Mathematical Induction

Part Two

Let P be some property. The ***principle of mathematical induction*** states that if

If it starts true…

$$\boldsymbol{P(0) \text{ is true}}$$

…and it stays true…

and

$$\boldsymbol{\forall k \in \mathbb{N}.\ (P(k) \to P(k+1))}$$

then

$$\boldsymbol{\forall n \in \mathbb{N}.\ P(n)}$$

…then it's always true.

Theorem: The sum of the first n powers of two is $2^n - 1$.

***Proof*:** Let $P(n)$ be the statement "the sum of the first n powers of two is $2^n - 1$." We will prove, by induction, that $P(n)$ is true for all $n \in \mathbb{N}$, from which the theorem follows.

For our base case, we need to show $P(0)$ is true, meaning that the sum of the first zero powers of two is $2^0 - 1$. Since the sum of the first zero powers of two is zero and $2^0 - 1$ is zero as well, we see that $P(0)$ is true.

For the inductive step, assume that for some $k \in \mathbb{N}$ that $P(k)$ holds, meaning that

$$2^0 + 2^1 + \ldots + 2^{k-1} = 2^k - 1. \qquad (1)$$

We need to show that $P(k + 1)$ holds, meaning that the sum of the first $k + 1$ powers of two is $2^{k+1} - 1$. To see this, notice that

$$\begin{aligned}
2^0 + 2^1 + \ldots + 2^{k-1} + 2^k &= (2^0 + 2^1 + \ldots + 2^{k-1}) + 2^k \\
&= 2^k - 1 + 2^k \qquad \textit{(via (1))} \\
&= 2(2^k) - 1 \\
&= 2^{k+1} - 1.
\end{aligned}$$

Therefore, $P(k + 1)$ is true, completing the induction. ■

Induction in Practice

- Typically, a proof by induction will not explicitly state $P(n)$.
- Rather, the proof will describe $P(n)$ implicitly and leave it to the reader to fill in the details.
- Provided that there is sufficient detail to determine
  - what $P(n)$ is;
  - that $P(0)$ is true; and that
  - whenever $P(k)$ is true, $P(k+1)$ is true,

  the proof is usually valid.

Theorem: The sum of the first n powers of two is $2^n - 1$.

Proof: By induction.

For our base case, we'll prove the theorem is true when $n = 0$. The sum of the first zero powers of two is zero, and $2^0 - 1 = 0$, so the theorem is true in this case.

For the inductive step, assume the theorem holds when $n = k$ for some arbitrary $k \in \mathbb{N}$. Then

$$\begin{aligned} 2^0 + 2^1 + \dots + 2^{k-1} + 2^k &= (2^0 + 2^1 + \dots + 2^{k-1}) + 2^k \\ &= 2^k - 1 + 2^k \\ &= 2(2^k) - 1 \\ &= 2^{k+1} - 1. \end{aligned}$$

So the theorem is true when $n = k+1$, completing the induction. ■

Variations on Induction: **Starting Later**

Induction Starting at 0

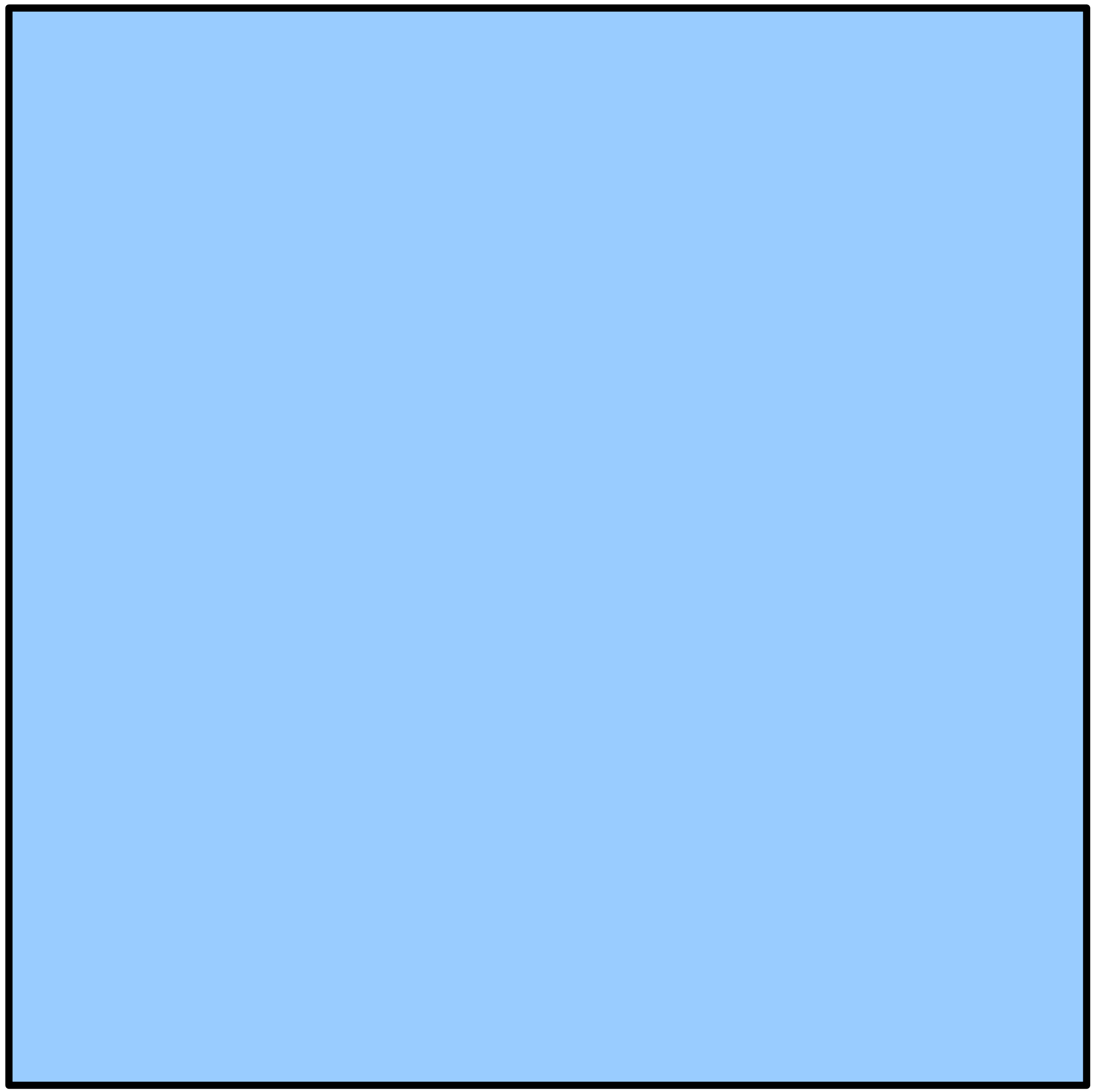
- To prove that $P(n)$ is true for all natural numbers greater than or equal to 0:
  - Show that $P(0)$ is true.
  - Show that for any $k \geq 0$, that
    if $P(k)$ is true, then $P(k+1)$ is true.
  - Conclude $P(n)$ holds for all natural numbers greater than or equal to 0.

Induction Starting at $\boldsymbol{m}$

- To prove that $P(n)$ is true for all natural numbers greater than or equal to $\boldsymbol{m}$:
  - Show that $P(\boldsymbol{m})$ is true.
  - Show that for any $k \geq \boldsymbol{m}$, that if $P(k)$ is true, then $P(k+1)$ is true.
  - Conclude $P(n)$ holds for all natural numbers greater than or equal to $\boldsymbol{m}$.

Variations on Induction: **Bigger Steps**

Subdividing a Square



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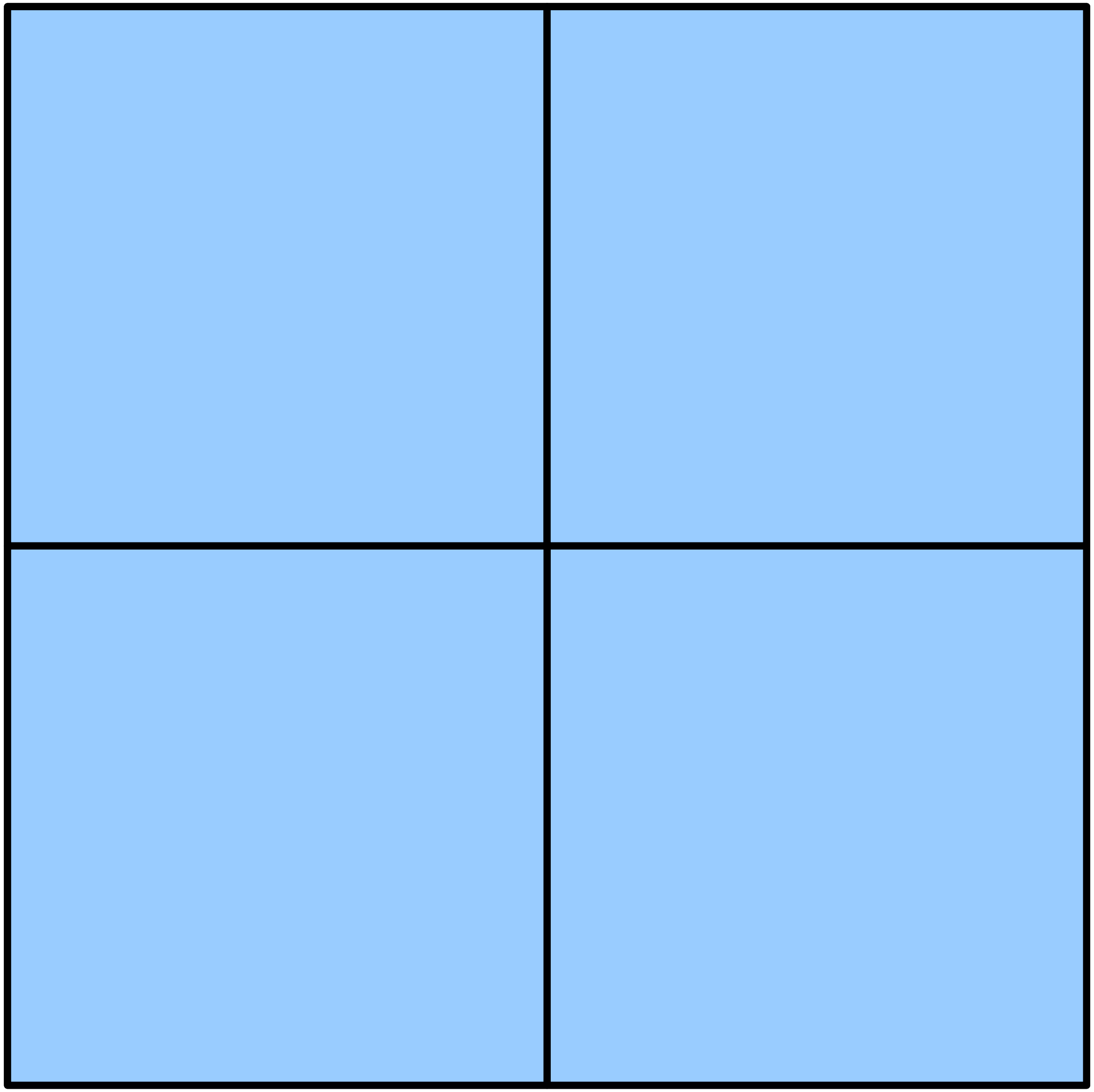

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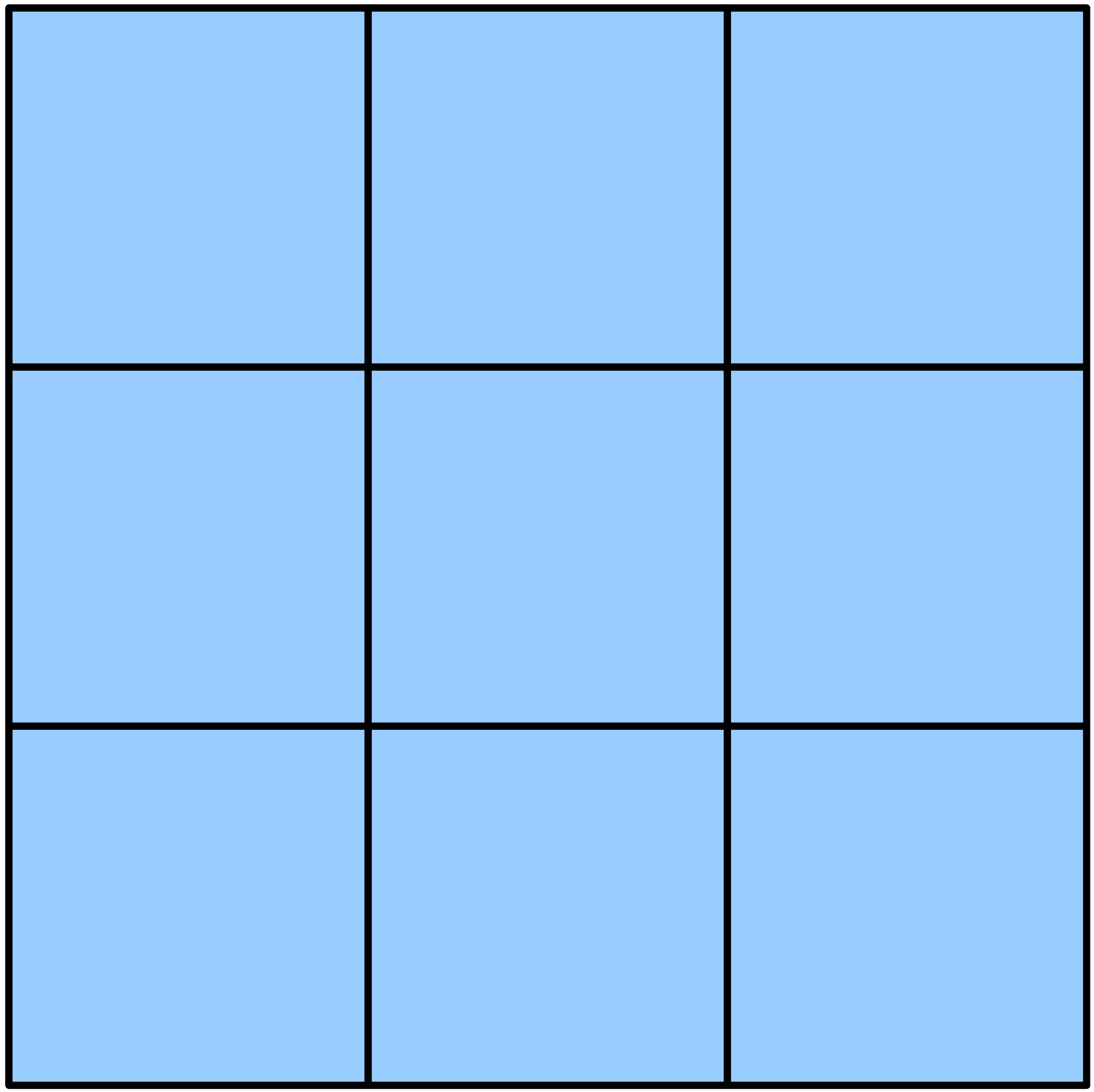

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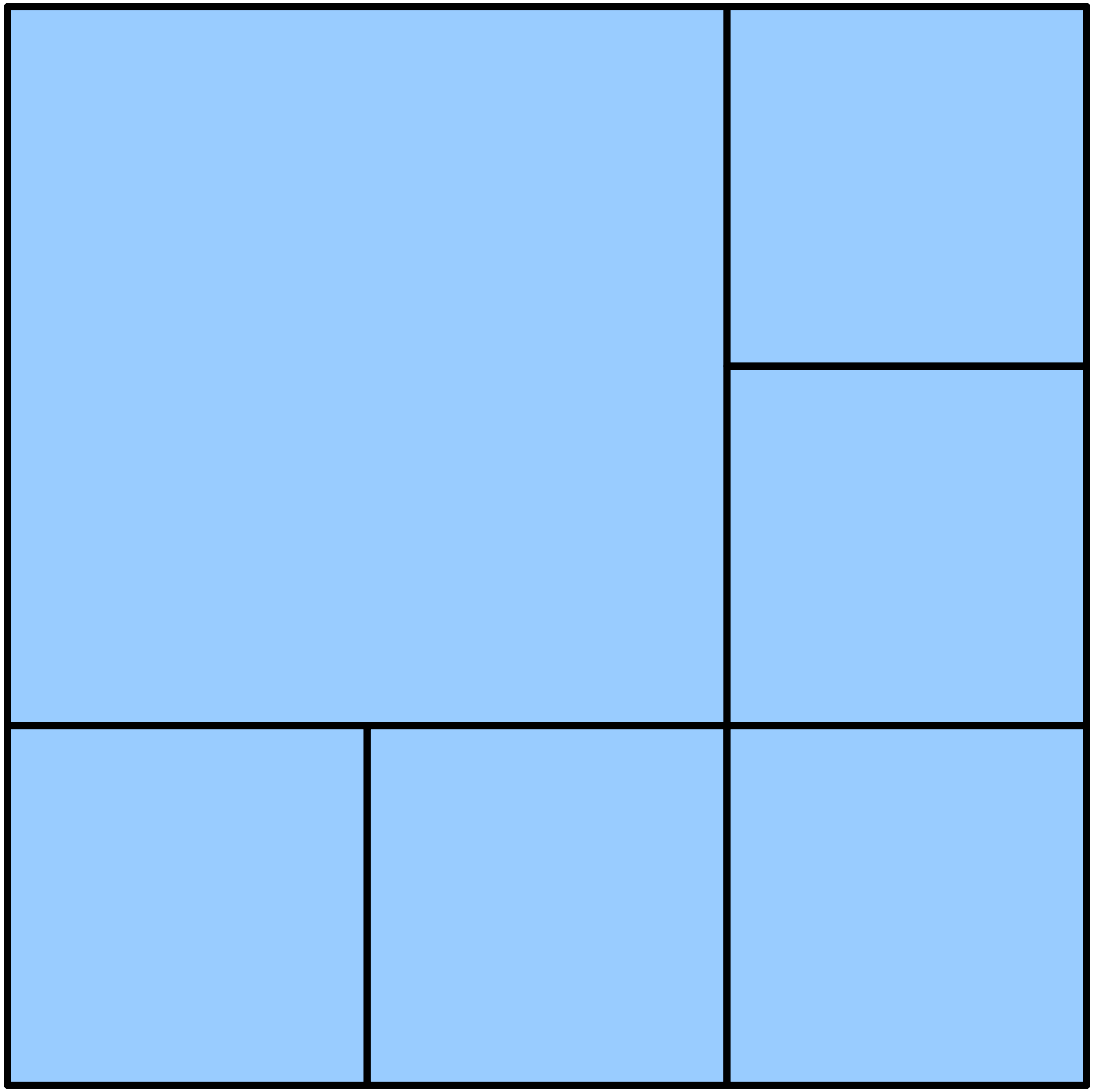

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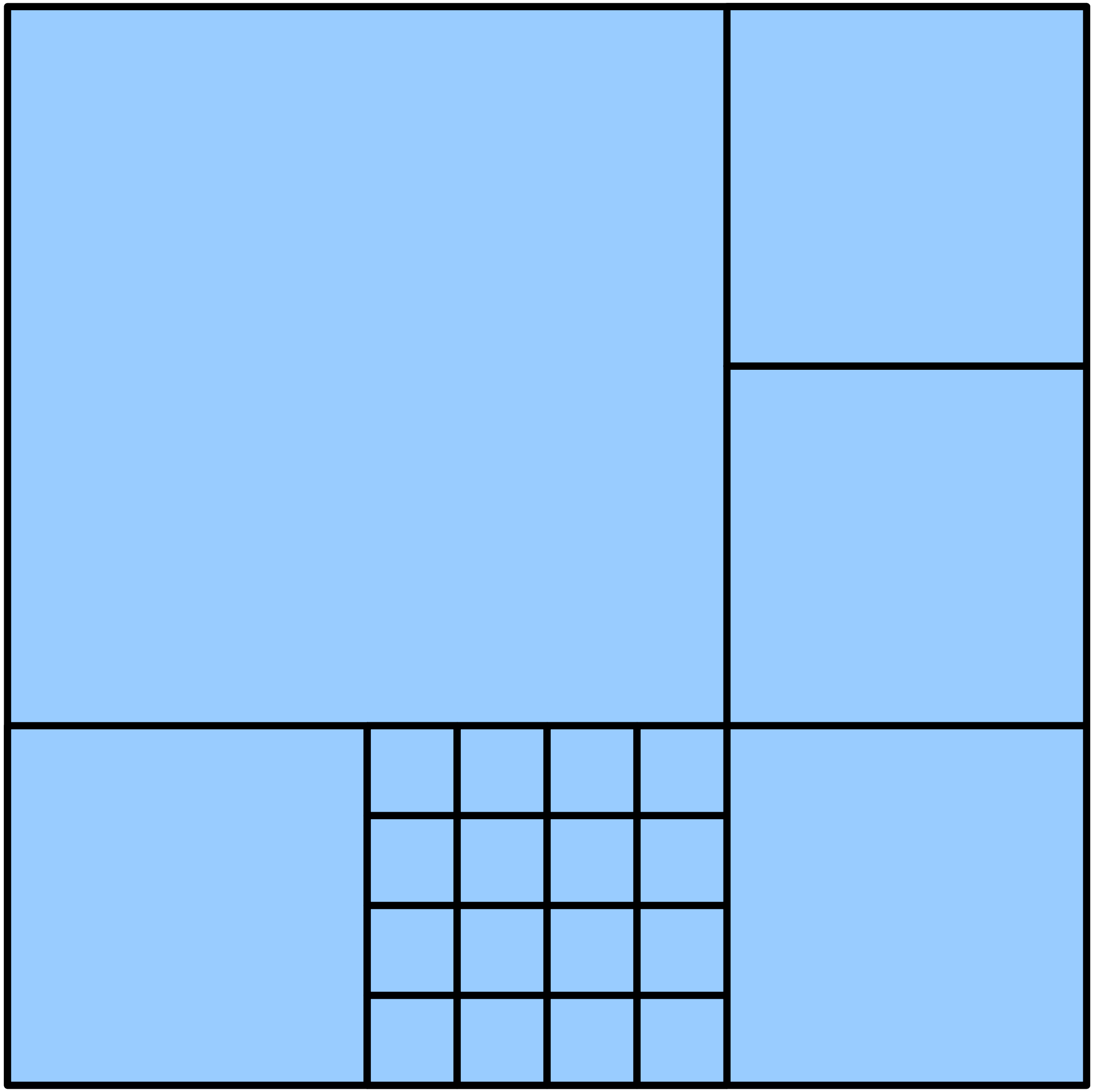

For what values of n can a square be subdivided into n squares?

1 2 3 4 5 6 7 8 9 10 11 12

1 ~~2~~ ~~3~~ 4 ~~5~~ 6 7 8 9 10 11 12

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$$\boxed{1}$$

1 ~~2~~ ~~3~~ 4 ~~5~~ 6 7 8 9 10 11 12

1	2
4	3

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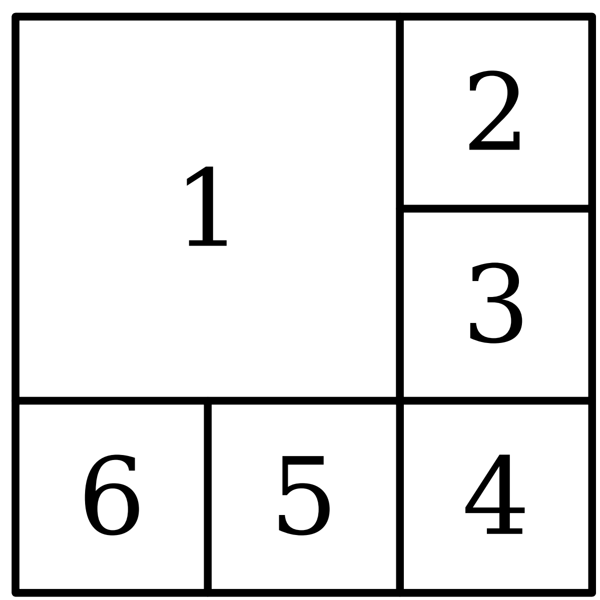

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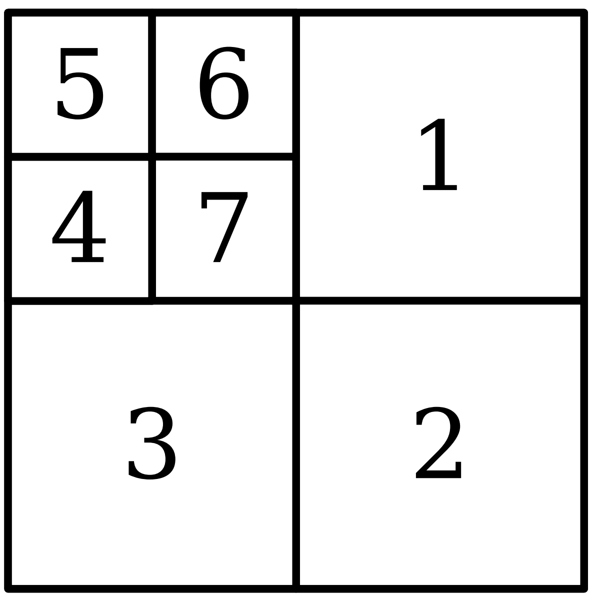

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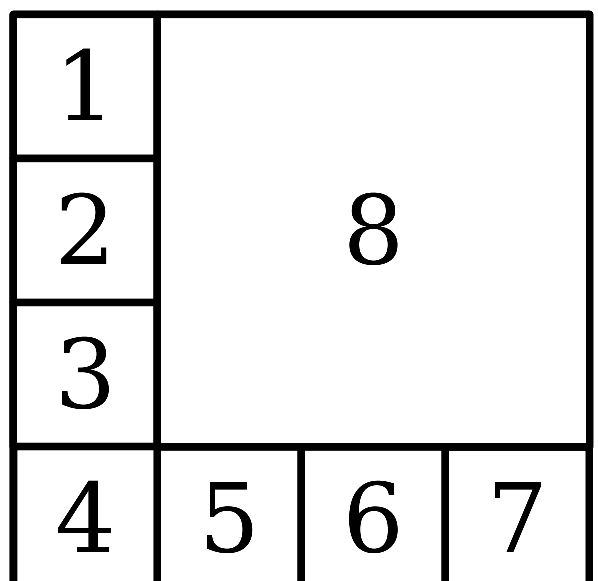

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1	2	3
8	9	4
7	6	5

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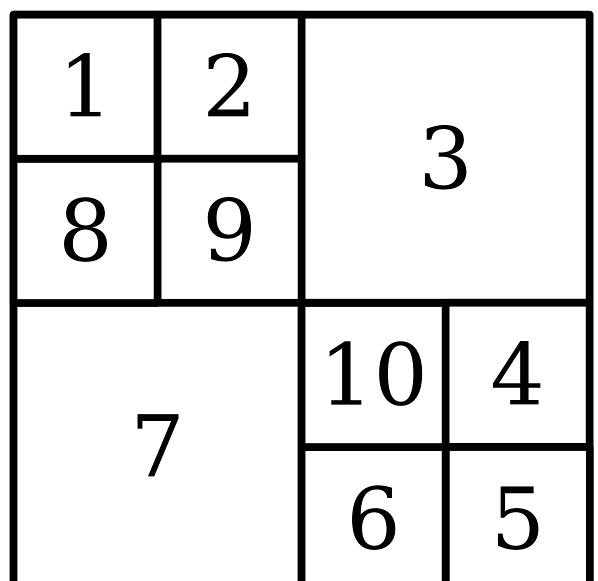

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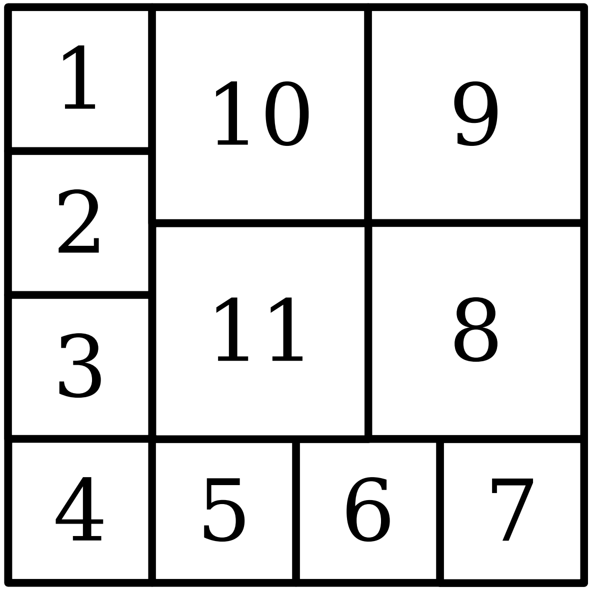

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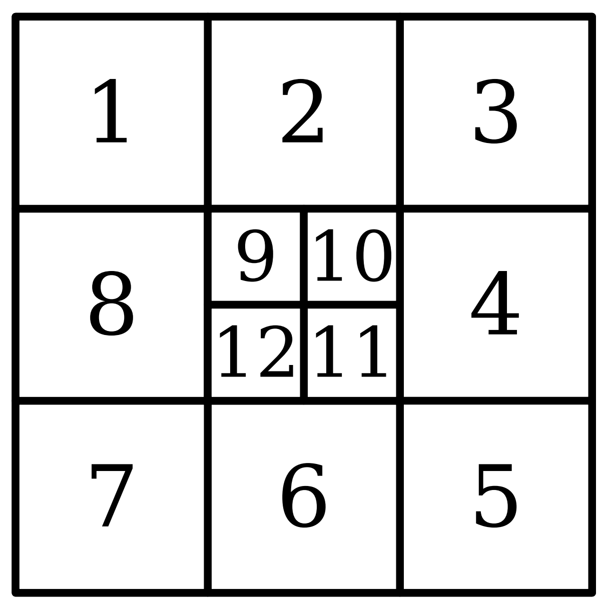

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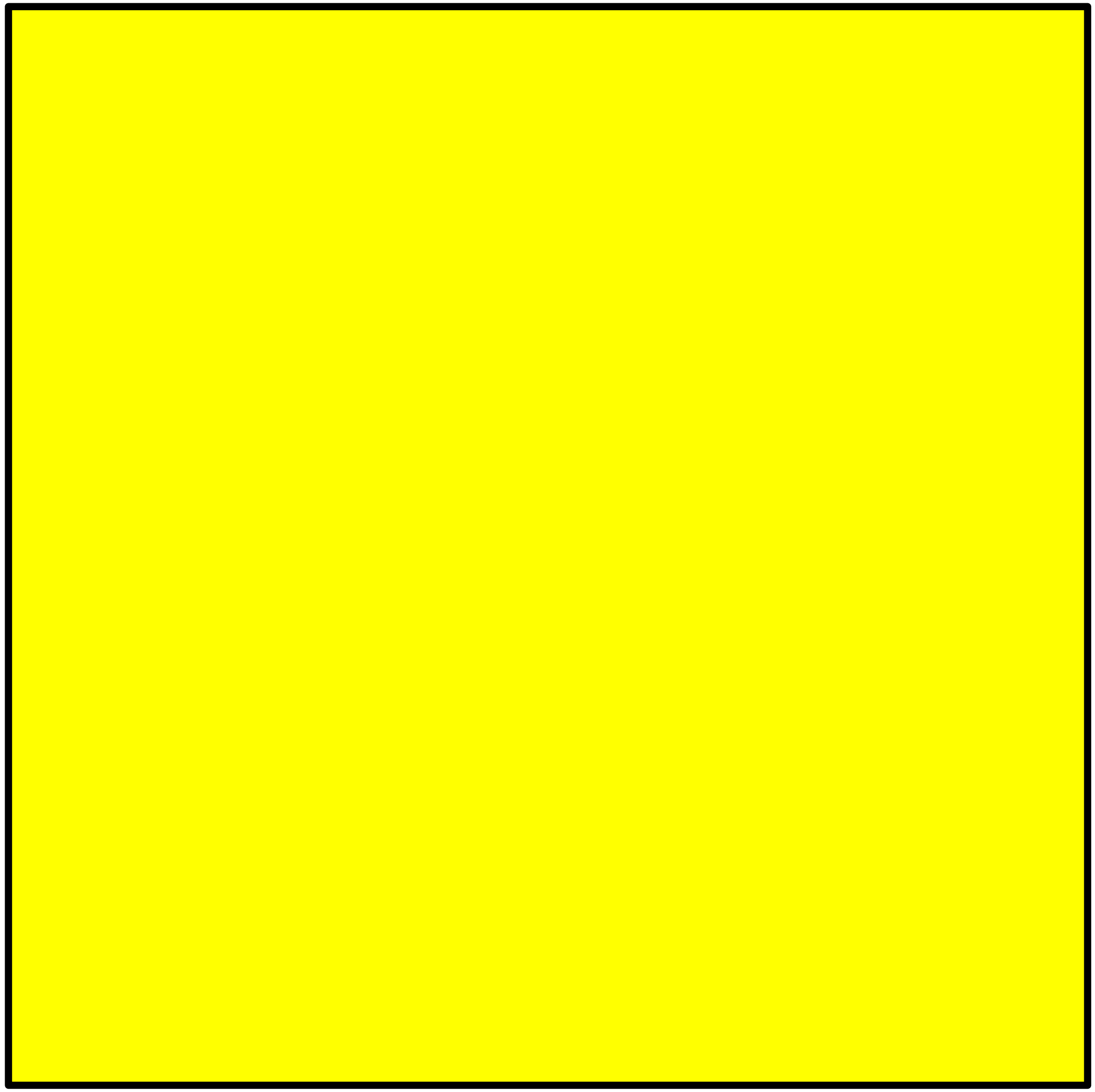

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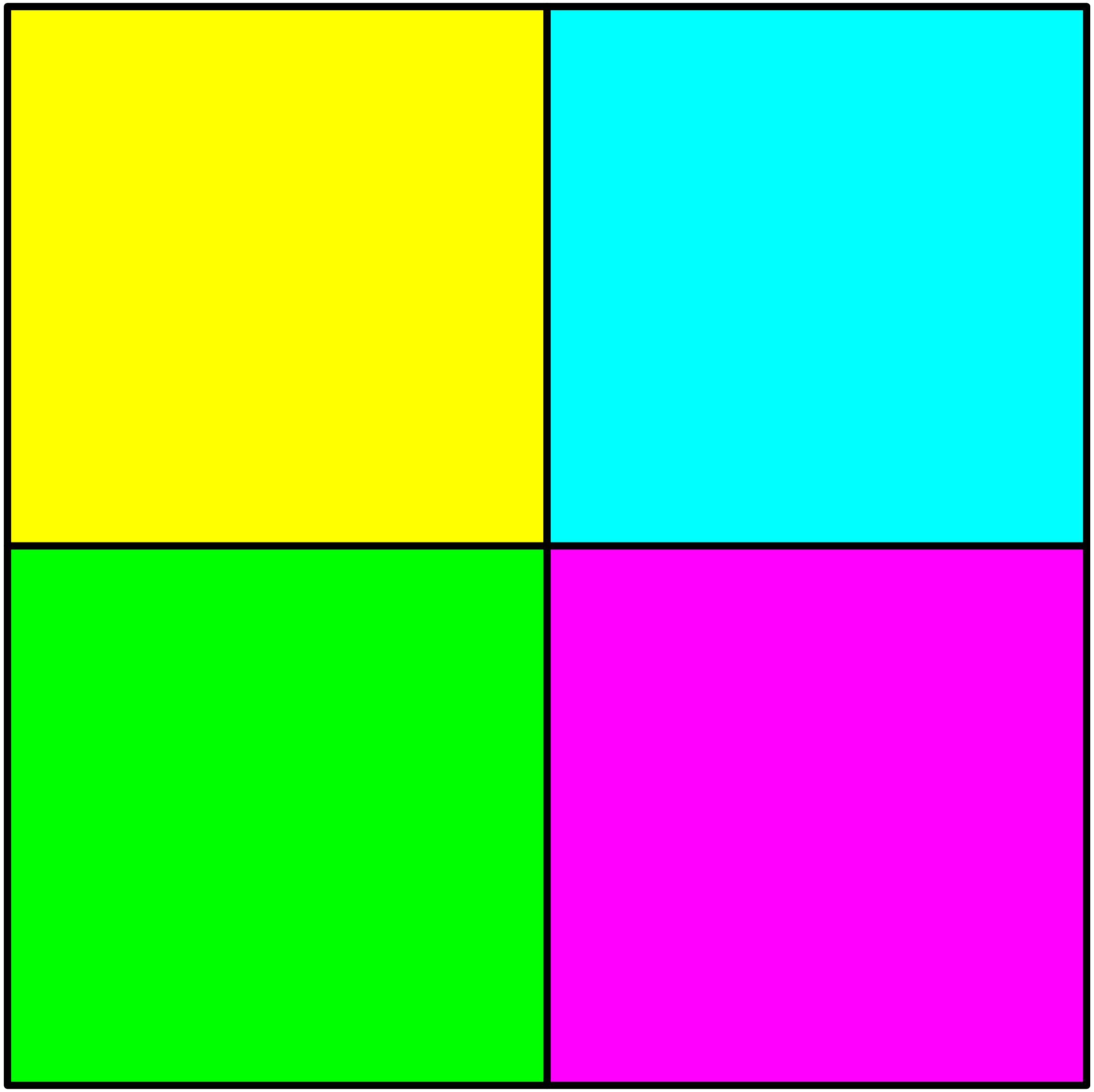

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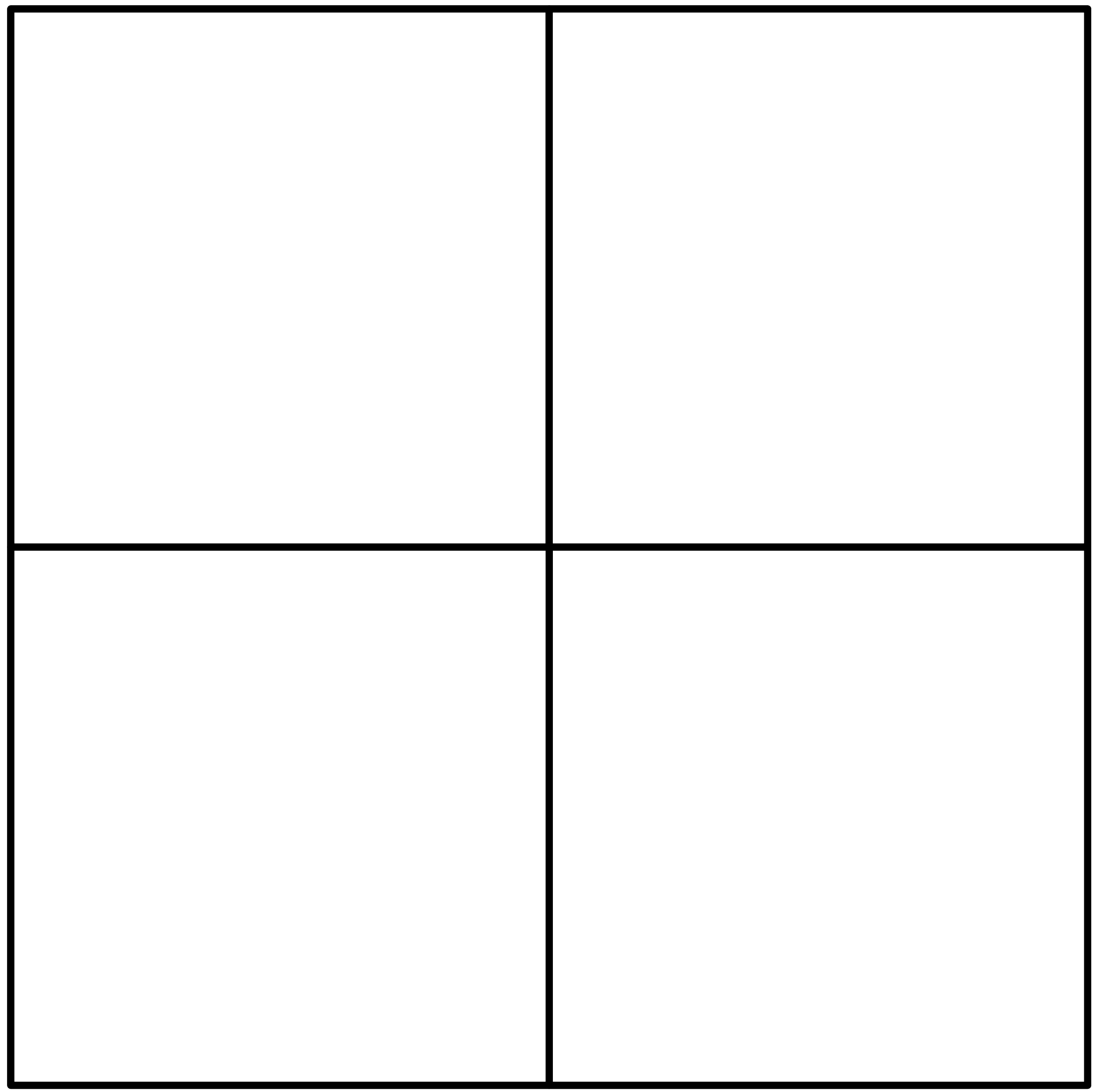

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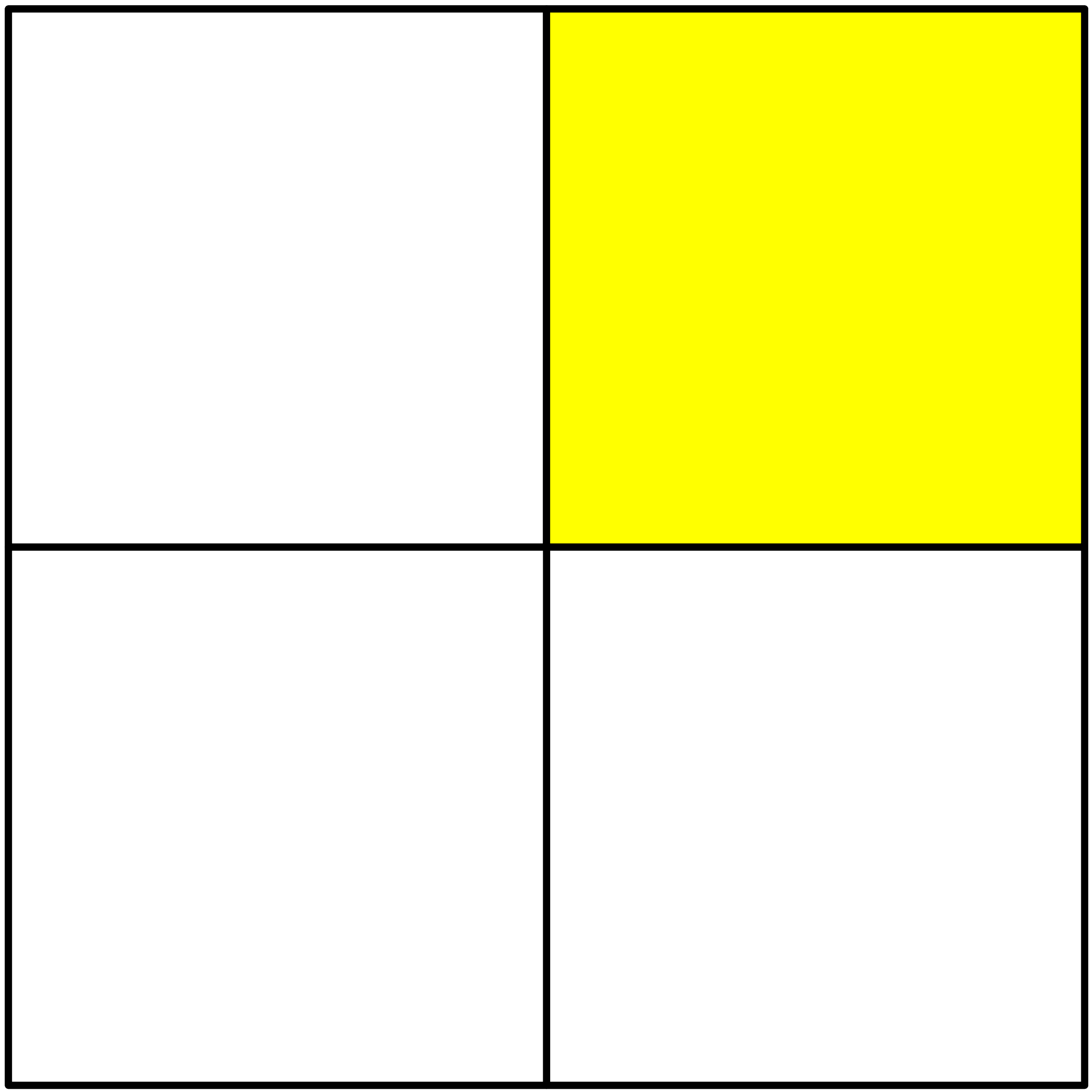

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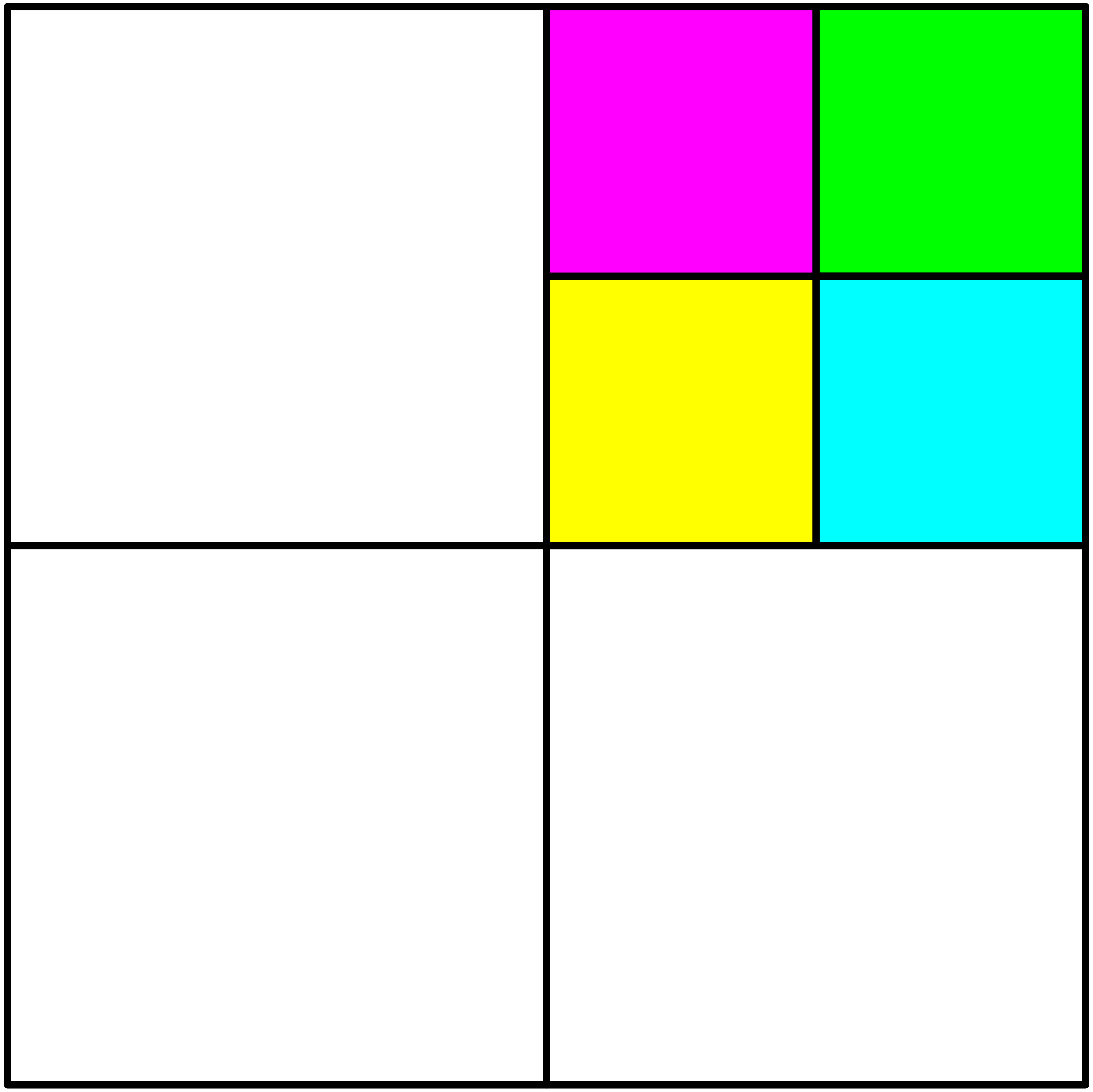

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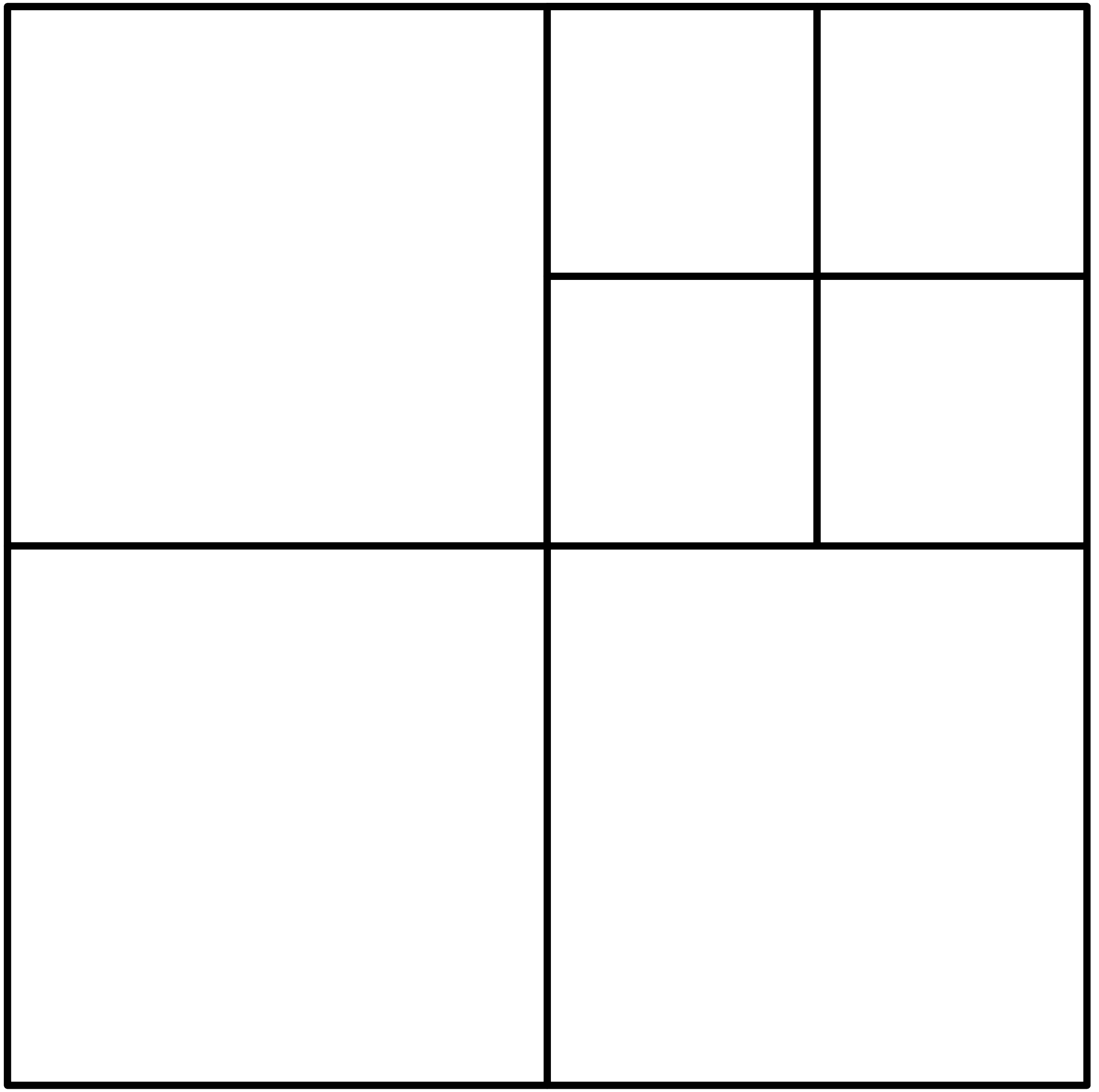

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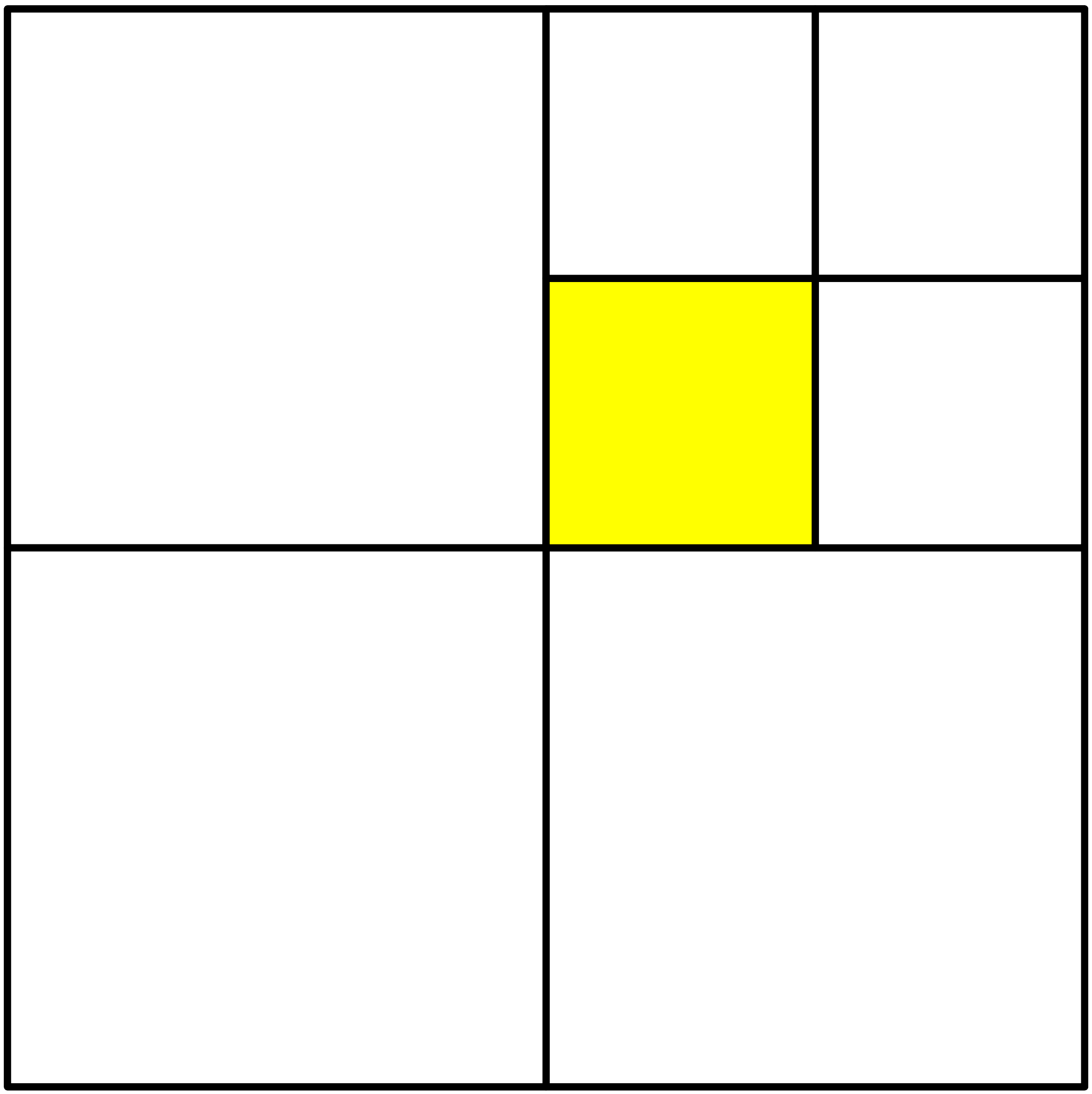

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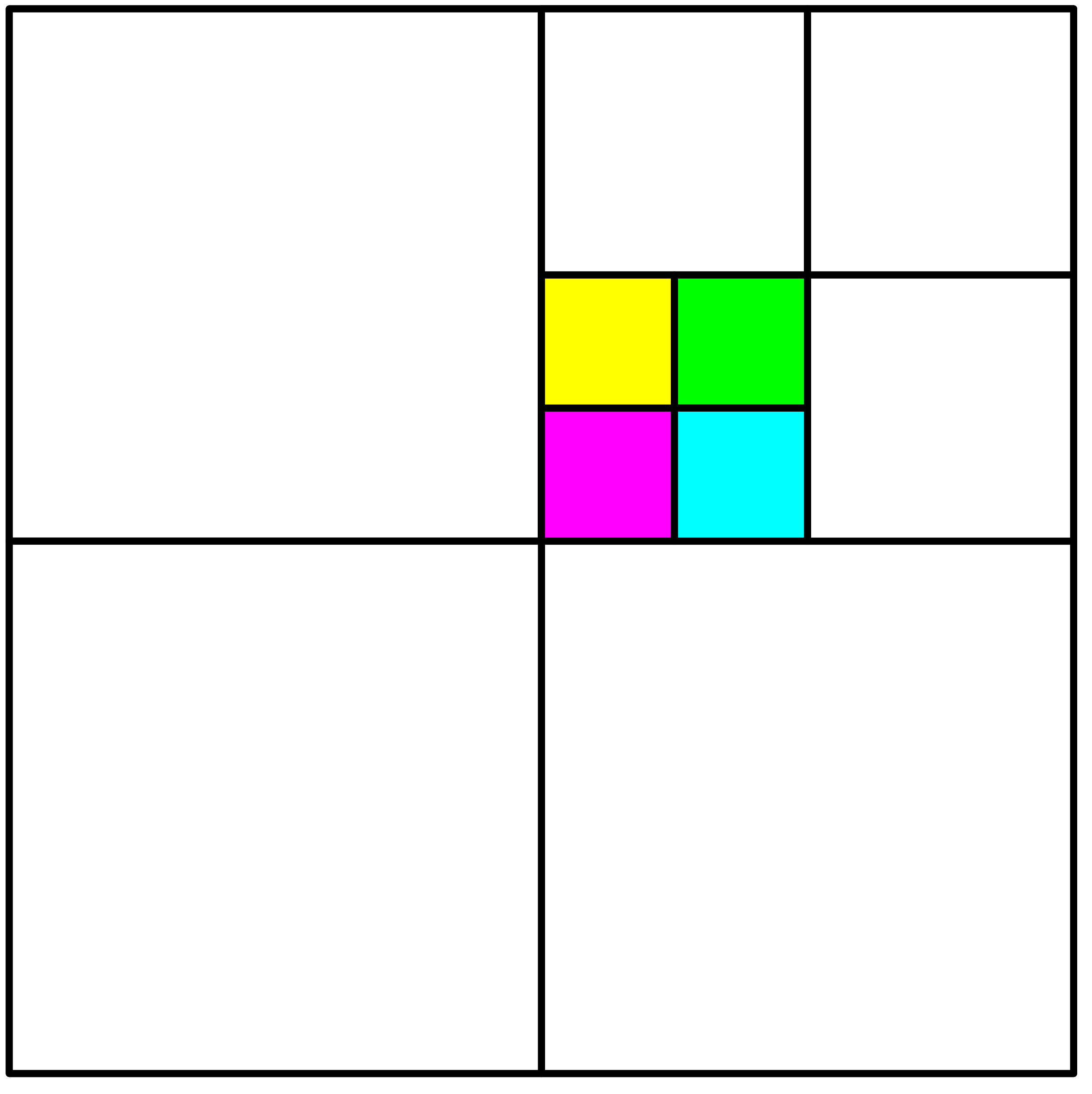

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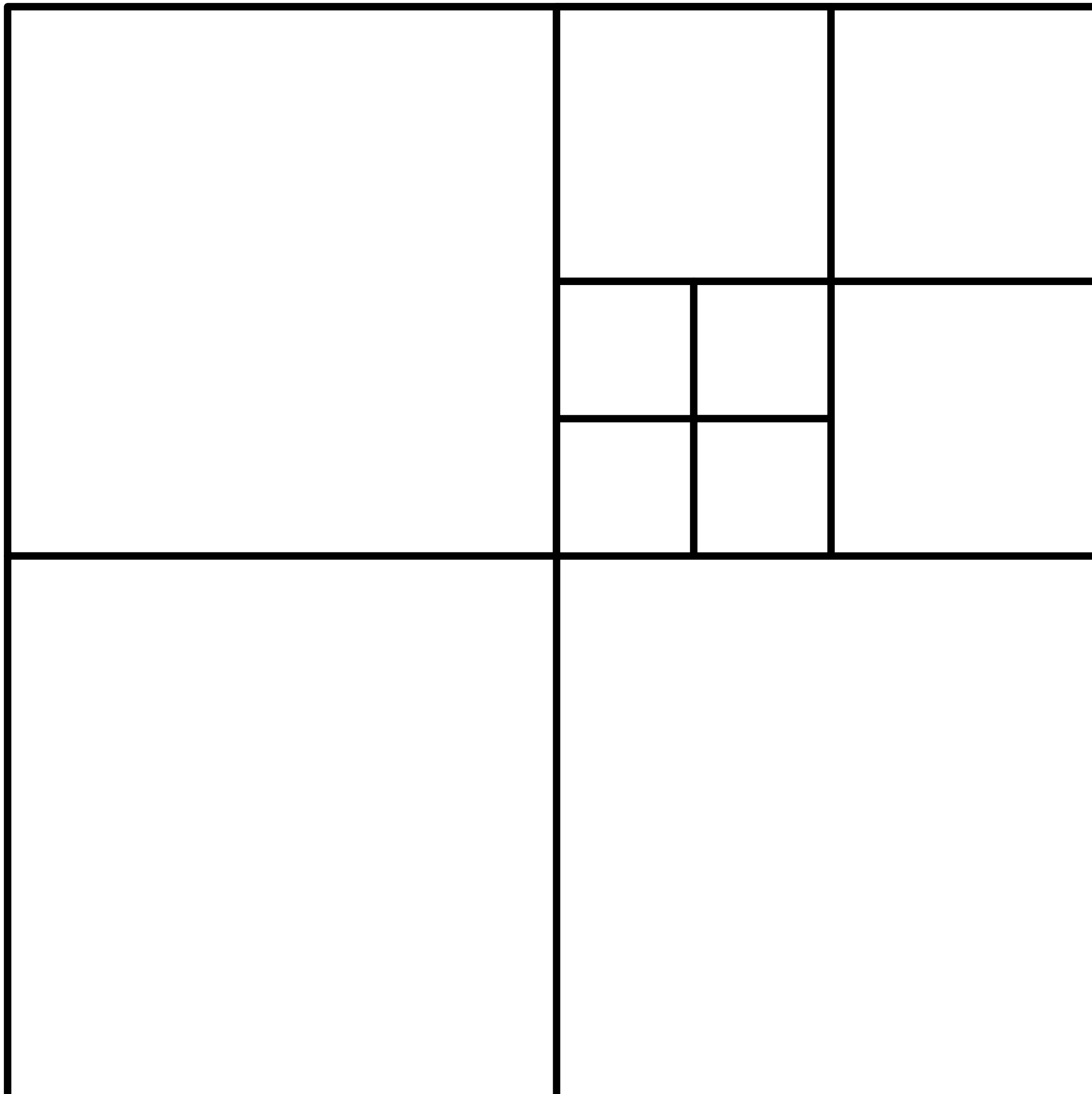

The Key Insight

- If we can subdivide a square into n squares, we can also subdivide it into $n + 3$ squares.
- Since we can subdivide a bigger square into 6, 7, and 8 squares, we can subdivide a square into n squares for any $n \geq 6$:
  - For multiples of three, start with 6 and keep adding three squares until n is reached.
  - For numbers congruent to one modulo three, start with 7 and keep adding three squares until n is reached.
  - For numbers congruent to two modulo three, start with 8 and keep adding three squares until n is reached.

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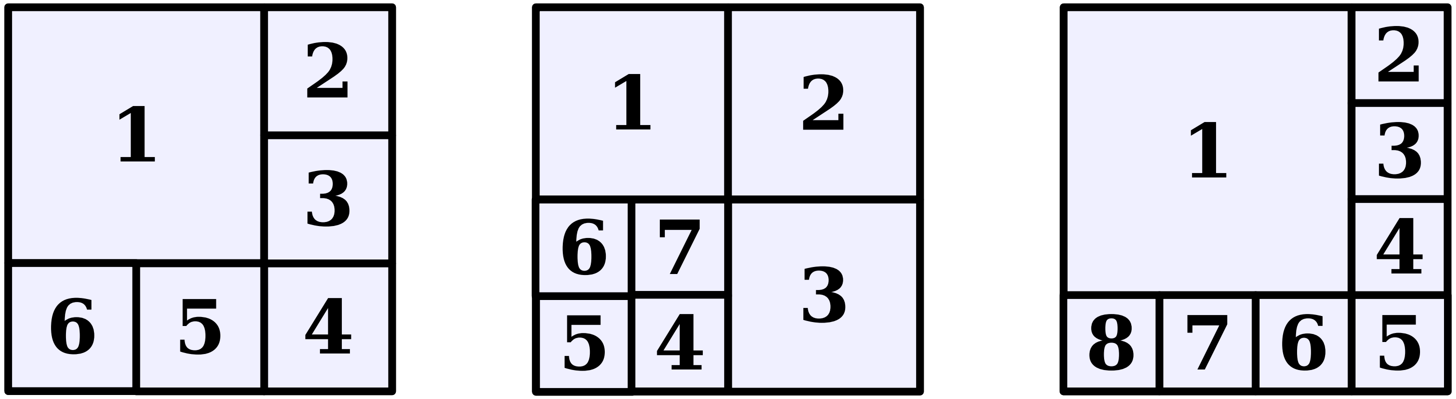

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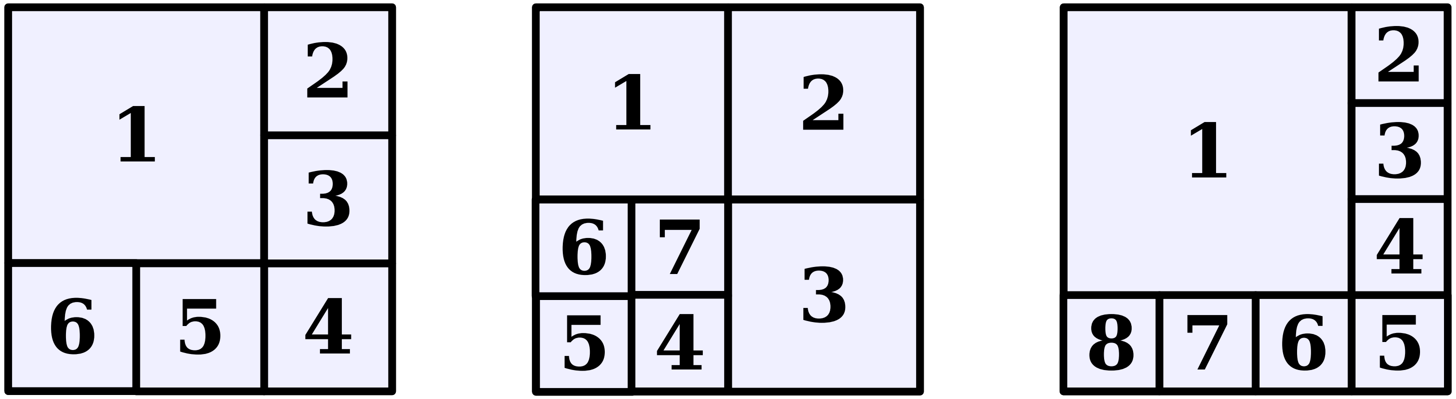

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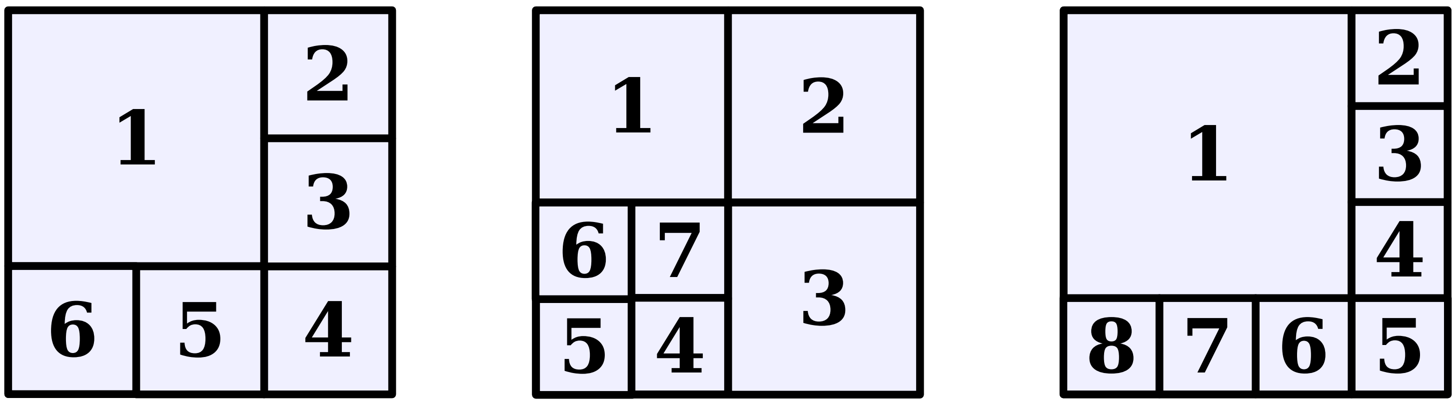

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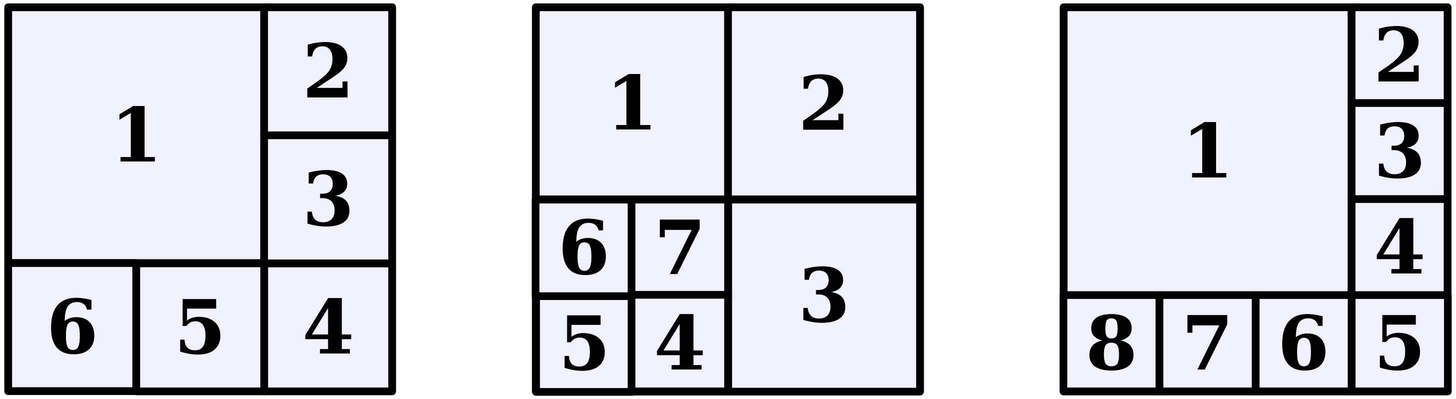

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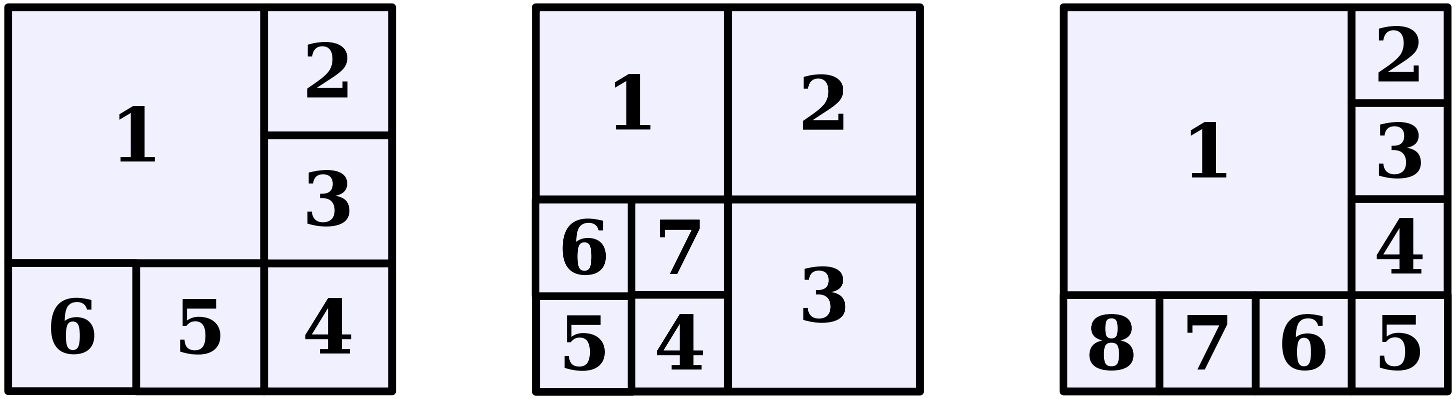

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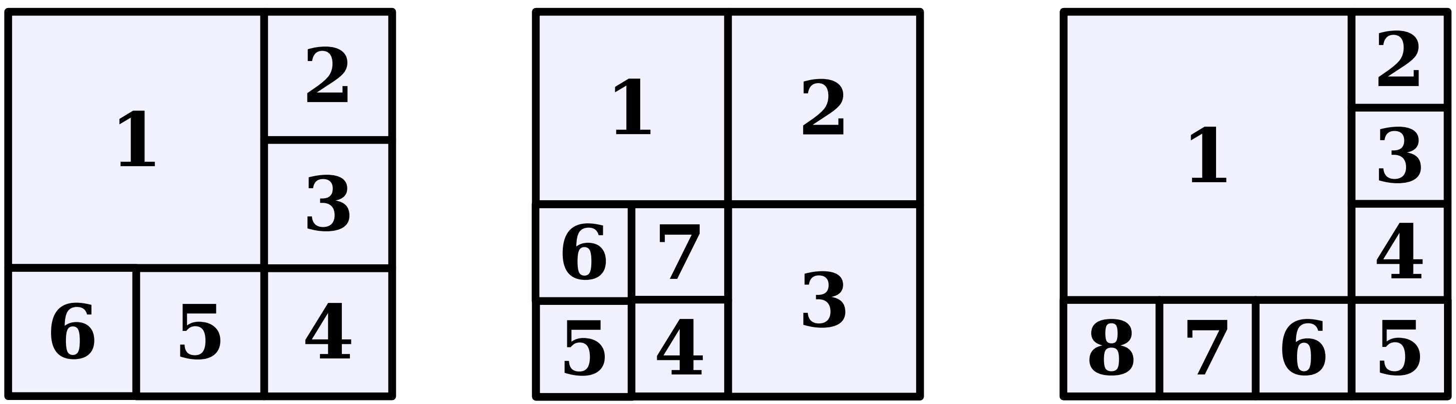

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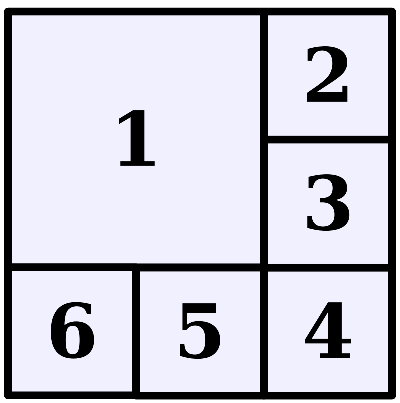

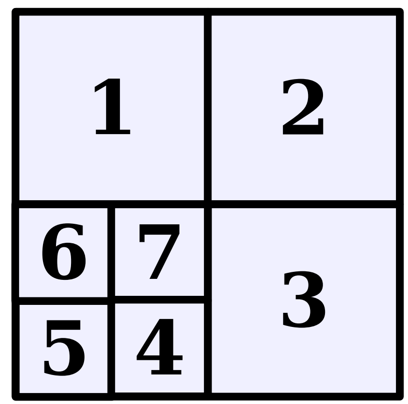

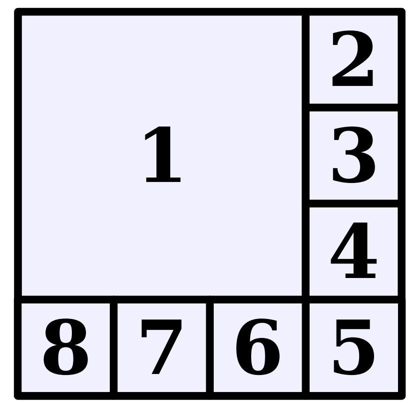

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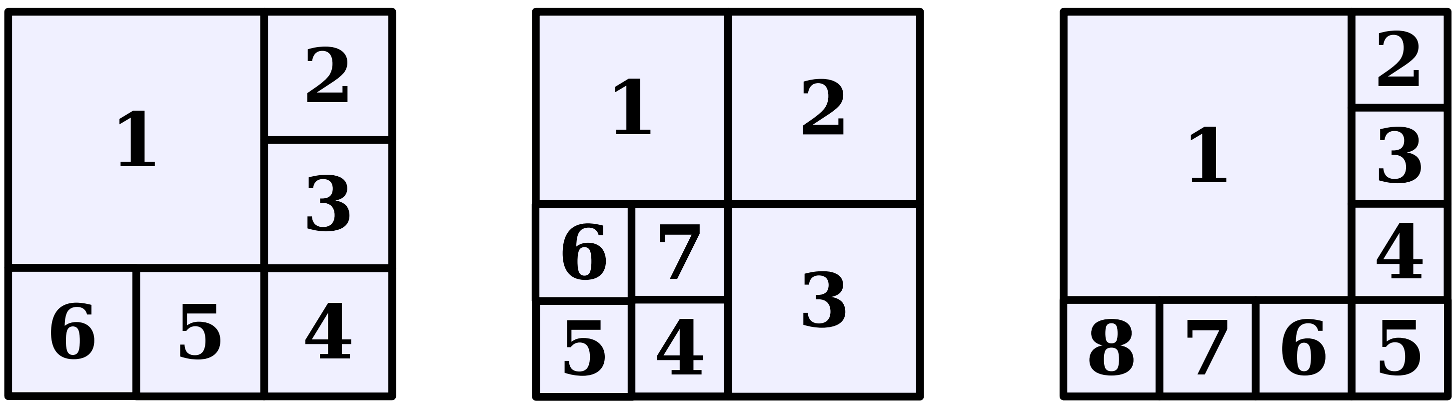

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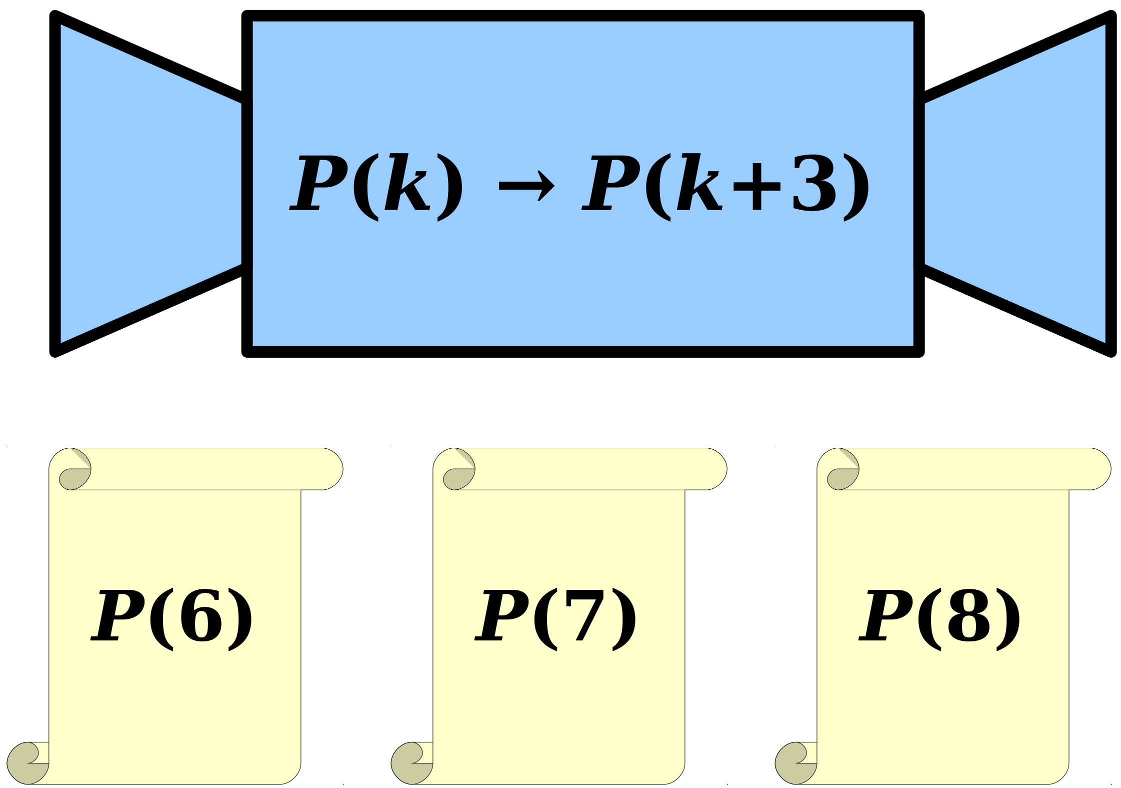
Why This Works

- This induction has three consecutive base cases and takes steps of size three.
- Thinking back to our "induction machine" analogy:



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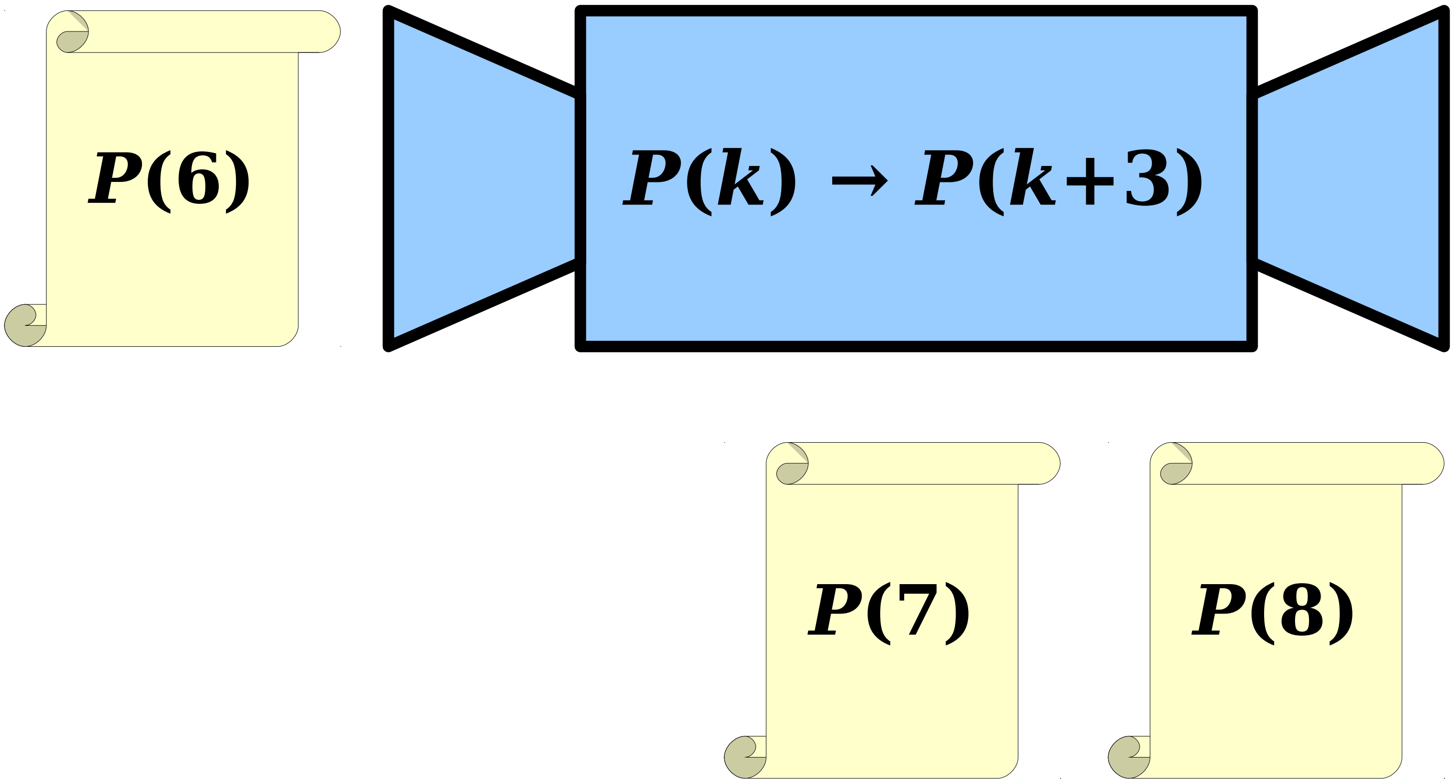

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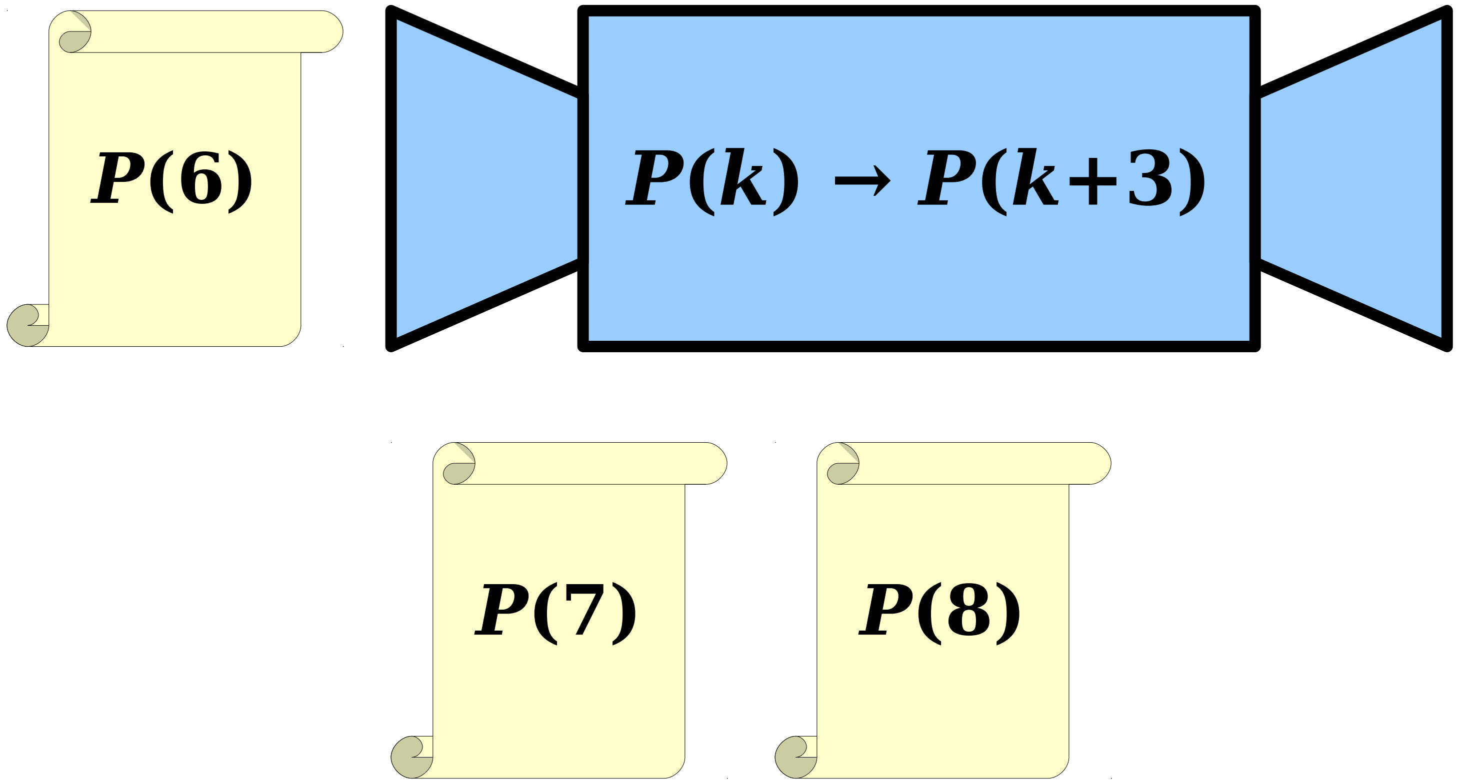

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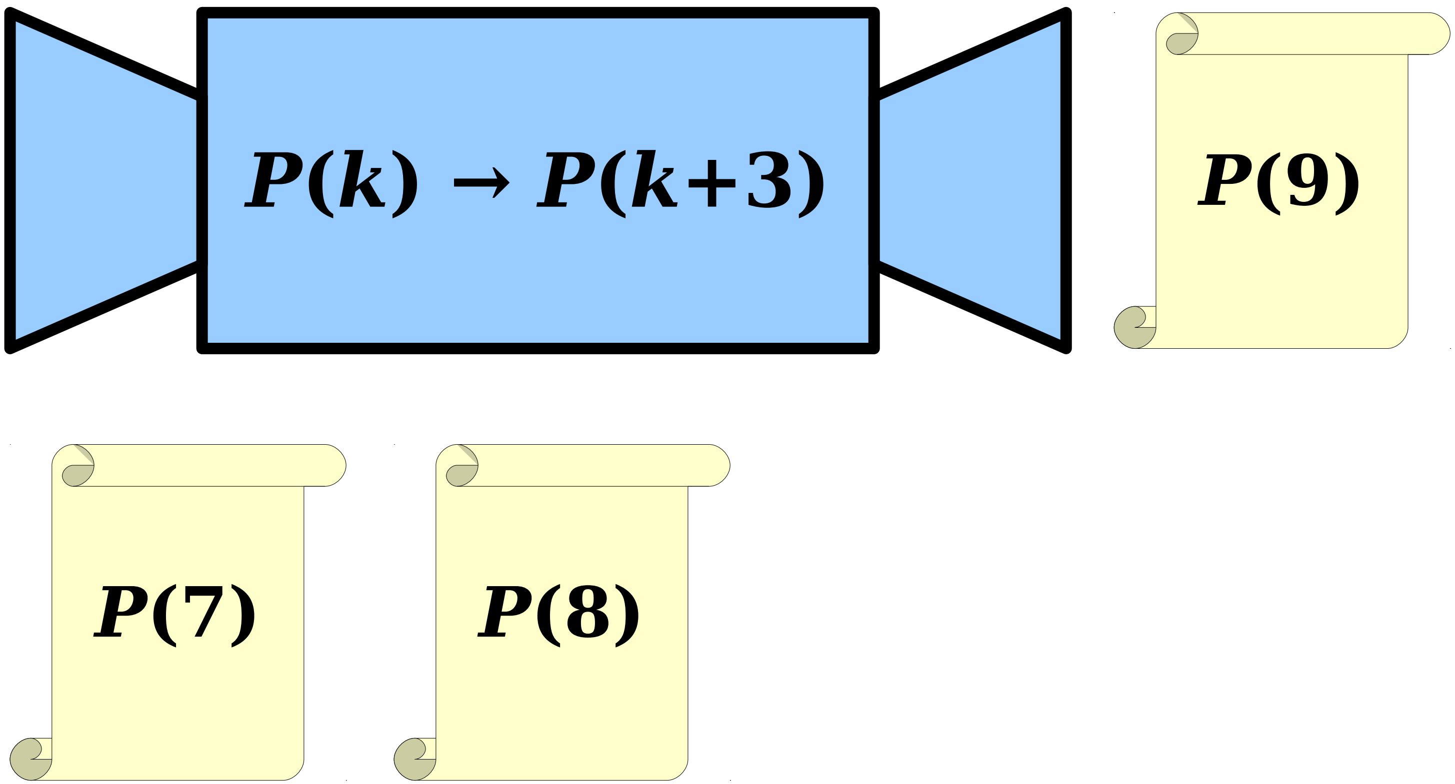

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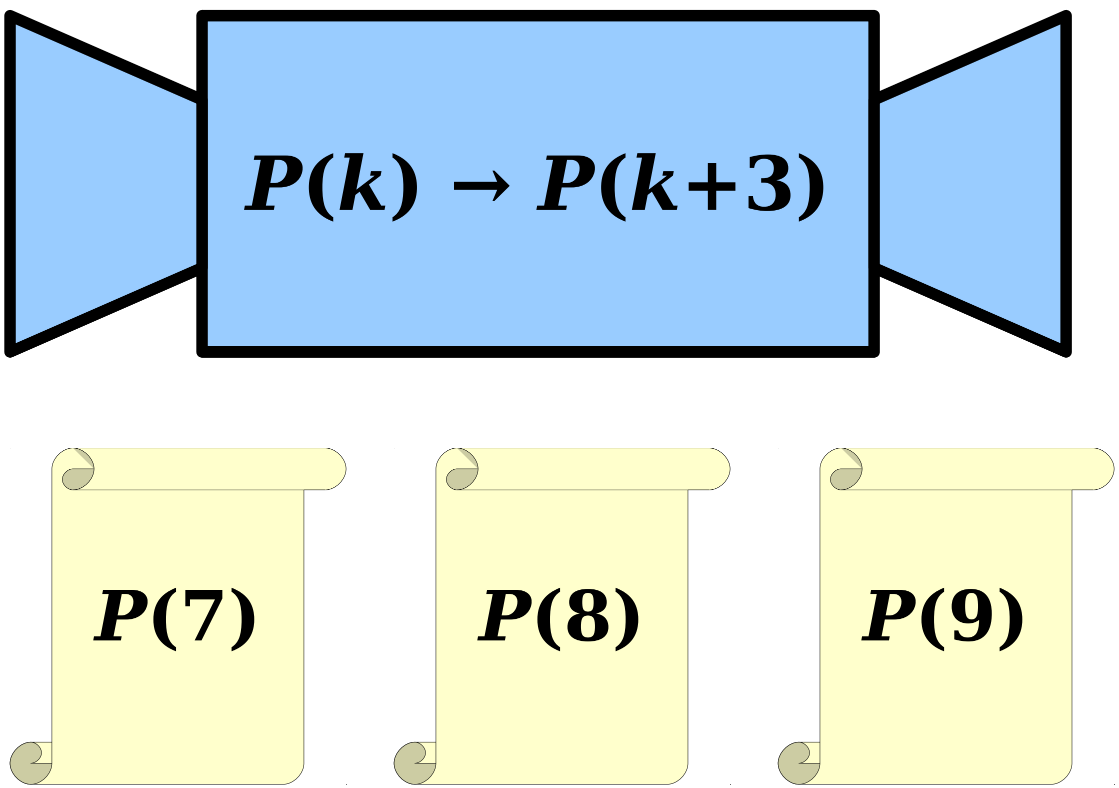

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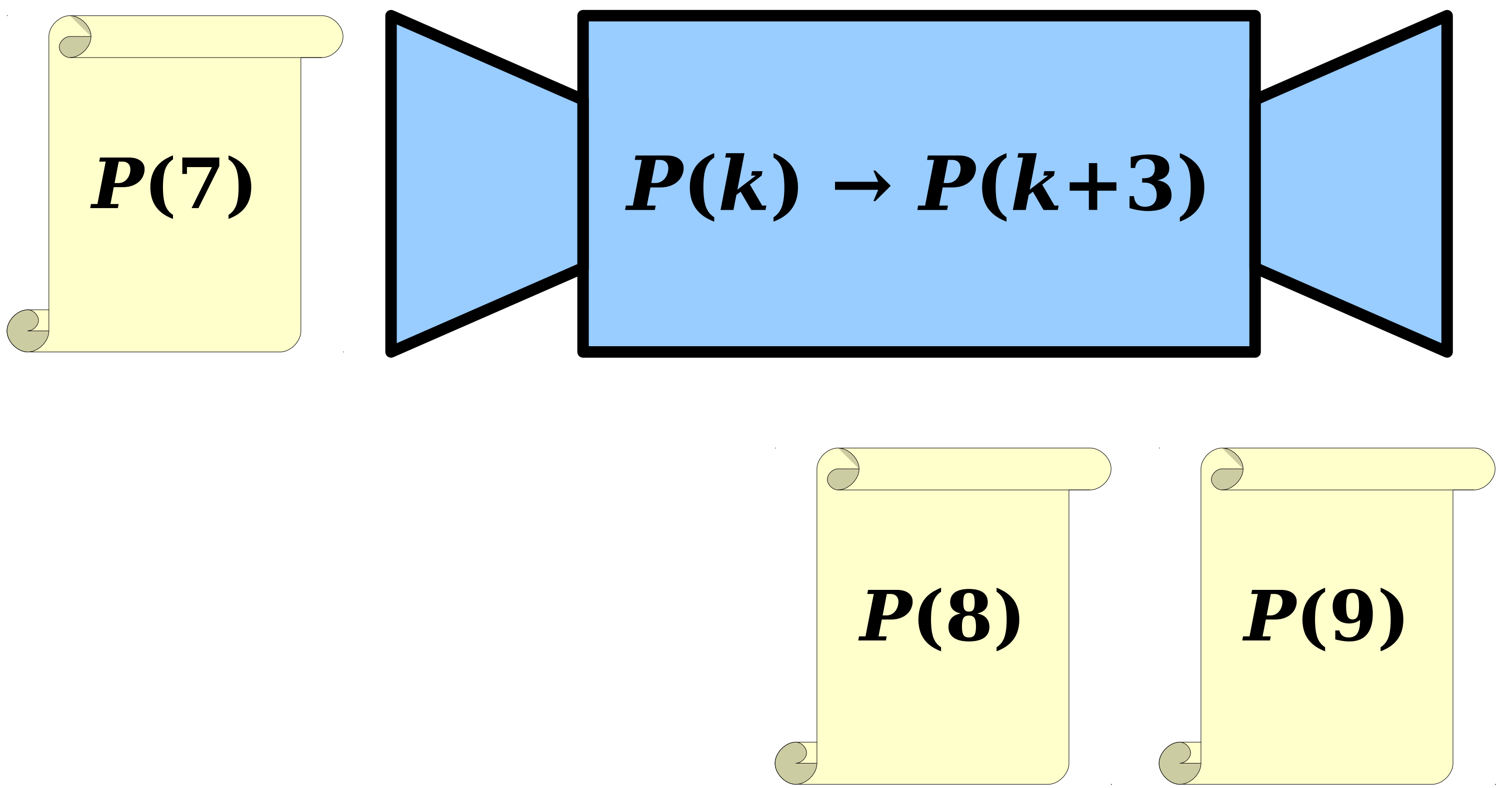

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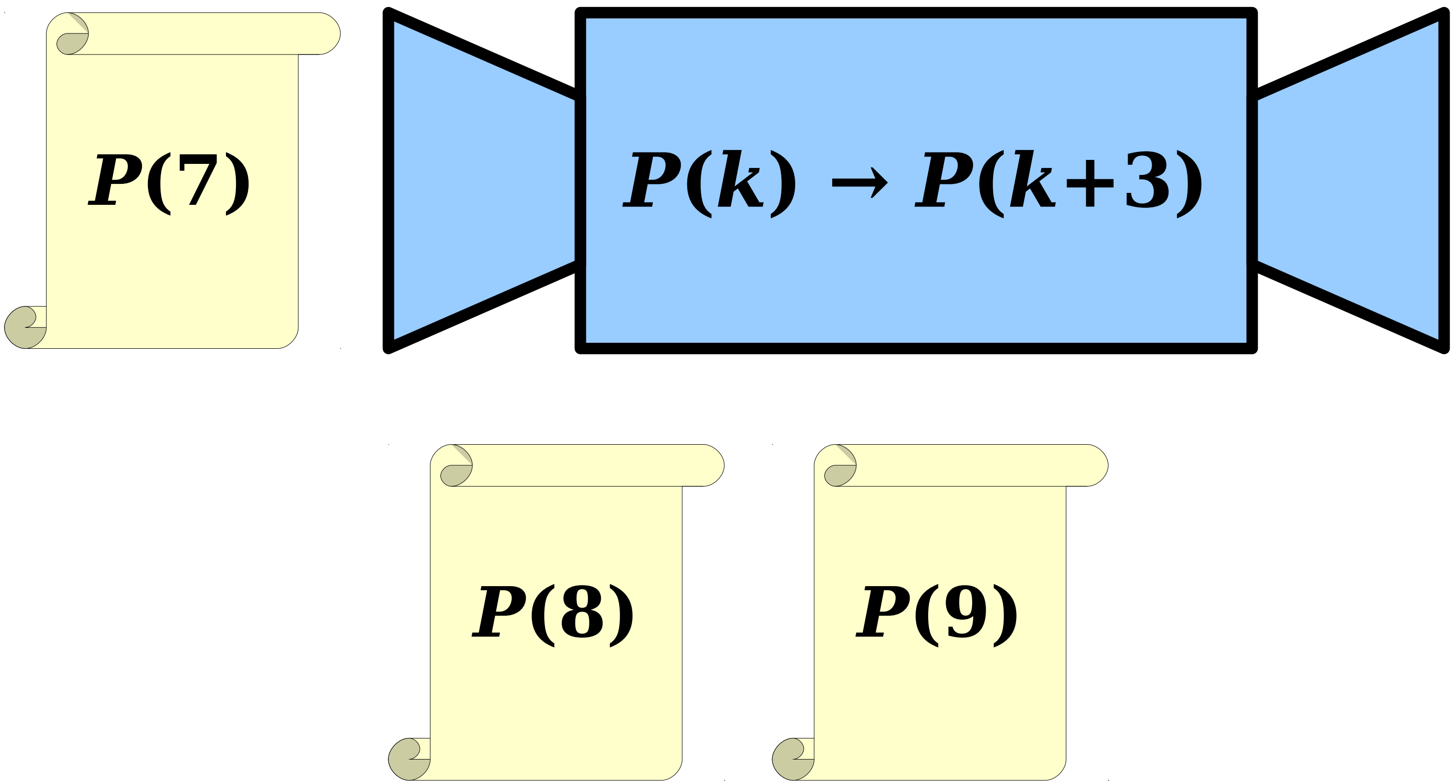

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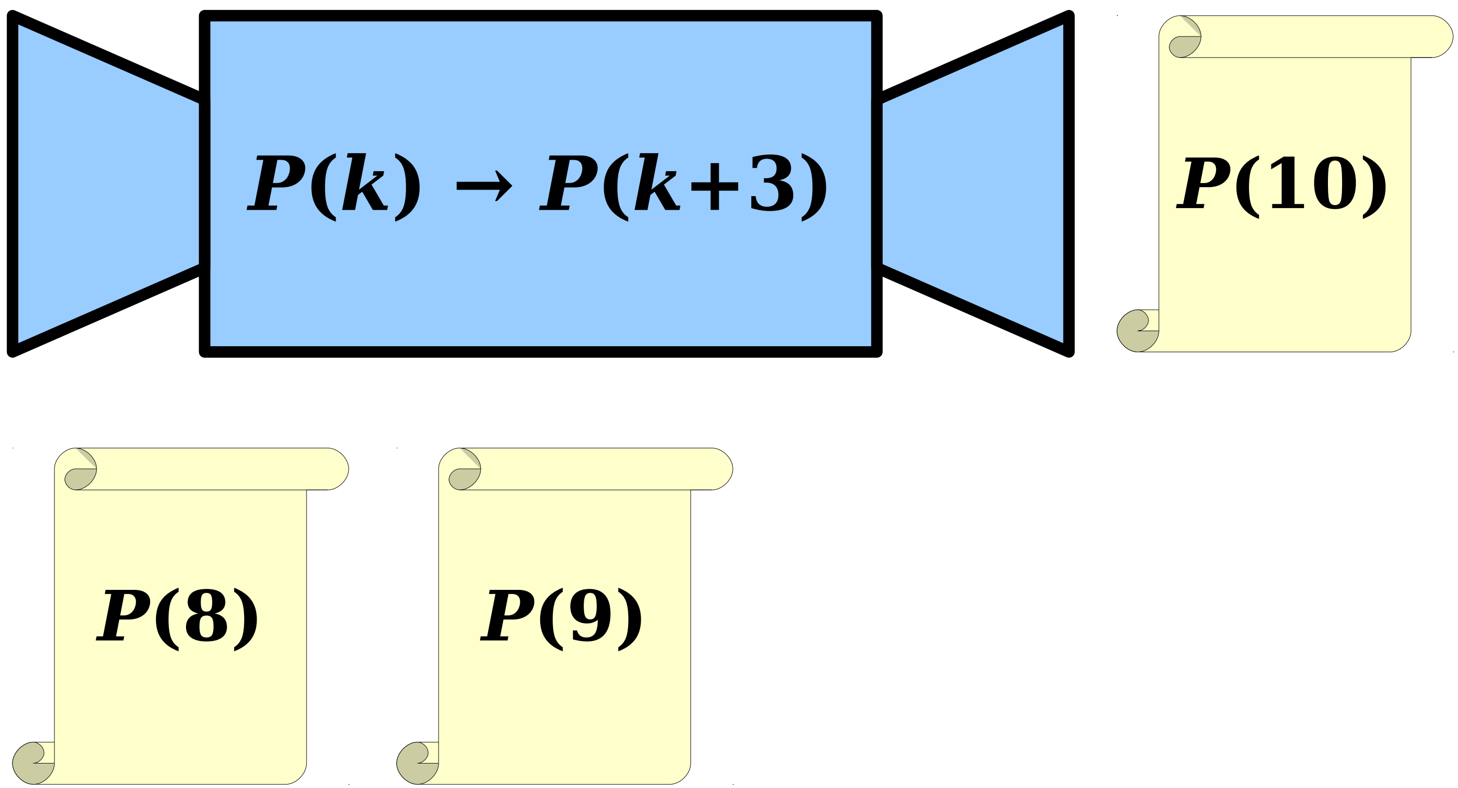

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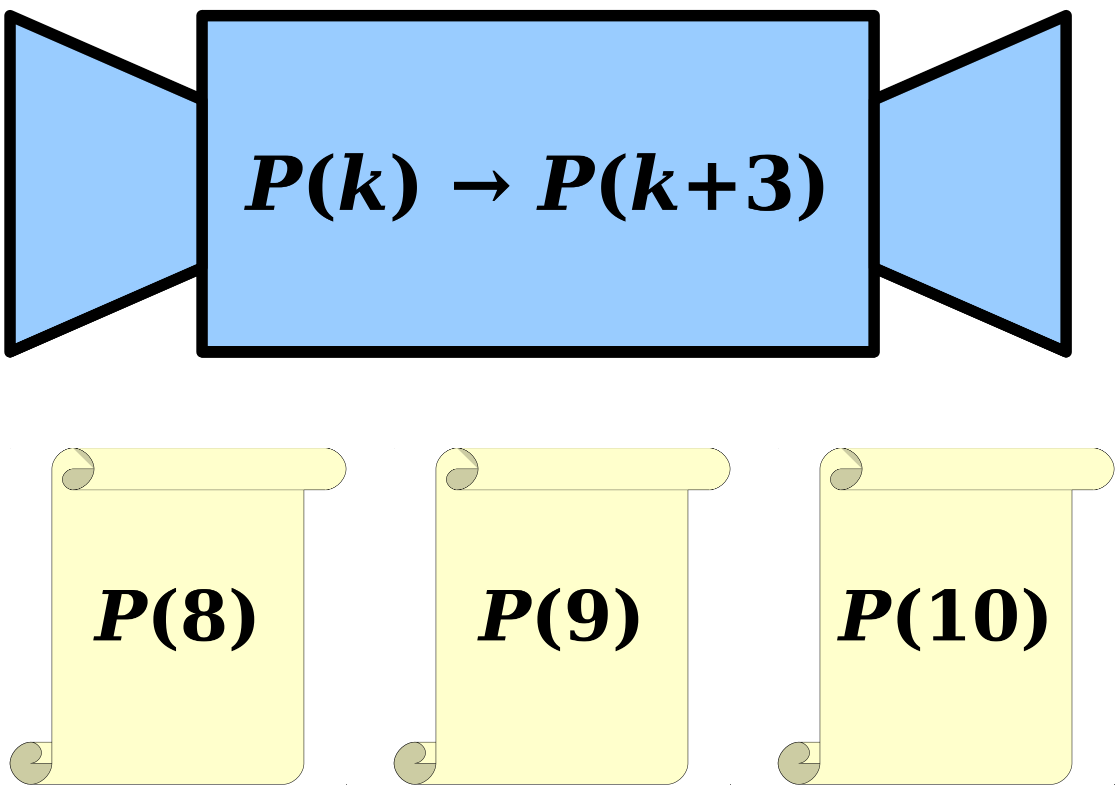

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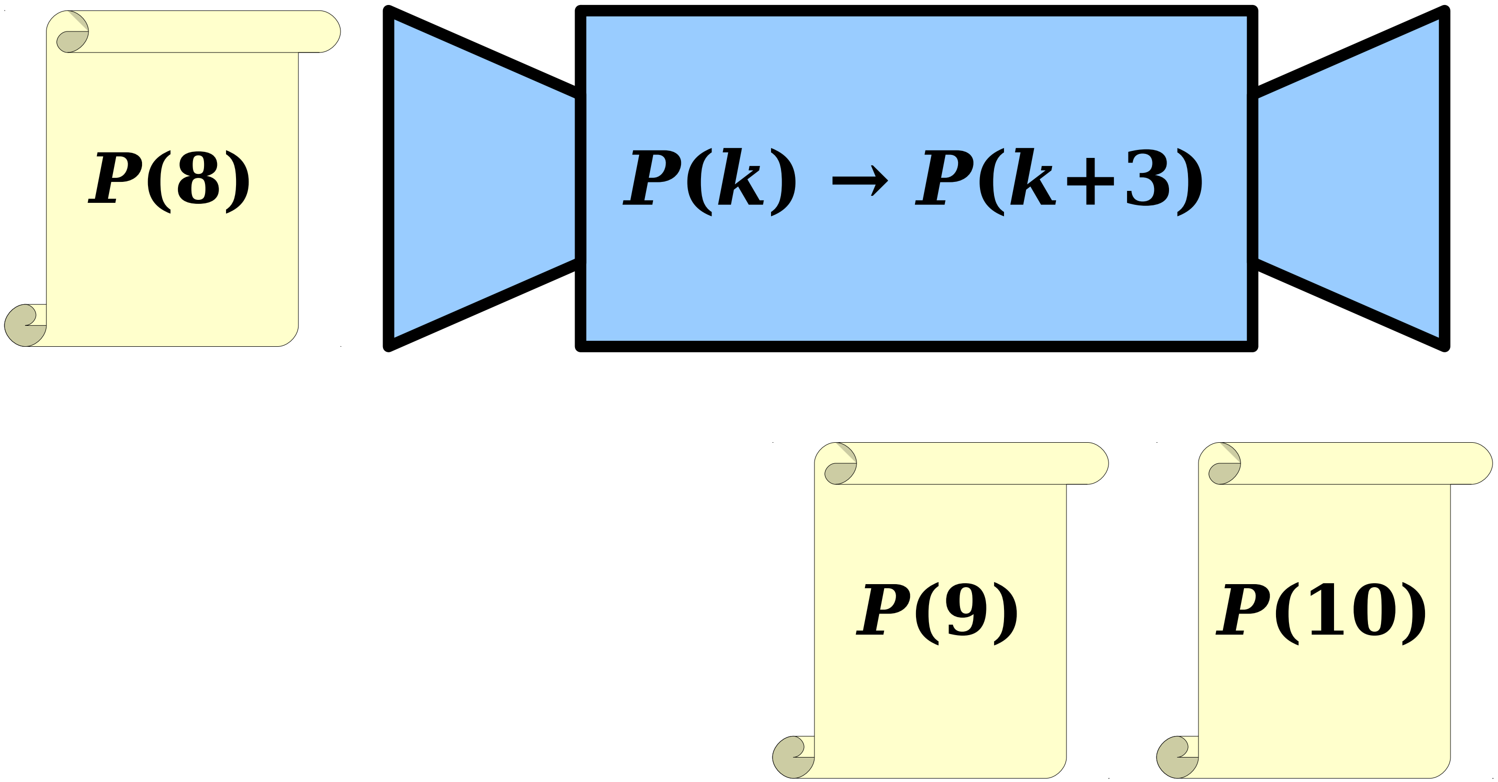

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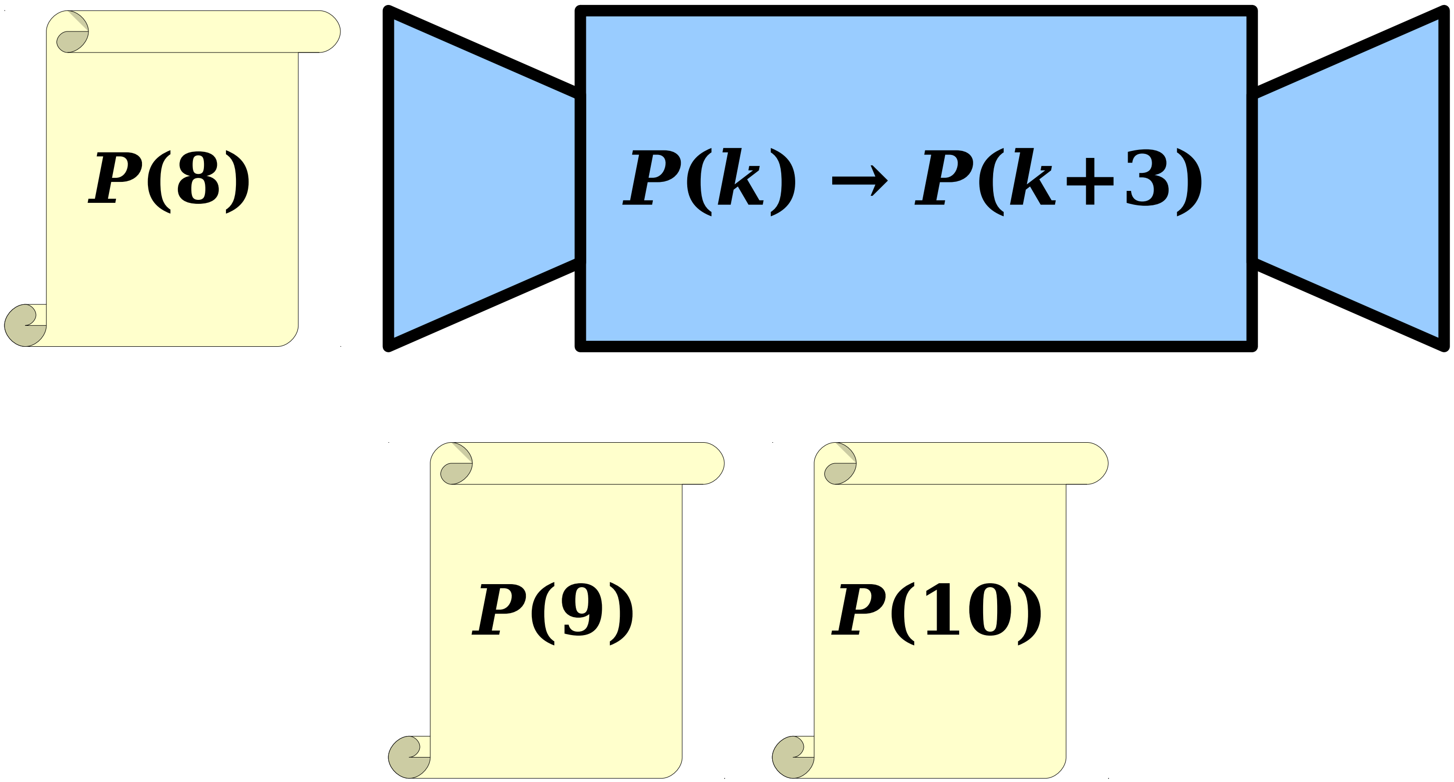

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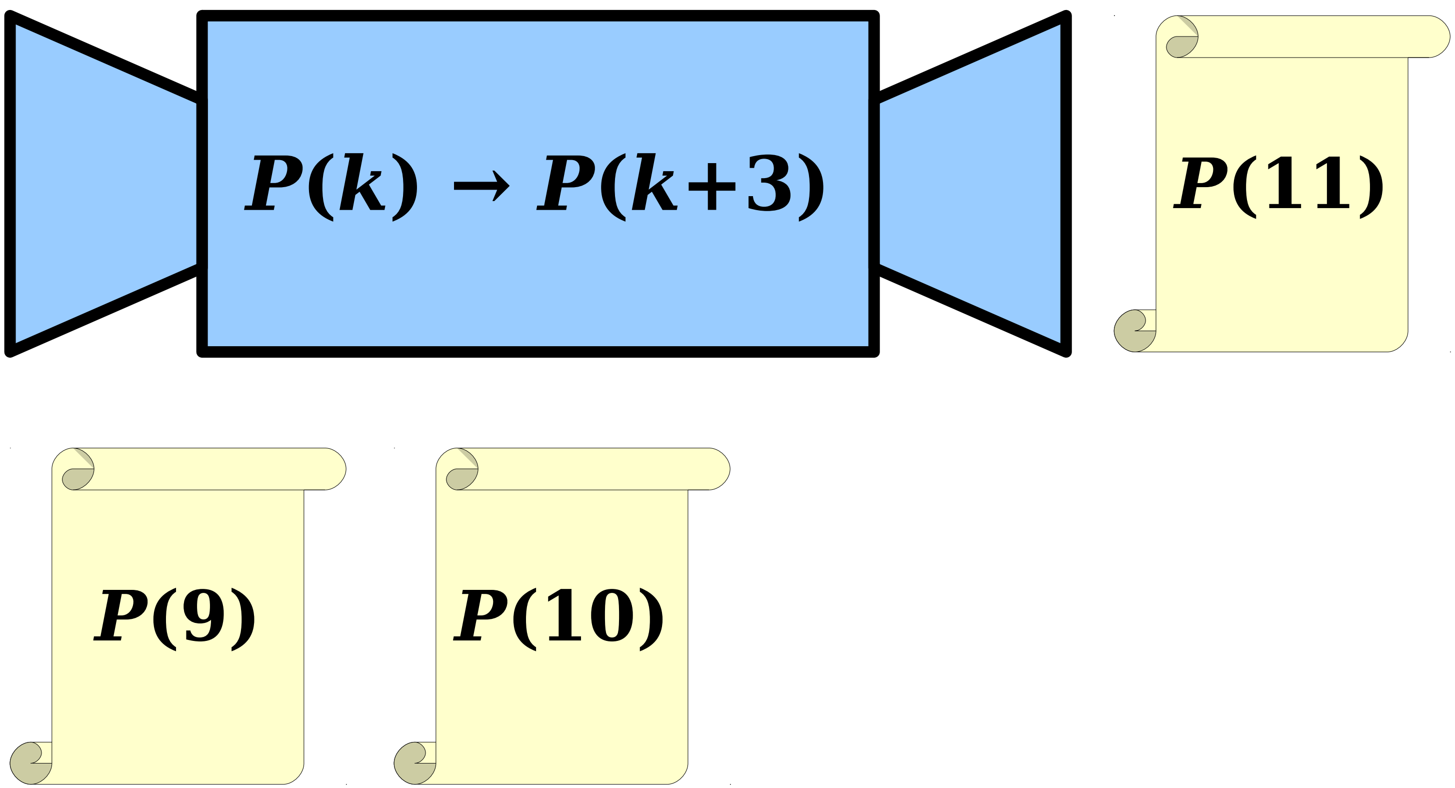

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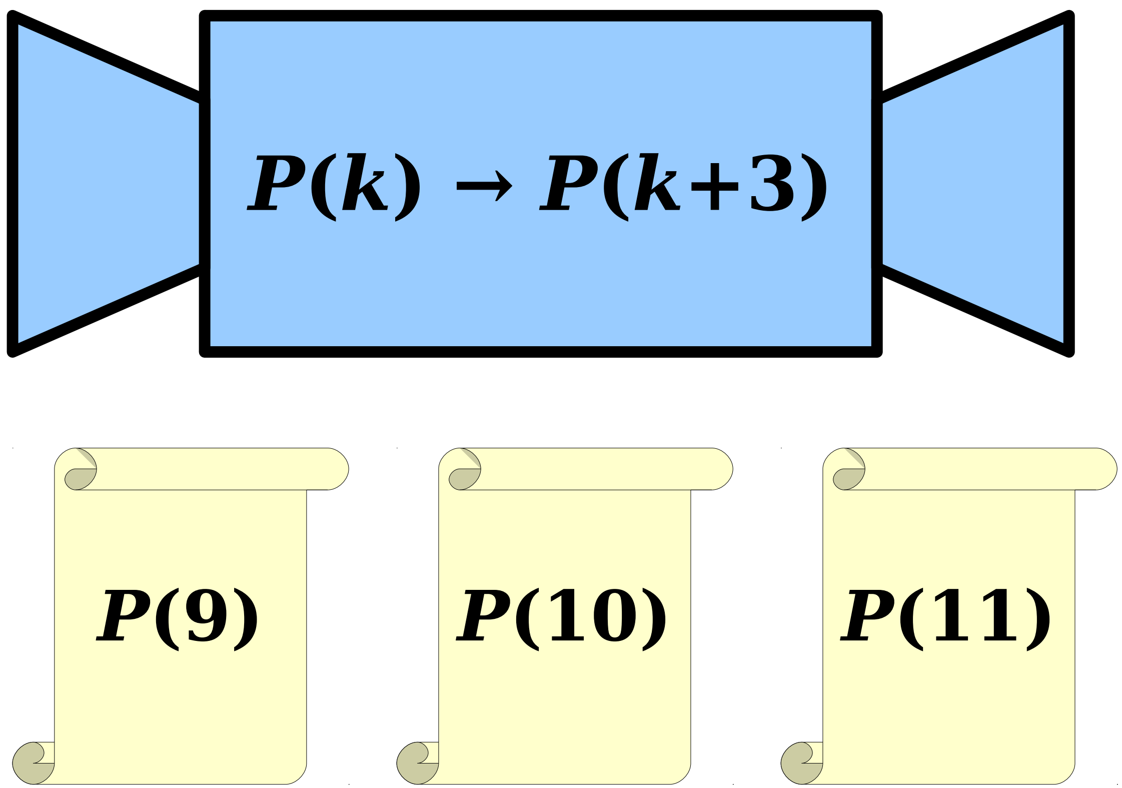


Generalizing Induction

- When doing a proof by induction,
  - feel free to use multiple base cases, and
  - feel free to take steps of sizes other than one.
- Just be careful to make sure you cover all the numbers you think that you're covering!
  - We won't require that you prove you've covered everything, but it doesn't hurt to double-check!

Time-Out for Announcements!

Midterm Reminder

- Our first midterm is this upcoming Monday from 7PM – 10PM.
- Room locations divvied up by last (family) name:
  - `Abd - Lin`: Go to **Cubberly Auditorium**.
  - `Liu - Raj`: Go to **370-370**.
  - `Ram - Zhu`: Go to **420-040**.
- Closed-book, closed-computer, limited-notes.
  - You can have a double-sided 8.5” × 11” sheet of notes when you take the exam.
- Covers material from PS1 – PS3 and Lectures 00–08.
- Need to take the exam at an alternate time for reasons other than OAE? Contact us *immediately*!

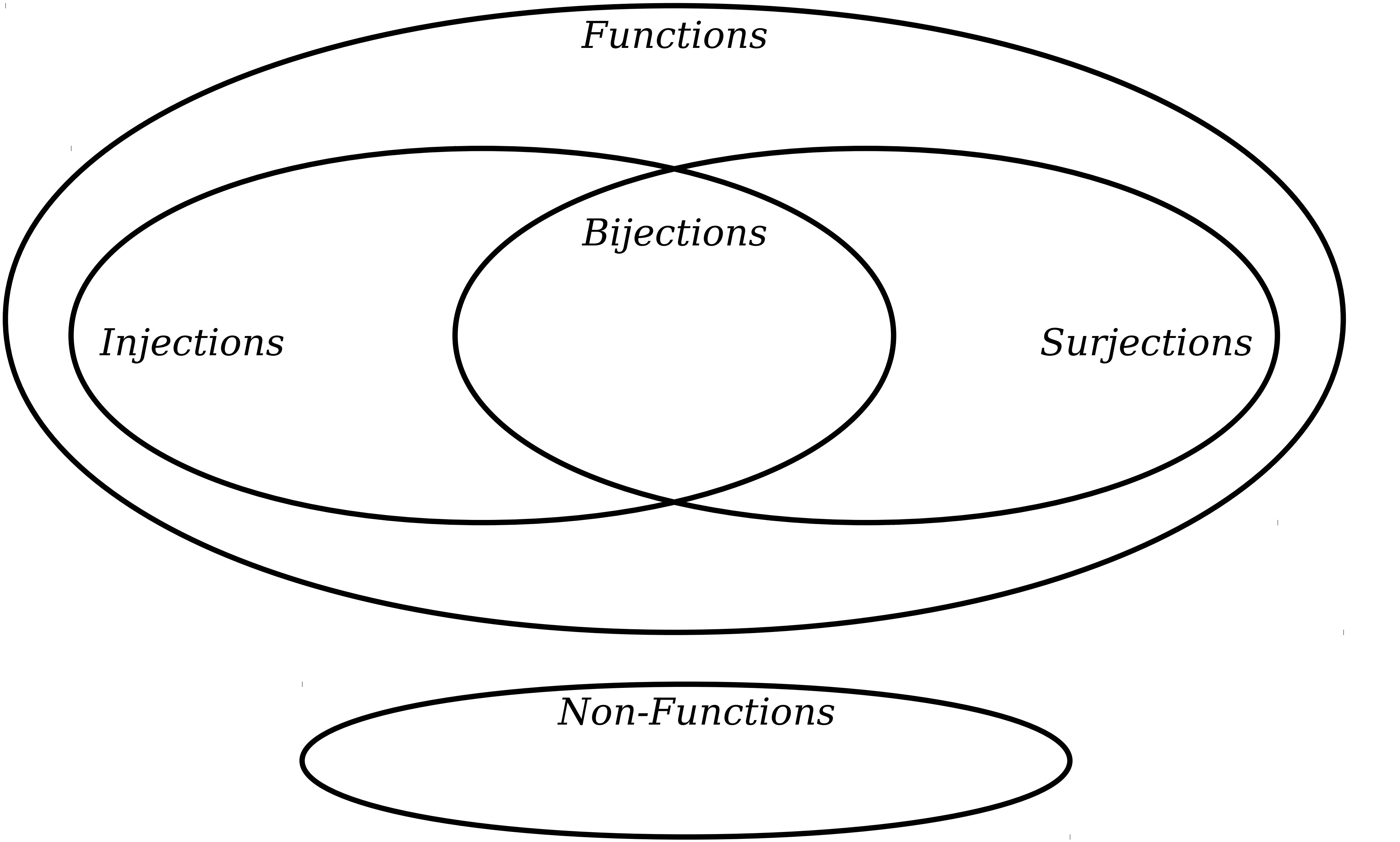
Practice Opportunities

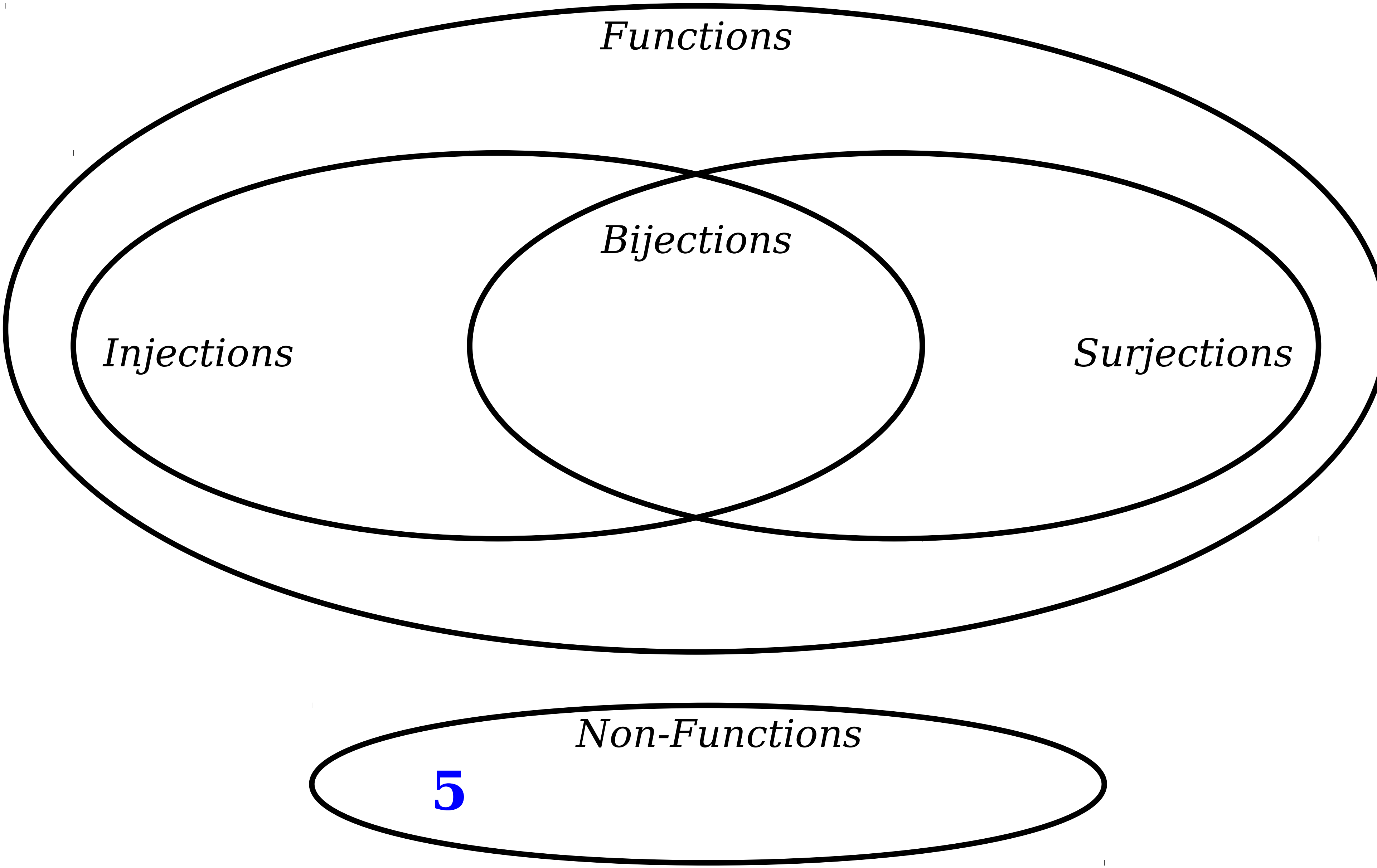
- There's a practice midterm ***tonight*** from 7PM – 10PM in Bishop Auditorium. Purely optional, but highly recommended!
- Solutions to Extra Practice Problems 1 have been released.
- Extra Practice Problems 2 has just been posted.
- Want more review on certain topics? Let us know what you want to see more of!

GTGTC

- Girls Teaching Girls To Code (GTGTC) is holding a Code Camp on campus on April 9th.
- Are you interested in running a one-hour programming section on a topic of your choice? If so, apply using **this link**.
- The deadline to apply ***tonight***. Sorry for the delay in announcing this!
- This is a phenomenal program. Highly recommended!

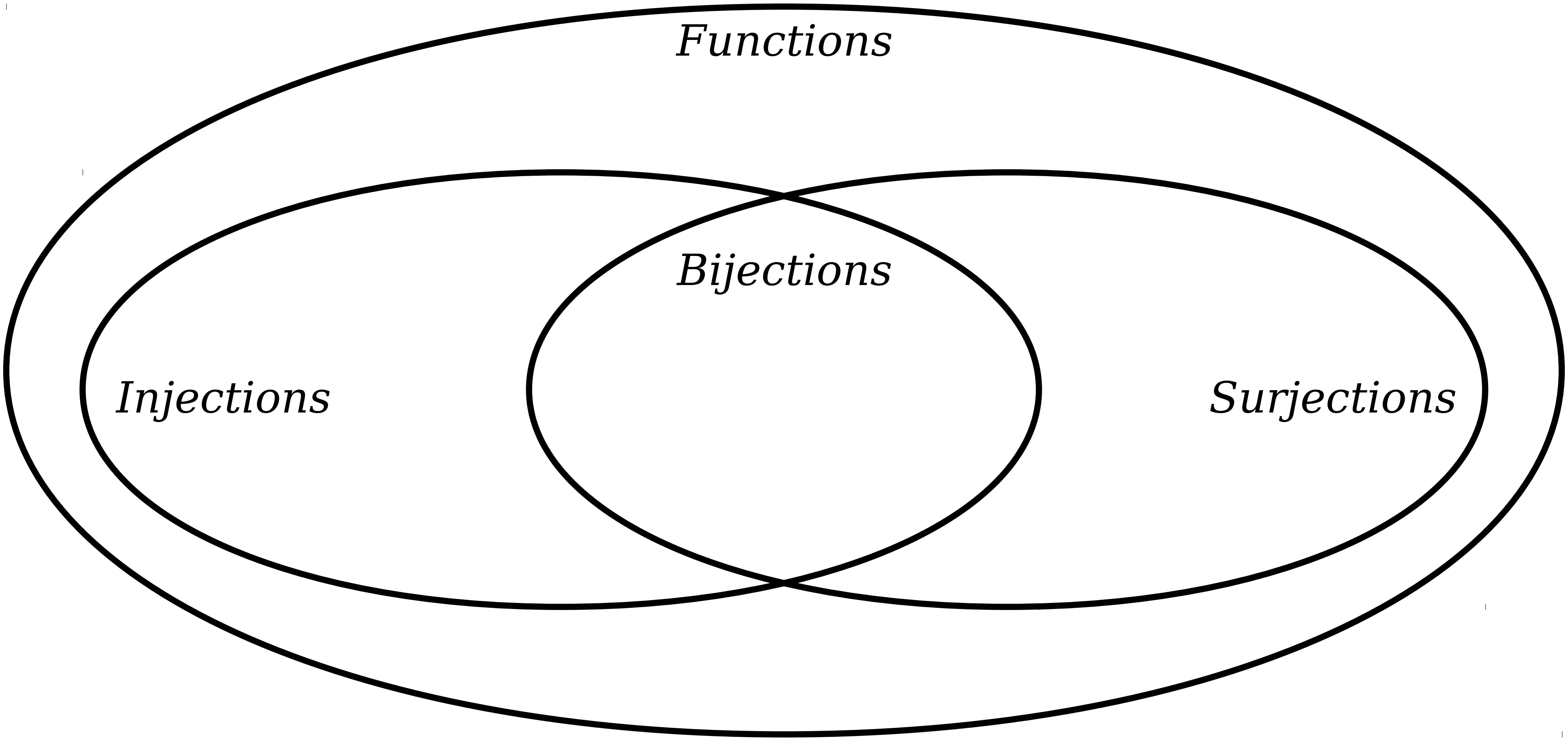
Problem Set Three: Common Mistakes

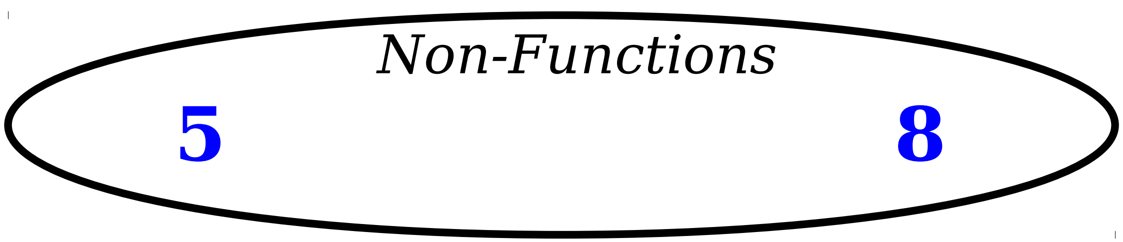

Functions
Bijections
Injections
Surjections
Non-Functions



5. $f : \mathbb{R} \rightarrow \mathbb{N}$ defined as $f(n) = n^2$

What is $f(½)$?





8. $f : \mathbb{R} \to \mathbb{R}$ defined as $f(n) = \sqrt{n}$

What is f(-1)?

A function ***must*** be defined for every element of the domain.

A function ***must*** only produce outputs that are elements of the codomain.

Theorem: If $A \subseteq B$, then $|A| \leq |B|$.

Proof: We will show that there is an injection $f : A \to B$. To do so, consider any a_1, a_2 where $f(a_1) = f(a_2)$. Because B contains all the elements of A, plus possibly some other elements, we see that $a_1 = a_2$, so f is injective, as required. ■

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What is this function f?

Without knowing anything about f other than its domain and codomain, it's impossible to prove anything about f!

Theorem: $|\wp(\mathscr{U})| \leq |\mathscr{U}|$.

Proof: Since $\mathscr{U}$ contains everything, we know that $\wp(\mathscr{U}) \in \mathscr{U}$. Therefore, $|\wp(\mathscr{U})| \leq |\mathscr{U}|$ ■.

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Does this line of reasoning work?

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Does this line of reasoning work?

Be careful not to confuse the $\in$ and $\subseteq$ relations – they're not the same thing!

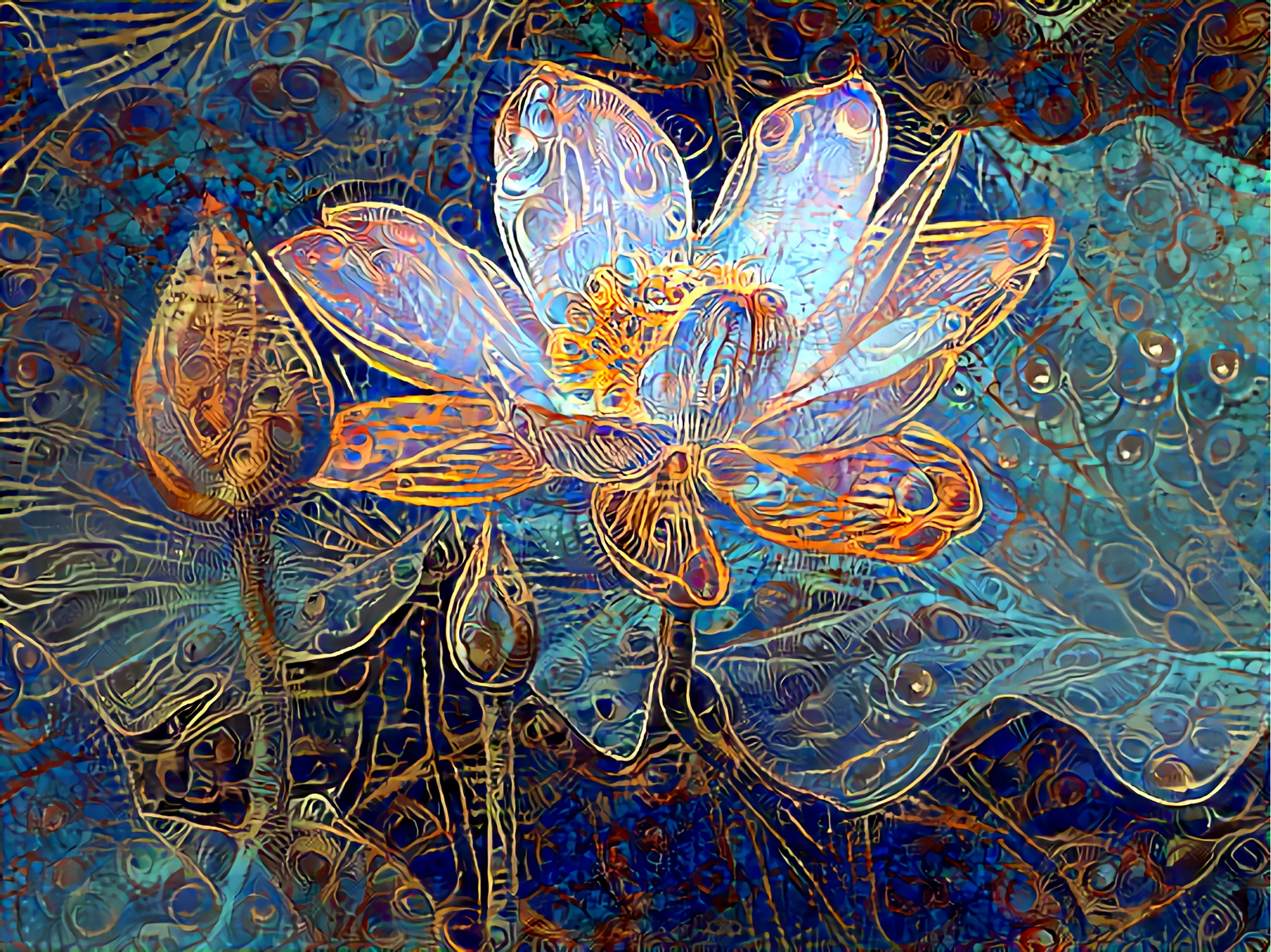
Your Questions

“How do you feel about the future of AI? All those important people in tech are scaring me.”

I think it depends on what you're worried about. If you're worried about a robot apocalypse, I'd heed the advice of Prof. Andrew Ng: “worrying about a robot apocalypse is like worrying about overpopulation on Mars.” It's something we eventually need to think about, but we're nowhere close yet.

If you're worried about people losing their jobs as everything gets automated and what impact that will have on society, then I think you have a good reason to be concerned. Prof. Fei-Fei Li, whose opinions I respect, is worried there's not enough empathy in the field of AI. I think the “right” way to address this is to get more cross-talk between AI researchers and social scientists (and to broaden participation in AI in general).

That said, modern AI has its upsides…



Check out the original paper here:
http://arxiv.org/abs/1508.06576

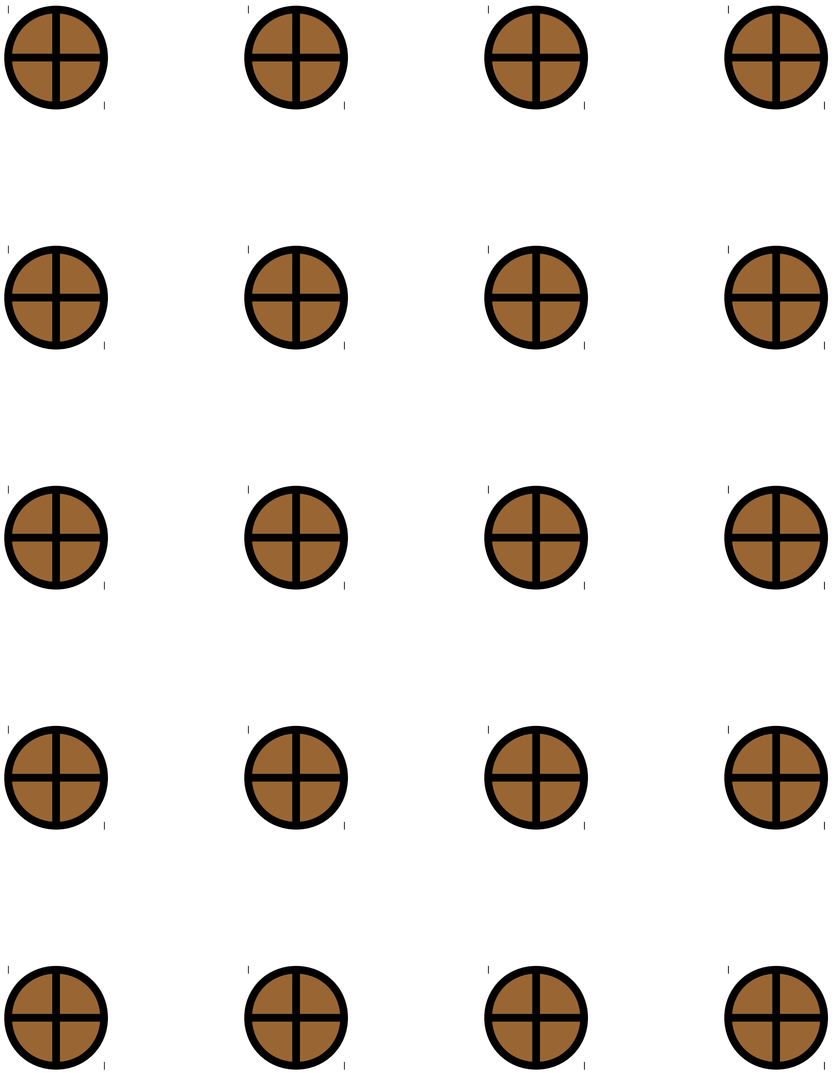
There's an implementation online at
https://github.com/jcjohnson/neural-style.

Back to CS103!

A Motivating Question: ***Rat Mazes***

Rat Mazes

- Suppose you want to make a rat maze consisting of an $n \times m$ grid of pegs with slats between them.



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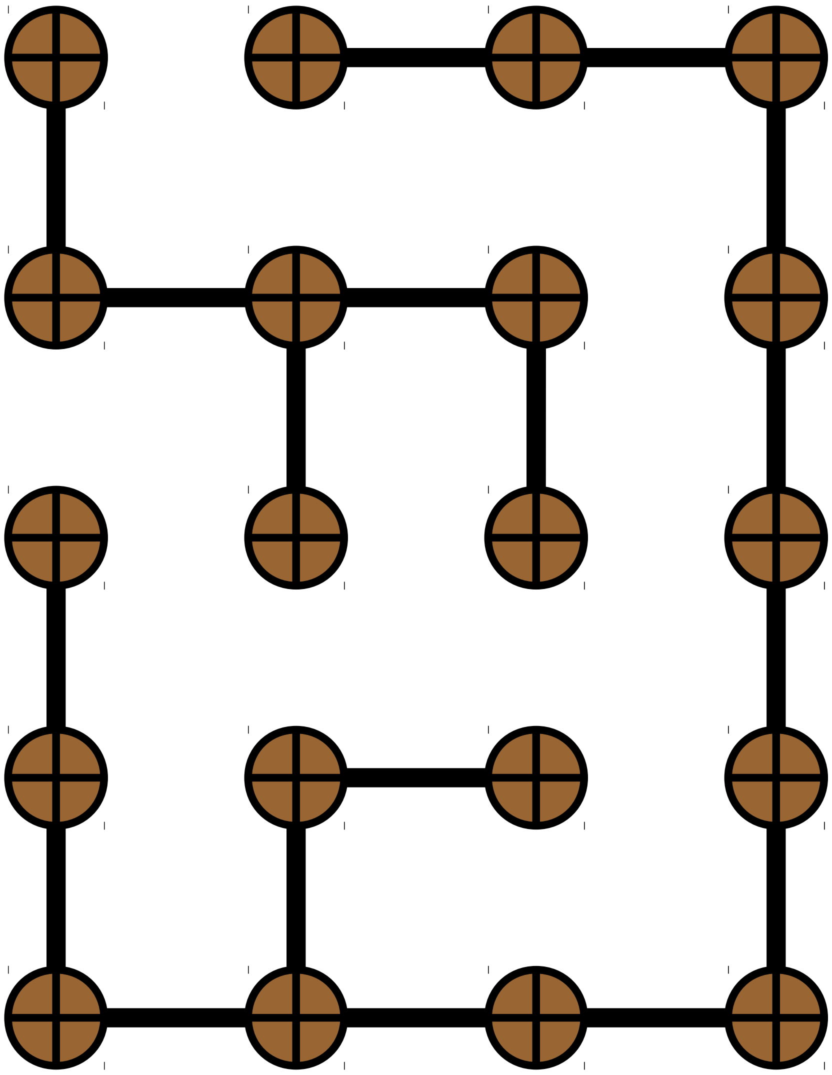
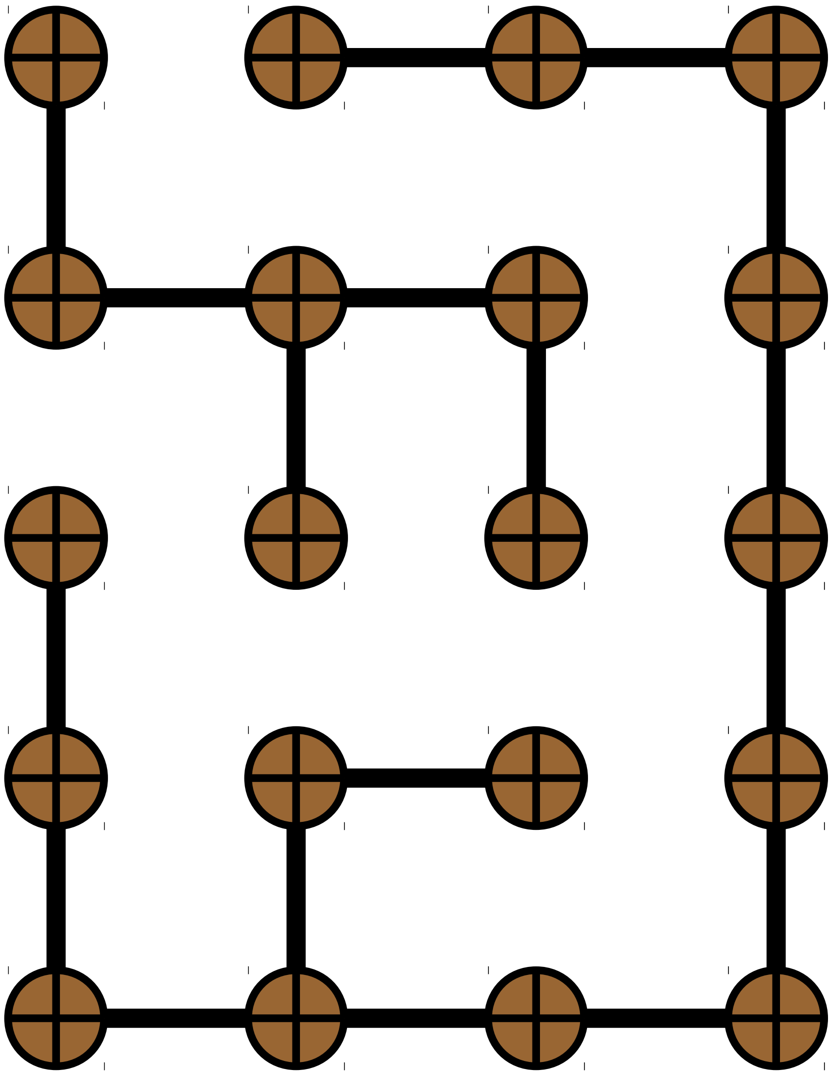


Rat Mazes

- Suppose you want to make a rat maze consisting of an $n \times m$ grid of pegs with slats between them.
- The maze should have these properties:
  - There is one entrance and one exit in the border.
  - Every spot in the maze is reachable from every other spot.
  - There is exactly one path from each spot in the maze to each other spot.



Question: If you have an $n \times m$ grid of pegs, how many slats do you need to make?

A Special Type of Graph: ***Trees***

Trees

- A ***tree*** is a connected, nonempty graph with no simple cycles.

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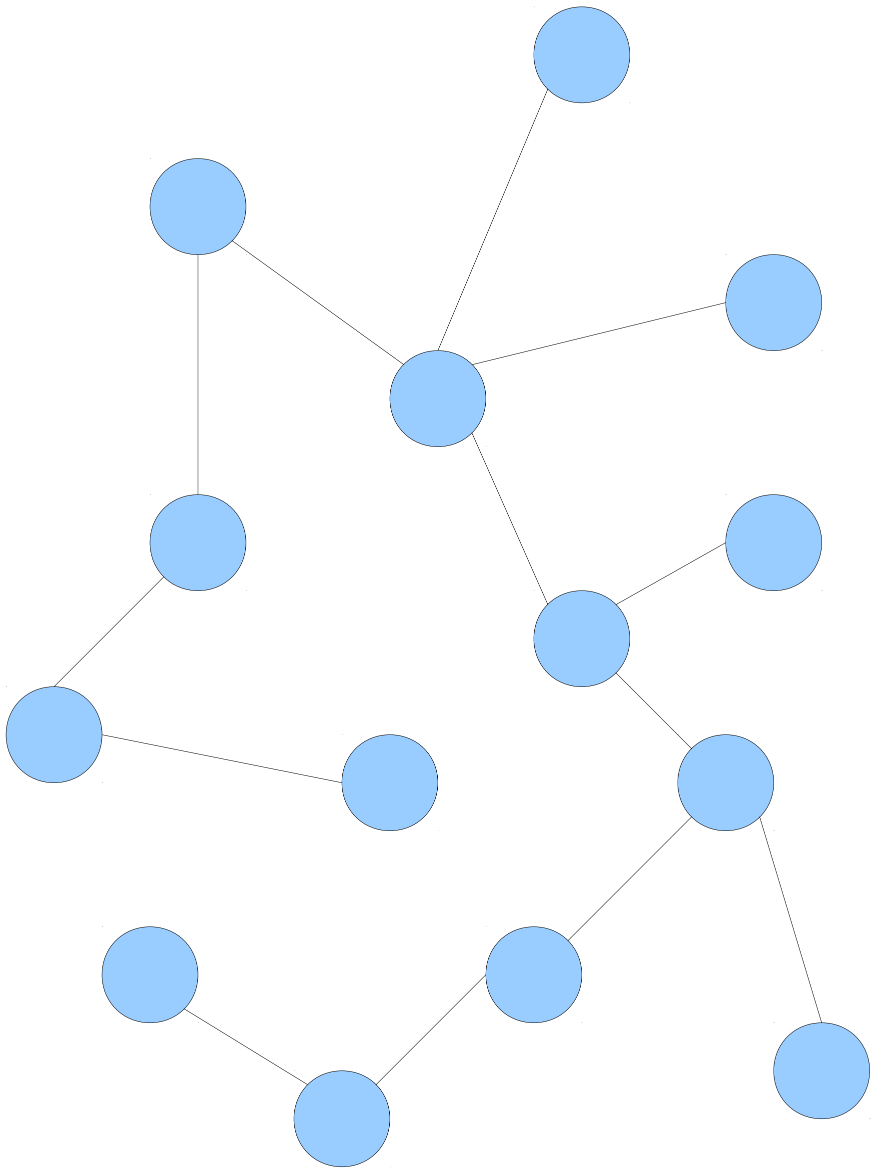

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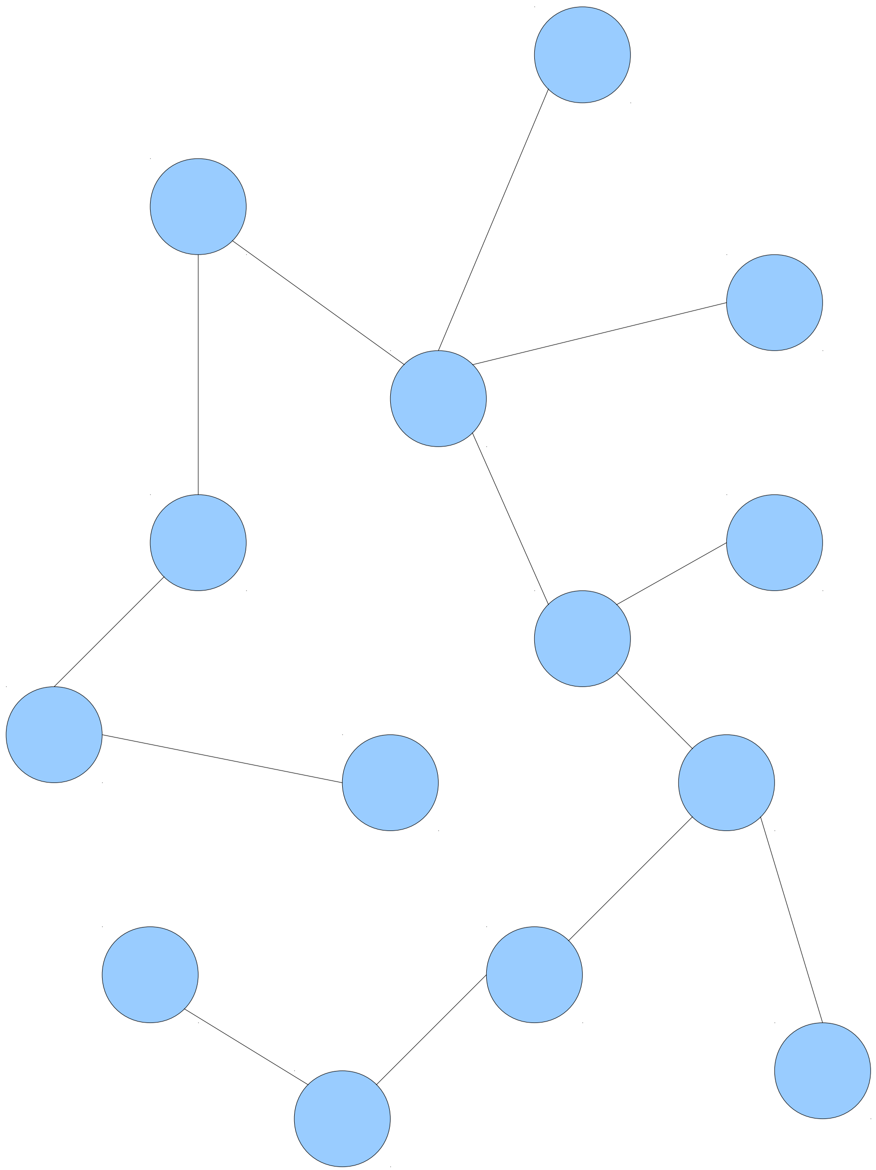
- A ***tree*** is a connected, nonempty graph with no simple cycles.
- Trees have tons of nice properties:
  - They're ***maximally acyclic*** (adding any missing edge creates a simple cycle)



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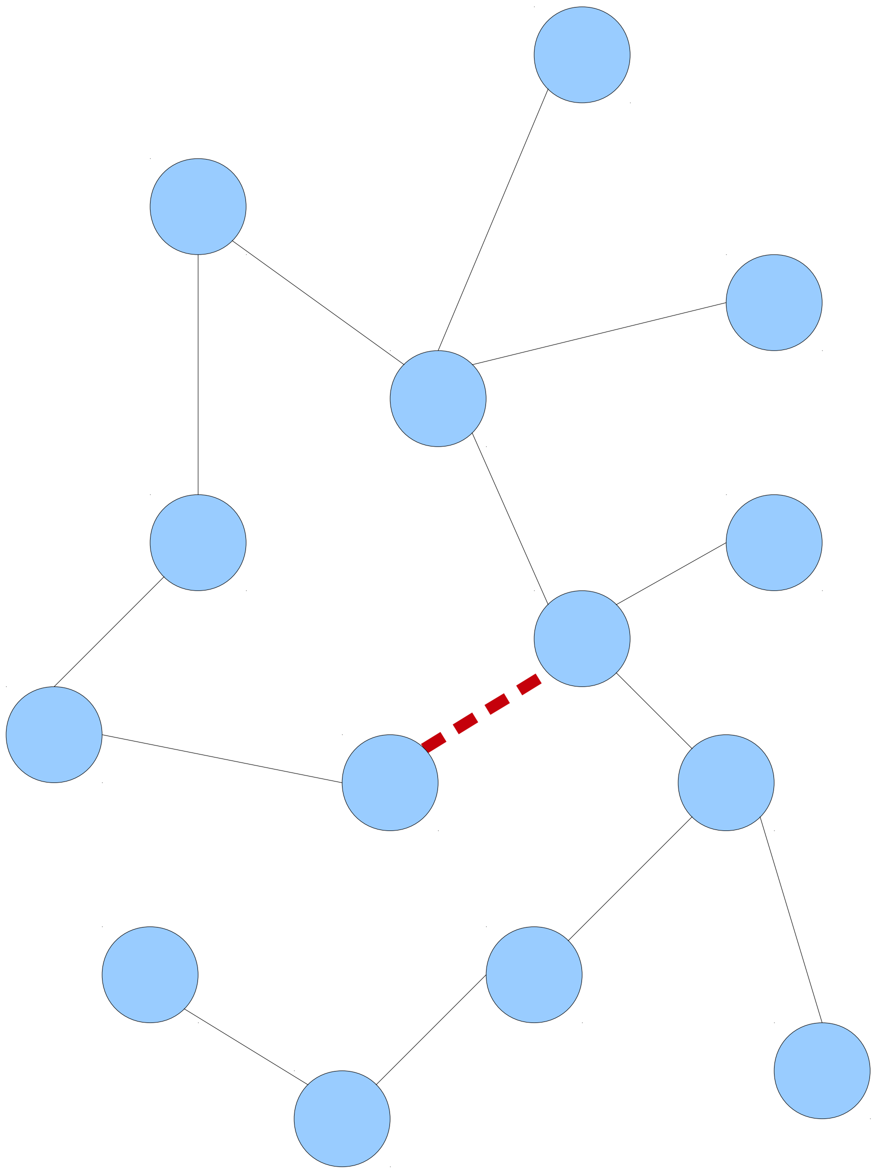

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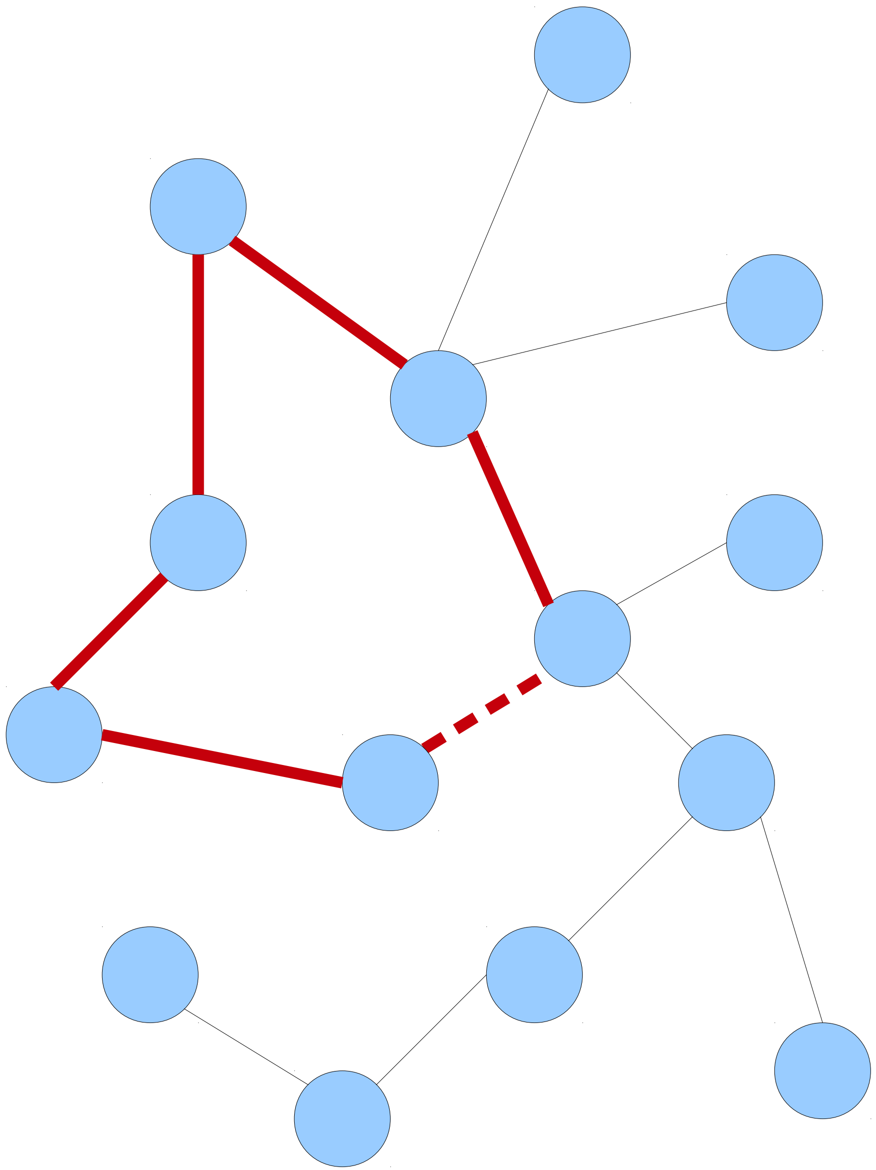

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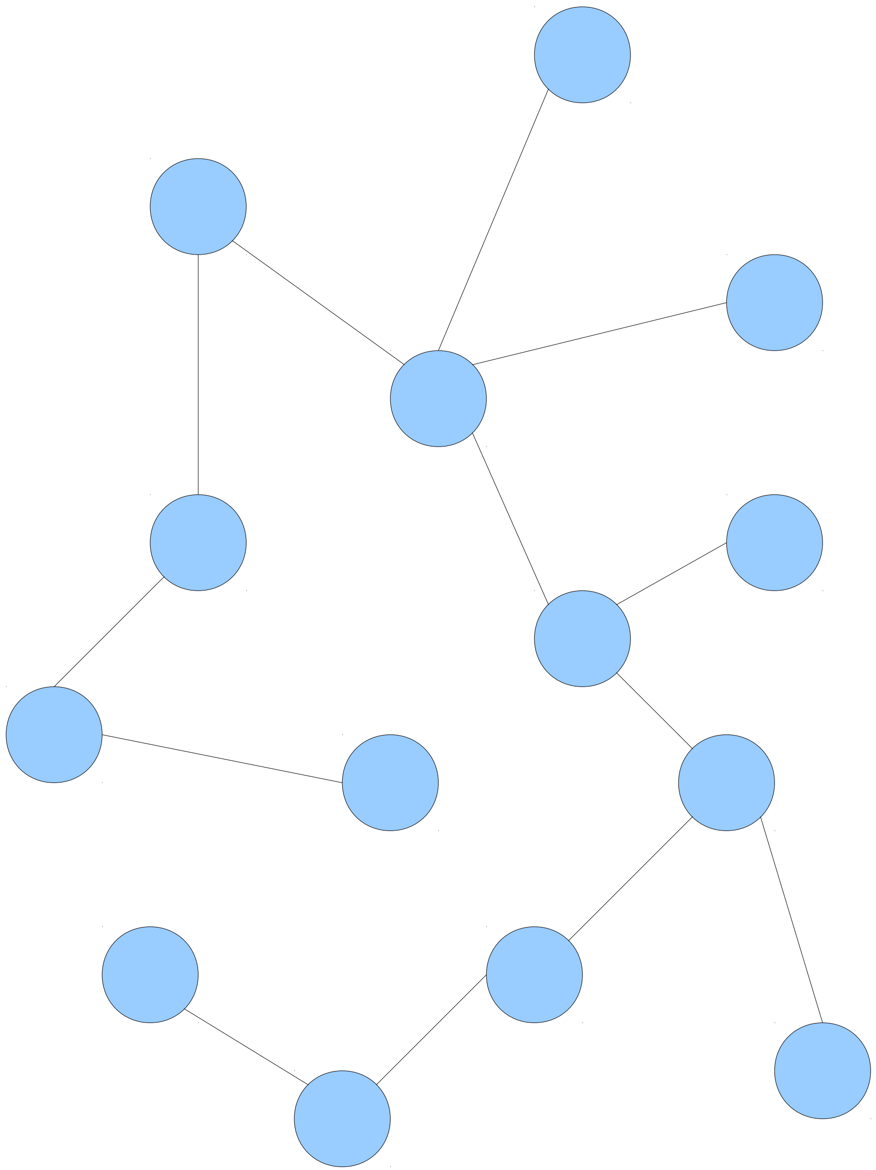

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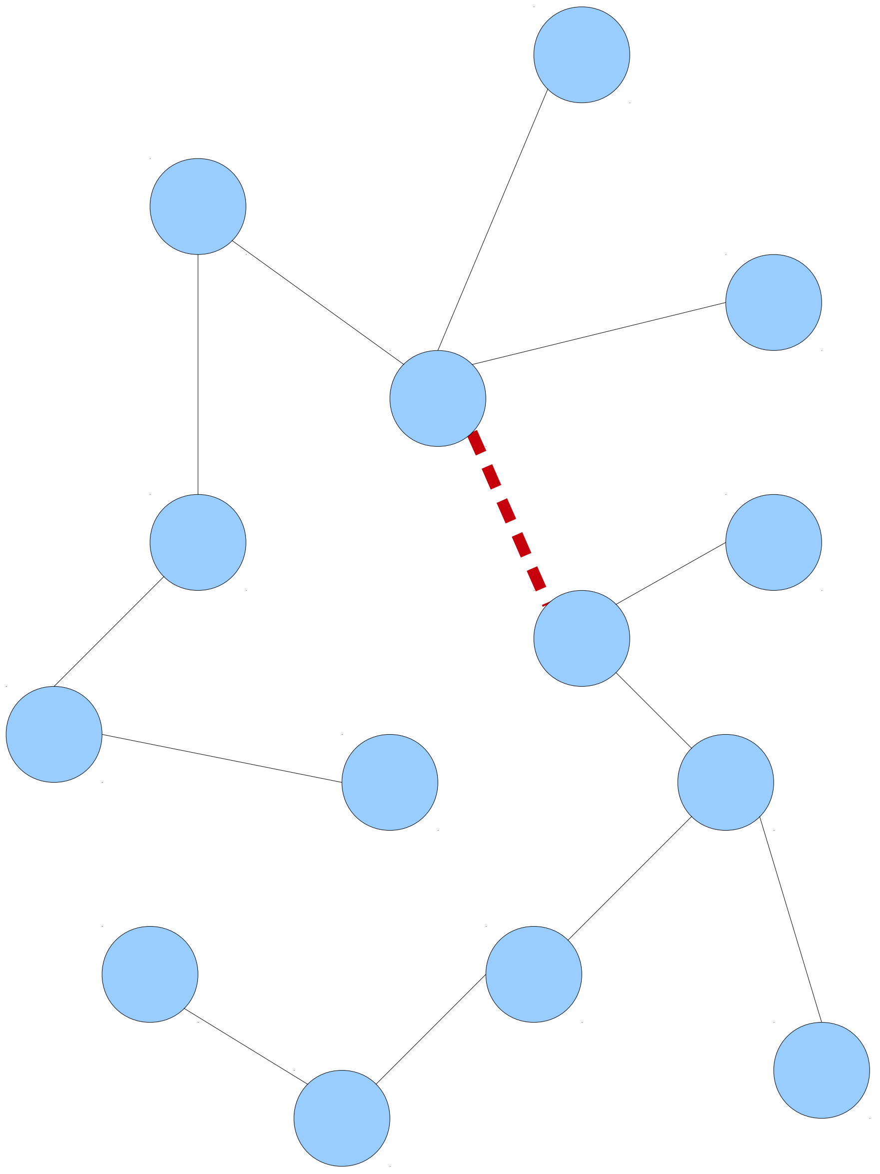

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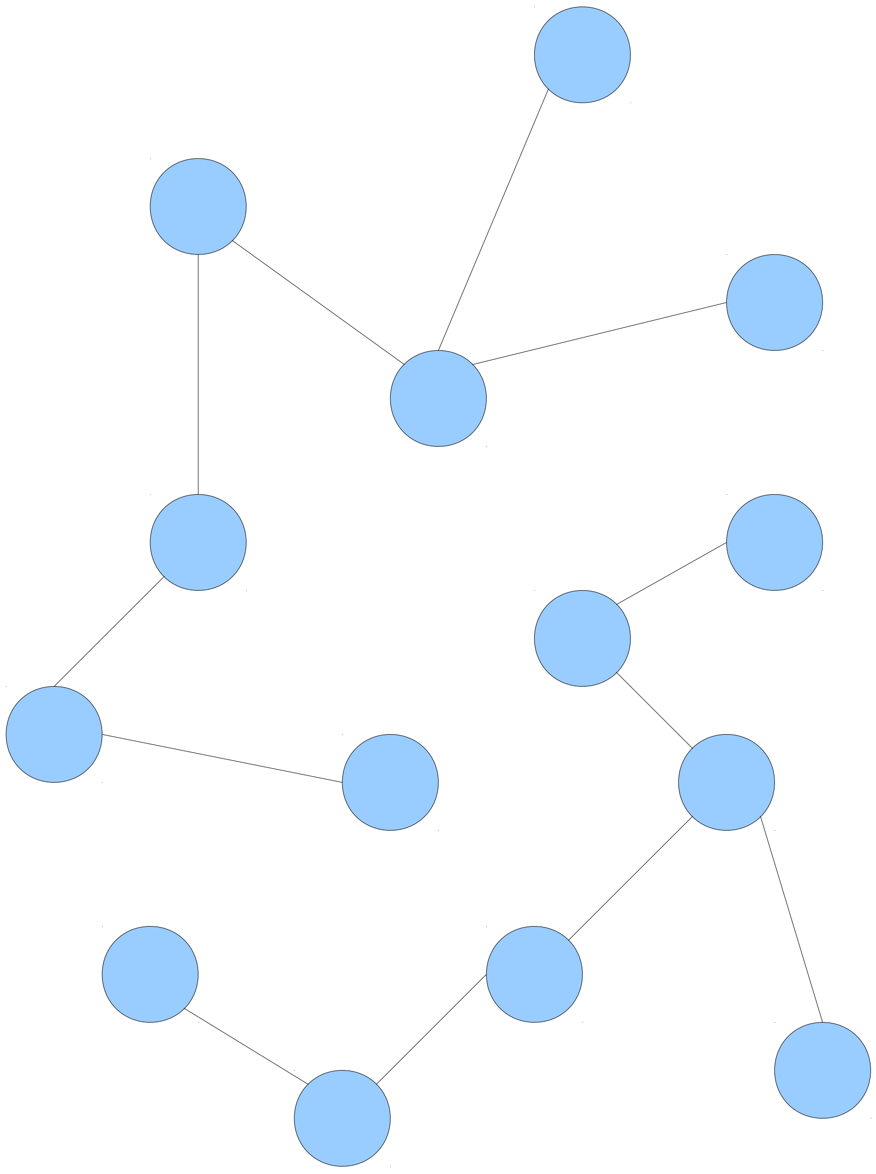

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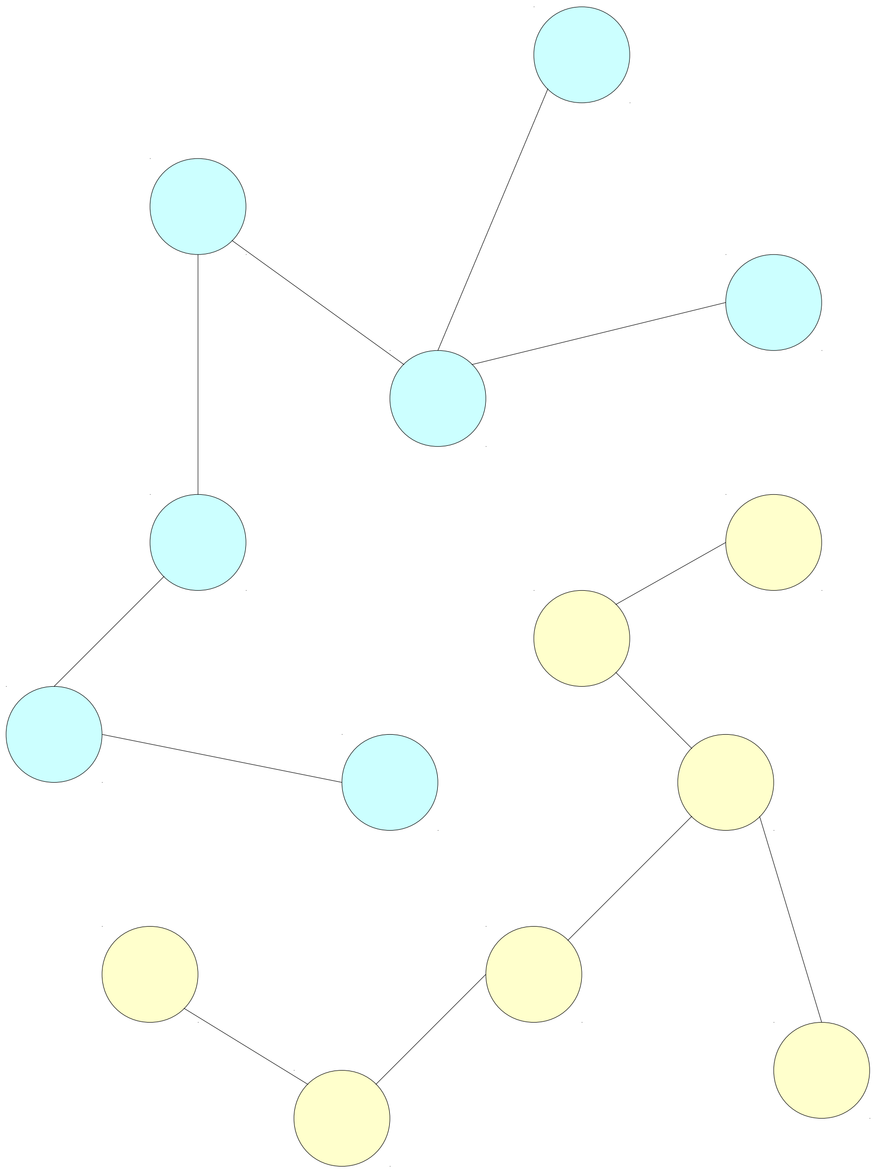

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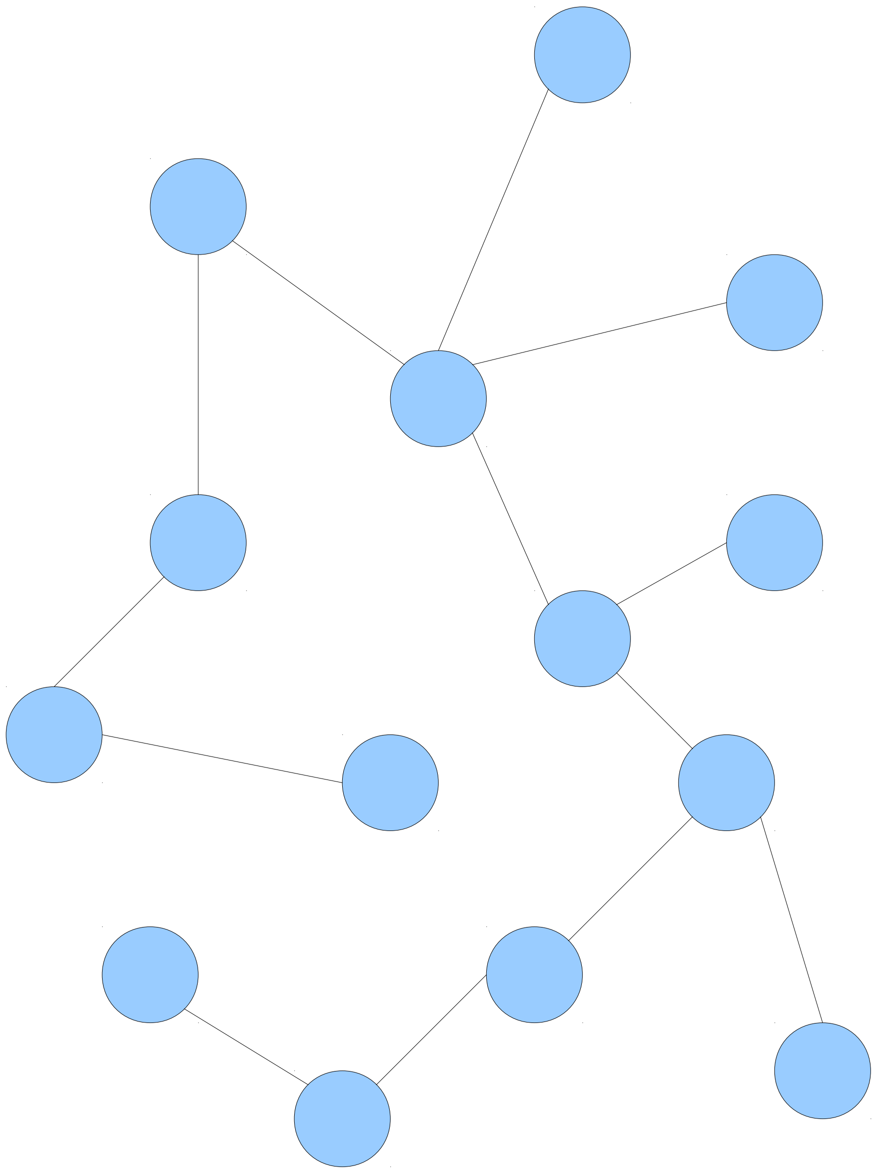
- Proofs of these results are in the course reader if you're interested. They're also great exercises.



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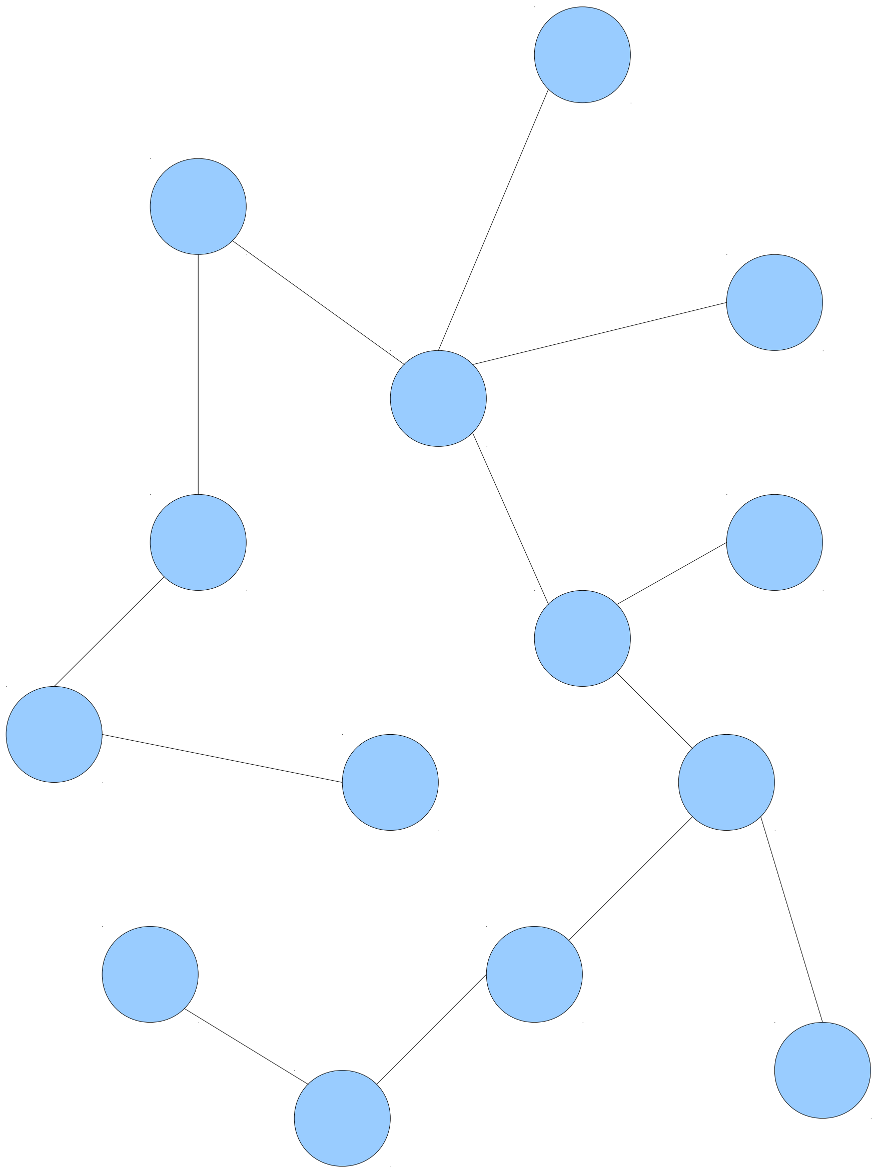
- ***Theorem:*** If T is a tree with at least two nodes, then deleting any edge from T splits T into two nonempty trees T_1 and T_2.
- ***Proof:*** Left as an exercise to the reader. ☺



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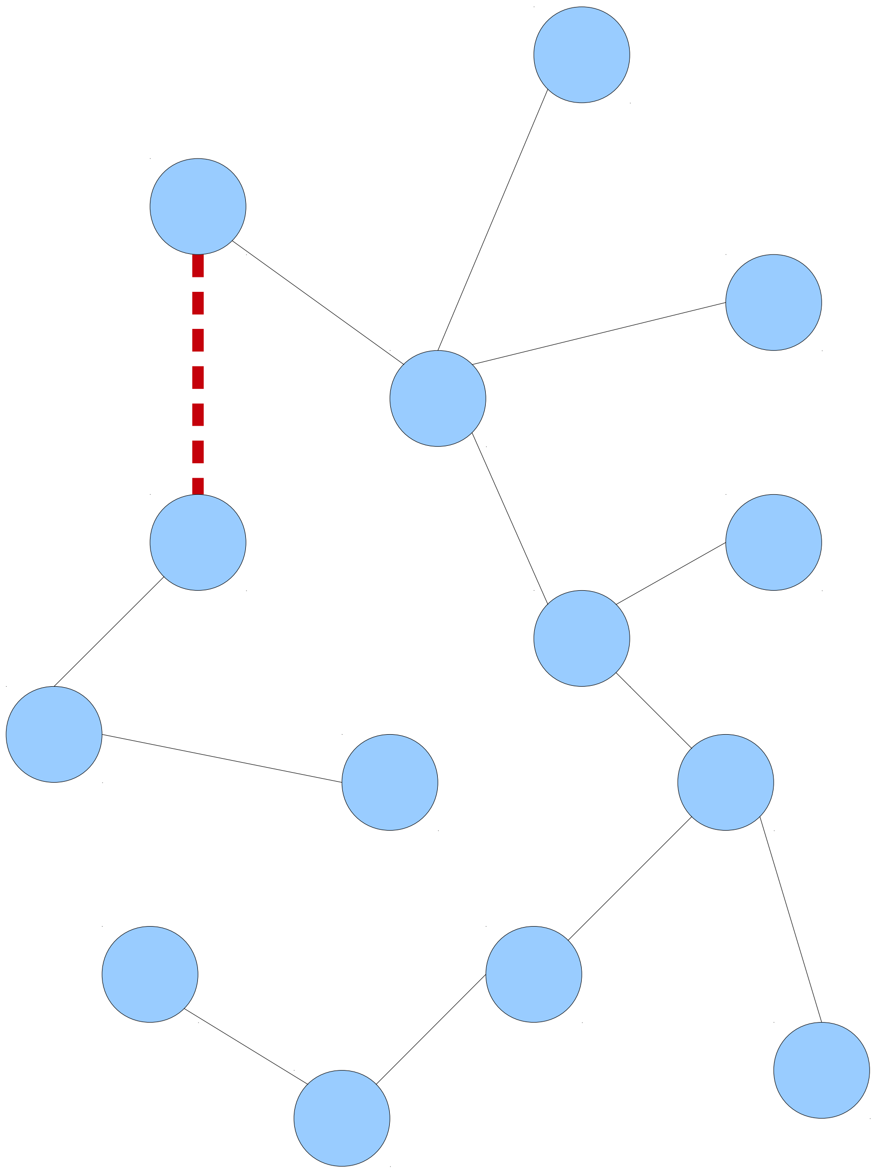

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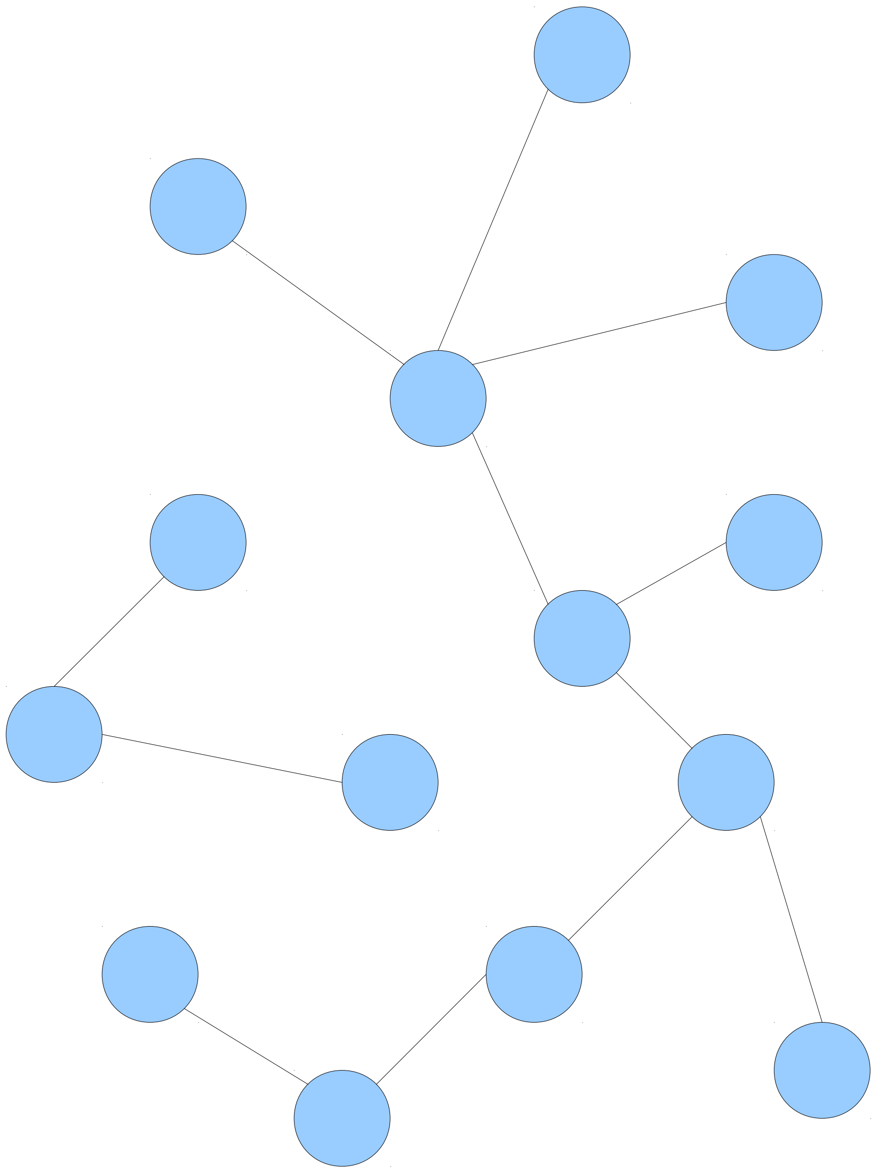

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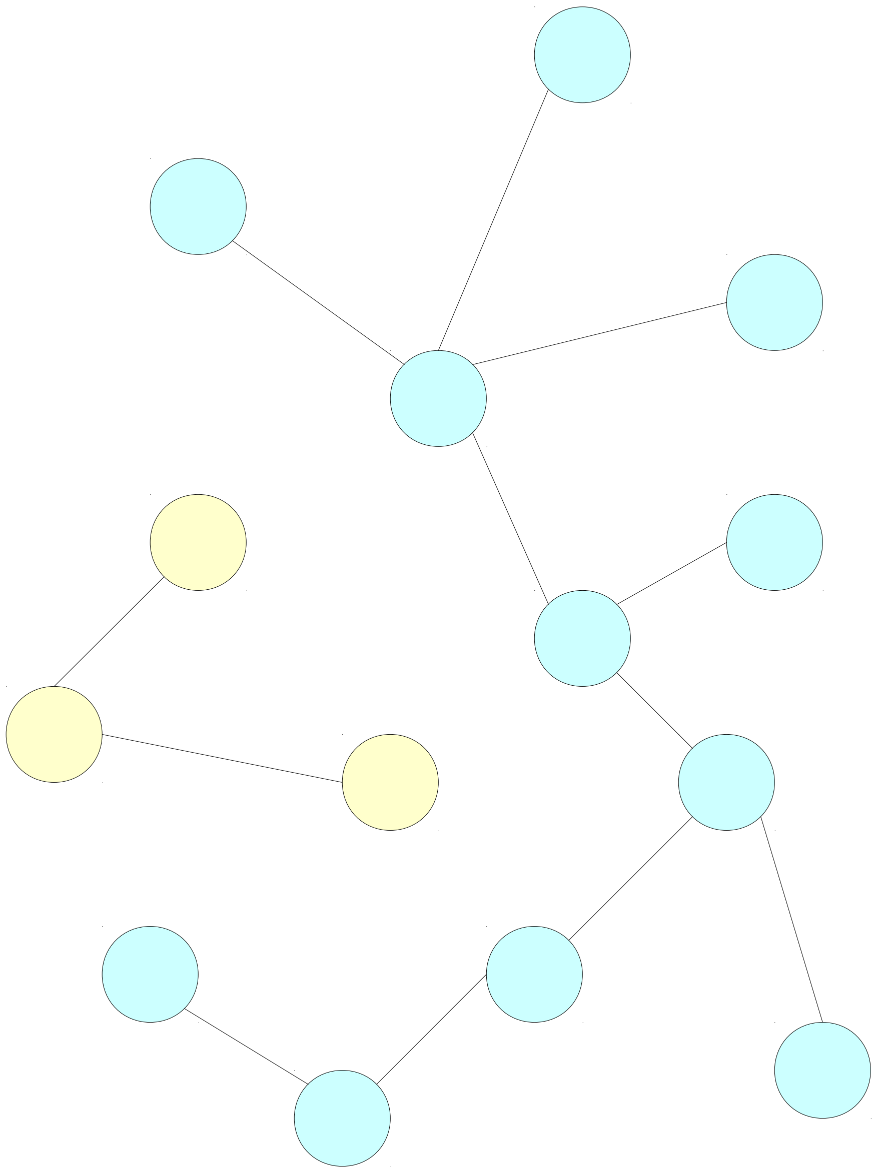

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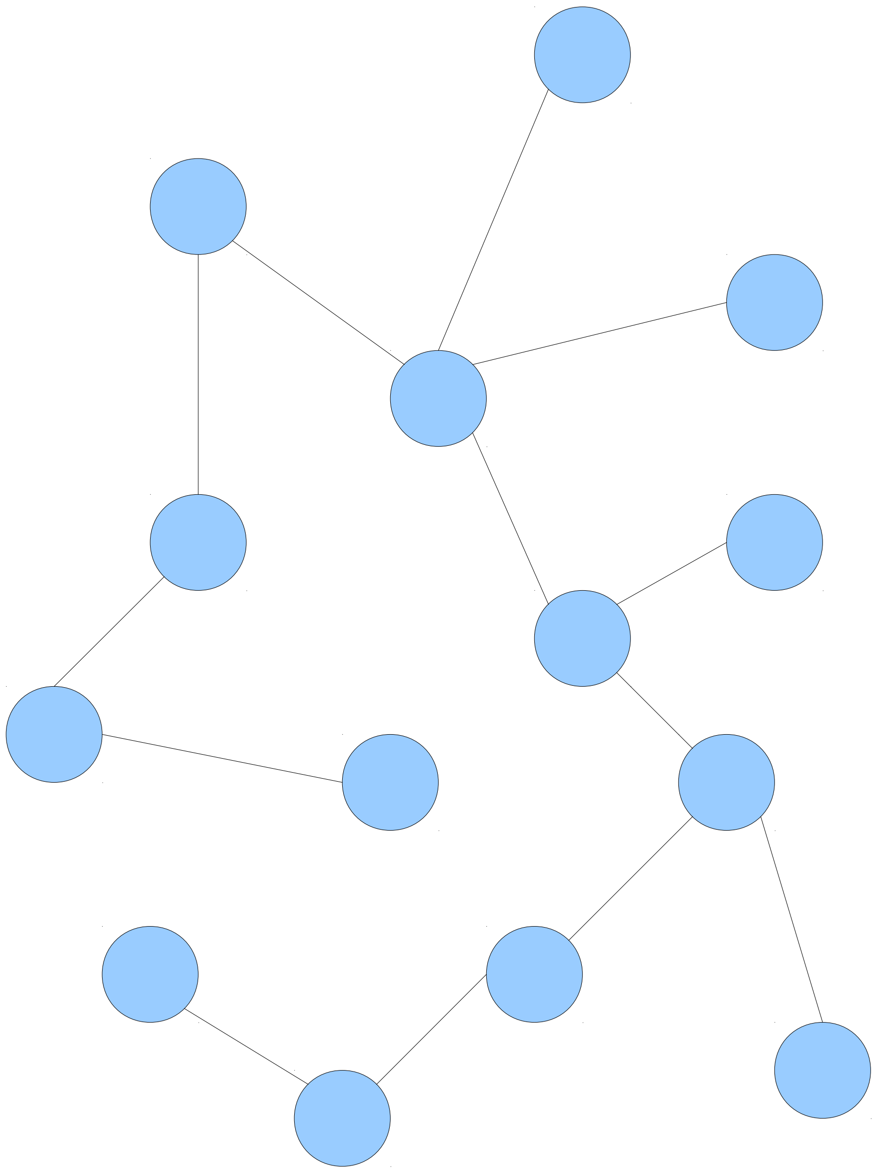

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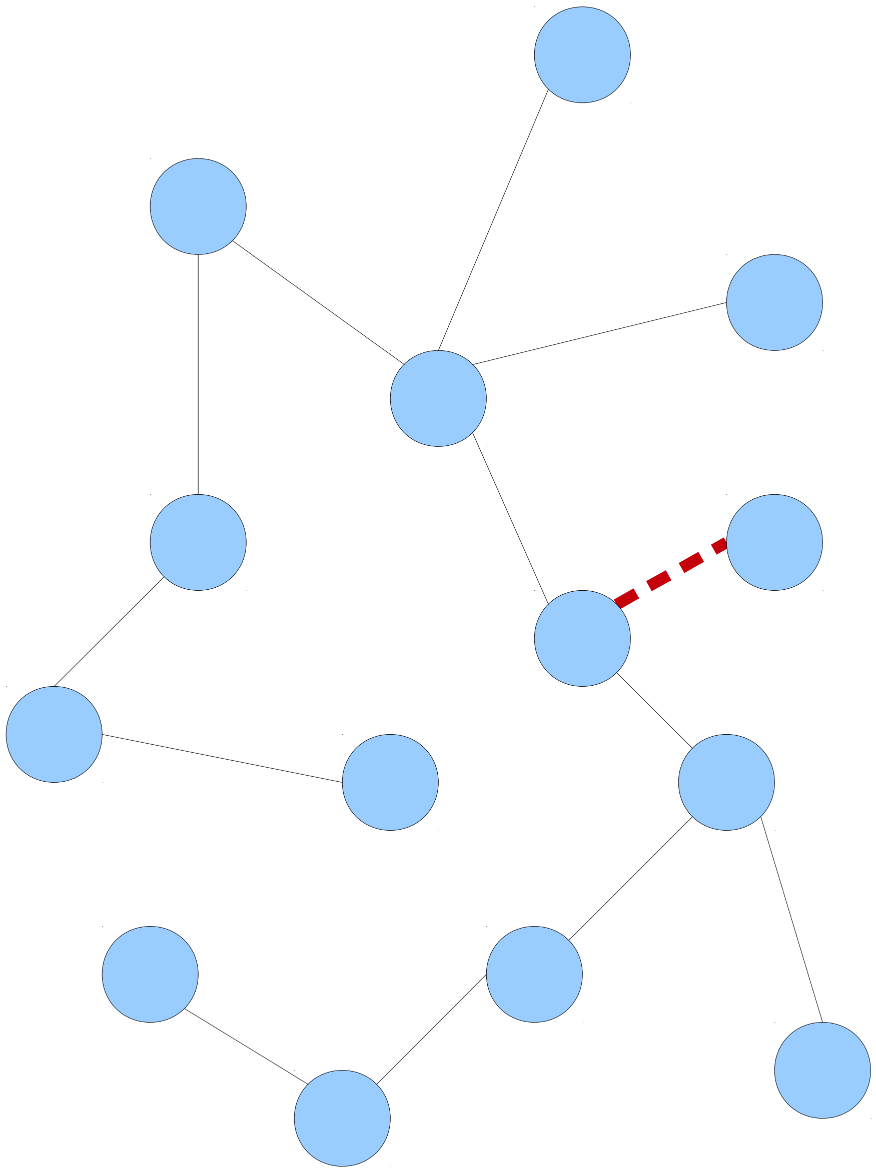

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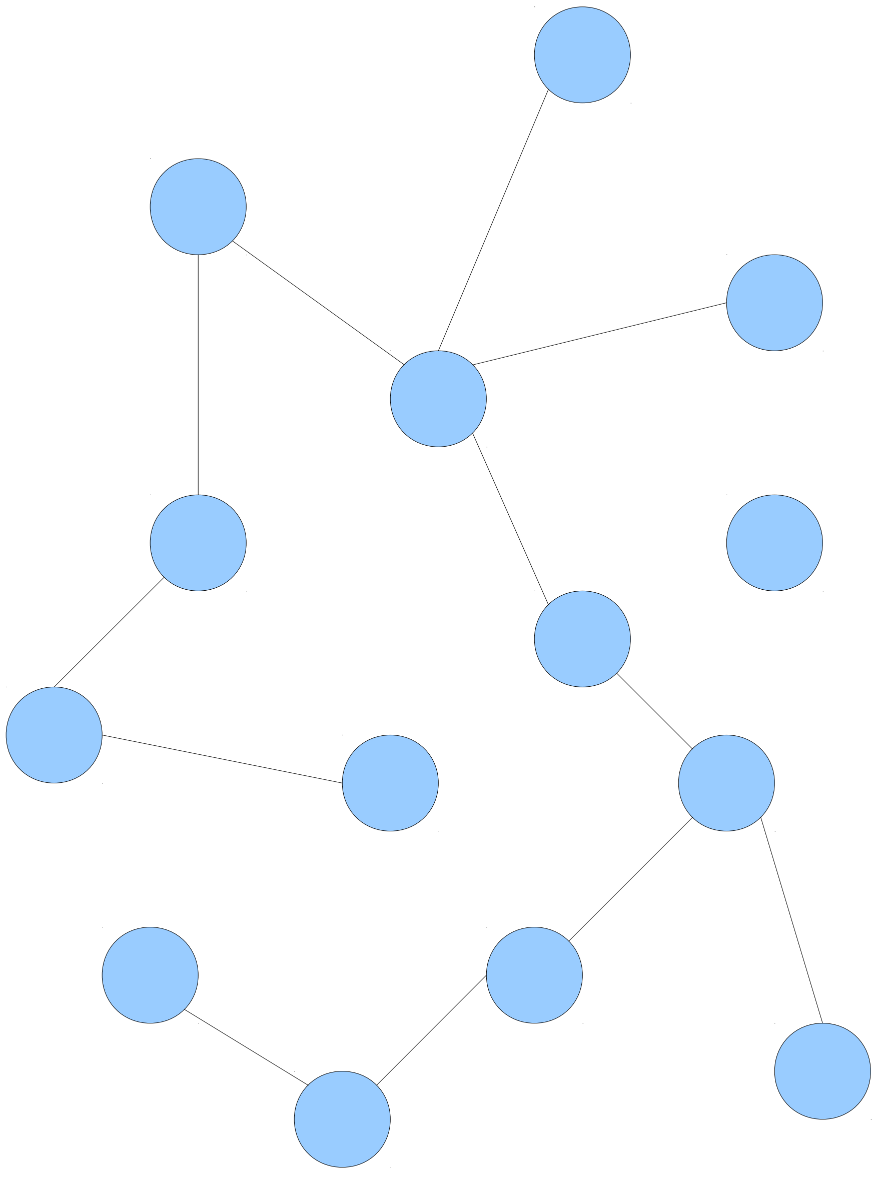

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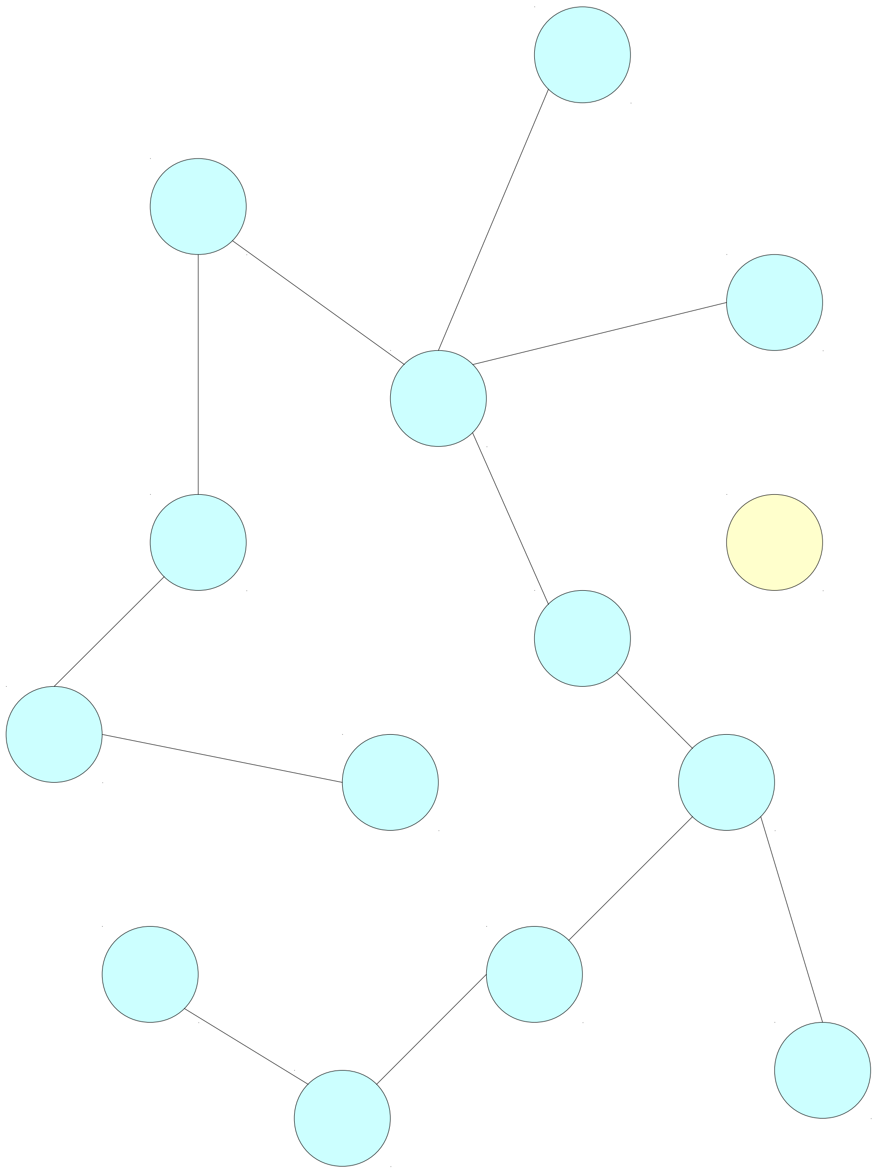

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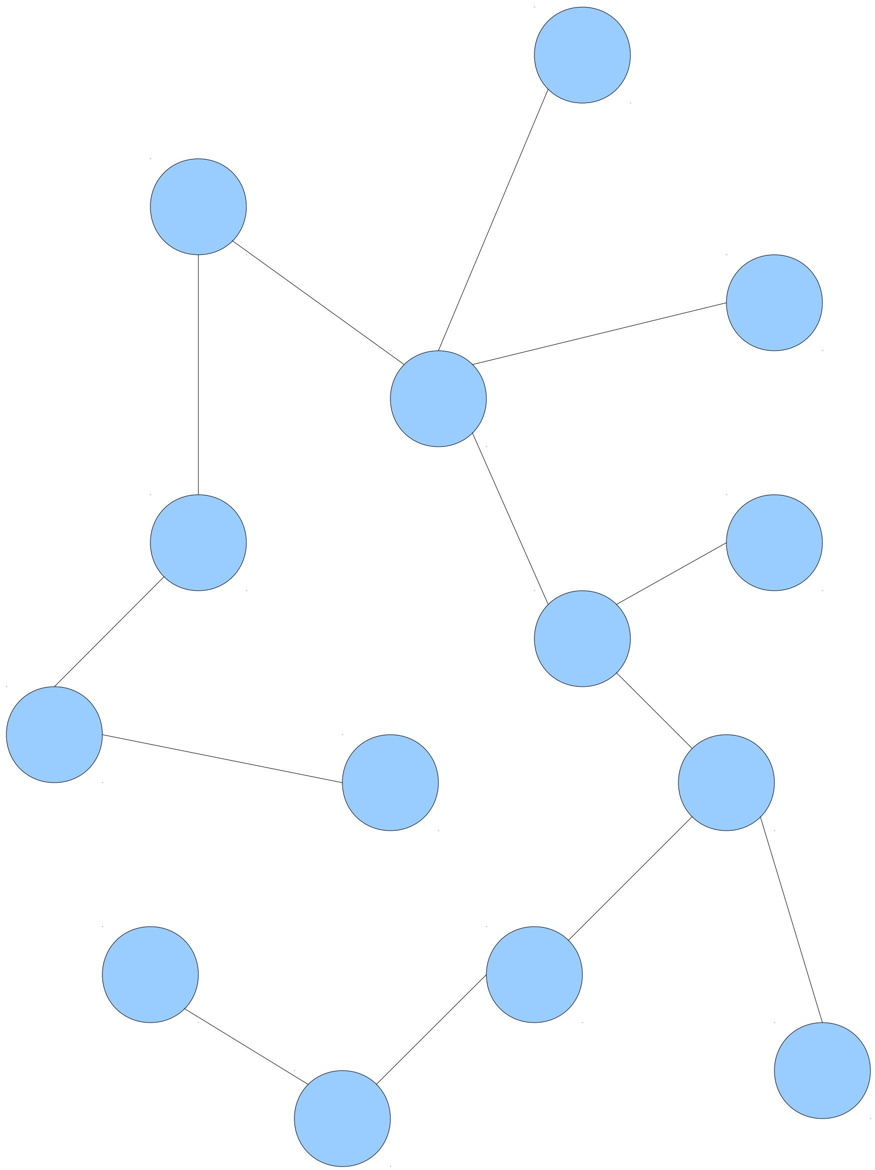
- ***Theorem:*** If T is a tree with $n \geq 1$ nodes, then T has exactly n-1 edges.
- ***Proof:*** Up next!



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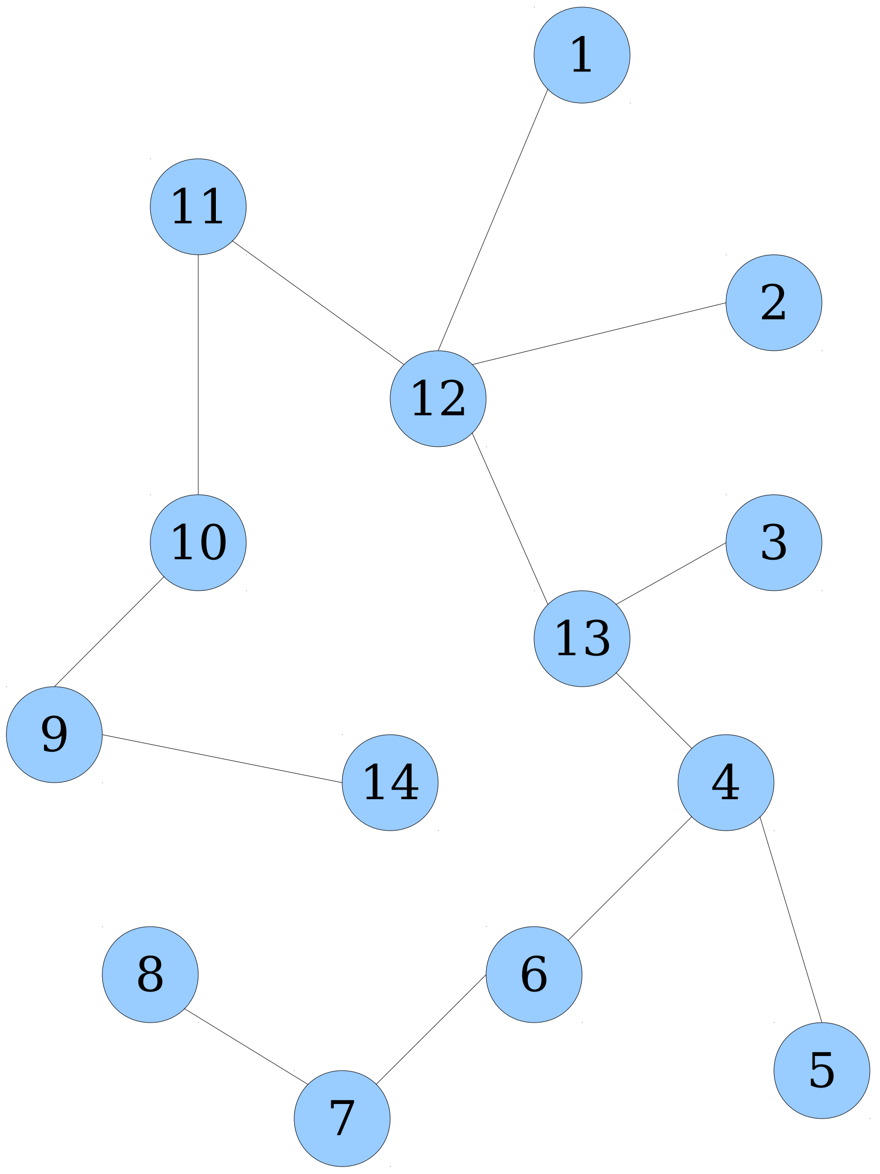

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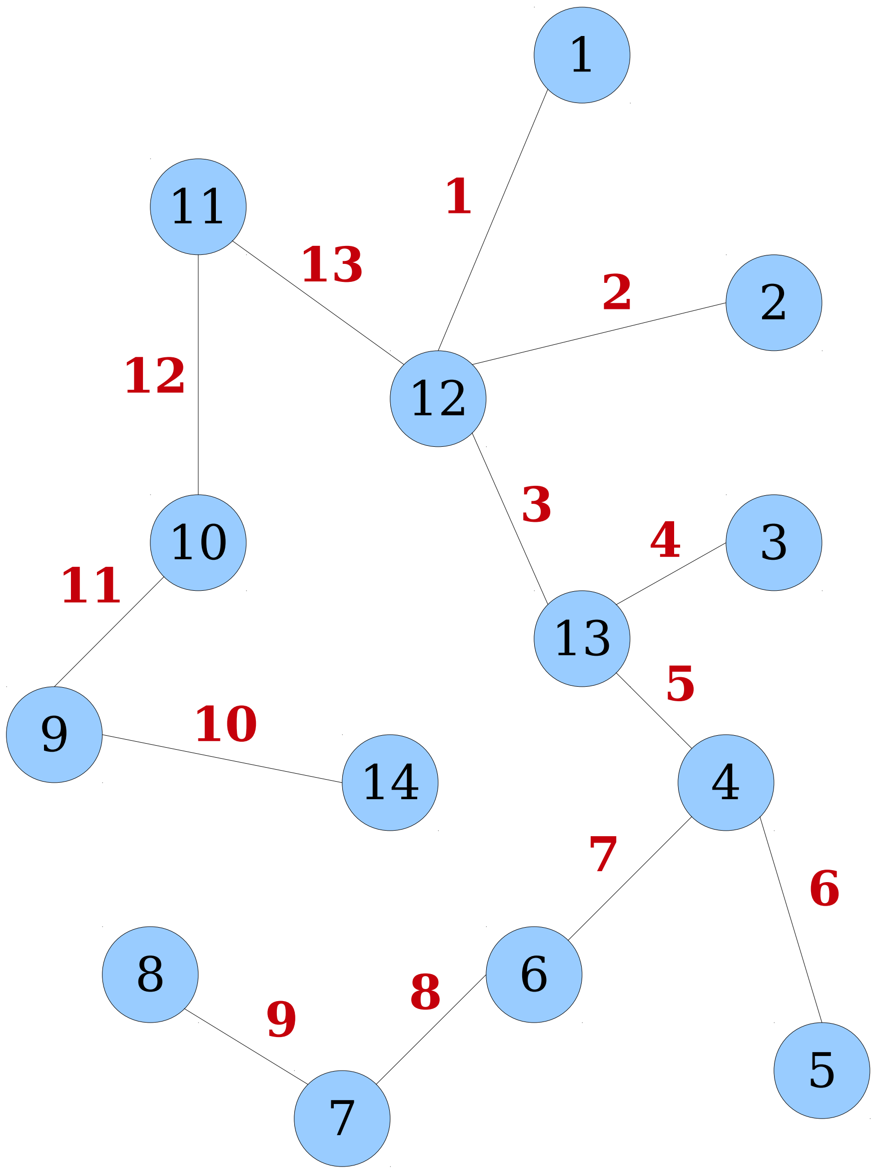

Assume any tree with at most k nodes has one more node than edge.

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Consider an arbitrary tree with $k+1$ nodes.

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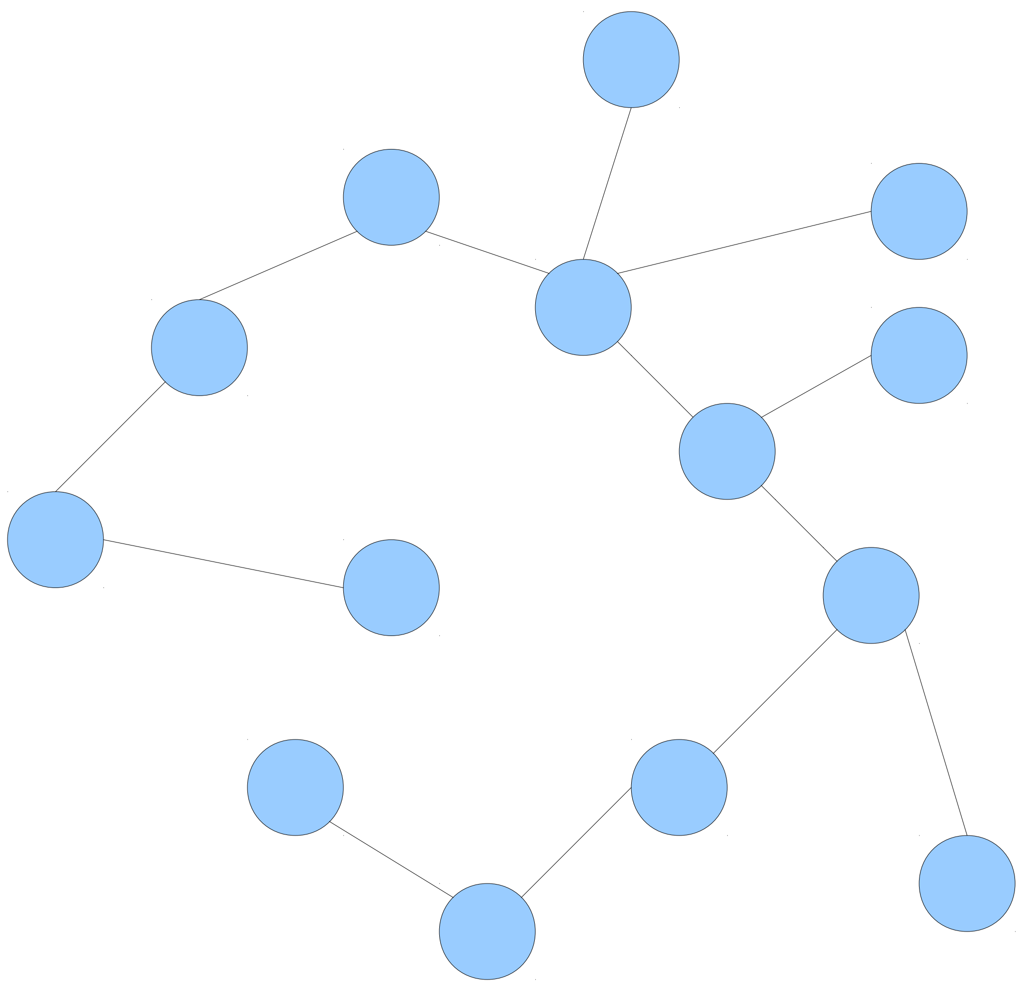

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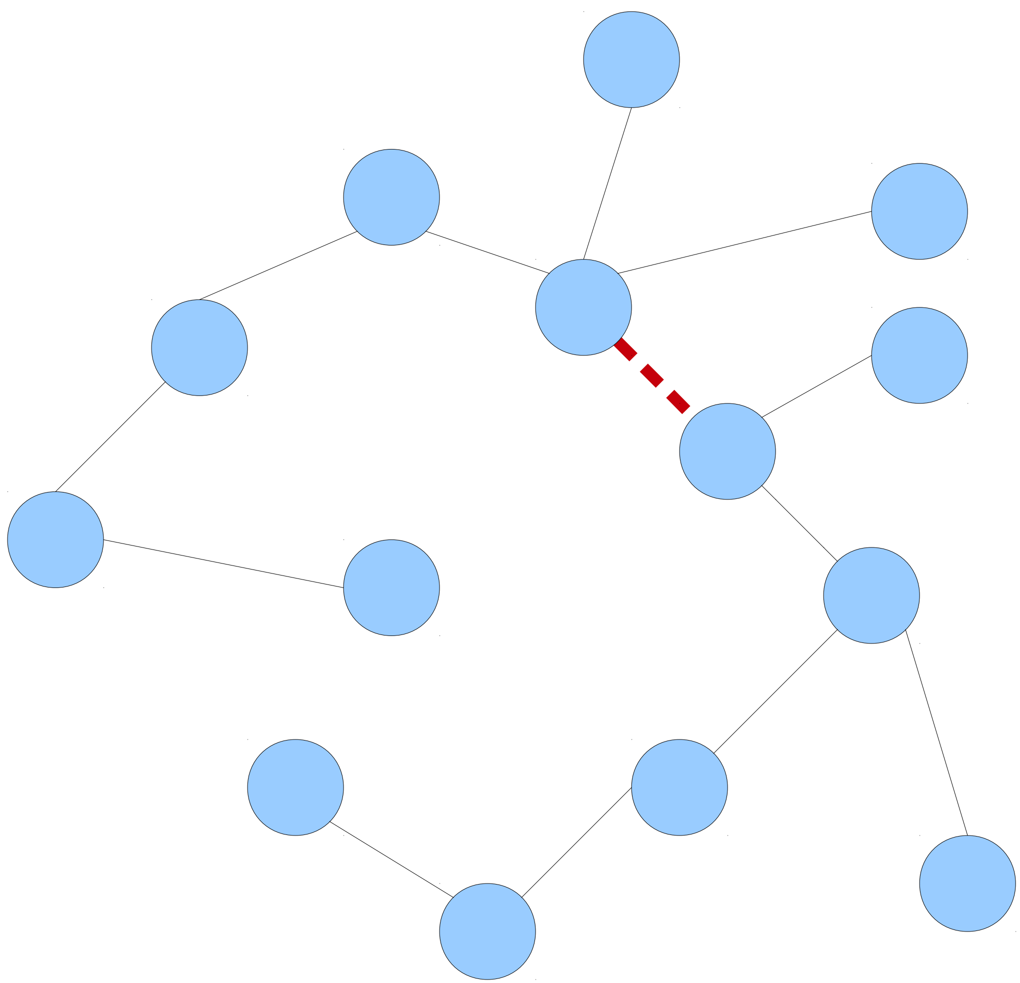

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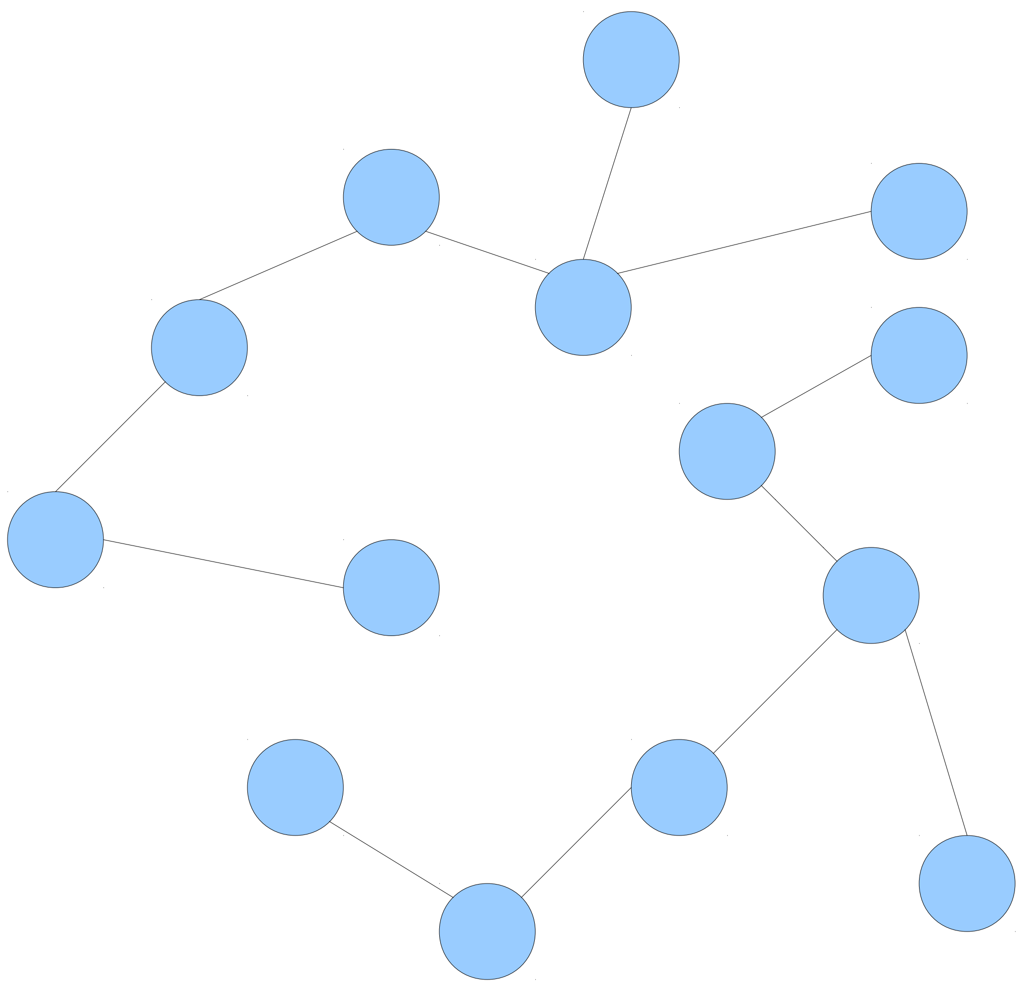

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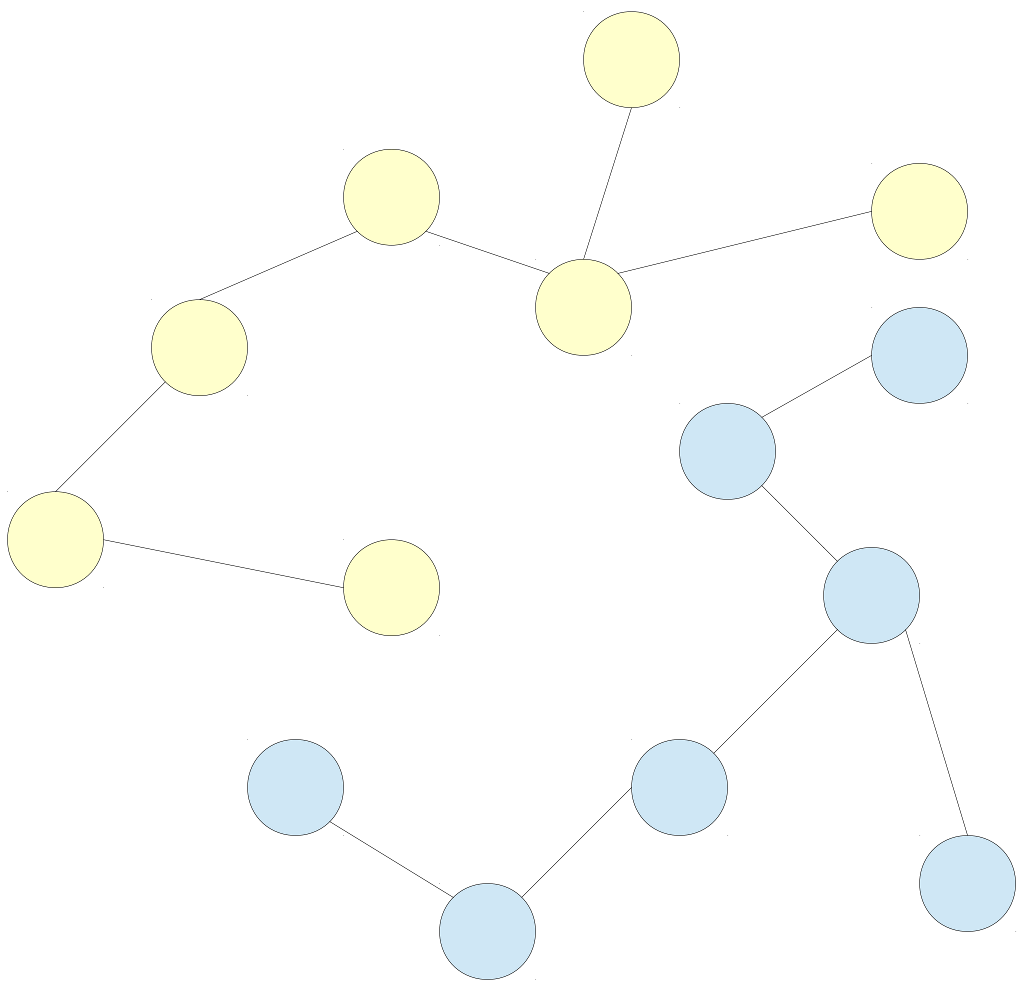

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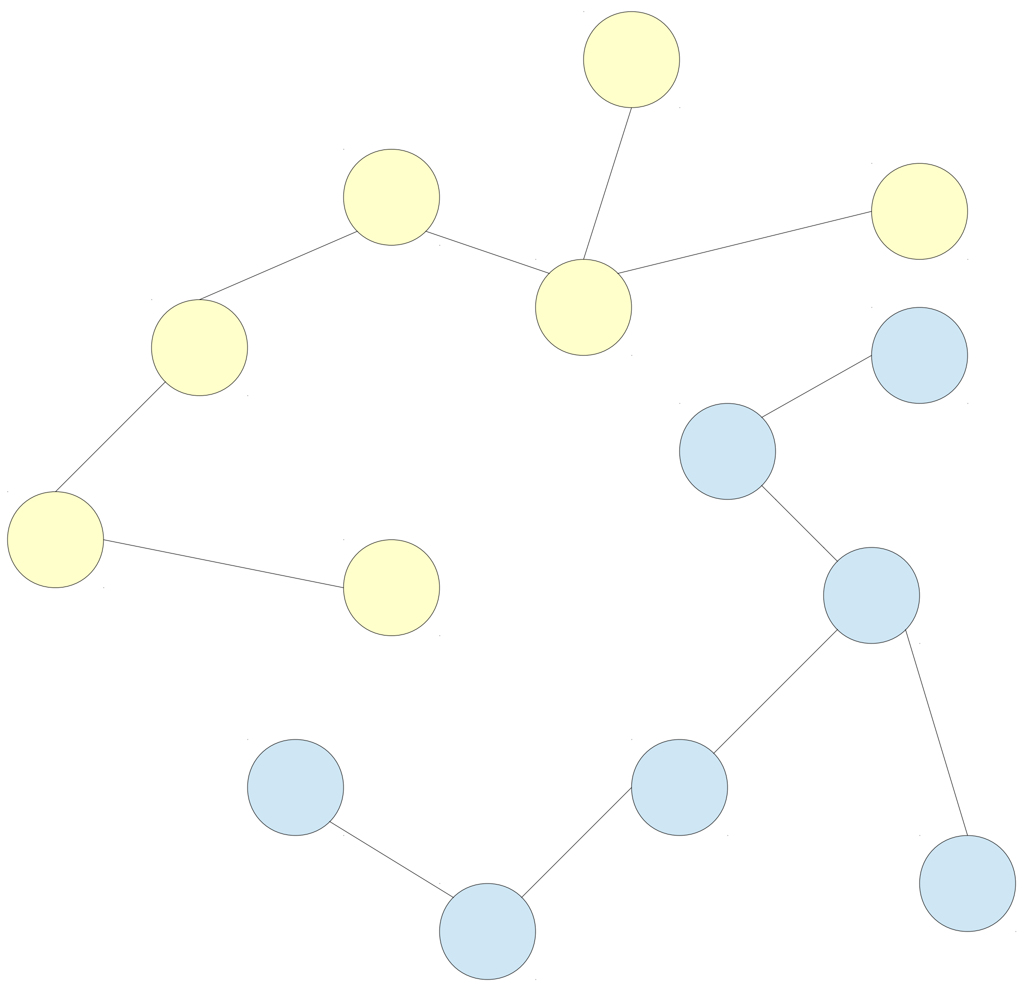
Consider an arbitrary tree with $k+1$ nodes.

Suppose there are r nodes in the yellow tree.



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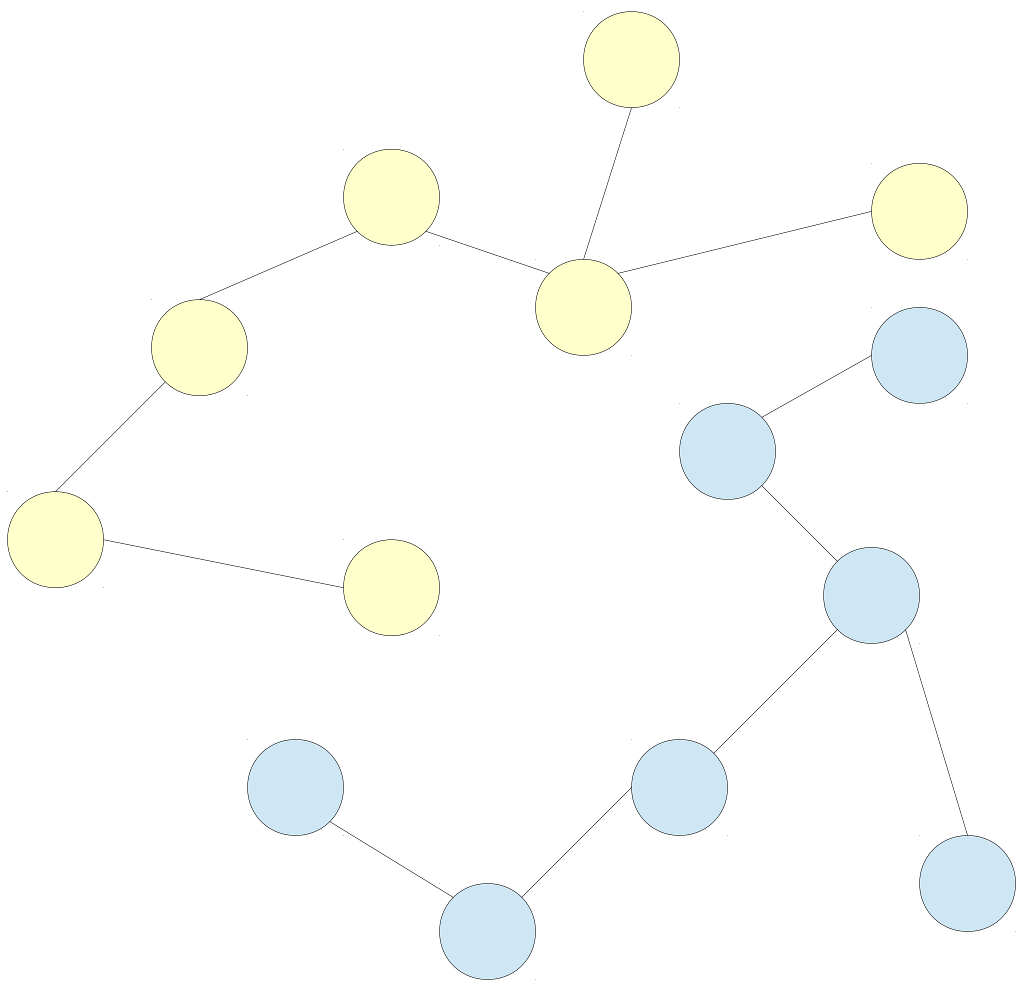
Consider an arbitrary tree with $k+1$ nodes.

Suppose there are r nodes in the yellow tree.

Then there are $(k+1)-r$ nodes in the blue tree.



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Consider an arbitrary tree with $k+1$ nodes.

Suppose there are r nodes in the yellow tree.

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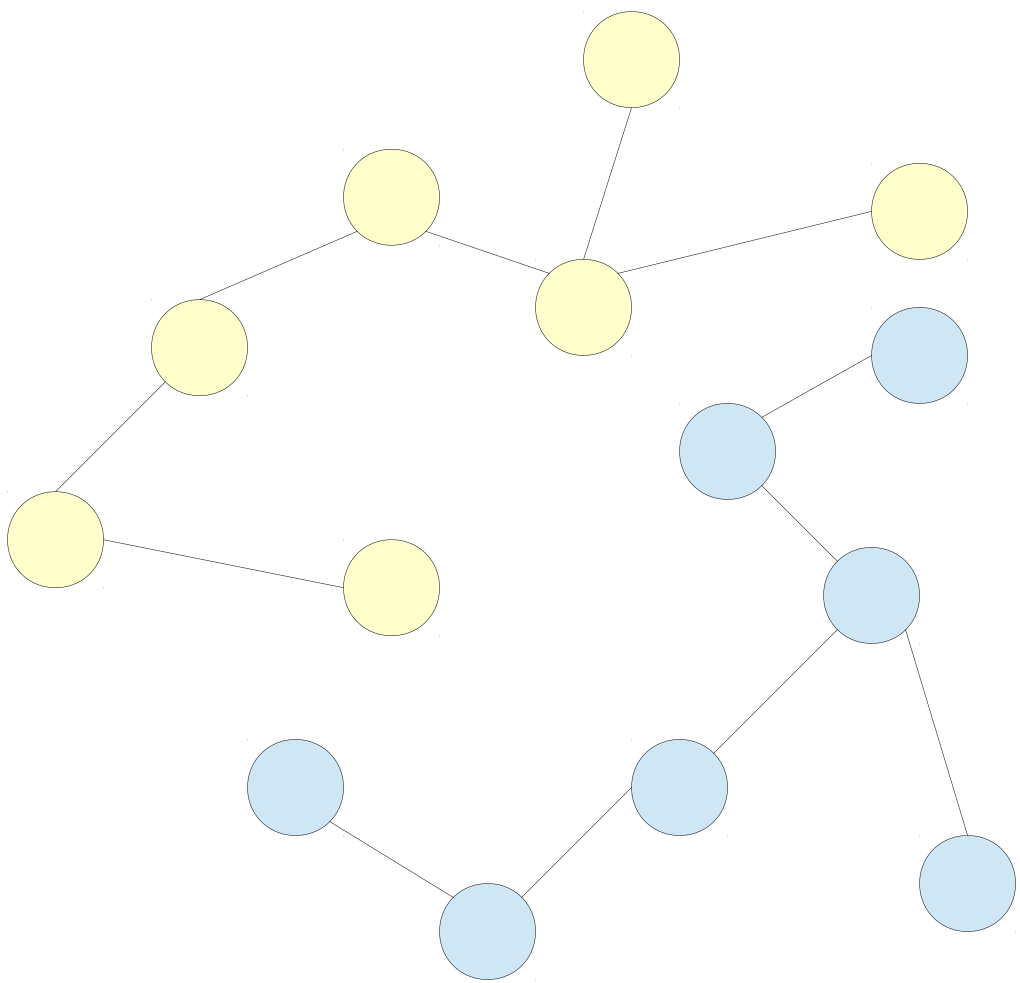
There are $r-1$ edges in the yellow tree and $k-r$ edges in the blue tree.



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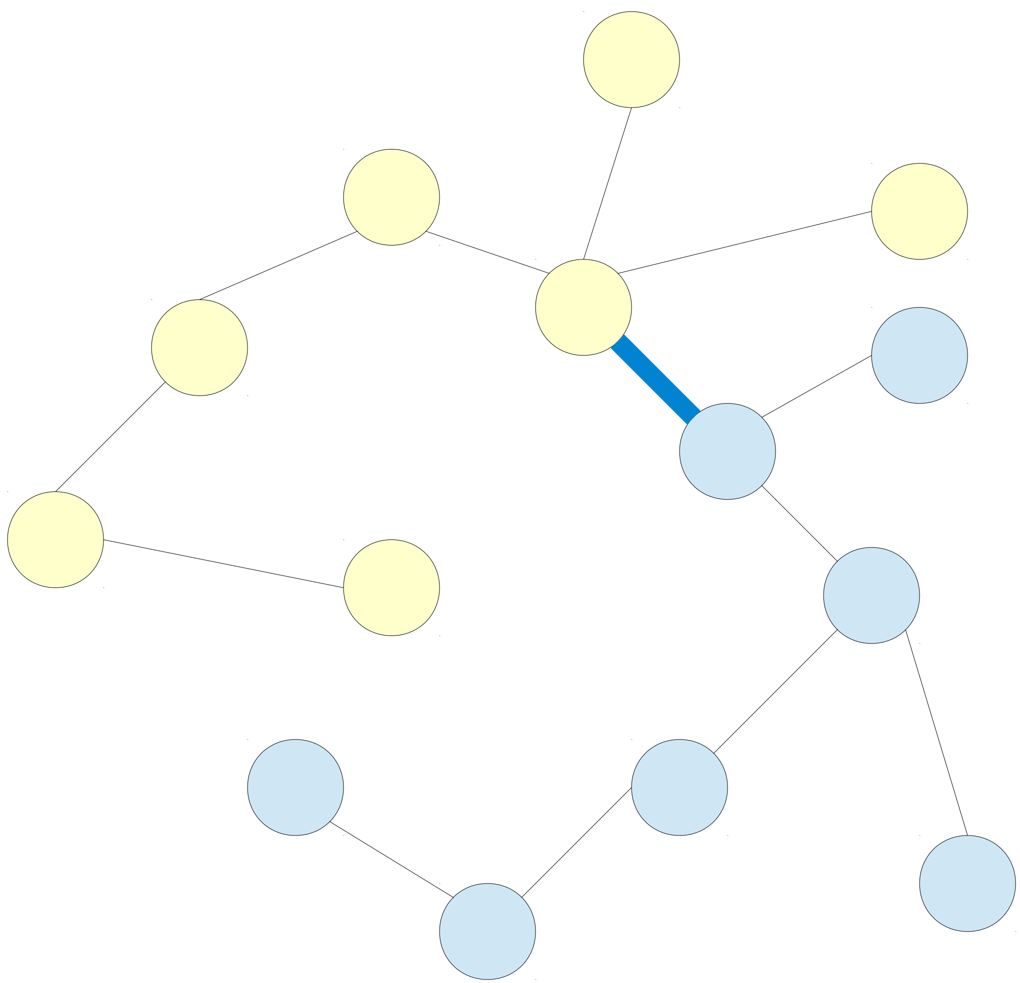

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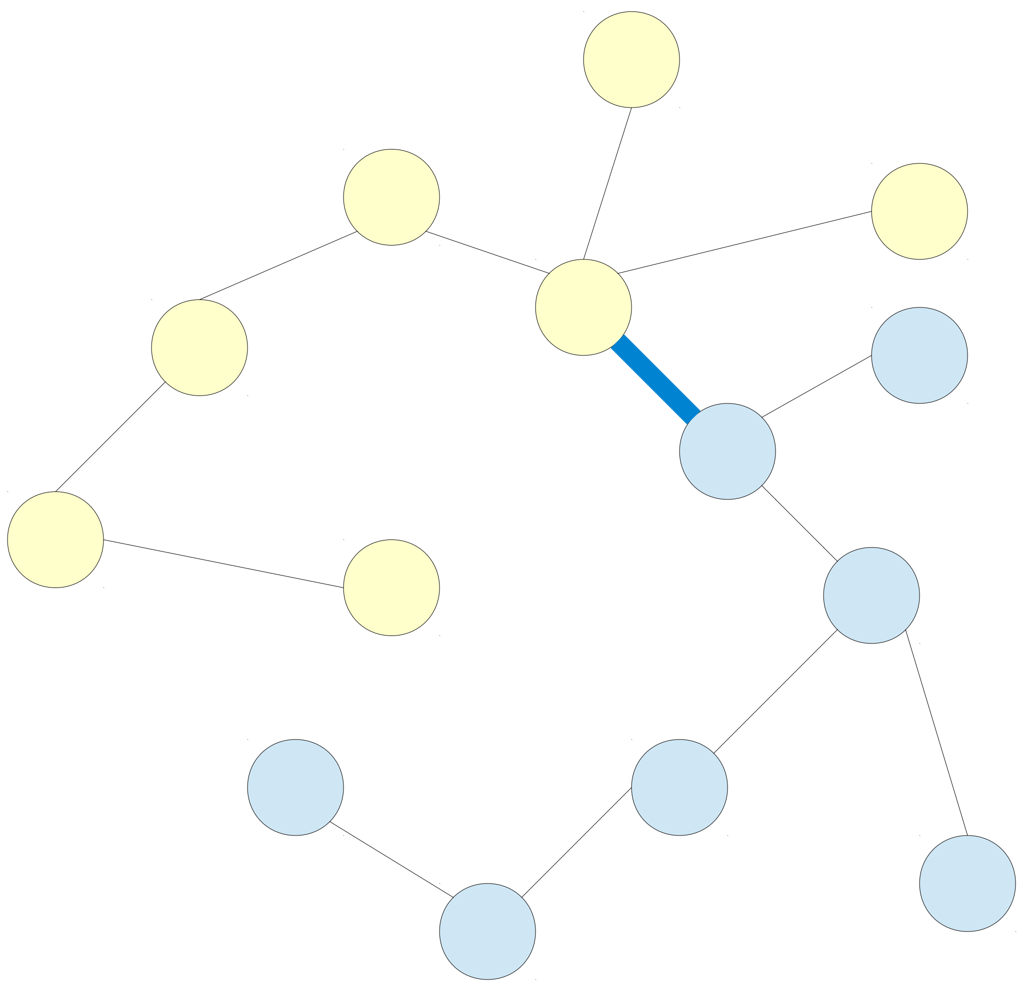

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Suppose there are r nodes in the yellow tree.

Then there are $(k+1) - r$ nodes in the blue tree.

There are $r - 1$ edges in the yellow tree and $k - r$ edges in the blue tree.

Adding in the initial edge we cut, there are $r - 1 + k - r + 1 = k$ edges in the original tree.

Theorem: If T is a tree with $n \geq 1$ nodes, then T has n-1 edges.

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Proof: Let $P(n)$ be the statement "any tree with n nodes has n-1 edges." We will prove by induction that $P(n)$ holds for all $n \geq 1$, from which the theorem follows.

As a base case, we will prove $P(1)$, that any tree with 1 node has 0 edges.

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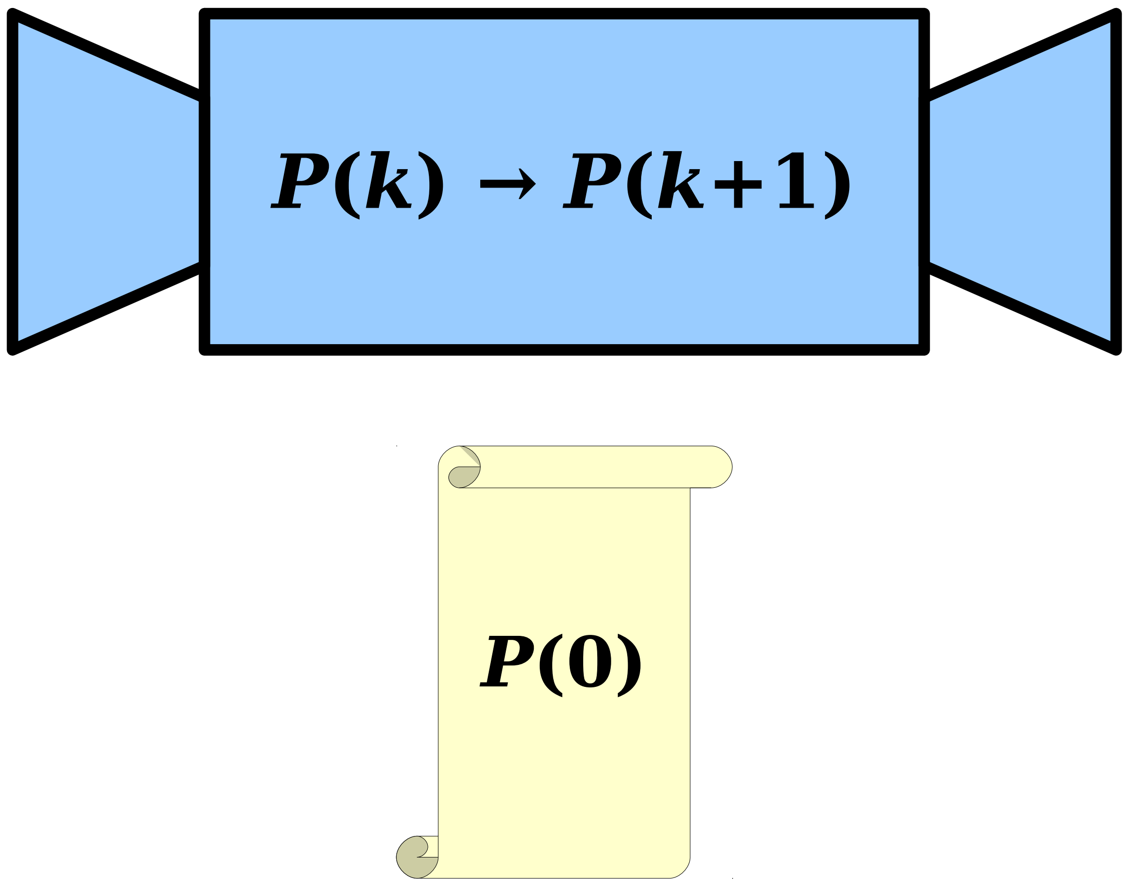
Complete Induction

- If the following are true:
  - $P(0)$ is true, and
  - If $P(0), P(1), P(2), \ldots, P(k)$ are true, then $P(k+1)$ is true as well.

  then $P(n)$ is true for all $n \in \mathbb{N}$.
- This is called the ***principle of complete induction*** or the ***principle of strong induction***.
  - (This also works starting from a number other than 0; just modify what you're assuming appropriately.)

Review: Induction as a Machine



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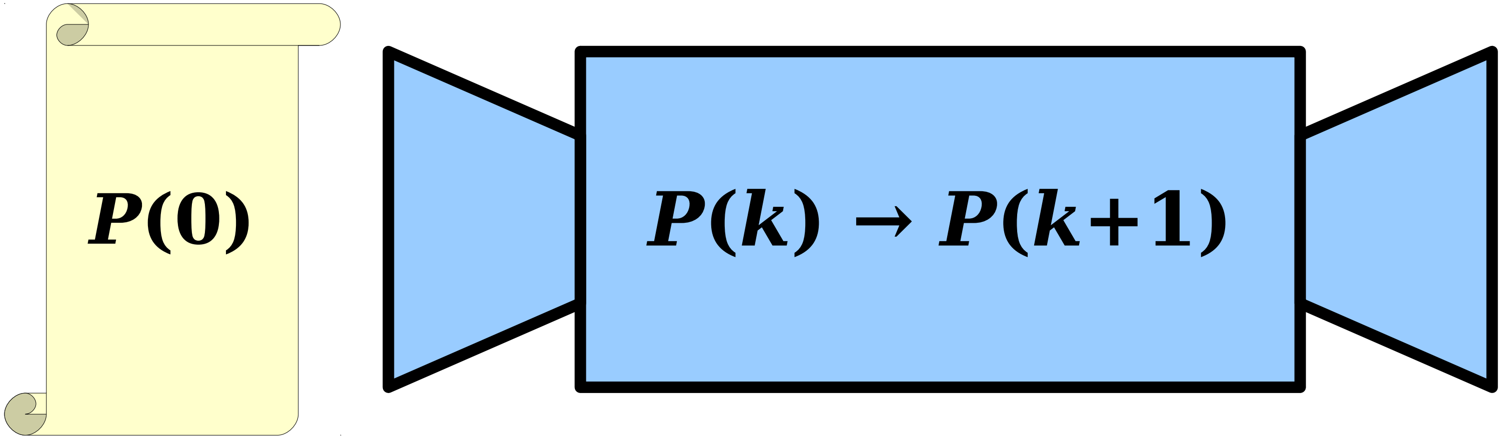

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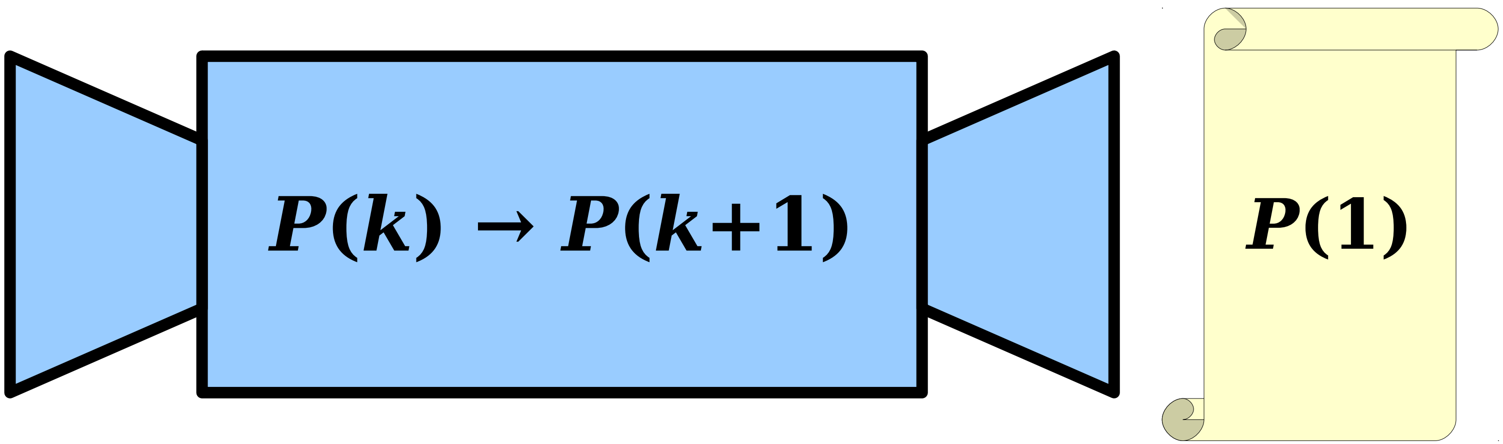

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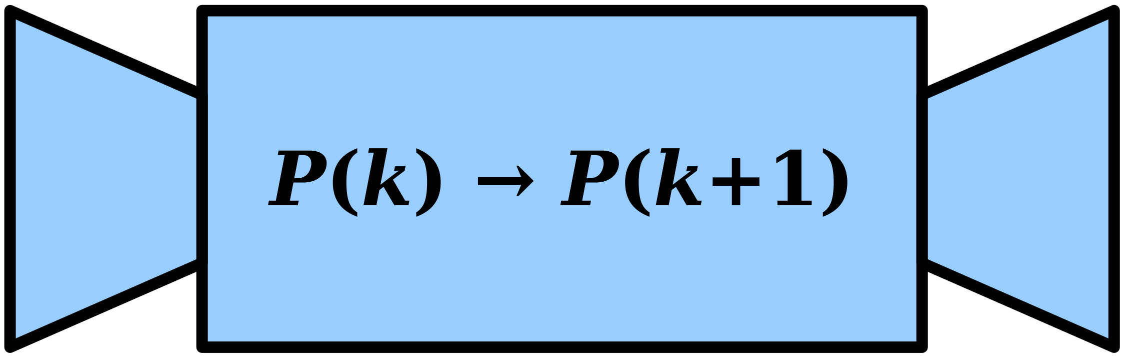

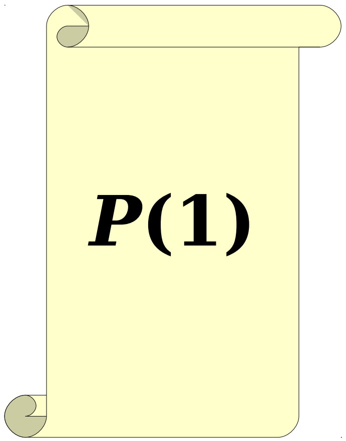

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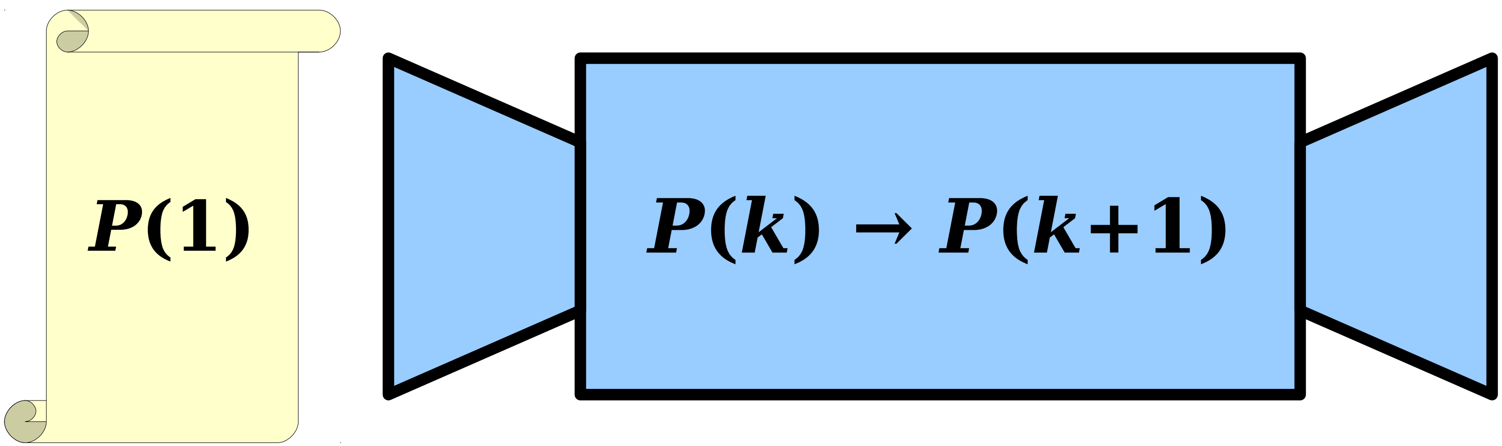

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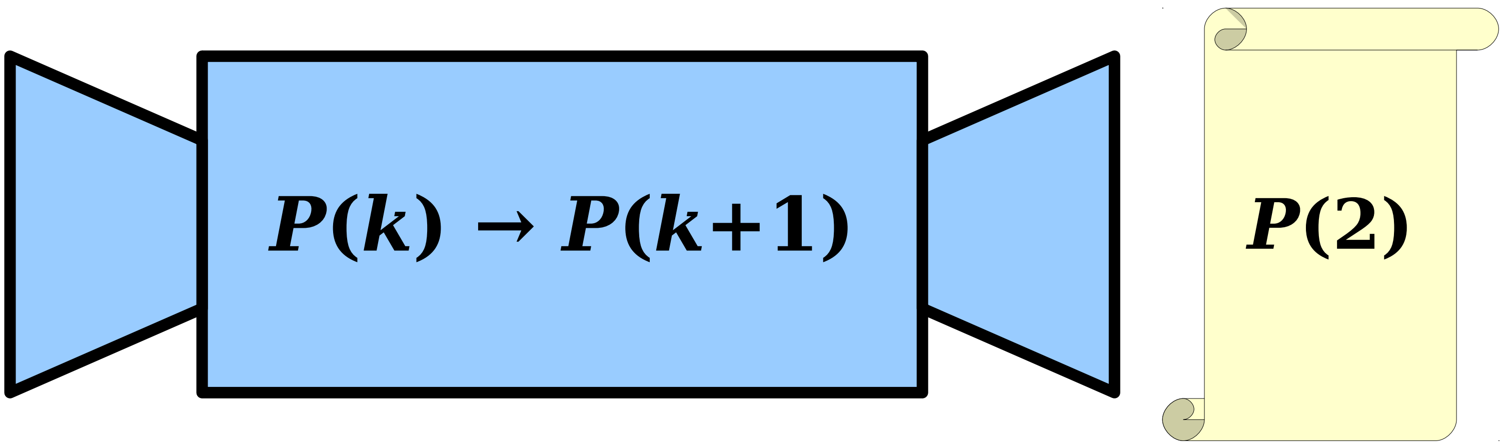

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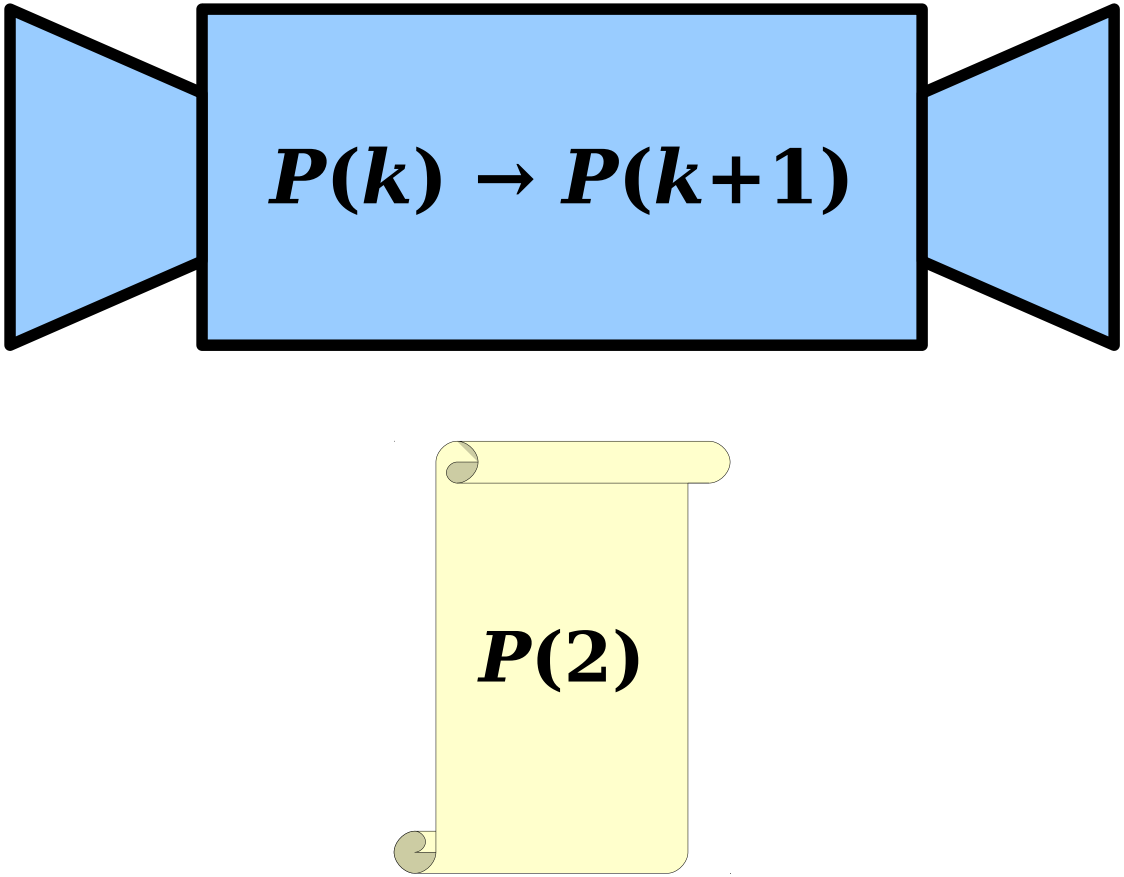

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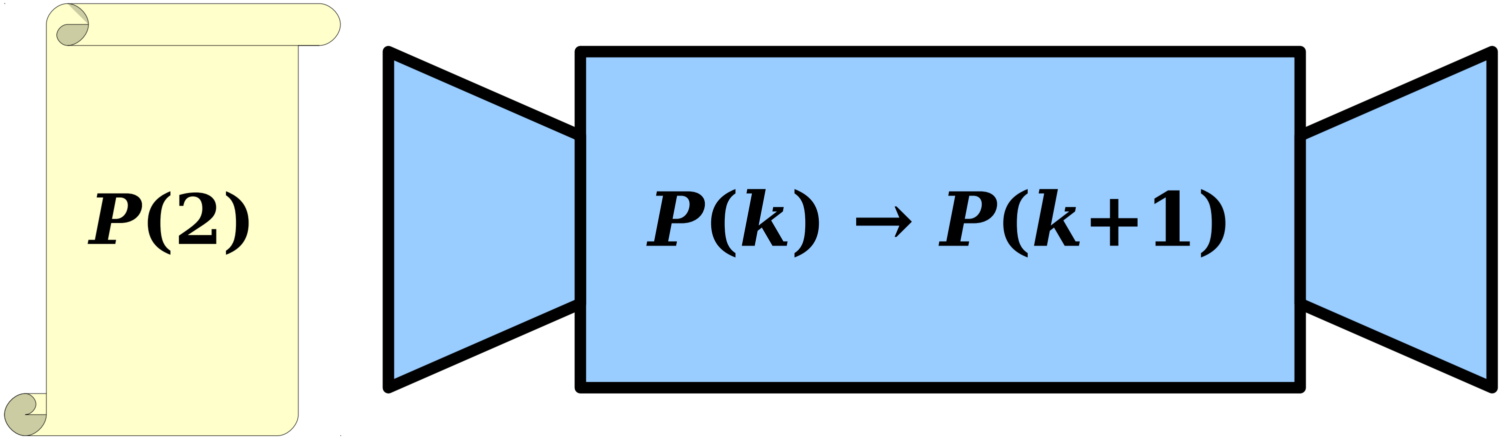

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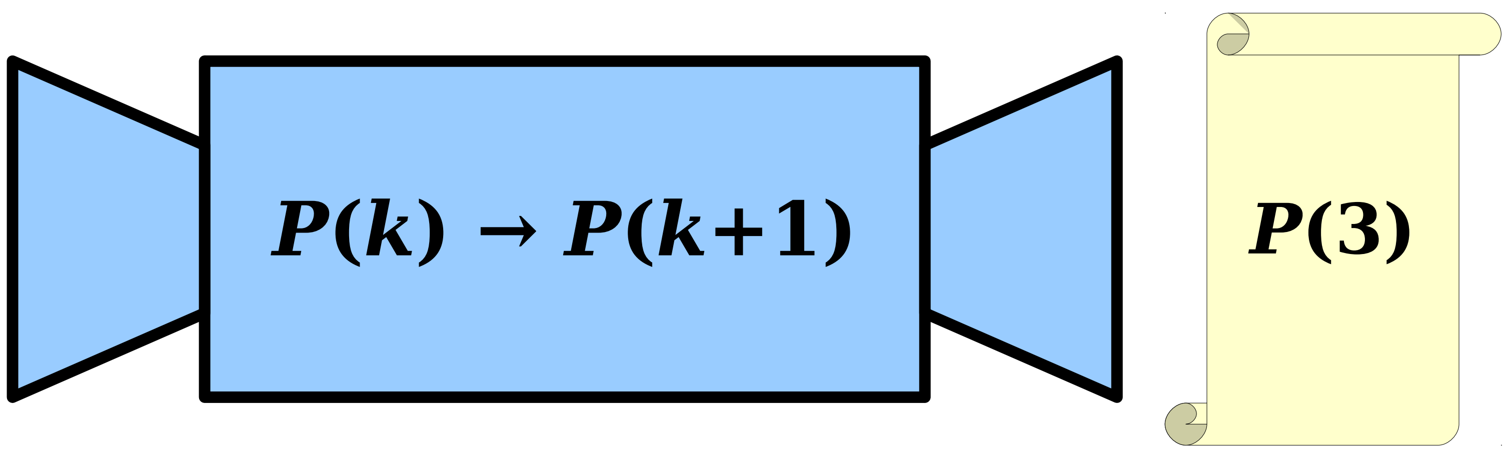

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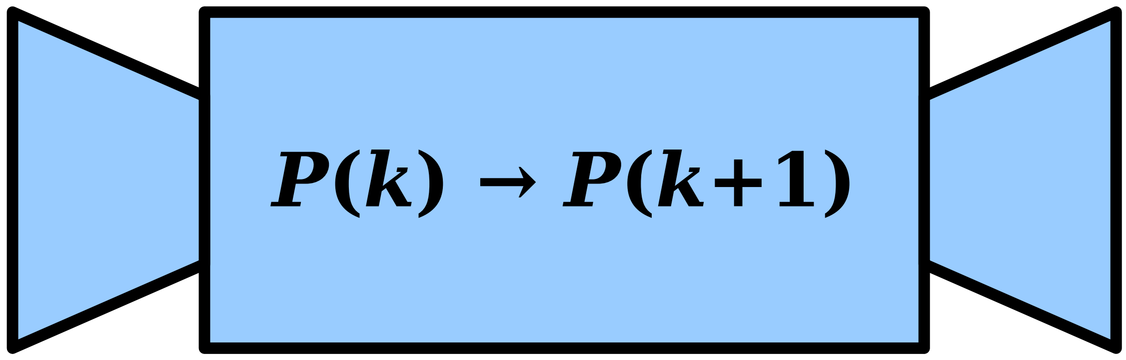

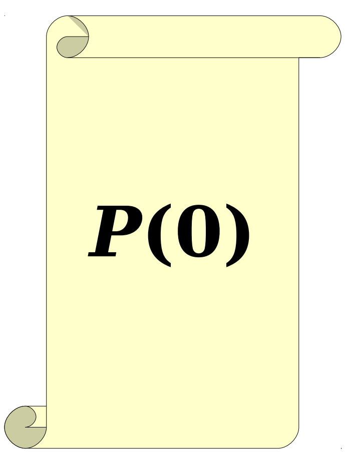

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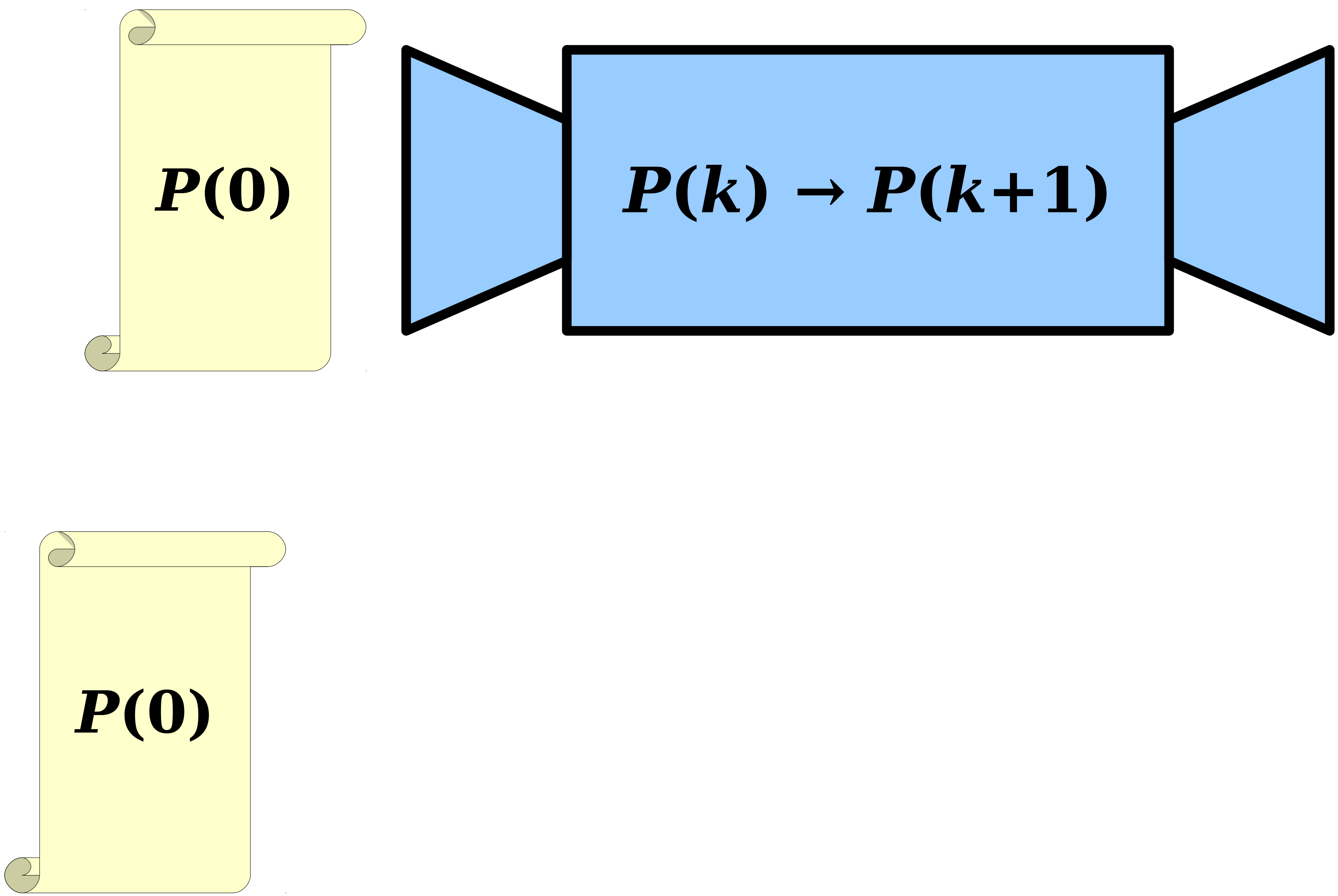

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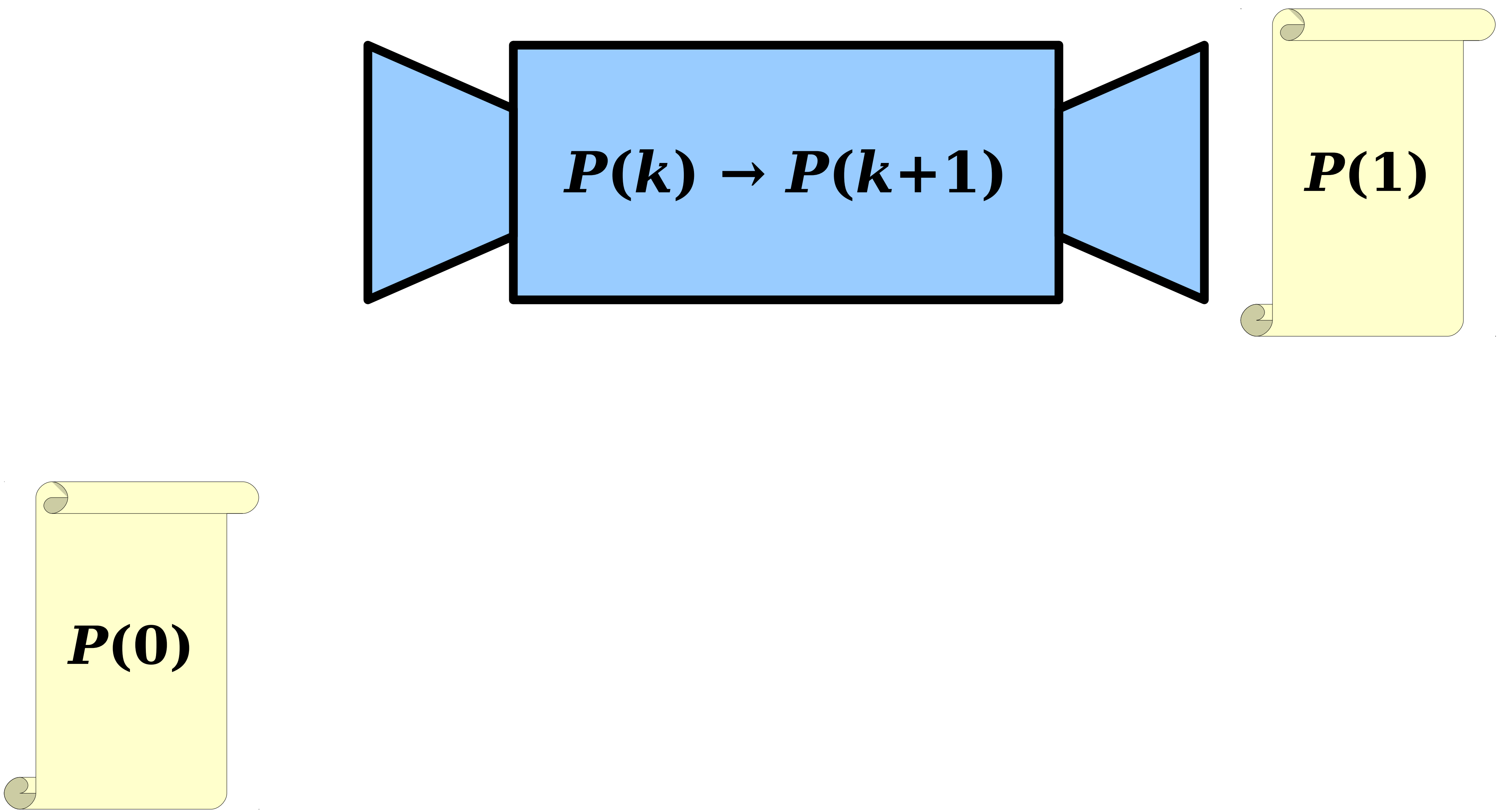

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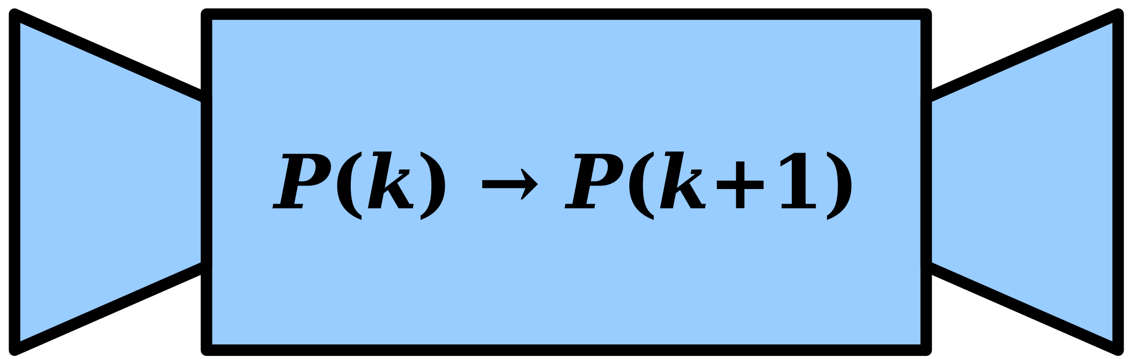

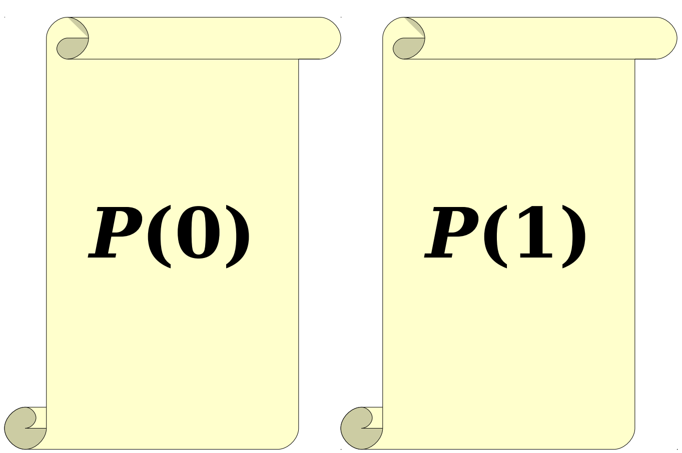

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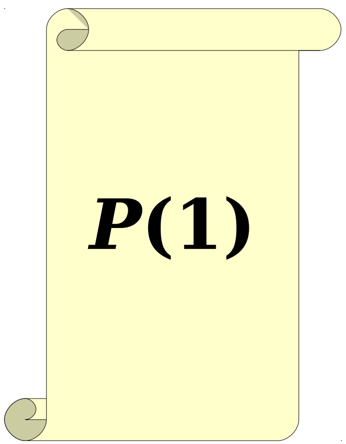

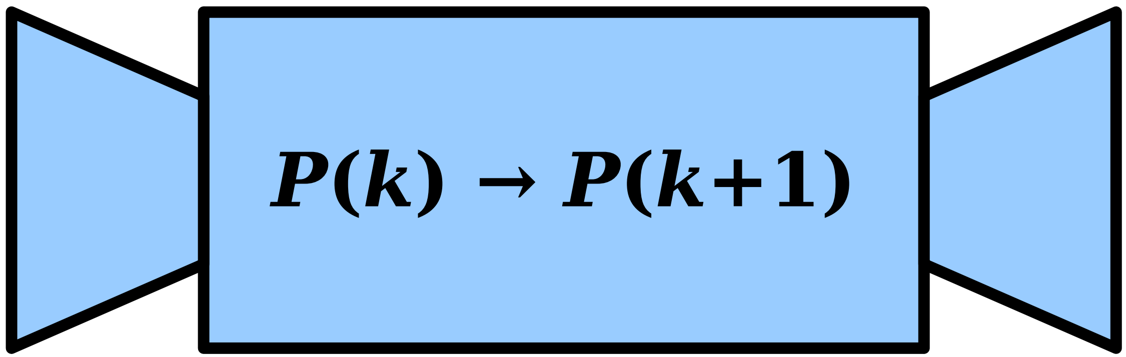

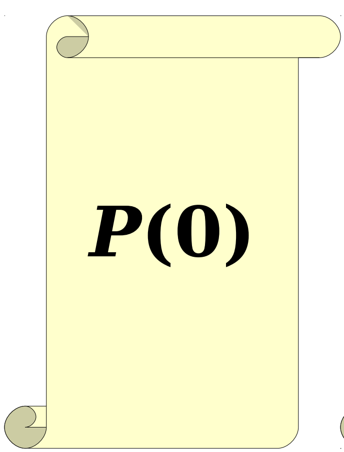

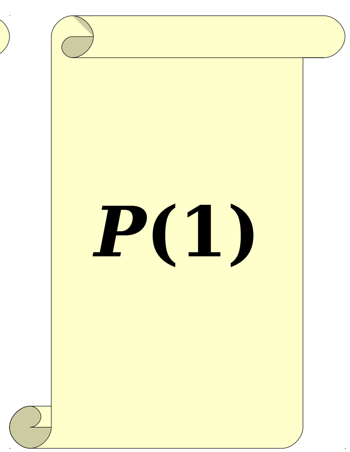

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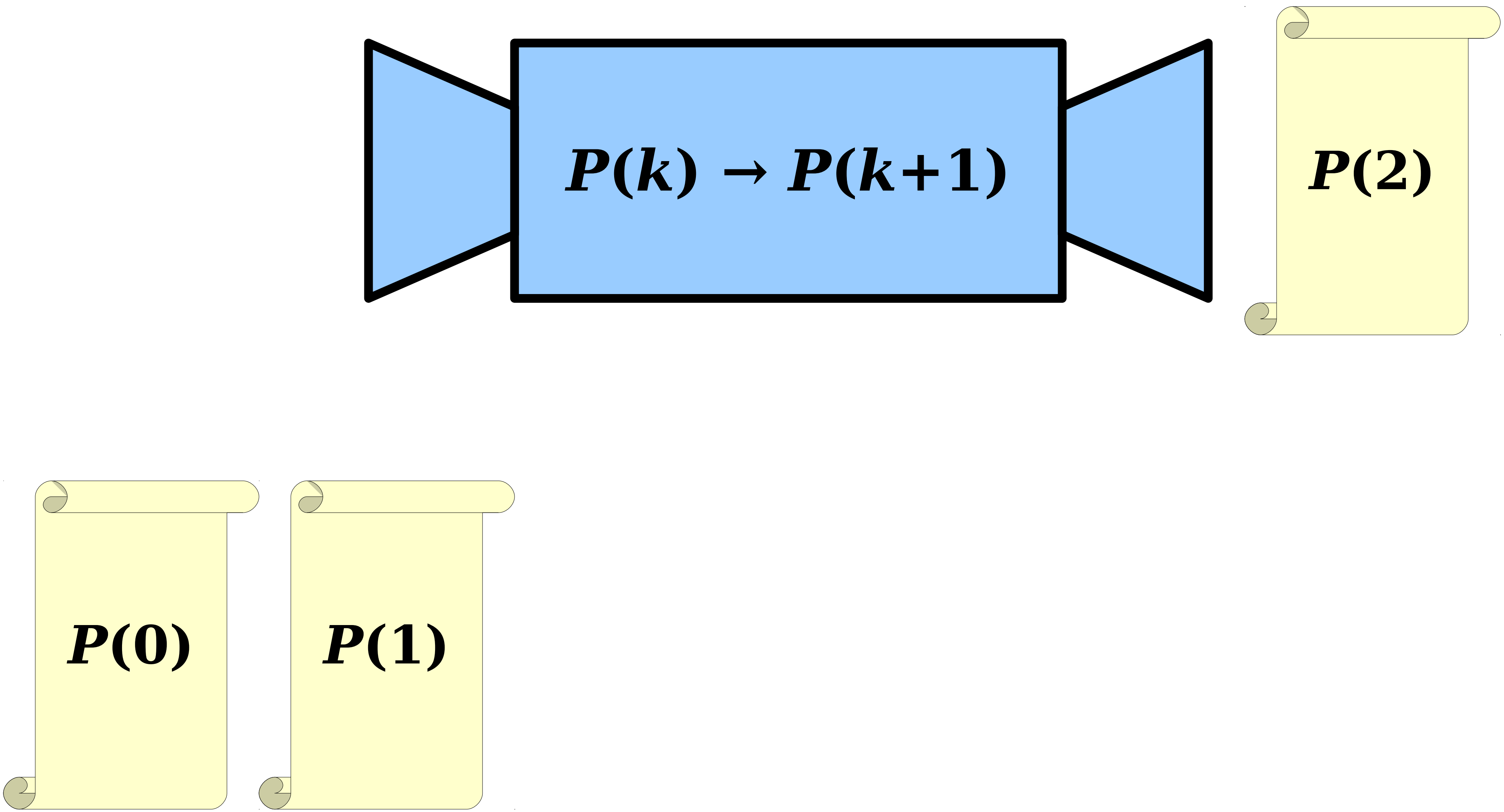

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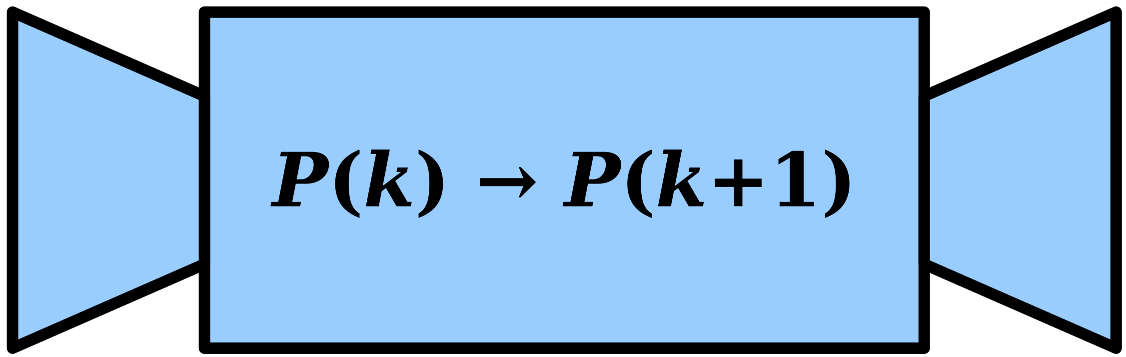

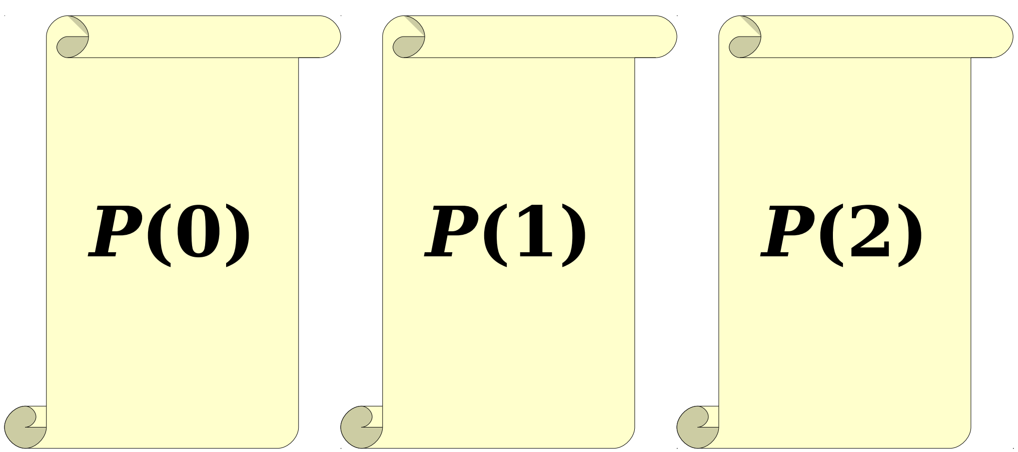

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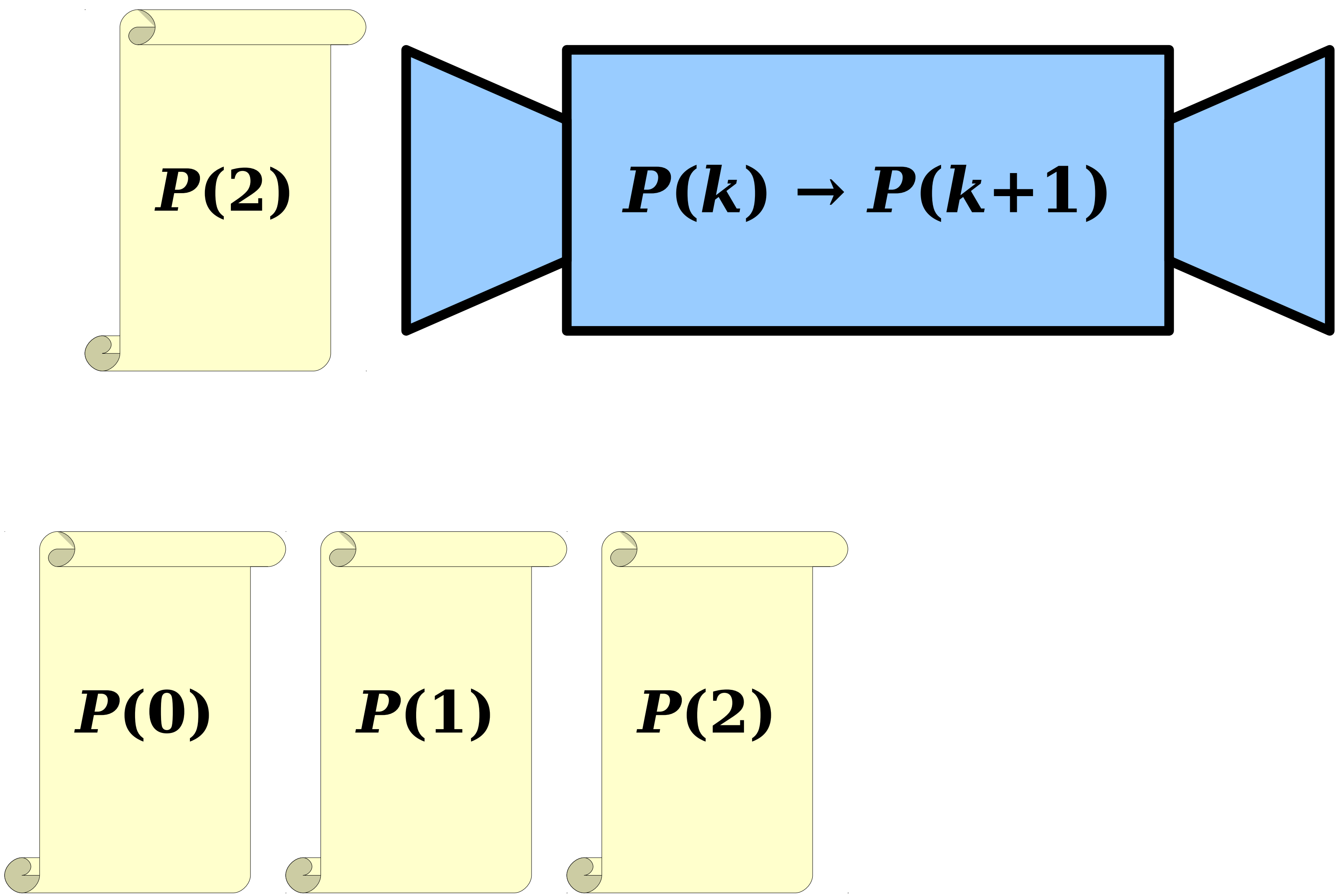

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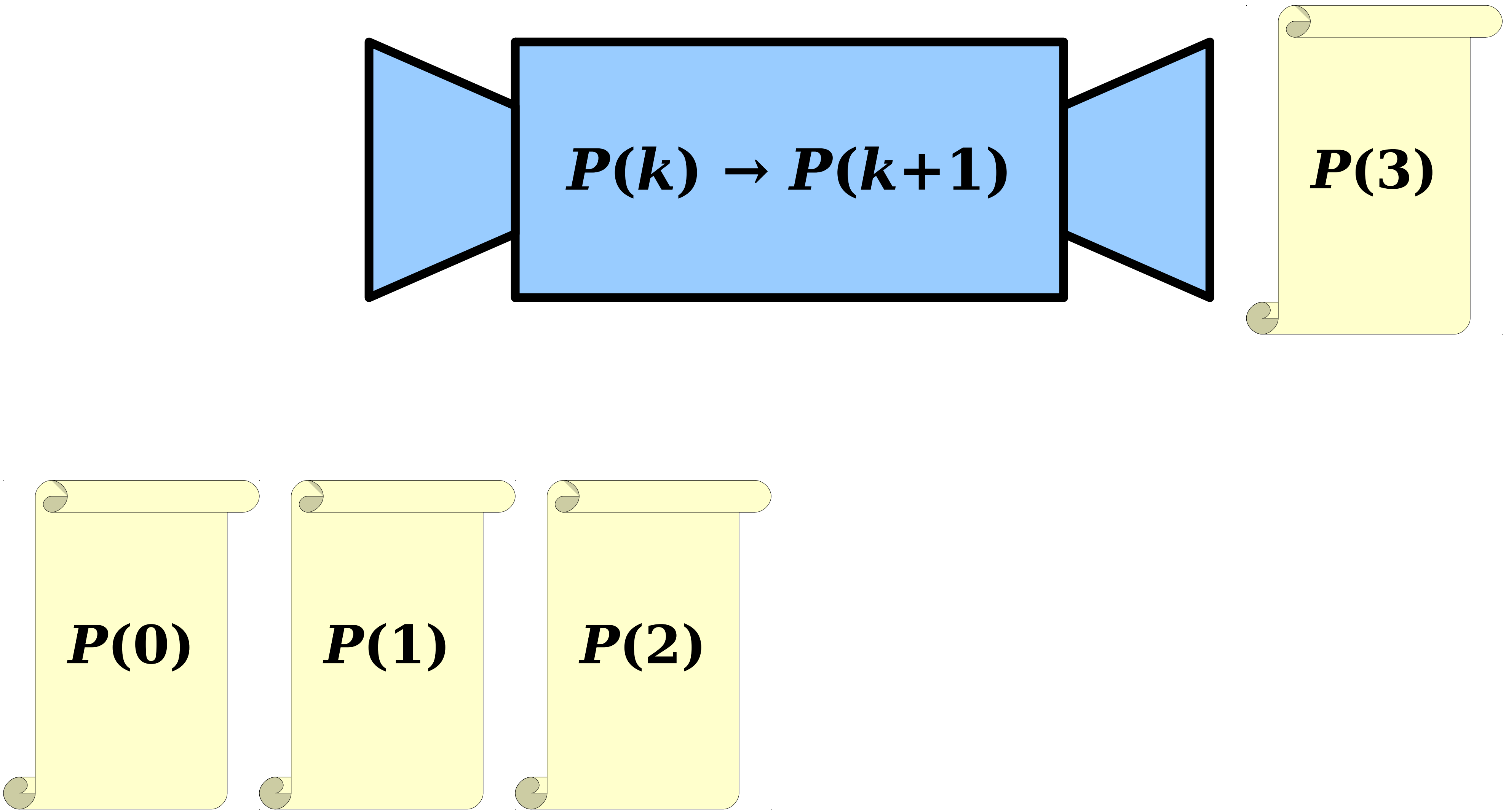

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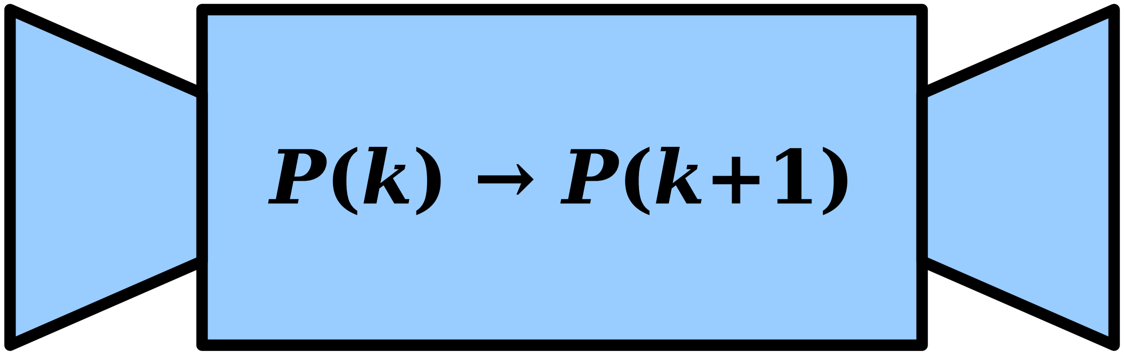

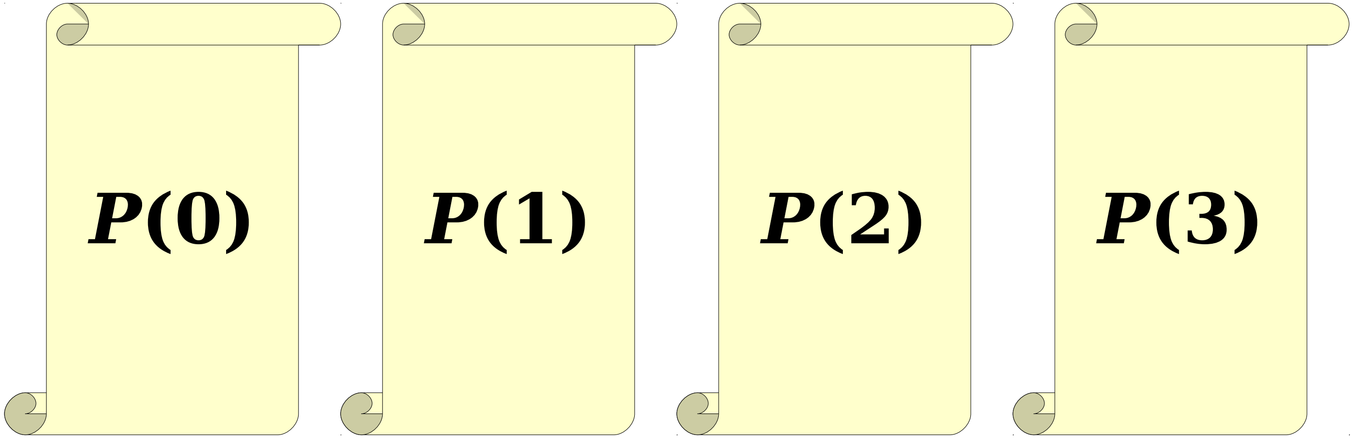

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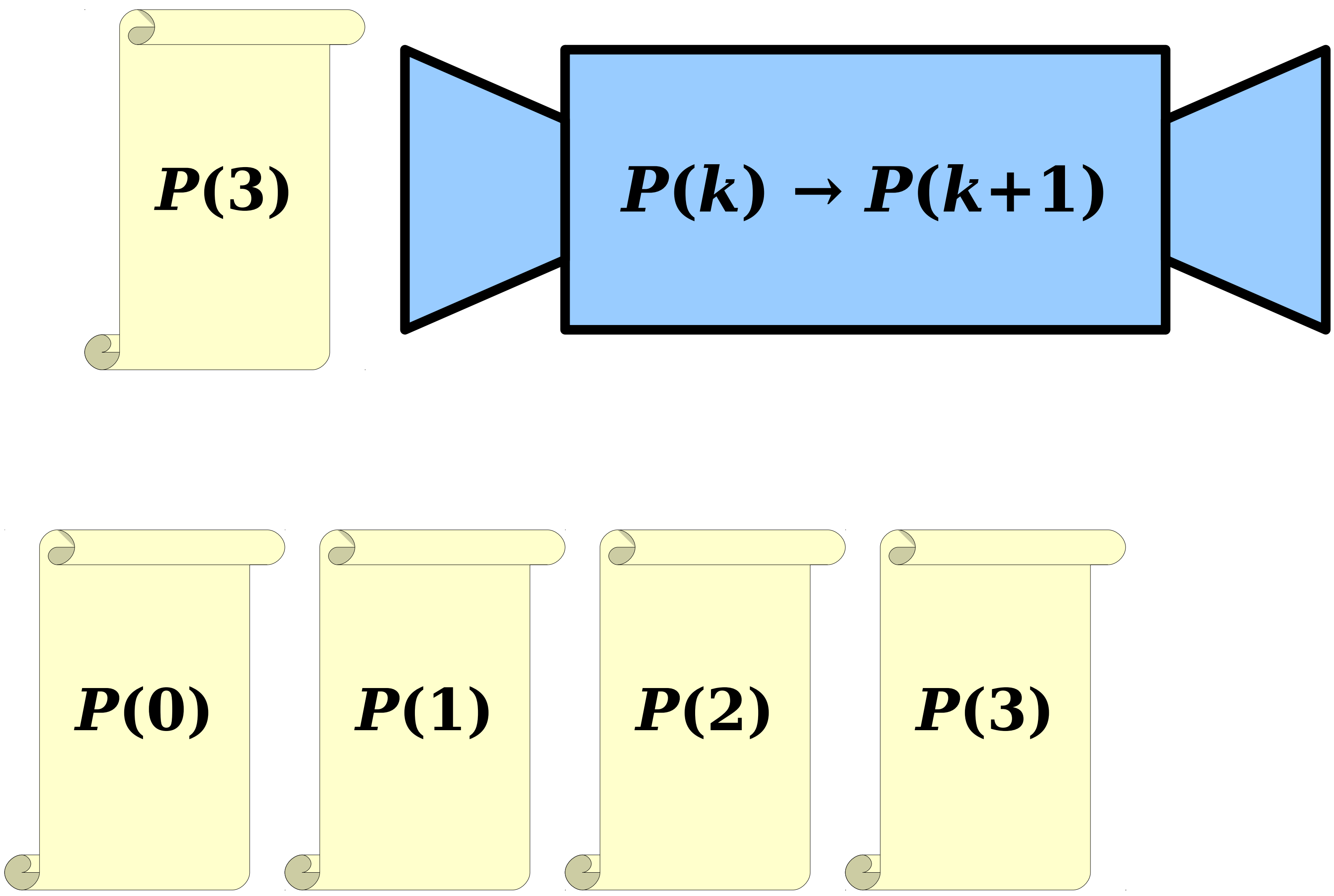

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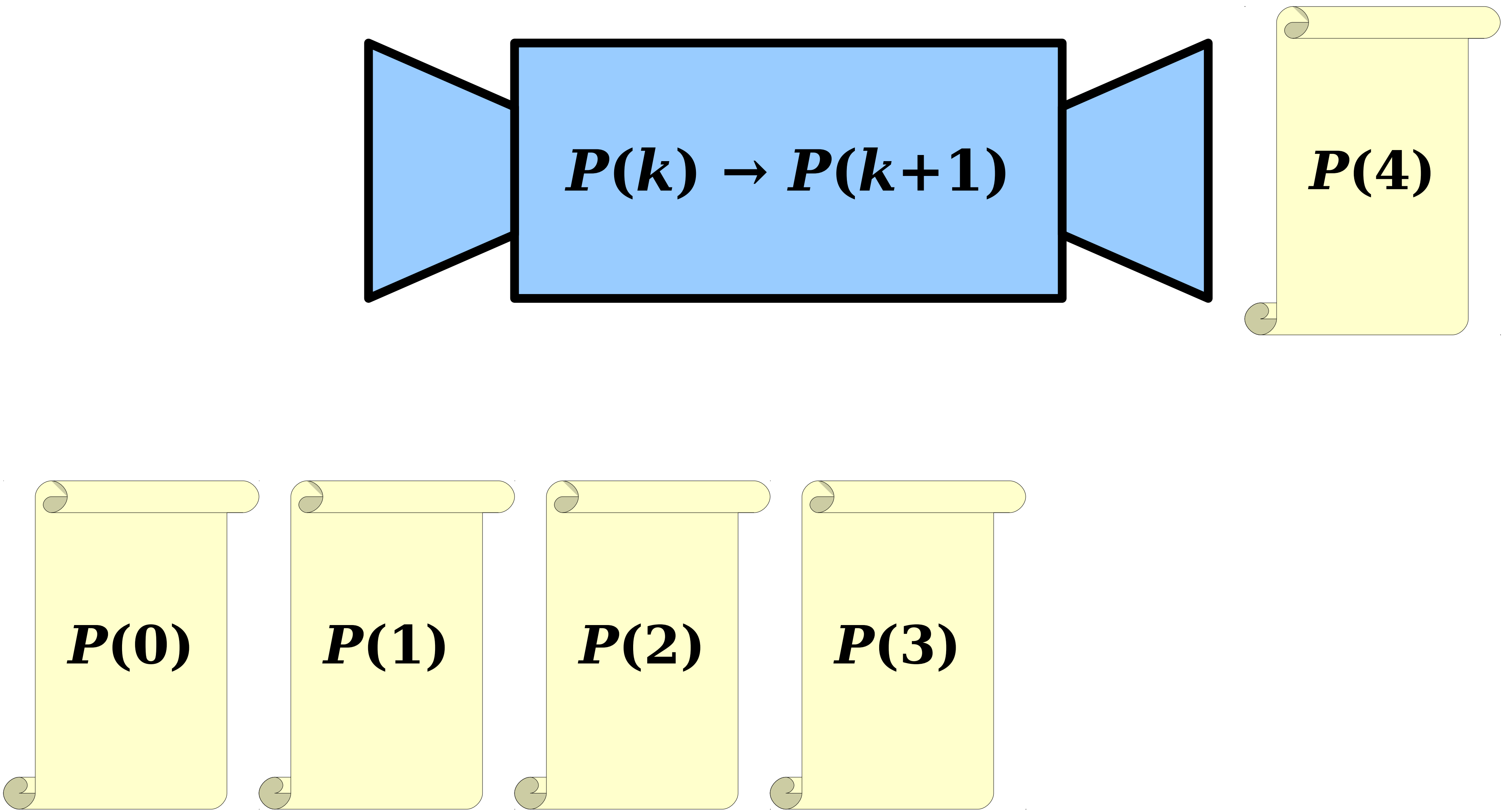

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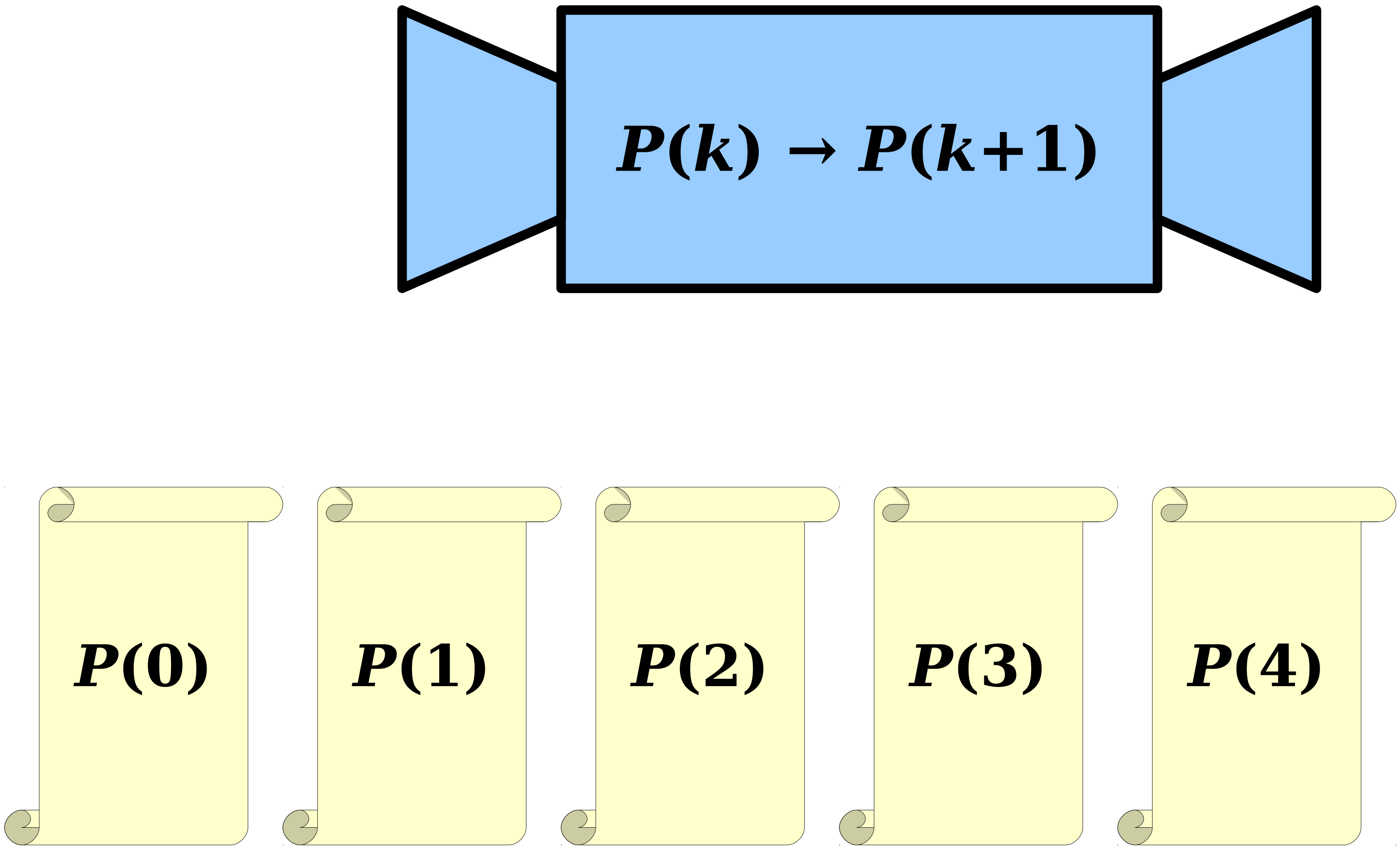

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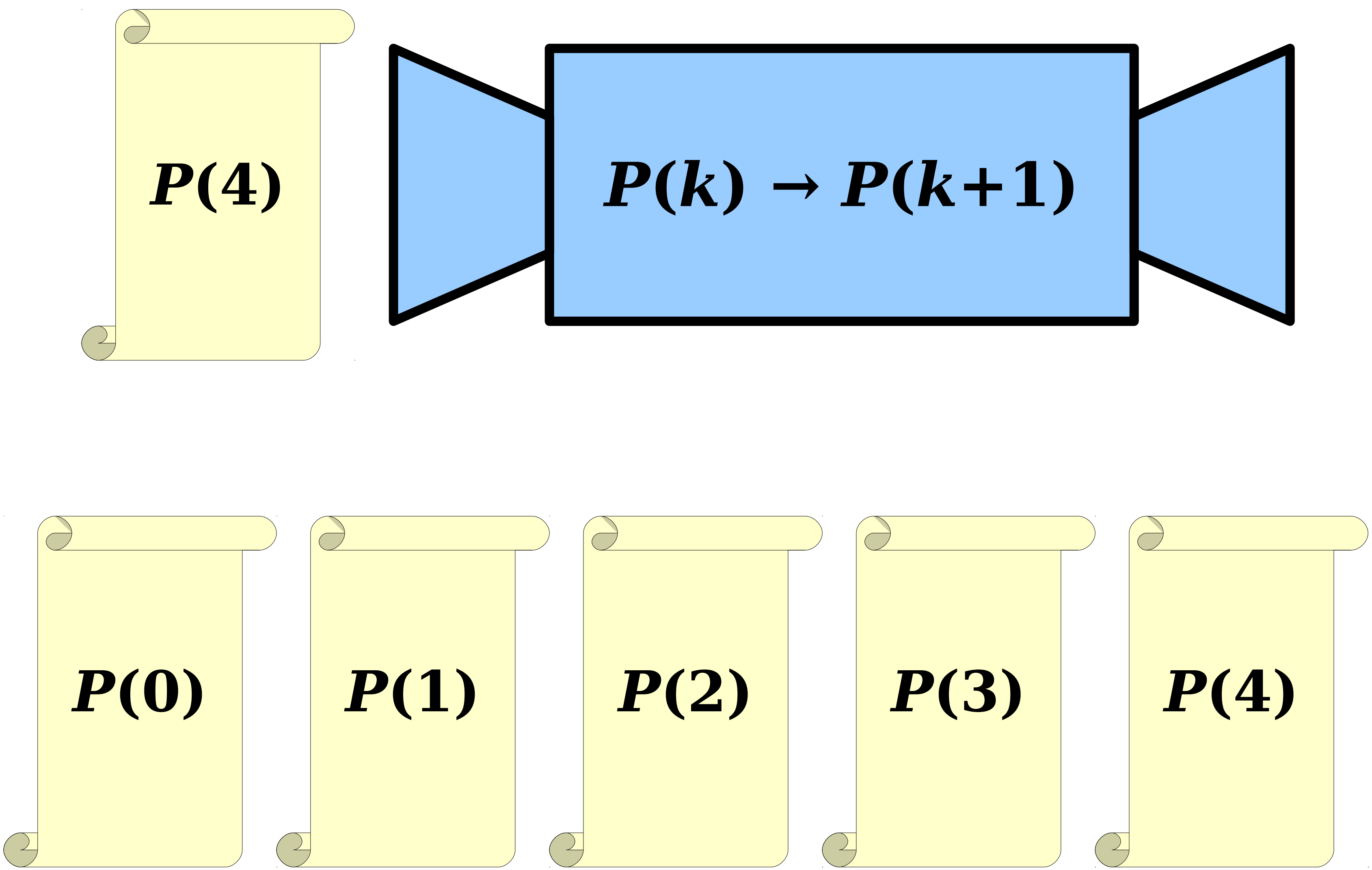

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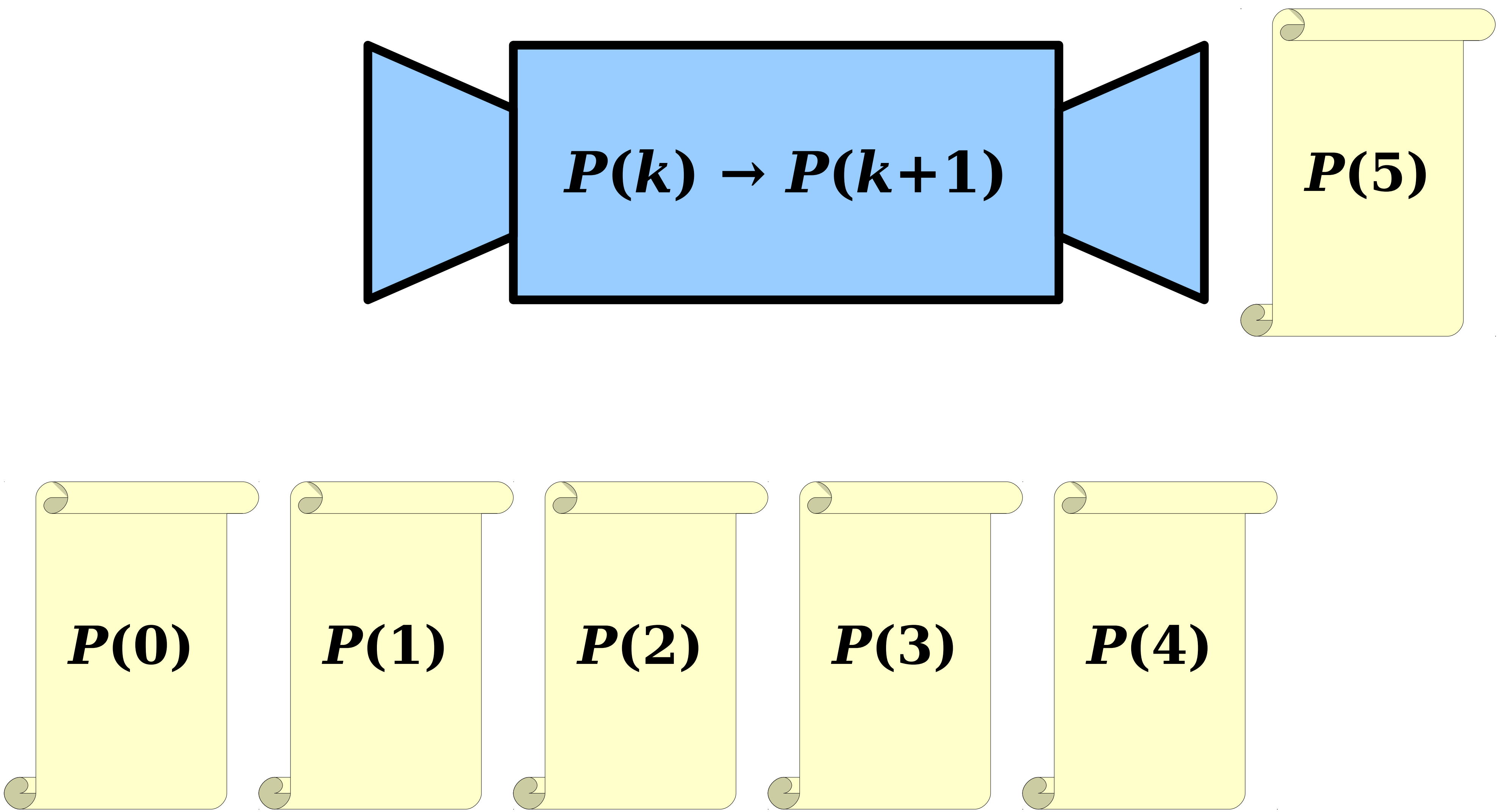

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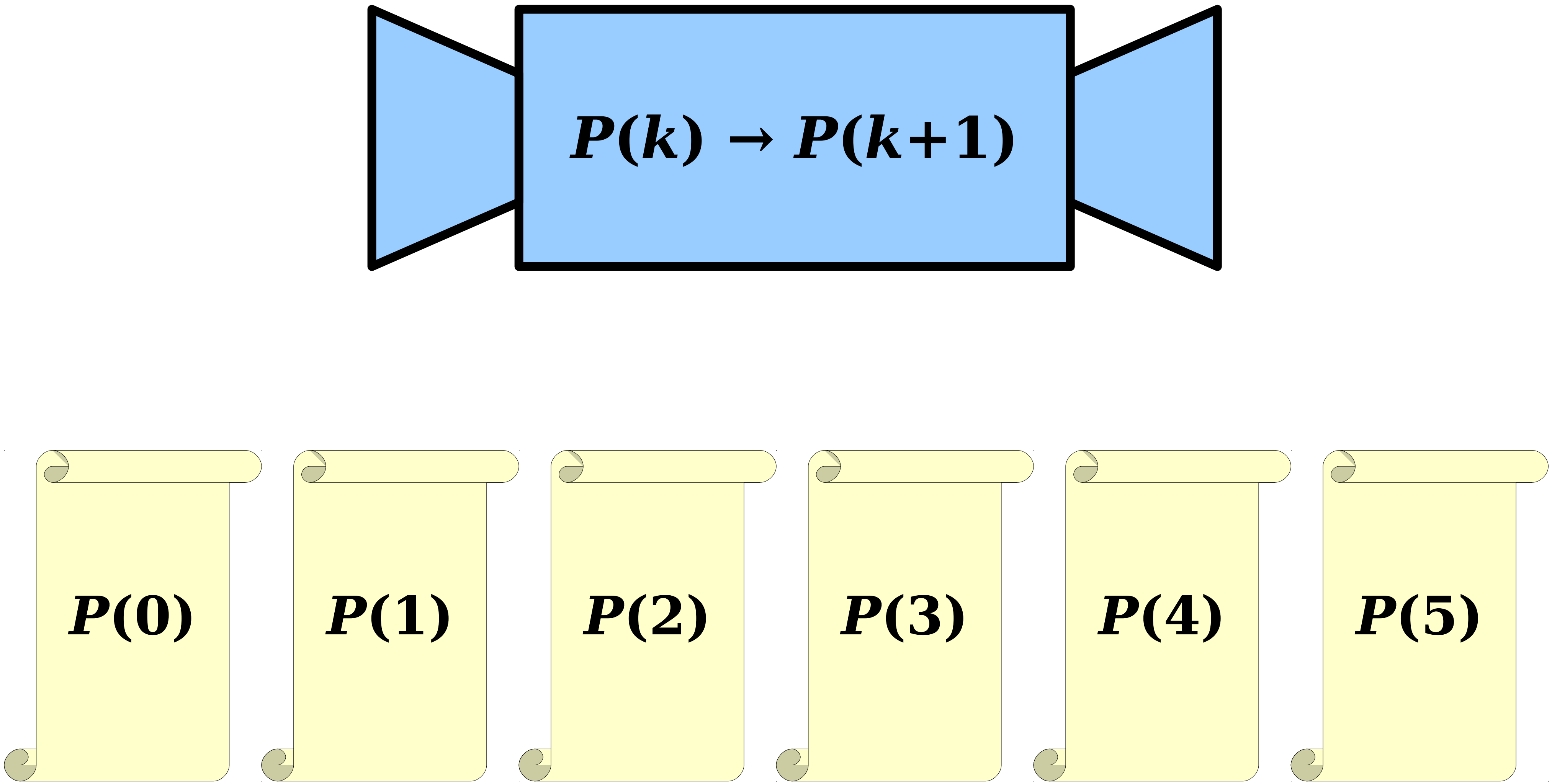
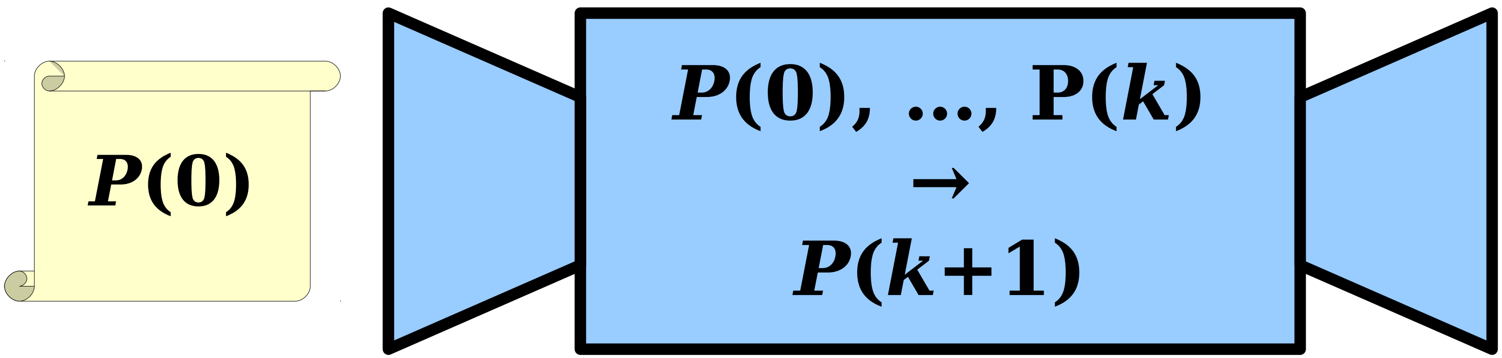


Intuiting Complete Induction

$$P(0), \ldots, P(k) \rightarrow P(k+1)$$

$P(0)$

Intuiting Complete Induction



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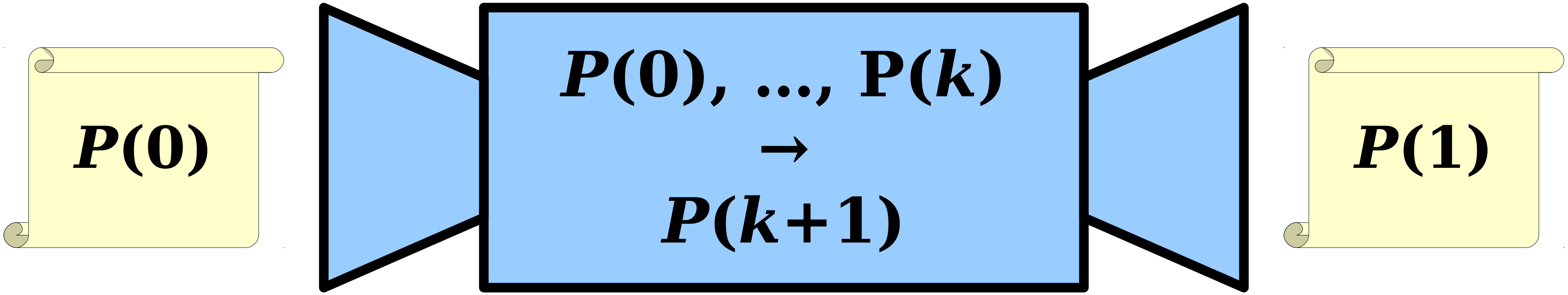
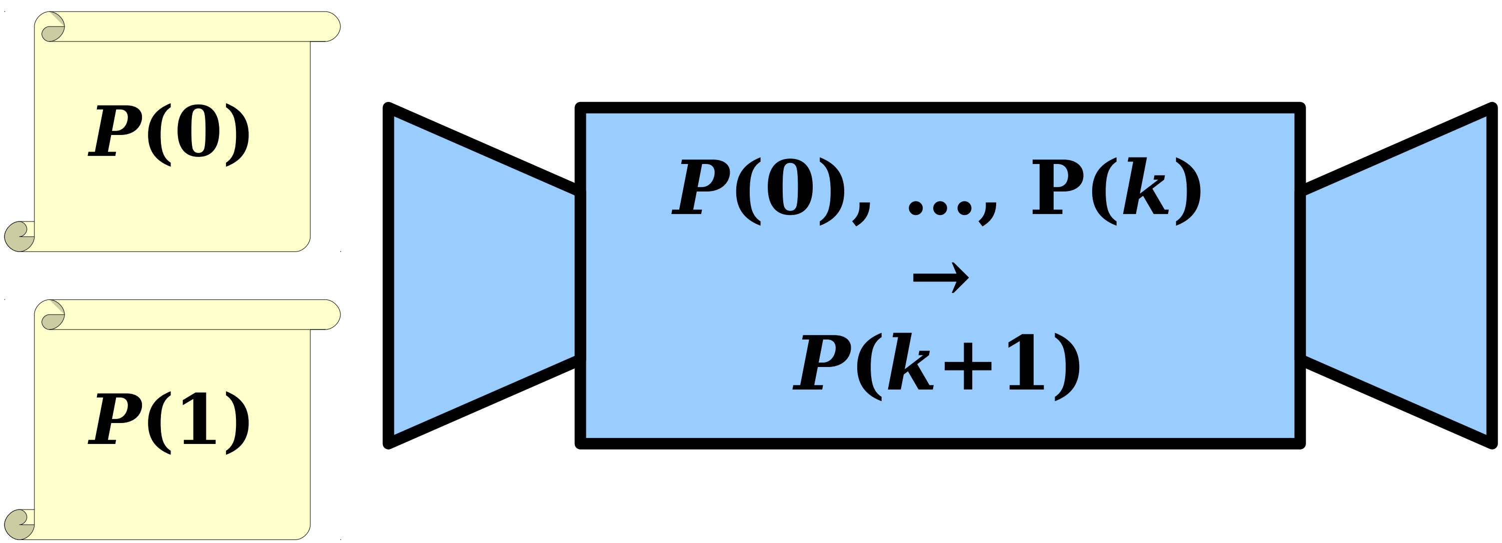


Intuiting Complete Induction

$$P(0), \ldots, P(k) \to P(k+1)$$

$P(0)$ $P(1)$

Intuiting Complete Induction



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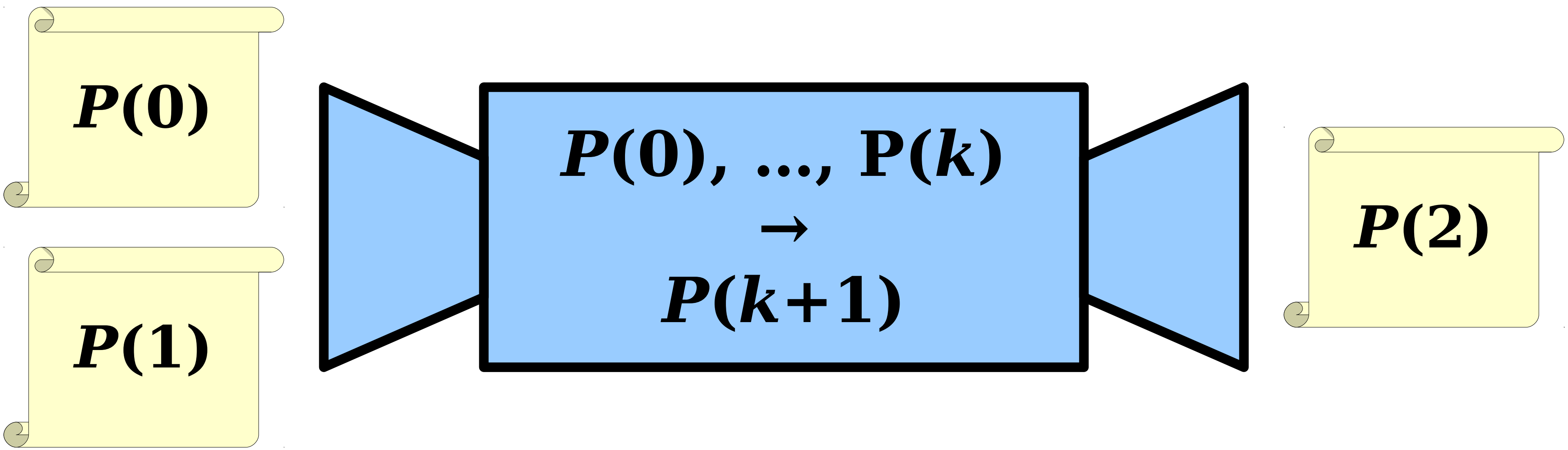

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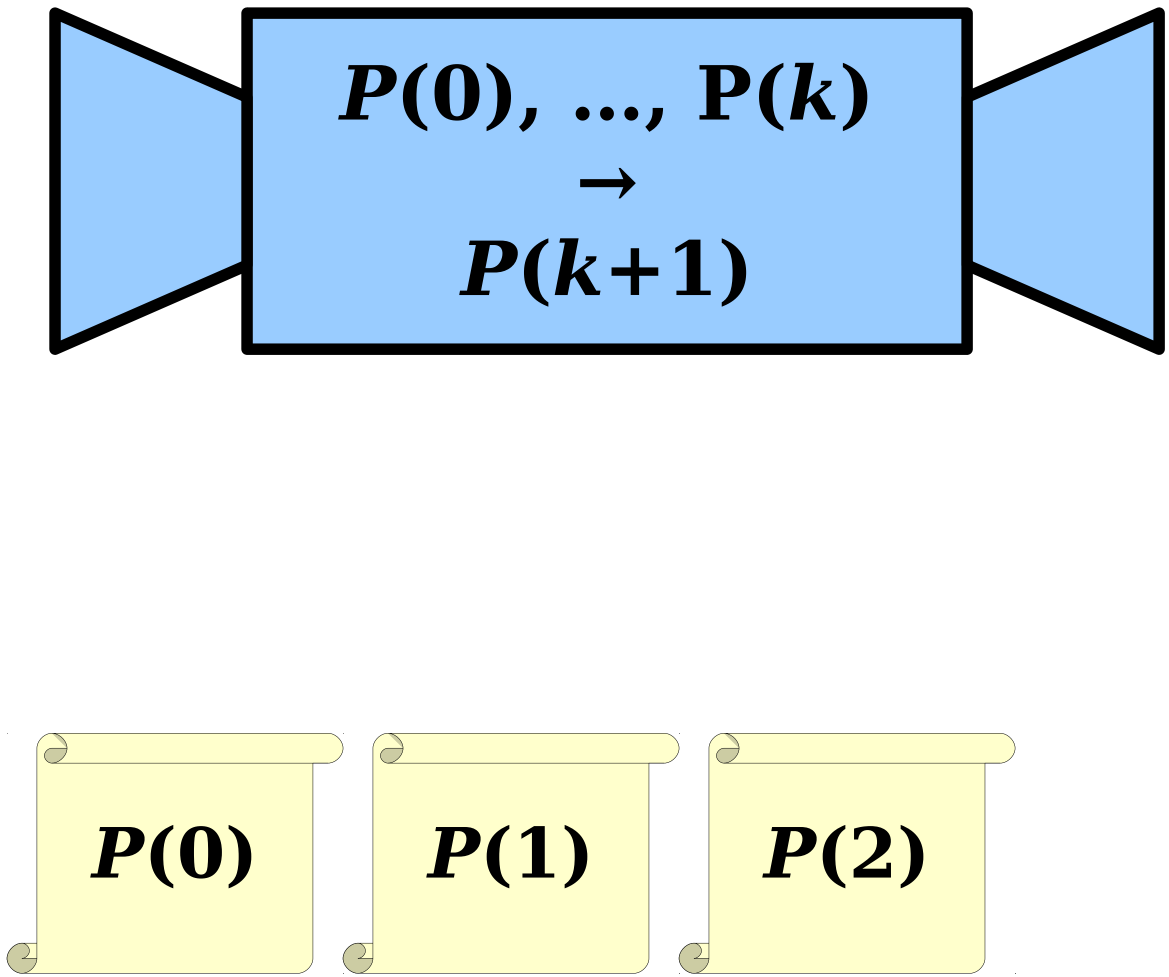

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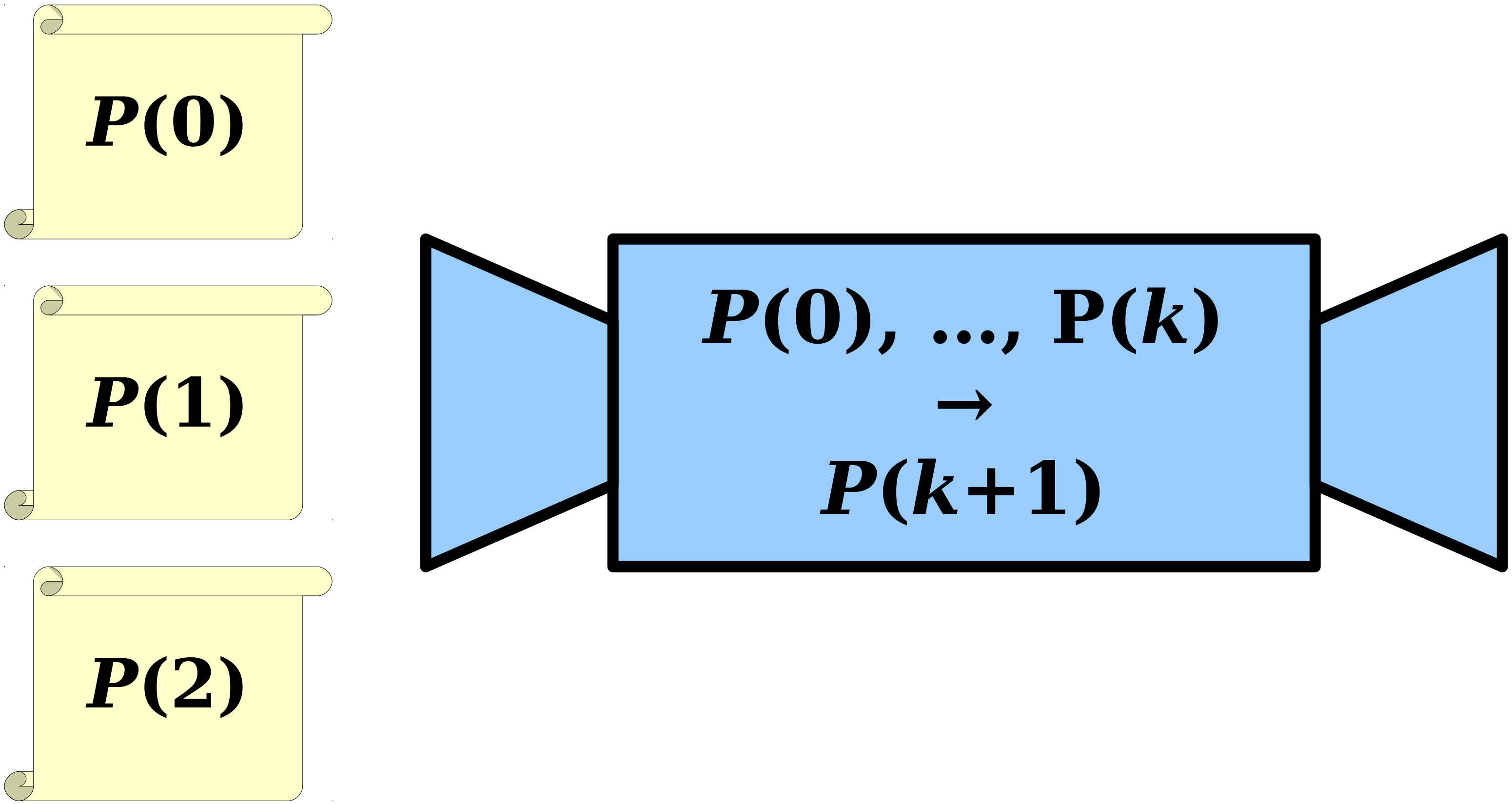

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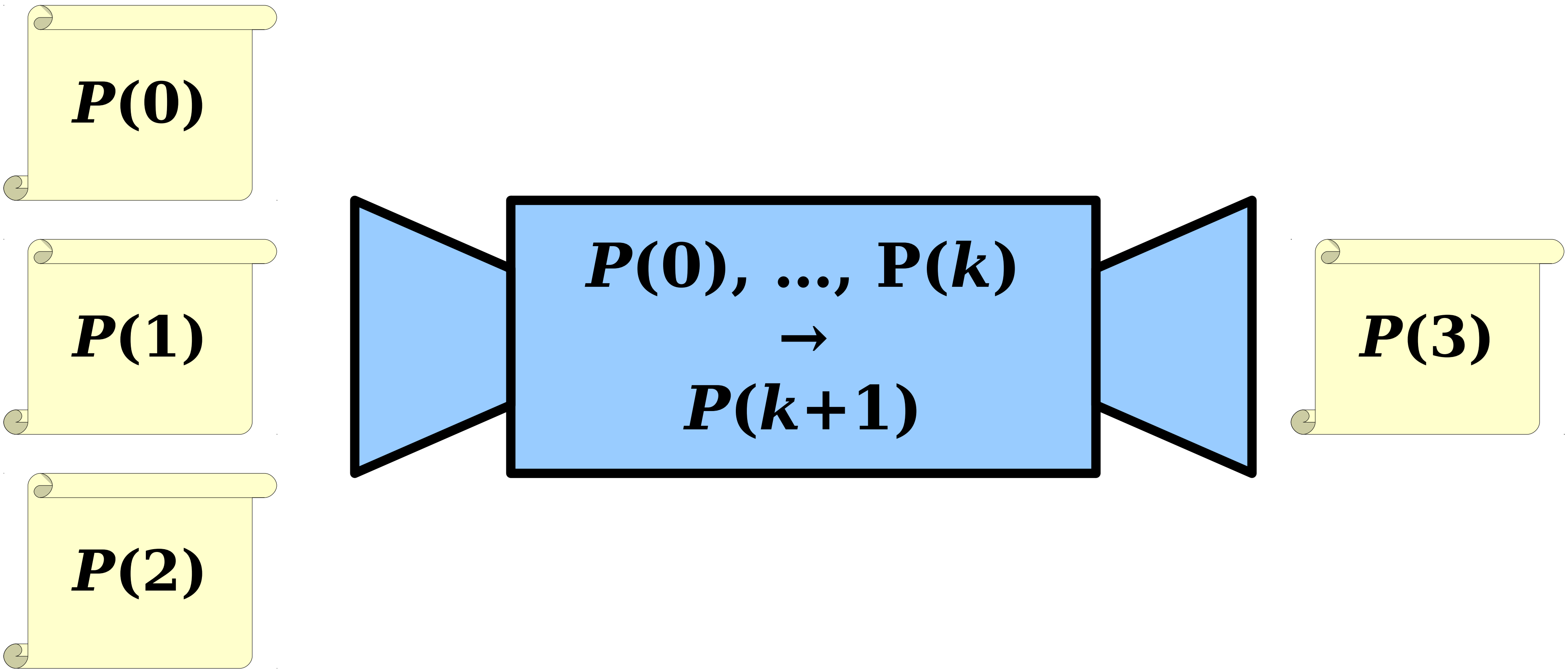

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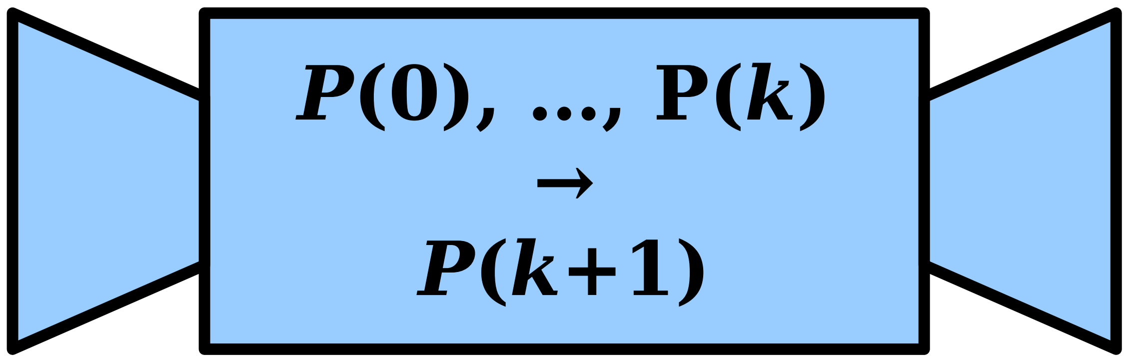
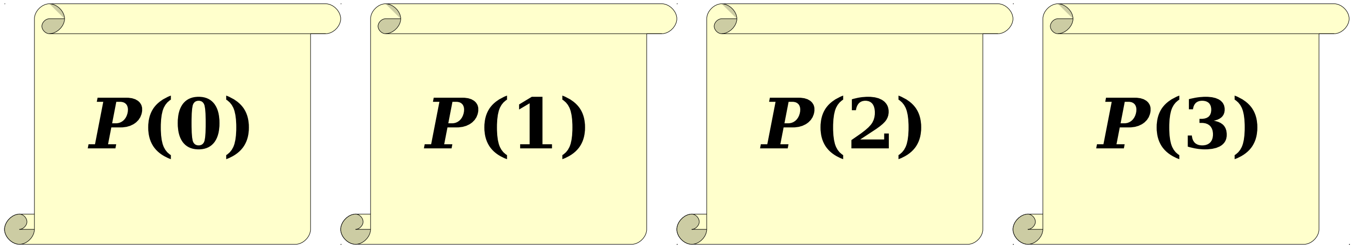


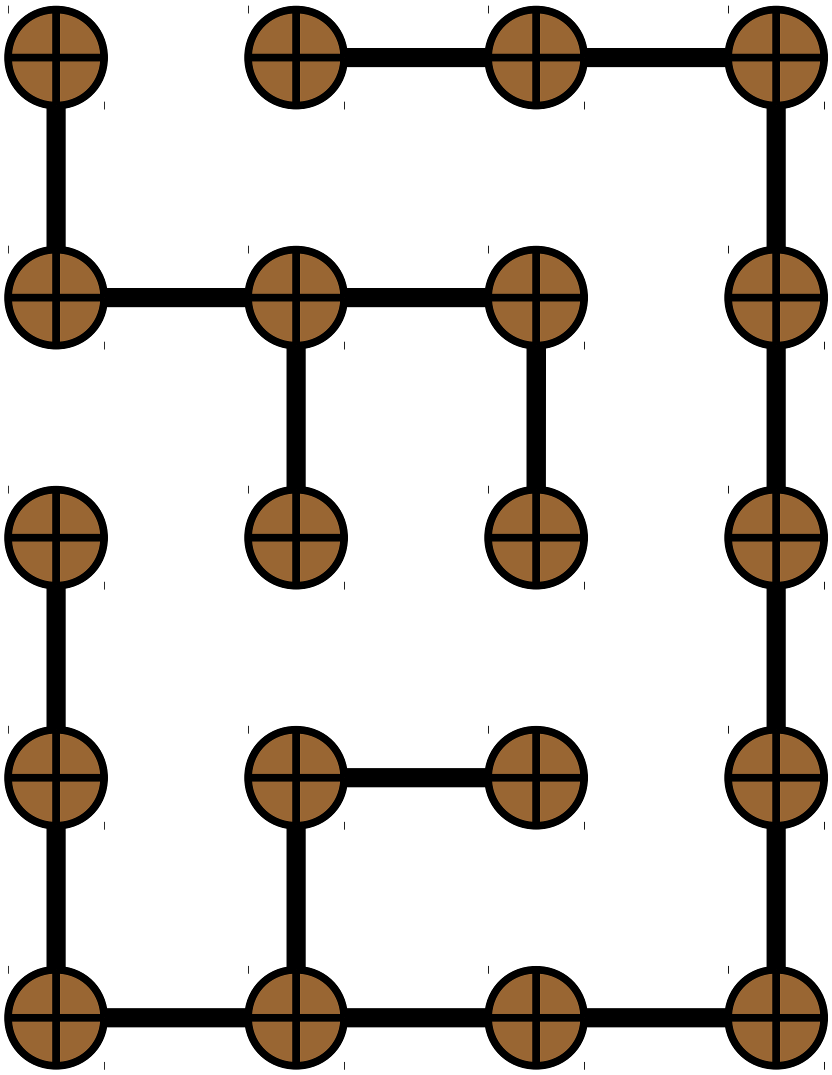


When Use Complete Induction?

- Normal induction is good for when you are shrinking the problem size by exactly one.
  - Peeling one final term off a sum.
  - Making one weighing on a scale.
  - Considering one more action on a string.
- Complete induction is good when you are shrinking the problem, but you can't be sure by how much.
  - In the previous example, if we delete a random edge, we can't know in advance how big the resulting trees will be.

Rat Mazes

- Suppose you want to make a rat maze consisting of an $n \times m$ grid of pegs with slats between them.
- ***Question:*** How many slats do you need to create?



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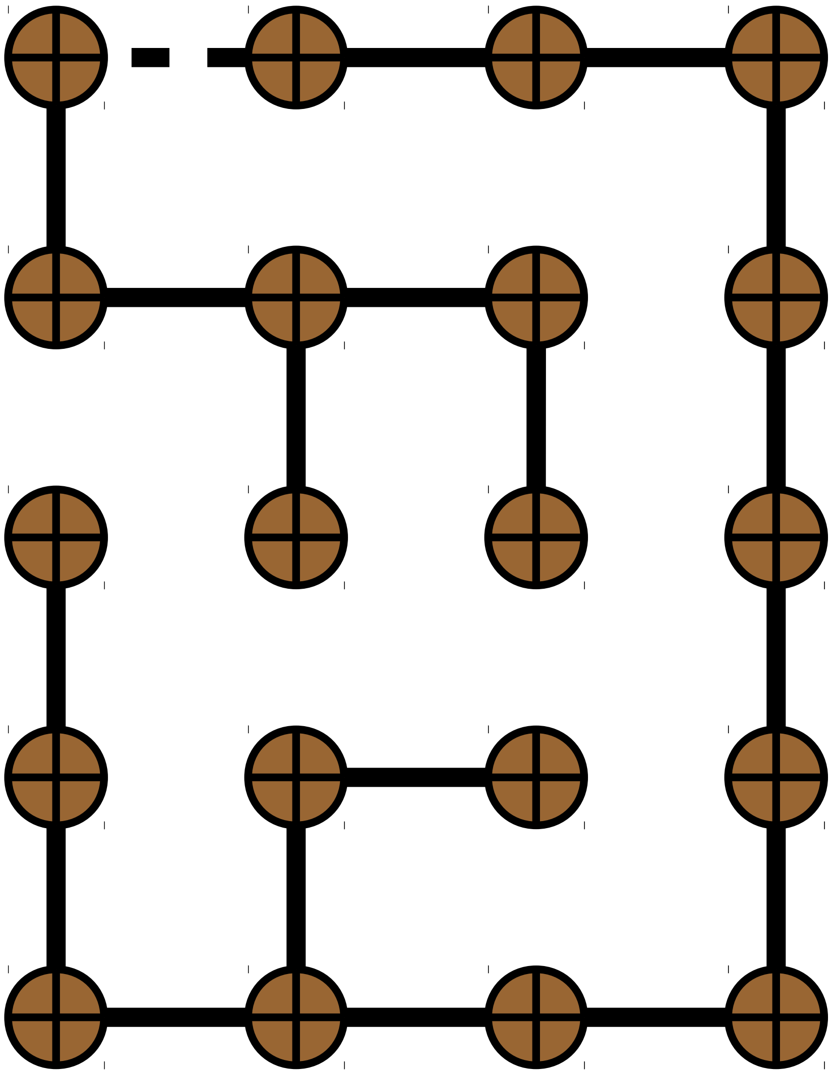

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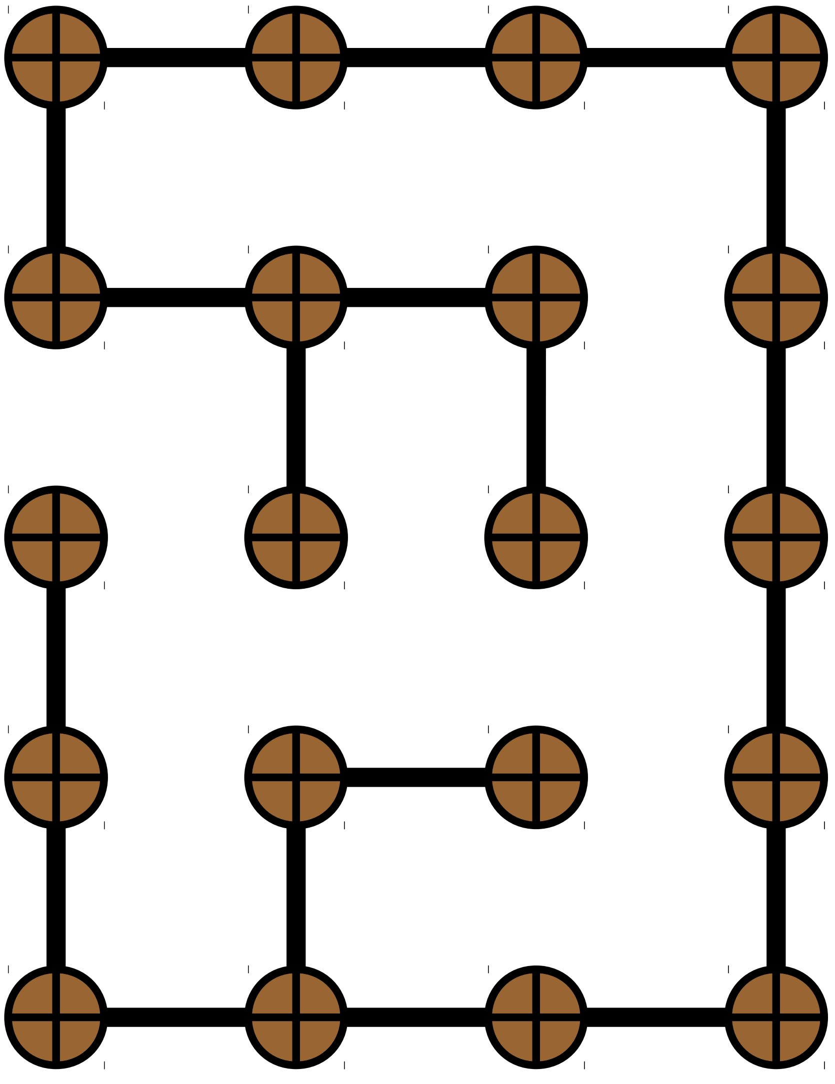

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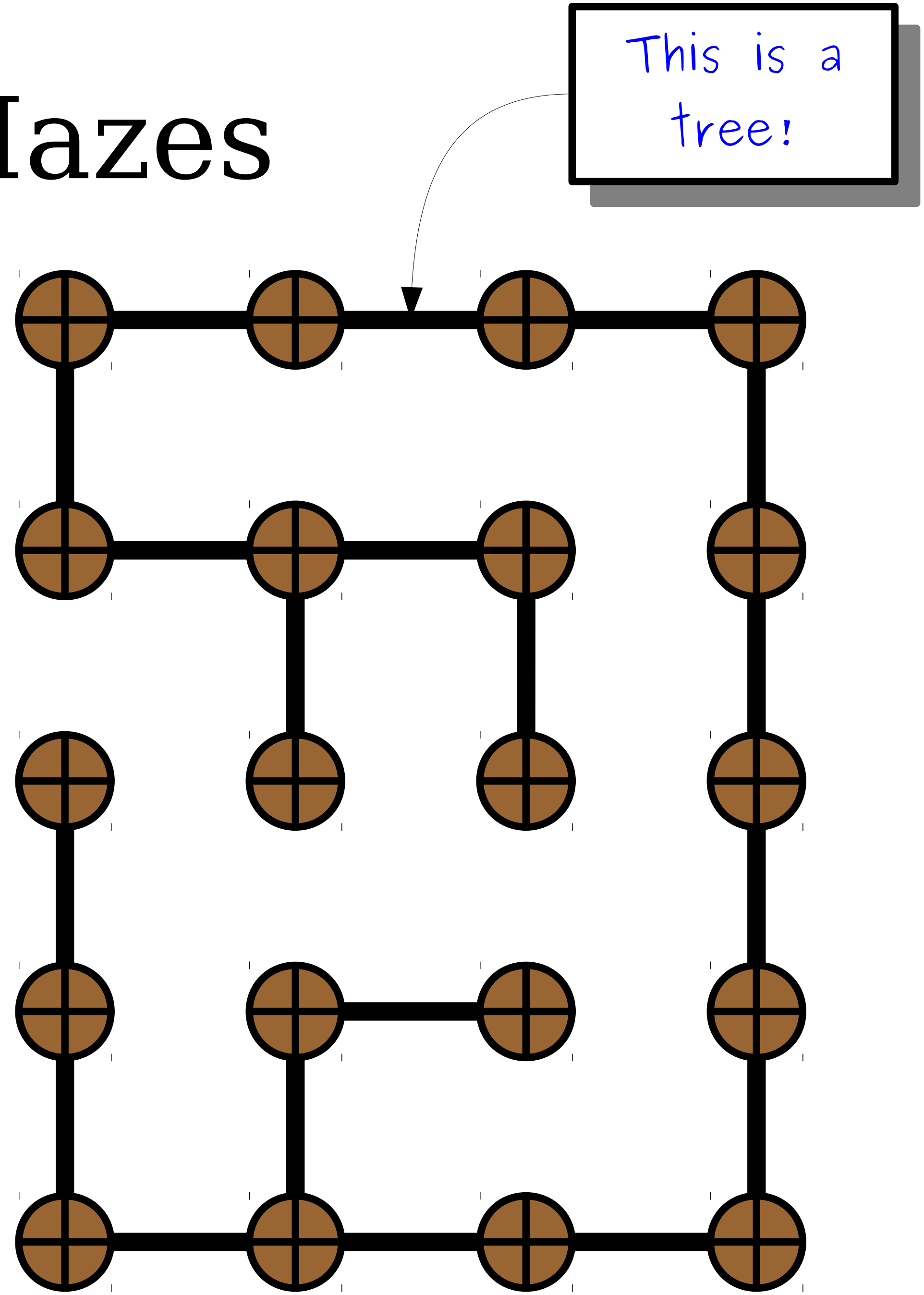
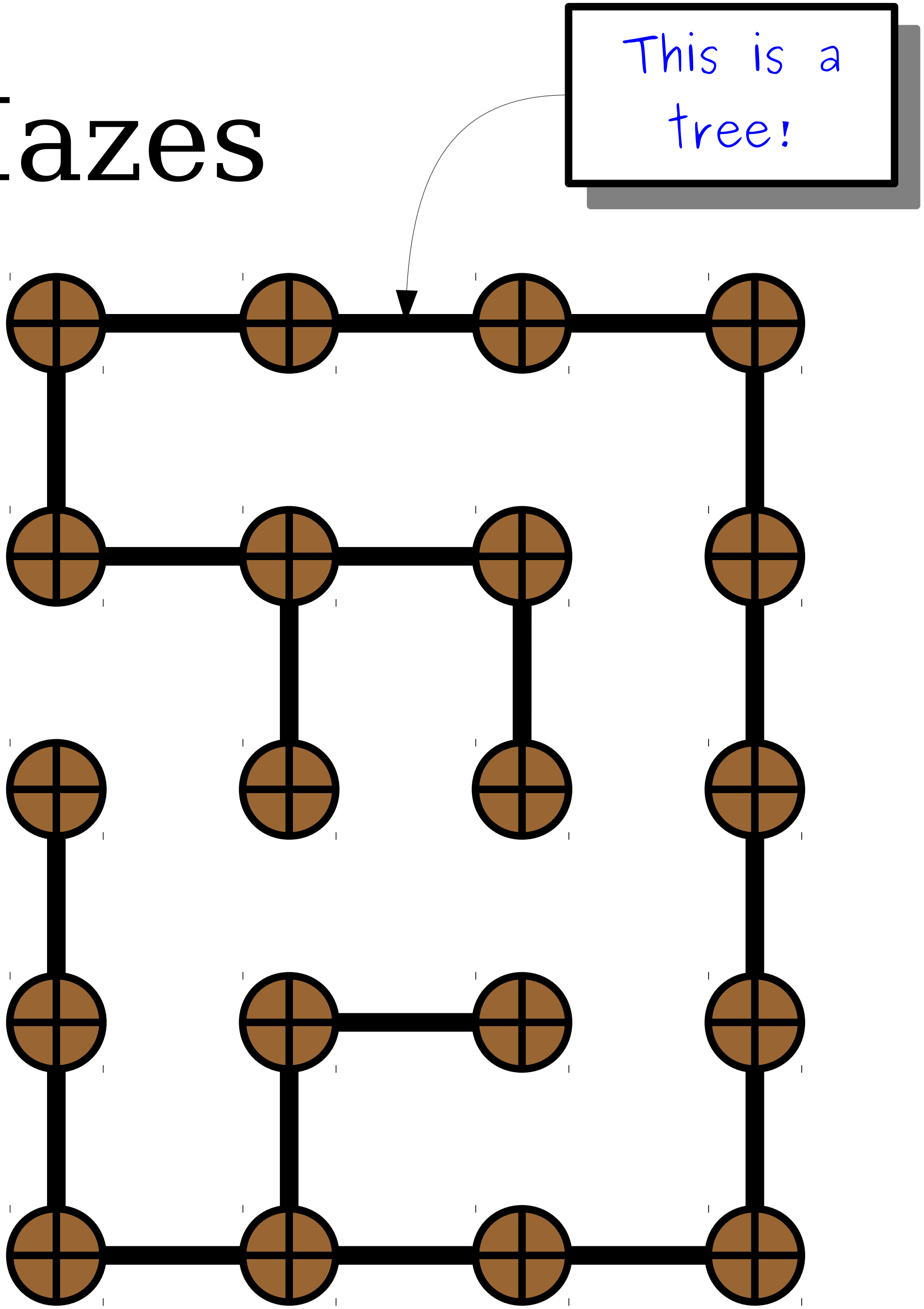


Rat Mazes



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- ***Question:*** How many slats do you need to create?
- ***Answer:*** $mn - 2$.

For more on trees, take CS161 / 261 / 267!

Next Time

- **Formal Language Theory**
  - How are we going to formally model computation?
- **Finite Automata**
  - A simple but powerful computing device made entirely of math!
- **DFAs**
  - A fundamental building block in computing.