Corso di Modelli e Analisi delle Prestazioni nelle Reti – AA 2004/2005
prova scritta – 22 settembre 2005– parte A

E1 Consider a Markov chain X_n with states 1, 2 and 3, $X(0) = 3$, and transition matrix

$$P = \begin{pmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.2 & 0.6 \\ 1 & 0 & 0 \end{pmatrix}$$

(a) Compute the steady-state probabilities and the average recurrence times of all states.

(b) Compute mean and variance of the first passage time from state 3 to state 1.

(c) Compute mean and variance of the first passage time from state 1 to state 3.

(d) Compute $P[X(1) = 1, X(3) = 1|X(2) = 2]$ and $P[X(2) = 2|X(1) = 1, X(3) = 1]$.

E2 Consider a queue where packets arrive according to a Poisson process with rate $\lambda = 1$ packet per second. All packets in the queue are transmitted when either of the following events occurs: (i) there are two packets in the queue, or (ii) there is one packet in the queue and its waiting time reaches two seconds. Transmission is instantaneous, i.e., the queue empties every time there is a packet arrival when one packet is already in the queue, or when the only packet in the queue has been there for enough time.

(a) Compute the fraction of time in which the queue is empty.

(b) Compute the average packet delay (i.e., the average time a packet spends in the queue).

E3 Consider a frequency division transmission system in which the number of channels is so large that the probability they are all occupied is negligible. Such system receives connection requests according to a Poisson process with rate $\lambda = 100$ calls per hour, and the duration of each call is exponential with mean 6 minutes. Let $X(t)$ be the number of occupied channels at time t.

(a) Compute the average of $X(t)$ at $t = 6, 10$ minutes and for $t = \infty$.

(b) Compute $P[X(t) = 10]$ for $t = 6$ and $t = \infty$

(c) Repeat the previous calculations assuming that the call duration is uniformly distributed in $[2, 10]$ (minutes)

E4 Consider a Go-Back-N protocol over a two-state Markov channel with transition probabilities 0.99 (from the good state to itself) and 0.1 (from the bad state to the good state). The packet error probability is 1 for a bad slot and 0 for a good slot, respectively. The round-trip time is $m = 2$ slots, i.e., a packet that is erroneous in slot t will be retransmitted in slot $t + 2$.

(a) Compute the throughput of the protocol for an error-free feedback channel

(b) Compute the throughput of the protocol for a feedback channel subject to iid errors with probability 0.1.

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T1 State and prove the elementary renewal theorem.

T2 Prove that in a Markov chain the period is a class property.

T3 Consider a random walk over the non-negative integers with the following transition probabilities: $P_{01} = 1$, $P_{i,i+1} = p, P_{i,i-1} = q, i > 0$, with $p + q = 1$. Study its behavior, and in particular characterize its recurrence or transiency and derive the steady-state distribution.