact numerical estimates, using sample ACVFs.

- 🕑 Durbin-Levinson
- 🕑 Innovations

  We'll be using this one, since it's more popular nowadays,
  and it's implemented by the `forecast()` function from Workshop 4.

MMSE: ARMA(1,1)

$$\mathrm{MSE} = \mathrm{E}\left[\left(Y - P(Y|W)\right)^2\right]$$

$$MSE = \mathrm{E}\big[(X_{N+1} - P_N X_{N+1})^2\big] =$$

$$=$$

$$\approx$$

This term is small,
and decreases as N gets larger