Moving Average Processes

STAT 464 / 864 | Fall 2024
Discrete Time Series Analysis
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We learned something Today, in Time Series 🕛

Linear Filters!
Passes certain qualities of the data and eliminates others
Imagine listening to someone behind a closed window:
The window filters out high-frequency noises like consonants, and passes low-frequency info like vowel sounds.

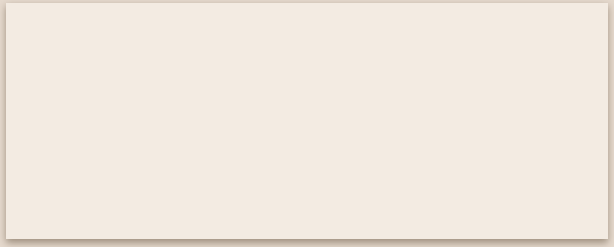
Causality:
The past is far behind us, the future doesn't exist

A Proposition:
Put in a stationary series → get out a stationary series!
Know the input ACVF → know the output ACVF!

What do we tell quin?

Moving Average Models | Order 1

Definition: A time series $\{X_t\}$ is a Moving Average process of Order 1, or an **MA(1) Process**, if:

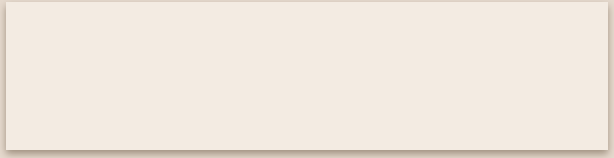


For some parameter $\theta \neq 0$

Causal? 🕑 **Yes**

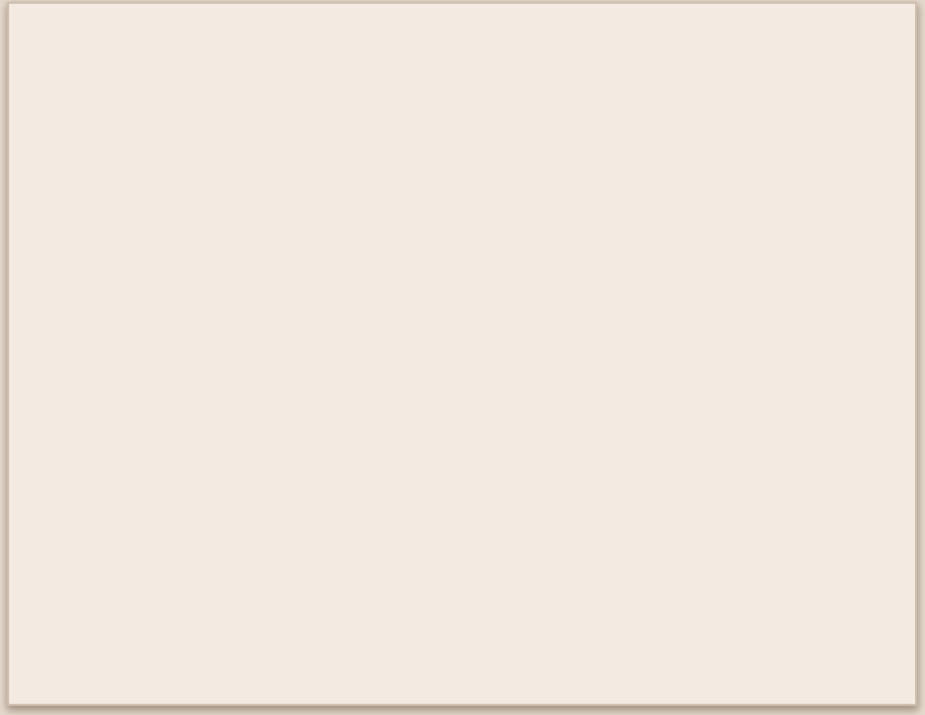
Linear Process? **Yes**

Filter?

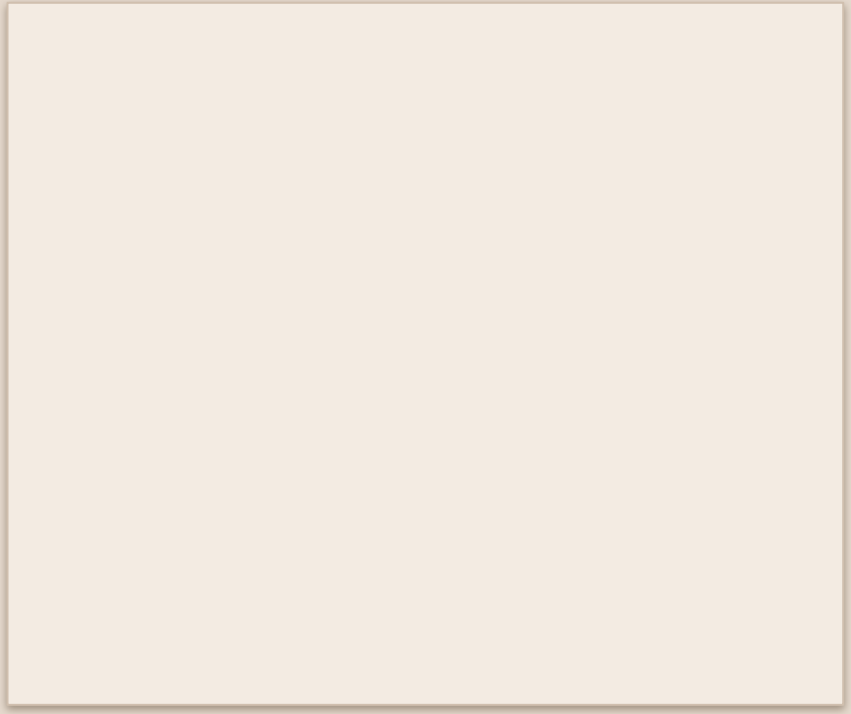


MA(1) Processes | Expectation

$\{Z_t\}$ is white noise →



MA(1) Processes | Variance σ_X^2



Uncorrelated $\{Z_t\}$

MA(1) Processes | Autocovariance

Moving Average Models | Order q > 1

MA(q) Process:

As before, the expectation is zero.

MA(q) Processes | Variance

MA(q) Processes | Covariance

MA(q) Processes | Correlation

$$\gamma(h) = \begin{cases} \sigma^2(1 + \theta_1^2 + \cdots + \theta_q^2) & h = 0 \\ \sigma^2(\theta_1 + \theta_1\theta_{1+h} \cdots + \theta_{q-1}\theta_{q-1+h}) & |h| = 1, 2, \ldots, q \\ 0 & |h| > q \end{cases}$$

We learned something last time, in Time Series 🕛

What did we tell quin??