Mathematical Logic

Part Two

A Note on the Career Fair

Recap from Last Time

Recap So Far

- A ***propositional variable*** is a variable that is either true or false.
- The ***propositional connectives*** are as follows:
  - Negation: $\neg p$
  - Conjunction: $p \land q$
  - Disjunction: $p \lor q$
  - Implication: $p \to q$
  - Biconditional: $p \leftrightarrow q$
  - True: $\top$
  - False: $\bot$

Take out a sheet of paper!

What's the truth table for the → connective?

What's the negation of $p \to q$?

New Stuff!

First-Order Logic

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What is First-Order Logic?

- ***First-order logic*** is a logical system for reasoning about properties of objects.
- Augments the logical connectives from propositional logic with
  - ***predicates*** that describe properties of objects,
  - ***functions*** that map objects to one another, and
  - ***quantifiers*** that allow us to reason about multiple objects.

Some Examples

$$Likes(You, Eggs) \wedge Likes(You, Tomato) \rightarrow Likes(You, Shakshuka)$$

$$AreNetflixingAndChilling(You, Him) \wedge \neg WatchesNetflix(You) \wedge \neg WatchesNetflix(Him)$$

$$IsStranded(MattDamon) \rightarrow WillBringHome(MattDamon)$$

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$$AreNetflixingAndChilling(You, Him) \wedge \neg WatchesNetflix(You) \wedge \neg WatchesNetflix(Him)$$

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$$AreNetflixingAndChilling(You, Him) \land \neg WatchesNetflix(You) \land \neg WatchesNetflix(Him)$$

$$IsStranded(MattDamon) \to WillBringHome(MattDamon)$$

These blue terms are called **constant symbols**. Unlike propositional variables, they refer to *objects*, not propositions.

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The red things that look like function calls are called *predicates*. Predicates take objects as arguments and evaluate to true or false.

$$Likes(You, Eggs) \land Likes(You, Tomato) \rightarrow Likes(You, Shakshuka)$$

$$AreNetflixingAndChilling(You, Him) \land \neg WatchesNetflix(You) \land \neg WatchesNetflix(Him)$$

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What remains are traditional propositional connectives. Because each predicate evaluates to true or false, we can connect the truth values of predicates using normal propositional connectives.

Reasoning about Objects

- To reason about objects, first-order logic uses ***predicates***.
- Examples:
  - *ExtremelyCute*(*Quokka*)
  - *DeadlockEachOther*(*Democrats*, *Republicans*)
- Predicates can take any number of arguments, but each predicate has a fixed number of arguments (called its ***arity***).
  - The arity and meaning of each predicate are typically specified in advance.
- Applying a predicate to arguments produces a proposition, which is either true or false.

First-Order Sentences

- Sentences in first-order logic can be constructed from predicates applied to objects:

$$LikesToEat(V, M) \land Near(V, M) \to WillEat(V, M)$$

$$Cute(t) \to Dikdik(t) \lor Kitty(t) \lor Puppy(t)$$

$$x < 8 \to x < 137$$

The notation **$x < 8$** is just a shorthand for something like ***LessThan*(*x*, 8)**. Binary predicates in math are often written like this, but symbols like **<** are not a part of first-order logic.

Equality

- First-order logic is equipped with a special predicate **=** that says whether two objects are equal to one another.
- Equality is a part of first-order logic, just as $\to$ and $\neg$ are.
- Examples:

$$TomMarvoloRiddle = LordVoldemort$$

$$MorningStar = EveningStar$$

- Equality can only be applied to ***objects***; to state that two ***propositions*** are equal, use $\leftrightarrow$.

For notational simplicity, define $\neq$ as

$$x \neq y \quad \equiv \quad \neg(x = y)$$

Let's see some more examples.

$$FavoriteMovieOf(You) \neq FavoriteMovieOf(Her) \wedge$$
$$StarOf(FavoriteMovieOf(You)) = StarOf(FavoriteMovieOf(Her))$$

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These purple terms are *functions*. Functions take objects as input and produce objects as output.

$$\mathit{FavoriteMovieOf}(\mathit{You}) \neq \mathit{FavoriteMovieOf}(\mathit{Her}) \wedge$$
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Functions

- First-order logic allows ***functions*** that return objects associated with other objects.
- Examples:

$$LengthOf(path)$$

$$MedianOf(x, y, z)$$

$$x + y$$

- As with predicates, functions can take in any number of arguments, but each function has a fixed arity.
  - As with predicates, the arity and interpretation of functions are specified in advance.
- Functions evaluate to ***objects***, not ***propositions***.
- There is no syntactic way to distinguish functions and predicates; you'll have to look at how they're used.

Objects and Predicates

- When working in first-order logic, be careful to keep objects (actual things) and predicates (true or false) separate.
- You cannot apply connectives to objects:

⚠ $\mathit{Venus} \to \mathit{TheSun}$ ⚠

- You cannot apply functions to propositions:

⚠ $\mathit{StarOf}(\mathit{IsRed}(\mathit{Sun}) \land \mathit{IsGreen}(\mathit{Mars}))$ ⚠

- Ever get confused? *Just ask!*

One last (and major) change

$$\forall x.\ (IsPuppy(x) \to IsAdorable(x))$$

$$\exists a.\ (IsAdorable(a) \land \neg IsPuppy(a))$$

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These green terms are called *quantifiers*. The symbol ∀ is read "for all," and the symbol ∃ is read "there exists"

$$\forall x.\ (IsPuppy(x) \to IsAdorable(x))$$

$$\exists a.\ (IsAdorable(a) \land \neg IsPuppy(a))$$

The teal terms are *variables*. Each quantifier introduces a variable that it "quantifies over."

$$\forall x.\ (IsPuppy(x) \to IsAdorable(x))$$

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In the first case, we're saying that ***for any*** choice of x, if x is a puppy, then x is adorable.

$$\forall x.\ (IsPuppy(x) \to IsAdorable(x))$$

$$\exists a.\ (IsAdorable(a) \land \neg IsPuppy(a))$$

The teal terms are *variables*. Each quantifier introduces a variable that it "quantifies over."

In the first case, we're saying that ***for any*** choice of x, if x is a puppy, then x is adorable.

In the second, we're saying that ***there exists*** some choice of *a* where *a* is adorable and *a* is not a puppy.

$$\forall x.\ (IsPuppy(x) \to IsAdorable(x))$$

(*"All puppies are adorable"*)

$$\exists a.\ (IsAdorable(a) \land \neg IsPuppy(a))$$

(*"Something is adorable but not a puppy"*)

Quantifiers

- The biggest change from propositional logic to first-order logic is the use of ***quantifiers***.
- A ***quantifier*** is an operator that expresses that some statement is true for some or all choices that could be made.

“For any natural number n,
n is even iff n^2 is even”

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n is even iff n^2 is even"

$$\forall n.\ (n \in \mathbb{N} \to (Even(n) \leftrightarrow Even(n^2)))$$

"For any natural number n,
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∀ is the ***universal quantifier***
and says "for any choice of n,
the following is true."

The Universal Quantifier

- A statement of the form $\boldsymbol{\forall x.\ \psi}$ asserts that for ***every*** choice of x, the statement ψ is true when we plug in that choice of x.
- Examples:

$$\forall v.\ (Puppy(v) \to Cute(v))$$

$$\forall x.\ (IsMillennial(x) \to IsSpecial(x))$$

$$Tallest(SK) \to \forall x.\ (SK \neq x \to ShorterThan(x, SK))$$

Some muggles are intelligent.

Some muggles are intelligent.

$$\exists m.\ (Muggle(m) \land Intelligent(m))$$

Some muggles are intelligent.

$$\boxed{\exists m.}\ (\mathit{Muggle}(m) \land \mathit{Intelligent}(m))$$

∃ is the ***existential quantifier*** and says "for some choice of m, the following is true."

The Existential Quantifier

- A statement of the form $\boldsymbol{\exists x.\ \psi}$ asserts that ***there is some*** choice of x where ψ is true when we plug in that x.
- Examples:

$$\exists x.\ (Even(x) \land Prime(x))$$

$$\exists x.\ (TallerThan(x, me) \land LighterThan(x, me))$$

$$(\exists x.\ Appreciates(x, me)) \to Happy(me)$$

Variables and Quantifiers

- Each quantifier has two parts:
  - the variable that is introduced, and
  - the statement that's being quantified.
- The variable introduced is scoped just to the statement being quantified.

$$(\forall x.\ Loves(You, x)) \to (\forall y.\ Loves(y, You))$$

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$$\underbrace{(\forall x.\ Loves(You, x))}_{\text{The variable } x \text{ just lives here.}} \to \underbrace{(\forall y.\ Loves(y, You))}_{\text{The variable } y \text{ just lives here.}}$$

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Variables and Quantifiers

- Each quantifier has two parts:
  - the variable that is introduced, and
  - the statement that's being quantified.
- The variable introduced is scoped just to the statement being quantified.

$$\underbrace{(\forall x.\ \mathit{Loves}(\mathit{You}, x))}_{\text{The variable } x \text{ just lives here.}} \to \underbrace{(\forall x.\ \mathit{Loves}(x, \mathit{You}))}_{\text{A different variable, also named } x \text{, just lives here.}}$$

Operator Precedence (Again)

- When writing out a formula in first-order logic, the quantifiers ∀ and ∃ have precedence just below ¬.
- The statement

  $$\forall x.\ P(x) \lor R(x) \to Q(x)$$

  is parsed like this:

  $$\big(\ (\forall x.\ P(x)\ )\ \lor\ R(x)\ \big)\ \to\ Q(x)$$

- This is syntactically invalid because the variable x is out of scope in the back half of the formula.
- To ensure that x is properly quantified, explicitly put parentheses around the region you want to quantify:

  $$\forall x.\ (P(x) \lor R(x) \to Q(x))$$

Time-Out for Announcements!

Checkpoints Graded

- The PS1 checkpoint assignment has been graded.
- Review your feedback on GradeScope. Contact the staff (via Piazza or by stopping by office hours) if you have any questions.
- Some notes:
  - ***Make sure to list your collaborators through GradeScope.*** There's a space to list your collaborators when you submit the assignment. If you forget to do this, *they won't get credit for the assignment*!
  - ***Make sure to check your grade ASAP.*** For the reason listed above, make sure you have a grade recorded. If not, contact the course staff. Plus, that way, you can take our feedback into account when writing up your answers to the rest of the problem set questions.
- Best of luck on the rest of the problem set!

WiCS Casual CS Dinner

- WiCS (Women in Computer Science) will be holding a Casual CS Dinner next **Wednesday, January 20** in the Women's Community Center at 6PM.
- These events are fantastic and are a great way to meet cool people, find study groups, and get Good Life Advice.
- Highly recommended!

Join us at our upcoming info session to learn more about

DIVERSITYBASE

an organization dedicated to supporting people from underrepresented backgrounds in Computer Science with a focus on understanding the intersectionality of those backgrounds

THURSDAY, JANUARY 14TH at 6PM

OLD UNION 200

FOR MORE INFO, CONTACT:
PATRICIA PEROZO (PPEROZO@STANFORD.EDU)

Your Questions

"How did you first learn discrete math? Any advice for first-timers?"

I first learned discrete math when I took CS103 many years back! The class was a lot of work, but I learned a ton!

My advice: don't settle for a partial understanding of a topic. If you "kinda" get something, ask questions about it! Don't be afraid of the concepts you don't understand. Take them head-on. You'll learn so much if you do.

"If you can prove that the contrapositive of a theorem is false, does that count as a proof that the theorem is false? (I understand that a true contrapositive is sufficient to prove a theorem true, but I want to know if it is necessary)"

Yep! A statement is logically equivalent to its contrapositive, so if you can disprove the contrapositive, you've disproven the original statement.

"What are your favorite books? Any reading recommendations?"

I highly recommend "Whistling Vivaldi" by Claude Steele. This book talks about stereotype threat, its causes, and how to address it. It's fundamentally changed the way I teach my classes. It's also a quick and fascinating read.

If you like long-form essays, read "Scott and Scurvy," the story of how we lost the cure to scurvy due to scientific progress in other areas. It's a fascinating read and will change the way you think about scientific progress.

I'd also recommend "Command and Control: Nuclear Weapons, The Damascus Incident and the Illusion of Nuclear Safety" by Eric Schlosser. It's a fascinating look at the history of nuclear weapons and institutional dysfunction. Fun fact – some of the ridiculous scenes from "Dr. Strangelove" and "War Games" actually happened.

Back to CS103!

Translating into First-Order Logic

Translating Into Logic

- First-order logic is an excellent tool for manipulating definitions and theorems to learn more about them.
- Applications:
  - Determining the negation of a complex statement.
  - Figuring out the contrapositive of a tricky implication.

Translating Into Logic

- ***Translating statements into first-order logic is a lot more difficult than it looks.***
- There are a lot of nuances that come up when translating into first-order logic.
- We'll cover examples of both good and bad translations into logic so that you can learn what to watch for.
- We'll also show lots of examples of translations so that you can see the process that goes into it.

Using the predicates

- $Puppy(p)$, which states that p is a puppy, and
- $Cute(x)$, which states that x is cute,

write a sentence in first-order logic that means “all puppies are cute.”

An Incorrect Translation

All puppies are cute!

$$\forall x.\ (Puppy(x) \land Cute(x))$$

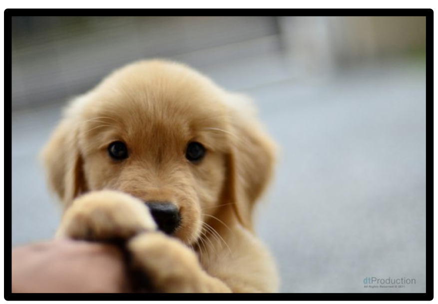
An Incorrect Translation

All puppies are cute!

$$\forall x.\ (Puppy(x) \land Cute(x))$$

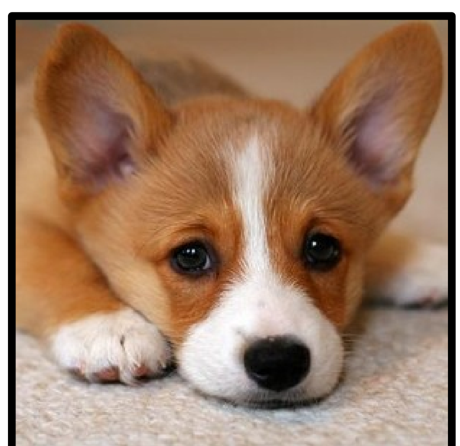
This should work for <u>*any*</u> choice of x, including things that aren't puppies.

An Incorrect Translation



All puppies are cute!

$\forall x.\ (Puppy(x) \land Cute(x))$



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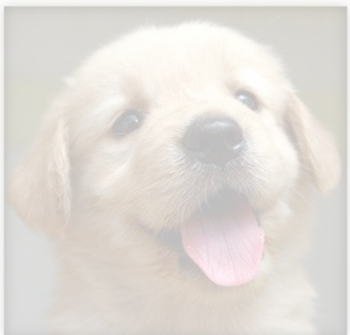
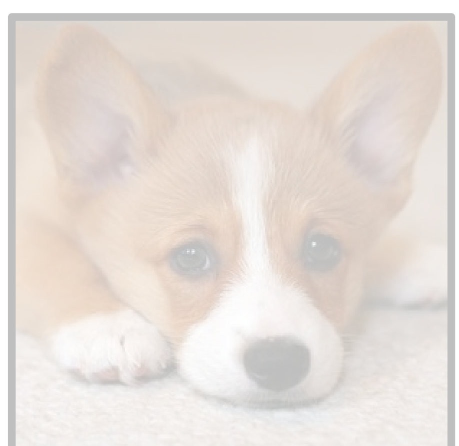


All puppies are cute!

$$\forall x.\ (\mathit{Puppy}(x) \land \mathit{Cute}(x))$$



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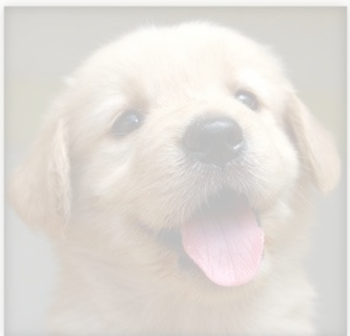

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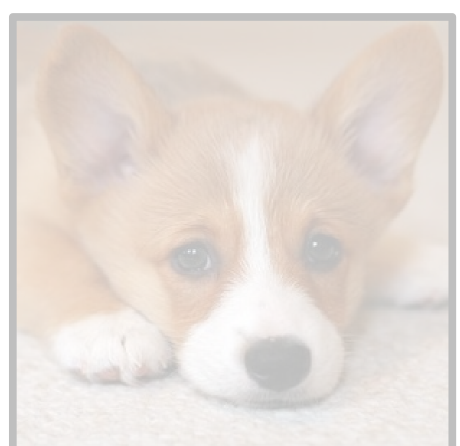

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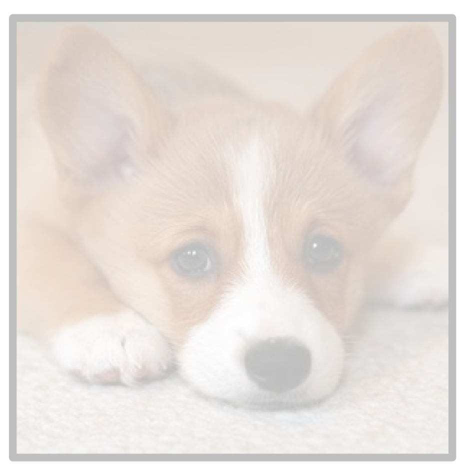
$$\forall x. \, (\cancel{Puppy(x)} \land Cute(x))$$



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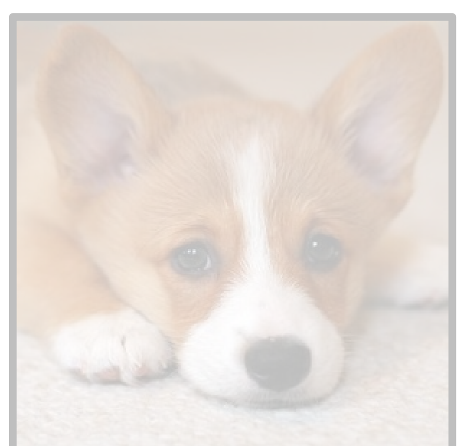

This should work for <u>*any*</u> choice of x, including things that aren't puppies.

An Incorrect Translation

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$\forall x.\ (\cancel{Puppy(x) \land Cute(x)})$

A statement of the form

$$\forall x.\ \psi$$

is true only when ψ is true for <u>every</u> choice of x.

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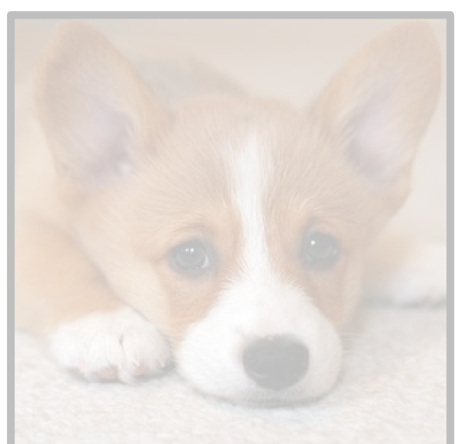
All puppies are cute!

$\cancel{\forall x.\ (Puppy(x) \land Cute(x))}$



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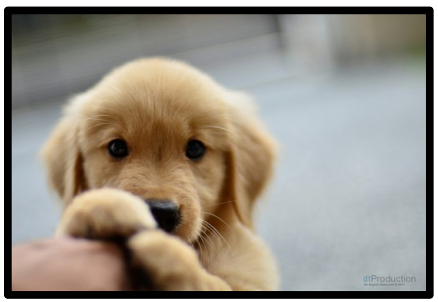
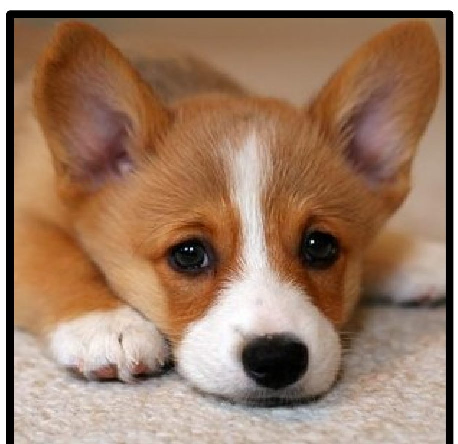


$\cancel{\forall x.\ (\mathit{Puppy}(x) \land \mathit{Cute}(x))}$



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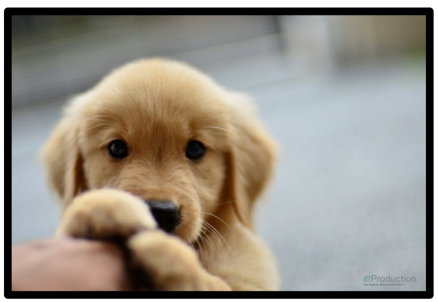

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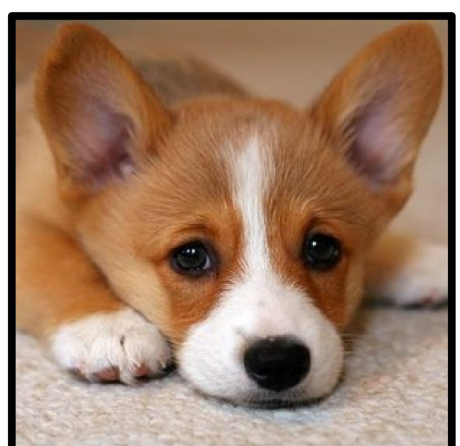

This first-order statement is false even though the English statement is true. Therefore, it can't be a correct translation.

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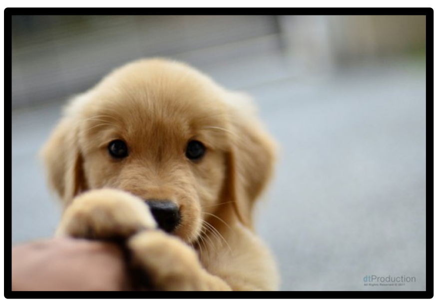
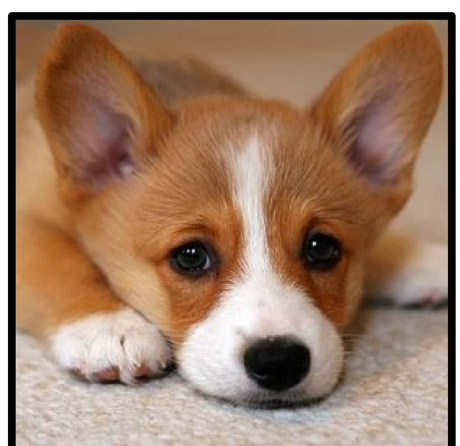


$$\forall x.\ (\color{blue}{Puppy(x)} \land Cute(x))$$



The issue here is that this statement asserts that everything is a puppy. That's too strong of a claim to make.

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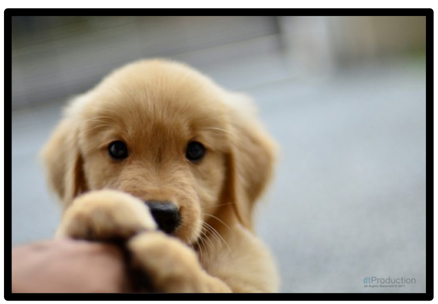

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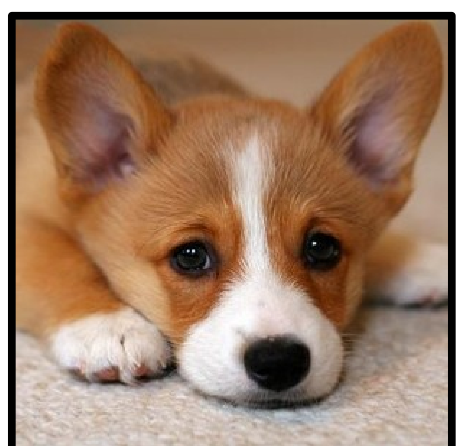

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This should work for <u>any</u> choice of x, including things that aren't puppies.

A Better Translation

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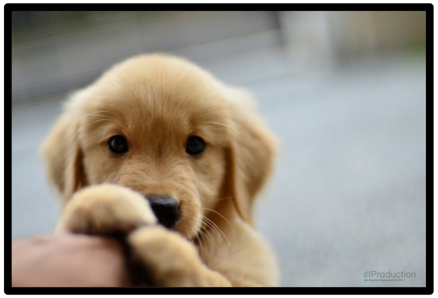

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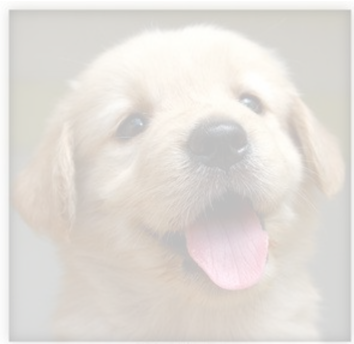

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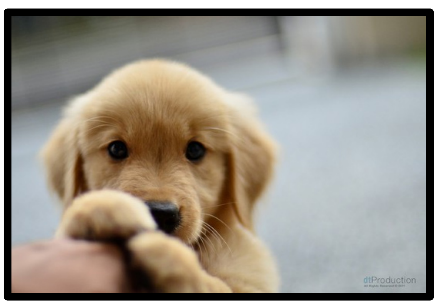

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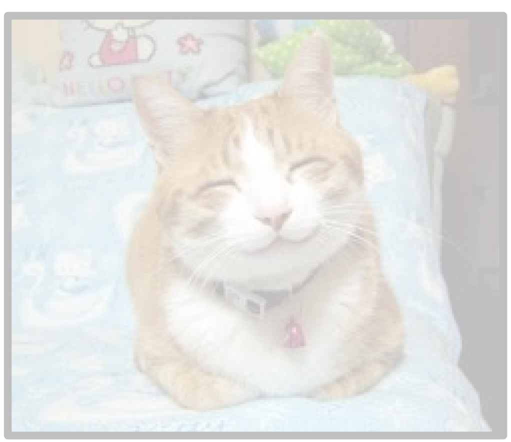

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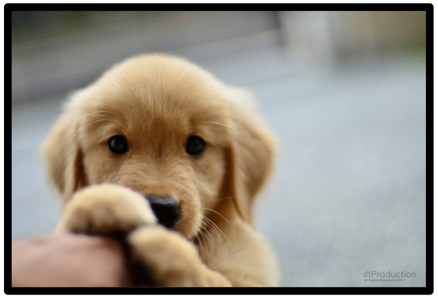

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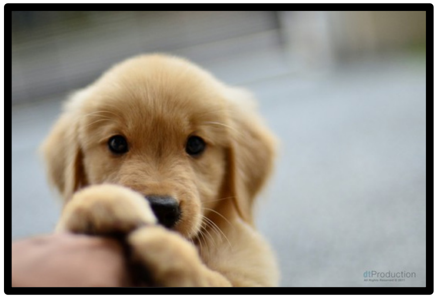

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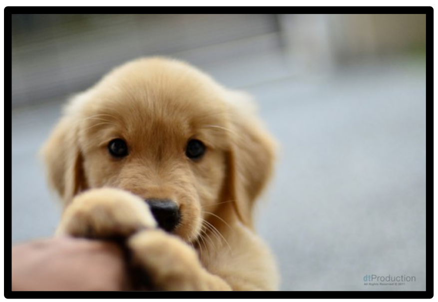

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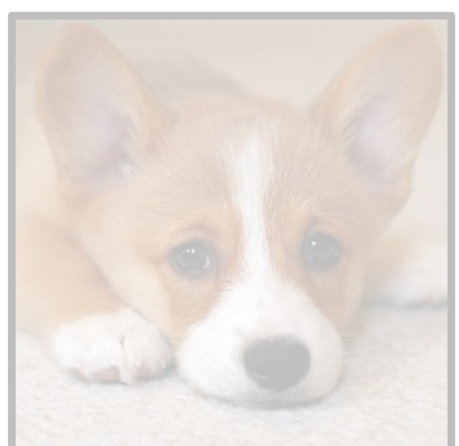

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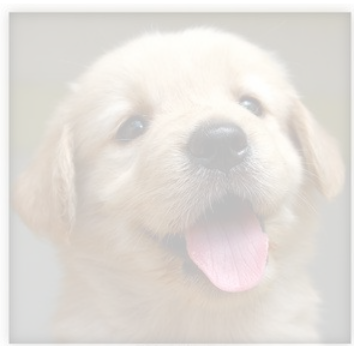

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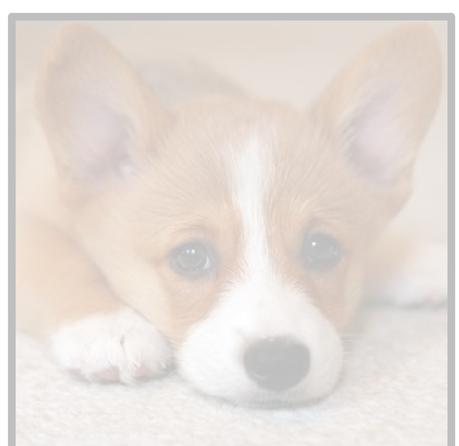

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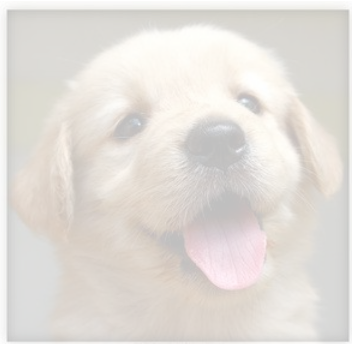

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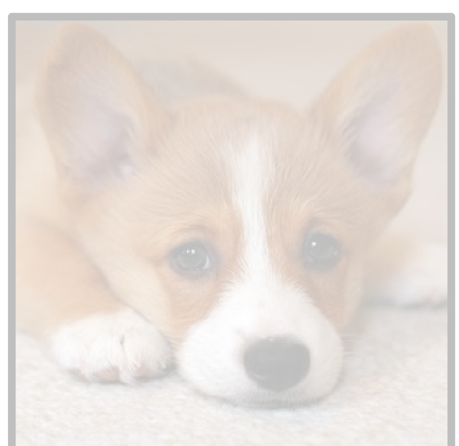

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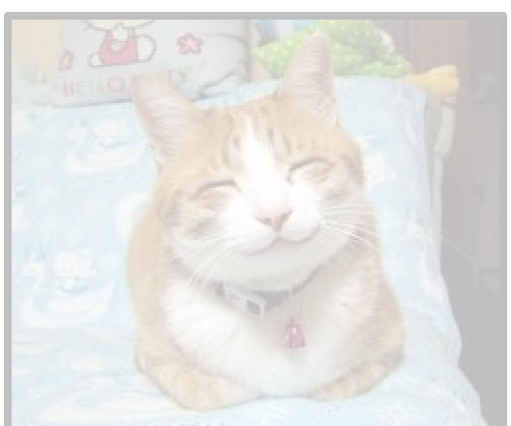

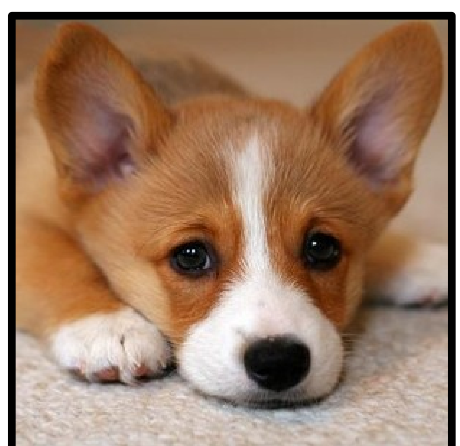

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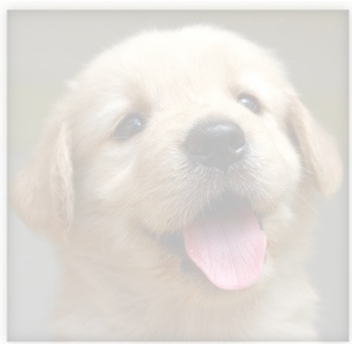

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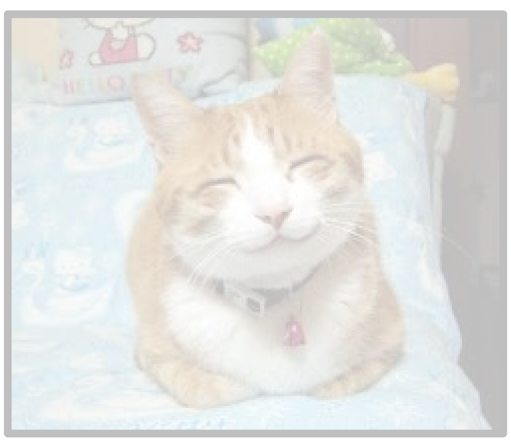

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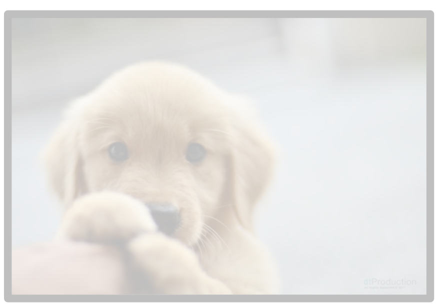

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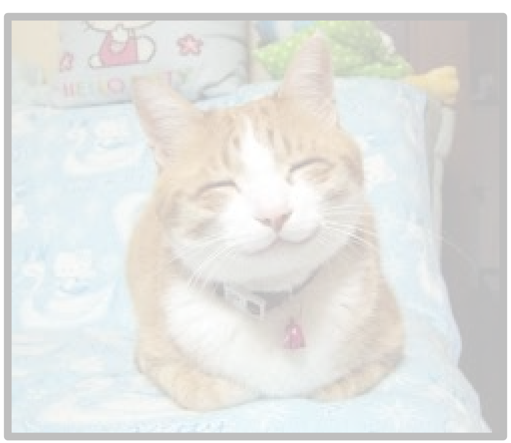

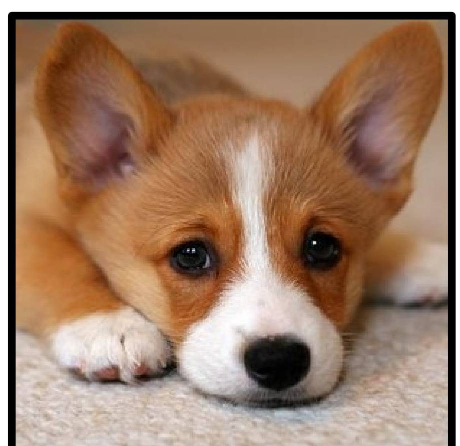

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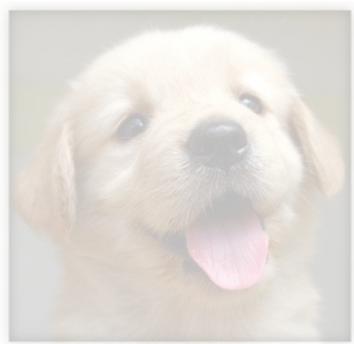

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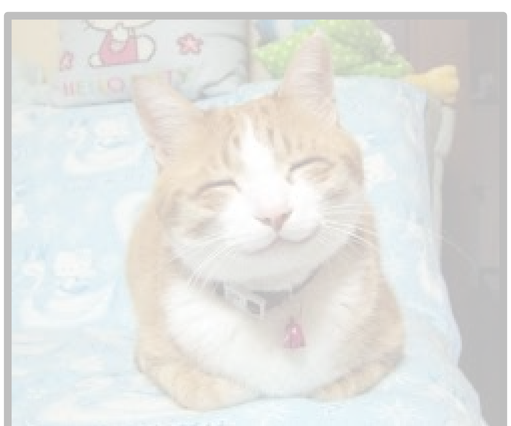

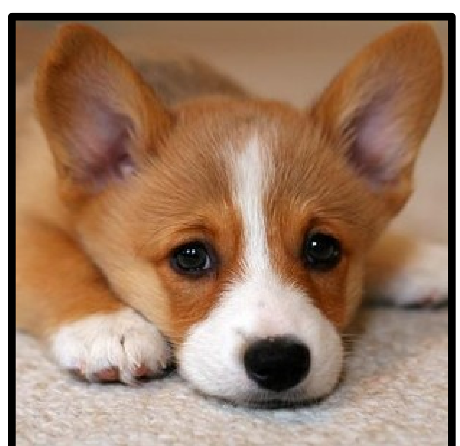

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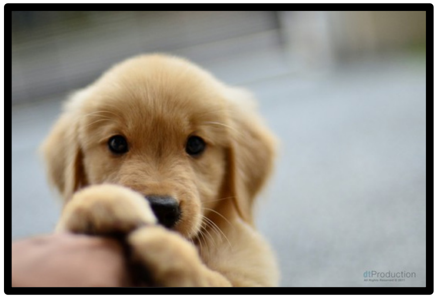

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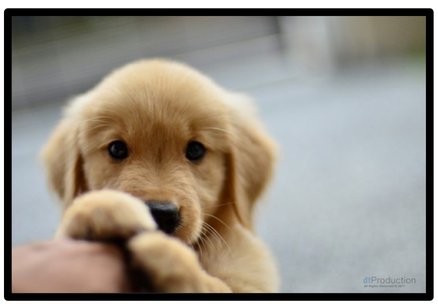

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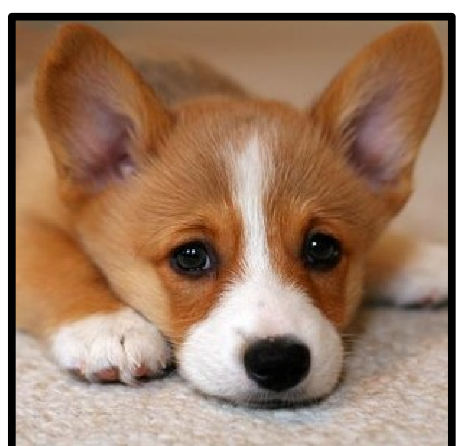

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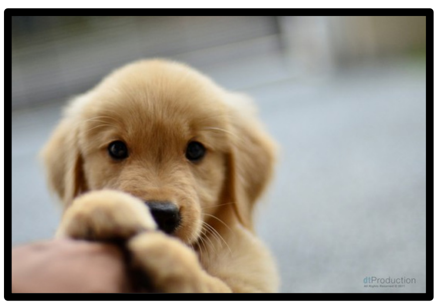

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A statement of the form

$\forall x. \, \psi$

is true only when ψ is true for *every* choice of x.

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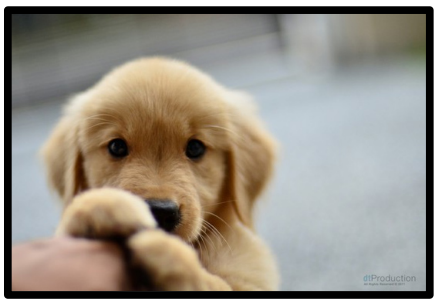

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“All *P*'s are *Q*'s”

translates as

$$\forall x.\ (P(x) \to Q(x))$$

Useful Intuition:

Universally-quantified statements are true unless there's a counterexample.

$$\forall x.\ (P(x) \to Q(x))$$

If x is a counterexample, it *must* have property P but not have property Q.

Using the predicates

- $Blobfish(b)$, which states that b is a blobfish, and
- $Cute(x)$, which states that x is cute,

write a sentence in first-order logic that means “some blobfish is cute.”

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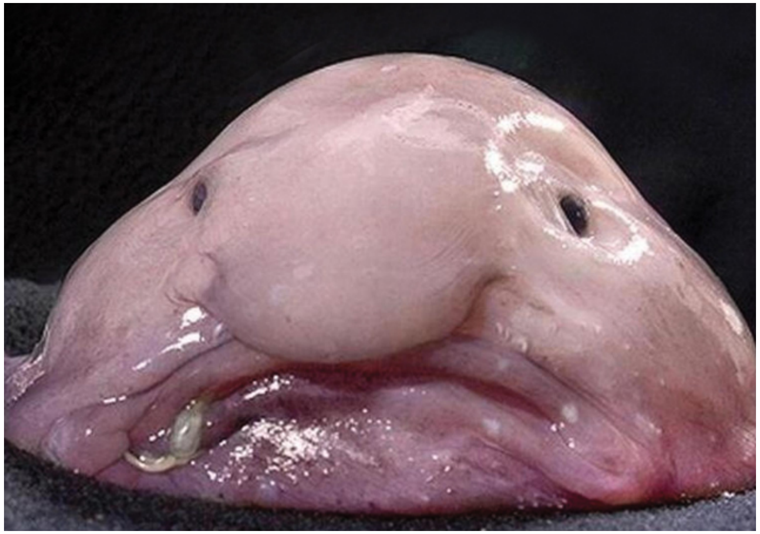

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An Incorrect Translation

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$$\exists x.\ (Blobfish(x) \to Cute(x))$$

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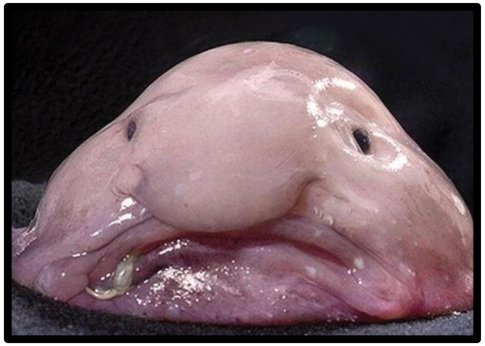

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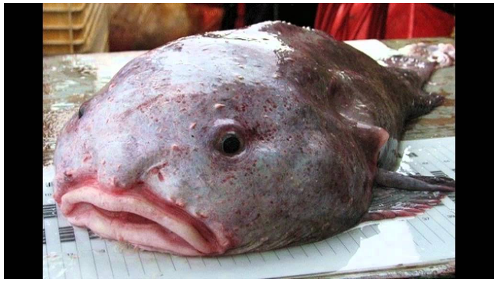

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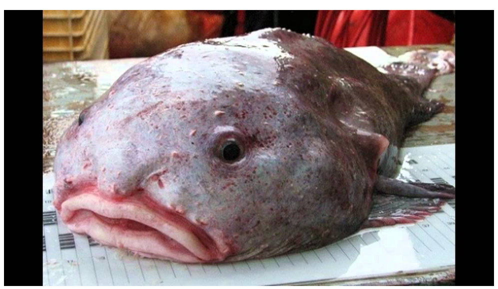

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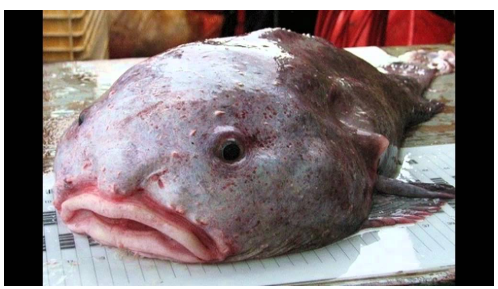

This first-order statement is true even though the English statement is false. Therefore, it can't be a correct translation.

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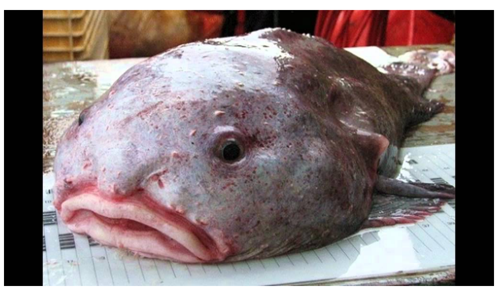

The issue here is that implications are true whenever the antecedent is false. This statement "accidentally" is true because of what happens when x isn't a blobfish.

A Correct Translation

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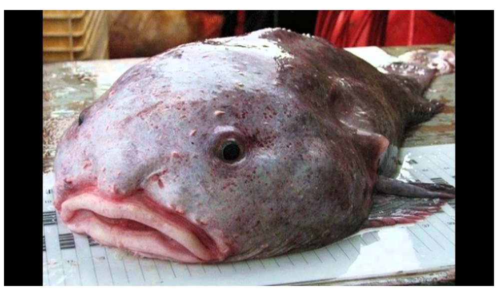

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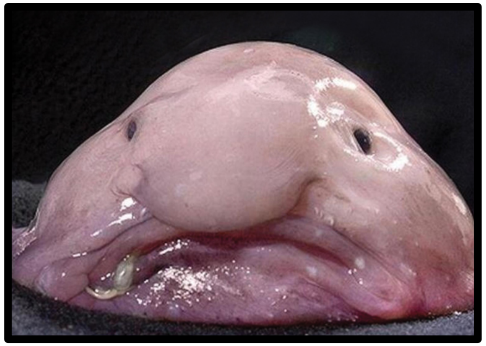

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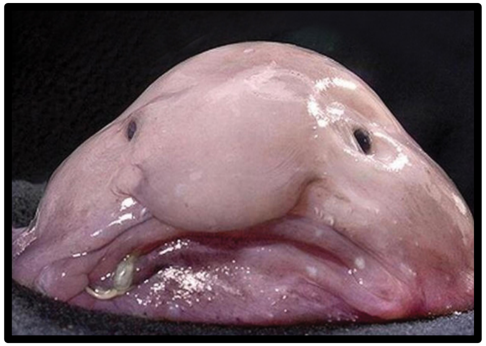

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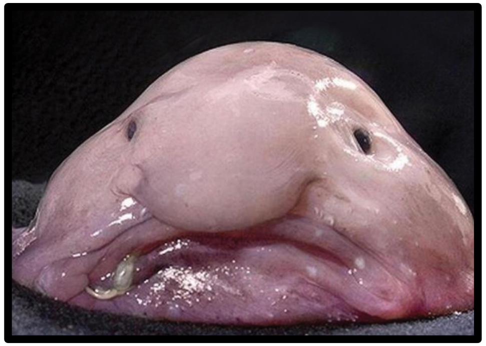

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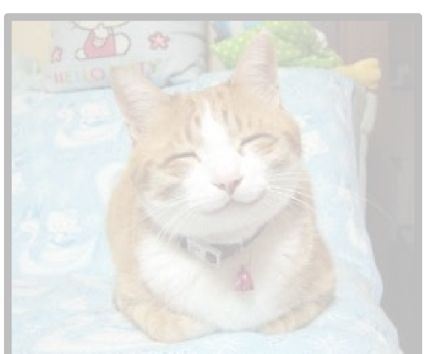

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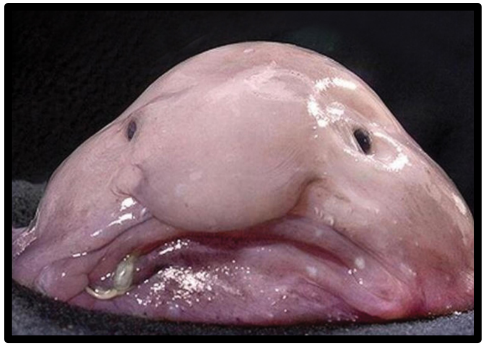

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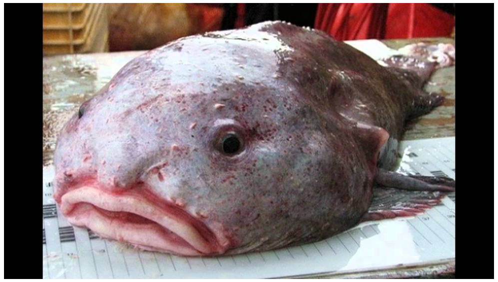

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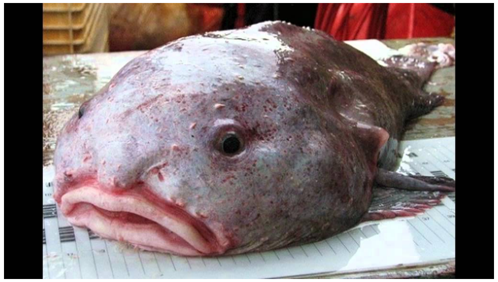

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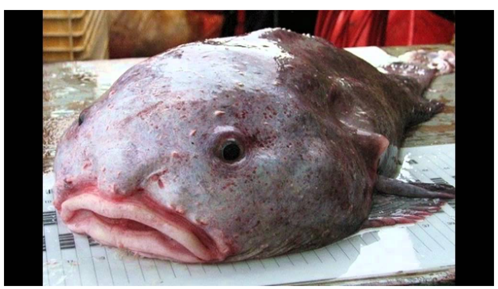

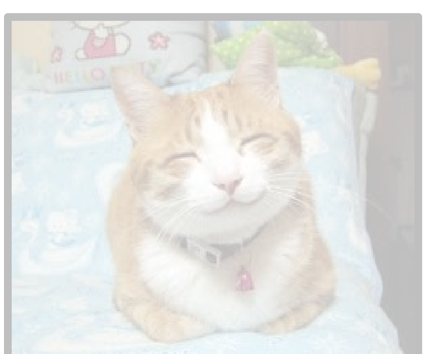

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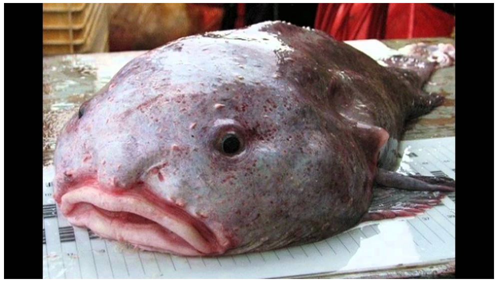

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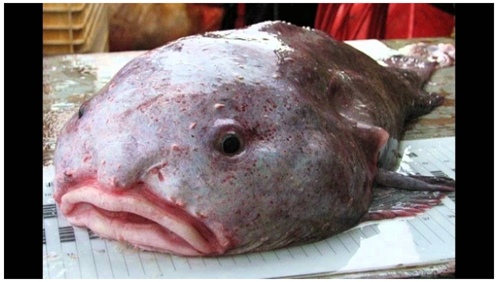

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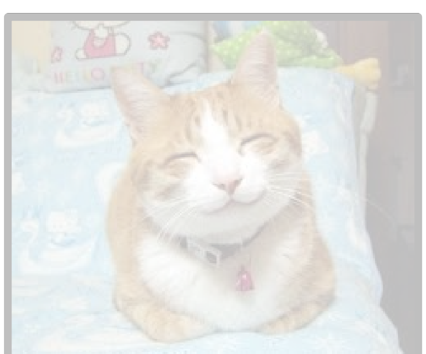

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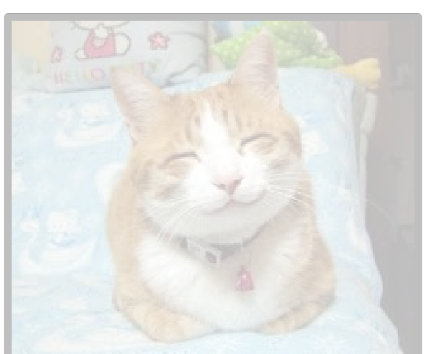

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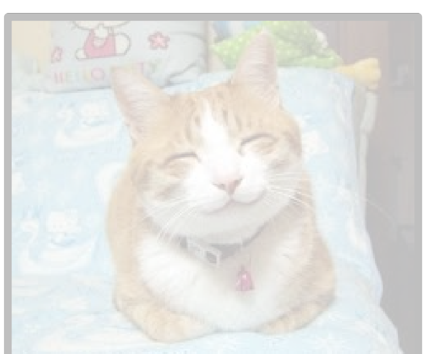

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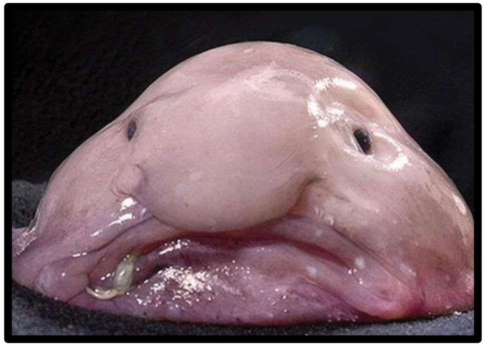

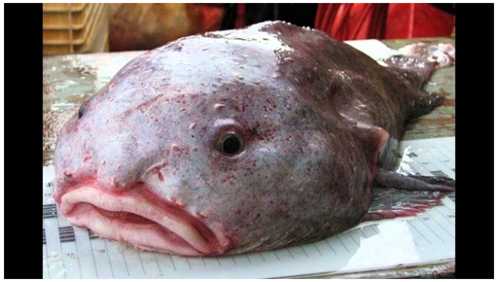

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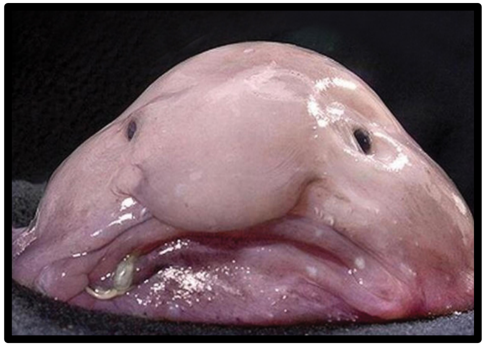

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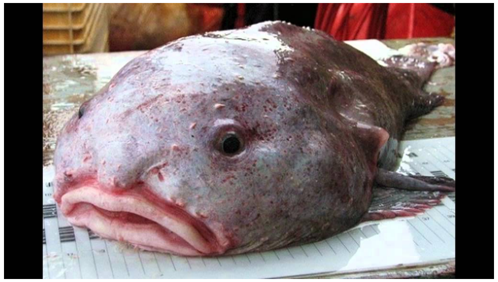

A statement of the form

$$\exists x.\ \psi$$

is true only when ψ is true for <u>*some*</u> choice of x.

“Some *P* is a *Q*”

translates as

$$\exists x.\ (P(x) \land Q(x))$$

Useful Intuition:

Existentially-quantified statements are false unless there's a positive example.

$$\exists x.\ (P(x) \land Q(x))$$

If x is an example, it *must* have property P on top of property Q.

Good Pairings

- The ∀ quantifier *usually* is paired with →.
- The ∃ quantifier *usually* is paired with ∧.
- In the case of ∀, the → connective prevents the statement from being *false* when speaking about some object you don't care about.
- In the case of ∃, the ∧ connective prevents the statement from being *true* when speaking about some object you don't care about.