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$\boxed{\text{ES 1}}$

| Working | Not W | WORK | Not W |

$1/\alpha$ $\quad$ $1/\beta$

1)
$$P(\text{not-working}) = \frac{1/\beta}{1/\alpha + 1/\beta} = \frac{1/19\alpha}{1/19\alpha + \frac{1}{\alpha}} = \frac{\frac{1}{19\alpha}}{\frac{1+19}{19\alpha}} = \frac{1}{19\alpha} \cdot \frac{19\alpha}{20} = \frac{1}{20}$$

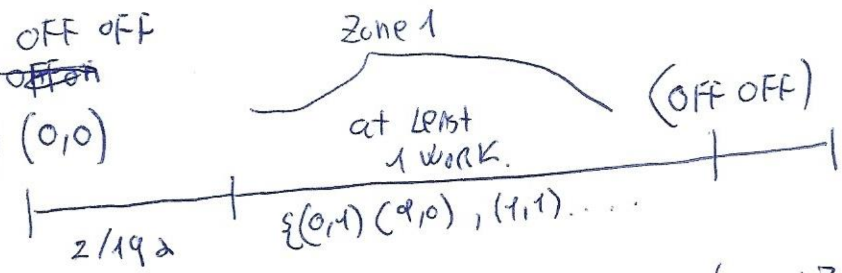
$$P(\text{Both Not Working}) = \left(\frac{1}{20}\right)^2$$

TIME of Processor 1 $\quad$ time of Processor 2

$\exp(1/19\alpha)$ $\quad$ $\exp(1/19\alpha)$

mean

$$\text{EXP}[\text{System down}] = \frac{1}{19\alpha}/2 = \frac{2}{19\alpha}$$ =?

2)


$$E[\text{cycle}] = \frac{\cancel{1/\alpha}\ 2/19\alpha}{P[\text{System down}]} = \frac{2}{19\alpha} \Big/ \left(\frac{1}{20}\right)^2$$

so, in the zone 1, (at Least 1 Work) $= \frac{2}{19\alpha} \Big/ \left(\frac{1}{20}\right)^2 - \frac{2}{19\alpha}$ =?

3) $P(\text{Working}) = \frac{19}{20}$

1 Processor : $250\text{Mbps} \cdot \frac{19}{20} = 237{,}5$ Mbps

2 Processor : $2 \cdot 237{,}5 = 475$ Mbps

NO

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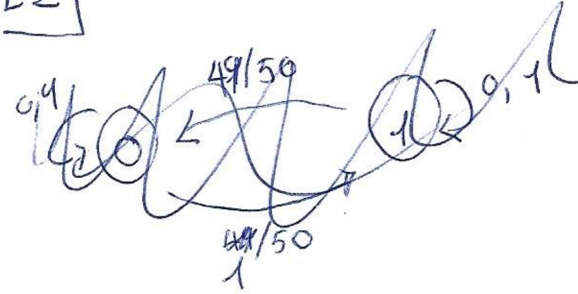
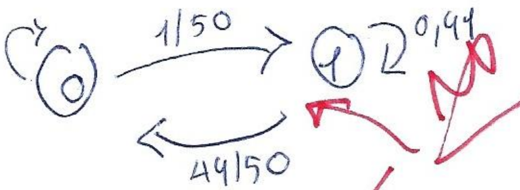
0 = good

1 = Bad

$\boxed{E2}$



$$P = \begin{array}{c|cc} & 0 & 1 \\ 0 & 0{,}9 & \frac{49}{50} \\ 1 & 1/50 & 0{,}1 \end{array}$$



NO

$$\frac{P_{01}}{P_{01} + P_{10}} = 0{,}9$$ YES

$$P = \begin{array}{c|cc} & 0 & 1 \\ 0 & 49/50 & 1/50 \\ 1 & 0{,}09 & 0{,}91 \end{array}$$

0,98   0,02

$$P^2 = \begin{array}{c|cc} & 0 & 1 \\ 0 & 0{,}96 & 0{,}037 \\ 1 & 0{,}9\cancel{1} & 0{,}82 \end{array}$$

$$\begin{bmatrix} \frac{49}{50} & \frac{1}{50} \end{bmatrix} \begin{bmatrix} \frac{49}{50} \\ 0{,}09 \end{bmatrix} = 0{,}967$$

$$\begin{bmatrix} \frac{49}{50} & \frac{1}{50} \end{bmatrix} \begin{bmatrix} 1/50 \\ 0{,}91 \end{bmatrix}$$

$$\begin{bmatrix} 0{,}09 & 0{,}91 \end{bmatrix} \begin{bmatrix} \overset{0{,}98}{\cancel{49/50}} \\ 0{,}91 \end{bmatrix}$$

$$\begin{bmatrix} 0{,}09 & 0{,}91 \end{bmatrix} \begin{bmatrix} 0{,}02 \\ 0{,}91 \end{bmatrix}$$

2) $\cancel{\dfrac{P_{10}(m)}{P_{10}(m) + m \cdot P_{01}} = \dfrac{0{,}037}{0{,}037 \cdot 2 \cdot 0{,}98}}$ NO

1) without Any protocol the PROB = $\pi_0 = 0{,}9$ OK

3) $$\frac{(1-\delta)\, P_{10}(m)}{(1-\delta)\, P_{10}(m) + 2\left[(1-\delta) \cdot P_{01} + \delta\, P_{01}(m) + \delta \cdot P_{10}(m)\right]}$$

$$= \frac{0{,}9 \cdot 0{,}91}{0{,}9 \cdot 0{,}91 + 2\left(0{,}9 \cdot \frac{1}{50} + 0{,}1 \cdot 0{,}037 + 0{,}1 \cdot 0{,}9\right)} =$$

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$\lambda_1 = 1$
$\lambda_2 = 1$

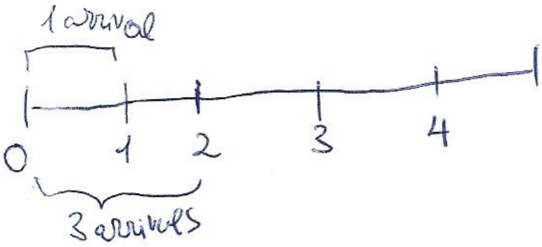
è possibile li parametro 4!

Es 4

- $P[X_1(1)=1 \mid \underbrace{X_1(2)+X_2(2)}_{X(2) \text{ with } \lambda=\lambda_1+\lambda_2} = 3]$

$P(X_1(1)=1 \mid X(2)=3) = P[P_0(\lambda_1) \overset{k}{=} 1 \mid P_0(\underset{2}{\overset{4}{2(\lambda_1+\lambda_2)}}) \overset{n}{=} 3) = \binom{3}{1}\left(\frac{1}{2}\right)^1\left(1-\frac{1}{2}\right)^{3-1=2}$ NO

- $P(X_1(2)+X_2(2)=3 \mid X_1(1)=1) =$

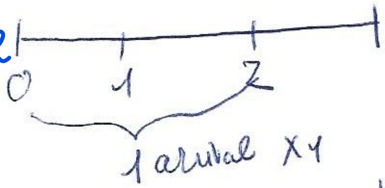


$P(\underbrace{X_1(2)+X_2(2)}_{X(2) \text{ with } \lambda=2[\lambda_1+\lambda_2]} = 2)$ NO $\longrightarrow P(X_1(2)+X_2(2)=2)$

$P(P_0(\underbrace{2(\lambda_1+\lambda_2)}_{\substack{\lambda \\ \| \\ 2(1+1)=4}})=2) = \frac{\lambda^2 e^{-\lambda}}{2!} = \frac{(2(\lambda_1+\lambda_2))^2 \cdot e^{-2(\lambda_1+\lambda_2)}}{2!}$ NO

$P[\underbrace{X(1)}_{\text{with } \lambda=2} = 1 / \underset{\lambda=2}{X_1(2)} = 1) \cdot \binom{1}{1}\left(\frac{1\cdot 2}{2\cdot 2}\right)^1\left(1-\frac{1}{2}\right)^0$

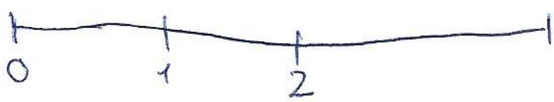
- $P[X_1(1)+X_2(1)=1 \mid X_1(2)=1]$



$\|$

$P(X_2(1)=1) = P_0(P_0(\lambda_1)=1) = \frac{\lambda_1^{1} \cdot e^{-\lambda_1}}{1!} = \lambda_1 \cdot e^{-\lambda_1} = 1 \cdot e^{-1}$ NO

- $P(X_1(2)=1 \mid \underbrace{X_1(1)+X_2(1)}_{X(1) \text{ con } \lambda=\lambda_1+\lambda_2}=1) = P(X_1(2)=1 \mid X(1)=1)$



ES 3 (hc)   (ES 4)

Average first passage.

$P(X_1(2)=1 \mid X(1)=1)$

$P(A|B) = \dfrac{P(A\cap B)}{P(B)}$

$\Rightarrow \dfrac{P(B\cap A)}{P(B)}$

$\dfrac{P(B|A)\cdot P(A)}{P(B)}$

$$= \frac{P(X(1)=1 \mid X_1(2)=1)\; P(X_1(2)=1)}{\cancel{P(B)}\; P(X(1)=1)} \Leftarrow \text{BAYES}$$

$P(X(1)=1 \mid X_1(2)=1) = e^{-1}$

$P(X_1(2)=1) = P(P_0(2\lambda_1)=1) = \dfrac{\lambda_1 \cdot e^{-\lambda_1}}{1!}$ (with $\lambda_1 = 1$)   NO

$P(X(1)=1)$

$\Downarrow$

$P(X_1(1)+X_2(1)=1) = P(P_0(2(\lambda_1+\lambda_2))=1) = \dfrac{(2\lambda_1+\lambda_2)^1 \cdot e^{-2(\lambda_1+\lambda_2)}}{1}$

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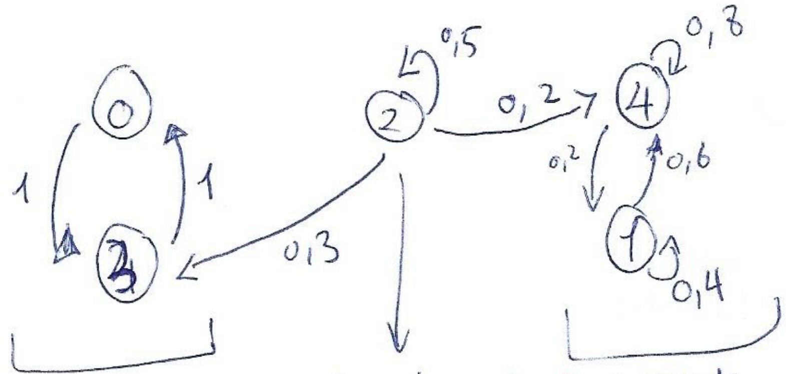
$\boxed{ES\ 3}$



Positive recurrent ~~state~~ class

PERIOD = 2

trasient.

POI · Recurrent Aperiodic

4, 1 0, 3 → Recurrent

2 → trasient

$$\lim_{n\to+\infty} P^n = \begin{array}{c|c} & \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 0\\1\\2\\3\\4 \end{array} & \begin{bmatrix} x & 0 & 0 & x & 0 \\ 0 & 3/4 & 0 & 0 & 1/4 \\ x & 3/10 & 0 & x & 1/10 \\ x & 0 & 0 & x & 0 \\ 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} \end{bmatrix} \end{array}$$

π of 4 1 :

$$P = \begin{array}{c|c} & \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1\\4 \end{array} & \begin{bmatrix} 0 & 0{,}4 & 0 & 0 & 0{,}6 \\ 0 & 0{,}2 & 0 & 0 & 0{,}8 \end{bmatrix} \end{array}$$

$$\begin{cases} 0{,}4\pi_1 + 0{,}2\pi_4 = \pi_1 \\ 0{,}6\pi_1 + 0{,}8\pi_4 = \pi_4 \\ \pi_1 + \pi_4 = 1 \end{cases}$$

$$\begin{cases} \frac{4}{10}\pi_1 + \frac{2}{10}\pi_4 = \pi_1 & \rightarrow 4\pi_1 + 2\pi_4 = 10\pi_1 \rightarrow \frac{2\pi_4}{2} = \frac{6\pi_1}{2} \rightarrow \pi_4 = 3\pi_1 \\ \frac{6}{10}\pi_1 + \frac{8}{10}\pi_4 = \pi_4 & \\ \pi_1 + \pi_4 = 1 & \rightarrow \pi_1 + 3\pi_4 = 1 \rightarrow \frac{4\pi_4}{4} = \frac{1}{4} \qquad \pi_1 = \frac{3}{4} \end{cases}$$

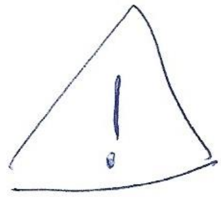
$P(2$ ASORBED BY $4)$?

$P(\text{"2" state is ASORBED BY the class } \overset{4-1}{(4,1)}) \cdot (P \text{ Limiting } \overset{4-1}{(4,1)}) =$

$$\frac{0{,}2}{0{,}5} = \frac{2}{5}$$

$$P(2 \text{ ASORBED BY } 4) = \frac{2}{5} \cdot \pi_4 = \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{10}$$

$$P(2 \quad '' \quad '' \quad 1) = \frac{2}{5} \cdot \pi_1 = \frac{2}{5} \cdot \frac{3}{4} = \frac{3}{10}$$



BEHIND →

(SORRY FOR the MISTAKE)

$$\lim_{n\to+\infty} \frac{1}{n}\sum_{k=0} P^k = \begin{bmatrix} x & 0 & 0 & x & 0 \\ 0 & 3/4 & 0 & 0 & 1/4 \\ x_1 & 3/10 & 0 & x_2 & 1/10 \\ x & 0 & 0 & x & 0 \\ 0 & 3/4 & 0 & 0 & 1/4 \end{bmatrix}$$

is $= \frac{1}{2}$ For $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is Double Stochastic.

$= P(\text{ASORBING})$

$$x_2 = x_1 = \frac{1}{2}\cdot\frac{0,3}{0,5} = \frac{1}{2}\cdot\frac{3}{5} = \frac{3}{10}$$

c) Average Recc time:

$m_0 = \cancel{1/\pi_1}$ $\quad 1/\pi_0$

$m_1 = \cancel{1/\pi_2}\ \cancel{+\infty}$ $\quad 1/\pi_1$

$m_2 = \cancel{1/\pi_2}$ $\quad +\infty$

$m_3 = \cancel{1/\pi_4}$ $\quad 1/\pi_3$

$m_4 = \cancel{1/}$ $\quad 1/\pi_4$

T1

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1

$$\lim_{t\to+\infty} \frac{N(t)}{t}$$

$N(t) \to$ NUMBER of Renewal time in the $[0,t]$

Consider:

$S_{N(t)}$ — t — $S_{N(t)+1}$

$$S_{N(t)} = \sum_{i=0}^{N(t)} x_i$$

$$S_{N(t)+1} = S_{N(t)} + X_{i+1}$$

We have : $S_{N(t)} \leq t < S_{n(t)+1}$

↓ Divide $N(t)$

$$\frac{S_{N(t)}}{N(t)} < \frac{t}{N(t)} < \frac{S_{N(t)+1}}{N(t)},$$

let's consider the follow Limit

$$(*) = \lim_{N\to+\infty} \frac{S(N)}{N} = \lim_{N\to+\infty} \frac{X_1 + X_2 + X_3 + \dots X_N}{N}$$

SUM of N° Random VARIABLE. / N° for $n \to +\infty$

Give As Result the Expection of the Random VARIABLE. For the LARGE number LAW.

$$= E[X] = \mu.$$

$\lim_{t\to\infty} N(t) = \infty$ w.p.1

~~For this reason~~

$\lim_{t\to+\infty} \frac{S_{n(t)}}{N(t)}$ has the SAME Behavior of $(*)$ if ~~N~~ $t \to +\infty \to$ limit $= \mu$

$\lim_{t\to+\infty} \frac{S_{n(t)+1}}{N(t)} = \mu$

For the th POLICE ~~LAW~~ of LIMIT : $\boxed{\lim_{t\to+\infty} \frac{t}{N(t)} = \mu.}$

↓

~~the ORIGINAL~~ the start Limit =

$$\boxed{\lim_{t\to+\infty} \frac{N(t)}{t} = \frac{1}{\mu}}$$ w.p.1

T2

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2

if i is recurrent $\rightarrow$ j is recurrent $\Leftarrow$ class propety
and $i \leftrightarrow j$

(*)
$$\boxed{\sum_{n=0}^{+\infty} P_{ii}^{n} = +\infty \leftrightarrow i \text{ is recurrent}}$$
we need this propety for proof.

we know that i and j are communicated beetween them:

$\exists\, m, n > 0$ such that : $P_{ji}^{n} > 0$; $P_{ij}^{m} > 0$

why?,

Let $u > 0$:

$$\sum_{u=0}^{+\infty} P_{jj}^{n+m+u} \geq \sum_{u} P_{ji}^{m} \cdot P_{ij}^{n} \cdot P_{ii}^{u}$$
(is always true)

$$P_{ji}^{m} \cdot P_{ij}^{n} \underbrace{\sum_{u=0}^{+\infty} P_{ii}^{u}}_{+\infty \text{ for the propety } (*)}$$
$\leftarrow$ P_{ji}^{m} and P_{ij}^{n} don't depend from u

$\cap$
$\dot{}$
$> +\infty$

$$\boxed{\sum_{u=0}^{+\infty} P_{jj}^{n+m+u} \geq +\infty} \longrightarrow j \text{ is recurrent.}$$

13

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S

$P[X(s)=k \mid X(t)=n) =$

1) $\boxed{0<s<t\ ;\ 0<k\leq n}$

$P[X(s)=k \mid X(t)=n] = \dfrac{P(X(s)=k \cap X(t)=n)}{P(X(t)=n)}$

$P(A|B)=\dfrac{P(A \cap B)}{P(B)}$.

$= \dfrac{P(X(s)=k\ ,\ [X(t)-X(s)]=n-k)}{P(X(t)=n)}$

indep!,

Remider
$P(P_o(\lambda)=k) = \dfrac{\lambda^k e^{-\lambda}}{k!}$

$$= \frac{\dfrac{(\lambda s)^k \cdot e^{-\lambda s}}{k!} \cdot \dfrac{[\lambda(t-s)]^{n-k} \cdot e^{-\lambda(t-s)}}{(n-k)!}}{\dfrac{(\lambda n)^n \cdot e^{-\lambda n}}{n!}} \rightarrow \binom{n}{k}$$

order
them

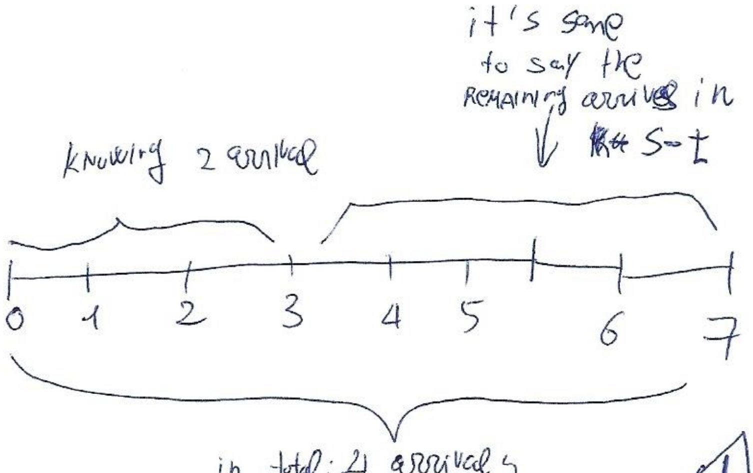
$$\binom{n}{k}\left(\frac{s}{t}\right)^k\left(1-\frac{s}{t}\right)^{n-k}.$$

2] $\boxed{s>t>0\ ;\ k>n>0}$

ex g: $P(X(\overset{s}{7})=4 \mid X(\overset{t}{3})=2)$

$\parallel$

$P(X(7-3)=4-2)$



⚠ Behind →

more General : $P[X(s)=k \mid X(t)=n) \overset{s\geq t,\ k>n}{=} P(X(s-t)=k-n)$

$$P(X(s-t) = k-n) = \cancel{\frac{[\lambda(s-t)]^{k-n} e^{-\lambda(s-t)}}{(k-n)!}}$$

$$= \frac{[\lambda(s-t)]^{k-n} e^{-\lambda(s-t)}}{(k-n)!}$$