Brockwell & Davis | Section 1.3.3 | General approach to time series modelling

Modeling **Xt** is like dissecting a pen.

$$X_t = \boxed{m_t} + \boxed{s_t} + \boxed{Y_t} \qquad (\star)$$

Remove the **frame** giving it its **shape**

$$X_t - \boxed{\hat{m}_t} = \boxed{s_t} + \boxed{Y_t}$$

Polynomial regression
MA smoothing
Exponential smoothing

Take off the **spring** pushing **back & forth**

$$X_t - \boxed{\hat{m}_t} - \boxed{\hat{s}_t} = \boxed{Y_t}$$

Harmonic regression
Season (S1) method

Examine the tube of **residual** ink

$$X_t - \boxed{\hat{m}_t} - \boxed{\hat{s}_t} = \boxed{\hat{Y}_t}$$

WN Hypothesis test
Autocorrelation Function

Put the **spring** back on the tube

$$X_t - \boxed{\hat{m}_t} = \boxed{\hat{s}_t} + \boxed{\hat{Y}_t}$$

Put the **body** of the pen back on

$$X_t = \boxed{\hat{m}_t} + \boxed{\hat{s}_t} + \boxed{\hat{Y}_t}$$

= unknown

= estimated

Unit 1 Finale

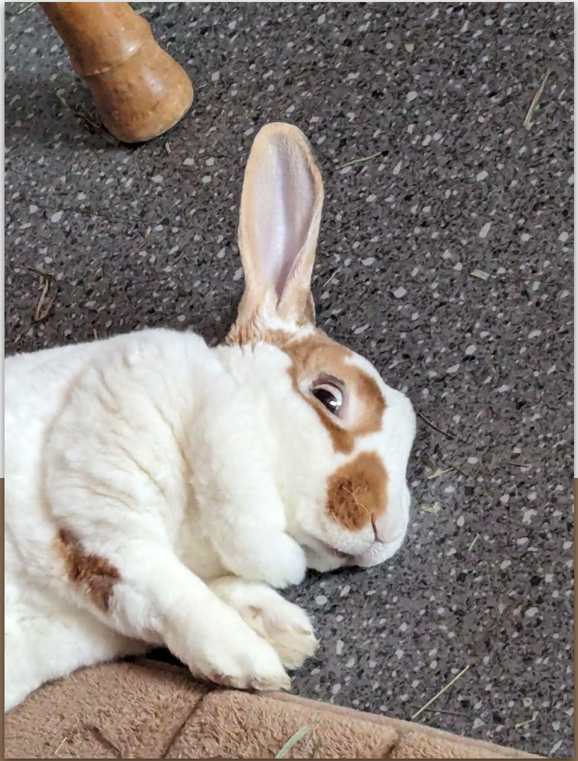
Classical Time Series Modelling

STAT 464 / 864 | Fall 2024

Discrete Time Series Analysis

Skyepaphora Griffith, Queen's University

We learned something last time, in Time Series 🕛



ACVF of a classical-decomposition (⋆) time series is the same as the ACVF of its noise

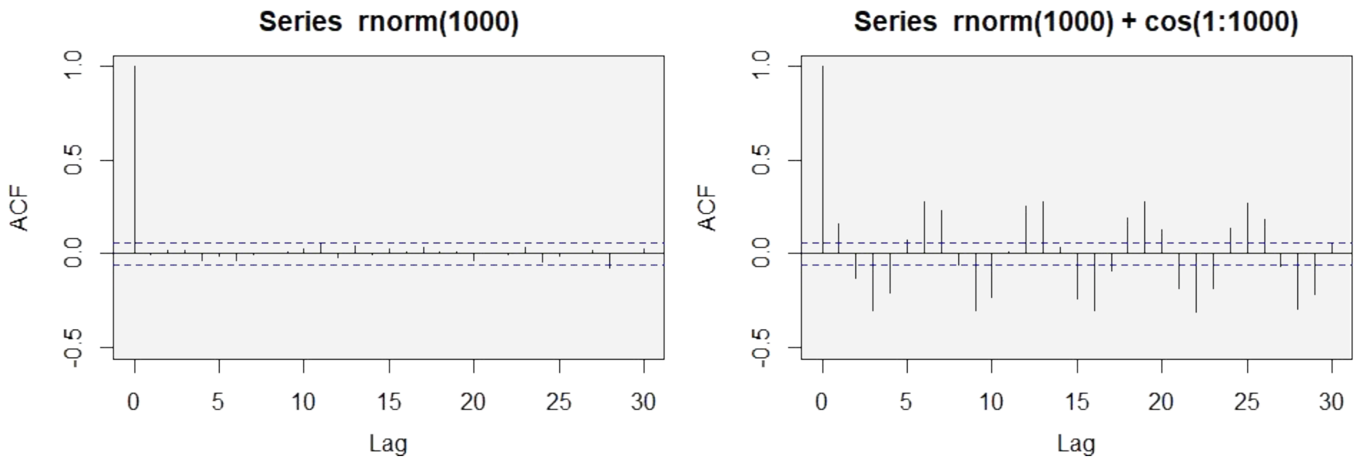
Not true for sample ACVFs

Theoretically, trend/seasonality don't affect ACVF
In practice, they can influence the sample ACVF

How to test if a time series is white noise!
If ACF doesn't look enough like the indicator fn: 1(h=0)
Then reject the hypothesis that Xt ~ wn

What did we tell quin?

ACFs in R



- Function `acf()` plots 2 horizontal dashed blue lines at $\pm 1.96/\sqrt{N}$
- $\hat{\rho}(h)$ that fall outside blue lines = evidence $\{X_t\}$ is not **wn**$(0, \sigma^2)$

Interpretation: If we compute $\hat{\rho}$ for white noise at **m** different lags,

We expect $\hat{\rho}$ to fall outside the blue lines $m \times \alpha$ times

Something Skye forgot to say about ∇ !

We don't actually do anything on this page
consider it a page for notes ¯_(ツ)_/¯

Applying ∇ to a polynomial

Output: Polynomial with reduced degree

Applying ∇ to a polynomial

Output: Polynomial with reduced degree

Consider the polynomial $$m_t = c_0 + c_1 t + c_2 t^2 + \cdots + c_k t^k$$

$$\begin{aligned}
\nabla m_t &= m_t - m_{t-1} \\
&= [c_0 + c_1 t + c_2 t^2 + \cdots + c_k t^k] \\
&\quad - [c_0 + c_1(t-1) + c_2(t-1)^2 + \cdots + c_k(t-1)^k] \\
&= [c_0 + c_1 t + c_2 t^2 + \cdots \boxed{+ c_k t^k]} \\
&\quad - [c_0 + c_1(t-1) + c_2(t-1)^2 + \cdots \boxed{+ c_k t^k} + c_k \boxed{p_t}]
\end{aligned}$$

Something Skye forgot to say about ∇ !

$$m_t = c_0 + c_1 t + c_2 t^2 + \cdots + c_k t^k$$

- 🕑 The polynomial $\mathbf{p_t}$ is of degree strictly less than **k**
- 🕑 Input polynomial of degree **k** → output polynomial of smaller degree
- 🕑 Applying ∇ **k**-many times will reduce a **k**th degree polynomial to a **constant**

  $\nabla^k m_t$ = some constant
- 🕑 ∇^k will eliminate polynomial trends up to degree **k**

Space for notes

Linear Filters

Definition: A linear filter is a sequence of coefficients $\{a_j\}_{j\in\mathbb{Z}}$

Satisfying $$\sum_{j=-\infty}^{\infty} |a_j| < \infty$$ (absolute summability)

Input process Y_t **Output process**

Space for notes

Typical uses for Linear Filters

1) Shape desirable properties in the output Xt

   Example:

   a filter might pass/block select range of frequencies

   low-pass / high-pass / band-pass

   Another example:

   The linear filter ∇ can be used to eliminate polynomial trends

2) Represent various time series models, themselves

Space for notes 🕛 🕑 🕞

Linear Filters we've seen

$$X_t = \sum_{j=-\infty}^{\infty} a_j Y_{t-j}$$

Difference operator

MA smoother with parameter q

Space for notes 🕛 🕑 🕞

🕛 **End of Unit 1!** 🕛 **Classical Time Series Modelling**

What do we tell quin?