

例 6.9

依 $E(X_i) = \mu, V(X_i) = \sigma^2 = E(X_i^2) - \mu^2$,

則 $E(\bar{X}) = \mu, V(\bar{X}) = \frac{\sigma^2}{n} = E(\bar{X}^2) - \mu^2$

$$\begin{aligned} E(\hat{\theta}_1) &= E\left(\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n}\right) = \frac{1}{n} E\left(\sum_{i=1}^n X_i^2 - n\bar{X}^2\right) \\ &= \frac{1}{n} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \frac{n-1}{n} \sigma^2 \end{aligned}$$

$$\begin{aligned} E(\hat{\theta}_2) &= E\left(\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n}\right) = \frac{1}{n-1} E\left(\sum_{i=1}^n X_i^2 - n\bar{X}^2\right) \\ &= \frac{1}{n-1} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2) = \sigma^2 \end{aligned}$$

因此, $\hat{\theta}_2 = \sum_{i=1}^n (X_i - \bar{X})^2 / (n-1)$ 為母體變異數 σ^2 之不偏估計量, 而 $\hat{\theta}_1 = \sum_{i=1}^n (X_i - \bar{X})^2 / n$ 為母體變異數 σ^2 之偏誤估計量。