Phys. 580

Project Proposal

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Magnetic responses of superconducting films

Background:

Superconductors are renowned, at least in one aspect, for the famous Messiner effect as well as Abrikosov lattice of vortices induced by external magnetic field. For bulk superconducting materials, such effects are almost perfectly described by London equations as well as linear Ginzberg-Landau theory. But in thin films, or mesoscopic samples of superconductors, due to various size effects, the magnetic response becomes different (e.g. non-quantized vortex, giant vortex, *et al*.). Theorists have been actively modelling these phenomena since 90s of last century, and have gained quite a few successes. In the project, I would like to repeat one or some of these successful models by means of C++ programming.

Steps:

1. Model magnetic response of 2-dimensional infinitely large superconducting film in external magnetic field.
2. (Optional) Model magnetic response of 2-dimensional superconducting film of finite size (still in macroscopic regime).
3. (Optional) Model magnetic response of 2-Dimensional superconducting film of finite size (mesoscopic regime).
4. (Optional) Model magnetic response of 3-Dimensional superconducting disk (high symmetry) of finite height (macroscopic regime) and finite size of cross section (mesoscopic regime).
5. (Optional) Model magnetic response of 3-Dimensinal superconducting film of finite size (all three dimensions in mesoscopic regime).

Tools:

3D-Maxwell equation:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \oint \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0} \int \rho d\tau$$

$$\nabla \cdot \vec{B} = 0 \qquad \oint \vec{B} \cdot d\vec{s} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{s}$$

Second order Ginzburg-Landau equation:

$$f = f_N + \alpha|\Psi|^2 + \beta|\Psi|^4 + \frac{1}{2m}\left|(-i\hbar\nabla - \frac{2e}{c}\vec{A})\Psi\right|^2 + \frac{H^2}{8\pi} - \frac{\vec{H}\cdot\vec{H}_0}{4\pi}$$

$$\alpha\Psi + \beta\Psi|\Psi|^2 + \frac{1}{4m}(i\hbar\nabla + \frac{2e}{c}\vec{A})\Psi = 0$$

$$(i\hbar\nabla\Psi + \frac{2e}{c}\vec{A}\Psi)\cdot\hat{n} = ia\Psi$$

Where f is Gibbs free energy of superconductors, and Ψ is both the order parameter of GL theory and the wave function of superconducting electrons. $\vec{H}_0$ is the external magnetic field, and $\vec{H}$ is the magnetic field inside the superconducting sample, α and β are tunable parameters, whose sign will decide the phase of the sample.

Reference:

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[3] B. Rosenstein, *et al.* Rev. Mod. Phys. **82**, 109 (2010).
[4] M.Tinkham. *Introduction to superconductivity.* 2nd edition. McGraw-Hill, Inc (1996).
[5] P. Muller, *et al. The physics of superconductors. Springer* (1997).