Stochastic Processes – AY 2020/2021
written test – August 27, 2021 – part A (90 minutes)

E1 Consider a node with two identical processors. Each processor alternates between working and failure states for exponential durations with average $1/\alpha = 19000$ seconds (working) and $1/\beta = 1/(19\alpha)$ (failure). The node is able to process data at 1 Gbps when both processors are working, at 250 Mbps when only one is working, and can process no data when both processors are not working.

(a) Compute the fraction of the time during which the node does not handle any traffic (i.e., both processors are not working), and the average duration of a period of time during which no traffic is handled

(b) by considering an appropriate renewal cycle, compute the average duration of the time interval during which the node is able to process data without interruptions (i.e., there is always at least one processor working)

(c) compute the average data processing speed of the node in Gbps

E2 Consider a two-state Markov channel, where the steady-state probability that the channel is in the bad state is 0.1 and the average number of consecutive good slots is 50. The packet error probability is 1 for a bad slot and 0 for a good slot, respectively. The round-trip time is $m = 2$ slots, i.e., a packet that is erroneous in slot t is retransmitted in slot $t + 2$ (if a retransmission protocol is used).

(a) Compute the throughput that could be obtained if packets were directly transmitted over the channel without using any protocol

(b) compute the throughput of a Go-Back-N protocol that transmits packets over the Markov channel described above, in the presence of an error-free feedback channel

(c) compute the throughput of a Go-Back-N protocol that transmits packets over the Markov channel described above, with a feedback channel subject to iid errors with probability $\delta = 0.1$.

E3 Consider a Markov chain with the following transition matrix (states are numbered from 0 to 4):

$$P = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0.4 & 0 & 0 & 0.6 \\ 0 & 0 & 0.5 & 0.3 & 0.2 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 & 0.8 \end{pmatrix}$$

(a) Draw the transition diagram, identify the classes, classify the states, and compute the probabilities of absorption in all recurrent classes starting from each transient state

(b) compute $\lim_{n\to\infty} P^n$ and $\lim_{n\to\infty} \frac{1}{n}\sum_{k=0}^{n-1} P^k$

(c) compute the average recurrence time for all states, and the average first passage time from any state to state 4.

E4 Consider two independent Poisson processes $X_1(t)$ e $X_2(t)$, where $X_i(t)$ is the number of arrivals for process i during $[0, t]$. The average number of arrivals per unit time of the two processes is $\lambda_1 = 1$ and $\lambda_2 = 1$, respectively.

(a) Compute $P[X_1(1) = 1|X_1(2) + X_2(2) = 3]$ and $P[X_1(2) + X_2(2) = 3|X_1(1) = 1]$

(b) Compute $P[X_1(1) + X_2(1) = 1|X_1(2) = 1]$ and $P[X_1(2) = 1|X_1(1) + X_2(1) = 1]$