Corso di Modelli e Analisi delle Prestazioni nelle Reti – AA 2005/2006
prova scritta – 12 dicembre 2006– parte A (90 minuti)

E1 Consider a Markov chain with the following transition matrix (states are numbered from 0 to 5):

$$P = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0.1 & 0 & 0.7 & 0 & 0.2 \\ 0 & 0.3 & 0.5 & 0 & 0.2 & 0 \\ 0 & 0.7 & 0 & 0.2 & 0 & 0.1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0.1 & 0 & 0.7 \end{pmatrix}$$

(a) Draw the transition diagram, classify the states, and identify the classes.

(b) Compute $\lim_{n\to\infty} P^n$.

(c) Compute $\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} P^k$.

(d) Compute $P[X_4 = 5, X_2 = 3 | X_3 = 1, X_1 = 3]$.

E2 Consider a network node that under normal conditions can handle a traffic equal to 1 Gbps. This node works normally for an exponential time with mean $99T$, and then enters an alarm state, during which its capacity is reduced to 250 Mbps. After being for a time T in the alarm state, the node is instantaneously repaired, and starts again to work normally.

(a) Compute the fraction of time the node spends in the alarm state, and the average traffic handled (suppose that the queues are always full, i.e., there are always packets to transmit).

(b) Suppose now that once entering the alarm state the node completely stops working after an exponential time of mean $2T$, unless it is repaired earlier (as before, the repair requires exactly a time T from when the node enters the alarm state). If the node stops working, it needs to be replaced, and this takes a time $20T$, during which the node cannot handle any traffic (note that this replacement is different from the simple repair considered in the previous case). Compute: (i) the average time between two subsequent substitutions, (ii) the fraction of time in which the node is not working, and (iii) the average system throughput.

E3 Consider two independent Poisson processes $X_1(t)$ and $X_2(t)$, where $X_i(t)$ is the number of arrivals for process i during $[0, t]$. The average number of arrivals per unit time of the two processes is $\lambda_1 = 0.5$ and $\lambda_2 = 1$, respectively.

(a) Compute $P[X_1(3) = 1 | X_1(3) + X_2(3) = 3]$ and $P[X_1(3) + X_2(3) = 3 | X_1(3) = 1]$.

(b) Compute $P[X_1(2) = 1 | X_1(3) = 3]$ e $P[X_1(3) = 3 | X_1(2) = 1]$.

E4 Consider a two-state Markov channel with transition probabilities 0.98 (from the good state to itself) and 0.1 (from the bad state to the good state). The packet error probability is 1 for a bad slot and 0 for a good slot, respectively.

(a) Compute the throughput (average number of successes per slot) of a protocol that transmits packets directly on the channel, with no retransmissions.

(b) Compute the throughput of a Go-Back-N protocol if the round-trip time is $m = 2$ slots (i.e., a packet that is erroneous in slot t is retransmitted in slot $t+2$), in the presence of an error-free feedback channel.

(c) Same as in the previous point, with the difference that now the feedback channel is subject to iid errors with probability 0.1.

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T1 State and prove the elementary renewal theorem.

T2 Prove that if states i and j of a Markov chain communicate and i is recurrent, then j is also recurrent.

T3 Prove that a Markov chain with a finite number of states cannot have any null recurrent state.