Eliminating Trends

Fits & Filters

STAT 464 / 864 | Fall 2024

Discrete Time Series Analysis

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Eliminating m_t

$$X_t = \boxed{m_t} + \boxed{s_t} + \boxed{Y_t} \quad (\star)$$

Once m_t is accounted for,
we can examine the data's remaining periodic structures

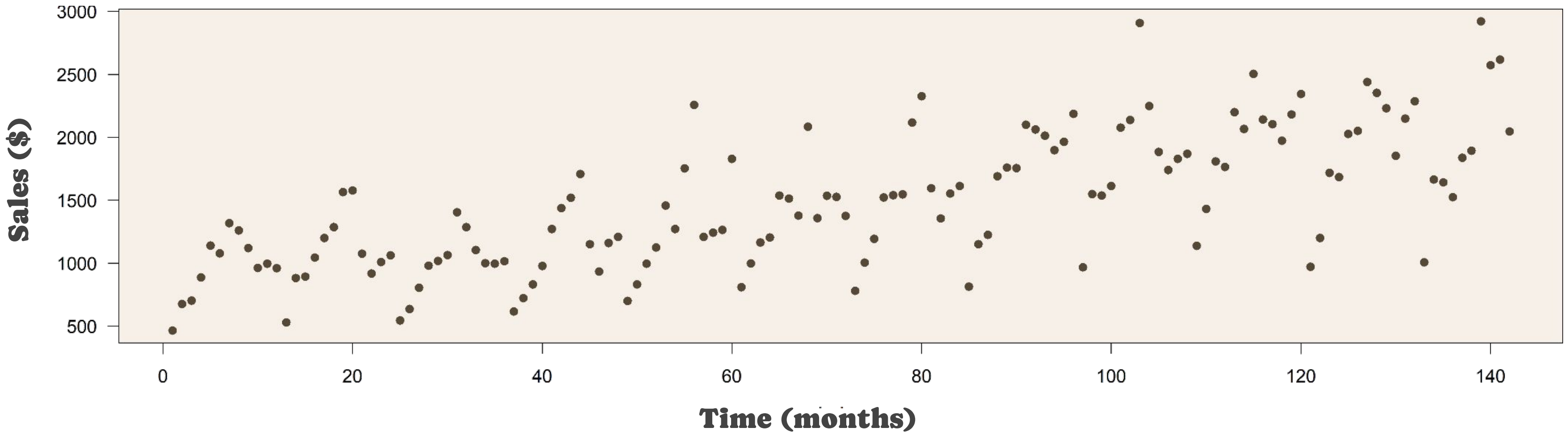

Fig 1: Australian Red Wine Sales from Jan. 1980 to Oct. 1991

Notice the **fluctuations** in sales are independent from the **general upward trend**

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Method 1
Polynomial Regression
(Similar to linear models)

Method 2
Moving Average (MA)
Smoothing Filters

Method 3
Exponential
Smoothing

Polynomial Regression

$$X_t = \boxed{m_t} + \boxed{s_t} + \boxed{Y_t} \qquad (\star)$$

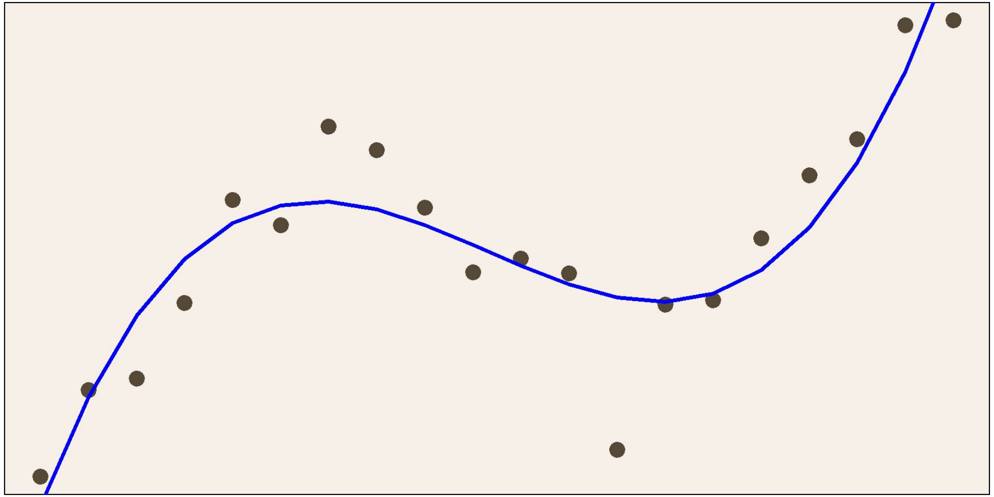
Assume m_t is well fit by a polynomial of some order $p \geq 1$

$$m_t = a_0 + a_1 t + a_2 t^2 + \ldots + a_p t^p$$

Use **linear regression** to estimate unknown coefficients $\{a_k\}$

Estimates chosen to minimize:

$$\sum_{t=1}^{N} \left(x_t - (a_0 + \ldots + a_p t^p)\right)^2$$



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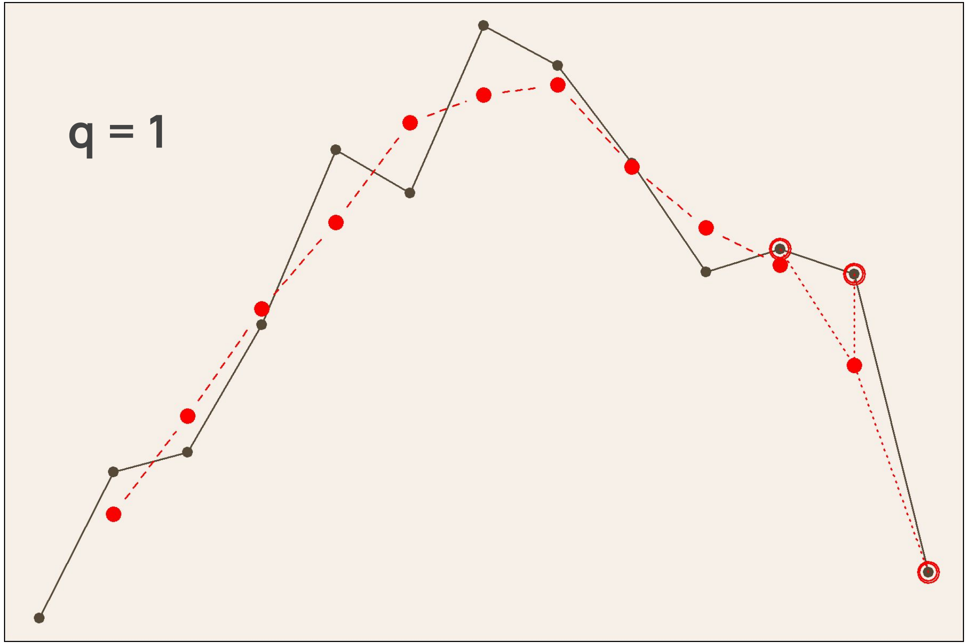
Moving Average (MA) Smoothing Filters

"Smooths" the series by estimating points using an average of surrounding data

- 🕑 Choose a **time-bandwidth** non-negative integer q
- 🕑 Get average of points in a $(2q+1)$-diameter window, centered at t

$$\hat{m}_t = \frac{1}{2q+1} \sum_{j=-q}^{q} x_{t-j}$$



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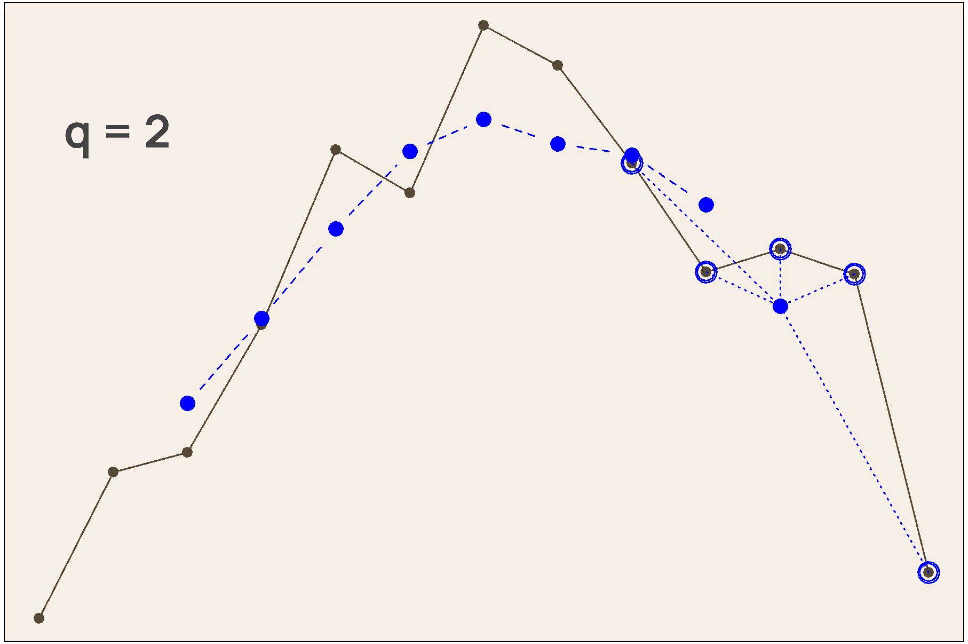
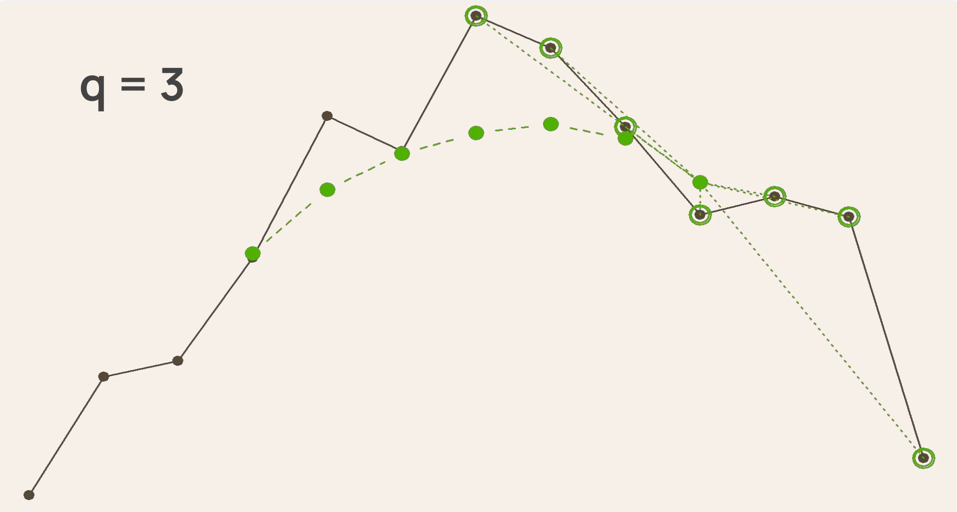


MA Smoothers : Endpoint Issues

Near endpoints of series,

$t \in [1, q]$ and $t \in [N - q + 1, N]$

The estimate $\hat{m}_t$ uses timepoints we don't get to observe



Possible Solutions:

1) pad" the ends with copies of x_1 and x_N (ITSMR does this)
2) set the missing data to 0
3) shorten window towards boundaries → only ever covers observed values

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MA-Smoothers: Choice of q

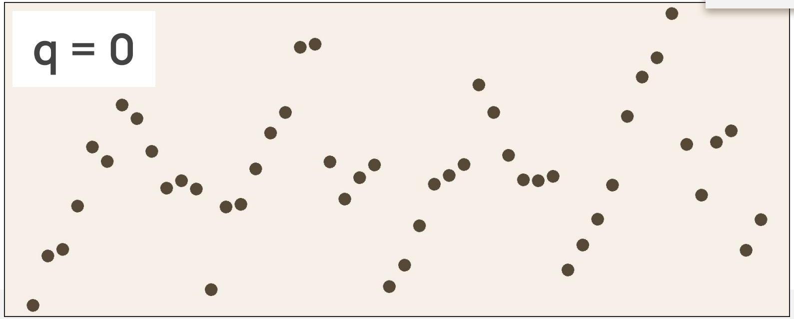
Too small: Not smooth enough.

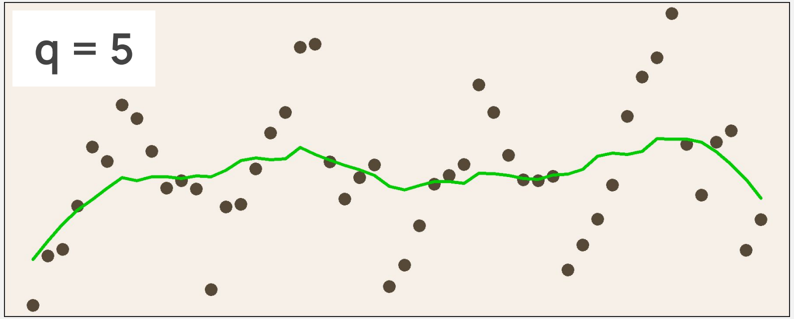
Extreme: If q = 0 you're doing nothing

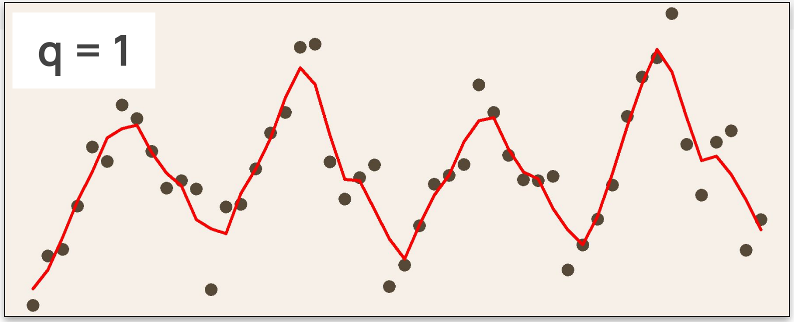
Too big: smooth but lose apparent evolution of trend over time

Extreme: If $q \geq N$, you're just taking the mean at all t (flattening effect!)

Just right: smallest q capable of smoothing significant trends







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Exponential Smoothing: Derivation

Let $\alpha \in [0,1]$. Then

$\hat{m}_1 = x_1$. For $t \geq 2$, $\hat{m}_t = \alpha x_t + (1-\alpha)\hat{m}_{t-1}$

Note that

$$\begin{aligned}
\hat{m}_t &= \alpha x_t + (1-\alpha)\hat{m}_{t-1} \\
&= \alpha x_t + (1-\alpha)\left(\alpha x_{t-1} + (1-\alpha)\hat{m}_{t-2}\right) \\
&= \alpha x_t + \alpha(1-\alpha)x_{t-1} + (1-\alpha)^2\hat{m}_{t-2} \\
&\ \ \vdots \\
&= \alpha x_t + \alpha(1-\alpha)x_{t-1} + \alpha(1-\alpha)^2 x_{t-2} + \dots + \alpha(1-\alpha)^{t-2}x_2 \\
&\qquad + (1-\alpha)^{t-1}\underbrace{x_1}_{\text{this is } \hat{m}_1} \\
&= \sum_{j=0}^{t-2} \alpha(1-\alpha)^j x_{t-j} + (1-\alpha)^{t-1}x_1
\end{aligned}$$

- The weights $\alpha(1-\alpha)^j$ decrease exponentially as **j** increases

  ie) as we go further into the past

Let $\alpha \in [0,1]$. Then

$\hat{m}_1 = x_1$. For $t \geq 2$, $\hat{m}_t = \alpha x_t + (1-\alpha)\hat{m}_{t-1} = \sum_{j=0}^{t-2} \alpha(1-\alpha)^j x_{t-j} + (1-\alpha)^{t-1} x_1$

- In the case $\alpha = 0$, we have $\hat{m}_t = \hat{m}_1 = x_1 \quad \forall t$

  → you never take current value into account

- In the case $\alpha = 1$, we have $\hat{m}_t = x_t$ ...so nothing happened

- Note $\hat{m}_t$ is computed only from the past relative to t

  → this smoother is one-sided

  → it behaves in the spirit of **forecasting**

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