**Corso di Modelli e Analisi delle Prestazioni nelle Reti – AA 2006/2007**
**prova scritta – 05 settembre 2007 – parte A**

E1 Consider a web server which receives download requests according to a Poisson process with rate $\lambda = 20$ requests per second. Each requests, after a fixed processing time of 20 ms, triggers the transfer of a file with size uniformly distributed between 1 and 2 MBytes. A request is said to be "active" from when it arrives to when the corresponding file transfer is completed. Assume that the server capacity, in terms of how many simultaneous requests it can handle, is infinite, and that the transfer data rate for each file is 100 Mbit/s, regardless of the number of files that are being transferred at any given time.

(a) Assuming that the server is switched on at time $t = 0$, when does the statistics of the number of active requests in the system reach its steady-state condition? In such condition, express $P[k$ active requests$]$

(b) Given that in an interval of duration $T$ the system received $N$ requests, find the probability that at the end of such interval there are no active requests in the two cases (b1) $T = 0.1$ s, $N = 2$ and (b2) $T = 1$ s, $N = 20$.

E2 Consider a Markov chain with the following transition matrix (states are numbered from 0 to 4):

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0.6 \\ 0.5 & 0 & 0 & 0.2 & 0.3 \\ 0 & 0 & 0.6 & 0 & 0.4 \end{pmatrix}$$

(a) Classify the states and identify the classes

(b) Compute the probabilities of absorption in all recurrent classes starting from each transient state

(c) Compute $\lim_{n\to\infty} P^n$ and $\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} P^k$

(d) Compute the average recurrence times for all states

E3 A fair coin is flipped until two consecutive heads (HH) or two consecutive tails (TT) are observed.

(a) Compute the probability that the game ends with sequence TT

(b) Compute the probability that the game ends with sequence TT given that the first flip is H

(c) Respond to the two questions above if the coin is unfair, where the probability that a flip returns H is $p = 1/4$

E4 Consider a two-state Markov channel with transition probabilities 0.98 (from the good state to itself) and 0.1 (from the bad state to the good state). The packet error probability is 1 for a bad slot and 0 for a good slot, respectively.

(a) Compute the throughput of a Go-Back-N protocol if the round-trip time is $m = 2$ slots (i.e., a packet that is erroneous in slot $t$ is retransmitted in slot $t + 2$), in the presence of an error-free feedback channel

(b) Consider now a system where a channel behavior as in point (a) (Markov model for the forward channel and error-free feedback channel) alternates with a channel behavior in which the forward channel is subject to iid errors with probability $\varepsilon = 0.01$. In particular, the channel follows the Markov model for a geometric number of slots with mean 1000000 slots, then follows the iid model for a geometric number of slots with mean 2000000 slots, then again the Markov model and so on. Compute the overall average throughput of the Go-Back-N protocol in this case.

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T1 Prove that a Markov chain with a finite number of states cannot have any null recurrent state.

T2 Prove that for a renewal process $E[S_{N(t)+1}] = E[X](M(t) + 1)$.

T3 Prove that for a Markov chain the $n$-step transition probabilities, $P_{ij}^{(n)}$, satisfy the relationship

$$P_{ij}^{(n)} = \sum_m P_{im}^{(k)} P_{mj}^{(n-k)}, k = 0, 1, \ldots, n$$