Difference Operators

STAT 464 / 864 | Fall 2024

Discrete Time Series Analysis

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We learned something today, in Time Series 🕐

Autocovariance: $\gamma_X(h) \stackrel{\text{def}}{=} \text{Cov}(X_t, X_{t+h})$,

for all t (time independent)

compares two observations/RVs distanced in time

Stationary:

frequency structure doesn't change over time,

(neither does mean or variance)

kinda feels like MC escher

What do we tell quin??

The Difference Operator: ∇

Basic operation: $\nabla X_t \stackrel{\text{def}}{=} X_t - X_{t-1}$

Lag-h difference: $\nabla_h X_t \stackrel{\text{def}}{=} X_t - X_{t-h}$

Powers denote repeat applications

Example:

Applying ∇ to a polynomial

Output: Polynomial with reduced degree

Consider the polynomial $m_t = c_0 + c_1 t + c_2 t^2 + \cdots + c_k t^k$

Detour: Binomial Theorem

Binomial expansion of $(t-1)^k$

Last term of expansion: $j=k$

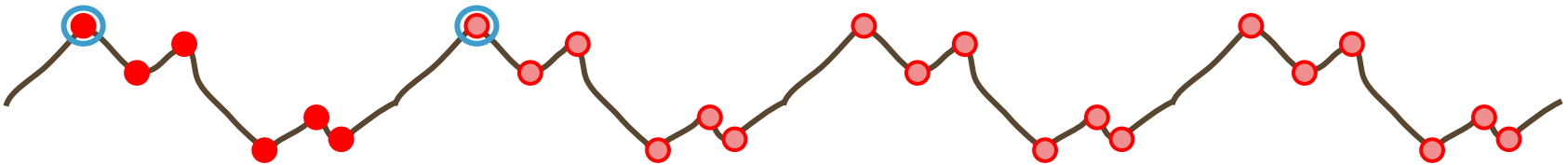
So, $(t-1)^k$ is t^k, plus

some polynomial with **degree < k**

Applying ∇ to a Seasonal Component

Suppose $\{X_t\}$ has a seasonal component s_t with period d

$$\nabla_d s_t = \boxed{s_t} - \boxed{s_{t-d}} = 0$$



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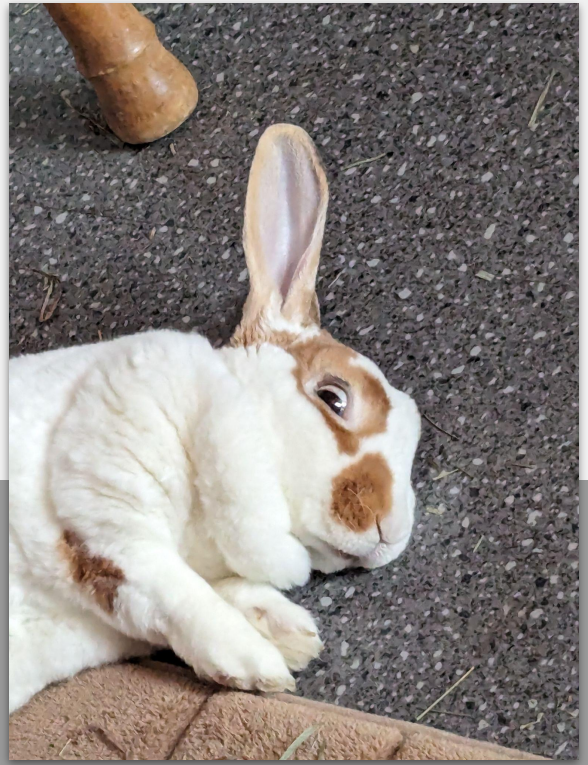

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