Corso di Modelli e Analisi delle Prestazioni nelle Reti – AA 2006/2007
prova scritta – 24 settembre 2007– parte A (90 minuti)

E1 Consider a Markov chain X_n with the following transition matrix (states are numbered from 0 to 2):

$$P = \begin{pmatrix} 0.2 & 0.4 & 0.4 \\ 0.5 & 0.5 & 0 \\ 0.4 & 0.4 & 0.2 \end{pmatrix}$$

(a) Draw the transition diagram, and find the probability distribution of X_1, X_2 and X_{500}, given $X_0 = 0$

(b) Compute the average first passage times from states 0, 1 and 2 to state 2.

(c) Let $W_{ij}^{(n)} = E\left[\sum_{k=0}^{n-1} I\{X_k = j\}\,\middle|\, X_0 = i\right]$ be the average number of visits to state j during the first n time slots, given that the chain starts in state i. Compute $W_{0j}^{(3)}$ and $W_{0j}^{(5000)}$ for $j = 0, 1, 2$.

E2 Consider a network node with two outgoing links L_1 and L_2. The two links have a capacity of 1 Mbps each, and are fed by two separate queues Q_1 and Q_2 which can contain one packet each. Assume that the node receives two packet flows, modeled as independent Poisson processes with rates $\lambda_1 = \lambda_2 = \lambda = 500$ pkt/s, and that flow λ_i is related only to link L_i (and therefore queue Q_i), $i = 1, 2$. Two packets are transmitted simultaneously when both queues are full. In particular, when there is a packet in only one of the queues, the node waits until the other queue also receives a packet, and only then does it send both packets, each on its own link. (Note that a time interval in which both queues are empty is followed by one in which one queue is empty and the other is full, which is in turn followed by another in which both packets are being transmitted.) Furthermore, assume that when Q_i is full (i.e., whene there is a packet waiting to be transmitted) any arrival of flow λ_i is rejected. All packets are 1000 bits long.

(a) Compute the throughput of the node, in terms of total bps transmitted.

(b) Compute the fraction of the total traffic that is rejected.

(c) Repeat the previous calculations if the length of the packets, instead of being fixed to 1000 bits, has exponential duration with mean 1000 bits. In this case, assume that the two queues are considered empty (and therefore the system again can accept incoming packets) only when *both* packets have been completely transmitted. (I.e., the queue that had the shorter packet will be able to accept packets only when the other queue is also empty.)

E3 Consider a network node with two incoming links, through which packets are received according to two independent Poisson processes with rates $\lambda_1 = \lambda_2 = 500$ packets per second.

(a) Compute the probability that in a 3-ms interval the node receives two packets from the first link and one from the second.

(b) Compute the probability that in a 3-ms interval the node receives three packets (in total).

(c) Compute the probability that in a 3-ms interval the node receives two packets from the first link, given that it received three packets in total.

E4 Consider a two-state Markov channel with transition probabilities 0.98 (from the good state to itself) and 0.1 (from the bad state to the good state). The packet error probability is 1 for a bad slot and 0 for a good slot, respectively.

(a) Compute the throughput of a Go-Back-N protocol if the round-trip time is $m = 2$ slots (i.e., a packet that is erroneous in slot t is retransmitted in slot $t + 2$), in the presence of an error-free feedback channel

(b) Consider now a system where a channel behavior as in point (a) (Markov model for the forward channel and error-free feedback channel) alternates with a channel behavior in which the forward channel is subject to iid errors with probability $\varepsilon = 0.01$. In particular, the channel follows the Markov model for a geometric number of slots with mean 1000000 slots, then follows the iid model for a geometric number of slots with mean 2000000 slots, then again the Markov model and so on. Compute the overall average throughput of the Go-Back-N protocol in this case.

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T1 Prove that if states i and j of a Markov chain communicate and i is recurrent, then j is also recurrent.

T2 For a Poisson process of rate λ, prove that the interarrival times are iid exponential with mean $1/\lambda$

T3 Prove that for a renewal process $E[S_{N(t)+1}] = E[X](M(t) + 1)$.