<u>another example</u>: GBN with limited retransmissions.

* model : . m slots round trip

. if N erroneous transmissions, packet is discarded.

. we need to keep memory of the number of failed transmissions:

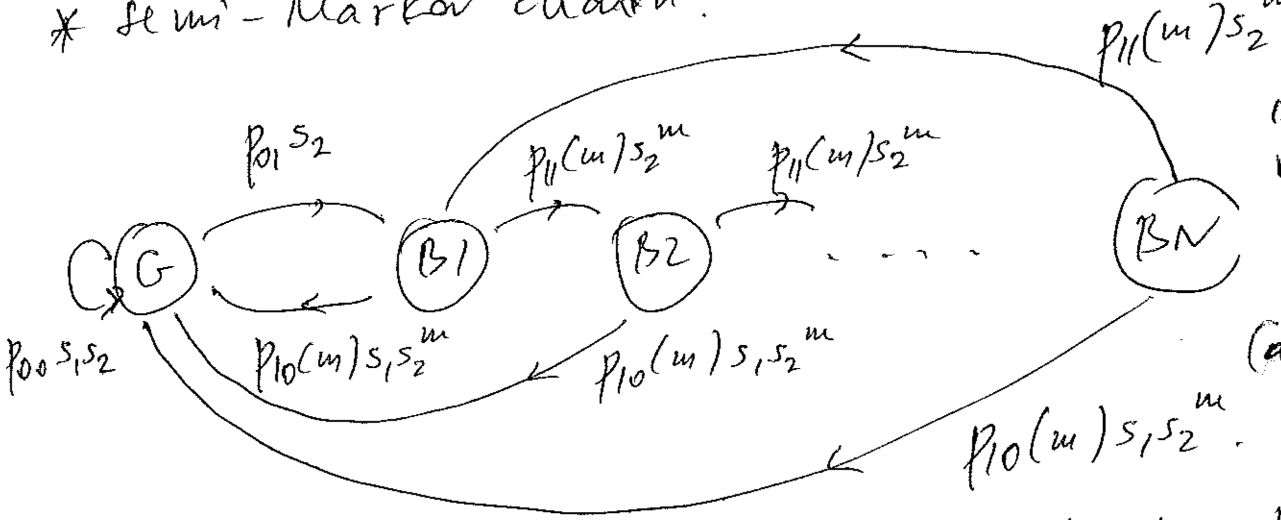
$X(t) = G$ if success att

$= B_j$ if j-th failure att.

state G: last tx was correct

B_i: i-th consecutive failure for a given packet.

* semi-Markov chain:



check whether S_2^m or S_2 (depends on protocol)

note: we can assign the success metric either to the transitions entering G or to those exiting G.

let $\beta_{10}(s_1, s_2) = p_{10}(m) s_1 s_2^m$; $\beta_{11}(s_1, s_2) = p_{11}(m) s_2^m$

$\beta_{00}(s_1, s_2) = p_{00} s_1 s_2$; $\beta_{01}(s_1, s_2) = p_{01} s_2$

We can use flow-graph reduction techniques since (i) metrics on different transitions are indep., and (ii) metrics are additive, and (iii) the transform of the distribution of the sum is the product of the transforms.

Therefore, we have that:

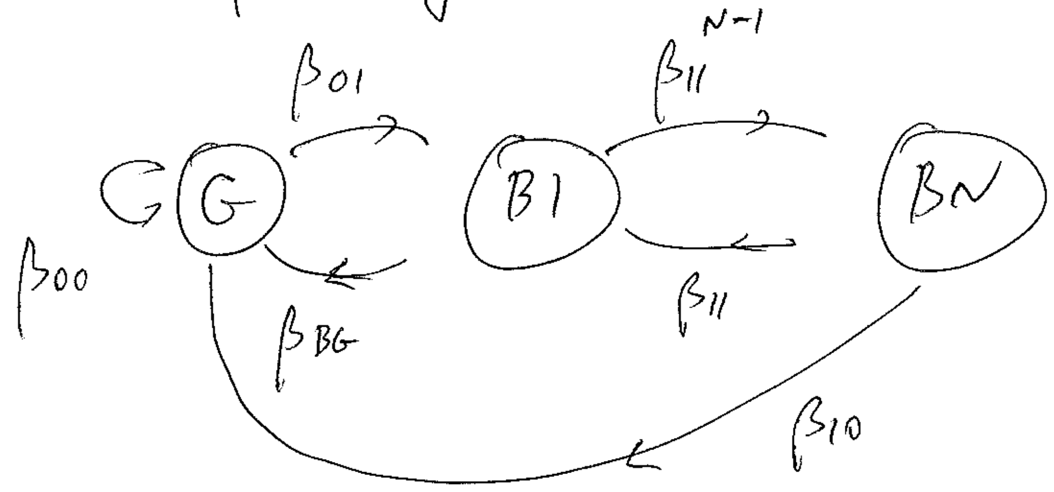
$$A \xrightarrow{\ \beta_1\ } B \xrightarrow{\ \beta_2\ } C \equiv A \xrightarrow{\ \beta_1\beta_2\ } C$$

and:

$$\begin{tikzcd} \arrow[r, "X"] & A \arrow[r, bend left, "\beta_1"] & B \arrow[l, bend left, "\beta_2"] \arrow[r, dash, "Y"] & {} \end{tikzcd} \equiv \begin{tikzcd} \arrow[r, "X"] & A \arrow[r, dash, "\frac{\beta_1}{1-\beta_2}"] & B \arrow[r, dash, "Y"] & {} \end{tikzcd}$$

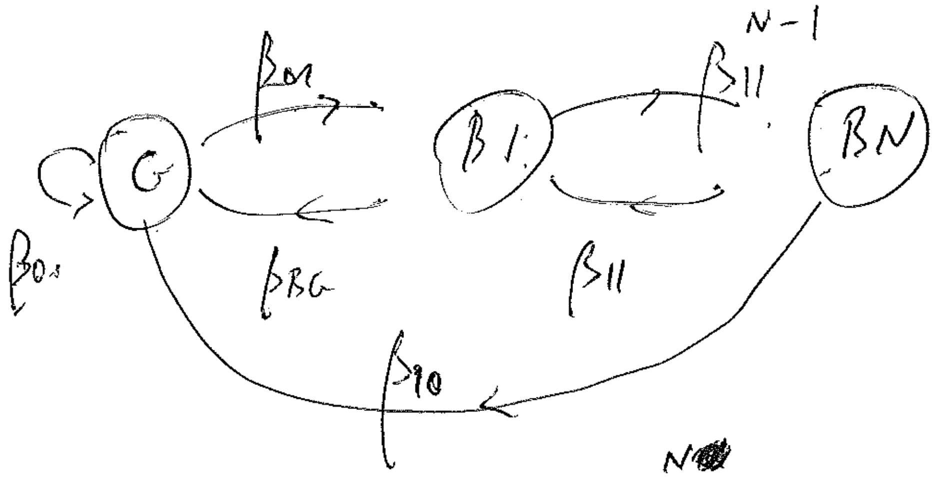
$$\left(Y = \beta_1 (X + \beta_2 Y) \Rightarrow Y = \frac{\beta_1 X}{1-\beta_2} \right)$$

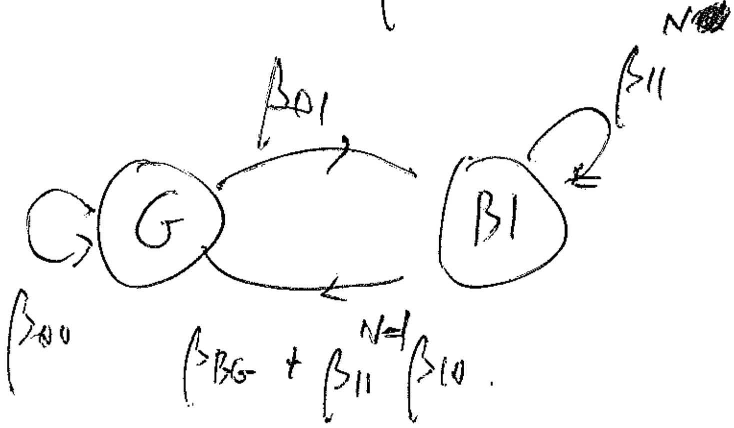
The following reductions can be made:

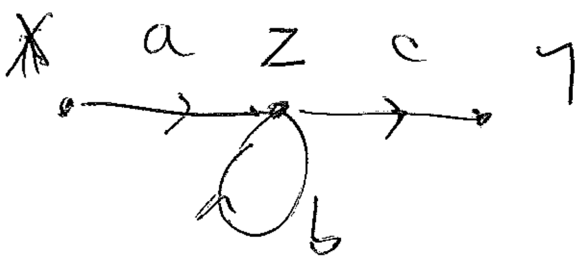


where $\beta_{BG} = \beta_{10} + \beta_{11}\beta_{10} + \ldots + \beta_{11}^{N-2}\beta_{10} =$

$$= \beta_{10} \cdot \sum_{k=0}^{N-2} \beta_{11}^{k} = \beta_{10} \cdot \frac{1-\beta_{11}^{N-1}}{1-\beta_{11}}$$







$$z = ax + bz$$

$$z = \frac{ax}{1-b}$$

$$\equiv \quad x \xrightarrow{\frac{ac}{1-b}} y$$

$$y = x \cdot \frac{ac}{1-b}$$

$$\frac{\beta_{01} \cdot \left[\beta_{BG} + \beta_{11}^{N-1}\beta_{10}\right]}{1-\beta_{11}^{N}}$$

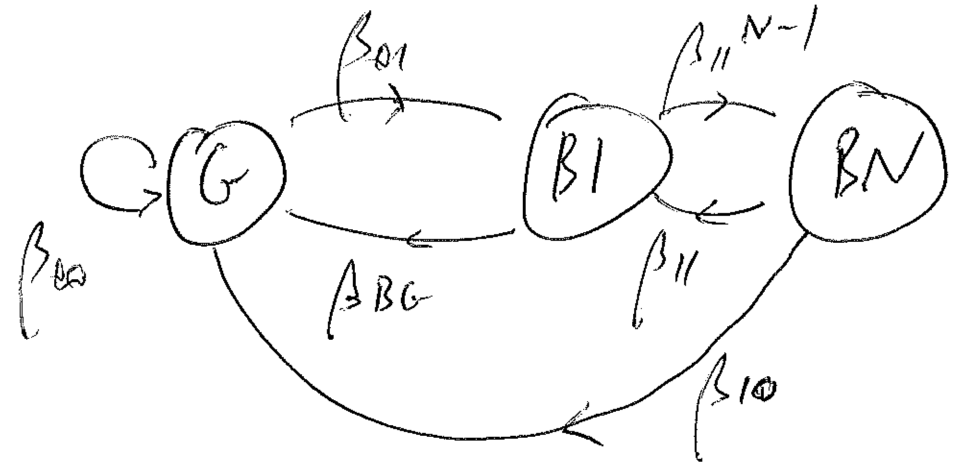
$$\beta_{00} + \frac{\beta_{01}\left[\beta_{BG} + \beta_{11}^{N-1}\beta_{10}\right]}{1-\beta_{11}^{N}} = B(s_1, s_2)$$

$$B(s_1, s_2) = \beta_{00} + \beta_{01} \, \frac{\beta_{10} \cdot \frac{1-\beta_{11}^{N-1}}{1-\beta_{11}} + \beta_{10}\beta_{11}^{N-1}}{1-\beta_{11}^{N}} =$$

$$= \beta_{00} + \beta_{01}\beta_{10} \cdot \frac{1 - \cancel{\beta_{11}^{N-1}} \cancel{+ \beta_{11}^{N-1}} - \beta_{11}^{N}}{(1-\beta_{11})(1-\beta_{11}^{N})} =$$

$$= \beta_{00} + \frac{\beta_{01}\beta_{10}}{1-\beta_{11}}$$

$$B(1,1) = p_{00} + \frac{p_{01} \cdot p_{10}(m)}{1 - p_{11}(m)} = 1 \qquad \underline{\underline{\text{OK}}}$$



$$\Psi(s_1,s_2) = \begin{bmatrix} \beta_{00} & \beta_{01} & 0 \\ \beta_{BG} & 0 & \beta_{11}^{N-1} \\ \beta_{10} & \beta_{11} & 0 \end{bmatrix}$$

"Funzioni di primo passaggio" $\Theta(s_1,s_2)$.

$$\Theta_{ij}(s_1,s_2) = \sum_{k\neq j} \Psi_{ik}(s_1,s_2)\,\Theta_{kj}(s_1,s_2) + \Psi_{ij}(s_1,s_2) \qquad i\neq j.$$

<u>per es.</u> per $j = BN = 2$ $(G=0,\ B1=1)$

$$\cancel{\Theta_{G,BN} = \Psi_{G,G}\cdot\Theta_{G,BN} + \Psi_{G,B1}\cdot\Theta_{B1,BN}}$$

$$\cancel{\Theta_{B1,BN} = \Psi_{B1,G}\,\Theta_{G,BN} + \Psi_{B1,BN}}$$

$$\Theta_{02} = \Psi_{00}\,\Theta_{02} + \Psi_{01}\,\Theta_{12} + \overset{=0.}{\cancel{\Psi_{02}}}$$

$$\Theta_{12} = \Psi_{10}\,\Theta_{02} + \underbrace{\cancel{\Psi_{11}\,\Theta_{12}}}_{=0} + \Psi_{12}$$

$$\begin{cases} \vartheta_{02} = \psi_{00}\vartheta_{02} + \psi_{01}\vartheta_{12} \\ \vartheta_{12} = \psi_{10}\vartheta_{02} + \psi_{12} \end{cases}$$

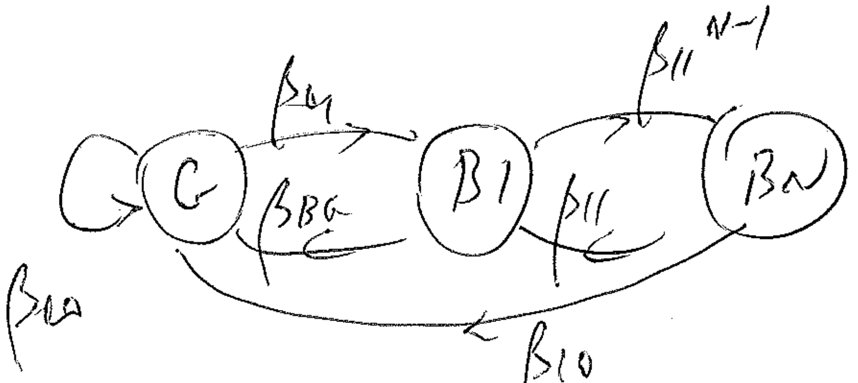
$$\vartheta_{02} = \psi_{00}\vartheta_{02} + \psi_{01}\psi_{10}\vartheta_{02} + \psi_{01}\psi_{12}$$

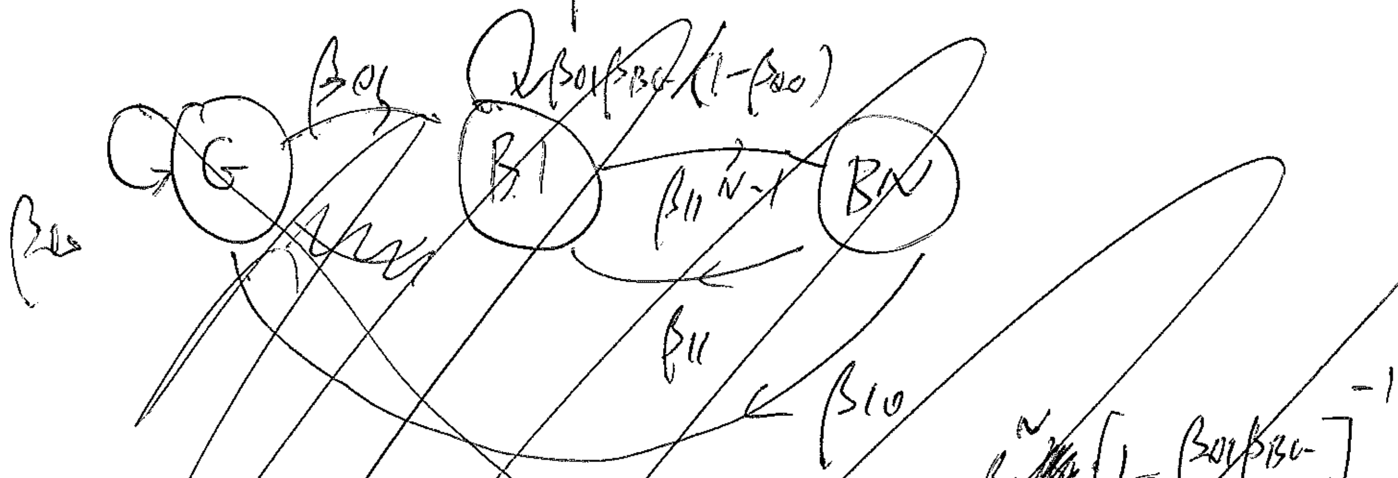
$$\vartheta_{02} = \frac{\psi_{01}\psi_{12}}{1 - \psi_{01}\psi_{10} - \psi_{00}} =$$

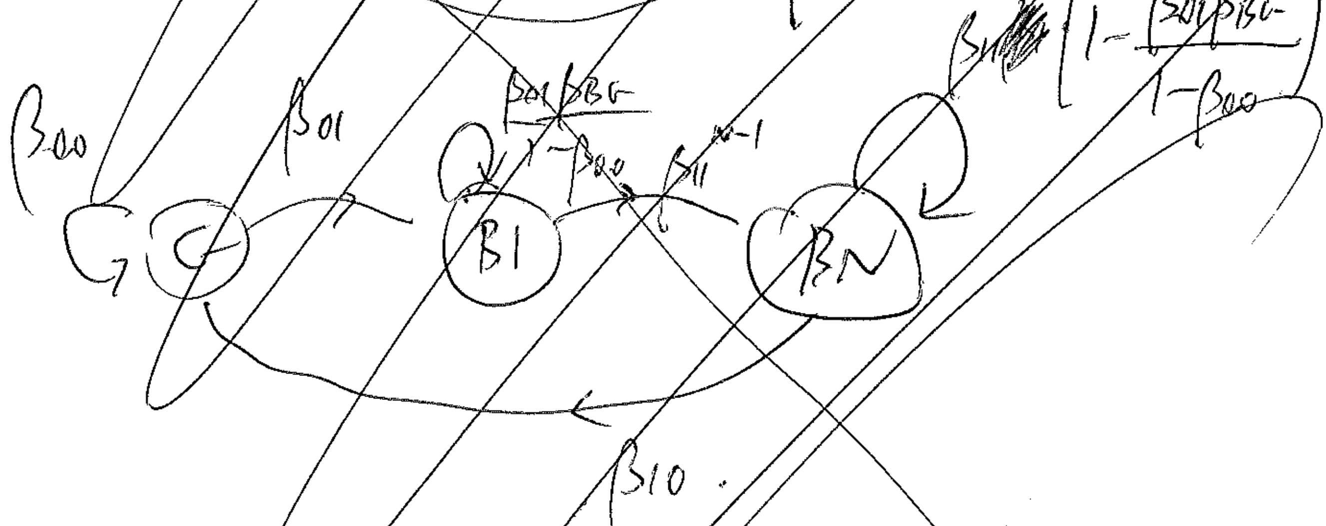
$$= \frac{\beta_{01}\beta_{11}^{N-1}}{1 - \beta_{01}\beta_{BG} - \beta_{00}}$$

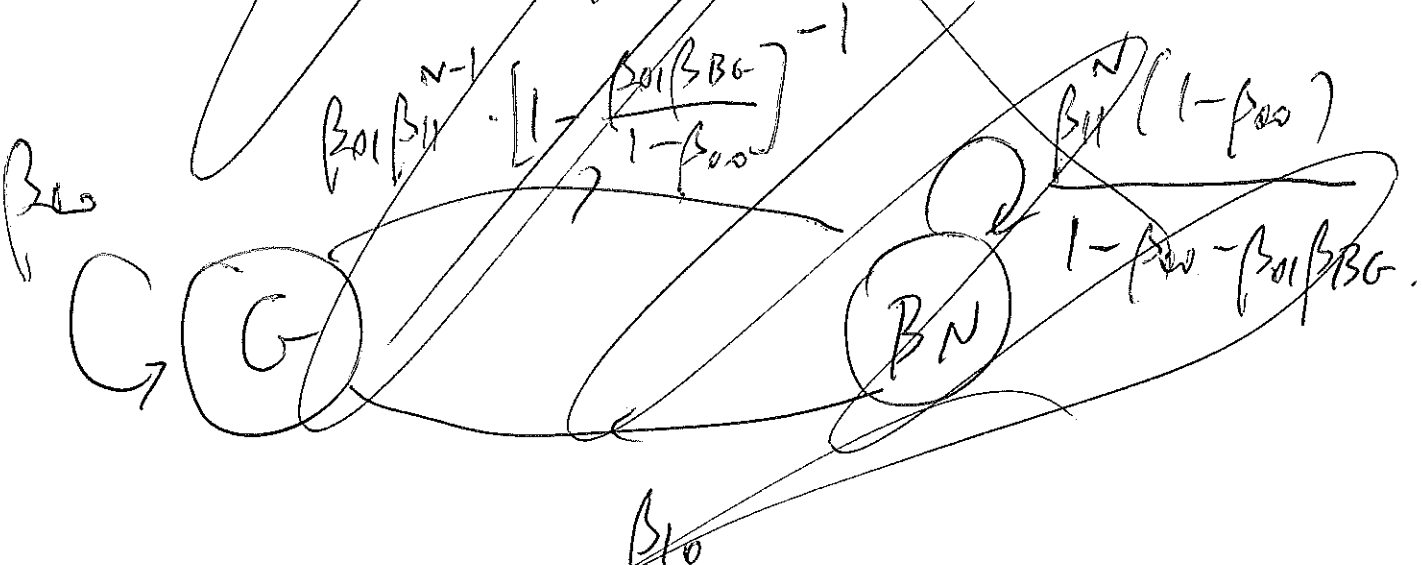
$$\vartheta_{12} = \psi_{10}\vartheta_{02} + \psi_{12} = \frac{\beta_{BG}\beta_{01}\beta_{11}^{N-1}}{1 - \beta_{01}\beta_{BG} - \beta_{00}} + \beta_{11}^{N-1} =$$

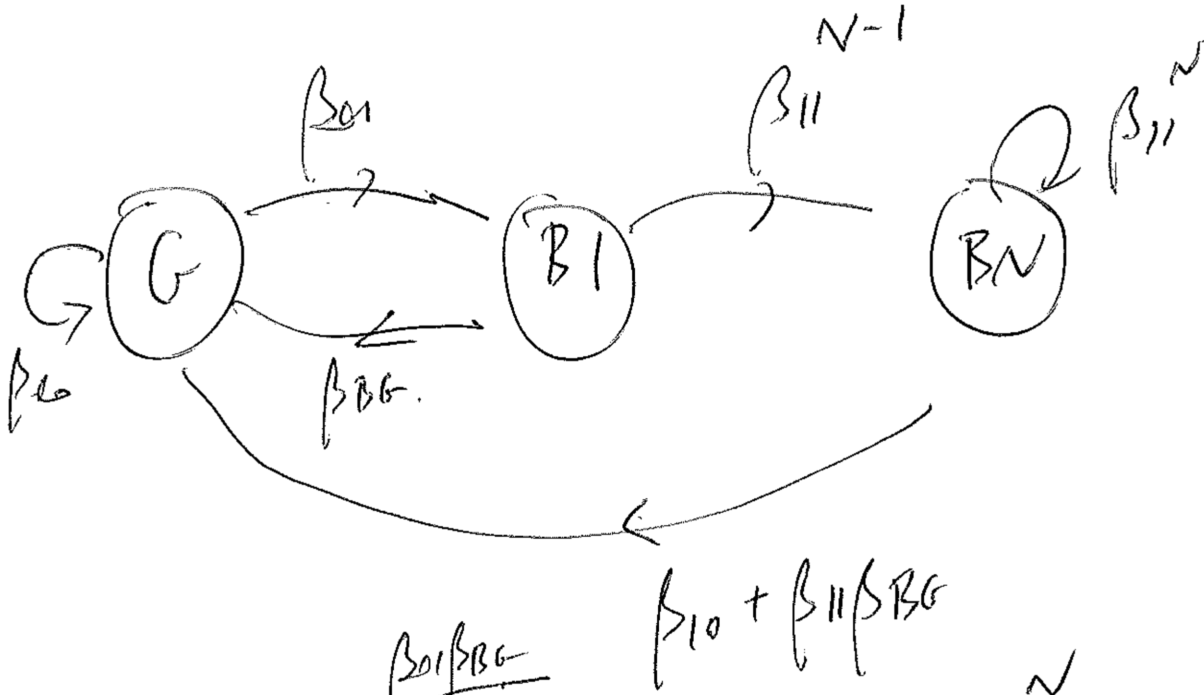
$$= \frac{\beta_{11}^{N-1}(1 - \beta_{00})}{1 - \beta_{01}\beta_{BG} - \beta_{00}}$$

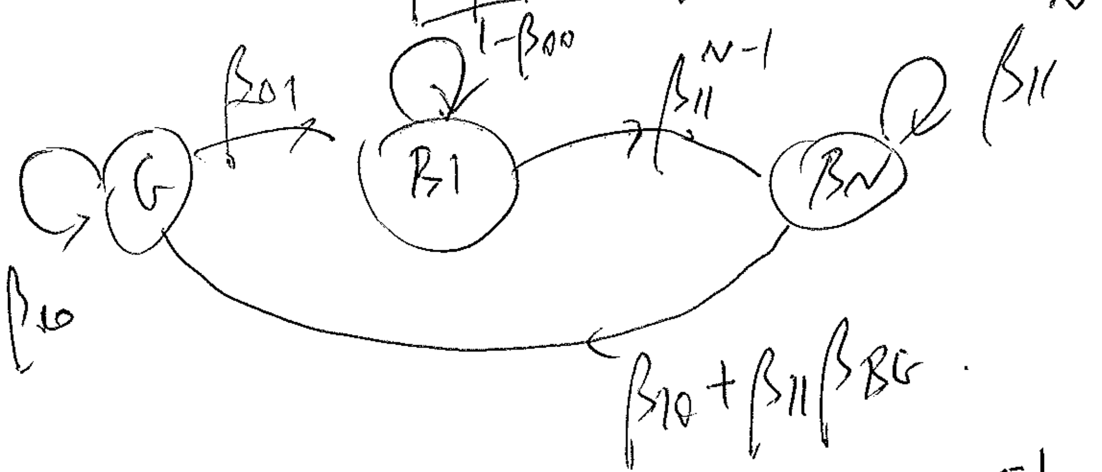

G
B1
BN
β_{01}
β_{11}^{N-1}
β_{BG}
β_{11}
β_{00}
β_{10}

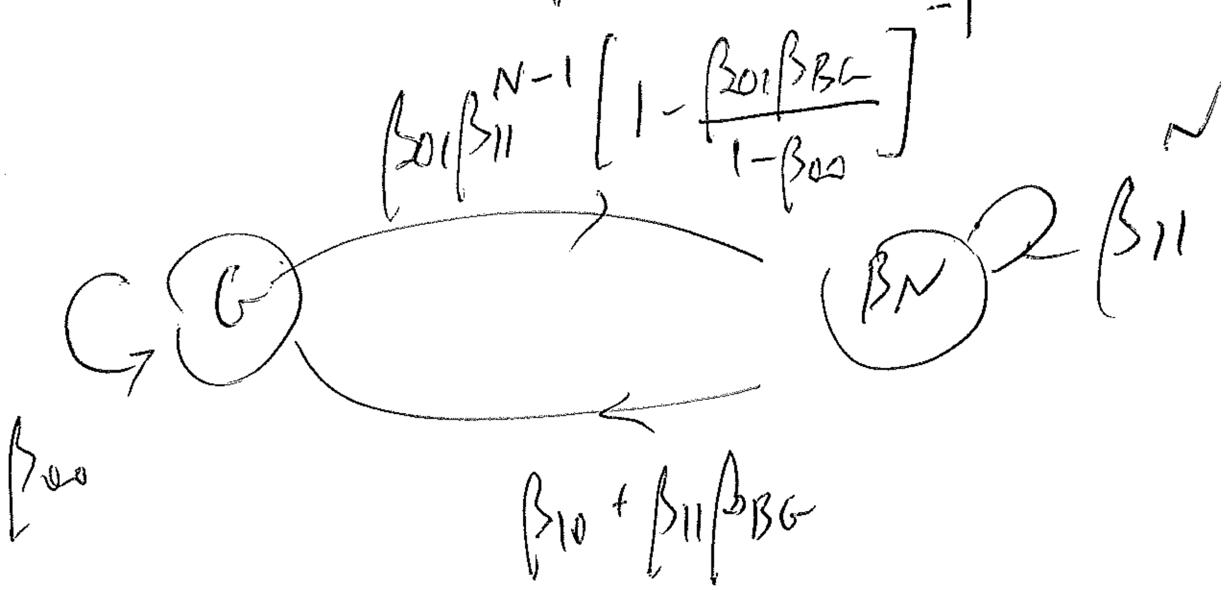

G
B1
BN
β_{01}
$\beta_{01}\beta_{BG}/(1-\beta_{00})$
β_{11}^{N-1}
β_{11}
β_{00}
β_{10}

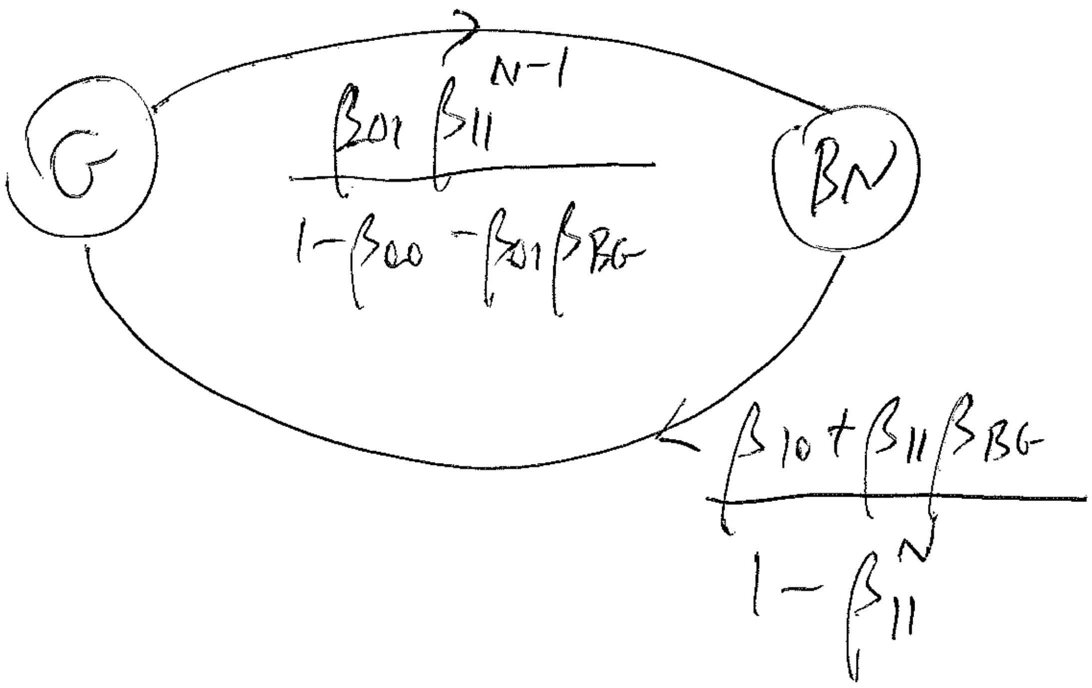

G
B1
BN
β_{00}
β_{01}
$\frac{\beta_{01}\beta_{BG}}{1-\beta_{00}}$
β_{11}^{N-1}
$\beta_{11}^{N}\left[1-\frac{\beta_{01}\beta_{BG}}{1-\beta_{00}}\right]^{-1}$
β_{10}.


G
BN
β_{00}
$\beta_{01}\beta_{11}^{N-1}\cdot\left[1-\frac{\beta_{01}\beta_{BG}}{1-\beta_{00}}\right]^{-1}$
$\frac{\beta_{11}^{N}(1-\beta_{00})}{1-\beta_{00}-\beta_{01}\beta_{BG}}$.
β_{10}


β_{01}
β_{11}^{N-1}
β_{11}^{N}
G
B1
BN
β_{00}
β_{BG}.
$\beta_{10} + \beta_{11}\beta_{BG}$


$\frac{\beta_{01}\beta_{BG}}{1-\beta_{00}}$
β_{01}
β_{11}^{N-1}
β_{11}^{N}
G
B1
BN
β_{00}
$\beta_{10} + \beta_{11}\beta_{BG}$.


$\beta_{01}\beta_{11}^{N-1}\left[1 - \frac{\beta_{01}\beta_{BG}}{1-\beta_{00}}\right]^{-1}$
β_{11}^{N}
G
BN
β_{00}
$\beta_{10} + \beta_{11}\beta_{BG}$


G
BN
$\frac{\beta_{01}\beta_{11}^{N-1}}{1-\beta_{00}-\beta_{01}\beta_{BG}}$
$\frac{\beta_{10}+\beta_{11}\beta_{BG}}{1-\beta_{11}^{N}}$

$$\theta_{00} = \psi_{00} + \psi_{01}\theta_{10} + \underbrace{\cancel{\psi_{02}\theta_{20}}}_{=0}$$

$$\theta_{10} = \psi_{10} + \underbrace{\cancel{\psi_{11}}}_{=0}\cancel{\theta_{10}} + \psi_{12}\theta_{20}$$

$$\theta_{20} = \psi_{20} + \psi_{21}\theta_{10} + \underbrace{\cancel{\psi_{22}\theta_{20}}}_{=0}.$$

$$\theta_{20} = \psi_{20} + \psi_{21}\psi_{10} + \psi_{21}\psi_{12}\theta_{20}$$

$$\theta_{20} = \frac{\psi_{20} + \psi_{21}\psi_{10}}{1-\psi_{21}\psi_{12}} =$$

$$= \frac{\beta_{10} + \beta_{11}\beta_{BG}}{1-\beta_{11}^{N}} =$$

OK