Artificial Intelligence

Lecture 5：Knowledge and Reasoning

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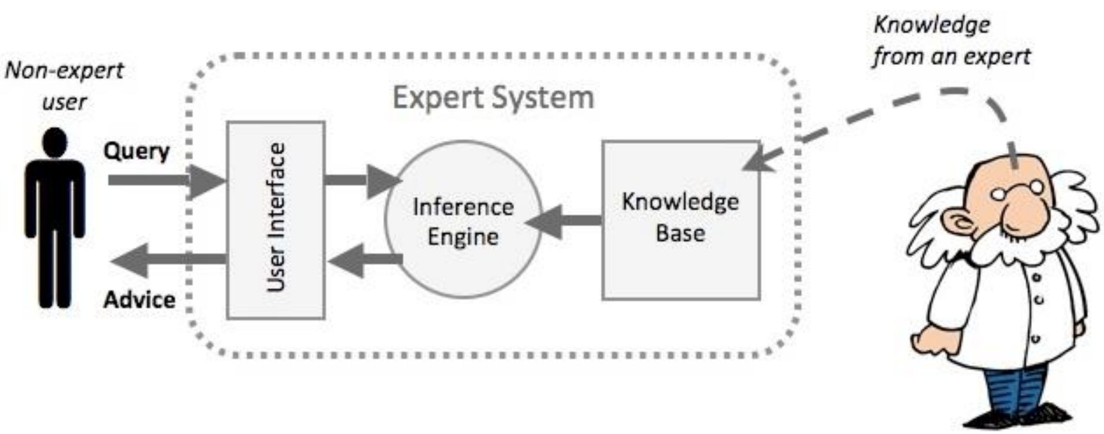
Credits: AI Courses in Berkeley & JHU

Outline

- Knowledge-based Agent
- Propositional Logic
- First-Order Logic
- Knowledge Representation



Expert system

Knowledge graph

Knowledge-based Agents

- A knowledge-based agent is composed of a knowledge base and an inference mechanism.
- Knowledge base: a set of sentences in a knowledge representation language that is defined by its syntax and semantics.
  - Propositional logic
  - First-order logic
- Inference: derive new sentences from old.

Knowledge-based Agents

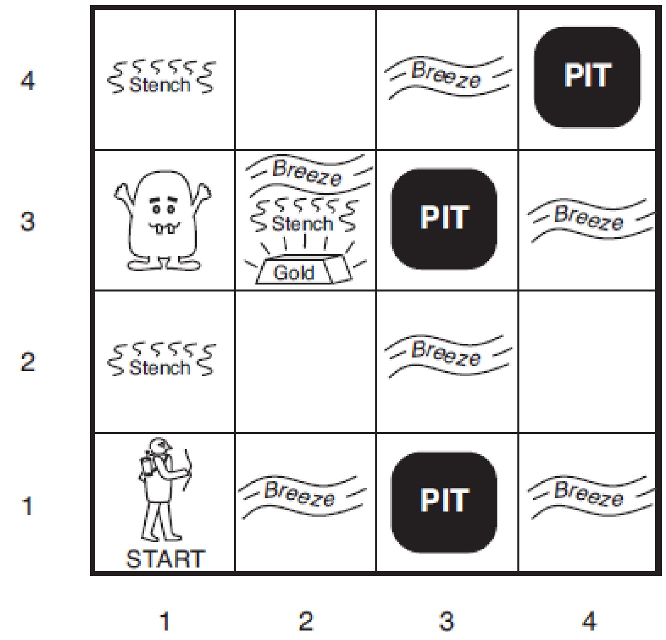
function KB-AGENT(*percept*) **returns** an *action*
  persistent: KB, a knowledge base
    t, a counter, initially 0, indicating time

  TELL(KB, MAKE-PERCEPT-SENTENCE($percept, t$))
  $action \leftarrow$ ASK(KB, MAKE-ACTION-QUERY(t))
  TELL(KB, MAKE-ACTION-SENTENCE($action, t$))
  $t \leftarrow t + 1$
  return *action*

- The agent must be able to
  - represent states, actions, etc.
  - incorporate new percepts
  - update internal representations of the world
  - deduce hidden properties of the world
  - deduce appropriate actions

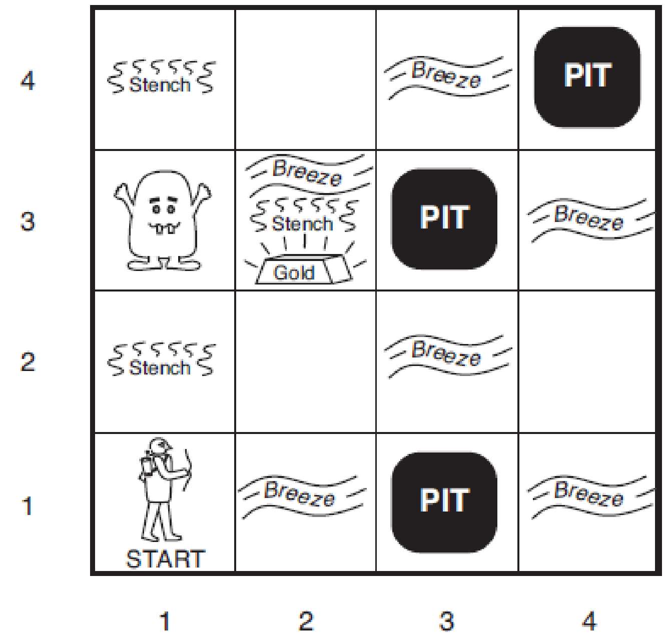
Example: The Wumpus World

- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow
- Environment
  - squares adjacent to wumpus are stench
  - squares adjacent to pit are breezy
  - glitter iff gold is in the same square
  - shooting kills wumpus if you are facing it
  - shooting uses up the only arrow
  - grabbing picks up gold if in same square
  - releasing drops the gold in same square
- Actuators
  - Left turn, Right turn, Forward, Grab, Shoot, Climb
- Sensors
  - Stench, Breeze, Glitter, Bump, Scream



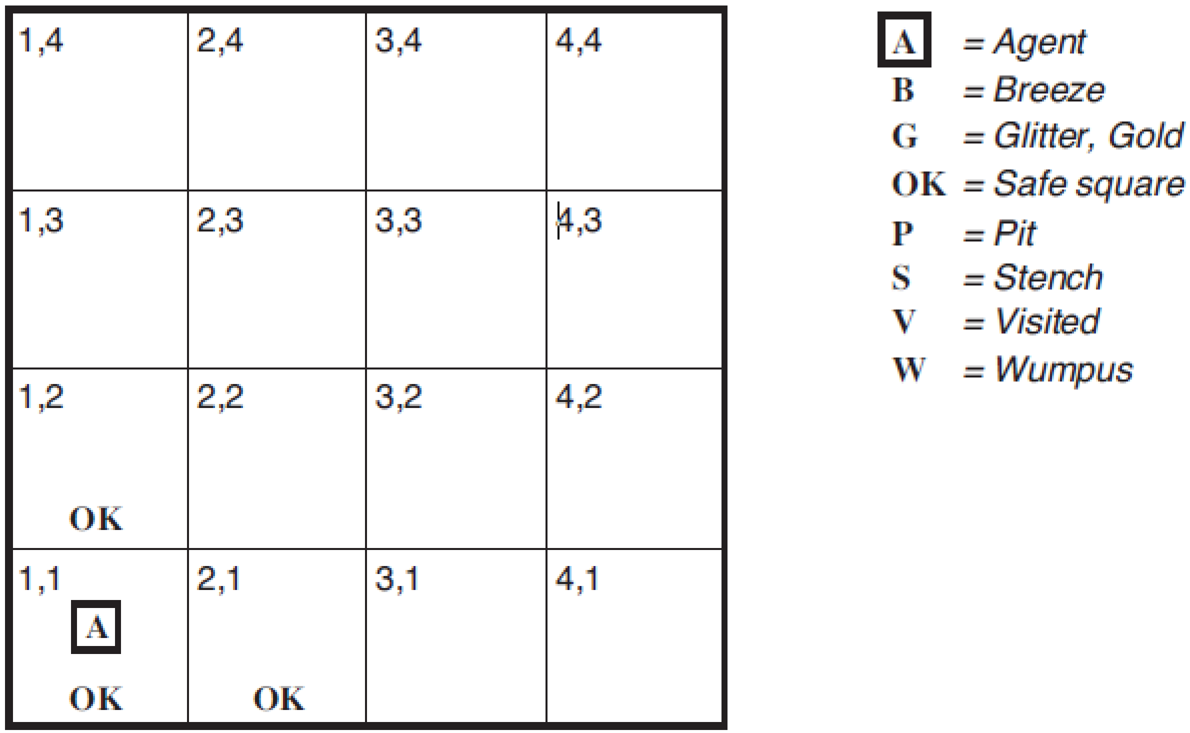
Example: The Wumpus World

- Observable?
  - No—only local perception
- Deterministic?
  - Yes—outcomes exactly specified
- Episodic?
  - No—sequential at the level of actions
- Static?
  - Yes—Wumpus and Pits do not move
- Discrete?
  - Yes
- Single-agent?
  - Yes—Wumpus is essentially a natural feature



Example: The Wumpus World

- A knowledge-based wumpus agent exploring the environment
  - Logical reasoning



$$[None, None, None, None, None].$$

- Stench, Breeze, Glitter, Bump, Scream

Example: The Wumpus World

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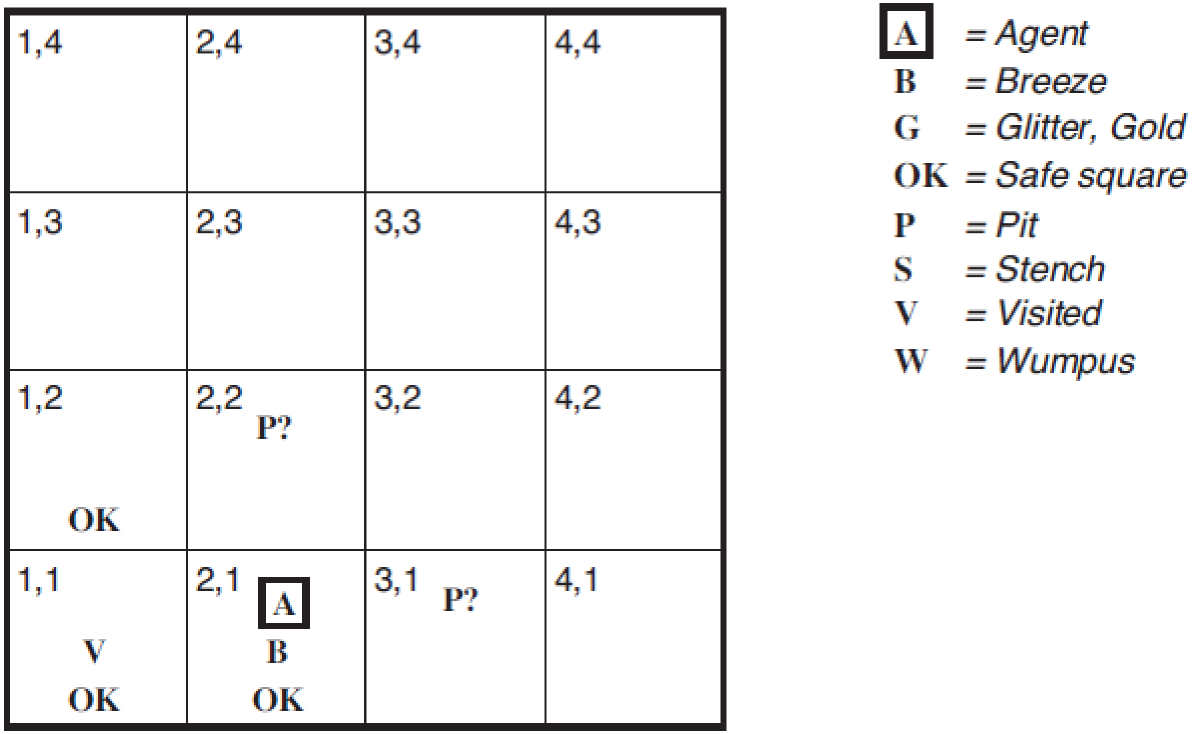

$$[None, Breeze, None, None, None]$$

Example: The Wumpus World

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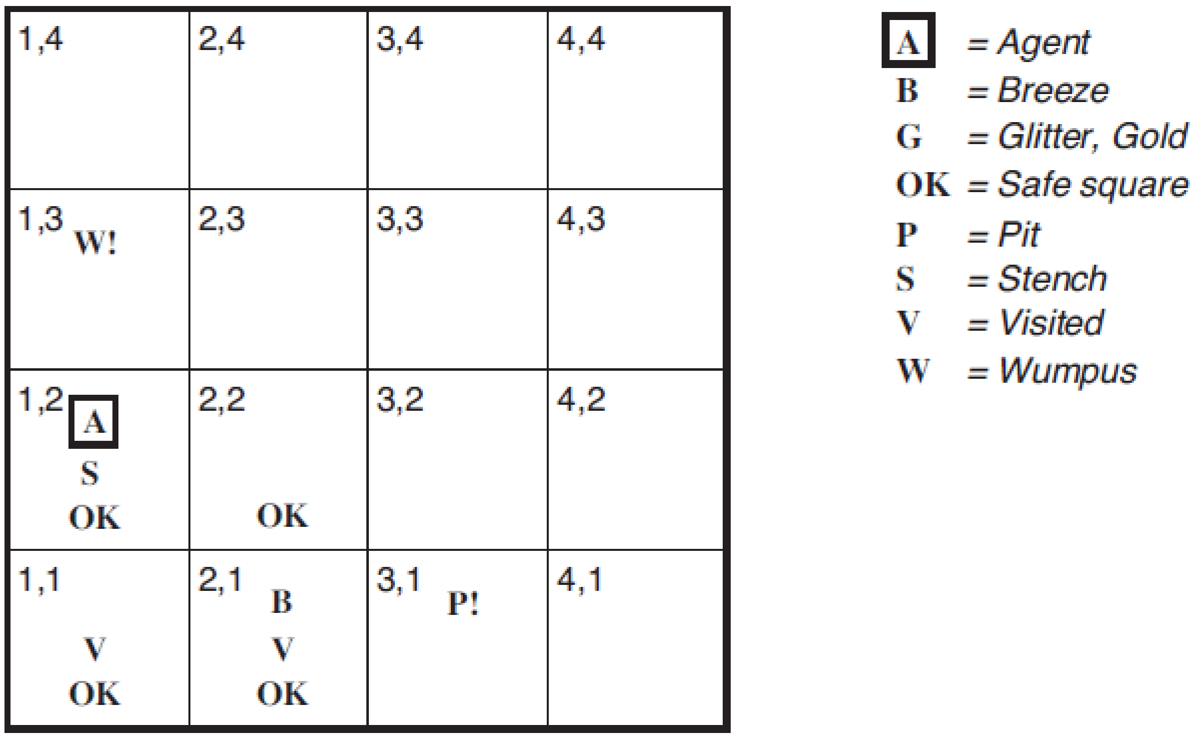

$$[Stench, None, None, None, None]$$

Example: The Wumpus World

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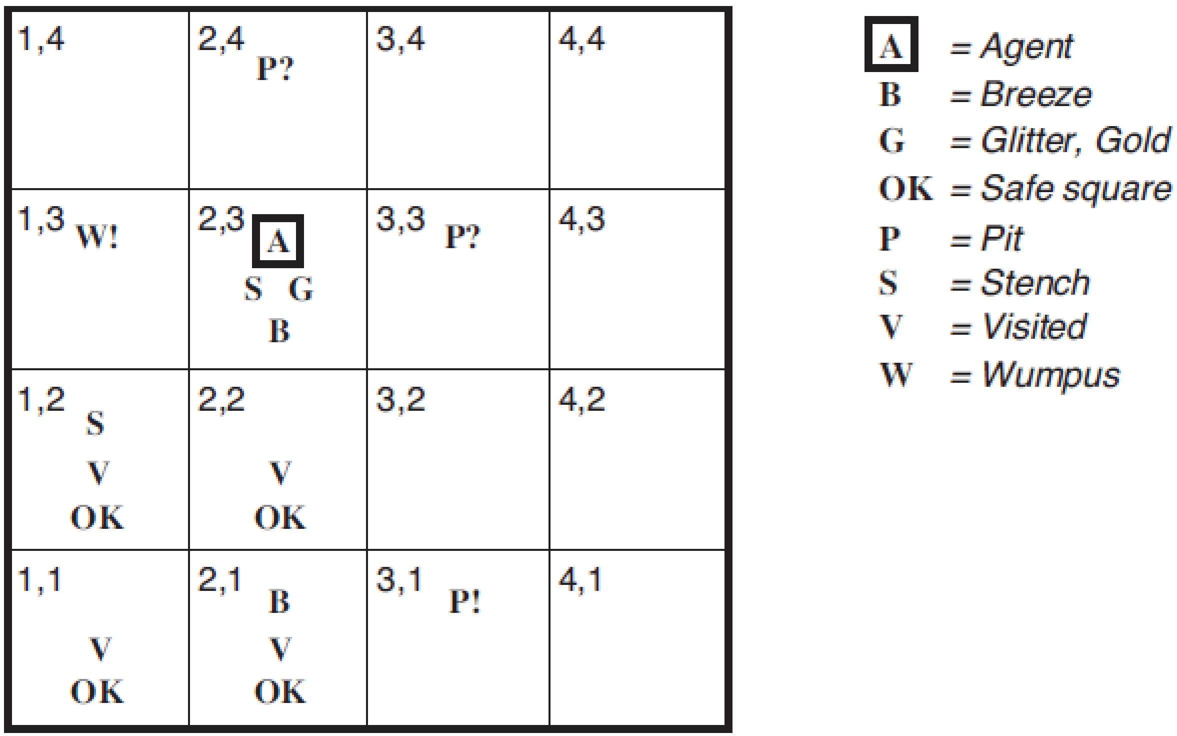


$$[Stench, Breeze, Glitter, None, None]$$

Logic in General

- Logics
  - Formal languages for representing information such that conclusions can be drawn
- Syntax
  - Defines the sentences in the language
- Semantics
  - Defines the truth of a sentence in a possible world (model)

- E.g., the language of arithmetic
  - $x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence
  - $x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number y
  - $x + 2 \geq y$ is true in a world where $x = 7,\ y = 1$
    $x + 2 \geq y$ is false in a world where $x = 0,\ y = 6$

Models

- A possible world is represented by a model
- Models are mathematical abstractions, each of which simply fixes the truth or falsehood of every relevant sentences.
- m is a model of a sentence α if α is true in model m.
- $M(\alpha)$: the set of all models of α.

Entailment

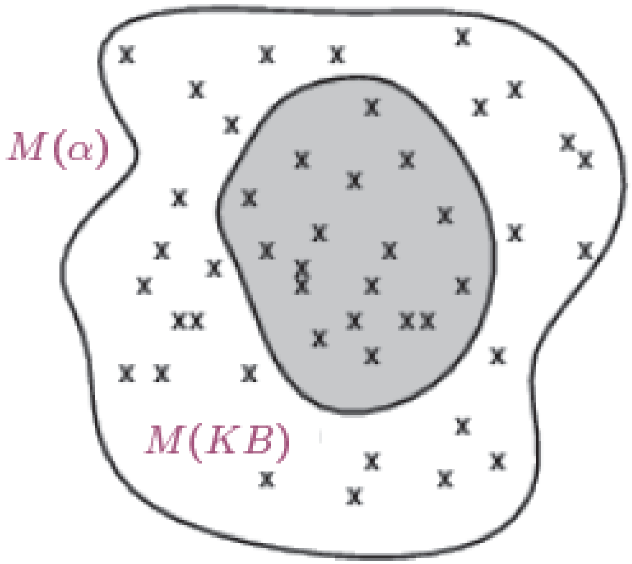
- Entailment: a sentence follows logically from another:

$$KB \models \alpha$$

- Knowledge base *KB* entails sentence α iff α is true in all models where *KB* is true

$$KB \models \alpha \text{ if and only if } M(KB) \subseteq M(\alpha)$$

- Eg. *x=0* entails *xy=0*



Entailment in the Wumpus World

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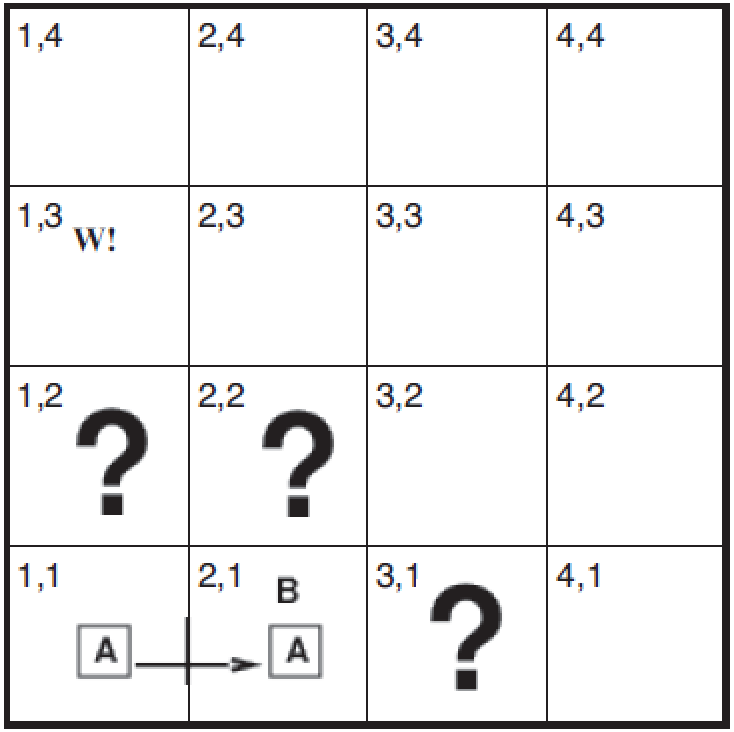
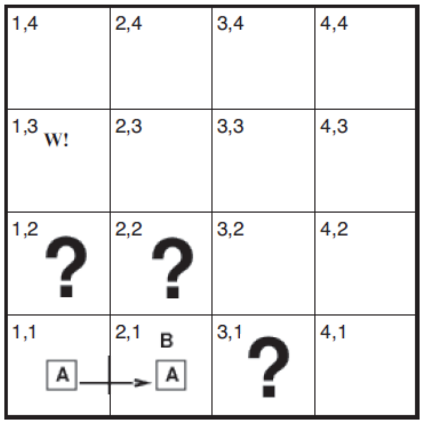
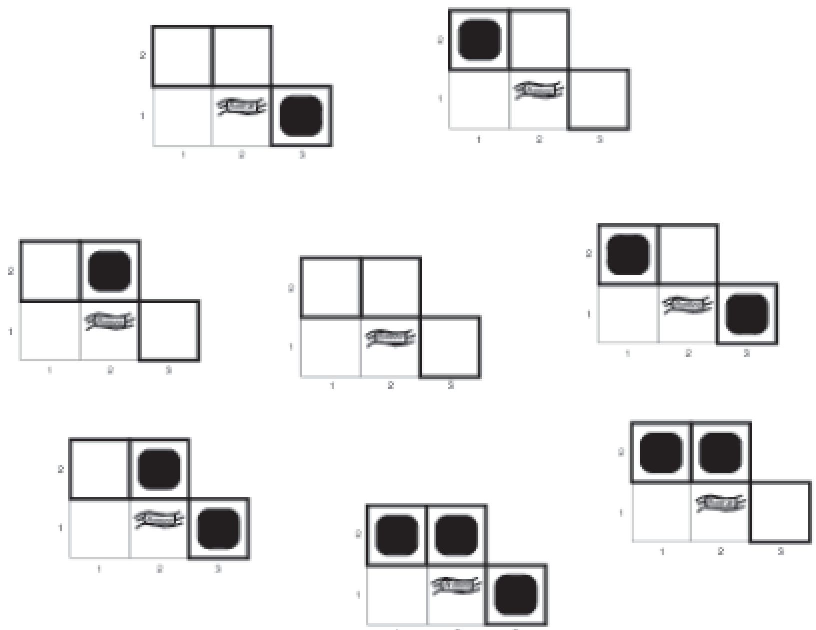


- Situation after detecting nothing in [1,1], moving right, breeze in [2,1]
- Consider possible models for all **?**, assuming only pits
- 3 Boolean choices $\Longrightarrow$ 8 possible models

Entailment in the Wumpus World

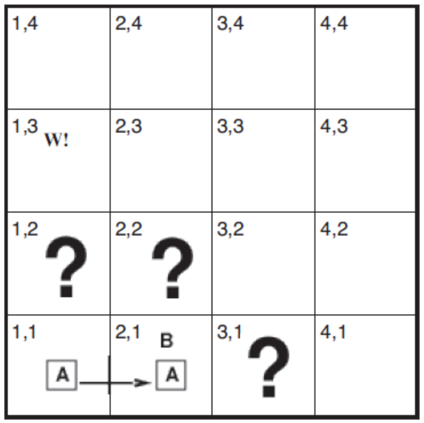
- Possible Wumpus Models

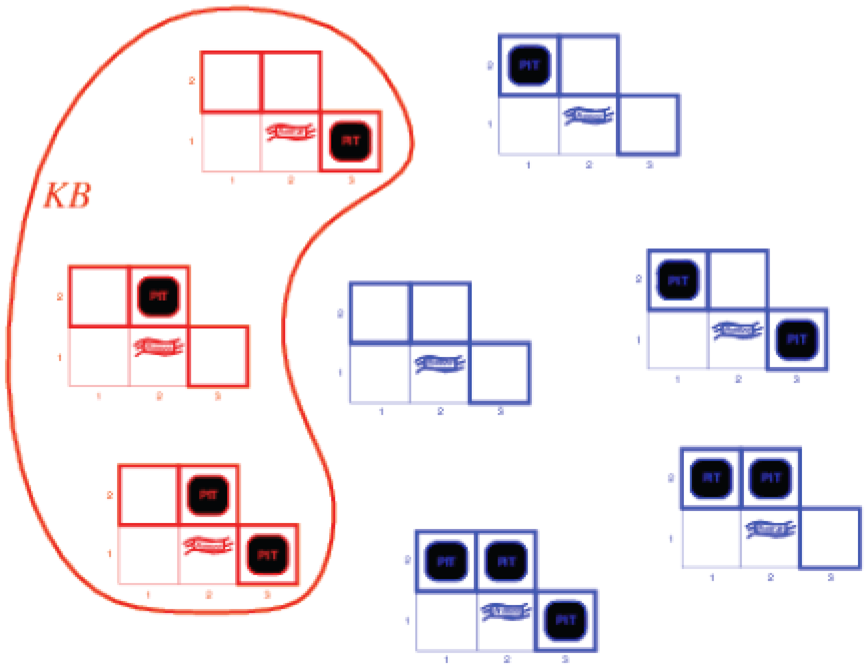




Entailment in the Wumpus World

- Valid Wumpus Models

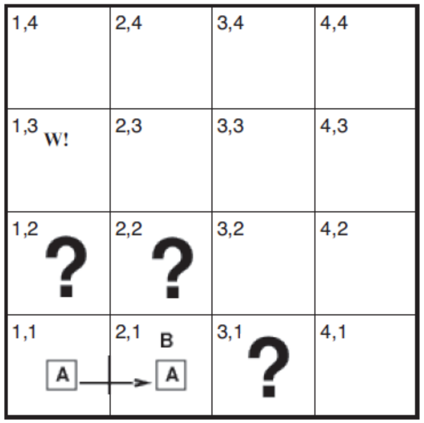


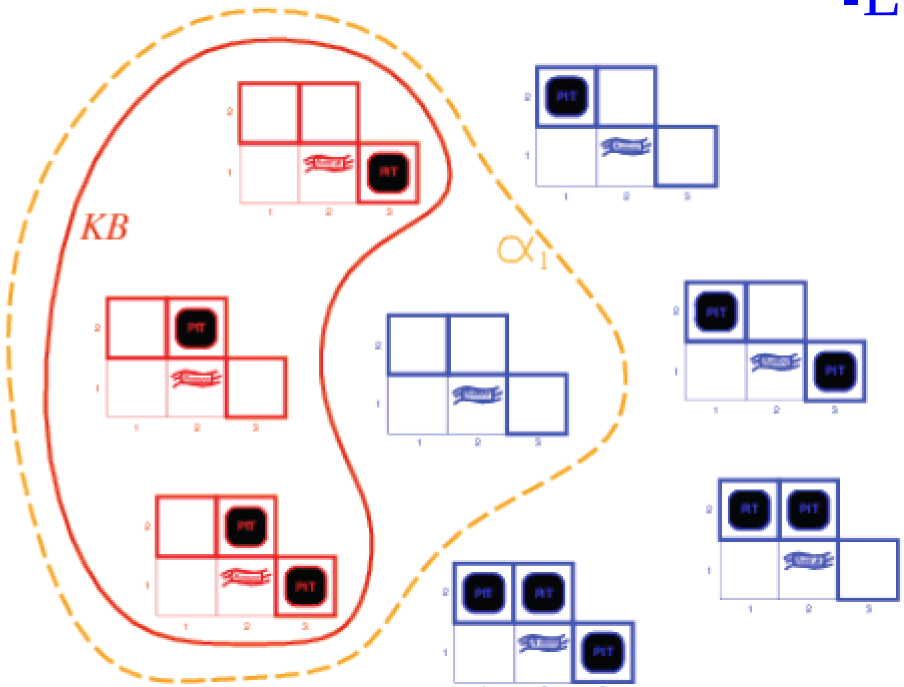


KB = wumpus-world rules + observations

Entailment in the Wumpus World

- Entailment





- Logical inference

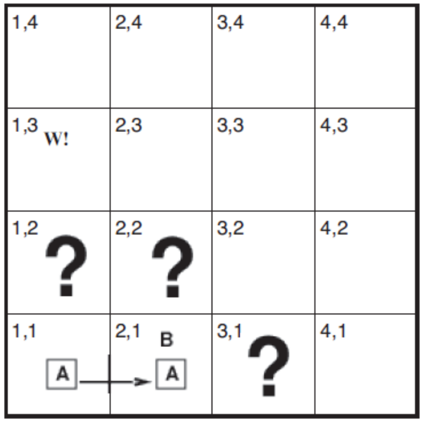
Model checking:
Enumerate the models, and check that α is true in every model in which *KB* is true

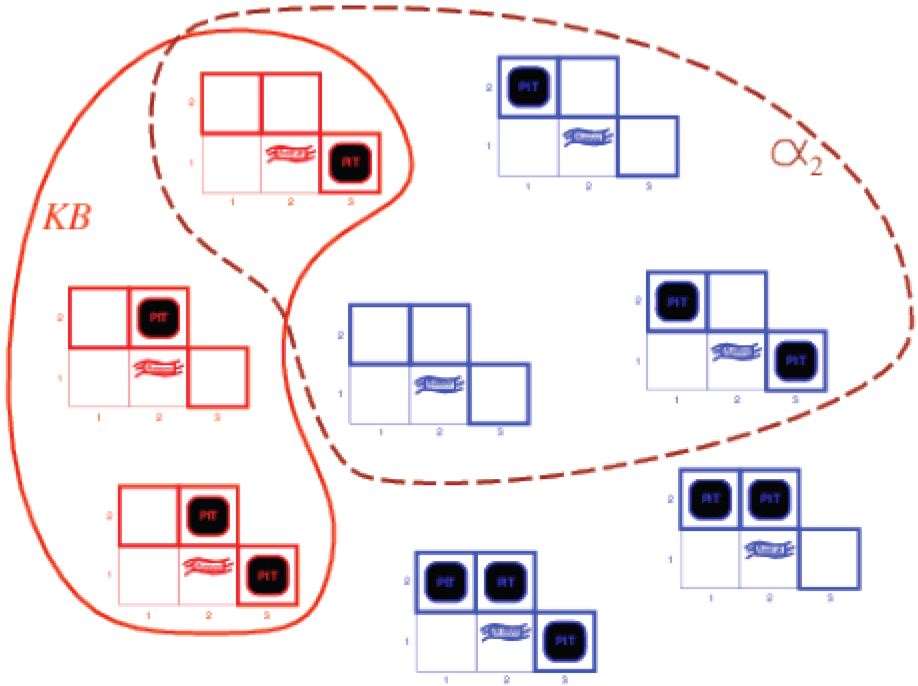
KB = wumpus-world rules + observations

α_1 = “[1,2] is safe”, $KB \models \alpha_1$, proved by model checking

Entailment in the Wumpus World

- Not Entailed





KB = wumpus-world rules + observations

α_2 = “[2,2] is safe”, $KB \not\models \alpha_2$

Inference

- If an inference algorithm *i* can derive α from *KB*

$$KB \vdash_i \alpha$$

- Soundness

i is sound if
whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

- Completeness

i is complete if
whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Propositional Logic: Syntax

logical connectives

- Propositional logic is the simplest logic—illustrates basic ideas
- The proposition symbols P_1, P_2 etc are sentences
- If P is a sentence, $\neg P$ is a sentence (negation)
- If P_1 and P_2 are sentences, $P_1 \wedge P_2$ is a sentence (conjunction)
- If P_1 and P_2 are sentences, $P_1 \vee P_2$ is a sentence (disjunction)
- If P_1 and P_2 are sentences, $P_1 \implies P_2$ is a sentence (implication)
- If P_1 and P_2 are sentences, $P_1 \Leftrightarrow P_2$ is a sentence (biconditional)

Propositional Logic: Syntax

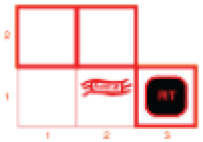
- BNF (Backus-Naur Form) grammar of sentences

$$
\begin{array}{rcl}
Sentence & \rightarrow & AtomicSentence \mid ComplexSentence \\
AtomicSentence & \rightarrow & True \mid False \mid P \mid Q \mid R \mid \ldots \\
ComplexSentence & \rightarrow & (\ Sentence\ ) \mid [\ Sentence\ ] \\
& \mid & \neg\ Sentence \\
& \mid & Sentence \land Sentence \\
& \mid & Sentence \lor Sentence \\
& \mid & Sentence \Rightarrow Sentence \\
& \mid & Sentence \Leftrightarrow Sentence \\
\\
\textsc{Operator Precedence} & : & \neg, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

Propositional Logic: Semantics

- Each model specifies true/false for each proposition symbol

  E.g. $m_1 = \{P_{1,2} = false,\ P_{2,2} = false,\ P_{3,1} = true\}$



  (with these symbols, 8 possible models, can be enumerated automatically)

- Rules for evaluating truth with respect to a model m:

$\neg P$	is true iff	P	is false		
$P_1 \wedge P_2$	is true iff	P_1	is true **and**	P_2	is true
$P_1 \vee P_2$	is true iff	P_1	is true **or**	P_2	is true
$P_1 \implies P_2$	is true iff	P_1	is false **or**	P_2	is true
i.e.,	is false iff	P_1	is true **and**	P_2	is false
$P_1 \Leftrightarrow P_2$	is true iff	$P_1 \implies P_2$	is true **and**	$P_2 \implies P_1$	is true

- Simple recursive process evaluates an arbitrary sentence, e.g.,
  $\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = true \wedge (false \vee true) = true \wedge true = true$

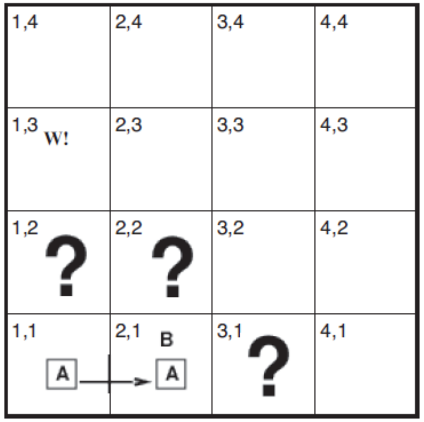
Truth Tables for Connectives

- Truth table

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

The Wumpus World

- Knowledge base
  - Let $P_{i,j}$ be true if there is a pit in $[i, j]$
    - observation $R_1 : \neg P_{1,1}$
  - Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.
  - "Pits cause breezes in adjacent squares"
    - rule $R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
    - rule $R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
    - observation $R_4 : \neg B_{1,1}$
    - observation $R_5 : B_{2,1}$
  - What can we infer about $P_{1,2}$, $P_{2,1}$, $P_{2,2}$, etc.?



The Wumpus World

- Model checking
  - Truth tables for inference

$R_1 : \neg P_{1,1}$
$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
$R_4 : \neg B_{1,1}$
$R_5 : B_{2,1}$

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Inference by Model Checking

```
function TT-ENTAILS?(KB, α) returns true or false
   inputs: KB, the knowledge base, a sentence in propositional logic
           α, the query, a sentence in propositional logic

   symbols ← a list of the proposition symbols in KB and α
   return TT-CHECK-ALL(KB, α, symbols, { })
```

```
function TT-CHECK-ALL(KB, α, symbols, model) returns true or false
   if EMPTY?(symbols) then
      if PL-TRUE?(KB, model) then return PL-TRUE?(α, model)
      else return true // when KB is false, always return true
   else do
      P ← FIRST(symbols)
      rest ← REST(symbols)
      return (TT-CHECK-ALL(KB, α, rest, model ∪ {P = true})
              and
              TT-CHECK-ALL(KB, α, rest, model ∪ {P = false }))
```

Figure 7.10 A truth-table enumeration algorithm for deciding propositional entailment. (TT stands for truth table.) PL-TRUE? returns *true* if a sentence holds within a model. The variable *model* represents a partial model—an assignment to some of the symbols. The keyword "**and**" is used here as a logical operation on its two arguments, returning *true* or *false*.

- Sound
- Complete
- $O(2^n)$

Inference

- Model checking:
  - Enumerating models and checking that α is true in every model in which KB is true.
- Theorem proving:
  - Applying rules of inference directly to the sentences in knowledge base to construct a proof of the desired sentence without consulting models.

Inference

- Logical equivalence

  - Two sentences are logically equivalent iff true in same models:

    $\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

$$
\begin{array}{rcll}
(\alpha \land \beta) & \equiv & (\beta \land \alpha) & \text{commutativity of } \land \\
(\alpha \lor \beta) & \equiv & (\beta \lor \alpha) & \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) & \equiv & (\alpha \land (\beta \land \gamma)) & \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) & \equiv & (\alpha \lor (\beta \lor \gamma)) & \text{associativity of } \lor \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \implies \beta) & \equiv & (\neg\beta \implies \neg\alpha) & \text{contraposition} \\
(\alpha \implies \beta) & \equiv & (\neg\alpha \lor \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \implies \beta) \land (\beta \implies \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \land \beta) & \equiv & (\neg\alpha \lor \neg\beta) & \text{De Morgan} \\
\neg(\alpha \lor \beta) & \equiv & (\neg\alpha \land \neg\beta) & \text{De Morgan} \\
(\alpha \land (\beta \lor \gamma)) & \equiv & ((\alpha \land \beta) \lor (\alpha \land \gamma)) & \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) & \equiv & ((\alpha \lor \beta) \land (\alpha \lor \gamma)) & \text{distributivity of } \lor \text{ over } \land
\end{array}
$$

Inference

- Validity
  - A sentence is valid if it is true in **all** models,
    e.g., $True$, $A \vee \neg A$, $A \implies A$, $(A \wedge (A \implies B)) \implies B$
  - Validity is connected to inference via the Deduction Theorem:
    $KB \models \alpha$ if and only if $(KB \implies \alpha)$ is valid
- Satisfiability
  - A sentence is satisfiable if it is true in **some** model
    e.g., $A \vee B$, C
  - A sentence is unsatisfiable if it is true in **no** models
    e.g., $A \wedge \neg A$
  - Satisfiability is connected to inference via the following:
    $KB \models \alpha$ if and only if $(KB \wedge \neg \alpha)$ is unsatisfiable

Inference: Proof by Resolution

- Resolution Rule:

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$

Inference: Proof by Resolution

- Conjunctive Normal Form

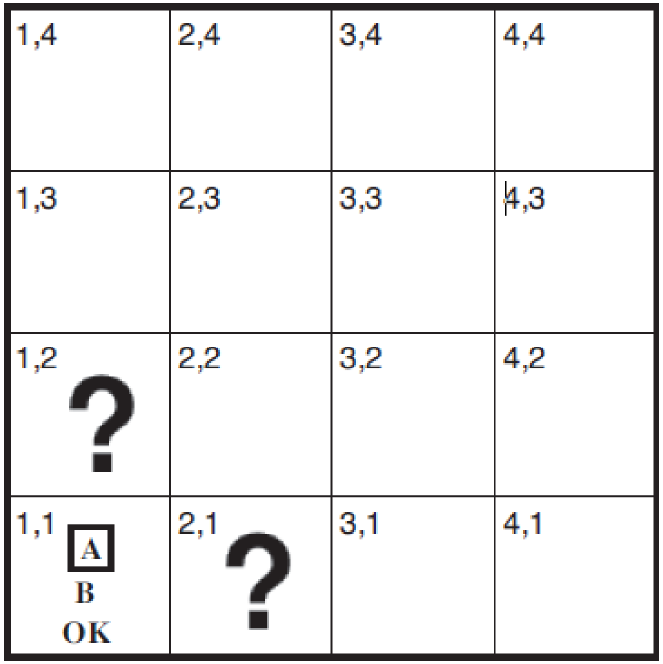
$$
\begin{aligned}
CNFSentence &\rightarrow Clause_1 \wedge \cdots \wedge Clause_n \\
Clause &\rightarrow Literal_1 \vee \cdots \vee Literal_m \\
Literal &\rightarrow Symbol \mid \neg Symbol \\
Symbol &\rightarrow P \mid Q \mid R \mid \ldots \\
HornClauseForm &\rightarrow DefiniteClauseForm \mid GoalClauseForm \\
DefiniteClauseForm &\rightarrow (Symbol_1 \wedge \cdots \wedge Symbol_l) \Rightarrow Symbol \\
GoalClauseForm &\rightarrow (Symbol_1 \wedge \cdots \wedge Symbol_l) \Rightarrow False
\end{aligned}
$$

conjunction of $\underbrace{\textbf{disjunctions} \text{ of } \textbf{literals}}_{\textbf{clauses}}$

E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

Inference: Proof by Resolution

- Example：The Wampus World



- Rules such as: “If breeze, then a pit adjacent.“

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

Inference: Proof by Resolution

- Example: The Wampus World

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \implies \beta) \wedge (\beta \implies \alpha)$.

$$(B_{1,1} \implies (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \implies B_{1,1})$$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

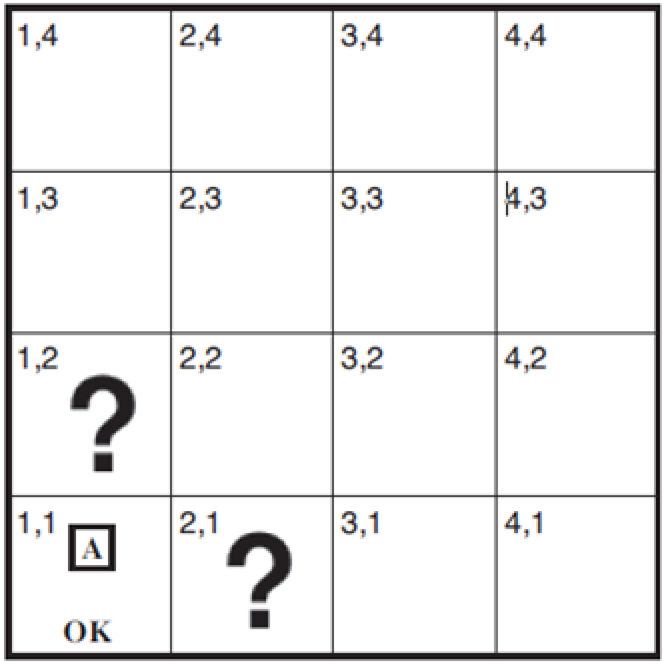
$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$$

4. Apply distributivity law ($\vee$ over $\wedge$) and flatten:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

Inference: Proof by Resolution

- Example： The Wampus World



- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1}))$
  reformulated as:
  $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$
- Observation: $\neg B_{1,1}$
- Goal: disprove: $\alpha = \neg P_{1,2}$
- Resolution

$$\frac{\neg P_{1,2} \vee B_{1,1} \quad \neg B_{1,1}}{\neg P_{1,2}}$$

- Resolution

$$\frac{\neg P_{1,2} \quad P_{1,2}}{false}$$

$\boxed{KB \models \alpha \text{ if and only if } (KB \wedge \neg\alpha) \text{ is unsatisfiable}}$

Inference: Proof by Resolution

- Resolution Algorithm

```
function PL-RESOLUTION(KB, α) returns true or false
  inputs: KB, the knowledge base, a sentence in propositional logic
          α, the query, a sentence in propositional logic

  clauses ← the set of clauses in the CNF representation of KB ∧ ¬α
  new ← { }
  loop do
     for each pair of clauses Ci, Cj in clauses do
        resolvents ← PL-RESOLVE(Ci, Cj)
        if resolvents contains the empty clause then return true
        new ← new ∪ resolvents
     if new ⊆ clauses then return false
     clauses ← clauses ∪ new
```

- Resolution is sound and complete for propositional logic

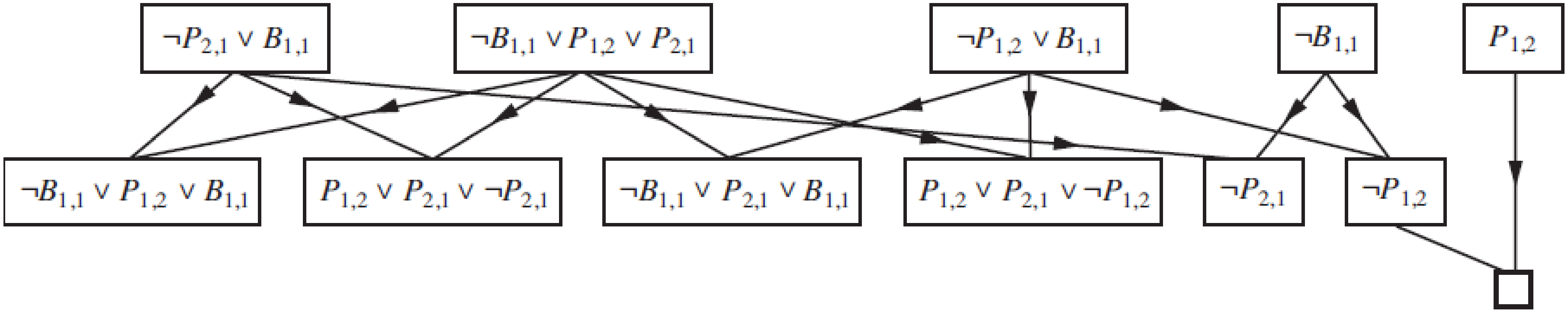
Inference: Proof by Resolution

- Example：The Wampus World
  - $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1}))$

    reformulated as:

    $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

  - Observation: $\neg B_{1,1}$
  - Goal: disprove: $\alpha = \neg P_{1,2}$



Inference:

- Definite clause:
  - A disjunction of literals of which exactly one is positive.
  - Eg. $\neg L_{1,1} \vee \neg Breeze \vee B_{1,1}$
- Horn clause:
  - A disjunction of literals of which at most one is positive.
- Goal clause:
  - Clauses with no positive literals.
- Horn clauses are closed under resolution.
  - If you resolve two Horn clauses, you get back a Horn clause.

Inference:

- Every definite clause can be written as
  - a conjunction of positive literals => a single positive literal

$$\neg L_{1,1} \vee \neg Breeze \vee B_{1,1} \qquad (L_{1,1} \wedge Breeze) \Rightarrow B_{1,1}$$

- Inference with Horn clauses can be done through forward chaining or backward chaining algorithms.
- These algorithms are very natural and run in linear time

Inference: Forward Chaining

- Forward chaining algorithm

```
function PL-FC-ENTAILS?(KB, q) returns true or false
  inputs: KB, the knowledge base, a set of propositional definite clauses
          q, the query, a proposition symbol
  count ← a table, where count[c] is the number of symbols in c's premise
  inferred ← a table, where inferred[s] is initially false for all symbols
  agenda ← a queue of symbols, initially symbols known to be true in KB

  while agenda is not empty do
     p ← POP(agenda)
     if p = q then return true
     if inferred[p] = false then
        inferred[p] ← true
        for each clause c in KB where p is in c.PREMISE do
           decrement count[c]
           if count[c] = 0 then add c.CONCLUSION to agenda
  return false
```

  - Forward chaining is sound and complete.

Inference: Forward Chaining

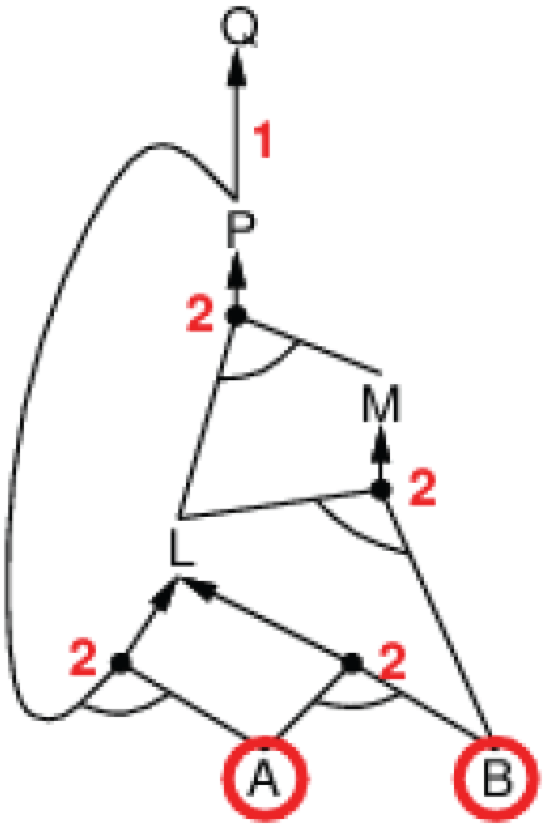
- Example

  - Given

    $$P \implies Q$$
    $$L \wedge M \implies P$$
    $$B \wedge L \implies M$$
    $$A \wedge P \implies L$$
    $$A \wedge B \implies L$$
    $$A$$
    $$B$$

  - Agenda: A, B

  - Annotate horn clauses with number of premises

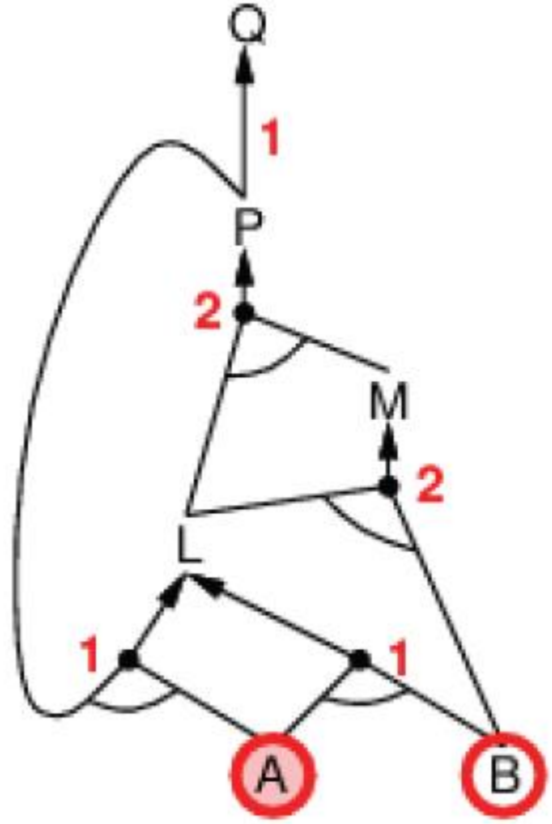


AND-OR Graph

Inference: Forward Chaining

- Example

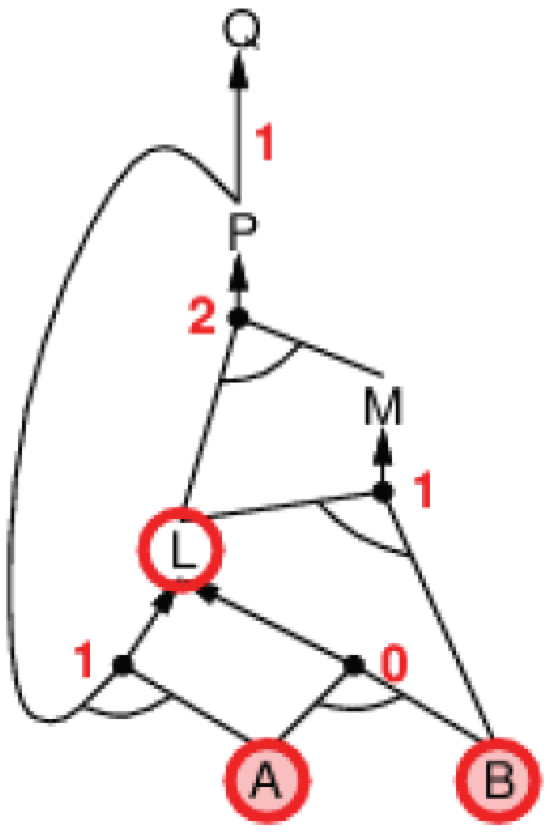
  - Process agenda item A
  - Decrease count for horn clauses in which A is premise



Inference: Forward Chaining

- Example

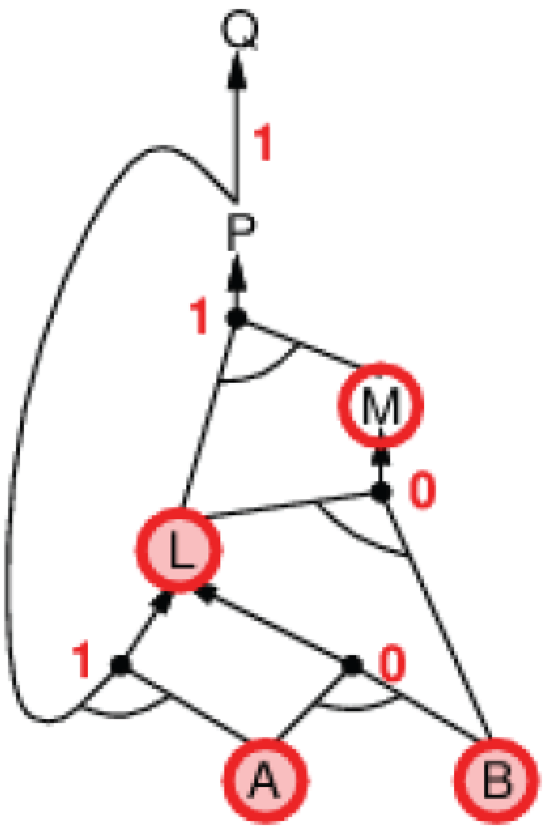
  - Process agenda item B
  - Decrease count for horn clauses in which B is premise
  - $A \wedge B \implies L$ has now fulfilled premise
  - Add L to agenda



Inference: Forward Chaining

- Example

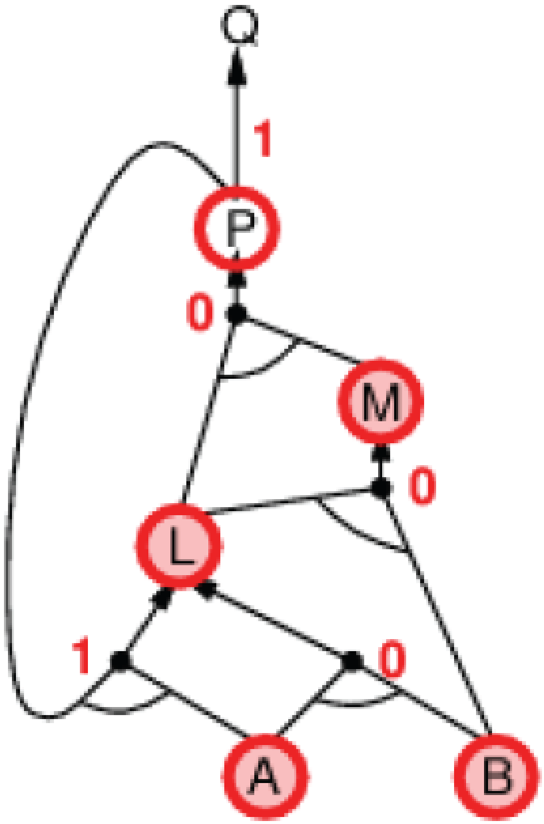
  - Process agenda item L
  - Decrease count for horn clauses in which L is premise
  - $B \wedge L \implies M$ has now fulfilled premise
  - Add M to agenda



Inference: Forward Chaining

- Example

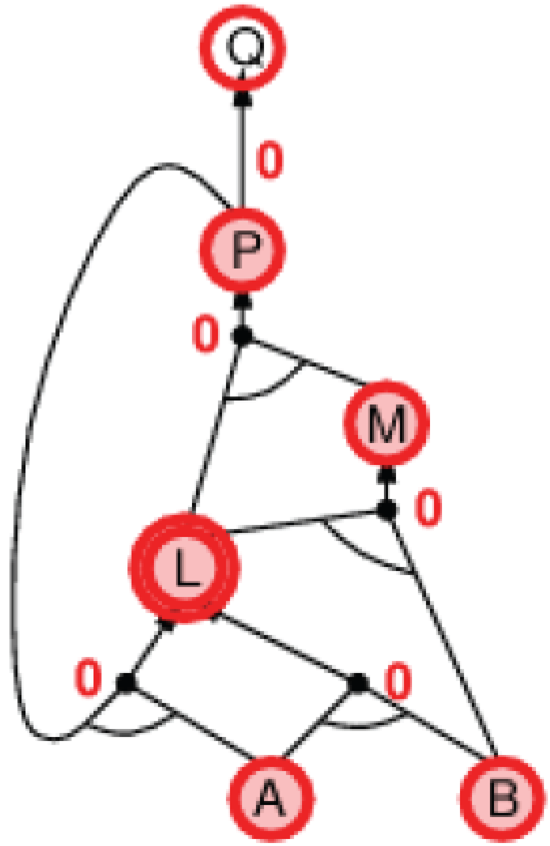
  - Process agenda item M
  - Decrease count for horn clauses in which M is premise
  - $L \wedge M \implies P$ has now fulfilled premise
  - Add P to agenda



Inference: Forward Chaining

- Example

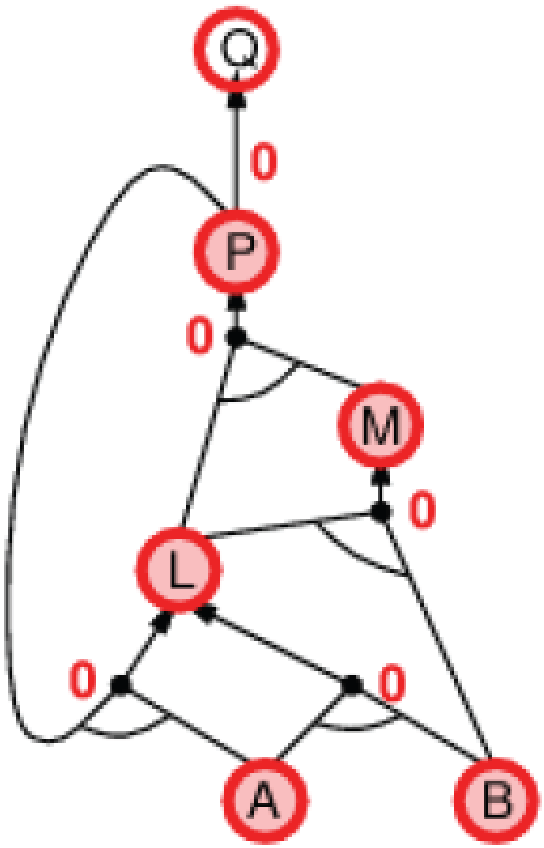
  - Process agenda item P
  - Decrease count for horn clauses in which P is premise
  - $P \implies Q$ has now fulfilled premise
  - Add Q to agenda
  - $A \wedge P \implies L$ has now fulfilled premise



Inference: Forward Chaining

- Example

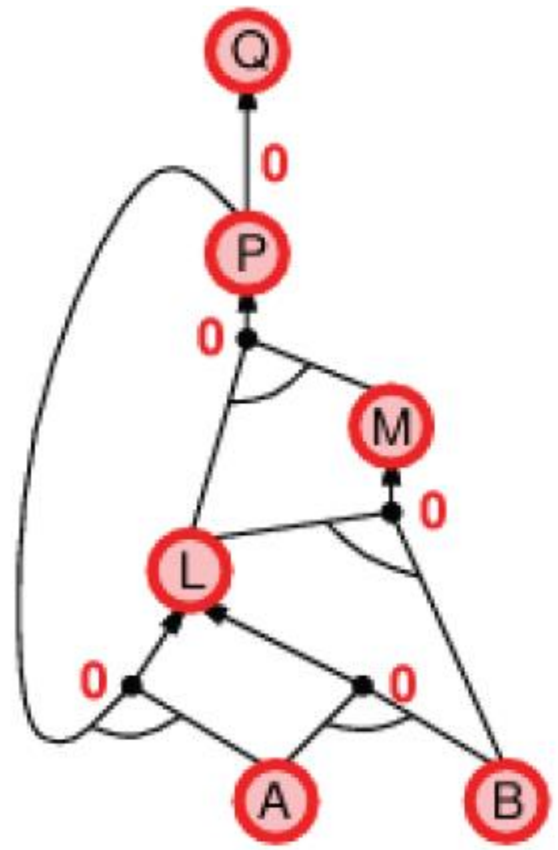
  - Process agenda item P
  - Decrease count for horn clauses in which P is premise
  - $P \implies Q$ has now fulfilled premise
  - Add Q to agenda
  - $A \wedge P \implies L$ has now fulfilled premise
  - But L is already inferred



Inference: Forward Chaining

- Example

  - Process agenda item Q
  - Q is inferred
  - Done



More Logics

▪

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief
Fuzzy logic	facts + degree of truth	known interval value

Higher-order logic:

relations and functions operate not only on objects, but also on relations and functions

First-Order Logic

- Propositional logic: world contains **facts**
- First-order logic: the world contains **objects**, **relations**, and **functions**
- Objects: people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries . . .
- Relations: red, round, bogus, prime, multistoried . . ., brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, . . .
- Functions: father of, best friend, third inning of, one more than, end of . . .

First-Order Logic

- Syntax

  - Constants: $KingJohn, 2, UCB, \ldots$
  - Predicates: $Brother, >, \ldots$
  - Functions: $Sqrt, LeftLegOf, \ldots$
  - Variables: $x, y, a, b, \ldots$
  - Connectives: $\wedge \ \vee \ \neg \ \Longrightarrow \ \Leftrightarrow$
  - Equality: $=$
  - Quantifiers: $\forall \ \exists$

First-Order Logic

- Syntax

  - Atomic sentence = $predicate(term_1, \ldots, term_n)$
    or $term_1 = term_2$
  - Term = $function(term_1, \ldots, term_n)$
    or $constant$ or $variable$
  - E.g., $Brother(KingJohn, RichardTheLionheart)$
    $> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))$

First-Order Logic

- Syntax

  - Complex sentences are made from atomic sentences using connectives

$$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \implies S_2, \quad S_1 \Leftrightarrow S_2$$

  - For instance
    - $Sibling(KingJohn, Richard) \implies Sibling(Richard, KingJohn)$
    - $>(1,2) \vee \leq(1,2)$
    - $>(1,2) \wedge \neg>(1,2)$

First-Order Logic

- Semantics

  - Sentences are true with respect to a model and an interpretation

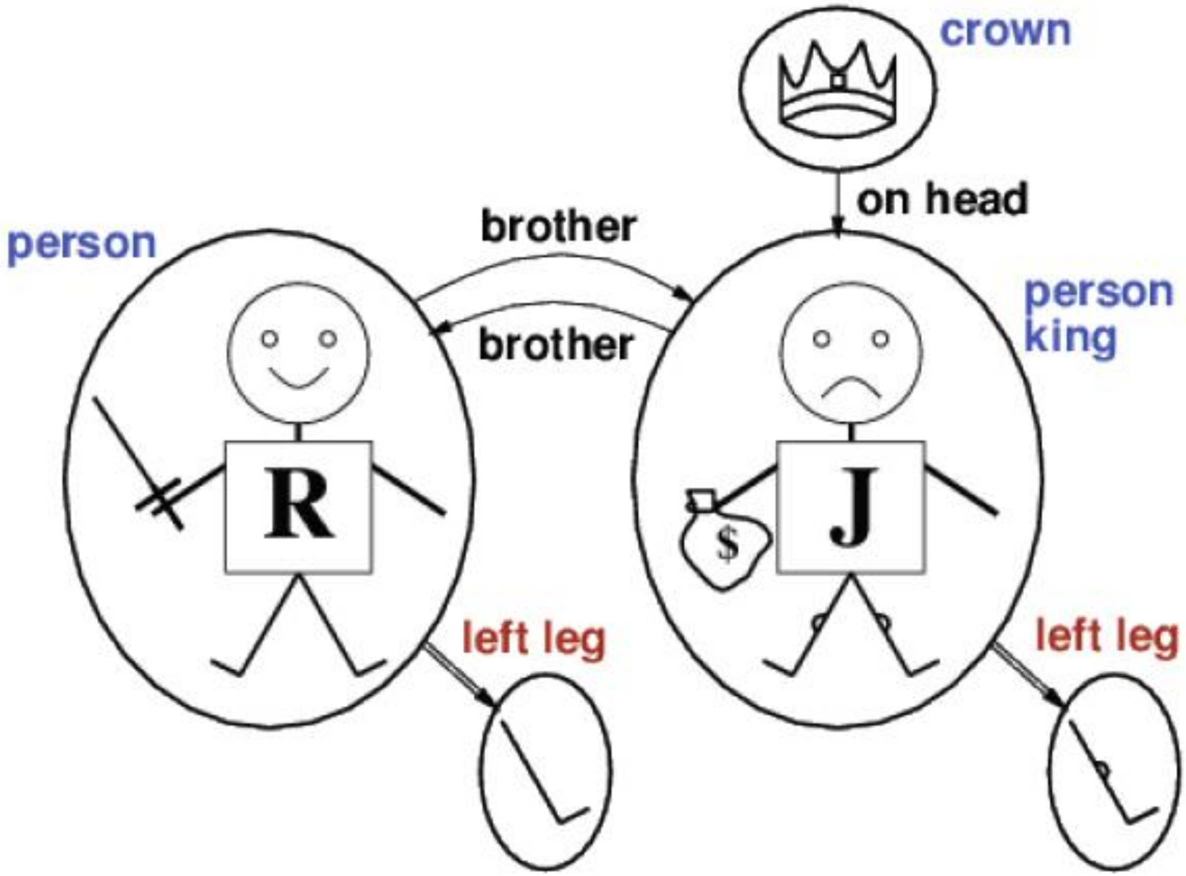
  - Model contains ≥ 1 objects (domain elements) and relations among them

  - Interpretation specifies referents for
    - constant symbols → objects
    - predicate symbols → relations
    - function symbols → functional relations

  - An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
    iff the objects referred to by $term_1, \ldots, term_n$
    are in the relation referred to by $predicate$

First-Order Logic

- An example



Knowledge Representation

- Type of knowledge
  - Objects
  - Events
  - Procedures
  - Relations
  - Mental states
  - Meta knowledge

Knowledge Representation

- Properties of Representation Systems
  - Representational adequacy
    - ability to represent the required knowledge
  - Inferential adequacy
    - ability to manipulate knowledge
    - produce new knowledge
    - ability to direct inference methods into productive directions
    - ability to respond with limited resources (time, storage)
  - Acquisitional efficiency
    - ability to acquire new knowledge
    - ideally, automatically

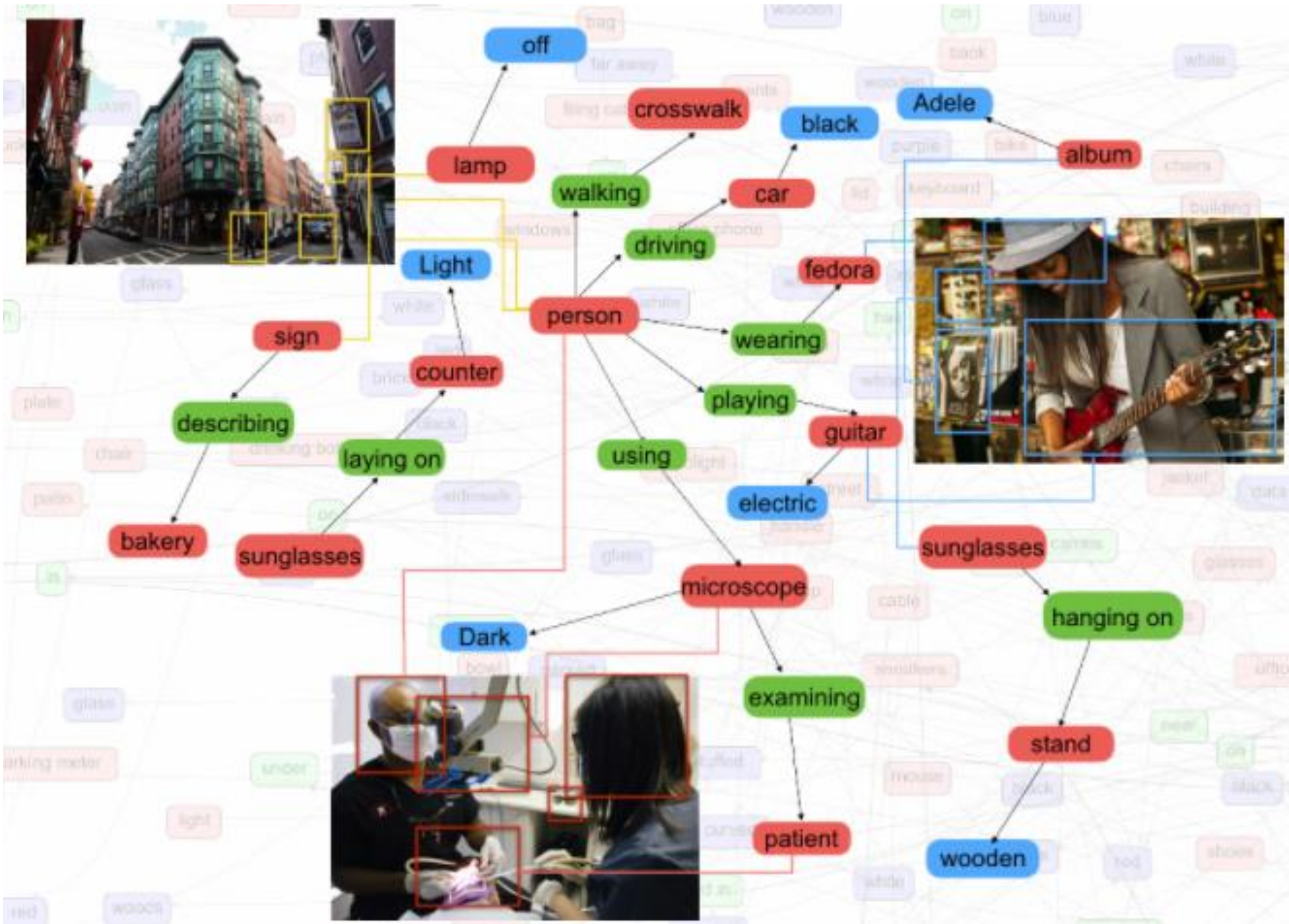
Knowledge Representation

- Knowledge Graph

https://googleblog.blogspot.com/2012/05/introducing-knowledge-graph-things-not.html

Knowledge Representation

- Knowledge Graph



https://visualgenome.org/

Assignments

- Reading assignment:
  - Ch. 7.1-7.5, Ch. 8.1-8.2

- Homework 3:
  - Due by Mar. 28, 2022.