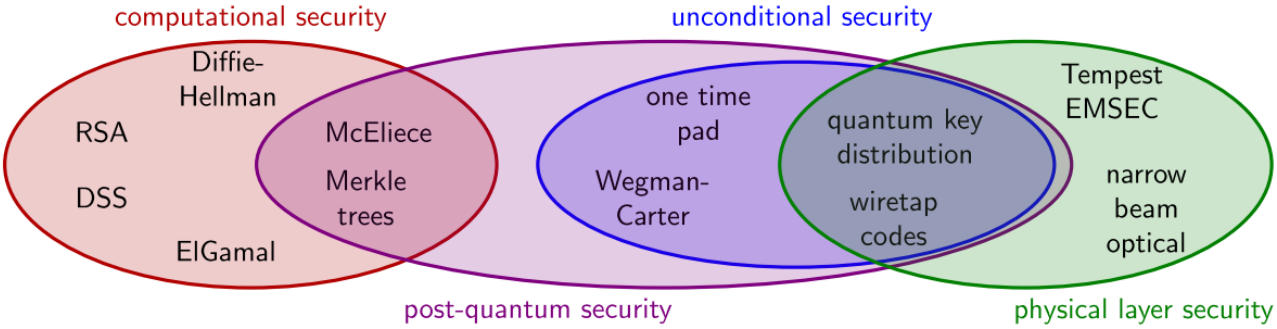
Risposte domande IS

Definizioni base:



Computational security systems can be broken by an attacker with enough computational power
Post-quantum security systems have not been shown breakable by quantum computers in short time
In unconditional security, the attacker is not better off at guessing by observing the protocol communications. However, in designing the system, (statistical) knowledge of the attacker channel is often required

Unconditional security: immune to compute power and to any form of attack. L'unico per cui vale questa cosa è One Time Pad.

Computational security: Secure if the amount of computing power required to break the encryption is so large that no one can get enough compute power necessary to break it.

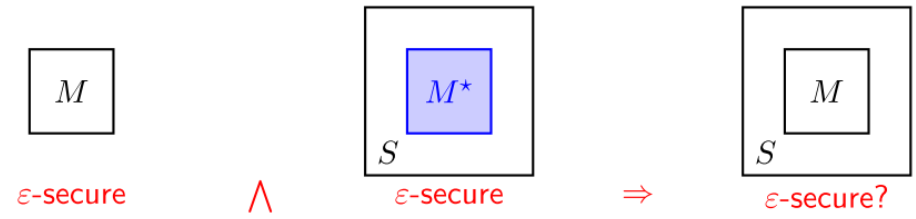
1)What is a composition theorem?

Context and motivation:

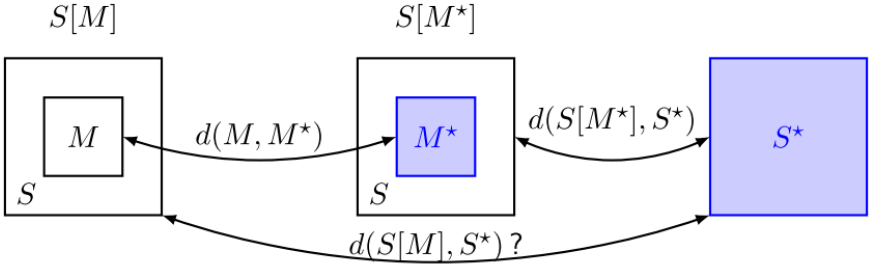
Consider a security mechanism S that makes use of another mechanism M, and denote this occurrence by $S[M]$.
Let $S[M^\star]$ denote the same mechanism S where M is replaced by its ideal counterpart $M^\star$.

The *composability* question

Is it possible to derive the security of $S[M]$ from those of M and $S[M^\star]$?



And let $S^\star$ denote the ideal counterpart of S (which need not use M nor $M^\star$).



Teorema:

Due to variational distance the triangular inequality è usata perché è one norm between statisical distribution. Upper bound perché a distinguisher that interact with M and M* is more effective than a distinguisher that do the same through S.

The composition theorem

Theorem (unconditional)

If M is ε_1-unconditionally secure and $S[M^\star]$ is ε_2-unconditionally secure, then $S[M]$ is $(\varepsilon_1 + \varepsilon_2)$-unconditionally secure

Proof.

Follows from the triangular inequality property of distinguishability. In fact:

$$\begin{aligned} d(S[M], S^\star) &\leq d(S[M], S[M^\star]) + d(S[M^\star], S^\star) \\ &\leq d(M, M^\star) + d(S[M^\star], S^\star) \leq \varepsilon_1 + \varepsilon_2 \end{aligned}$$

□

By repeatedly applying the above result, we can generalize to N-fold uses of M in S

Corollary

If M is ε_1-unconditionally secure and $S[M^\star]$ is ε_2-unconditionally secure, then $S[M^N]$ is $(N\varepsilon_1 + \varepsilon_2)$-unconditionally secure

2)We extended composition theorem to computational theorem? What is asymptotic version formulation in composition theorem?

Theorem (computational, concrete)

If M is (ε_1, T_0)-computationally secure and $S[M^\star]$ is (ε_2, T_0)-computationally secure, then $S[M]$ is $(\varepsilon_1 + \varepsilon_2, T_0)$-computationally secure

In the asymptotic form, the asymptotic security is retained even if M is used in S a number of times that is upper bounded by a polynomial in n, as follows

Theorem (computational, asymptotic)

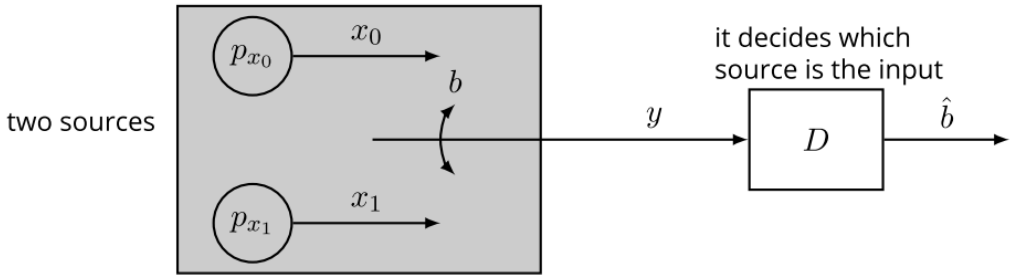
In the asymptotic formulation, if $\{M_n\}$ is computationally secure and $S_n[M_n^\star]$ is computationally secure, then for any polynomial $p(\cdot)$, $S_n[M_n^{p(n)}]$ is computationally secure

How many times do we use (m) in asymptotic version in polynomial? M è usato in S un numero di volte <= polinomio in n

3)What type of distance is this? Variational Distance

Context:

Variable distinguishers (or binary hypothesis testing)



A distinguisher between two random variables x_0 and x_1 is a system D that is allowed to observe a realization of y without knowing in advance if $b = 0$ or $b = 1$ and should then guess which one holds

- x_0 and x_1 are characterized by their PMDs p_{x_0}, p_{x_1}
- D is composed of a decision function $g : \mathcal{Y} \mapsto \{0, 1\}$, i.e. $\hat{b} = g(y)$

It is a common situation in security (e.g., intrusion detection, authenticity verification, etc.)

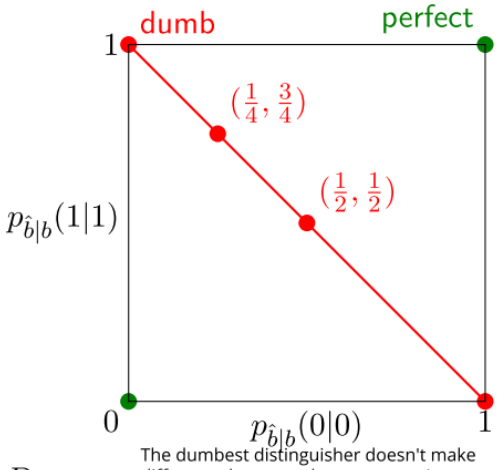
Distinguisher performance

The performance of a distinguisher D is given by the pair of correct decision probabilities

$$\left(p_{\hat{b}|b}(0|0), p_{\hat{b}|b}(1|1)\right)$$

or complementarily by the pair of error probabilities

$$\left(p_{\hat{b}|b}(1|0), p_{\hat{b}|b}(0|1)\right)$$



The dumbest distinguisher doesn't make difference between the two cases. It says "the input is always 0 or its always 1" so its dumb

We define the distinguishability between x_0 and x_1 with D as

$$d_D(x_0, x_1) = |p_{\hat{b}|b}(0|0) + p_{\hat{b}|b}(1|1) - 1| = |p_{\hat{b}|b}(1|0) + p_{\hat{b}|b}(0|1) - 1|$$

Note that $d_D(x_0, x_1) = 1$ for a perfect distinguisher while $d_D(x_0, x_1) = 0$ for a dumb distinguisher

even if its always wrong its good for us because we can just invert it

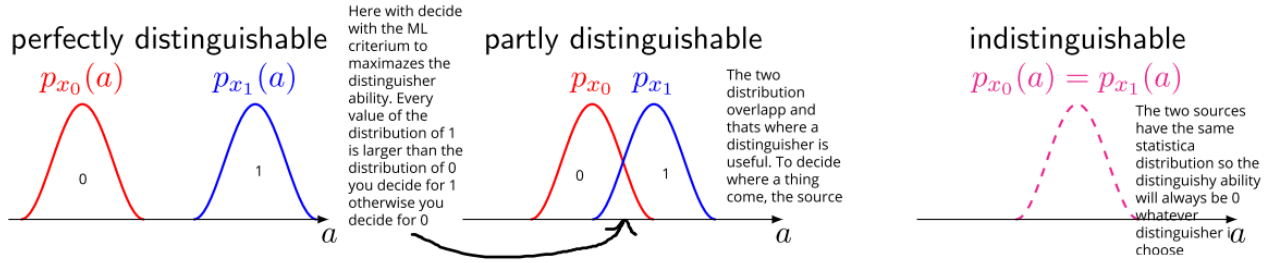
Indistinguishability and statistical distance

It is not always possible to find a perfect or even a good distinguisher

Definition (unconditional)

Two variables x_0 and x_1 are said to be ε-unconditionally indistiguishable if, for any distinguisher D, it is $d_D(x_0, x_1) \leq \varepsilon$

Unconditonal distinguishability is a measure of statistical distance between two variables



The distinguisher that maximizes $d_D(x_0, x_1)$ is the ML estimator of b from observation y
(Maximum likelihood detection)

Risposta:

Variational statistical distance

Definition

The variational distance between two rvs x, y with alphabet $\mathcal{A}$ is defined as
(random variables; A is the set of the possible messages)

$$d_{\mathsf{V}}(x, y) = \frac{1}{2} \sum_{a \in \mathcal{A}} |p_x(a) - p_y(a)|$$

It is a 1-norm distance between their PMD, and it holds

$$\text{(indistinguishable)} \quad 0 \leq d_{\mathsf{V}}(x, y) \leq 1 \quad \text{(perfectly distinguishable)}$$

Relationship with distinguishability

$$\sup_D d_D(x, y) = d_{\mathsf{V}}(x, y)$$

4)Can you describe the scheme protocols in 2G mobile? 2G è GSM (primo appello scritto)

Security services

GSM security was designed to provide the following services:

user privacy against attackers trying to identify and/or trace a specific user's location

access control against network usage by unauthorized entities

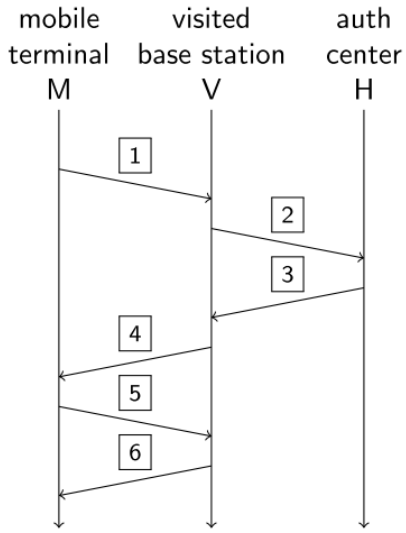
user authentication against billing frauds

user data secrecy against eavesdropping on the radio channel

. . . no data integrity protection

A3,A8 sono solo funzioni. User A ha una sim con IdA e Ka(=Km = master key).

GSM mobile user authentication protocol



1 $M \to V$: $\mathrm{id}_M, \mathrm{id}_H$

2 $V \to H$: $\mathrm{id}_M, \mathrm{id}_V$

3 H : for $n = 1, \ldots, N$:

generate random challenge (128 bit) $c_n \sim \mathcal{U}(\mathcal{C})$

compute expected response (32 bit) $\hat{r}_n = A3(k_M, c_n)$

compute session key (64 bit) $\hat{k}'_n = A8(k_M, c_n)$

$H \to V$: $[c_1, \hat{r}_1, \hat{k}'_1, \ldots, c_N, \hat{r}_N, \hat{k}'_N]$

4 $V \to M$: c_1

5 M : compute response $r_1 = A3(k_M, c_1)$

compute session key $k'_1 = A8(k_M, c_1)$

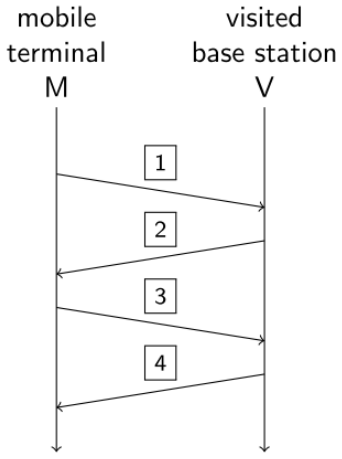
$M \to V$: r_1

6 V : if $r_1 = \hat{r}_1$, accept M and generate temporary $\mathrm{id}'_{M,1}$

$V \to M$: $[\mathrm{id}_V, \mathrm{id}'_{M,1}]$

Re authentication with same V

With the same VLR V:



1 $M \to V$: $\mathrm{id}'_{M,n}, \mathrm{id}_V$

2 $V \to M$: c_{n+1}

3 M : compute $r_{n+1} = A3(k_M, c_{n+1})$ and $k'_{n+1} = A8(k_M, c_{n+1})$

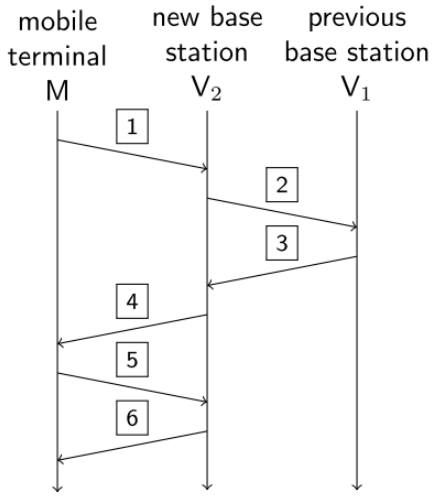
$M \to V$: r_{n+1}

4 V : if $\hat{r}_{n+1} = \hat{r}_{n+1}$, accept M and generate temporary $\mathrm{id}'_{M,n+1}$

$V \to M$: $[\mathrm{id}_V, \mathrm{id}'_{M,n+1}]$

Switching to another V

Handover from a VLR V_1 to another VLR V_2:



1. $M \to V_2$: $\mathrm{id}'_{M,n}, \mathrm{id}_{V_1}$
2. $V_2 \to V_1$: $\mathrm{id}'_{M,n}, \mathrm{id}_{V_1}$
3. $V_1 \to V_2$: $\mathrm{id}'_{M,n}, \mathrm{id}_M, [c_{n+1}, \hat{r}_{n+1}, \hat{k}'_{n+1}, \ldots, c_N, \hat{r}_N, \hat{k}'_N]$
4. $V_2 \to M$: c_{n+1}
5. M : compute $r_{n+1} = A3(k_M, c_{n+1})$ and $k'_{n+1} = A8(k_M, c_{n+1})$
   $M \to V_2$: r_{n+1}
6. V_2 : if $r_{n+1} = \hat{r}_{n+1}$, accept M and generate temporary $\mathrm{id}'_{M,n+1}$
   $V_2 \to M$: $[\mathrm{id}_{V_2}, \mathrm{id}'_{M,n+1}]$

LFSR=Linear Feedbak Shift Register. Importante sapere che ci sono 3 LFSR, State & Key di 64 bit

GSM encryption

GSM provides 4 encryption modes: A5/0 (none), A5/1 (good), A5/2 (weak), A5/3 (strong)
A5/1 is a binary stream cipher

$$\mathcal{A}_u = \mathcal{A}_x = \mathcal{A}_z = \mathcal{A}_k = \mathcal{A}_s = \mathbb{B} = \{0, 1\}$$

The global state comprises the state of the 3 LFSRs (19-bit, 22-bit, and 23-bit), both state and key are 64-bit long

$$\mathcal{K} = \mathcal{A}_k^{\ell_k} = \mathbb{B}^{\ell_k} \quad , \quad \mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2 \times \mathcal{S}_3 \quad , \quad \boldsymbol{s}_n = [\boldsymbol{s}_{1,n}, \boldsymbol{s}_{2,n}, \boldsymbol{s}_{3,n}] \quad , \quad \mathcal{S}_i = \mathbb{B}^{\ell_i}$$

$$\boldsymbol{s}_{i,n} = [s_{i,n}(0), \ldots, s_{i,n}(\ell_i - 1)] \quad , \quad \ell_1 = 19, \ell_2 = 22, \ell_3 = 23 \quad , \quad \ell_s = \ell_1 + \ell_2 + \ell_3 = 64$$

Key stream generation

$$h : z_n = s_{1,n}(\ell_1 - 1) \oplus s_{2,n}(\ell_2 - 1) \oplus s_{3,n}(\ell_3 - 1)$$

depends on current state (XOR the last bit from each LFSR), not on key.

State update each register i advances only if $s_{i,n}(c_i) = s_{j,n}(c_j)$ for some other $j \neq i$
(at each step either all or only two LFSRs advance)

5)What are the limitations of 2G? What kind of information

The GSM cipher A5/1: vulnerabilities

Specific vulnerabilities

- State update of A5/1 is not one-to-one
- Long time with the same BSC, states will concentrate
- 64 bit key / state are too short

→

Biased birthday state guessing attack

1. precompute the 64-bit outputs that correspond to the most likely states
2. observe until any of them appears in the actual transmission
3. the state is known

The security level of k'_M was initially set to 54 bits (with 10-bit zero padding), then extended to 64 actual bits

GSM security vulnerabilities

In the authentication protocol

- No authentication of V to M $\Rightarrow$ M will respond to any challenge
- Weakness of the A3 function: for some $c_n = \gamma_i$, r_n leaks information about k_M
- A3 is used in a time invariant way

→

k_M recovery attack

- By simulating a fake base station in the vicinity of the victim mobile,
- or by directly accessing the victim SIM (phone resellers, repair shops, ...)

an attacker can submit challenges $\{\gamma_i\}$ and recover k_M (aka SIM cloning)

In the security negotiation

- Negotiation of the encryption mechanism (which A5/X) is carried out between V and M without H being aware
- M cannot enforce a minimum security level

→

Security downgrade attack

- a forged V' can force a low security level (A5/2) or sometimes none (A5/0)

UMTS: Il protocollo del 3g, Implemente anche user data integrity, mutual authentication e key management. Utilizzare uno stronger crpytographic mechanisms. H calcola oltre che a challenge, response anche session key per encryption/data mac/anonymity (with function f3,f4,f5). Poi uno step sequence number, update secuirity parameters, compute A+IP tag and network auth token. Nello step [5] M si calcola il tag e va verificare che sia lo stesso di quello fatto da H se è vero allora accetta V e poi calcola la risposta da mandare a V. V calcola la risposta, se check ok allora genera idm,1 e manda a M.

6)What is the secret key rate? Talk about secret key capacity

Memoryless: A communication channel for which the statistical properties of the output signal at a time t are determined only by the input signal transmitted at this moment t of time (and consequently do not depend on the signal transmitted prior to or after the moment t). A channel in which we have the same statistical distribution. Xyz are indipendent not from each other but from previous and next session.

Secret key rate means that how many secret bits that we can transmit over some given wiretap channel under the assumpution that they are received secret and correctly by our counterpart.

The secret key capacity, which is the least upper bound of the key generation rate in the secret key agreement.

Memoryless sources

Consider n-symbol sequences, $\mathbf{x} = [x_1, \ldots, x_n]$ (and similarly for $\mathbf{y}$ and $\mathbf{z}$) and memoryless sources, $p_{\mathbf{xyz}}(\mathbf{a}, \mathbf{b}, \mathbf{c}) = \prod_{i=1}^{n} p_{xyz}(a_i, b_i, c_i)$

Definition

$R_\mathsf{k} \geq 0$ is an achievable secret key rate for the source p_{xyz} if, $\forall n$, there exist: key spaces $\{\mathcal{K}_n\}$ and schemes $f_{\mathsf{A},n}(\cdot,\cdot)$ and $f_{\mathsf{B},n}(\cdot,\cdot)$ such that

- cardinality: $|\mathcal{K}_n| \geq 2^{nR_\mathsf{k}}$
- correctness: $\lim_{n\to\infty} \mathrm{P}\left[k_\mathsf{A} \neq k_\mathsf{B}\right] = 0$
- secrecy: $\lim_{n\to\infty} I(k_\mathsf{A}, k_\mathsf{B}; \mathbf{z}, c_\mathsf{A}, c_\mathsf{B}) = 0$
- uniformity: $\lim_{n\to\infty} nR_\mathsf{k} - H(k_\mathsf{A}) = 0$

Definition

The secret key capacity of the memoryless source p_{xyz} is

$$C_\mathsf{k} = \sup\{R_\mathsf{k} \,:\, R_\mathsf{k} \text{ is an achievable secret key rate}\}$$

Schemes=Algoritmi o protocolli.

Cardinality: Means that we can retrieve nRk bits from n symbol.

Correctness: Error probability of a mismatch for the keys goes to 0 in asymptotic sense.

Secrecy: We bring to 0 the mutual information between the keys and everything the attacker observes.

21)Talk and explain memoryless channels, what results do we have?

Memoryless channels

By considering n-symbol sequences, $\boldsymbol{x} = [x_1, \ldots, x_n]$ (and similarly for $\boldsymbol{y}$ and $\boldsymbol{z}$) and memoryless channels, $p_{\boldsymbol{yz}|\boldsymbol{x}}(\boldsymbol{b}, \boldsymbol{c}|\boldsymbol{a}) = \prod_{i=1}^{n} p_{yz|x}(b_i, c_i|a_i)$, we define:

Definition

$R_\mathsf{s} \geq 0$ is an achievable secrecy rate for memoryless channel $p_{yz|x}$ if, $\forall n \geq n_0$, there exist: a message set $\mathcal{M}_n$, an encoder and decoder such that

- $|\mathcal{M}_n| \geq 2^{nR_\mathsf{s}}$
- $\lim_{n\to\infty} \mathrm{P}\left[\hat{u} \neq u\right] = 0$
- $\lim_{n\to\infty} I(u; \boldsymbol{z}) = 0$

Definition

The secrecy capacity of memoryless channel $p_{yz|x}$ is

$$C_\mathsf{s} = \sup\{R_\mathsf{s} \,:\, R_\mathsf{s} \text{ is an achievable secrecy rate}\}$$

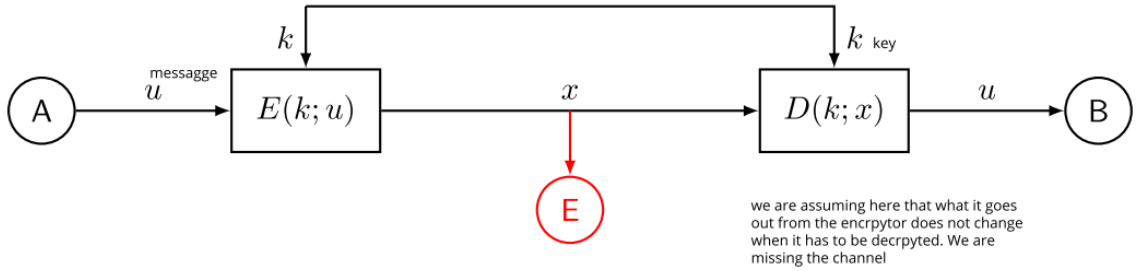
7)What is the best layer to apply a secret? Physical layer? Why is physical layer the best?

We should do physical layer security since wireless communcations are prone to access to physical layer and any device is a potential eavesdropper so we need a way to protect the information at physical layer. At physical layer we can levarage diversity and randomness(like noise) of the channels to provide security.

Physical layer secrecy means that we need to provide unconditional secrecy at the physical layer.

A channel is a probabilistic transformation. Qui stiamo assumendo che quello che esce da E non cambia quando viene decriptato da D. Ci manca il canale quindi ecco perché dobbiamo usare il physical layer(motivation perché dobbiamo usare il wiretap channel).

Unconditional secrecy [Shannon, '49]



Kerchoff's Assumption

E knows:

- the functions $C(\cdot;\cdot)$, $D(\cdot;\cdot)$
- the distributions $p_u(\cdot)$, $p_k(\cdot)$

Secrecy of u is only based on hiding the key k

Perfect secrecy

u statistically independent of x

$$p_u(\alpha) = p_{u|x}(\alpha|\beta) \quad , \quad I(u;x) = 0$$

Theorem

Perfect secrecy requires $H(k) \geq H(u)$

8)Can you give an example why we use to protect application layer data?

5G?

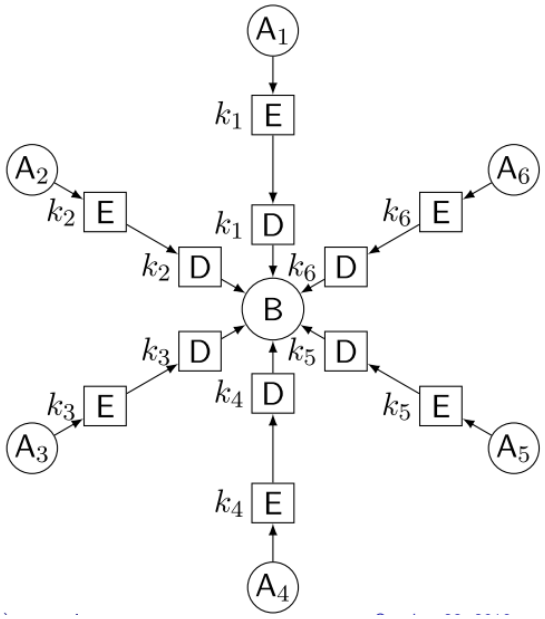
9)What is public key encryption? What are the requirements for the public key?

TLDR: chiave pubblica per tutti per cifrare, chiave privata del proprietario per decifrare. Si utilizza nella firma digitale. Quello che è un requisito fondamentale è il one way function cioè easy to compute e hard to invert(no reversibility). Asymmetric encryption è digital signature

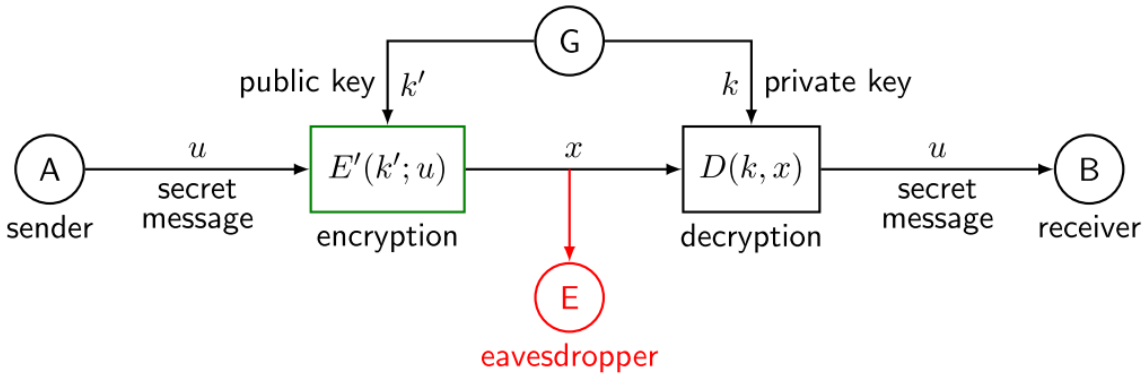
Consider the problem of a single user B having to receive confidential messages $u_1, \ldots, u_N$ from each of N different sources A_i, so that B obtains message u_i but any A_i cannot learn any message u_j, $j \neq i$.

With a symmetric encryption mechanism $(\mathcal{M}, \mathcal{X}, \mathcal{K}, E, D, p_k, p_u)$, B must agree and share a different key k_i with any A_i

Can we build a mechanism where B uses a single key k_B ?



General model of an asymmetric encryption system



La firma però è diversa perché in quel caso firmi con la chiave privata e poi verifichi con quella pubblica.

private key $k \in \mathcal{K}$ private key space
public key $k' \in \mathcal{K}'$ public key space

(reparametrized) encryption map $E' : \mathcal{K}' \times \mathcal{M} \mapsto \mathcal{X}$
$E_{k'} : \mathcal{M} \mapsto \mathcal{X} \quad E_{k'}(u) \doteq E(k', u)$

decryption map $D : \mathcal{K} \times \mathcal{X} \mapsto \mathcal{M}$
$D_k : \mathcal{X} \mapsto \mathcal{M} \quad D_k(x) \doteq D(k, x)$

Keys are random with joint probability mass distribution $p_{kk'} : \mathcal{K} \times \mathcal{K}' \mapsto [0, 1]$
typically $(k, k') \not\sim \mathcal{U}(\mathcal{K} \times \mathcal{K}')$ are uniform but not independent
often $k \sim \mathcal{U}(\mathcal{K})$ is random and uniform, $k' = f(k)$ is computed with $f : \mathcal{K} \mapsto \mathcal{K}'$ deterministic
The encryption system is completely specified as:

$$\mathcal{S} = (\mathcal{M}, \mathcal{X}, \mathcal{K}, \mathcal{K}', E', D, p_u, p_{kk'})$$

General assumptions

- (perfect reliability)The receiver must be able to recover the secret message perfectly

$$D_c = E_c^{-1} = (E'_{c'})^{-1} \quad \forall c \in \mathcal{K}, c' \in \mathcal{K}' \ : \ p_{kk'}(c, c') > 0 \quad (\text{or } c' = f(c))$$

- (Kerchoff's assumption) The eavesdropper knows the system $\mathcal{S}$ (in particular the maps $E'(\cdot,\cdot)$ and $D(\cdot,\cdot)$)

Where does secrecy come from?

Secrecy can only be computational and is based on the following requirements

1. it is hard to derive k from k' (i.e., f is one-way)
2. it is hard to derive u from (k', x) (i.e., $E'_{k'}$ is one-way)
3. it is hard to derive k from (u, x) (i.e., $D(\cdot, x)$ is one-way)

One-way function: definitions

One-way functions are a fundamental tool in many computationally secure mechanisms and their analysis. They are informally referred to as "easy to compute and hard to invert".

Definition (concrete)

A function $f : \mathcal{X} \mapsto \mathcal{Y}$ is said to be $(\varepsilon_0, T_0; \varepsilon_1, T_1)$-one-way if

(easy to compute) there exists a probabilistic algorithm $\mathtt{A}$ such that

$$\forall x \in \mathcal{X} \quad , \quad \mathrm{P}\left[\{\mathtt{A}[x] \to f(x)\} \cap \{T_{\mathtt{A}} \leq T_1\}\right] \geq 1 - \varepsilon_1$$

(hard to invert) for any probabilistic algorithm $\mathtt{B}$

$$\forall y \in \mathcal{Y}, \forall x \in f^{-1}(y) \quad , \quad \mathrm{P}\left[\{\mathtt{B}[y] \to x\} \cap \{T_{\mathtt{B}} \leq T_0\}\right] \leq \varepsilon_0$$

A deterministic variant for the easy to compute requires that there exists a deterministic algorithm $\mathtt{A}$ such that $T_{\mathtt{A}} \leq T_1$ and $\mathtt{A}[x] \to f(x)$, $\forall x \in \mathcal{X}$

10)How can we say k prime(public key) is compatible with one another?

???

11)What is a universal hashing function? Why do we need universal hashing function? What is its purpose? (Purpose = MAC)

Universal hashing (in a randomized algorithm or data structure) refers to selecting a hash function at random from a family of hash functions with a certain mathematical property.

Universal hashing

For unconditionally secure A+IP, we take $\{T_k(\cdot)\}_{k\in\mathcal{K}}$ to be a ε-almost strongly universal$_2$ family of hashing functions $\mathcal{M} \to \mathcal{T}$, for some parameter $\varepsilon \in (0, 1)$, that is

1. (uniform mapping) $\forall u \in \mathcal{M}, t \in \mathcal{T}$, and with $\mathcal{K}_{u\to t} = \{k \in \mathcal{K} \ : \ T_k(u) = t\}$ it must be

$$|\mathcal{K}_{u\to t}| \le \varepsilon\,|\mathcal{K}|$$

the set of indeces k that map u into t

2. (uniform collisions) $\forall u_1 \neq u_2 \in \mathcal{M}$, and with $\mathcal{K}_{u_1u_2} = \{k \in \mathcal{K} \ : \ T_k(u_1) = T_k(u_2)\}$ it must be

$$|\mathcal{K}_{u_1u_2}| \le \varepsilon\,|\mathcal{K}|$$

3. (uniform pairwise mapping) $\forall u_1 \neq u_2 \in \mathcal{M}$ and $\forall t_1, t_2 \in \mathcal{T}$, it must be

$$|\mathcal{K}_{u_1\to t_1} \cap \mathcal{K}_{u_2\to t_2}| \le \varepsilon\,|\mathcal{K}_{u_1\to t_1}|$$

(actually, property 3 $\Rightarrow$ property 2)

Message authentication codes (MACs)

A Message authentication code (MAC) is a symmetric mechanism providing authentication and integrity protection of the "tag-appending" kind

$$x = (u, t) \quad , \quad t = T(k, u)$$

that offers computational security.

Security requirements

It must be hard for F to find t, given u but not k, possibly even under

known message attacks (KMA) F has observed previous $x_i = (u_i, t_i)$ signed with the same k, i.e. $t_i = T(k, u_i)$

chosen message attacks (CMA) F can choose $u_1, \ldots, u_n \neq u$, have them signed with the same k, i.e. $t_i = T(k, u_i)$ and observe the resulting $x_i = (u_i, t_i)$

Possiamo usare D come Tag Function T?

- In encryption/decryption |X|>= |M| since Ek is injective.
- In MAC the tag space depends only on security level and not length of the message so it's not simple to use a decryption function as tag function

Possiamo usare T come D? Sì ma il messaggio u potrebbe essere troppo corto o troppo lungo. Soluzione per questo è usare CBC-Mac o Hash and Sign.

What is a reasonable cardinality for the tag space?

- If you have one bit and you sign with one an attacker can randomly guess with 50% of success probability so you have to sign with a lot of bit to have a prob success fro attacker of 1/n bit to get correct signature
- If you have a long message you don't need a signature with same length of the message. You just need to sign with enough bit to guarantee security

Message authentication codes (MACs)

The requirements for $T(\cdot,\cdot)$ are similar to those for the decryption function $D(\cdot,\cdot)$ in a symmetric encryption scheme

Can we use a well designed $D : \mathcal{K} \times \mathcal{X} \mapsto \mathcal{M}$ as tag computation function $T : \mathcal{K} \times \mathcal{M} \mapsto \mathcal{T}$?

There is a difference:

- in encryption/decryption, $|\mathcal{X}| \geq |\mathcal{M}|$ because each E_k is injective, and typically $|\mathcal{X}| \approx |\mathcal{M}|$
- in MACs, $H(t)$ and $|\mathcal{T}|$ are dictated by the target security level, and nearly independent of $H(u)$, $|\mathcal{M}|$

We could use as T_k a block cipher decryptor with output range $\mathcal{T}$, but the authentic message u may be too long / short.

Two possible solutions

CBC-MAC pad or split u into blocks $u_1, \ldots, u_n$ of proper length, then compute their tags

Hash & Sign compress u with a hash function $h : \mathcal{M} \mapsto \mathcal{T}$ before computing the tag

12)How does the handshake work in TLS? What is its purpose?

The purpose is to create a secure connection with server and choose system keys.

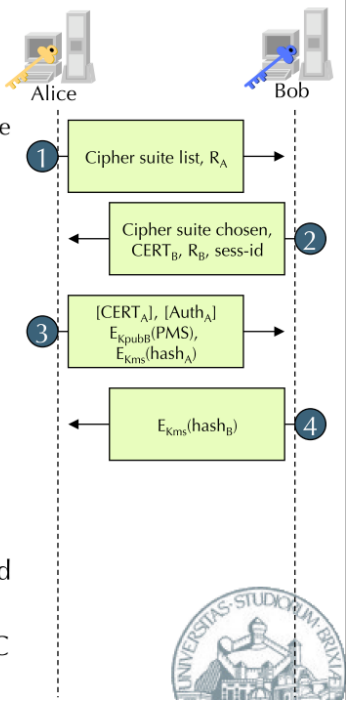
Cipher suite: a set of algorithms that help secure a network connection.

Alice A è il client. Bob B è il server. Vogliamo setuppare una session key km(master key) come seed per una o più connession keys e far autenticare B da A (e volendo anche viceversa).

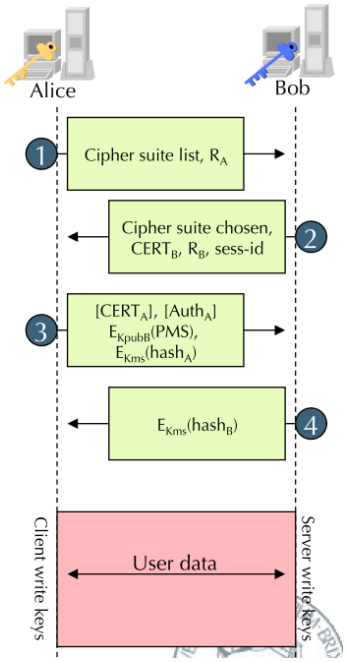
1) A=>B: Cipher suite list, numero casuale Ra
2) B=>A: Cipher suite scelto, numero casuale Rb, CERTb come certificazione server, id sessione sess-id
3) A=>B:
    - Manda CERTa come certificazione client e AuthA
    - Genera un numero casuale come pre master secret PMS e lo critta con la chiave pubblica di B (ottenuta dal certificato di B). EkpubB(PMS)
    - PMS viene usata per trovare la master secret key Kms=fPMS(Ra,Rb) dove f è una MAC function
    - Fa un hash di tutti i messaggi che ha mandato e critta tutto ciò con la master secret key Kms. Ekms(hashA)
4) B=>A: Il server si può calcolare la Kms e critta l'hash di tutti i messaggi che ha mandato lui con questa chiave. Ekmas(hashB)

TLS Handshake protocol: high level overview

- Goals
  - Authentication of B to A and, optionally, of A to B
  - Setup of ephemeral ***session*** key K_{ms} (*Master Secret*), as a "seed" for one or more ***connection*** keys
- In TLS, A (*initiator*) is the ***client***, while B (*responder*) is the ***server***
- The Handshake protocol messages are transported by the TLS Record protocol
- Each record message can contain more than one handshake messages
  - For example, record message (2) usually contains messages "server_hello, certificate, [certificate_request], server_hello_done"
- R_A, R_B: random numbers
- $Auth_A = SIG_{KprivA}(hash(previous\ messages))$
- PMS: pre-master secret, random number chosen by A
- $K_{ms} = f_{PMS}(R_A, R_B)$, where *f* is a MAC function derived from both SHA1 and MD5
- $hash_{A/B} = g_{Kms}(\{client/server\}, previous\ messages)$, where *g* is another MAC function derived from both SHA1 and MD5



- At this point we have a ***TLS session***
  - Authentication [optionally mutual]
    - A authenticates B by the fact that B can prove they are able to decrypt PMS (i.e., calculate the correct K_{ms}) with the private key associated to $CERT_B$
    - Optionally, B authenticates A through $Auth_A$
  - TLS session, identified by (K_{ms}, sess-id). K_{ms} is a 384 bit (48 byte) string
- One or more ***TLS connections*** can now be instantiated, by deriving the necessary ephemeral keys from K_{ms}. User traffic will be protected by these TLS connections inside the TLS record protocol
- By key expansion, ***three key pairs*** are derived from K_{ms}
  - Client write MAC, client write, client IV (K_{cm}, K_c, IV_c)
  - Server write MAC, server write, server IV (K_{sm}, K_s, IV_s)
  - Key expansion is similar to the one used to derive K_{ms} from PMS, i.e., it is based on a MAC function that works on K_{ms}, R_A, R_B
  - Note that there are three keys ***for each of the two directions***



13)What is the difference between Digital signature and message authentication code? What services do they provide?

These types of cryptographic primitive can be distinguished by the security goals they fulfill (in the simple protocol of "appending to a message"):

- **Integrity:** Can the recipient be confident that the message has not been accidentally modified?
- **Authentication:** Can the recipient be confident that the message originates from the sender?

- **Non-repudiation:** Non-repudiation is the assurance that an entity cannot deny a previous commitment or action
  Il non ripudio si riferisce alla condizione secondo la quale l'autore di una dichiarazione non potrà negare la paternità e la validità della dichiarazione stessa. Grazie a questa proprietà, un'entità che ha firmato alcune informazioni non può in un secondo momento negare di averle firmate

	Hash	MAC	Digital signature
Integrity	Yes	Yes	Yes
Authentication	No	Yes	Yes
Non-repudiation	No	No	Yes
Kind of keys	None	Symmetric	Asymmetric

A (unkeyed) hash of the message, if appended to the message itself, only protects against accidental changes to the message (or the hash itself), as an attacker who modifies the message can simply calculate a new hash and use it instead of the original one. So this only gives integrity. If the hash is transmitted over a different, protected channel, it can also protect the message against modifications. This is sometimes be used with hashes of very big files (like ISO-images), where the hash itself is delivered over HTTPS, while the big file can be transmitted over an insecure channel.

A message authentication code (MAC) (sometimes also known as keyed hash) protects against message forgery by anyone who doesn't know the secret key (shared by sender and receiver).This means that the receiver can forge any message – thus we have both integrity and authentication (as long as the receiver doesn't have a split personality), but not non-repudiation. Also an attacker could replay earlier messages authenticated with the same key(known message attacks), so a protocol should take measures against this (e.g. by including message numbers or timestamps). (Also, in case of a two-sided conversation, make sure that either both sides have different keys, or by another way make sure that messages from one side can't sent back by an attacker to this side.) MACs can be created from unkeyed hashes (e.g. with the HMAC construction), or created directly as MAC algorithms.

A (digital) signature is created with a private key, and verified with the corresponding public key of an asymmetric key-pair. Only the holder of the private key can create this signature, and normally anyone knowing the public key can verify it. Digital signatures don't prevent the replay attack mentioned previously. There is the special case of designated verifier signature, which only ones with knowledge of another key can verify, but this is not normally meant when saying "signature".So this provides all of integrity, authentication, and non-repudiation.

If there is a shared key (such as in the MAC setting) one user could deny having produced a MAC tag on the message as the key is shared- external parties are not sure which owner of the MAC key could have produced the tag. Whereas with digital signatures, each key is unique to each user, so there is non-repudiation as only one user could produce a valid signature.

14)What is the difference between message source authentication and entity source authentication?

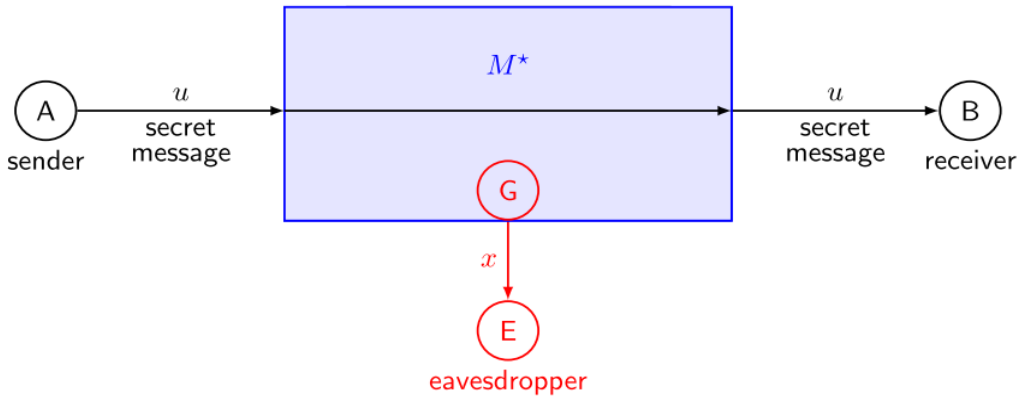
Message source authentication is the assurance that a given entity was the original source of the received data.

Entity source authentication is the assurance that a given entity is involved and currently active in a communication session. Message authentication provides assurance on the original source and does not tell us anything about when the message was sent or who is sending it now.

Consider the following scenario with data origin authentication but no entity authentication: Alice could sign a message and send the message and signature to Bob. Bob could then forward the message and signature to Charlie, who can verify that the original source of the signature was Alice (as there is message authentication) but not that they are communicating with Alice now. Hence there is no entity authentication. To get entity authentication, we need some kind of freshness. For example, for Charlie to be assured about Bob's identity and current activity, Charlie could request Bob produce a valid signature on a fresh nonce, generated by Charlie. This ensures that Bob is currently active and that the signature came from him. Obviously, this may be vulnerable to man in the middle attacks (Bob could forward the none to Alice to sign, and return the signature to Charlie), but there are things that can be done about this (for example, by asking Bob to sign the challenge along with a session ID: the session ID is unlikely to be the same as the session between Bob and Alice, so Alice would not sign).

15)What is the definition of perfect secrecy? Why is it indipendent?

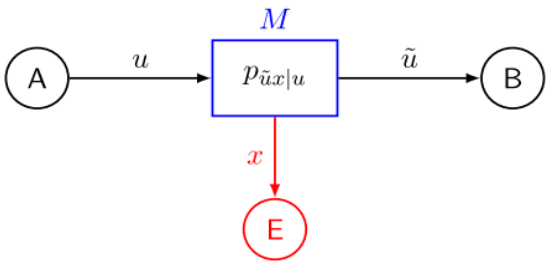
Ideal world model



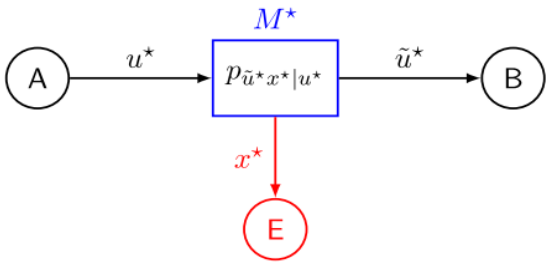
In the ideal counterpart of encryption, the secret message u is directly delivered, unmodified to B, and the message observed by E is generated independently from u

The best we can hope for, is an encryption system M that is statistically identical to its ideal counterpart $M^\star$

real



ideal



$$
\begin{aligned}
p_{\tilde{u}x|u}(b,c|a) &= p_{\tilde{u}^\star x^\star|u^\star}(b,c|a) \\
(\text{independence}) &= p_{\tilde{u}^\star|u^\star}(b|a)p_{x^\star}(c) \\
(\text{correctness}) &= \delta(a,b)p_{x^\star}(c)
\end{aligned}
$$

0-unconditional significa che è undistiguishable dalla controparte ideale.

Definition

An encryption system is perfect if it provides 0-unconditional security based on indistinguishability, i.e. the plaintext is statistically independent of the ciphertext

$$p_{x|u}(b|a) = p_x(b) \quad \forall a \in \mathcal{M}, b \in \mathcal{X}$$

or equivalently

$$p_{ux}(a,b) = p_u(a)p_x(b) \quad \forall a \in \mathcal{M}, b \in \mathcal{X}$$
$$p_{u|x}(a|b) = p_u(a) \quad \forall a \in \mathcal{M}, b \in \mathcal{X}$$

In a system with perfect secrecy, since $p_{u|x} = p_u$ the optimal informed guessing strategy coincides with the optimal ignorant guessing

Perfect secrecy is the notion that, given an encrypted message (or ciphertext) from a perfectly secure encryption system (or cipher), absolutely nothing will be revealed about the unencrypted message (or plaintext) by the ciphertext. In terms of probabilities, it means that the probability distribution of the possible plaintexts is independent of the ciphertext. This is obviously a desirable property of a cipher, and perfectly secret ciphers do exist: e.g. One-time pad.

Quello che offre perfect secrecy è il one time pad. La perfect secrecy va offerta nel physical layer con il wiretap channel.

Indipendent? Lo sono perché in questo modo non è possibile recuperare il plaintext dal chiphertext. In realtà sono solo statisticamente indipendenti perché sono related dalla chiave quindi se qualcuno non conosce la chiave x e u devono apparire statisticamente indipendenti.

Chain rules for (conditional) entropy and mutual info

The above relationships can be generalized to any collection of rvs $x_1, \ldots, x_n, y_1, \ldots, y_m, z_1, \ldots, z_\ell$ as the following chain rules:

1. $H(x_1, \ldots, x_n, z_1, \ldots, z_\ell | y_1, \ldots, y_m) \geq H(x_1, \ldots, x_n | y_1, \ldots, y_m)$, entropy increases with more conditioned variables
2. $H(x_1, \ldots, x_n | y_1, \ldots, y_m, z_1, \ldots, z_\ell) \leq H(x_1, \ldots, x_n | y_1, \ldots, y_m)$, entropy decreases with more conditioning variables
3. $H(x_1, \ldots, x_n, z_1, \ldots, z_\ell | y_1, \ldots, y_m) =$
   $H(x_1, \ldots, x_n | y_1, \ldots, y_m, z_1, \ldots, z_\ell) + H(z_1, \ldots, z_\ell | y_1, \ldots, y_m)$, conditional entropy split

Necessary condition for perfect secrecy

Theorem

A necessary condition for perfect secrecy and decodability is that

$$H(k) \geq H(u)$$

Proof.

Assume perfect secrecy holds, that is u is independent of x. Then,

$$\begin{aligned} H(u) &= H(u|x) && \text{by independence of } u, x \\ &\leq H(u,k|x) && \text{by chain rule 1} \\ &= H(u|x,k) + H(k|x) && \text{by chain rule 3} \\ &= H(k|x) && \text{by perfect decodability} \\ &\leq H(k) && \text{by chain rule 2} \end{aligned}$$

Necessary condition for perfect secrecy (cont.)

Corollary

In a system with perfect secrecy for all message distributions p_u we have

$$\log_2 |\mathcal{K}| \geq H(k) \geq \log_2 |\mathcal{M}|$$

Proof.

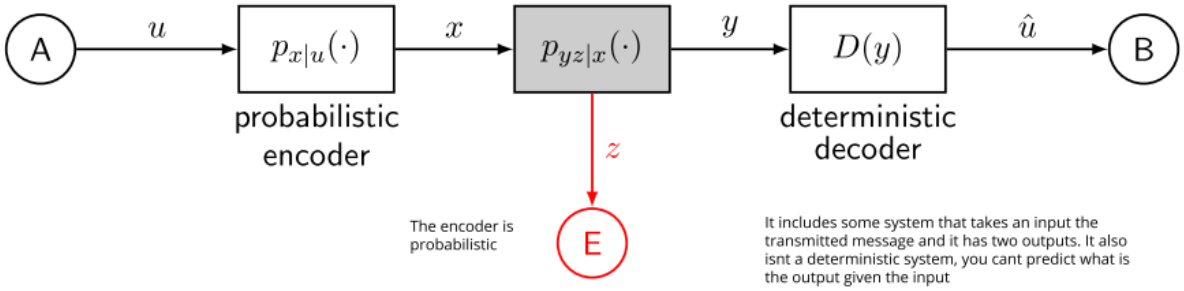
$H(k) \leq \log_2 |\mathcal{K}|$ is the upper bound for entropy.
From the previous theorem $H(k) \geq H(u)$ must hold for any p_u.
In particular, for uniform $u \sim \mathcal{U}(\mathcal{M})$, where $H(u) = \log_2 |\mathcal{M}|$.

La stessa chiave k non può essere riutilizzata senza sacrificare la perfect secrecy. Se ci provi, l'entropia di u aumenta mentre quella di k rimane uguale e viola la condizione necessaria della perfect secrecy.

16)What is a wiretap channel in physical layer?

It is a system that takes as input the transmitted message and it has two outputs. It is not a deterministic system, you can't predict what is the output given the input(due to noise,error, loss etc). We have to use the joint conditional distribution given the input. Our aim is to have perfect reliability(output lo stesso di quello trasmesso) and secrecy.

The wiretap channel [Wyner, '75]



We aim for reliable transmissions to B, and secrecy with respect to E

perfect reliability: $\hat{u} = u$

perfect secrecy: z, u statistically independent

The attacker can see only z and z must be indipendent from the message x

In terms of unconditional distinguishability from the ideal counterpart

reliability is measured by the error probability $\mathrm{P}\left[\hat{u} \neq u\right] = d_{\mathsf{V}}(p_{\hat{u}|u}, p_{u|u})$

secrecy is measured by the mutual information $I(u; z) = \mathrm{D}\left(p_{uz} \| p_u p_z\right)$

The finite and the asymptotic view

In general, the ideal case is not achievable, we can take either of two views

Finite view

We must seek a tradeoff among

amount of information: $H(u)$

reliability: $\mathrm{P}\left[u \neq \hat{u}\right] \leq \varepsilon$

secrecy: $I(u; z) \leq \delta$

Asymptotic view: secrecy capacity

By processing blocks of length n, $\boldsymbol{u} = [u_1, \ldots, u_n]$, $\boldsymbol{x} = [x_1, \ldots, x_n]$ and letting $n \to \infty$, we seek the secrecy capacity

$$C_{\mathsf{s}} = \lim_{n\to\infty} \max_{p_{\boldsymbol{u}}, p_{\boldsymbol{x}|\boldsymbol{u}}, D} \left[\tfrac{1}{n} H(\boldsymbol{u})\right]$$

subject to the constraints:

reliability: $\lim_{n\to\infty} P\left[\boldsymbol{u} \neq \hat{\boldsymbol{u}}\right] = 0$

(strong) secrecy: $\lim_{n\to\infty} I(\boldsymbol{u}; \boldsymbol{z}) = 0$

or (weak) secrecy: $\lim_{n\to\infty} \frac{1}{n} I(\boldsymbol{u}; \boldsymbol{z}) = 0$

Il wiretap channel offre perfect reliability e perfect secrecy

17)How do we solve a problem in wiretap channel?

18)How do we order the wiretap channel?

1. è come avere due canali in fila dove uno introduce dell'errore nell'input e l'output è y. Poi c'è un altro canale in cui aggiunge altro distortion e l'output è z. B osserva una versione meno distorta dell'input.

2. Si osserva un'altra y'. Il canale Py/x ha la stessa distribuzione statitstica(stessa quantità di noise ecc dal punto di vista statistico) di quello in 1. È stocastico perché non abbiamo veramente y tra A ed E ma una cosa dal punto di vista statistico uguale.

3. Per ogni encoder che metto nel wiretap channel confronto la mutual information con un sistema a cascata tra wiretap channell ed encoder.

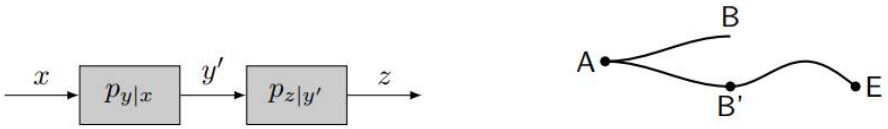
4. no encoders qui ed è il peggiore perché la mutual information è maggiore tra X e Y che tra X e Z.

Channel orderings

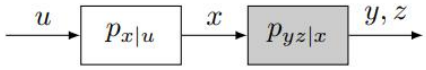
We consider the following channel orderings, in decreasing order of strength

1. channel A $\to$ E is physically degraded with respect to A $\to$ B if z is independent of x given y

2. channel A $\to$ E is (stochastically) degraded with respect to A $\to$ B if z is independent of x given some other y', with $p_{y'|x} = p_{y|x}$



3. channel A $\to$ E is more noisy than A $\to$ B if for any precoder $u \to x$ with u independent of (y, z) given x, we have $I(u; y) > I(u; z)$



4. channel A $\to$ E is less capable than A $\to$ B if, for any x, we have $I(x; y) > I(x; z)$

19)How does McElice Cryptosystem work in encryption?

Criptosistema assimetrico che usa randomizzazione nell'encrytion. Fa parte della post-quantum cryptography perché è immune dagli attacchi che usano lo Shor Algorithm. Lo Shor Algorithm è un algoritmo che calcola efficientemente l'ordine di ogni elemento x in un gruppo G.

Matrice è usata per generare il codeword. Information word = input message tutti con lunghezza n. Easy problem è polinomiale. Hard Problem è esponenziale b^ è stima dei messaggi transmessi. Questo punto diventa easy se matrice è in forma canonica. Non è possibile però trovare facilmente la forma canonica di una matrice partendo da una matrice non in forma canonica.

B^l è set of binary information with length l.

The McEliece cryptosystem [McEliece, '78]

Based on NP problem

minimum Hamming distance (mHd) decoding of binary codes

In a (n, ℓ, t) linear binary FEC code (e.g., Goppa codes) with

- n codeword length
- ℓ code dimension = information word length
- t maximum nr. of correctable errors

easy given an information word $\boldsymbol{b} \in \mathbb{B}^\ell$ and a generating matrix $\boldsymbol{G} \in \mathbb{B}^{n \times \ell}$, compute the codeword $\boldsymbol{c} = \boldsymbol{G}\boldsymbol{b} \in \mathbb{B}^n$

hard given a received word (not necessarily a codeword) $\tilde{\boldsymbol{c}} \in \mathbb{B}^n$ and a generating matrix $\boldsymbol{G} \in \mathbb{B}^{n \times \ell}$, compute $\hat{\boldsymbol{b}} = \arg\min_{\boldsymbol{\beta} \in \mathbb{B}^\ell} d_{\mathsf{H}}(\tilde{\boldsymbol{c}}, \boldsymbol{G}\boldsymbol{\beta})$

easy given a received word (not necessarily a codeword) $\tilde{\boldsymbol{c}} \in \mathbb{B}^n$ and a generating matrix $\boldsymbol{G} \in \mathbb{B}^{n \times \ell}$ in canonical form, compute $\hat{\boldsymbol{b}} = \arg\min_{\boldsymbol{\beta} \in \mathbb{B}^\ell} d_{\mathsf{H}}(\tilde{\boldsymbol{c}}, \boldsymbol{G}\boldsymbol{\beta})$

S è una matrice casuale singolare(Una matrice singolare è una matrice quadrata con determinante uguale a zero e non è invertibile).

The McEliece cryptosystem

Key generation

1. B chooses $\boldsymbol{G} \in \mathbb{B}^{n\times\ell}$ canonical generating matrix of a (n, ℓ, t) Goppa code
2. generates $\boldsymbol{S} \in \mathbb{B}^{\ell\times\ell}$ non singular
3. generates $\boldsymbol{P} \in \mathbb{B}^{n\times n}$ a permutation matrix (exactly one '1' in each row and column)
4. computes $\boldsymbol{S}^{-1}, \boldsymbol{P}^{-1}$, and $\boldsymbol{G}' = \boldsymbol{P}^{-1}\boldsymbol{G}\boldsymbol{S}^{-1} \in \mathbb{B}^{n\times\ell}$ noncanonical generating matrix of an equivalent (n, ℓ, t) Goppa code

private key $k = (\boldsymbol{G}, \boldsymbol{P}, \boldsymbol{S})$, $\mathcal{K} = \mathbb{B}^{n\times\ell} \times \mathbb{B}^{n\times n} \times \mathbb{B}^{\ell\times\ell}$

public key $k' = f(k) = (\boldsymbol{G}', t)$, $\mathcal{K}' = \mathbb{B}^{n\times\ell} \times \mathbb{N}$

e è un vettore(error pattern with no more than t errors). Wh è Hamming Weight del vettore e(numero di elementi non zero nel vettore). Tutte le parole nel cipher space x sono fatte così. In 1) di decryption rimuovi la permutazione e in 2) u' è la distanza minima nella decodifica di x'.

The McEliece cryptosystem

Encryption by A (public key, probabilistic)

$$\mathcal{M} = \mathbb{B}^{\ell} \quad , \quad \mathcal{X} = \mathbb{B}^{n}$$

A generates a random $\boldsymbol{e} \in \mathbb{B}^n$ such that $w_{\mathsf{H}}(\boldsymbol{e}) \leq t$ (i.e., a correctable error pattern)

$$E'_{k'} \; : \; \boldsymbol{x} = \boldsymbol{G}'\boldsymbol{u} + \boldsymbol{e}$$

Decryption by B (private key)

$\hat{u} = D(k, x) = D(\boldsymbol{G}, \boldsymbol{P}, \boldsymbol{S}, x)$ is computed as follows

1. B computes $\boldsymbol{x}' = \boldsymbol{P}\boldsymbol{x}$
2. solves the mHd decoding of $\boldsymbol{x}'$ in the Goppa code with canonical $\boldsymbol{G}$, i.e.,

$$\boldsymbol{u}' = \arg\min_{\boldsymbol{\beta}\in\mathbb{B}^{\ell}} d_{\mathsf{H}}(\boldsymbol{x}', \boldsymbol{G}\boldsymbol{\beta})$$

3. computes $\hat{\boldsymbol{u}} = \boldsymbol{S}\boldsymbol{u}'$

The McEliece cryptosystem

Correctness

We prove that $\hat{\boldsymbol{u}} = \boldsymbol{u}$

$$\begin{aligned} \boldsymbol{x}' &= \boldsymbol{P}\boldsymbol{x} \\ &= \boldsymbol{P}(\boldsymbol{G}'\boldsymbol{u} + \boldsymbol{e}) \\ &= \boldsymbol{P}\boldsymbol{P}^{-1}\boldsymbol{G}\boldsymbol{S}^{-1}\boldsymbol{u} + \boldsymbol{P}\boldsymbol{e} \\ &= \boldsymbol{G}\boldsymbol{S}^{-1}\boldsymbol{u} + \boldsymbol{e}' \\ &= \boldsymbol{G}\boldsymbol{u}' + \boldsymbol{e}' \end{aligned}$$

where $\boldsymbol{u}' = \boldsymbol{S}^{-1}\boldsymbol{u}$ is an information word, too, and $\boldsymbol{e}' = \boldsymbol{P}\boldsymbol{e}$ has $w_{\mathsf{H}}(\boldsymbol{e}') = w_{\mathsf{H}}(\boldsymbol{e}) \leq t$, so it is a correctable error pattern, too.
Therefore the mHd decoding of $\boldsymbol{x}'$ with $\boldsymbol{G}$ is $\boldsymbol{u}'$ and

$$\hat{\boldsymbol{u}} = \boldsymbol{S}\boldsymbol{u}' = \boldsymbol{S}\boldsymbol{S}^{-1}\boldsymbol{u} = \boldsymbol{u}$$

The McEliece cryptosystem

Security

$x = E'_{k'}(u)$ is one-way given x and noncanonical G', it is hard to find u (mHd decoding)
$k' = f(k)$ is one-way given the non canonical $\boldsymbol{G}' = \boldsymbol{P}^{-1}\boldsymbol{G}\boldsymbol{S}^{-1}$ it is hard to factor it into $\boldsymbol{P}, \boldsymbol{G}, \boldsymbol{S}$ with a canonical $\boldsymbol{G}$

20)How does Elgamal Cryptosystem work in encryption and in signature? Is Elgamal probalistic? Where does the probabilistic come from?

The Elgamal cryptosystem [Elgamal, '85]

Based on NP problem

finite logarithm

In a group $(\mathbb{G}, \circ)$, we denote $\alpha \overset{n}{\circ} = \underbrace{\alpha \circ \cdots \circ \alpha}_{n \text{ times}}$

easy given $\alpha \in \mathbb{G}, n \in \mathbb{N}$, compute $\beta = \alpha \overset{n}{\circ}$

hard given $\alpha, \beta \in \mathbb{G}$, find $n \in \mathbb{N}$ such that $\alpha \overset{n}{\circ} = \beta$

Key generation

Let $(\mathbb{G}, \circ)$ be a group with a primitive element $\alpha \in \mathbb{G}$, i.e. such that $\forall \beta \in \mathbb{G}, \exists n : \alpha \overset{n}{\circ} = \beta$.

private key space $\mathcal{K} = \{1, \ldots, |\mathbb{G}| - 1\} \subset \mathbb{N}$

public key space $\mathcal{K}' = \mathbb{G}$

Let $(\mathbb{G}, \circ)$ and α be publicly known. B generates $k \sim \mathcal{U}(\mathcal{K})$, then computes $k' = f(k) = \alpha \overset{k}{\circ}$

The Elgamal cryptosystem

Encryption by A (public key, probabilistic)

$\mathcal{M} = \mathbb{G}$, $\mathcal{X} = \mathbb{G}^2$

A generates $b \sim \mathcal{U}(\mathcal{K})$

Unlike RSA the encryption is probabilistic

$$x = E'_{k'}(u, b) = (x_1, x_2) \quad , \quad \begin{cases} x_1 = \alpha \overset{b}{\circ} \\ x_2 = u \circ (k' \overset{b}{\circ}) \end{cases}$$

a operating himself b times

Decryption by B (private key)

B need not know b

$$\hat{u} = D_k(x) = D_k(x_1, x_2) = x_2 \circ \left((x_1 \overset{k}{\circ}) \overset{-1}{\circ} \right)$$

where $\cdot \overset{-1}{\circ}$ denotes the inverse in $(\mathbb{G}, \circ)$

Probabilistico sì e dipende da finite log problem

The finite logarithm problem

A strong requirement for security of the Elgamal encryption is that "exponentiation" $f_\alpha(n) = \alpha \overset{n}{\circ}$ is a one-way function of n and this depends on the choice of the group $(\mathbb{G}, \circ)$. If computing $\circ$ has linear complexity in $\ell = \log |\mathbb{G}|$, exponentiation can be computed with complexity $O(\ell^2)$, by iterative squaring and multiplying

For a general group $(\mathbb{G}, \circ)$

Consider the computation of $y = x \overset{n}{\circ}$, with $n < |\mathbb{G}|$.
Let $\boldsymbol{b} = [b_0, b_1, \ldots, b_{\ell-1}]$ be the binary representation of n

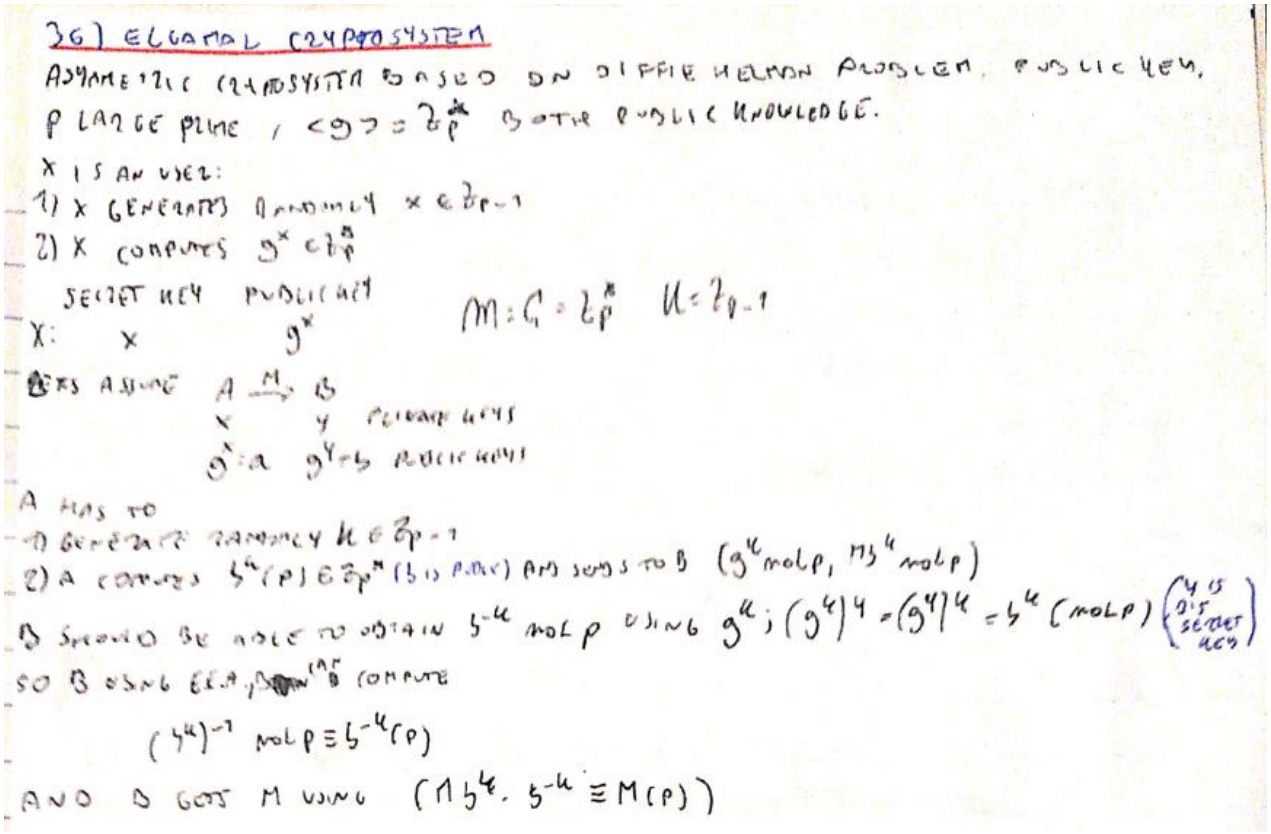
$$n = \sum_{i=0}^{\ell-1} b_i 2^i$$

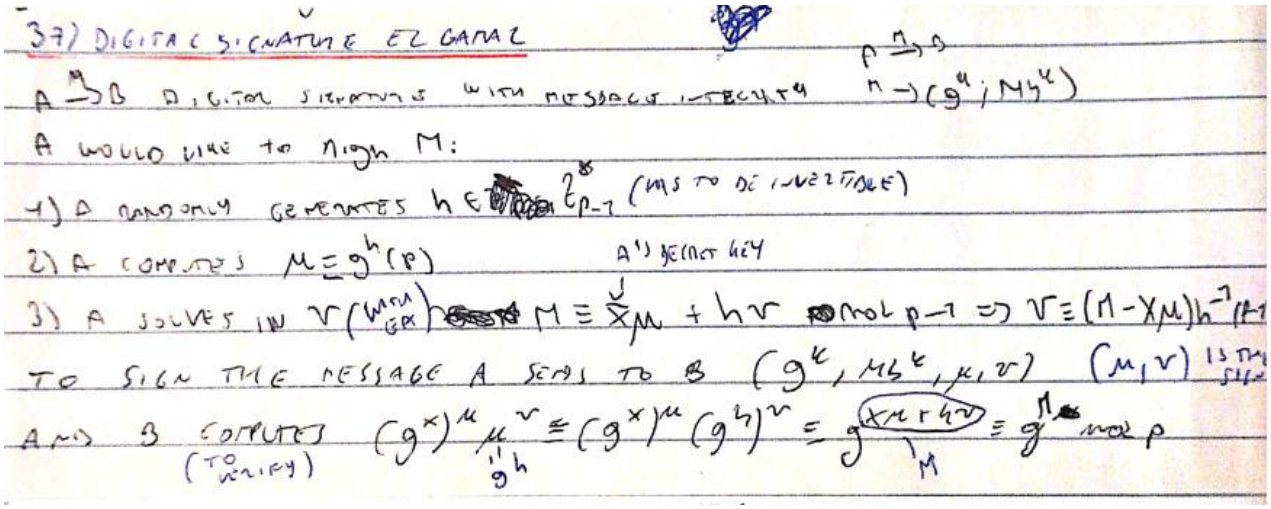
Then $y = x \overset{n}{\circ} = x \overset{\sum_{i=0}^{\ell-1} b_i 2^i}{\circ} = \left(x \overset{b_0 2^0}{\circ}\right) \circ \cdots \circ \left(x \overset{b_{\ell-1} 2^{\ell-1}}{\circ}\right)$

Iterative square and multiply

$c \leftarrow e$ (identity in $\mathbb{G}$)
$a \leftarrow x$
for $i = 0$ to $\ell - 1$ **do**
  if $b_i = 1$ **then**
    $c \leftarrow c \circ a$
  end if
  $a \leftarrow a \circ a$
end for
$y \leftarrow c$

Vedi slide languasco





23)What do we require in an entity authentication protocol?

General model for entity authentication

An entity A (called the prover) wants to prove his identity to another entity B (called the verifier), typically through an iteractive protocol. At the end of the protocol, the entity B must decide whether he trusts A (accept) or not (reject)

Attack scenario

Masquerade A malicious F wants to pose as A while interacting with B

Requirements

Correctness If A is honest, B accepts hime with high probability

Security / robustness (to false provers) If the prover is not honest (i.e., it is some F posing as A) it is hard for F to be accepted

Non transferability (against malicious verifiers) Even after the protocol has taken place between A and B it is hard for B to pose as A in an exchange with another entity C and be accepted

24)Give an example of an entity that does offer non-transferability and which doesn't

Hash password does not offer non transferability

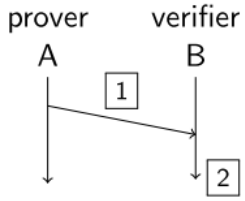
Hashed password authentication protocols

its an improve of the previous one

entities the prover A, the verifier B

tools a hash function $h : \mathcal{W} \mapsto \mathcal{T}$

setup A chooses a password $w_A \in \mathcal{W}$ and securely delivers it to B
B computes $t_A = h(w_A)$ and stores a copy of (id_A, t_A) in database $\mathcal{D}$



1 $A \to B : \ u = (u_1, u_2) = (\mathrm{id}_A, w_A)$

2 B : computes $\tilde{t} = h(u_2)$ and checks if $(u_1, t) \in \mathcal{D}$ and, if so, accepts A

the verify is the hash not the password

Improvement

- w_A no longer stored in clear

Weaknesses

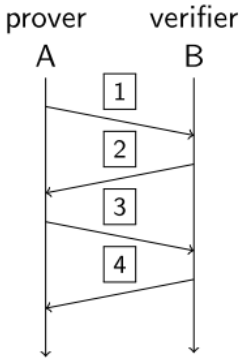
- transferability, as B learns w_A
- w_A still transmitted in clear
- a forger could carry on a 2nd preimage attack

Zero knowledge e challenge response + A + IP

Challenge-response protocols with symmetric A+IP

entities the prover A, the verifier B

tool a symmetric message A+IP mechanism, of the tag appending type with key k_A and tag function $T(\cdot, \cdot)$



1 $A \to B : \ u_1 = \mathrm{id}_A$

2 B : generates a random challenge $r \sim \mathcal{U}(\mathcal{R})$
$B \to A : \ u_2 = r$

3 A : builds $u_3 = \mathrm{id}_A, r$
signs $t_3 = T(k_A, u_3)$
$A \to B : \ t_3$

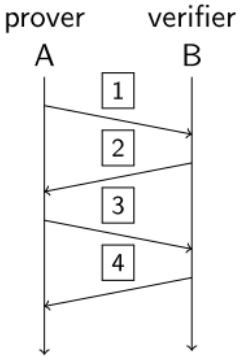
4 B : verifies whether $V(k_A, u_3, t_3) = (r, \mathrm{ok})$ and, if so, accepts A

The challenge r must be changed at every run of the protocol, otherwise a dishonest prover F, pretending to be A, can replay 1 and 3 even without knowing k_A, and would be accepted

Challenge-response protocols with asymmetric A+IP

entities the prover A, the verifier B

tool a digital signature mechanism, with keys k_A, k'_A; a certificate c_A for k'_A and k'_B



1 A → B : $u_1 = \mathrm{id}_A$

2 B : generates a random challenge $r_B \sim \mathcal{U}(\mathcal{R})$
B → A : $u_2 = r_B$

3 A : generates a random challenge $r_A \sim \mathcal{U}(\mathcal{R})$
builds $u_3 = [\mathrm{id}_B, r_B, r_A]$ signs $x_3 = S(k_A, u_3)$
A → B : x_3

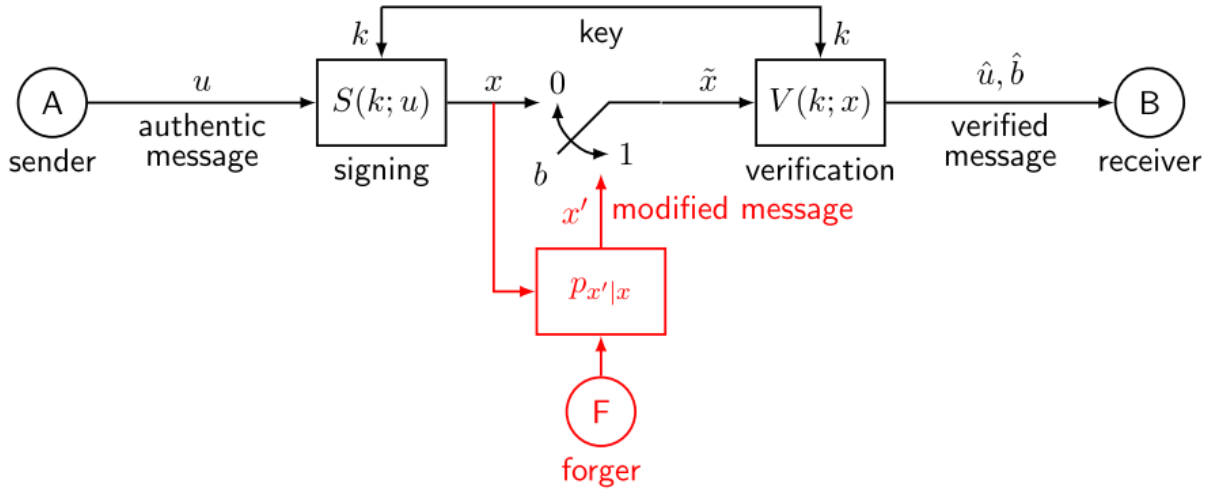
4 B : verifies whether $V(k'_A, x_3) = ([\mathrm{id}_B, r_B, r_A], \mathrm{ok})$ and, if so, accepts A

The challenge r must be changed at every run of the protocol, otherwise an dishonest prover F, pretending to be A, can replay 1 and 3 even without knowing k_A, and would be accepted

25)Talk about integrity protection, what are the requirements for symmetrics encryption?

Integrity protection makes it possible to detect whether a message was intecepted and modified.

General model of the integrity protection problem



Illegitimate modification (alteration) attack

F can block x and wants to replace it with x' such that $\hat{u} \neq u$ and $\hat{b} = 0$

Symmetric encryption is a type of encryption where only one key (a secret key) is used to both encrypt and decrypt. Stessa chiave condivisa che serve per scambiare in maniera sciura (DiffieHellmann). La lunghezza della chiave è related alla robustness.

26)In Digital signature using RSA, how to generate keys? What is needed for public and private keys? How to sign and verify messages?

Based on NP problems

- integer factorization
  - easy given $p, q \in \mathbb{Z}$, compute $n = pq$
  - hard given $n \in \mathbb{Z}$, find $p, q \in \mathbb{Z}$ such that $pq = n$
- finite logarithm and finite root
  - easy given $n \in \mathbb{Z}$, $x, d \in \mathbb{Z}_n$ compute $y = x^d \bmod n$ (finite exponential)
  - hard given $n \in \mathbb{Z}$, $x, y \in \mathbb{Z}_n$ find $d \in \mathbb{Z}_n$ such that $x^d \bmod n = y$
  - hard given $n \in \mathbb{Z}$, $d, y \in \mathbb{Z}_n$ find $x \in \mathbb{Z}_n$ such that $x^d \bmod n = y$

The RSA cryptosystem

Key generation (ℓ-bit)

B chooses $p, q < 2^{\ell/2}$ primes
computes $n = pq$, $\varphi = (p-1)(q-1)$
chooses $d \in \mathbb{Z}_n$ such that $\gcd(\varphi, d) = 1$
computes $e \in \mathbb{Z}_n$ such that $ed = 1 \pmod{\varphi}$

private key $k = (p, q, d)$, $\mathcal{K} = \mathbb{Z}_{2^\ell}^3$
public key $k' = (n, e)$, $\mathcal{K}' = \mathbb{Z}_{2^\ell}^2$

Encryption by A (public key)

$\mathcal{M} = \mathcal{X} = \mathbb{Z}_n$
$E' : \mathcal{K}' \times \mathcal{M} \mapsto \mathcal{X}$
$x = E'(k', u) = E'(n, e, u) = u^e \bmod n$

Decryption by B (private key)

$D : \mathcal{K} \times \mathcal{X} \mapsto \mathcal{M}$
$\hat{u} = D(k, x) = D(n, d, x) = x^d \bmod n$

The RSA signature scheme

Necessity of hashing

Besides existential forgery, hashing also prevents the following chosen message attack

- F knows one pair $x = (t, u)$ with $t = T(k, u) = u^d \bmod n$ and aims to forge $x' = (u', t')$, for some target forged message u', with $t' = T(k, u') = (u')^d \bmod n$.
- he computes $u_1 = u'u \bmod n$ and lures A into signing u_1, obtaining

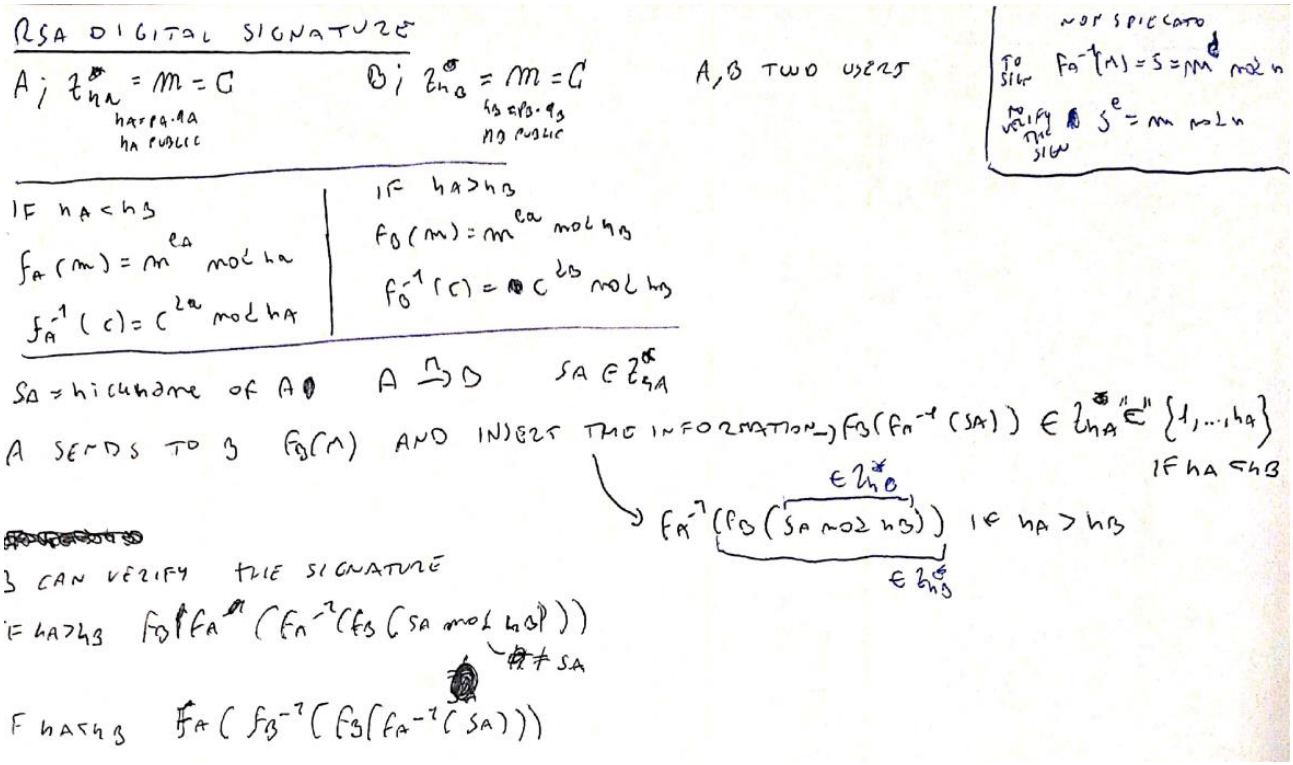
$$t_1 = (u_1)^d = (u'u)^d = (u')^d u^d = t't \pmod{n}$$

- he derives $t' = t_1 t^{-1} \bmod n$

With hashing the attacker can compute

$$h(u')h(u) \bmod n = v'v \bmod n = v_1 = h(u_1)$$

but can not derive u_1 (preimage reistance)



Mancano un po' di cose, sicuramente qualcosa sul **distance bounding (lecture 20)** perché è stato chieso al primo appello quindi ci fa qualche domanda.

Distance bounding cryptographic protocols

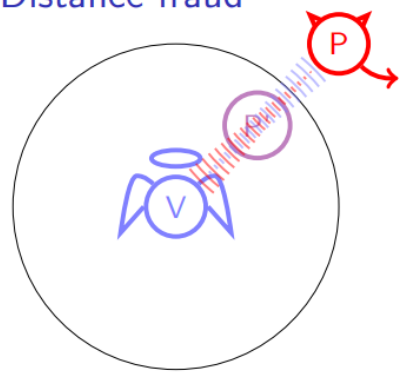
General formulation

1. preliminary setup: sharing a long-term cryptographic secret
2. initialization phase: sharing a temporary session key
3. timing phase: many (single bit) challenge-response iterations
4. verification phase: checking answers vs key, and times vs distance bound

- very short response process times (~ 1 ns)
- cryptographic computations at initialization

Verifier $\mathcal{V}$		Prover $\mathcal{P}$
$x_{\mathcal{VP}}$		$x_{\mathcal{VP}}$
Initialization phase		
$N_{\mathcal{V}} \leftarrow \{0,1\}^m$	$\xrightarrow{N_{\mathcal{V}}}$	$N_{\mathcal{P}} \leftarrow \{0,1\}^m$
	$\xleftarrow{N_{\mathcal{P}}}$	
$a_0 = f_{x_{\mathcal{VP}}}(N_{\mathcal{V}}, N_{\mathcal{P}})$		$a_0 = f_{x_{\mathcal{VP}}}(N_{\mathcal{V}}, N_{\mathcal{P}})$
$a_1 = \mathsf{Enc}_{a_0}(x_{\mathcal{VP}})$		$a_1 = \mathsf{Enc}_{a_0}(x_{\mathcal{VP}})$
Distance-bounding phase		
for $i = 1, \ldots n$		
pick $c_i \in \{0,1\}$		
Start Clock	$\xrightarrow{c_i}$	if $c_i \notin \{0,1\}$, halt
	$\xleftarrow{r_i}$	else $r_i = (a_{c_i})_i$
Stop Clock		
Verification phase		
Check that $\Delta t_i < t_{\mathsf{max}} \quad \forall i = 1, \ldots, n$		
Verify r_i		

Distance fraud



scenario a dishonest prover P is far from the verifier V

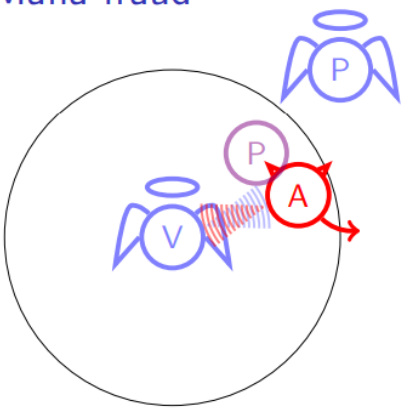
aim P attempts to convince V that P is close to V

attack P reduces the response computation time, or guesses the challenge in advance

Requirements against distance fraud

- The response computation time must be $\tau_c \leq (d' - d)/c \ll t_{\max}$
- The response must depend on the challenge, which must be unpredictable $c_i \sim U(\{0, 1\})$
- The challenge and response waveforms must have short duration

Mafia fraud



scenario an honest prover P is far from the verifier V
a malicious attacker A is close to V

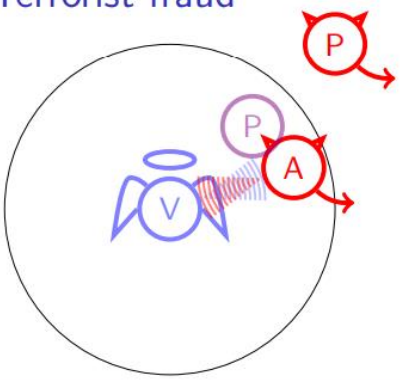
aim A attempts to convince V that P is close to V

attack A intercepts c_i and answers r_i in place of P
or forwards c_i to P and uses P as an oracle

Requirements against mafia fraud

- The response must depend on some secret shared between P and V
- The shared secret must be derived from P's credentials
- The shared secret must be renewed in each session

Terrorist fraud



scenario a dishonest prover P is far from the verifier V
a malicious attacker A is close to V

aim P and A collude and attempt to convince V that P is close to V
without P giving his/her long term credentials to A (otherwise it would be unstoppable)

attack P shares with A his/her temporary secret, so A can compute r_i and answer immediately

Requirements against terrorist fraud

- The response must depend on some secret shared between P and V
- The shared secret must leak out significant information from P's credentials (but the pairs (c_i, r_i) must not)

Altre cose:

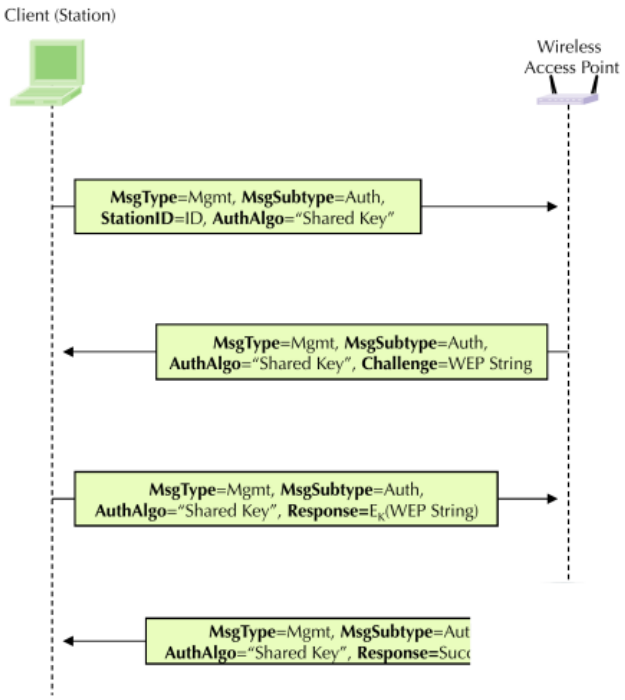
Wireless LAN (Lecture 22)

WEP: goals and main components

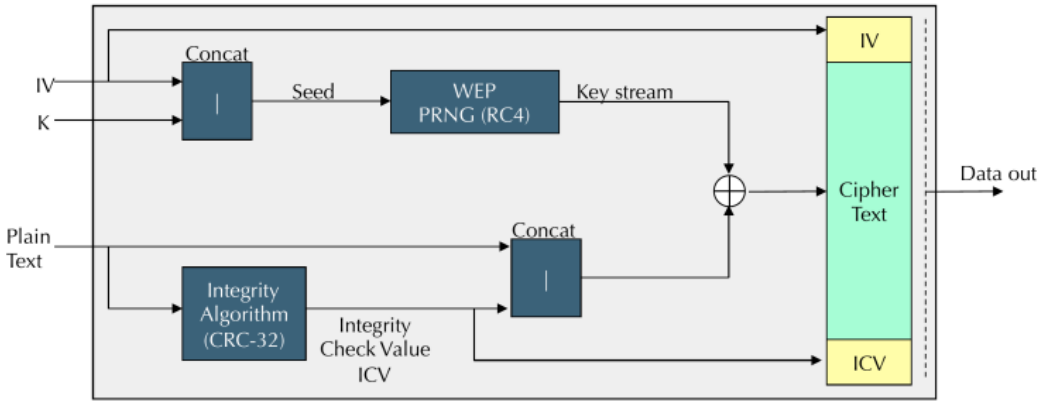
- Security at layer-2: cryptographic protection of most of the fields of pseudo-ethernet frames
- Goals
  - Authentication
  - Confidentiality
  - Integrity protection
- Main components
  - A pre-shared key *K*, shared among all APs and STAs that make up a WLAN: security goals are "shared" within a (potentially large and dynamic) group
    - In theory the standard specifies the possibility of using four different pre-shared keys, thus dividing users and APs in four different sub-groups
    - The same key is used for authentication and confidentiality
  - A very simple authentication protocol
  - A single cryptographic primitive: RC4
  - A non-crypto primitive for integrity protection: CRC-32

WEP: authentication protocol

- Authentication is one way (client to network)
- STA proves knowledge of *K* by encrypting the challenge with RC4
- As we will see, there are several security pitfalls to this approach
  - Authentication is not mutual
  - The way RC4 is used in WEP makes the authentication protocol leak security information that should not be exposed
- The protocol is easily breakable (remember: RC4, if used properly, ***is not***)



The use of RC4 in WEP



- K: the pre-shared key between all APs and STAs in a given WLAN
  - 40 or 104 bit
- IV: 24 bit
  - To generate different key-streams, the IV should be different for each packet [think about what would happen if an intruder could get a hold of c_1 (with $c_1=k_1 \oplus m_1$) and c_2 (with $c_2=k_2 \oplus m_2$), with $\mathbf{k_1=k_2}$]
- ICV: CRC-32, 32 bit (non-crypto, linear function)

The main problems with WEP

- K is a ***group key***
  - The system is not scalable
  - Key management is nearly impossible: what happens when a client leaves the group for good?
  - All clients are equal WRT authentication (same key)
- RC4 is used incorrectly
  - WEP design choices
    - Wireless devices ⇒ reduced computing power ⇒ stream cipher
    - Problem: how do we keep transmitter and receiver in sync
    - How does WEP address the issue: ***different (random) IVs for each packet***
  - The crux of the issue
    - The IV space is too small: the probability of collision is too high
- The authentication protocol is particularly ill-designed
  - Non mutual
  - It uses the same RC4-based engine (as it is used for confidentiality) as a MAC. The same key and IV space are used in both cases
- Integrity protection is left to a non-crypto function
  - It is relatively easy to create colliding messages

One Time Pad

One-time pad

Let $(\mathbb{G}, \circ)$ be a finite group, [e.g., $(\mathbb{Z}_N, + \bmod N)$]. A one-time pad (OTP) over $(\mathbb{G}, \circ)$ is the encryption system described by

$$\text{equal spaces} \qquad \mathcal{M} = \mathcal{X} = \mathcal{K} = \mathbb{G}$$

$$\text{uniform key} \qquad k \sim \mathcal{U}(\mathbb{G}) \quad \Leftrightarrow \quad p_k(a) = \frac{1}{|\mathbb{G}|} \, \forall a \in \mathbb{G}$$

$$\text{encrypt by add} \quad E(a, b) = b \circ a$$

$$\text{decrypt by subtract} \quad D(a, c) = c \circ a^{-1}$$

Example

Let $\mathbb{G} = \mathbb{B}^N$, with $\mathbb{B} = \{0, 1\}$, $N = 5$, and $\circ =$ bitwise XOR. Then, e.g.,

$$u = 01101, k = 10110 \quad \Rightarrow \quad x = u \circ k = 11011$$

B can recover the message with $k^{-1} = k = 10110$

$$u = x \circ k^{-1} = 01101$$

Secrecy of one-time pad

Theorem

The one-time pad offers perfect reliability and perfect secrecy for any message distribution

Proof.

Perfect reliability is guaranteed by the existence and uniqueness of $k^{-1} \in \mathbb{G}$.
As regards perfect secrecy, we prove that $p_{u,x}(b,c) = p_u(b)p_x(c), \forall b \in \mathcal{M}, c \in \mathcal{X}$. In fact,

$$\begin{aligned} p_{u,x}(b,c) &= \mathrm{P}\left[u = b, x = c\right] = \mathrm{P}\left[u = b, k = b^{-1} \circ c\right] \\ &= p_u(b)p_k(b^{-1} \circ c) = p_u(b)/|\mathcal{K}| \\ p_x(c) &= \sum_{b \in \mathcal{M}} p_{u,x}(b,c) = \sum_{b \in \mathcal{M}} p_u(b)/|\mathcal{K}| = 1/|\mathcal{K}| \end{aligned}$$

Observe that this result holds for any $p_u(\cdot)$. □

One time pad authentication

We aim for ε-unconditionally secure authentication against forging, i.e.

$$\mathrm{P}\left[S_{\mathsf{f}}; M, A\right] \leq \varepsilon \quad , \quad \forall A \in \mathcal{A}_{\mathsf{f}}$$

A possible solution is a mechanism M of the tag appending type, described by

equal tag and key spaces $\mathcal{T} = \mathcal{K}$ — the tag and the key are the same thing

uniform distributed key $k \sim \mathcal{U}(\mathcal{K}) \quad \Leftrightarrow \quad p_k(a) = \dfrac{1}{|\mathcal{K}|} \, \forall a \in \mathcal{K}$

sign by appending the key $t = T(k,u) = k \quad , \quad x = (u,k)$

verify by checking the key $\hat{b} = \begin{cases} 0 & , \text{ if } \tilde{t} = k \\ 1 & , \text{ if } \tilde{t} \neq k \end{cases}$ — if its the same you accept the message, if not you discard the message

Correctness

Trivially, it is of the tag appending type

One time pad authentication

Security

x' is the forged message

Consider the class $\mathcal{A}_\mathsf{f}$ of forging attacks, where x' is independent of k and observe that

Message forgery is the sending of a message to deceive the recipient as to whom the real sender is. Forgery -> messaggio contraffatto per ingannare il destinatario su chi sia il vero mittente

$$S_\mathsf{f} = \left\{\hat{u} = u', \hat{b} = 0\right\} = \left\{t' = k\right\}$$

$$\text{so that} \quad \mathrm{P}\left[S_\mathsf{f}\right] = \mathrm{P}\left[t' = k\right] = \sum_{a \in \mathcal{K}} \mathrm{P}\left[t' = a, k = a\right]$$

$$\text{(by independence)} \quad = \sum_{a \in \mathcal{K}} p_{t'}(a) p_k(a) = \sum_{a \in \mathcal{K}} p_{t'}(a) \frac{1}{|\mathcal{K}|} = \frac{1}{|\mathcal{K}|}$$

its not possible to get perfect authentication(epsilon=0 is not possible) with this method. If i increase the key entropy(and key space) i can go close to 0 so close to the perfect authentication

yielding ε-unconditional security, for $\varepsilon \geq 1/|\mathcal{K}|$, that is, if $H(k) \geq \log_{1/2} \varepsilon$

OTP authentication cannot offer integrity protection

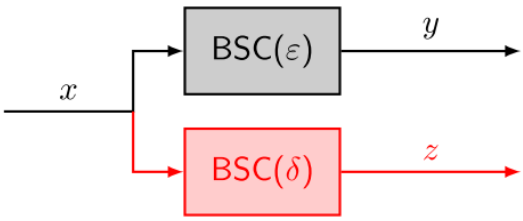
Trivial attack from $\mathcal{A}_\mathsf{m}$: F blocks $x = (u, k)$, replaces u with u', transmists $x' = (u', k)$
B verifies that $t' = k$ and accepts u'

problema, b verifica solo che k=t', non controlla che u (il messaggio codificato) sia cambiato

AWGN, BSC

Secrecy capacity for the wiretap BSC

Let the channels from A to B and from A to E be memorlyess binary symmetric with error rates ε and δ, respectively



If $\left|\varepsilon - \frac{1}{2}\right| < \left|\delta - \frac{1}{2}\right|$
legitimate channel is more noisy
(e.g., $0 < \delta < \varepsilon < \frac{1}{2}$)

$$C_\mathsf{s} = 0$$

no secrecy is possible

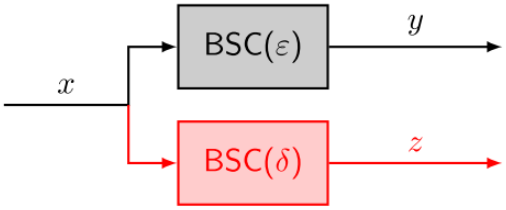
If $\left|\varepsilon - \frac{1}{2}\right| > \left|\delta - \frac{1}{2}\right|$
eavesdropper channel is more noisy
(e.g., $0 < \varepsilon < \delta < \frac{1}{2}$)

$$C_\mathsf{s} = C_\mathsf{AB} - C_\mathsf{AE} = h_2(\delta) - h_2(\varepsilon)$$

where
$h_2(\varepsilon) = \varepsilon \log_{1/2} \varepsilon + (1 - \varepsilon) \log_{1/2}(1 - \varepsilon)$

Secret key capacity for the wiretap BSC

Let the channels from A to B and from A to E be memorlyess binary symmetric with error rates ε and δ, respectively



y and z are indipendent. The probability they are different is the sigma

For all values of ε, δ the secret key capacity is attained with $x \sim \mathcal{U}(\{0,1\})$ and reverse reconciliation B $\to$ A: it yields

$$C_{\mathsf{k}} = I(x;y) - I(y;z) = h_2(\varepsilon) - h_2(\gamma)$$

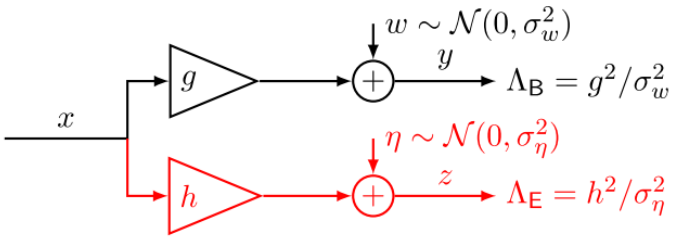
where $\gamma = \varepsilon + \delta - 2\varepsilon\delta$ and $h_2(\varepsilon) = \varepsilon \log_{1/2} \varepsilon + (1-\varepsilon)\log_{1/2}(1-\varepsilon)$.

Observe that $\left|\varepsilon - \frac{1}{2}\right| > \left|\gamma - \frac{1}{2}\right|$ for all $\delta \in (0,1)$, so that $C_k > 0$ unless the channel from A to E is perfect.

Secrecy capacity for the wiretap AWGN channel

Let the channels A $\to$ B and A $\to$ E be additive white Gaussian noise

g legitimate channel
h drop channel



here im fixing the input power, we put a bound

If $\Lambda_{\mathsf{E}} \geq \Lambda_{\mathsf{B}}$
legitimate channel is degraded

$$C_{\mathsf{s}} = 0$$

no secrecy is possible

If $\Lambda_{\mathsf{E}} < \Lambda_{\mathsf{B}}$
eavesdropper channel is degraded

shannon formula

$$C_{\mathsf{s}} = C_{\mathsf{AB}} - C_{\mathsf{AE}} = \frac{1}{2}\log_2 \frac{1+\Lambda_{\mathsf{B}} P}{1+\Lambda_{\mathsf{E}} P}$$

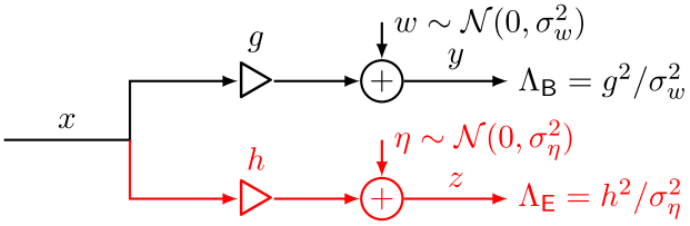
and it is achieved with $x \sim \mathcal{N}(0, P)$.

$$\lim_{P\to\infty} C_{\mathsf{s}} = \frac{1}{2}\log_2 \frac{\Lambda_{\mathsf{B}}}{\Lambda_{\mathsf{E}}}$$

P is the maximal power

Secret key capacity for the wiretap AWGN channel

Let the channels A $\to$ B and A $\to$ E be additive white Gaussian noise



For all values of Λ_B, Λ_E the secret key capacity is achieved with $x \sim \mathcal{N}(0, P)$ and reverse reconciliation. It is given by

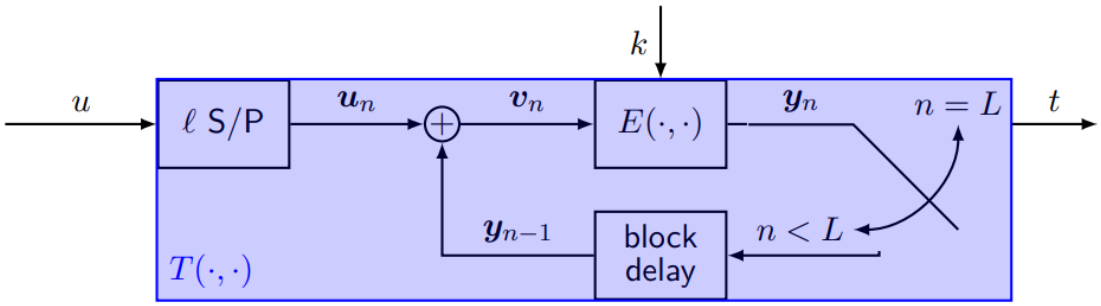
$$C_k = I(x;y) - I(y;z) = \frac{1}{2}\log_2 \frac{1-\rho_{yz}^2}{1-\rho_{xy}^2} = \frac{1}{2}\log_2 \frac{1+(\Lambda_B+\Lambda_E)P}{1+\Lambda_E P}$$

Observe that $C_k > 0$ for all $\Lambda_E < \infty$, that is unless the channel from A to E is noiseless.

$$\lim_{P\to\infty} C_k = \frac{1}{2}\log_2\left(1+\frac{\Lambda_B}{\Lambda_E}\right) \quad , \quad C_k \asymp \frac{\Lambda_B P}{2\ln 2} \quad \text{as } P \to 0$$

CBC MAC

Cipher block chaining MAC (CBC-MAC)



- Choose block length ℓ and tag space $\mathcal{T} = \mathcal{A}^\ell$ according to target security level:

$$|\mathcal{T}| \geq 1/\mathrm{P}\left[S_f\right], 1/\mathrm{P}\left[S_m\right]$$

- Choose a block encryptor $E : \mathcal{K} \times \mathcal{T} \mapsto \mathcal{T}$
- Make $(\mathcal{T}, +)$ a group, and the message space $\mathcal{M} = \cup_{L\in\mathcal{L}} \mathcal{T}^L$

Security of CBC-MAC

If $E(.,.)$ is secure, i.e., a pseudo random permutation (computationally indistinguishable from ideal random permutation)

- and $\mathcal{M}$ is prefix-free, i.e., no message u can be the prefix of another, different message u' (e.g., all messages in $\mathcal{M}$ have the same length), then CBC-MAC is computationally secure against forging and modification
- but $\mathcal{M}$ is not prefix-free, there is a known message modification attack which succeeds deterministically, can be avoided by prepending the length L to message u

Deterministic KMA attack

F observes two messages, $x = (u, t)$ with $u = (u_1, \ldots, u_L)$ and $x' = (u', t')$ with $u' = (u'_1, \ldots, u'_{L'})$, then constructs $x'' = (u'', t'')$ with $u'' = (u_1, \ldots, u_L, u'_1 - t, u'_2, \ldots, u'_{L'}) \in \mathcal{T}^{L+L'}$ and $t'' = t'$. Then at verification

$$y''_i = y_i \ , \quad i = 1, \ldots, L \quad \Rightarrow \quad y''_{L+1} = E_k(u''_{L+1} + y''_L) = E_k(u'_1 - t + y_L) = E_k(u'_1) = y'_1$$

$$\Rightarrow \quad y''_{L+i} = y'_i \ , \quad i = 2, \ldots, L' \quad \Rightarrow \quad t'' = t'$$

Elliptic curve DSA

Elliptic curve DSA

An important variant of the Elgamal signature is the elliptic curve version

Uses an elliptic curve group $(\mathcal{E}, \circ)$ on a field $\mathbb{F} = \mathbb{Z}_p$ with p prime, with cardinality $|\mathcal{E}| = q$ and a primitive point $P \in \mathcal{E}$, and mixes operations on both $\mathbb{Z}_q$ and $\mathcal{E}$.

The use of elliptic curve cryptography increases the security wrt DSA for the same key length. Hence ECDSA is very appropriate when short keys are needed (especially the public key that must be distributed)

Elliptic curve DSA

Key generation

private key space $\mathcal{K} = \{1, \ldots, q-1\}$, public key space $\mathcal{K}' = \mathcal{E}$

A randomly generates $k \sim \mathcal{U}(\mathcal{K})$, computes $k' = P \overset{k}{\circ}$

Denote by $c_1(Q) \in \mathbb{F}$ the first coordinate of a point $Q \in \mathbb{F}^2$

Signing (private, probabilistic)

$$\mathcal{T} = \mathcal{K}^2 \ , \ \mathcal{V} = \mathbb{Z}_q$$

A generates $r \sim \mathcal{U}(\mathcal{K})$

hashes message $v = h(u)$

computes $t = (t_1, t_2) \in \mathcal{T}$

$$\begin{cases} t_1 = c_1(P \overset{r}{\circ}) \bmod q \\ t_2 = (v + kt_1) r^{-1} \bmod q \end{cases}$$

Verification (public key)

B computes

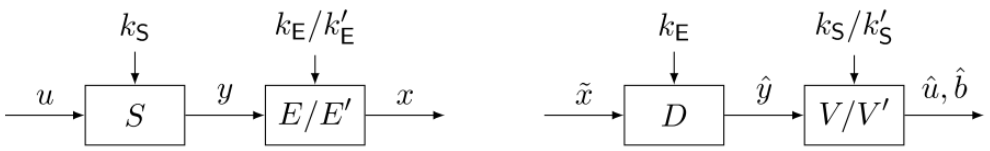
$$\begin{cases} m = t_2^{-1} h(u) \bmod q \\ n = t_2^{-1} t_1 \bmod q \\ Q = (P \overset{m}{\circ}) \circ (k' \overset{n}{\circ}) \end{cases}$$

$$\hat{b} = \begin{cases} 0 & , \text{ if } c_1(Q) = t_1 \pmod q \\ 1 & , \text{ otherwise} \end{cases}$$

Sign-then-encrypt

What if we want to both keep our message secret and guarantee is authenticity and integrity (aka build a secure channel) ?

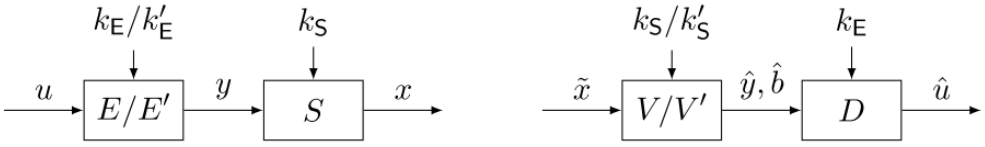
One possible solution is to first sign the information message, then encrypt the signed message



- the two mechanisms (E, D) and (S, V) can be separately designed, each with its target security requirements
- needs two distinct key pairs
- can use symmetric or asymmetric mechanisms both for signature and encryption
- was used in the Transport Layer Security (TLS) Record protocol
- vulnerable to padding oracle attacks

Encrypt-then-sign

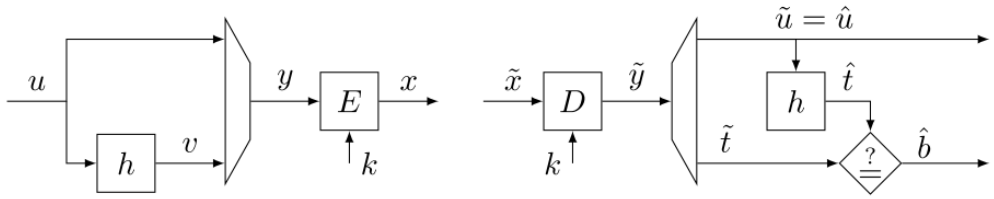
Another possible solution is to first encrypt the information message, then sign the encrypted message



- the two mechanisms (E, D) and (S, V) can be separately designed, each with its target security requirements
- needs two distinct key pairs
- if a message is not accepted ($\hat{b} = 1$), do not decrypt it: save workload and avoid padding oracle attacks
- can use symmetric or asymmetric mechanisms both for signature and encryption
- used in the Internet Protocol Security (IPSec) Encapsulating Security Payload (ESP) protocol

Hash-then-encrypt

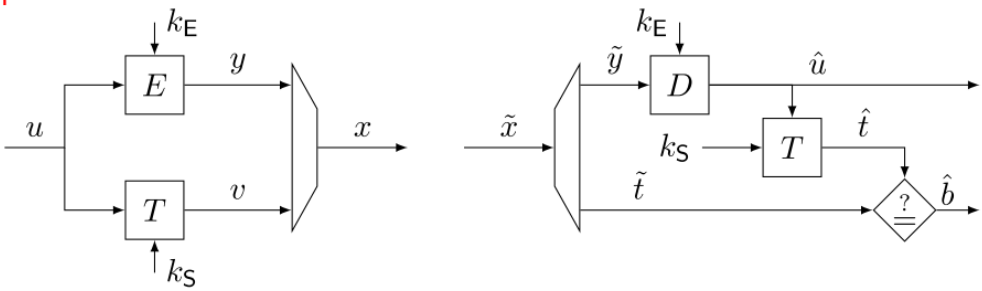
Another possible solution is to first hash the information message, then encrypt the concatenation of message and its hash



- encryption acts also as signature
- needs only one key pair
- cannot use asymmetric mechanisms (public encryption $\Rightarrow$ public signing, public verification $\Rightarrow$ public decryption)
- used in the ill-famed IEEE 802.11 Wired Equivalent Privacy (WEP) protocol (epic fail: the hash was a linear CRC code and the encryption was the RC4 additive stream cipher)

Sign-and-encrypt

Another possible solution is to compute the authentication tag on the plaintext, then append it to the ciphertext

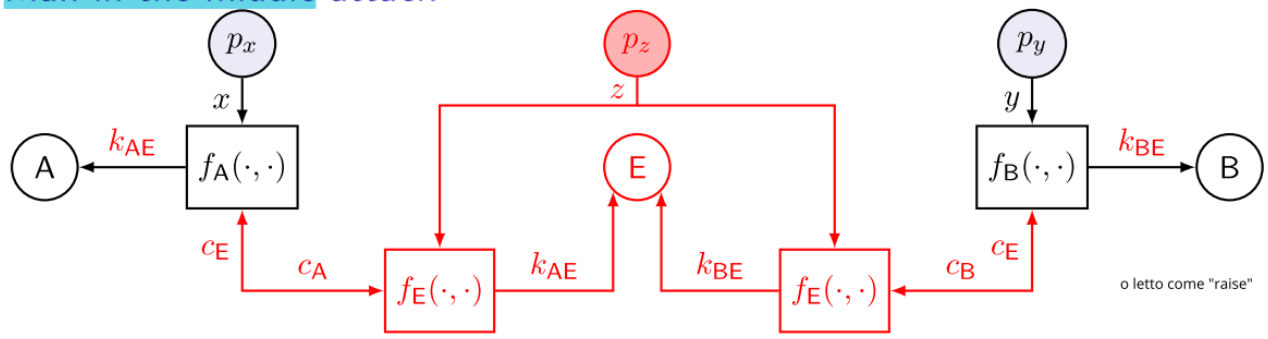


- the two mechanisms (E, D) and T can be separately designed, each with its target security requirements
- needs two distinct key pairs
- can use symmetric or asymmetric mechanisms
- in asymmetric signature, cryptographic hashing is needed to avoid revealing $u = T'(k', t)$ (beside preventing existential forgery)

Man in the middle attack

Man-in-the-middle attack

this attack only work online

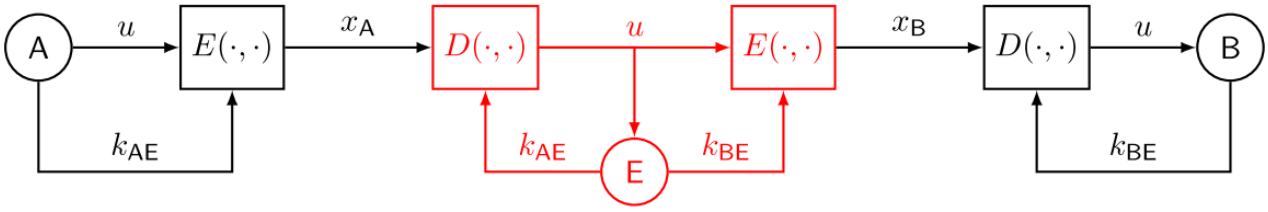


g is public

- E generates $z \in \mathbb{Z}_n \setminus \{0\}$ and computes $c_E = g \overset{z}{\circ}$
- E intercepts c_A, c_B and replaces each with c_E Ca,b,e sono le comunicazioni
- A receives c_E, computes $k_{AE} = c_E \overset{x}{\circ} = g \overset{xz \bmod n}{\circ}$
- B receives c_E, computes $k_{BE} = c_E \overset{y}{\circ} = g \overset{yz \bmod n}{\circ}$
- E can compute $k_{AE} = c_A \overset{z}{\circ} = g \overset{xz \bmod n}{\circ}$ as well as $k_{BE} = c_B \overset{z}{\circ} = g \overset{yz \bmod n}{\circ}$

Man-in-the-middle attack

If the attack succeeds, E can violate the symmetric protocol between A and B without them knowing



In order to avoid the man-in-the-middle attack, messages c_A, c_B must be authenticated and integrity protected

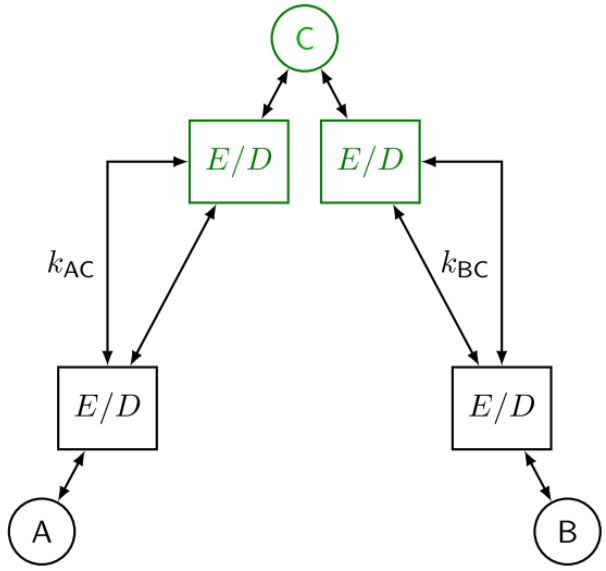
Is this a "chicken and egg" problem? No, can use digital signature

Forward secrecy

Even if E later learns the authentication provate keys, he will no longer be able to retrieve k_A

Needham-Schroeder symmetric protocol

Needham-Schroeder symmetric protocol



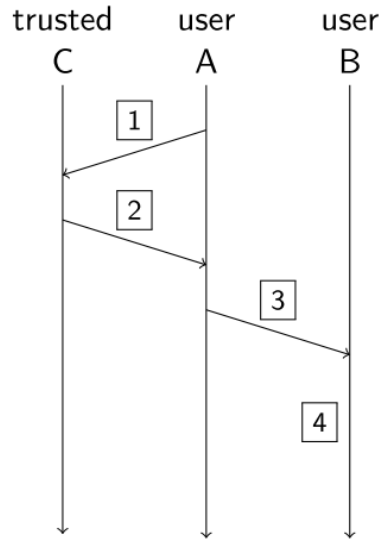
entities two parties A and B, a trusted third party C

tools a symmetric encryption mechanism $E(\cdot,\cdot)$ with keys shared between A and C, and between B and C; random generators at all entities

aim to securely distribute a key k_{AB} between A and B for a symmetric mechanism

Needham-Schroeder symmetric protocol (cont.)

phase I: key distribution



nonce= used only once

1 A : generates nonce n_A
A → C : $u_1 = (\mathrm{id}_A, \mathrm{id}_B, n_A)$

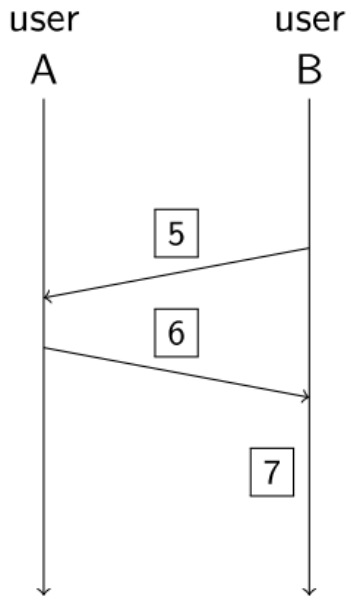
2 C : generates $k_{AB} \sim \mathcal{U}(\mathcal{K})$
encrypts $x_2 = E_{k_{BC}}([\mathrm{id}_A, k_{AB}])$
and $x_3 = E_{k_{AC}}([n_A, \mathrm{id}_B, k_{AB}, x_2])$
C → A : x_3

3 A : decrypts $[n_A, \mathrm{id}_B, k_{AB}, x_2] = D_{k_{AC}}(x_3)$
checks n_A and id_B
A → B : x_2

4 B : $[\mathrm{id}_A, k_{AB}] = D_{k_{BC}}(x_2)$

Needham-Schroeder symmetric protocol (cont.)

phase II: key confirmation



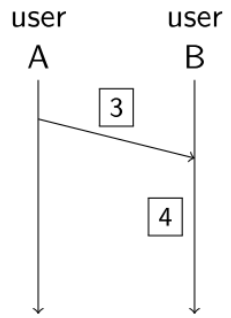
5 B : generates random $r_B \sim \mathcal{U}(\mathcal{R})$
encrypts $x_4 = E_{k_{AB}}(r_B)$
$B \to A$: x_4

6 A : decrypts $r_B = D_{k_{AB}}(x_4)$
computes $r_A = r_B - 1$
encrypts $x_5 = E_{k_{AB}}(r_A)$
$A \to B$: x_5

7 B : decrypts $r_A = D_{k_{AB}}(x_5)$
checks if $r_A = r_B - 1$

The Dennning-Sacco attack

Assume that E has learnt an old k_{AB}^{old} somehow and that he had recorded the corrsponding session between A and B
Then, E can replay message 3 to B



3 $E \to B$: x_2^{old}

4 B : $[id_A, k_{AB}^{old}] = D_{k_{BC}}(x_2^{old})$

B will use k_{AB}^{old} as if it were a good new key shared with A
solution needs a nonce known to B, too