

習題 9.

$$1) S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n-1}}$$

$$= \sqrt{\frac{1289 - 6 \times 14.33}{5}} = \sqrt{10.38} = 3.22$$

$\therefore \sigma$  之點估計為 3.22

2)  $1 - \alpha = 0.90, \frac{\alpha}{2} = 0.05, n-1 = 5$

$$\chi_{\frac{\alpha}{2}}^2(n-1) = \chi_{0.05}^2(5) = 11.07$$

$$\chi_{1-\frac{\alpha}{2}}^2(n-1) = \chi_{0.95}^2(5) = 1.15$$

$\therefore \sigma$  之 90% 信賴區間為

$$\left( \sqrt{\frac{5 \times 10.38}{\chi_{0.05}^2(5)}}, \sqrt{\frac{5 \times 10.38}{\chi_{0.95}^2(5)}} \right) = \left( \sqrt{\frac{51.9}{11.07}}, \sqrt{\frac{51.9}{1.15}} \right) = (2.17, 6.72)$$

習題 20

$\mu_1$  為投資組合一之獲利率之平均數,  $\mu_2$  為投資組合二之獲利率之平均數,  $n_1 = 9, \bar{x} = 7.67, S_1 = 9.27, n_2 = 9, \bar{y} = 6.78, S_2 = 21.15$

1) 因  $\sigma_1^2 \neq \sigma_2^2$

$$v = \frac{\left( \frac{9.27^2}{9} + \frac{21.15^2}{9} \right)}{\frac{\left( \frac{9.27^2}{9} \right) + \frac{\left( \frac{21.15^2}{9} \right)}{2}} = 10.96 \approx 11$$

$$df = (16.06, 17.87) \approx$$

$\therefore \mu_1 - \mu_2$  之 95% 信賴區間為

$$(\bar{x} - \bar{y}) \pm t_{\frac{\alpha}{2}}(v) \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$= (7.67 - 6.78) \pm t_{0.025}(11) \sqrt{\frac{9.27^2}{9} + \frac{21.15^2}{9}}$$

$$= 0.89 \pm 2.201 \times 2.70 = 0.89 \pm 5.94$$

2)  $1 - \alpha = 0.90, \chi_{\frac{\alpha}{2}}^2(n_1-1) = \chi_{0.05}^2(8) = 15.51, \chi_{1-\frac{\alpha}{2}}^2(n_1-1) = \chi_{0.95}^2(8) = 2.73$

$\therefore \sigma_1^2$  之 90% 信賴區間為

$$\left( \frac{8 \times 9.27^2}{\chi_{0.05}^2(8)}, \frac{8 \times 9.27^2}{\chi_{0.95}^2(8)} \right) = \left( \frac{687.46}{15.51}, \frac{687.46}{2.73} \right) = (44.32, 251.81)$$

3)  $1 - \alpha = 0.90, F_{\frac{\alpha}{2}}(n_1-1, n_2-1) = F_{0.05}(8, 8) = 3.44, F_{1-\frac{\alpha}{2}}(n_1-1, n_2-1) = F_{0.95}(8, 8)$

$\therefore \frac{\sigma_1^2}{\sigma_2^2}$  之 90% 信賴區間為

$$= \frac{1}{F_{0.95}(8, 8)} = 0.29$$

$$\left( \frac{S_1^2}{S_2^2} \times \frac{1}{F_{\frac{\alpha}{2}}(n_1-1, n_2-1)}, \frac{S_1^2}{S_2^2} \times \frac{1}{F_{1-\frac{\alpha}{2}}(n_1-1, n_2-1)} \right)$$

$$= \left( \frac{9.27^2}{21.15^2} \times \frac{1}{3.44}, \frac{9.27^2}{21.15^2} \times \frac{1}{0.29} \right)$$

$$= (0.06, 0.66)$$