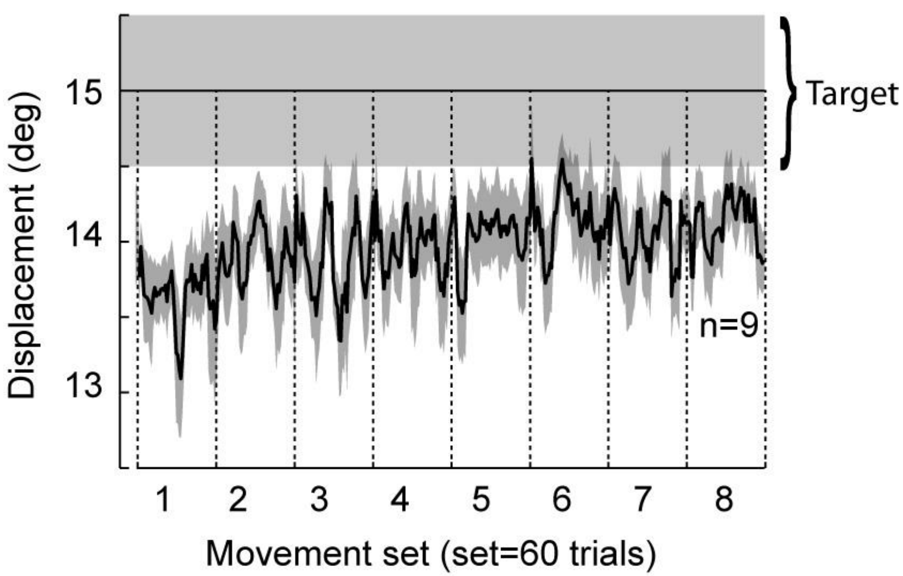
Why is saccadic gain less than one?

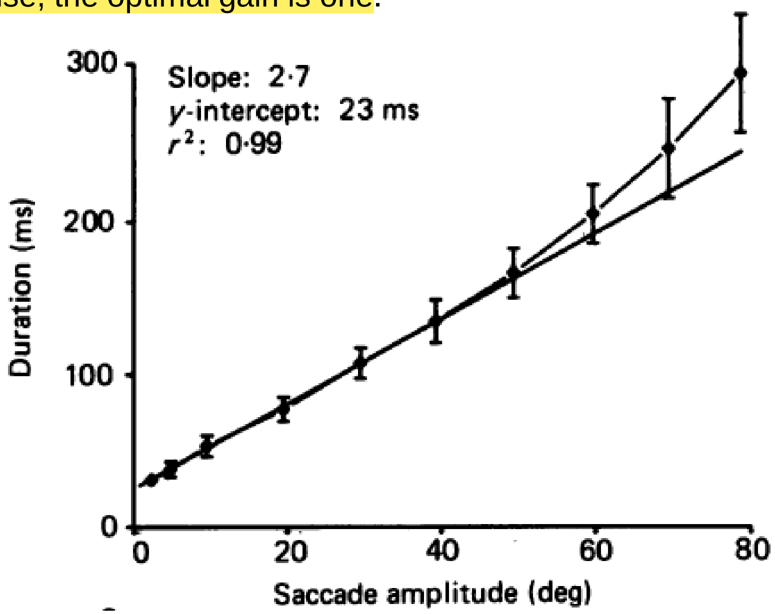
It is common to find that saccades to targets tend to fall slightly short. On the left figure, we have data from n=9 subjects who made saccades to a target at 15 degrees (the size of the target was 1 degree). The saccades were short by about 0.7 degrees. Indeed, most of these saccades were followed by a second saccade that brought the target on to the fovea. The 'undershoot' appears to be a deliberate strategy. In this exercise, we will consider why this might be happening.

On the right figure, we have the relationship between saccade amplitude and duration. Duration of a saccade is approximately a linear function of amplitude. However, note how the variability of duration is not constant, but increases as amplitude increases.

In our exercise, we will try to find the optimum saccade gain so to minimize a cost that depends on two terms: the distance of the eye to the target, and the time it takes to make a saccade. That is, we will try to find the gain the brings the eyes as close as possible to the target, while minimizing the time it takes to get there. Our objective is to show that even if the weight associated with minimizing the distance to target is infinite, a system with signal-dependent noise has an optimal gain that is less than one. However, for a system with Gaussian noise, the optimal gain is one.



Chen-Harris et al. (2008)



Collewijn et al. (1988)

What is the optimum gain when motor commands are corrupted by signal dependent noise?

Position of the eye $x = u + \varepsilon \qquad \varepsilon \sim N\left(0, c^2 u^2 \phi\right) \qquad \phi \sim N\left(0,1\right)$

$$x = u\left(1 + c\phi\right)$$

Duration of the saccade $t = a + bx = a + bu\left(1 + c\phi\right)$

$$J = \lambda\left(g - x\right)^2 + t^2$$

Objective is to find the motor command u that brings the eye as close as possible to the goal, as soon as possible.

1. Find the motor command u that minimizes E[J].
2. For a given goal g, what is the ‘gain’ of the system as lambda goes to infinity?

What is the optimum gain when motor commands are corrupted by Gaussian noise?

Position of the eye $x = u + \varepsilon$ $\qquad \varepsilon \sim N\left(0, \sigma^2\right)$

Duration of the saccade $t = a + bx = a + b\left(u + \varepsilon\right)$

$$J = \lambda\left(g - x\right)^2 + t^2$$

Objective is to find the motor command u that brings the eye as close as possible to the goal, as soon as possible.

1. Find the motor command u that minimizes E[J].
2. For a given goal g, what is the ‘gain’ of the system as lambda goes to infinity?