Difference Operators

STAT 464 / 864 | Fall 2024

Discrete Time Series Analysis

Skyepaphora Griffith, Queen's University

The Difference Operator: ∇

Basic operation: $\nabla X_t \stackrel{\text{def}}{=} X_t - X_{t-1}$

Lag-h difference: $\nabla_h X_t \stackrel{\text{def}}{=} X_t - X_{t-h}$

Powers denote repeat applications

Example:

$$\begin{aligned}\nabla_3^2 X_t = \nabla_3(\nabla_3 X_t) &= \nabla_3 \nabla_3 X_t \\ &= \nabla_3(X_t - X_{t-3}) \\ &= (X_t - X_{t-3}) - (X_{t-3} - X_{t-6}) \\ &= X_t - 2X_{t-3} + X_{t-6}\end{aligned}$$

Space for notes 🕐🕑🕒

Applying ∇ to a Seasonal Component

Suppose $\{X_t\}$ has a seasonal component s_t with period d

$$\nabla_d s_t = s_t - s_{t-d}$$
$$= 0$$