Single Antenna Interference Cancellation for 8PSK Signals in EGPRS

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***Abstract*—In this paper, we explore a method for interference suppression in EGPRS (also commonly known as EDGE) systems, wherein 8PSK is used as the modulation for the desired signal and the interferer is modulated using GMSK. Previous methods for single antenna interference cancellation (SAIC) have concentrated on the case when both the desired and interfering signals are GMSK modulated, or have relied on highly complex methods such as joint demodulation. We derive a simple method that exploits the redundancy inherent in the interferer signal to suppress it. The interference suppression method causes a well-modeled nonlinear distortion to the 8PSK signal. This non-linear distortion is handled using a modified equalizer that deals with a time-varying input symbol constellation applied to a matrix channel. Simulation results using the method show great promise.**

I. INTRODUCTION

Interference cancellation for GSM systems has received significant attention in recent years as a means to increasing the system capacity. In particular, emphasis has been placed on Single Antenna Interference Cancellation (SAIC) methods since they can perform the function of interference suppression without the use of multiple receive antennas, thereby saving cost on the handsets. SAIC has been studied by standardization bodies [1], and performance requirements that require the use of such receivers have been set [2]. The specification concentrates on the case when the signal is GMSK modulated, as is the case with GSM voice, circuit-switched data, and packet data services using GPRS. With EGPRS or EDGE, GMSK and 8PSK modulations are permitted, but most SAIC methods concentrate on the case when the desired signal and the interferer are GMSK modulated. In such a case, the interferer modulation is essentially real-valued, and this can be exploited by the receiver to cancel it.

A number of publications have explored different methods for SAIC. In [3], the method essentially treats the real and imaginary parts of the signal as antenna branches and uses the correlation of the signal on the two branches to suppress the interference. The authors in [3] calculate performance bounds for interference cancellation, but do not explicitly evaluate the performance of a receiver. The one-dimensional nature of the GMSK interferer lends itself to a signal that is highly correlated among the two virtual antenna branches, and thus lends itself to suppression via a matrix filter. An allusion to a similar principle can be found in [4]. The same approach has been used in [5] to cancel a general interferer using multiple receive antennas. A similar approach is used in [6], where a widely linear filter, that operates on the signal and its complex conjugate is used to suppress the interference. In [7], the authors use a filter to project the interferer signal onto one dimension, and use the signal along the orthogonal dimension for further equalization to recover the desired signal. Again, the one-dimensional nature of the interferer signal lends itself well to the projection operation. In [8], [9], and in many other publications, joint detection is used as a method to perform SAIC. Finally, [10] and [11] have exploited the constant envelope nature of the GMSK interferer to perform interference suppression.

Link and System performance of SAIC has been evaluated by a number of authors. In [12] and [13], the system performance of a SAIC algorithm is evaluated in terms of improvement in terms of voice and data capacity of the system. Link and system performance results are provided in [4]. Similar evaluations are found in [14] and [15].

As stated earlier, most of the literature has concentrated on cases where both the desired signal and the interferer signal are GMSK modulated. Some authors have considered the case when the desired signal is 8PSK modulated. In [16] and [9], joint detection of the desired and interferer signals is used to suppress the interferer with an 8PSK-modulated desired signal. The joint detection method exhibits significantly higher complexity than the SAIC methods that exploit the 1-D nature of the GMSK interferer, and this complexity is considerably increased with 8PSK modulated signals. In [10], the constant envelope nature of the GMSK interferer is exploited to perform interference suppression even for 8PSK-modulated desired signals. However, the method used in [10] does not work well when the interferer passes through a dispersive channel due to the fact that the assumption of a constant envelope does not hold. In [17] and [18], dual antennas are used at the mobile station to suppress interference for the 8PSK mode of EGPRS.

It is clear that the one-dimensional nature of the GMSK interference is what enables most SAIC methods. If the interferer signal is GMSK modulated, a receiver should be able to exploit the structure inherent in this signal to suppress the interference, irrespective of the modulation of the desired signal. Thus, in this paper, we examine a method to cancel GMSK interference when the desired signal is 8PSK modulated. We will exploit the one-dimensional nature of the GMSK interferer to perform such interference suppression, this is in contrast to [10], wherein the constant envelope property of the interferer signal is used. In GSM/EGPRS, the GMSK and 8PSK modulations use different progressive rotation factors to enable blind detection of modulation. When SAIC methods that exploit the 1-D nature of GMSK are used for interference suppression, a non-linear distortion accrues on the remaining 8PSK signal due to residual rotation, and any postprocessing will need to handle this distortion. In this paper, we adapt the virtual multiple antenna method analogous to that used in [3] and [4] to perform interference suppression. We find that the residual distortion, which has intersymbol interference, is handled using a modified equalizer that deals with a time-varying input constellation, and a matrix channel response. We concentrate on the DFSE equalizer due to its reduced complexity [19]. However, the DFSE requires compaction of the channel impulse response in order to minimize the effects of error propagation. The projective method [7] is also amenable to cancellation of GMSK interference with an 8PSK signal.

In an EGPRS system, packet data can be modulated via GMSK or 8PSK, and the interferer could be a GSM voice, circuit-data, GPRS or EDGE (GMSK or 8PSK signal), and the interferer could vary from frame to frame due to frequency hopping. Thus, a significant number of situations will encounter an 8PSK-modulated desired signal and a GMSK modulated interferer. There will also be cases when both the desired and interferer signals are 8PSK modulated, the method in this paper does not apply in that case. Also, in EGPRS, a coded radio block is interleaved over four radio frames. With frequency hopping, it is likely that a different interferer will be encountered on each radio frame. Even if interferer suppression is done only when the interferer is GMSK modulated (which is expected to be a majority), it is clear that a significant improvement in performance can be obtained at both the link level and at the system level. In this paper, a system level evaluation has not been performed, but is an avenue for future research.

The rest of the paper is organized as follows: In Section II, we modify the virtual antenna method to derive the main algorithm for interference suppression. In Section III, we present some preliminary link simulation results for the virtual multiple antenna method, and compare them with the performance of conventional receivers. A significant gain in link performance is observed. We conclude in Section IV. In the appendix, we present a treatment of how the projection method can be adapted to perform the required interference suppression.

II. Derivation of Interference Cancellation Method

In EGPRS, two modulation schemes are used: GMSK, which can be approximated as a $\frac{\pi}{2}$-shifted BPSK scheme [20], and $\frac{3\pi}{8}$-shifted 8PSK. The transmitted signal in each case can be approximated as:

$$\begin{aligned} t_{GMSK}(n) &= i(n)e^{\frac{j\pi n}{2}}, i(n) = \pm 1 & (1)\\ t_{8PSK}(n) &= s(n)e^{\frac{j3\pi n}{8}}, s(n) = e^{\frac{j2\pi m}{8}}, m = 0, \ldots 7 & (2)\end{aligned}$$

When the desired signal is 8PSK-modulated and the interferer is GMSK modulated, the received signal consists of an 8PSK desired signal and a GMSK interferer. It is approximated by

$$\begin{aligned} r(n) &= \sum_{k=0}^{L-1} \tilde{h}(k)s(n-k)e^{\frac{j3\pi(n-k)}{8}} \\ &\quad + \sum_{k=0}^{L-1} \tilde{g}(k)i(n-k)e^{\frac{j\pi(n-k)}{2}} + w(n), \end{aligned} \tag{3}$$

where $\tilde{h}(k)$ is the impulse response of the channel encountered by the desired signal, $s(n) \in \{e^{\frac{j2\pi m}{8}}\}$, $\tilde{g}(k)$ is the impulse response of the channel encountered by the interferer signal and $i(n) \in \{+1, -1\}$, and $w(n)$ is additive noise. By a rearrangement of terms, and defining $h(k) = \tilde{h}(k)e^{\frac{-j3\pi k}{8}}$, $g(k) = \tilde{g}(k)e^{\frac{-j\pi k}{2}}$, we get

$$\begin{aligned} r(n) &= \{\sum_{k=0}^{L-1} h(k)s(n-k)\}e^{\frac{j3\pi n}{8}} \\ &\quad + \{\sum_{k=0}^{L-1} g(k)i(n-k)\}e^{\frac{j\pi n}{2}} + w(n). \end{aligned} \tag{4}$$

In order to reveal the one-dimensional nature of the interferer's GMSK signal, a progressive rotation $e^{\frac{-j\pi n}{2}}$ is applied to the received signal. Thus, the rotated received signal is given by

$$\begin{aligned} t(n) &= r(n)e^{\frac{-j\pi n}{2}} \\ &= \{\sum_{k=0}^{L-1} h(k)s(n-k)\}e^{\frac{-j\pi n}{8}} \\ &\quad + \sum_{k=0}^{L-1} g(k)i(n-k) + u(n) & (5)\\ &= \sum_{k=0}^{L-1} h(k)e^{\frac{-j\pi k}{8}} s(n-k)e^{\frac{-j\pi(n-k)}{8}} \\ &\quad + \sum_{k=0}^{L-1} g(k)i(n-k) + u(n) & (6)\\ &= \sum_{k=0}^{L-1} \hat{h}(k)\hat{s}(n-k) \\ &\quad + \sum_{k=0}^{L-1} g(k)i(n-k) + u(n), & (7)\end{aligned}$$

where

$$\hat{h}(k) = h(k)e^{\frac{-j\pi k}{8}} \tag{8}$$
$$\hat{s}(n) = s(n)e^{\frac{-j\pi n}{8}}. \tag{9}$$

The real and imaginary parts of $t(n)$ are treated as signals received on two different virtual antennas, the correlation of the interfering signal is exploited to cancel the interferer. A linear multidimensional filter is used to suppress the vector interference signal. After interference suppression, we find that the desired 8PSK signal is nonlinearly distorted due to the residual rotation on it, and we will describe methods by which this can be handled in an equalizer that follows the interference cancellation mechanism.

We rewrite equation 7 in matrix form to obtain

$$\begin{bmatrix} t_I(n) \\ t_Q(n) \end{bmatrix} = \sum_{k=0}^{L-1} \begin{bmatrix} \hat{h}_I(k) & -\hat{h}_Q(k) \\ \hat{h}_Q(k) & \hat{h}_I(k) \end{bmatrix} \begin{bmatrix} \hat{s}_I(n-k) \\ \hat{s}_Q(n-k) \end{bmatrix} + \sum_{k=0}^{L-1} \begin{bmatrix} g_I(k) \\ g_Q(k) \end{bmatrix} i(n-k) + \sum_{k=0}^{L-1} \begin{bmatrix} u_I(n) \\ u_Q(n) \end{bmatrix}. \tag{10}$$

This equation can be rewritten using a matrix representation of the symbols and a vector representation of the rotation. This yields

$$\mathbf{t}(n) = \sum_{k=0}^{L-1} \mathbf{H}(k)\mathbf{S}(n-k)\theta(n-k) + \sum_{k=0}^{L-1} \mathbf{g}(k)i(n-k) + \mathbf{u}(n) \tag{11}$$

where

$$\mathbf{t}(n) = \begin{bmatrix} t_I(n) \\ t_Q(n) \end{bmatrix} \tag{12}$$
$$\mathbf{H}(k) = \begin{bmatrix} \hat{h}_I(k) & -\hat{h}_Q(k) \\ \hat{h}_Q(k) & \hat{h}_I(k) \end{bmatrix} \tag{13}$$
$$\theta(k) = \begin{bmatrix} \cos(\frac{\pi k}{8}) \\ \sin(\frac{\pi k}{8}) \end{bmatrix} \tag{14}$$
$$\mathbf{S}(n) = \begin{bmatrix} s_I(n) & -s_Q(n) \\ s_Q(n) & s_I(n) \end{bmatrix} \tag{15}$$
$$\mathbf{g}(k) = \begin{bmatrix} g_I(k) \\ g_Q(k) \end{bmatrix} \tag{16}$$
$$\mathbf{u}(n) = \begin{bmatrix} u_I(n) \\ u_Q(n) \end{bmatrix}. \tag{17}$$

The vector impairment signal $\mathbf{q}(n)$ is given by

$$\mathbf{q}(n) = \sum_{k=0}^{L-1} \mathbf{g}(k)i(n-k) + \mathbf{u}(n), \tag{18}$$

where $i(n)$ are real-valued. As indicated in [3] and [5], the vector impairment signal can suppressed using a 2-D filter. We denote $\mathbf{A}(n),\ n = 0, \ldots, M-1$ as the matrix coefficients of the 2-D filter that is so used. When $\mathbf{t}(n)$ is passed through this 2-D filter, the resultant output can be expressed as

$$\mathbf{v}(n) = \sum_{m=0}^{Q-1} \mathbf{P}(m)\mathbf{S}(n-m)\theta(n-m) + \mathbf{x}(n) \tag{19}$$

where $\mathbf{x}(n)$ denotes the whitened impairment signal and $\mathbf{P}(n)$ is obtained by the matrix convolution of $\mathbf{A}(n)$ and $\mathbf{H}(n)$.

Once the equivalent *matrix* channel $\mathbf{P}(n)$ is known, a trellis-based equalizer can be executed using postulates of the symbols. The metric is given by

$$M = \mathbf{e}^T\mathbf{e} \tag{20}$$
$$\mathbf{e} = \mathbf{v}(n) - \hat{\mathbf{v}}(n) \tag{21}$$
$$\hat{\mathbf{v}}(n) = \sum_{m=0}^{Q-1} \mathbf{P}(m)\tilde{\mathbf{S}}(n-m)\theta(n-m) \tag{22}$$

where $\tilde{\mathbf{S}}$ are hypotheses of the symbol sequence. Note that the state space of the equalizer is completely governed by the cardinality of the $\tilde{\mathbf{S}}$, which is 8 in this case, and the length of the equivalent channel Q, as is the case for the conventional equalizer. Also, the $\tilde{\mathbf{S}}$ are just an abstraction of the actual symbols themselves.

In order to use a trellis-based equalizer of reduced complexity such as a DFSE, it is necessary to constrain the 'energy' of $\mathbf{P}(m)$ among the first few 'taps', the aim being to minimize the effects of error propagation due to erroneous past symbols. Since $\mathbf{P}(m)$ is a matrix channel, this suggests that a matrix prefilter be used. Many matrix prefilters have been evaluated in the literature. For example, [21] explores channel-shortening prefilters for MIMO channels, [22] explores a minimum-phase prefilter for MIMO channels, and [23] explores a MIMO prefilter that maximizes the SNR. Any of these prefilters may be used to constrain the energy of the matrix impulse response.

III. PERFORMANCE

In this section, we evaluate the performance of the Virtual Antenna Method for GMSK interference cancellation with an 8PSK desired signal. As per the EGPRS standards [24], the desired signal is 8PSK modulated with a midamble of 26 symbols. We consider the cases of one interferer and two interferers with a dominant interferer ratio (DIR) of 10 dB. The interferer symbols are random, which is intended to model an asynchronous system. The radio channel for the desired signal and the interferers is assumed to be a Typical Urban channel with a vehicular speed of 3 kph, i.e., a TU3 channel. The modulation and coding scheme used is MCS-7, which employs a coding rate of about 3/4. The interference canceling receiver also uses 2x oversampling of the received signal. We use practical algorithms for synchronization and parameter estimation, including the channel response of the desired signal and the matrix filter used to suppress the interference. We use a minimum phase matrix prefilter [22] for compaction of the impulse response of the signal after interference suppression. The resultant signal after interference suppression is passed through a modified DFSE-based equalizer that uses two MLSE

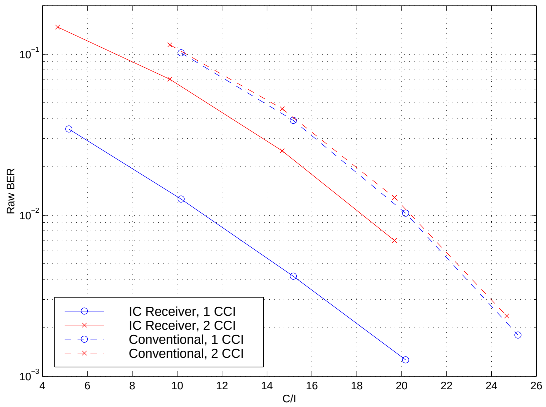

Fig. 1. Performance of the Interference Cancellation Method in TU3 Channel: BER Results

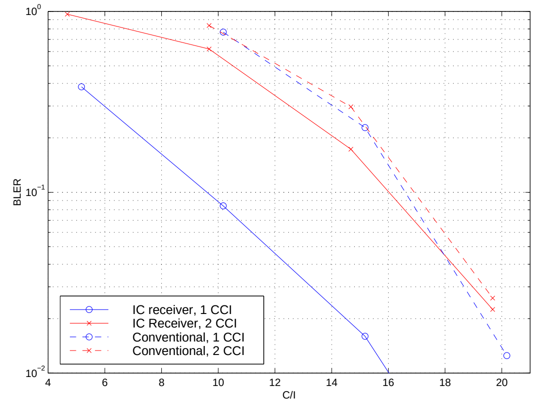

Fig. 2. Performance of the Interference Cancellation Method in TU3 Channel: BLER Results for MCS-7

taps. For comparison purposes, we also simulate the performance of a conventional receiver that treats the interference as noise. This receiver also uses a DFSE-based equalizer that models two MLSE taps, and uses a prefilter for compaction of the channel response.

In Figure 1, we show the raw bit error rate (BER) vs. the carrier-to-interference (C/I) ratio for the interference canceling receiver and for the the conventional receiver. We observe that the Interference Canceling (IC) receiver shows an improvement of 9 dB (measured at 1% raw BER) over the conventional receiver for the case of a single interferer. The gain is 2.1 dB for the case of two interferers. In Figure 2, we show the coded block error rate (BLER) for the same cases. The gains obtained by the IC receiver over the conventional receiver, measured at 10% BLER, are 7 dB and 0.9 dB, respectively.

As seen in the figures, the interference canceling method is clearly able to suppress one interferer effectively. However, with two interferers, the gain of the interference canceling mechanism is significantly reduced. The sensitivity of the performance of 8-PSK modulation to residual interference that cannot be canceled, and to errors in estimation of the different parameters, appears to be the main reason for this loss. With the use of a more efficient prefilter, e.g., the max-SNR prefilter of [23], we expect an improvement in performance. However, it is to be noted that 8-PSK modulation will typically be used in parts of the system where the signal quality is high, and the likelihood of seeing many dominant interferers is low. Thus, it can be reasonably expected that the proposed interference cancellation mechanism will provide gains in practical system deployments.

IV. Conclusions

In this paper, we presented methods for Single Antenna Interference Cancellation for an 8PSK-modulated desired signal and GMSK-modulated interference. We elucidated methods based on projection and virtual antennas. Performance results for the virtual antenna method showed significant gains with respect to a conventional receiver. The interference suppression mechanism has the potential to improve EGPRS system performance since the users see lower interference and can thus support higher data rates due to the link adaptation inherent in EGPRS. A system simulation including the Interference Cancellation scheme presented in this paper is an avenue for further research. Performance evaluation of the projection method is also an avenue for further work.

References

[1] 3GPP TR 45.903, ‘DRAFT Feasibility Study on Single Antenna Interference Cancellation (SAIC) for GSM Networks (Release 6.0.1)’ Dec. 2004.

[2] 3GPP TS 45.005 v6.8.0 (2005-01), Radio Access Network; Radio Transmission and Reception (Release 6).

[3] H. Trigui, and D.T.M. Slock, ‘Performance bounds for cochannel interference cancellation within the current GSM standard,’ Signal Processing 80 (2000), pp. 1335-1346.

[4] M. Olsson, A. Brostrom, S. Craig, and H. Arslan, ‘Single antenna interference rejection in GSM/EDGE networks,’ VTC 2004-Spring, 17-19 May 2004, pp.1698-1702.

[5] D. Astely and B. Ottersten, ‘MLSE and Spatio-Temporal Interference Rejection Combining with Antenna Arrays,’ in Proc. EUSIPCO-98, 11th European Signal Processing Conference, September 1998.

[6] J.C. Olivier and W. Kleynhans, ‘Single antenna interference cancellation for synchronised GSM networks using a widely linear receiver,’ IET Commun. Vol. 1, No. 1, Fenb. 2007, pp. 131-136.

[7] R. Meyer, W. H. Gerstacker, R. Schober, and J. B. Huber, ‘A Single Antenna Interference Cancellation Algorithm for Increased GSM Capacity,’ IEEE Trans. Wireless Comm., vol. 5, no. 7, July 2006, pp. 1616-1621.

[8] P.A. Ranta, A. Hottinen, and Z.-C. Honkasalo, ‘Co-channel Interference Cancellation Receiver for TDMA Mobile Systems’ Proc. IEEE ICC 95, Seattle, WA, June 1995, pp. 1721.

[9] P. A. Hoeher, S. Badri-Hoeher, W. Xu, and C. Krakowski, ‘Single-Antenna Co-Channel Interference Cancellation for TDMA Cellular Systems,’ IEEE Wireless Communications Magazine, April 2005, pp. 30-37.

[10] D. Hui and R. Ramésh, ‘Maximum Likelihood Sequence Estimation in the Presence of Constant Envelope Interference,’ VTC 2003-Fall, Oct. 2003, pp. 1060-1064.

[11] M. Pukkila, G. P. Mattellini, P. A. Ranta, ‘Constant modulus single antenna interference cancellation for GSM,’ VTC 2004-Spring, May 2004, pp. 584-588.

[12] M. Moisio and S. Nikkarinen, ‘Capacity gains of single antenna interference cancellation in GSM,’ PIMRC 2004, Sept. 2004, pp. 2700-2704.

[13] M. Moisio and K. Aschan, ‘The effect of single-antenna interference cancellation on GPRS performance,’ 1st International Symposium on Wireless Communication Systems, Sept. 2004, pp. 1-5.

[14] A. A. Mostafa, ‘Single antenna interference cancellation (SAIC) method in GSM network,’ VTC 2004-Fall, Sept. 2004, pp. 3748-3752.

[15] A. Mostafa, R. Kobylinksi, I. Kostanic and M. Austin, ‘Single antenna interference cancellation (SAIC) for GSM networks,’ VTC 2003-Fall, Oct. 2003, pp. 1089-1093.

[16] A. Hafeez, D. Hui and H. Arslan, 'Interference cancellation for EDGE via two-user joint demodulation,' VTC 2003-Fall, Oct. 2003, pp. 1025-1029.
[17] H. Wu, R. Heddergott, W. Xu, S. Badri-Hoeher, and P. A. Hoeher, 'Employing Dual Antenna Interference Suppression Techniques in GSM 8PSK Downlink,' PIMRC 2006, Sept. 2006, pp. 1-5.
[18] R. Kobylinski, A. Ghosh, A. Mostafa, J. Whitehead, 'EDGE terminal with interference cancellation and spatial diversity processing,' 2005 International Conference on Wireless Networks, Communications and Mobile Computing, June 2005, pp. 884-889.
[19] A. Duel-Hallen and C. Heegard, 'Delayed Decision-Feedback Sequence Estimation,' IEEE Trans. Commun., vol. COM-37, pp. 428-436, May 1989.
[20] P. Jung, 'Laurenté Representation of Binary Digital Continuous Phase Modulated Signals with Modulation Index 1/2 Revisited' IEEE Trans. Commun., vol. COM-42, pp. 221-224, Feb/Mar/Apr 1994.
[21] N. Al-Dhahir, 'FIR Channel-Shortening Equalizers for MIMO ISI Channels,' IEEE Transactions on Communications, vol. 49, no. 2, February 2001, pp. 213-225.
[22] R. Ramésh, D. Hui, A.-R. Hafeez, H. Arslan, 'Prefilter design for low-complexity equalization of MIMO systems,' VTC 2004-Spring, May 2004, pp. 871-875.
[23] A. Hafeez, R. Ramésh and D. Hui, 'Maximum SNR prefiltering for MIMO systems,' 2005 IEEE 6th Workshop on Signal Processing Advances in Wireless Communications, June 2005, pp.186-190.
[24] A. Furuskär et al., 'EDGE Enhanced Data Rates for GSM and TDMA136 Evolution,' in IEEE Personal Communications, June 1999, Vol. 6, No. 3.

APPENDIX

In this appendix, we we present a theoretical treatment of how the projection method can be adapted to our purposes. In the projection method, a filter is used to direct the interferer signal along the imaginary axis. The real part of the filtered signal then has significantly reduced interference. We have not evaluated the projection method in this paper, but have presented the theoretical derivation to demonstrate its applicability to interference cancellation for 8PSK signals with GMSK interference.

Following the discussion in [7], an aptly chosen complex filter with coefficients $p(n)$ can be found such that the real part of the combined filter $\{p(n) * g(n)\}$ is zero, where * denotes the convolution operator. This can clearly be done if the real part of $p(n)$ is chosen equal to the imaginary part of $g(n)$ and the imaginary part of $p(n)$ is chosen equal to the real part of $g(n)$. Thus, the real part of $z(n) = \{t(n) * p(n)\}$ will have no contribution due to the interference $i(n)$. Thus, we ideally have

$$x(n) = \mathrm{Re}\{p(n) * t(n)\} \tag{23}$$

$$= \sum_{k=0}^{M-1} d_r(k)\mathrm{Re}\{\hat{s}(n-k)\} - d_I(k)\mathrm{Im}\{\hat{s}(n-k)\} + \hat{w}_R(n). \tag{24}$$

Here, $p(n)$ is complex, whereas $d_R(n)$ and $d_I(n)$, the real and imaginary parts of the filtered impulse response of the channel of the desired signal, are real. The real part of the filtered noise is also shown.

Each slot received in an EGPRS system has a midamble of known symbols. Over this midamble, $s(n)$ is known, and the knowledge of these can be used to determine the best values of $p(n)$, $d_R(n)$ and $d_I(n)$ as solutions to an optimization problem that minimizes the squared magnitude of the error

$$e(n) = \mathrm{Re}\{p(n) * t(n)\} - \sum_{k=0}^{M-1} d_R(k)\mathrm{Re}\{\hat{s}(n-k-\tau)\} - d_I(k)\mathrm{Im}\{\hat{s}(n-k-\tau)\}. \tag{25}$$

Note that τ is an appropriately chosen delay value that models the extra delay due to the filtering by $p(n)$ and can be determined appropriately as part of the optimization process. The optimization problem can be solved directly, or by using an adaptation algorithm such as the LMS algorithm or the RLS algorithm. The number of taps in the resultant channel response given by $d_R(n)$ and $d_I(n)$ can be chosen to suit a subsequent equalization algorithm used to determine the values of $s(n)$ over the slot (apart from the midamble). To avoid degenerate values, $d_R(0)$ can be set to 1.

Once the values of $p(n), \tau, d_R(n)$ and $d_I(n)$ have been determined, an equalization method is used in order to determine the best estimate of $s(n)$ from $x(n)$. The equalization process is similar to a traditional equalizer with some differences. In a traditional equalizer, the signal is given by

$$r(n) = \sum_{k=0}^{N-1} h(k)s(n-k) + w(n) \tag{26}$$

where all quantities may be complex. A trellis-based equalizer uses postulates of $s(n)$ to define states and uses the branch metric

$$E(n) = |r(n) - \sum_{k=0}^{N-1} h(k)s_P(n-k)|^2 \tag{27}$$

to extend the trellis, where the subscript P refers to a postulate. In an MLSE equalizer, all of the N values of $s_P(n)$ are postulated within the trellis. In a DFSE-based equalizer, some of the values are postulated within the trellis, and the rest are used from delayed decisions on previous symbols. In our case, the signal to be demodulated has a slightly different structure, as shown in equation (24). An equalizer for this signal will postulate values of $s(n)$ to determine the states of the trellis, but values of $\hat{s}(n)$, defined as per equation (9) will be used in the calculation of the branch metrics given by

$$E(n) = |x(n) - \sum_{k=0}^{M-1} d_R(k)\mathrm{Re}\{\hat{s}_P(n-k)\} - d_I(k)\mathrm{Im}\{\hat{s}_P(n-k)\}|^2. \tag{28}$$

A MLSE-based equalizer or a DFSE-based equalizer can be used. In the case of a DFSE-based equalizer, care must be taken to ensure that the appropriate rotation as per equation (9) is applied to past decisions on $s(n)$. With a DFSE equalizer, it is also important to ensure that the final channel response given by $d_R(n)$ and $d_I(n)$ has most of its energy concentrated in the first few taps, in order to prevent error propagation. By suitably defining the metric in the optimization process used to calculate $p(n), d_R(n)$ and $d_I(n)$, a prefilter can be found that performs the necessary energy compaction. Details of such a derivation are left as an exercise to the reader.