

ST362 Project

2024-11-24

Installing packages

```
#Install and import package to deal with the categorical variable, region.  
if (!require(fastDummies)) install.packages('fastDummies', dependencies = TRUE)
```

```
## Loading required package: fastDummies
```

```
## Warning: package 'fastDummies' was built under R version 4.4.2
```

```
library(fastDummies)  
if (!require(leaps)) install.packages('leaps', dependencies = TRUE)
```

```
## Loading required package: leaps
```

```
## Warning: package 'leaps' was built under R version 4.4.2
```

```
library(leaps)
```

Useful functions (minimize code chunks for readability)

```
# Modified Box-cox by Dr Sunny Wang. You can minimize this code chunk.  
box.cox <- function(x,y,intercept=TRUE, ylim=NULL,  
                    lambda =seq (-1, 1, len=21), transform.x=FALSE,verbose =TRUE, make.plot=TRUE)  
{  
  good.cases <- (y>0)  
  y <- y[good.cases]  
  x <- as.matrix(x)  
  x <- x[good.cases, , drop=F]  
  
  g <- exp(mean(log(y)))  
  
  if(transform.x)  
  {  
    x.pos <- vector(mode= "logical", length=ncol(x))  
    for(j in 1:ncol(x))  
      x.pos[j] <- (min(x[j]) > 0)  
    x.name <- dimnames(x)[[2]]  
    if (mode(x.name)=="NULL")  
      x.name <- paste("X",1:ncol(x),sep="")  
  }  
}
```

```

log.lik <- vector(mode = "numeric", length = length(lambda))

for(i in 1:length(lambda))
{
  if(lambda[i] != 0)
  {
    z <- y^lambda[i]
    if (transform.x)
    {
      x.new <- x
      x.new.name <- x.name
      for(j in 1:ncol(x))
      {
        if(x.pos[j])
        {
          x.new[,j] <- x[,j]^lambda[i]
          x.new.name[j] <- paste(x.name[j], "^", lambda[i], sep = "")
        }
      }
    }
  }
  else
  {
    z <- log(y)
    if(transform.x)
    {
      x.new <- x
      x.new.name <- x.name
      for(j in 1:ncol(x))
      {
        if(x.pos[j])
        {
          x.new[,j] <- log(x[,j])
          x.new.name[j] <- paste("log(", x.name[j], ")", sep = "")
        }
      }
    }
  }

  if(transform.x)
  {
    dimnames(x.new) <- list(NULL, x.new.name)
    reg <- lsfit(x.new, z, intercept = intercept)
  }
  else
  {
    reg <- lsfit(x, z, intercept = intercept)
  }

  if(verbose)
  {
    cat("Lambda:", lambda[i], "\n")
    ls.print(reg)
  }
}

```

```

    res <- reg$residuals
    ss.res <- sum(res^2)/g^(2.0*(lambda[i] - 1.0 ))
    if (lambda[i] !=0.0)
      ss.res <- ss.res/lambda[i]^2
    log.lik[i] <- -length(y)/2.0*log(ss.res)
  }

  if(make.plot)
  {
    if(mode(ylim)=="NULL")
      ylim <- range(log.lik)
    plot(lambda,log.lik,ylim=ylim, ylab="Log Likelihood", type="b", lty=1)
    abline(max(log.lik)-3.8416/2,0,lty=2)
  }

  return(log.lik)
}

```

```

# Mallow's Ck by Dr Sunny Wang
leaps.ck <- function (X, Y, nbest =3)
{
  # call leaps using C_k criterion, make plot of C_k versus k,
  # and return leaps output in a matrix for nicer output.
  # The matrix has two columns: for k and for C_k. The row labels
  # contain the regression labels.
  leaps.out <- leaps(X,Y,method="Cp",nbest=nbest)

  leaps.mat <- cbind(leaps.out$size,round(leaps.out$Cp,digits=4))

  colnames(leaps.mat) <- c("k","C_k")

  leaps.mat <- cbind(leaps.out$which, leaps.mat)

  return(leaps.mat)
}

```

```

#Import the csv file
HAPPY <- read.csv("2019_happiness_cleaned.csv")
# Remove rank, as we will be using score as the response variable
HAPPY <- subset(HAPPY, select = -c(rank))
head(HAPPY)

```

```

##           region score   gdp support life_expectancy freedom generosity
## 1 Western Europe 7.769 1.340   1.587           0.986   0.596   0.153
## 2 Western Europe 7.600 1.383   1.573           0.996   0.592   0.252
## 3 Western Europe 7.554 1.488   1.582           1.028   0.603   0.271
## 4 Western Europe 7.494 1.380   1.624           1.026   0.591   0.354
## 5 Western Europe 7.488 1.396   1.522           0.999   0.557   0.322
## 6 Western Europe 7.480 1.452   1.526           1.052   0.572   0.263
## corruption
## 1           0.393

```

```
## 2      0.410
## 3      0.341
## 4      0.118
## 5      0.298
## 6      0.343
```

```
#Create dummy variables
```

```
HAPPY2 <- dummy_cols(HAPPY, select_columns = "region")
```

```
# Drop the `region`, because we have converted it into dummy variables
```

```
# Drop the `Australia and New Zealand` region, because we need to ensure  $t(X) \%*\% X$  will be non-singular
```

```
HAPPY2 <- subset(HAPPY2, select = -c(region, `region_Australia and New Zealand`))
```

```
head(HAPPY2)
```

```
##   score   gdp support life_expectancy freedom generosity corruption
## 1 7.769 1.340   1.587         0.986   0.596       0.153     0.393
## 2 7.600 1.383   1.573         0.996   0.592       0.252     0.410
## 3 7.554 1.488   1.582         1.028   0.603       0.271     0.341
## 4 7.494 1.380   1.624         1.026   0.591       0.354     0.118
## 5 7.488 1.396   1.522         0.999   0.557       0.322     0.298
## 6 7.480 1.452   1.526         1.052   0.572       0.263     0.343
##   region_Central and Eastern Europe region_Eastern Asia
## 1                                0                        0
## 2                                0                        0
## 3                                0                        0
## 4                                0                        0
## 5                                0                        0
## 6                                0                        0
##   region_Latin America and Caribbean region_Middle East and Northern Africa
## 1                                0                        0
## 2                                0                        0
## 3                                0                        0
## 4                                0                        0
## 5                                0                        0
## 6                                0                        0
##   region_North America region_Southeastern Asia region_Southern Asia
## 1                        0                        0                        0
## 2                        0                        0                        0
## 3                        0                        0                        0
## 4                        0                        0                        0
## 5                        0                        0                        0
## 6                        0                        0                        0
##   region_Sub-Saharan Africa region_Western Europe
## 1                        0                        1
## 2                        0                        1
## 3                        0                        1
## 4                        0                        1
## 5                        0                        1
## 6                        0                        1
```

```
# Model building
```

```
null_model <- lm(score~1, HAPPY2)
```

```
summary(null_model)
```

```
##
## Call:
## lm(formula = score ~ 1, data = HAPPY2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.35087 -0.88587 -0.00887  0.76413  2.33513
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.43387    0.09104   59.69  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.111 on 148 degrees of freedom
```

```
# Currently: Not an optimal model. Needs to add interaction effects.
full_model <- lm(score~., HAPPY2)
summary(full_model)
```

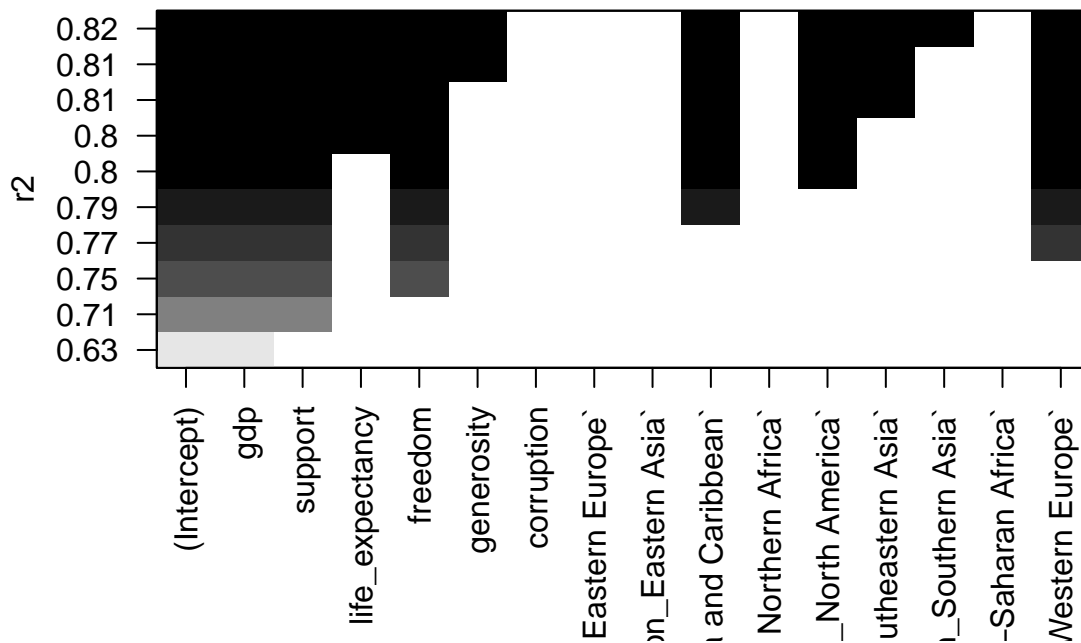
```
##
## Call:
## lm(formula = score ~ ., data = HAPPY2)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.68392 -0.23628  0.01338  0.31126  1.35117
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      3.00279    0.56716   5.294 4.80e-07
## gdp              0.94996    0.22823   4.162 5.62e-05
## support          0.82305    0.23187   3.550 0.000534
## life_expectancy  0.50483    0.40263   1.254 0.212096
## freedom          1.55711    0.39453   3.947 0.000128
## generosity       0.83556    0.50924   1.641 0.103200
## corruption       0.09014    0.58136   0.155 0.877018
## 'region_Central and Eastern Europe' -0.59467    0.39890  -1.491 0.138383
## 'region_Eastern Asia' -0.80082    0.42272  -1.894 0.060333
## 'region_Latin America and Caribbean' -0.24174    0.40515  -0.597 0.551742
## 'region_Middle East and Northern Africa' -0.72920    0.40309  -1.809 0.072701
## 'region_North America' -0.01242    0.49671  -0.025 0.980082
## 'region_Southeastern Asia' -1.12474    0.41535  -2.708 0.007661
## 'region_Southern Asia' -1.02342    0.44144  -2.318 0.021955
## 'region_Sub-Saharan Africa' -0.77216    0.43922  -1.758 0.081044
## 'region_Western Europe' -0.08745    0.37190  -0.235 0.814460
##
## (Intercept)      ***
## gdp              ***
## support          ***
## life_expectancy
## freedom          ***
## generosity
## corruption
```

```
## 'region_Central and Eastern Europe'
## 'region_Eastern Asia'
## 'region_Latin America and Caribbean'
## 'region_Middle East and Northern Africa'
## 'region_North America'
## 'region_Southeastern Asia'
## 'region_Southern Asia'
## 'region_Sub-Saharan Africa'
## 'region_Western Europe'
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4907 on 133 degrees of freedom
## Multiple R-squared:  0.8247, Adjusted R-squared:  0.805
## F-statistic: 41.72 on 15 and 133 DF,  p-value: < 2.2e-16
```

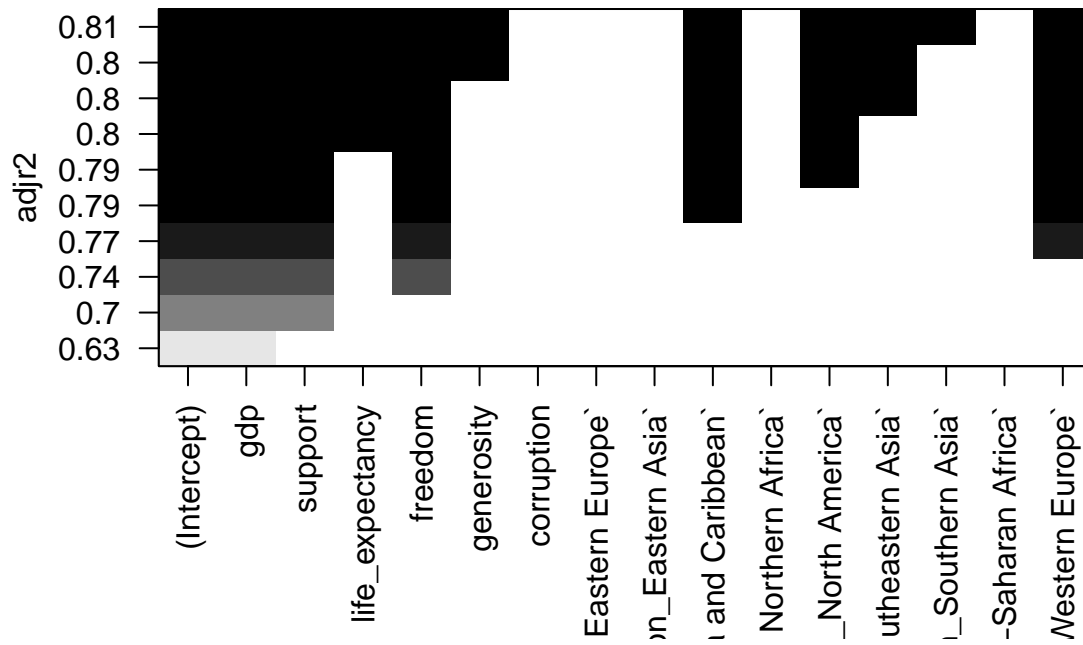
Variable selection methods

```
# Best subset selection: forward
result.leaps.forward <- regsubsets(score~., data=HAPPY2, nvmax=10, method="forward") # run best subsets

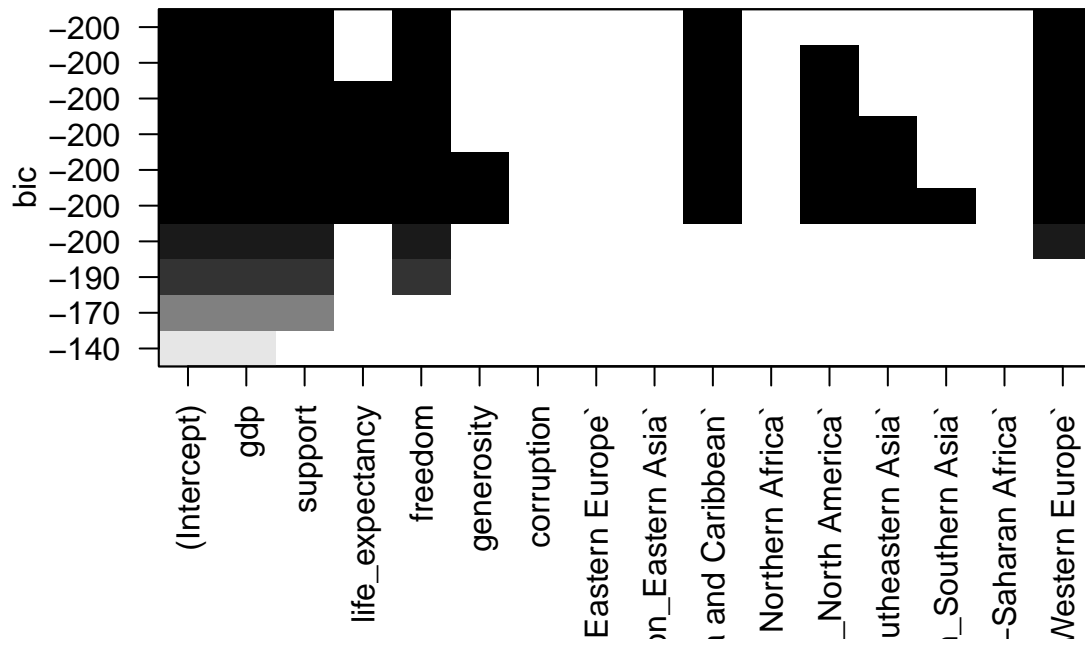
# Maybe ignore R^2 for now, not preferable criteria
# Note that AIC isn't a thing for this
plot(result.leaps.forward, scale="r2")
```



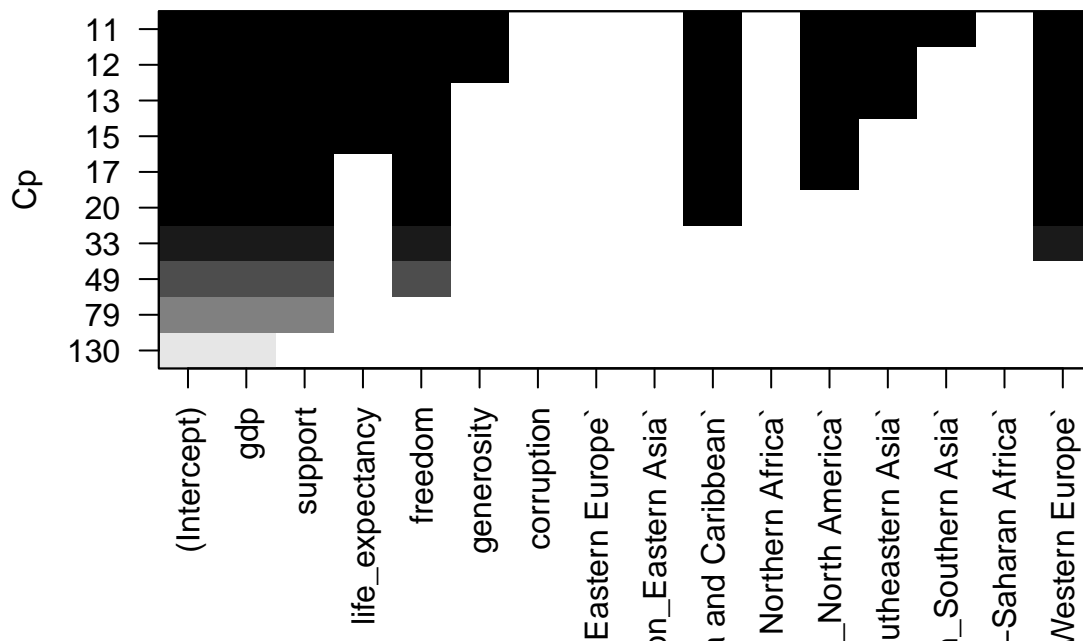
```
plot(result.leaps.forward, scale="adjr2")
```



```
plot(result.leaps.forward, scale="bic")
```



```
plot(result.leaps.forward, scale="Cp")
```

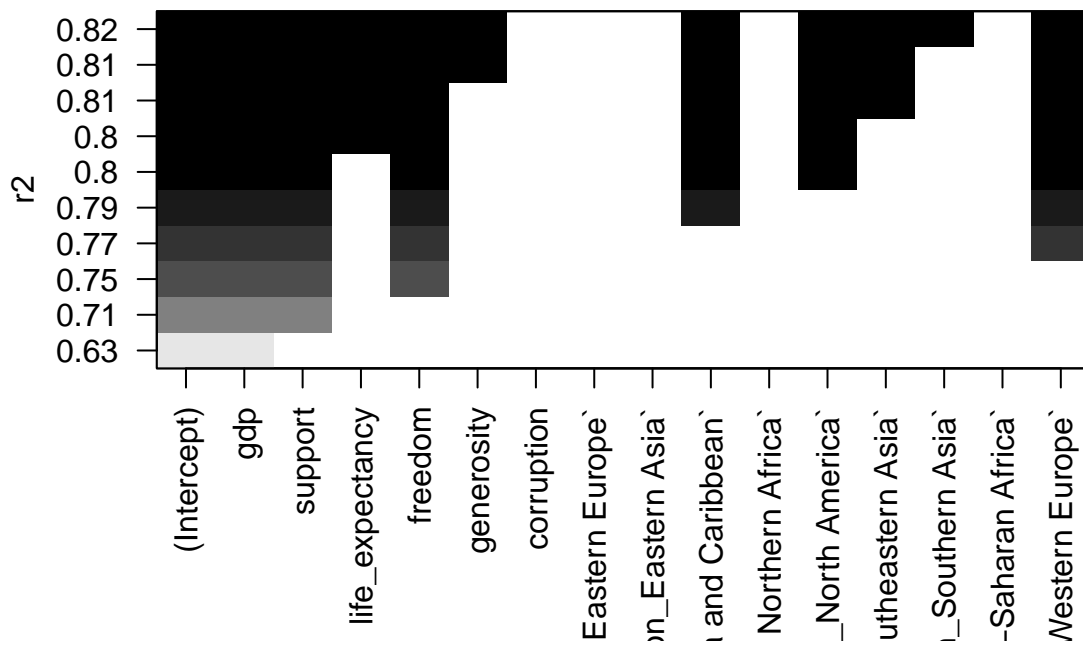



Keep this code chunk, run later for each regsubset

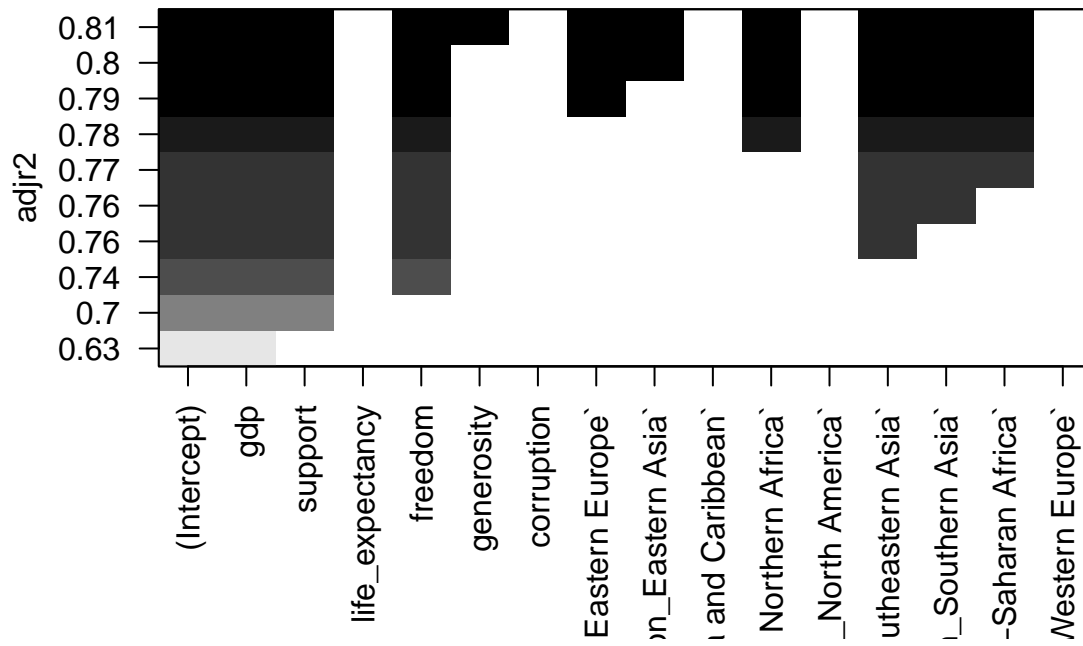
```
# reg_summary <- summary(result.leaps.forward) # get a summary of the models
# plot(reg_summary$bic, xlab = "Number of Variables", ylab = "BIC", type = "b")
#
# # Extract coefficients for the model with, say, 4 predictors
# coef(result.leaps.forward, 4)

# Best subset selection: backward
result.leaps.backward <- regsubsets(score~., data=HAPPY2, nvmax=10, method="backward")

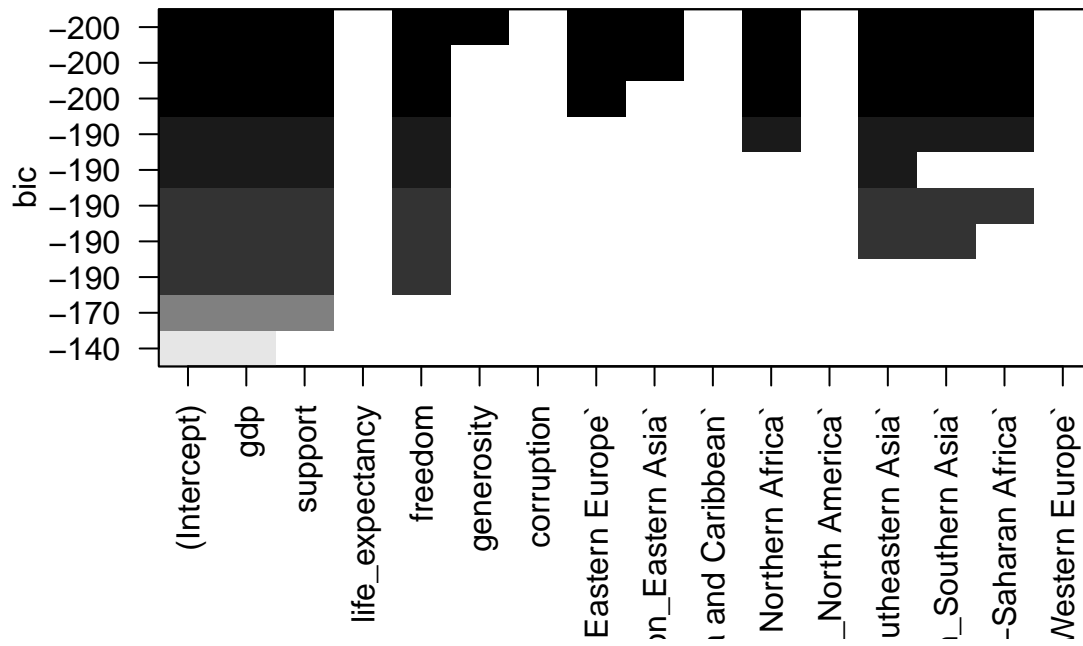
# Maybe ignore R^2 for now, not preferable criteria
# Note that AIC isn't a thing for this
plot(result.leaps.forward, scale="r2")
```



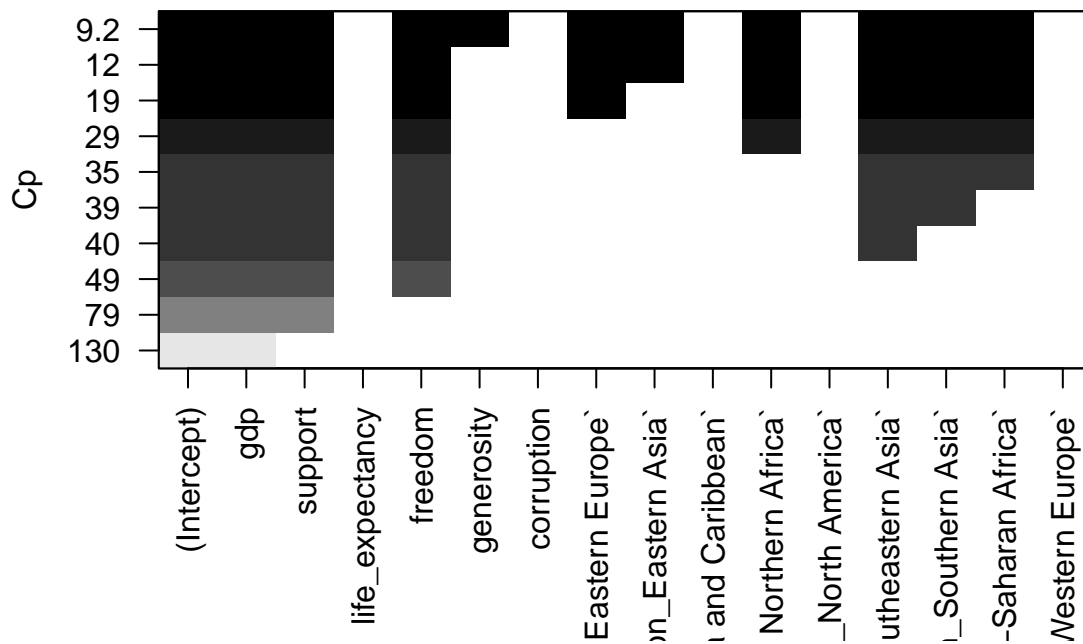
```
plot(result.leaps.backward, scale="adjr2")
```



```
plot(result.leaps.backward, scale="bic")
```

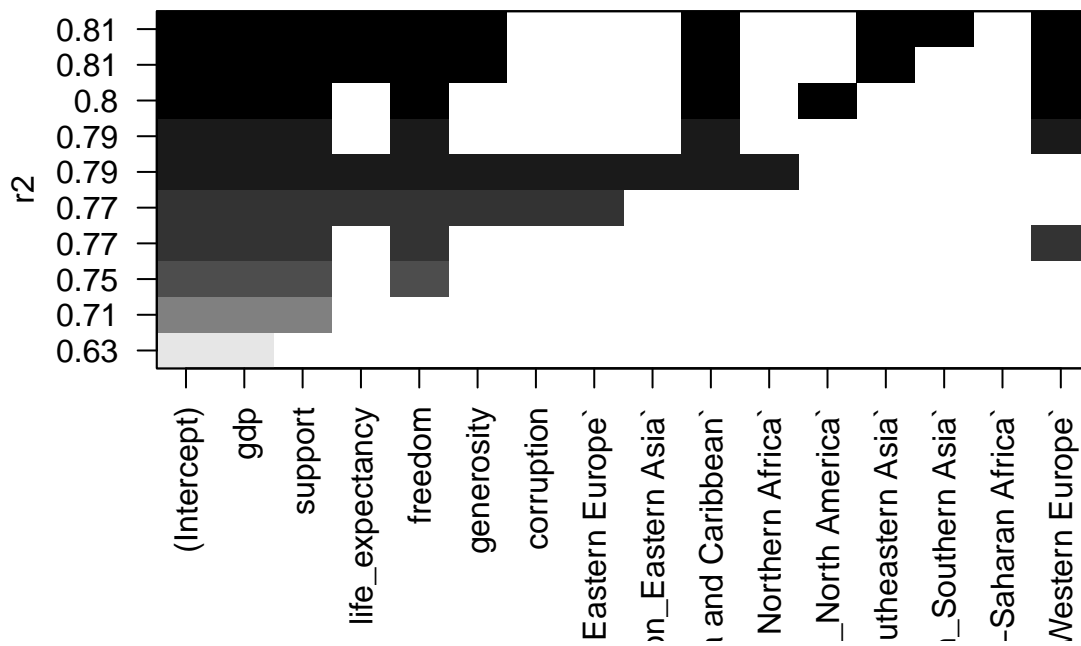


```
plot(result.leaps.backward, scale="Cp")
```

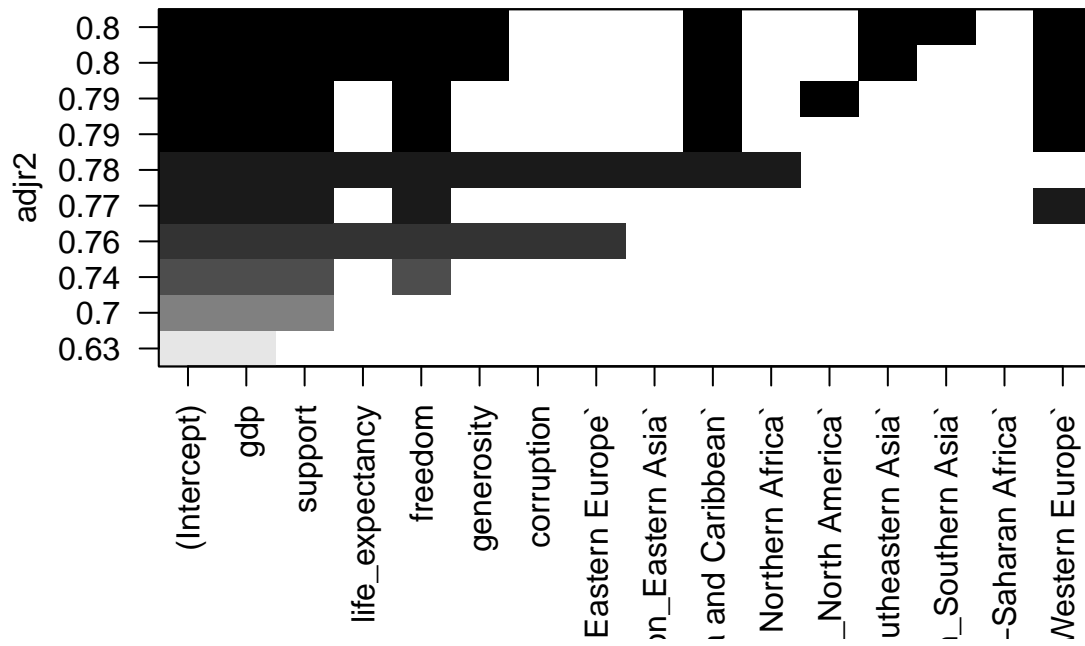


```
# Best subset selection: seqrep
# Not sure if this is hybrid
result.leaps.seqrep <- regsubsets(score~., data=HAPPY2, nvmax=10, method="seqrep")

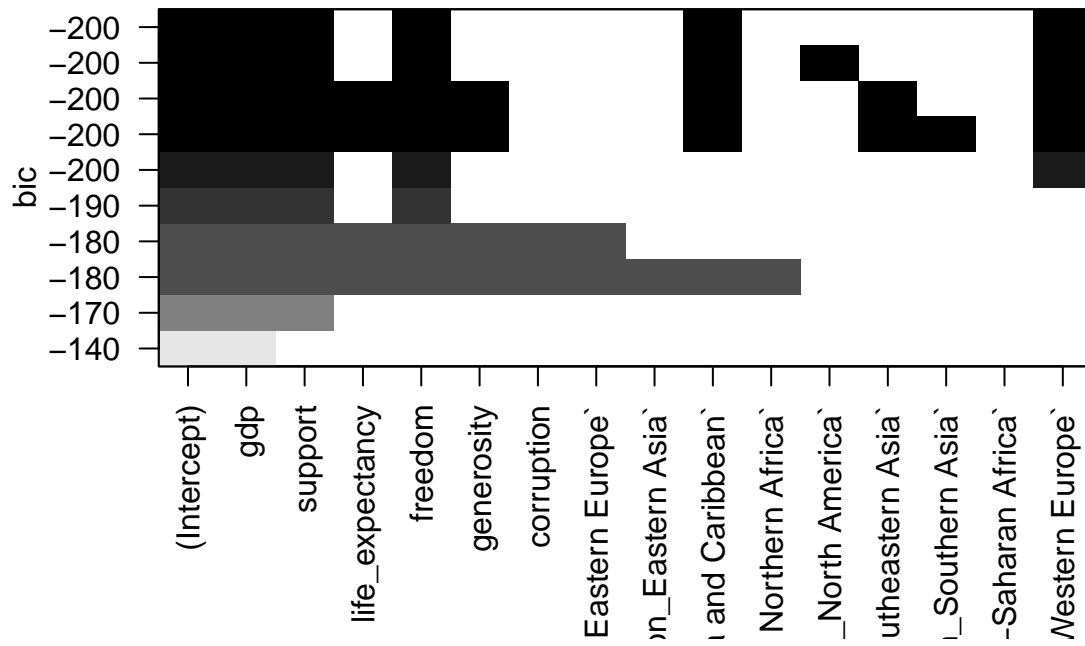
# Maybe ignore R^2 for now, not preferable criteria
# Note that AIC isn't a thing for this
plot(result.leaps.seqrep, scale="r2")
```



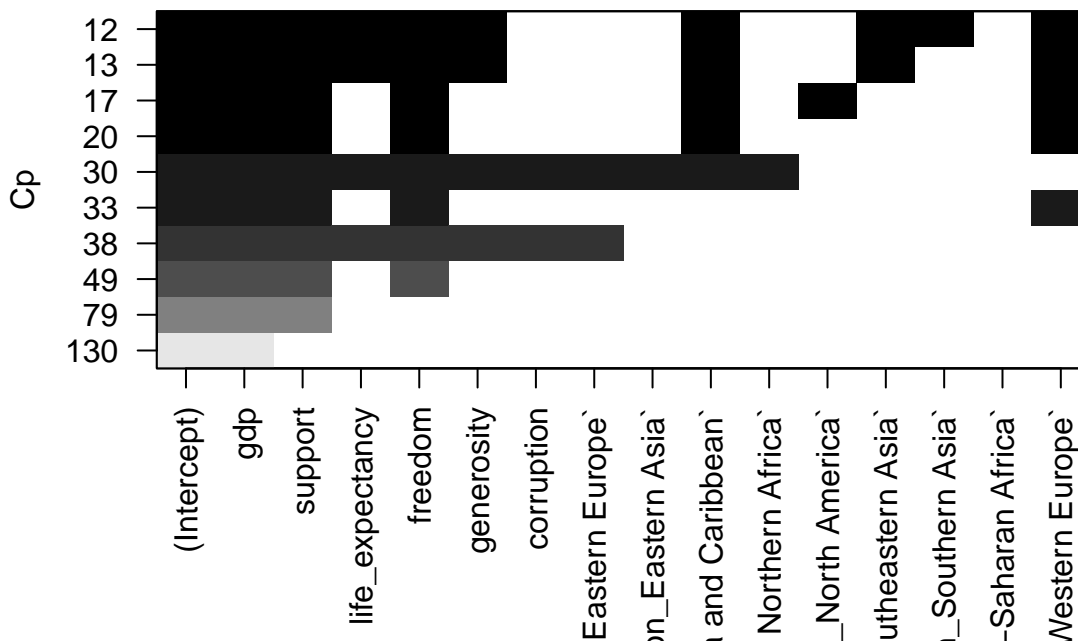
```
plot(result.leaps.seqrep, scale="adjr2")
```



```
plot(result.leaps.seqrep, scale="bic")
```



```
plot(result.leaps.seqrep, scale="Cp")
```

```
# Look at last result for AIC and variables to select
step_forward <- step(null_model, scope=list(lower=null_model, upper=full_model), direction="forward")
```

```
## Start: AIC=32.43
## score ~ 1
##
##
## Df Sum of Sq RSS AIC
## + gdp 1 115.918 66.842 -115.441
## + life_expectancy 1 111.071 71.688 -105.011
## + support 1 109.084 73.675 -100.938
## + 'region_Sub-Saharan Africa' 1 59.467 123.293 -24.218
## + freedom 1 56.937 125.822 -21.192
## + 'region_Western Europe' 1 49.547 133.212 -12.689
## + corruption 1 30.800 151.960 6.931
## + 'region_Southern Asia' 1 6.043 176.717 29.420
## + 'region_Latin America and Caribbean' 1 5.977 176.782 29.475
## + 'region_North America' 1 5.527 177.233 29.854
## <none> 182.760 32.430
## + generosity 1 1.259 181.501 33.400
## + 'region_Middle East and Northern Africa' 1 0.844 181.916 33.740
## + 'region_Central and Eastern Europe' 1 0.656 182.104 33.894
## + 'region_Eastern Asia' 1 0.406 182.353 34.098
## + 'region_Southeastern Asia' 1 0.246 182.514 34.229
##
## Step: AIC=-115.44
## score ~ gdp
```

```

##
##
## Df Sum of Sq RSS AIC
## + support 1 13.2887 53.553 -146.47
## + freedom 1 13.1433 53.699 -146.06
## + life_expectancy 1 8.2425 58.599 -133.05
## + 'region_Latin America and Caribbean' 1 6.2652 60.577 -128.10
## + 'region_Middle East and Northern Africa' 1 6.0814 60.761 -127.65
## + 'region_Western Europe' 1 6.0071 60.835 -127.47
## + corruption 1 3.6444 63.197 -121.80
## + generosity 1 3.2987 63.543 -120.98
## + 'region_Eastern Asia' 1 1.4108 65.431 -116.62
## <none> 66.842 -115.44
## + 'region_Southern Asia' 1 0.7307 66.111 -115.08
## + 'region_North America' 1 0.6459 66.196 -114.89
## + 'region_Sub-Saharan Africa' 1 0.4968 66.345 -114.55
## + 'region_Central and Eastern Europe' 1 0.3928 66.449 -114.32
## + 'region_Southeastern Asia' 1 0.3664 66.476 -114.26
##
## Step: AIC=-146.47
## score ~ gdp + support
##
## Df Sum of Sq RSS AIC
## + freedom 1 7.8187 45.734 -167.98
## + 'region_Western Europe' 1 5.3503 48.203 -160.15
## + corruption 1 4.6081 48.945 -157.87
## + life_expectancy 1 3.9684 49.585 -155.94
## + generosity 1 3.0025 50.551 -153.06
## + 'region_Latin America and Caribbean' 1 2.8045 50.749 -152.48
## + 'region_Middle East and Northern Africa' 1 2.1837 51.369 -150.67
## + 'region_Central and Eastern Europe' 1 1.4899 52.063 -148.67
## + 'region_Eastern Asia' 1 0.9078 52.645 -147.01
## <none> 53.553 -146.47
## + 'region_North America' 1 0.6689 52.884 -146.34
## + 'region_Southeastern Asia' 1 0.5063 53.047 -145.88
## + 'region_Southern Asia' 1 0.2782 53.275 -145.24
## + 'region_Sub-Saharan Africa' 1 0.0530 53.500 -144.62
##
## Step: AIC=-167.98
## score ~ gdp + support + freedom
##
## Df Sum of Sq RSS AIC
## + 'region_Western Europe' 1 4.4002 41.334 -181.06
## + life_expectancy 1 3.0861 42.648 -176.39
## + 'region_Southeastern Asia' 1 2.5275 43.207 -174.45
## + 'region_Latin America and Caribbean' 1 2.1018 43.633 -172.99
## + corruption 1 1.3239 44.411 -170.36
## + 'region_Middle East and Northern Africa' 1 0.7429 44.992 -168.42
## + generosity 1 0.7047 45.030 -168.30
## <none> 45.734 -167.98
## + 'region_Southern Asia' 1 0.5570 45.177 -167.81
## + 'region_Eastern Asia' 1 0.4833 45.251 -167.57
## + 'region_North America' 1 0.4459 45.289 -167.44
## + 'region_Central and Eastern Europe' 1 0.3204 45.414 -167.03
## + 'region_Sub-Saharan Africa' 1 0.2357 45.499 -166.75

```

```

##
## Step: AIC=-181.06
## score ~ gdp + support + freedom + 'region_Western Europe'
##
##
## Df Sum of Sq RSS AIC
## + 'region_Latin America and Caribbean' 1 3.5778 37.756 -192.55
## + life_expectancy 1 1.9820 39.352 -186.38
## + 'region_Southeastern Asia' 1 1.6734 39.661 -185.21
## + 'region_North America' 1 0.9019 40.432 -182.34
## <none> 41.334 -181.06
## + 'region_Sub-Saharan Africa' 1 0.5447 40.790 -181.03
## + 'region_Southern Asia' 1 0.4803 40.854 -180.80
## + generosity 1 0.2129 41.121 -179.82
## + corruption 1 0.1715 41.163 -179.68
## + 'region_Middle East and Northern Africa' 1 0.1512 41.183 -179.60
## + 'region_Eastern Asia' 1 0.1188 41.215 -179.49
## + 'region_Central and Eastern Europe' 1 0.0007 41.334 -179.06
##
## Step: AIC=-192.55
## score ~ gdp + support + freedom + 'region_Western Europe' + 'region_Latin America and Caribbean'
##
##
## Df Sum of Sq RSS AIC
## + 'region_North America' 1 1.21185 36.545 -195.41
## + 'region_Southeastern Asia' 1 0.95865 36.798 -194.38
## + life_expectancy 1 0.94043 36.816 -194.30
## + generosity 1 0.71969 37.037 -193.41
## <none> 37.756 -192.55
## + corruption 1 0.45336 37.303 -192.35
## + 'region_Central and Eastern Europe' 1 0.38216 37.374 -192.06
## + 'region_Southern Asia' 1 0.28895 37.467 -191.69
## + 'region_Sub-Saharan Africa' 1 0.05012 37.706 -190.74
## + 'region_Eastern Asia' 1 0.02672 37.730 -190.65
## + 'region_Middle East and Northern Africa' 1 0.02564 37.731 -190.65
##
## Step: AIC=-195.41
## score ~ gdp + support + freedom + 'region_Western Europe' + 'region_Latin America and Caribbean' +
## 'region_North America'
##
##
## Df Sum of Sq RSS AIC
## + life_expectancy 1 0.89665 35.648 -197.11
## + 'region_Southeastern Asia' 1 0.79000 35.755 -196.66
## + 'region_Central and Eastern Europe' 1 0.60508 35.939 -195.89
## + generosity 1 0.50012 36.044 -195.46
## <none> 36.545 -195.41
## + corruption 1 0.29502 36.250 -194.61
## + 'region_Southern Asia' 1 0.26699 36.278 -194.50
## + 'region_Sub-Saharan Africa' 1 0.06759 36.477 -193.68
## + 'region_Eastern Asia' 1 0.00627 36.538 -193.43
## + 'region_Middle East and Northern Africa' 1 0.00213 36.542 -193.41
##
## Step: AIC=-197.11
## score ~ gdp + support + freedom + 'region_Western Europe' + 'region_Latin America and Caribbean' +
## 'region_North America' + life_expectancy
##

```

```

##                               Df Sum of Sq    RSS      AIC
## + 'region_Southeastern Asia' 1  0.87899 34.769 -198.83
## <none>                        35.648 -197.11
## + generosity                 1  0.45034 35.198 -197.00
## + 'region_Southern Asia'     1  0.35227 35.296 -196.59
## + 'region_Central and Eastern Europe' 1  0.34589 35.302 -196.56
## + corruption                 1  0.25875 35.389 -196.19
## + 'region_Eastern Asia'      1  0.07005 35.578 -195.40
## + 'region_Sub-Saharan Africa' 1  0.05635 35.592 -195.34
## + 'region_Middle East and Northern Africa' 1  0.00213 35.646 -195.12
##
## Step: AIC=-198.83
## score ~ gdp + support + freedom + 'region_Western Europe' + 'region_Latin America and Caribbean' +
## 'region_North America' + life_expectancy + 'region_Southeastern Asia'
##
##                               Df Sum of Sq    RSS      AIC
## + generosity                 1  0.89159 33.877 -200.70
## + 'region_Southern Asia'     1  0.48644 34.282 -198.93
## <none>                        34.769 -198.83
## + corruption                 1  0.21083 34.558 -197.73
## + 'region_Central and Eastern Europe' 1  0.16293 34.606 -197.53
## + 'region_Eastern Asia'      1  0.12277 34.646 -197.35
## + 'region_Middle East and Northern Africa' 1  0.01996 34.749 -196.91
## + 'region_Sub-Saharan Africa' 1  0.00048 34.768 -196.83
##
## Step: AIC=-200.7
## score ~ gdp + support + freedom + 'region_Western Europe' + 'region_Latin America and Caribbean' +
## 'region_North America' + life_expectancy + 'region_Southeastern Asia' +
## generosity
##
##                               Df Sum of Sq    RSS      AIC
## + 'region_Southern Asia'     1  0.65657 33.221 -201.62
## <none>                        33.877 -200.70
## + 'region_Central and Eastern Europe' 1  0.27326 33.604 -199.91
## + 'region_Eastern Asia'      1  0.16982 33.708 -199.45
## + corruption                 1  0.06972 33.808 -199.01
## + 'region_Middle East and Northern Africa' 1  0.02521 33.852 -198.81
## + 'region_Sub-Saharan Africa' 1  0.00595 33.871 -198.72
##
## Step: AIC=-201.61
## score ~ gdp + support + freedom + 'region_Western Europe' + 'region_Latin America and Caribbean' +
## 'region_North America' + life_expectancy + 'region_Southeastern Asia' +
## generosity + 'region_Southern Asia'
##
##                               Df Sum of Sq    RSS      AIC
## <none>                        33.221 -201.62
## + 'region_Eastern Asia'      1  0.219768 33.001 -200.60
## + 'region_Central and Eastern Europe' 1  0.173175 33.048 -200.39
## + 'region_Sub-Saharan Africa' 1  0.085270 33.135 -200.00
## + 'region_Middle East and Northern Africa' 1  0.065386 33.155 -199.91
## + corruption                 1  0.046356 33.174 -199.82

```

```
step_backward <- step(null_model, scope=list(lower=null_model, upper=full_model), direction="backward")
```

```
## Start: AIC=32.43
## score ~ 1
```

```
step_both <- step(null_model, scope=list(lower=null_model, upper=full_model), direction="both")
```

```
## Start: AIC=32.43
## score ~ 1
```

```
##
##
```

	Df	Sum of Sq	RSS	AIC
## + gdp	1	115.918	66.842	-115.441
## + life_expectancy	1	111.071	71.688	-105.011
## + support	1	109.084	73.675	-100.938
## + 'region_Sub-Saharan Africa'	1	59.467	123.293	-24.218
## + freedom	1	56.937	125.822	-21.192
## + 'region_Western Europe'	1	49.547	133.212	-12.689
## + corruption	1	30.800	151.960	6.931
## + 'region_Southern Asia'	1	6.043	176.717	29.420
## + 'region_Latin America and Caribbean'	1	5.977	176.782	29.475
## + 'region_North America'	1	5.527	177.233	29.854
## <none>			182.760	32.430
## + generosity	1	1.259	181.501	33.400
## + 'region_Middle East and Northern Africa'	1	0.844	181.916	33.740
## + 'region_Central and Eastern Europe'	1	0.656	182.104	33.894
## + 'region_Eastern Asia'	1	0.406	182.353	34.098
## + 'region_Southeastern Asia'	1	0.246	182.514	34.229

```
##
```

```
## Step: AIC=-115.44
```

```
## score ~ gdp
```

```
##
##
```

	Df	Sum of Sq	RSS	AIC
## + support	1	13.289	53.553	-146.47
## + freedom	1	13.143	53.699	-146.06
## + life_expectancy	1	8.242	58.599	-133.05
## + 'region_Latin America and Caribbean'	1	6.265	60.577	-128.10
## + 'region_Middle East and Northern Africa'	1	6.081	60.761	-127.65
## + 'region_Western Europe'	1	6.007	60.835	-127.47
## + corruption	1	3.644	63.197	-121.80
## + generosity	1	3.299	63.543	-120.98
## + 'region_Eastern Asia'	1	1.411	65.431	-116.62
## <none>			66.842	-115.44
## + 'region_Southern Asia'	1	0.731	66.111	-115.08
## + 'region_North America'	1	0.646	66.196	-114.89
## + 'region_Sub-Saharan Africa'	1	0.497	66.345	-114.55
## + 'region_Central and Eastern Europe'	1	0.393	66.449	-114.32
## + 'region_Southeastern Asia'	1	0.366	66.476	-114.26
## - gdp	1	115.918	182.760	32.43

```
##
```

```
## Step: AIC=-146.47
```

```
## score ~ gdp + support
```

```
##
##
```

	Df	Sum of Sq	RSS	AIC
## + freedom	1	7.8187	45.734	-167.98
## + 'region_Western Europe'	1	5.3503	48.203	-160.15
## + corruption	1	4.6081	48.945	-157.87

```

## + life_expectancy      1    3.9684 49.585 -155.94
## + generosity           1    3.0025 50.551 -153.06
## + 'region_Latin America and Caribbean' 1    2.8045 50.749 -152.48
## + 'region_Middle East and Northern Africa' 1    2.1837 51.369 -150.67
## + 'region_Central and Eastern Europe' 1    1.4899 52.063 -148.67
## + 'region_Eastern Asia' 1    0.9078 52.645 -147.01
## <none>                  53.553 -146.47
## + 'region_North America' 1    0.6689 52.884 -146.34
## + 'region_Southeastern Asia' 1    0.5063 53.047 -145.88
## + 'region_Southern Asia' 1    0.2782 53.275 -145.24
## + 'region_Sub-Saharan Africa' 1    0.0530 53.500 -144.62
## - support              1   13.2887 66.842 -115.44
## - gdp                  1   20.1220 73.675 -100.94
##
## Step: AIC=-167.98
## score ~ gdp + support + freedom
##
##
## Df Sum of Sq  RSS    AIC
## + 'region_Western Europe' 1    4.4002 41.334 -181.06
## + life_expectancy         1    3.0861 42.648 -176.39
## + 'region_Southeastern Asia' 1    2.5275 43.207 -174.45
## + 'region_Latin America and Caribbean' 1    2.1018 43.633 -172.99
## + corruption              1    1.3239 44.411 -170.36
## + 'region_Middle East and Northern Africa' 1    0.7429 44.992 -168.42
## + generosity              1    0.7047 45.030 -168.30
## <none>                    45.734 -167.98
## + 'region_Southern Asia' 1    0.5570 45.177 -167.81
## + 'region_Eastern Asia' 1    0.4833 45.251 -167.57
## + 'region_North America' 1    0.4459 45.289 -167.44
## + 'region_Central and Eastern Europe' 1    0.3204 45.414 -167.03
## + 'region_Sub-Saharan Africa' 1    0.2357 45.499 -166.75
## - freedom                 1    7.8187 53.553 -146.47
## - support                 1    7.9641 53.699 -146.06
## - gdp                     1   17.5299 63.264 -121.64
##
## Step: AIC=-181.06
## score ~ gdp + support + freedom + 'region_Western Europe'
##
##
## Df Sum of Sq  RSS    AIC
## + 'region_Latin America and Caribbean' 1    3.5778 37.756 -192.55
## + life_expectancy         1    1.9820 39.352 -186.38
## + 'region_Southeastern Asia' 1    1.6734 39.661 -185.21
## + 'region_North America' 1    0.9019 40.432 -182.34
## <none>                    41.334 -181.06
## + 'region_Sub-Saharan Africa' 1    0.5447 40.790 -181.03
## + 'region_Southern Asia' 1    0.4803 40.854 -180.80
## + generosity              1    0.2129 41.121 -179.82
## + corruption              1    0.1715 41.163 -179.68
## + 'region_Middle East and Northern Africa' 1    0.1512 41.183 -179.60
## + 'region_Eastern Asia' 1    0.1188 41.215 -179.49
## + 'region_Central and Eastern Europe' 1    0.0007 41.334 -179.06
## - 'region_Western Europe' 1    4.4002 45.734 -167.98
## - freedom                 1    6.8686 48.203 -160.15
## - support                 1    7.7607 49.095 -157.42

```

```

## - gdp                                1    11.5976 52.932 -146.21
##
## Step: AIC=-192.55
## score ~ gdp + support + freedom + 'region_Western Europe' + 'region_Latin America and Caribbean'
##
##                                Df Sum of Sq    RSS    AIC
## + 'region_North America'        1     1.2118 36.545 -195.41
## + 'region_Southeastern Asia'     1     0.9587 36.798 -194.38
## + life_expectancy                1     0.9404 36.816 -194.30
## + generosity                    1     0.7197 37.037 -193.41
## <none>                          1     37.756 -192.55
## + corruption                    1     0.4534 37.303 -192.35
## + 'region_Central and Eastern Europe' 1     0.3822 37.374 -192.06
## + 'region_Southern Asia'        1     0.2889 37.467 -191.69
## + 'region_Sub-Saharan Africa'    1     0.0501 37.706 -190.74
## + 'region_Eastern Asia'          1     0.0267 37.730 -190.65
## + 'region_Middle East and Northern Africa' 1     0.0256 37.731 -190.65
## - 'region_Latin America and Caribbean' 1     3.5778 41.334 -181.06
## - support                       1     5.2740 43.030 -175.06
## - freedom                       1     5.8600 43.616 -173.05
## - 'region_Western Europe'        1     5.8763 43.633 -172.99
## - gdp                           1    13.1580 50.914 -150.00
##
## Step: AIC=-195.41
## score ~ gdp + support + freedom + 'region_Western Europe' + 'region_Latin America and Caribbean' +
##         'region_North America'
##
##                                Df Sum of Sq    RSS    AIC
## + life_expectancy                1     0.8967 35.648 -197.11
## + 'region_Southeastern Asia'     1     0.7900 35.755 -196.66
## + 'region_Central and Eastern Europe' 1     0.6051 35.939 -195.89
## + generosity                    1     0.5001 36.044 -195.46
## <none>                          1     36.545 -195.41
## + corruption                    1     0.2950 36.250 -194.61
## + 'region_Southern Asia'        1     0.2670 36.278 -194.50
## + 'region_Sub-Saharan Africa'    1     0.0676 36.477 -193.68
## + 'region_Eastern Asia'          1     0.0063 36.538 -193.43
## + 'region_Middle East and Northern Africa' 1     0.0021 36.542 -193.41
## - 'region_North America'        1     1.2118 37.756 -192.55
## - 'region_Latin America and Caribbean' 1     3.8878 40.432 -182.34
## - support                       1     5.2617 41.806 -177.36
## - freedom                       1     5.4514 41.996 -176.69
## - 'region_Western Europe'        1     6.5371 43.082 -172.89
## - gdp                           1    12.0619 48.606 -154.91
##
## Step: AIC=-197.11
## score ~ gdp + support + freedom + 'region_Western Europe' + 'region_Latin America and Caribbean' +
##         'region_North America' + life_expectancy
##
##                                Df Sum of Sq    RSS    AIC
## + 'region_Southeastern Asia'     1     0.8790 34.769 -198.83
## <none>                          1     35.648 -197.11
## + generosity                    1     0.4503 35.198 -197.00
## + 'region_Southern Asia'        1     0.3523 35.296 -196.59

```

```

## + 'region_Central and Eastern Europe'      1    0.3459 35.302 -196.56
## + corruption                               1    0.2587 35.389 -196.19
## - life_expectancy                           1    0.8967 36.545 -195.41
## + 'region_Eastern Asia'                     1    0.0701 35.578 -195.40
## + 'region_Sub-Saharan Africa'               1    0.0564 35.592 -195.34
## + 'region_Middle East and Northern Africa'  1    0.0021 35.646 -195.12
## - 'region_North America'                   1    1.1681 36.816 -194.30
## - 'region_Latin America and Caribbean'     1    2.8048 38.453 -187.82
## - support                                   1    4.3400 39.988 -181.99
## - gdp                                       1    4.5661 40.214 -181.15
## - 'region_Western Europe'                  1    5.1822 40.830 -178.88
## - freedom                                  1    5.2254 40.873 -178.73
##
## Step: AIC=-198.83
## score ~ gdp + support + freedom + 'region_Western Europe' + 'region_Latin America and Caribbean' +
##       'region_North America' + life_expectancy + 'region_Southeastern Asia'
##
##                                     Df Sum of Sq    RSS      AIC
## + generosity                        1    0.8916 33.877 -200.70
## + 'region_Southern Asia'           1    0.4864 34.282 -198.93
## <none>                             1    34.769 -198.83
## + corruption                       1    0.2108 34.558 -197.73
## + 'region_Central and Eastern Europe' 1    0.1629 34.606 -197.53
## + 'region_Eastern Asia'            1    0.1228 34.646 -197.35
## - 'region_Southeastern Asia'       1    0.8790 35.648 -197.11
## + 'region_Middle East and Northern Africa' 1    0.0200 34.749 -196.91
## + 'region_Sub-Saharan Africa'      1    0.0005 34.768 -196.83
## - life_expectancy                  1    0.9856 35.755 -196.66
## - 'region_North America'           1    0.9922 35.761 -196.64
## - 'region_Latin America and Caribbean' 1    2.1597 36.929 -191.85
## - 'region_Western Europe'          1    4.1910 38.960 -183.87
## - support                          1    4.2843 39.053 -183.51
## - gdp                             1    4.3896 39.159 -183.11
## - freedom                          1    6.0826 40.852 -176.81
##
## Step: AIC=-200.7
## score ~ gdp + support + freedom + 'region_Western Europe' + 'region_Latin America and Caribbean' +
##       'region_North America' + life_expectancy + 'region_Southeastern Asia' +
##       generosity
##
##                                     Df Sum of Sq    RSS      AIC
## + 'region_Southern Asia'           1    0.6566 33.221 -201.62
## <none>                             1    33.877 -200.70
## + 'region_Central and Eastern Europe' 1    0.2733 33.604 -199.91
## - 'region_North America'           1    0.6934 34.571 -199.68
## + 'region_Eastern Asia'            1    0.1698 33.708 -199.45
## + corruption                       1    0.0697 33.808 -199.01
## - generosity                       1    0.8916 34.769 -198.83
## + 'region_Middle East and Northern Africa' 1    0.0252 33.852 -198.81
## + 'region_Sub-Saharan Africa'      1    0.0059 33.871 -198.72
## - life_expectancy                  1    0.9357 34.813 -198.64
## - 'region_Southeastern Asia'       1    1.3202 35.198 -197.00
## - 'region_Latin America and Caribbean' 1    2.5335 36.411 -191.95
## - 'region_Western Europe'          1    3.2762 37.154 -188.94

```



```

## - support                1    4.4015 38.279 -184.50
## - freedom                1    4.5582 38.436 -183.89
## - gdp                    1    4.9492 38.827 -182.38
##
## Step:  AIC=-201.61
## score ~ gdp + support + freedom + 'region_Western Europe' + 'region_Latin America and Caribbean' +
##       'region_North America' + life_expectancy + 'region_Southeastern Asia' +
##       generosity + 'region_Southern Asia'
##
##                               Df Sum of Sq    RSS    AIC
## <none>                                33.221 -201.62
## - 'region_North America'              1    0.6285 33.849 -200.82
## - 'region_Southern Asia'              1    0.6566 33.877 -200.70
## + 'region_Eastern Asia'                1    0.2198 33.001 -200.60
## + 'region_Central and Eastern Europe'  1    0.1732 33.048 -200.39
## + 'region_Sub-Saharan Africa'          1    0.0853 33.135 -200.00
## + 'region_Middle East and Northern Africa' 1    0.0654 33.155 -199.91
## + corruption                          1    0.0464 33.174 -199.82
## - generosity                          1    1.0617 34.282 -198.93
## - life_expectancy                     1    1.0624 34.283 -198.92
## - 'region_Southeastern Asia'           1    1.5533 34.774 -196.81
## - 'region_Latin America and Caribbean' 1    2.2489 35.470 -193.85
## - 'region_Western Europe'              1    2.9493 36.170 -190.94
## - support                             1    4.0325 37.253 -186.54
## - gdp                                 1    4.6917 37.912 -183.93
## - freedom                             1    4.8155 38.036 -183.44

```