Single Antenna Interference Cancellation Algorithm Based on Lattice Reduction

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Abstract —Single antenna co-channel interference (CCI) cancellation is a challenge in mobile communication systems, especially for downlink systems. A novel single antenna CCI cancellation algorithm based on lattice reduction (LR) and decision feedback equalization scheme is proposed in this paper. The idea behind the proposed algorithm is to form a multi-channel multi-user detection model through over-sampling and then adopts a decision feedback detection method based on LR. Complexity analysis results show that the complexity of the proposed algorithm is quadratic with the memory length of the channels and is nearly independent of the modulation order. Furthermore, simulation results are also presented to indicate the bit error rate performance of the proposed algorithm and its robustness with respect to channel estimation errors.

Index Terms—Co-channel interference, single antenna, interference cancellation, lattice reduction

I. Introduction

Co-channel interference (CCI) is a common phenomenon in cellular radio communication networks as a result of frequent reuse. It is one of the factors limiting system capacity. To reduce performance degradation, the co-channel interference cancellation is necessary, and has attracted the attention of many researchers for more than 20 years. In particular, interference cancellation is a challenging task in the receiver equipped with one antenna, such as the downlink time division multiple access system. The corresponding solutions are known as single antenna interference cancellation (SAIC).

The SAIC techniques can generally be classified as either linear filter-based (LFB) or multi-user detection (MUD) methods [1]. The basic principle behind LFB methods is to design a linear filter to cancel CCI. A linear or nonlinear inter-symbol interference (ISI) equalizer is then applied [2]-[4]. However, these approaches can only cancel an interference signal. Moreover, the constellation of modulated signals should be one-dimensional (e.g., binary phase-shift keying (BPSK) and Gaussian minimum shift keying (GMSK)). In contrast to LFB methods, MUD methods detect all co-channel signals simultaneously. The optimal multi-user detector in terms of maximum likelihood is the jointly maximum likelihood sequence estimation (JMLSE) method [5]. However, this method is difficult to implement because of its high complexity. To reduce the complexity, reduced state trellis-based joint detection algorithms have been proposed [6], [7]. Unfortunately, the computational complexity is still growing exponentially with the modulation order of co-channel signals. Thus, this method is unsuitable for high-order modulation signals. H. Arslan [8] proposed a low-complexity, successive SAIC method based on the power difference of co-channel signals. However, the performance of this method declines sharply when the powers of co-channel signals are comparable. Miller [9] presented a co-channel data estimation method using a set of partitioned Viterbi detectors, whereas the performance loss is unacceptable given only one antenna. To improve the performance, Wei Jiang [10] proposed several iterative SAIC algorithms by combining the traditional SAIC methods with turbo equalization with the help of channel coding.

The challenge in SAIC techniques lies in designing a low-complexity and high-performance SAIC algorithm that is suitable for co-channel signals with comparable powers and high-order modulation. In the last decades, near-optimal detectors with lower complexity have been proposed in multiple-input and multiple-output (MIMO) systems. Decision feedback equalization (DFE) and Successive Interference Cancellation (SIC) has been widely used in multiuser detection in MIMO and CDMA systems [11]-[15]. Recently, lattice reduction (LR) aided detection has proven to be an attractive solution for near optimal MIMO detection with low complexity [16].

In this paper, a new SAIC algorithm based on lattice reduction and DFE is thus proposed. Unlike existing works on SAIC, the contribution of our work has two aspects. One is to convert a single channel-received signal model to a virtual multiple channel signal model, which is fundamental in the proposed SAIC algorithm. The other is to design a signal detection method based on the LR algorithm and the DFE scheme. Essentially, the proposed SAIC algorithm is a nonlinear scheme, and can effectively cancel interference signals. In addition, our analysis and simulations show that the complexity of the proposed algorithm for the block fading channel is quadratic with the channel length and is independent of modulation order.

The remainder of this paper is organized as follows. The co-channel signal model is described in the next

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section, specifically the construction of the equivalent virtual multi-channel discrete signal model. The principles of LR and of the proposed SAIC method are presented in section III. The numerical results and analysis are provided in section IV. Conclusions are drawn in the final section.

II. Modeling of Co-Channel Signals with a Single Antenna Receiver

Given a scenario with multiple co-channel signals, $K-1$ co-channel interferers along with a desired user's signal are captured by a receiver equipped with an antenna. The received baseband co-channel signals can be expressed as

$$y(t)=\sum_{k=1}^{K}x_k(t)+v(t) \tag{1}$$

where $x_1(t)$ and $x_k(t), 2\le k\le K$ are the desired signal and the interferer signals, respectively, and $v(t)$ is the white Gaussian noise with variance σ^2. This noise is independent of the co-channel signals.

Co-channel signal $x_k(t)$ is a digital modulation signal and can be expressed as

$$x_k(t)=\sum_{l}s_k(l)h_k(t-lT_s),\ k=1,2,\cdots,K \tag{2}$$

where T_s is the symbol period and $s_k(l)$ is the transmitted modulation symbol sequence of the kth co-channel signal. The modulation type of $s_k(l)$ is restricted to $MQAM, M=4,8,16$. The modulation symbol alphabet is denoted by Φ, that is, $s_k(l)\in\Phi$. $h_k(t)$ is the equivalent channel impulse response, which consists of a pulse shaping filter, a physical transmission channel, and a receiving filter. Without loss of generality, the effective memory length of $h_k(t)$ is assumed to be *L* symbol periods.

The received signal $y(t)$ is over-sampled *P* times per symbol, that is, $P\ge 2$. The discrete time model can be represented as

$$\begin{aligned} y_i(n) &= y(t)\,|_{t=nT+iP/T_s} \\ &= \sum_{k=1}^{K}\sum_{l=0}^{L}s_k(n-l)h_{k,i}(l)+v_i(n),\quad 0\le i<P \end{aligned} \tag{3}$$

where $y_i(n)=y(nT_s+iT_s/P)$ is the discrete form of $y(t)$ and $h_{k,i}(l)=h_k(lT_s+iT_s/P)$ is the discrete equivalent channel. $v_i(n)=v(nT_s+iT_s/P)$ denotes the discrete channel noise. The noise power and $h_{k,i}(l)$ are presumably known at the receiver. The purpose of this study is to develop an effective method to derive the desired signal $s_1(n)$ from $y_i(n)$.

A simple method for detecting $s_1(n)$, based on the signal model (3), is to use the Wiener filter for its low complexity and optimality in the sense of the minimum mean square error (MMSE). Assuming that the filter length is *D*, thus the linear filter output can be expressed as

$$z(n)=\sum_{i=0}^{P-1}\sum_{k=0}^{D-1}a_{i,k}y_i(n-k) \tag{4}$$

The MSE of $z(n)$ is

$$MSE=E\{|z(n)-s_1(n-\tau)|^2\} \tag{5}$$

where τ is the filter delay. The filter coefficients $a_{i,k}$ can be obtained via the Wiener–Khinchin algorithm.

The estimation results are then given by

$$s_1(n-\tau)=q_\Phi(z(n)) \tag{6}$$

where $q_\Phi\{\cdot\}$ denotes quantization with respect to Φ.

It is well known that linear filter method is simple, but its performance is affected by the power of interference. Unfortunately, the linear filter method lowers the detection performance significantly when the powers of the co-channel signals are comparable. Hence, the direct use of the linear filter algorithm is unfavorable, and new methods should be developed.

III. Interference Cancellation Based on Multi-User Detection

To improve the detection performance, we investigate the LR-aided SAIC method in this section. LR-aided linear detection and nonlinear detection is widely studied in MIMO communications in recent years. However, these methods cannot be applied directly to the signal model expressed by (3). Thus, we need to convert this model from a scalar to a vector form.

A. Virtual Multiple Channel Signal Model

The signal model expressed by (3) can be transformed into the following vector form

$$\boldsymbol{y}(n)=\sum_{l=0}^{L}\mathbf{H}(l)\boldsymbol{s}(n-l)+\boldsymbol{v}(n) \tag{7}$$

where $\mathbf{H}(l)\triangleq(h_{k,i}(l))_{P\times K}$, $\boldsymbol{y}(n)\triangleq[y_1(n),\cdots,y_P(n)]^T$, $\boldsymbol{s}(n)\triangleq[s_1(n),\cdots,s_K(n)]^T$, $\boldsymbol{v}(n)\triangleq[v_1(n),\cdots,v_P(n)]^T$, and subscript *T* denotes transpose.

To detect $s_1(n-\tau)$, an observing window of the received signal $\{y_i(n),\ldots,y_i(n-D+1)\}$ is required according to equation (4). By denoting $\boldsymbol{Y}(n)$, $\boldsymbol{S}(n)$, and $\boldsymbol{V}(n)$ as

$$\begin{cases} \boldsymbol{Y}(n)\triangleq \begin{bmatrix}\boldsymbol{y}^T(n) & \boldsymbol{y}^T(n-1) & \cdots & \boldsymbol{y}^T(n-D+1)\end{bmatrix}^T \\ \boldsymbol{S}(n)\triangleq \begin{bmatrix}\boldsymbol{s}^T(n) & \boldsymbol{s}^T(n-1) & \cdots & \boldsymbol{s}^T(n-D-L+1)\end{bmatrix}^T \\ \boldsymbol{V}(n)\triangleq \begin{bmatrix}\boldsymbol{v}^T(n) & \boldsymbol{v}^T(n-1) & \cdots & \boldsymbol{v}^T(n-D+1)\end{bmatrix}^T \end{cases}$$

then formula (3) can be rewritten in the following vector form,

$$\boldsymbol{Y}(n) = \mathbf{H}\boldsymbol{S}(n) + \boldsymbol{V}(n) \tag{8}$$

where $\mathbf{H}$ is a $PD \times K(D+L)$ block-Toeplitz matrix, that is,

$$\boldsymbol{H} \triangleq \begin{bmatrix} \boldsymbol{H}(0) & \boldsymbol{H}(1) & \cdots & \boldsymbol{H}(L) & 0 & \cdots & 0 \\ 0 & \boldsymbol{H}(0) & \boldsymbol{H}(1) & \cdots & \boldsymbol{H}(L) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \boldsymbol{H}(0) & \boldsymbol{H}(1) & \cdots & \boldsymbol{H}(L) \end{bmatrix}$$

We regard equation (8) as the virtual multiple channel model or virtual MIMO model because this signal model is similar in form to that of multiple physical receiving channels, such as MIMO. Given the virtual multiple channel signal model, the SAIC problem may be treated as a delay-formed multi-channel MUD problem. In contrast to the Wiener filter method, we consider the detection of vector $\boldsymbol{S}(n)$, which contains the desired signal $s_1(n-\tau)$.

B. LR-aided Symbol Detection

The fundamental principle of LR-aided MIMO detection is explained as follows. For a given time *n*, equation (8) can be regarded as a $PD \times K(L+D)$ MIMO model, which can be expressed briefly as

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{S} + \boldsymbol{V} \tag{9}$$

where the matrix $\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2, \cdots, \boldsymbol{h}_{K(L+D)}]$ and each column of $\boldsymbol{H}$ can be considered as a basis for the $K(L+D)$ dimensional lattice. The transmitted M-ary quadrature amplitude modulation (MQAM) symbol can be regarded as a complex integer, and the symbol vector $\boldsymbol{S}$ belongs to the set $\mathbb{CZ}^{K(L+D)}$. This set is the $K(L+D)$ dimensional complex integer space. Hence, lattice ℓ generates all possible noiseless received signals

$$\ell \triangleq \left\{ \sum_{m=1}^{K(L+D)} \boldsymbol{h}_m z_m : z_m \in \mathbb{CZ}^{K(L+D)} \right\} \tag{10}$$

Any element of the lattice can be represented as $\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{Z}$ given $\boldsymbol{Z} \in \mathbb{CZ}^{K(L+D)}$.

Matrices $\boldsymbol{H}$ and $\boldsymbol{H}\boldsymbol{T}$ span the same lattice if and only if $\boldsymbol{T}$ is unimodular [17], i.e., the elements of matrix $\boldsymbol{T}$ are all integers and $\det(\boldsymbol{T}) = \pm 1$. If the lattice basis is better conditioned, the lattice-based linear detectors (e.g., zero-forcing (ZF) and MMSE detectors) are increasingly reliable. Thus, the main issue is how to find the matrix $\boldsymbol{T}$ so as to make $\boldsymbol{H}\boldsymbol{T}$ well conditioned. Fortunately, the complex Lenstra–Lenstra–Lovász (CLLL) algorithm [18] offers an excellent solution.

After obtaining $\boldsymbol{T}$ by performing the CLLL algorithm, formula (9) can be rewritten as

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{T}\boldsymbol{T}^{-1}\boldsymbol{S} + \boldsymbol{V} = \tilde{\boldsymbol{H}}\tilde{\boldsymbol{S}} + \boldsymbol{V} \tag{11}$$

where $\tilde{\boldsymbol{S}} = \boldsymbol{T}^{-1}\boldsymbol{S}$ is an integer symbol vector. Hence, we can detect $\tilde{\boldsymbol{S}} \in \mathbb{CZ}^{K(L+D)}$ based on $\tilde{\boldsymbol{H}}$ through the ZF or MMSE algorithm instead of detecting $\boldsymbol{S} \in \Phi$ based on $\boldsymbol{H}$. For linear ZF detection, the received signal $\boldsymbol{Y}$ is multiplied by the pseudo-inverse of matrix $\tilde{\boldsymbol{H}}$ to obtain the estimation of $\tilde{\boldsymbol{S}}$, that is,

$$\hat{\tilde{\boldsymbol{S}}} = \tilde{\boldsymbol{H}}^{+}\boldsymbol{Y} = \tilde{\boldsymbol{S}} + \tilde{\boldsymbol{H}}^{+}\boldsymbol{V} \tag{12}$$

Given the non-orthogonal channel matrix $\tilde{\boldsymbol{H}}$, ZF detection suffers from enhanced noise. MMSE detection minimizes the overall error containing noise and other interferers and can perform better than ZF detection. LR-aided MMSE detection (LR-MMSE) is achieved by applying LR to the extended channel matrix $\underline{\boldsymbol{H}}$ and the extended received vector $\underline{\boldsymbol{Y}}$ [19], that is,

$$\underline{\boldsymbol{H}} = \begin{pmatrix} \boldsymbol{H} \\ \sigma I_{K(L+D)} \end{pmatrix}, \qquad \underline{\boldsymbol{Y}} = \begin{pmatrix} \boldsymbol{Y} \\ \sigma I_{K(L+D)} \end{pmatrix}$$

The result of the LR-MMSE detection is

$$\hat{\tilde{\boldsymbol{S}}} = q\{\tilde{\underline{\boldsymbol{H}}}^{+}\underline{\boldsymbol{Y}}\} \tag{13}$$

where $\hat{\tilde{\boldsymbol{S}}} \in \mathbb{CZ}^{K(L+D)}$, $q\{\cdot\}$ denotes an integer rounding operation. The final detection result for the transmission symbols is then obtained by

$$\hat{\boldsymbol{S}} = q_{\Phi}\{\boldsymbol{T}\hat{\tilde{\boldsymbol{S}}}\} \tag{14}$$

Given that $\boldsymbol{S}$ contains $s_1(n-\tau)$, $\hat{s}_1(n-\tau)$ can thus be obtained directly from $\hat{\boldsymbol{S}}$.

C. Application of DFE to the Symbol Detection Process

According to Subsection B, LR-aided linear detection can be applied directly in SAIC based on the virtual MIMO model. However, we notice that in this virtual multiple channel model, the input symbol vector $\boldsymbol{S}(n) = (\boldsymbol{s}^T(n) \ \cdots \ \boldsymbol{s}^T(n-L-D+1))^T$ has a peculiar structure; that is, $\boldsymbol{S}(n)$ is composed of $\boldsymbol{s}(n)$ and its *L*+*D*-1 delay versions, $\boldsymbol{s}(n-k)$, $k = 1, 2, \cdots, L+D-1$. A natural problem brought by above observation is how to utilize the peculiar structure of $\boldsymbol{S}(n)$ to improve detection performance.

Noticing that $\boldsymbol{s}(n-1), \cdots, \boldsymbol{s}(n-L-D+1)$ are the time delay versions of $\boldsymbol{s}(n)$, a decision feedback mechanism can be adopted to improve detection performance and to reduce complexity. We divide $\boldsymbol{S}(n)$ into two parts, namely, the forward-moving part $\boldsymbol{S}_f(n)$ and feedback part $\boldsymbol{S}_b(n)$ as follows

$$\boldsymbol{S}_f(n) \triangleq \left[\boldsymbol{s}^T(n), \ldots, \boldsymbol{s}^T(n-\tau) \right]^T$$

$$\boldsymbol{S}_b(n) \triangleq \left[\boldsymbol{s}^T(n-\tau-1), \cdots, \boldsymbol{s}^T(n-L-D+1) \right]^T$$

At time index n, $\boldsymbol{S}_b(n)$ is detected by the previous data vector $\boldsymbol{Y}(n-1),\cdots,\boldsymbol{Y}(n-L)$. Thus, the detection results can be used to cancel the ISI of $\boldsymbol{s}(n-\tau-1),\cdots,\boldsymbol{s}(n-L-D+1)$ in $\boldsymbol{Y}(n)$. Based on the reduced ISI data vector, a reliable $\hat{\boldsymbol{s}}(n-\tau)$ can be obtained through LR-based detection methods such as LR-MMSE.

According to the aforementioned description, expression (9) can be expressed as

$$\boldsymbol{Y}'(n)=\boldsymbol{Y}(n)-\boldsymbol{H}_b\hat{\boldsymbol{S}}_b(n)=\boldsymbol{H}_f\boldsymbol{S}_f(n)+\boldsymbol{V}(n) \tag{15}$$

where $\boldsymbol{H}_b$ and $\boldsymbol{H}_f$ are the feedback and feed-forward channel matrices, respectively. That is,

$$\boldsymbol{H}_f \triangleq \boldsymbol{H}(:,1:K(\tau+1))$$

$$\boldsymbol{H}_b \triangleq \boldsymbol{H}(:,K(\tau+1)+1:K(D+L))$$

$\hat{\boldsymbol{S}}_b(n)$ is the feedback symbol vector obtained from the previous symbol detection process.

From expression (15), it can be known that $\boldsymbol{Y}'(n)$ equals $\boldsymbol{Y}(n)-\boldsymbol{H}_b\hat{\boldsymbol{S}}_b(n)$, which means that the part ISI is eliminated. Therefore, detecting $\boldsymbol{S}_f(n)$ based on $\boldsymbol{Y}'(n)$ is more reliable than the using $\boldsymbol{Y}(n)$.

D. LR-DFE Algorithm for SAIC

According to Subsection B and C, $\hat{\boldsymbol{S}}_f(n)$ can be derived from $\boldsymbol{Y}'(n)$ by using the method discussed in Subsection A. That is, if $\tilde{\boldsymbol{H}}_f=\boldsymbol{H}_f\boldsymbol{T}$ is the reduced basis, then $\tilde{\boldsymbol{S}}_f(n)=\boldsymbol{T}^{-1}\boldsymbol{S}_f(n)$.

To improve detection performance further, we apply a successive interference cancellation algorithm [20] to detect $\tilde{\boldsymbol{S}}_f(n)$. By conducting QR-decomposition on matrix $\tilde{\boldsymbol{H}}_f$, $\tilde{\boldsymbol{H}}_f=\mathbf{QR}$, we then obtain

$$\hat{\tilde{\boldsymbol{S}}}_f(n)=Q^T\boldsymbol{Y}'(n)=R\tilde{\boldsymbol{S}}_f(n)+Q^TV(n) \tag{16}$$

Due to the up-triangular structure of the *R*, elements of $\tilde{\boldsymbol{S}}_f(n)$ can be detected successively through interference cancellation. Once $\hat{\tilde{\boldsymbol{S}}}_f(n)$ is determined, the desired estimation result $\hat{\boldsymbol{s}}(n-\tau)$ can be obtained from $\hat{\boldsymbol{S}}_f(n)$ by

$$\hat{\boldsymbol{S}}_f(n)=q_\Phi\{\boldsymbol{T}\hat{\tilde{\boldsymbol{S}}}_f(n)\} \tag{17}$$

Because $\hat{\boldsymbol{S}}_f(n)$ contains $\boldsymbol{s}(n-\tau),\cdots,\boldsymbol{s}(n-L-D)$, the LR-based method detects $\boldsymbol{s}(n-\tau),\cdots,\boldsymbol{s}(n-L-D)$ simultaneously. The direct solution is to select $\hat{\boldsymbol{s}}(n-\tau)$ as the estimation of $\boldsymbol{s}(n-\tau)$ and discard $\boldsymbol{s}(n-\tau-1),\cdots,\boldsymbol{s}(n-L-D)$. Clearly, this method is suboptimal.

Given the peculiar structure of $\hat{\boldsymbol{S}}_f(n)$, $\hat{\boldsymbol{s}}(n-\tau)$ can be detected $L+D-\tau$ times by using $\boldsymbol{Y}'(n-L-D+\tau),\ldots,\boldsymbol{Y}'(n-1)$. However, some estimation results, e.g., $\hat{\boldsymbol{s}}(n-\tau)\mid_{\boldsymbol{Y}'(n-L-D+\tau)}$, are unreliable because these results contain insufficient information regarding $\boldsymbol{s}(n-\tau)$. Given that $\boldsymbol{Y}'(n-n_0),\ldots,\boldsymbol{Y}'(n-1)$ contain enough information regarding $\boldsymbol{s}(n-\tau)$, the corresponding estimation results can be used to improve the estimation result of $\boldsymbol{s}(n-\tau)$. In this case, n_0 denotes the number of effective detection results, and n_0 can be selected according to $\boldsymbol{H}_f$, $n_0<\tau$.

The optimal solution can be determined by combining the estimation results according to their detection probability. However, it is difficult to calculate the detection probability. Hence, we provide a low-complexity, suboptimal estimation of $\boldsymbol{s}(n-\tau)$

$$\hat{\boldsymbol{s}}'(n-\tau)=q_\Phi\{\frac{1}{n_0+1}\sum_{k=0}^{n_0}\hat{\boldsymbol{s}}(n-\tau)\mid_{\boldsymbol{Y}'(n-k)}\} \tag{18}$$

where $\hat{\boldsymbol{s}}(n-\tau)\mid_{\boldsymbol{Y}'(n-k)}$ represents the estimation of $\boldsymbol{s}(n-\tau)$ through using $\boldsymbol{Y}'(n-k)$.

Based on the analysis and discussion above, the LR-DFE SAIC algorithm is described in Table I.

TABLE I: THE LR-DFE SAIC ALGORITHM

Input: $y_i(n), 0\le n\le N$, τ, D, n_0

Output: $\hat{\boldsymbol{s}}(n)$

Initialize: $\hat{\boldsymbol{s}}(n)=0, n<0$

1. $\left[\underline{\tilde{\boldsymbol{H}}}_f,\boldsymbol{T}\right]\triangleq CLLL(\underline{\boldsymbol{H}}_f)$ //lattice reduction of the extend feed-forward channel matrix
2. **for** $n=\tau:N$
3. $\hat{\boldsymbol{S}}_b(n)\triangleq \left[\hat{\boldsymbol{s}}^T(n-\tau-1),\cdots,\hat{\boldsymbol{s}}^T(n-L-D+1)\right]^T$,
   $\boldsymbol{Y}'(n)\triangleq\boldsymbol{Y}(n)-\boldsymbol{H}_b\hat{\boldsymbol{S}}_b(n)$, $\underline{\boldsymbol{Y}}'(n)\triangleq\begin{bmatrix}\boldsymbol{Y}'(n)\\0_{1,K(\tau+1)}\end{bmatrix}$
   // get the extended feed-forward vector
4. $\hat{\tilde{\boldsymbol{S}}}_f(n)=q\{SIC(\underline{\boldsymbol{Y}}'(n))\}$
   // SIC detection on the lattice field
5. $\hat{\boldsymbol{S}}_f(n)=q_\Phi\{\boldsymbol{T}\hat{\tilde{\boldsymbol{S}}}_f(n)\}$
   //transform the detection result to the original field
6. $\hat{\boldsymbol{s}}(n-\tau)=q_\Phi\{\frac{1}{n_0+1}\sum_{k=0}^{n_0}\hat{\boldsymbol{s}}(n-\tau)\mid_{\boldsymbol{Y}'(n-k)}\}$
7. **end for**
8. **return** the estimated solution $\hat{\boldsymbol{s}}(n)$

Remark 1: The principle of SIC is similar to that of DFE. Hence, the proposed algorithm is mainly composed of two nested DFEs, which is the reason the proposed algorithm is named as the LR-DFE algorithm. The inner SIC detection is based on the transformed lattice basis $\underline{\widetilde{\boldsymbol{H}}}$ and can improve the estimation performance of $\widehat{\widetilde{\boldsymbol{S}}}_f(n)$. Given $\widehat{\boldsymbol{S}}_f(n) = \boldsymbol{T}\widehat{\widetilde{\boldsymbol{S}}}_f(n)$, thus the performance of $\widehat{\boldsymbol{S}}_f(n)$ is also enhanced. In addition, an improved estimation of $\widehat{\boldsymbol{S}}_f(n)$ implies that fewer error propagation appears in the outer DFE process.

Remark 2: The real and imaginary parts of the modulation symbols for MQAM are obtained from the set $\{-(\sqrt{M}-1),\cdots,-1,1,\cdots,\sqrt{M}-1\}$, which is not a consecutive integer set. Therefore, we should transfer the symbol constellation to a consecutive integer set at the beginning of the algorithm. The implementation of the transformation is detailed in Ref. [21].

Remark 3: The over-sampling parameter *P* is an important parameter in the proposed algorithm. The over-sampling factor $P \geq 2$ is necessary to fully utilize the excess bandwidth. From the perspective of the Nyquist theorem, *P* should be larger than 2. However, *P* should be equal or larger than the number of co-channel signals to construct an over-determined virtual MIMO system. Therefore, $P = \max\{2, K\}$ is suitable for practical applications.

E. Channel Estimation

Channel state information (CSI) and noise power is presumably known in the proposed algorithm. However, CSI should be estimated in practical systems. Channel estimation is more difficult in multiple-user scenario than in a single user scenario because more channel coefficients must be estimated. Given a slow fading channel, channels can be estimated via a joint least-square channel estimator when the training symbols of all the co-channel users are known to the receiver.

Assume that N_t is the number of training symbols of each user, $\boldsymbol{S}_t = [\boldsymbol{s}_{L:1}, \boldsymbol{s}_{L+1:2}, \ldots, \boldsymbol{s}_{N_t:N_t-L+1}]$ is the training symbol matrix that consists of the training symbols of all users, $\boldsymbol{Y}_t = [\boldsymbol{y}(L), \boldsymbol{y}(L+1), \ldots, \boldsymbol{y}(N_t)]$ is the received data matrix, and $\boldsymbol{H}_t \triangleq [\boldsymbol{H}(0), \boldsymbol{H}(1), \ldots, \boldsymbol{H}(L)]$ is the channel matrix to be estimated. According to equation (7), we can obtain the following equation

$$\boldsymbol{Y}_t = \boldsymbol{H}_t \boldsymbol{S}_t + \boldsymbol{V} \tag{19}$$

The least squares estimation of $\boldsymbol{H}_t$ is

$$\widehat{\boldsymbol{H}}_t = \boldsymbol{Y}_t \boldsymbol{S}_t^H (\boldsymbol{S}_t \boldsymbol{S}_t^H)^{-1} \tag{20}$$

Given the estimated channel, noise power can be easily determined via the power of the received signal minus the signal power of all users.

In some cases, the receiver can only acquire knowledge on the training sequence of the desired signal but none regarding the interferences. This scenario can be regarded as a semi-blind CSI estimation problem, as discussed in Ref. [22].

F. Complexity Analysis

The main computational complexity of the proposed algorithm lies in the preprocessing procedure and the detection procedure. The rough analysis of the complexity is as follows.

The preprocessing procedure includes the CLLL algorithm and QR decomposition. Notice that QR decomposition can be done in CLLL algorithm. Given that the size of $\boldsymbol{H}_f$ is $PD \times K(\tau+1)$, the average complexity of the CLLL algorithm is therefore $O(KP^3D^3(\tau+1)\log 2PD)$ [18].

The detection procedure contains three parts: SIC detection (i.e. $SIC(\underline{\boldsymbol{Y}}'(n))$), matrix multiplication (i.e. $\boldsymbol{T}\widehat{\widetilde{\boldsymbol{S}}}_f(n)$) and feedback process (i.e. $\boldsymbol{H}_b\widehat{\boldsymbol{S}}_b(n)$). For simplicity, we ignore the cost of rounding. Thus, the complexity of the detection procedure for each symbol is approximately $O(KPD(D+L))$.

For a slow or block fading channel, the channel can be regarded as time-invariant over a finite data block. Thus the preprocessing procedure need only be executed once in a data block. However, the detection procedure has to be executed for every symbol in a data block. Therefore, the average complexity for detecting each symbol is $O((KP^3D^3(\tau+1)\log 2PD)/N_d) + O(KPD(D+L))$, where N_d is the number of symbols in a data block.

Notice that the size of observing window *D* is mainly decided by channel length *L*, and *D* is usually slightly larger than *L*. Therefore, the complexity of the LR-DFE algorithm is quadratic with the memory length of the channel *L* and is nearly independent of the modulation order *M*. By contrast, the complexity of the JMLSE method is $O(M^{2L})$, which is exponentially increased with channel length *L*. Hence, the complexity of the proposed algorithm is much lower than that of the JMLSE method, especially in high-order modulation. The numerical comparison of the complexity for the some given parameters is presented in the next section.

IV. Numerical Results

The performance of the proposed LR-DFE algorithm was evaluated via numerical simulation using QPSK and 16QAM modulated signals (QPSK signal can be seen as a phase shifted 4QAM signal). Simulations were performed in Matlab. A desired signal and an interference signal are considered. A slow-fading frequency-selective channel similar to that in Ref. [8] is employed. $D = 6$, $n_0 = 2$, and $\tau = 5$ is set for the simulation. The desired user and the interference are similar in terms of pulse shape, symbol alphabet and transmission power. The desired signal-to-noise (SNR) power ratio is defined as $SNR = E_s / N_0$, where E_s denotes the average received energy per symbol of the desired signal and N_0 is the spectrum density of noise.

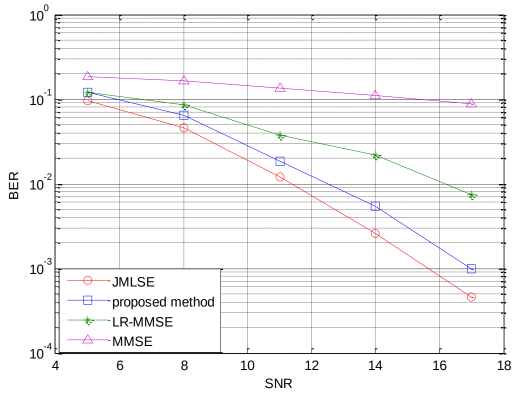

Fig. 1. BER performance vs. SNR for co-channel QPSK signals with perfect CSI

Fig. 1 plots bit error rate (BER) performance as a function of SNR for two QPSK co-channel signals. This performance is compared with that of the JMLSE [5] and MMSE detectors. The MMSE detector is a linear detection method that applies the MMSE detection algorithm directly to formula (5). The direct MMSE method can be regarded as a linear filter method that mitigates CCI and ISI. The JMLSE detector can achieve optimal performance using a joint Viterbi algorithm. Given the low SINR in this instance (SINR < 0 dB), the BER of MMSE is high. However, an improved channel matrix is obtained by utilizing the LR algorithm. Therefore, the LR-MMSE outperforms MMSE. Nonetheless, the proposed LR-DFE algorithm performs better than LR-MMSE. According to Fig. 1, the SNR loss when the LR-DFE algorithm is used is approximately 1.5 dB at a BER of 10^{-3}, unlike that when JMLSE is employed. Given the significant reduction in computational complexity, the proposed method is highly suitable for realistic application.

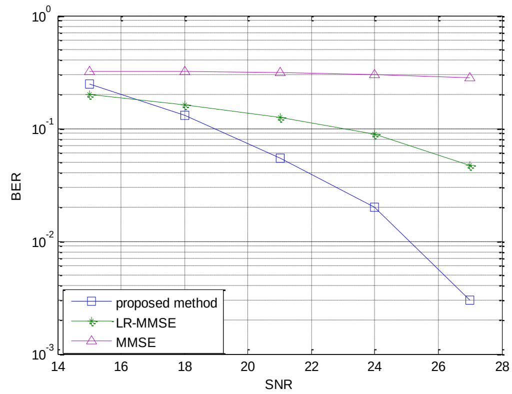

Fig. 2. BER performance vs. SNR for co-channel 16QAM signals with perfect CSI

Fig. 2 depicts BER performance as a function of SNR for two 16QAM co-channel signals. LR and decision feedback can considerably improve the performance of the MMSE detector. However, a high SNR is still required for the 16QAM co-channel signals. Because of a huge time consuming for the JMLSE detector in 16QAM case, JMLSE performance has not been obtained through simulation.

According to parameters set in simulations and complexity analysis in section III, Table II gives the average number of floating-point operations (flops) of three different methods for QPSK and 16QAM co-channel signals, where the number of flops equals 2 for complex addition and 6 for multiplication. GSM burst structure with $N_d = 61$ data symbols is used. From the Table II, we notice that though the complexity of the proposed LR-DFE method is 6 times of that of the MMSE method, the dramatic performance improvement is provided by the LR-DFE method (see Fig. 1 and Fig. 2). On the other hand, compared to the JMLSE method, the complexity of LR-DFE method is significantly reduced with small performance degradation.

TABLE II: THE NUMERICAL COMPARISON OF AVERAGE COMPLEXITY FOR DETECTING EACH SYMBOL (IN FLOPS)

	LR-DFE methtod	MMSE method	JMLSE method
M=4	3645.8	635.4	16432
M=16	3645.8	635.4	262336

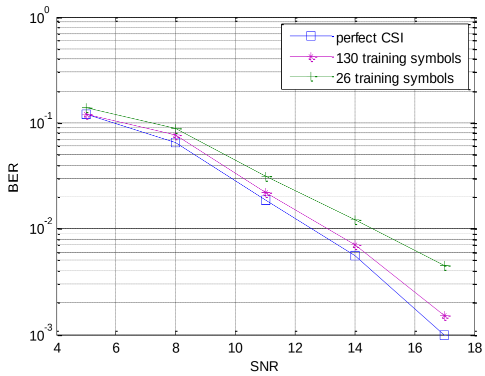

Fig. 3. BER performance vs. SNR for co-channel QPSK signals with channel estimation

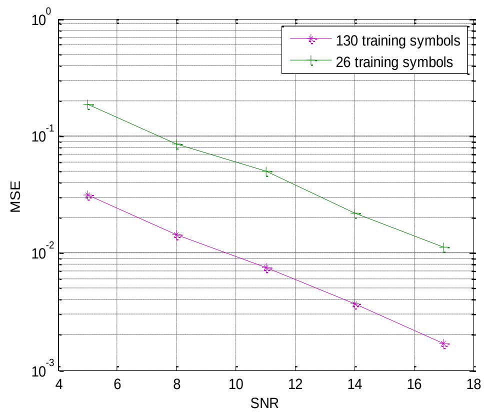

Fig. 4. MSE channel estimation vs. SNR for co-channel QPSK signals

Fig. 3 displays the BER performance of the proposed LR-DFE method with channel estimation, whereas Fig. 4 illustrates the performance of the channel estimator with different training symbols. The BER performance of the proposed LR-DFE method with 130 training symbols is nearly similar to that of the perfect CSI. Given 26 training symbols, the performance loss is about 2dB when the BER is equal to 10^{-2}. Therefore, the proposed LR-DFE algorithm is robust with respect to channel errors.

V. Conclusions

In this paper, we investigated SAIC in terms of lattice reduction. LR can be applied to SAIC based on the virtual MIMO model, which is constructed using the over-sampled received signal. Given the special construction of the virtual MIMO model, a low-complexity SAIC algorithm, i.e., the LR-DFE algorithm, was developed to effectively reduce the effects of CCI and ISI. The proposed algorithm is based on LR and the DFE scheme; thus, the complexity increases only quadratically with channel length and is nearly independent of the modulation order. The BER performance was compared with that of the traditional MMSE, JMLSE SAIC methods through computer simulation. The simulation results showed a loss of only 1.5 dB occurs for the QPSK signals at a BER 10^{-3}, compared to JMLSE method. Furthermore, the complexity is significantly reduced. In addition, the proposed algorithm is considerably superior to the MMSE method for the QPSK and 16QAM modulation signals. Finally, the simulations demonstrated the robustness of the proposed algorithm with respect to channel estimation error.

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