Correlation

A Time Series Analyst’s Crash Course

STAT 464 / 864 | Fall 2024
Discrete Time Series Analysis
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We learned something Fri/Mon, in Time Series 🕛

Cauchy Schwarz Inequality

$$\mathrm{Cov}(X, Y)^2 \leq \mathrm{Var}(X)\mathrm{Var}(Y)$$

Inner product space?

Functions **E / Cov / Var** of linear combos

What do we tell quin?

Covariance vs. Correlation

Drawback of using covariance as measure of dependence:

It depends on the units of the RVs.

Example:

If **X, Y** measured are in **cm** → **10X** and **10Y** are same quantities in **mm**

But notice: **Cov[10X, 10Y] = 100 Cov[X,Y]** → cov is not standardized.

Don't want the **measure** of dependence to change based on units

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Correlation | Definition & Properties

Correlation coefficient:

$$\rho(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}}$$

1) Symmetry (follows from symmetry of cov.)
2) Unitless, real valued on **[-1, 1]**
3) $\rho = \pm 1 \iff$ Y is a linear function of **X**,

   ie) **Y = a + bX**

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Correlation Property 3)

"Perfect" Correlation

③ $\rho(X,Y) = \pm 1$ if and only if Y is a linear function of X.

<u>Proof</u> Suppose $Y = a + bX$ for some nonzero constant b.

Then $$\begin{aligned} Cov(X,Y) &= Cov(X, a+bX) \\ &= Cov(X,a) + b\,Cov(X,X) \\ &= b\,Var(X) \end{aligned}$$

$$Var(Y) = Var(a+bX) = b^2 Var(X)$$

So $$\rho(X,Y) = \frac{b\,Var(X)}{\sqrt{Var(X)\,b^2\,Var(X)}} = \frac{b}{|b|} = \pm 1$$

Conversely, suppose $\rho(X,Y) = +1$. Let $\sigma_X^2 = Var(X)$ and $\sigma_Y^2 = Var(Y)$. Then

$$\begin{aligned} Var\left(\frac{X}{\sigma_X} - \frac{Y}{\sigma_Y}\right) &= \frac{Var(X)}{\sigma_X^2} + \frac{Var(Y)}{\sigma_Y^2} - 2\,Cov\left(\frac{X}{\sigma_X}, \frac{Y}{\sigma_Y}\right) \\ &= 1 + 1 - 2\rho(X,Y) = 0 \end{aligned}$$

$$\left(Cov\left(\frac{X}{\sigma_X}, \frac{Y}{\sigma_Y}\right) = \frac{1}{\sigma_X}\frac{1}{\sigma_Y} Cov(X,Y) \right)$$

So $\frac{X}{\sigma_X} - \frac{Y}{\sigma_Y}$ is a constant with prob. 1

$\Rightarrow$ Y is a linear function of X.

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We learned something Today, in Time Series 🕛

Correlation coefficient:

$$\rho(X, Y) = \frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}}$$

Correlation = Covariance,
standardized to scale of [-1, 1]

What do we tell quin?