A Tutorial on Interference Alignment

Farzad Talebi

Abstract

As an inevitable phenomenon in wireless networks, interference has been always an important concern in designing communication networks. There are some conventional interference management approaches which will be mentioned here. The important note about all these approaches is that using them, the sum-capacity of the network will be limited by interference, regardless of the number of users, or the approach itself cannot be generalized for more than two user cases. Interference alignment is a surprising approach with which the sum-capacity of the time-varying interference networks using limited resources e.g. time, frequency, can be increased linearly with the number of users. Some special cases in which interference alignment can be interpreted easily will be presented, and the cases of imperfect or limited channel state information, which is necessary for this approach, will also be provided. Finally, an overview of benefits and challenges of the interference alignment approach will be discussed.

I. Introduction

An interference channel is where several sender-receiver pairs share a common media so that transmission of information from one sender to its corresponding receiver interferes with communications between the other pairs [1]. In this tutorial we consider Additive White Gaussian Noise (AWGN) fading interference and X channels. Channel output of the interference channel is provided in (1). X channels have same outputs as interference channels but in this case it is intended that every transmitter has a specific message for every receiver, rather than just for its intended receiver. In Interference -or X- channels, interference can be a bigger concern than the noise, because if all users operate in high signal to noise ratio (SNR), noise will become less important but the interference will become more and more challenging. There are some practical interference management approaches that are already useful:

Decode the interference

When the interference is much stronger than the signal it can be decoded then subtracted from the desired signal, so it allows receiver to decode the desired signal. It is less common in practice due to complexity of multi-user detection [2]. This approach is supported by information theoretic results on "strong interference" [3] and an important note about this approach is that generalization to more than two-user case in this approach is not straight forward.

Orthogonalize

The practical approach to counter interferences which are as strong as signals is to orthogonalize signals and interferences in time, frequency or code. In this setting the degrees of freedom, which will be defined later, is 1 and if there are K users, available rate for each one will be $1/K\ log(\text{SNR}) + o(log(\text{SNR}))$, in which $o(log(\text{SNR}))$ represents a function of $log(\text{SNR})$ that does not depend on K.

Treat it as noise

Treating weak interference as noise is proven to be useful both practically and theoretically because it is known that introducing structure in weak interference does not help [2].

Degrees of Freedom

"Degrees of freedom" is an important capacity approximation in networks literature. To give a simple intuition of degrees of freedom of a network it is worthy to note that:

1- The degrees of freedom of a network may be interpreted as the number of resolvable signal space dimensions.
2- A network has d degrees of freedom if and only if the sum capacity of the network can be expressed as $d\ log(\text{SNR}) + o(log(\text{SNR}))$.
3- It is a capacity approximation that is accurate in high SNR's.

II. Interference Alignment

Interference alignment was first considered in [4] as a coding technique for the two-user Multiple-Input Multiple-Output (MIMO) X channel, where it was shown to achieve multiplexing gains strictly higher than that of the embedded MIMO interference channel (IC), multiple-access channel (MAC) and broadcast channel (BC) taken separately [5].
Authors in [2] introduce interference alignment as an approach to maximize interference–free space for the desired signal. It is shown that all the interference can be concentrated roughly into one half of the signal space at each receiver, leaving the other half available to the desired signal and free of interference [2].

On interference channels, the alignment can be accomplished for any number of users but as the number of users increases a larger signal space is needed for each user to recover nearly half of it [2]. As an example, for the three user interference channel we present the interference alignment that shows sum capacity per user can reach an upper bound of $1/2\ log(\text{SNR}) + o(log(\text{SNR}))$. When considering sum capacity for K users, it is equivalent to the pre–mentioned $K/2$ degrees of freedom for the sum capacity.

A) Interference alignment for three user interference channel

Consider K transmitter-receiver pair aiming to communicate using a common media. The output signal for k th user at time slot t is given by:

$$Y^{[k]}(t) = H^{[k1]}(t)X^{[1]}(t) + \ldots + H^{[kK]}(t)X^{[K]}(t) + Z^{[k]}(t) \tag{1}$$

where $k \in \{1, 2, \mathbf{L}, K\}$ is the user index, $Y^{[k]}(t)$ is the channel output signal for k th user, $X^{[i]}(t)$ and $H^{[ki]}(t)$ are transmitted signal of i th user and its gain to received signal of k th user, respectively. Here $Z^{[k]}(t)$ represents additive white Gaussian noise (AWGN). $H^{[ki]}(t)$'s are assumed to be drawn i.i.d. from continuous distributions, so they represent fast-fading channels. Authors in [2] showed that for $K = 3$ we are able to achieve $3n+1$ degrees of freedom over a $2n+1$ symbol extension of the channel. It means that by choosing large n we can approach $3/2$ degrees of freedom. For $n=1$, assume that users are transmitting their coded messages $\boldsymbol{x}$ along beamforming vectors $\mathbf{v}$. The aim of interference alignment here is by using full causal channel knowledge at transmitters [2], shape beamforming vectors such that interferences at each receiver are aligned so that the interference free subspace is maximized.

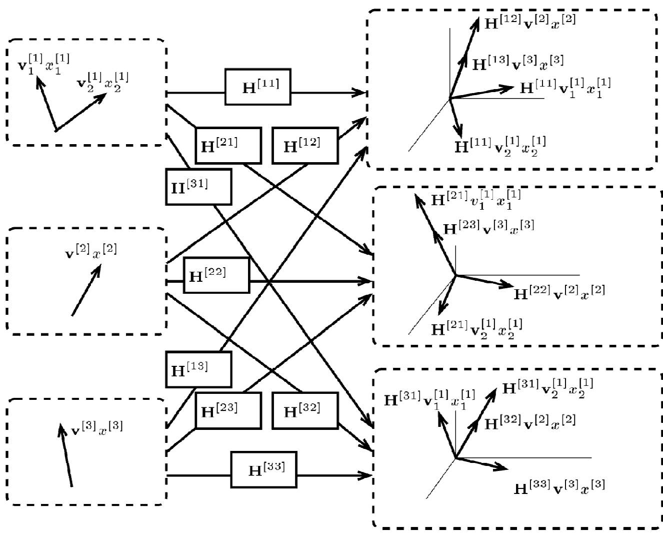

Figure 1. Interference alignment for three user interference channel over 3 symbol extension of thee channel, 4/3 degrees of freedom obtained [2].

Without loss of generality we can pick beamforming vector of user 2 as:

$$\mathbf{v}^{[2]} = \mathbf{1}_{3\times 1}, \tag{2}$$

now, other beamforming vectors can be derived by aligning other interferer signals (signals from user 3 in this case) to $\mathbf{v}^{[2]}$ at receiver 1, so the beamforming vector for the user 3 should be:

$$\mathbf{H}^{[12]} \mathbf{v}^{[2]} = \mathbf{H}^{[13]} \mathbf{v}^{[3]} \Rightarrow \mathbf{v}^{[3]} = \left(\mathbf{H}^{[13]}\right)^{-1} \mathbf{H}^{[12]} \mathbf{1}_{3\times 1}, \tag{3}$$

which allignes received vectors from users 2 and 3 in same subspace at receiver 1. It is a simple key point in the algebraic manupulations of interference allignment.

Similarly, at receiver 2, as we want that transmitter 1 to have two beamforming vectors $\mathbf{v}_1^{[1]}, \mathbf{v}_2^{[1]}$, the interference from transmitter 3 is alligned to one of the beamformign vectors of transmitter 1, so:

$$\mathbf{H}^{[23]}\mathbf{v}^{[3]} = \mathbf{H}^{[21]}\mathbf{v}_1^{[1]} \Rightarrow \mathbf{v}_1^{[1]} = \left(\mathbf{H}^{[21]}\right)^{-1}\mathbf{H}^{[23]}\left(\mathbf{H}^{[13]}\right)^{-1}\mathbf{H}^{[21]}\mathbf{1}_{3\times1}. \tag{4}$$

And finally at receiver 3, the interference from transmitter 2 should be alligned along one dimension of interference from transmitter 1, so:

$$\mathbf{H}^{[32]}\mathbf{v}^{[2]} = \mathbf{H}^{[31]}\mathbf{v}_2^{[1]} \Rightarrow \mathbf{v}_2^{[1]} = \left(\mathbf{H}^{[31]}\right)^{-1}\mathbf{H}^{[32]}\mathbf{1}_{3\times1}. \tag{5}$$

Figure 1 shows the alligned interference as derived above for all receivers.

In [2] the assumption of time–varying channels is necessary because the proof is base on continues random variables as channel gains. But, [6] and [7] prove same results for specific constant channels and some slow-fading cases, showing that the generalization of the idea is possible.

B) Interference alignment for 2x2 X-channel

In [8] authors extended their results on interference channel to a more general class of networks, X networks in which there are a pair of transmitter and receiver group in which every transmitter has an independent message for every receiver. This network is a generalized model for Broadcast Channel (BC), Multiple-Access Channel (MAC) and interference channel.

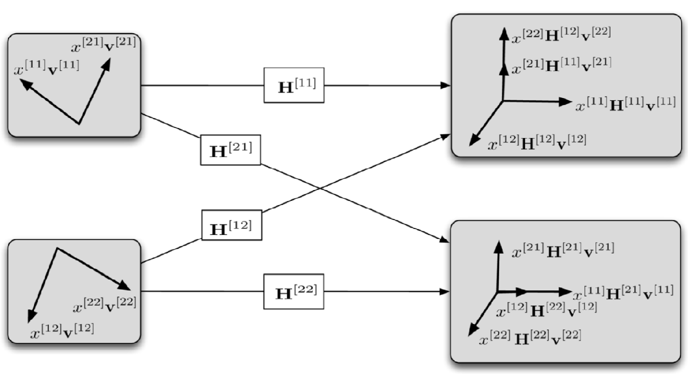

Figure 2. Interference alignment for 2x2 X-network [8].

Authors in [8] proved similar result for an $M \times N$ X-network, and it is that the degrees of freedom of such networks under fast-fading assumption is:

$$\frac{MN}{M+N-1} \tag{6}$$

where M is the number of transmitters and N represents number of receivers in the X-network. For the case $M = N = 2$ figure 2 shows how interference alignment works, details are very similar to the interference channel case so omitted here. Also it is shown that the results for interference channel can be derived directly from the results for X-network [8].

C) Limited or Imperfect Channel State Information

All results in [2] and [8] are based on full channel state information of all network channels at each transmitter, but [9] shows that only limited (quantized) amount of feedback is sufficient for this purpose. It shows that in an interference channel with M transmitter receiver pairs, each with P/M power constraint, over frequency selective channels represented by L taps, the number of feedback bits –about impulse response of channels- necessary to be transmitted by each receiver (to provide channel state information) is $M(L-1)log(P)$. So, the resource amount for feedback can be bounded in the case considered in [2].

In a cellular system, under the assumption of noisy channels estimation, [5] shows that the sum-capacity achievable using interference alignment when base stations share their information about channels is the same as when they don't cooperate even under moderately accurate knowledge of the channels [5].

III. Conclusions

In this tutorial we presented a brief overview of interference alignment approach to manage interference in wireless networks. It has been surprisingly seen that this approach can make time-varying interference networks have degrees of freedom increasing with the number of users sharing a common channel. It also mentioned that the approach works for slow fading networks and some constant channel networks. Also, as this approach requires channel state information about all network links at each transmitter, it is shown that bounded number of feedback bits is sufficient. But, in a case of cellular system, it is shown that the sum-capacity achievable by this approach can also be achieved (by a little performance loss) using some other simpler methods which don't need cooperation between nodes.

References

[1] A. B. Carleial, "Interference channels," *IEEE Trans. Inf. Theory*, vol. 24, no. 1, pp. 60–70, 1978.

[2] V. R. Cadambe and S. A. Jafar, "Interference Alignment and Degrees of Freedom of the K-User Interference Channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[3] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, pp. 49–60, Jan. 1981.

[4] M. Maddah-Ali, A. Motahari, and A. Khandani, “Signaling over MIMO multi-base systems: Combination of multi-access and broadcast schemes,” in *Proc. IEEE Int. Symp. Information Theory* (ISIT), Seattle, WA, USA, July 2006, pp. 2104–2108.

[5] R. Tresch, and M. Guillaud, “Cellular Interference Alignment with Imperfect Channel Knowledge,” in *Proc IEEE Int. Conf. on Communications, Workshops*, 2009, 14-18 June 2009, pp 1-5.

[6] V.R. Cadambe, S.A. Jafar, S. Shamai, Interference Alignment on the Deterministic Channel and Application to Fully Connected Gaussian Interference Networks,” *IEEE Trans. Inf. Theory,* Vol. 55, Issue 1, Jan. 2009 pp. 269 – 274.

[7] A. Ozgur, D. Tse, “Achieving Linear Scaling with Interference Alignment,” in *Proc International Symposium of Information Theory ISIT* 2009, Korea, Jun. 28-Jul 3, 2009.

[8] V.R. Cadambe, and S. A. Jafar, “Interference Alignment and the Degrees of Freedom of Wireless X Networks,” *IEEE Trans. Inf. Theory*. , Vol. 55, no. 9, Sep. 2009.

[9] J. Thukral, and H. B¨olcskei, “Interference Alignment with Limited Feedback,” in *Proc International Symposium of Information Theory ISIT* 2009, Korea, Jun. 28-Jul 3, 2009.