Dual-Baseline Verification: Technical Brief

Temporal Fairness Infrastructure for Algorithmic Systems

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1 Core Architecture

The dual-baseline verification framework addresses a fundamental gap in algorithmic fairness: proving not just that declared rules were followed, but that rule application remained consistent and equitable across stakeholders and time periods. This prevents both selective rule enforcement and post-hoc manipulation of criteria to achieve desired outcomes.

DUAL-BASELINE VERIFICATION

Input: Algorithmic Decision + Stakeholder Weights

$\downarrow$

BASELINE 1: SYMBOLIC CONSISTENCY
Verifies: declared_rule(input) $\rightarrow$ output
Proof: $\pi_1 = \text{SNARK}\{\text{rule\_applied}, \text{state\_delta}\}$

$\downarrow$

BASELINE 2: TEMPORAL CONSISTENCY
Verifies: $\text{similarity}(s(t), \text{weighted\_history}(s)) > \alpha$
Proof: $\pi_2 = \text{SNARK}\{\text{similarity\_score}, \text{time\_delta}\}$

$\downarrow$

Output: Recursive Proof Chain $\pi_1 \circ \pi_2 \circ \ldots \circ \pi_n$

Verification: Any party can cryptographically verify
both symbolic rule adherence AND temporal consistency

1.1 Mathematical Framework

Symbolic Consistency (Baseline 1):

$$\forall \text{ transition } t : \text{verify}(\text{rule\_declared}(t), \text{state\_delta}(t)) = \text{TRUE} \tag{1}$$

Temporal Consistency (Baseline 2):

$$\forall \text{ state } s(t) : \text{similarity}(s(t), \text{weighted\_history}(s)) > \alpha \tag{2}$$

where α = consistency threshold

Conceptual Recursive Structure:

$$\text{Proof\_chain}(t) = \text{Prove}(\pi_1(t) \wedge \pi_2(t) \wedge \text{Proof\_chain}(t-1)) \tag{3}$$

[Illustrates dependency chain but creates circular reference]

Cryptographic Implementation:

$$\text{Proof\_chain}(t) = \begin{cases} \text{Prove}(\pi_1(0) \wedge \pi_2(0)) & \text{if } t = 0 \\ \text{Prove}(\pi_1(t) \wedge \pi_2(t) \wedge H(\text{Proof\_chain}(t-1))) & \text{if } t > 0 \end{cases} \tag{4}$$

[Hash-chained for efficient verification - see Circuit Implementation §3.1]

where $H(\cdot)$ represents a cryptographic hash function (Poseidon in our implementation), enabling efficient verification without storing the complete proof history.

2 Implementation: Core Verification Logic

2.1 Type Definitions

```
// Simplified type definitions for implementation examples
type StateVector = number[];
type StakeholderInput = { stakeholderWeights: number[]; data: any };
type RuleParameters = { [key: string]: any };
type ConstraintSet = { verify: (prev: StateVector, next: StateVector) => boolean; getViolations: ()
     => string[] };

interface VerificationResult {
  isValid: boolean;
  expectedState: StateVector;
  actualState: StateVector;
  violatedConstraints: string[];
}
```

Listing 1: Core Type Definitions

2.2 Symbolic Consistency Verification (Baseline 1)

```
interface SymbolicTransition {
  previousState: StateVector;
  rule: GovernanceRule;
  input: StakeholderInput;
  nextState: StateVector;
  timestamp: number;
}

interface GovernanceRule {
  id: string;
  parameters: RuleParameters;
  apply: (state: StateVector, input: StakeholderInput) => StateVector;
  constraints: ConstraintSet;
}

// Example: Quadratic funding rule implementation
class QuadraticFundingRule implements GovernanceRule {
  apply(state: StateVector, input: StakeholderInput): StateVector {
```

```
    const contributions = input.stakeholderWeights;
    // Simplified quadratic funding calculation
    // Production implementation would handle edge cases and validation
    const sqrtSum = contributions.map(x => Math.sqrt(Math.max(0, x))).reduce((a,b) => a+b);
    const allocation = sqrtSum * sqrtSum;
    return [...state.slice(0,-1), allocation];
  }
}

function verifySymbolicConsistency(
  transition: SymbolicTransition
): VerificationResult {
  // Verify declared rule was actually applied
  const expectedState = transition.rule.apply(
    transition.previousState,
    transition.input
  );

  const isConsistent = vectorEquals(expectedState, transition.nextState);
  const constraintsSatisfied = transition.rule.constraints.verify(
    transition.previousState,
    transition.nextState
  );

  return {
    isValid: isConsistent && constraintsSatisfied,
    expectedState,
    actualState: transition.nextState,
    violatedConstraints: constraintsSatisfied ? [] :
      transition.rule.constraints.getViolations()
  };
}

function vectorEquals(a: StateVector, b: StateVector, epsilon: number = 1e-6): boolean {
  if (a.length !== b.length) return false;
  return a.every((val, i) => Math.abs(val - b[i]) < epsilon);
}
```

Listing 2: TypeScript Implementation of Symbolic Verification

2.3 Temporal Consistency Verification (Baseline 2)

```
interface TemporalScore {
  score: number;           // [0,1] similarity to historical behavior
  isConsistent: boolean;   // score > threshold
  temporalDrift: number;   // rate of change over time
  anomalyFlags: string[];  // detected irregularities
}

interface SimilarityPoint {
  similarity: number;
  timeWeight: number;
  temporalDistance: number;
}

function verifyTemporalConsistency(
  currentState: StateVector,
```

```
  historicalStates: StateVector[],
  timeWeights: number[],
  threshold: number = 0.85  // Derived from empirical analysis of legitimate
                            // algorithmic evolution patterns
): TemporalScore {
  // Calculate weighted similarity to historical behavior
  const similarities = historicalStates.map((state, i) => ({
    similarity: cosineSimilarity(currentState, state),
    timeWeight: timeWeights[i],
    temporalDistance: timeWeights.length - i
  }));

  // Weighted average prioritizing recent behavior
  const weightedSimilarity = similarities.reduce(
    (acc, { similarity, timeWeight }) =>
      acc + (similarity * timeWeight), 0
  ) / similarities.reduce((acc, { timeWeight }) => acc + timeWeight, 0);

  // Detect temporal drift patterns
  const drift = calculateTemporalDrift(similarities);
  const anomalies = detectBehavioralAnomalies(similarities, drift);

  return {
    score: weightedSimilarity,
    isConsistent: weightedSimilarity > threshold,
    temporalDrift: drift,
    anomalyFlags: anomalies
  };
}

function calculateTemporalDrift(similarities: SimilarityPoint[]): number {
  // Linear regression slope of similarity over time
  const n = similarities.length;
  const timePoints = similarities.map((_, i) => i);
  const simValues = similarities.map(s => s.similarity);

  const meanTime = timePoints.reduce((a,b) => a+b) / n;
  const meanSim = simValues.reduce((a,b) => a+b) / n;

  const numerator = timePoints.reduce((sum, t, i) =>
    sum + (t - meanTime) * (simValues[i] - meanSim), 0);
  const denominator = timePoints.reduce((sum, t) =>
    sum + Math.pow(t - meanTime, 2), 0);

  return denominator === 0 ? 0 : numerator / denominator; // Drift rate
}

function detectBehavioralAnomalies(
  similarities: SimilarityPoint[],
  drift: number
): string[] {
  const anomalies: string[] = [];
  const threshold = 0.3; // Similarity drop threshold

  // Check for sudden drops in similarity
  for (let i = 1; i < similarities.length; i++) {
    const drop = similarities[i-1].similarity - similarities[i].similarity;
    if (drop > threshold) {
      anomalies.push(`sudden_drop_t${i}`);
```

```
    }
  }

  // Check for excessive drift
  if (Math.abs(drift) > 0.1) {
    anomalies.push(drift > 0 ? 'positive_drift' : 'negative_drift');
  }

  return anomalies;
}

function cosineSimilarity(a: StateVector, b: StateVector): number {
  if (a.length !== b.length) return 0;

  const dotProduct = a.reduce((sum, val, i) => sum + val * b[i], 0);
  const magnitudeA = Math.sqrt(a.reduce((sum, val) => sum + val * val, 0));
  const magnitudeB = Math.sqrt(b.reduce((sum, val) => sum + val * val, 0));

  if (magnitudeA === 0 || magnitudeB === 0) return 0;
  return dotProduct / (magnitudeA * magnitudeB);
}
```

Listing 3: Temporal Consistency Algorithm

3 Zero-Knowledge Circuit Implementation

3.1 Dual-Baseline Verification Circuit

```
pragma circom 2.0.0;

template DualBaselineVerifier(n, historySize) {
    // Public inputs - visible to verifiers
    signal input previousStateHash;
    signal input currentStateVector[n];
    signal input declaredRule[4];
    signal input timeDelta;
    signal input stakeholderWeights[n];

    // Private inputs - hidden from verifiers
    signal private input previousStateVector[n];
    signal private input historicalStates[historySize][n];
    signal private input ruleParameters[8];
    signal private input nonce;

    // Outputs - verification results
    signal output isValid;
    signal output temporalScore;
    signal output stateHash;

    // Verify previous state hash matches
    component hasher = Poseidon(n);
    for (var i = 0; i < n; i++) {
        hasher.inputs[i] <== previousStateVector[i];
    }
    hasher.out === previousStateHash;

    // Baseline 1: Verify symbolic rule application
```

```
    component ruleVerifier = RuleApplicationVerifier(n);
    ruleVerifier.prevState <== previousStateVector;
    ruleVerifier.rule <== declaredRule;
    ruleVerifier.params <== ruleParameters;
    ruleVerifier.nextState <== currentStateVector;
    ruleVerifier.weights <== stakeholderWeights;

    // Baseline 2: Verify temporal consistency
    component temporalVerifier = TemporalConsistencyVerifier(n, historySize);
    temporalVerifier.currentState <== currentStateVector;
    temporalVerifier.historicalStates <== historicalStates;
    temporalVerifier.timeDelta <== timeDelta;

    // Combined verification logic
    isValid <== ruleVerifier.isValid * temporalVerifier.isValid;
    temporalScore <== temporalVerifier.consistencyScore;

    // Generate new state hash for chain continuity
    component newHasher = Poseidon(n + 1);
    for (var i = 0; i < n; i++) {
        newHasher.inputs[i] <== currentStateVector[i];
    }
    newHasher.inputs[n] <== nonce;
    stateHash <== newHasher.out;
}

template RuleApplicationVerifier(n) {
    signal input prevState[n];
    signal input rule[4];
    signal input params[8];
    signal input nextState[n];
    signal input weights[n];

    signal output isValid;

    // Note: Simplified example - production would implement specific rule logic
    // Verify weighted state transition follows declared rule
    component stateTransition = WeightedStateTransition(n);
    stateTransition.prevState <== prevState;
    stateTransition.rule <== rule;
    stateTransition.params <== params;
    stateTransition.weights <== weights;

    // Check computed next state matches actual next state
    component equalityChecker = VectorEquality(n);
    equalityChecker.a <== stateTransition.computedNextState;
    equalityChecker.b <== nextState;

    isValid <== equalityChecker.isEqual;
}

template TemporalConsistencyVerifier(n, historySize) {
    signal input currentState[n];
    signal input historicalStates[historySize][n];
    signal input timeDelta;

    signal output isValid;
    signal output consistencyScore;

```

```
    // Calculate similarity to historical behavior
    component similarities[historySize];
    for (var i = 0; i < historySize; i++) {
        similarities[i] = CosineSimilarity(n);
        similarities[i].a <== currentState;
        similarities[i].b <== historicalStates[i];
    }

    // Weighted average with temporal decay
    component weightedAvg = TemporalWeightedAverage(historySize);
    for (var i = 0; i < historySize; i++) {
        weightedAvg.values[i] <== similarities[i].similarity;
        weightedAvg.timeWeights[i] <== 1000 - (timeDelta * i);
    }

    consistencyScore <== weightedAvg.result;

    // Threshold check for consistency
    component threshold = GreaterThan(10);
    threshold.in[0] <== consistencyScore;
    threshold.in[1] <== 850; // 0.85 threshold scaled to integer

    isValid <== threshold.out;
}
```

Listing 4: Circom Circuit for Dual-Baseline Verification

4 Circuit Implementation Examples

The following simplified templates demonstrate how key operations translate to arithmetic constraints. Note that production implementations would require additional components and optimizations:

```
template CosineSimilarity(n) {
    signal input a[n];
    signal input b[n];
    signal output similarity;

    // Dot product calculation
    component dotProduct = DotProduct(n);
    dotProduct.a <== a;
    dotProduct.b <== b;

    // Magnitude calculations (production requires sqrt approximation)
    component magA = VectorMagnitude(n);
    component magB = VectorMagnitude(n);
    magA.vec <== a;
    magB.vec <== b;

    // Division via modular multiplicative inverse
    component divider = SafeDiv();
    divider.numerator <== dotProduct.out;
    divider.denominator <== magA.out * magB.out;

    similarity <== divider.out;
}

```

```
template QuadraticFundingVerifier(maxContributors) {
    signal input contributions[maxContributors];
    signal input numContributors; // Actual number used
    signal output allocation;

    // Square root sum calculation (simplified approximation)
    component sqrtSum = SqrtSum(maxContributors);
    sqrtSum.values <== contributions;
    sqrtSum.count <== numContributors;

    // Quadratic formula: (sqrt_sum)^2
    allocation <== sqrtSum.out * sqrtSum.out;
}

// Note: Components like DotProduct, VectorMagnitude, SafeDiv, etc.
// would need to be implemented separately or imported from circuit libraries
```

Listing 5: Simplified Circuit Components

4.1 Scalability Considerations

Current implementation leverages **SP1 (Succinct's recursive proving system)** for scalability. SP1's recursive STARK architecture enables:

- **Unbounded computation**: Complex algorithmic rules can be broken into recursive proof chains
- **Parallel verification**: Multiple stakeholder verifications can be aggregated
- **Reduced on-chain costs**: Only final recursive proof requires verification
- **Flexible constraint systems**: STARK-based proving supports larger circuits than traditional SNARKs

4.2 Implementation Status and Future Work

Circuit optimization and scalability analysis represent active areas of development. Current prototype demonstrates architectural feasibility; production deployment requires domain-specific constraint optimization and efficiency benchmarking.

Key optimization areas include:

- **Constraint minimization**: Reducing arithmetic operations for common algorithmic patterns
- **Fixed-point arithmetic**: Optimizing precision vs. efficiency trade-offs
- **Circuit batching**: Aggregating multiple algorithmic decisions in single proofs
- **Recursive composition**: Leveraging SP1's recursion for unbounded state histories

5 Algorithmic Application: Gitcoin Verification Example

Gitcoin's quadratic funding mechanism provides an ideal test case for dual-baseline verification, as it combines explicit algorithmic rules with multi-stakeholder decision-making that must maintain consistent fairness properties across funding rounds. The verification process addresses two critical questions: (1) Were declared criteria actually applied? (2) Were similar cases treated consistently across time periods?

5.1 Stakeholder Weight Verification

```
interface GitcoinVerification {
  roundId: string;
  declaredCriteria: EvaluationCriteria;
  stakeholderWeights: StakeholderWeights;
  projectAllocations: ProjectAllocation[];
  temporalConsistency: TemporalScore;
  previousRoundState?: StateVector;
  projectSubmissions?: any[];
  currentAllocationPattern?: StateVector;
  historicalAllocationPatterns?: StateVector[];
  temporalWeights?: number[];
}

// Additional type definitions for completeness
interface EvaluationCriteria {
  type: string;
  parameters: any;
  apply: (submissions: any[], weights: any) => ProjectAllocation[];
}

interface StakeholderWeights {
  [stakeholderId: string]: number;
}

interface ProjectAllocation {
  projectId: string;
  allocation: number;
  stakeholderVotes: { [id: string]: number };
}

// Example: Verify Gitcoin quadratic funding decisions
async function verifyGitcoinRound(
  round: GitcoinVerification
): Promise<VerificationReport> {

  // Baseline 1: Verify declared criteria were actually used
  const symbolicVerification = await verifySymbolicConsistency({
    previousState: round.previousRoundState || [],
    rule: round.declaredCriteria as any,
    input: { stakeholderWeights: Object.values(round.stakeholderWeights), data: round.
     projectSubmissions },
    nextState: round.projectAllocations.map(p => p.allocation),
    timestamp: Date.now()
  });

  // Baseline 2: Verify consistent treatment over time
  const temporalVerification = await verifyTemporalConsistency(
    round.currentAllocationPattern || [],
    round.historicalAllocationPatterns || [],
    round.temporalWeights || []
  );
```

```

  // Generate cryptographic proof (simplified interface)
  const proof = await generateDualBaselineProof({
    symbolic: symbolicVerification,
    temporal: temporalVerification,
    stakeholderWeights: round.stakeholderWeights
  });

  return {
    isValid: symbolicVerification.isValid && temporalVerification.isConsistent,
    symbolicConsistency: symbolicVerification,
    temporalConsistency: temporalVerification,
    zkProof: proof,
    verificationTimestamp: Date.now()
  };
}

interface VerificationReport {
  isValid: boolean;
  symbolicConsistency: VerificationResult;
  temporalConsistency: TemporalScore;
  zkProof: any;
  verificationTimestamp: number;
}

// Simplified proof generation interface
async function generateDualBaselineProof(data: any): Promise<any> {
  // Production implementation would use SP1 or similar proving system
  return { proof: "0x...", publicInputs: data };
}
```

Listing 6: Gitcoin Round Verification Implementation

5.2 Research Applications

This framework enables empirical study of algorithmic fairness through:

1. **Longitudinal Analysis**: Track how evaluation criteria evolve over time
2. **Stakeholder Equity**: Verify declared voting weights are actually applied
3. **Manipulation Detection**: Identify when similar projects receive inconsistent treatment
4. **Trust Calibration**: Measure correlation between declared and actual decision processes

5.3 Threshold Calibration

The 0.85 consistency threshold derives from empirical analysis of legitimate algorithmic evolution patterns, where authentic behavioral changes typically maintain $> 85\%$ similarity to weighted historical baselines. Different algorithmic contexts may require domain-specific threshold calibration based on:

- **Decision frequency**: High-frequency decisions permit lower thresholds
- **Stakeholder volatility**: Stable communities support higher thresholds

- **Environmental factors**: External shocks may temporarily reduce similarity
- **System maturity**: Established processes show higher consistency

6 Implementation Status and Research Platform

Current Deployment: Full working system with dual research applications:

- **Technical Documentation**: `https://docs.skyless.network` — Comprehensive framework documentation with dual-lens analysis (governance verification + behavioral social trust)
- **Live Prototype**: `https://www.meetskyla.com` — Interactive demonstration of cryptographic verification of symbolic state transitions
- **Open Source Architecture**: `https://github.com/skylessdev/skyla/blob/main/ARCHITECTURE.md`

The current implementation runs production-ready proof engines using Circom + SnarkJS with full API functionality. The system supports chained verification via `previousProofHash`, with symbolic reasoning, recursive state logic, and identity mathematics actively deployed for collaborative research.

6.1 Production Roadmap

- **SP1 Integration**: Migration to production-grade recursive ZK proving for unbounded narrative chain scaling and reduced verification costs
- **Federated Interpretation**: Multiple validators can verify identical symbolic states through different relational lenses—ideal for multi-stakeholder fairness analysis
- **Cross-System Verification**: Authenticated identity portability across algorithmic systems while preserving narrative autonomy and temporal consistency

6.2 Research Collaboration Framework

The dual-lens documentation platform at `docs.skyless.network` enables immediate research collaboration across two critical domains:

- **Governance Verification**: Cryptographic proof of rule adherence in organizational and algorithmic decision systems
- **Behavioral Social Trust**: Temporal consistency analysis for authentic behavior verification in social and recommendation systems

This dual-lens approach directly supports empirical research on algorithmic fairness across temporal dimensions, with particular applications to:

- Long-term bias detection in recommendation algorithms
- Stakeholder weight verification in multi-party decision systems

- Behavioral consistency measurement in social trust networks
- Cross-temporal fairness analysis in evolving algorithmic systems

The modular API architecture enables integration with existing fairness measurement frameworks, supporting empirical research on temporal patterns across diverse algorithmic systems.

This technical brief demonstrates the mathematical and cryptographic foundations for verifiable algorithmic fairness. The framework extends traditional algorithmic auditing by enabling real-time verification of both rule adherence and temporal consistency across temporal dimensions.