

Prediction intervals

$$(1 - \alpha) \times 100\%$$

STAT 464 / 864 | Fall 2024

Discrete Time Series Analysis

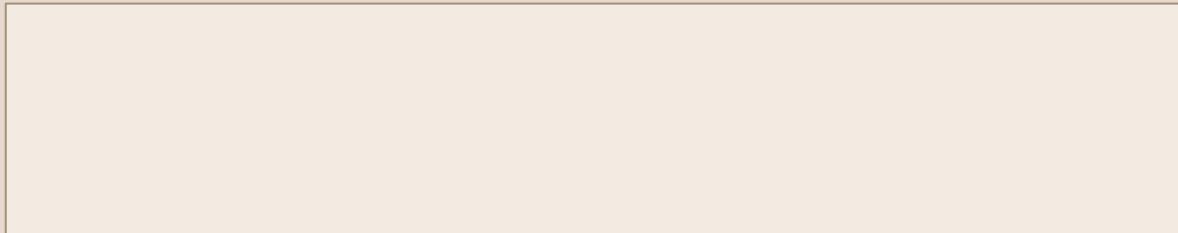
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Distribution of the innovations

- 1) The process $\{X_t\}$ is **Gaussian** (identically distributed **normal** RVs)
(True for ARMA models if their noise process $\{Z_t\}$ is Gaussian)
- 2) ACVF of $\{X_t\}$ is known

For h-step ahead prediction, the error $X_{N+h} - P_N X_{N+h}$

Is normally distributed with mean = 0, and variance



Prediction Intervals

Derivation

$(1 - \alpha/2)$ - quantile of
std. Normal distribution



Implications of knowing the ACVF

Suppose the ACVF $\gamma_X(h)$ is known \rightarrow we know exactly:

- ⌚ covariance matrix Γ of $\mathbf{X} := (X_N, \dots, X_1)$
- ⌚ Vector γ of covariances $\text{Cov}(X_{N+h}, \mathbf{X})$
- ⌚ The prediction $P_N X_{N+h} = a_1 X_N + \dots + a_N X_1$
where vector \mathbf{a} satisfies $\Gamma \mathbf{a} = \gamma$
- ⌚ $MSE = \gamma_X(0) - \mathbf{a}^T \gamma$

Remarks



In practice, assumptions 1 & 2 (Gaussian, known ACVF) aren't very accurate.

Can do checks / transformations → assumptions are approx. correct



Assumption 2: we almost always need to estimate the ACVF.

Get sample ACVF → input into innovations algorithm

→ get h-step ahead prediction, minimum MSE