Sum of Products - Truth Table

Example

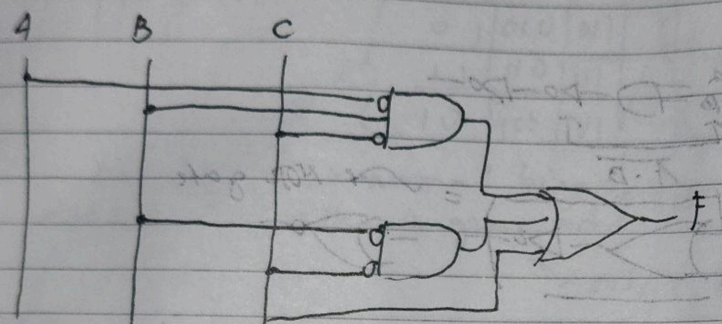
$F = \bar{A}B\bar{C} + \bar{B}\bar{C} + C$

$\bar{A}B\bar{C}$: 010

$\bar{B}\bar{C}$: $\bar{B} = 0$, $C = 0$, $A = 1 / 0$ ∴ 000, 100

C: $C = 1$, $A = 1$ or 0, $B = 1$ or 0

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

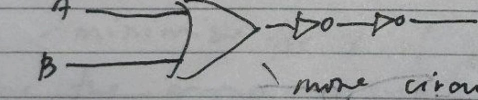


Using De Morgan's Theorem

$\overline{A \cdot B} = \bar{A} + \bar{B}$

$\overline{A + B} = \bar{A} \cdot \bar{B}$

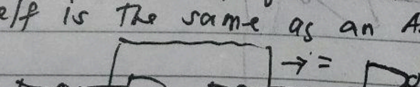
A + B



more circuit

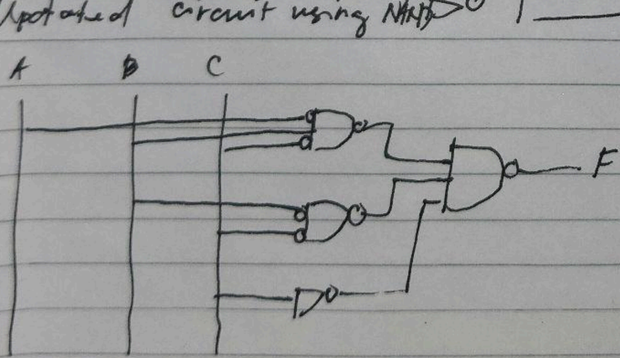
more propagation of data

- An OR gate by itself is the same as an AND gate w/ inverted inputs & outputs



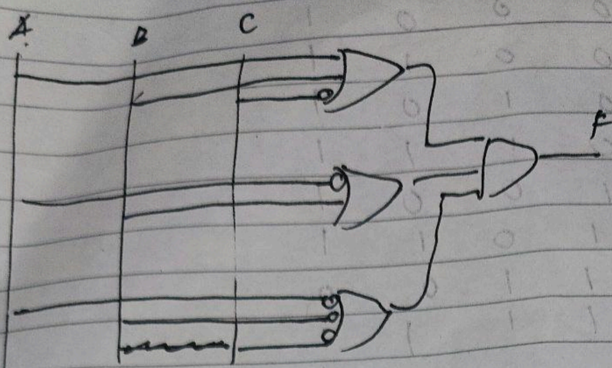
NAND gate

Updated circuit using NAND



Product of Sum to Truth Table

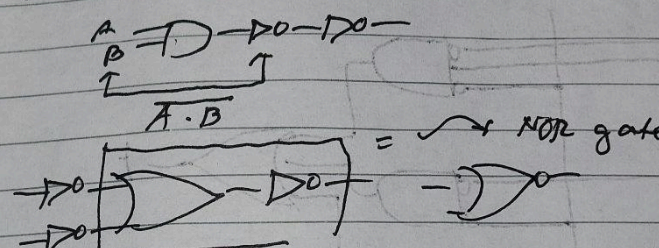
$F = (A+B+\bar{C})(\bar{A}+B)(\bar{A}+\bar{B}+\bar{C})$



A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

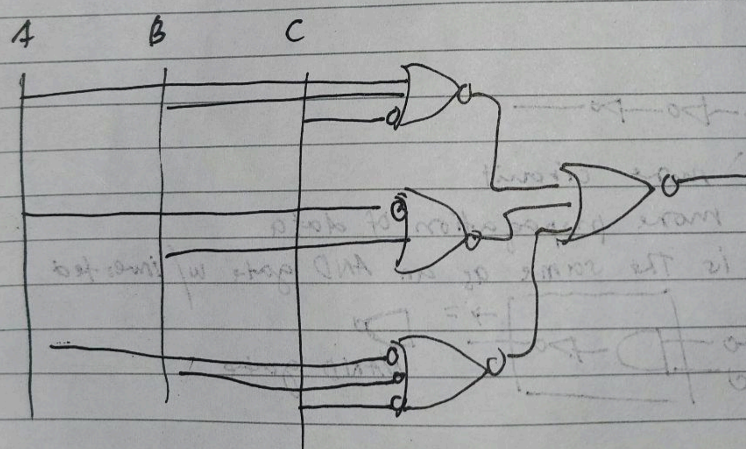
using De Morgan's

$\overline{A \cdot B} = \bar{A} + \bar{B}$



An AND gate by itself is the same as an OR gate w/ inverted inputs & outputs

Updated circuit using NOR gate



Karnaugh Maps (KMaps)

- used for minimizing Boolean expressions
- minimized expression
  - fewer gates
  - simpler circuits
  - less complex circuits
  - lower cost of implementation

2 variable

A \ B	0	1
0	00	01
1	10	11

3 variable

AB \ C	00	01	~~11~~	~~10~~
0 0	000	001		
0 1	010	011		
1 1	110	111		
1 0	100	101		

- cannot be in normal binary sequence
- must be written in gray code
  → only 1 bit changes at a time

A \ BC	00	01	11	10
0				
1				

Tips

1. group the 1's (SOP) or 0s (POS)
   - maximize the group sizes - size 1 or 2^n (1, 2, 4, 8, 16 ...)
     - select the largest group
   - minimize the # of groups by overlapping
2. determine the group product/sum expression
3. sum/product the groups

$X = AB\bar{C} + ABC + A\bar{B}C$

110 111 101

AB \ C	0	1
00	0	0
01	0	0
11	1	1
10	0	1

First group: 110, 111 → AB

2nd group: 111, 101 → A C

$X = AB + AC$

4 variable

$X = A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D} + AB\bar{C}D$

1011 1010 0011 0010 0001 0000 1101

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	0	1	0	0
10	0		1	1

$\bar{A}\bar{B} + AB\bar{C}D + \bar{B}C = F$

AB \ CD	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

$A\bar{B}C + B\bar{C} + \bar{D} = F$

DC \ BA	00	01	11	10
00		1		
01		1		1
11	1	1	1	1
10	1	1	1	1

$\bar{B}A + D + CB\bar{A} = F$

using Product of Sums

WX \ YZ	00	01	11	10
00	0		0	0
01	0		0	0
11	0			
10	0			

$\rightarrow (W + \bar{Y})$

$(Y + Z)$

$F = (W + Y')(Y + Z)$

Don't care conditions

$F(A, B, C, D) = \Sigma m(0, 1, 4, 5, 6, 10, 11, 13) + d(2, 3)$

X
θ

AB \ CD	00	01	11	10
00	1	1	X	X
01	1	1		1
11		1		
10			1	1

variable kmap

$f(A,B,C,D,E) = \sum(0,1,6,7,8,9,21,22,23,29,31)$

A = 0

BC \ DE	00	01	11	10
00	1 (0)	1 (1)	3	2
01	4	5	1 (7)	1 (6)
11	12	13	15	14
10	1 (8)	1 (9)	11	10

A = 1

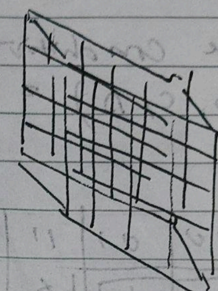
BC \ DE	00	01	11	10
00	16	17	19	18
01	20	1 (21)	1 (23)	1 (22)
11	28	1 (29)	1 (31)	30
10	24	25	27	26

$\bar{A}\bar{C}\bar{D}$

ACE

$\bar{B}CD$

A = 0



A = 1

$f(A,B,C,D,E) = \bar{A}\bar{C}\bar{D} + ACE + \bar{B}CD$

$f(A,B,C,D,E) = \sum m(0,16) + d(17,18,19,20,21,22,23)$

A = 0

BC \ DE	00	01	11	10
00	1 (0)	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

A = 1

BC \ DE	00	01	11	10
00	1 (16)	X (17)	X (19)	X (18)
01	X (20)	X (21)	X (23)	X (22)
11	28	29	31	30
10	24	25	27	26

$f(A,B,C,D,E) = \bar{B}\bar{C}\bar{D}\bar{E} + A\bar{B}$

Using NAND & NOR as Universal gates

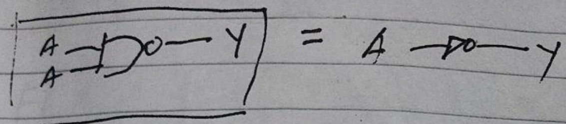
NAND - ~~all inputs are the same~~

$\cancel{Y = A.\ \overline{A \cdot A} = \overline{A}}$

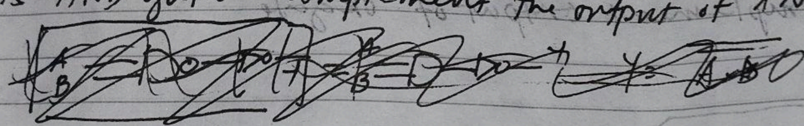
As NOT gate

- all inputs are the same

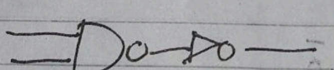
$Y = \overline{A} = \overline{A \cdot A}$



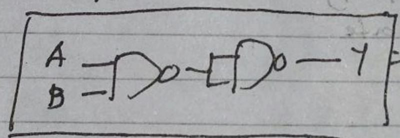
As AND gate - complement the output of AND gate



- use the NAND NOT gate to complement the NAND gate



$Y = \overline{\overline{A \cdot B}}$

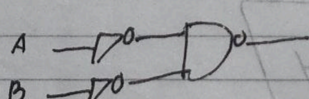


$= A \cdot B$

As an OR gate - use the NAND NOT gate to complement inputs

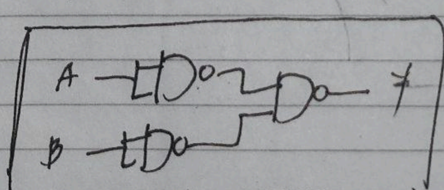
$Y = \overline{A \cdot B} = \overline{A} + \overline{B}$

OR $= A + B$

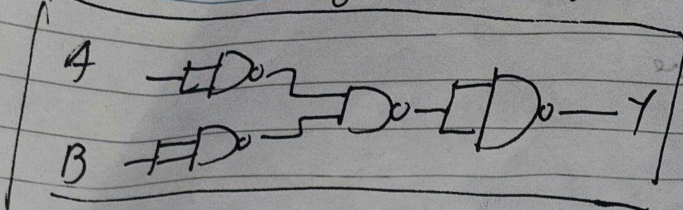


$Y = (\overline{\overline{A} \cdot \overline{B}})$

$= A + B$



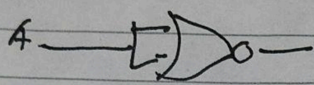
NAND as NOR gate - complement the output of OR gate



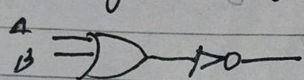
NOR as universal gate

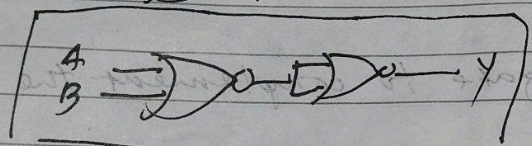
As NOT gate

$Y = \overline{A}$



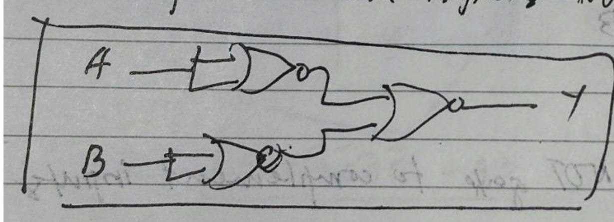
As OR gate - complement output of OR gate



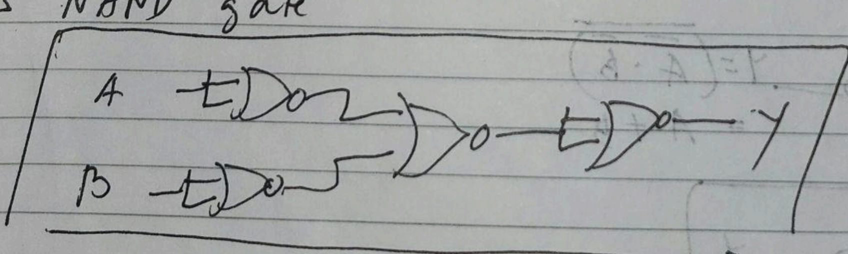


As AND gate $Y = \overline{\overline{A} + \overline{B}} = \overline{\overline{A}} \cdot \overline{\overline{B}}$

- complement the inputs NOR gate



As NAND gate



Tabular Method (Quine - McCluskey Minimization)

1. Prime Implicant
2. Essential Prime Implicant

- Implicant (group of 1s)
- Prime Implicant (largest group of 1s)
- Essential Prime Implicant (Prime Implicant having at least 1 minterms that can't be combined)

MSB LSB

$F(A,B,C,D) = \sum_m (0,1,3,7,8,9,11,15)$

0 - 0000
1 - 0001
3 - 0011
7 - 0111
8 - 1000
9 - 1001
11 - 1011
15 - 1111

1. group the minterms by their number of 1s

Grp #	Minterm Delegation	Binary Value A B C D
0	m_0	0 0 0 0
1	m_1	0 0 0 1
	m_8	1 0 0 0
2	m_3	0 0 1 1
	m_9	1 0 0 1
3	m_7	0 1 1 1
	m_{11}	1 0 1 1
4	m_{15}	

Group #	Matching pair	Binary Value A B C D
0	m_0, m_1	0 0 0 -
	m_0, m_8	- 0 0 0
1	m_1, m_3	0 0 - 1
	m_1, m_9	- 0 0 1
	m_8, m_9	1 0 0 -
2	m_3, m_7	0 - 1 1
	m_3, m_{11}	- 0 1 1
	m_9, m_{11}	1 0 - 1
3	m_7, m_{15}	- 1 1 1
	m_{11}, m_{15}	1 - 1 1

0	m_0, m_1, m_8, m_9	- 0 0 -	$\bar{B}\bar{C}$
	m_0, m_1, m_8, m_9	- 0 0 -	
	~~m_1, m_3, m_9, m_{11}~~		
1	$m_1\ m_3\ m_9\ m_{11}$	- 0 - 1	$\bar{B}D$
	$m_1\ m_9\ m_3\ m_{11}$	- 0 - 1	
2	$m_3\ m_7\ m_{11}\ m_{15}$	- - 1 1	CD
	$m_3\ m_{11}\ m_7\ m_{15}$	- - 1 1	

Prime Implicant Table

PI	Minterms	0	1	3	7	8	9	11	15
$\bar{B}\bar{C}$	0, 1, 8, 9	✓	✓			✓	✓		
$\bar{B}D$	1, 3, 9, 11		✓	✓			✓	✓	
CD	3, 7, 11, 15			✓	✓			✓	✓

Verify w KMap

AB \ CD	00	01	11	10
00	1	1	1	0
01	0	0	1	0
11	0	0	1	0
10	1	1	1	0

$Y = CD + \bar{B}\bar{C}$

Example 2 $F(W, X, Y, Z) = \sum m(2, 6, 8, 9, 10, 11, 14, 15)$

Grp #	Minterm Delegation	Binary Value W	X	Y	Z
0	2	0	0	1	0
	8	1	0	0	0
1	6	0	1	1	0
	9	1	0	0	1
	10	1	0	1	0
2	11	1	0	1	1
	14	1	1	1	0
4	~~8~~ 15	1	1	1	1

Grp #	Minterm Delegation	W	X	Y	Z
0	2, 6	0	-	1	0
	2, 10	-	0	1	0
	8, 9	1	0	0	-
	8, 10	1	0	-	0
1	6, 14	-	1	1	0
	9, 11	1	0	-	1
	10, 11	1	0	1	-
	10, 14	1	-	1	0
2	11, 15	1	-	1	1
	14, 15	1	1	1	-

Grp #	Minterm Delegation	W	X	Y	Z
0	2, 6, 10, 14	-	-	1	0
	2, 10, 6, 14	-	-	1	0
	8, 9, 10, 11	1	0	-	-
	8, 10, 9, 11	1	0	-	-
1	10, 11, 14, 15	1	-	1	-
	10, 14, 11, 15	1	-	1	-

Grp #	Minterm Delegation	Binary Value W	X	Y	Z
0	2, 6, 10, 14	–	–	1	0
	8, 9, 10, 11	1	0	–	–
1	10, 11, 14, 15	1	–	1	–

Prime Implicant Table

PI	2	6	8	9	10	11	14	15
YZ'	1	1			1		1	
WX'			1	1	1	1		
WY					1	1	1	1

$$f(W,X,Y,Z) = YZ' + WX' + WY$$