GPS-Denied Loitering about a Moving Target

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1 Project definition

The goal of this project is to control a fixed-wing UAV to follow a constant radius orbit about a moving ground target without using GPS information. For the scope of this project, we assume that using on-board sensors, we can obtain accurate measurements of altitude, attitude, heading, and LOS vector azimuth and elevation angles in the UAV vehicle 1 frame (unpitched, unrolled). Using these measurements, we assume that we can control the UAV using a desired roll angle command.

2 System Dynamics

In order to simplify our motion model, we treat the motion of the target like a wind vector, equal and opposite the motion of the target. Also, for simplicity, we derive the dynamics of the system using a constant-velocity model for the ground target.

Let the state be represented by

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ \psi \\ \chi \end{pmatrix},$$

where x and y are the position of the UAV relative to the target, ψ represents the angle between true North and the body x-axis of the UAV, and χ represents the angle between true North and the velocity vector of the UAV relative to the target.

These states evolve according to

$$\dot{\mathbf{x}} = \begin{pmatrix} V_g \cos\chi \\ V_g \sin\chi \\ \frac{g}{V_a} \tan\phi \\ \frac{g}{V_g} \tan\phi \end{pmatrix},$$

where V_g is the "ground velocity" of the UAV, or the velocity relative to the target, and V_a is the airpseed of the aircraft.

The relationship between V_g and V_a is given by

$$\mathbf{V_g} = \mathbf{V_a} + \mathbf{V_w}$$

Assuming $\mathbf{V_w}$ is actually just the negative of the target velocity, $\mathbf{V_t}$, the above equation becomes

$V_g \begin{pmatrix} \cos\chi \\ \sin\chi \end{pmatrix} = V_a \begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix} - \begin{pmatrix} v_{x_t} \\ v_{y_t} \end{pmatrix}$.

Using this equation, we get

$$\begin{aligned}
V_g \begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix}^\top \begin{pmatrix} \cos\chi \\ \sin\chi \end{pmatrix} &= V_a \begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix}^\top \begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix} - \begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix}^\top \begin{pmatrix} v_{x_t} \\ v_{y_t} \end{pmatrix} \\
V_g \cos(\chi - \psi) &= V_a - v_{x_t}\cos\psi - v_{y_t}\sin\psi \\
\chi - \psi &= \cos^{-1}\left(\frac{V_a - v_{x_t}\cos\psi - v_{y_t}\sin\psi}{V_g}\right) \\
\chi &= \psi + \cos^{-1}\left(\frac{V_a - v_{x_t}\cos\psi - v_{y_t}\sin\psi}{V_g}\right).
\end{aligned}$$

Similarly,

$$\begin{aligned}
V_g &= \|V_a \begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix} - \mathbf{V_t}\| \\
&= \sqrt{(V_a\cos\psi - v_{x_t})^2 + (V_a\sin\psi - v_{y_t})^2}.
\end{aligned}$$

3 Control Implementation

We will converge to an orbit about the target when $\dot\chi = \frac{V_g}{R}$, therefore, we can derive a feedforward term for our roll control according to

$$
\begin{aligned}
\dot{\chi} &= \frac{V_g}{R} \\
&= \frac{g}{V_g} \tan\phi \\
\phi_{ff} &= \tan^{-1}\left(\frac{V_g^2}{gR}\right) \\
&= \tan^{-1}\left(\frac{\left\|V_a \begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix} + \mathbf{V_w}\right\|^2}{gR_d}\right) \\
&= \tan^{-1}\left(\frac{\left(V_a\cos\psi - v_{x_t}^w\right)^2 + \left(V_a\sin\psi - v_{y_t}^w\right)^2}{gR_d}\right) .
\end{aligned}
$$

When the UAV is in orbit, the roll command should match ϕ_{ff} almost exactly. But when the UAV is not in the proper orbit, we need to augment the roll command to converge onto the proper orbit. We do so by defining γ to be the angle difference between the LOS vector and the heading vector, $\mathbf{V_g}$. This can be defined as

$$
\begin{aligned}
\gamma &= \alpha_{az} + \psi - \chi \\
&= \alpha_{az} - (\chi - \psi) \\
&= \alpha_{az} - \cos^{-1}\left(\frac{V_a - v_{x_t}\cos\psi - v_{y_t}\sin\psi}{V_g}\right)
\end{aligned}
$$

In a perfect orbit, this angle will be $\lambda\frac{\pi}{2}$, where λ is the direction of the orbit. But in order to converge to the proper orbit with the desired radius, we must adjust our current R by controlling γ away from $\lambda\frac{\pi}{2}$. If we need to decrease the current radius, we want to drive γ to zero and if we want to increase the orbit radius, we want to drive γ to π. The resulting controller is

$$
\begin{aligned}
\phi_c &= \phi_{ff} + k_{p_\gamma} e_\gamma + k_{d_\gamma} \dot{e}_\gamma \\
e_\gamma &= \gamma - \gamma_d \\
\gamma_d &= \lambda\left[\tan^{-1}\left(-\beta\left(R - R_d\right)\right) + \frac{\pi}{2}\right] ,
\end{aligned}
$$

where the desired value for γ is dependent upon our current radius, approaching 0 or π at when the radius error is large and converging to $\frac{\pi}{2}$ as the radius error goes to zero.

In order to compute V_g and χ we must estimate the target velocity. We initially assume that the target velocity and allow the reactive PD portion of our control approximate the appropriate trajectory for a constant radius orbit about the moving target. When the radius error is sufficiently small, we can use the output of our control to extract measurements of the target velocity according to

$$\begin{aligned}
\hat{V}_g &= \sqrt{|gR_d \tan\phi|} \\
\hat{\chi} &= \psi - \cos^{-1}\left(\frac{V_a - v_{x_t}\cos\psi - v_{y_t}\sin\psi}{V_g}\right) \\
\mathbf{\bar{V}_t} &= \mathbf{V_a} - \mathbf{\hat{V}_g} \\
&= V_a \begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix} - \hat{V}_g \begin{pmatrix} \cos\hat{\chi} \\ \sin\hat{\chi} \end{pmatrix} .
\end{aligned}$$

The measurement obtained using this model is then an input to a simple constant velocity Kalman filter.