

Estimating Seasonal Components

Harmonic Regression
& the S1 Method

STAT 464 / 864 | Fall 2024

Discrete Time Series Analysis

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We learned something Fri/Mon, in Time Series



Moving average (MA) smoothing,
average of points around time of interest
gets rid of high frequencies (low pass filter)

Some basic intro quarto and r stuff
Made our first document
Quarto interprets *code as code* and *text as text*!

Exponential Smoothing
Only uses past values (can forecast)

What did we tell quin?

Harmonic Regression

$$X_t = \cancel{m_t} + s_t + Y_t \quad (\star)$$

1

Assume s_t has the form
$$s_t = \sum_{k=1}^M a_k \cos\left(2\pi \frac{1}{d_k} t + \phi_k\right)$$

For the k^{th} seasonal component,

a_k Amplitude

d_k Period $\frac{1}{d_k} = \text{Frequency}$

ϕ_k Phase

(from textbook)

$$s_t = a_0 + \sum_{j=1}^k (a_j \cos(\lambda_j t) + b_j \sin(\lambda_j t)),$$

a_j and b_j are unknown parameters

λ_j = fixed frequencies of the form $2k\pi/d$ for integer k

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The S1 Method

(See textbook: Brockwell & Davis)

🕒 This method doesn't assume any particular behaviour within a cycle (unlike HR)

🕒 Step 1 is based on the fact:

(True for average) $\frac{1}{d} (s_t + s_{t+1} + \dots + s_{t+d-1})$ is the same for every t



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The S1 Method: Step 1

Apply MA smoother with window length = d

d odd \rightarrow set order $q = (d-1)/2$

d even

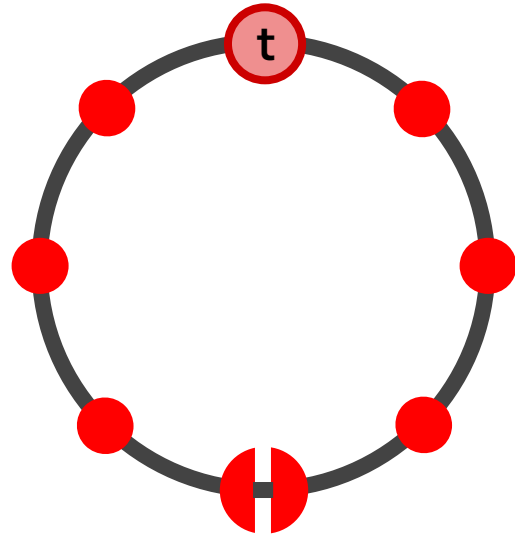
\rightarrow set $q = d/2$

\rightarrow set window length = $d+1$

\rightarrow modify MA smoother:

first/last value get weighted by $1/2$

$$s_{t-q} = s_{t+q}$$



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The S1 Method: Step 1

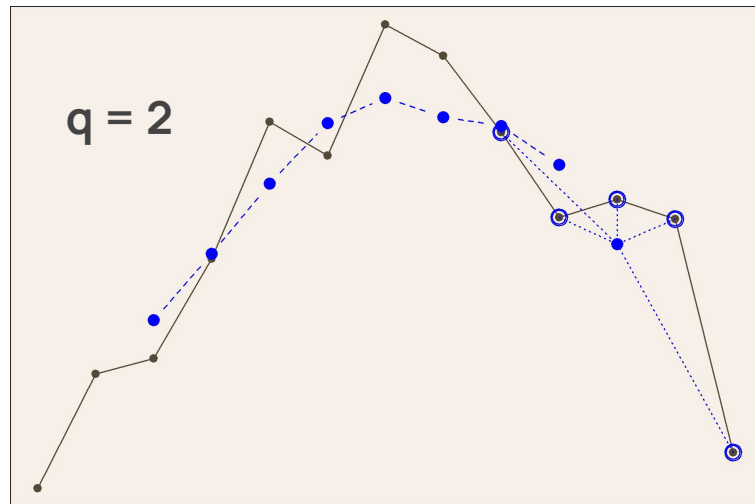
⌚ The smooth at time t is:

$$\hat{m}_t = \begin{cases} \frac{1}{d}(x_{t-q} + \cdots + x_t + \cdots + x_{t+q}) & d \text{ odd} \\ \frac{1}{d} \left(\frac{1}{2}x_{t-q} + x_{t-(q-1)} + \cdots + x_t \right. \\ \quad \left. + \cdots + x_{t+(q-1)} + \frac{1}{2}x_{t+q} \right) & d \text{ even} \end{cases}$$

⌚ **Effect of MA smoother:**
eliminates s_t , dampens noise

⌚ **Residuals:** $r_t = x_t - \hat{m}_t$
Should be dominated by s_t

Compute r_t only for $q < t \leq N - q$
(t for which we have a full window of data)



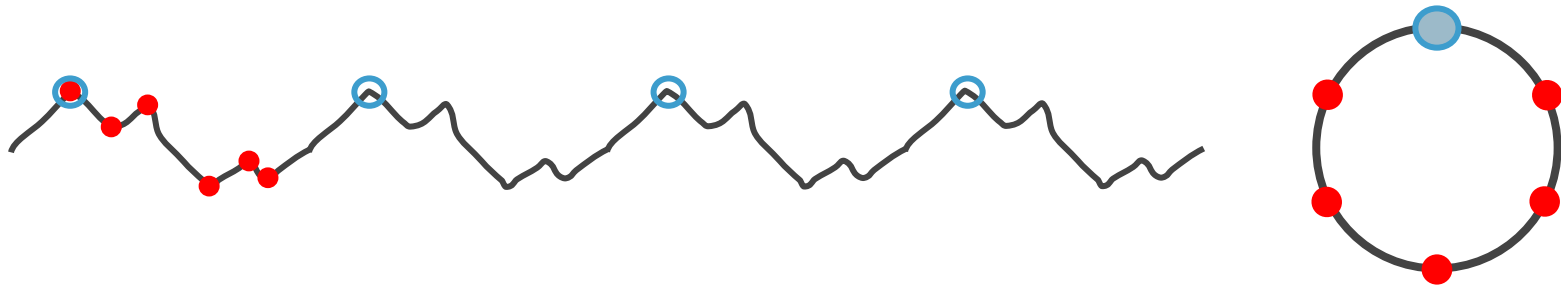
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The S1 Method: Step 2

Use residuals to estimate seasonal component within 1 cycle

For k in $\{1, \dots, d\}$, compute w_k : the average of all residuals in the set

$$\left\{ (x_{k+jd} - \hat{m}_{k+jd}) ; q < k + jd \leq N - q \right\}$$



→ w_k estimates k^{th} value of s_t in 1 cycle

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The S1 Method: Step 2

Use residuals to estimate seasonal component within 1 cycle


We're assuming

$$\sum_{t=1}^d s_t = 0$$

Final estimate

demeans the w_k :

$$\hat{s}_k = w_k - \frac{1}{d} \sum_{j=1}^d w_j$$

 Now the $\{\hat{s}_1, \dots, \hat{s}_d\}$ estimate one cycle.

 Clone it across the time span of the data, and you've estimated all of s_t

$$\hat{s}_k = \hat{s}_{k-d}, k > d.$$

Notes

- 1) again, you have to know d beforehand
- 2) Only works for 1 seasonal component

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Skye talk about this pls ♥

The *deseasonalized* data is then defined to be the original series with the estimated seasonal component removed, i.e.,

$$d_t = x_t - \hat{s}_t, \quad t = 1, \dots, n. \quad (1.5.14)$$

Finally, we reestimate the trend from the deseasonalized data $\{d_t\}$ using one of the methods already described. The program ITSM allows you to fit a least squares polynomial trend \hat{m} to the deseasonalized series. In terms of this reestimated trend and the estimated seasonal component, the estimated noise series is then given by

$$\hat{Y}_t = x_t - \hat{m}_t - \hat{s}_t, \quad t = 1, \dots, n.$$

The reestimation of the trend is done in order to have a parametric form for the trend that can be extrapolated for the purposes of prediction and simulation.

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We learned something Today, in Time Series 🕒



The S1 method

Eliminates seasonality

Gives us a model for the data's noise, Y_t

What do we tell quin?