ENGR-301L: Digital Signal Processing and Applications
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Laboratory #7: Linear Phase FIR Filter Design Using Window Functions

Lab Description: Finite Impulse Response (FIR) filters are popular in a wide variety of applications due to its design simplicity, ease of implementation, and, most of all, effective performance. In FIR filter design, a set of finite number of filter coefficients (also commonly known as *taps*) $\{h(n); n = 0, 1 \ldots N\}$ is used. Some performance issues are taken into account in FIR filters design to assure effectiveness.

1. The abrupt discontinuity of the finite number of filter coefficients causes an undesirable effect in the frequency response of FIR filters, known as the *Gibbs phenomenon*. To alleviate this problem, a tapering function $\{w(n)\}$ such as Hamming, Hanning, or Bartlett function is often multiplied to the ideal filter coefficients $\{h_d(n)\}$, i.e., $\{h(n) = h_d(n)\, w(n)\}$, to allow a smooth transition to zero.

2. It is important in some applications that the phase information of the source signal is to be maintained after filtering. To achieve that, linear-phase FIR filters are designed to generate a phase response that is linearly proportional to the frequency with a ratio/slope of $(N-1)/2$. Linear-phase FIR filters impose a constraint that filter coefficients are symmetric around the center coefficient, namely $h(n) = h(N - 1 - n)$.

Objective:

- Become familiar with linear-phase FIR filter design
- Generate FIR filter design specifications according to application requirements
- Understand window function effects on main-lobe width, side-lobe attenuation, and transition bandwidth

Linear Phase FIR Filter Design Procedure:

1. Determine the ideal linear phase FIR filter specification $H_d(e^{j\omega})$. Note that the half of the sampling frequency (F_{samp} / 2) in Hertz corresponds to π (radians) and the ideal filter frequency response within the passband is given by (1) and zero elsewhere.

$$H_d(e^{j\omega}) = e^{-\frac{j\omega(N-1)}{2}} \quad \omega_{p1} \le \omega \le \omega_{p2} \tag{1}$$

2. Derive the ideal impulse responses (filter coefficients) $\{h_d(n)\}$.

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}\ d\omega \tag{2}$$

3. Choose a tapering window function $\{w(\mathrm{n})\}$ to realize the actual FIR filter coefficients

$$h(n) = h_d(n)w(n) \quad \mathrm{n} = 0,1,\ldots(\mathrm{N}-1) \tag{3}$$

4. Perform FIR filtering using liner convolution: $y(n) = x(n)*h(n)$

Specific Goals:

You will evaluate the performance of FIR filter design through a test signal ***Sig3.xlsx***, saved in Excel workbook format. You are required to design and implement three different types of linear-phase FIR filters to perform the following tasks:

- design and implement a *low-pass* linear-phase FIR filter to retain the lowest frequency component in Sig3.xlsx

- design and implement a *high-pass* linear-phase FIR filter to retain the highest frequency component in Sig3.xlsx
- design and implement *a band-pass* linear-phase FIR filter to retain the middle frequency component in Sig3.xlsx

To retrieve data from an excel spreadsheet, you can use the Matlab command below:

```
[xdata, xname] = xlsread('Sig3.xlsx');
```

Where *xdata* contains the two column data and *xname* contains the first row text information. The sampling frequency can be deducted from the sampling time instants. *Note: Excel 97-2003 workbook files use a different extension, i.e.,'**. xls'**.*

1. Generate a 2x1 plot with the data on top and the power spectrum on the bottom.
2. Generate a 3x1 plot including $h(n)$, $|H(e^{j\omega})|$, and $Phase(H(e^{j\omega}))$; generate a 2x1 plot having $y(n)$, and $|Y(e^{j\omega})|$ to confirm your design for each type of filter described above.

Discuss the performance of the system response $|H(e^{j\omega})|$ with regard to:

- tapering function type - Rectangular (no tapering), Triangular (Bartlett), Hamming, and Hanning window
- number of FIR filter taps (N)
- phase response of the filter

Evaluate the effectiveness of filtering through power spectrum estimation (using dB scale), e.g., if the undesired signals are sufficiently attenuated as design.