Group 1: A Study of Tommy MIllions Pizza

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Introduction

A system study of Tommy Millions Pizza. A take away pizza retailer in Courtenay Place.

“We make pizza, sub sandwiches and coffee. We use good ingredients and try not to mess them up.”

Planning Process

Tommy Millions Pizza

- Good rate of arrivals
- Limited servers
- Service time needed to be a substantial fraction of the interarrival time
- FCFS M/M/1

Other Considerations:

- McDonalds Drive Through
- Self Services Checkouts at Countdown
- Petrol Stations
- and Movie Theaters

Data Criteria

Service Includes

- Arriving at Server
- Waiting place order
- Placing an Order
- Payment

Excludes

- Waiting in line behind person being served
- Waiting for order (receiving food)
- Coffee queue
- Pulled Pork queue

Data Criteria

Collection Times

- 21st, 23rd, 24th, and 27th April 2018
- 3rd of may 2018
- 12:00 - 14:00
- n = 273

Variables

- Date
- Arrival Time
- Service Start Time
- Service Finish Time

Notes:

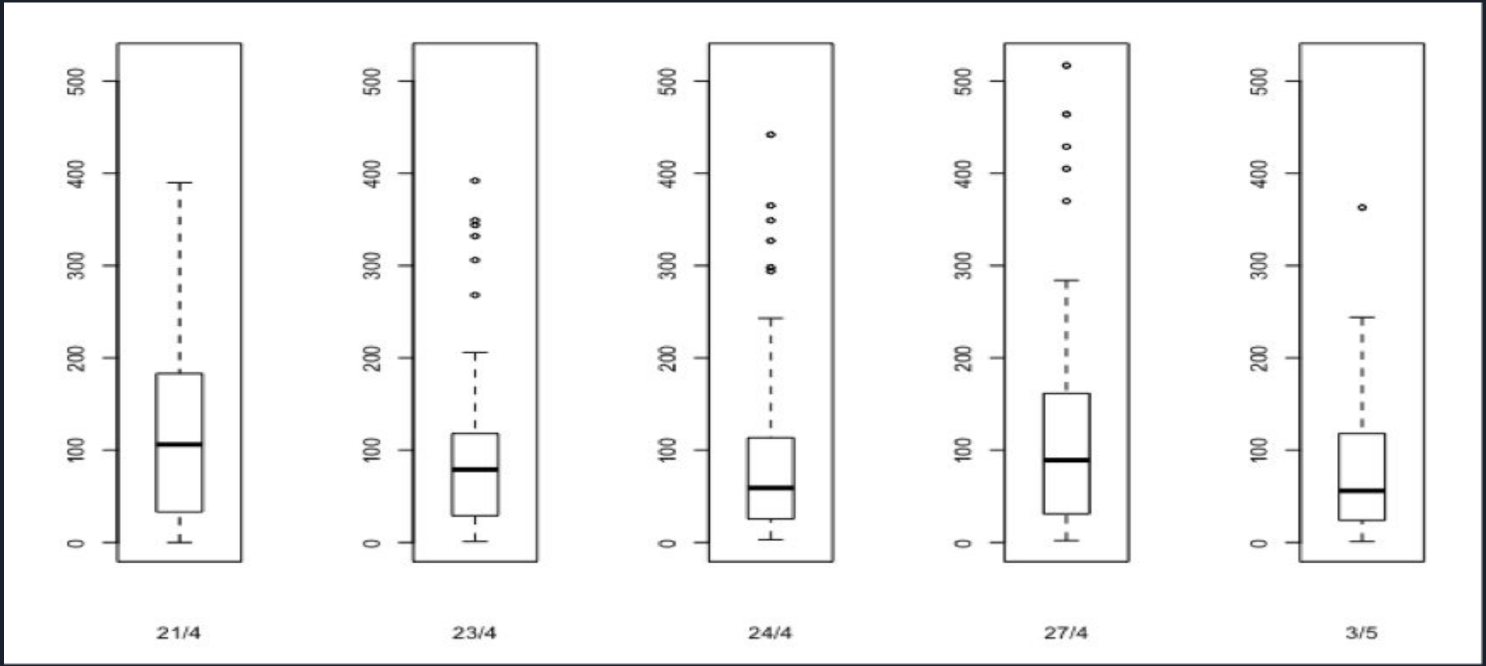
- School holidays
- Weekend

- If they do it might later in the day
- Hungover?

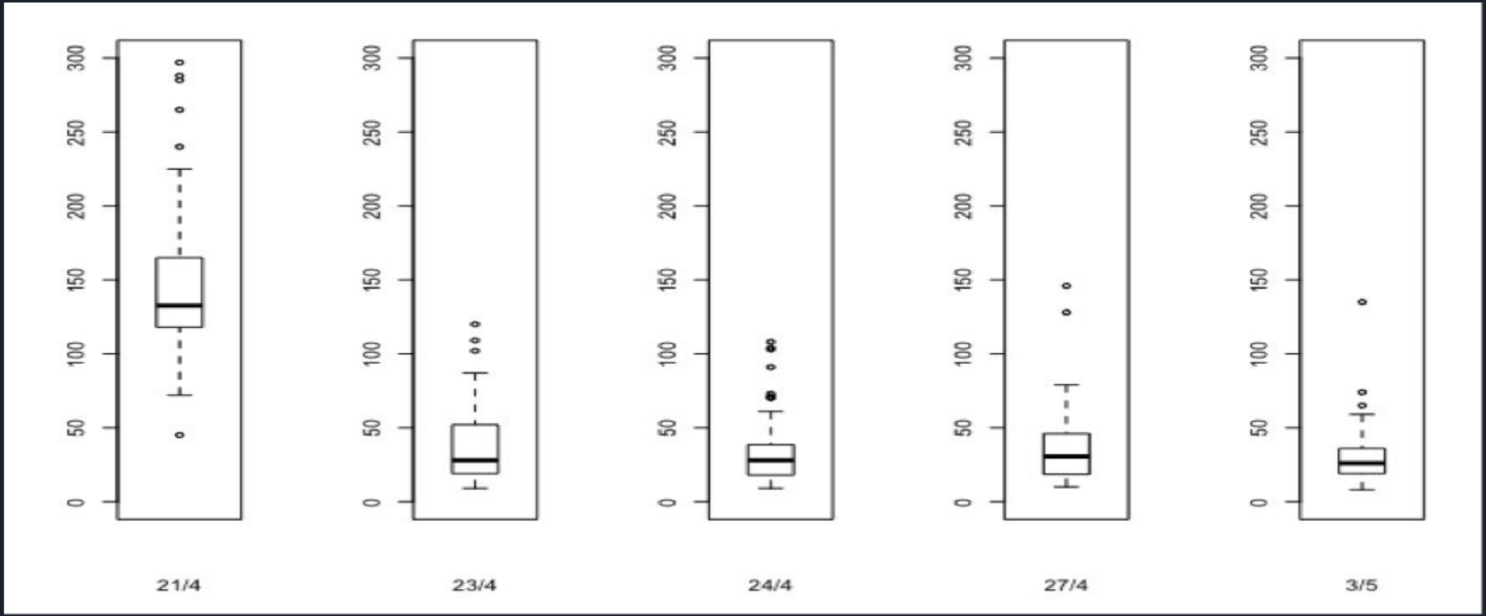
Data Collection

- No operational definitions
- Subjectivity about transitions in system state
- Limits of accuracy in time measurements

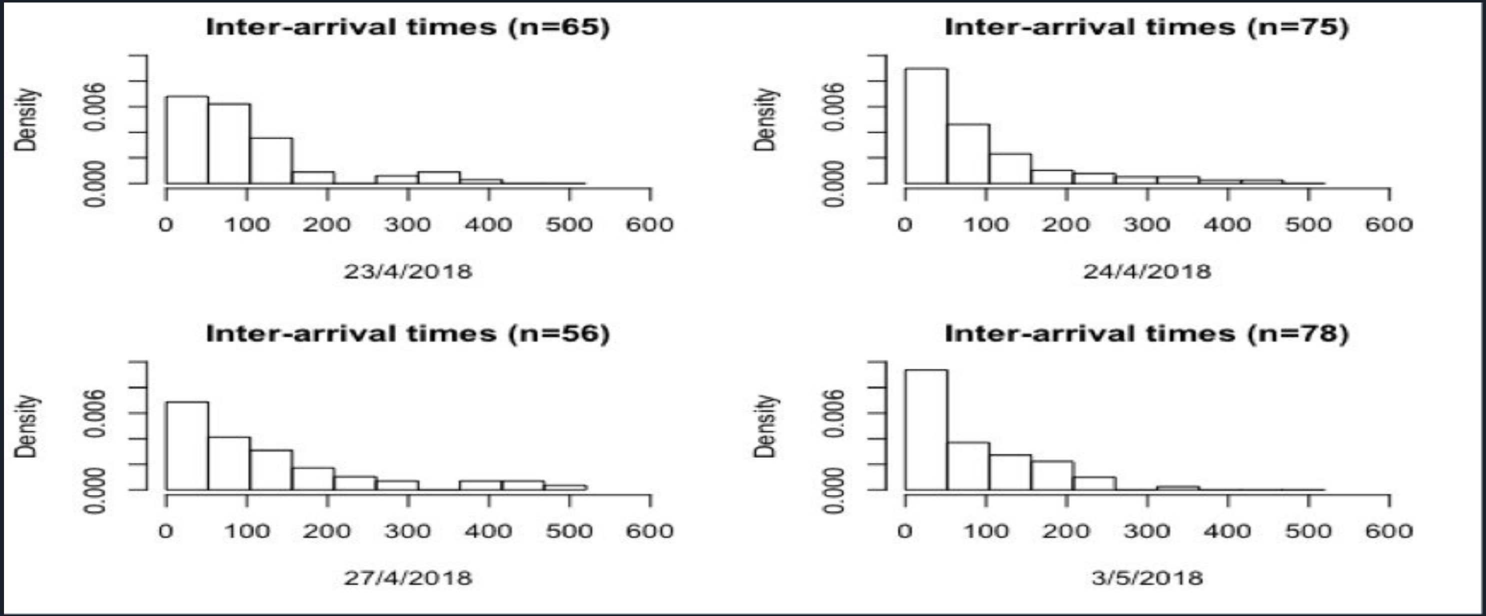
Data diagnostics: comparison of inter-arrival times



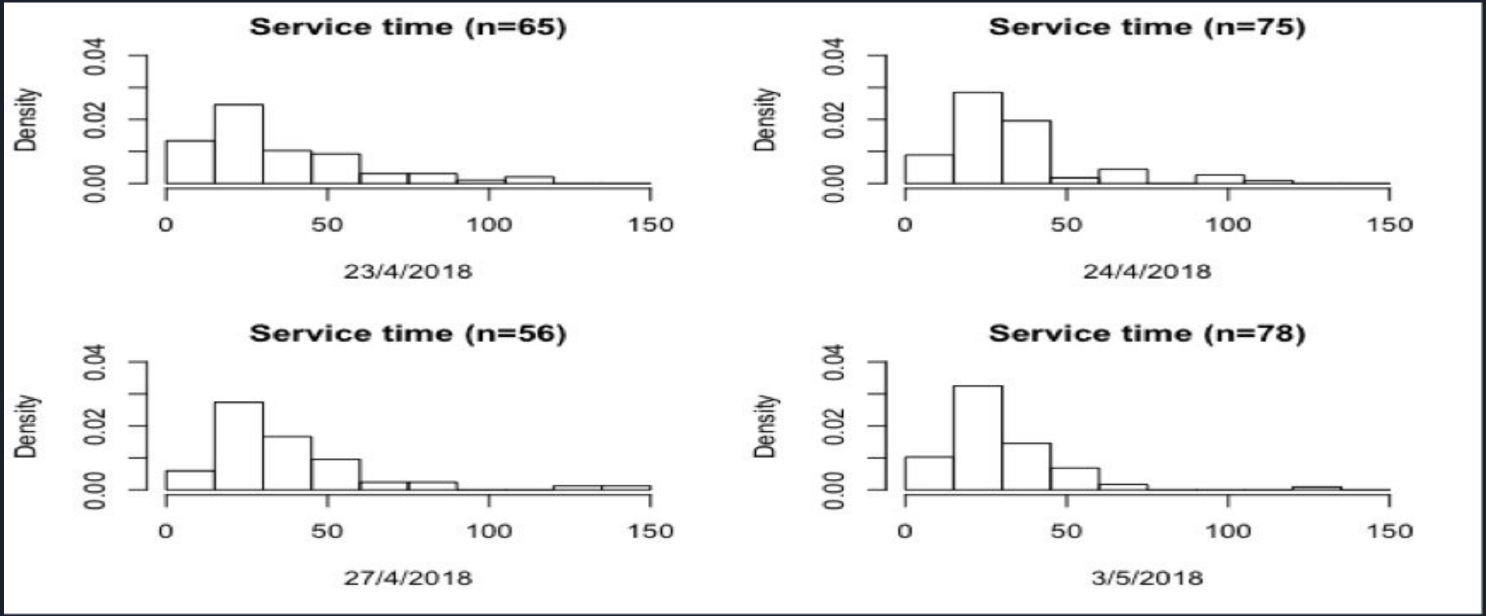
Data diagnostics: comparison of service times



Data diagnostics: distributions of inter-arrival times



Data diagnostics: distributions of inter-arrival times



Data Analysis

Load handwritten script

- Machine readable data
  - .csv file
- Parser data into two lists:
  1. Interarrival time list
  2. Service time list

Date	Arrive	Start	Finish
2018/4/23	12:12:47	12:12:47	12:13:44
2018/4/23	12:14:16	12:14:30	12:14:39
2018/4/23	12:14:32	12:14:39	12:14:54
2018/4/23	12:16:38	12:16:38	12:16:49
2018/4/23	12:17:17	12:17:17	12:17:32
2018/4/23	12:19:57	12:19:57	12:20:39
2018/4/23	12:20:17	12:20:39	12:21:03
2018/4/23	12:22:08	12:22:08	12:22:17
2018/4/23	12:22:27	12:22:27	12:23:50
2018/4/23	12:23:50	12:23:50	12:25:06
2018/4/23	12:29:34	12:29:34	12:31:23
2018/4/23	12:31:18	12:31:23	12:31:48
2018/4/23	12:35:46	12:35:46	12:36:06
2018/4/23	12:37:26	12:37:26	12:37:43
2018/4/23	12:37:32	12:37:43	12:38:02
2018/4/23	12:38:49	12:38:49	12:39:01
2018/4/23	12:40:01	12:40:01	12:40:27
2018/4/23	12:40:47	12:40:47	12:41:03
2018/4/23	12:40:51	12:41:03	12:41:25

Chi-square Goodness of Fit Test

1. State the hypothesis
   - Consistent Gamma, Erlang or Exponential
2. Analyse the sample data
   - Estimate the unknown Parameters
3. Interpret results

   Interarrival time:
   - Exponential distribution:
     - Chi-square test value is 4.395118 and the p-value is 0.355166

   Service time:
   - Erlang distribution:
     - Chi-square test value is 14.865949 and the p-value is 0.0049874767

Simulation Differences M/M/1 and M/G/1

- M/M/1 simulations have a exponential service time and arrival time
- M/G/1 simulation is like M/M/1 but has a general distribution for the service time
- Used Erlang distribution in the simulation because we have used it before
- These both require a shape variable k (alpha) and scale parameter mu (beta)
- Erlang distribution becomes a Gamma distribution when k is a real number
- Gamma becomes exponential when k is 1
- So... M/G/1 becomes M/M/1 when k is 1

Testing Different Values

- Different values of k (1-4)

As k increases so does the waiting time, average number of people in the system and chance there will be someone already in the system when there's a new arrival.

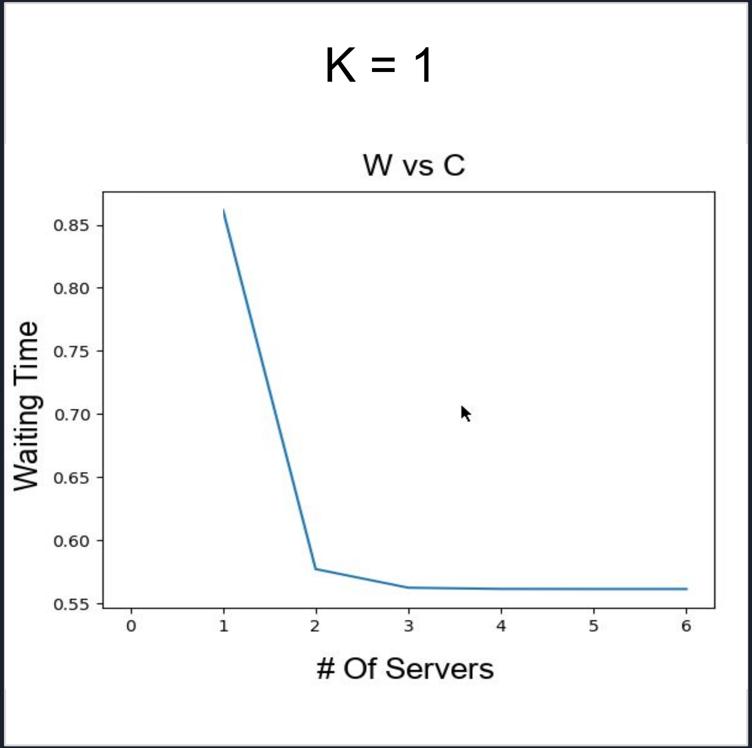
- Different numbers of servers (1-3)

The average time a customer spends in the system decreases as the number of servers increase.

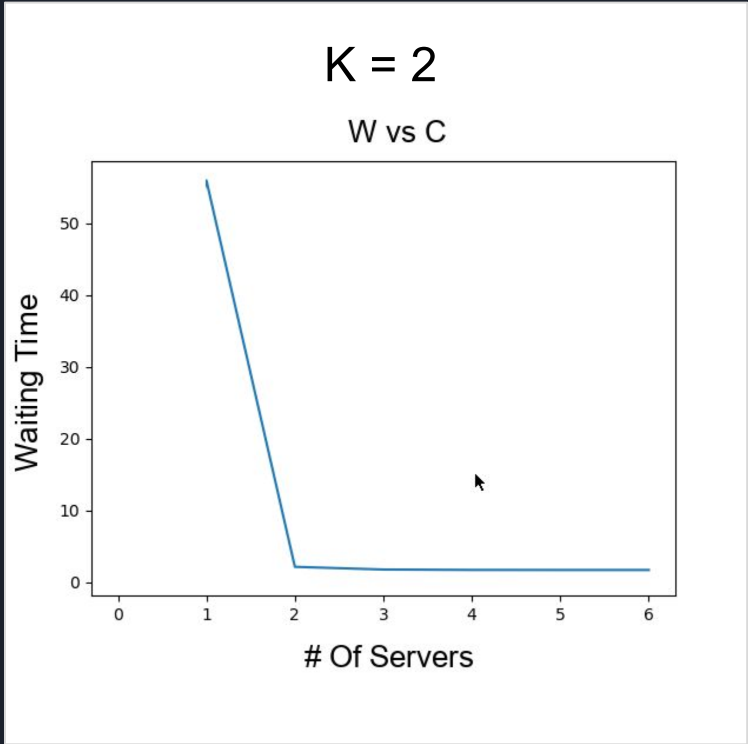
- Different numbers of jobs (10,100,1000,10000)

Not much change in values after 1000 jobs

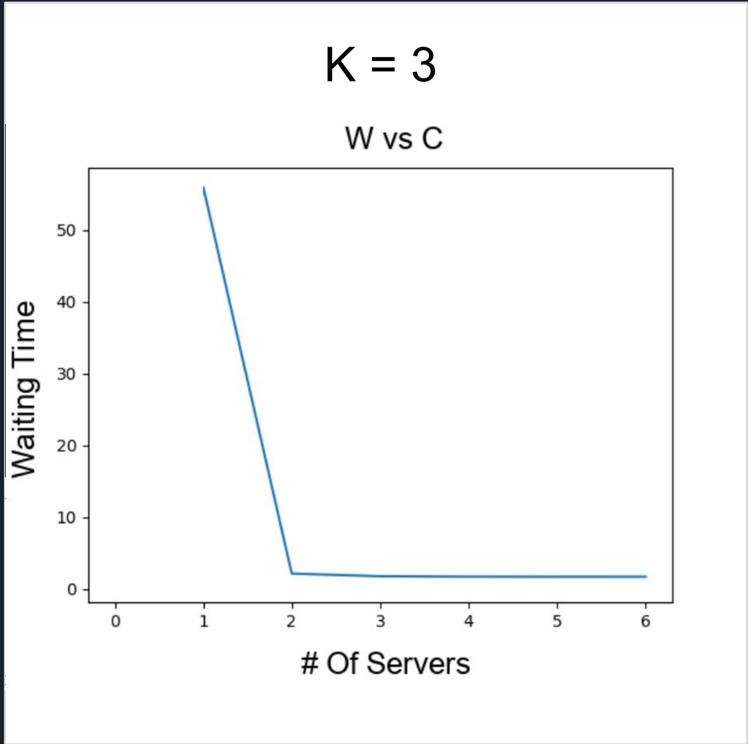
M/M/1


K = 1
W vs C
Waiting Time
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0
1
2
3
4
5
6
Of Servers

M/G/1 With Erlang Distribution



M/G/1 With Erlang Distribution



M/M/1 and M/G/1 Results

Estimates of values when k=1

Time in Line (W) : 0.867m

Average Number of Customers (L) : 0.538

Chance Person Has to Wait : 0.349

Lamb Effective : 0.620

Estimates of values when k=2

Time in Line (W) : 3.097m

Average Number of Customers (L) : 1.919

Chance Person Has to Wait : 0.698

Lamb Effective : 0.619

Note: When k becomes larger than 2 and we only have one server in the simulation, the service time becomes larger than the arrival times so the model breaks down.

Empirical Simulation

Simulating with empirical arrivals and service, c=1 server, and 10,000 jobs.

Interarrival time:

95.6194 (expected ~96)

Service time:

33.1556 (expected ~33)

Time spent in the queue:

12.3711 seconds

Empirical results are very close to the experimental data.

Simulated scenarios

Scenario 1: “Increase server capacity” (from c = 1 to c = 2)					
Expected W_q = 0.69 seconds					
$\pi(0) = 0.70$	$\pi(1) = 0.25$	$\pi(2) = 0.04$	$\pi(3) = 0.01$	$\pi(4) = 0.00$	$\pi(5^+) = 0.00$
Scenario 2: “Shock test” (increase lambda x 2)					
Expected W_q = 55 seconds					
$\pi(0) = 0.30$	$\pi(1) = 0.26$	$\pi(2) = 0.17$	$\pi(3) = 0.10$	$\pi(4) = 0.07$	$\pi(5^+) = 0.10$
Scenario 3: “Shock test” (increase lambda x 2.5, ie. close to mu)					
Expected W_q = 163 seconds					
$\pi(0) = 0.13$	$\pi(1) = 0.14$	$\pi(2) = 0.12$	$\pi(3) = 0.10$	$\pi(4) = 0.09$	$\pi(5^+) = 0.42$

Findings

- Waiting times under the existing system appears to be relatively efficient
- Increasing the servers from one to two is unlikely to be cost effective
- If the arrival rate doubled, under the existing system, it would lead to a waiting time for customers of around a minute

Thanks!

Acknowledgement to Mr. Millions for allowing us to study his pizza place.

QA