Corso di Modelli e Analisi delle Prestazioni nelle Reti – AA 2005/2006
prova scritta – 14 luglio 2006– parte A (90 minuti)

E1 Consider a Markov chain X_n with the following transition matrix (states are numbered from 0 to 2):

$$P = \begin{pmatrix} 0.2 & 0.4 & 0.4 \\ 0.5 & 0.5 & 0 \\ 0.4 & 0.4 & 0.2 \end{pmatrix}$$

(a) Draw the transition diagram, and find the probability distribution of X_1, X_2 and X_{500}, given $X_0 = 0$

(b) Compute the average first passage times from state 0 to states 0, 1 and 2.

(c) Let $W_{ij}^{(n)} = E\left[\sum_{k=0}^{n-1} I\{X_k = j\} \middle| X_0 = i\right]$ be the average number of visits to state j during the first n time slots, given that the chain starts in state i. Compute $W_{0j}^{(3)}$ and $W_{0j}^{(5000)}$ for $j = 0, 1, 2$.

E2 Consider a link with capacity 1 Mbps, shared among many users who collectively produce packets according to a Poisson process with rate $\lambda = 500$ packets per second. All packets are of the same length, equal to 1000 bits. The access protocol is an ideal CSMA, where a packet generated when the channel is idle gets immediate access, whereas a packet that finds the channel busy is rescheduled after an exponential time of average $100/\lambda$. (If this new access attempt again finds a busy channel the protocol keeps trying after random times until success.) Assume that the total traffic (new packets plus all retransmissions) can be approximated as Poisson with rate λ.

(a) Compute the throughput (average traffic handled) on the link.

(b) Compute the average access delay, from when a packet is generated to when it finally gets access to the channel.

(c) If a transmission corresponds to a gain of 1 unit and each failed access attempt (i.e., a packet finding the channel busy) corresponds to a cost of 0.2 units, compute the total gain of the system in units per second.

E3 Consider an exhibition where visitors arrive according to a Poisson process with rate $\lambda = 10$ customers per hour. Each visitor spends a time uniformly distributed between 20 and 30 minutes, and then leaves. The room in which the exhibition is shown is large enough to ensure there is never a need to block customers at the entrance due to too many people inside. The exhibition is open from 8 AM to 6 PM.

(a) Compute the probability that fewer than three visitors arrive during the first half hour.

(b) Compute the probability that at 8:15 AM there is only one visitor in the room.

(c) Compute the probability that at closing time (6 PM) the room is empty.

E4 Consider a two-state Markov channel with transition probabilities 0.98 (from the good state to itself) and 0.1 (from the bad state to the good state). The packet error probability is 1 for a bad slot and 0 for a good slot, respectively.

(a) Compute the throughput of a Go-Back-N protocol if the round-trip time is $m = 2$ slots (i.e., a packet that is erroneous in slot t is retransmitted in slot $t + 2$), in the presence of an error-free feedback channel

(b) Consider now a system where a channel behavior as in point (a) (Markov model for the forward channel and error-free feedback channel) alternates with a channel behavior in which the forward channel is subject to iid errors with probability $\varepsilon = 0.01$. In particular, the channel follows the Markov model for a geometric number of slots with mean 1000000 slots, then follows the iid model for a geometric number of slots with mean 2000000 slots, then again the Markov model and so on. Compute the overall average throughput of the Go-Back-N protocol in this case.

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T1 For a Poisson process of rate λ, prove that the interarrival times are iid exponential with mean $1/\lambda$.

T2 Prove that in a Markov chain the period is a class property.

T3 Prove that for a renewal process $E[S_{N(t)+1}] = E[X](M(t) + 1)$.