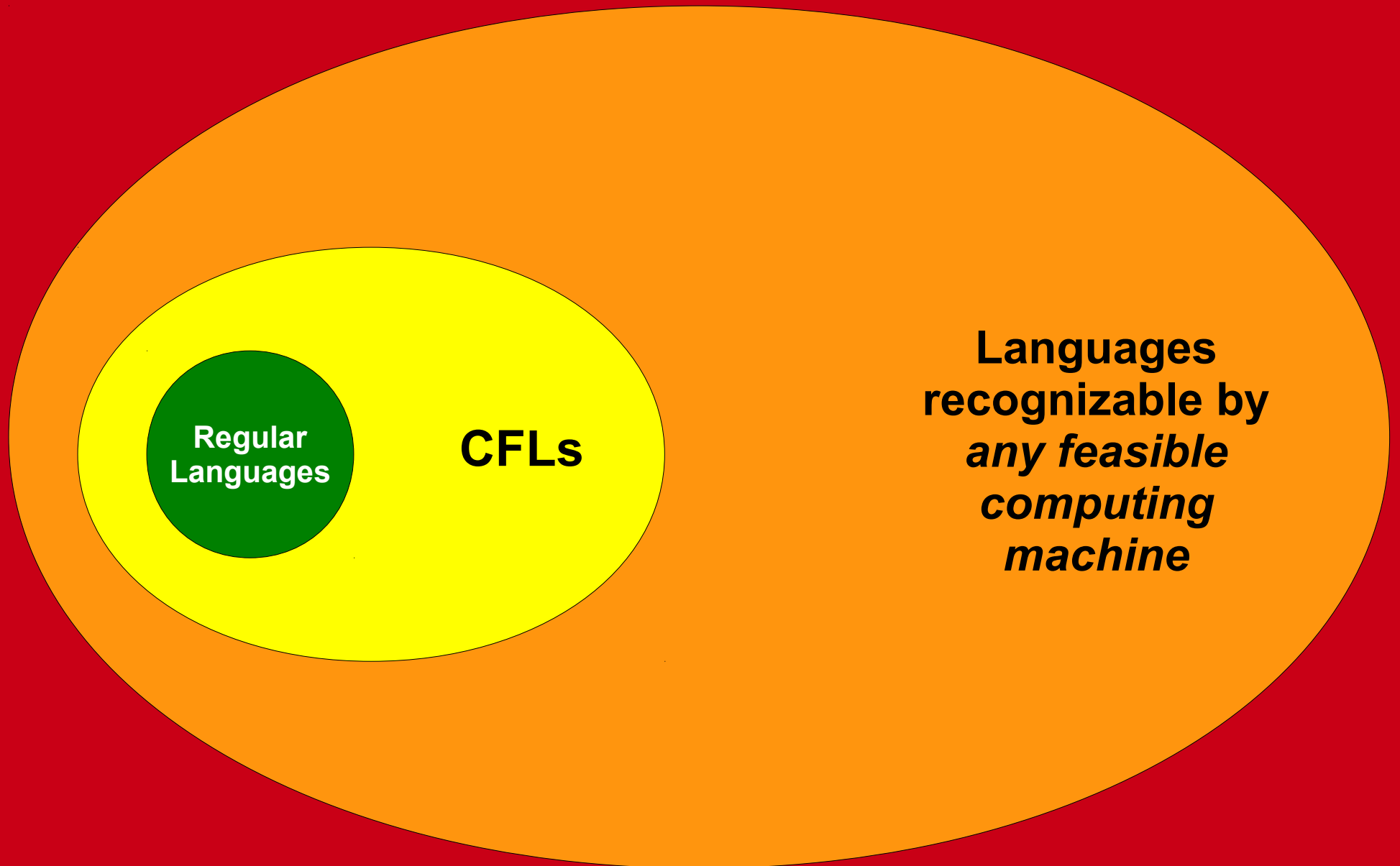


# Turing Machines

## Part One

What problems can we solve with a computer?



**All Languages**

That same drawing, to scale.

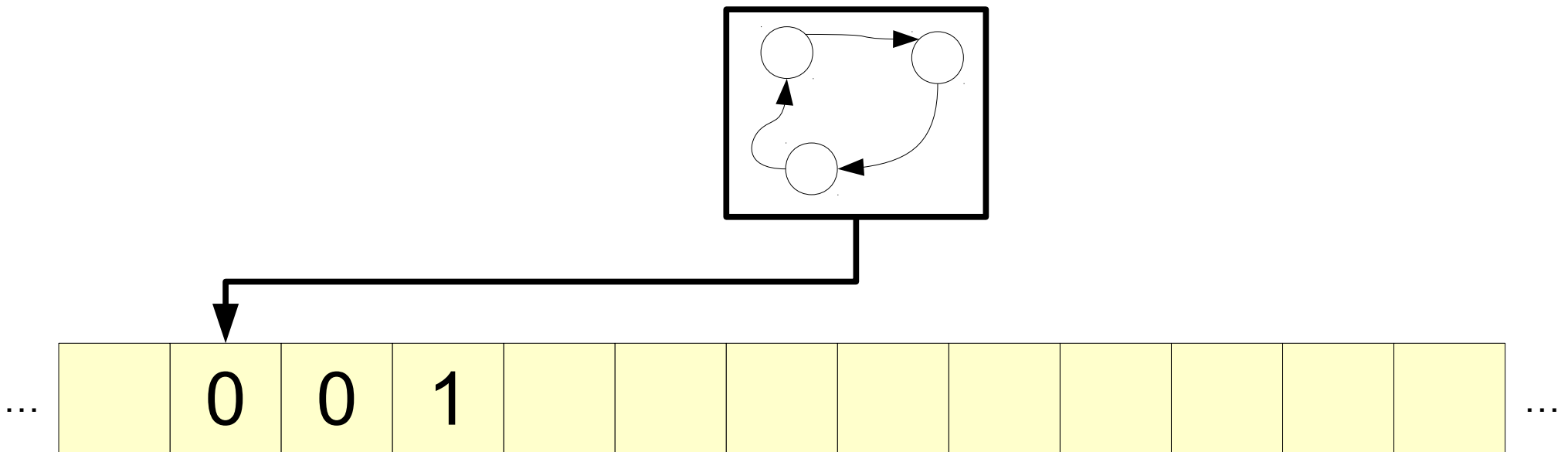
# The Problem

- Finite automata accept precisely the regular languages.
- We may need unbounded memory to recognize context-free languages.
  - e.g.  $\{ \mathbf{a}^n \mathbf{b}^n \mid n \in \mathbb{N} \}$  requires unbounded counting.
- How do we build an automaton with finitely many states but unbounded memory?

# A Brief History Lesson

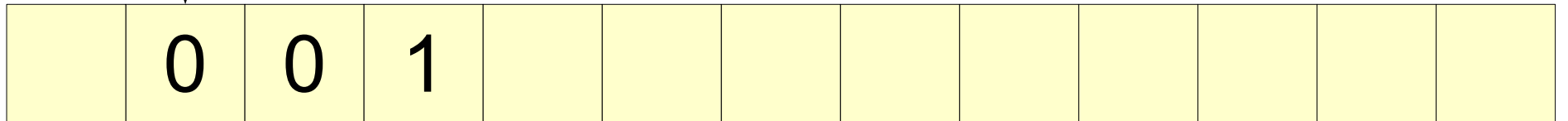
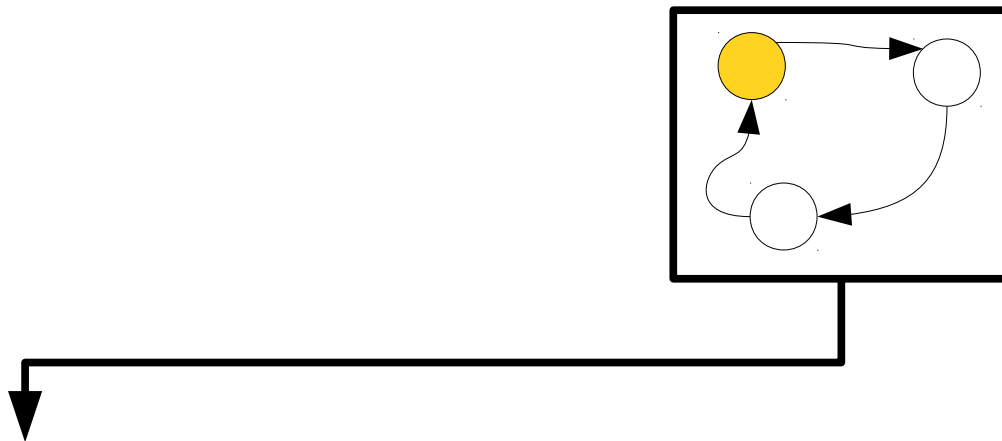
# A Better Memory Device

- A **Turing machine** is a deterministic finite automaton equipped with an **infinite tape** as its memory.
- The input is written on the tape when the computation begins, surrounded by infinitely many blank cells.
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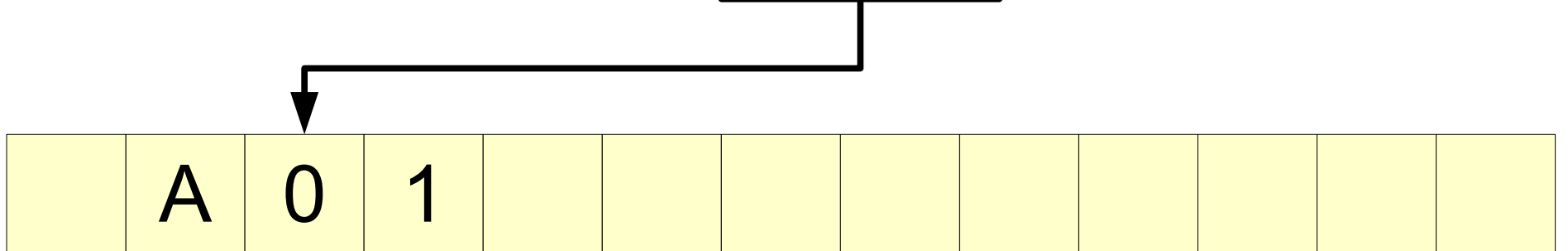
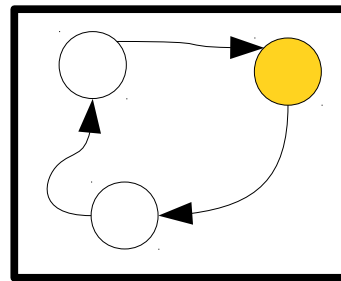
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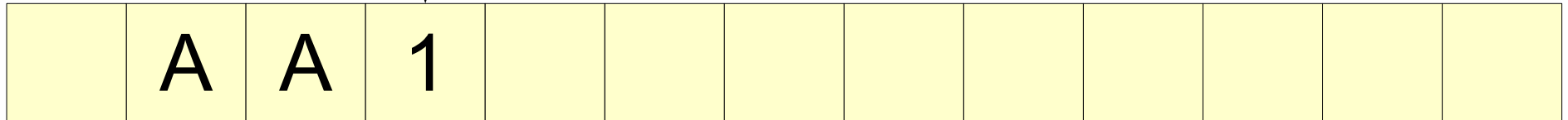
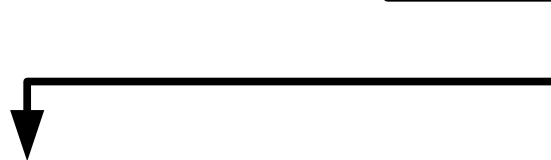
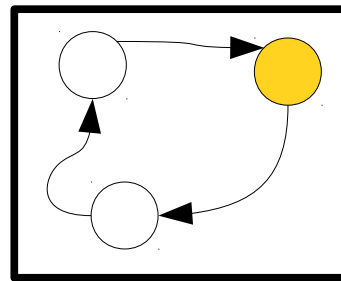
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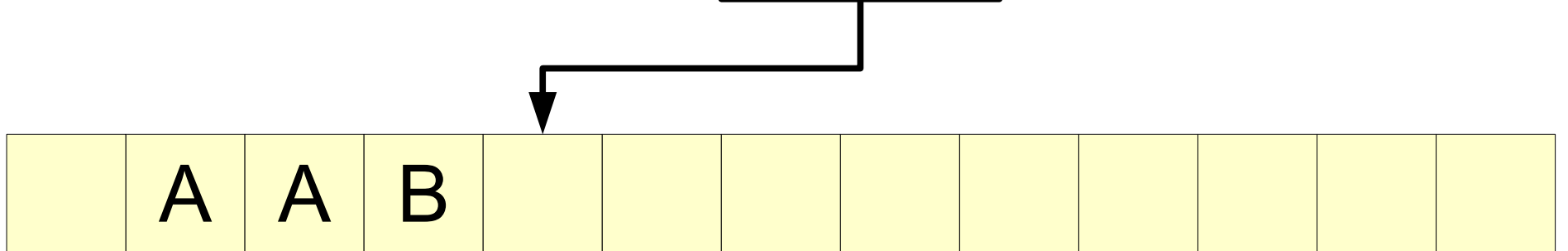
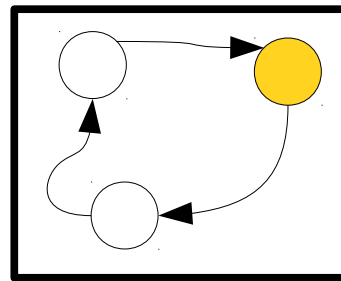
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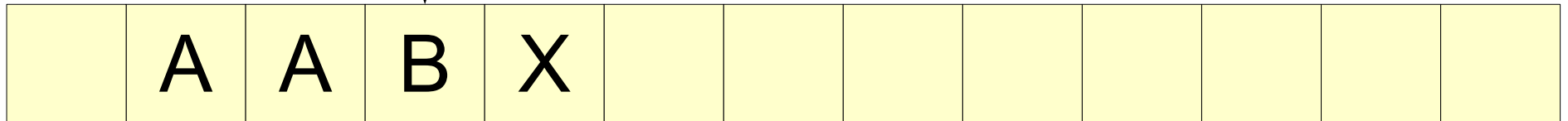
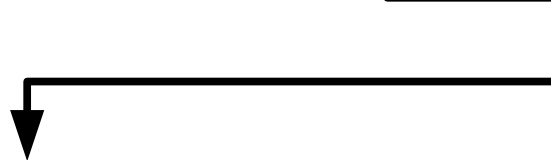
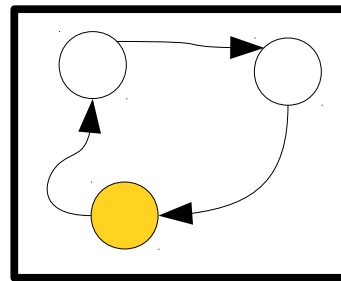
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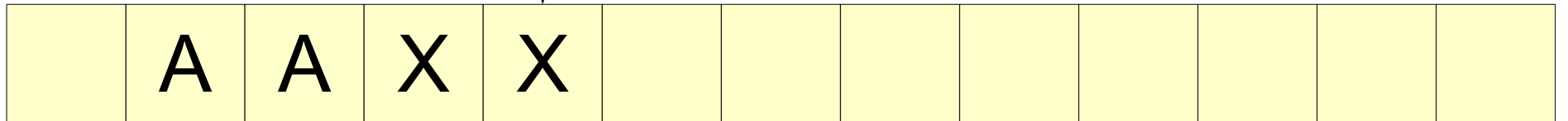
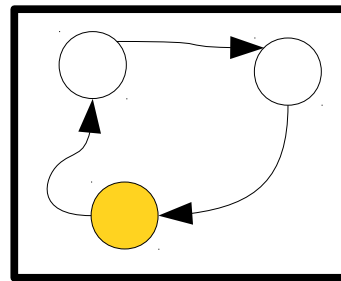
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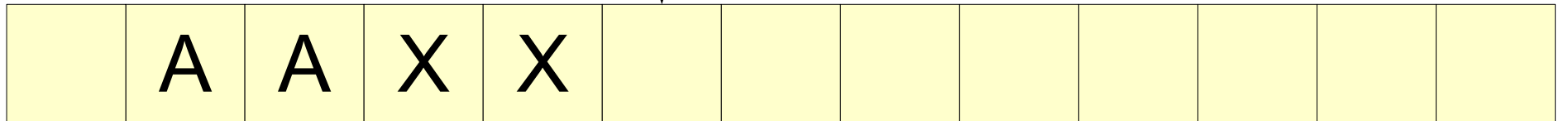
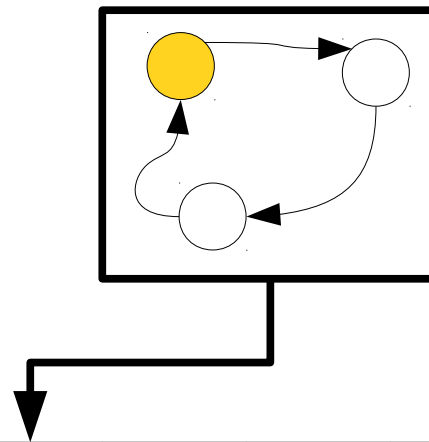
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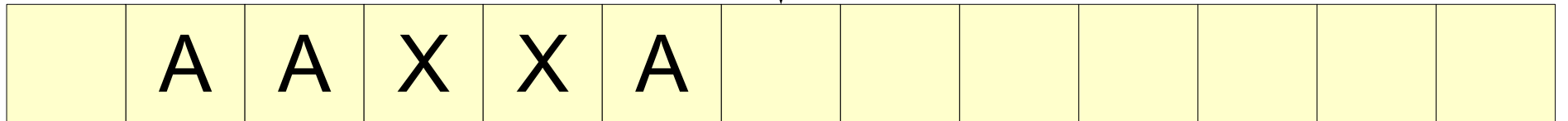
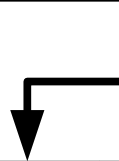
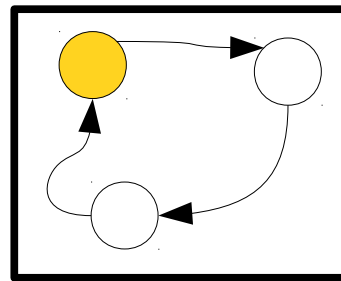
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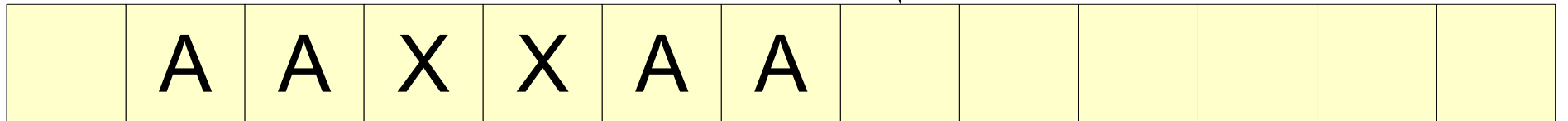
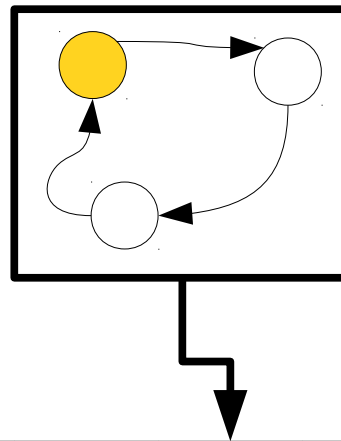
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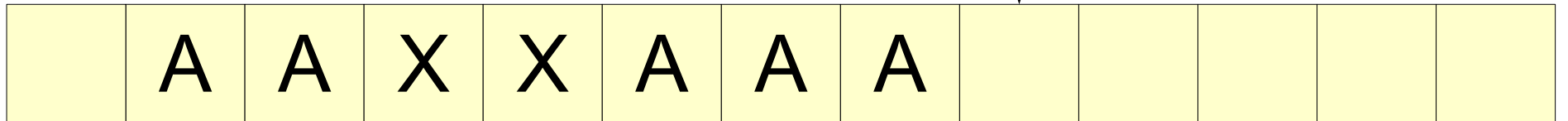
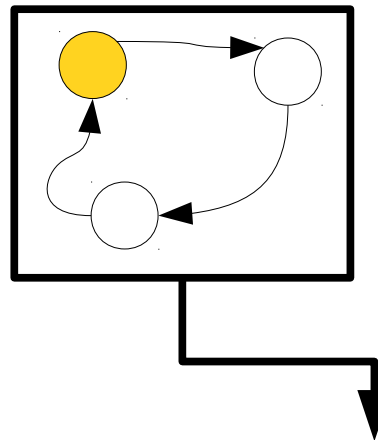
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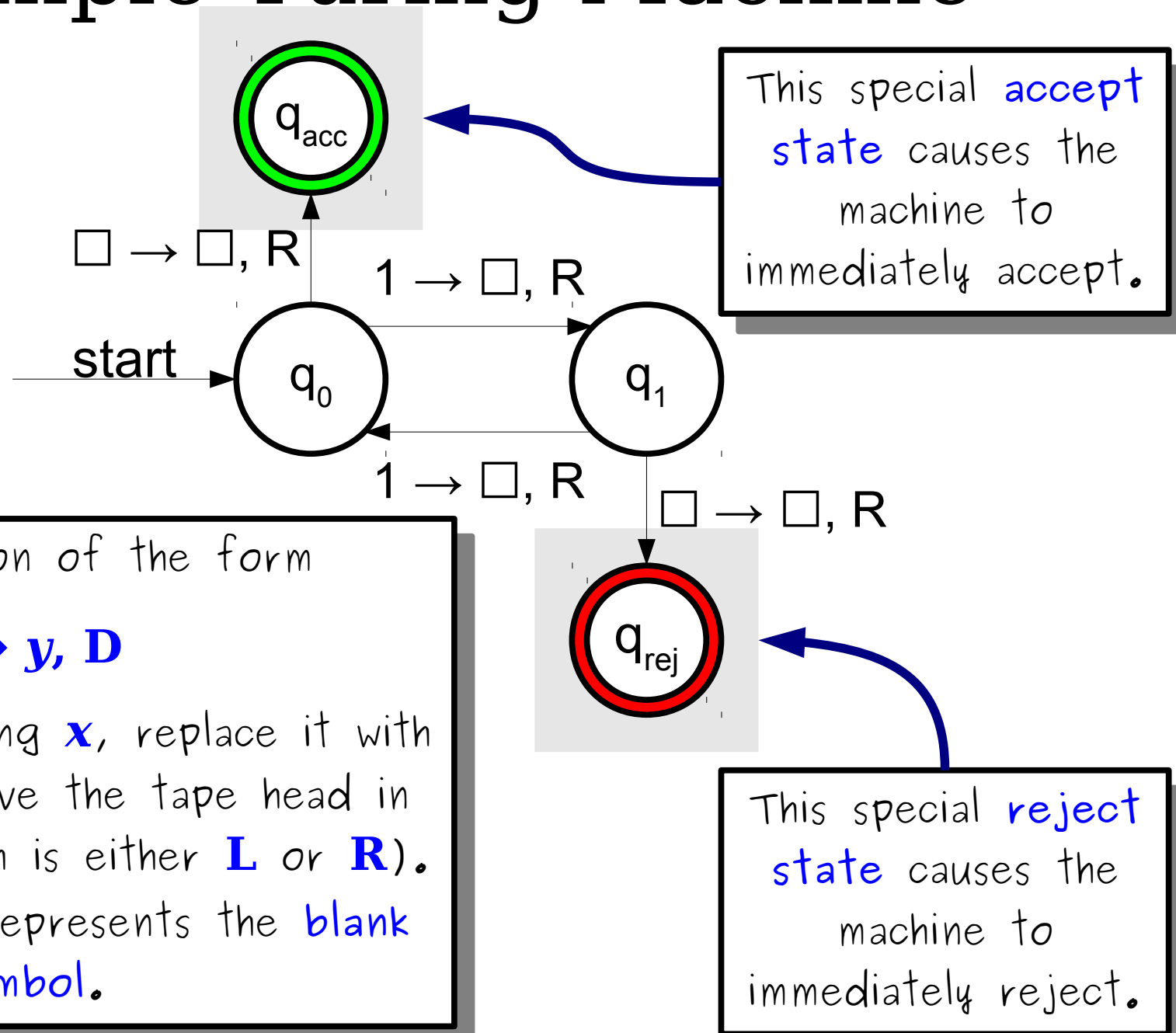
# The Turing Machine

- A Turing machine consists of three parts:
  - A *finite-state control* that issues commands,
  - an *infinite tape* for input and scratch space, and
  - a *tape head* that can read and write a single tape cell.
- At each step, the Turing machine
  - writes a symbol to the tape cell under the tape head,
  - changes state, and
  - moves the tape head to the left or to the right.

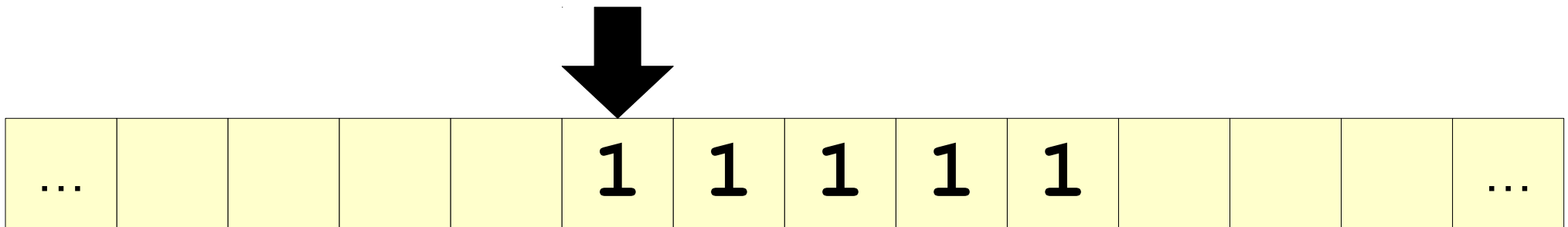
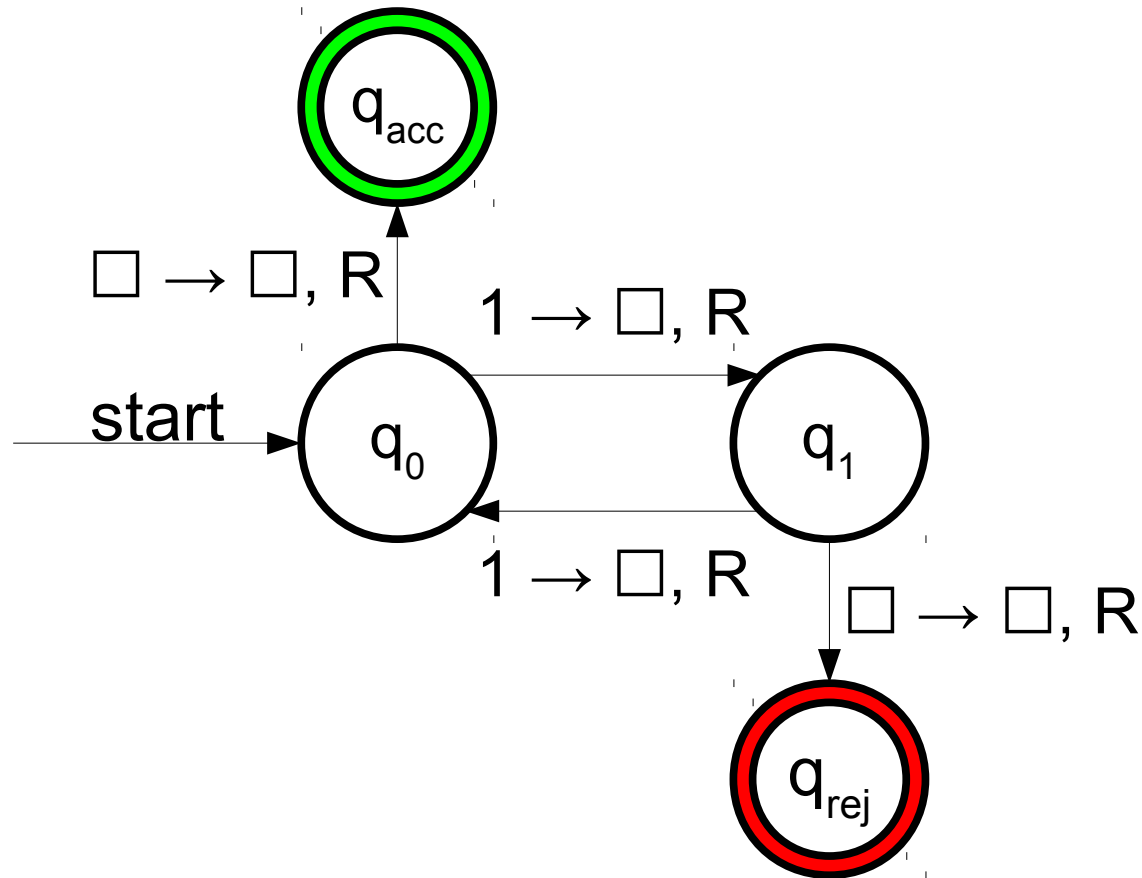
# Input and Tape Alphabets

- A Turing machine has two alphabets:
  - An **input alphabet**  $\Sigma$ . All input strings are written in the input alphabet.
  - A **tape alphabet**  $\Gamma$ , where  $\Sigma \subseteq \Gamma$ . The tape alphabet contains all symbols that can be written onto the tape.
- The tape alphabet  $\Gamma$  can contain any number of symbols, but always contains at least one **blank symbol**, denoted  $\square$ . You are guaranteed  $\square \notin \Sigma$ .
- At startup, the Turing machine begins with an infinite tape of  $\square$  symbols with the input written at some location. The tape head is positioned at the start of the input.

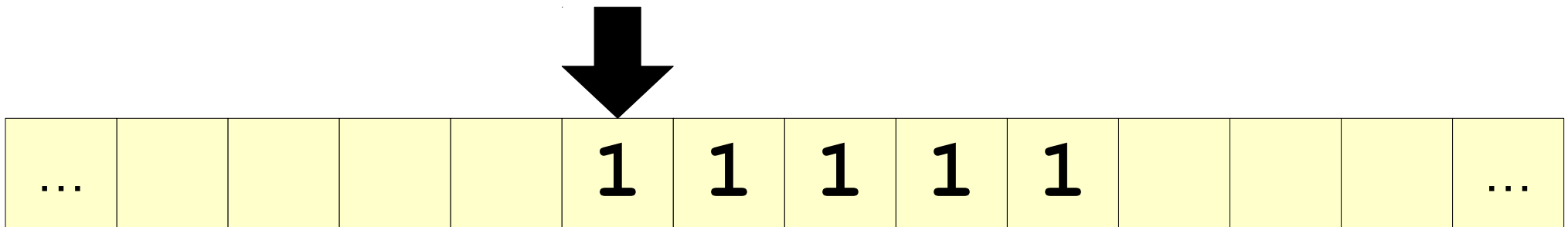
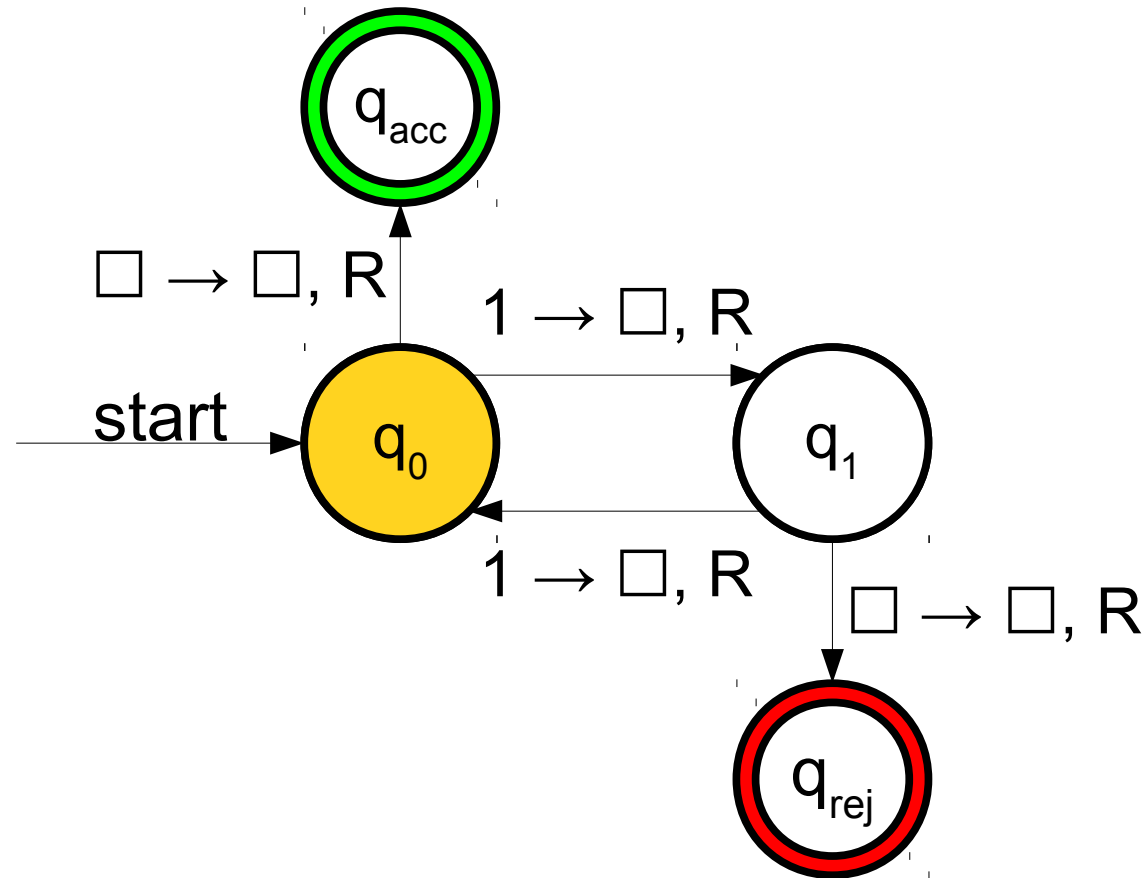
# A Simple Turing Machine



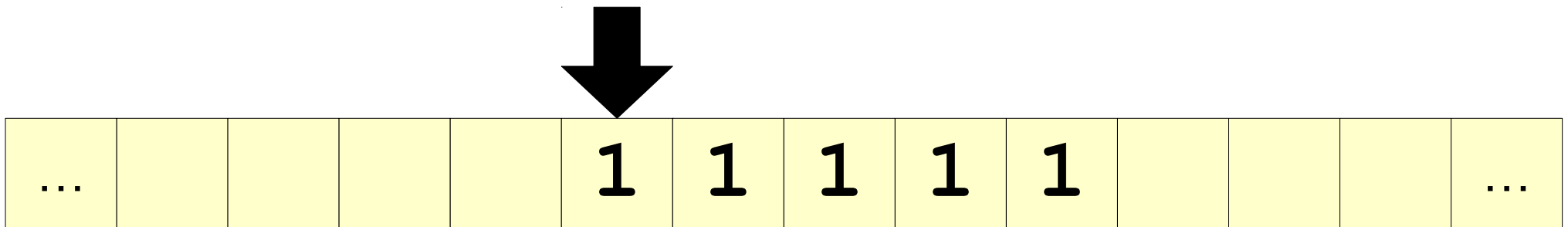
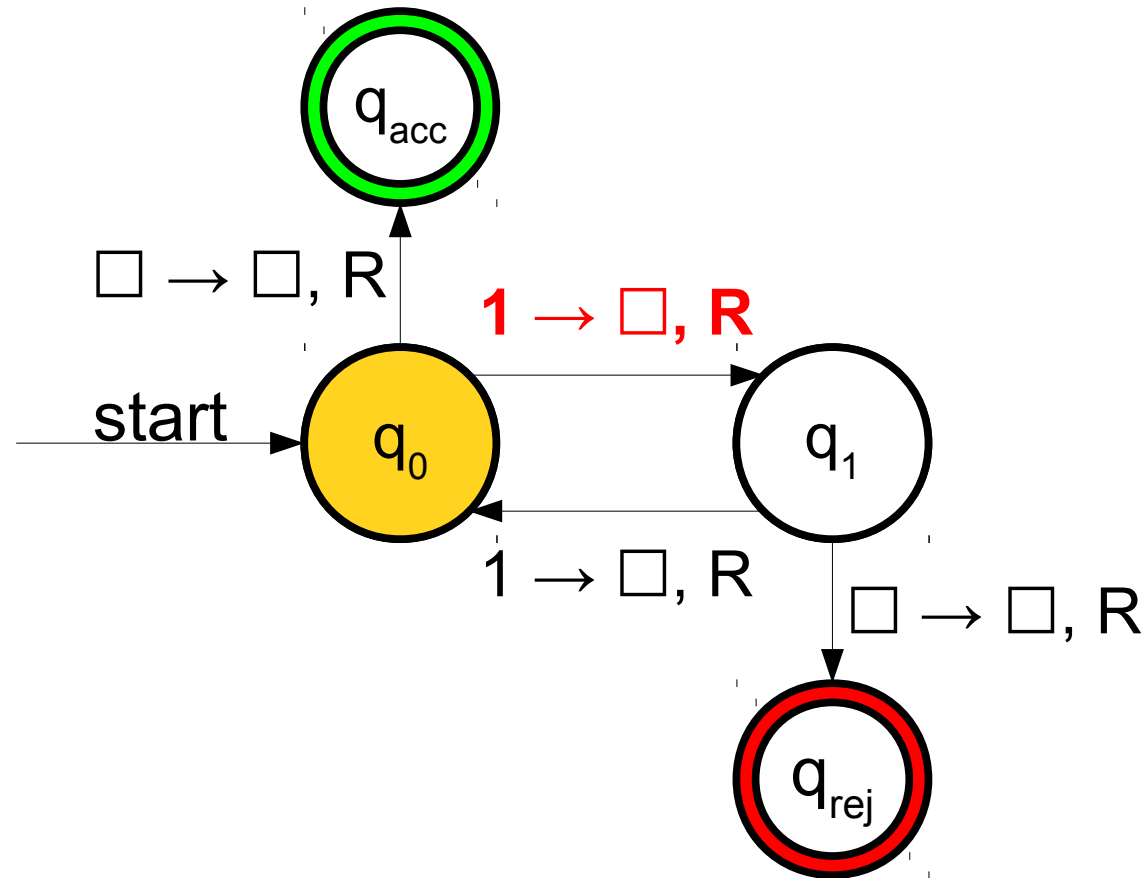
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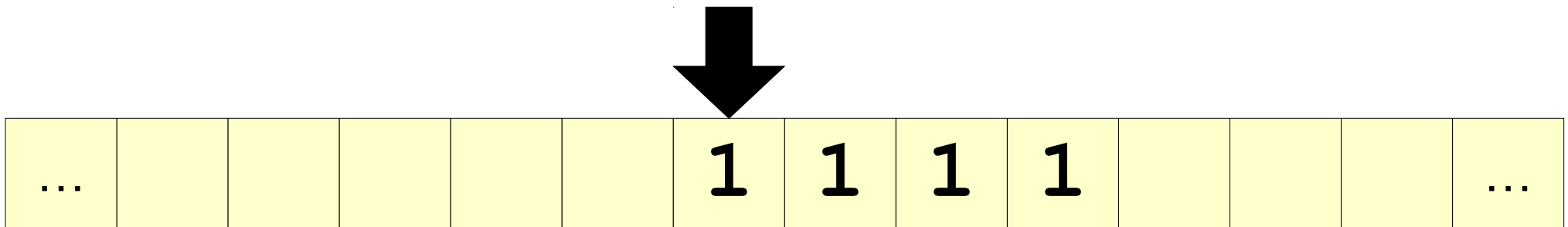
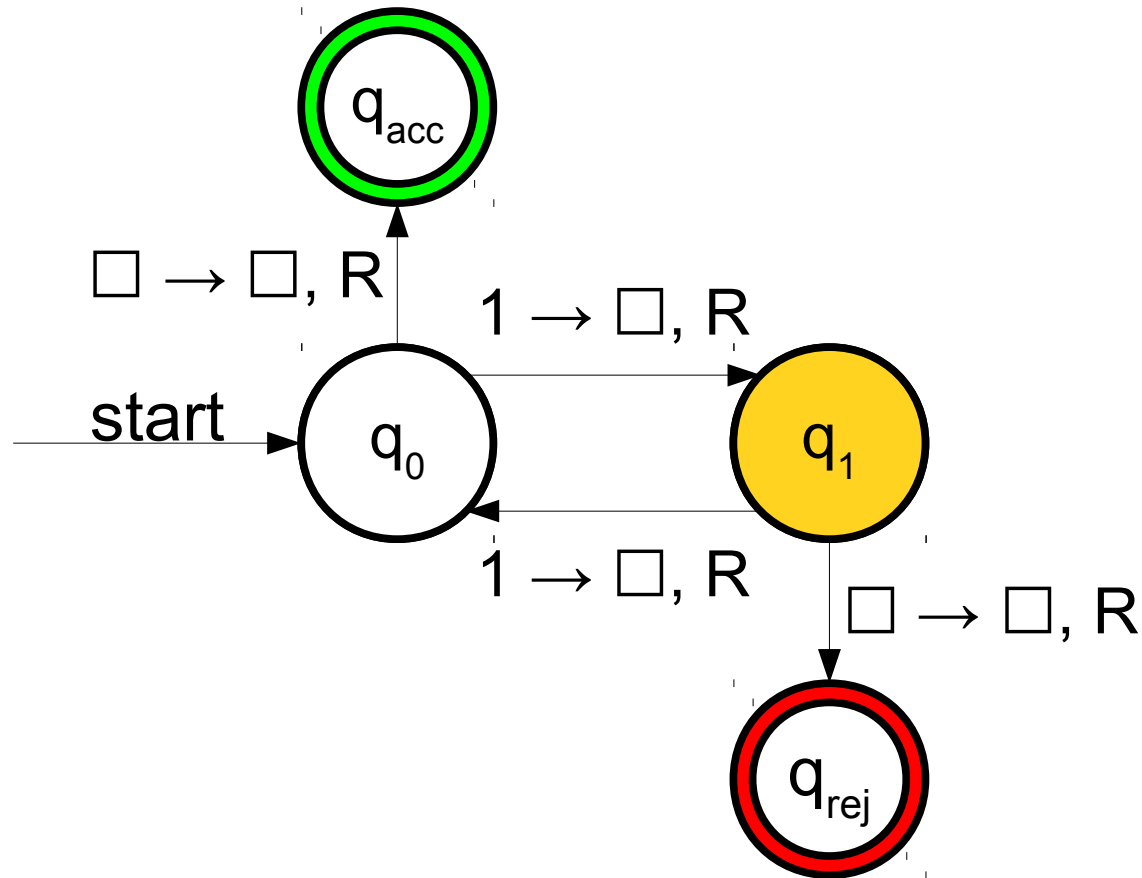
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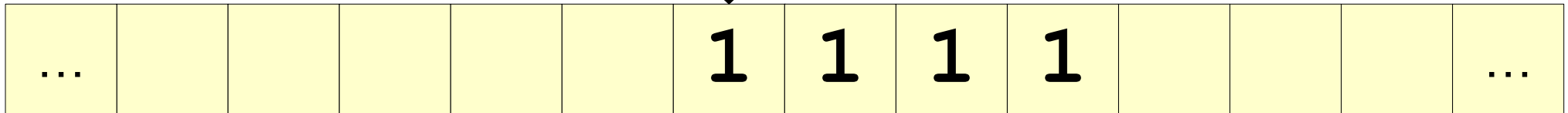
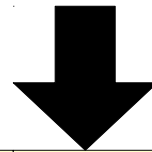
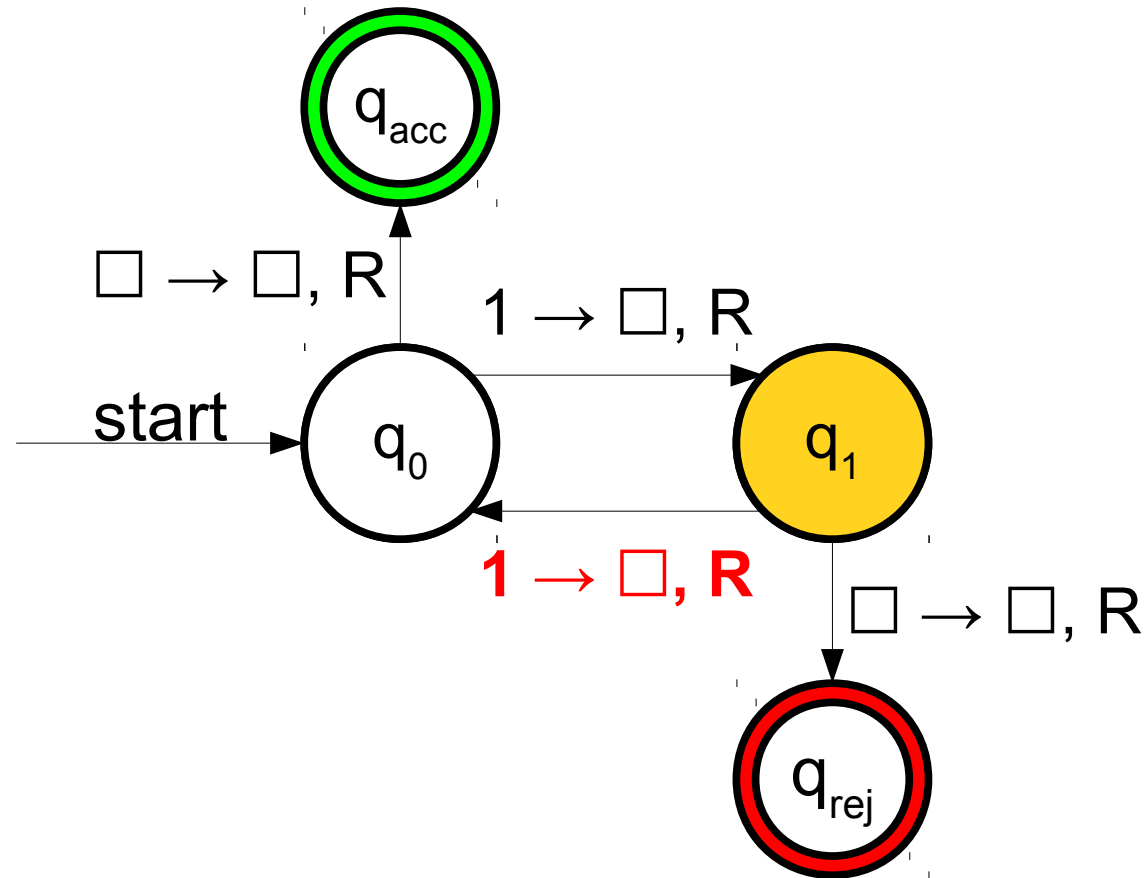


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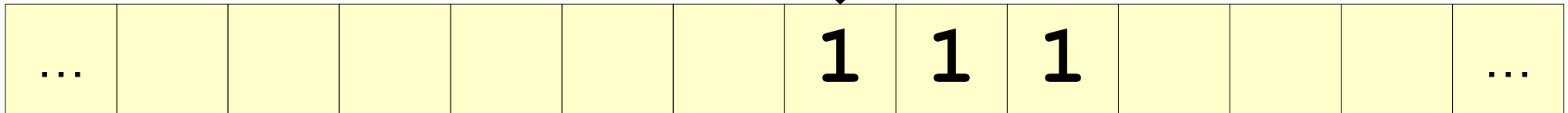
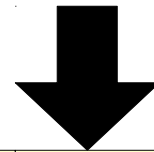
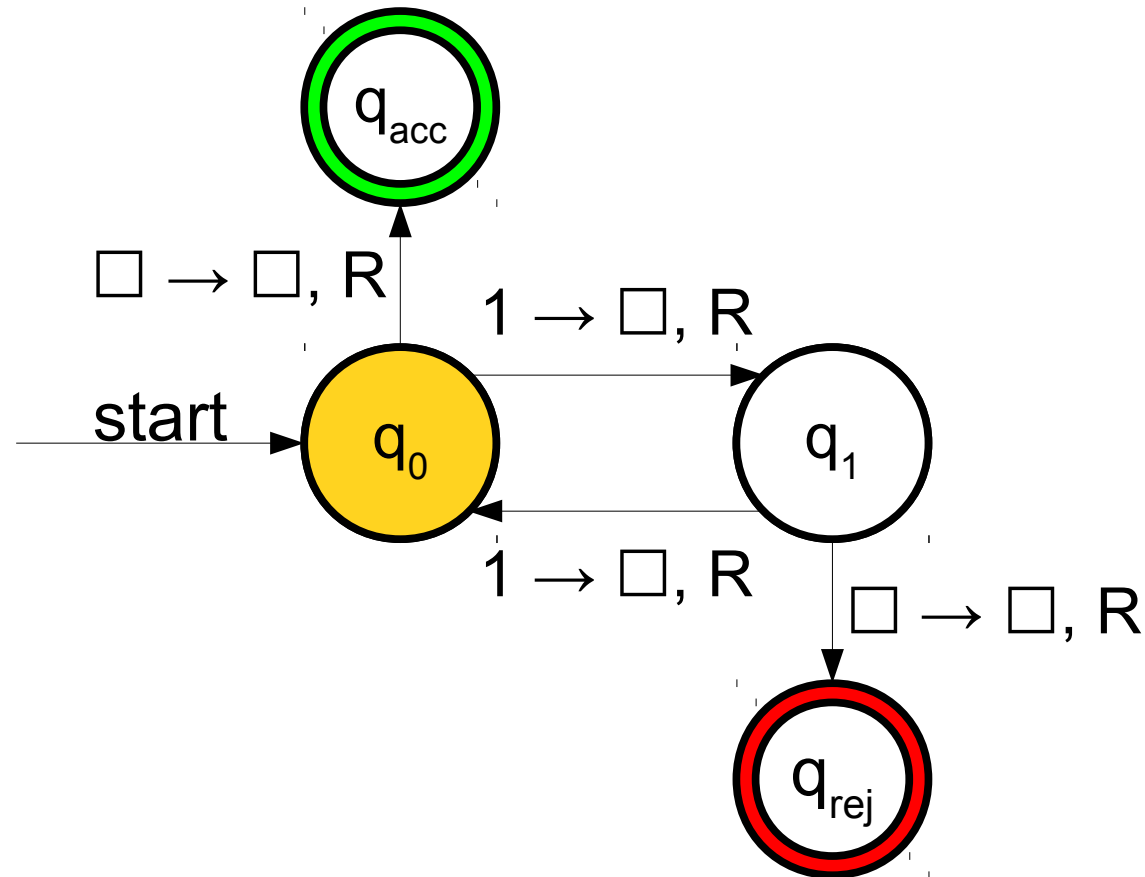




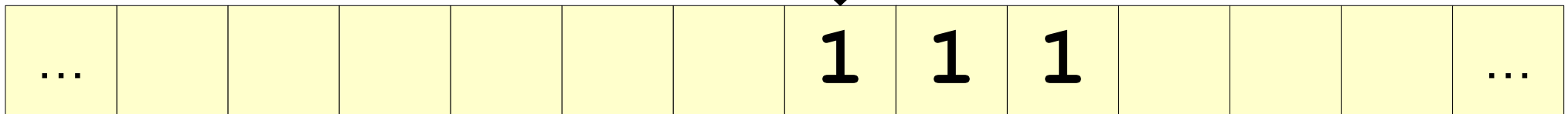
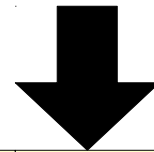
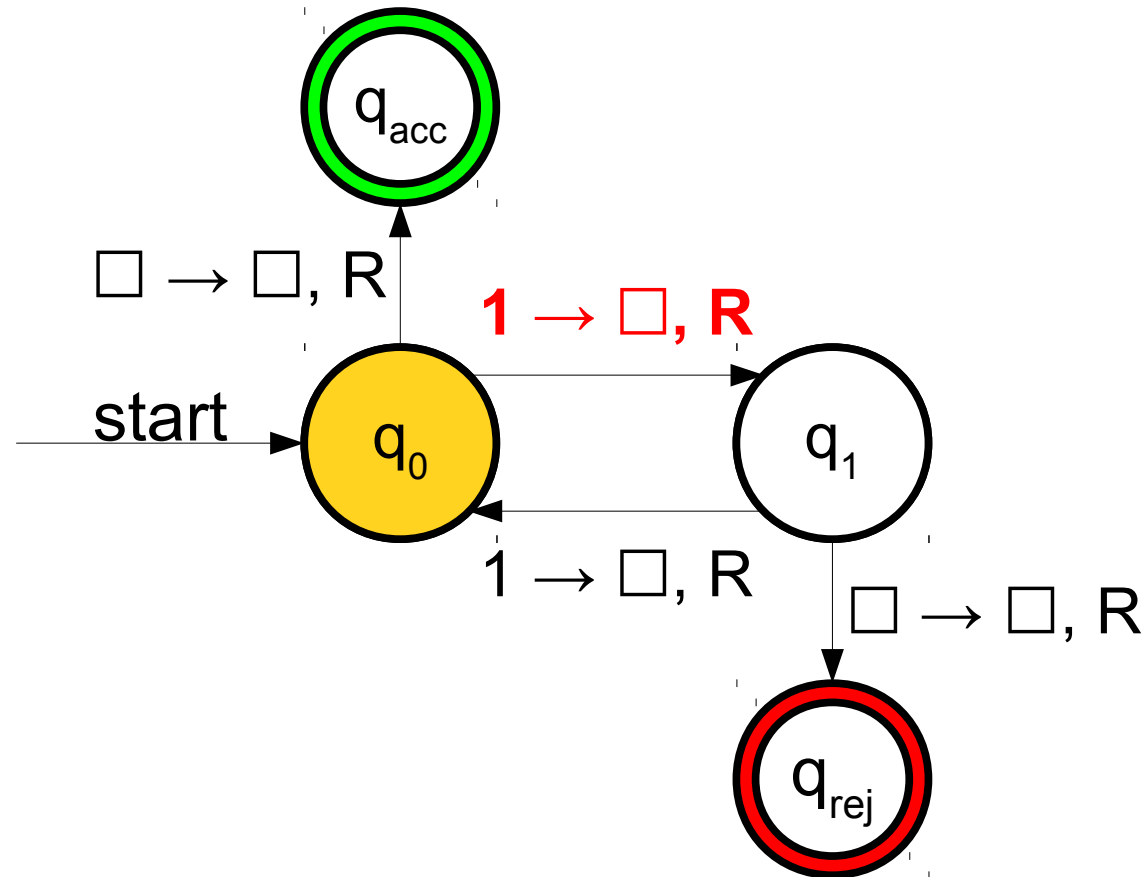
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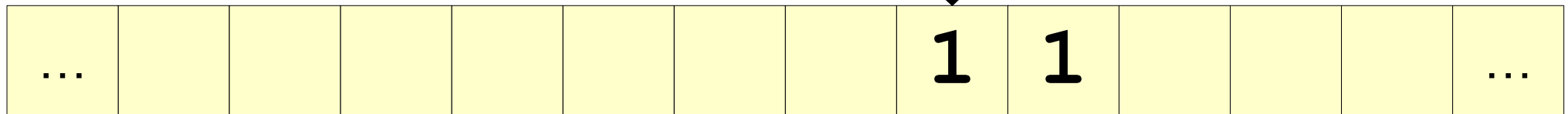
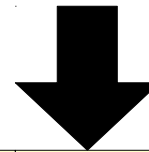
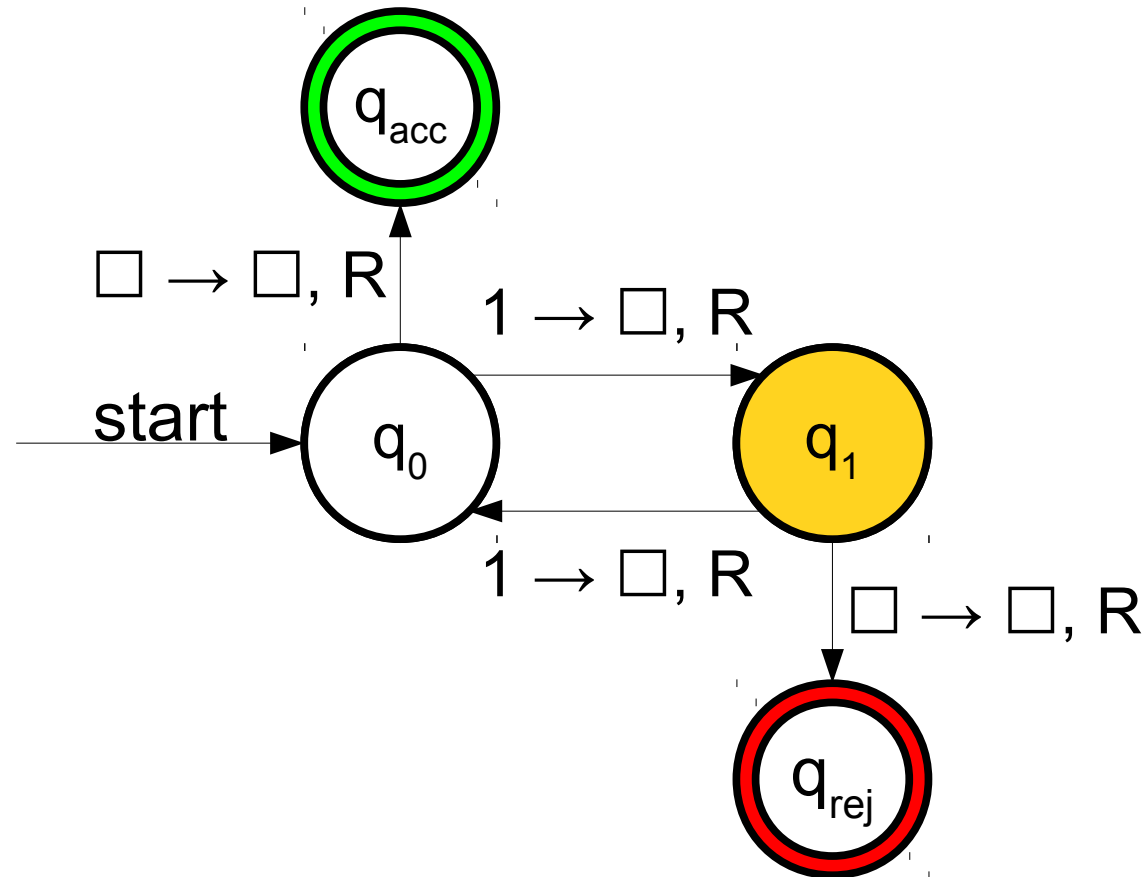
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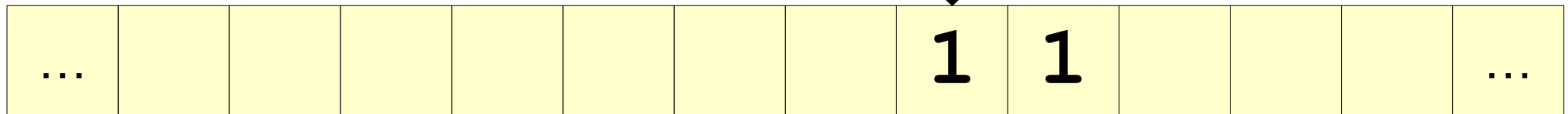
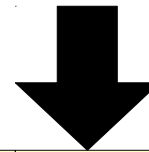
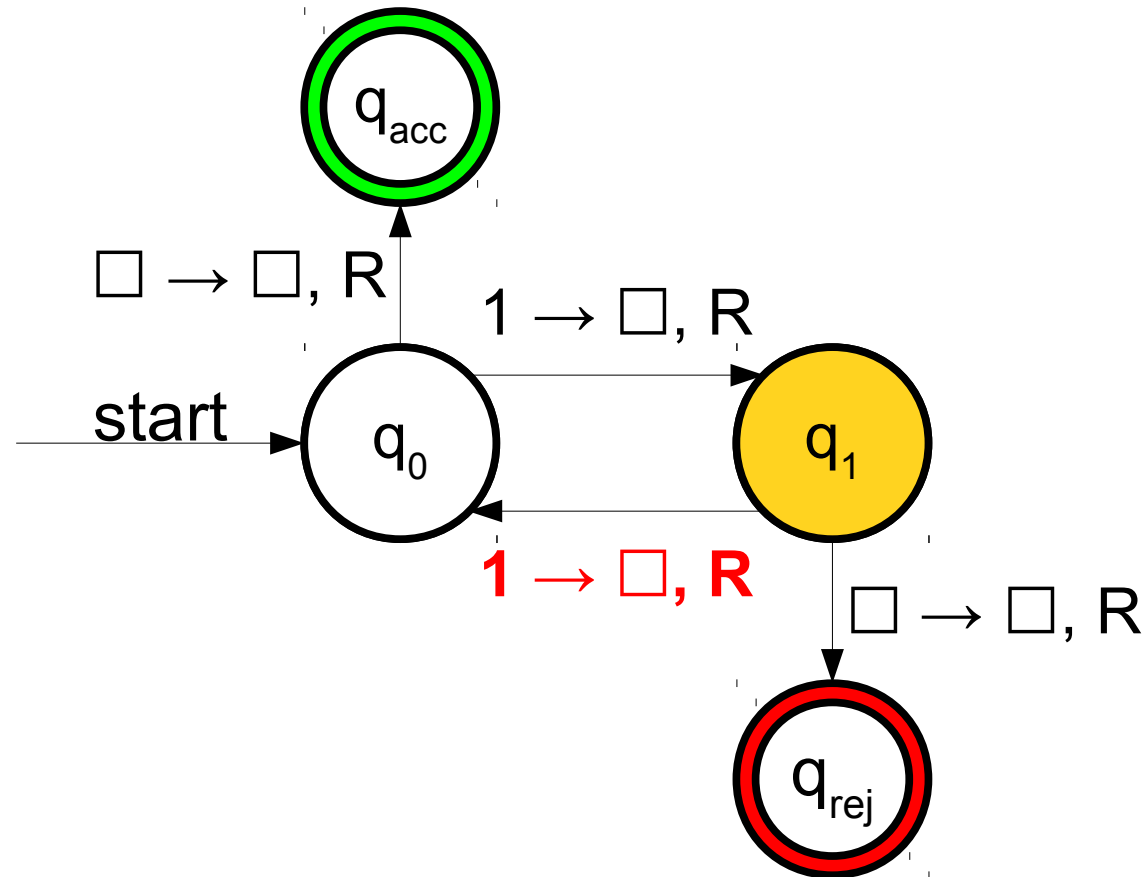
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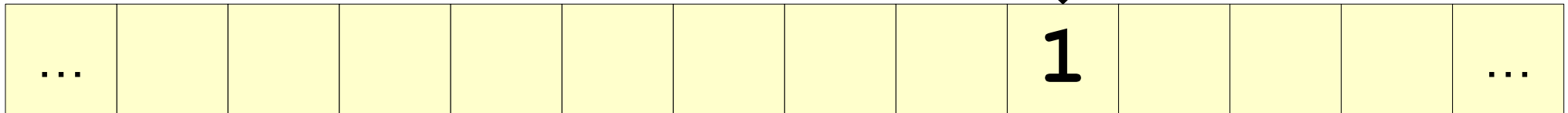
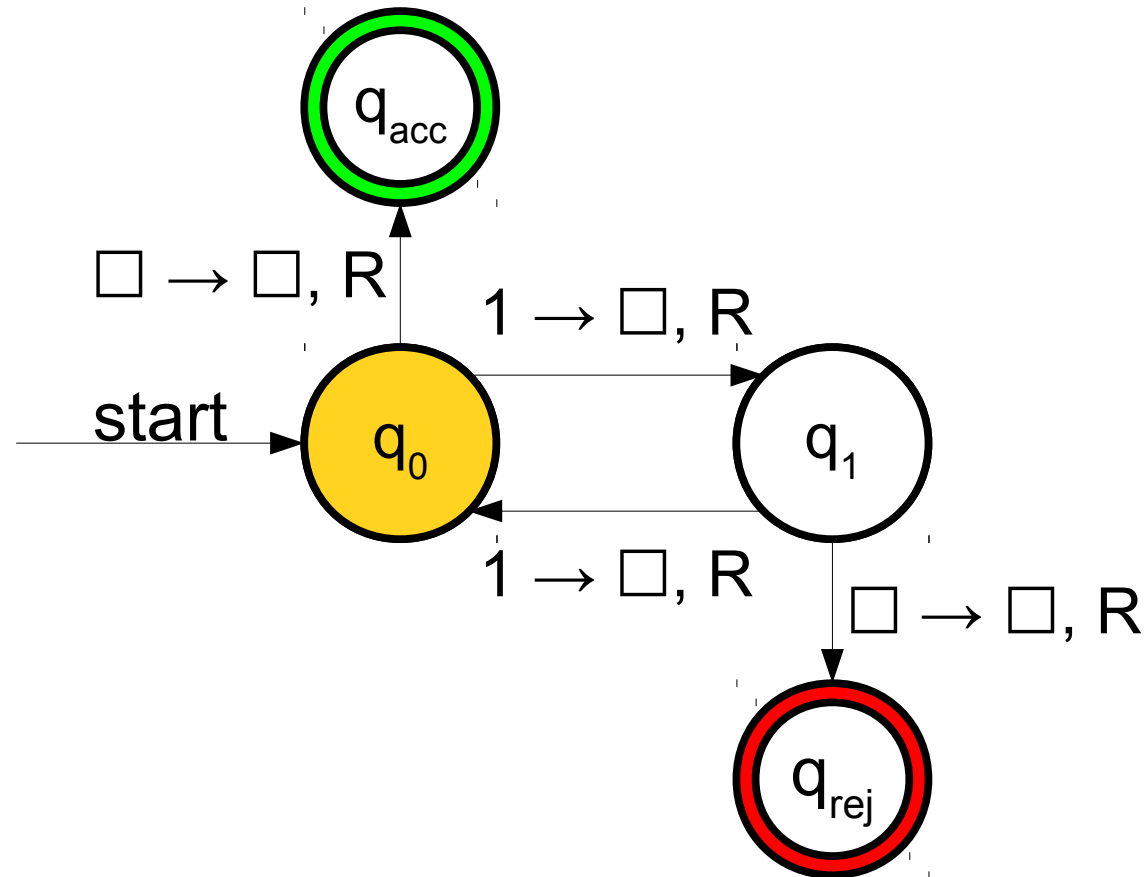
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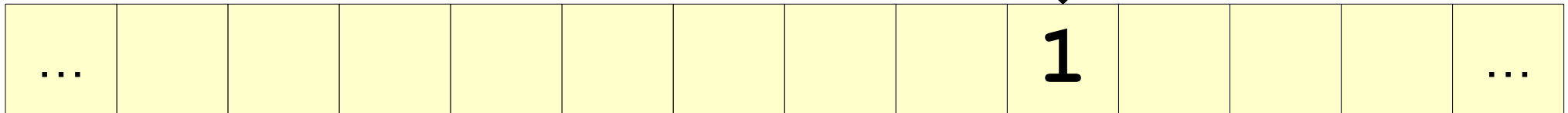
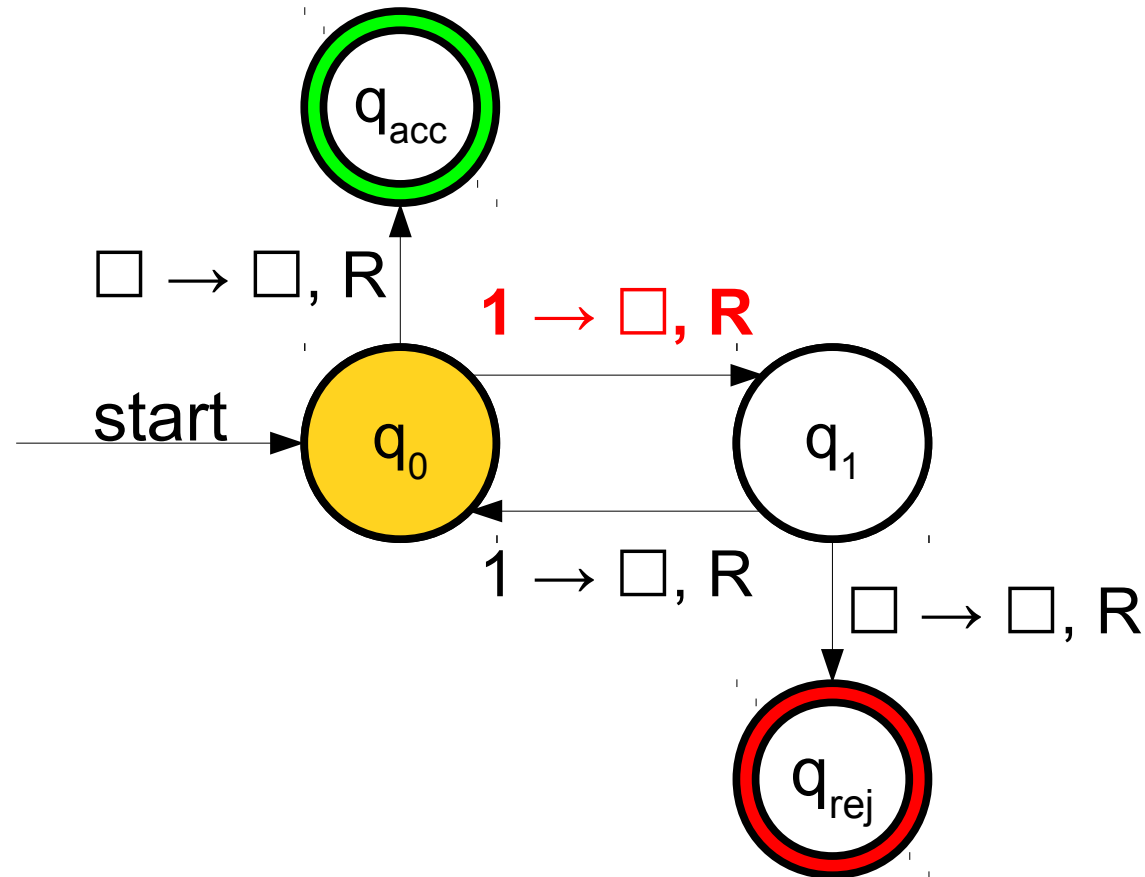
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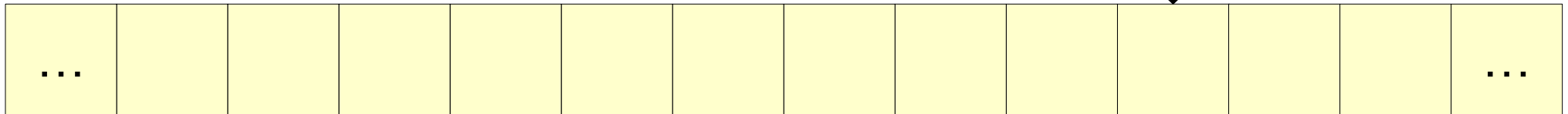
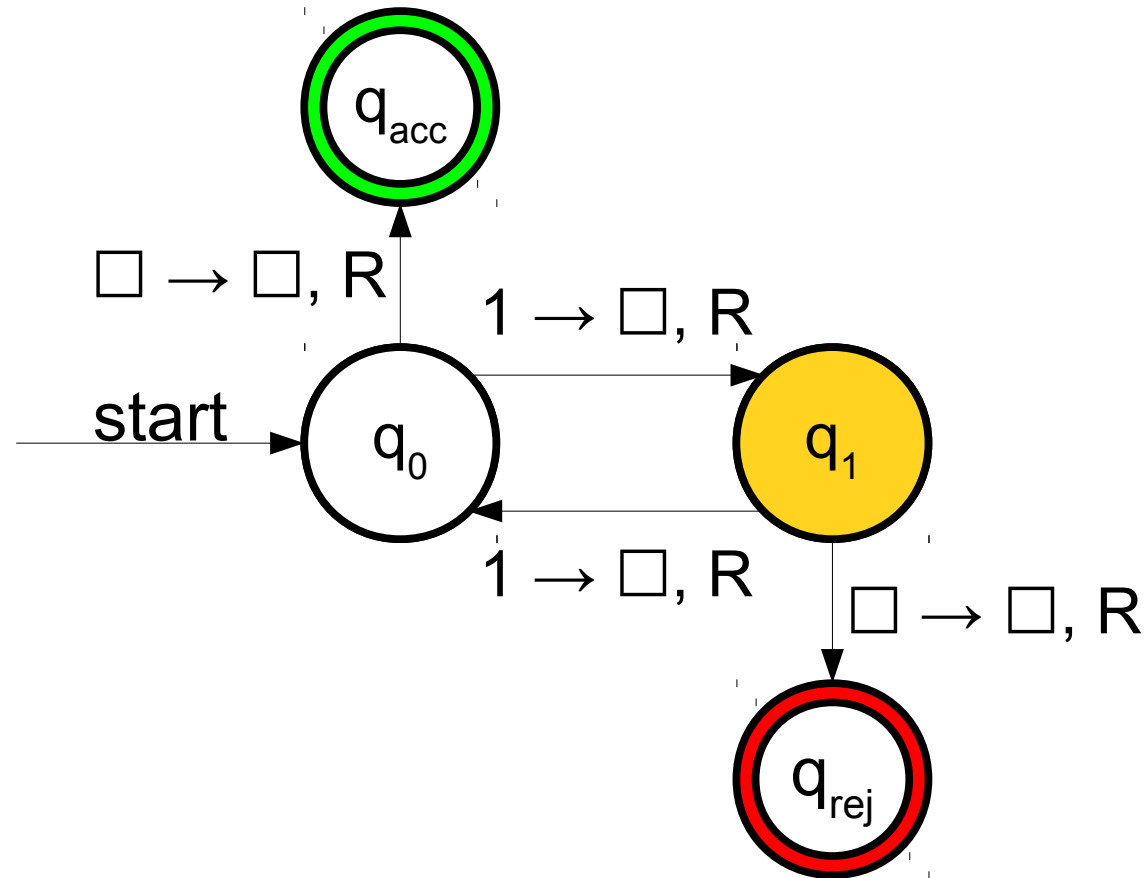
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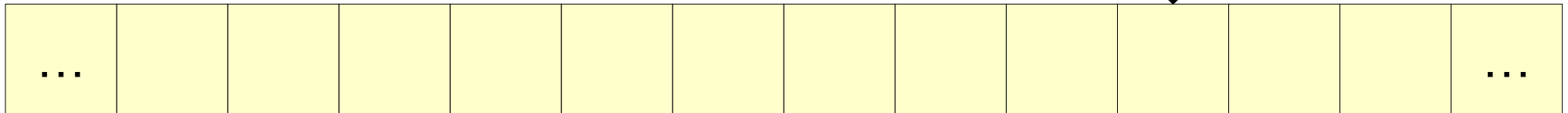
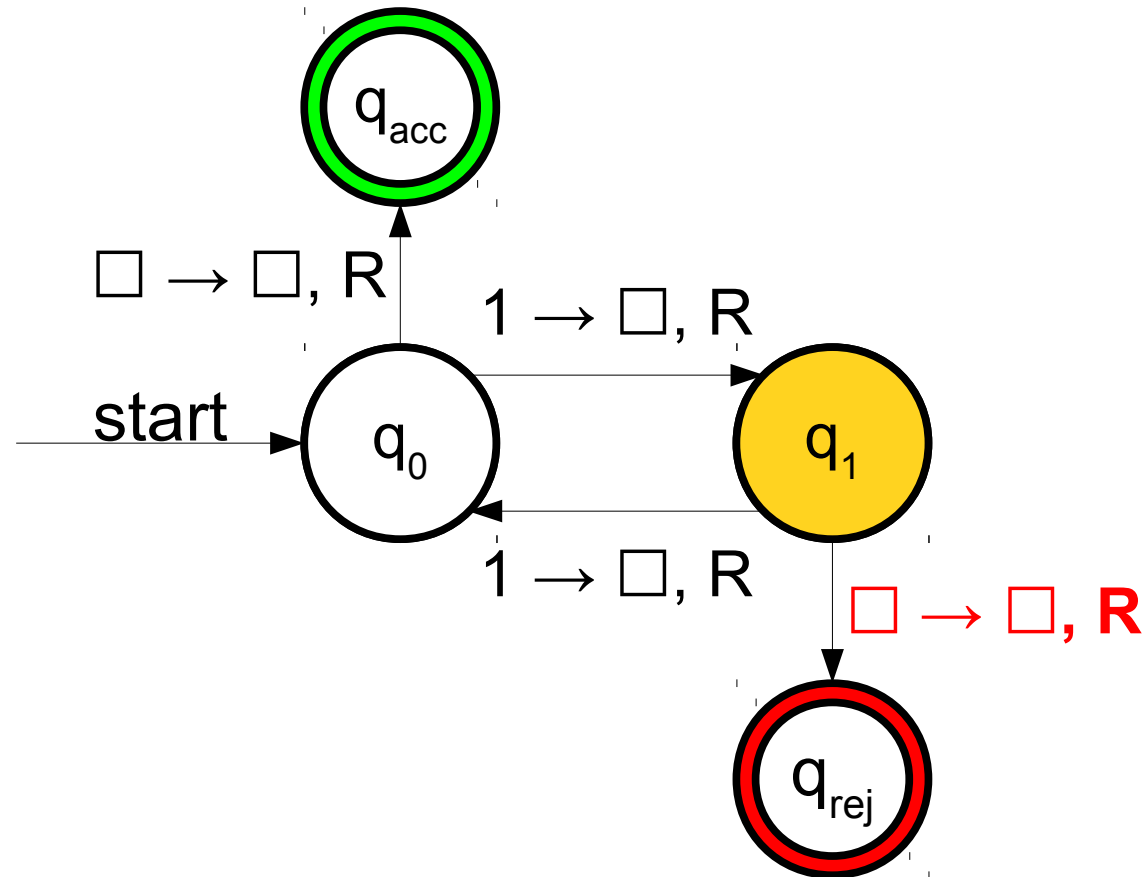


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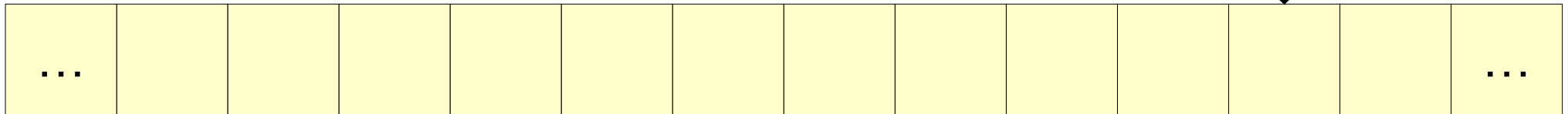
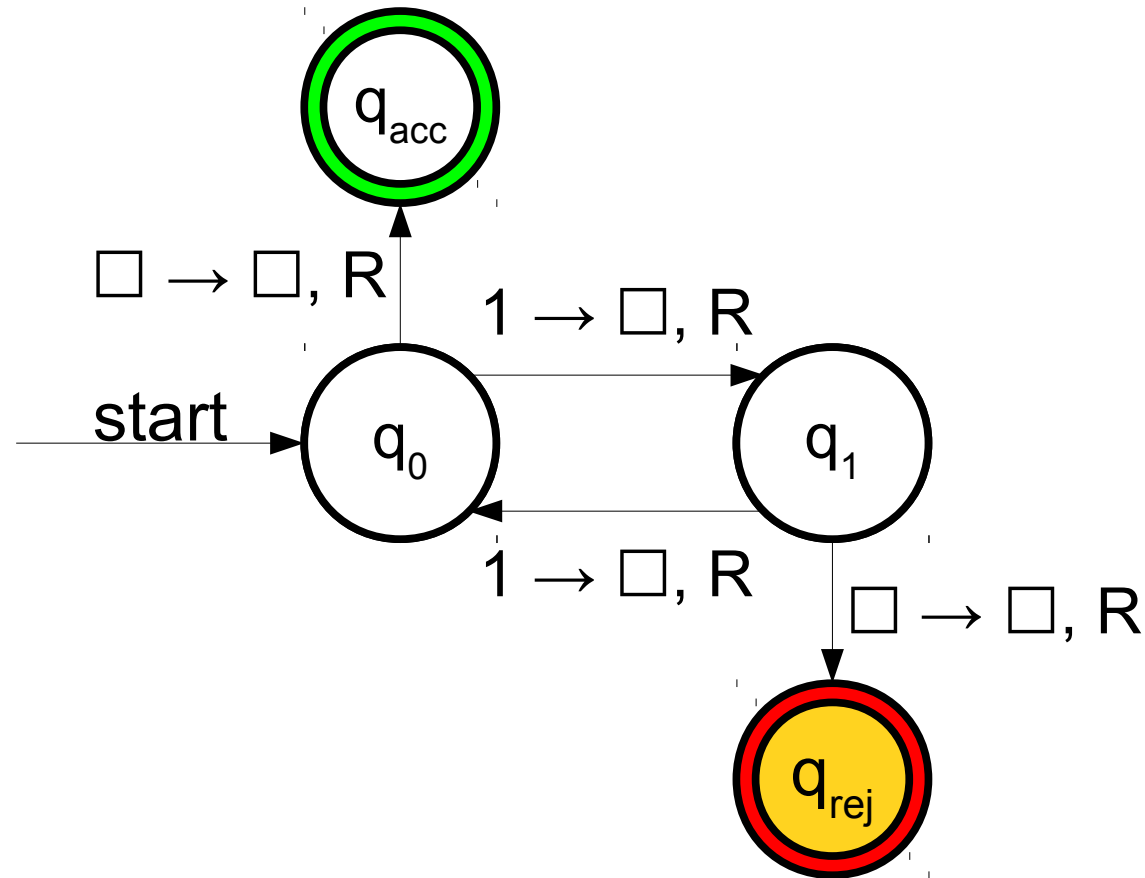




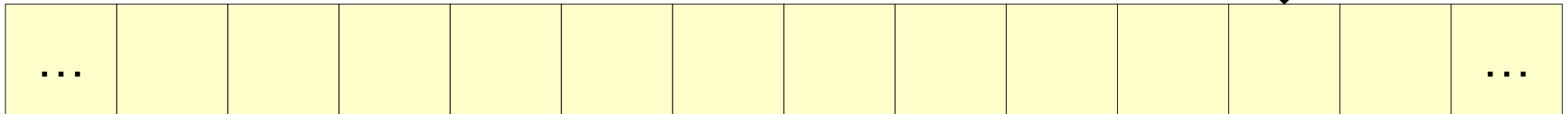
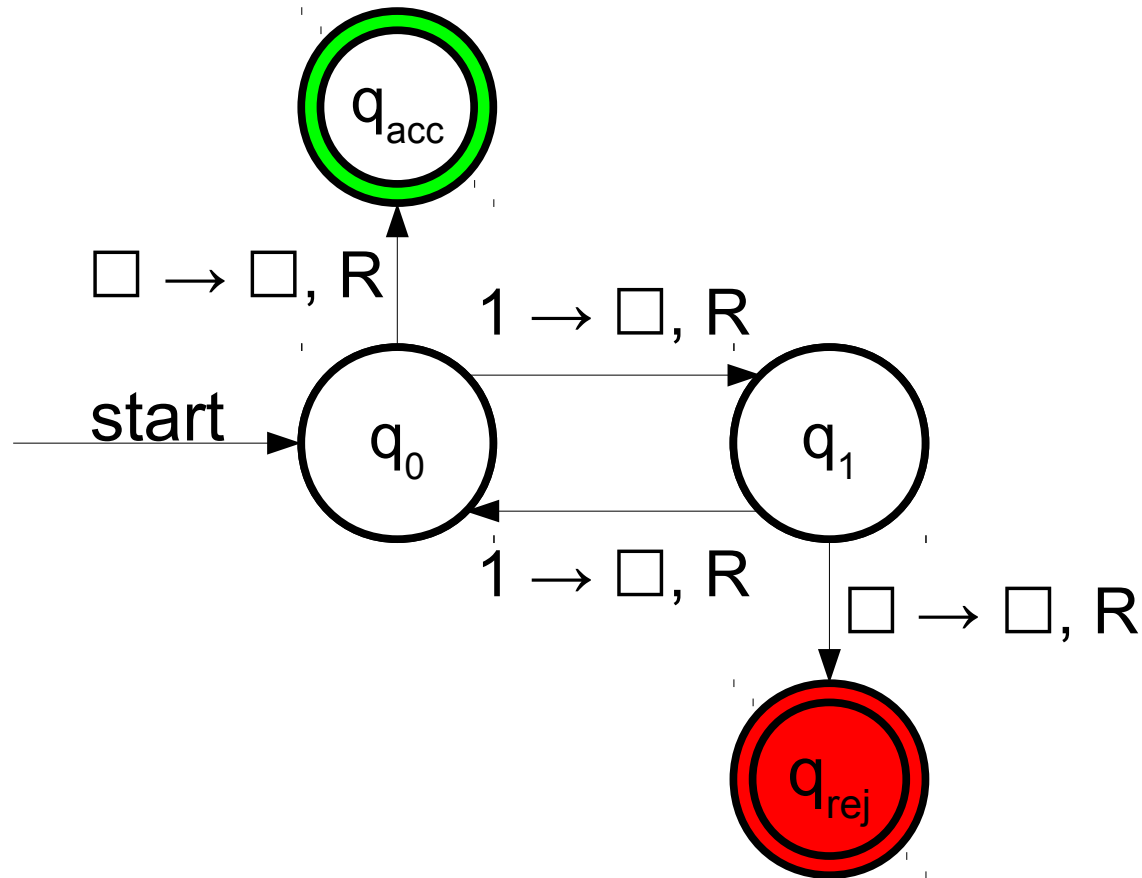
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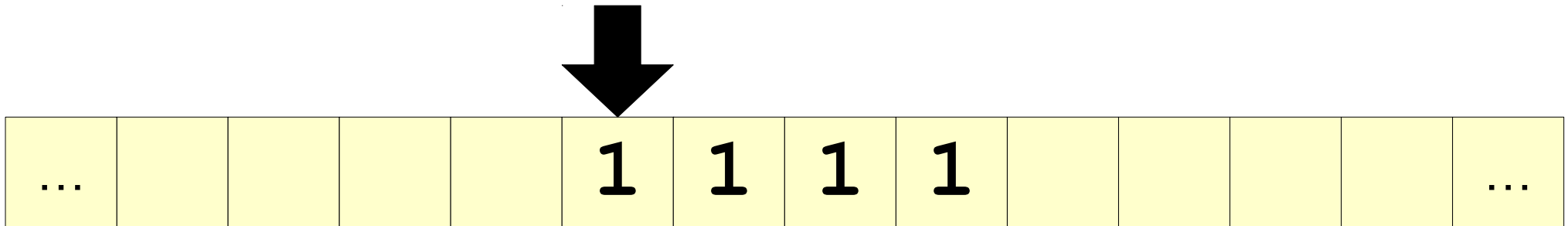
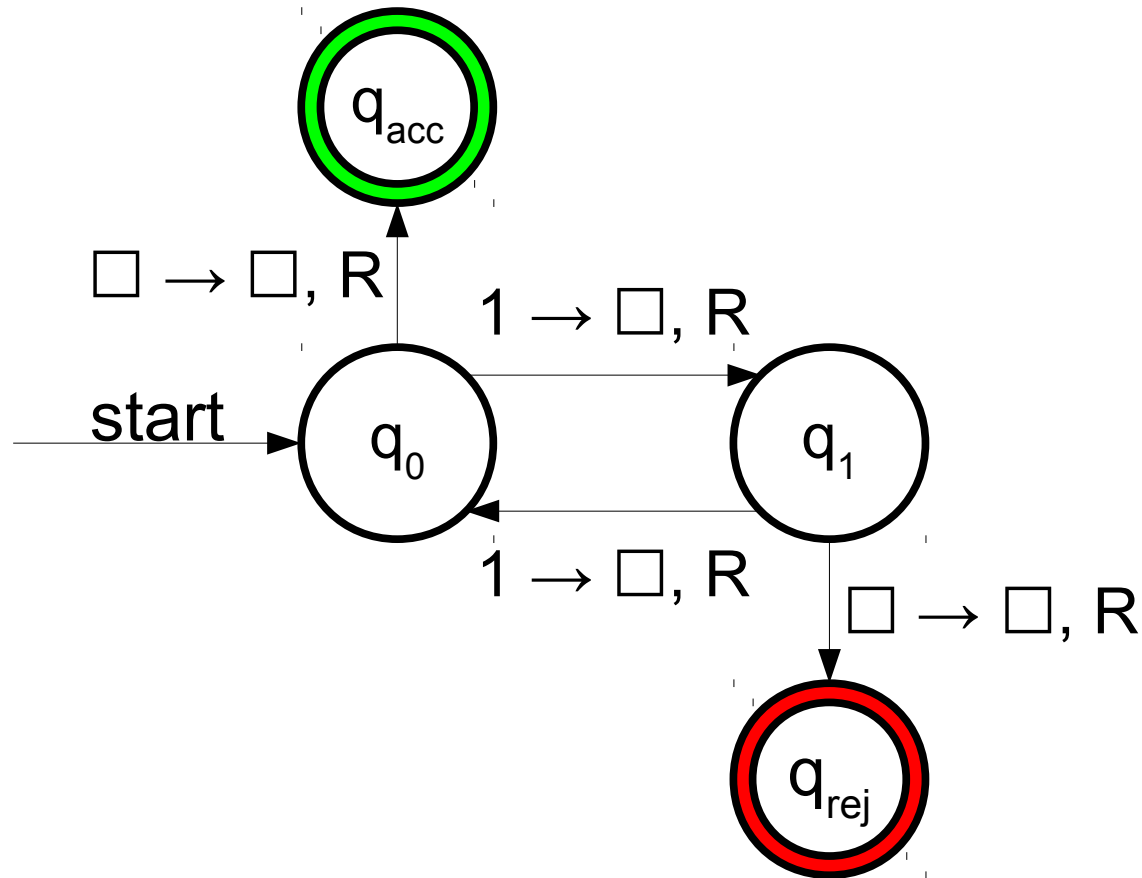
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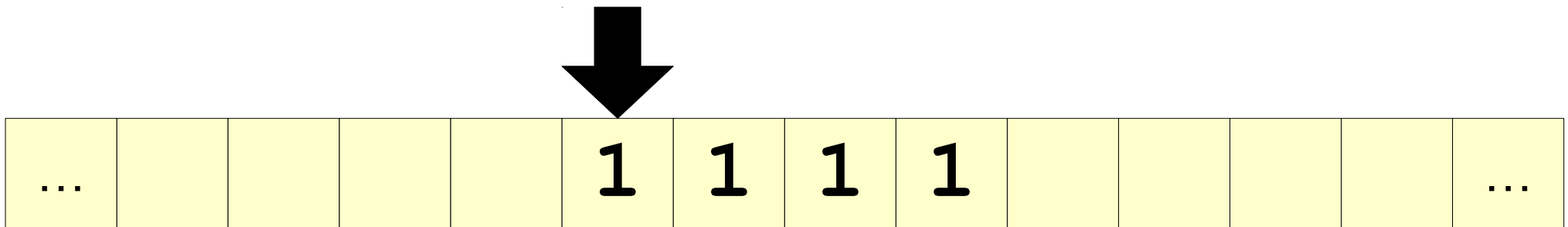
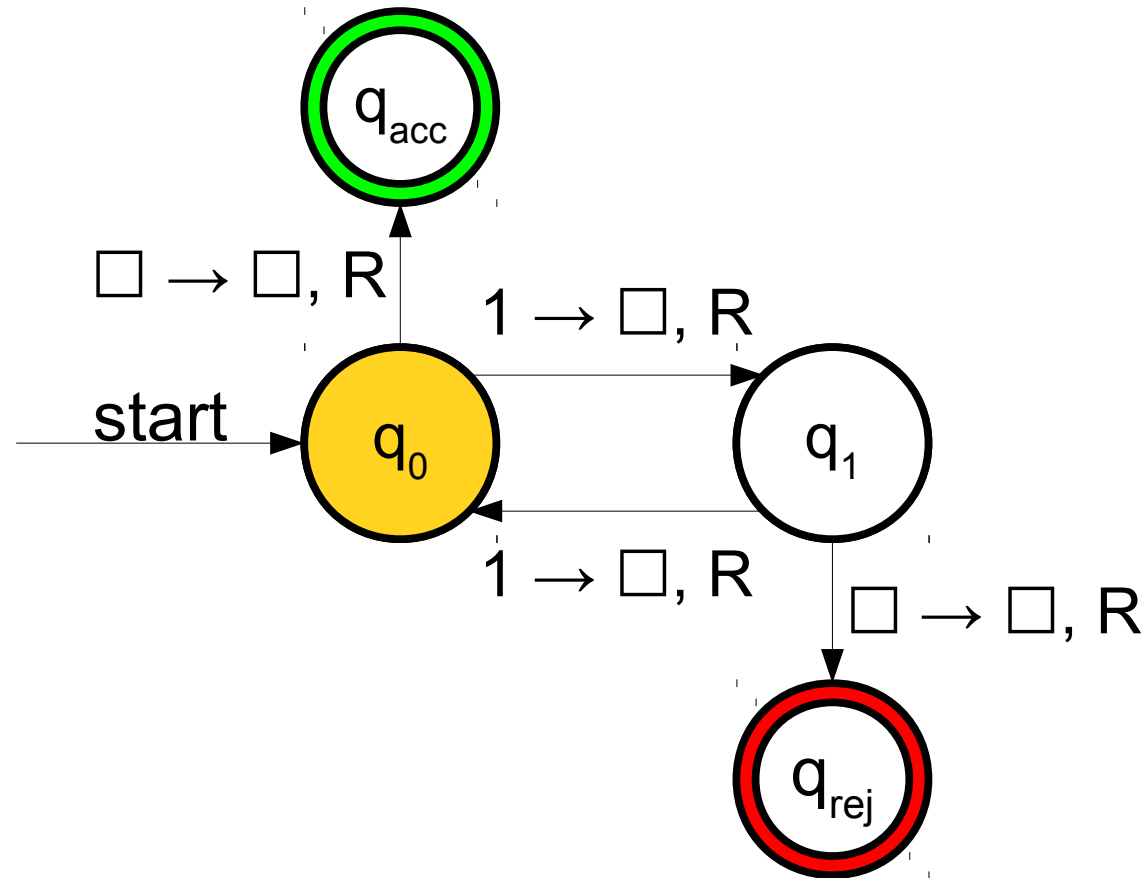
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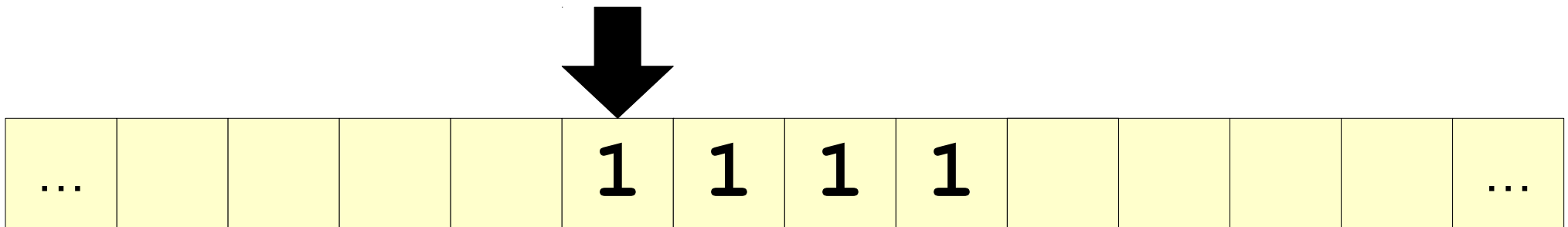
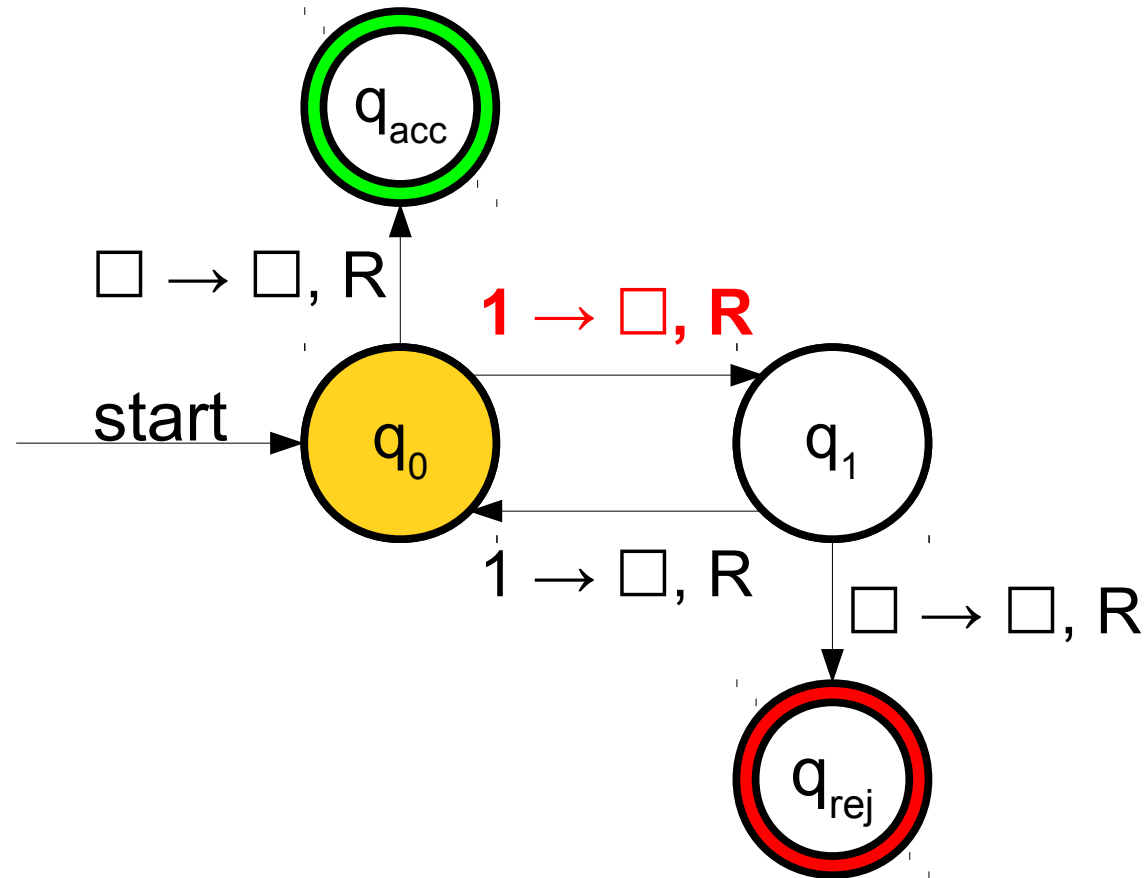
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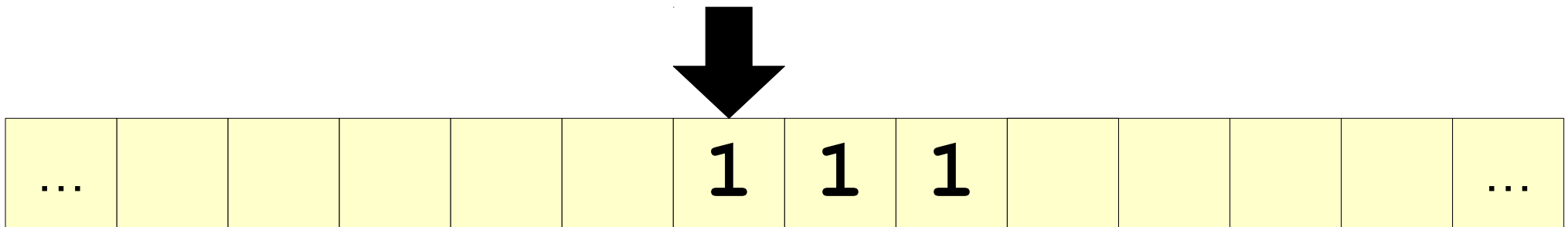
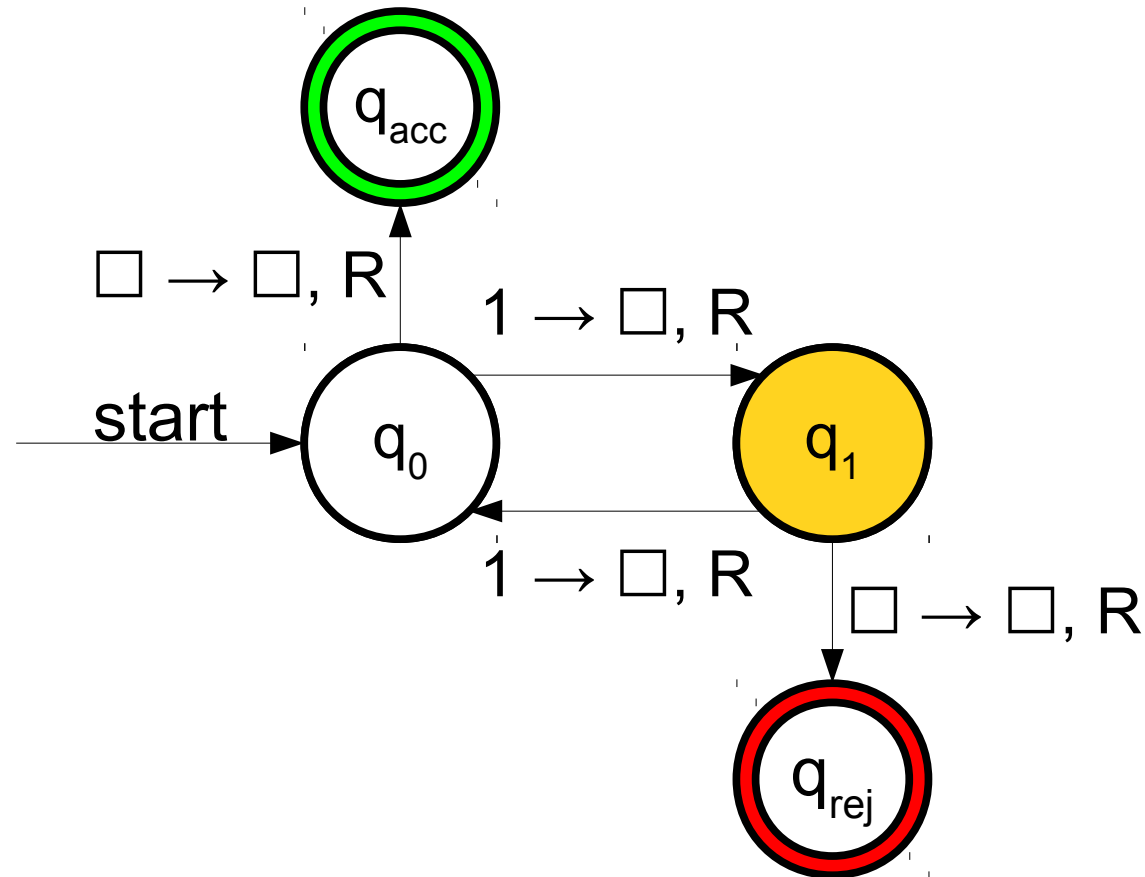
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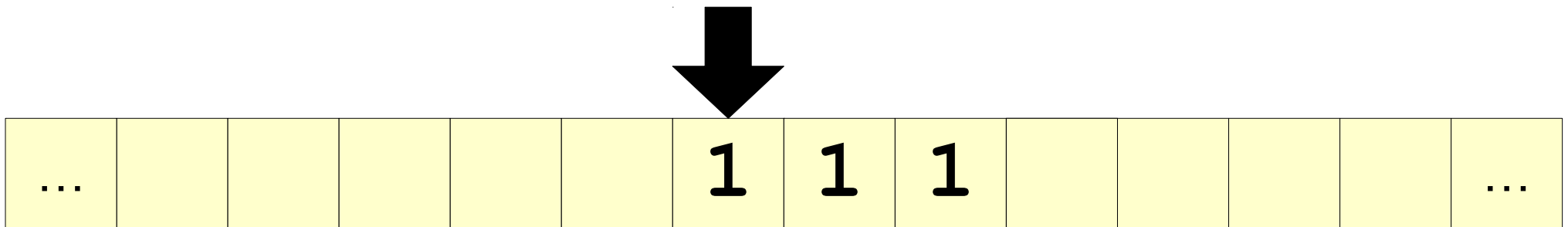
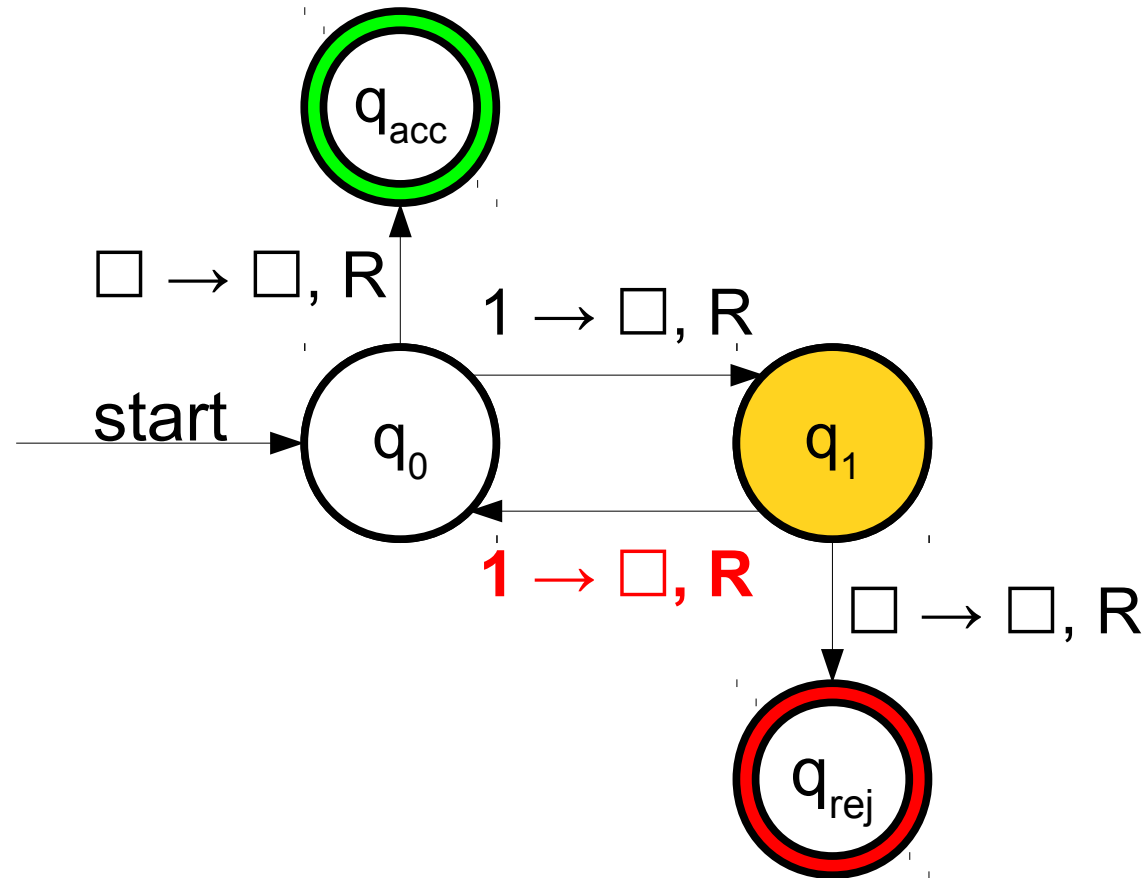
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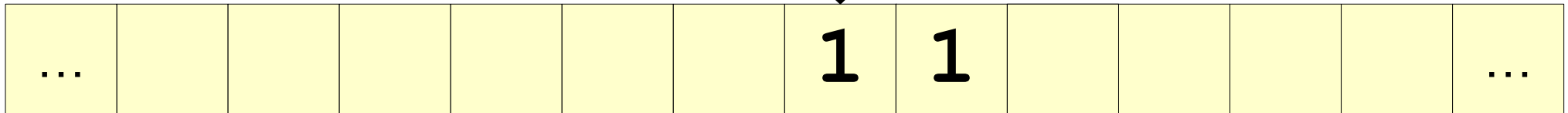
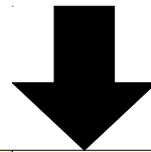
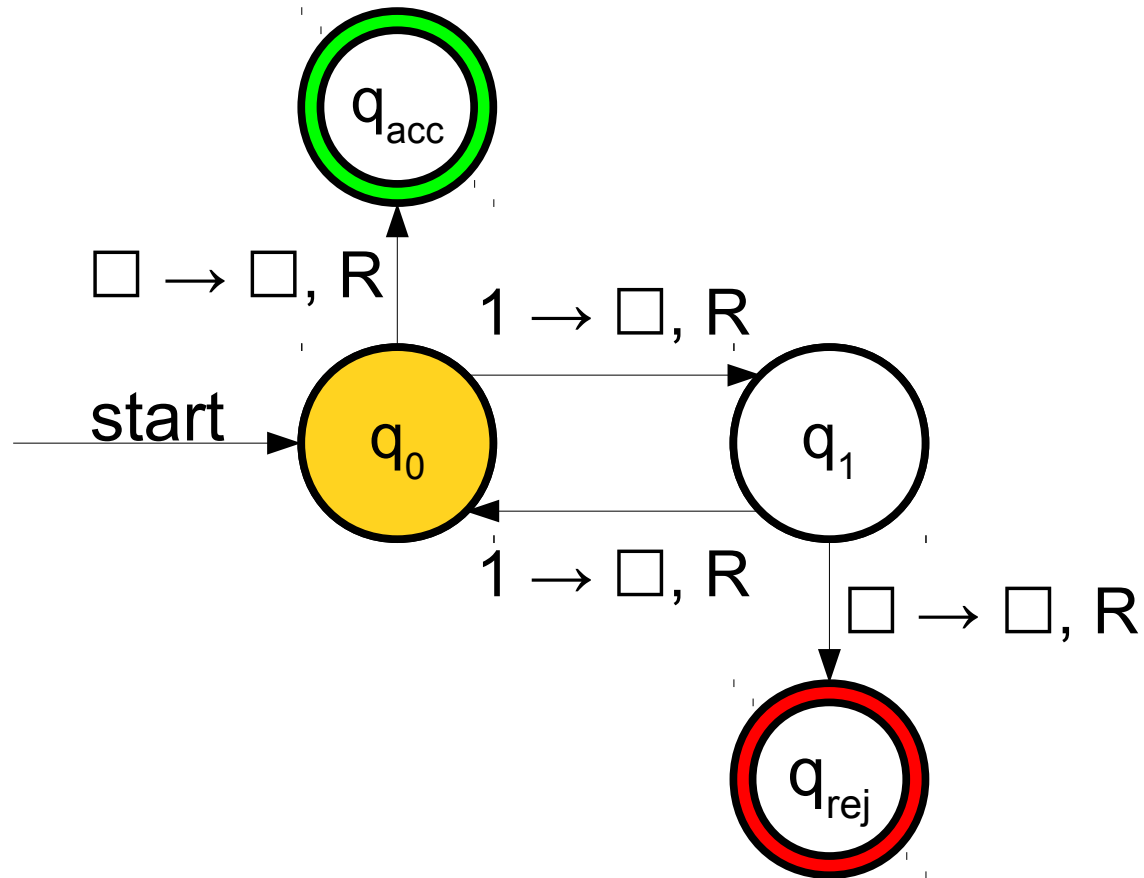


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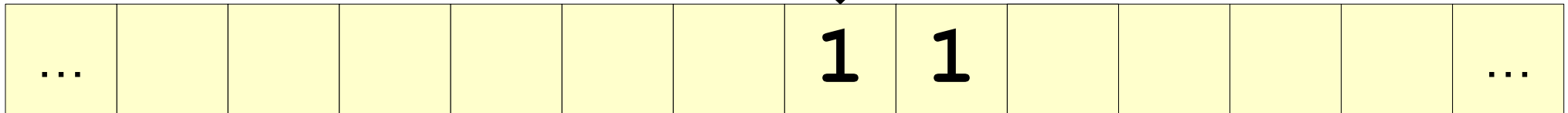
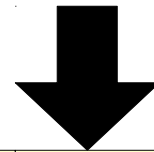
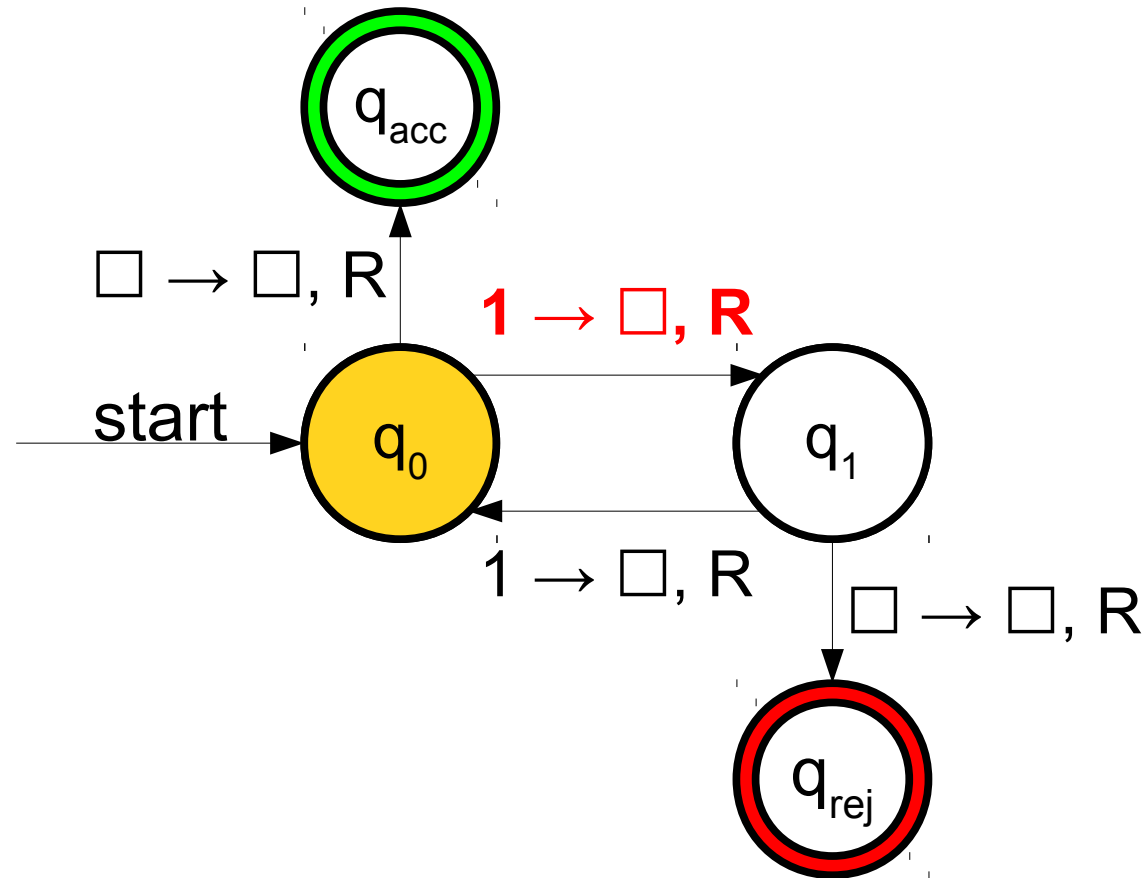




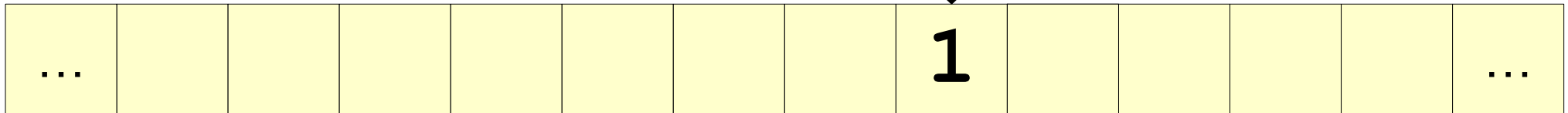
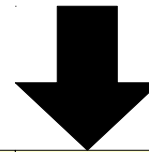
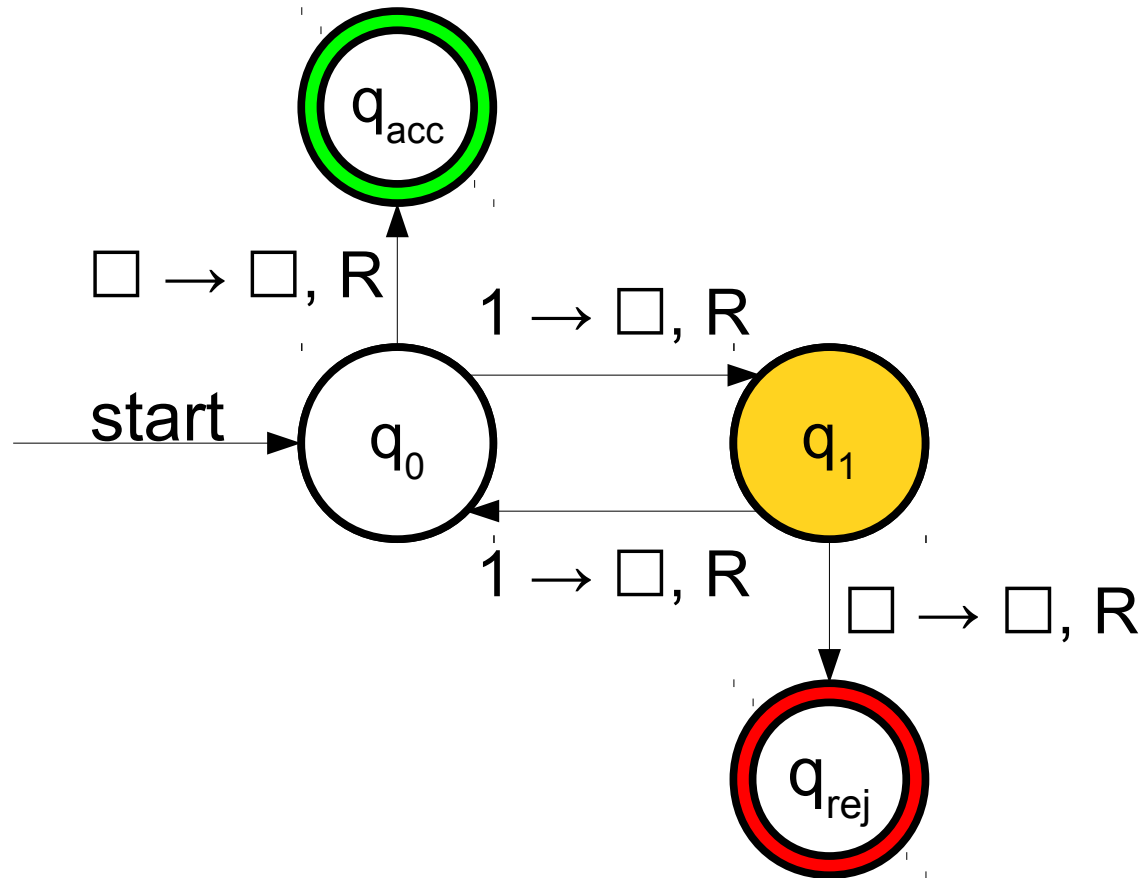
# A Simple Turing Machine



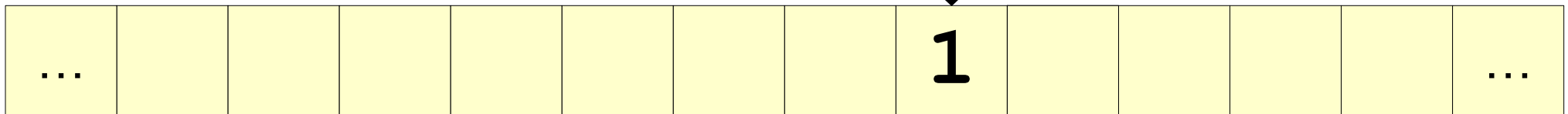
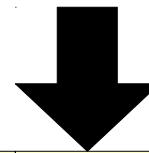
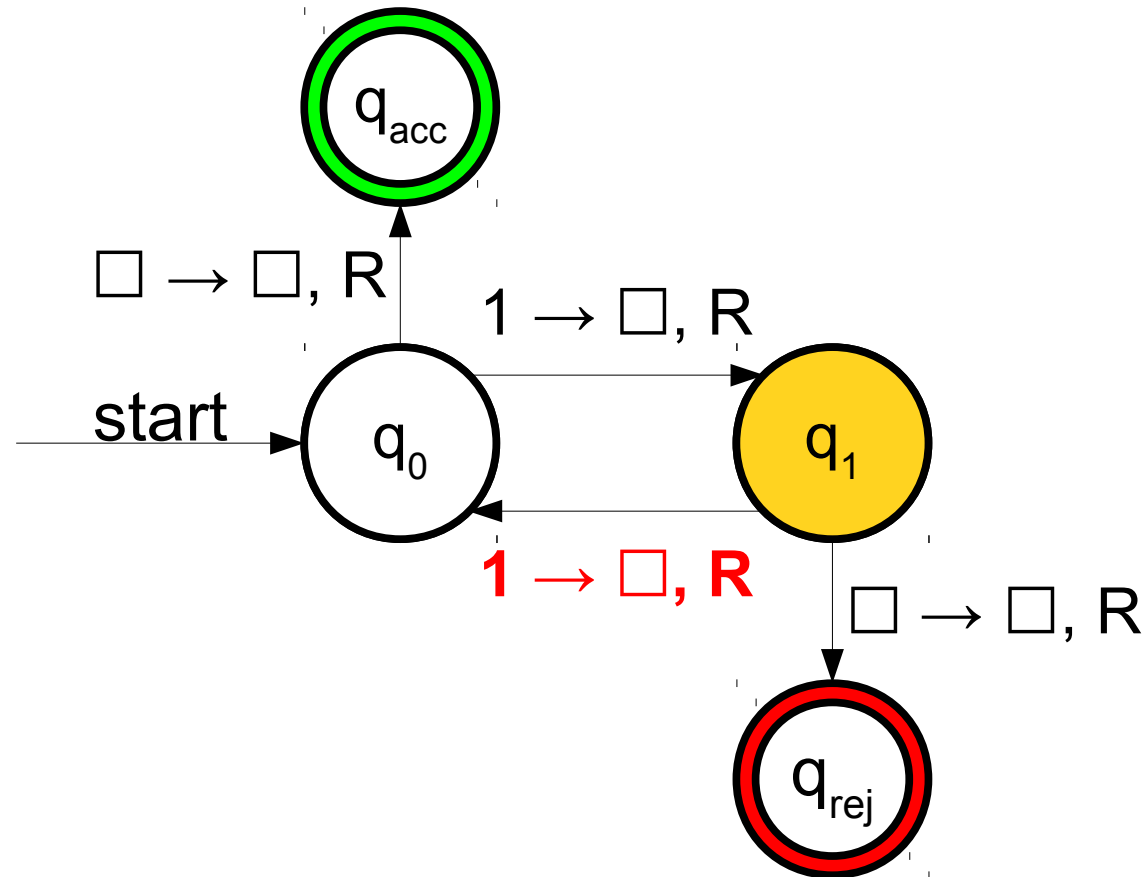
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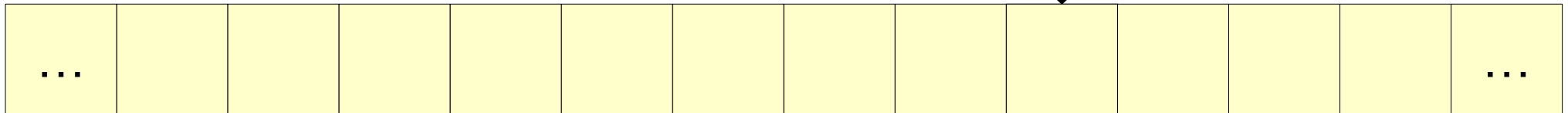
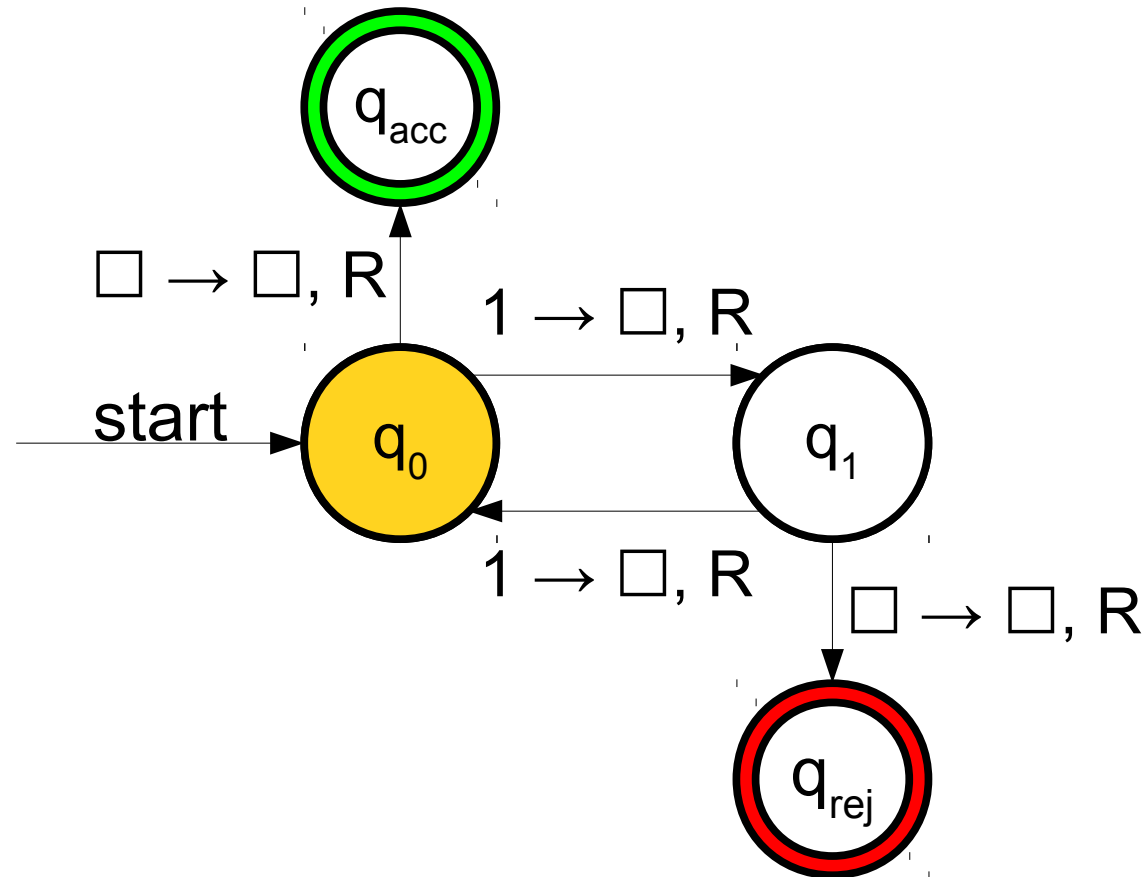
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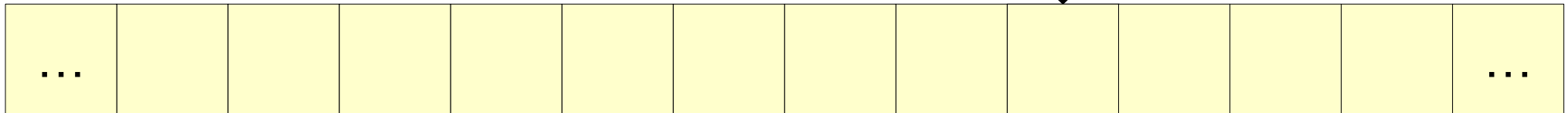
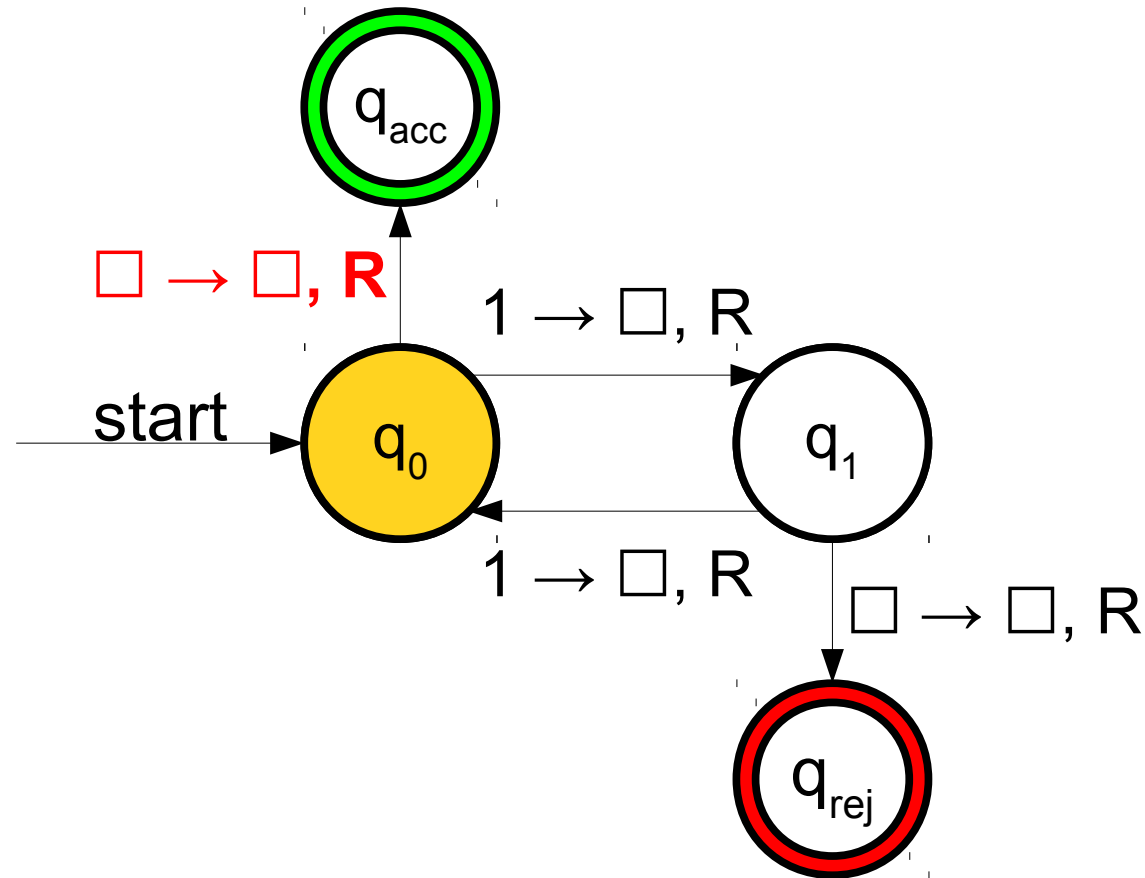
# A Simple Turing Machine



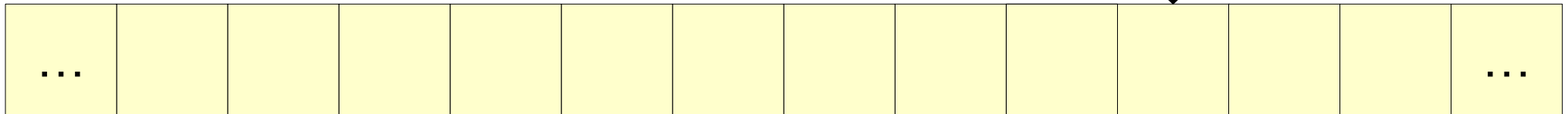
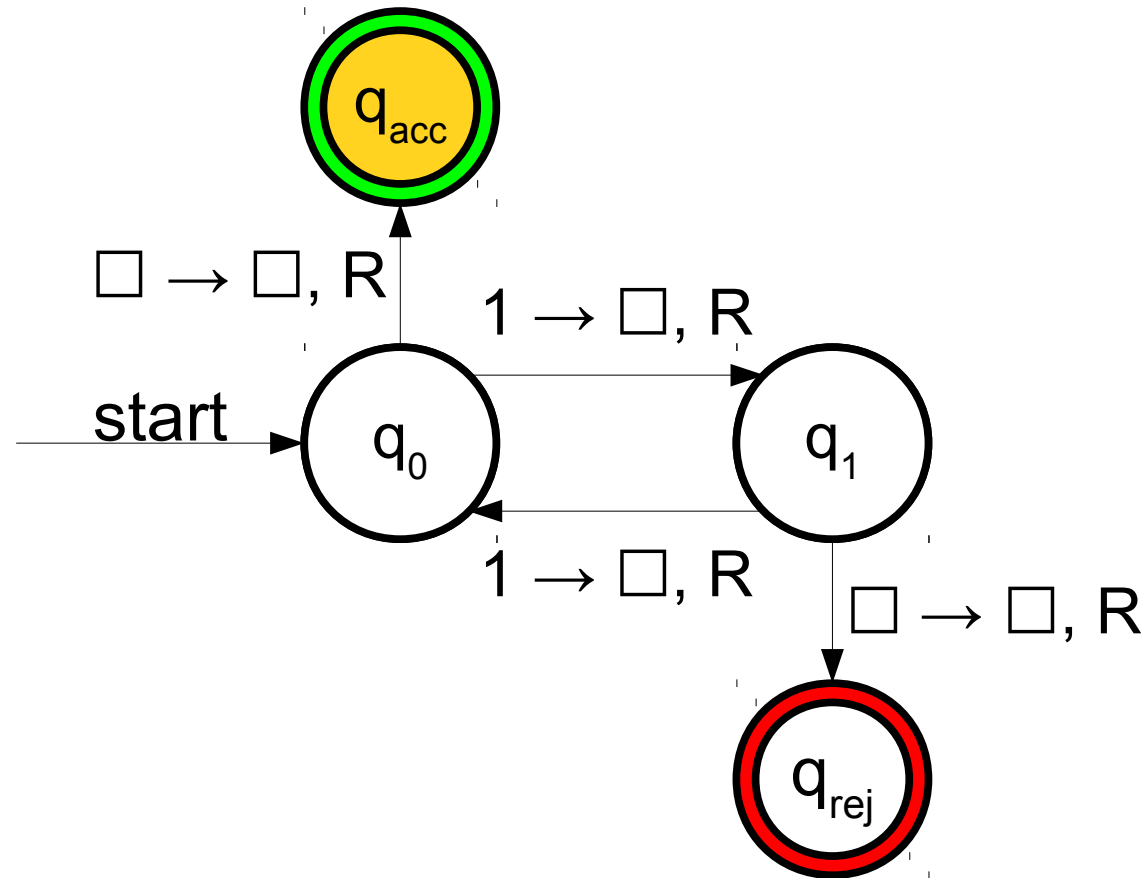
# A Simple Turing Machine



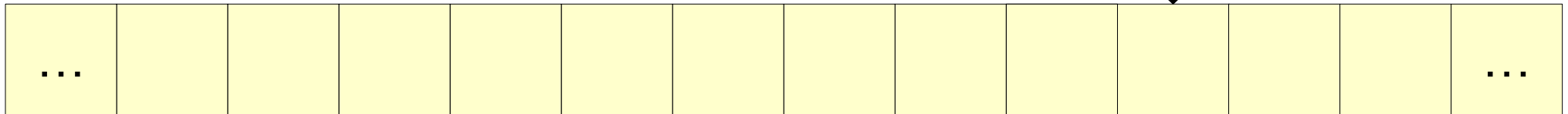
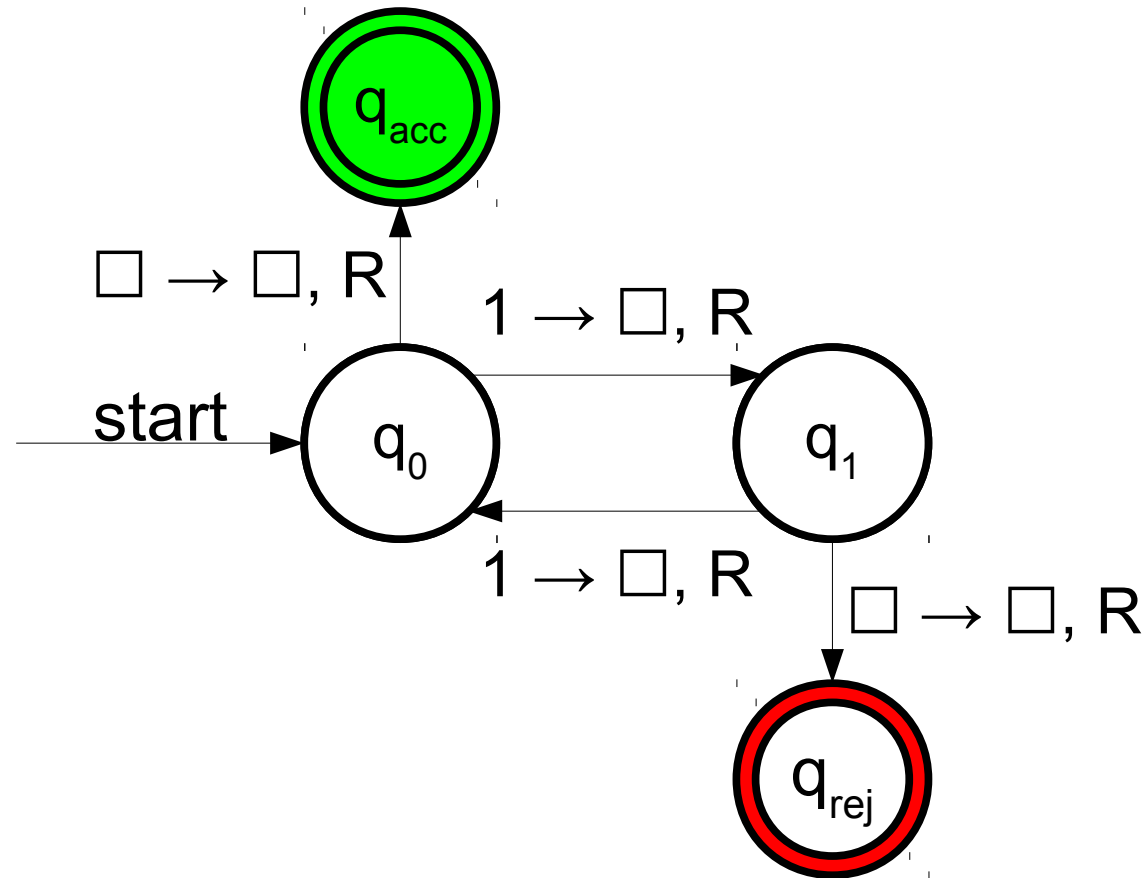
# A Simple Turing Machine



# A Simple Turing Machine



# A Simple Turing Machine





# Accepting and Rejecting States

- Unlike DFAs, Turing machines do not stop processing the input when they finish reading it.
- Turing machines decide when (and if!) they will accept or reject their input.
- Turing machines can enter infinite loops and never accept or reject; more on that later...

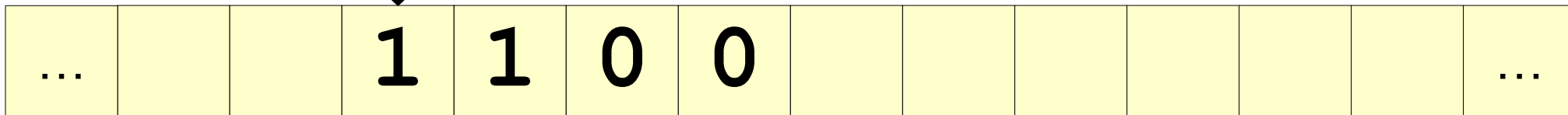
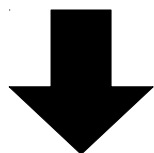
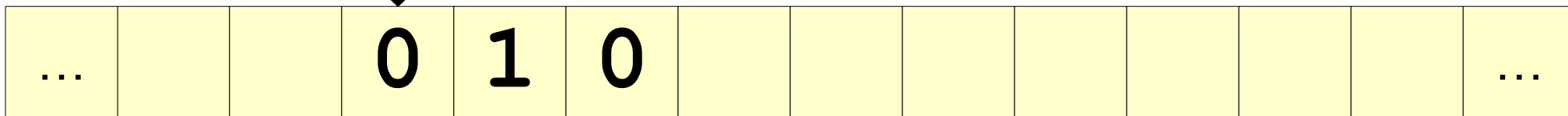
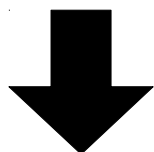
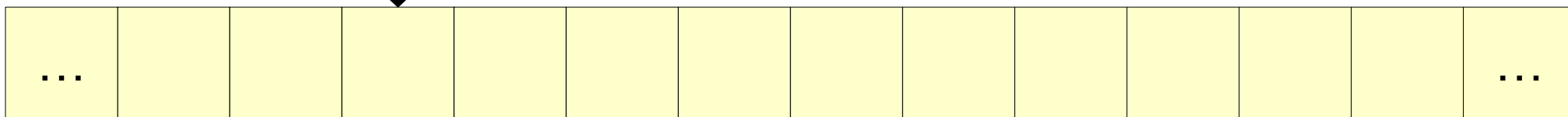
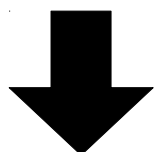
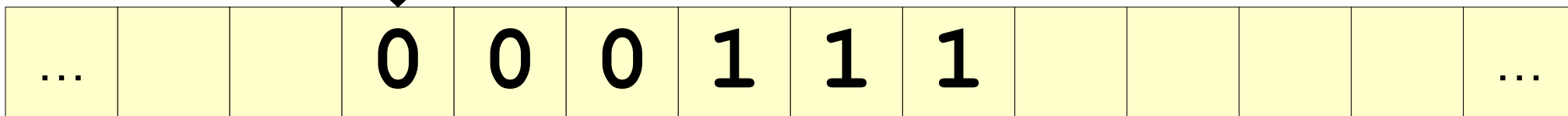
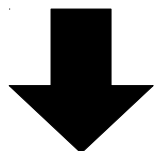
# Designing Turing Machines

- Despite their simplicity, Turing machines are very powerful computing devices.
- Today's lecture explores how to design Turing machines for various languages.

# Designing Turing Machines

- Let  $\Sigma = \{0, 1\}$  and consider the language  $L = \{0^n 1^n \mid n \in \mathbb{N}\}$ .
- We know that  $L$  is context-free.
- How might we build a Turing machine for it?

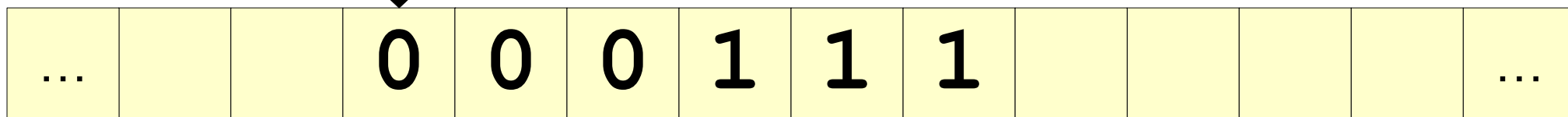
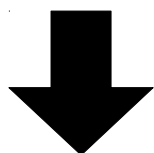
$$L = \{ \mathbf{0}^n \mathbf{1}^n \mid n \in \mathbb{N} \}$$



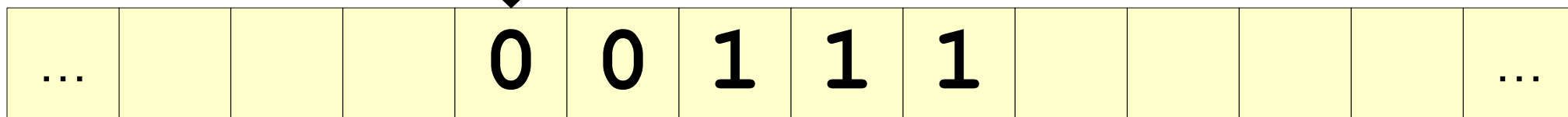
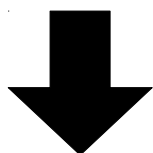
# A Recursive Approach

- The string  $\varepsilon$  is in  $L$ .
- The string  $0w1$  is in  $L$  iff  $w$  is in  $L$ .
- Any string starting with  $1$  is not in  $L$ .
- Any string ending with  $0$  is not in  $L$ .

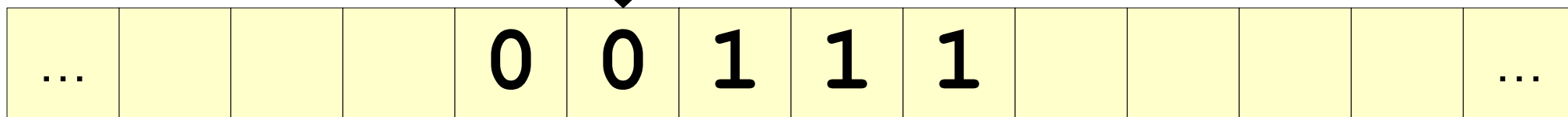
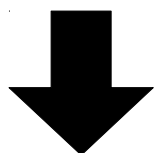
# A Sketch of the TM



# A Sketch of the TM

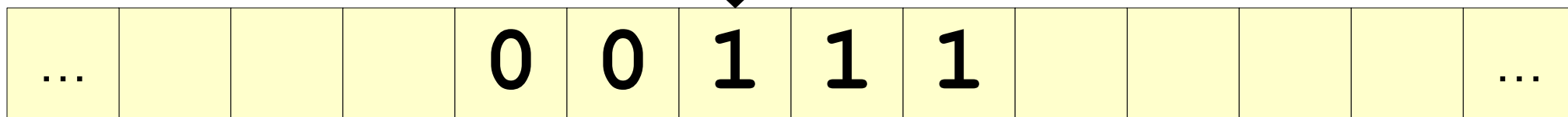
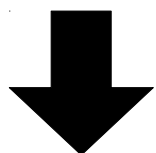


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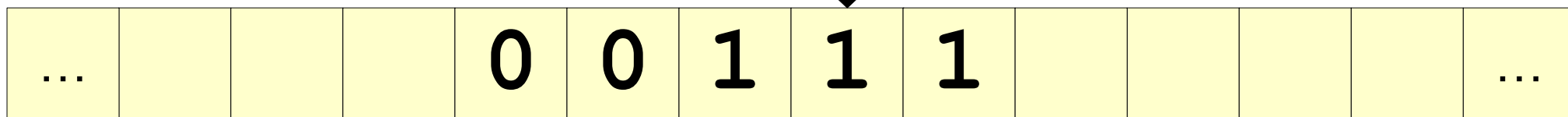
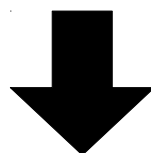




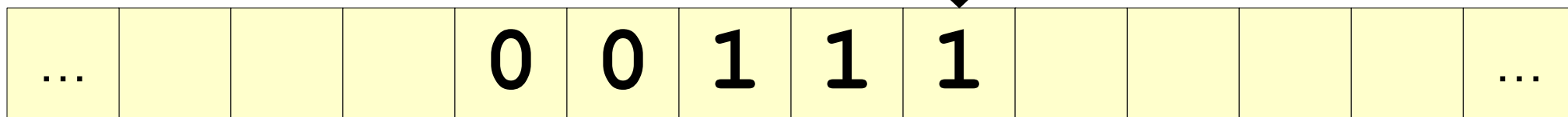
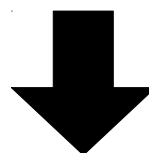
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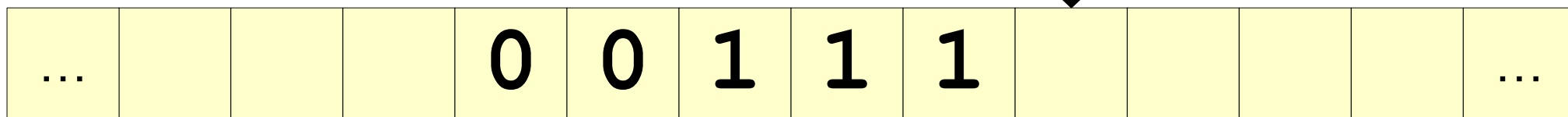
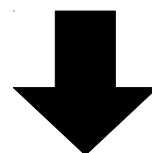
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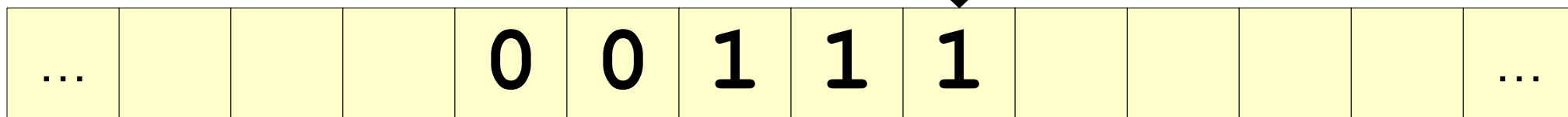
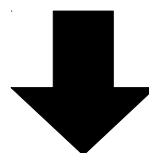
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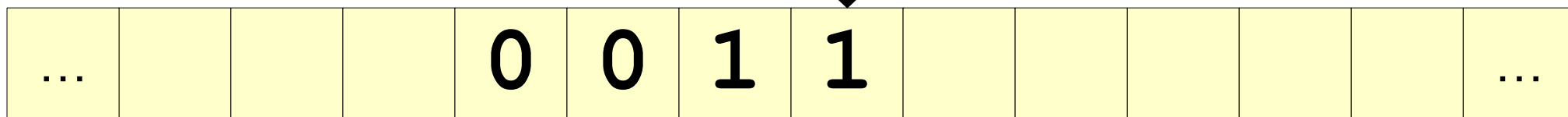
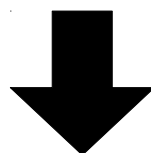
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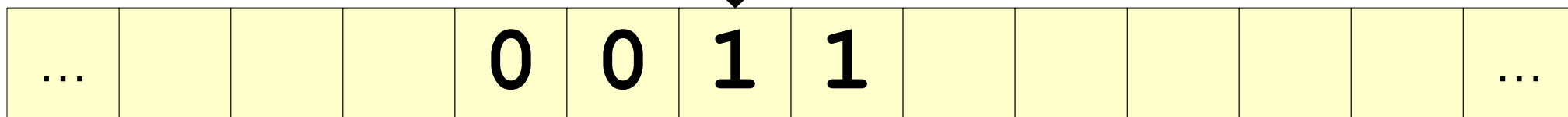
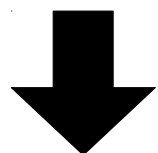
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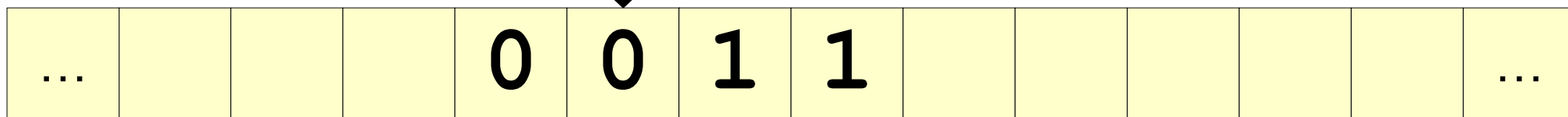
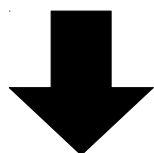
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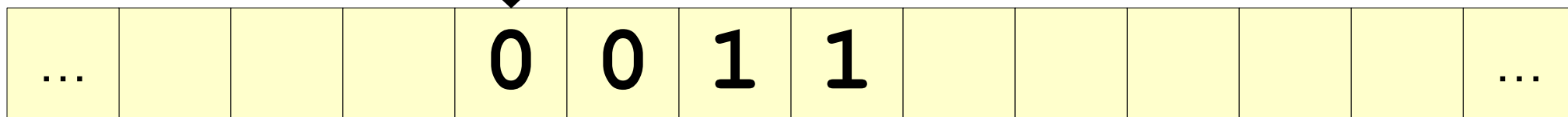
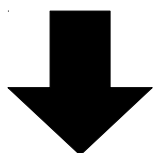


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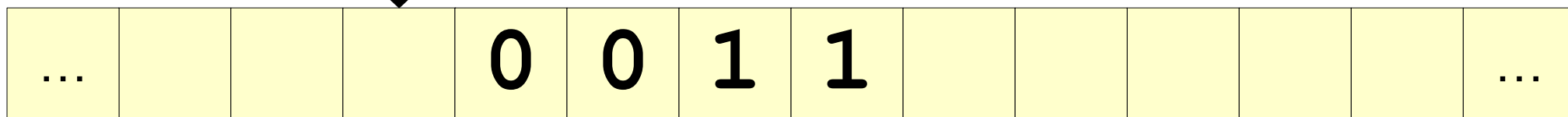
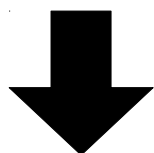




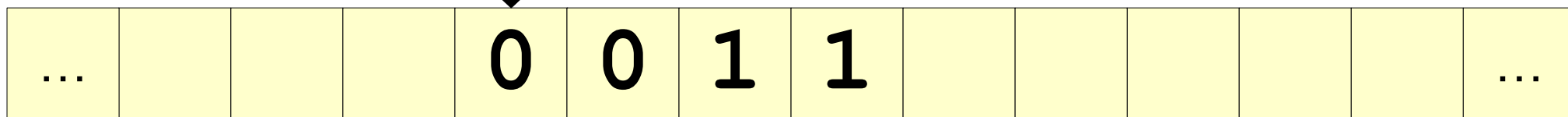
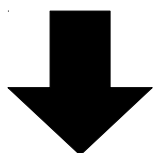
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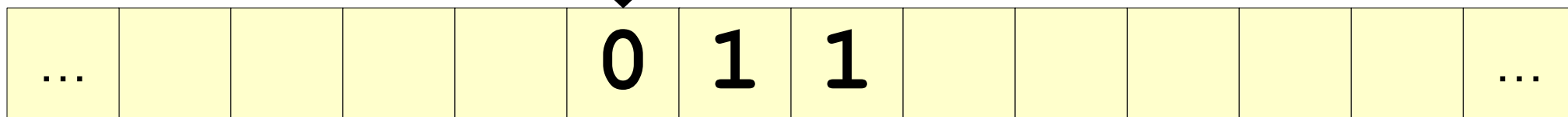
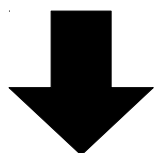
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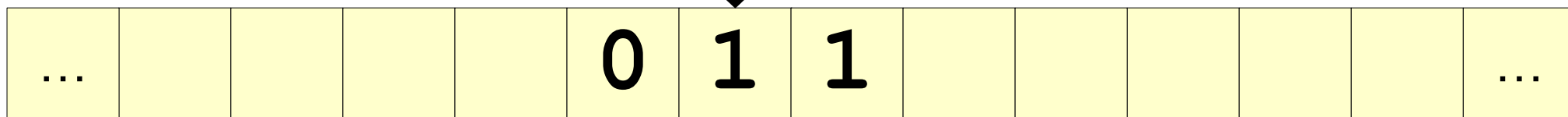
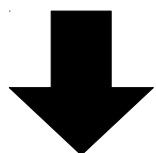
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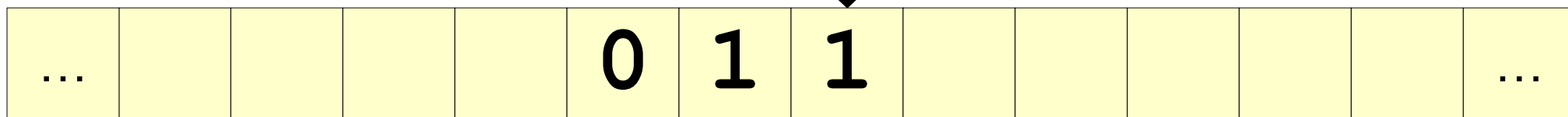
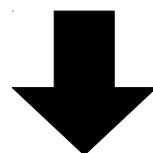
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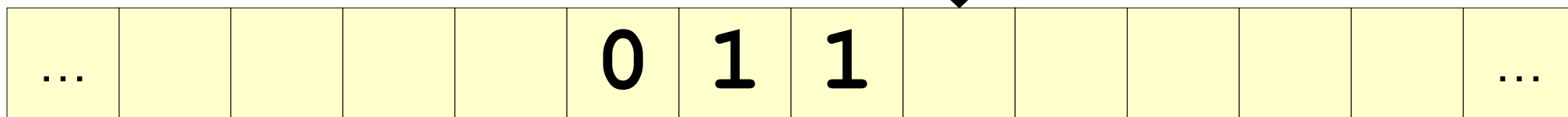
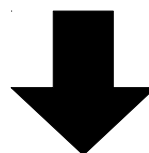
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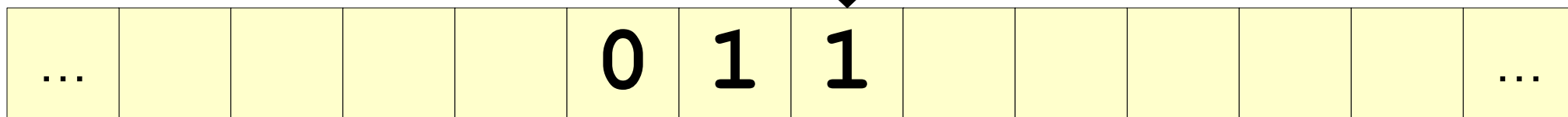
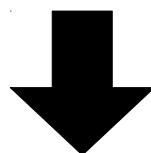
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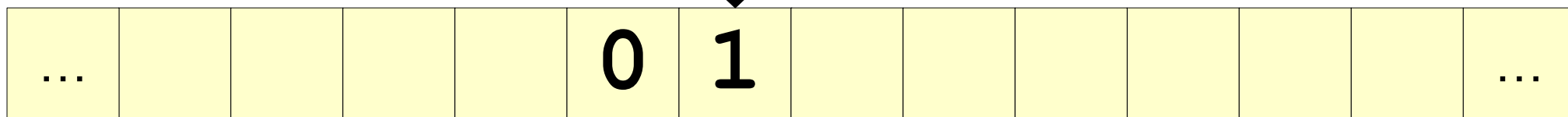
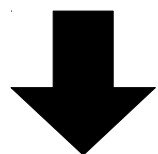


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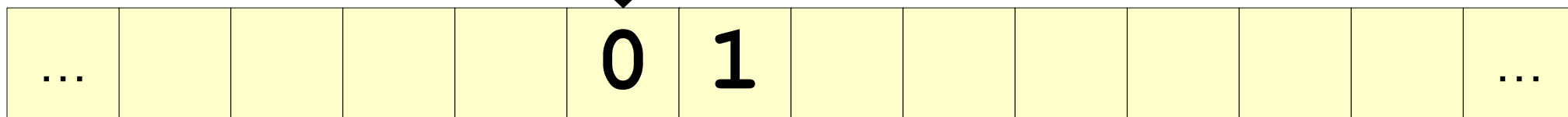
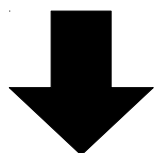




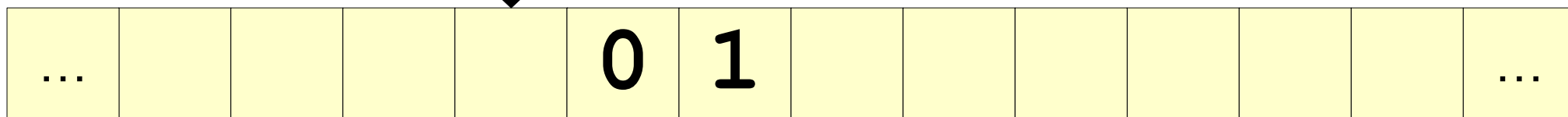
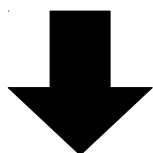
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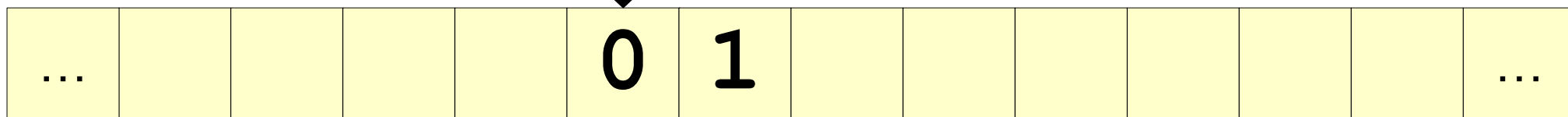
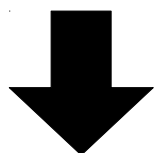
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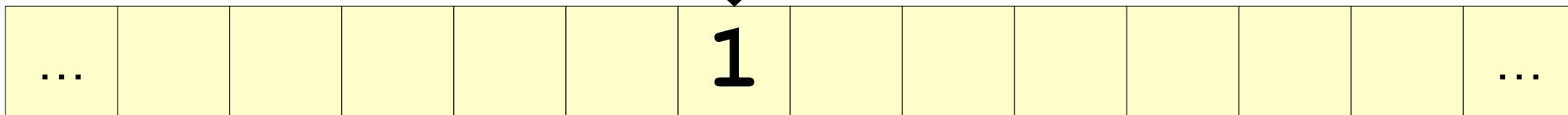
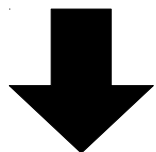
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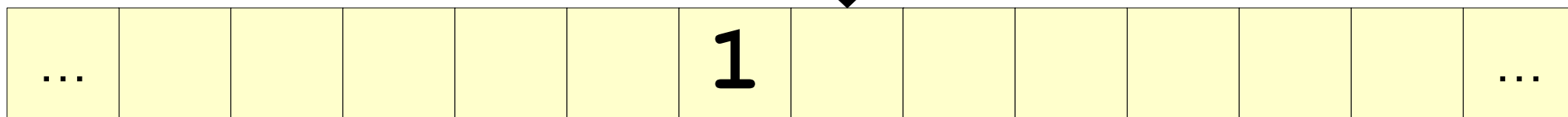
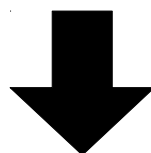
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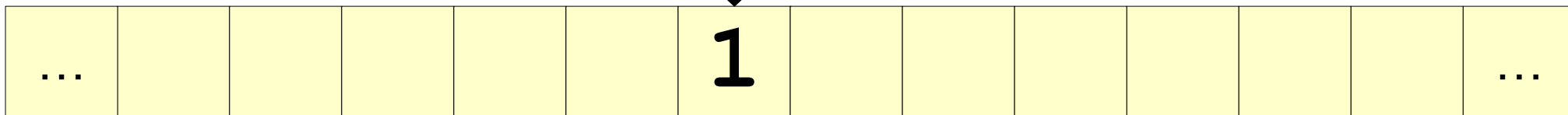
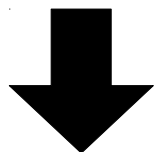
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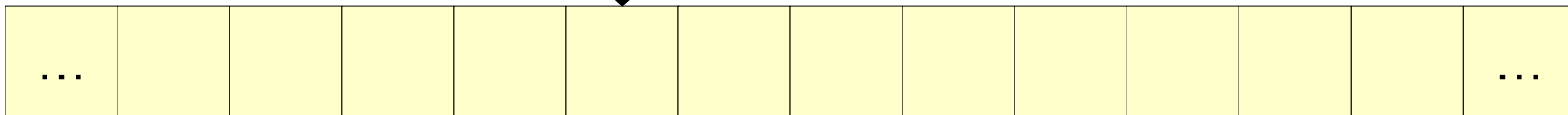
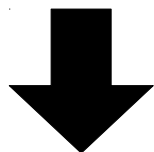
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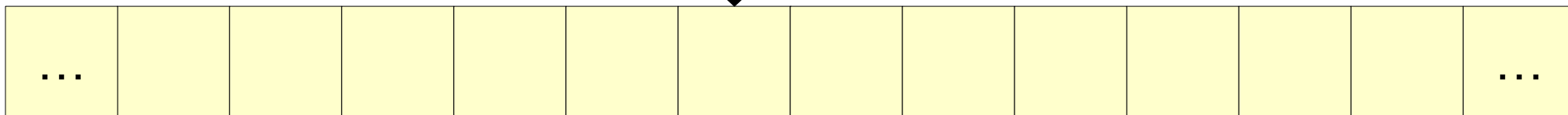
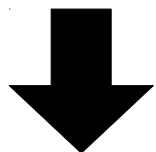


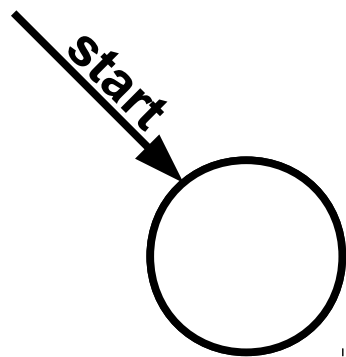
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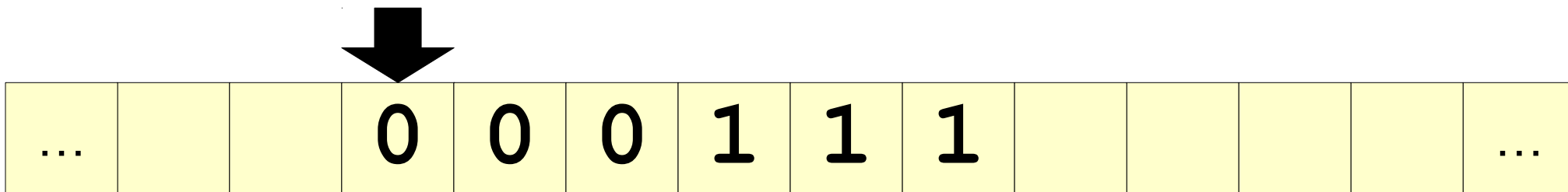
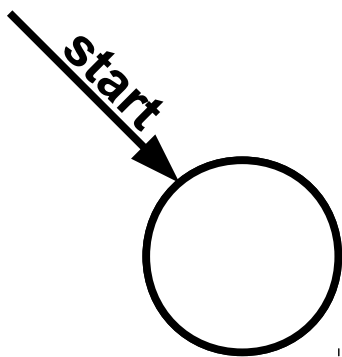


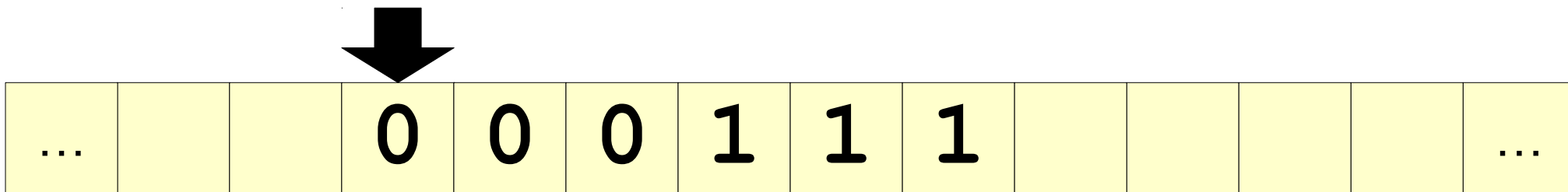
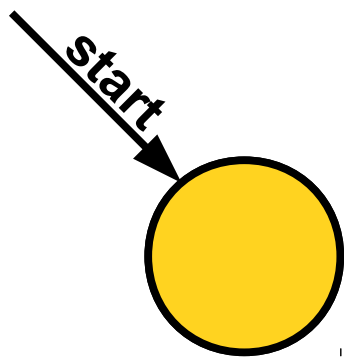


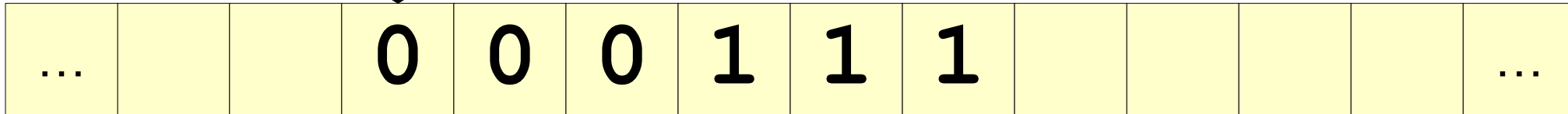
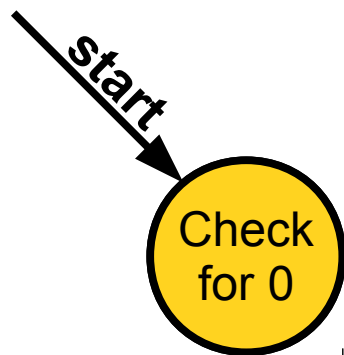
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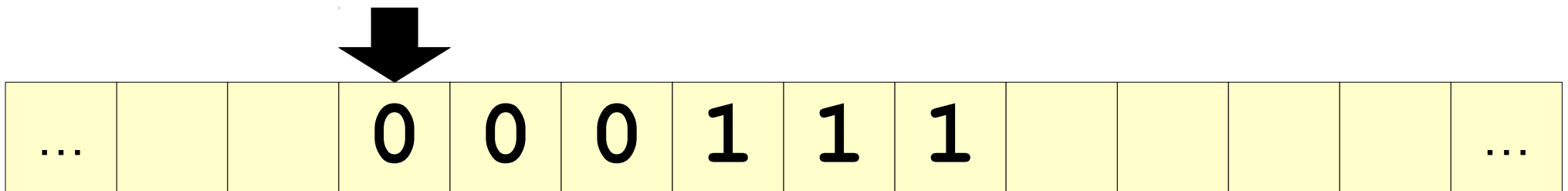
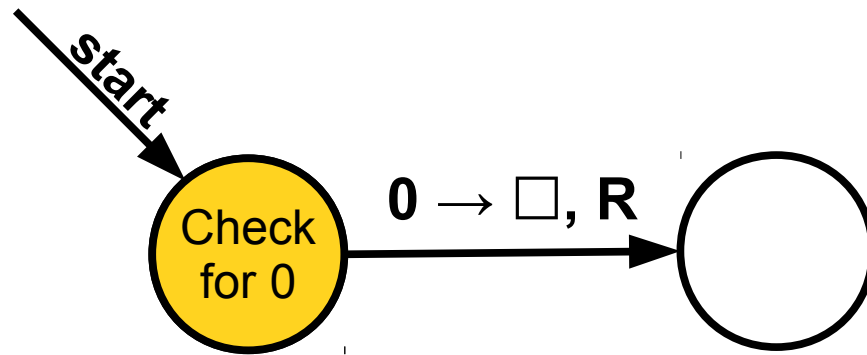


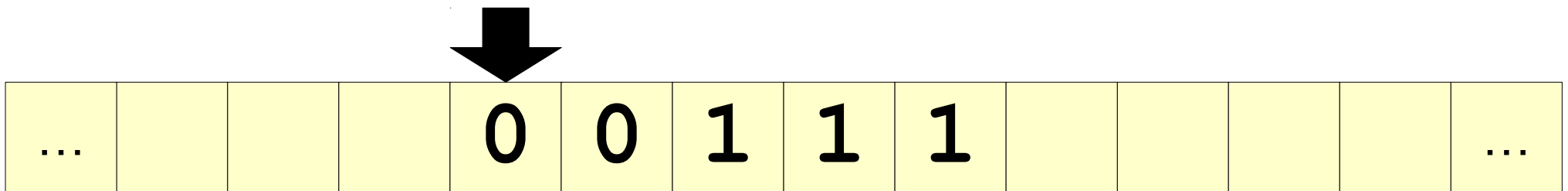
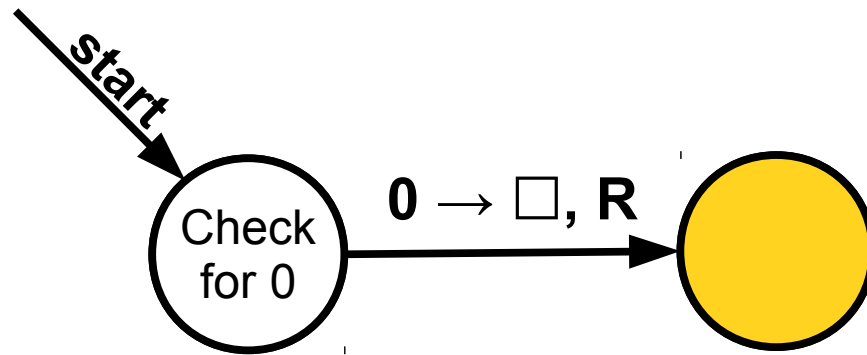


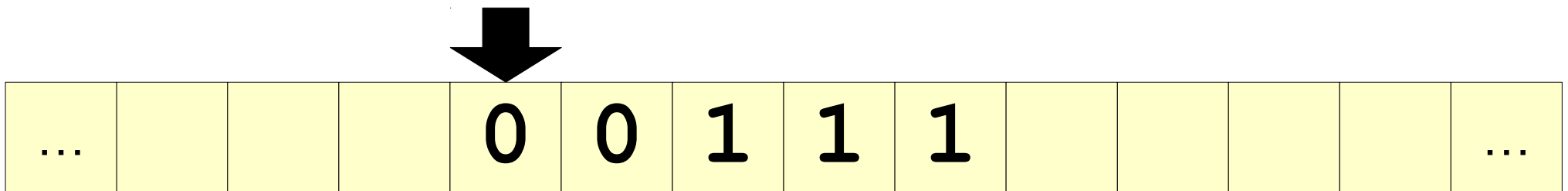
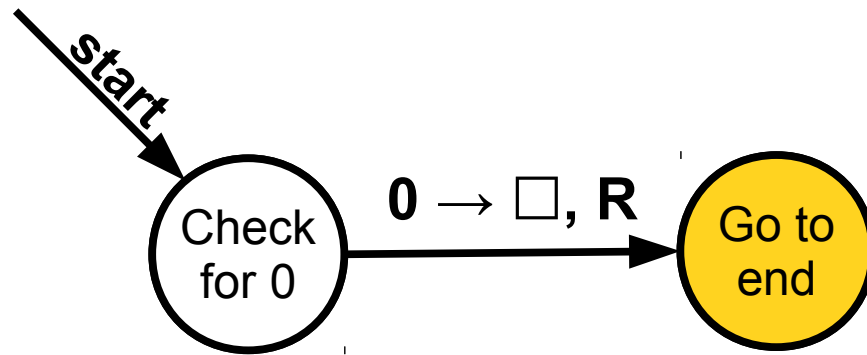




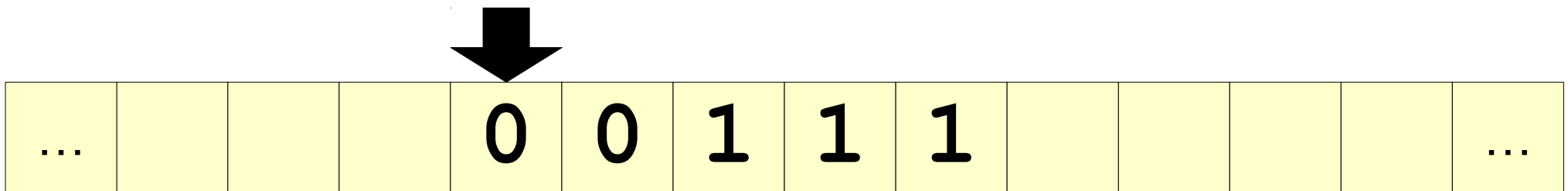
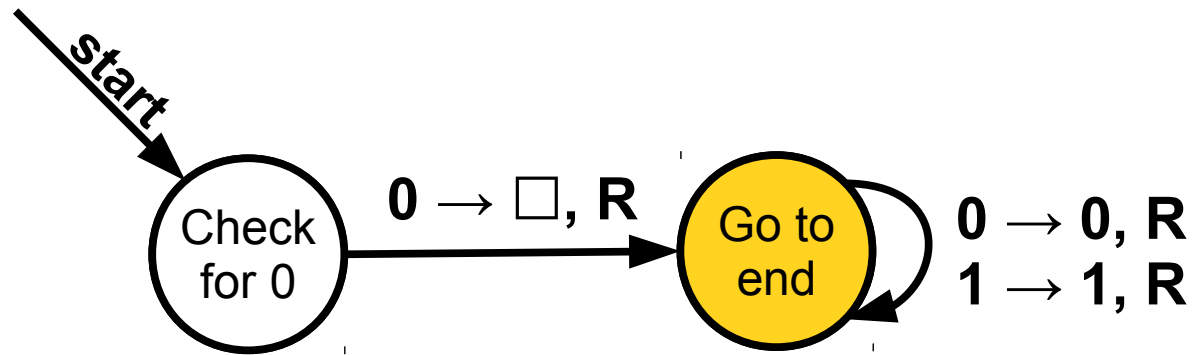


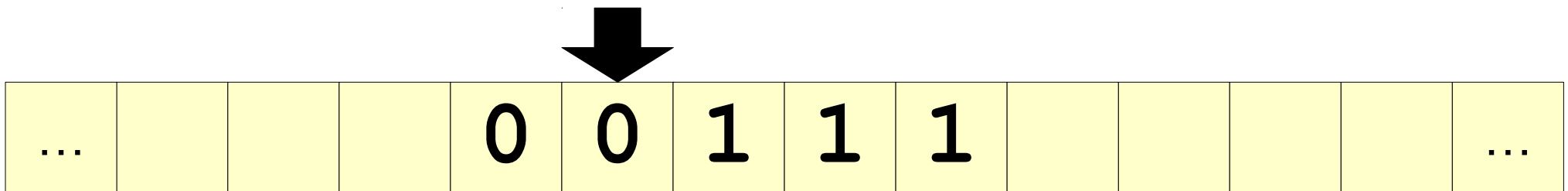
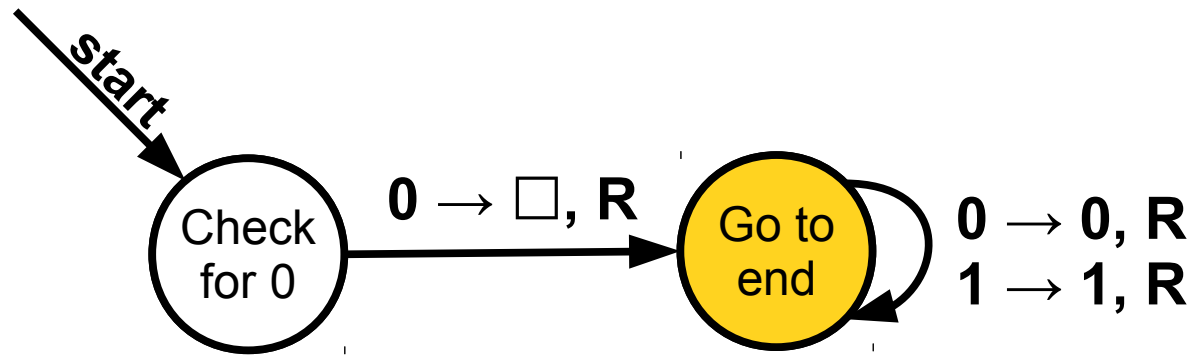


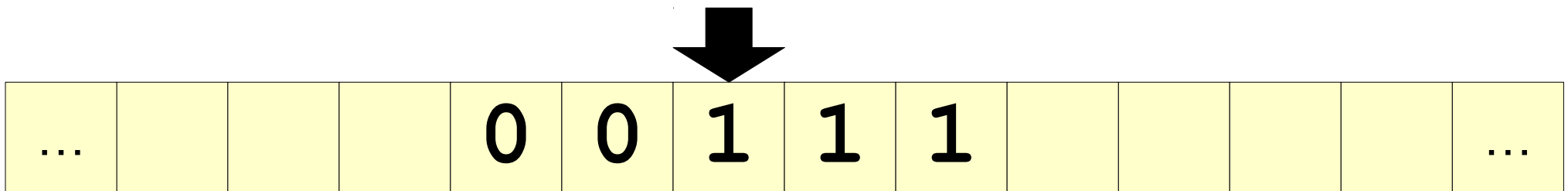
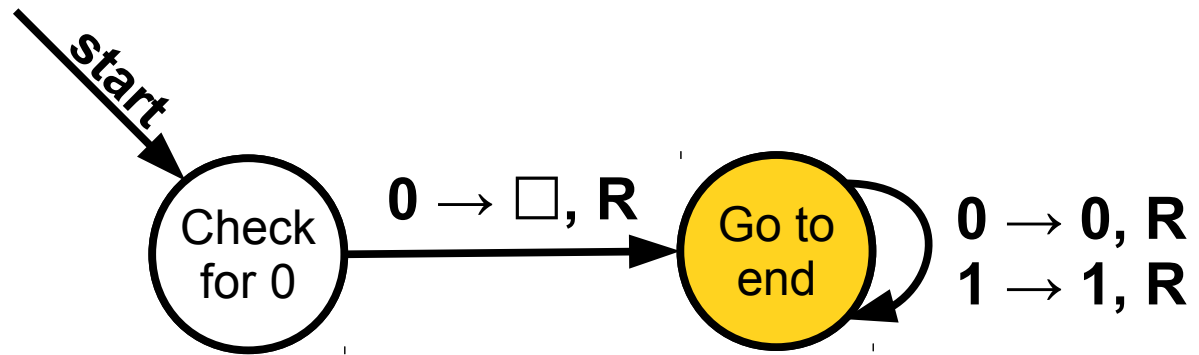


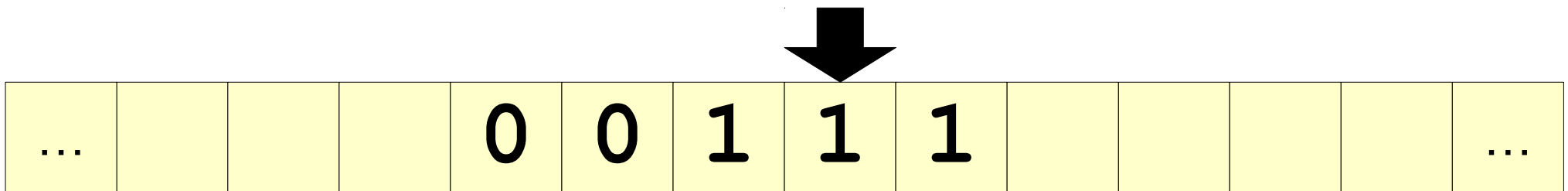
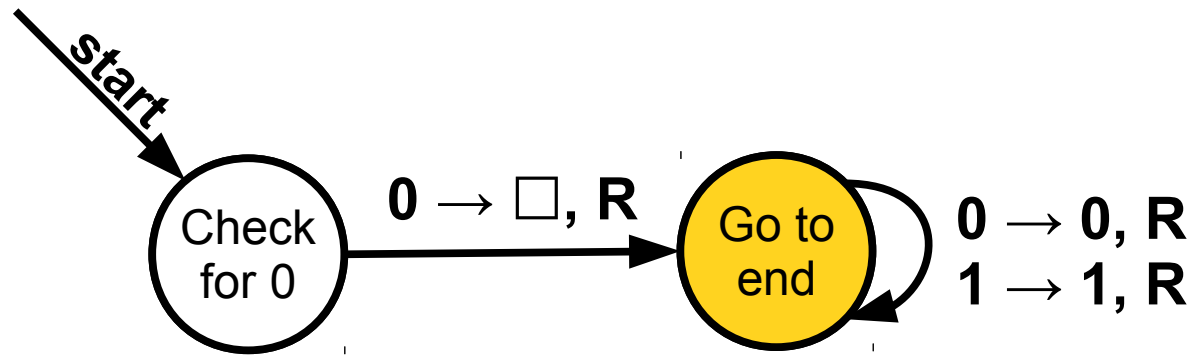


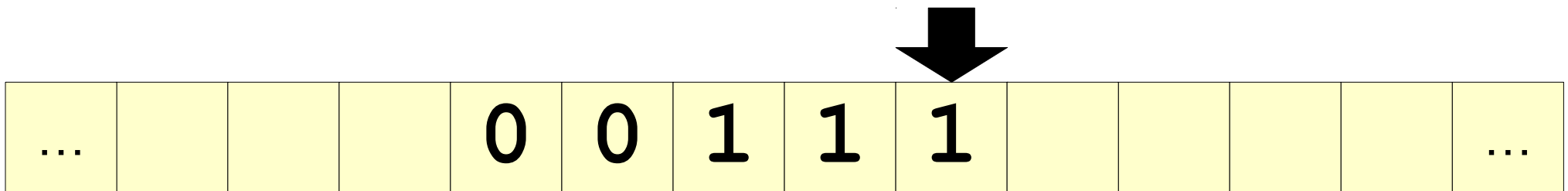
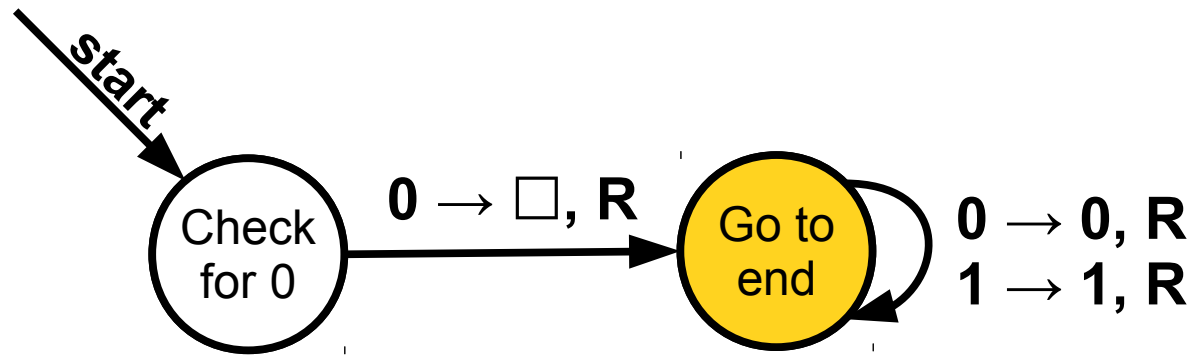


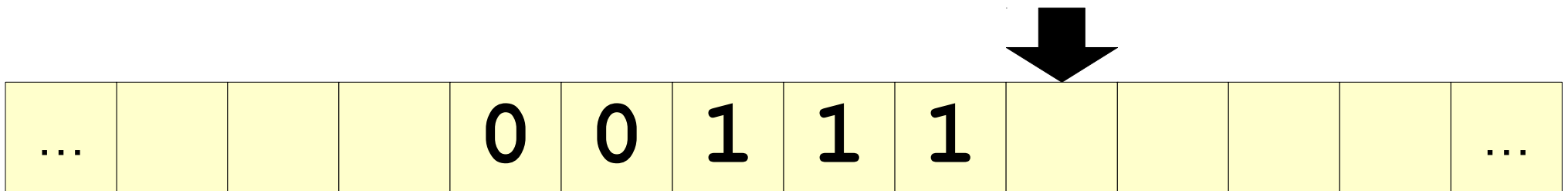
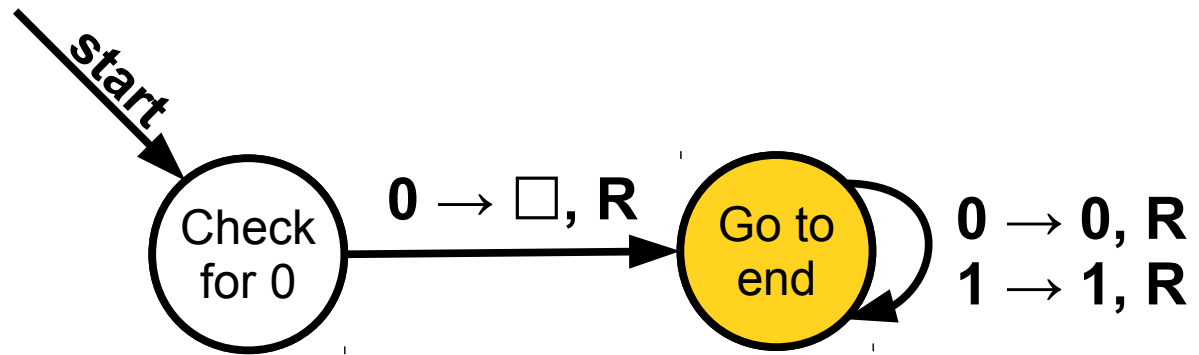


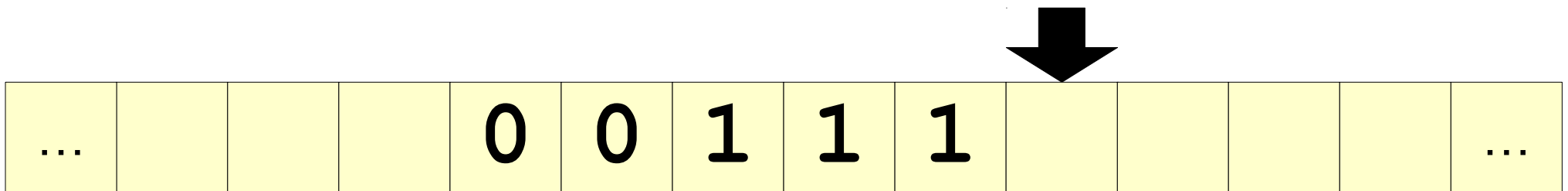
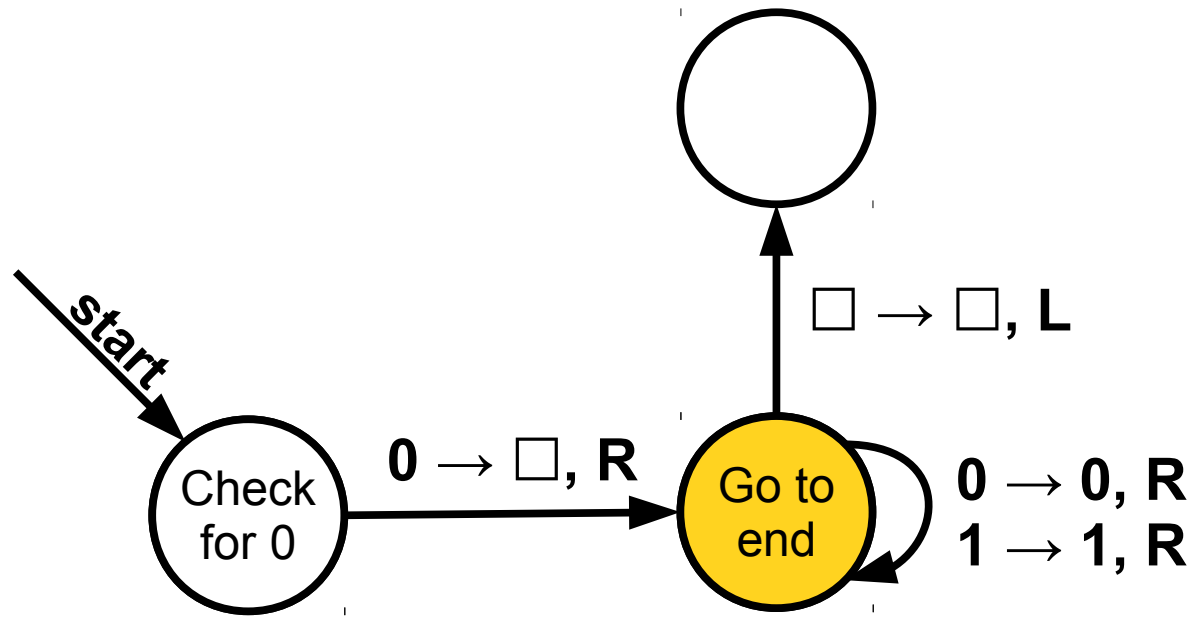


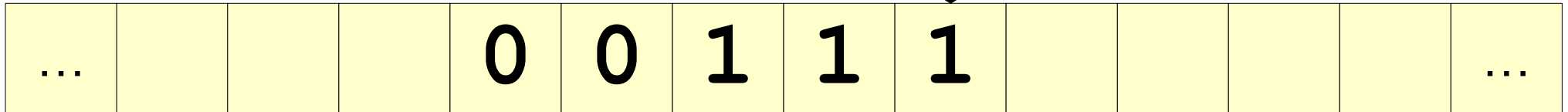
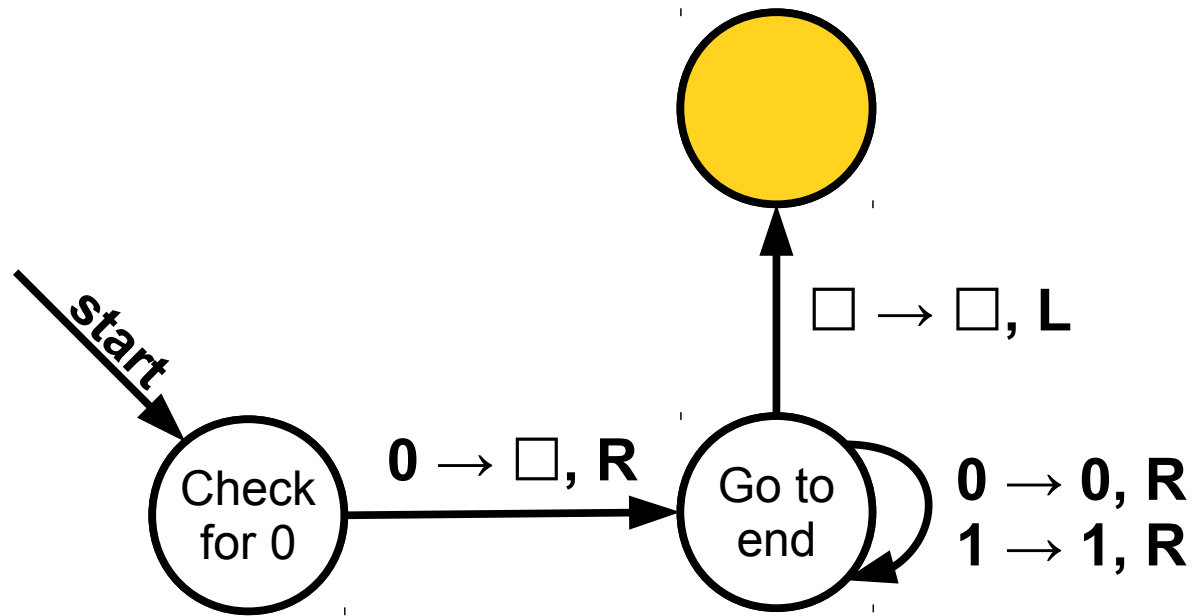




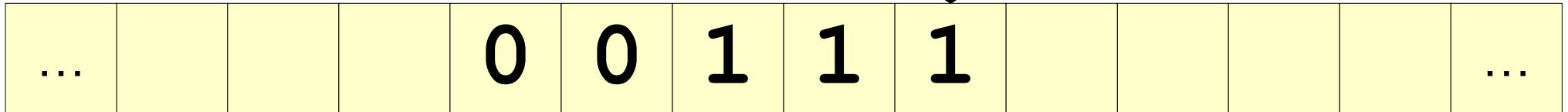
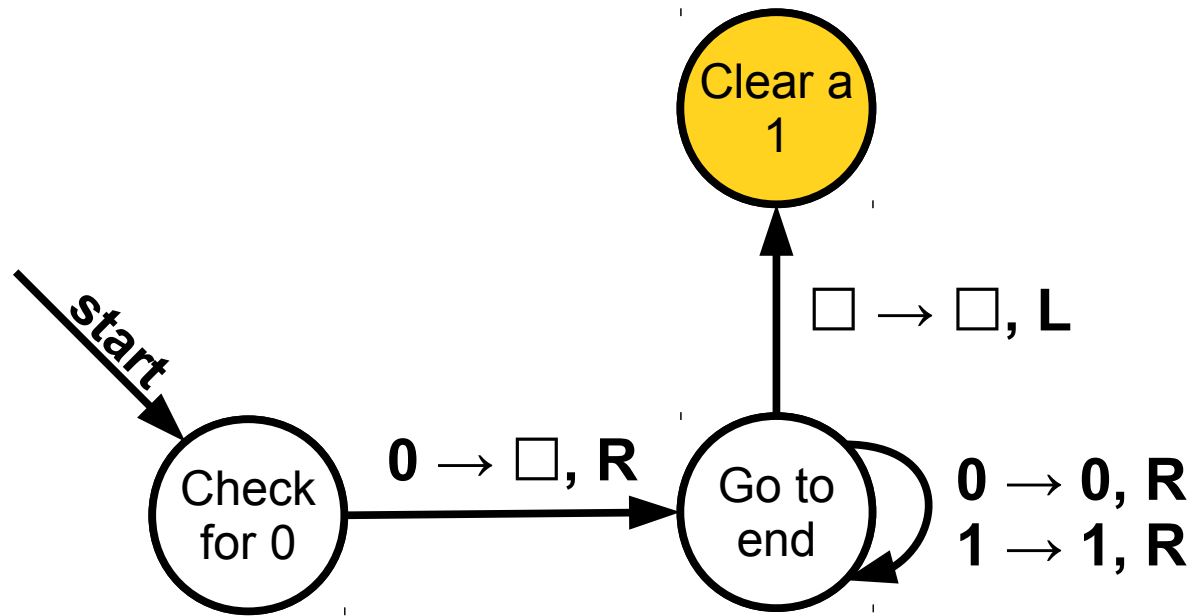


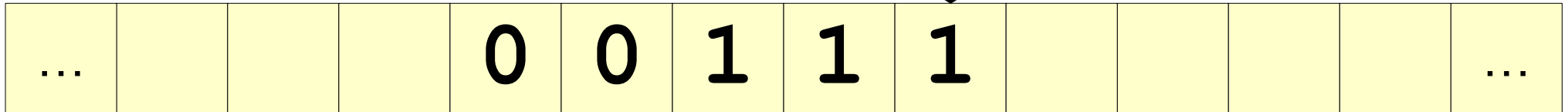
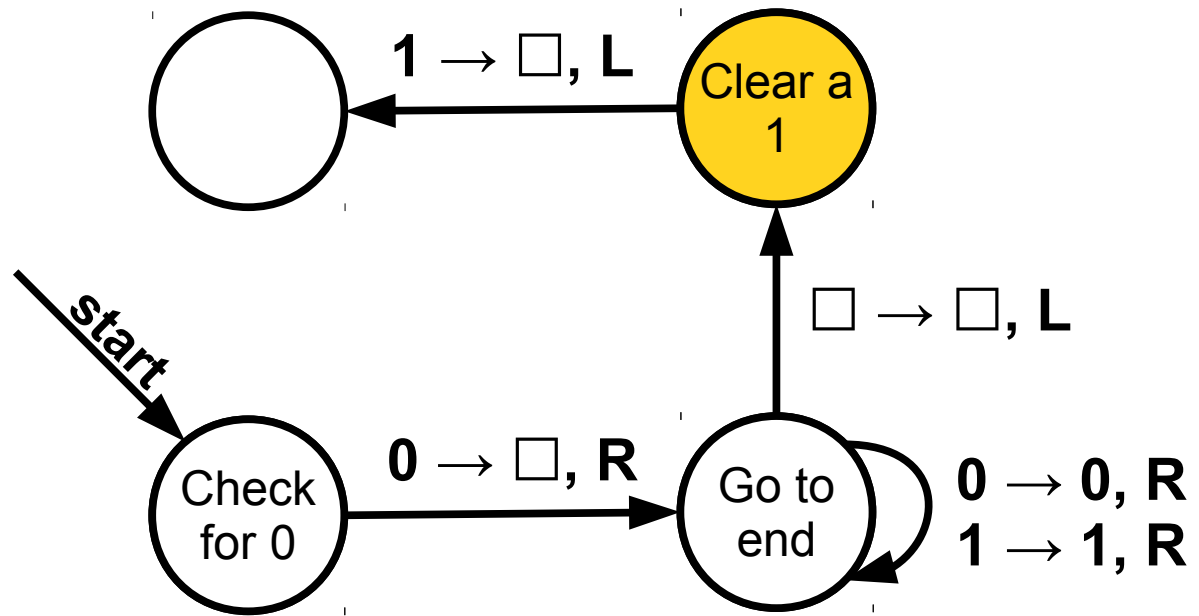


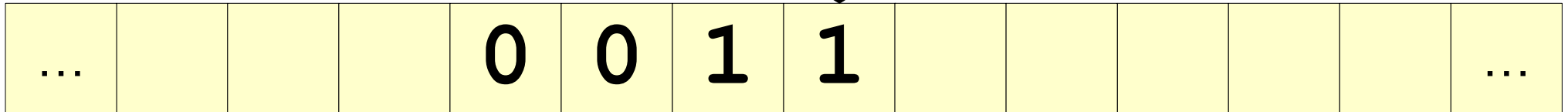
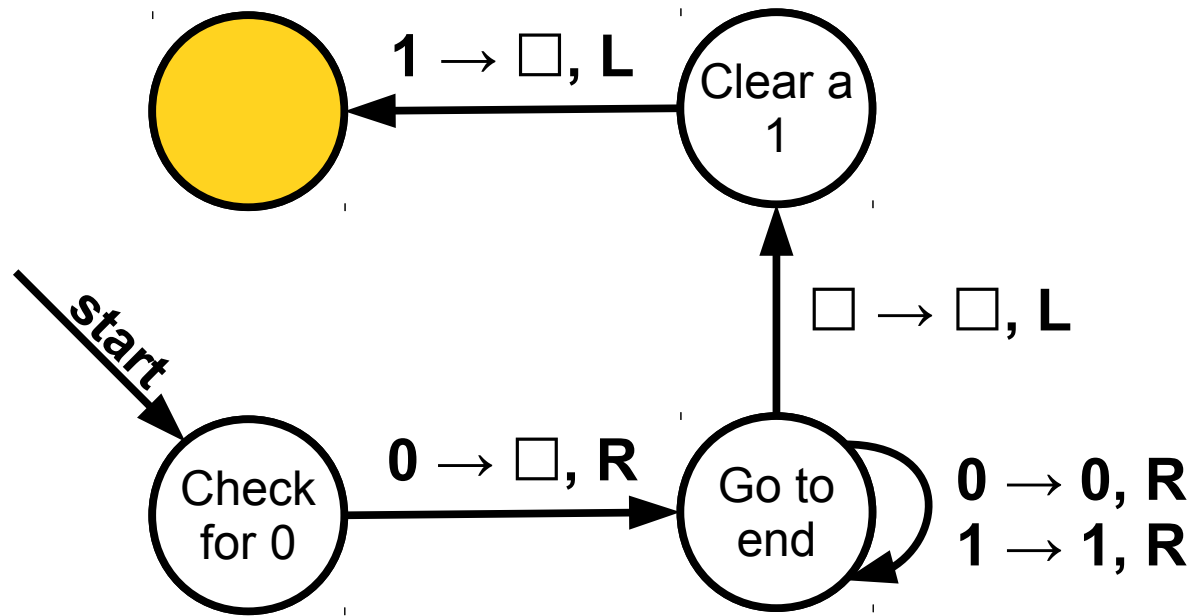


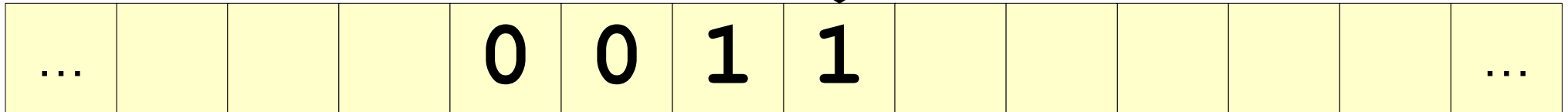
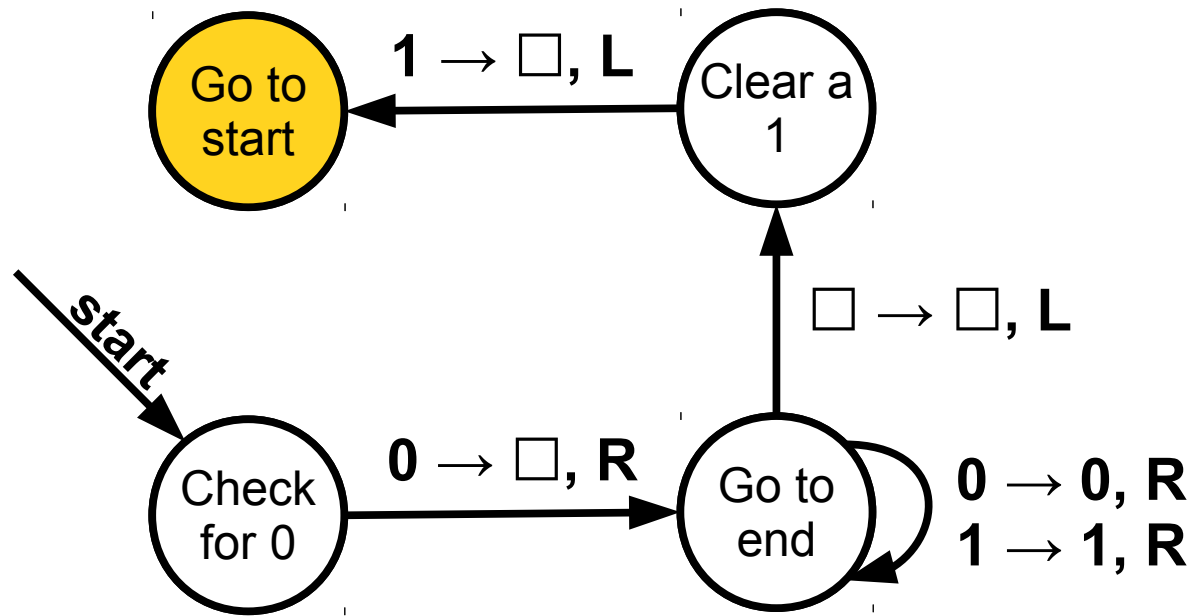


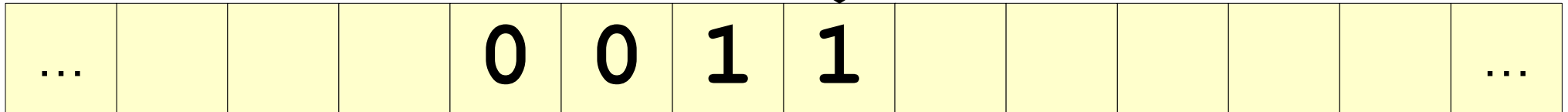
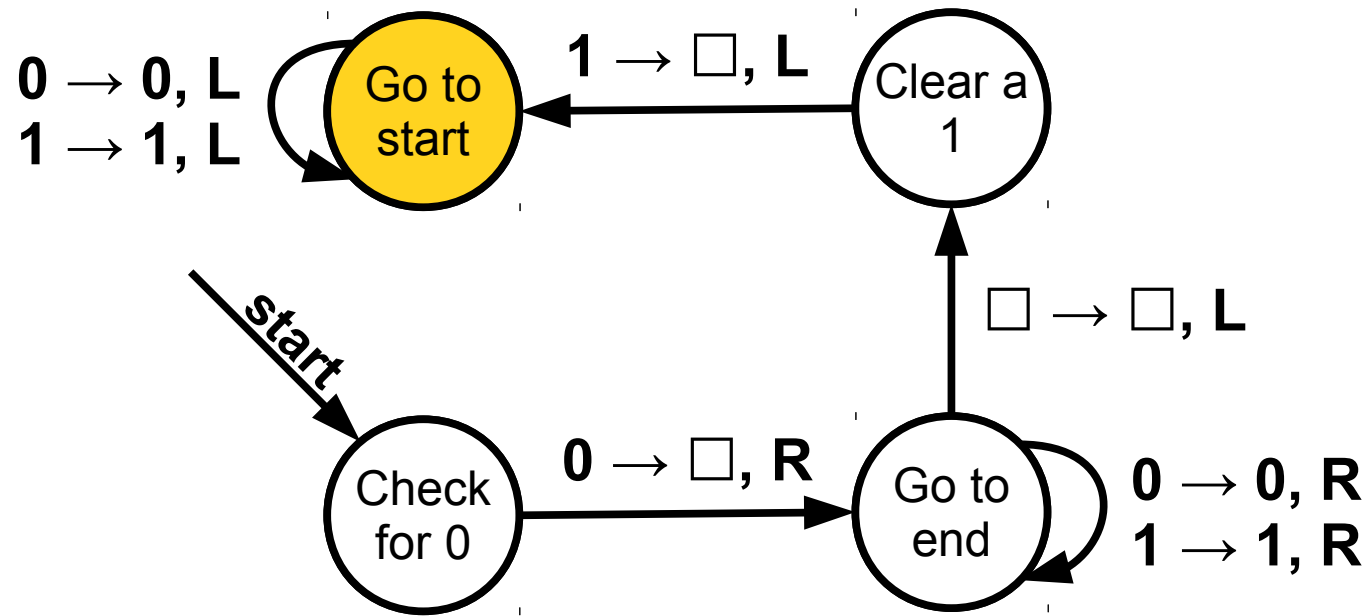


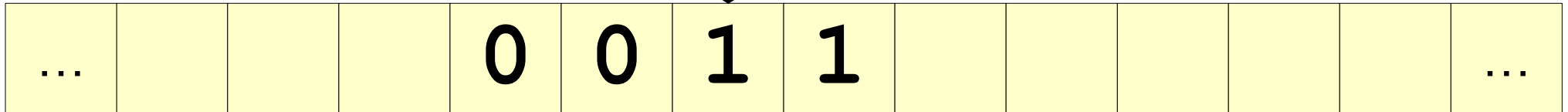
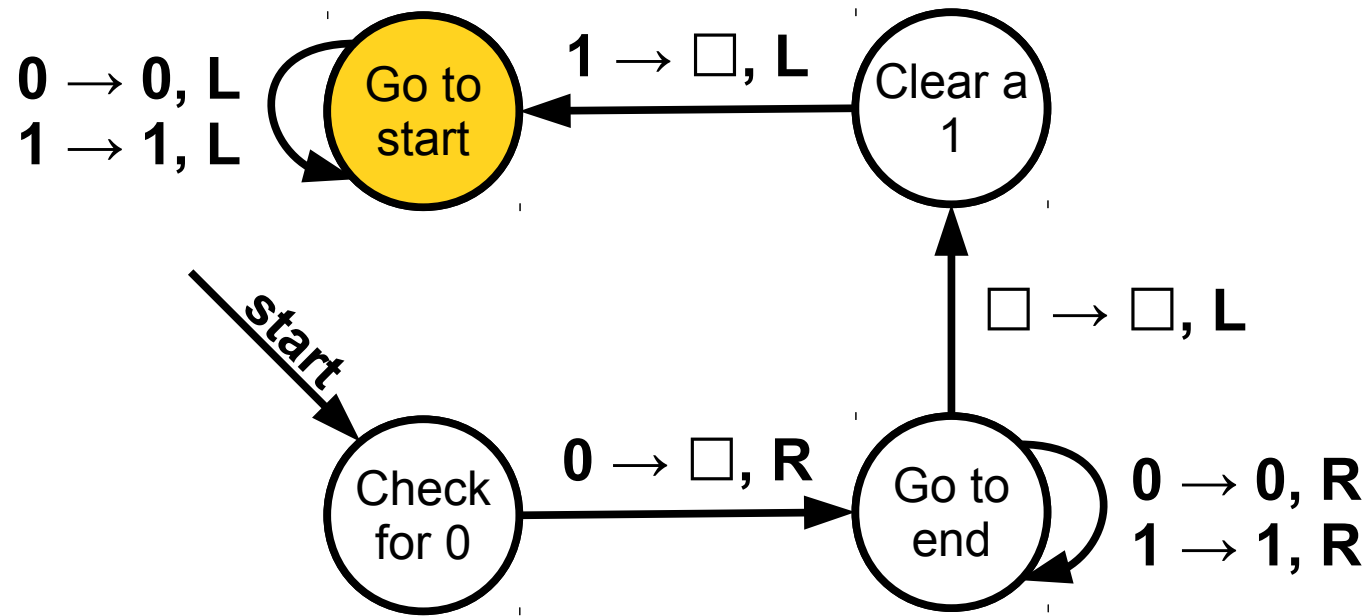


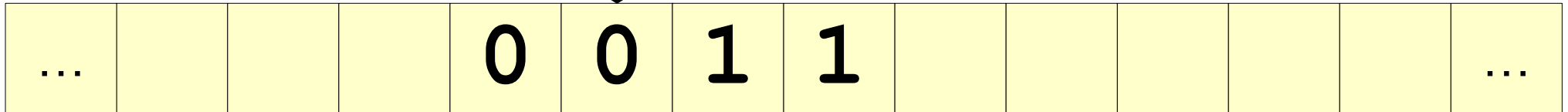
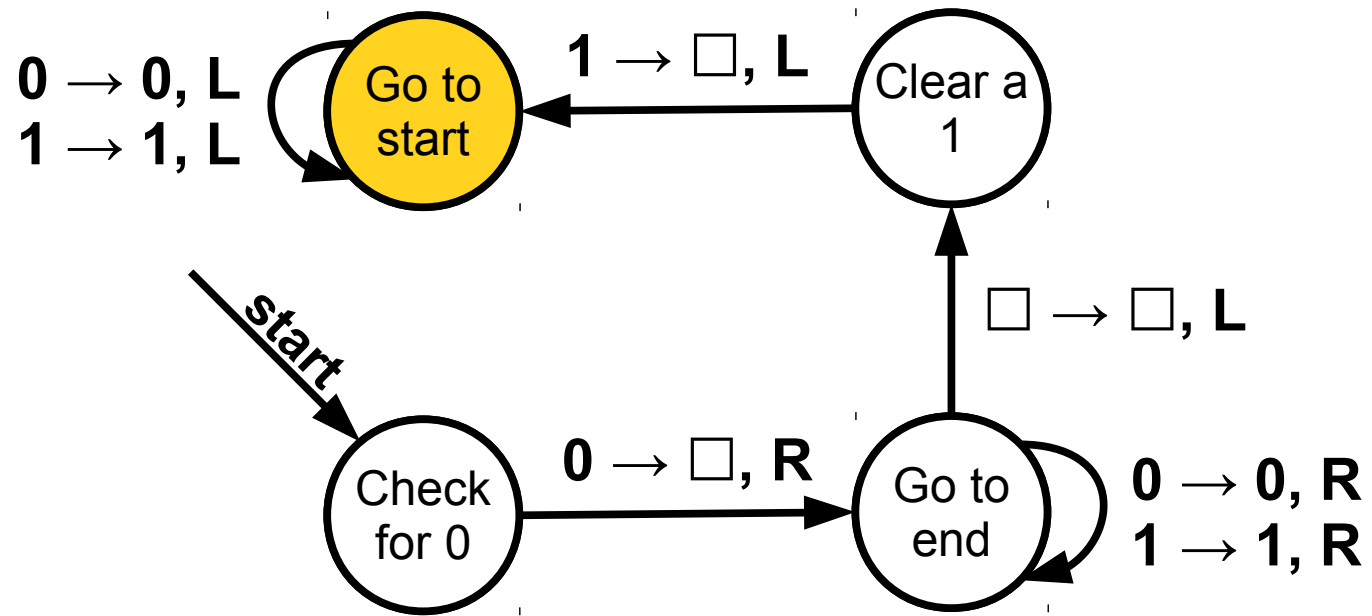


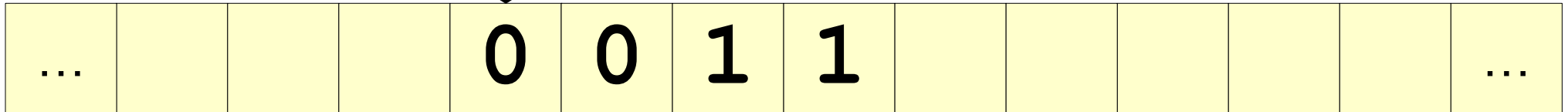
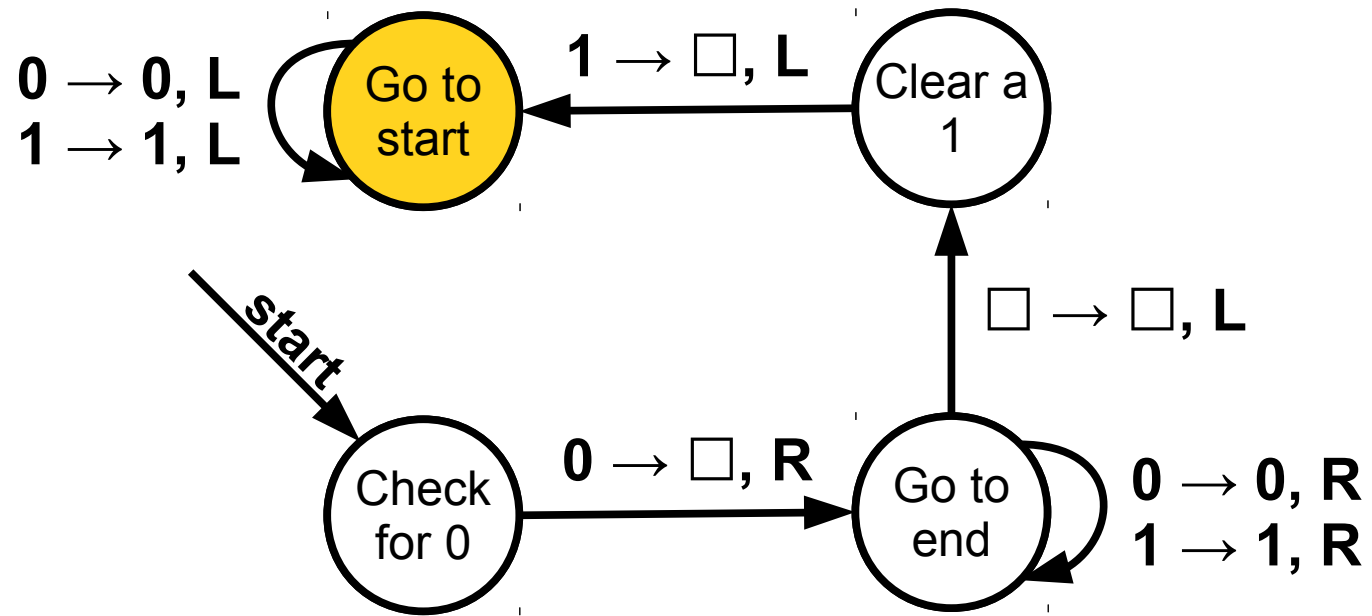




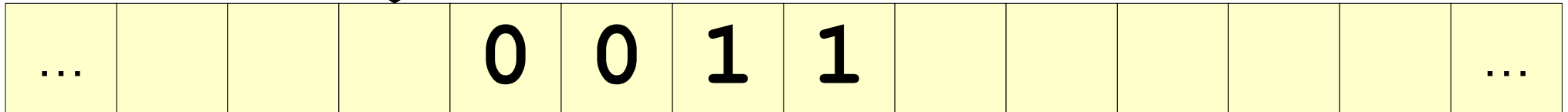
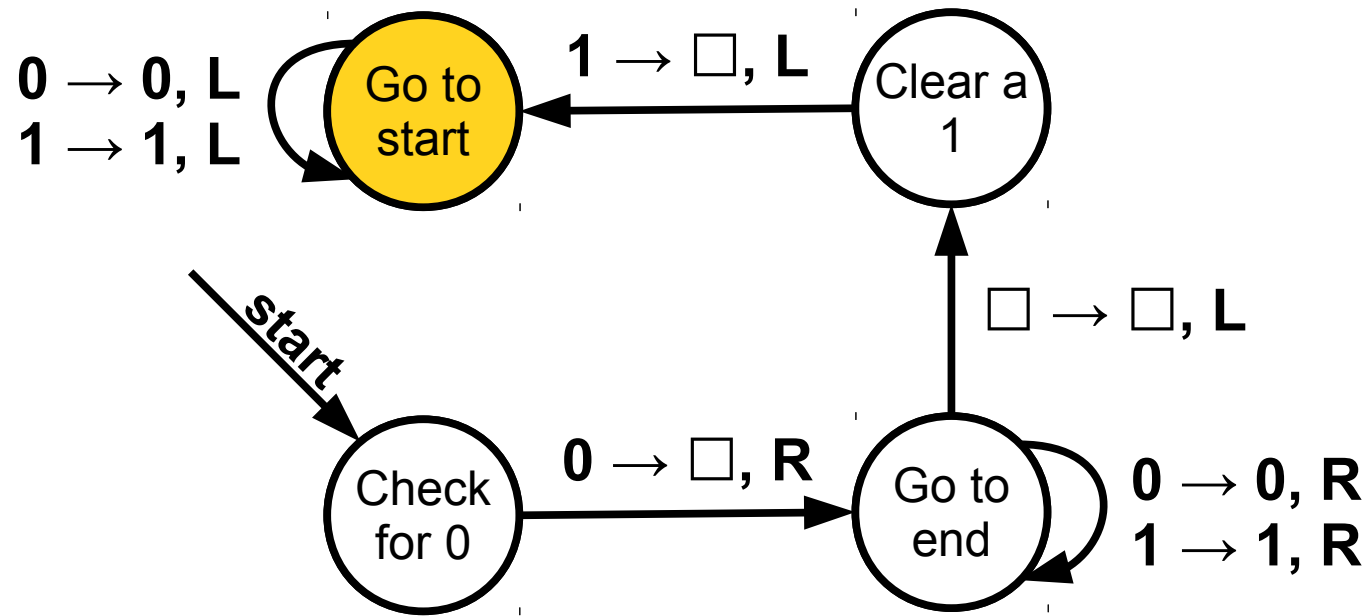


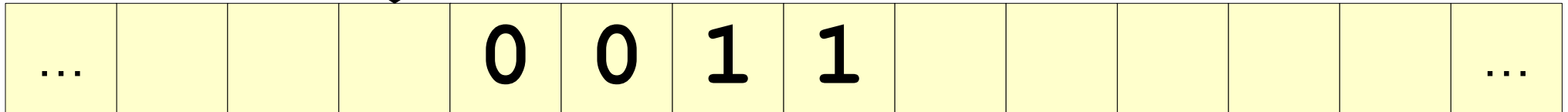
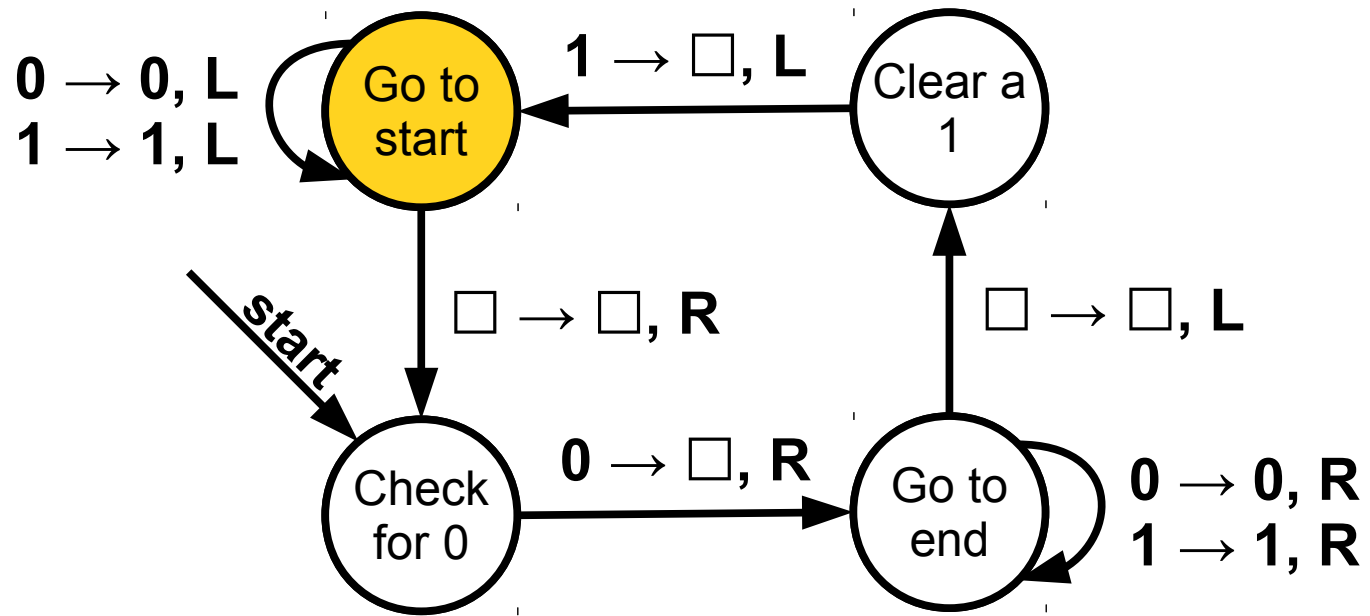


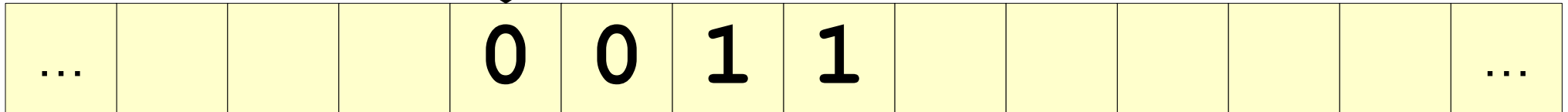
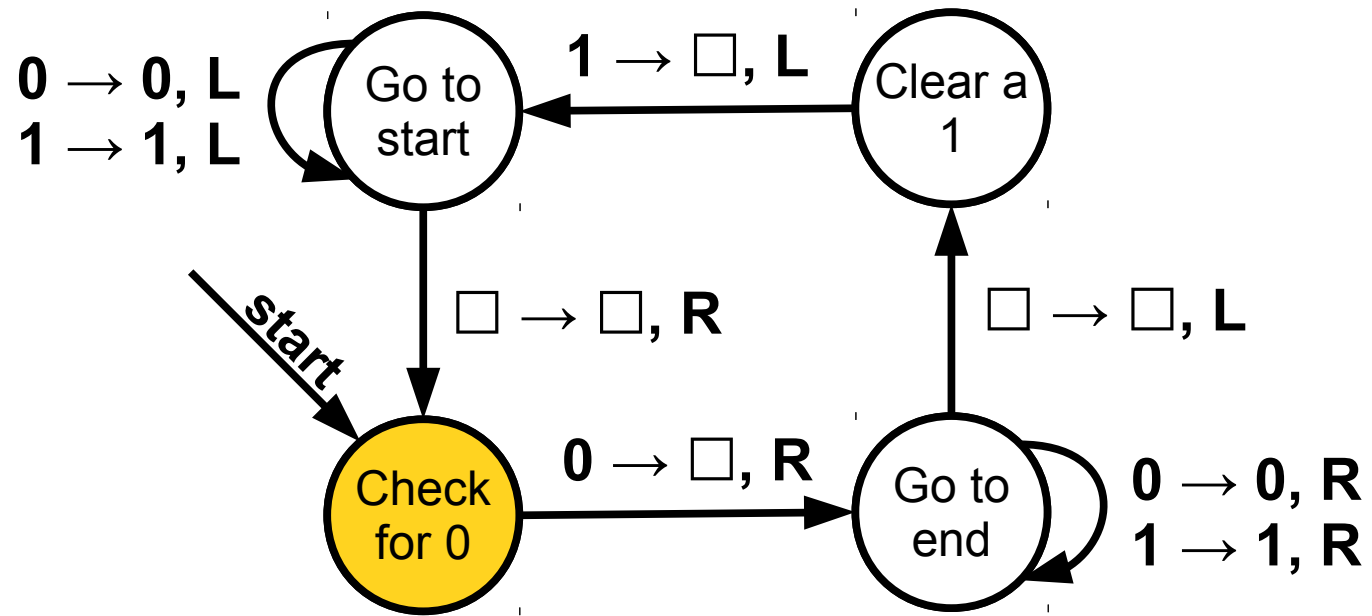


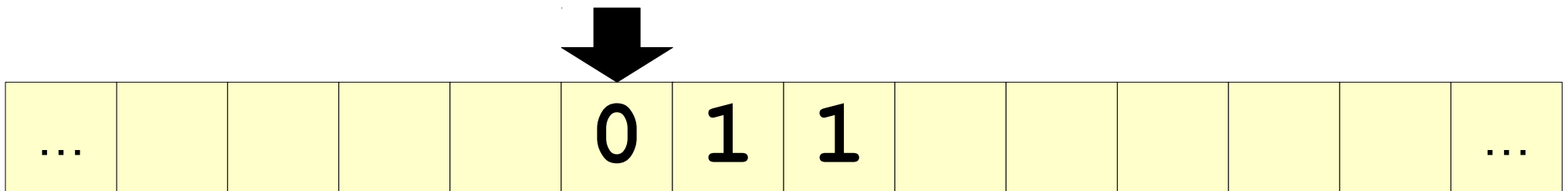
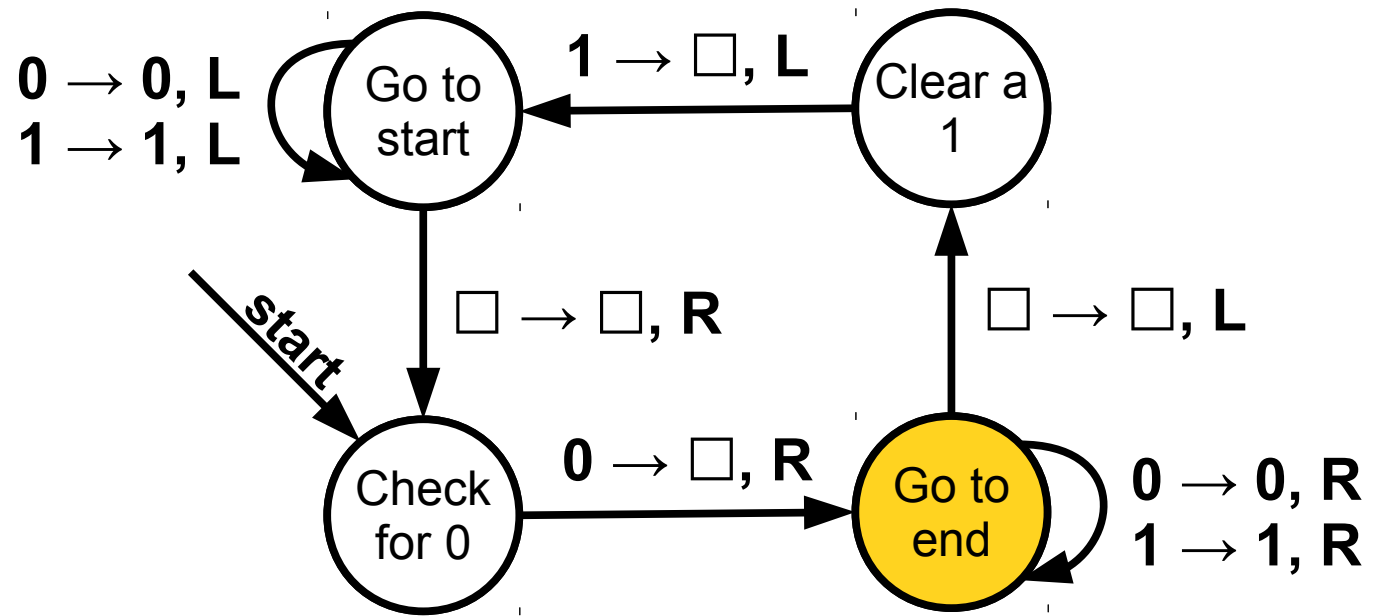


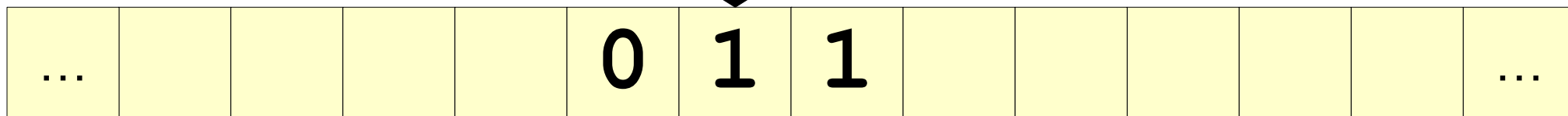
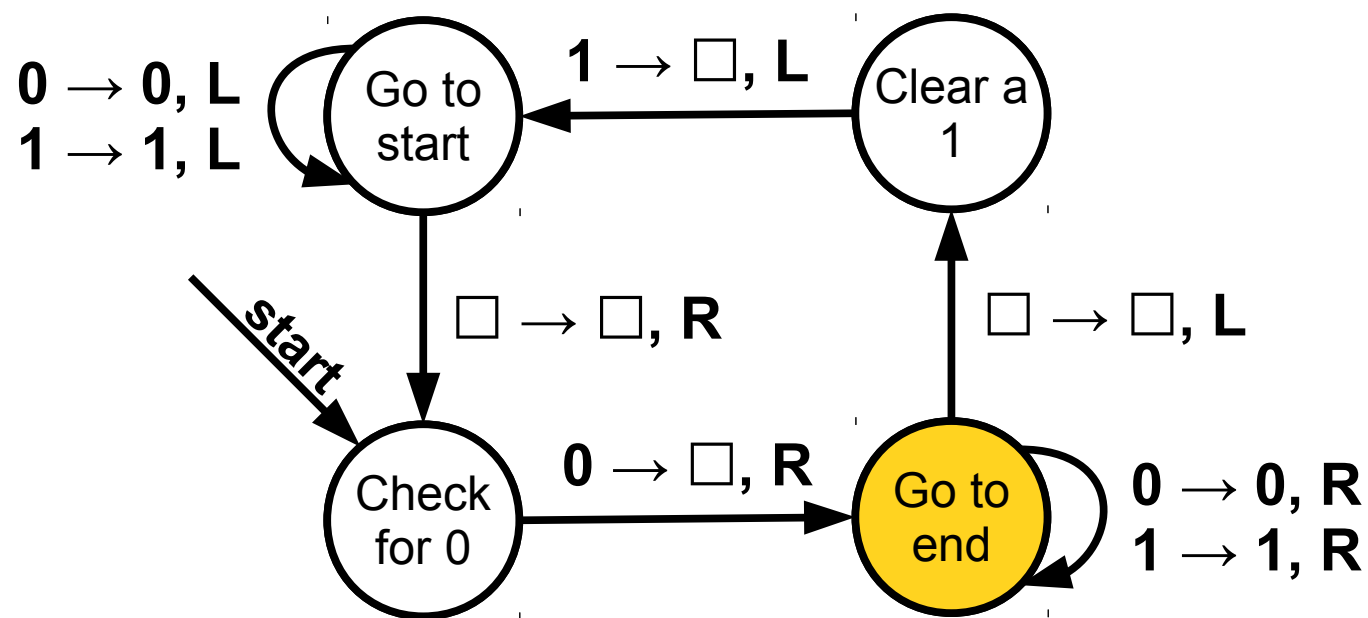


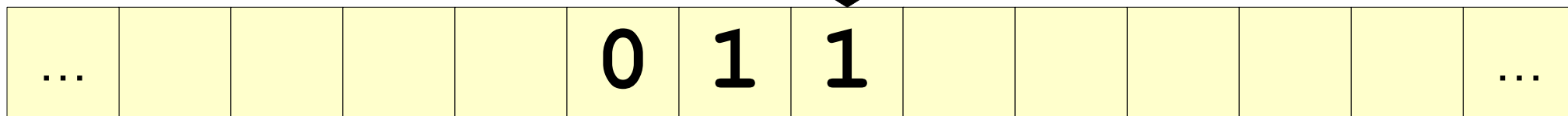
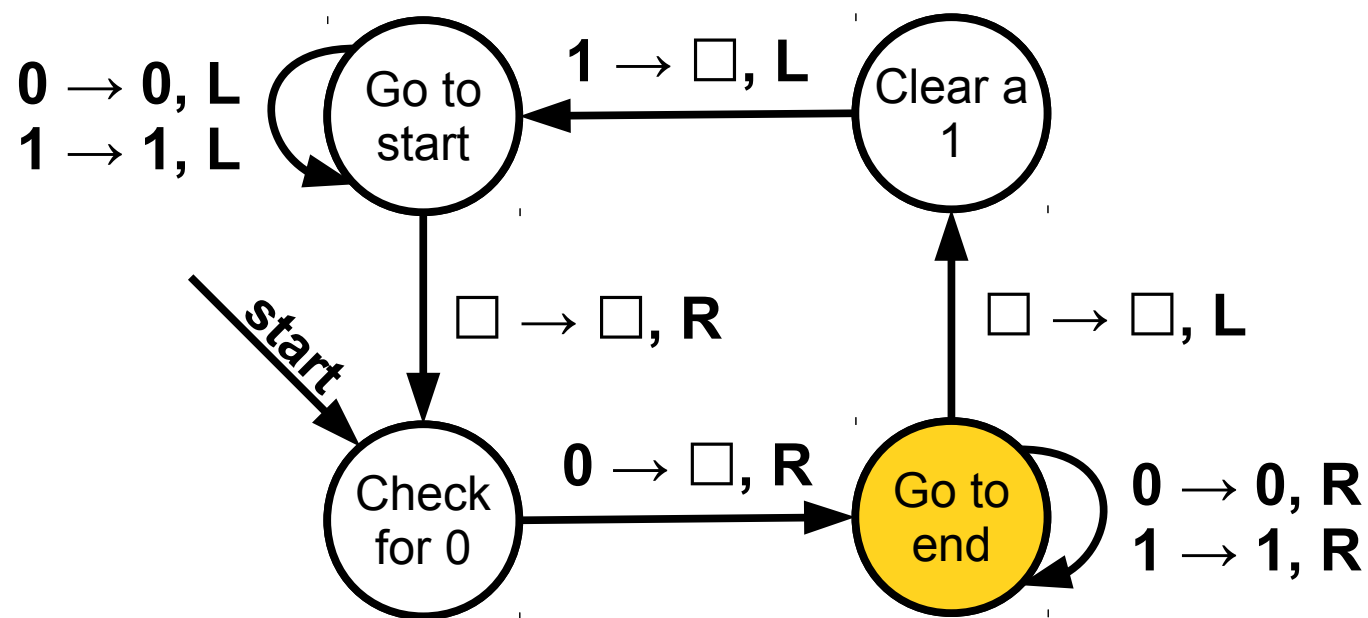


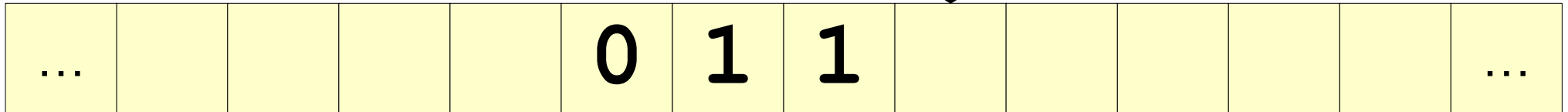
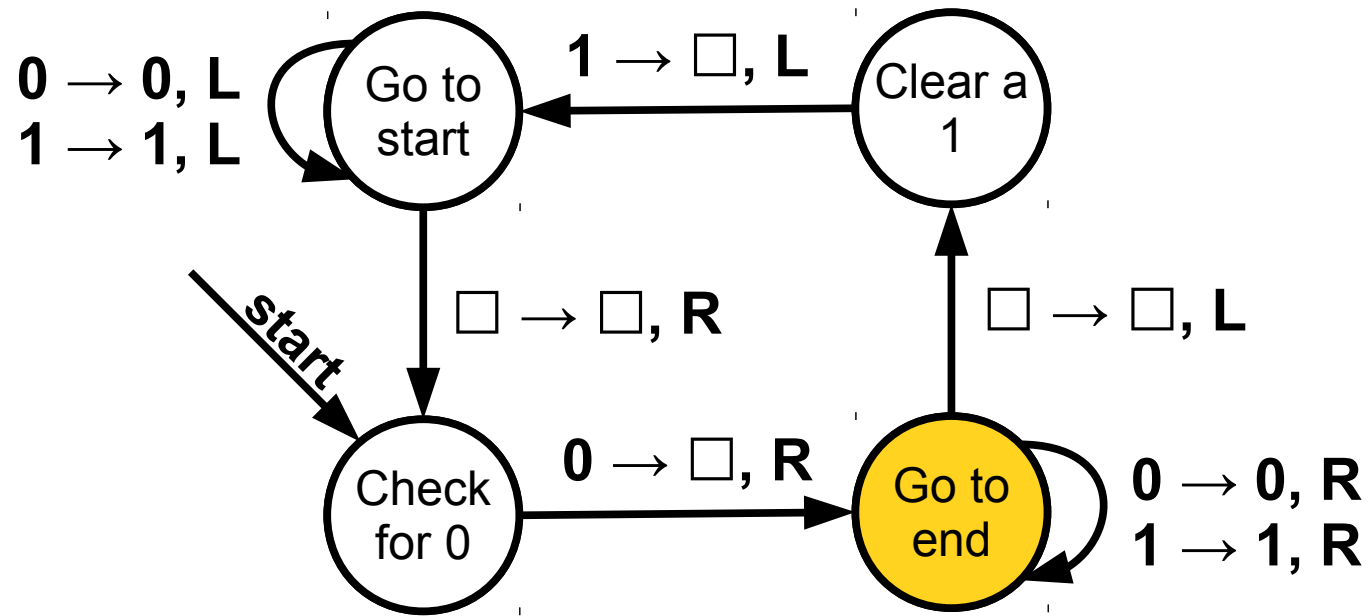


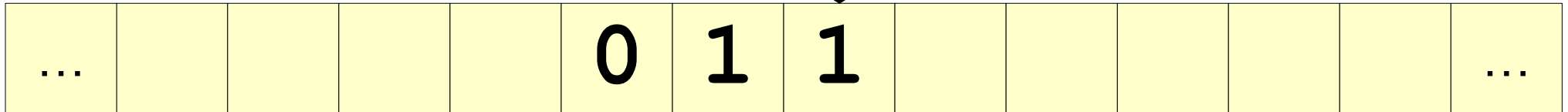
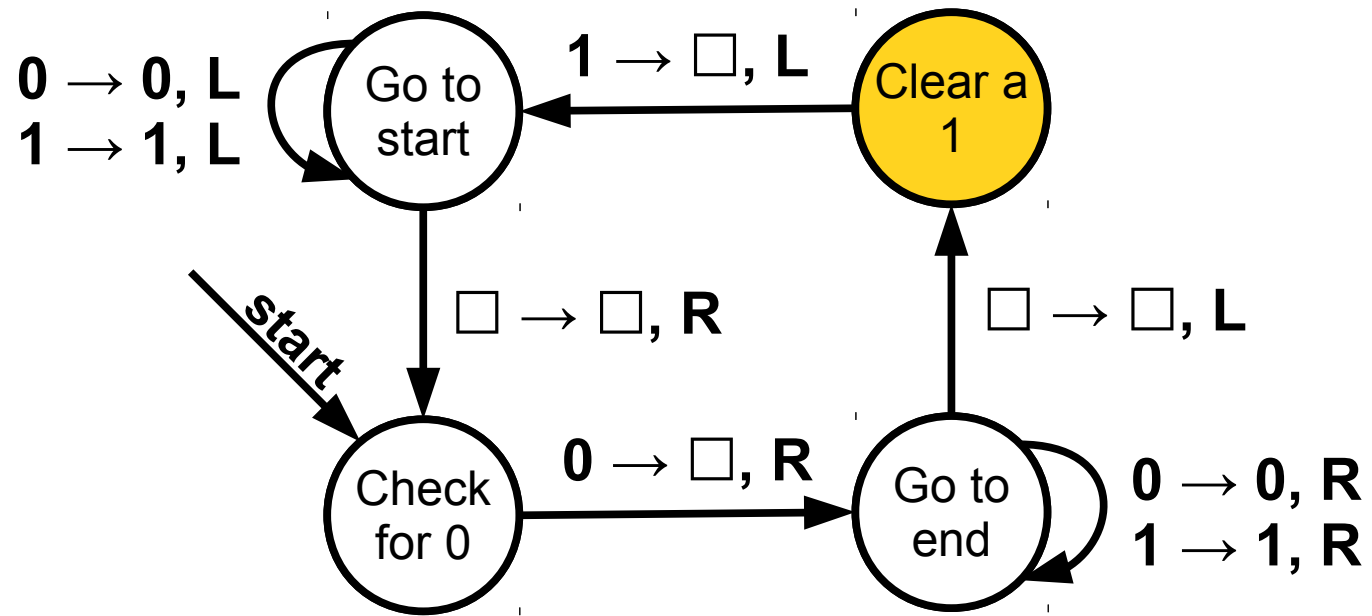




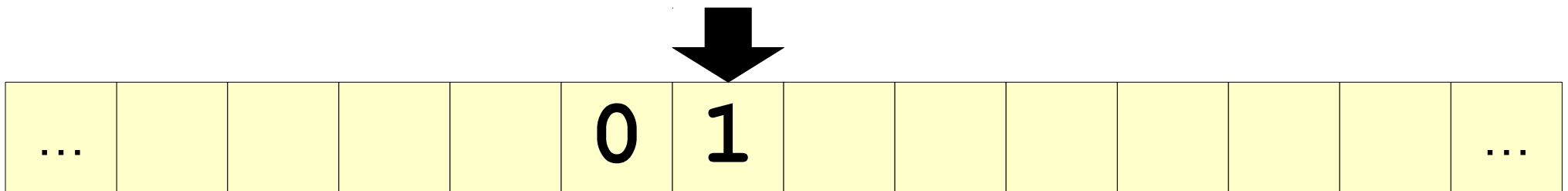
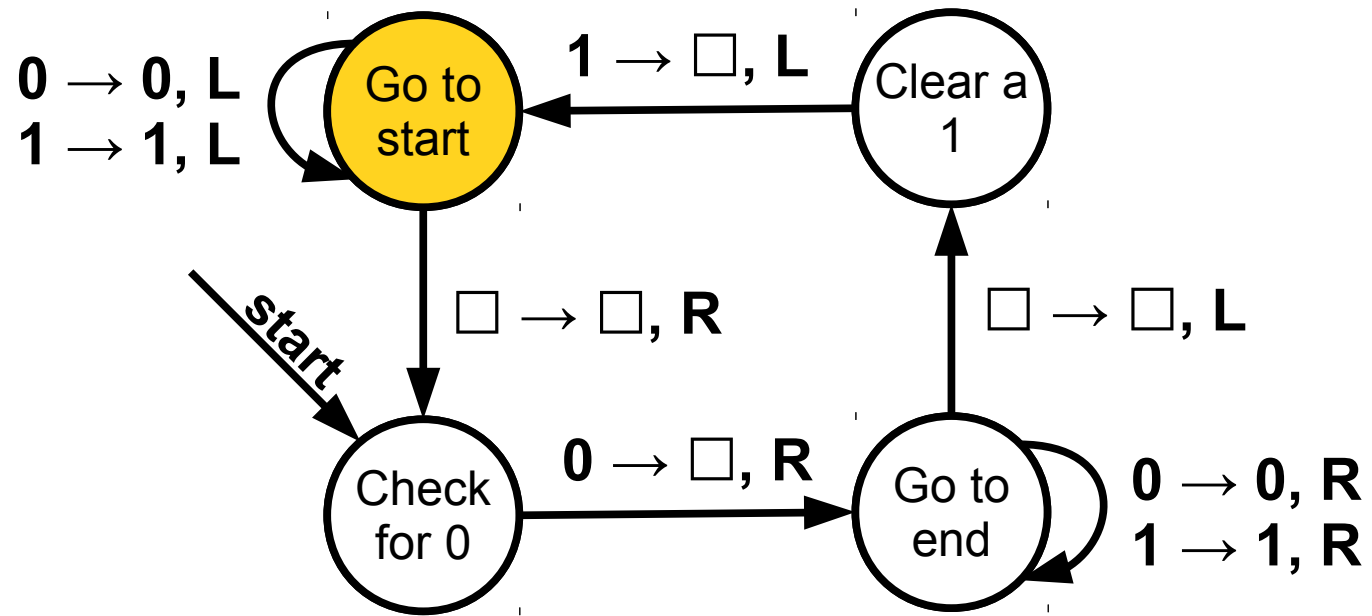


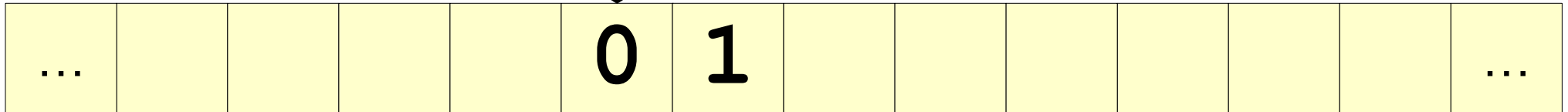
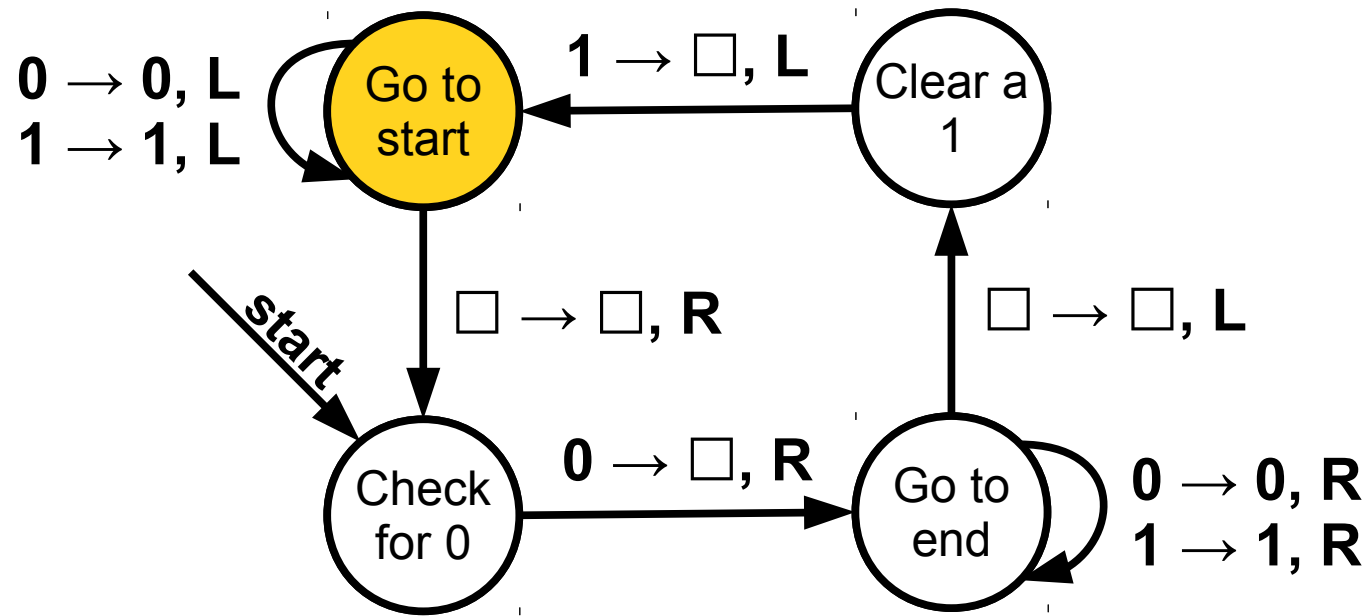


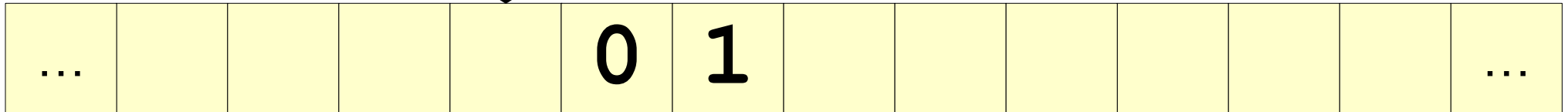
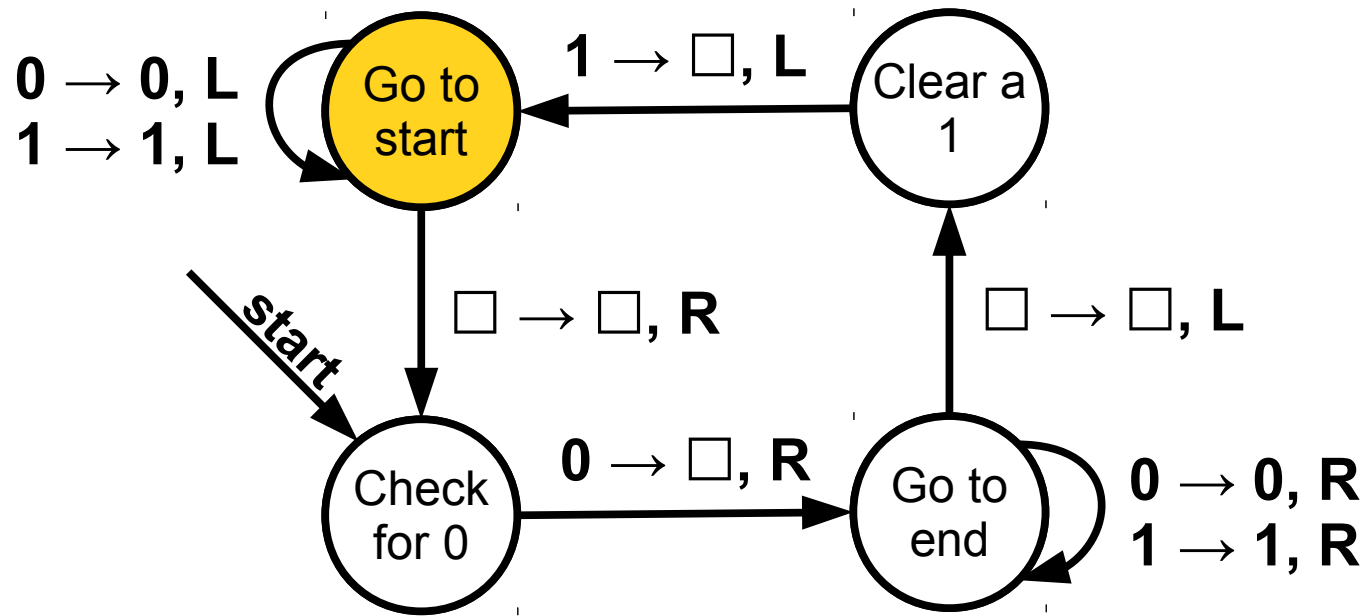


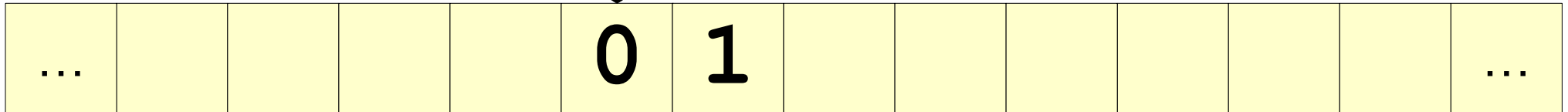
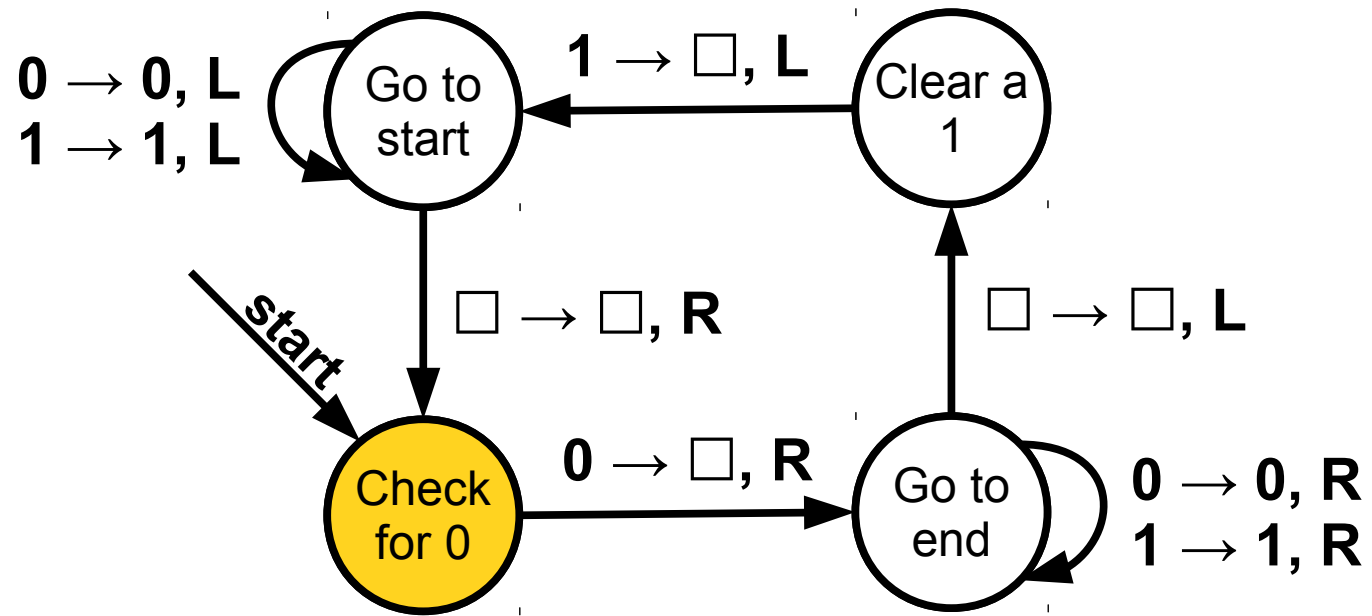


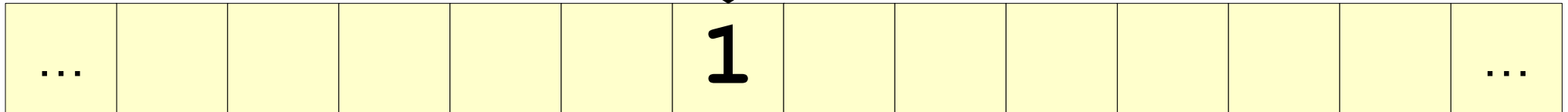
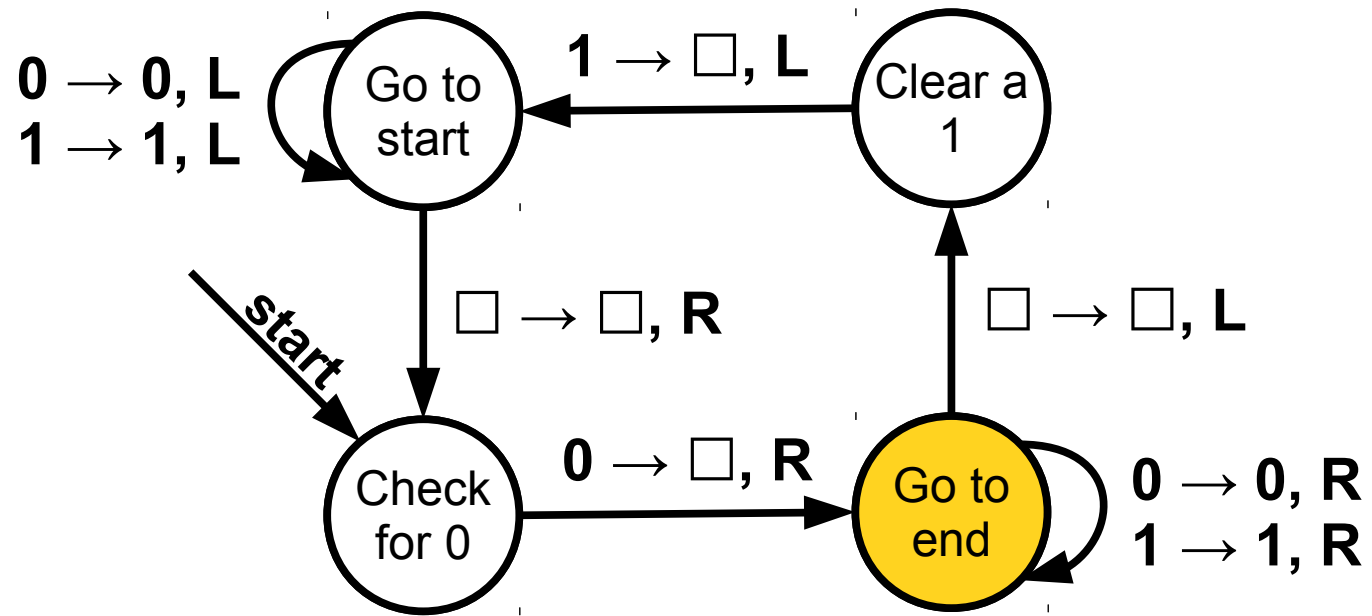


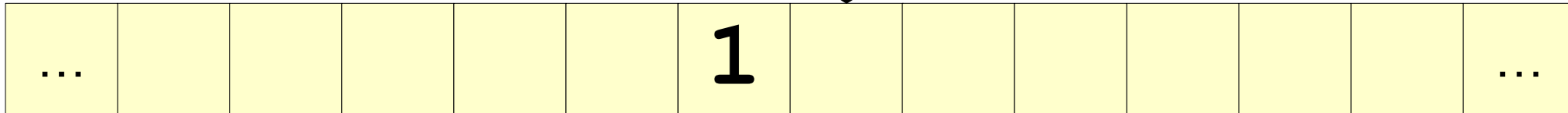
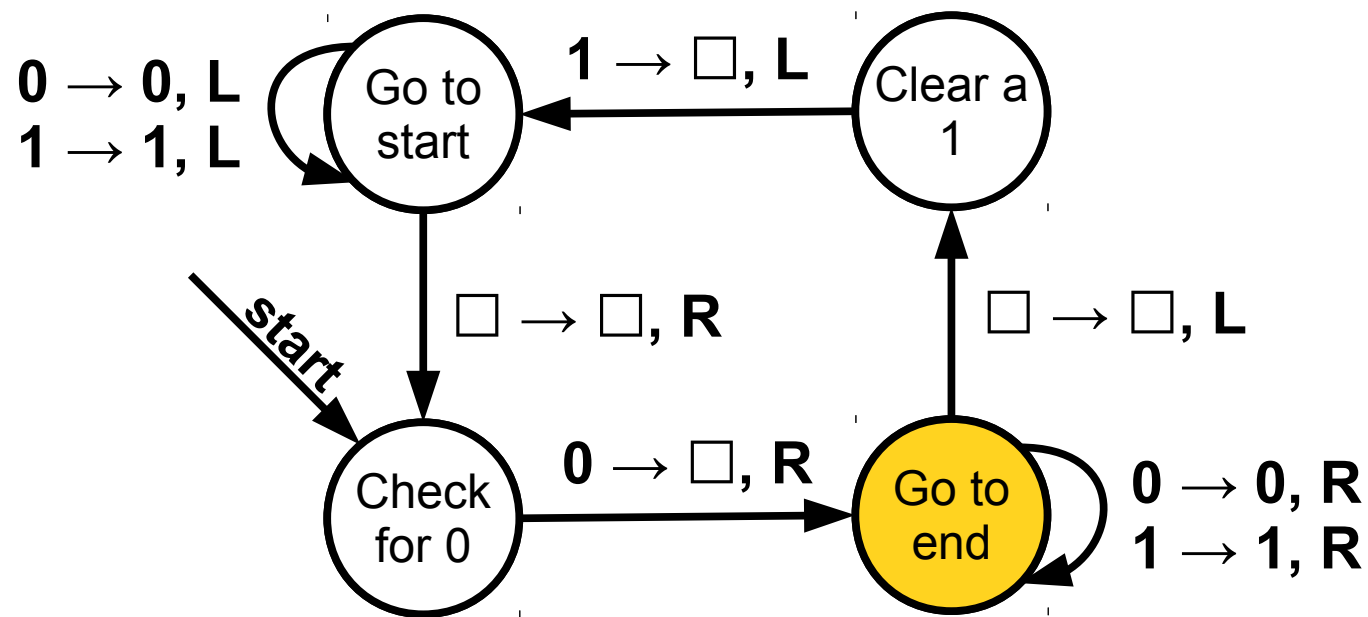


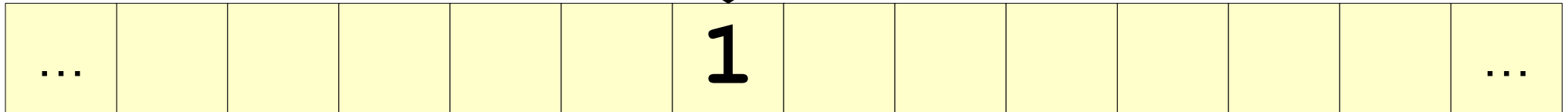
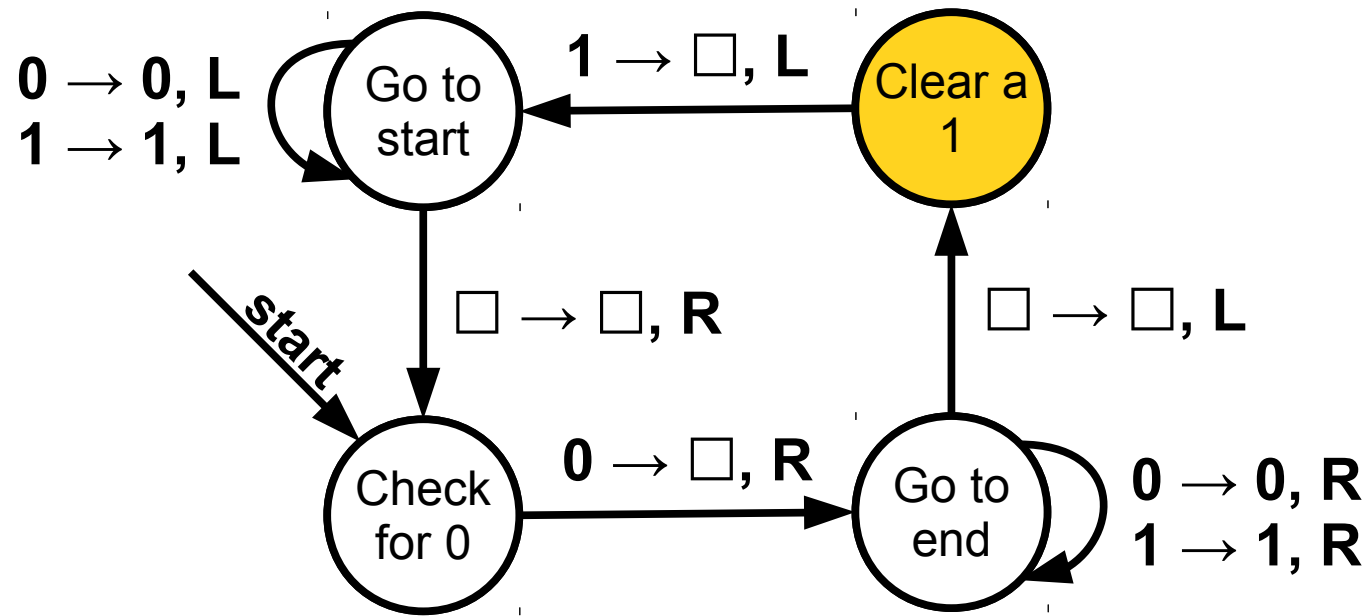


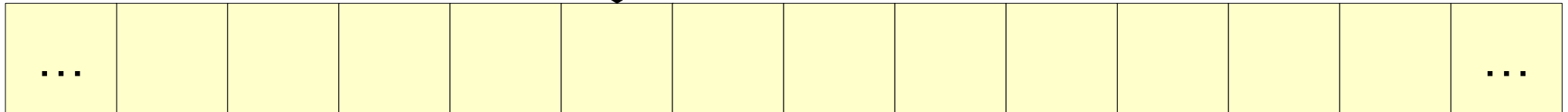
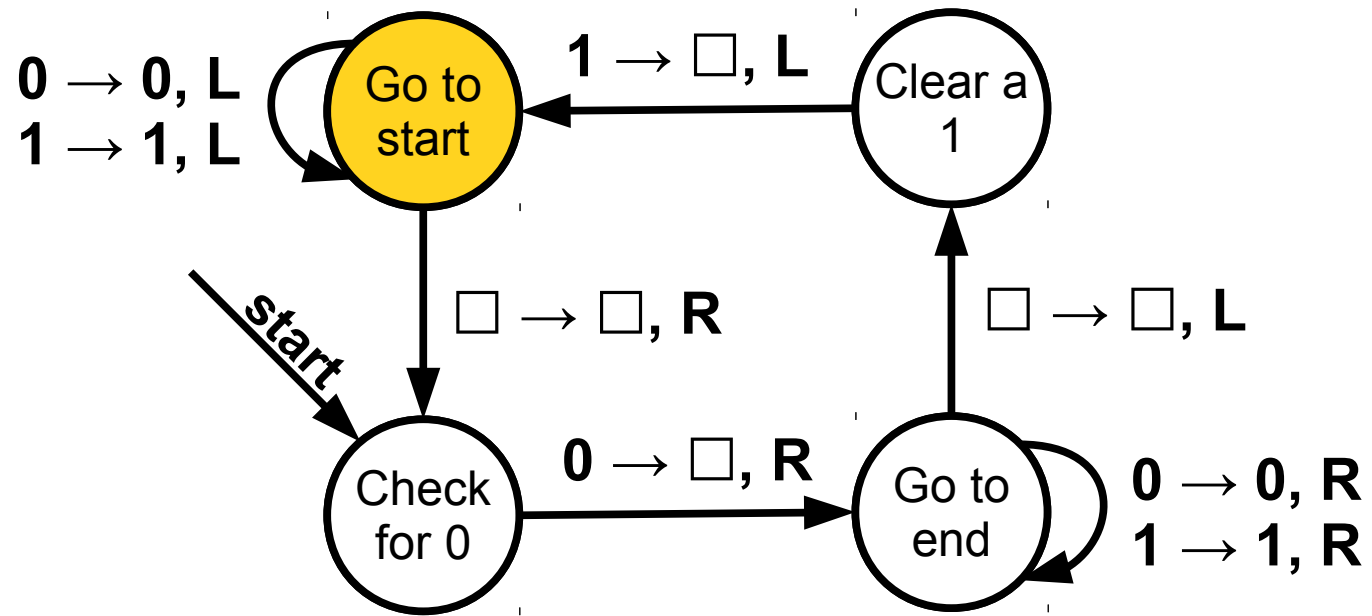




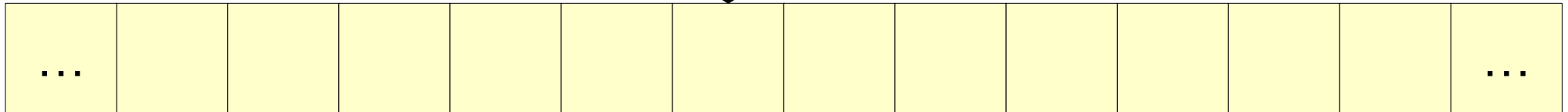
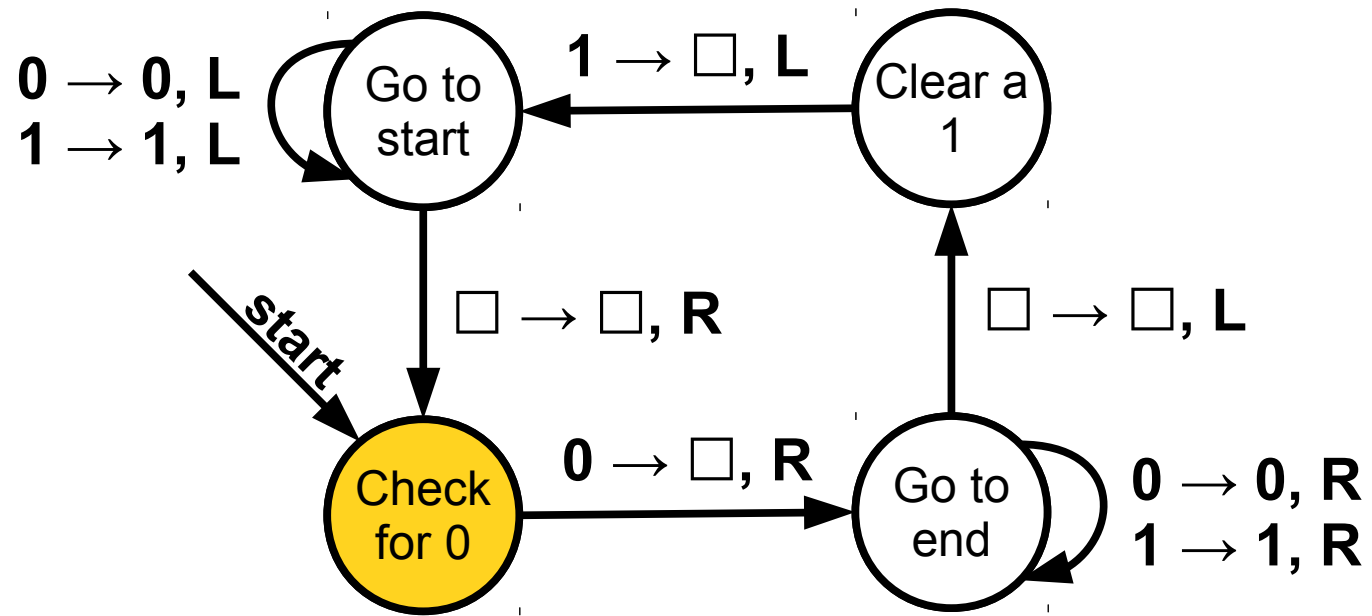


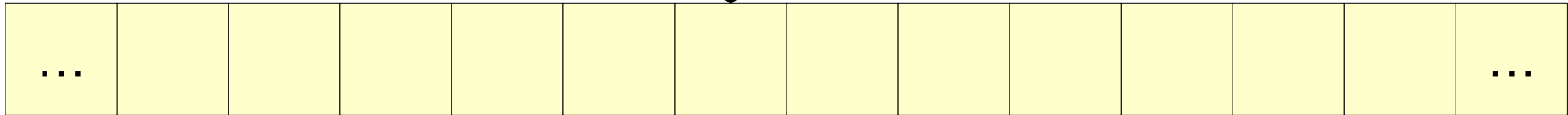


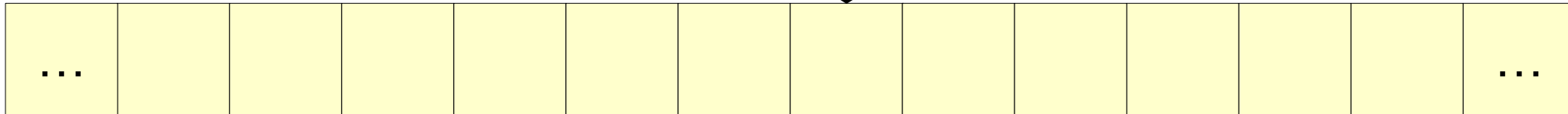
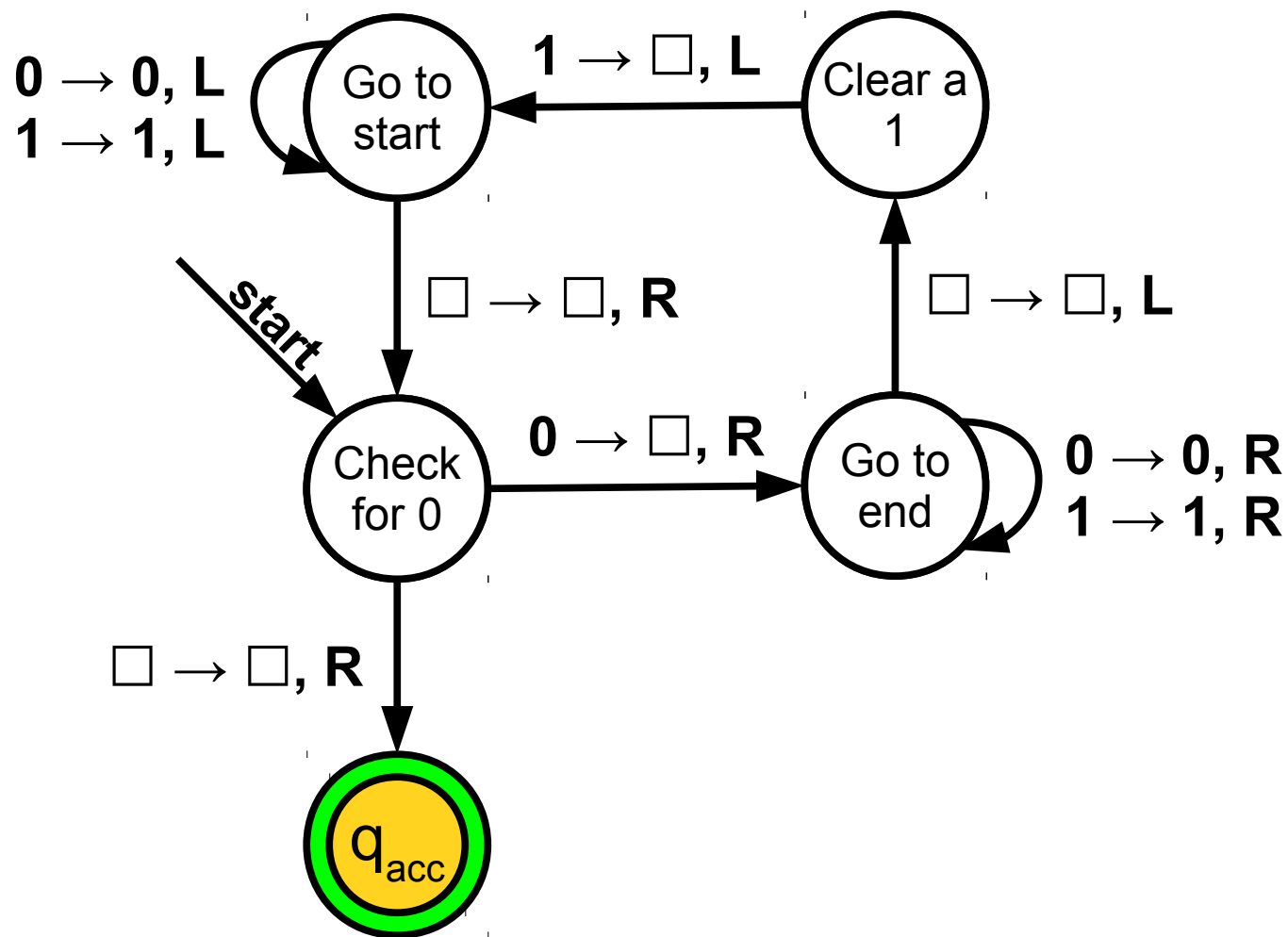


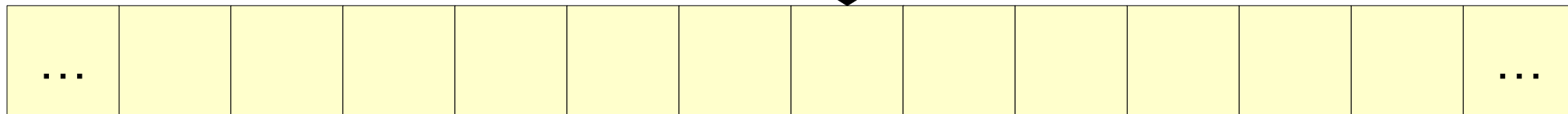
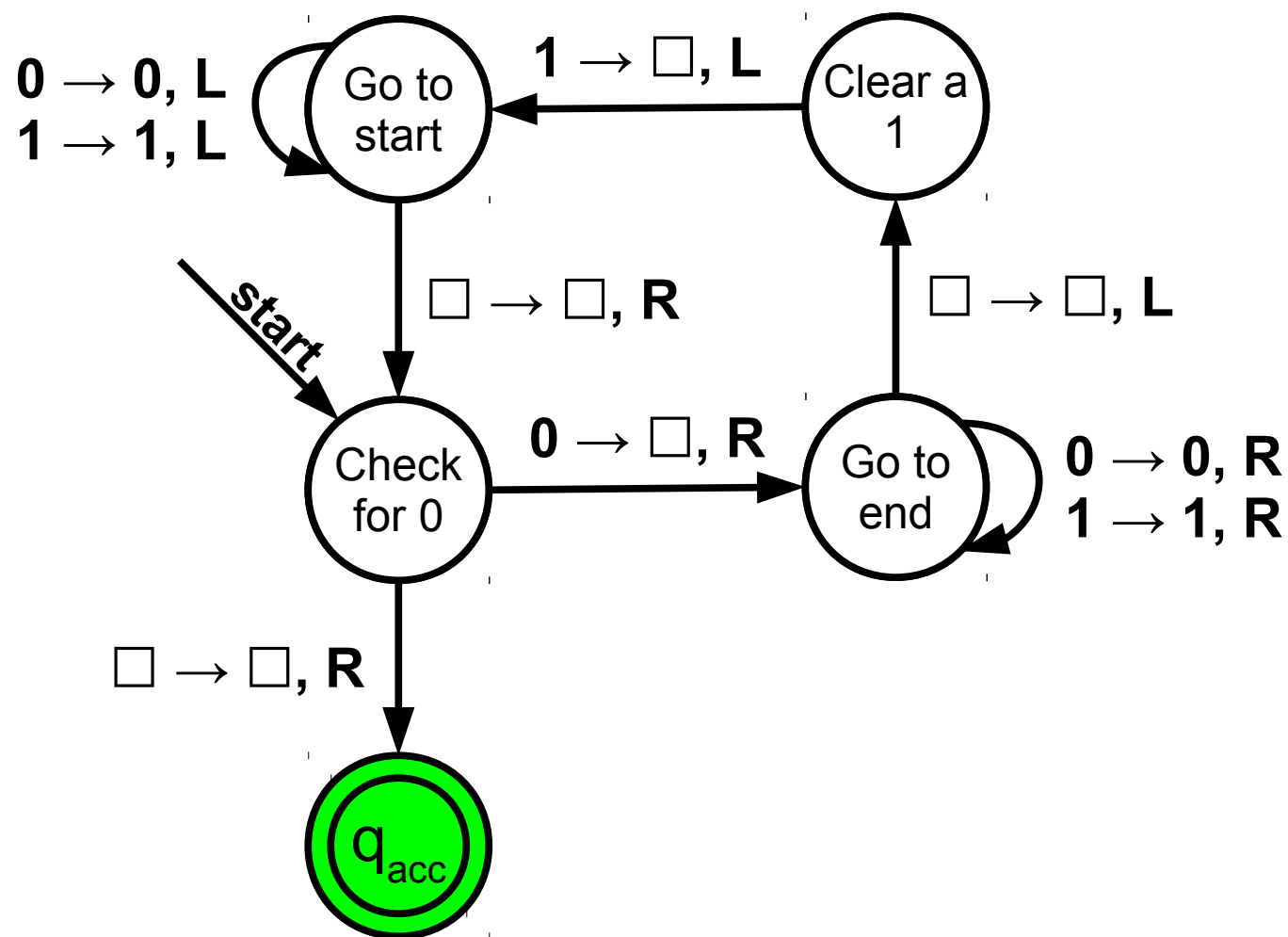


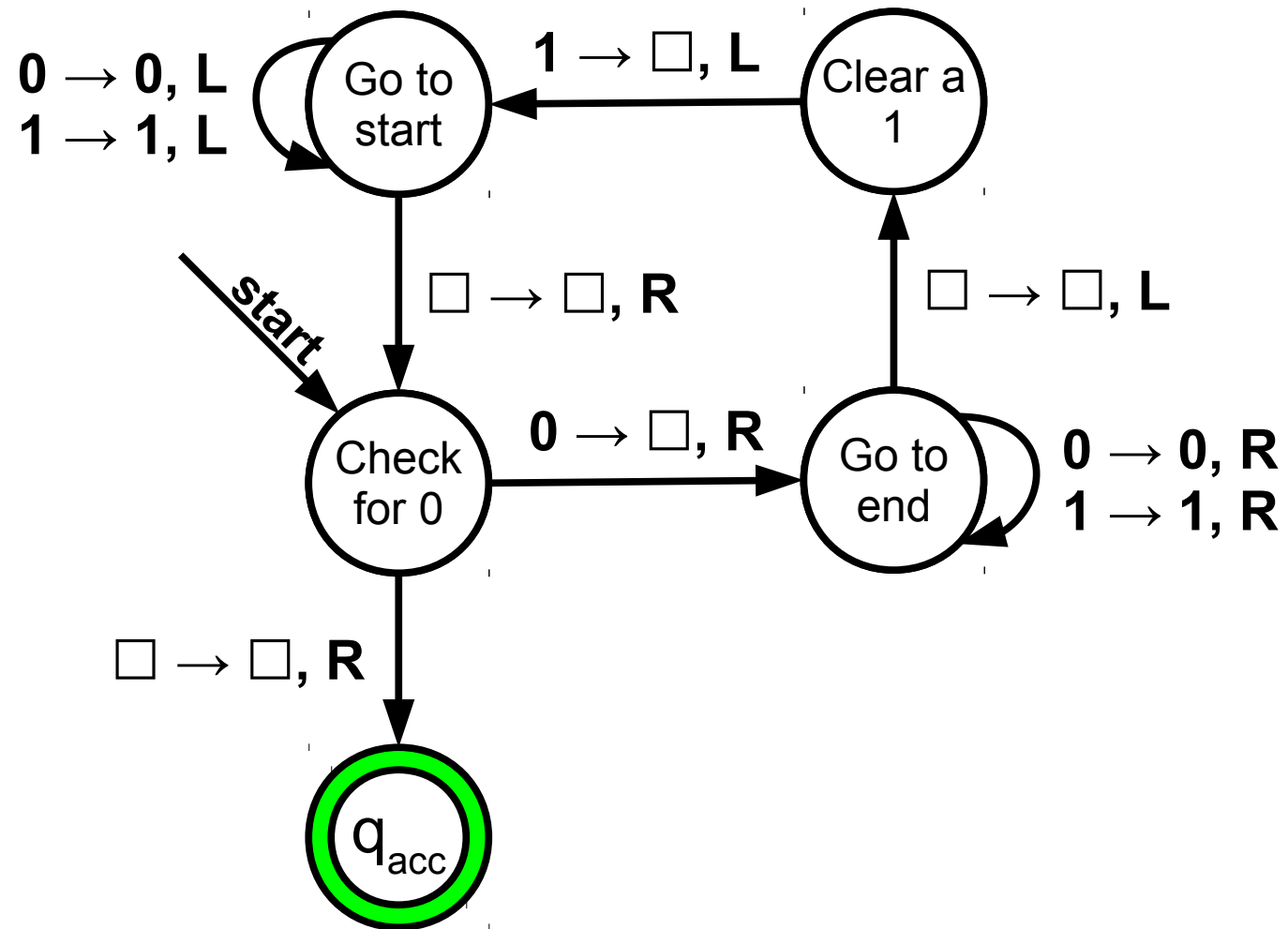


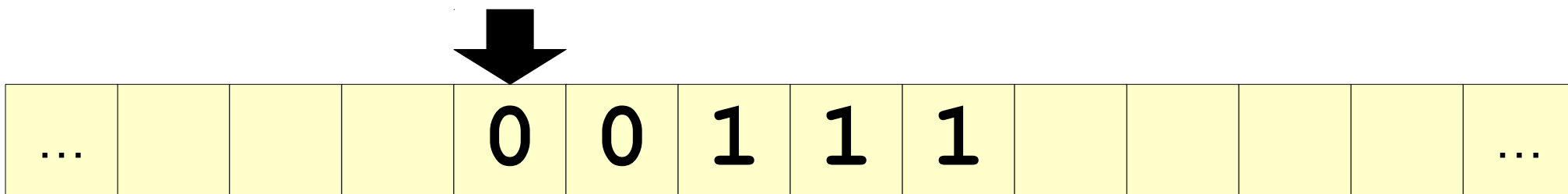
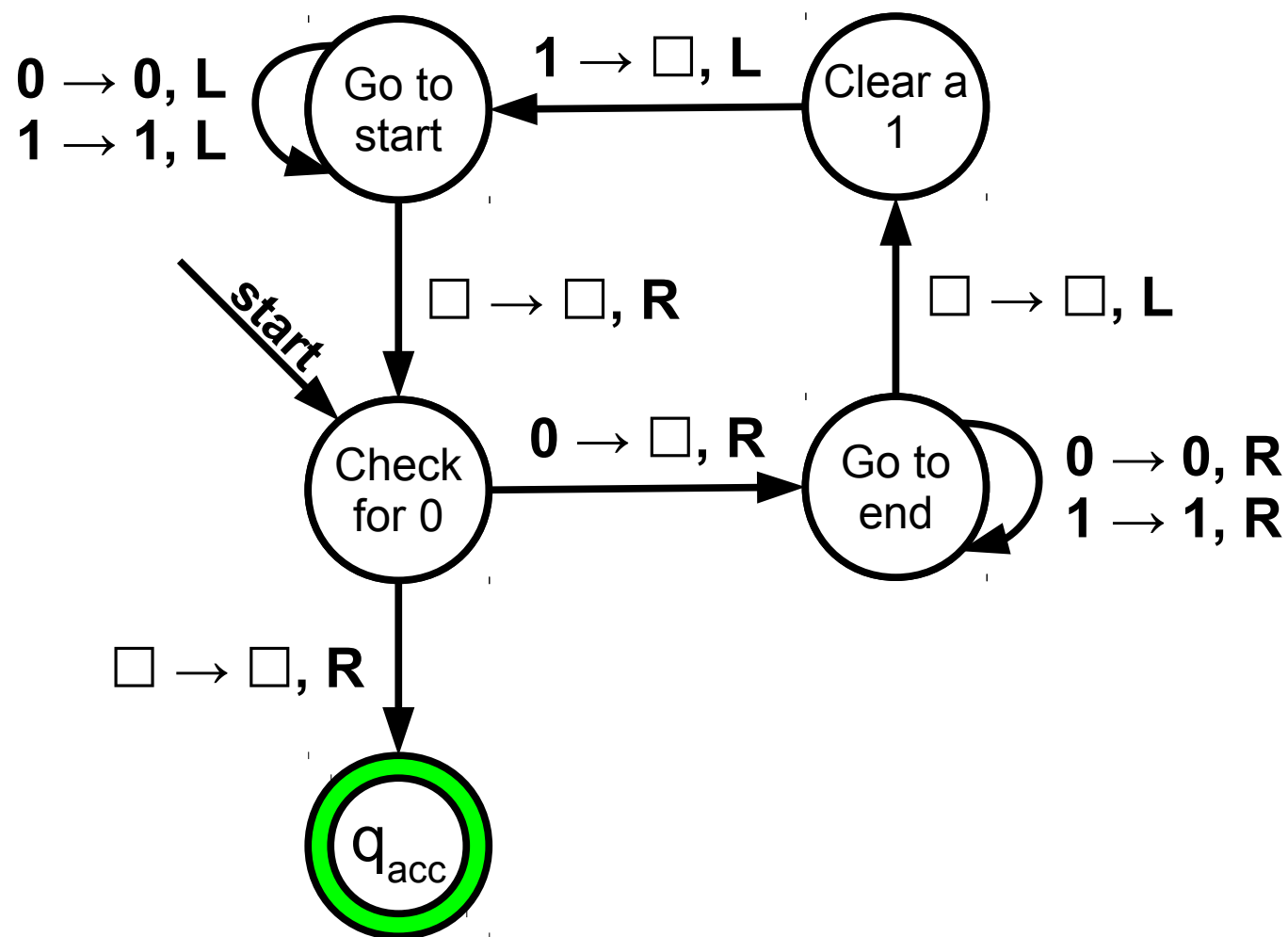


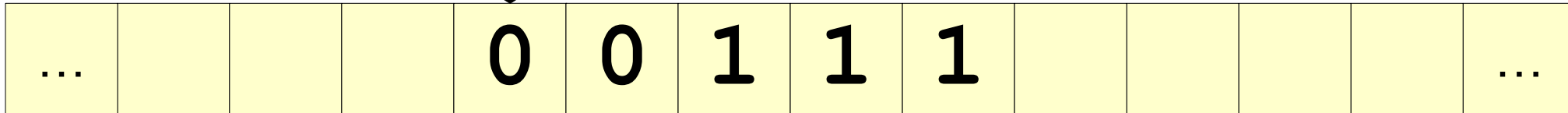


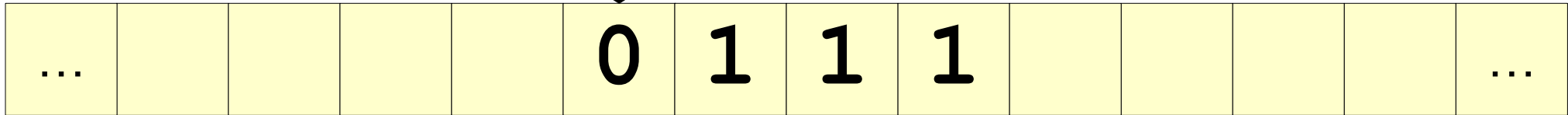
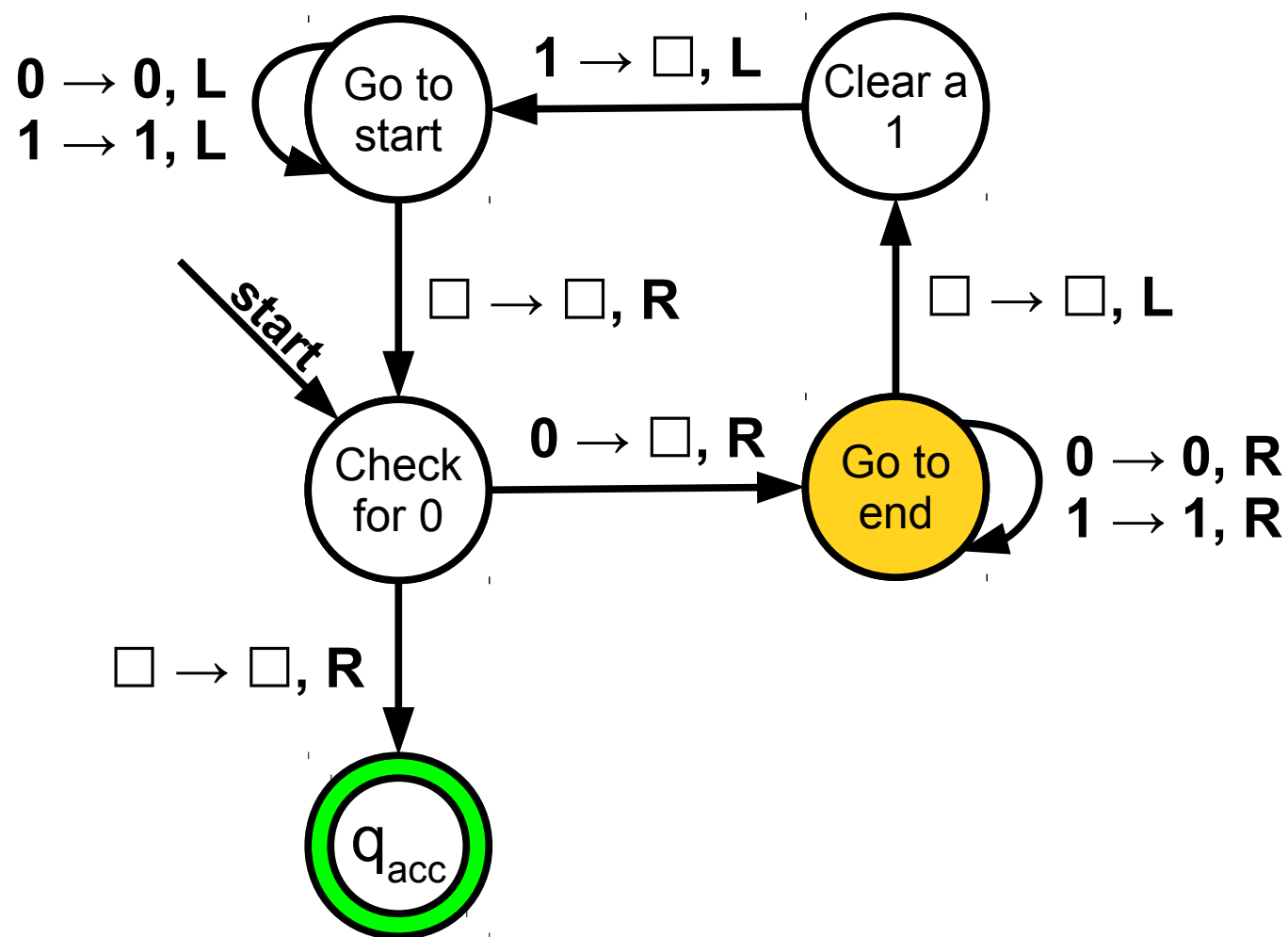




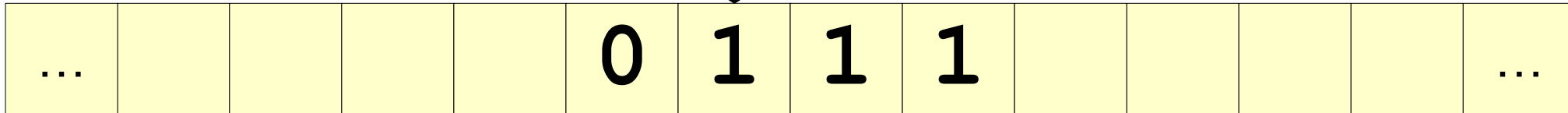
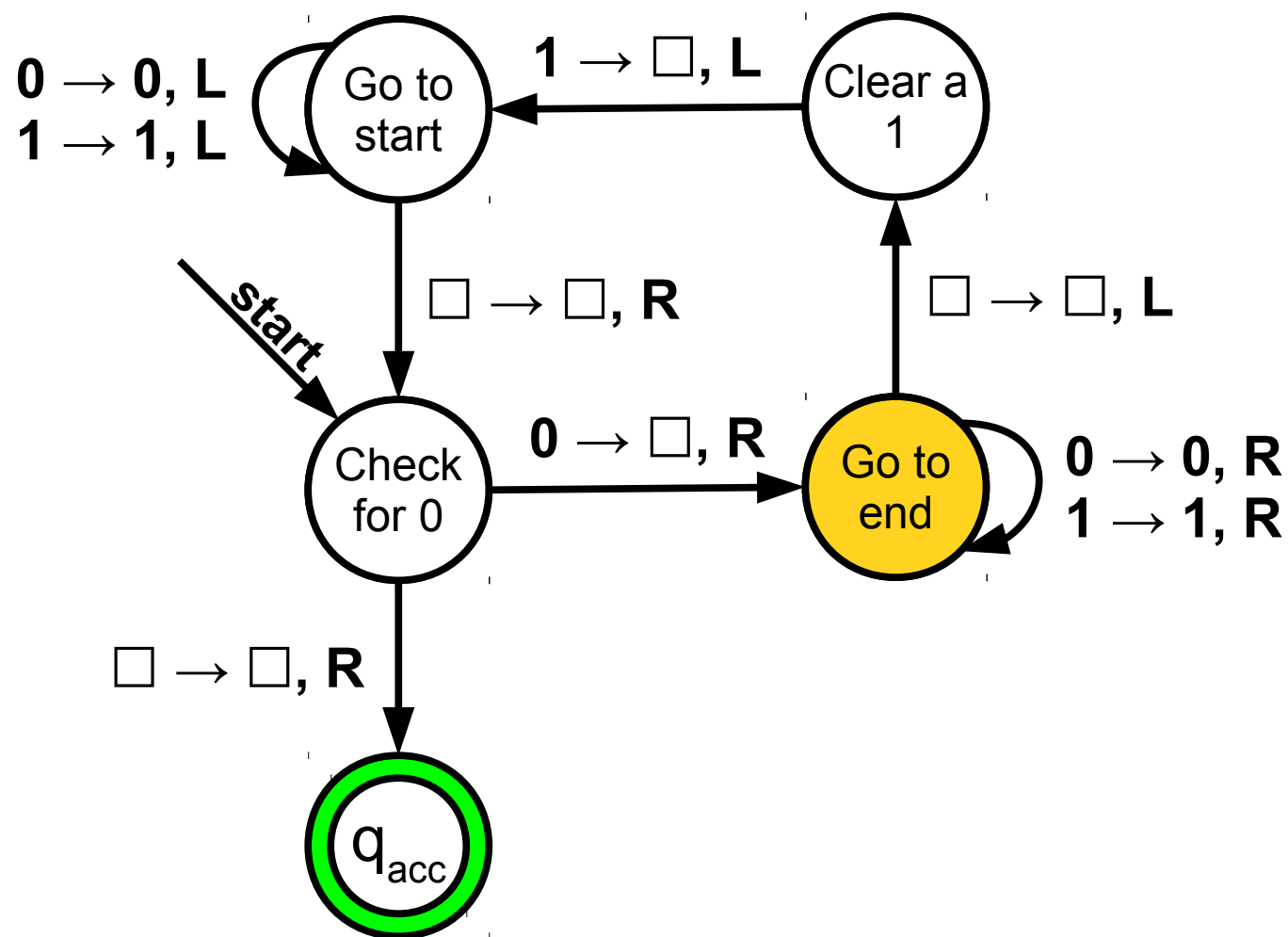


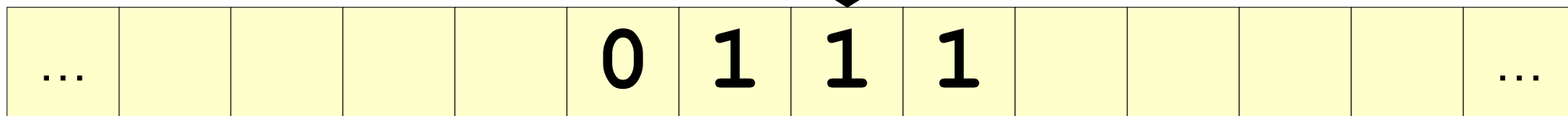
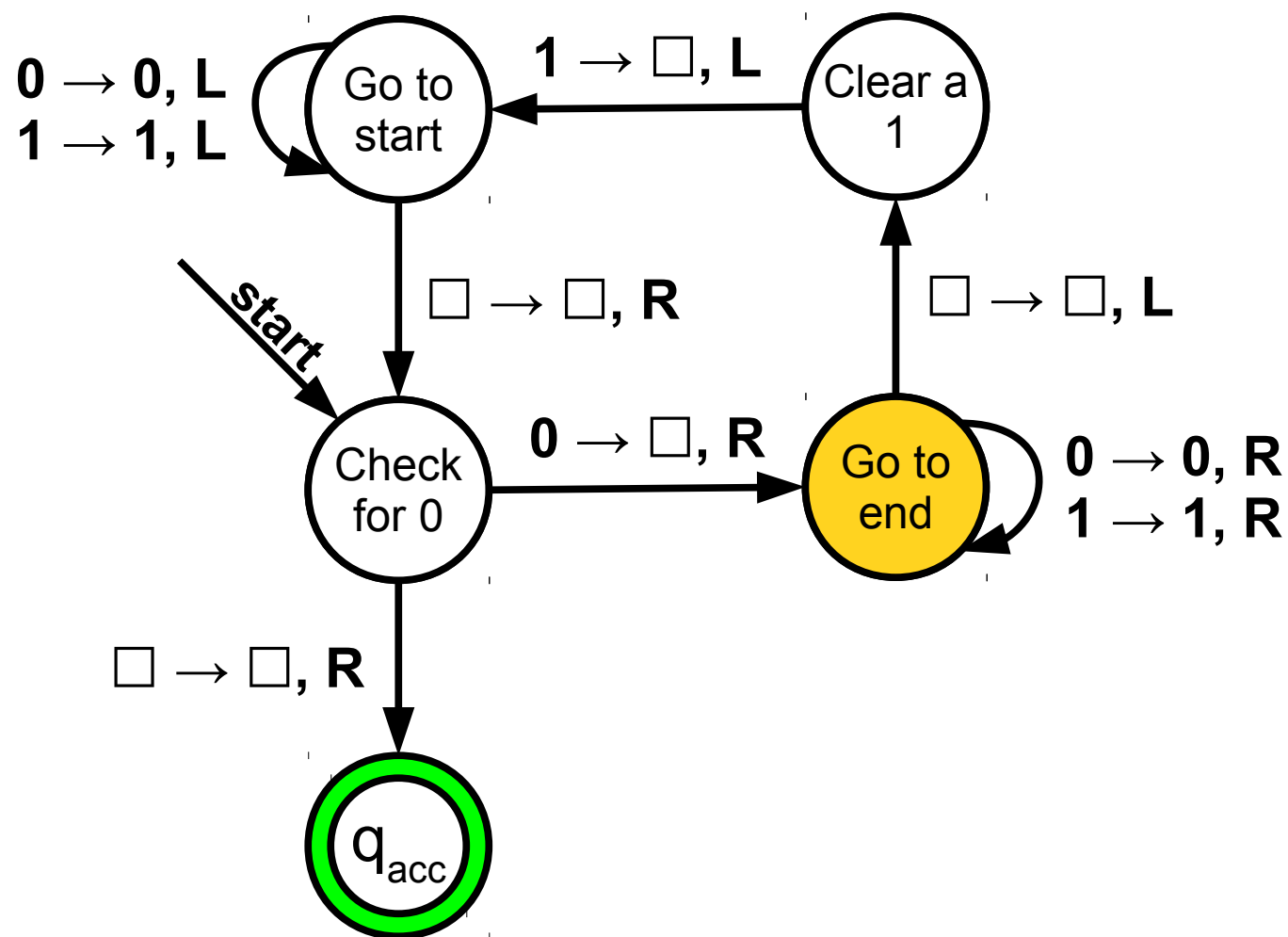


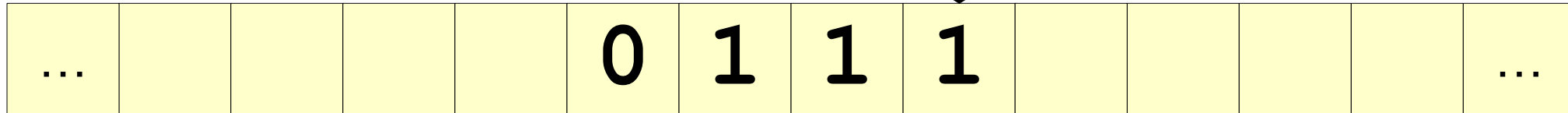
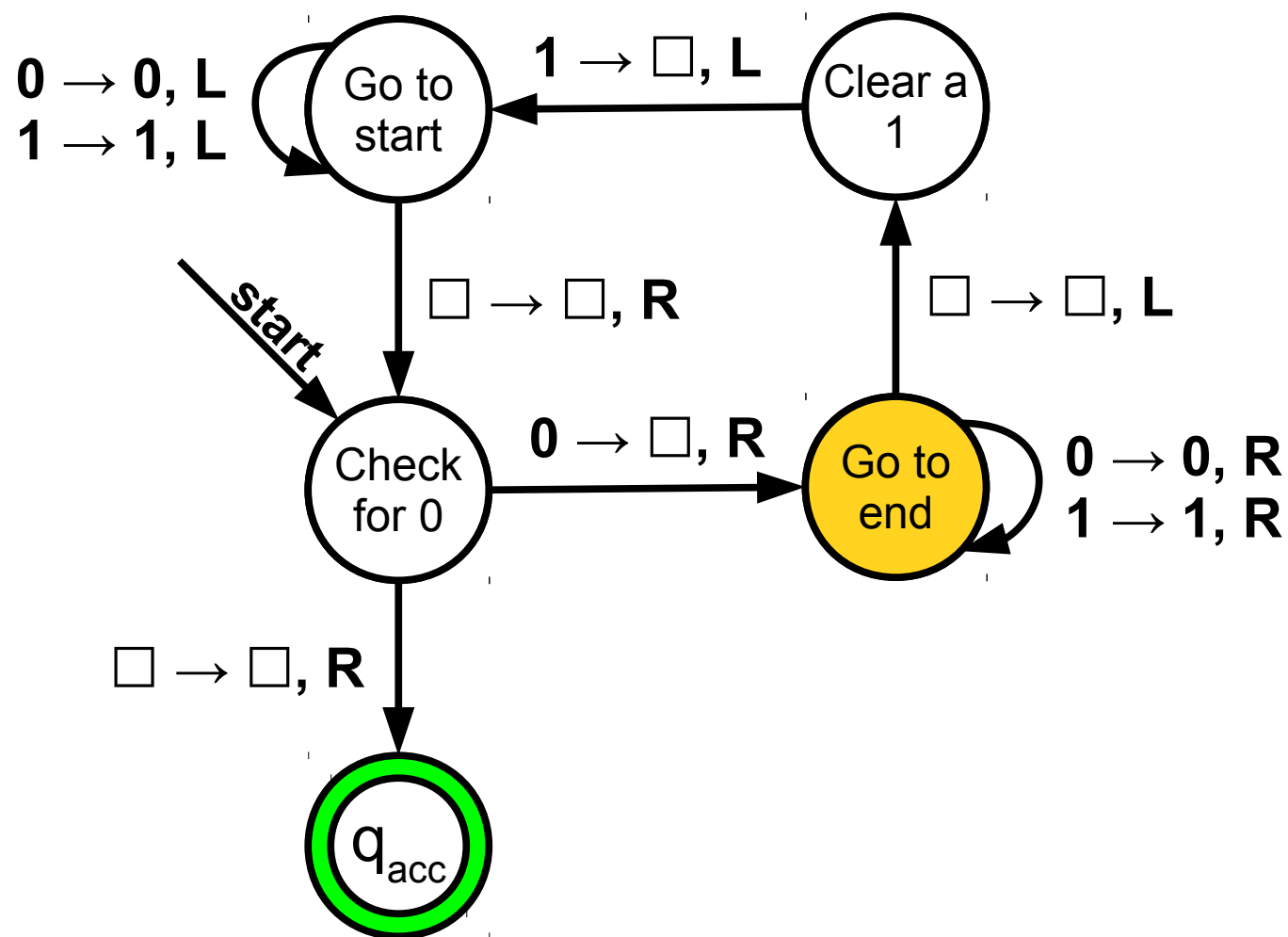


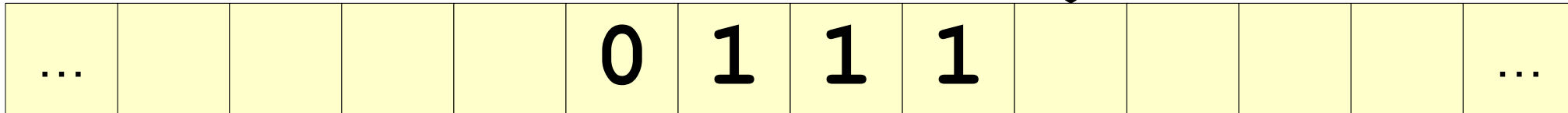
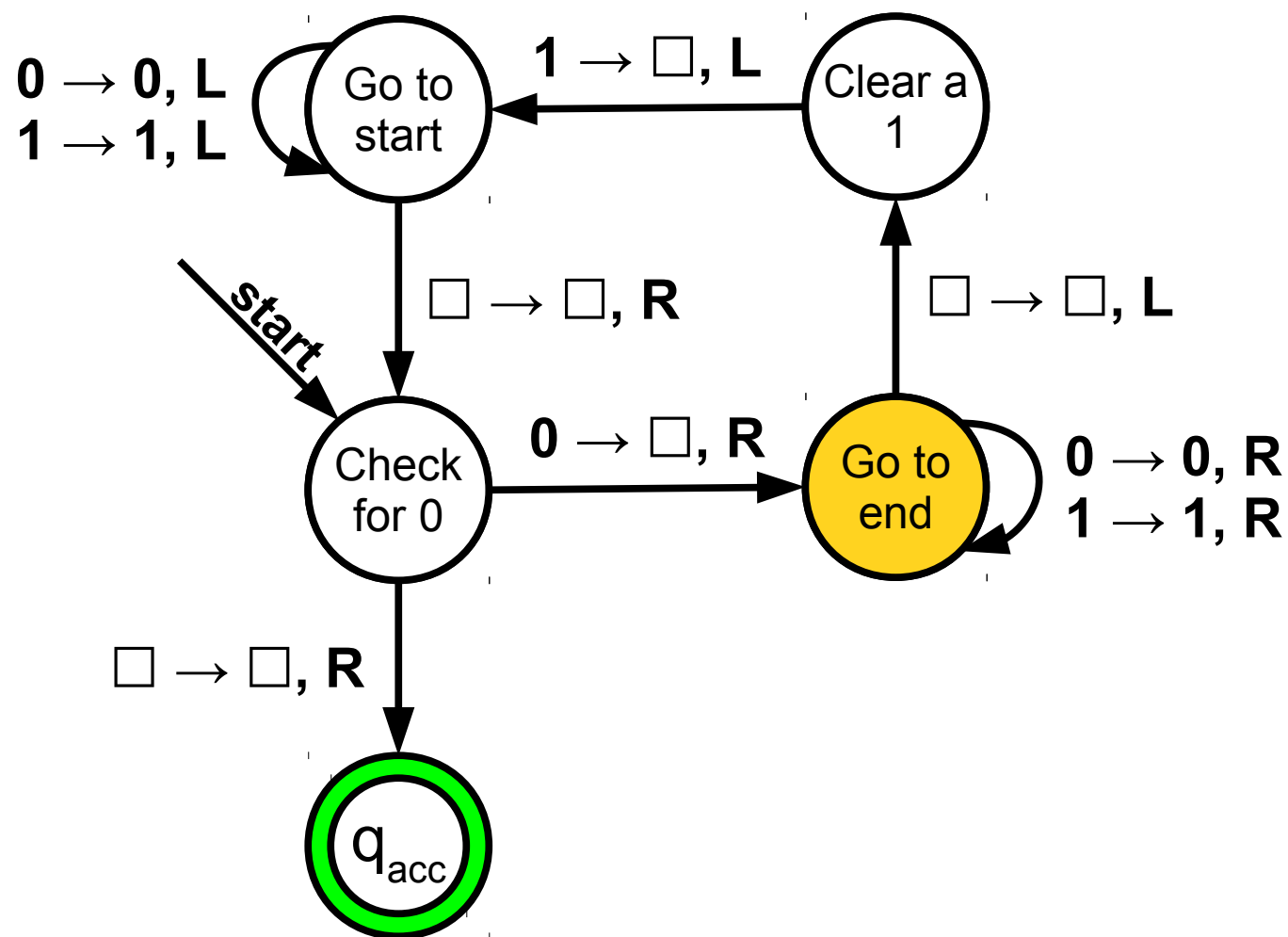


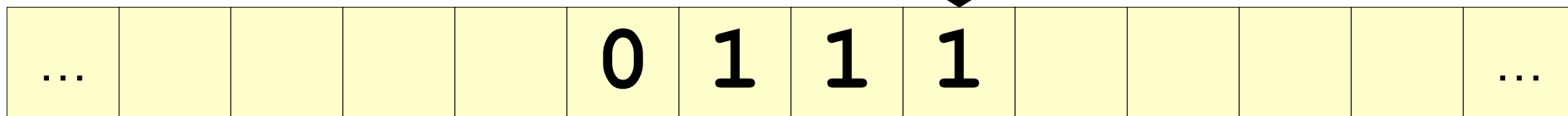
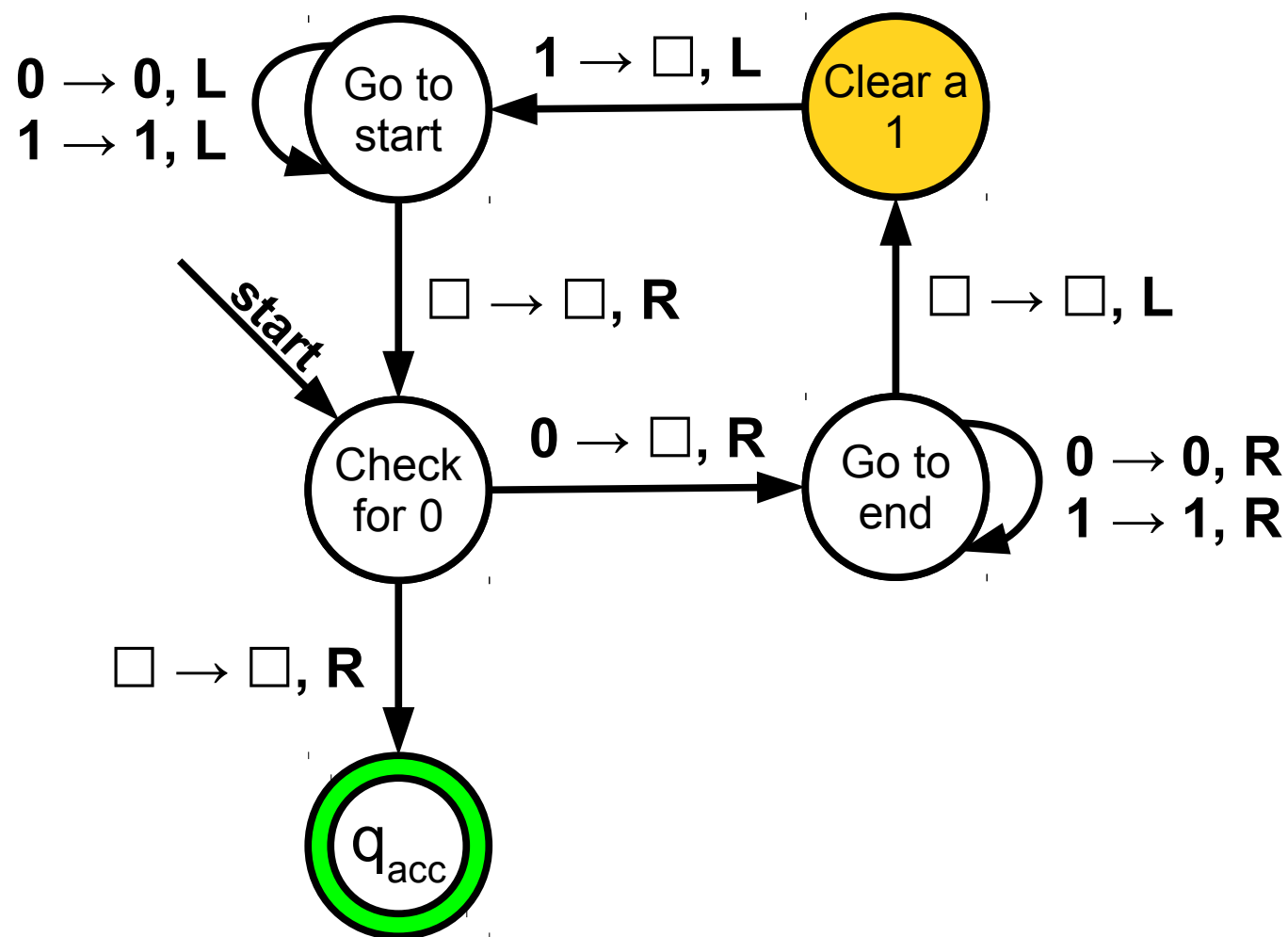


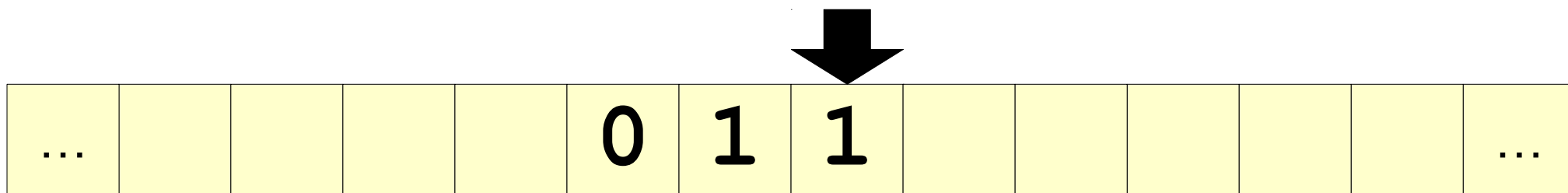
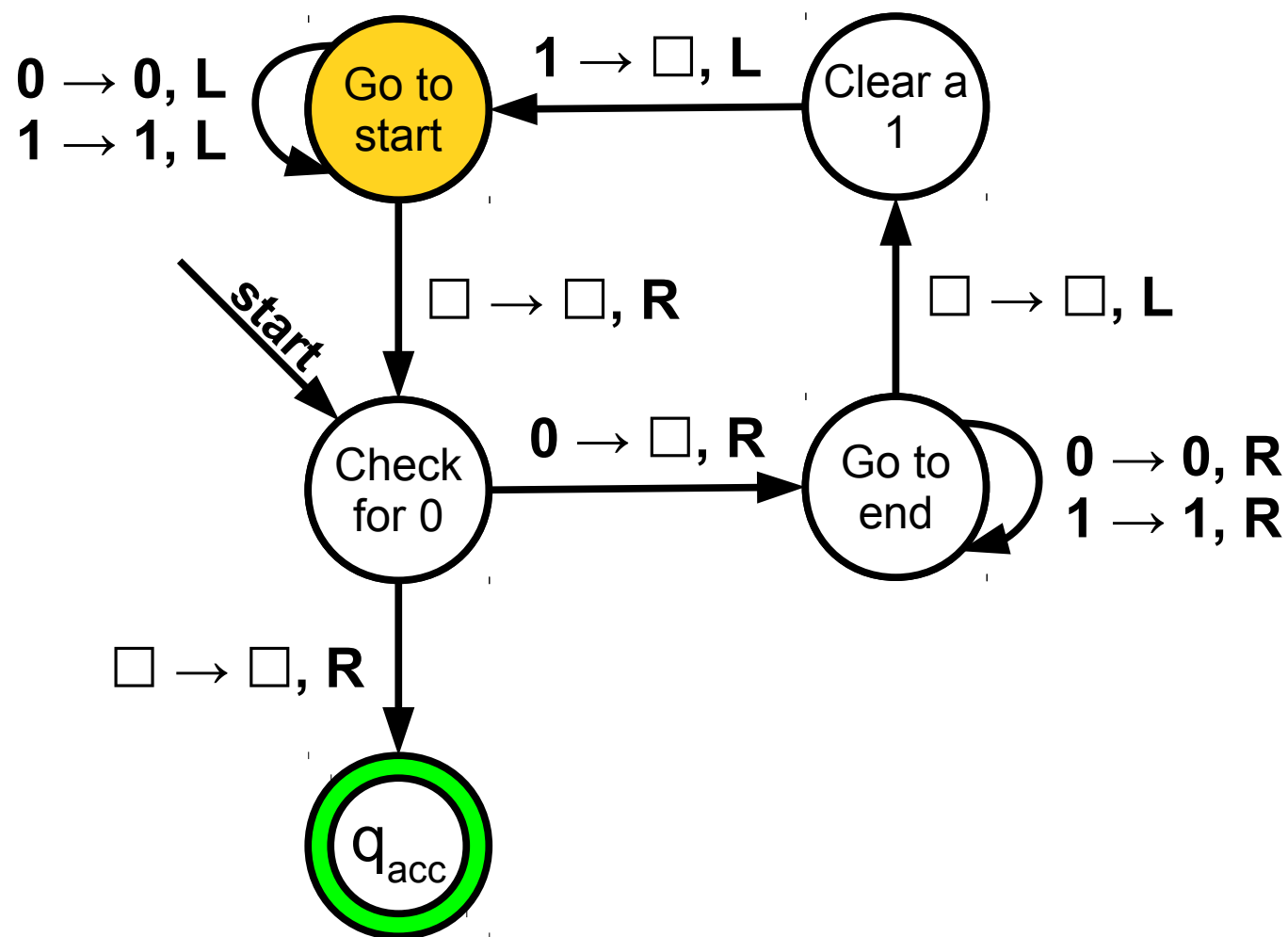


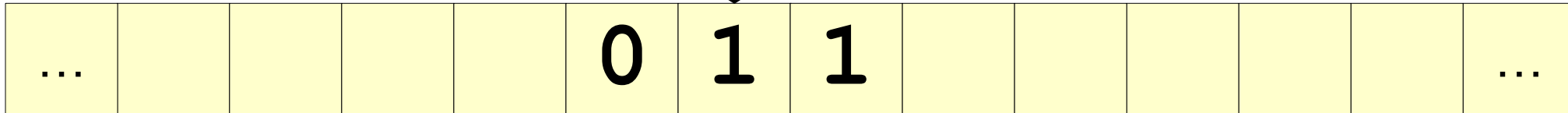
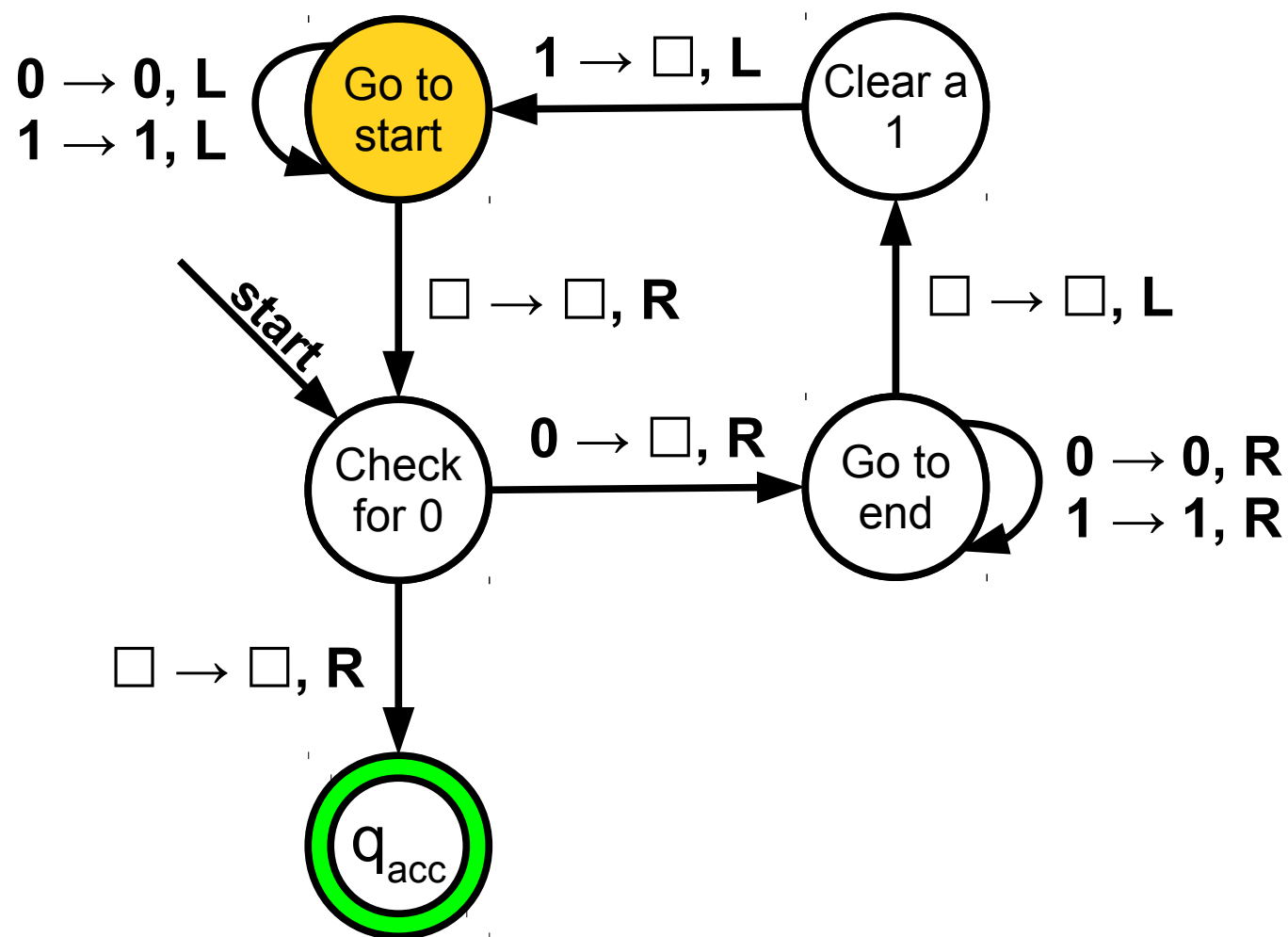


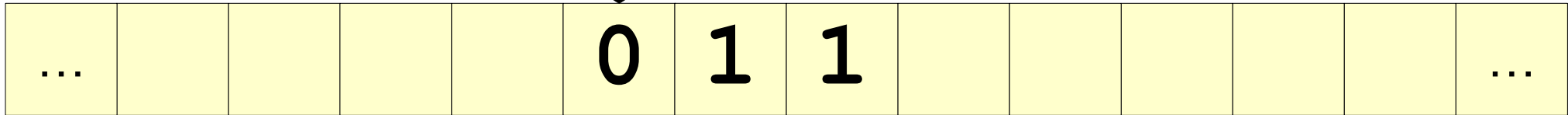
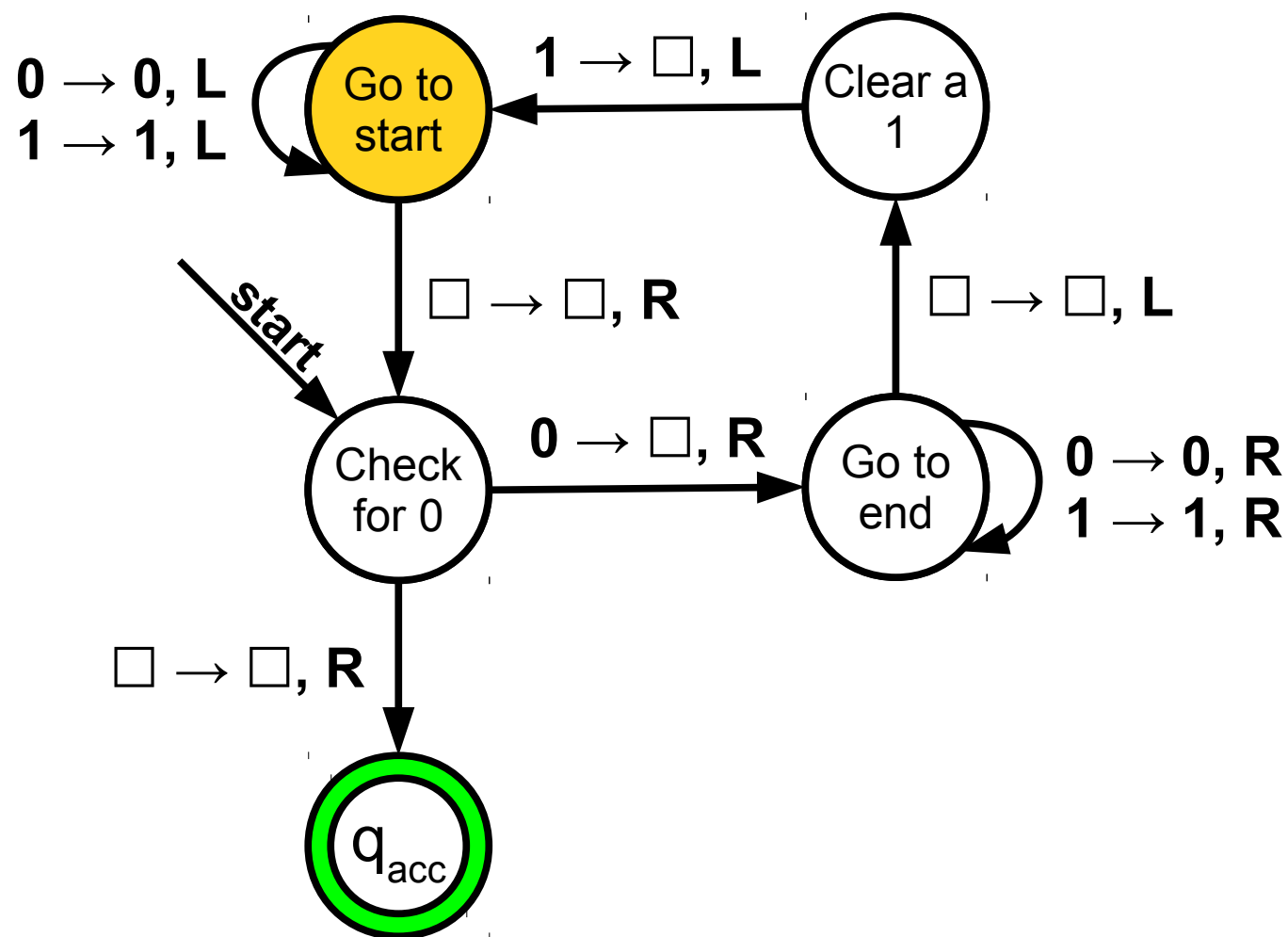




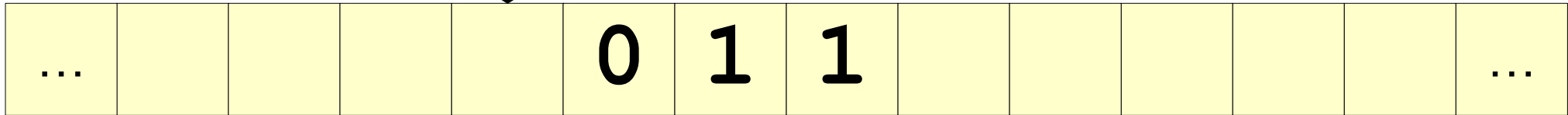
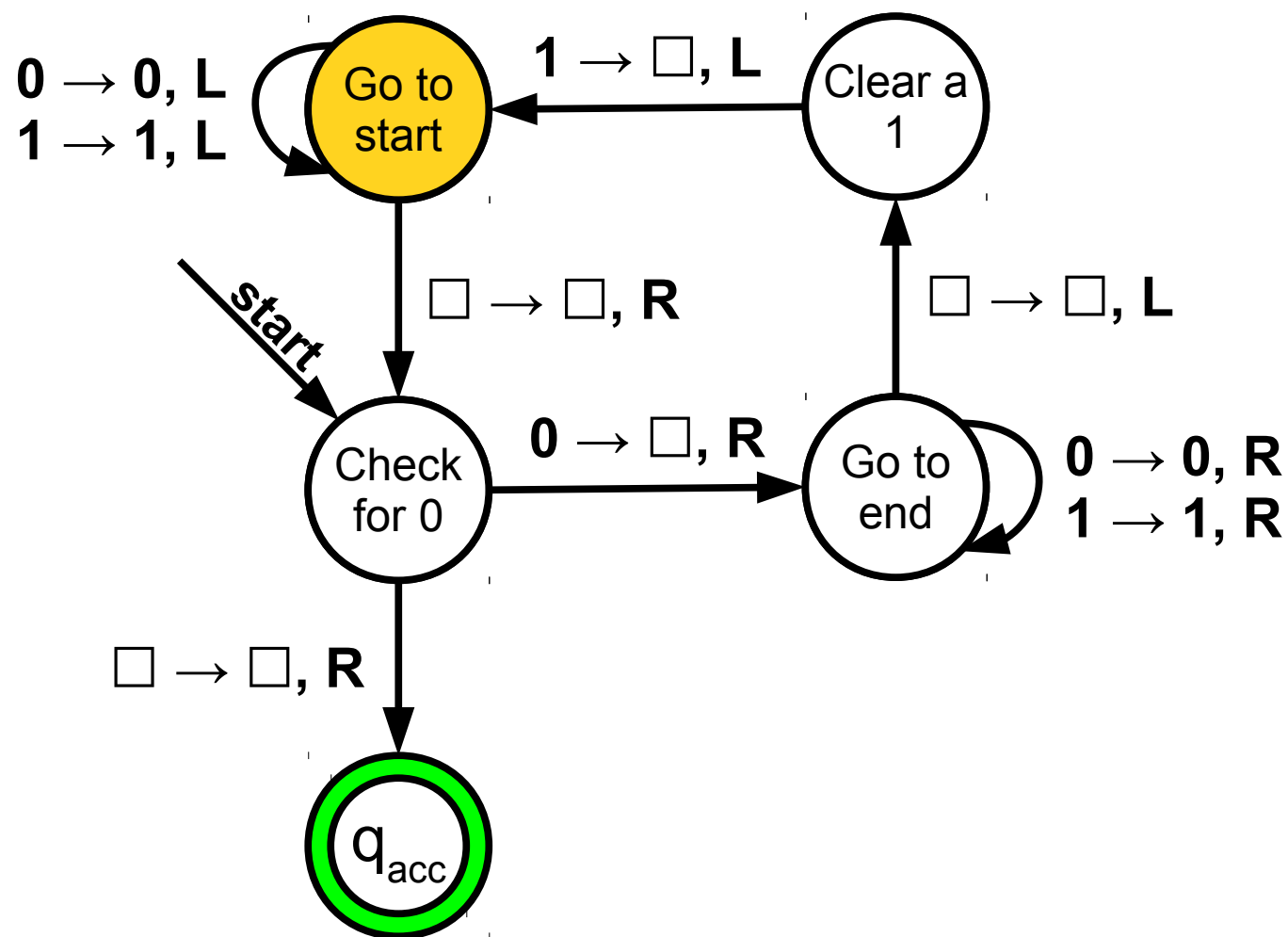


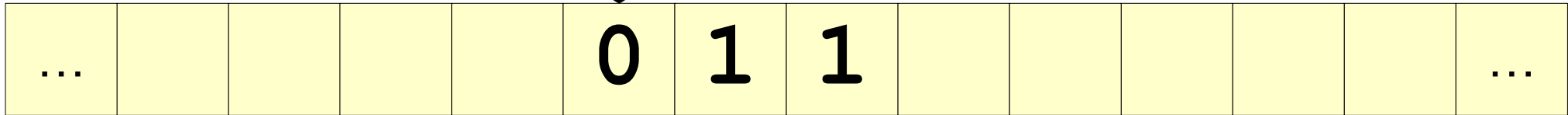
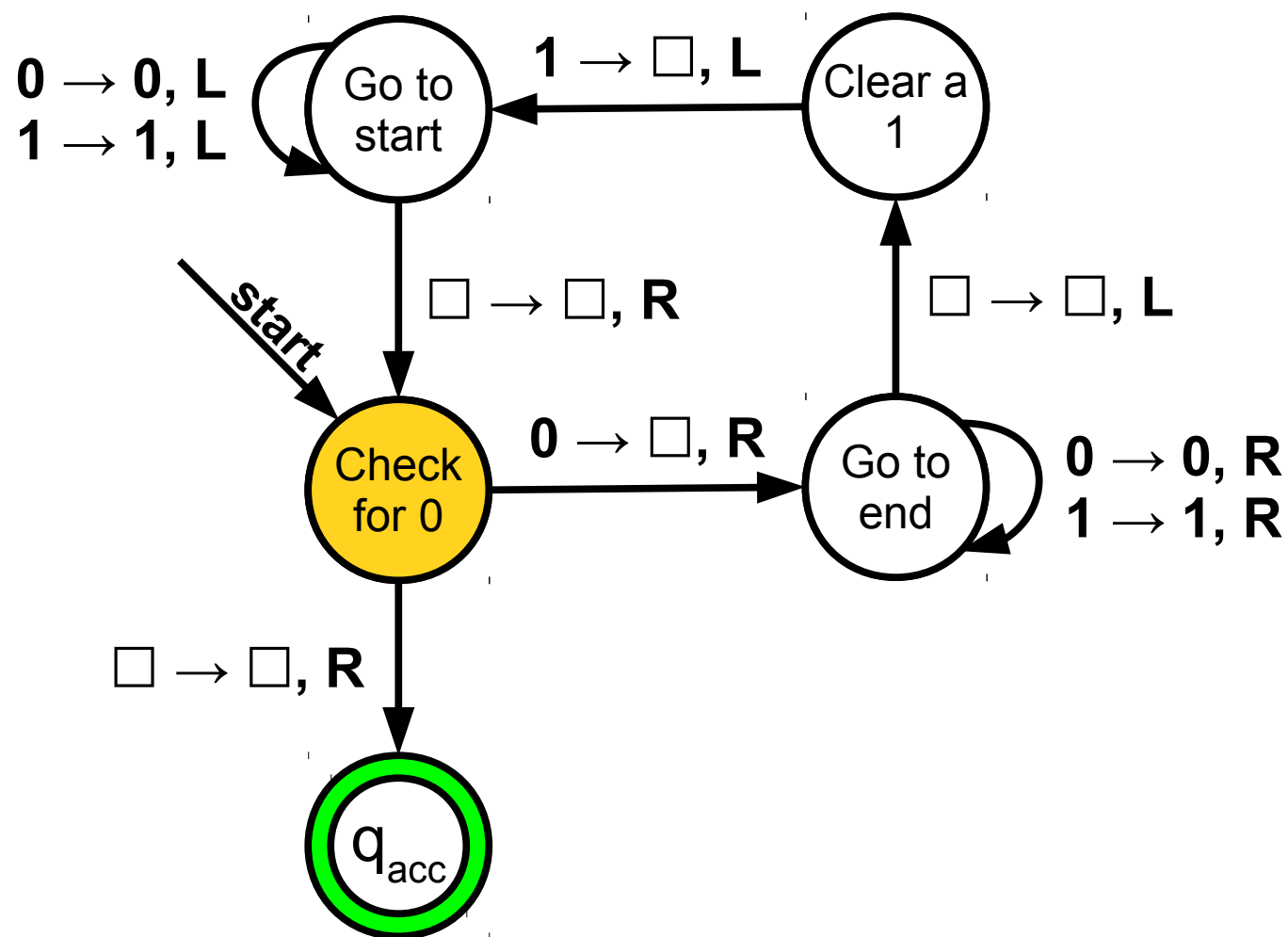


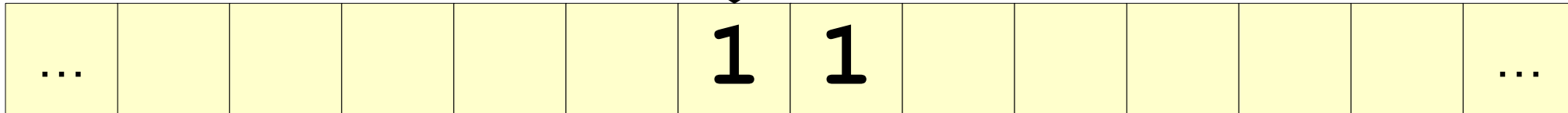
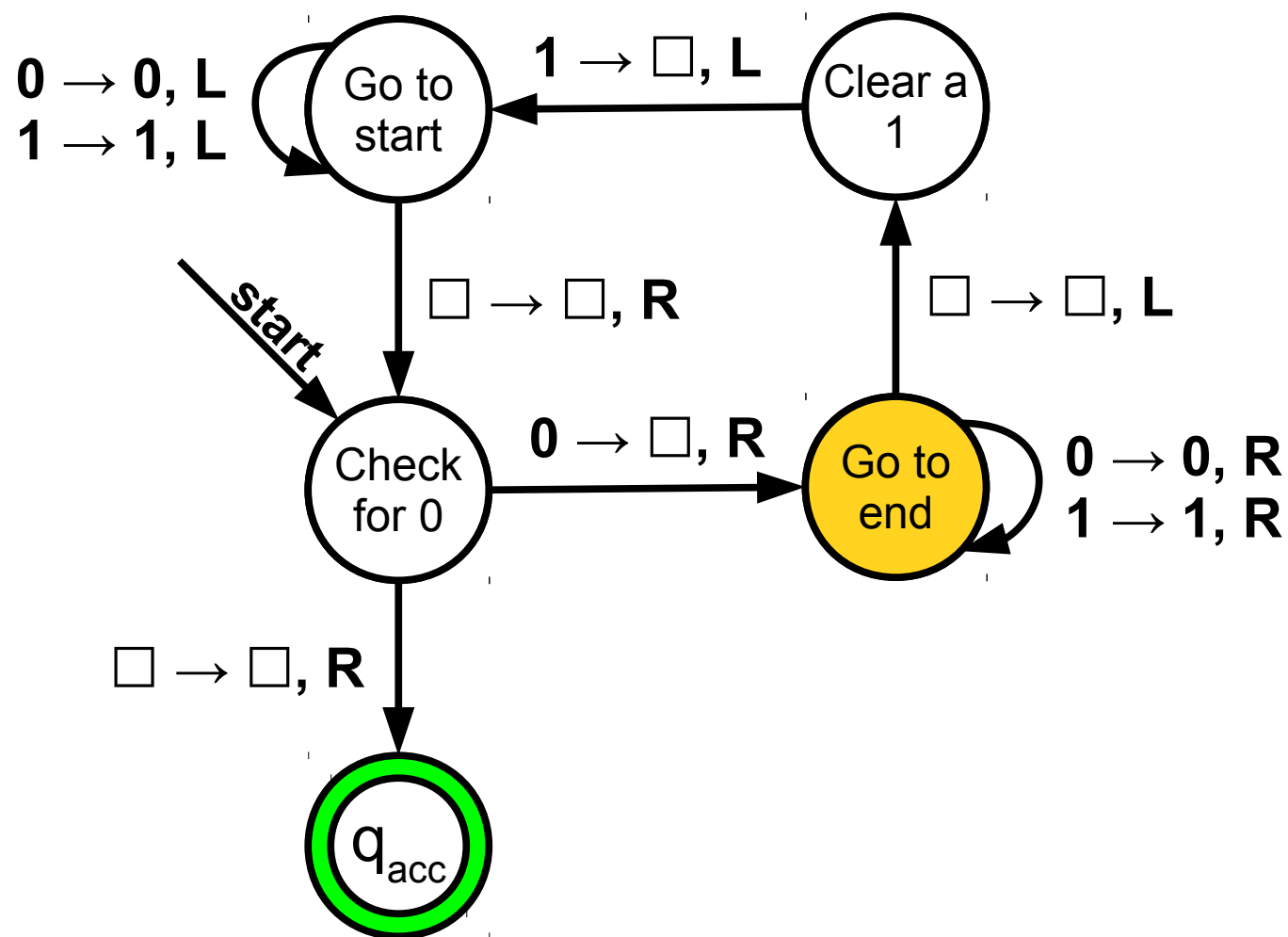


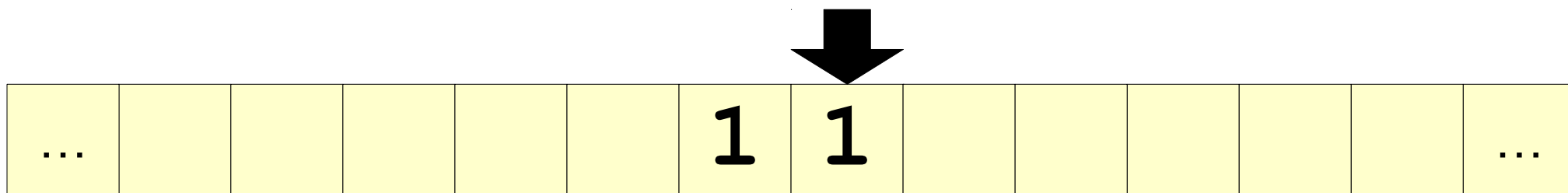
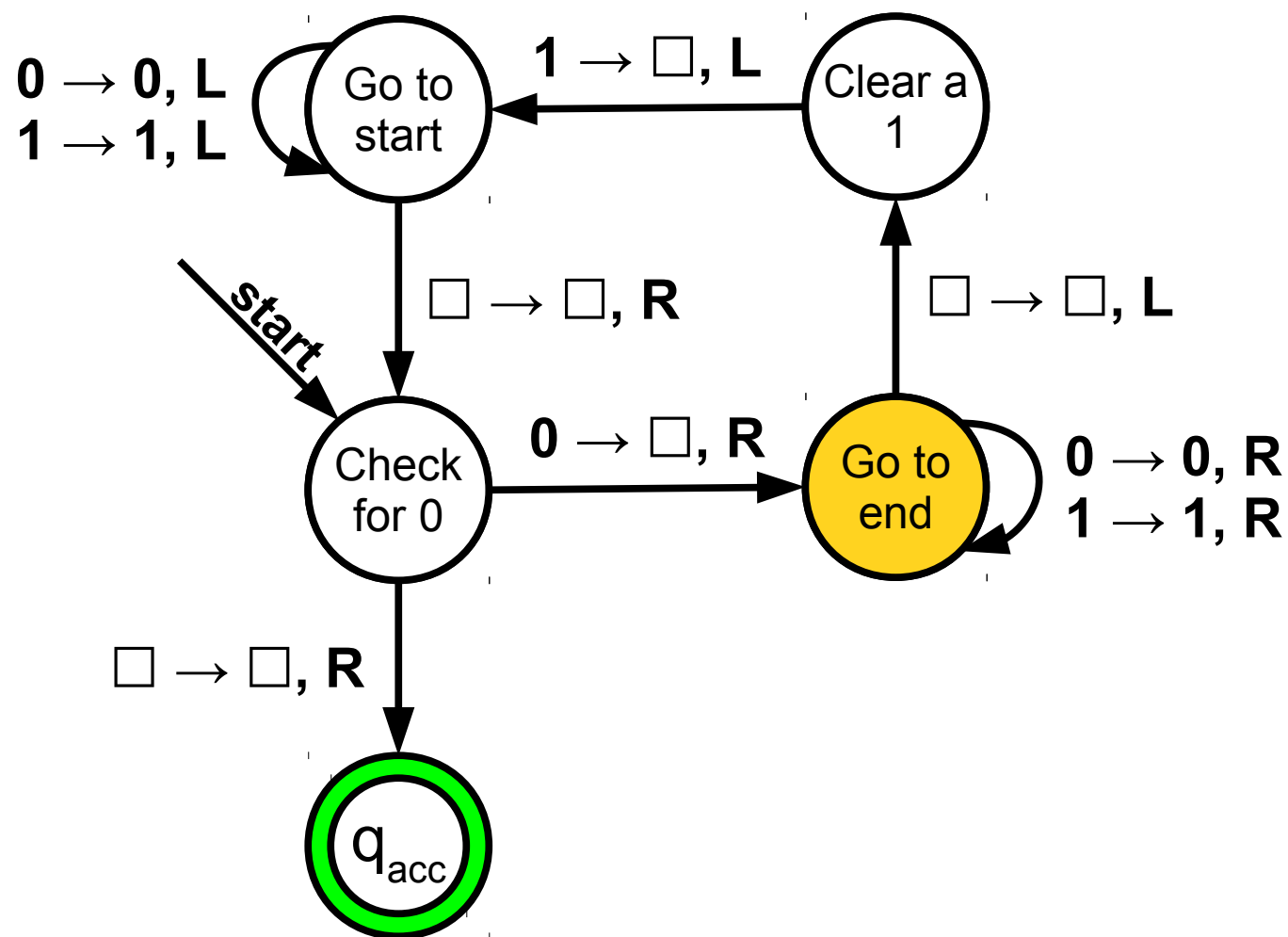


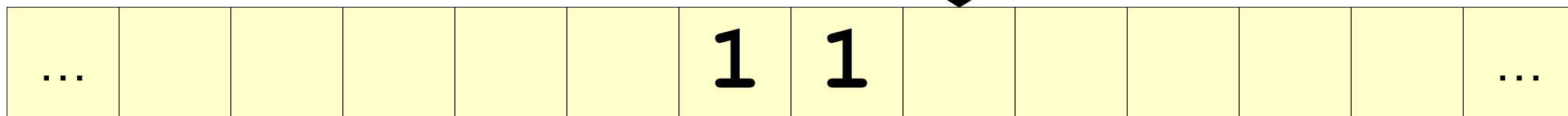
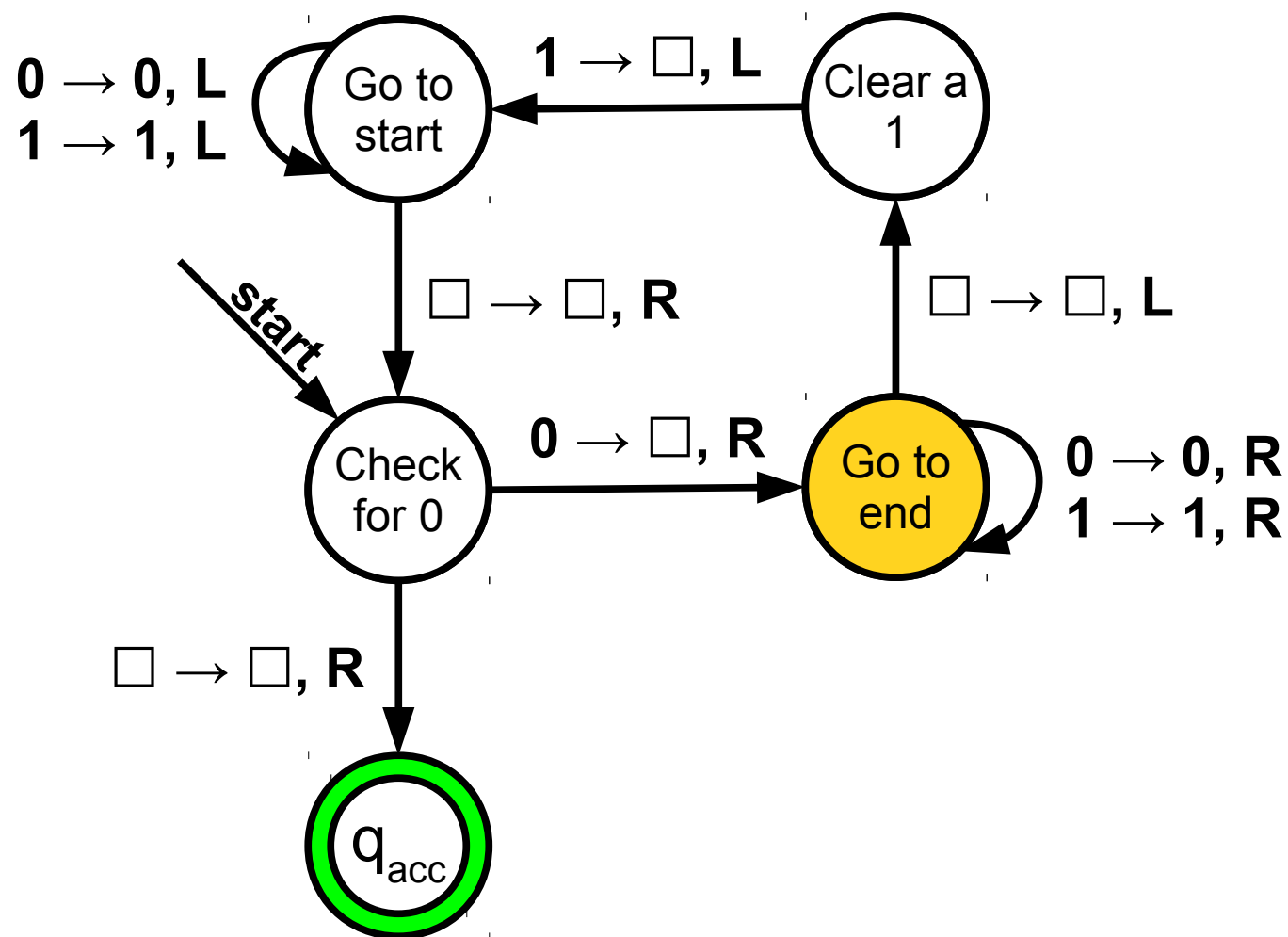


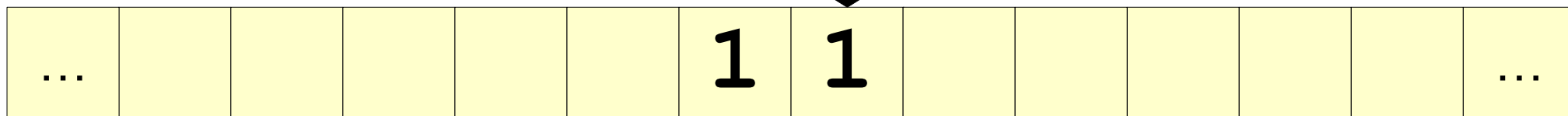
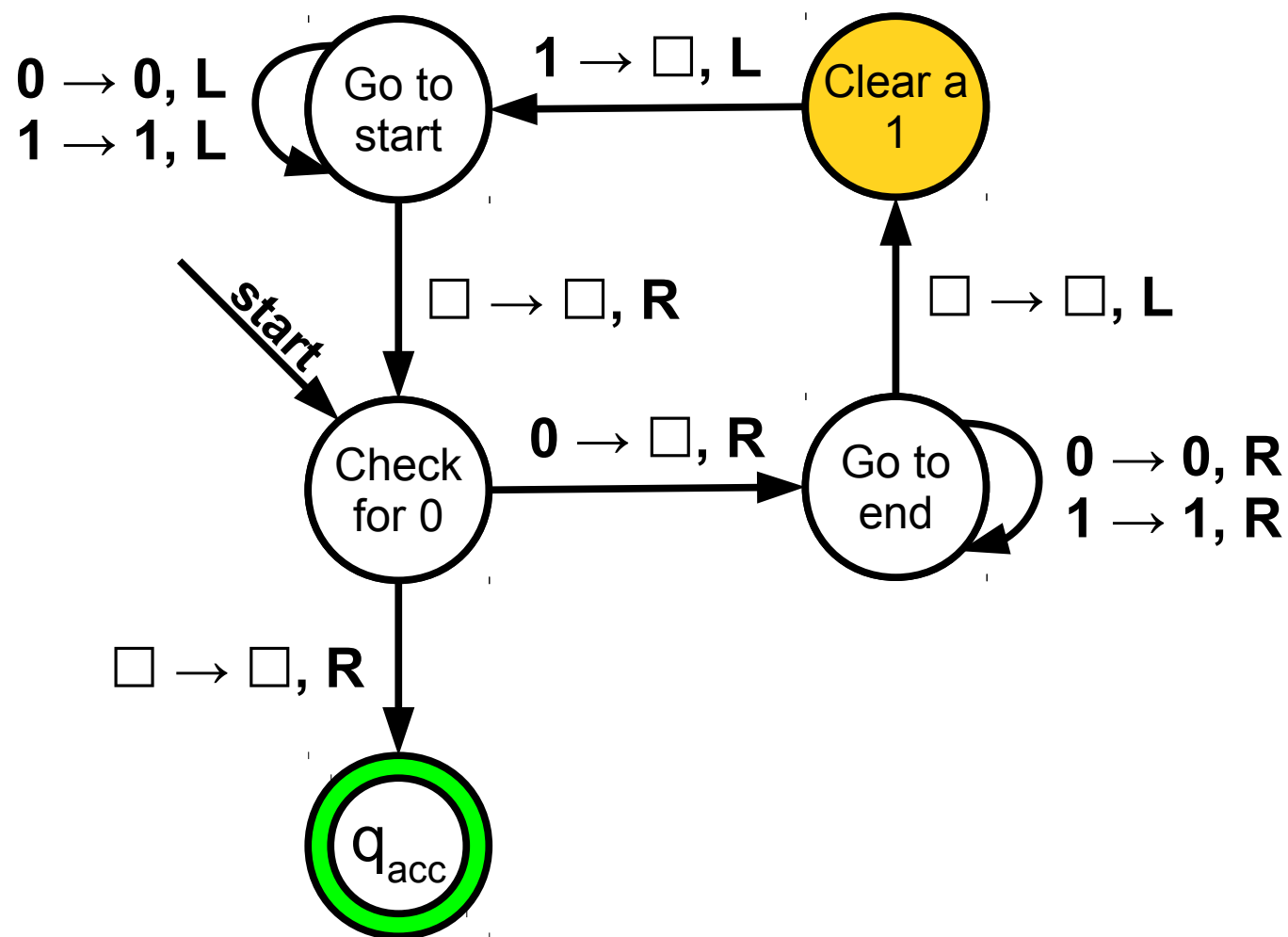


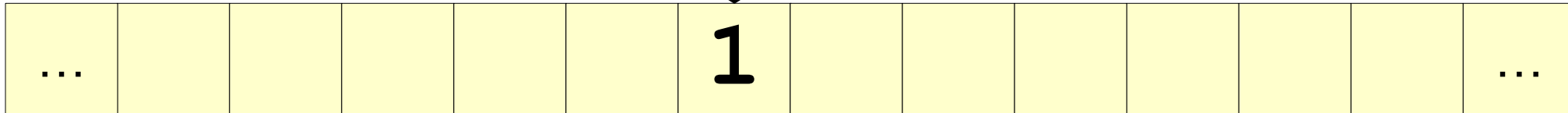
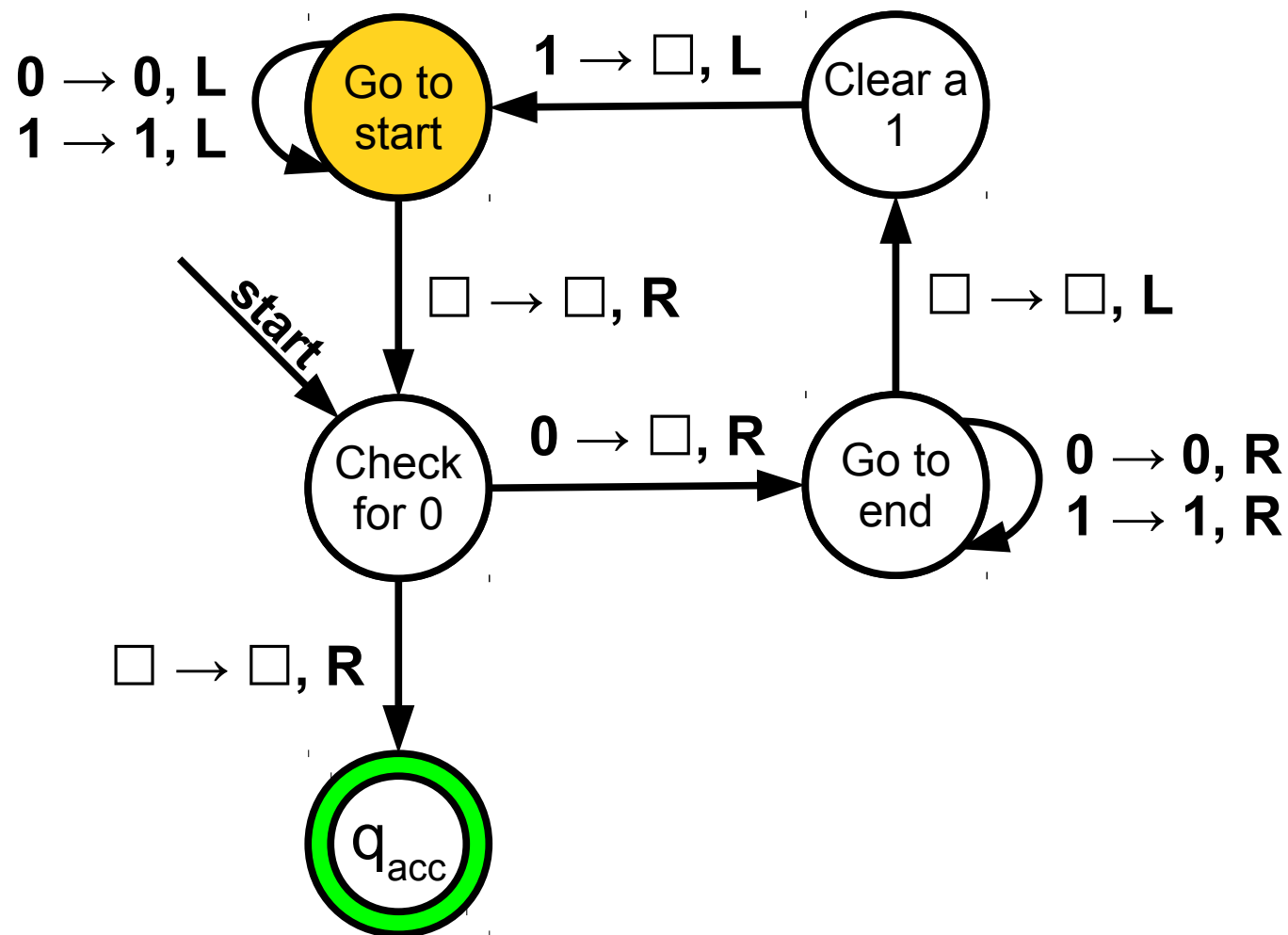


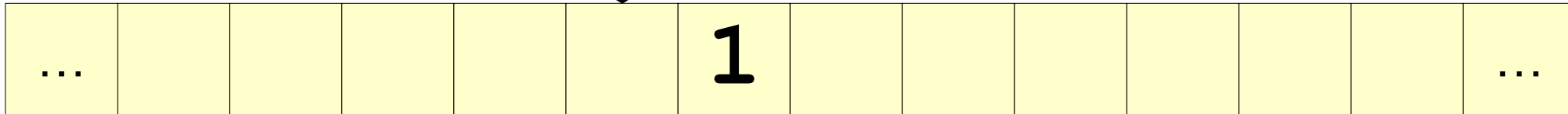
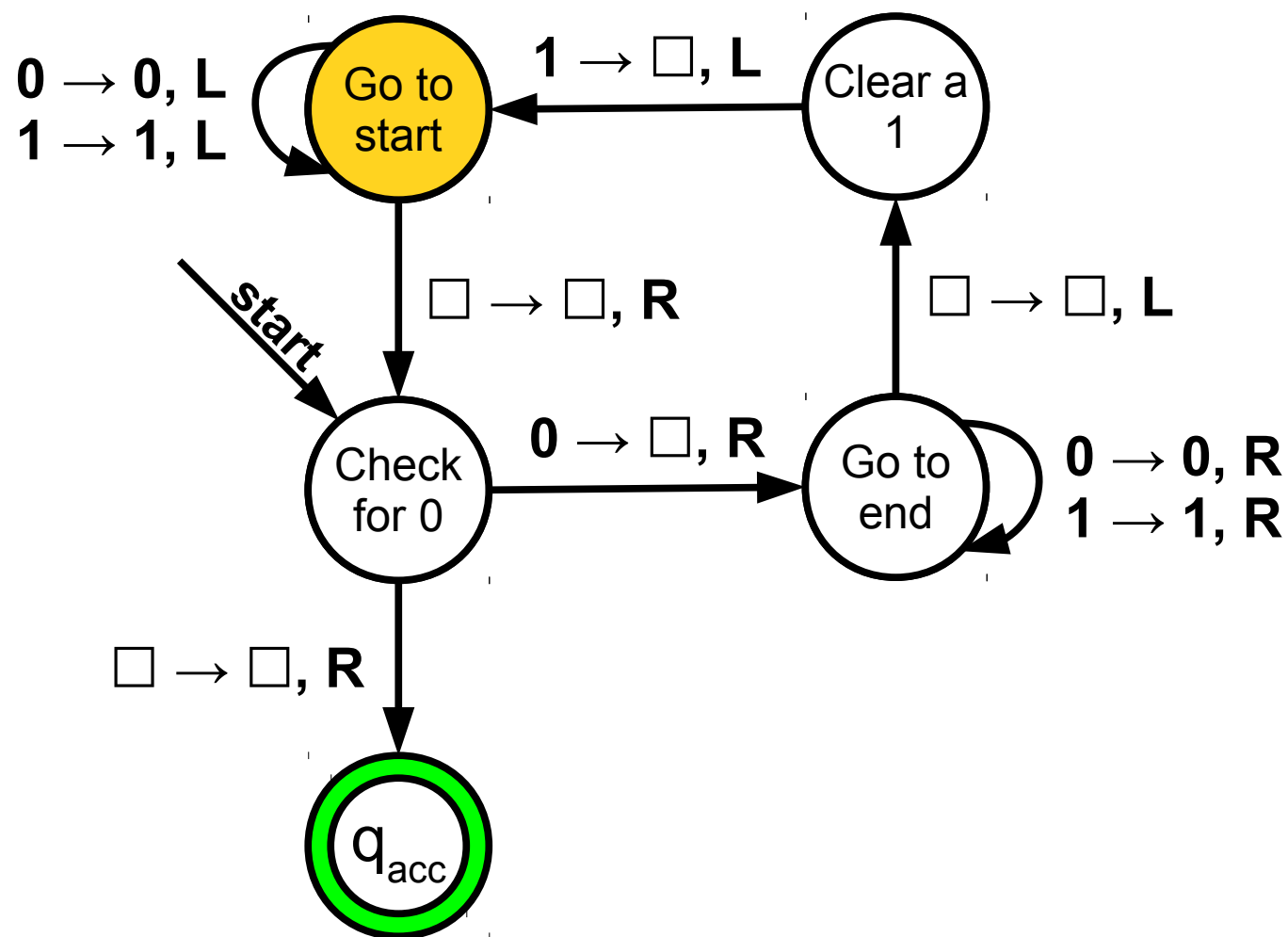




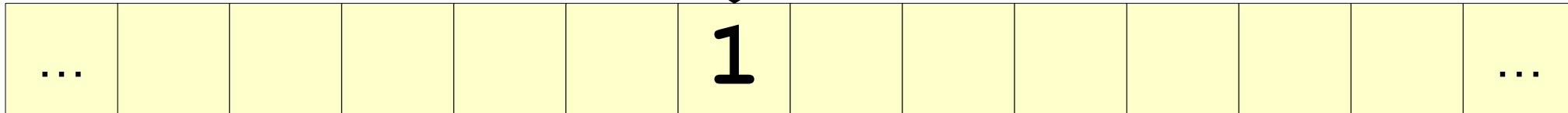
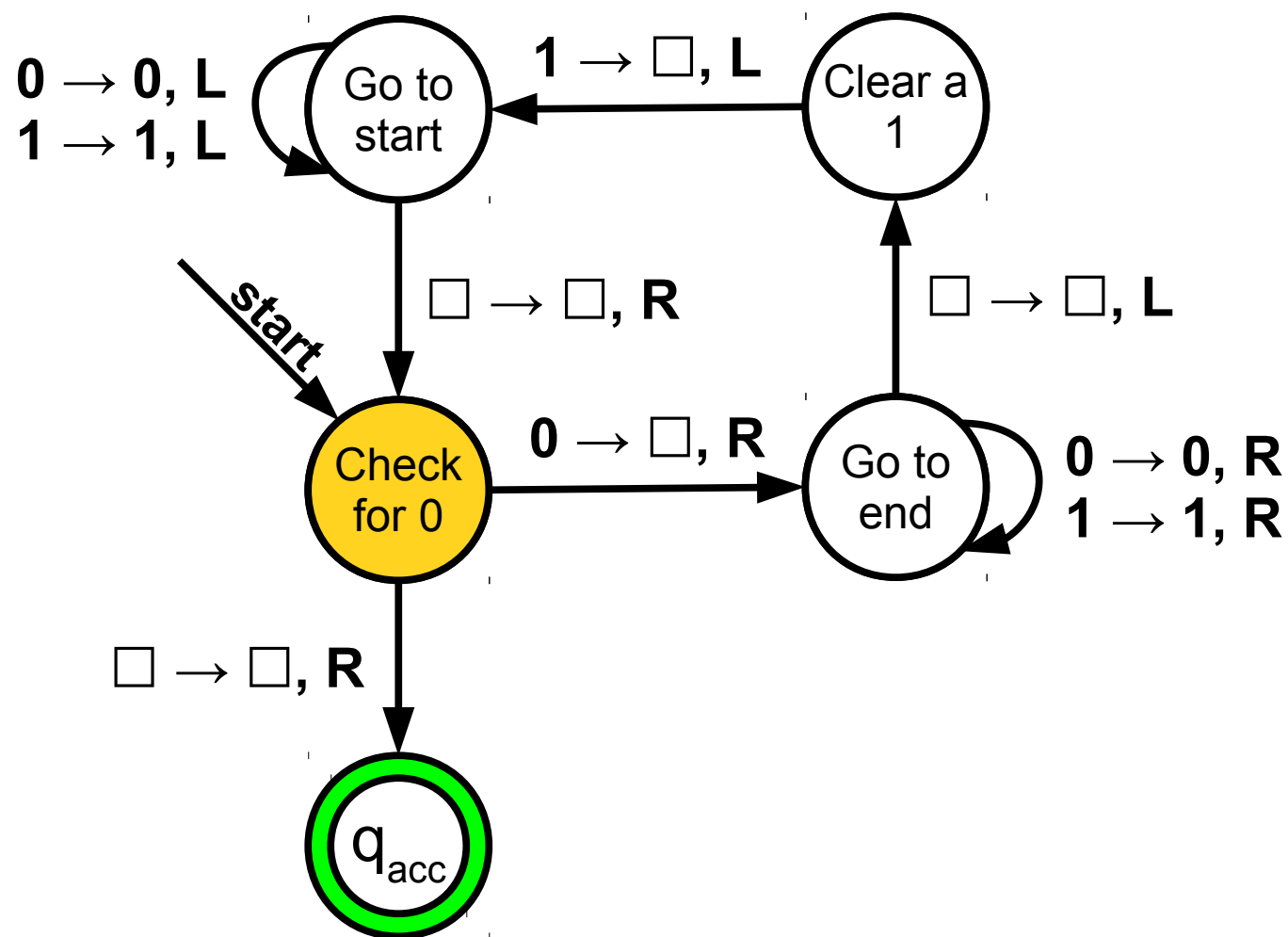


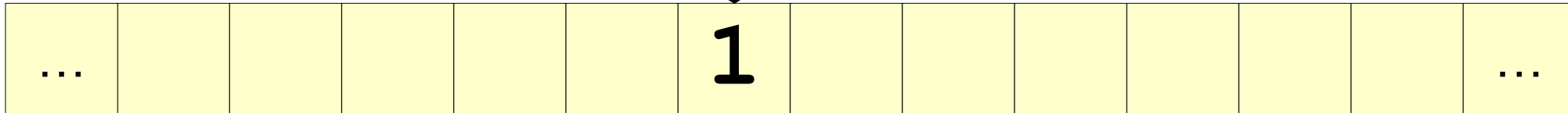
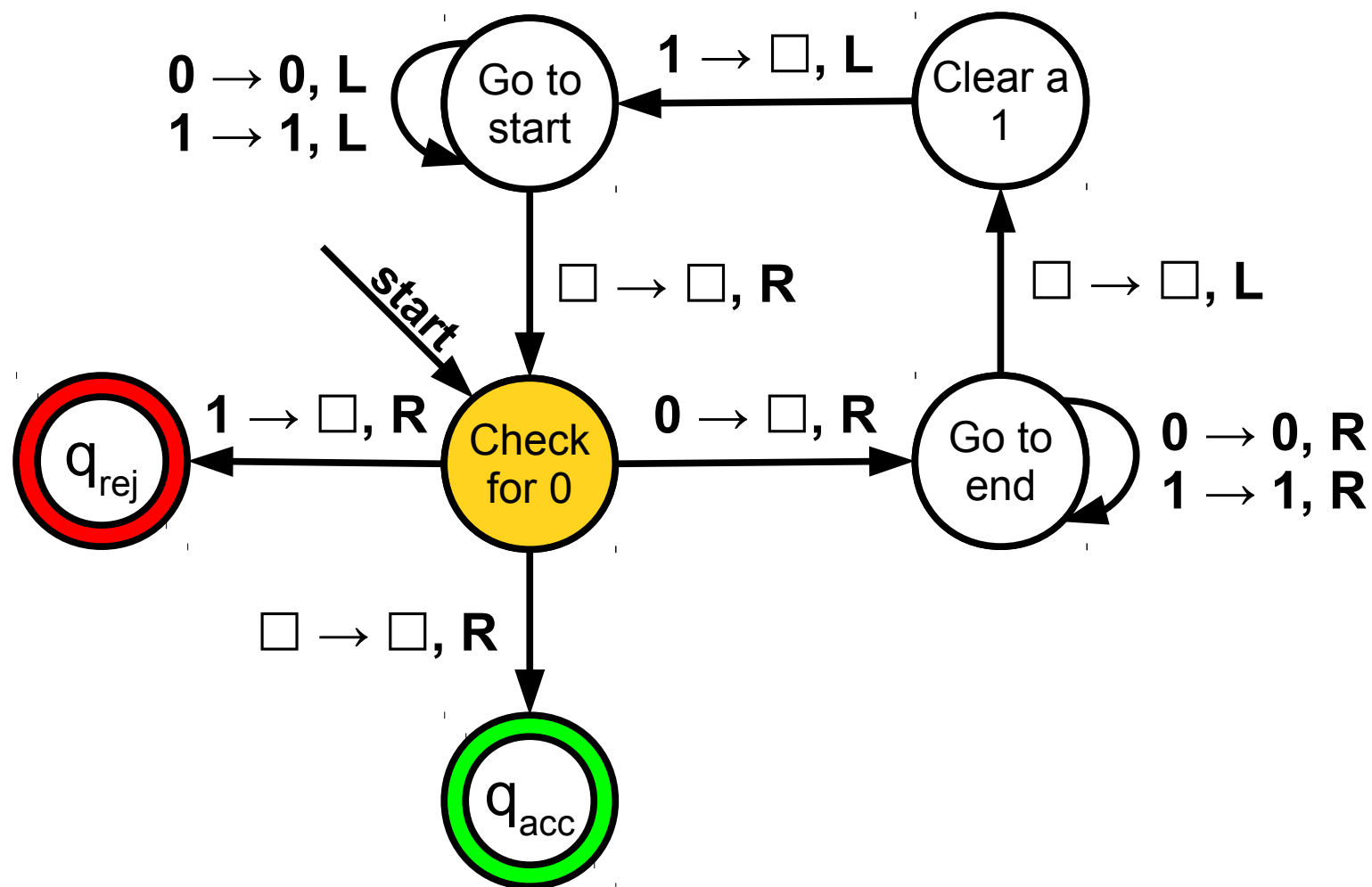


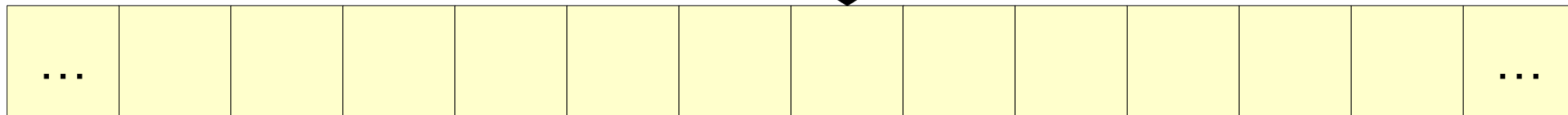
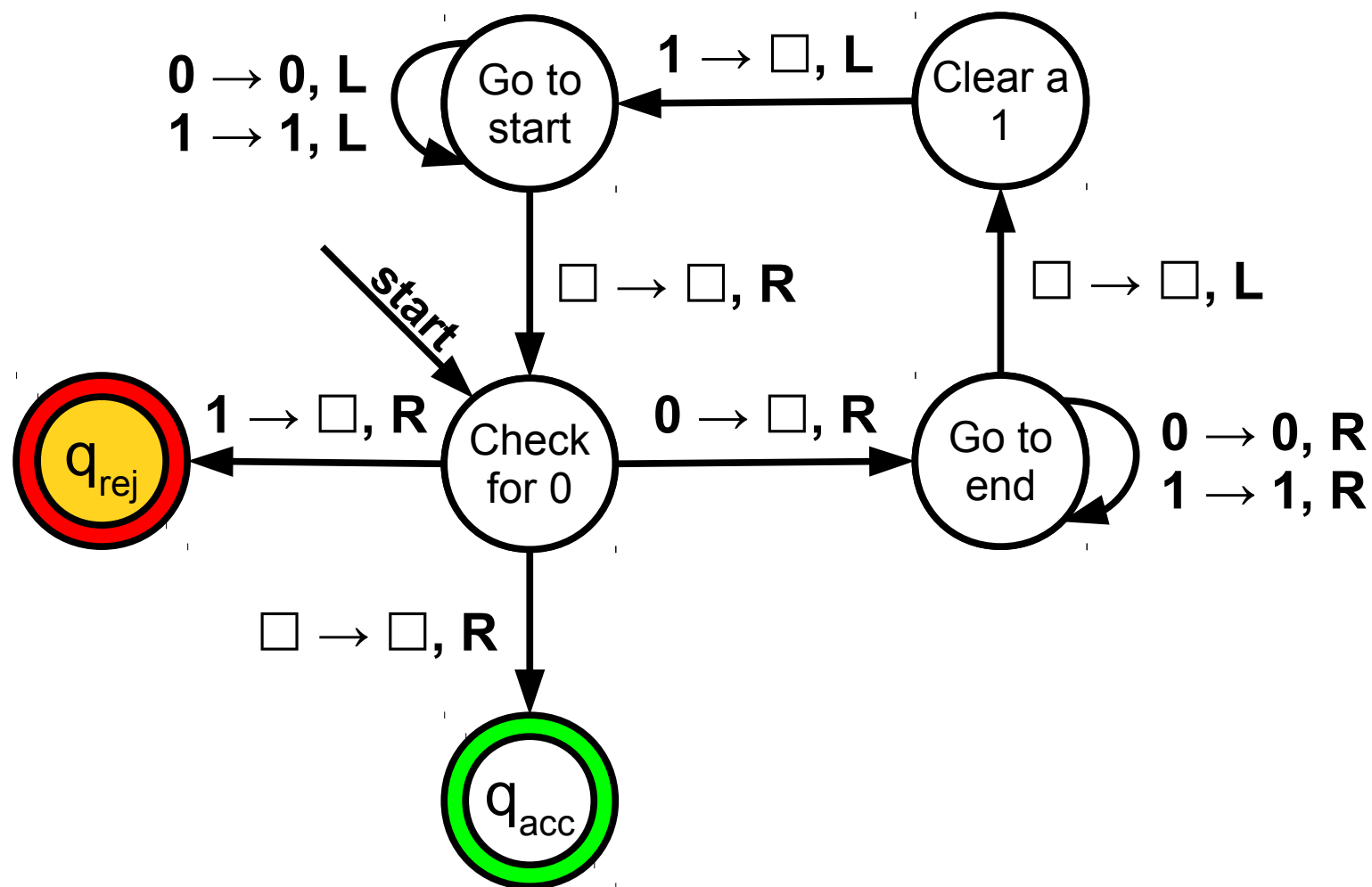


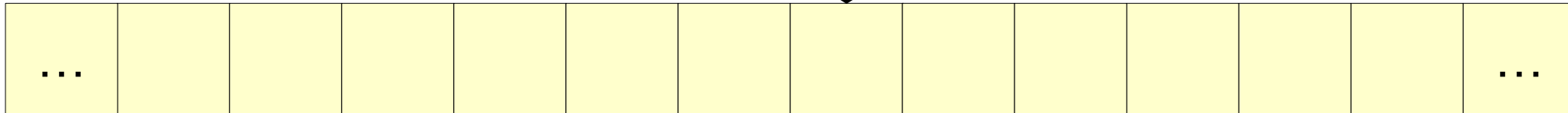
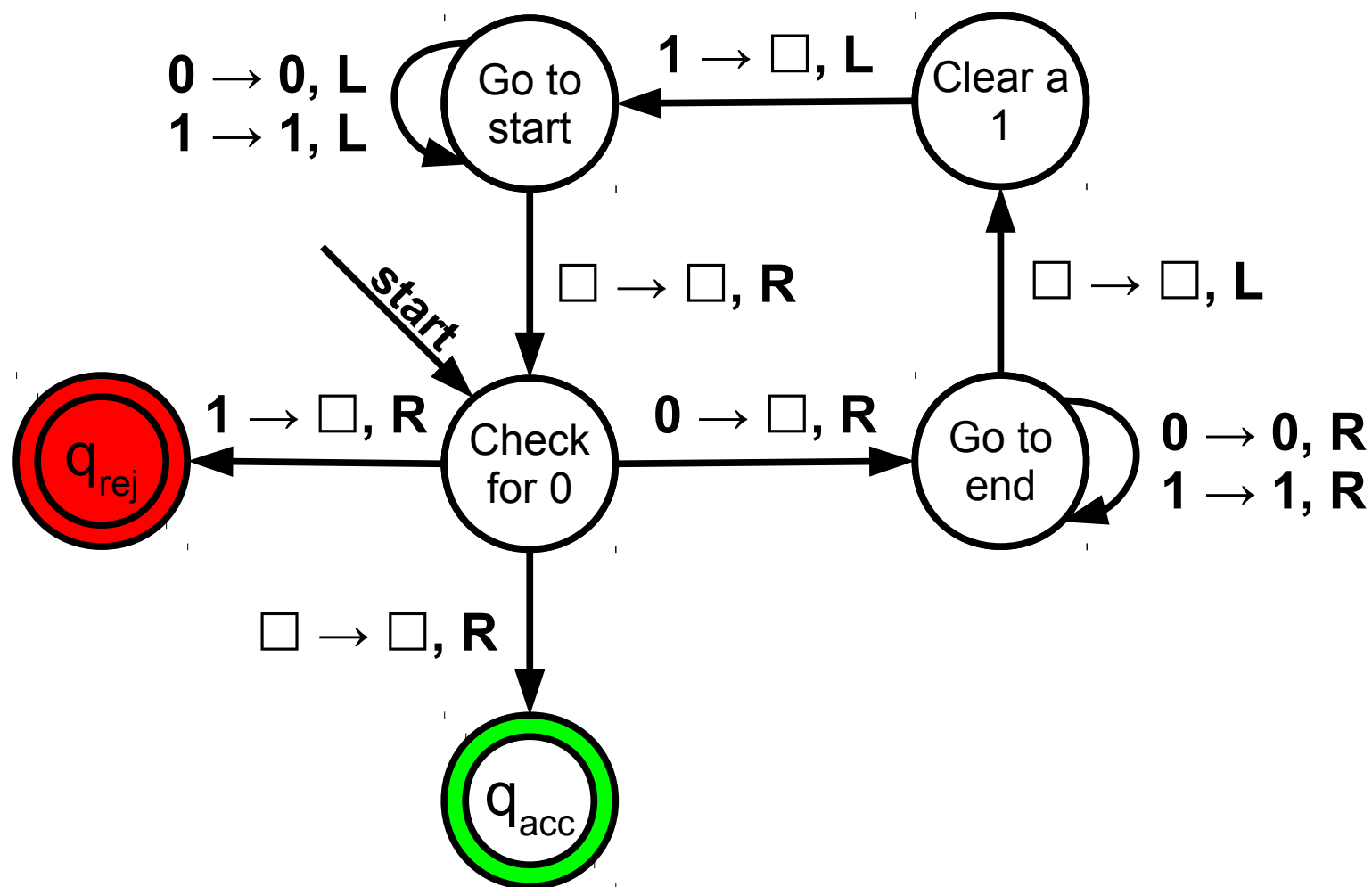


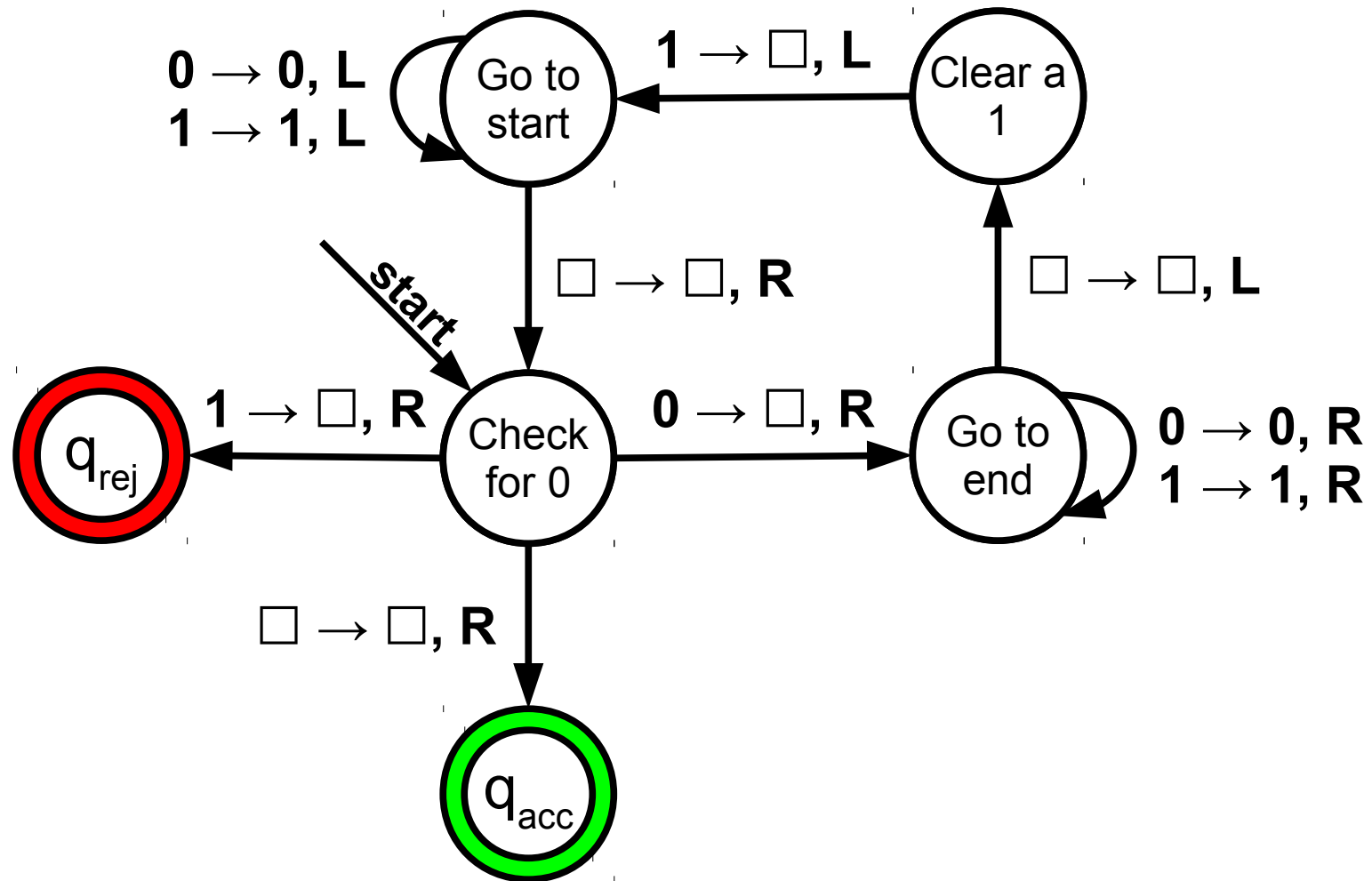


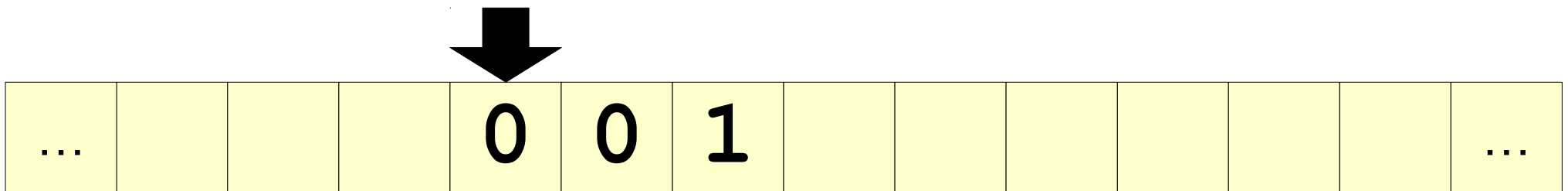
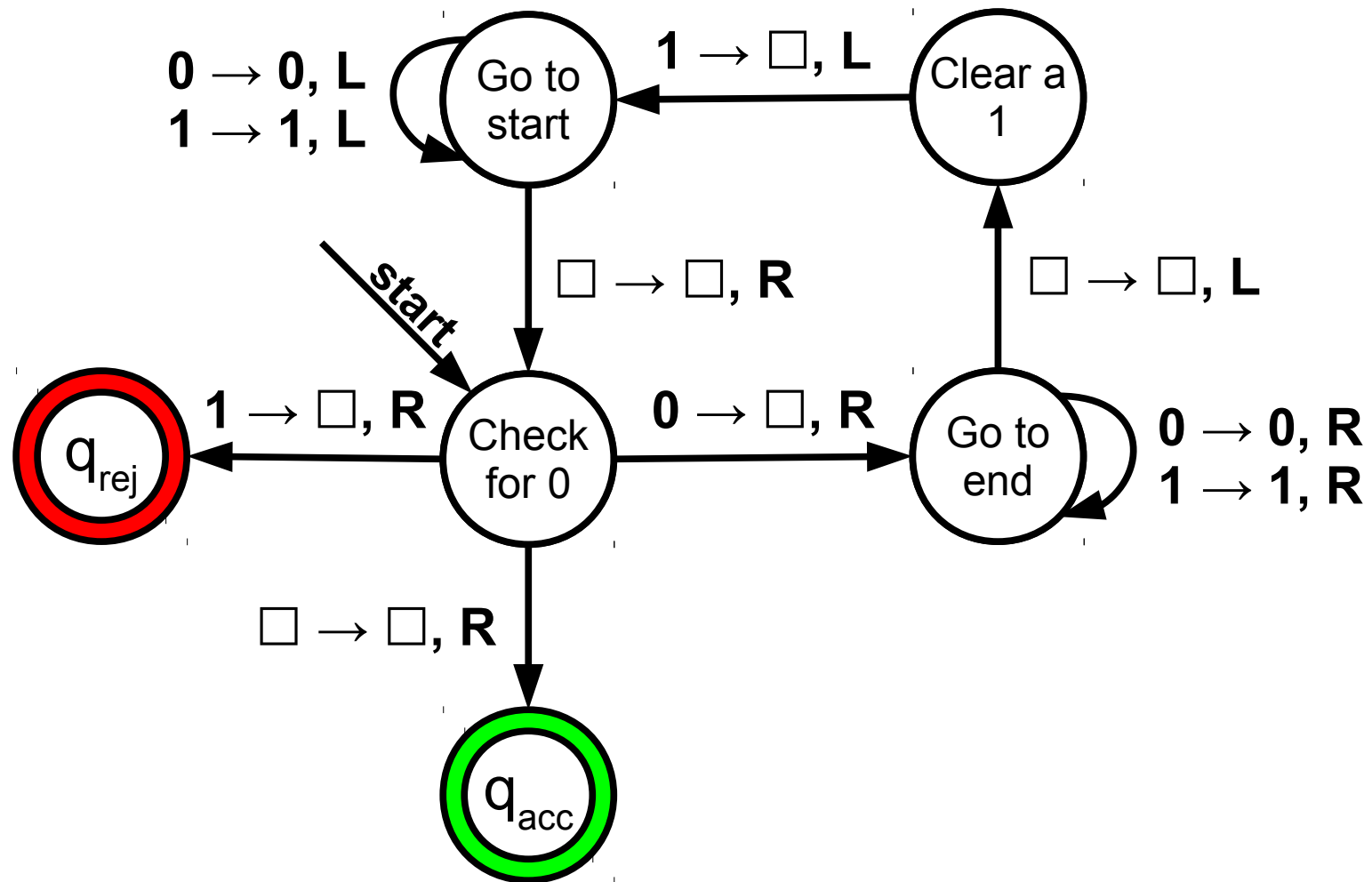


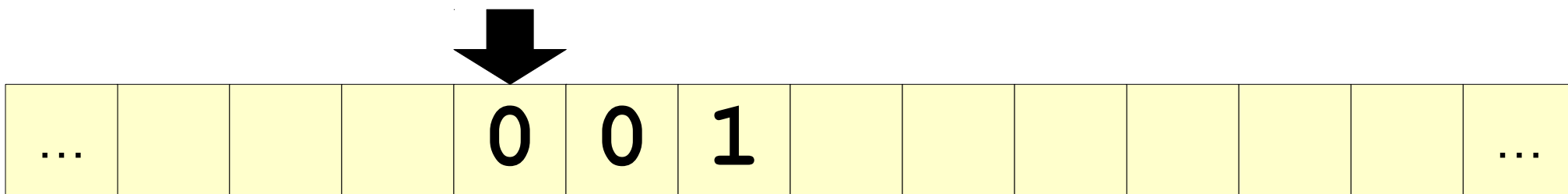
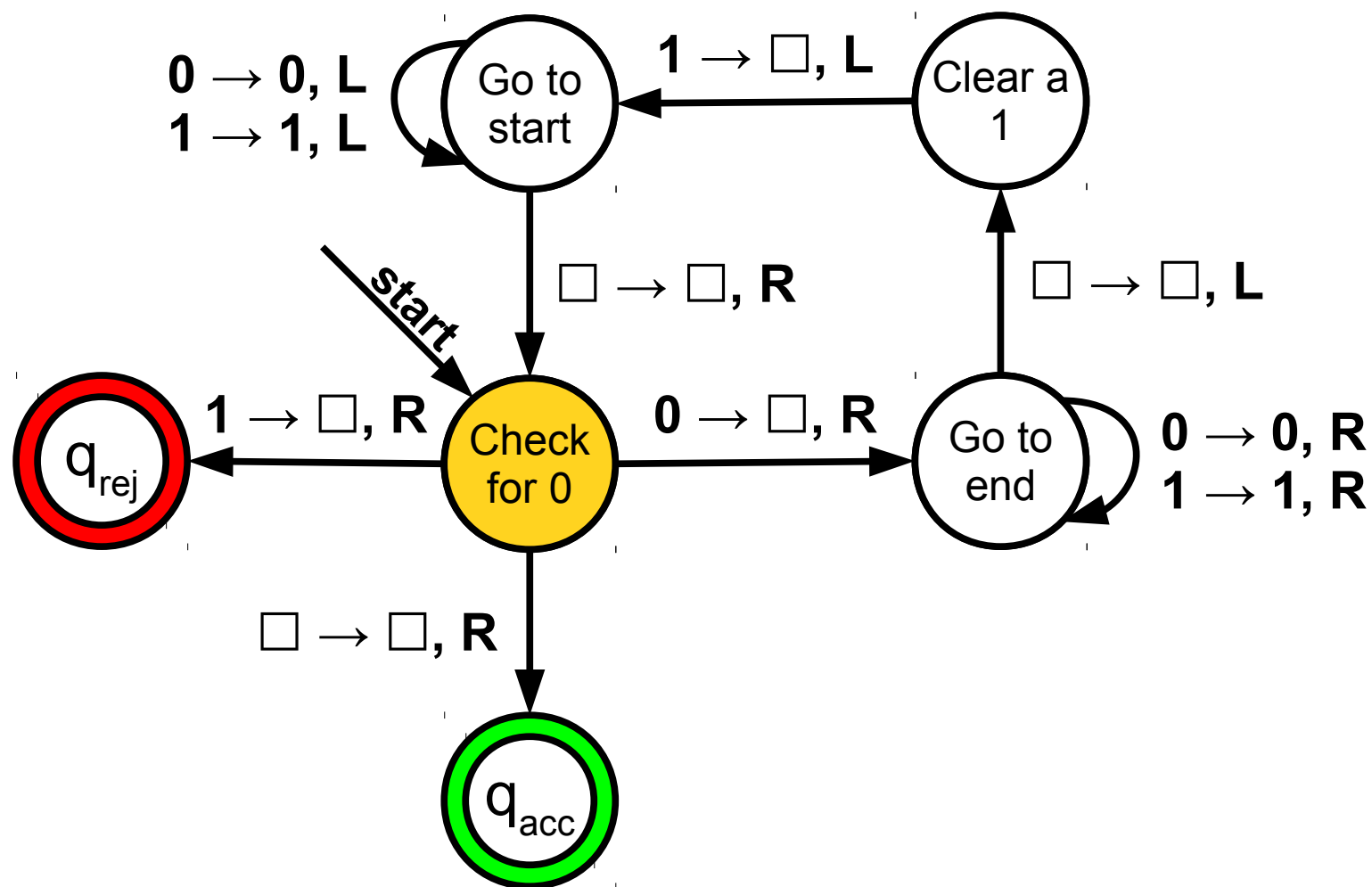


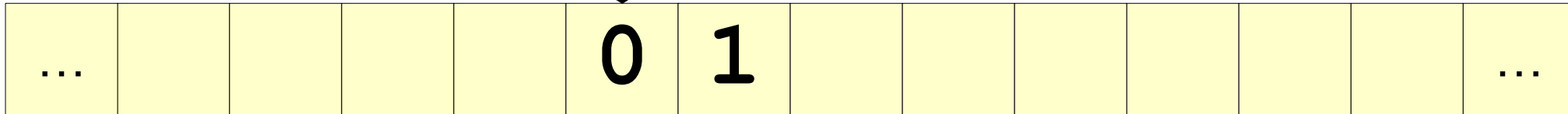
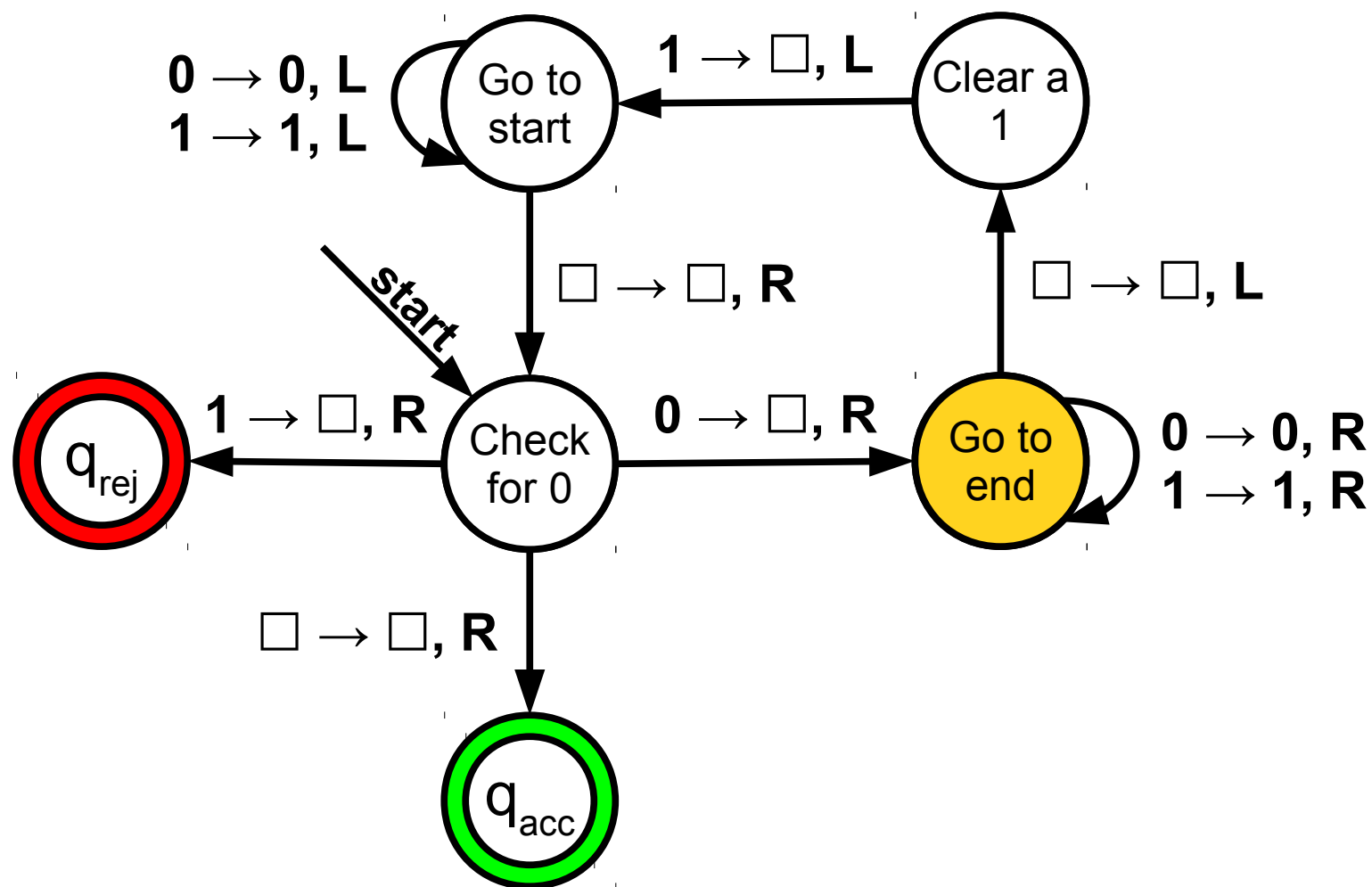




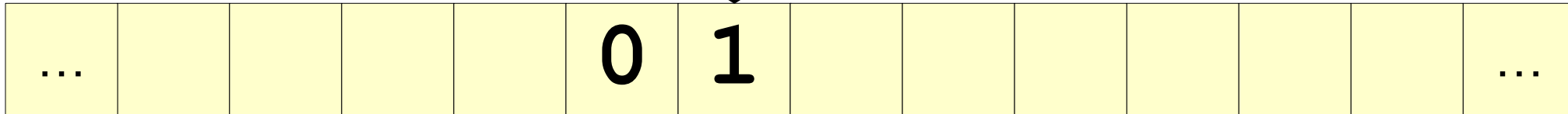
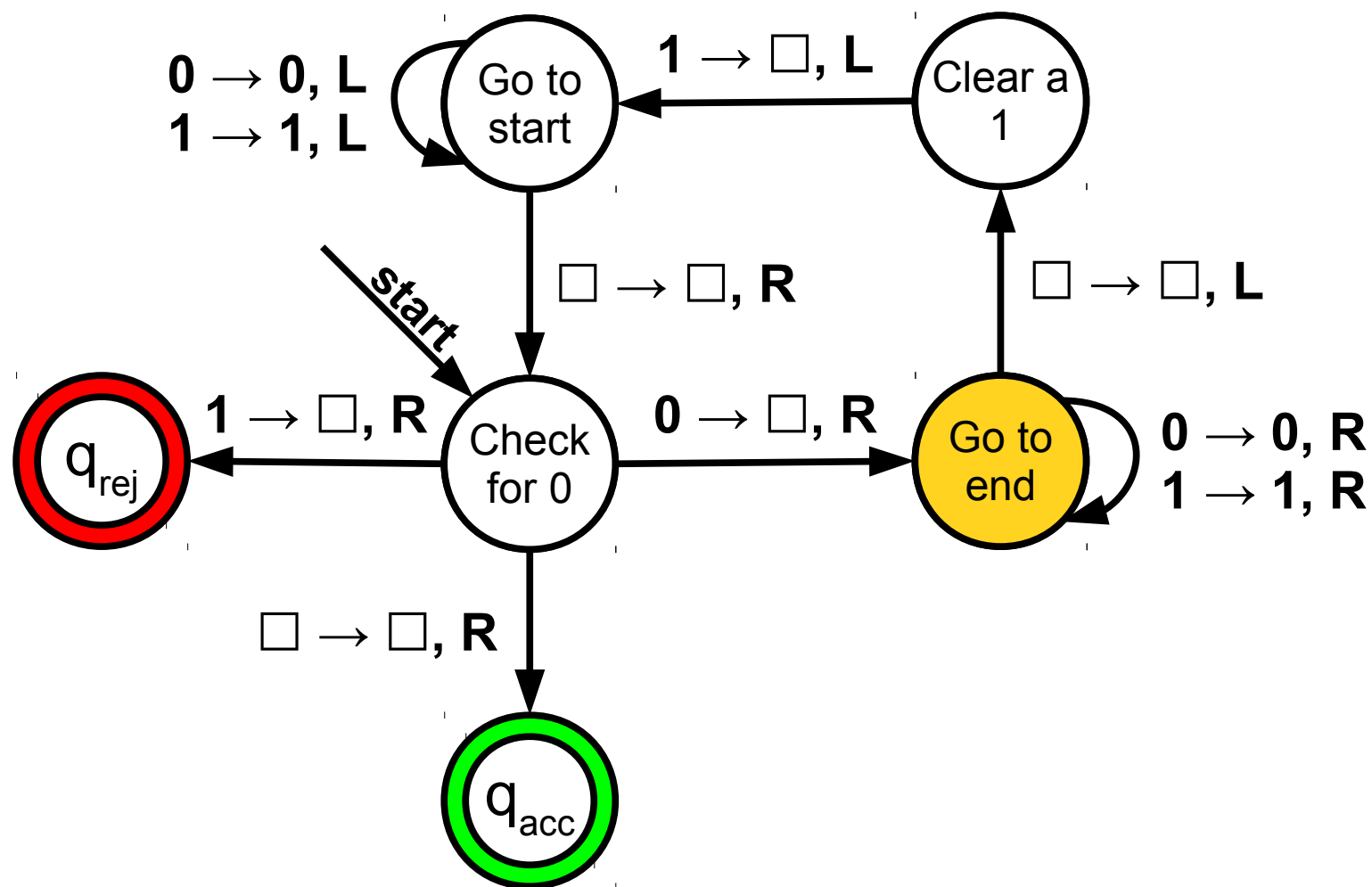


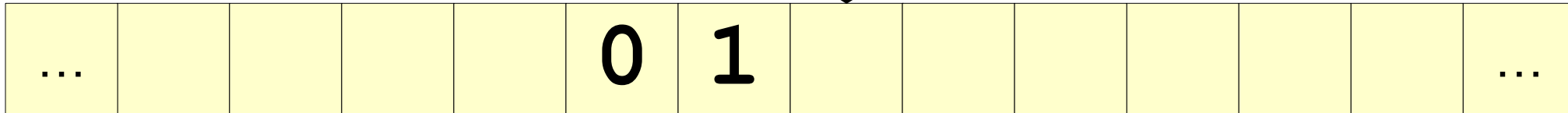
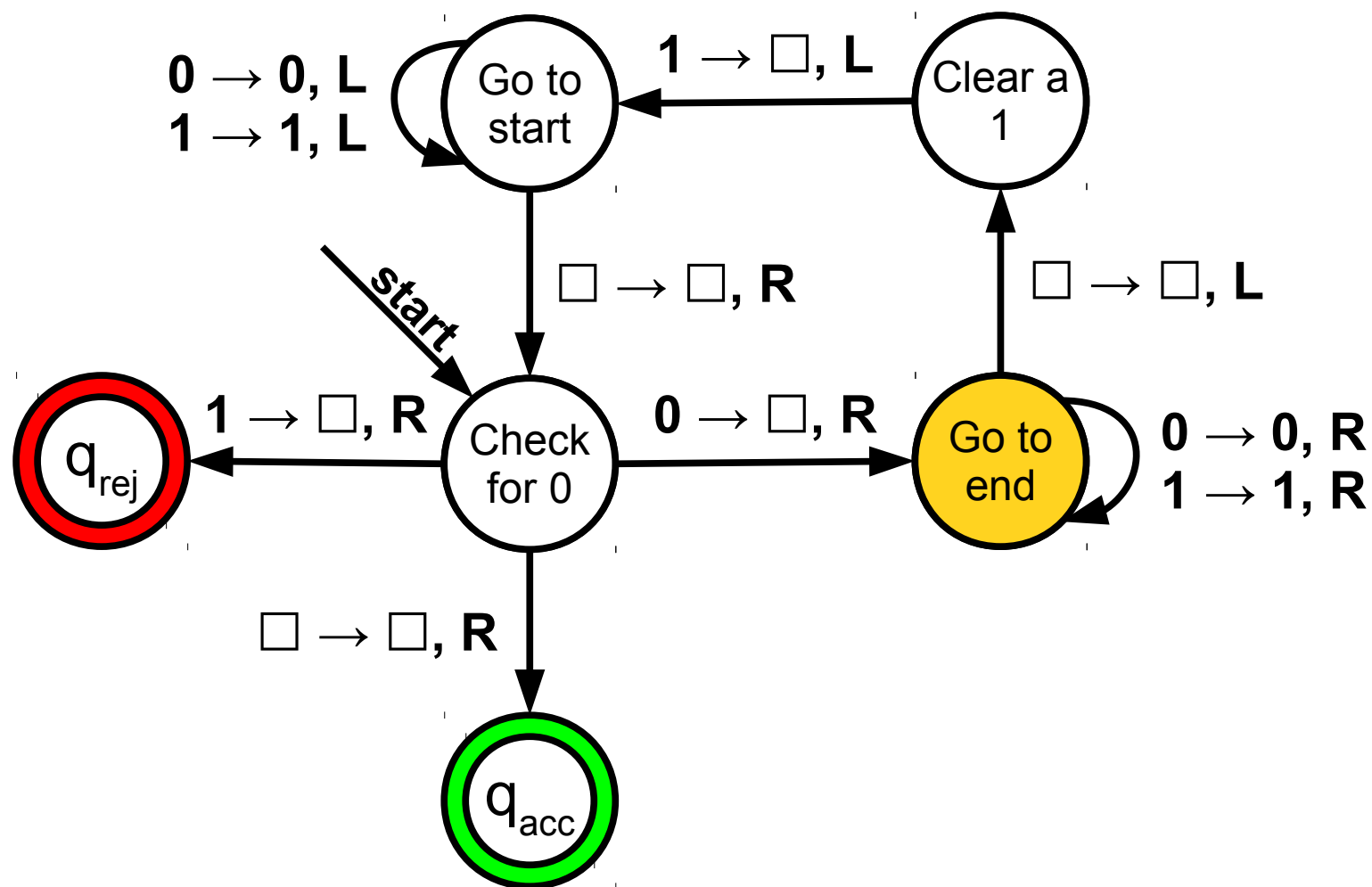


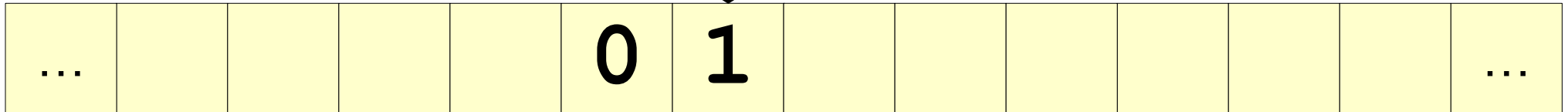
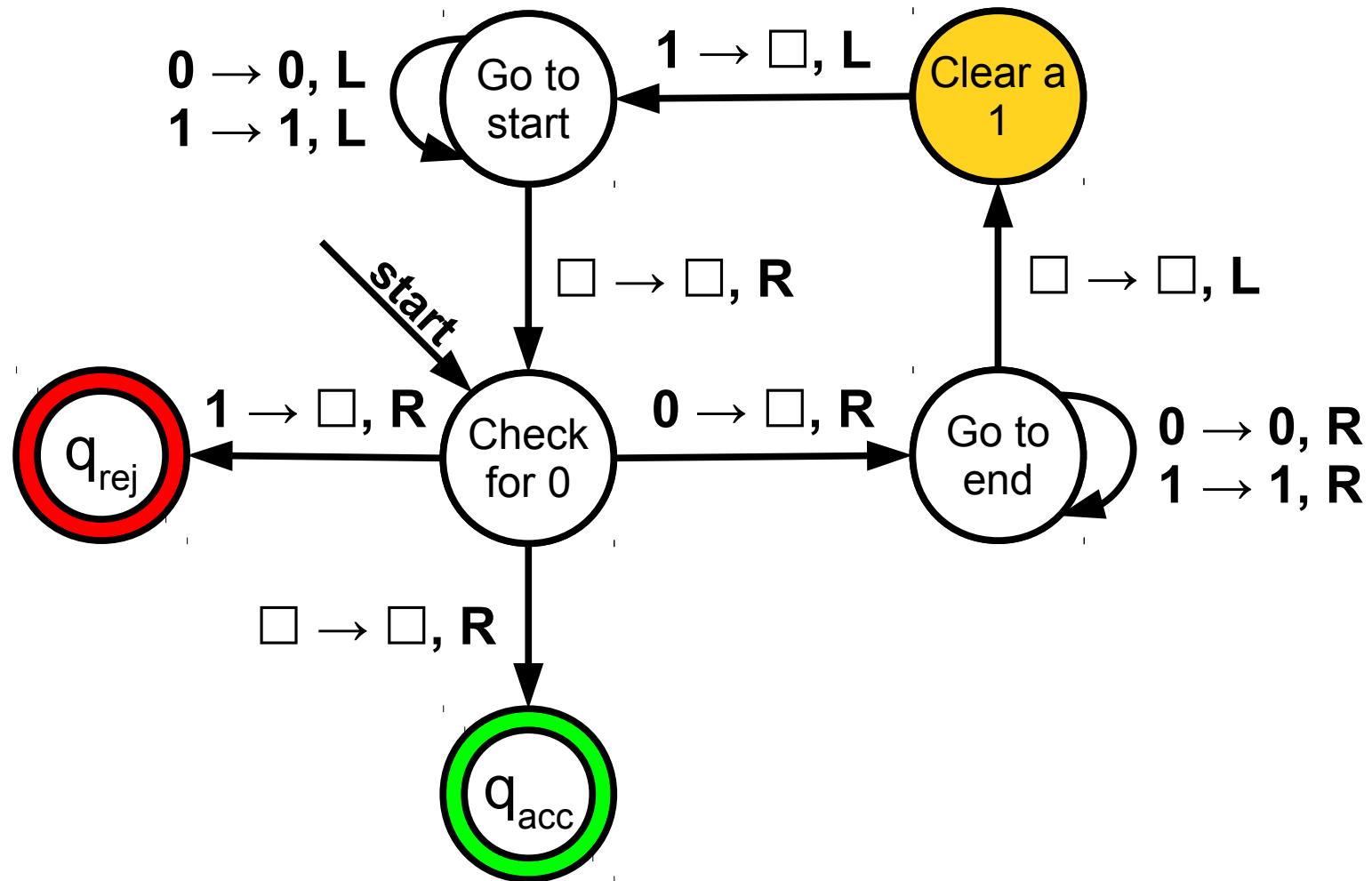


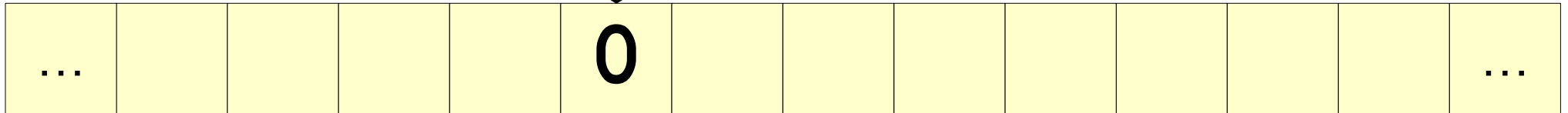
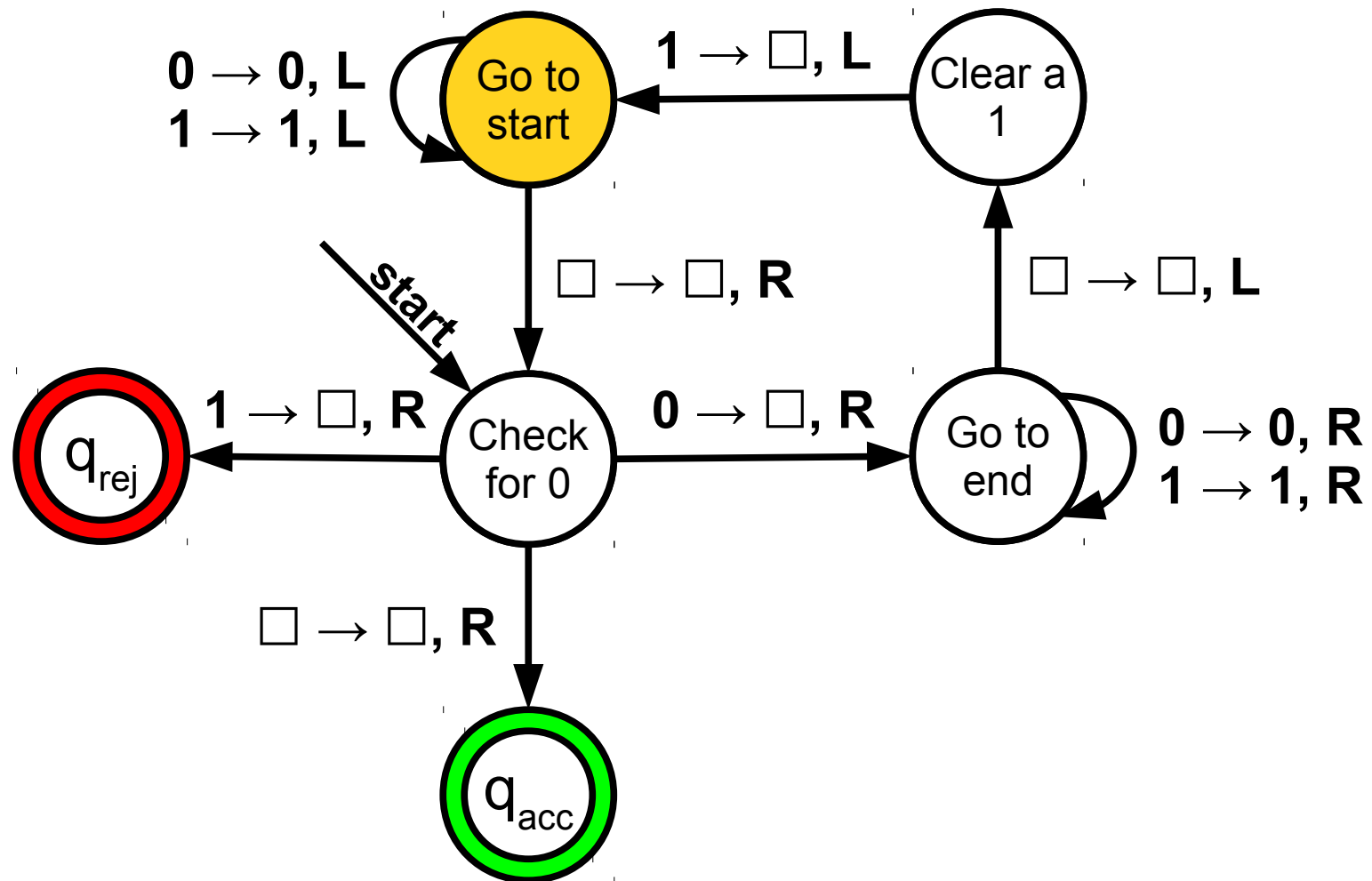


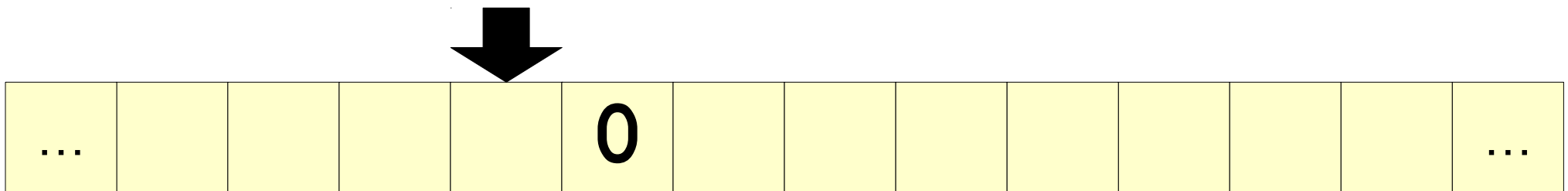
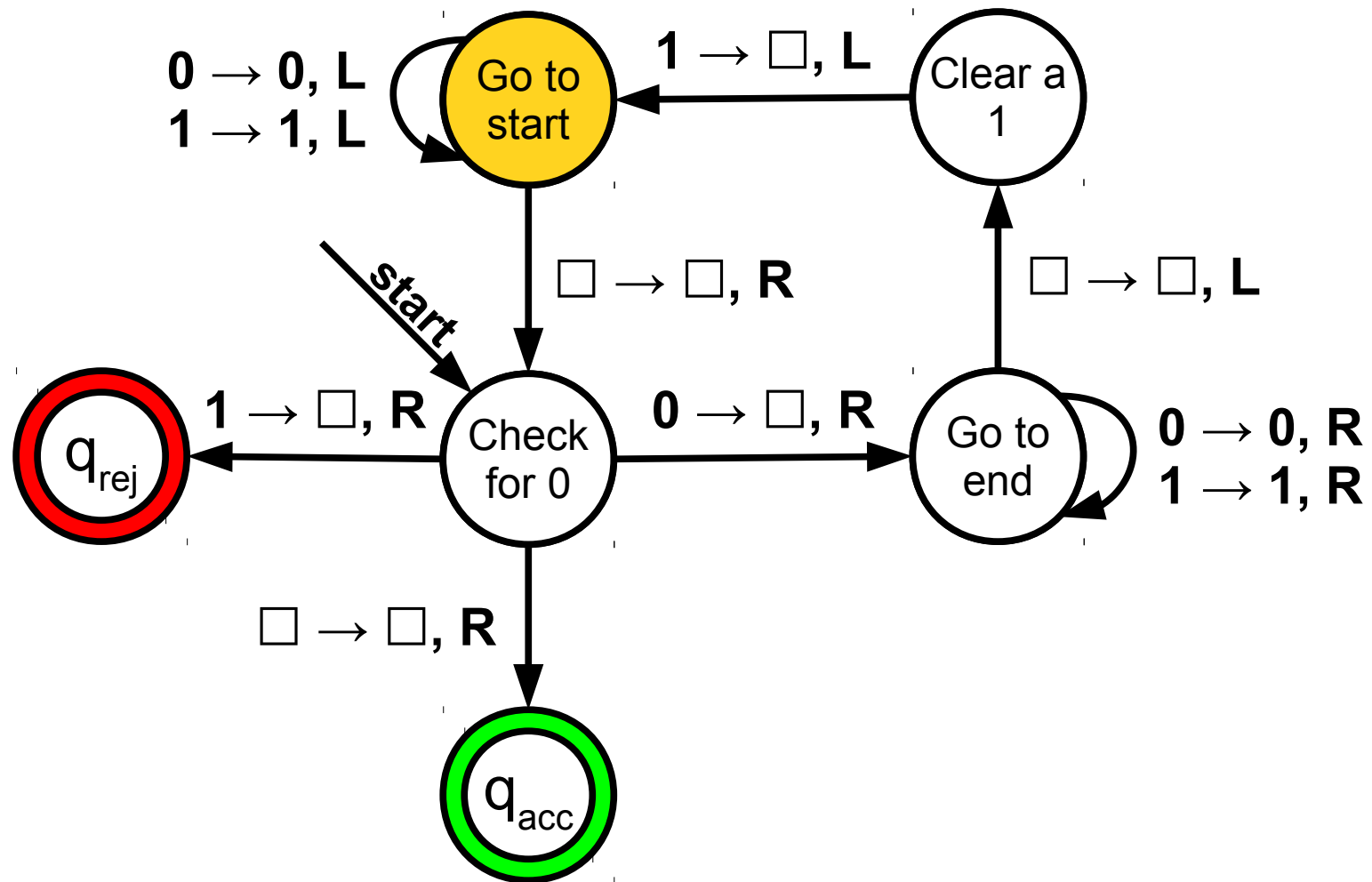


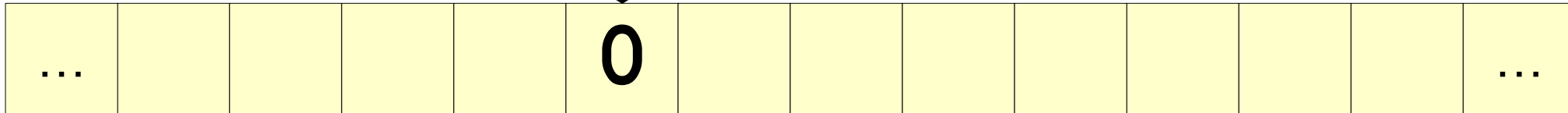
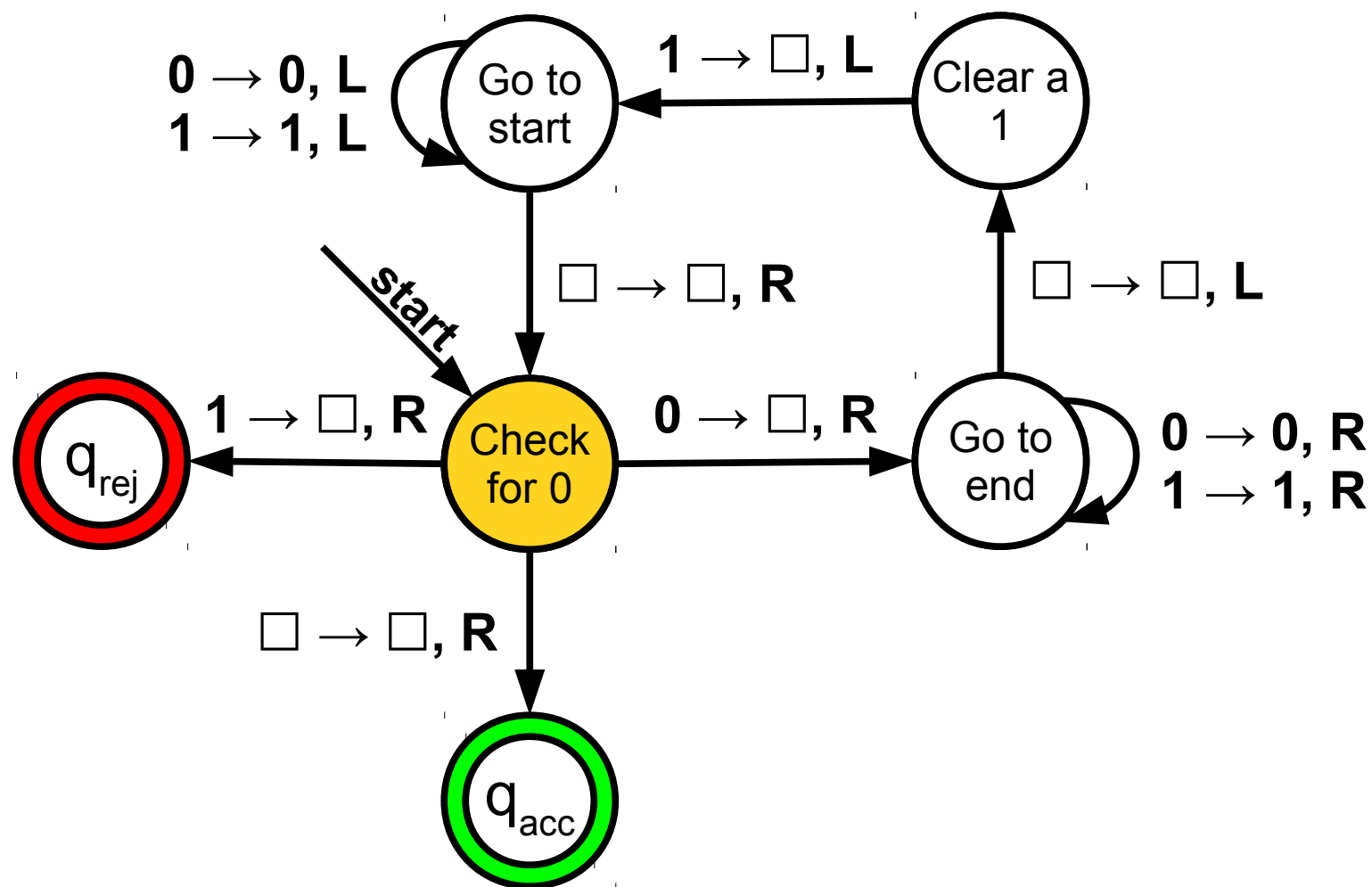


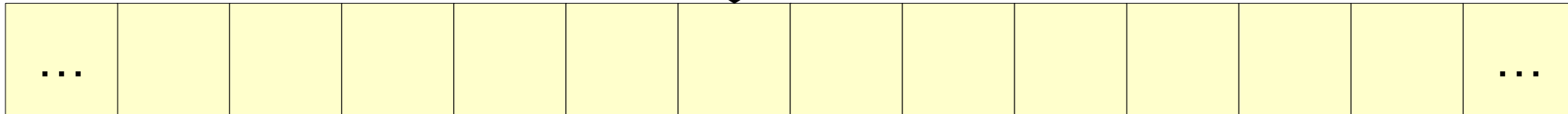
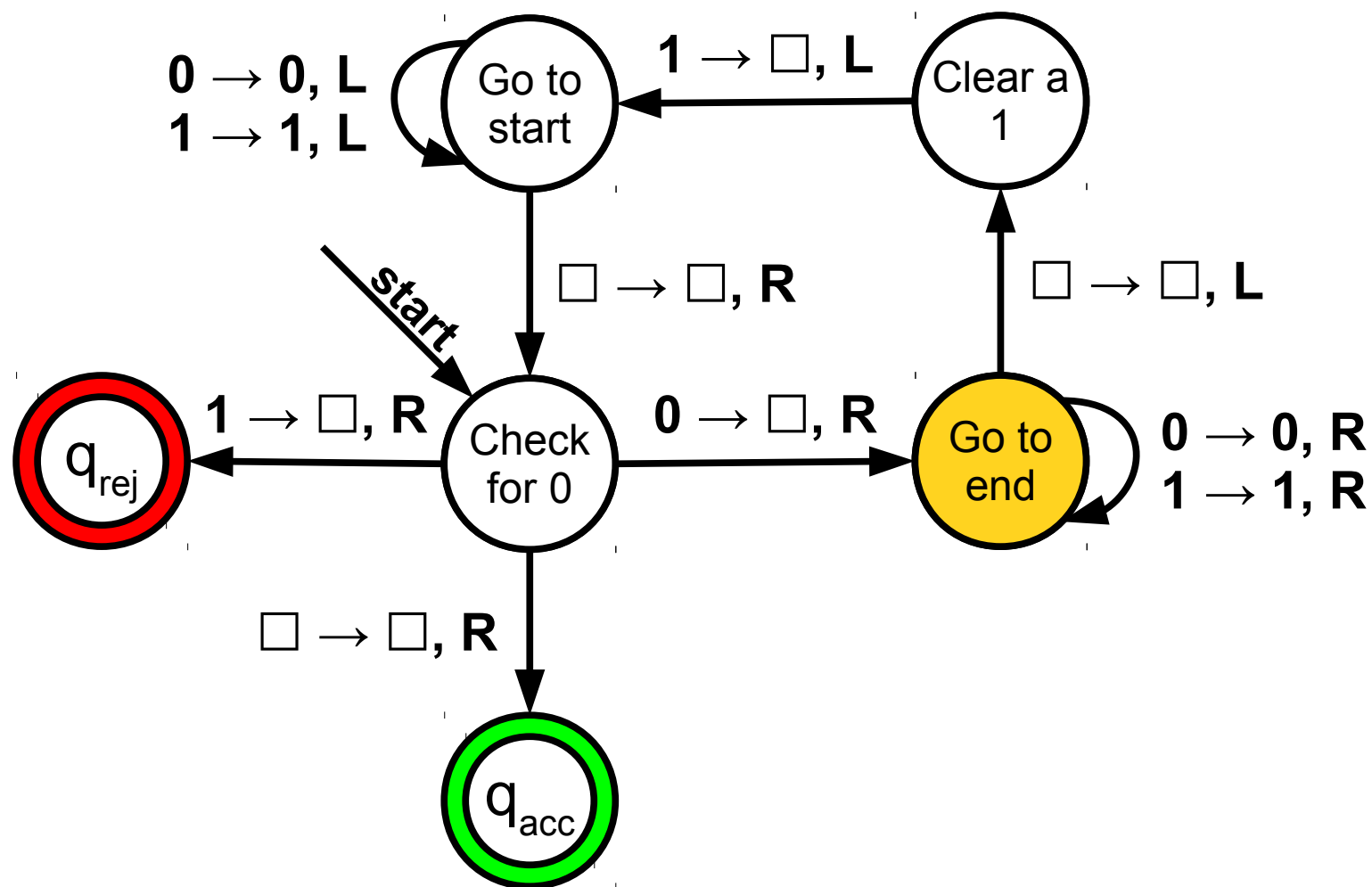


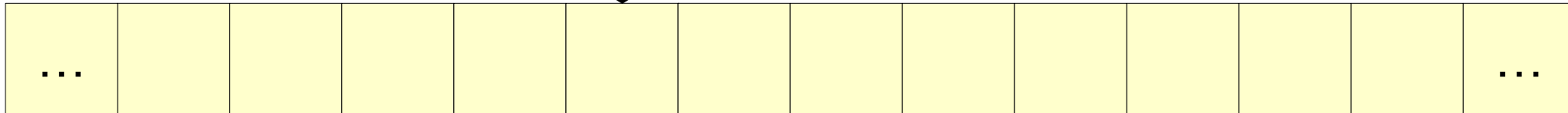
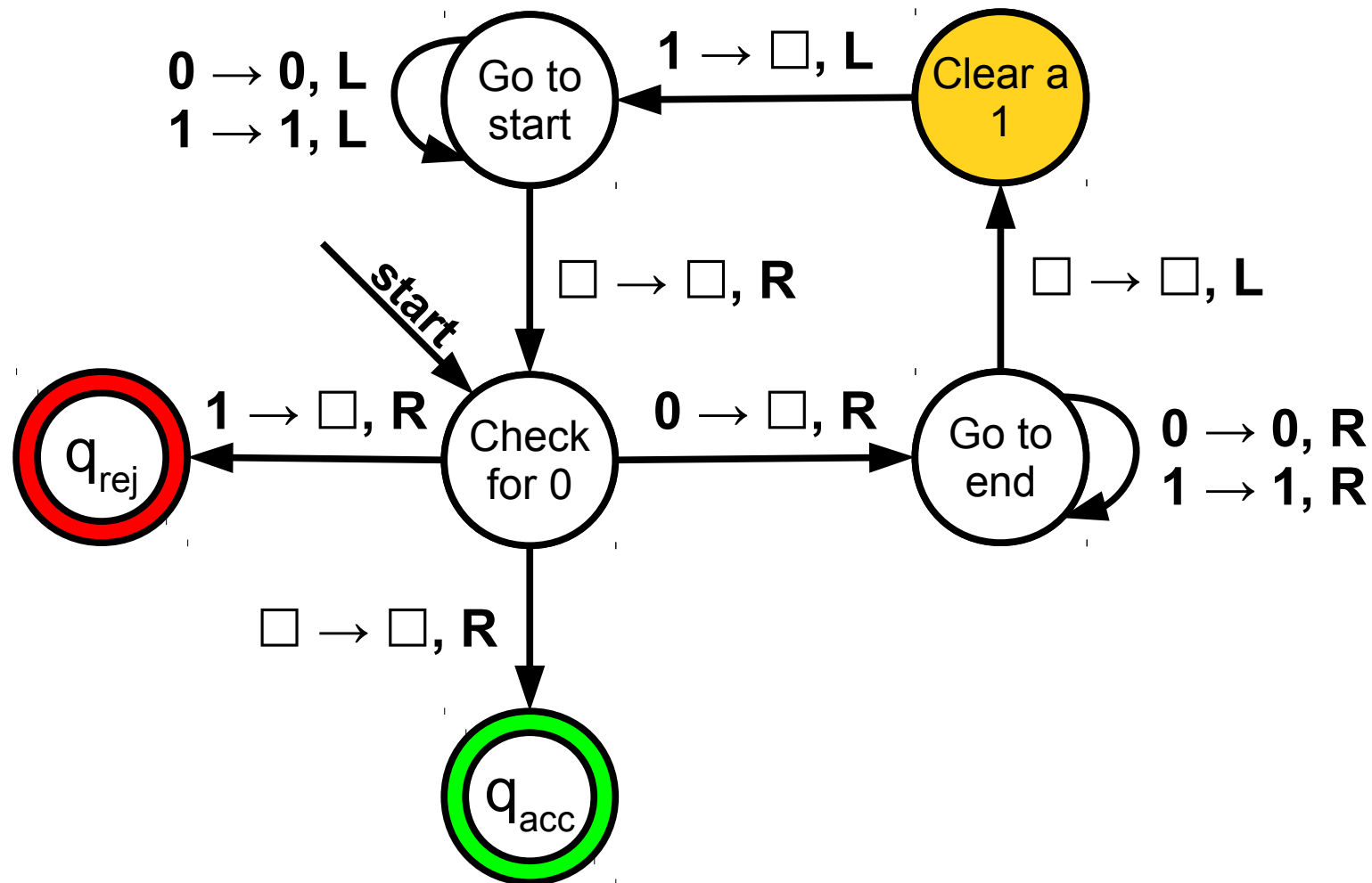








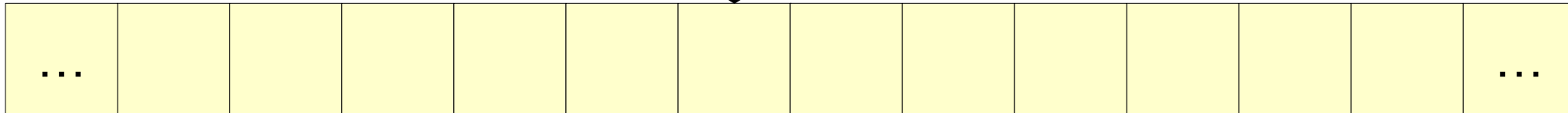
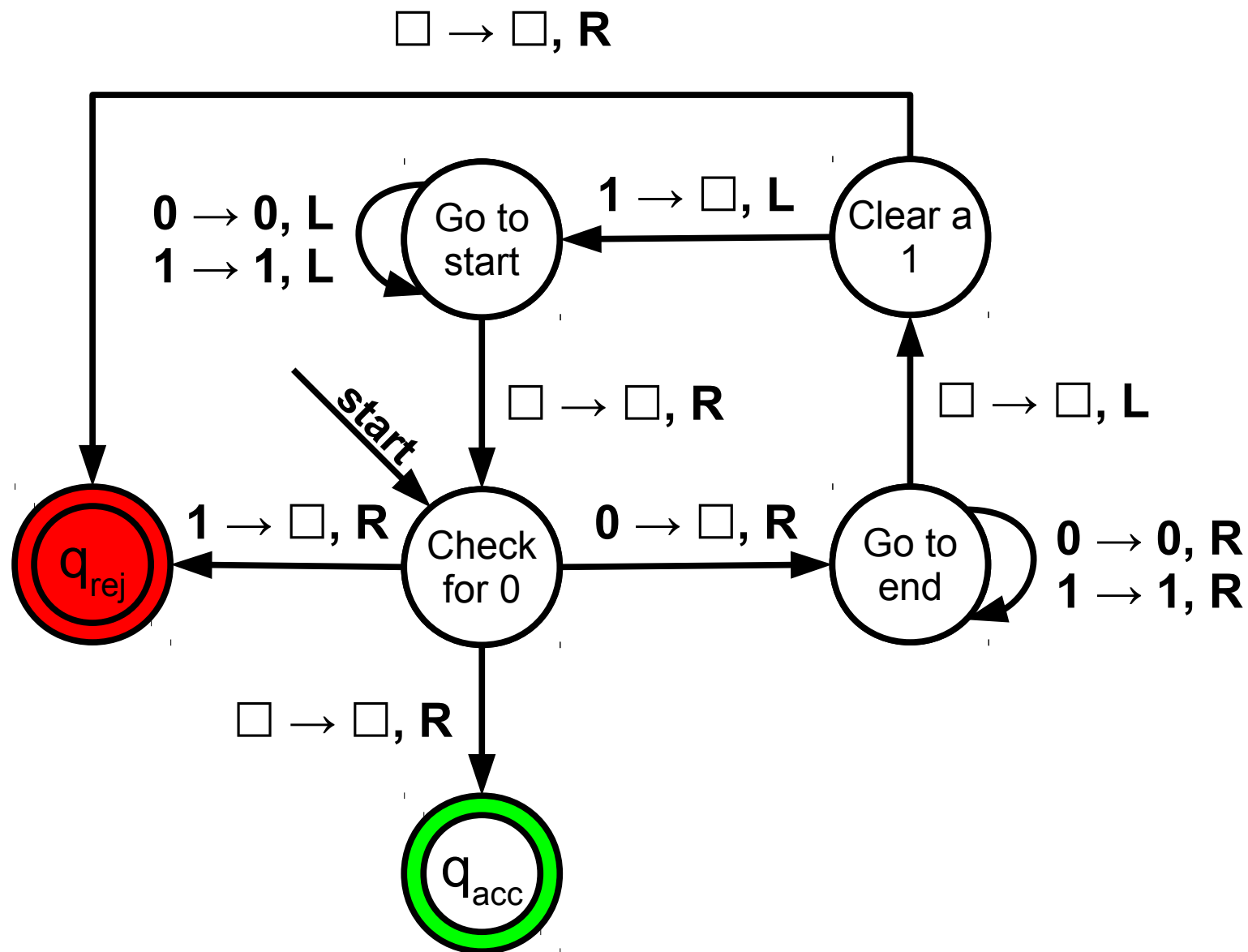


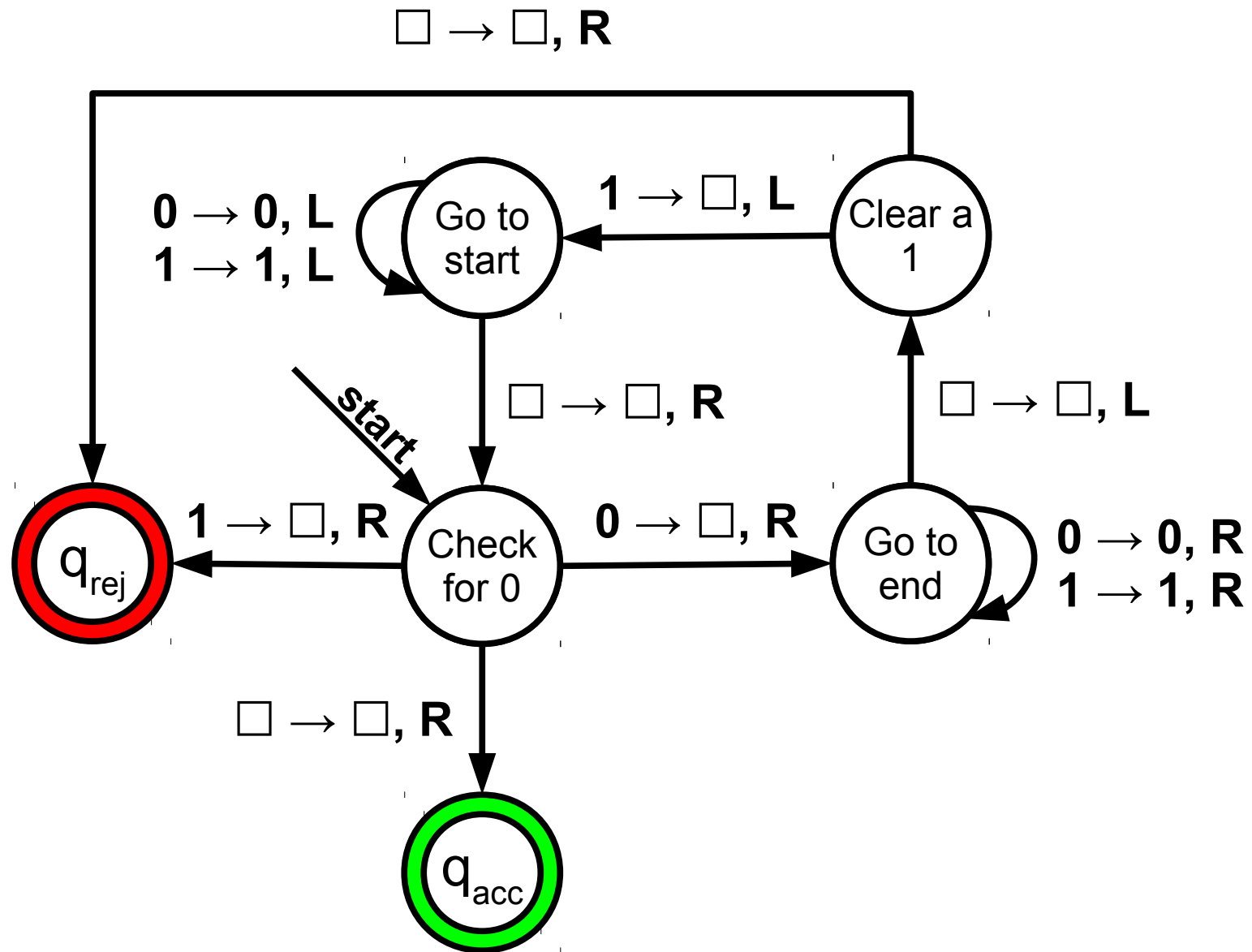


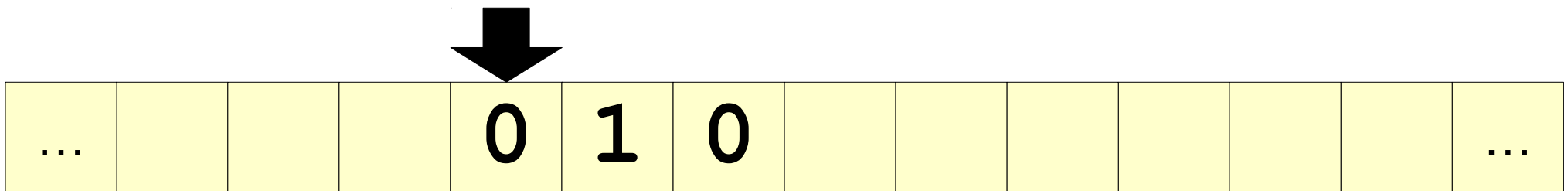
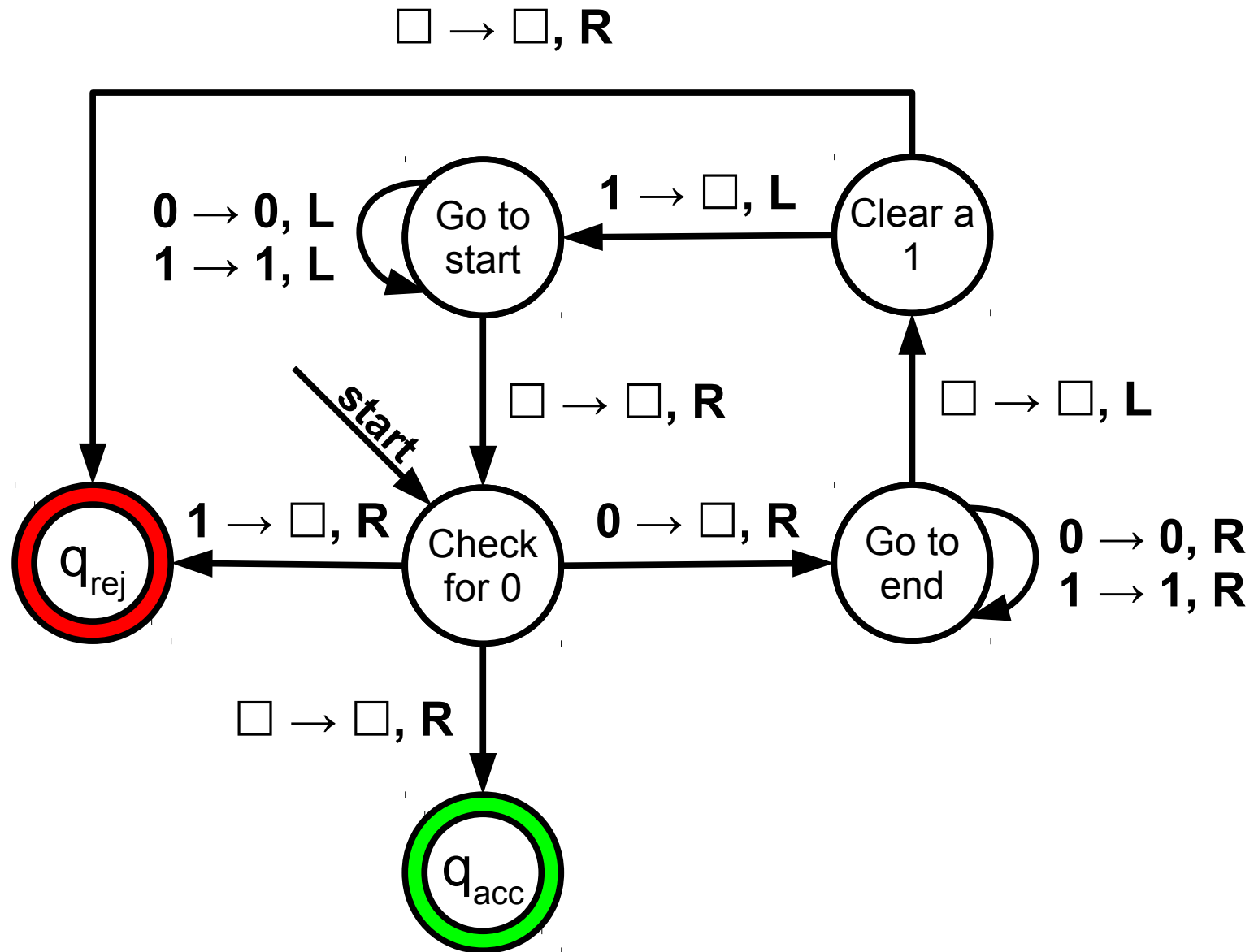


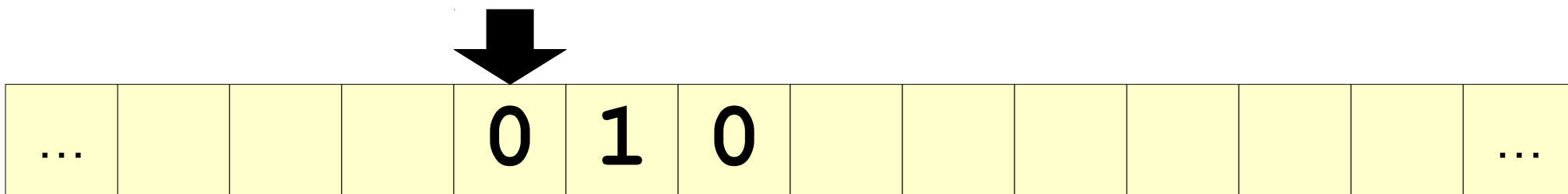
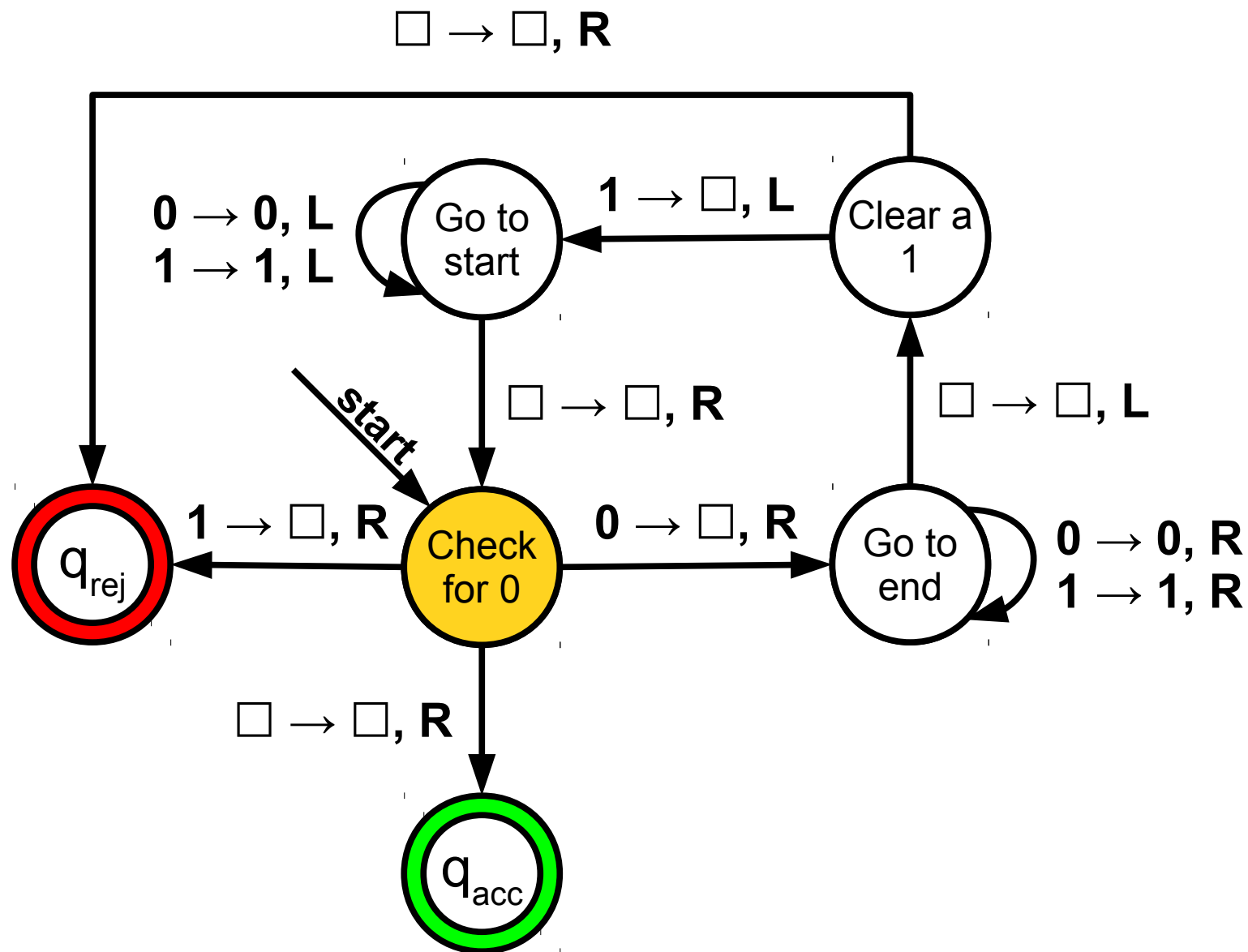


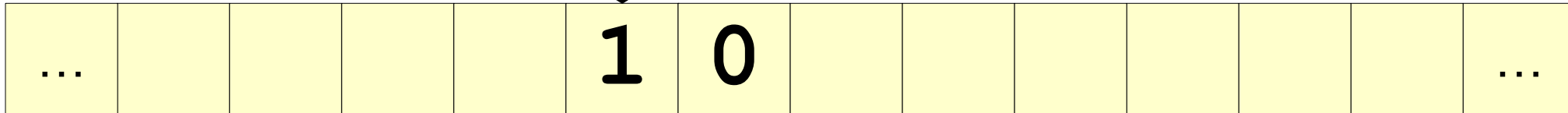
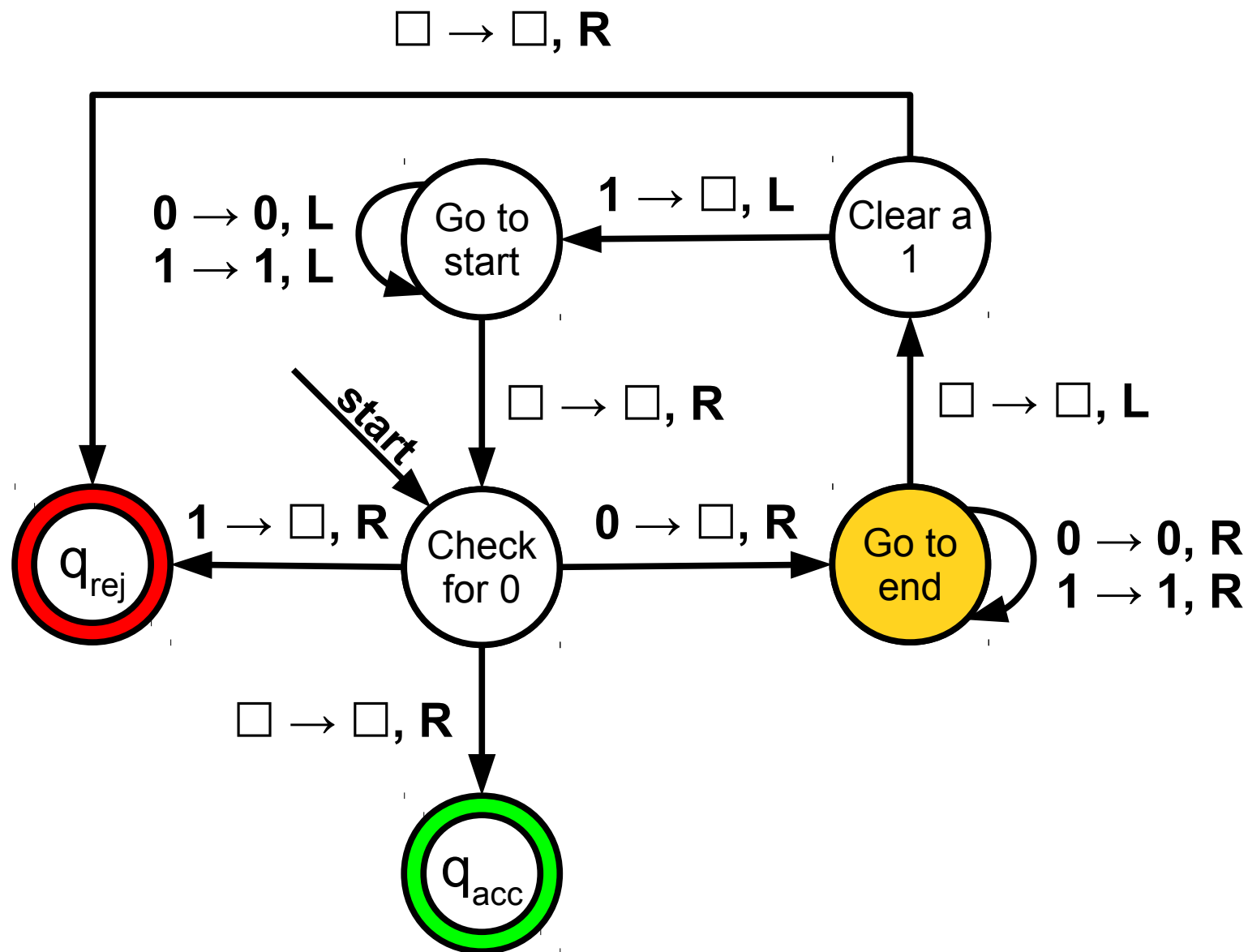


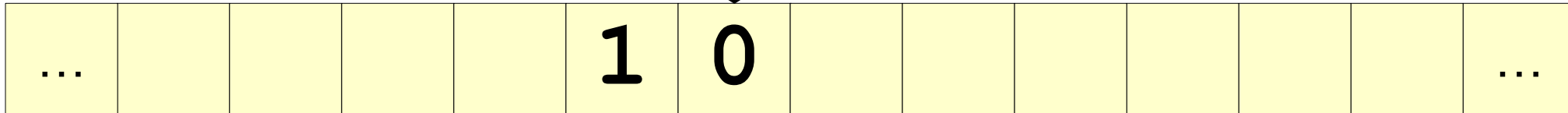
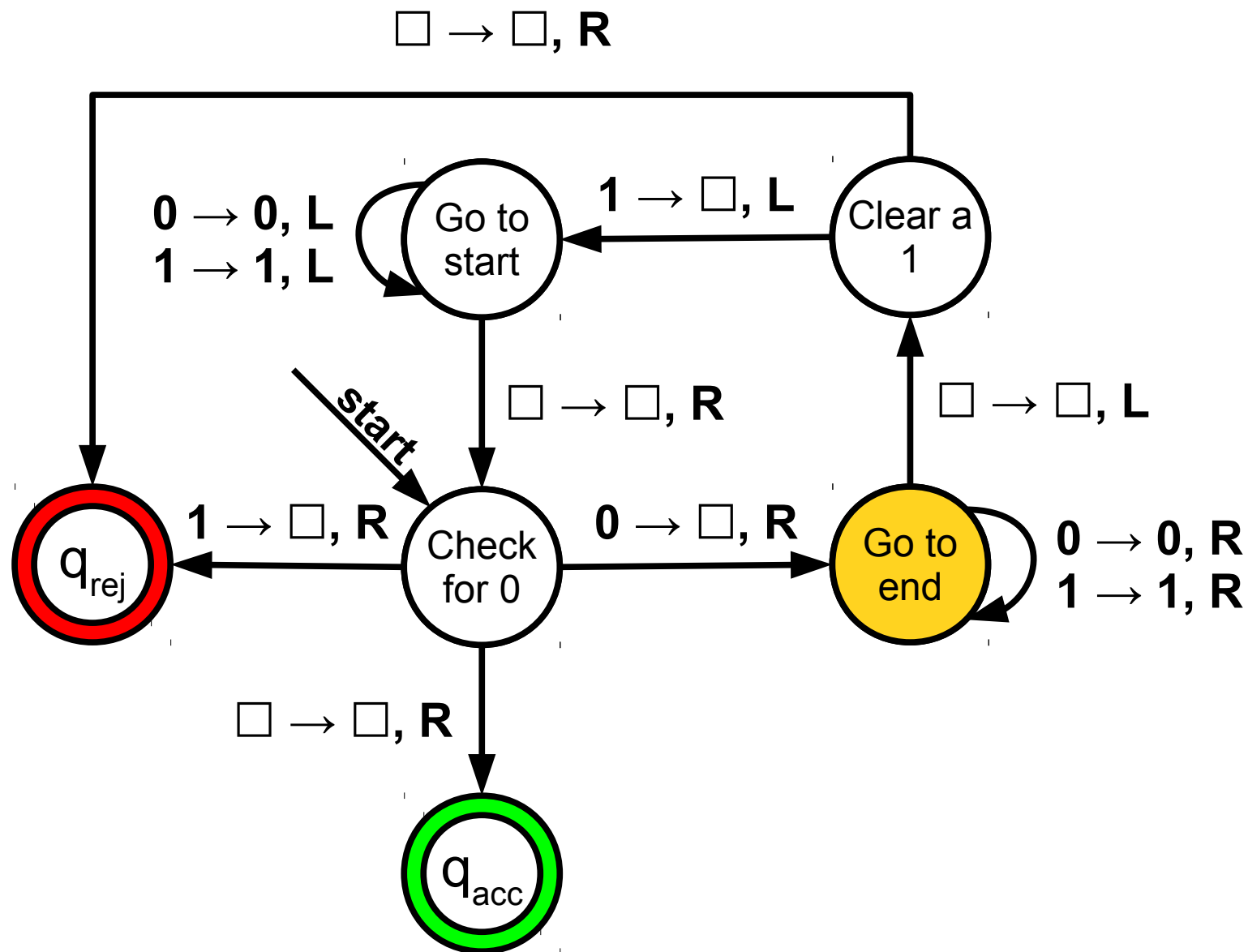




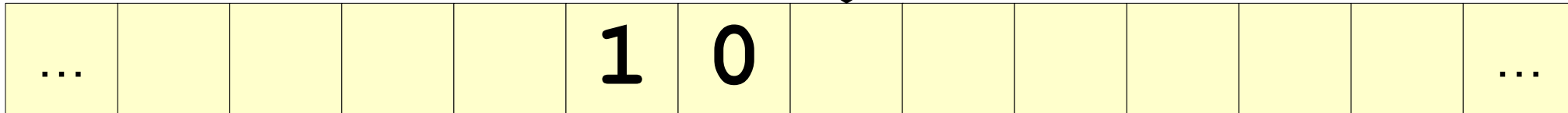




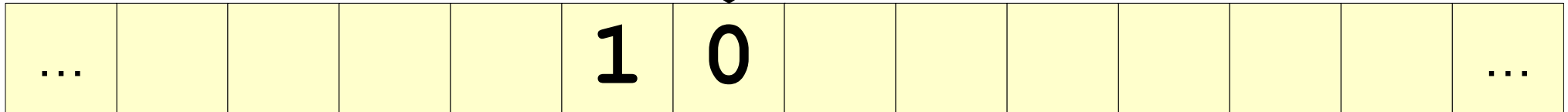
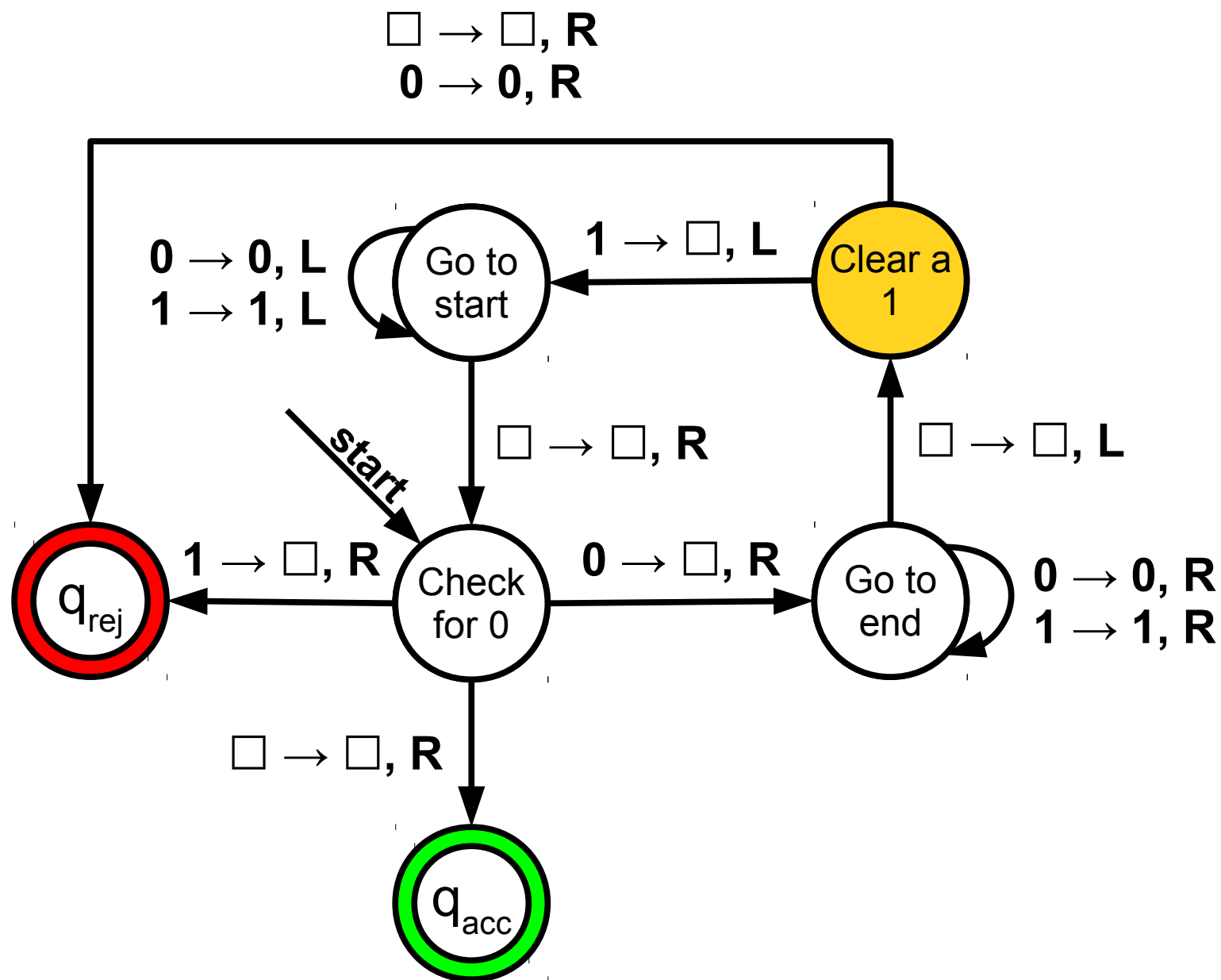


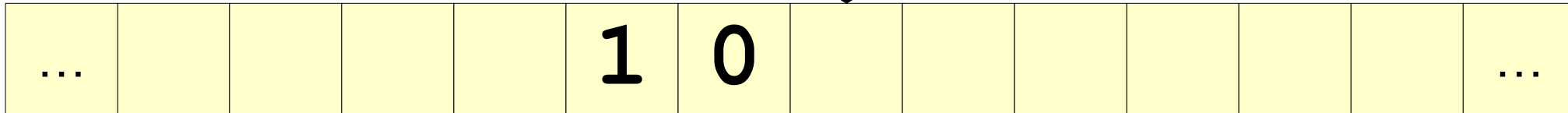
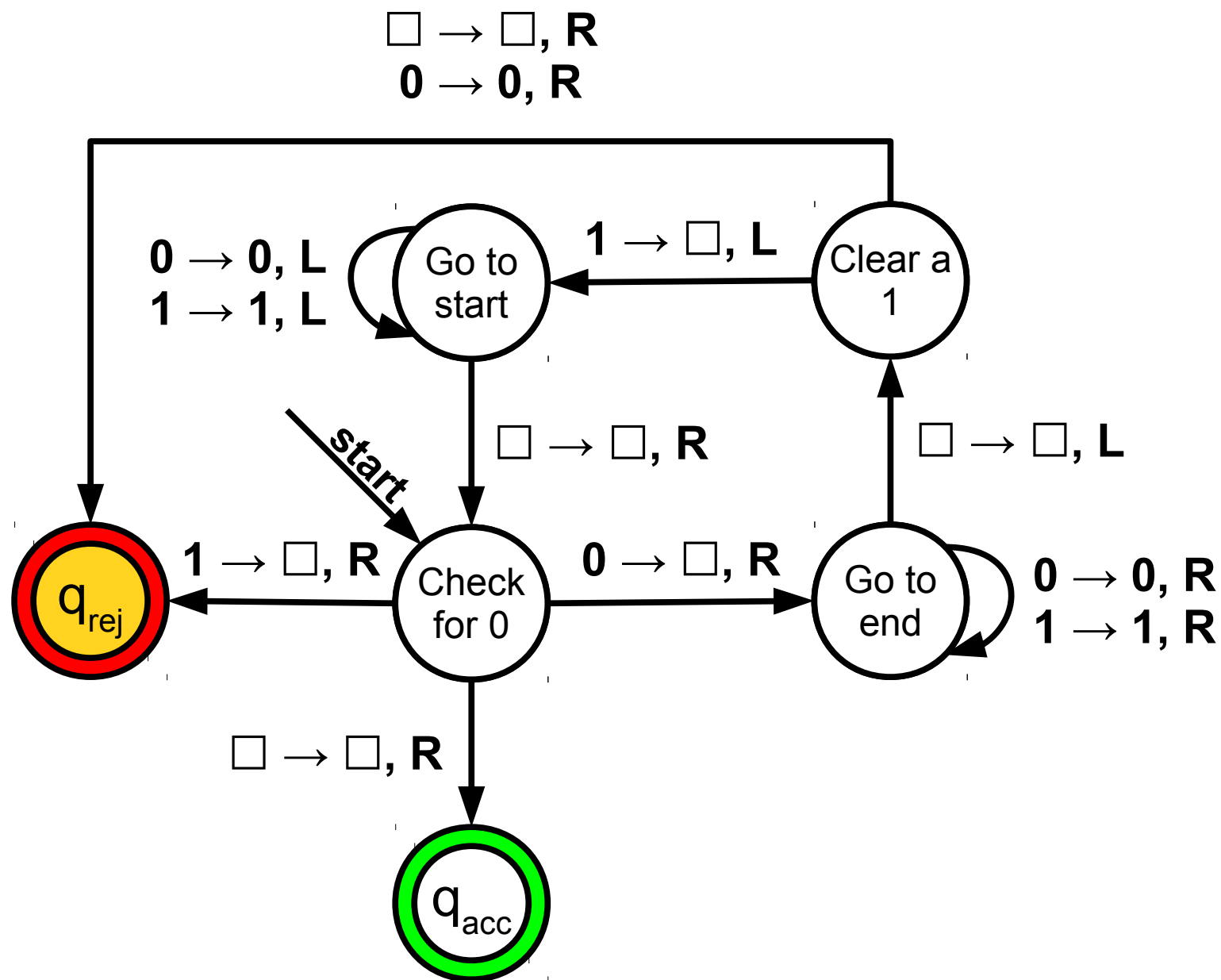


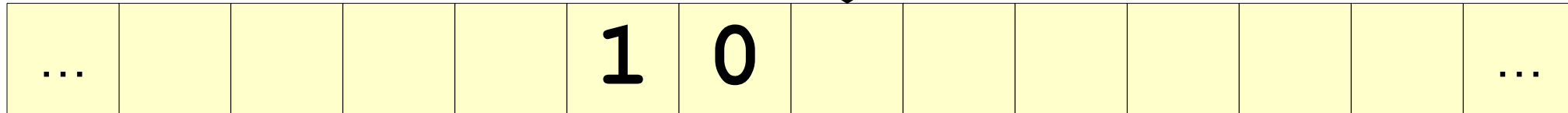
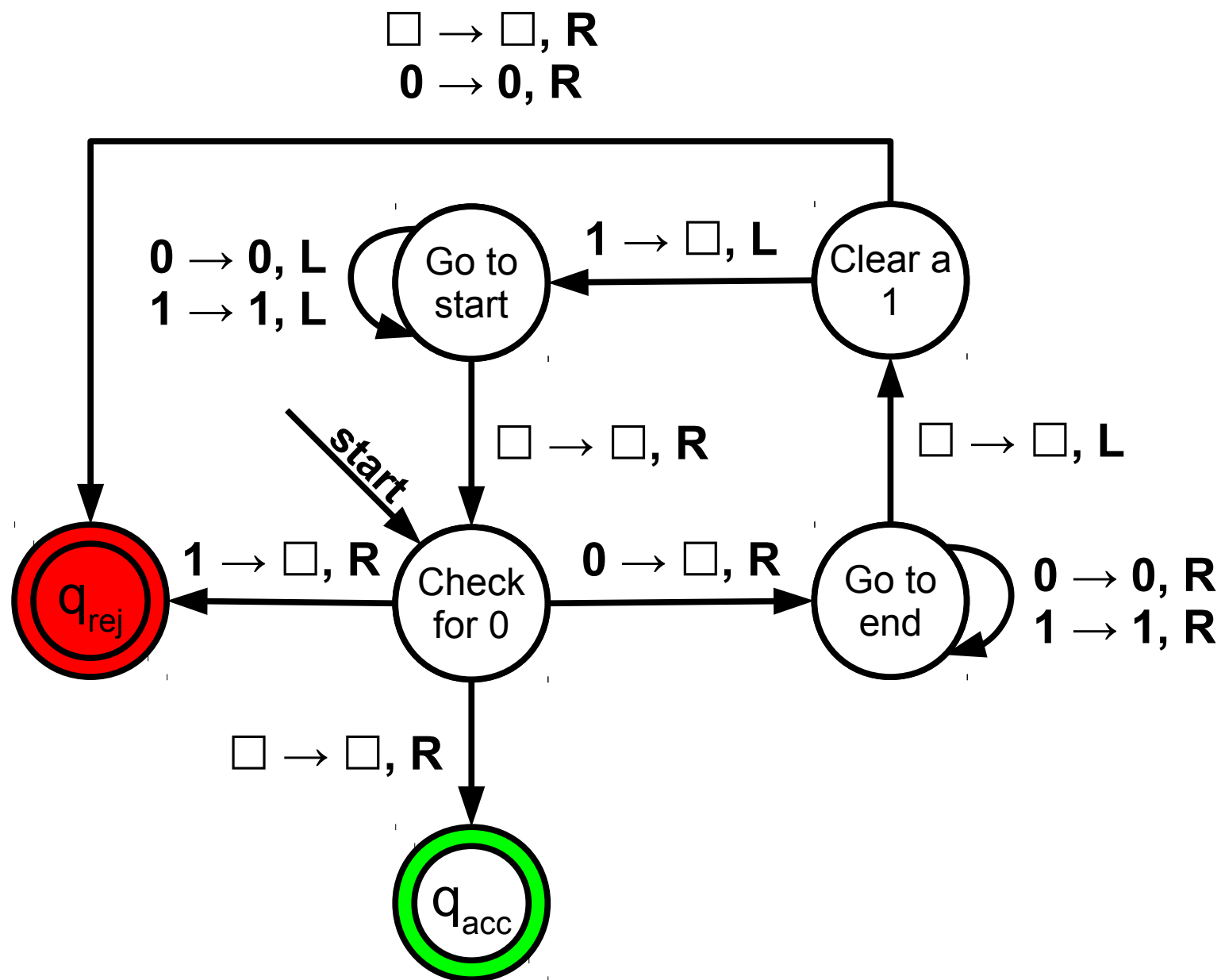


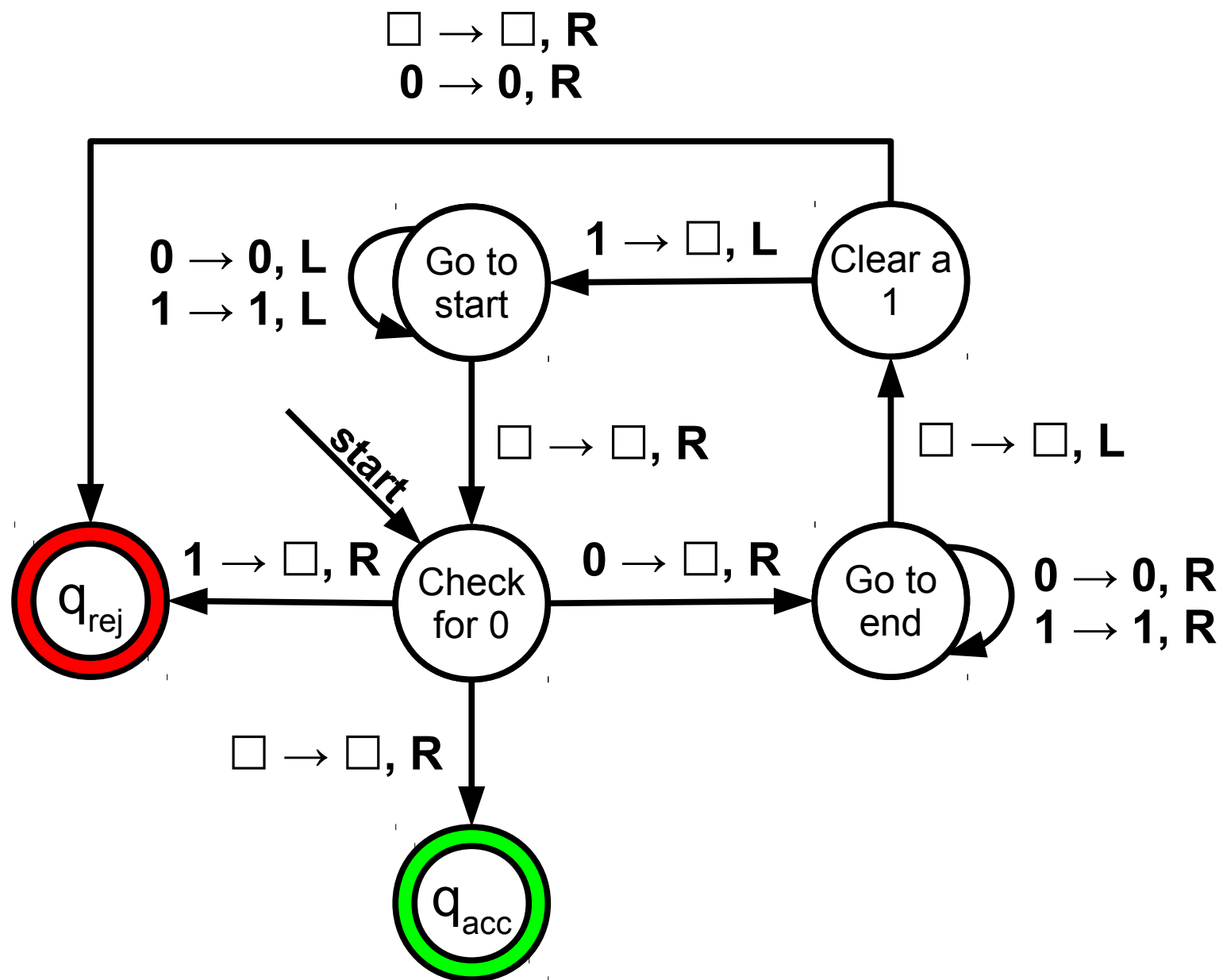












# Another TM Design

- We've designed a TM for  $\{0^n 1^n \mid n \in \mathbb{N}\}$ .
- Consider this language over  $\Sigma = \{0, 1\}$ :  
$$L = \{ w \in \Sigma^* \mid w \text{ has the same number of 0s and 1s} \}$$
- This language is also not regular, but it is context-free.
- How might we design a TM for it?

# A Caveat



...			0	0	0	1	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----



# A Caveat



...				0	0	1	1	1	1	0			...
-----	--	--	--	---	---	---	---	---	---	---	--	--	-----

# A Caveat



...				0	0	1	1	1	1	0			...
-----	--	--	--	---	---	---	---	---	---	---	--	--	-----

# A Caveat



...				0	0	1	1	1	1	0			...
-----	--	--	--	---	---	---	---	---	---	---	--	--	-----

# A Caveat



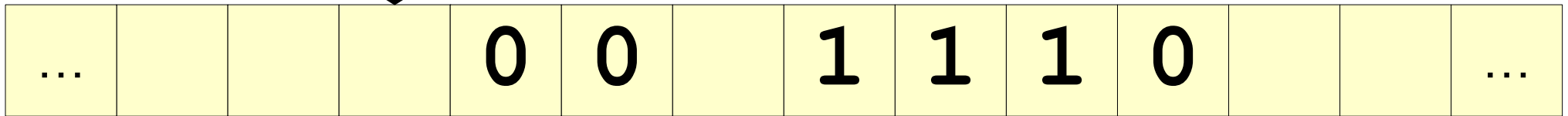
...				0	0		1	1	1	0			...
-----	--	--	--	---	---	--	---	---	---	---	--	--	-----

# A Caveat

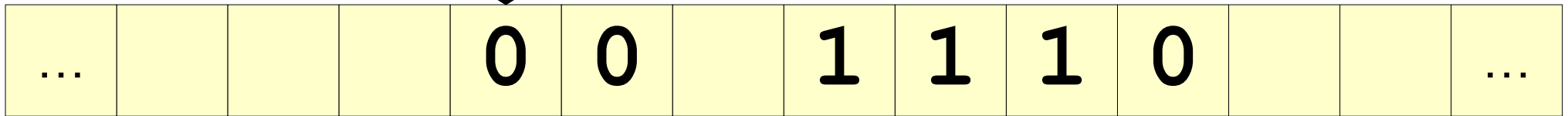


...				0	0		1	1	1	0			...
-----	--	--	--	---	---	--	---	---	---	---	--	--	-----

# A Caveat



# A Caveat



# A Caveat



...					0		1	1	1	0			...
-----	--	--	--	--	---	--	---	---	---	---	--	--	-----

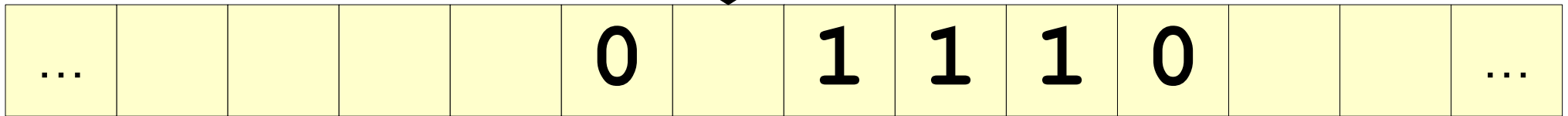


# A Caveat



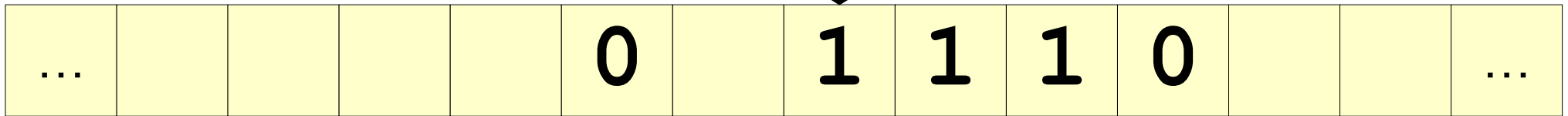
...					0		1	1	1	0			...
-----	--	--	--	--	---	--	---	---	---	---	--	--	-----

# A Caveat

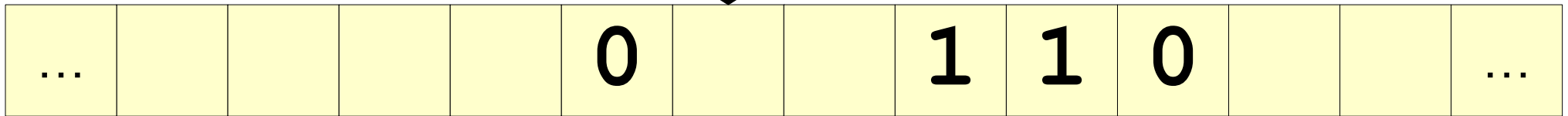


How do we know that  
this blank isn't one of  
the infinitely many  
blanks after our input  
string?

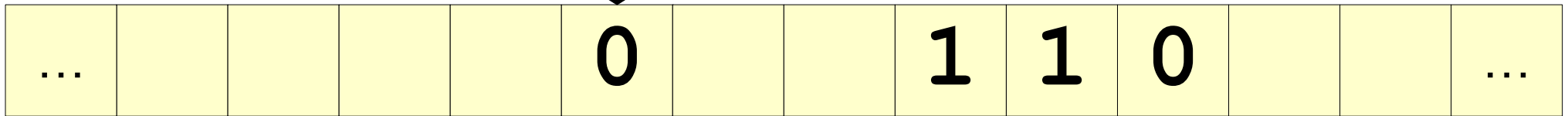
# A Caveat



# A Caveat



# A Caveat

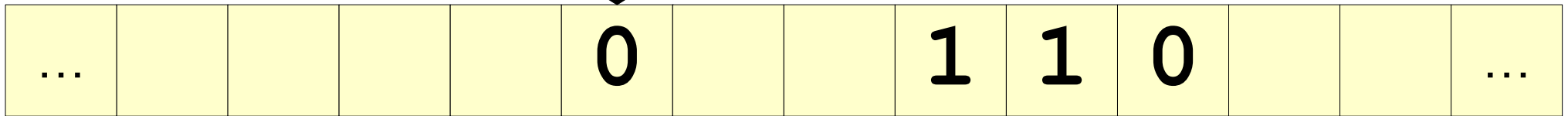


# A Caveat

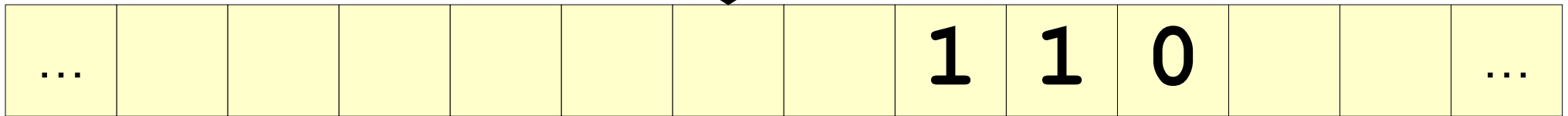


...					0			1	1	0			...
-----	--	--	--	--	---	--	--	---	---	---	--	--	-----

# A Caveat

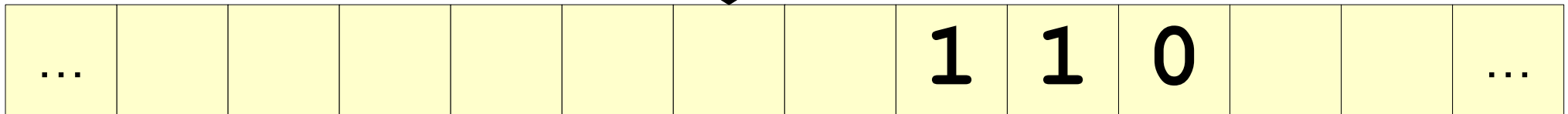


# A Caveat



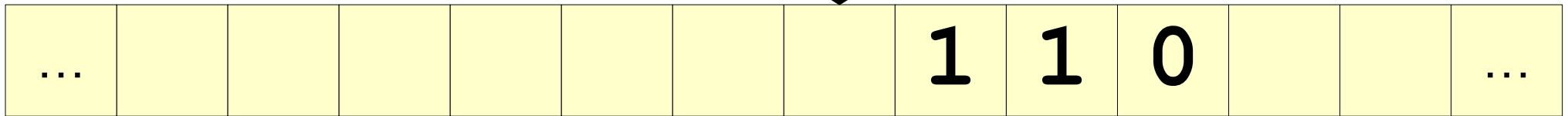


# A Caveat

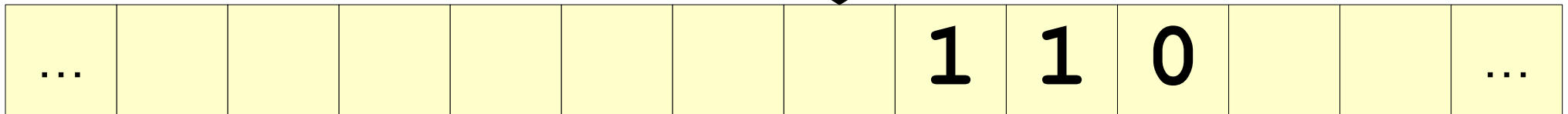


How do we know that  
this blank isn't one of  
the infinitely many  
blanks after our input  
string?

# A Caveat



# A Caveat



How do we know that  
this blank isn't one of  
the infinitely many  
blanks after our input  
string?

# The Solution



...			0	0	0	1	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			×	0	0	1	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			×	0	0	1	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			×	0	0	1	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			×	0	0	×	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----



# The Solution



...			x	0	0	x	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			x	0	0	x	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			×	0	0	×	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			x	0	0	x	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			x	0	0	x	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			x	x	0	x	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			x	x	0	x	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			x	x	0	x	1	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----



# The Solution



...			x	x	0	x	x	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			x	x	0	x	x	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			x	x	0	x	x	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			x	x	0	x	x	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			x	x	0	x	x	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			x	x	0	x	x	1	1	0			...
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# The Solution



...			x	x	0	x	x	1	1	0			...
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# The Solution



...			x	x	0	x	x	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----



# The Solution



...			x	x	x	x	x	1	1	0			...
-----	--	--	---	---	---	---	---	---	---	---	--	--	-----

# The Solution



...			x	x	x	x	x	1	1	0			...
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# The Solution



...			x	x	x	x	x	1	1	0			...
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# The Solution

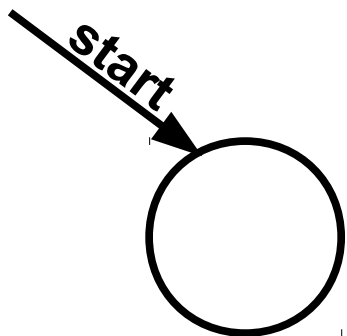


...			x	x	x	x	x	x	1	0			...
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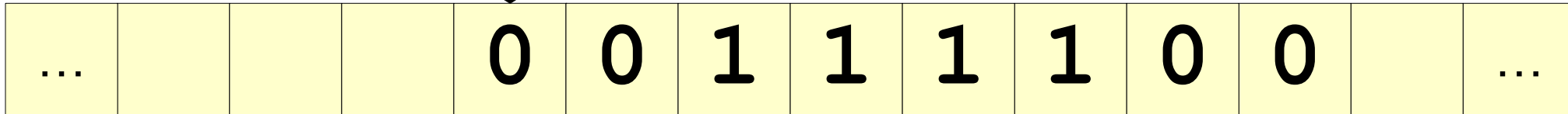
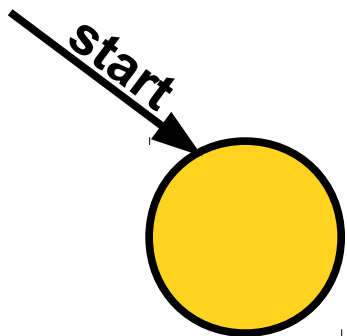
# The Solution

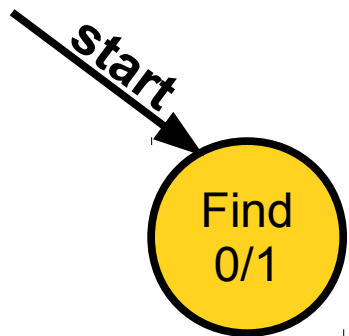


...			x	x	x	x	x	x	1	0			...
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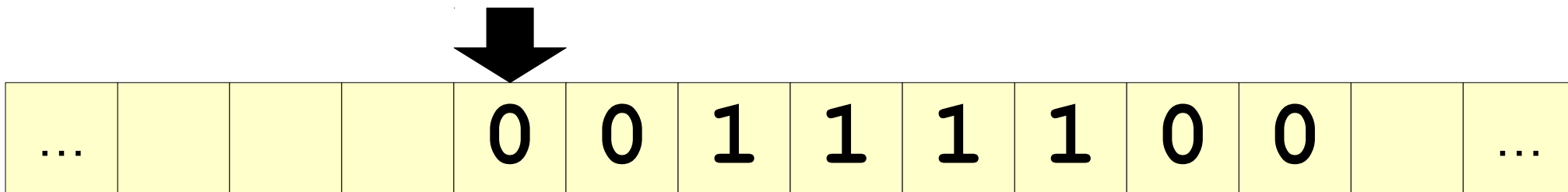
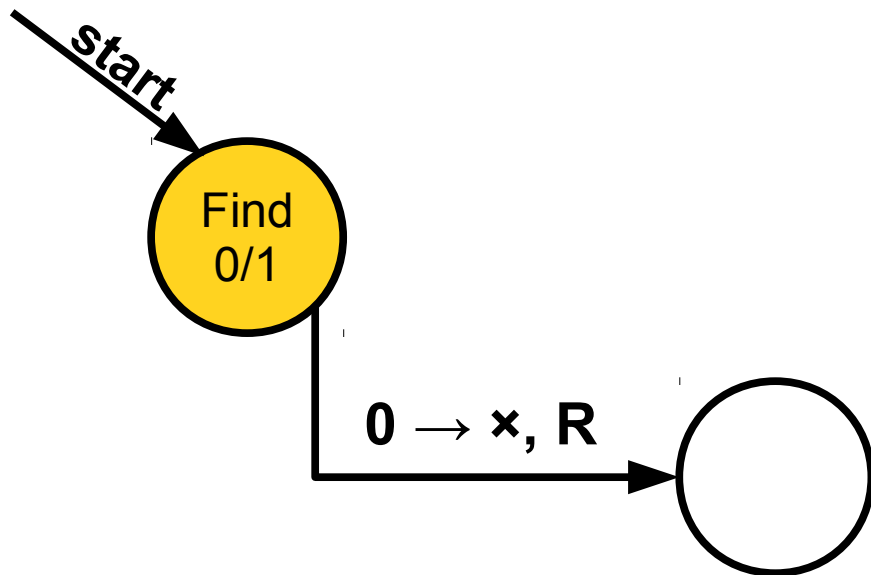
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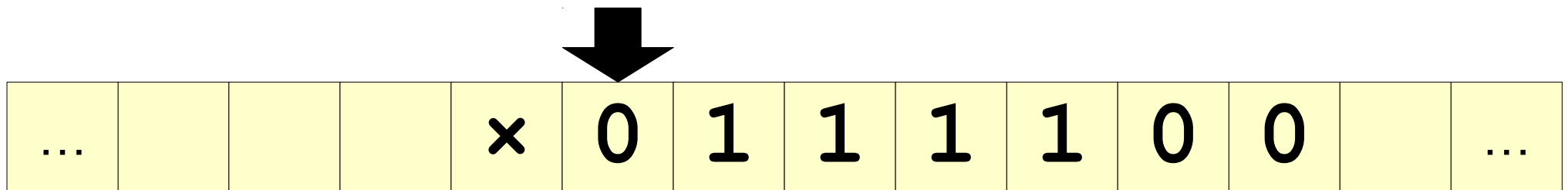
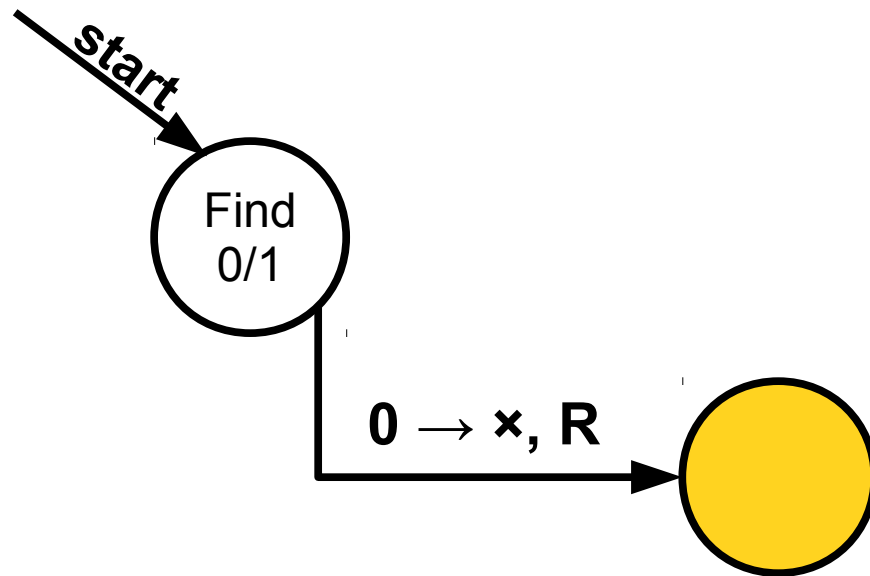


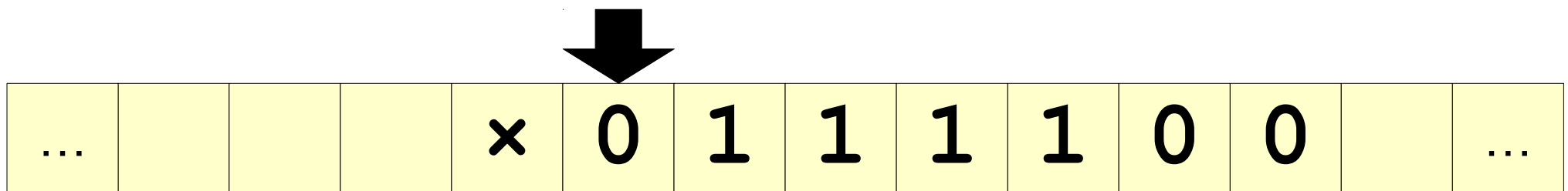
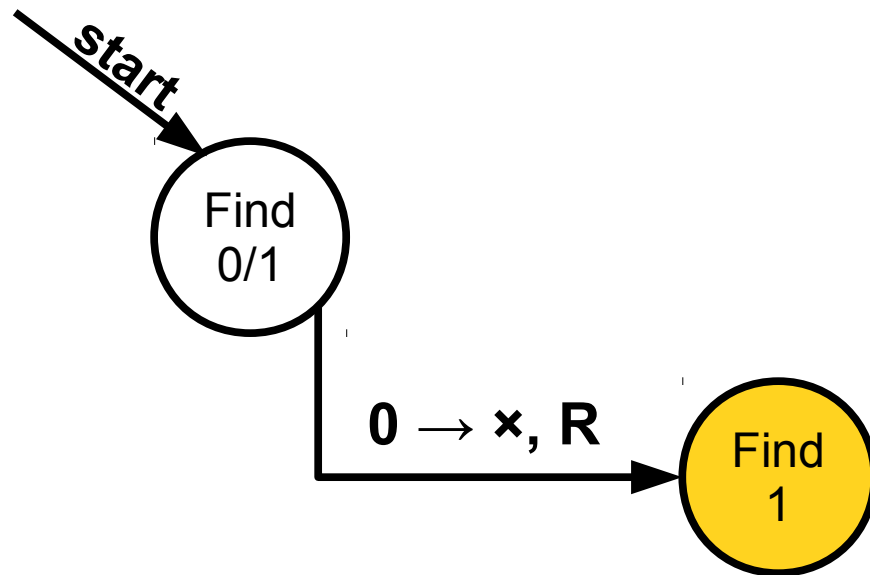


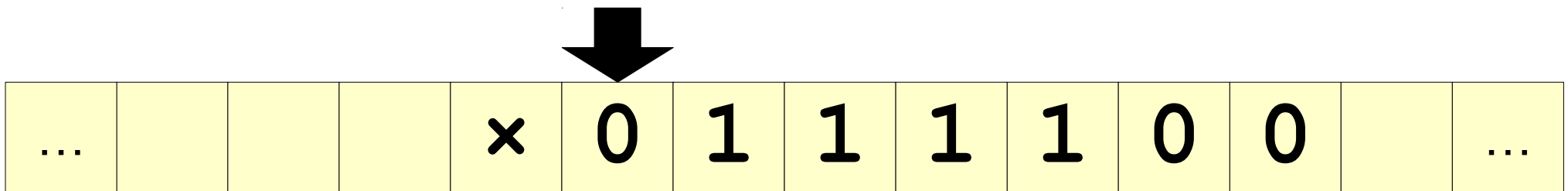
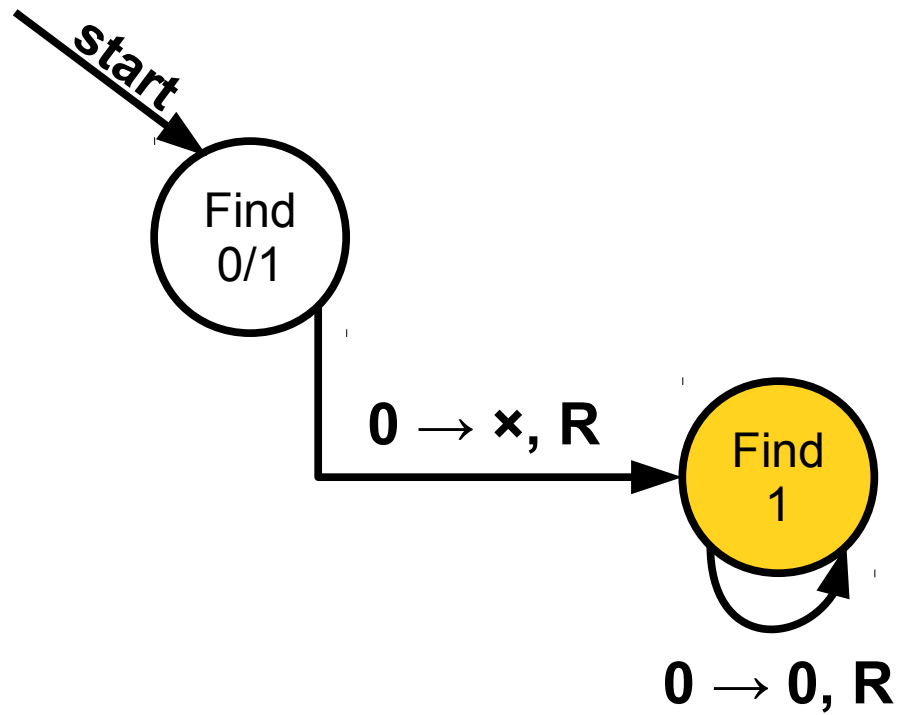
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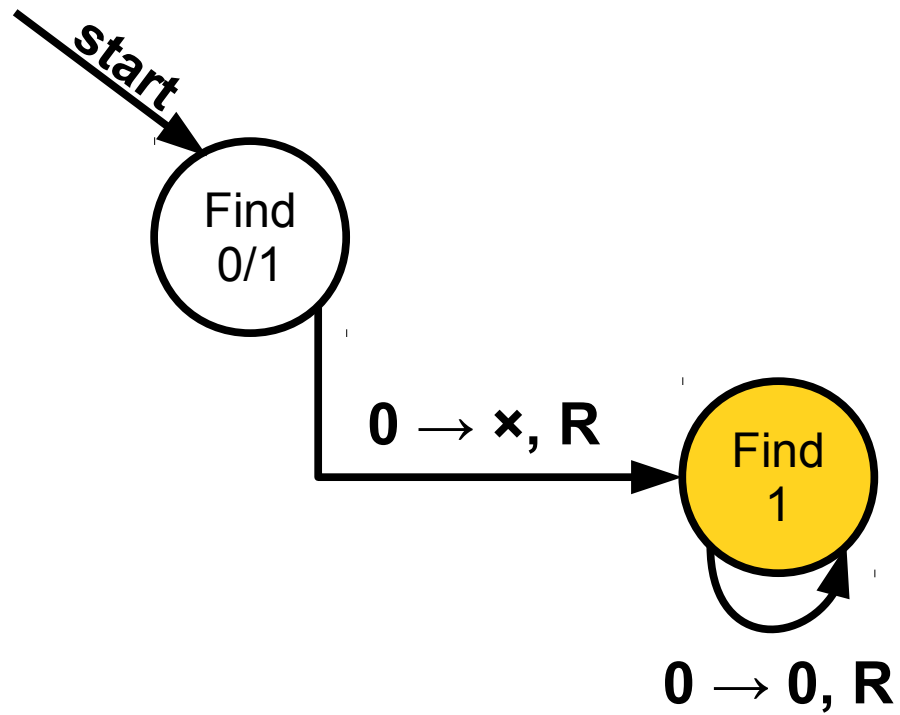




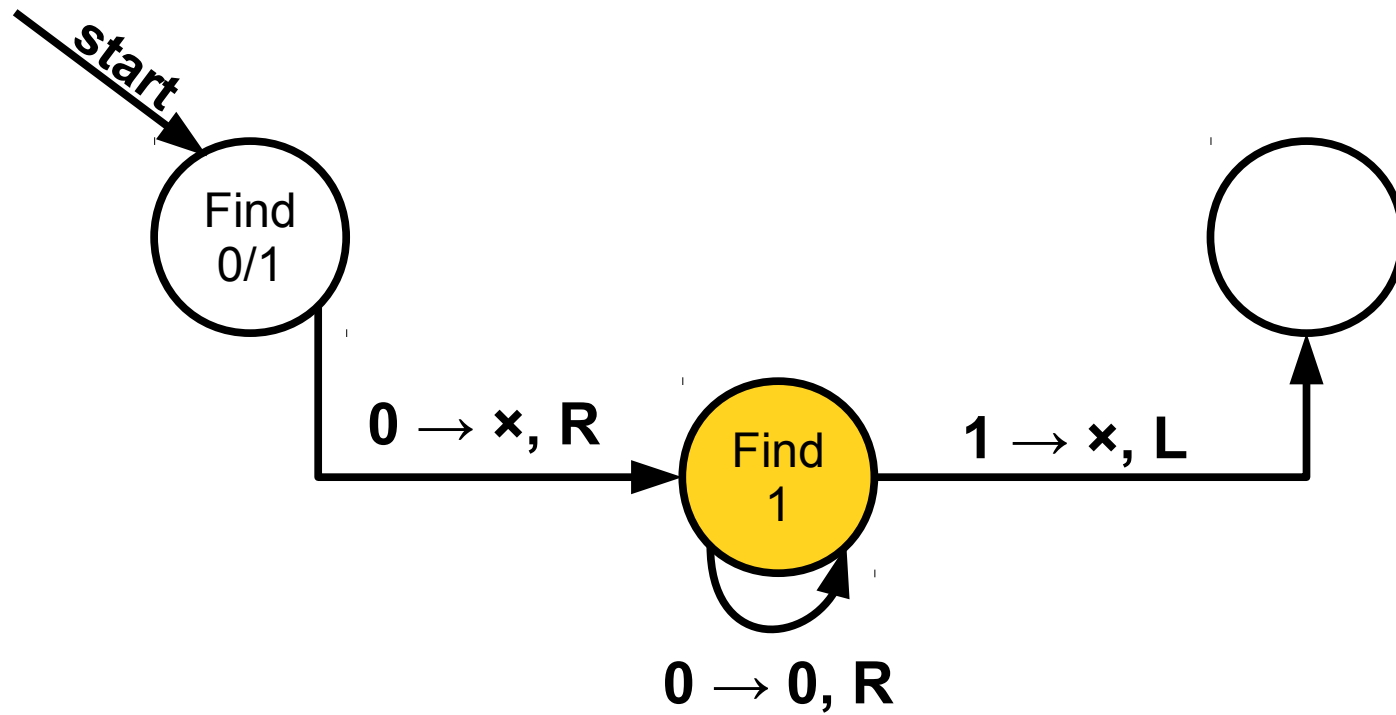


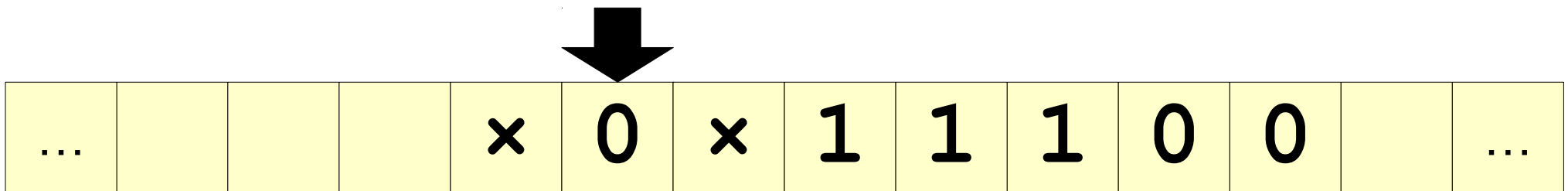
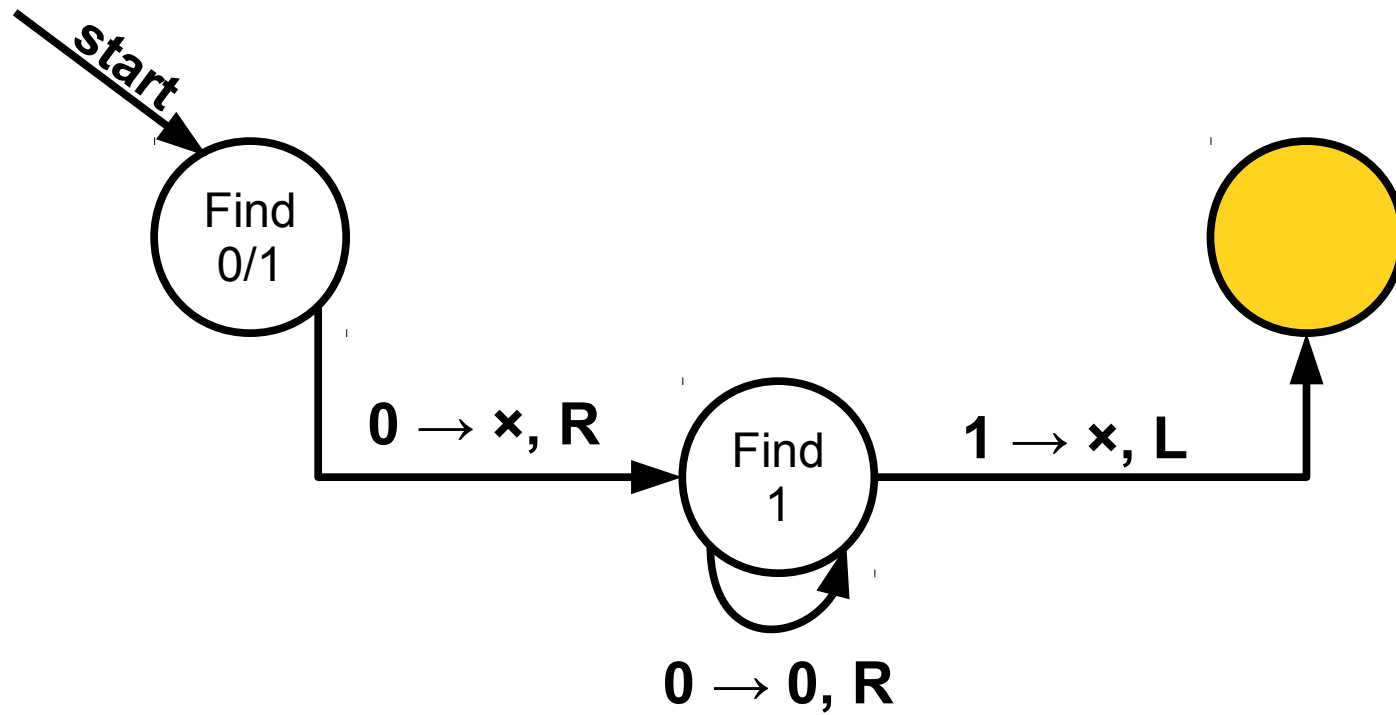


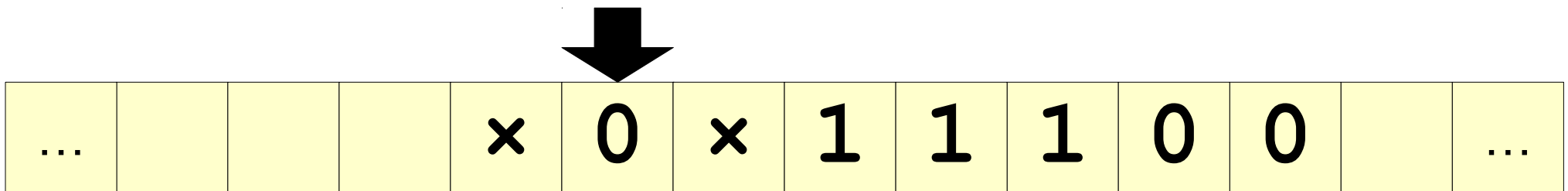
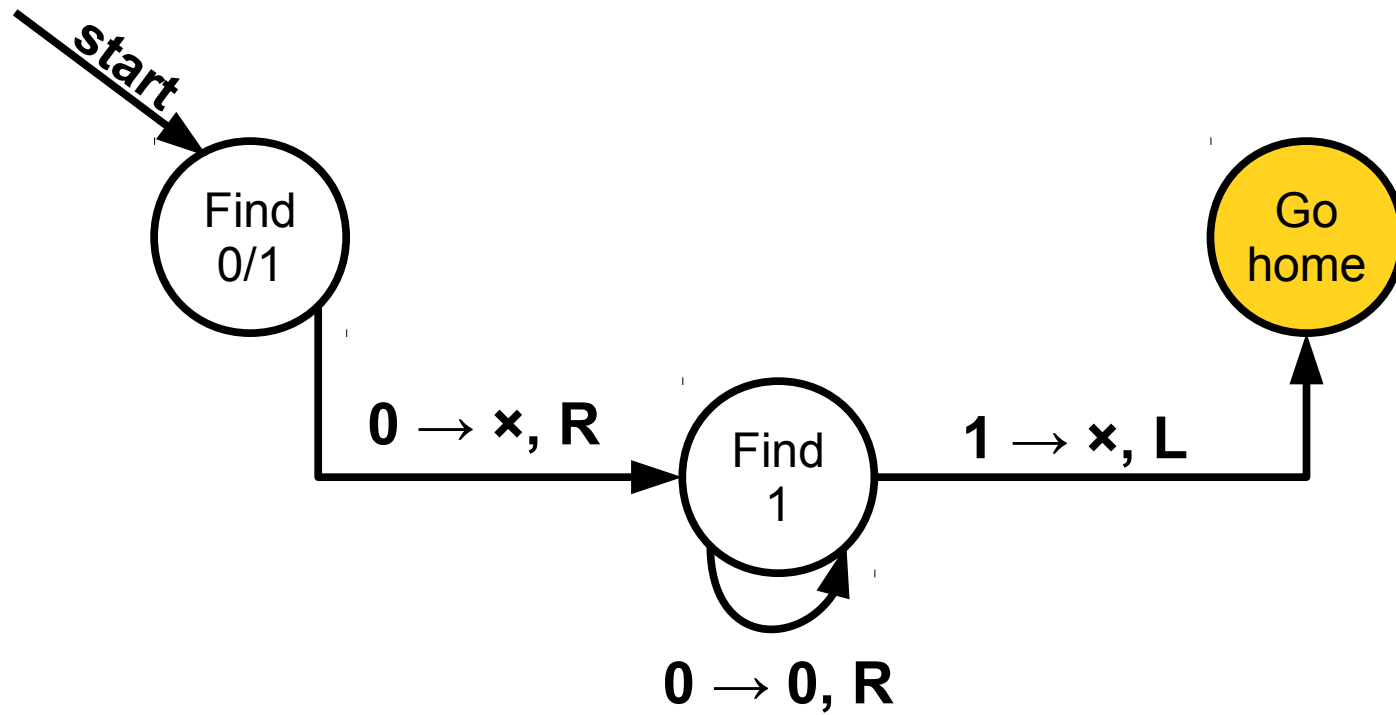




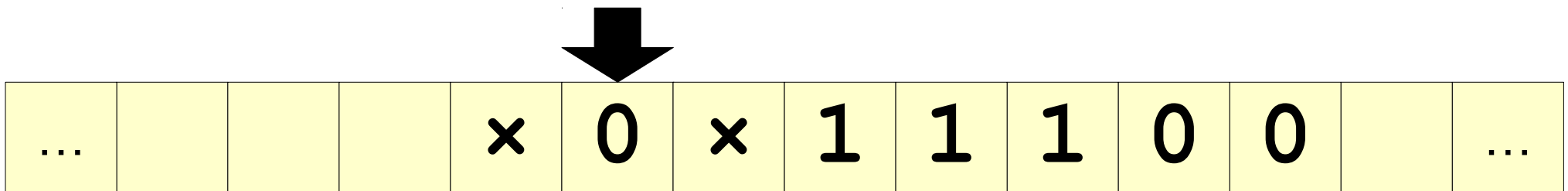
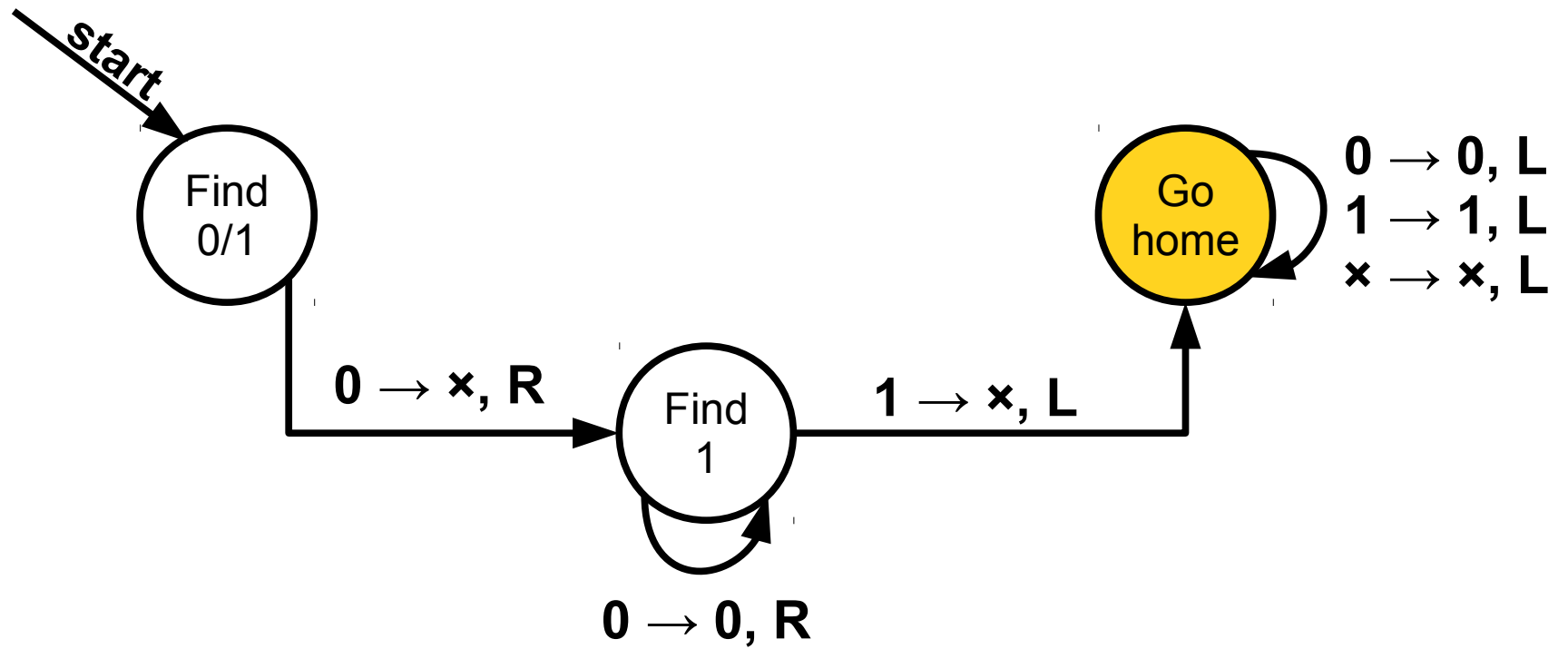
...				x	0	1	1	1	1	0	0		...
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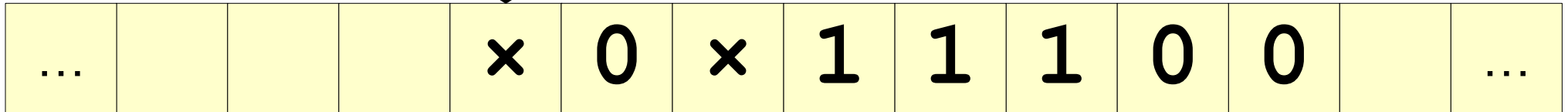
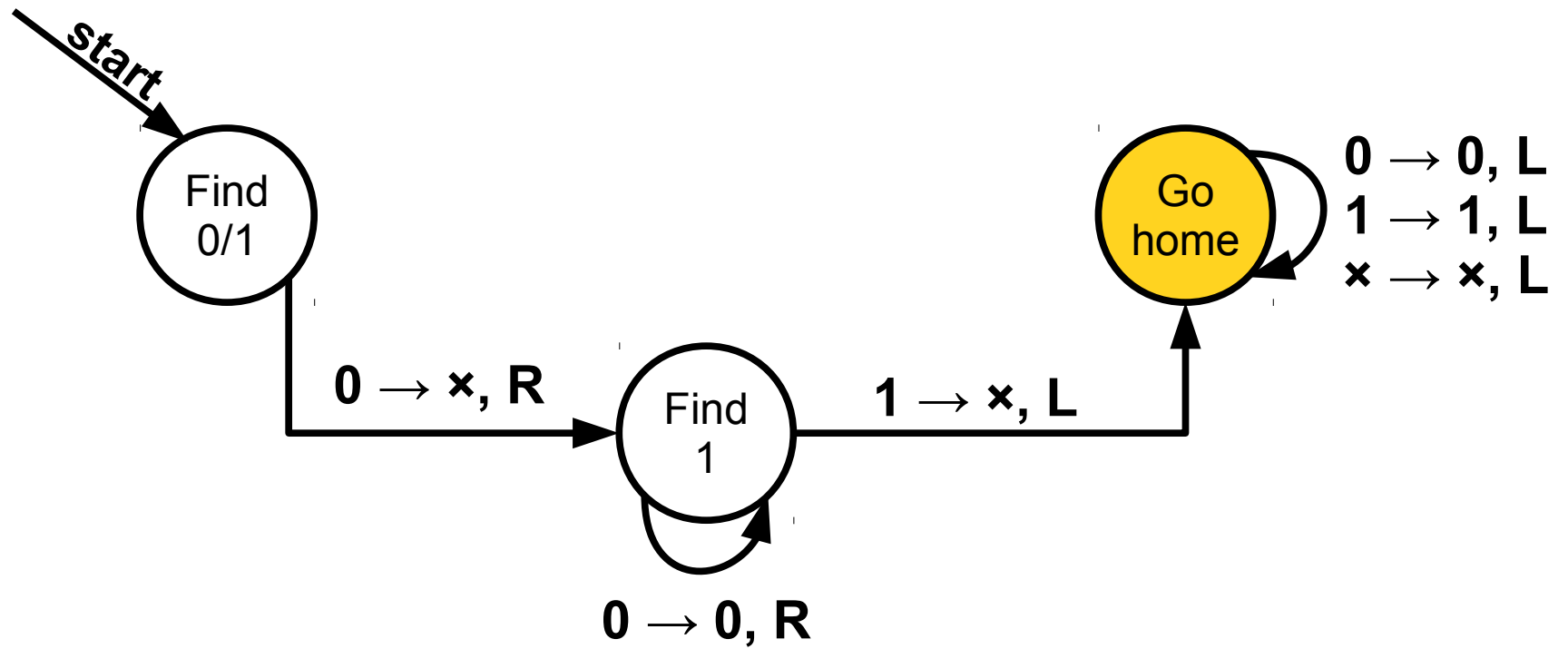


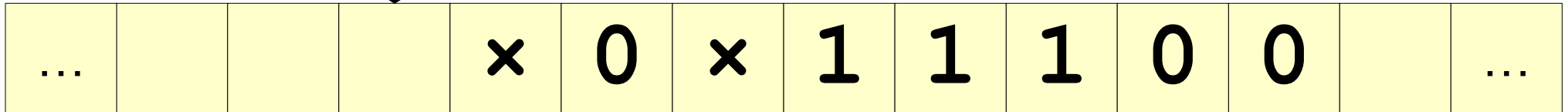
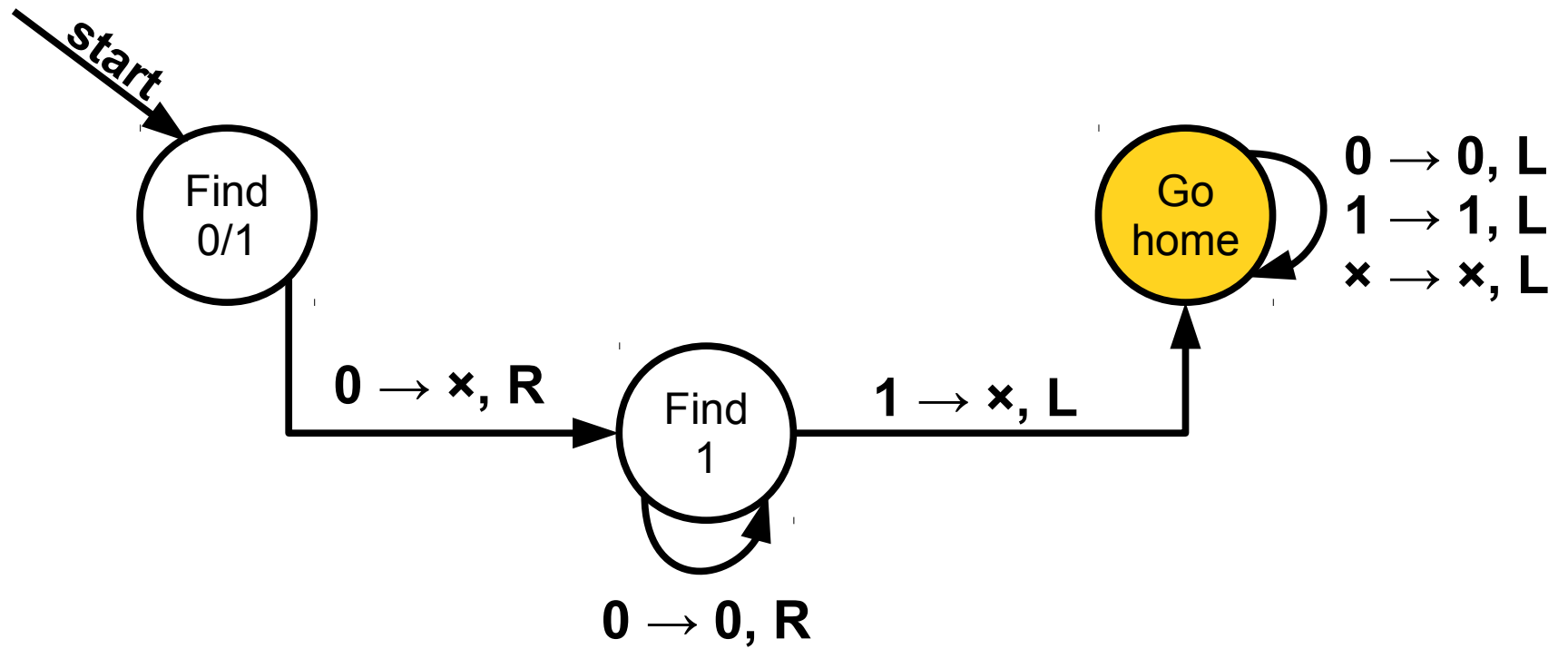


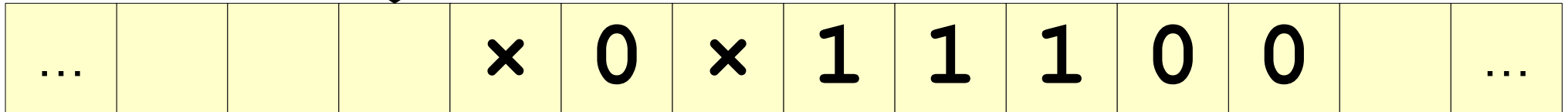
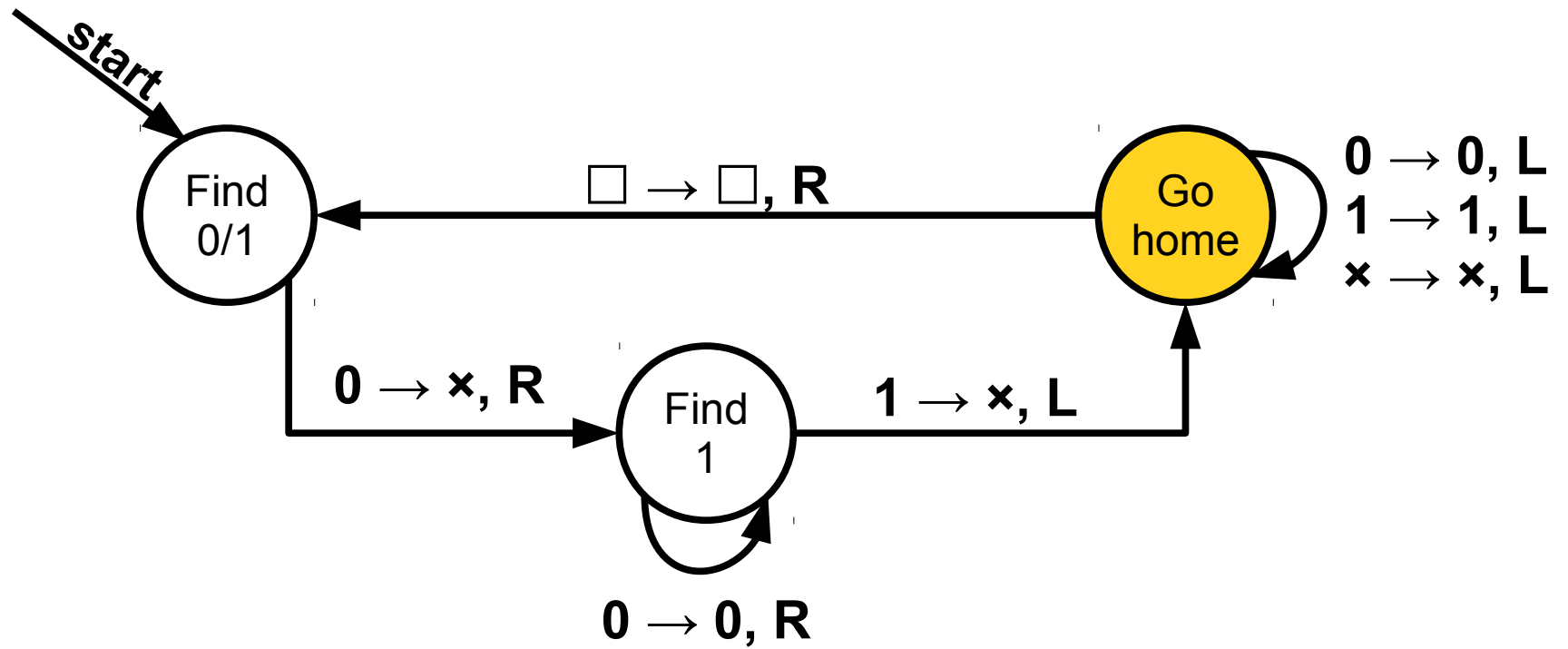


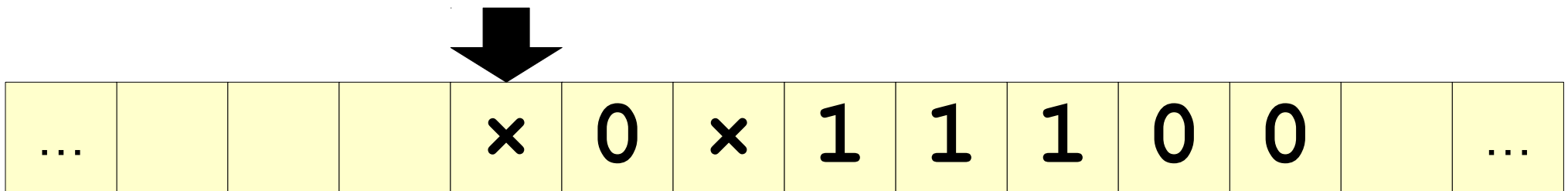
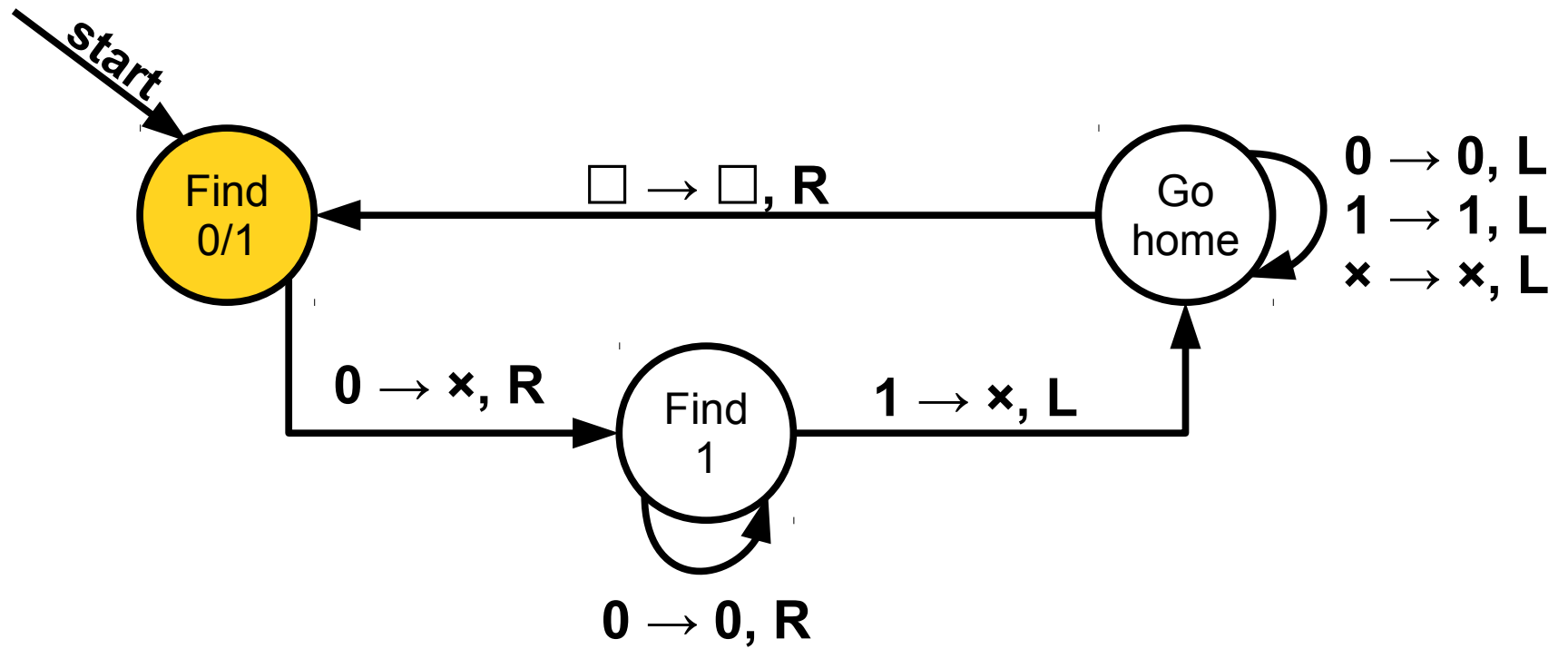


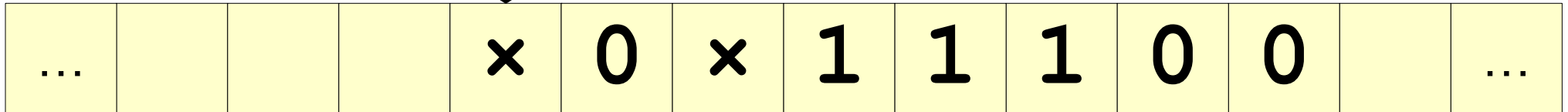
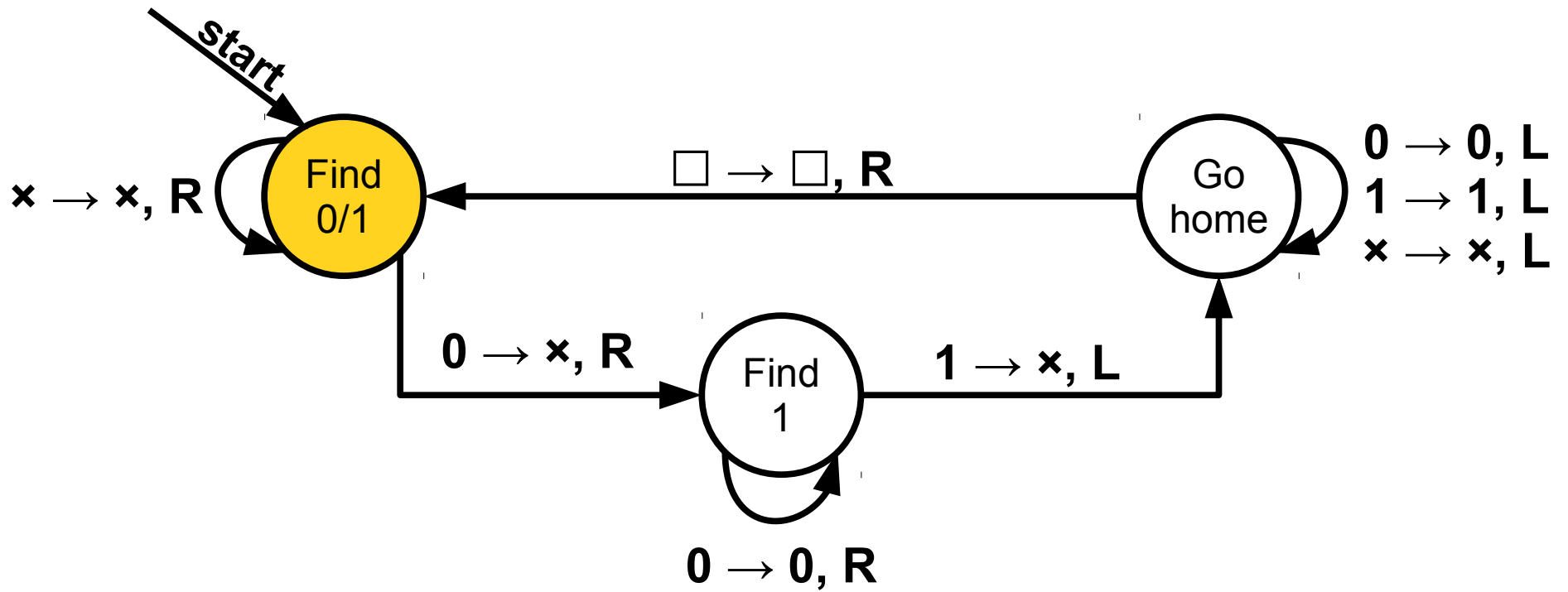


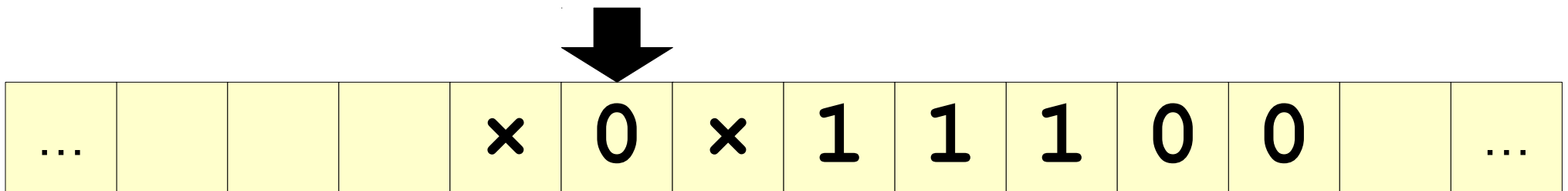
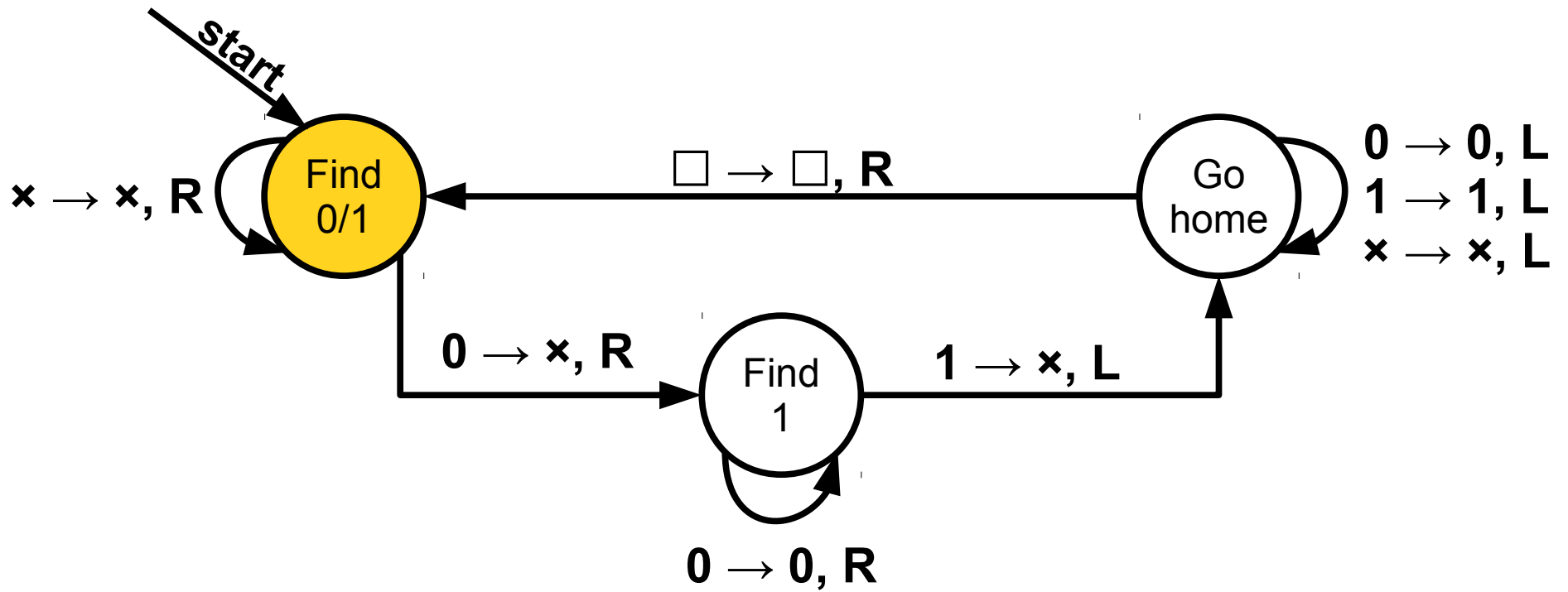


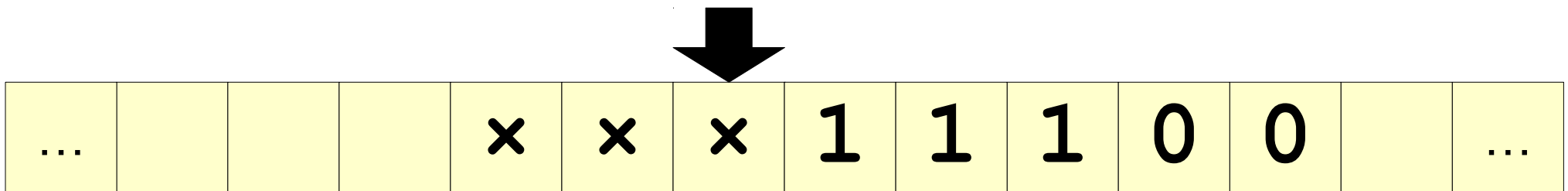
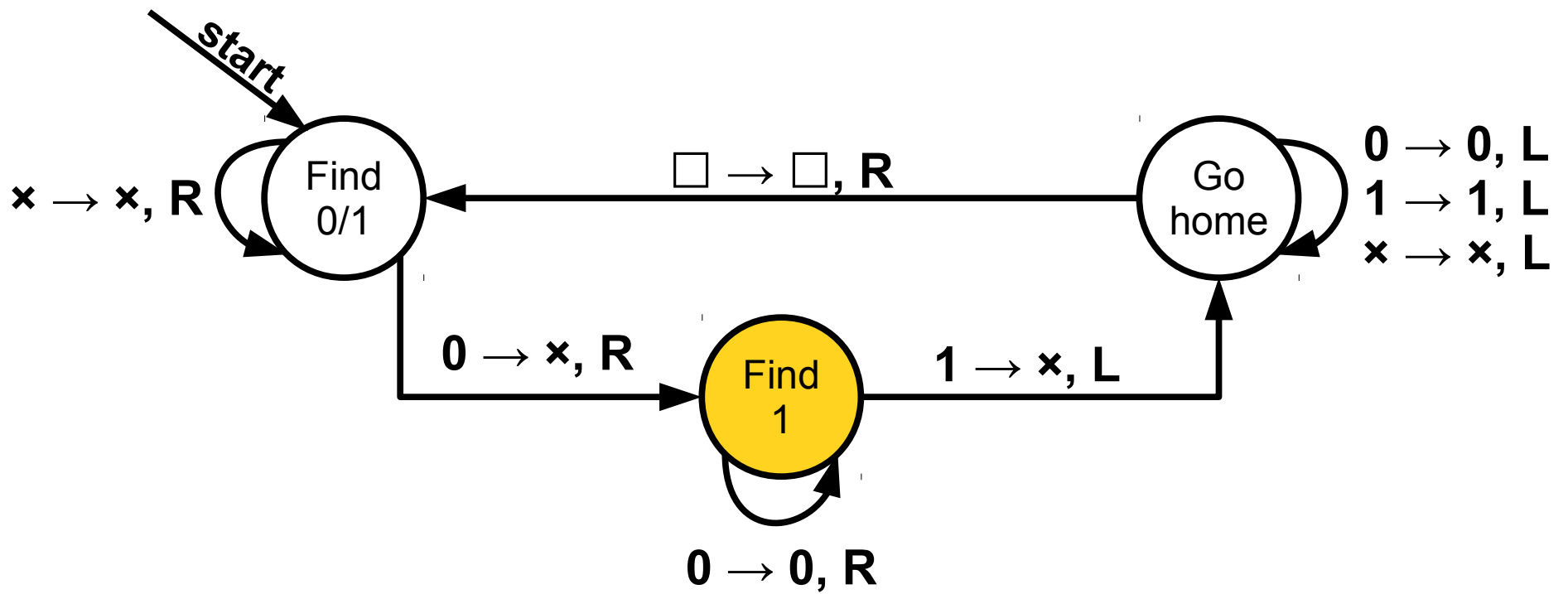




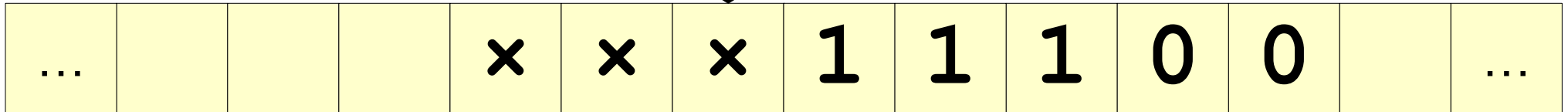
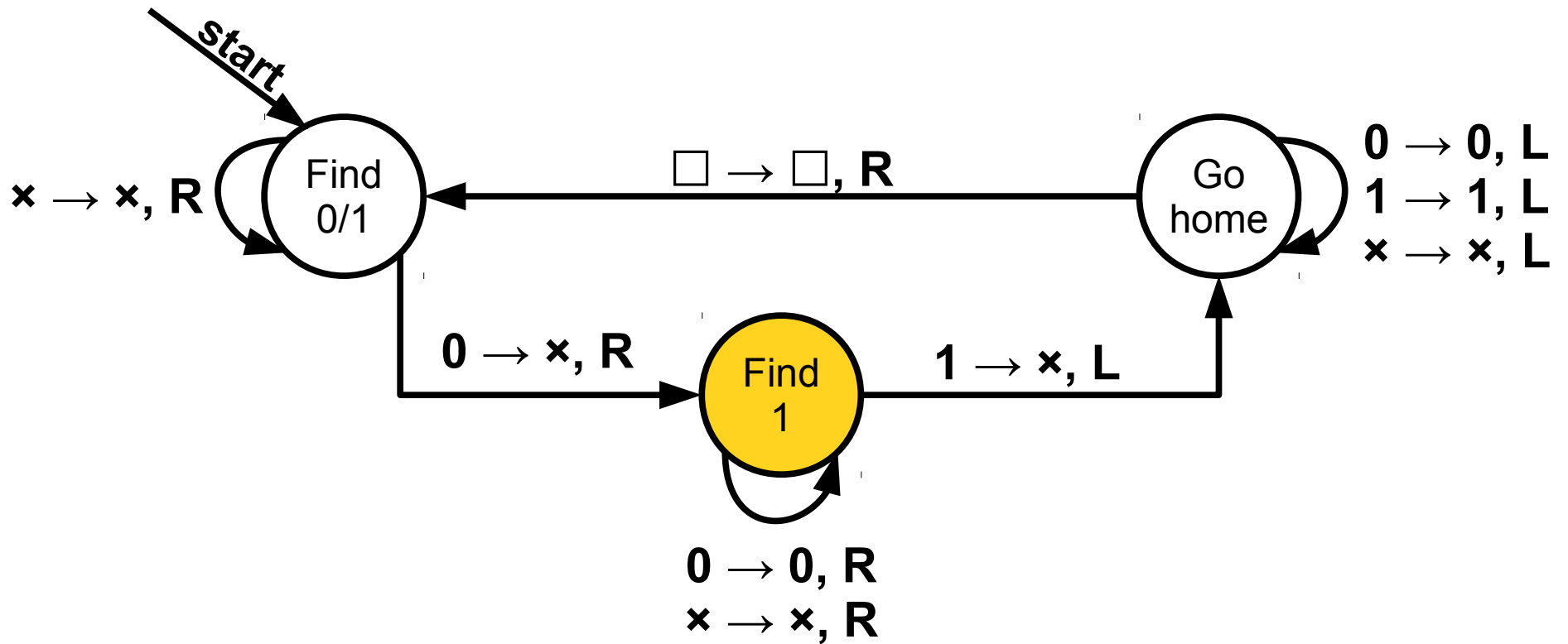


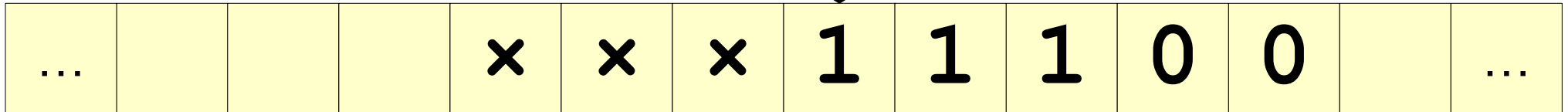
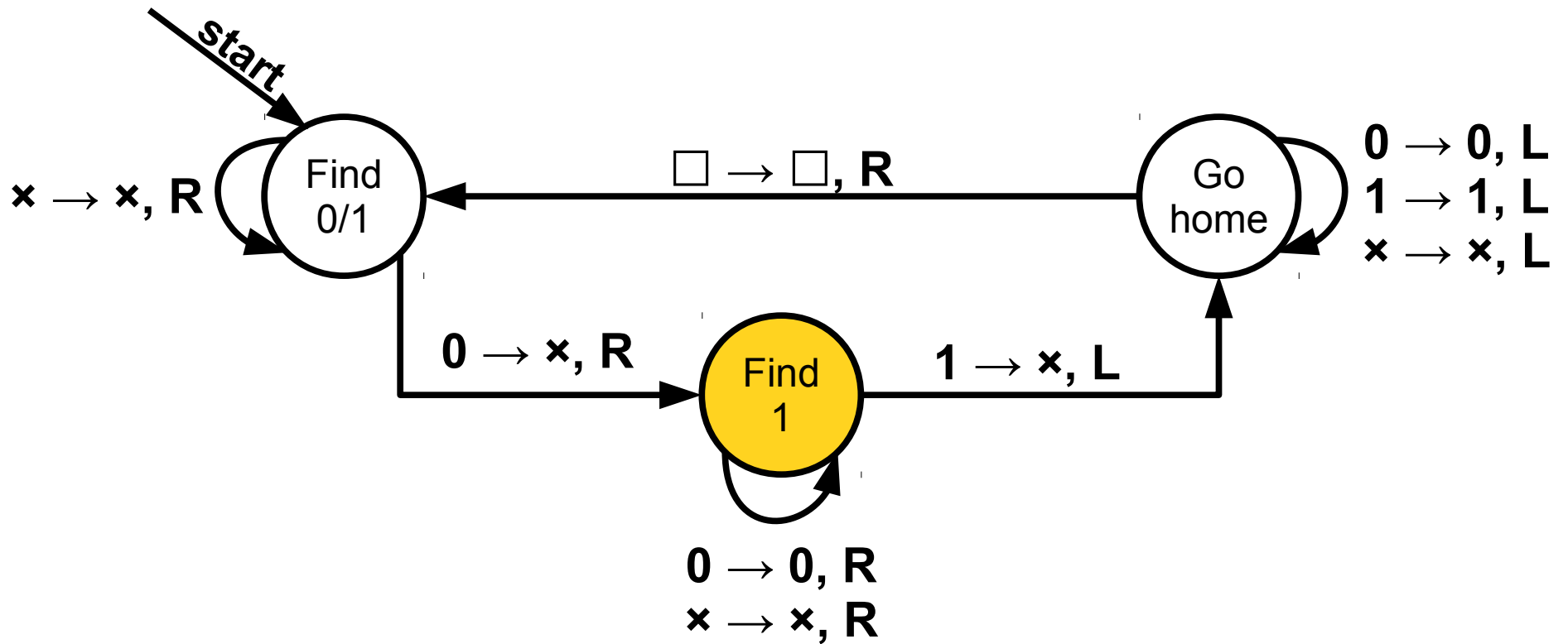


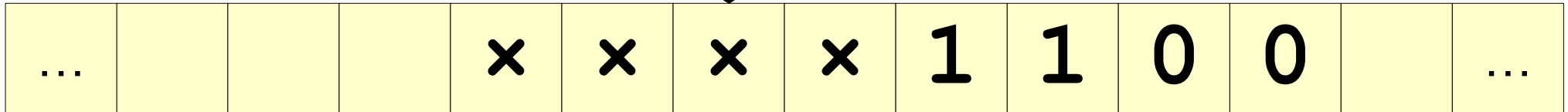
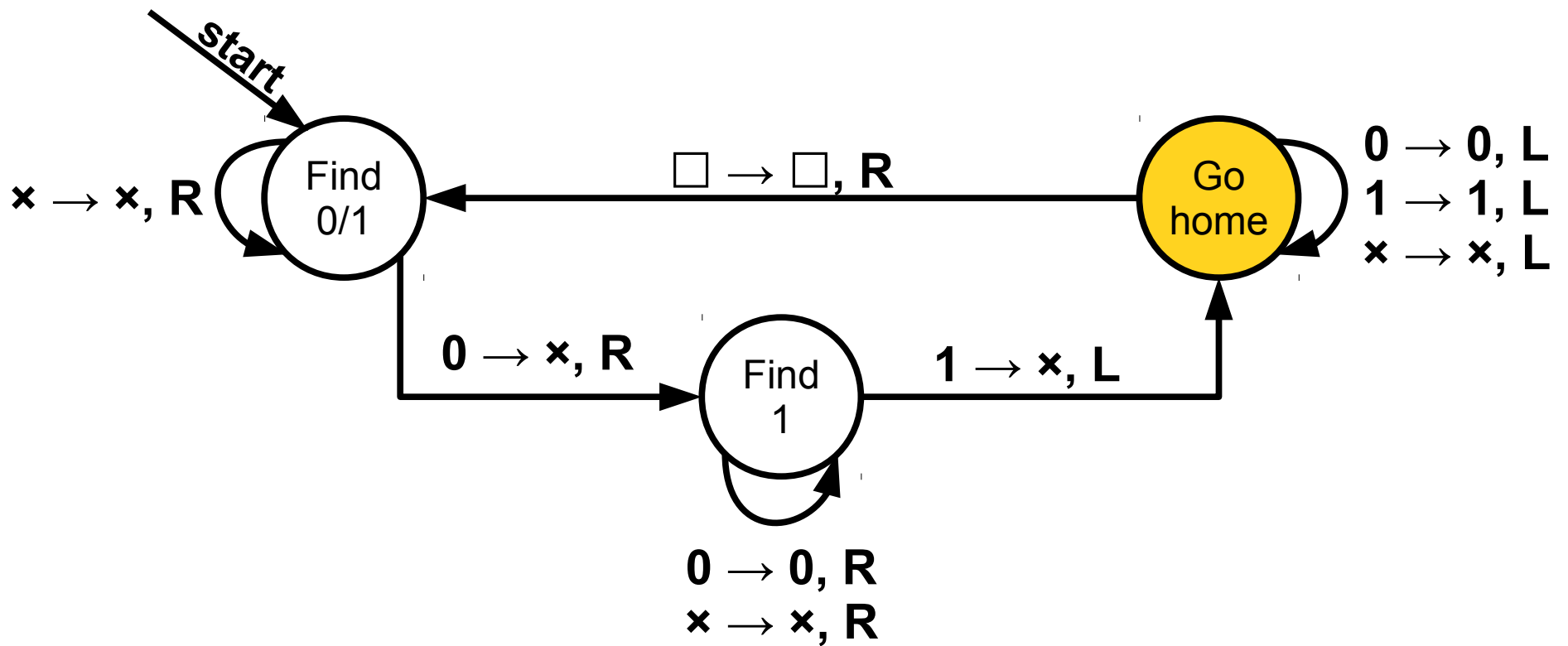


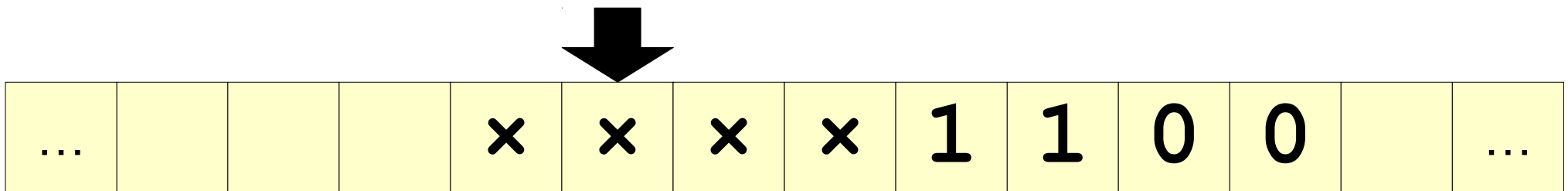
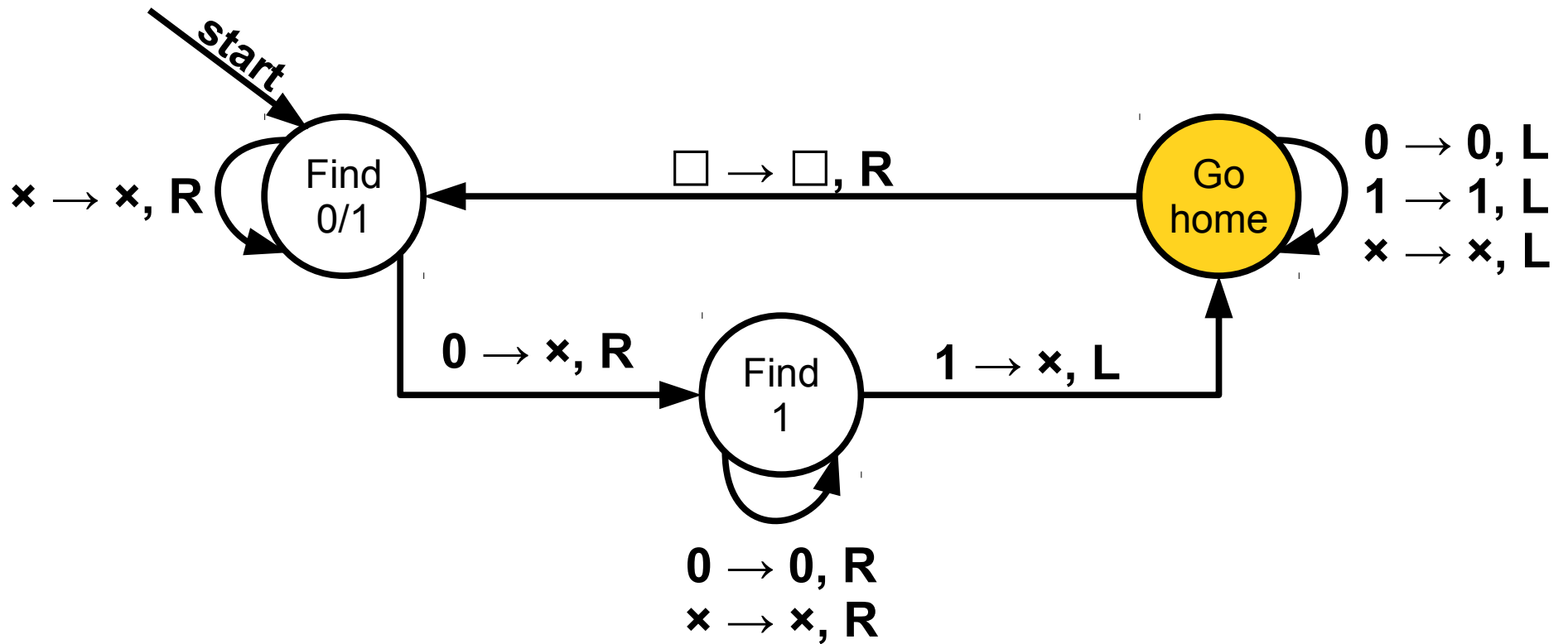


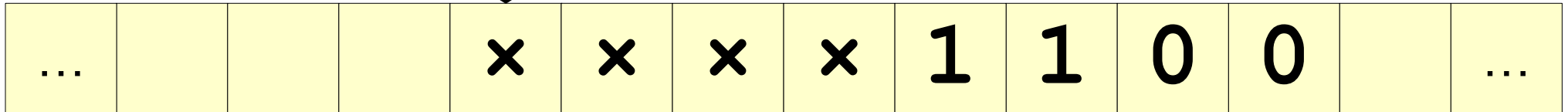
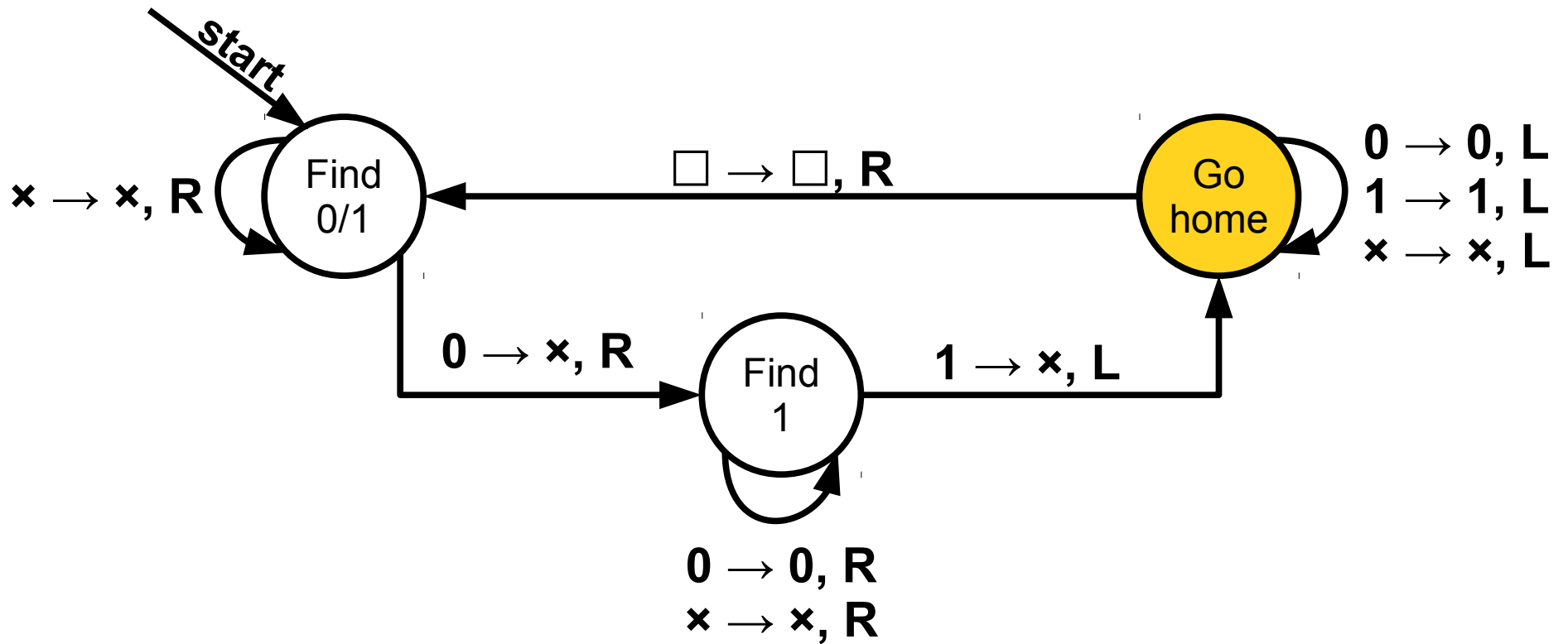


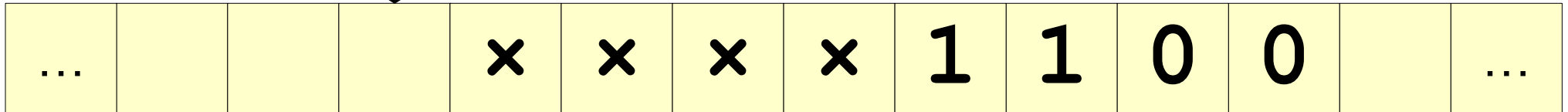
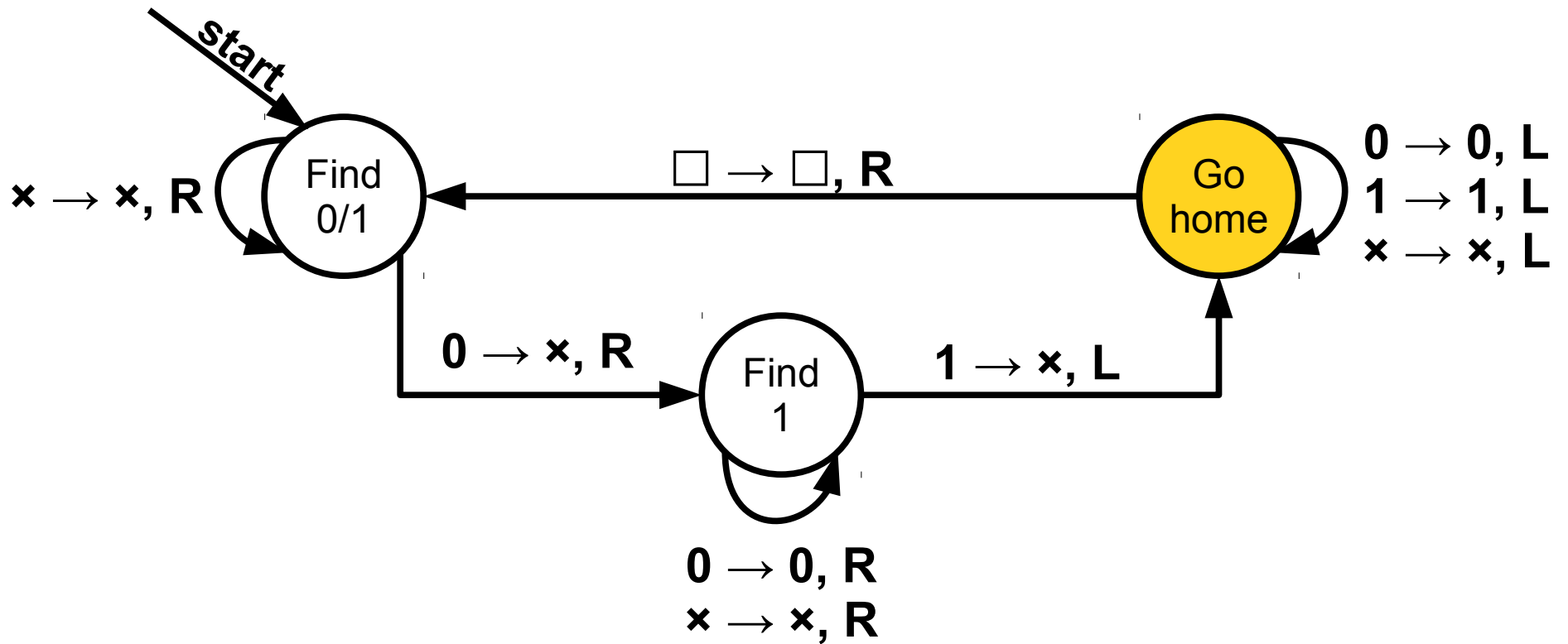


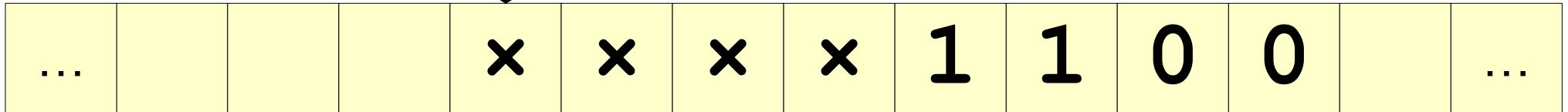
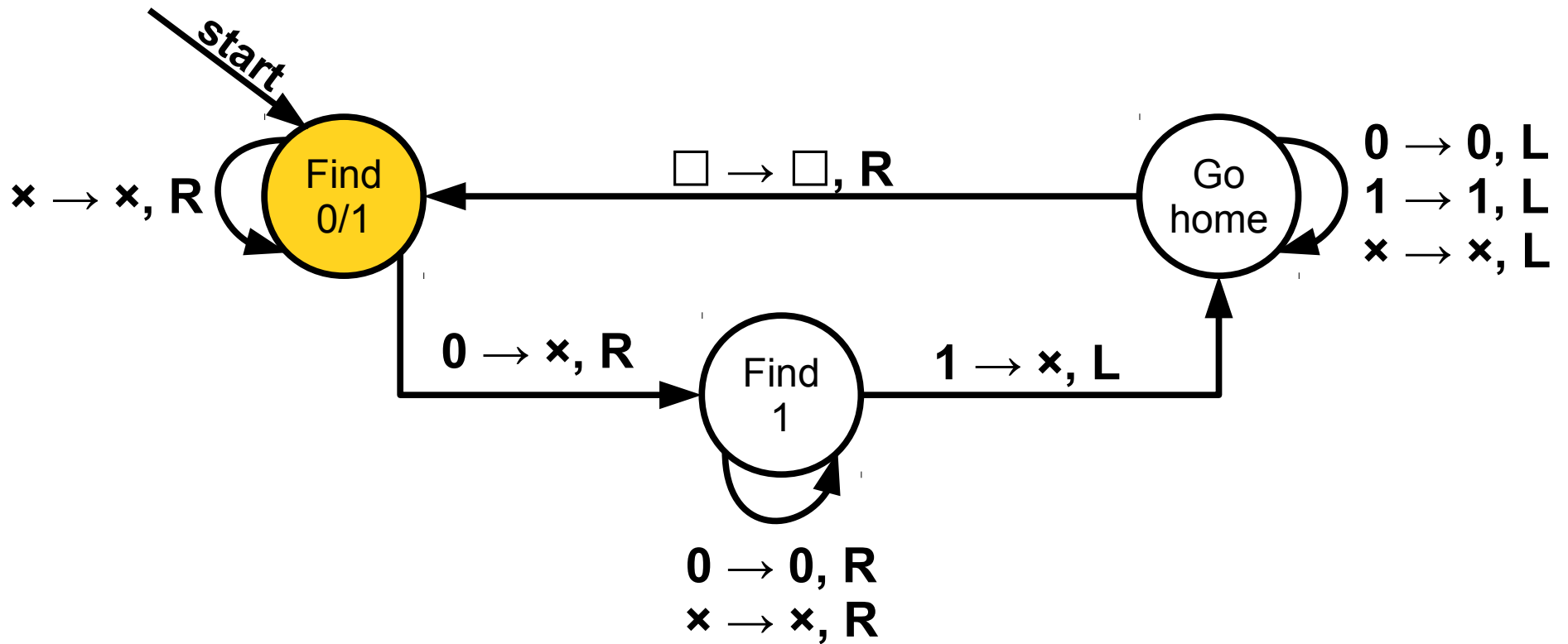


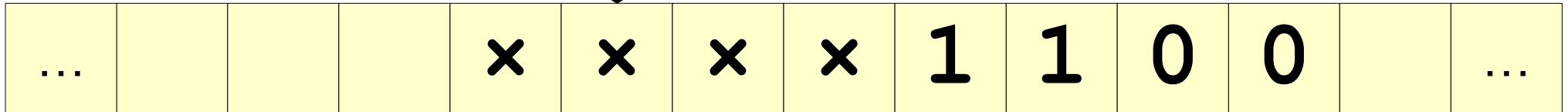
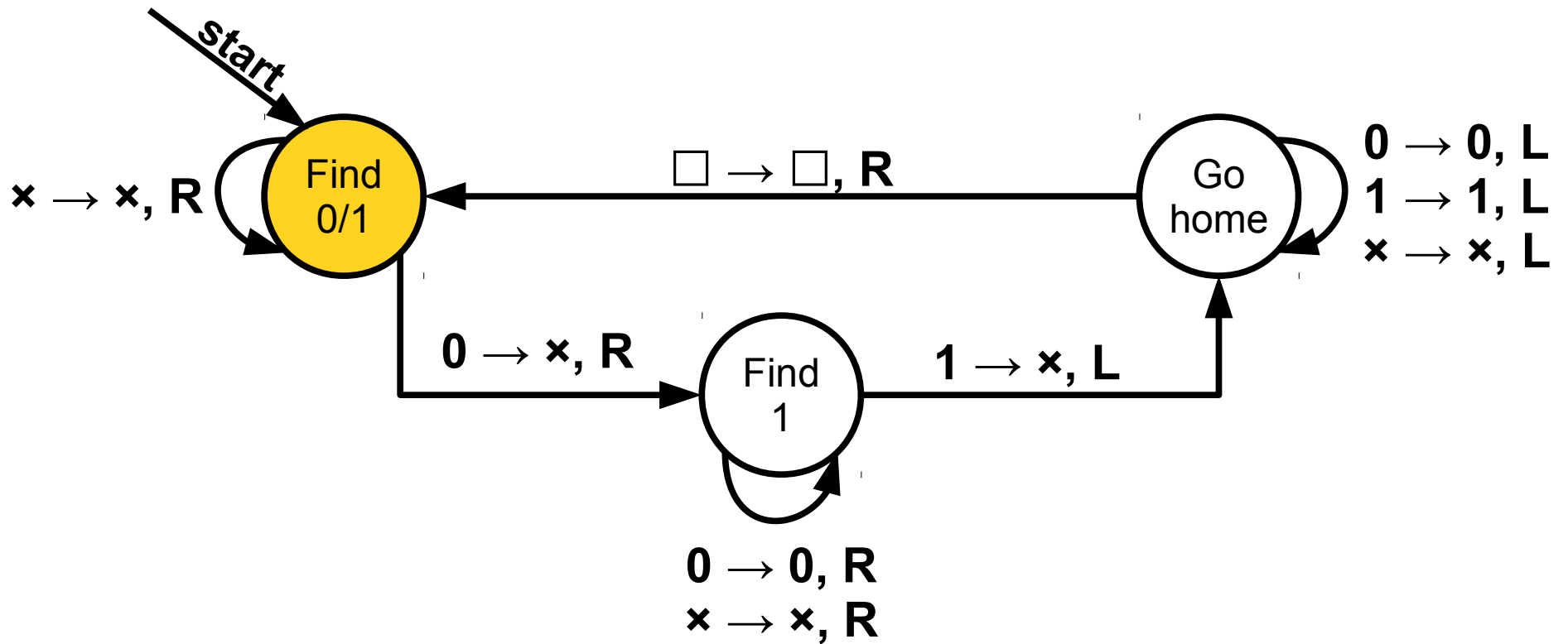




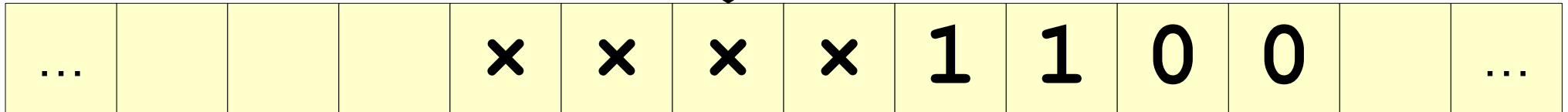
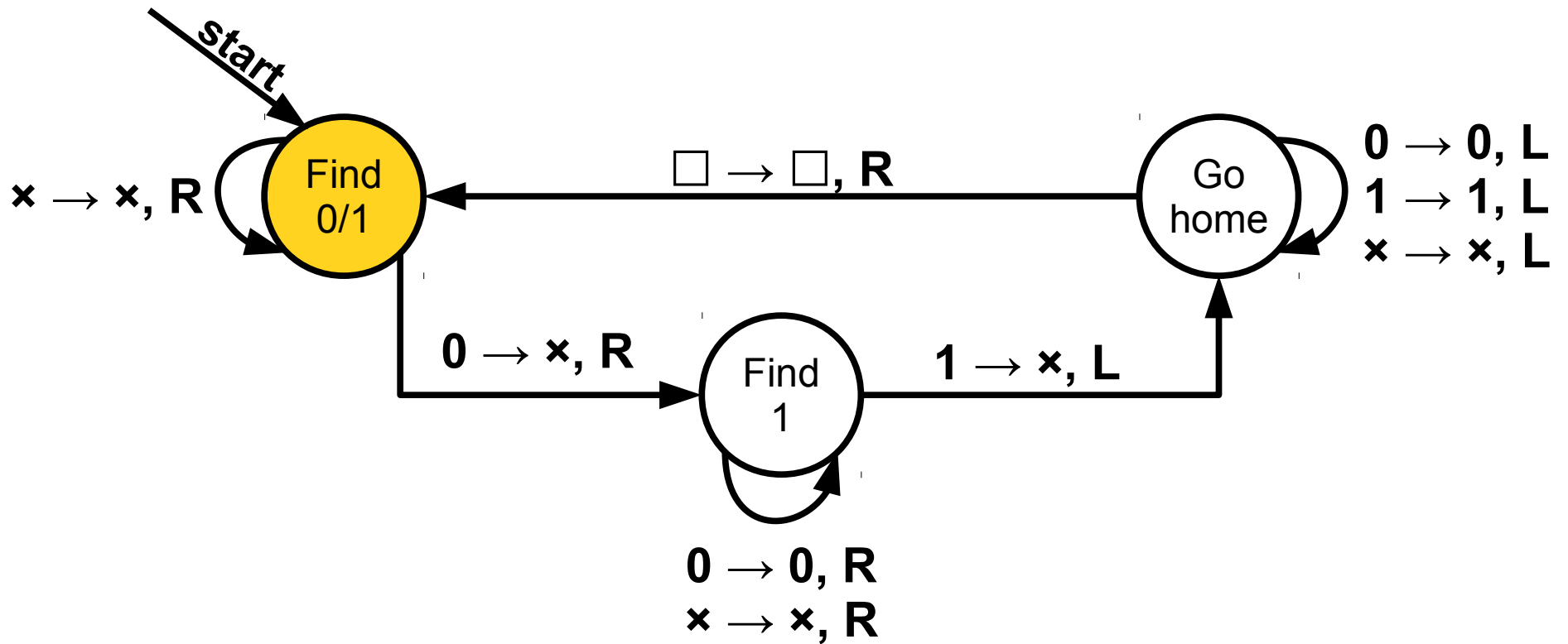


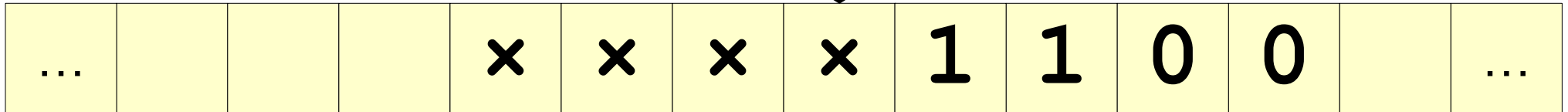
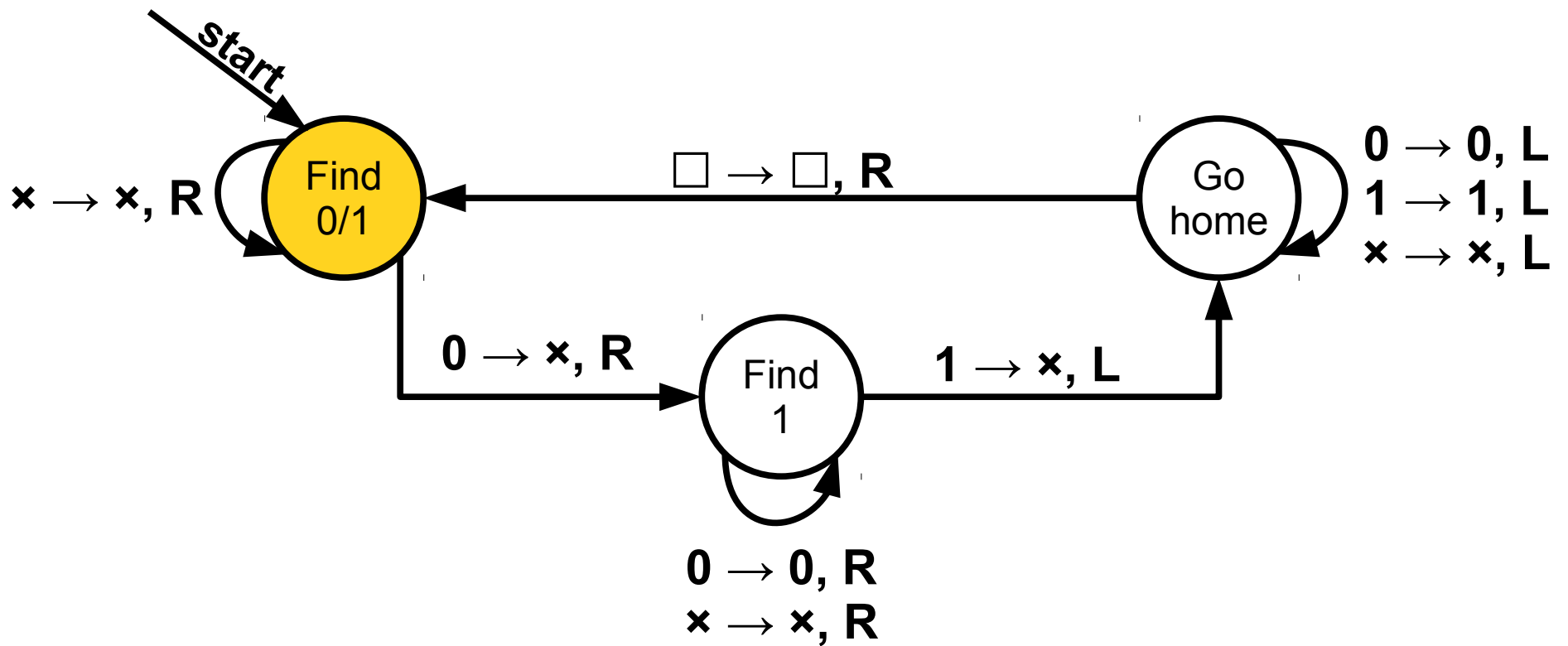


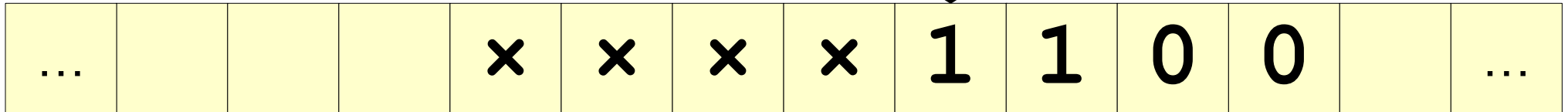
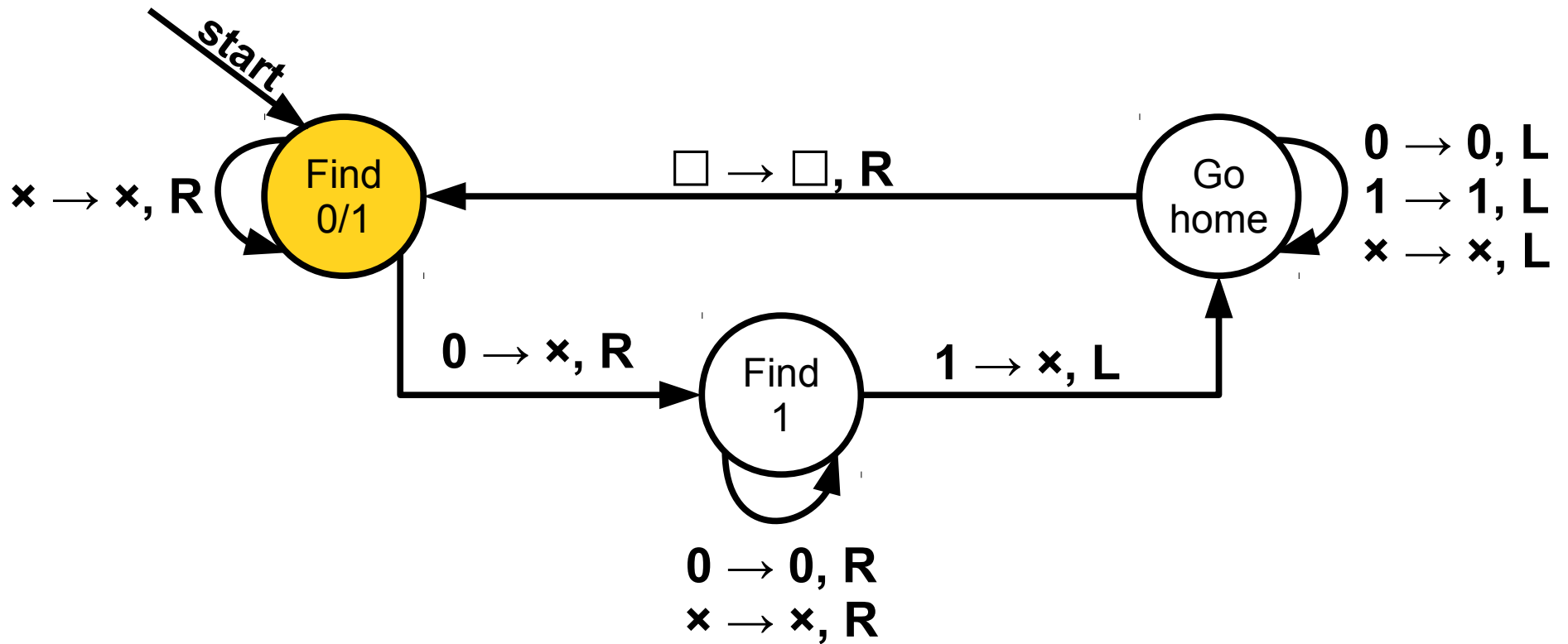


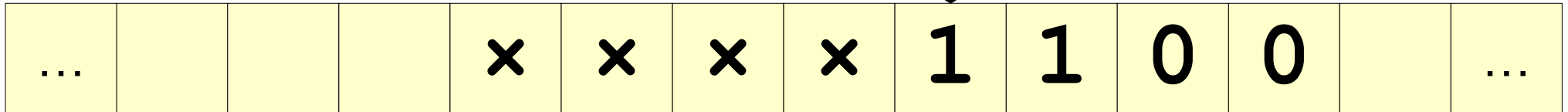
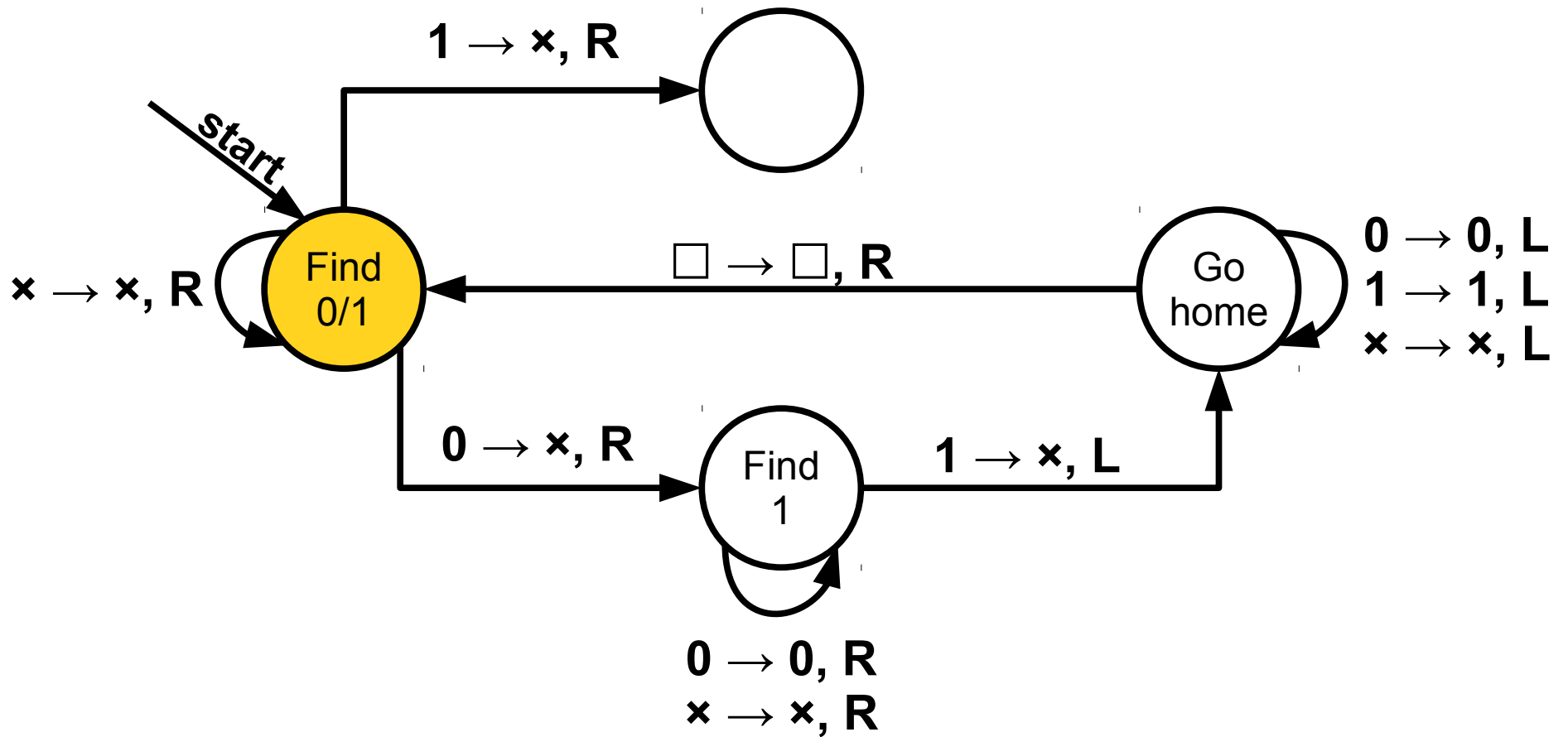


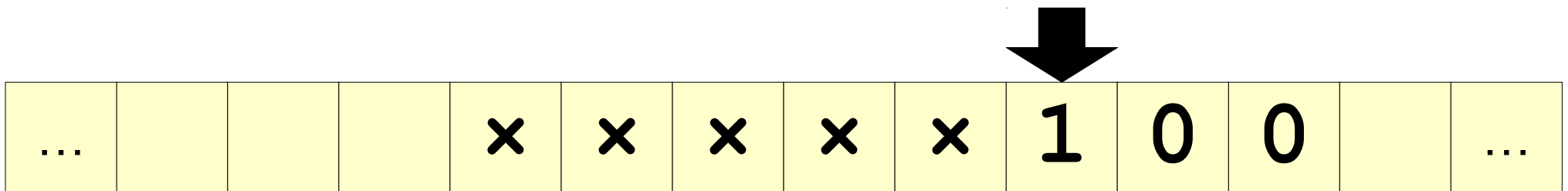
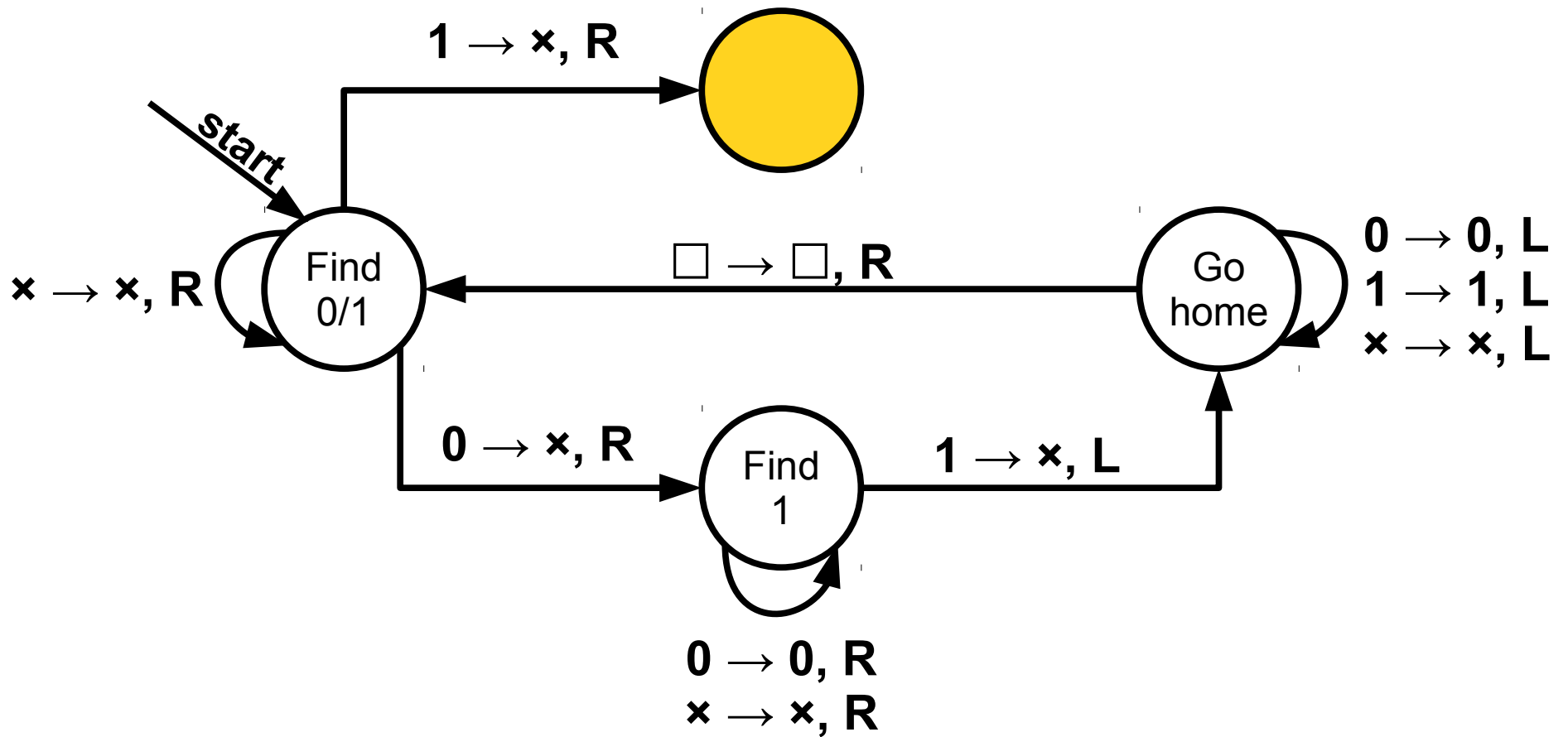


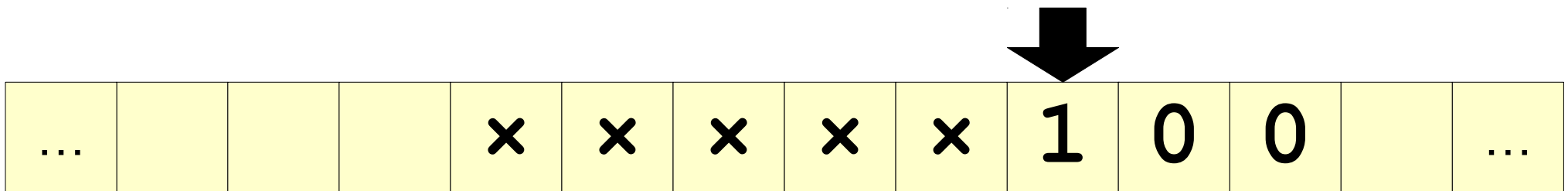
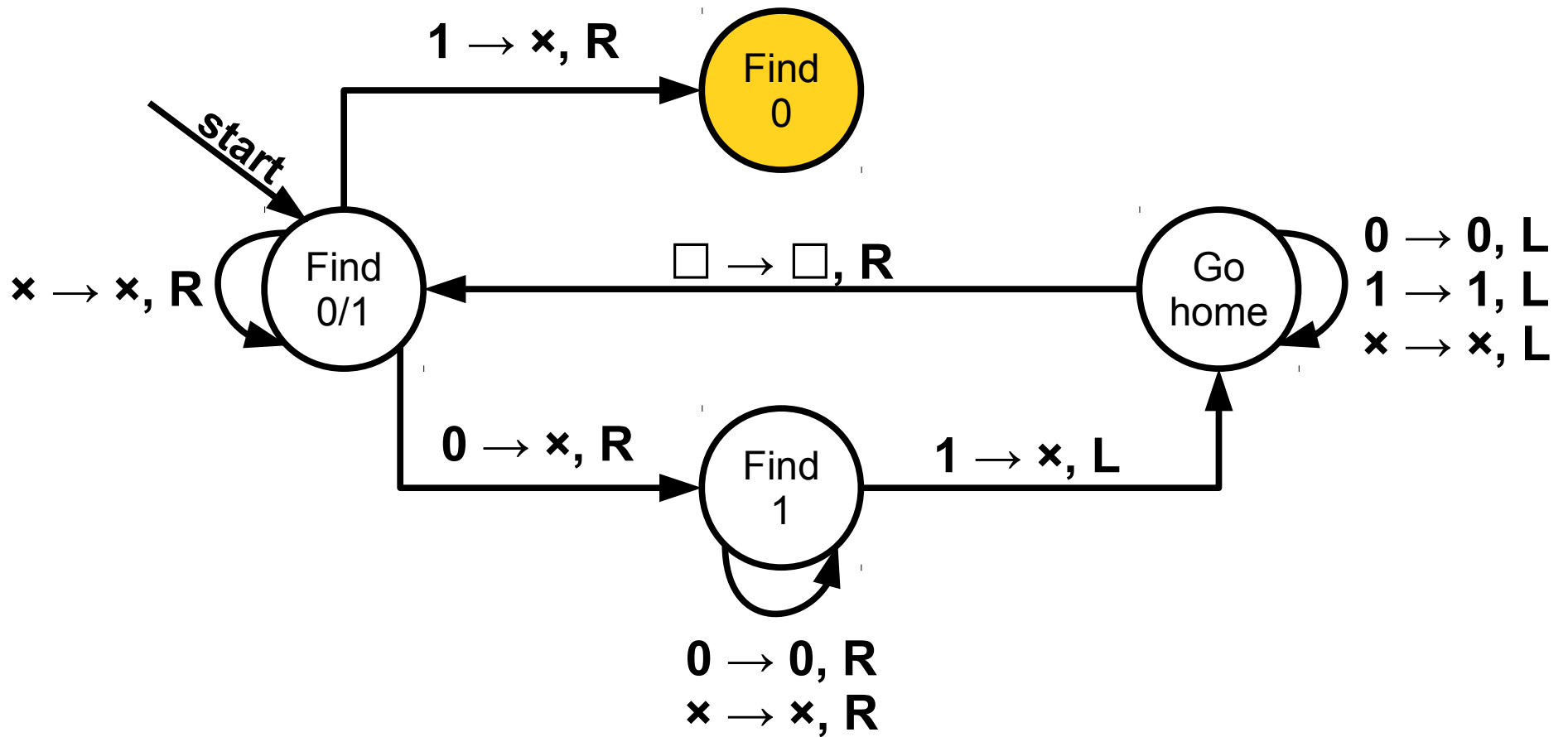


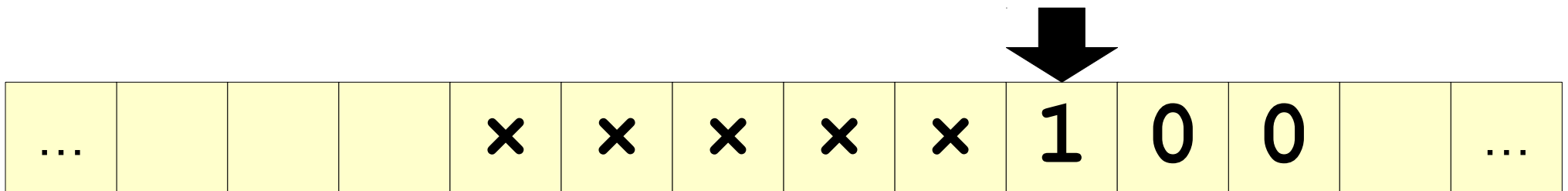
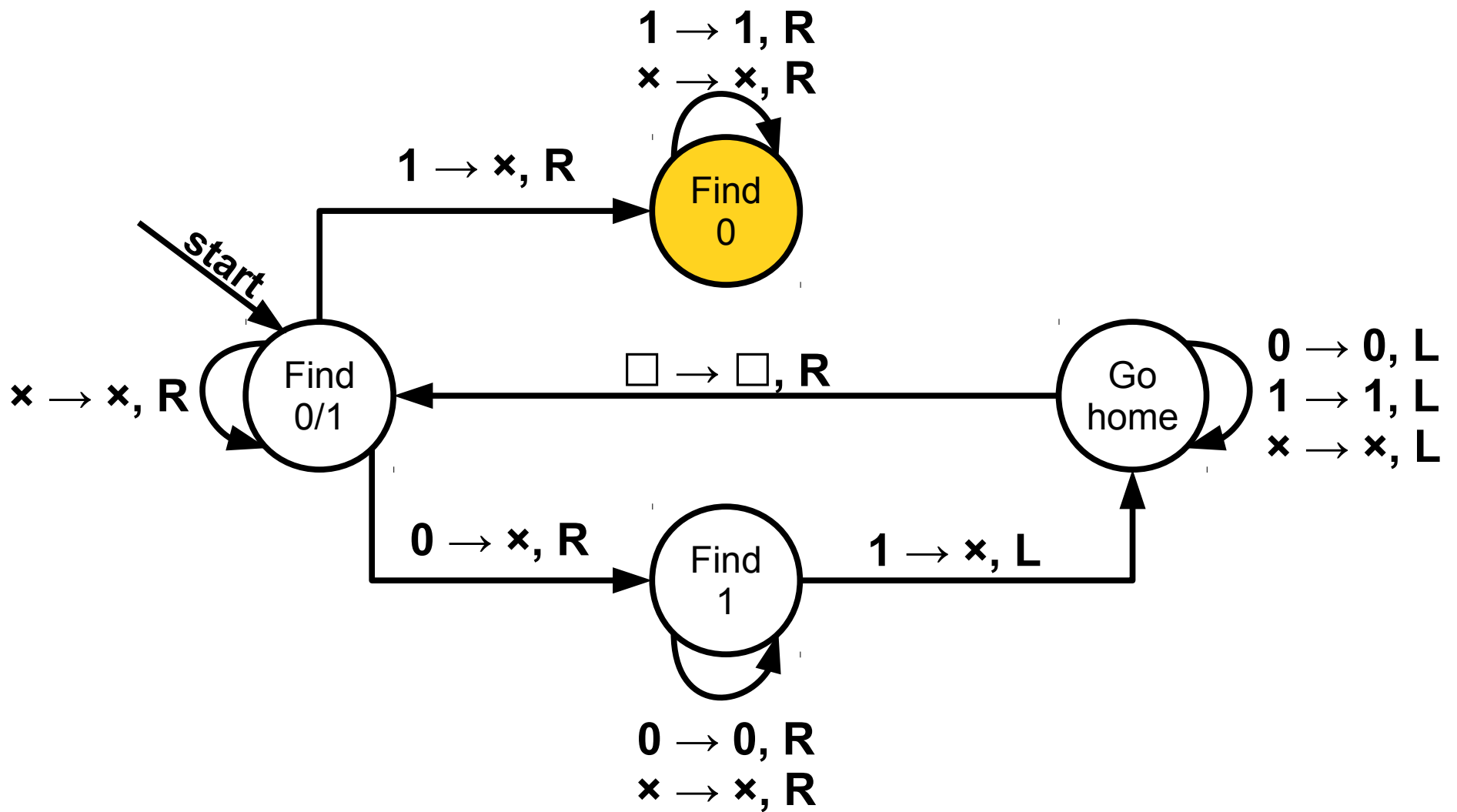


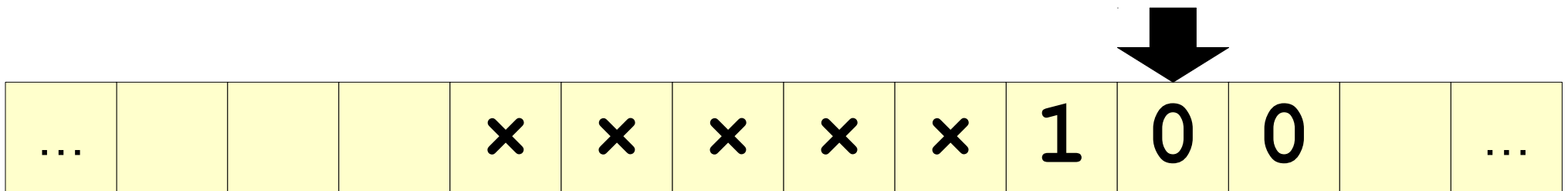
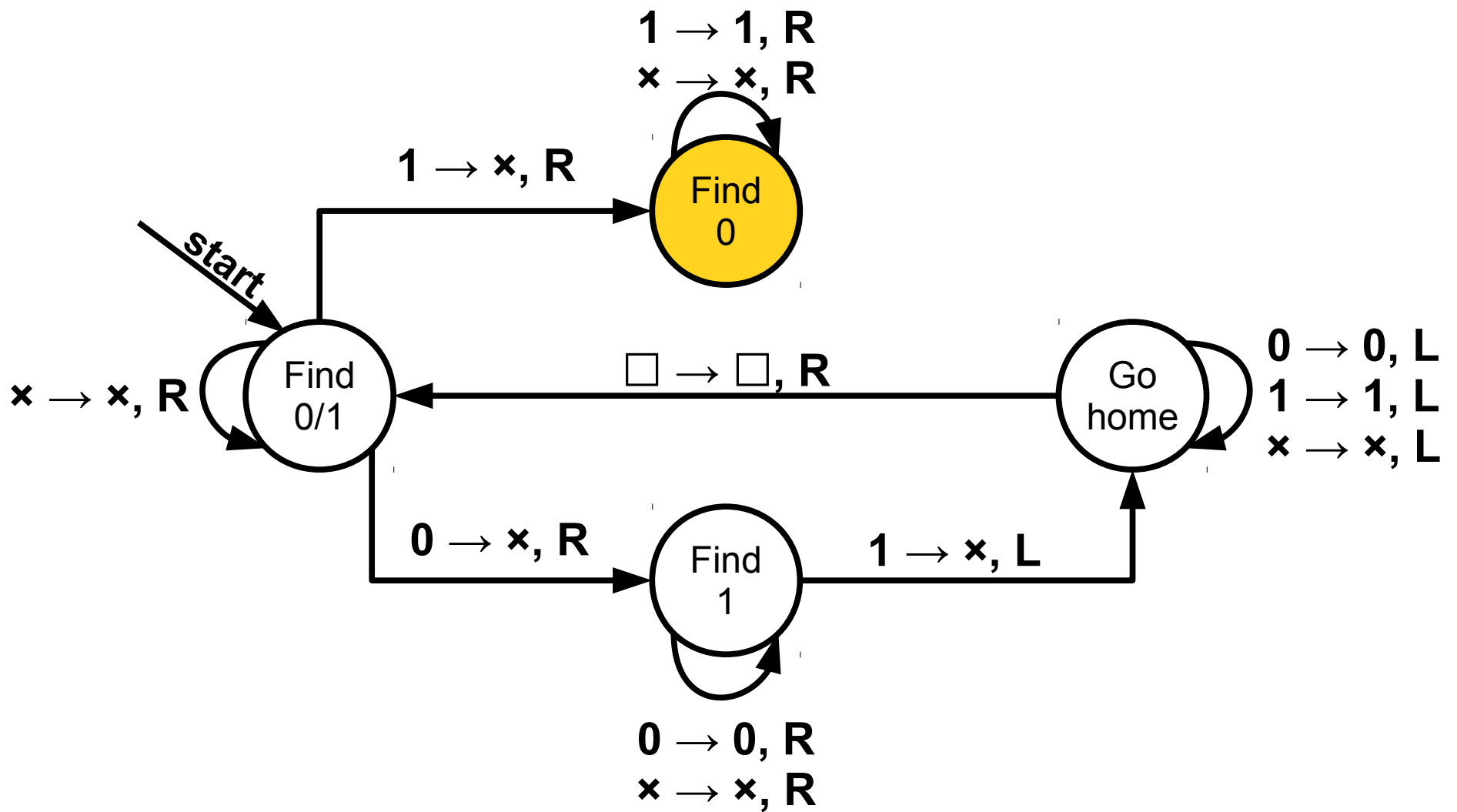




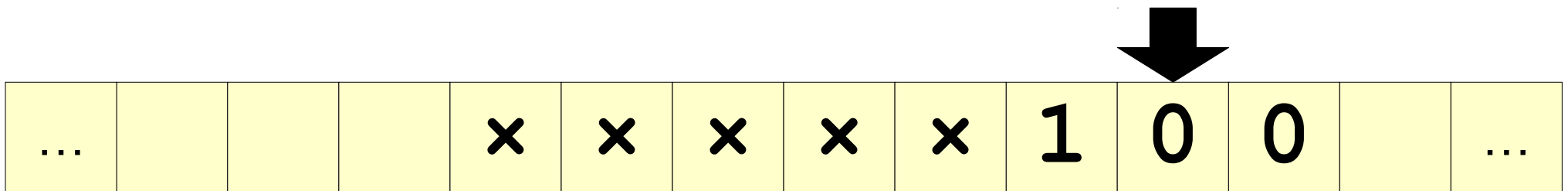
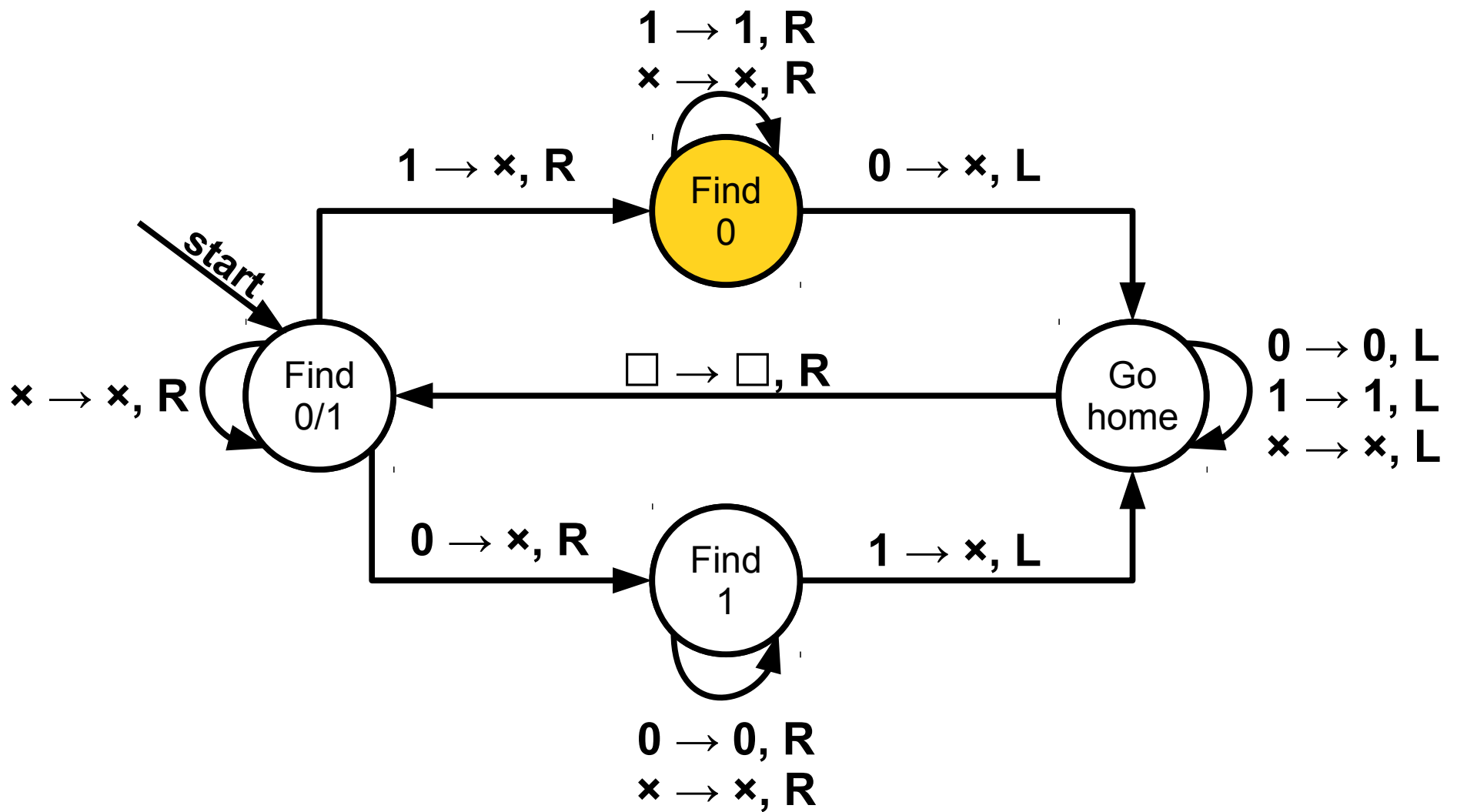


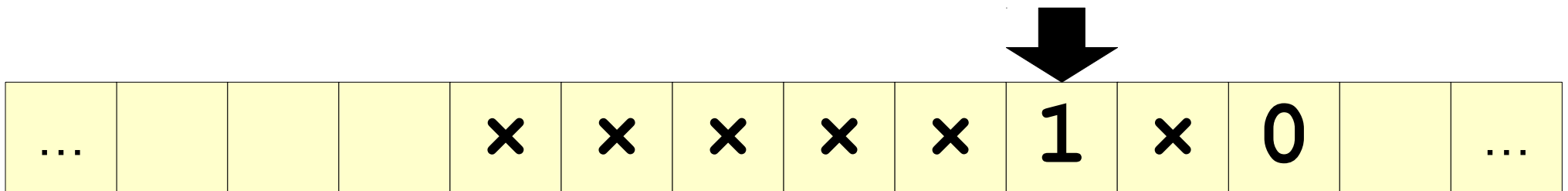
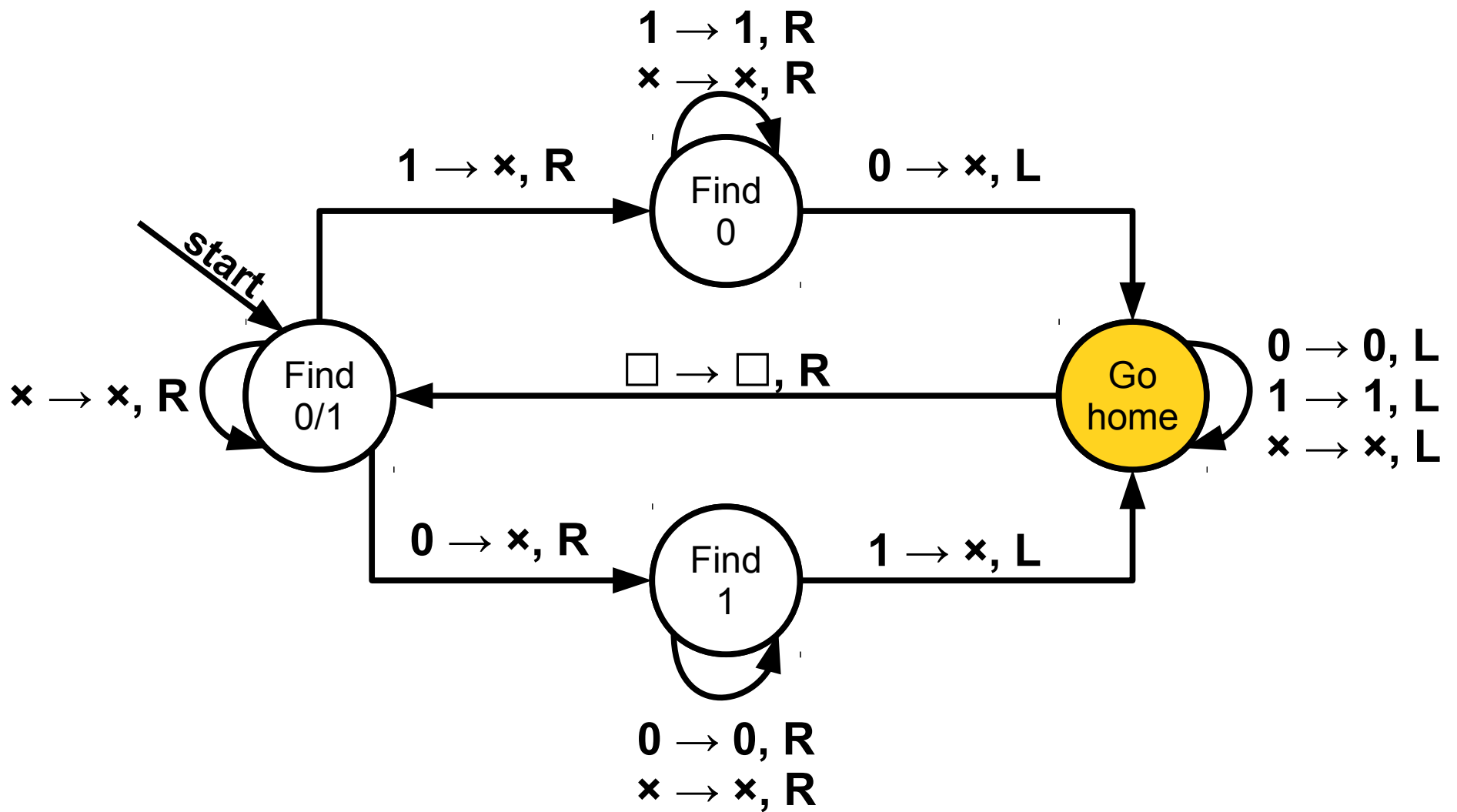


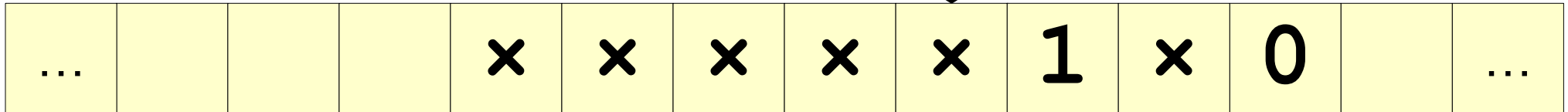
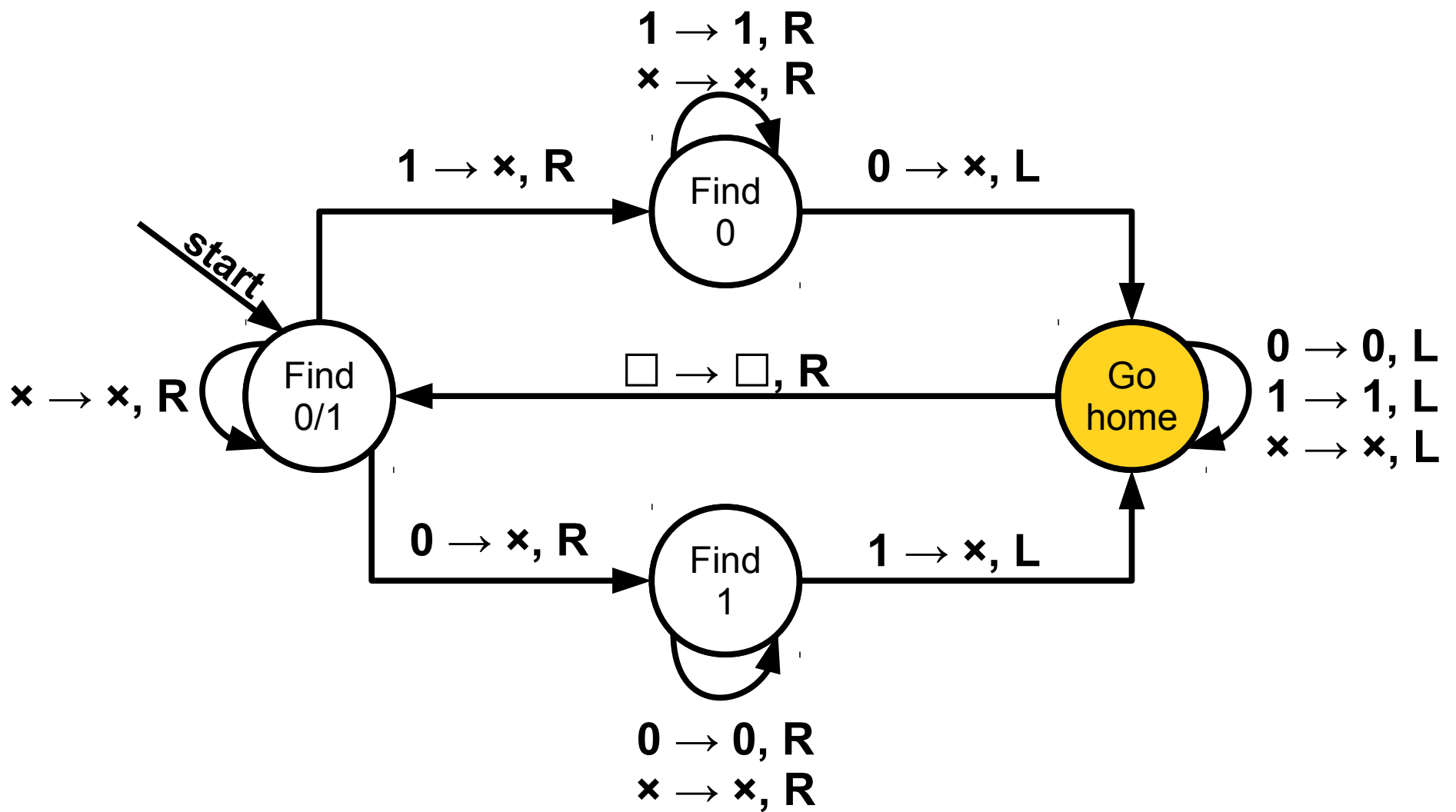


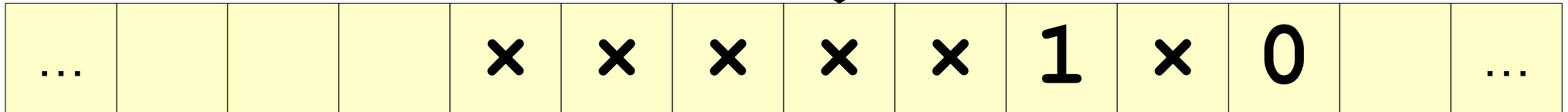
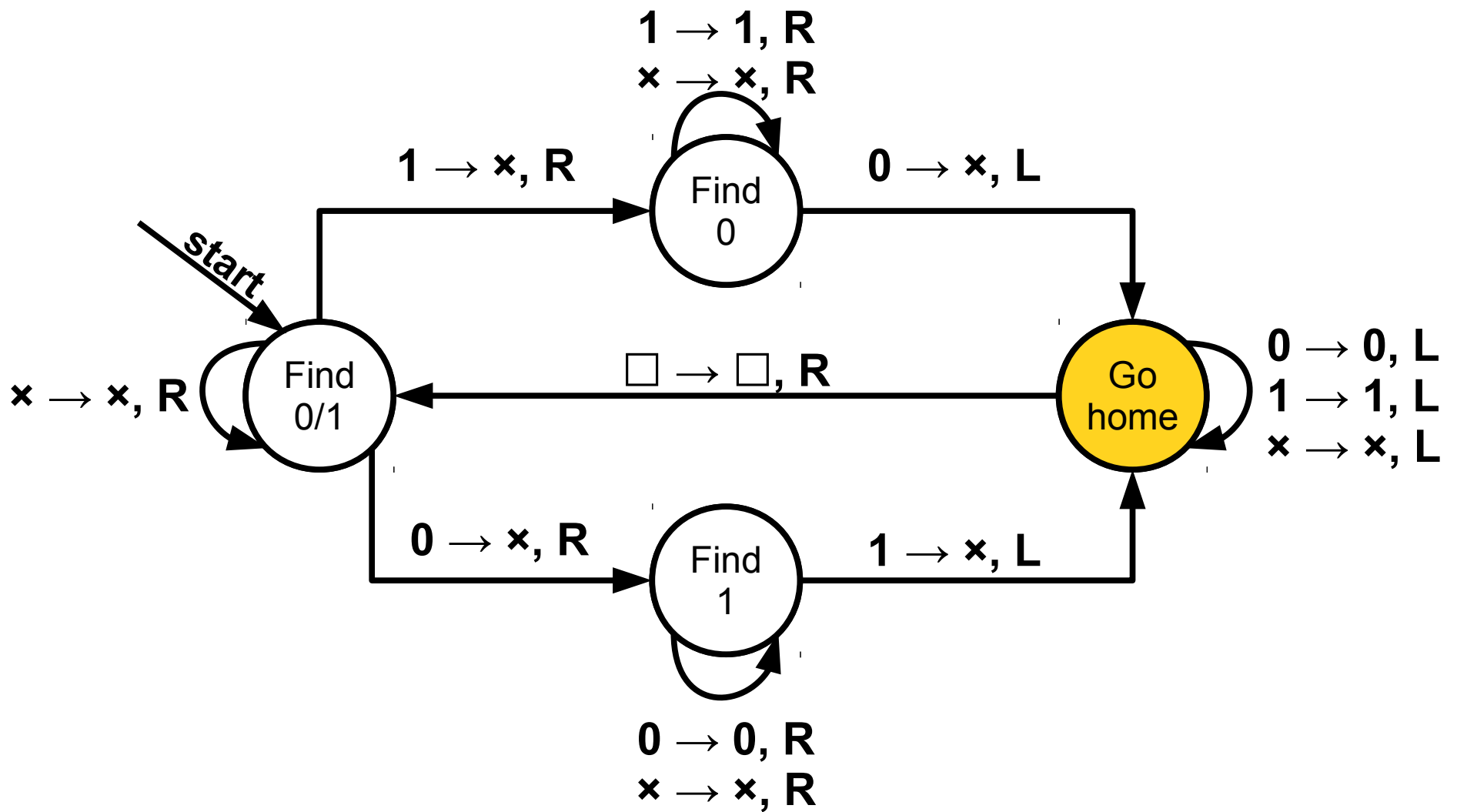


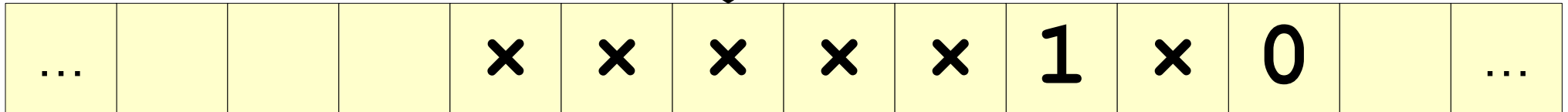
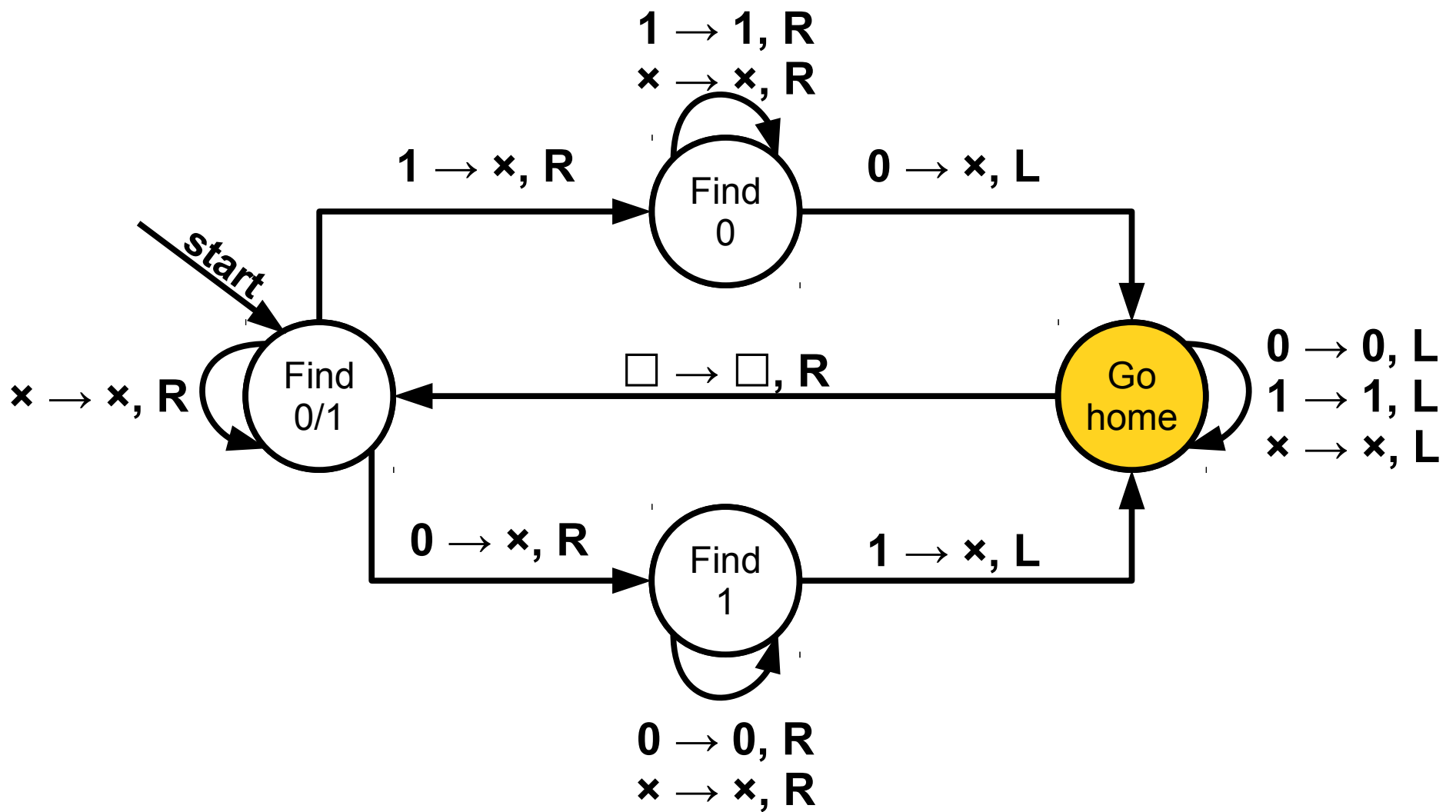


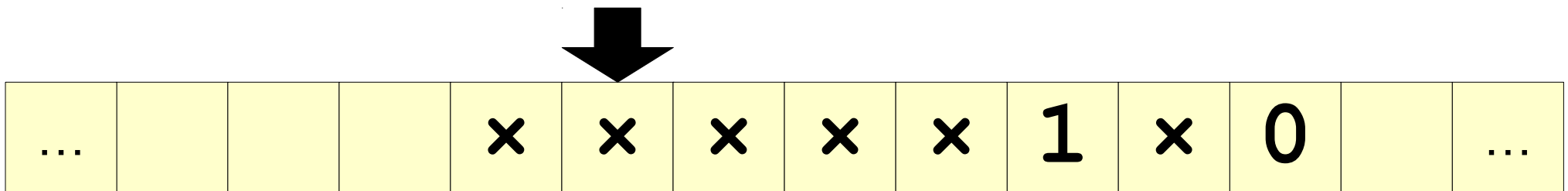
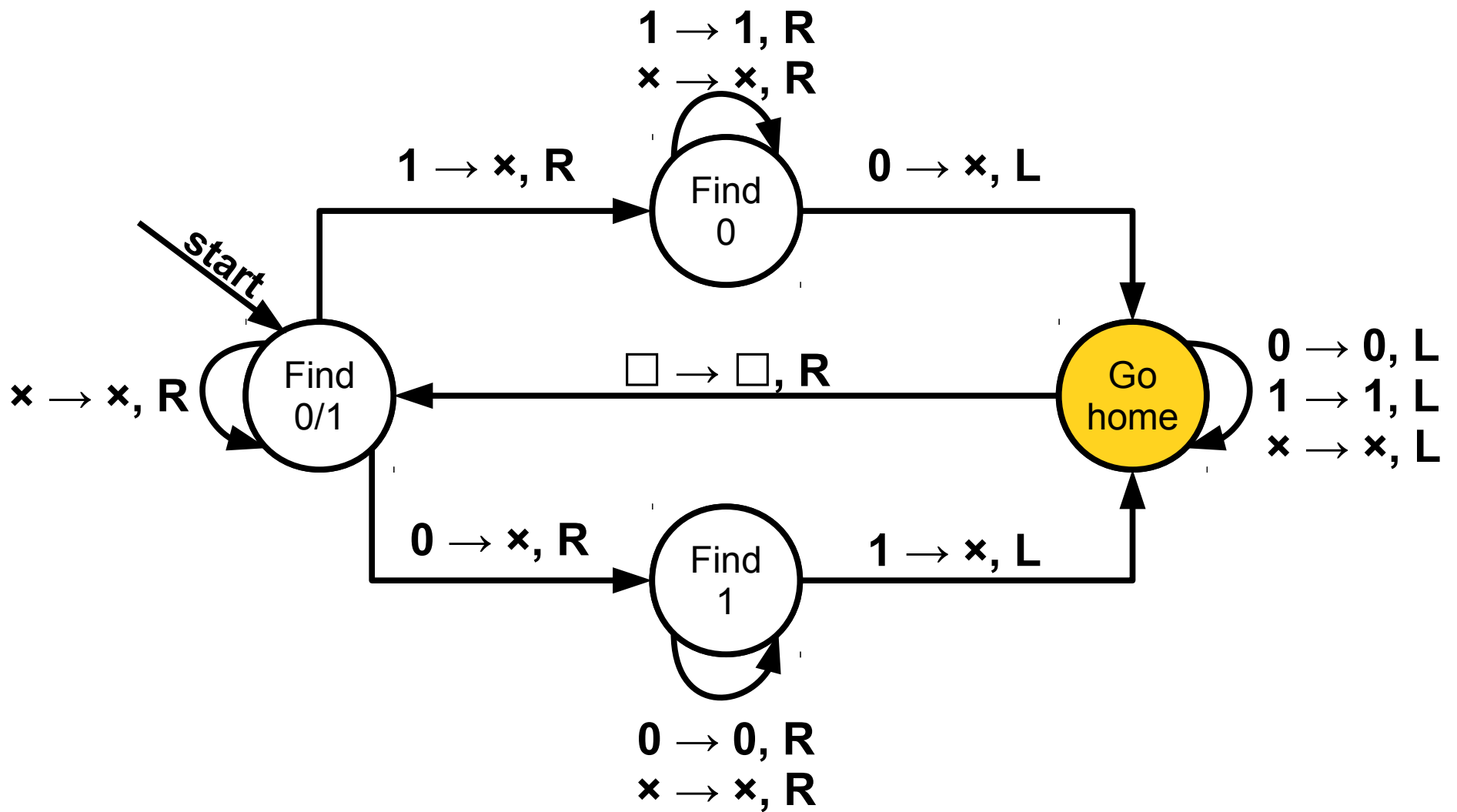


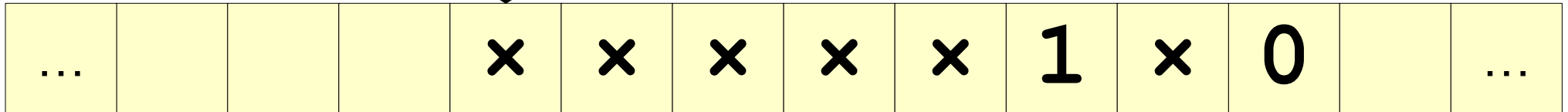
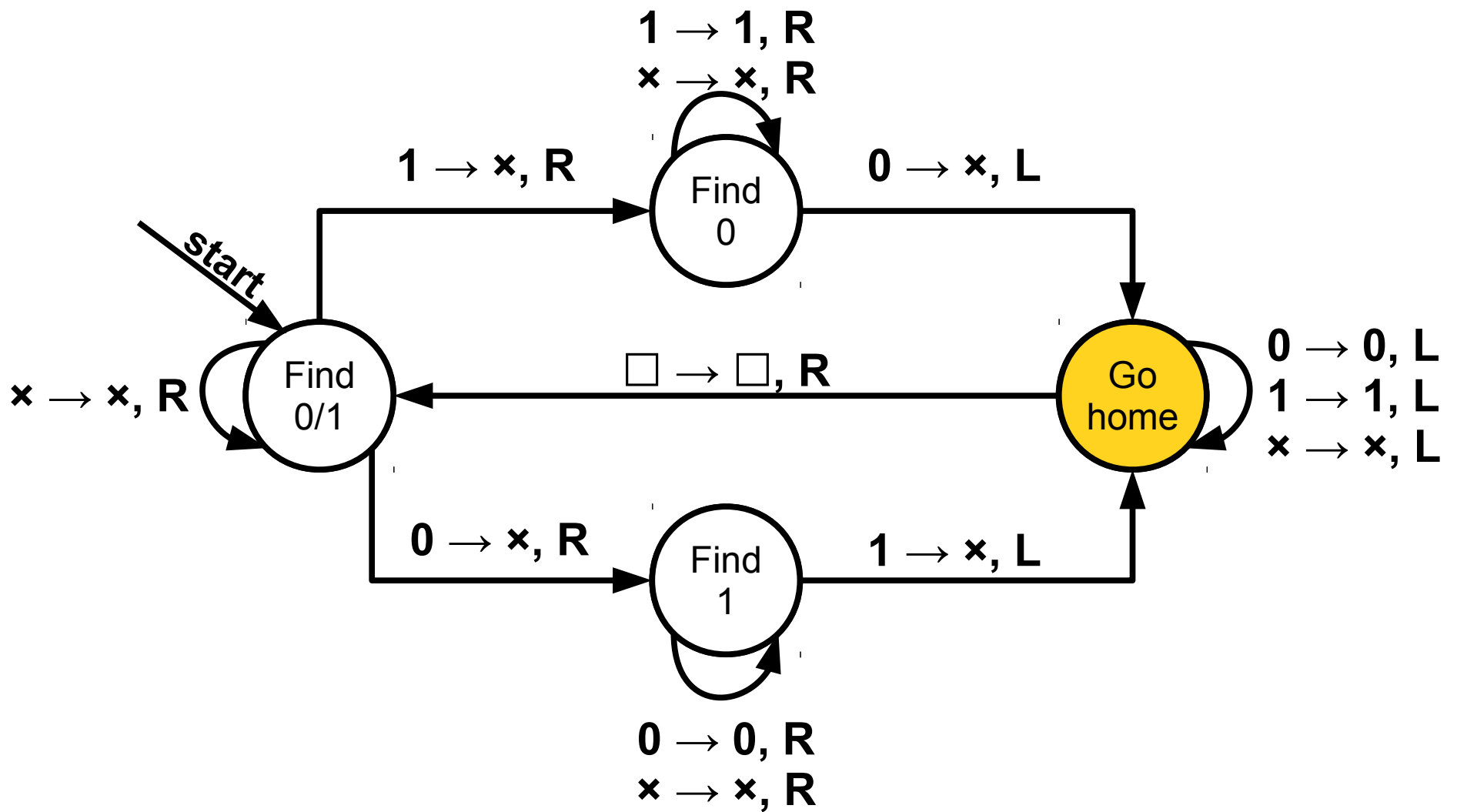


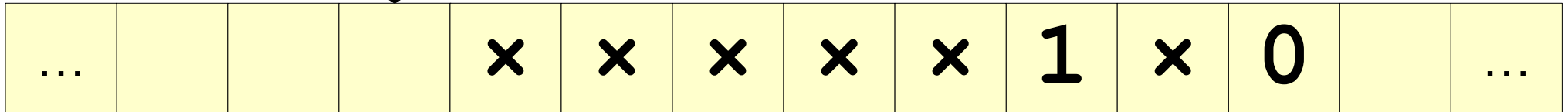
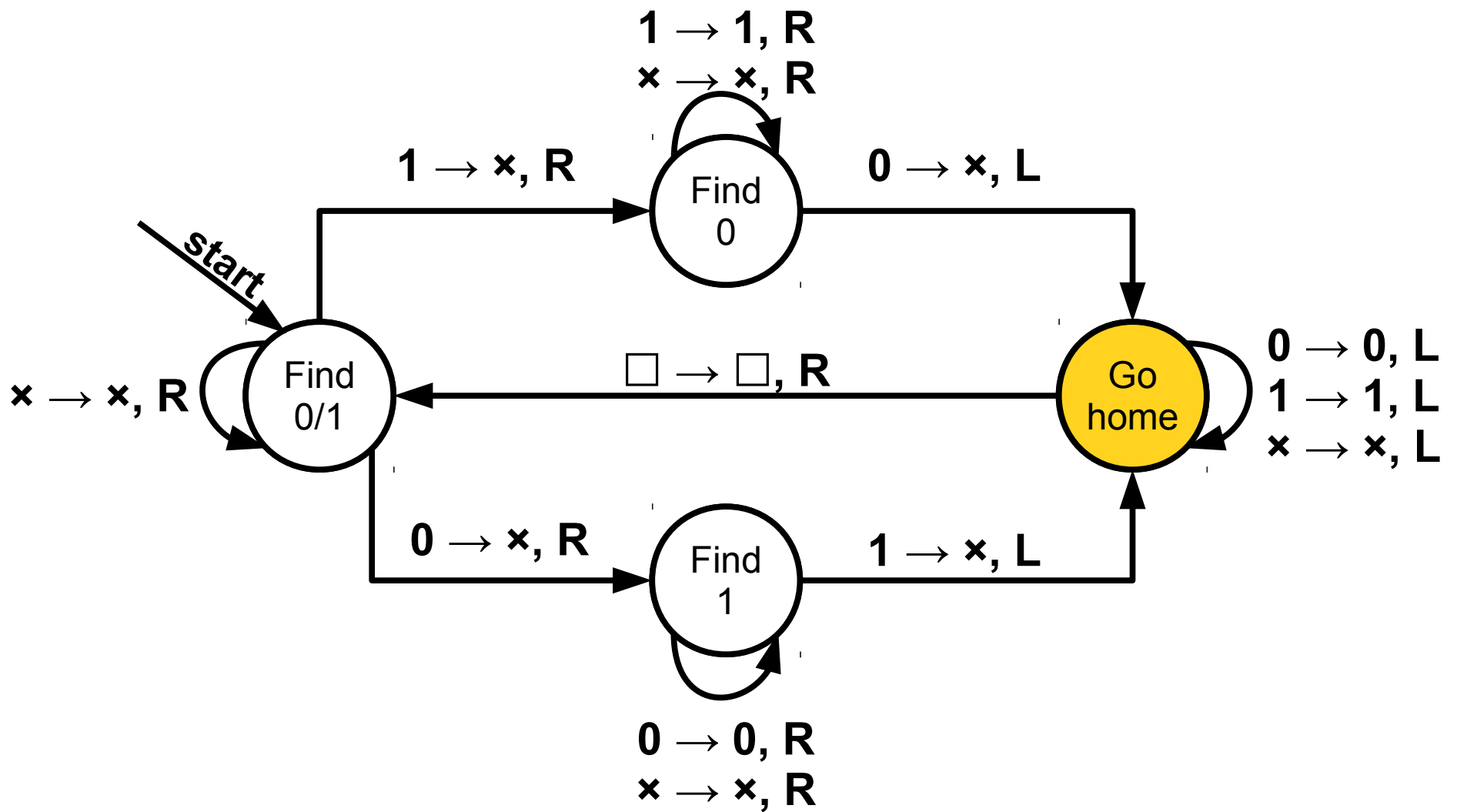




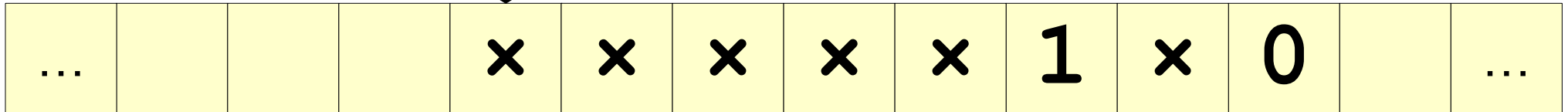
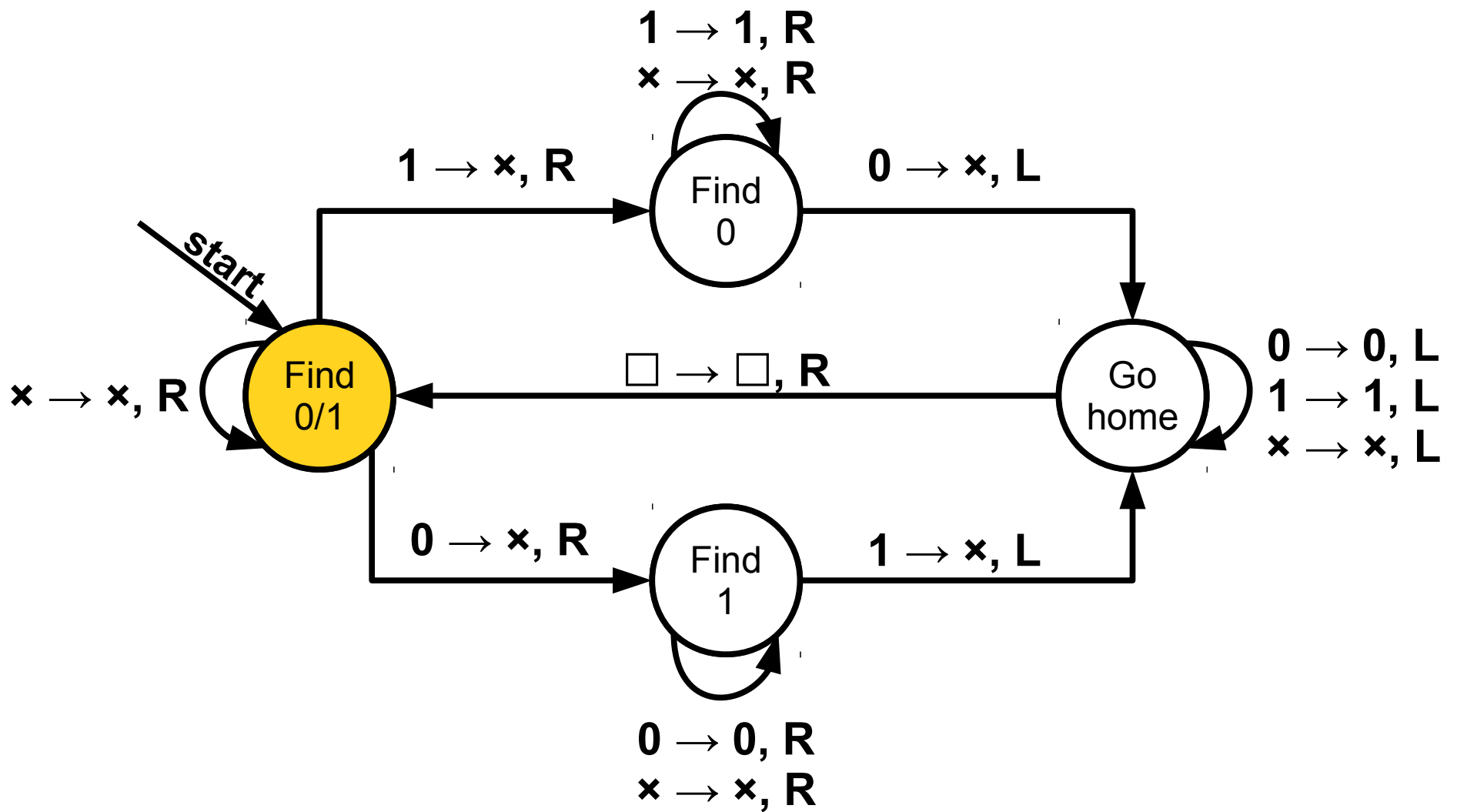


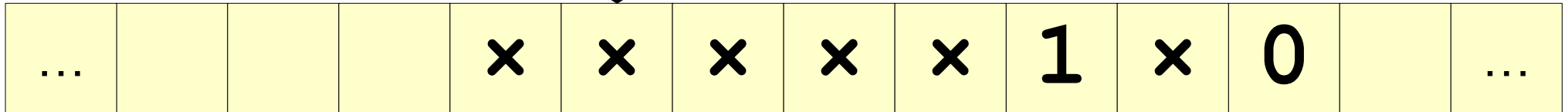
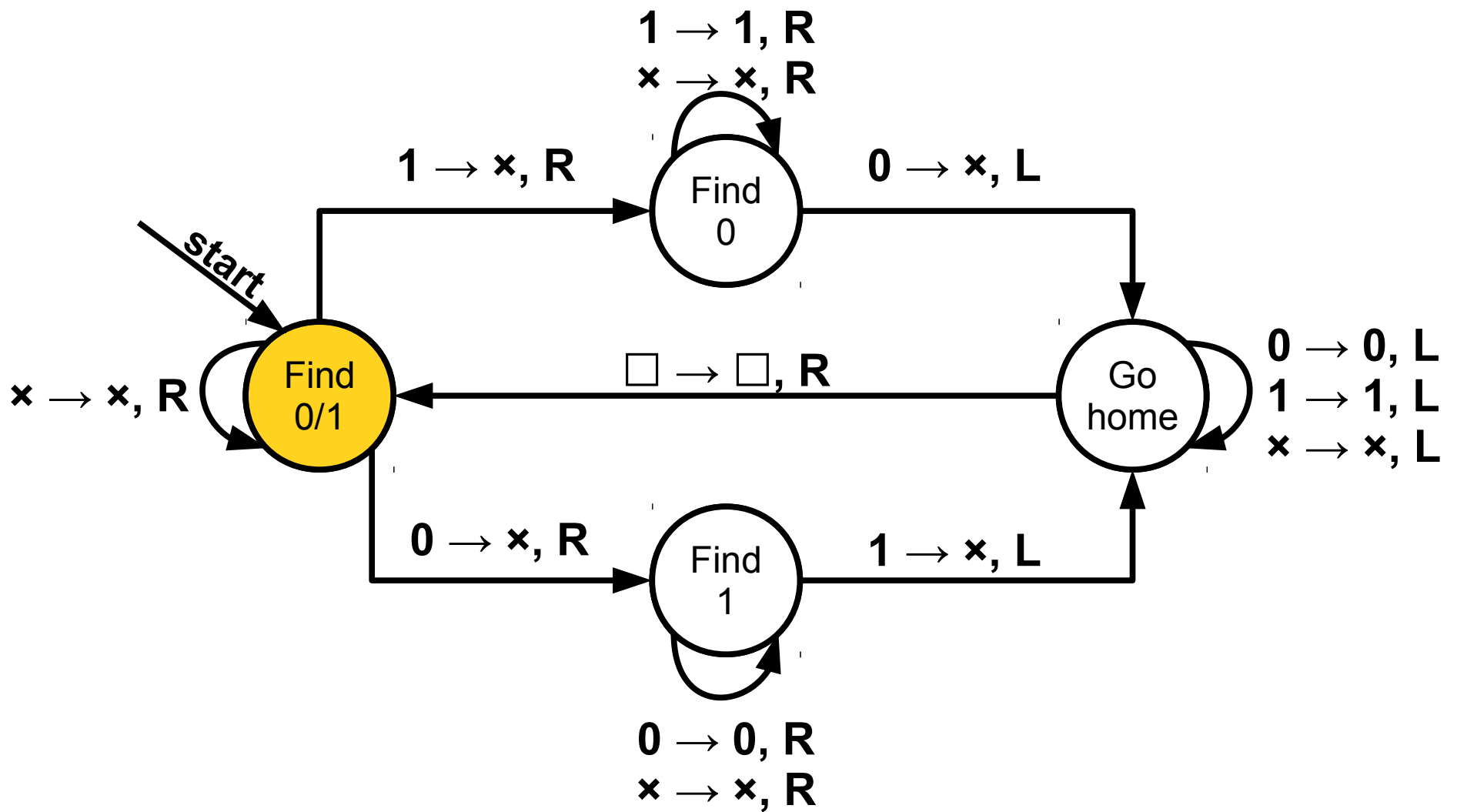


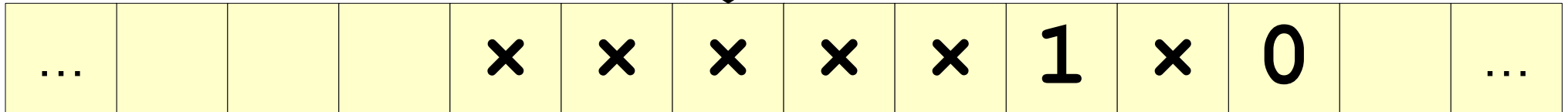
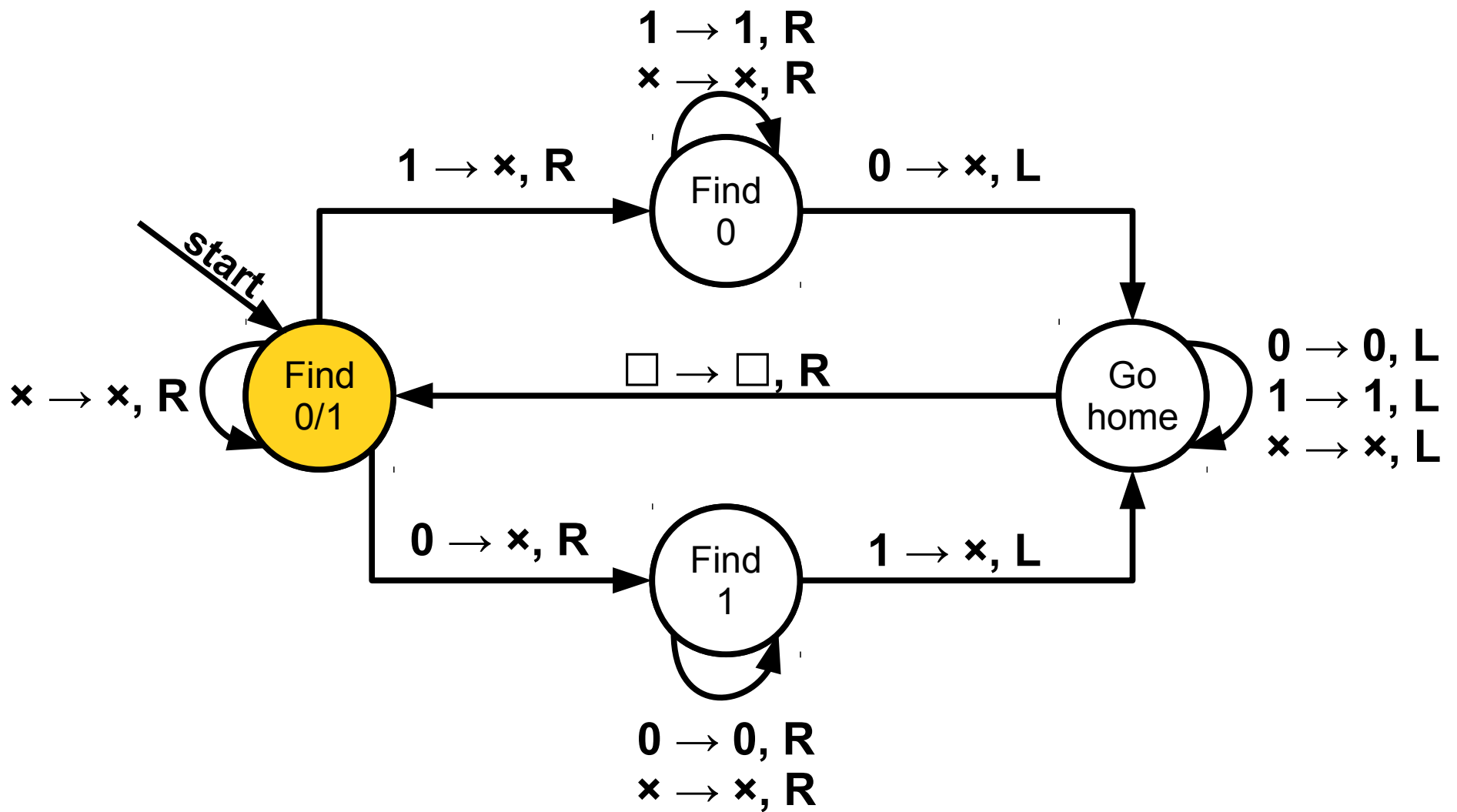


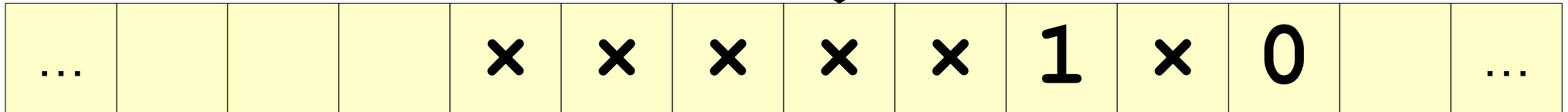
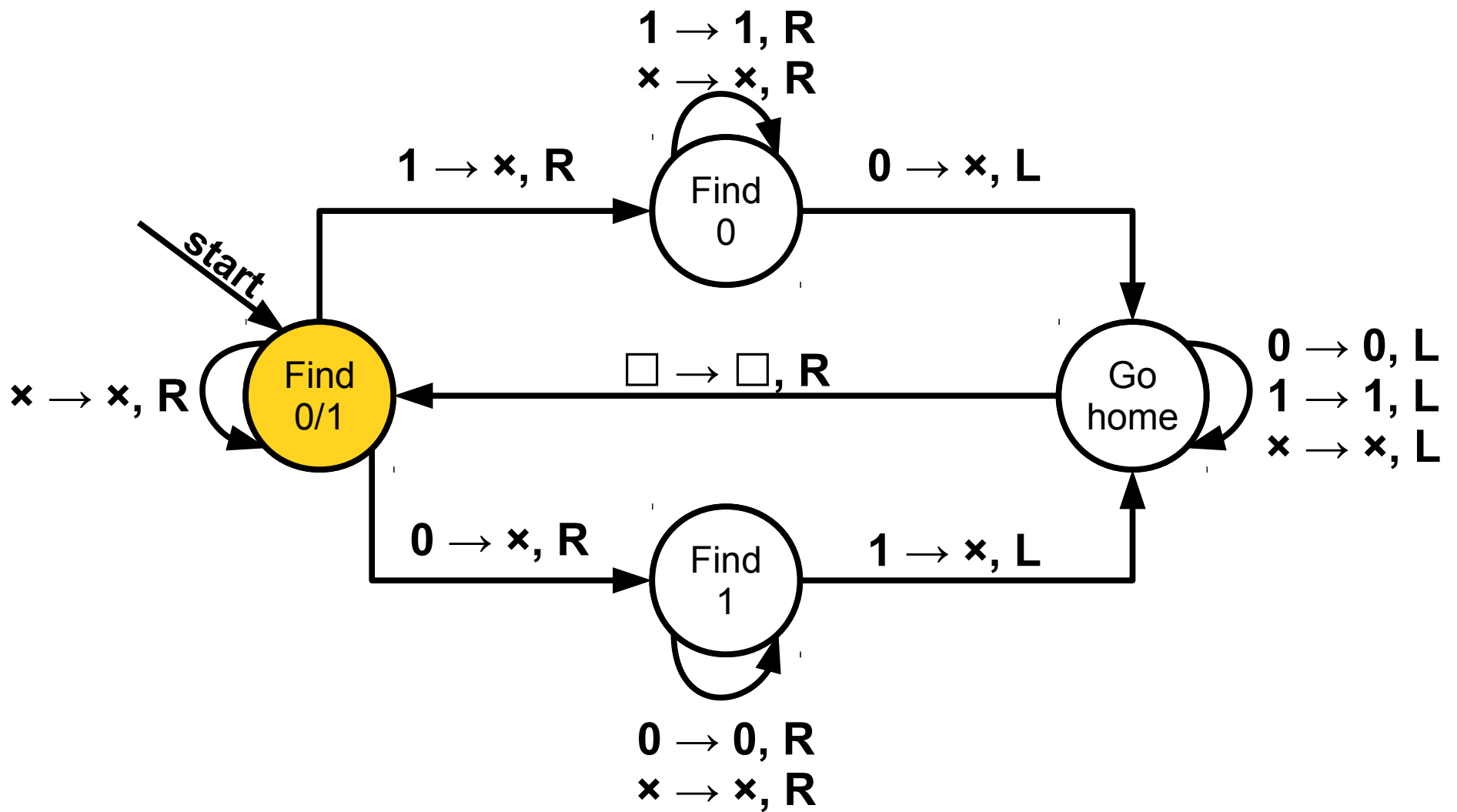


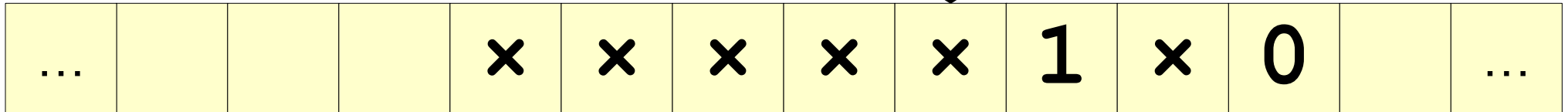
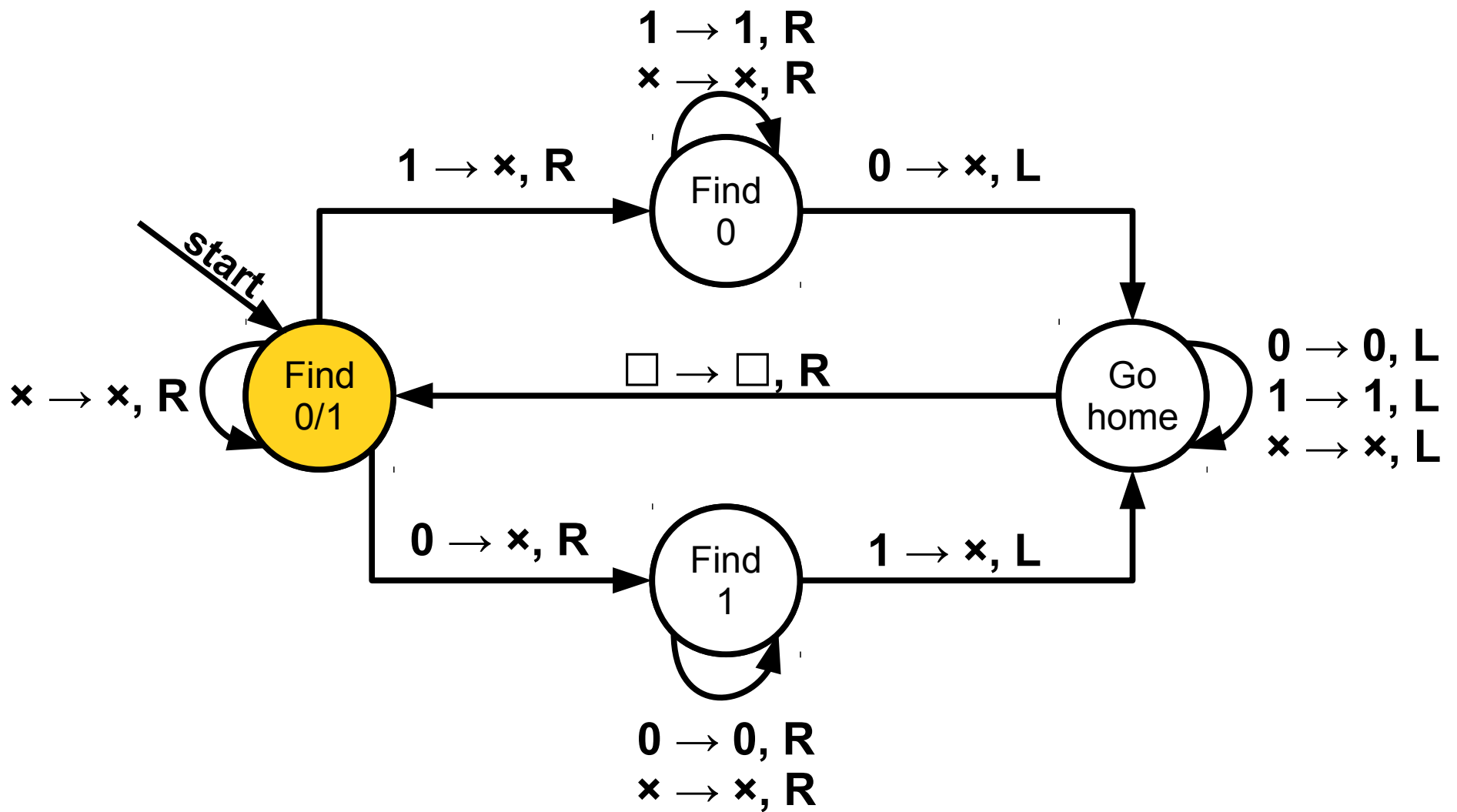


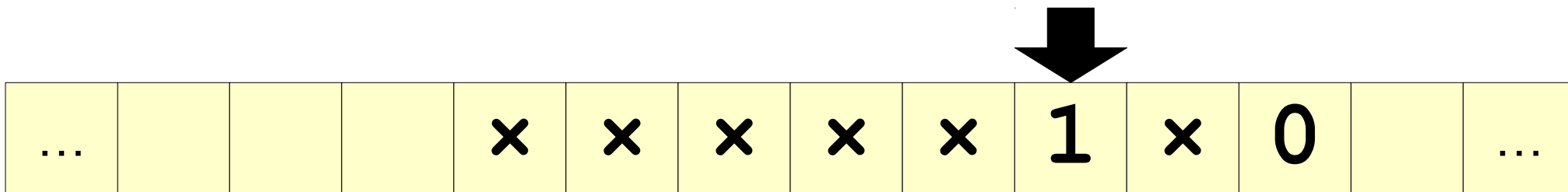
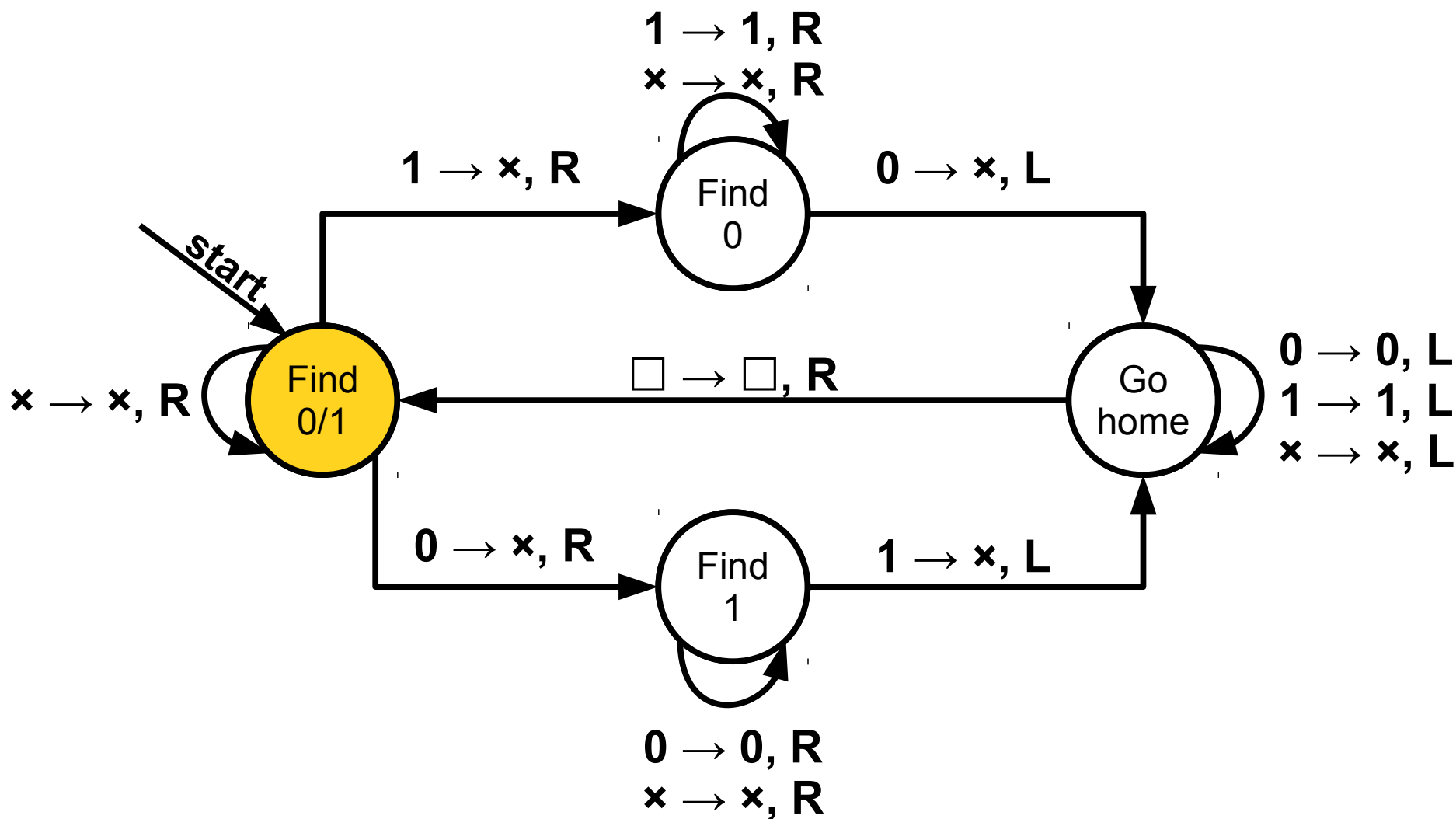


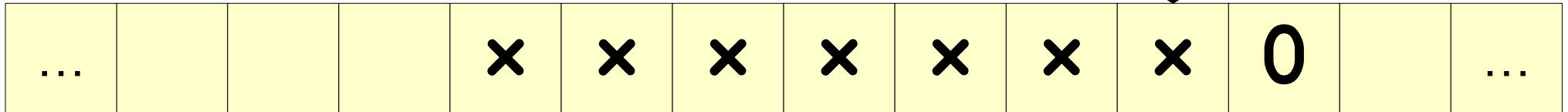
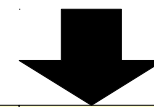
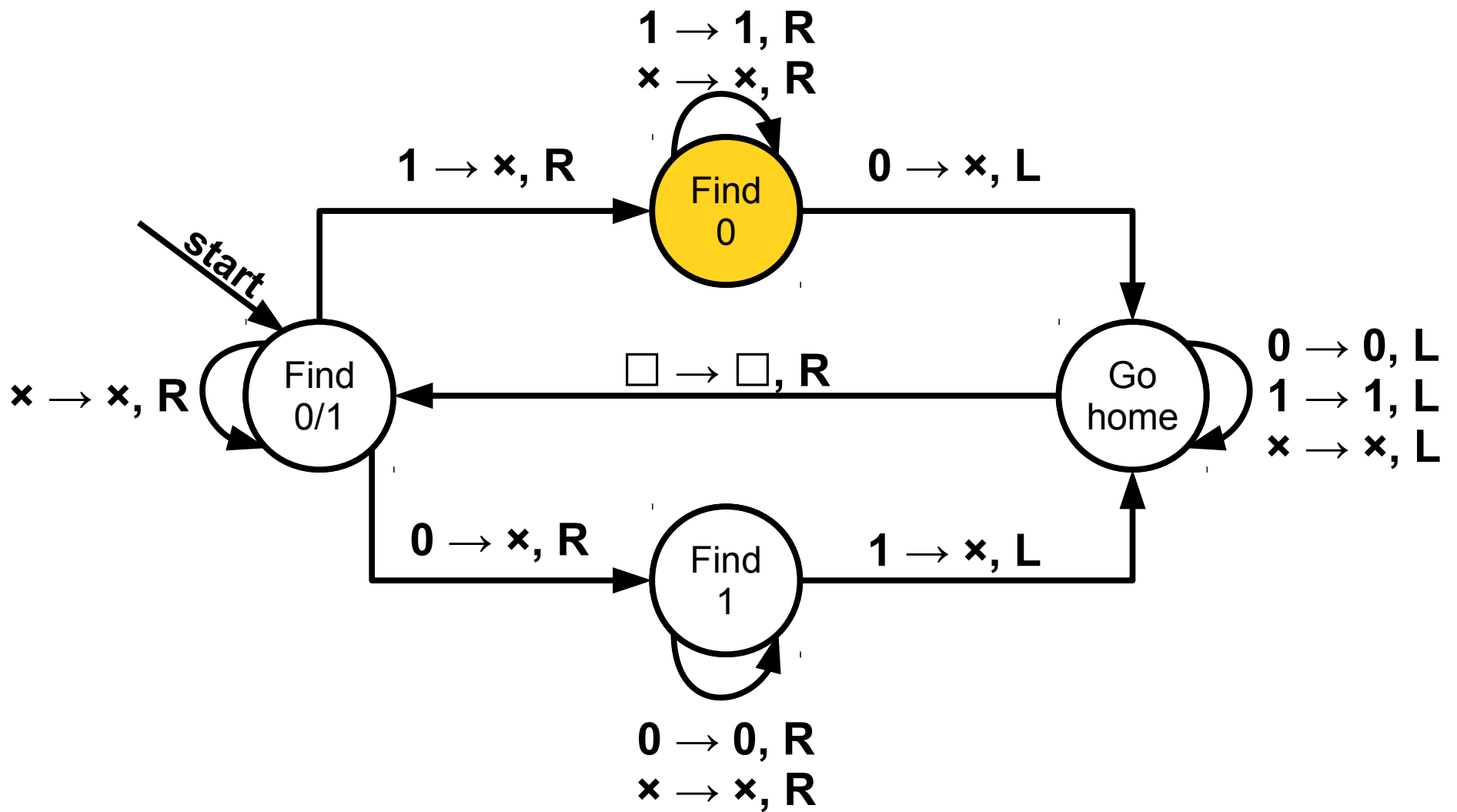






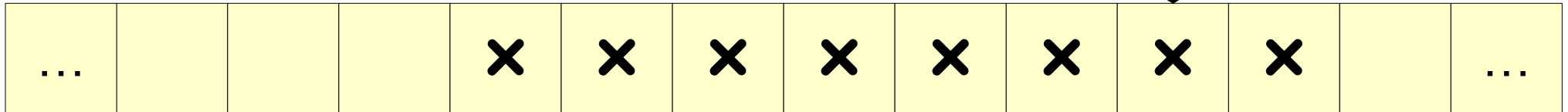
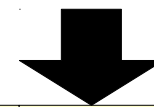
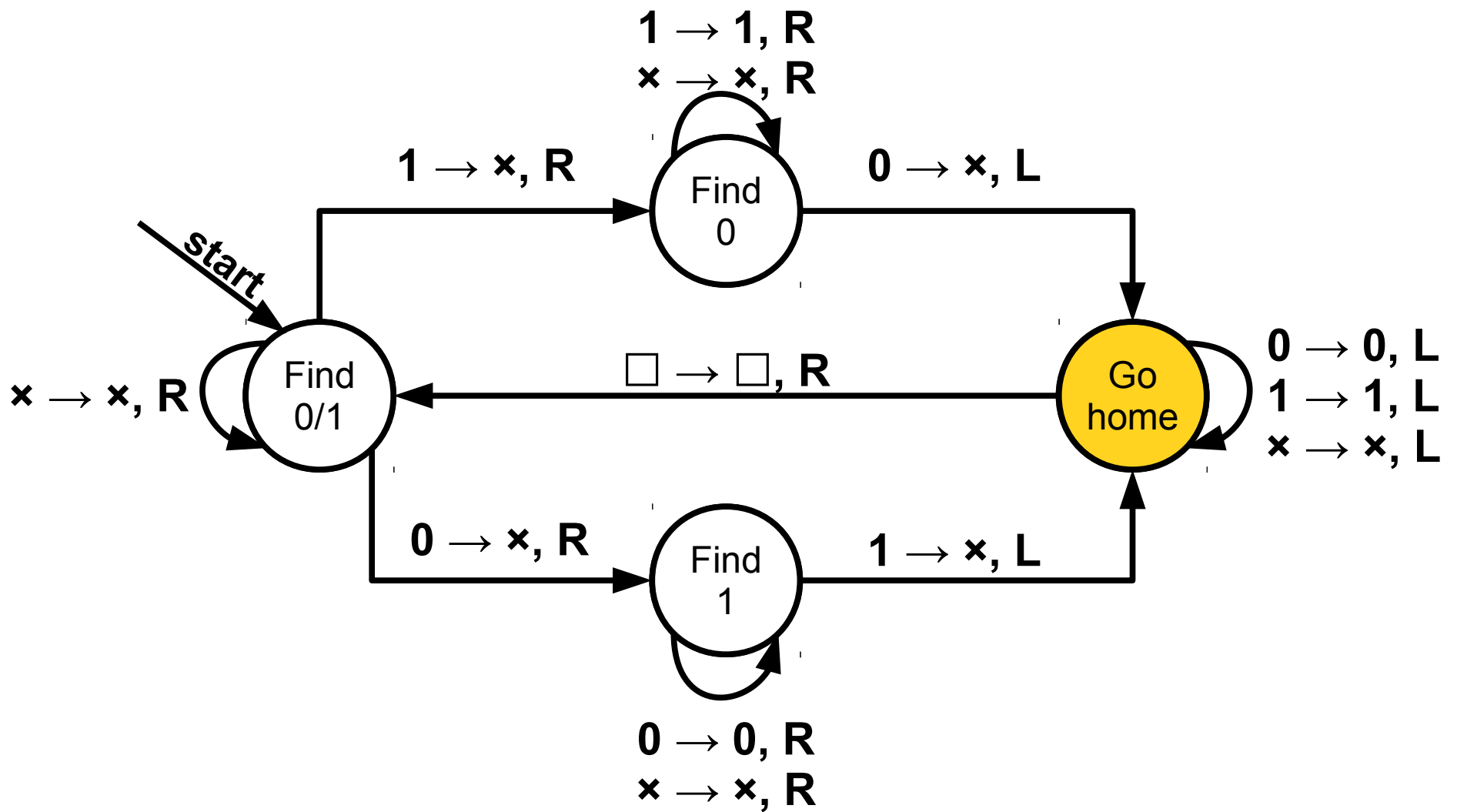










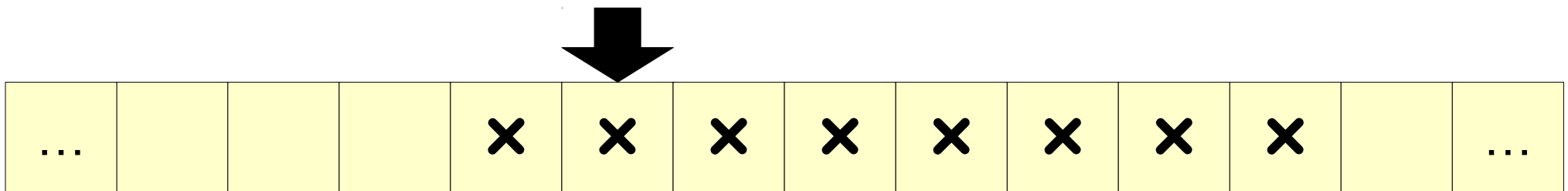
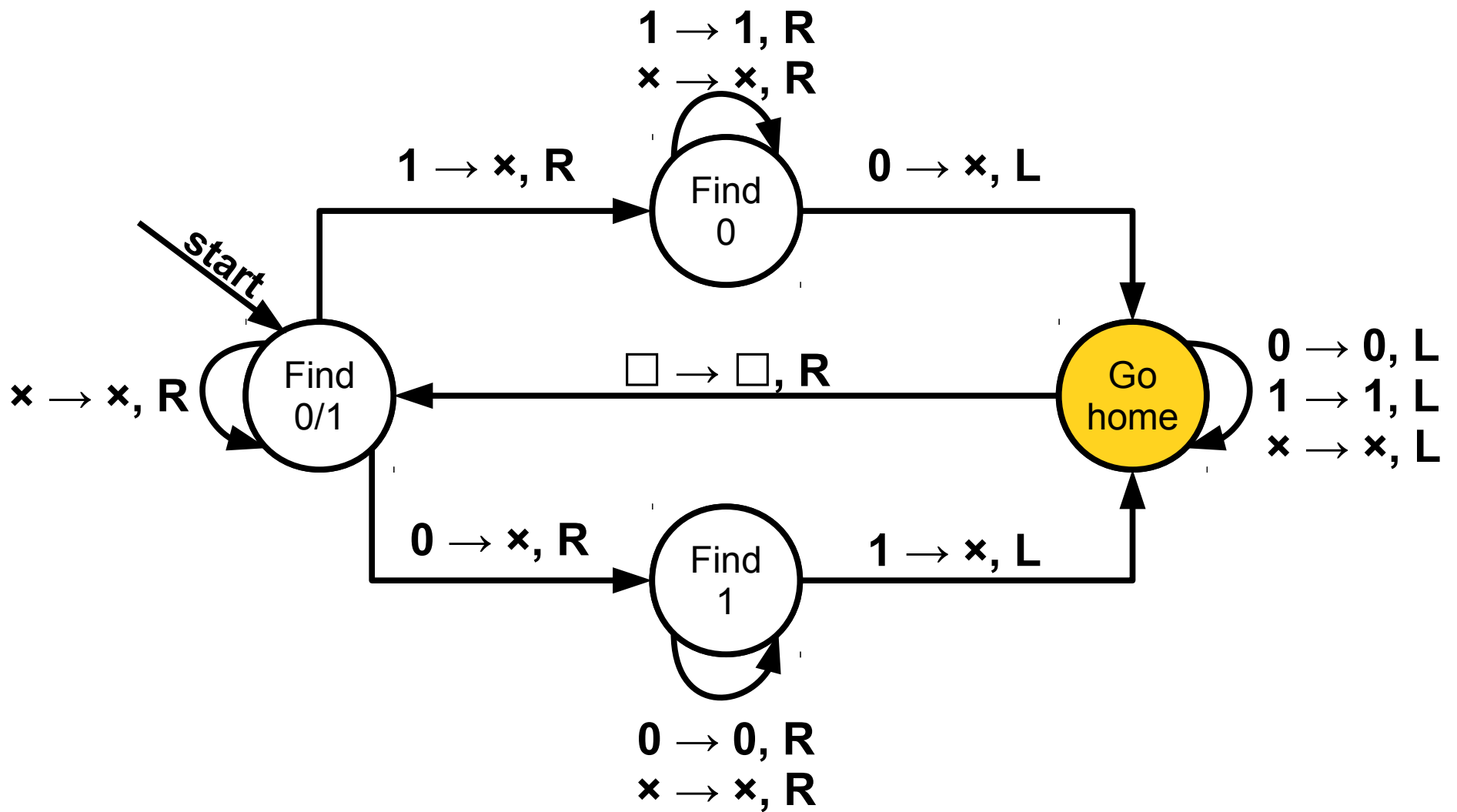










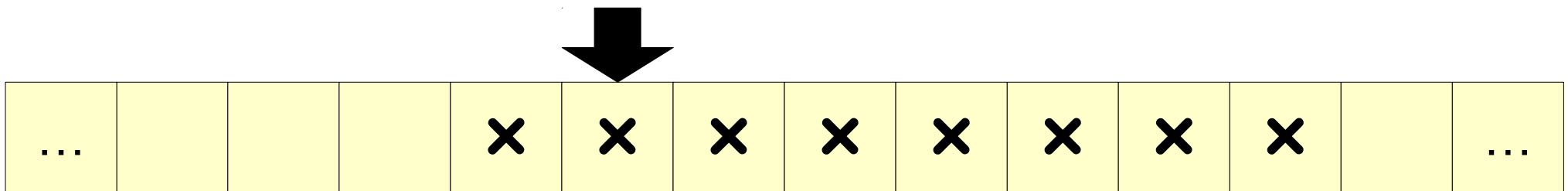
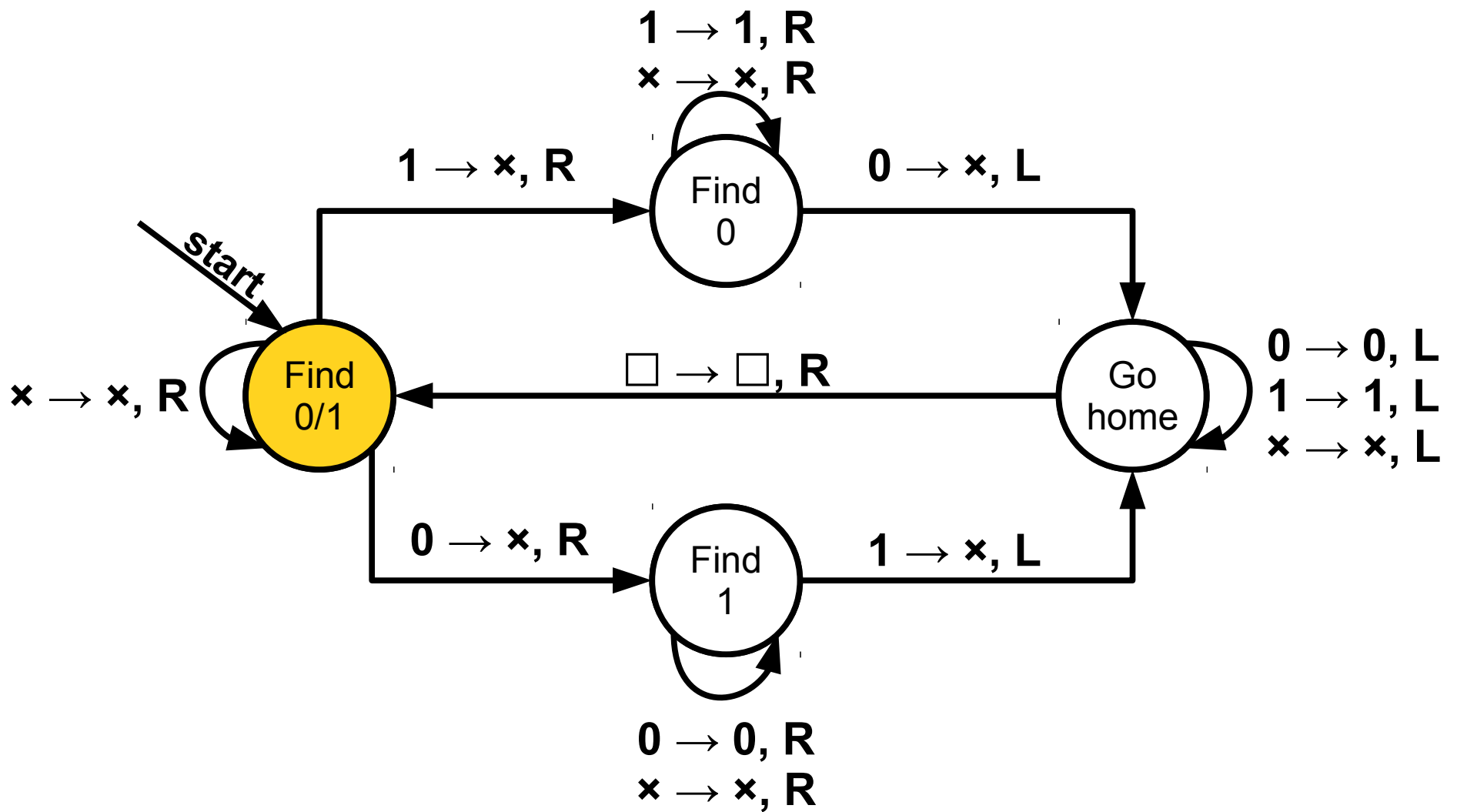












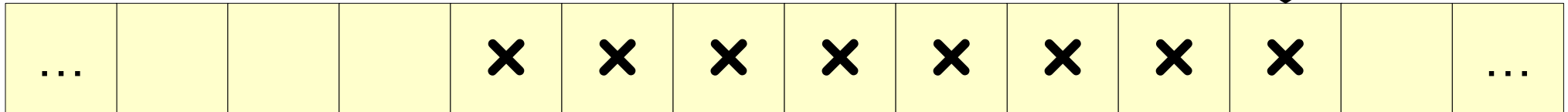
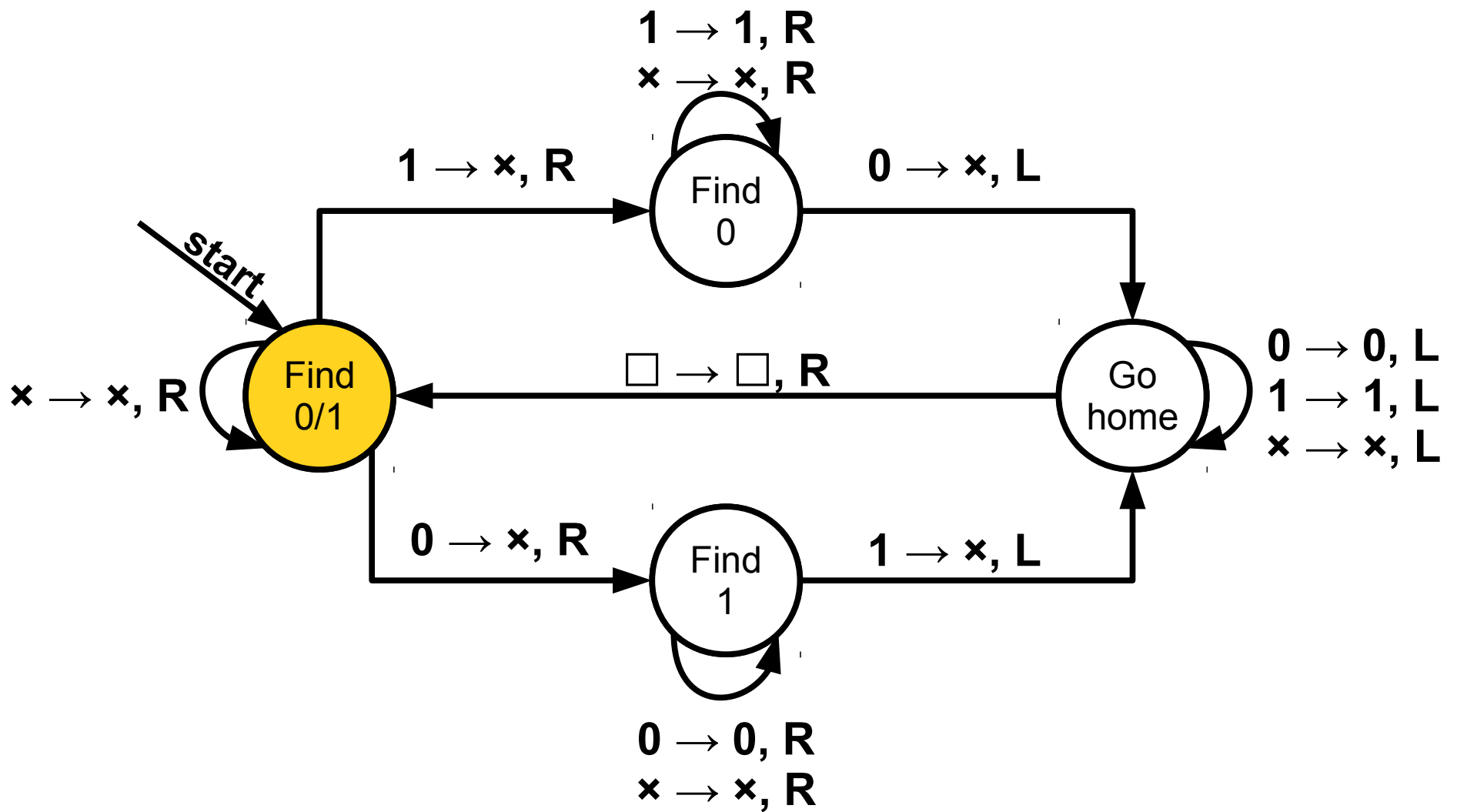








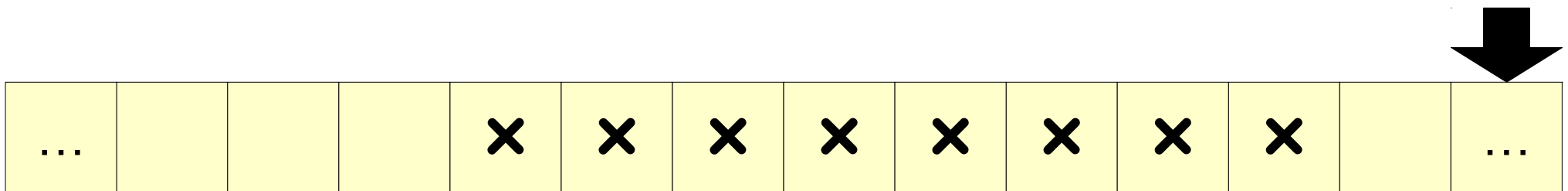
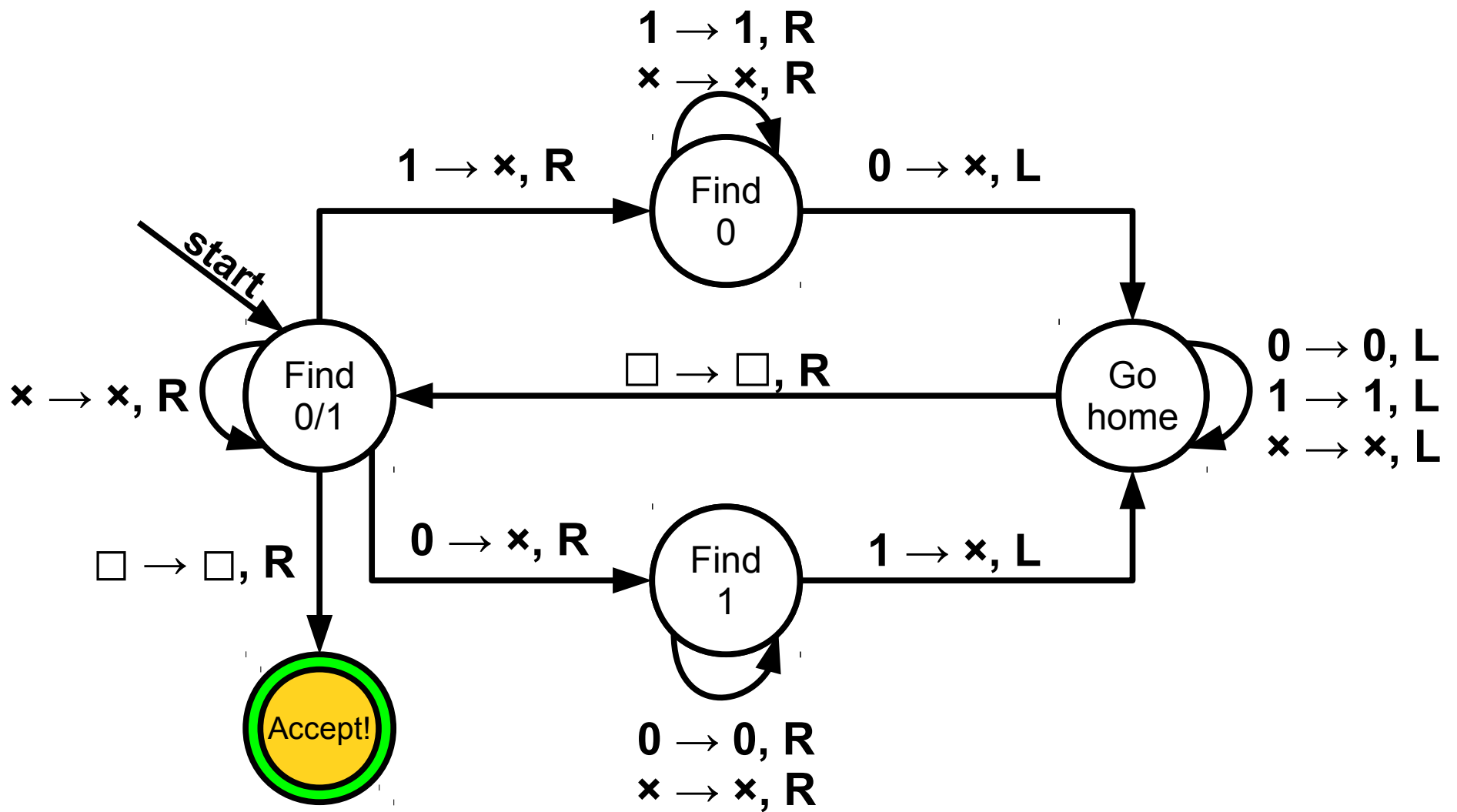


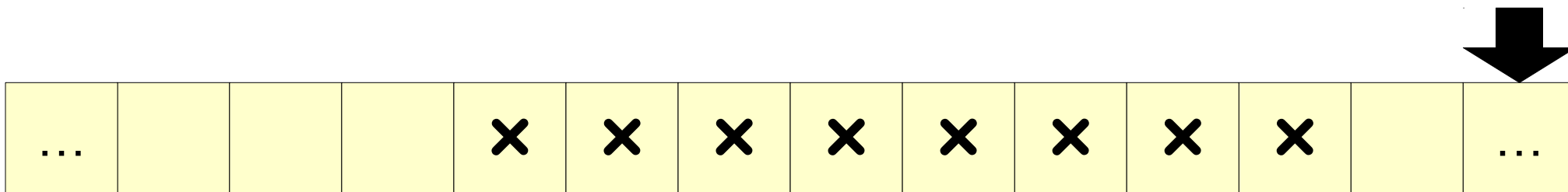
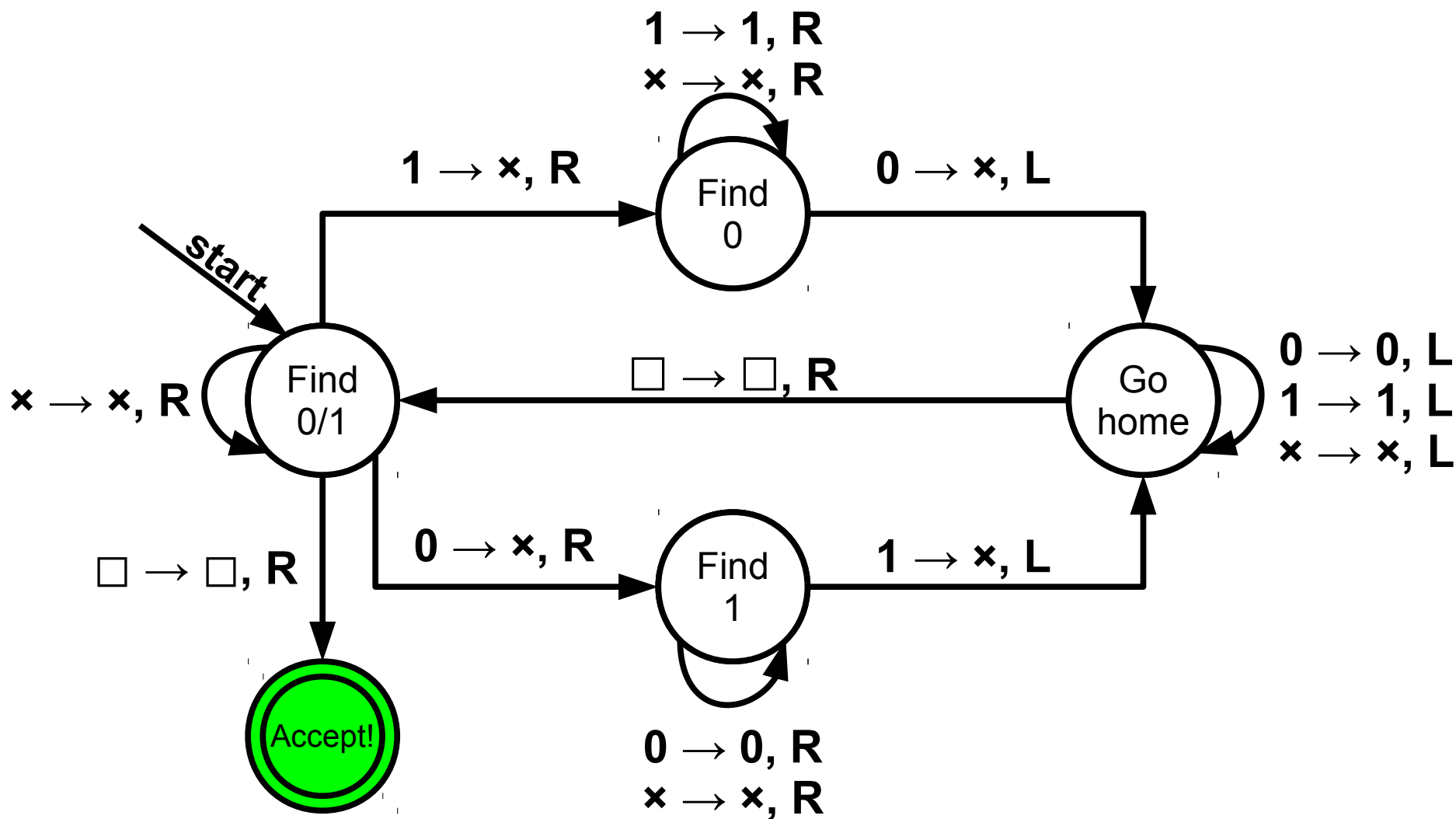


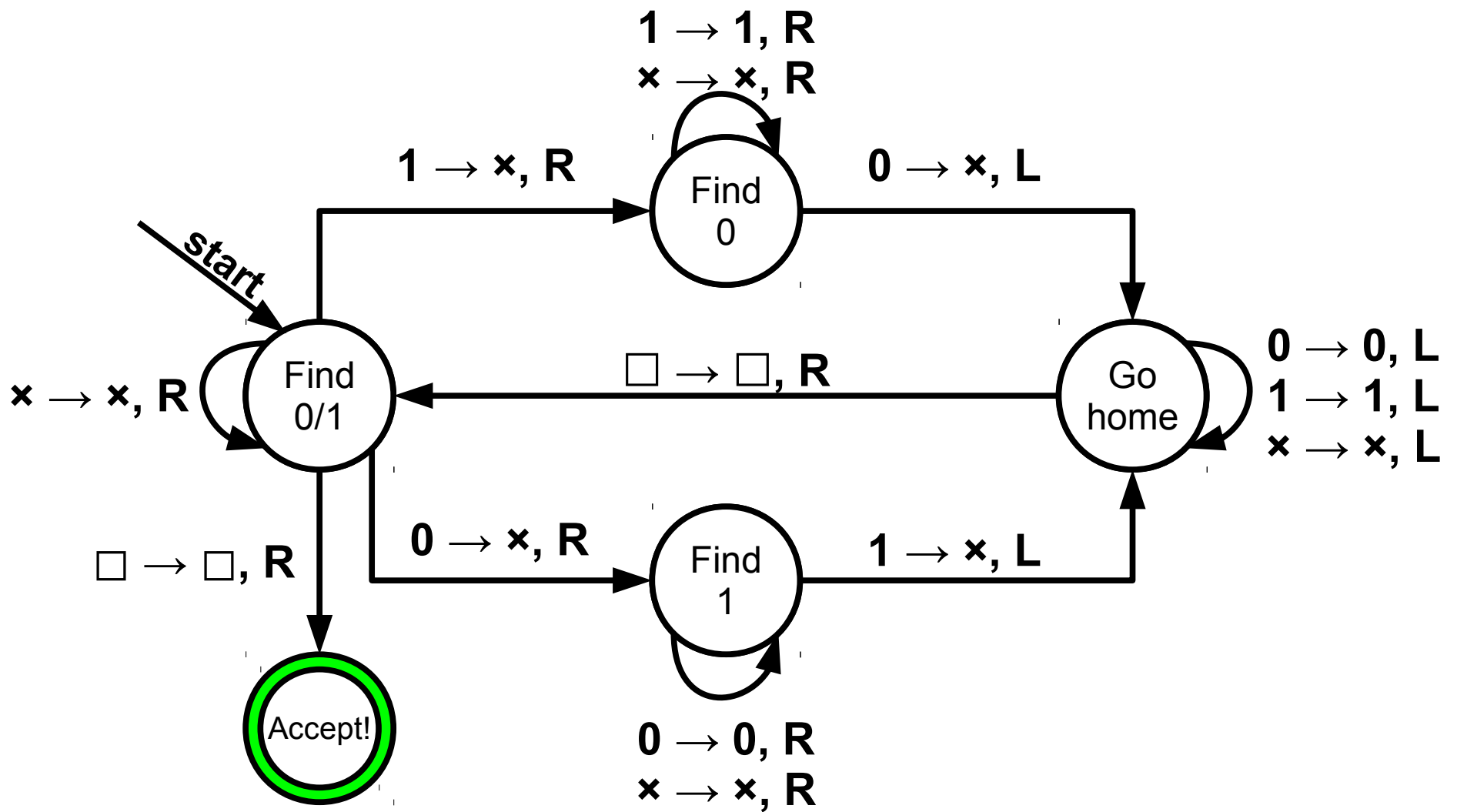


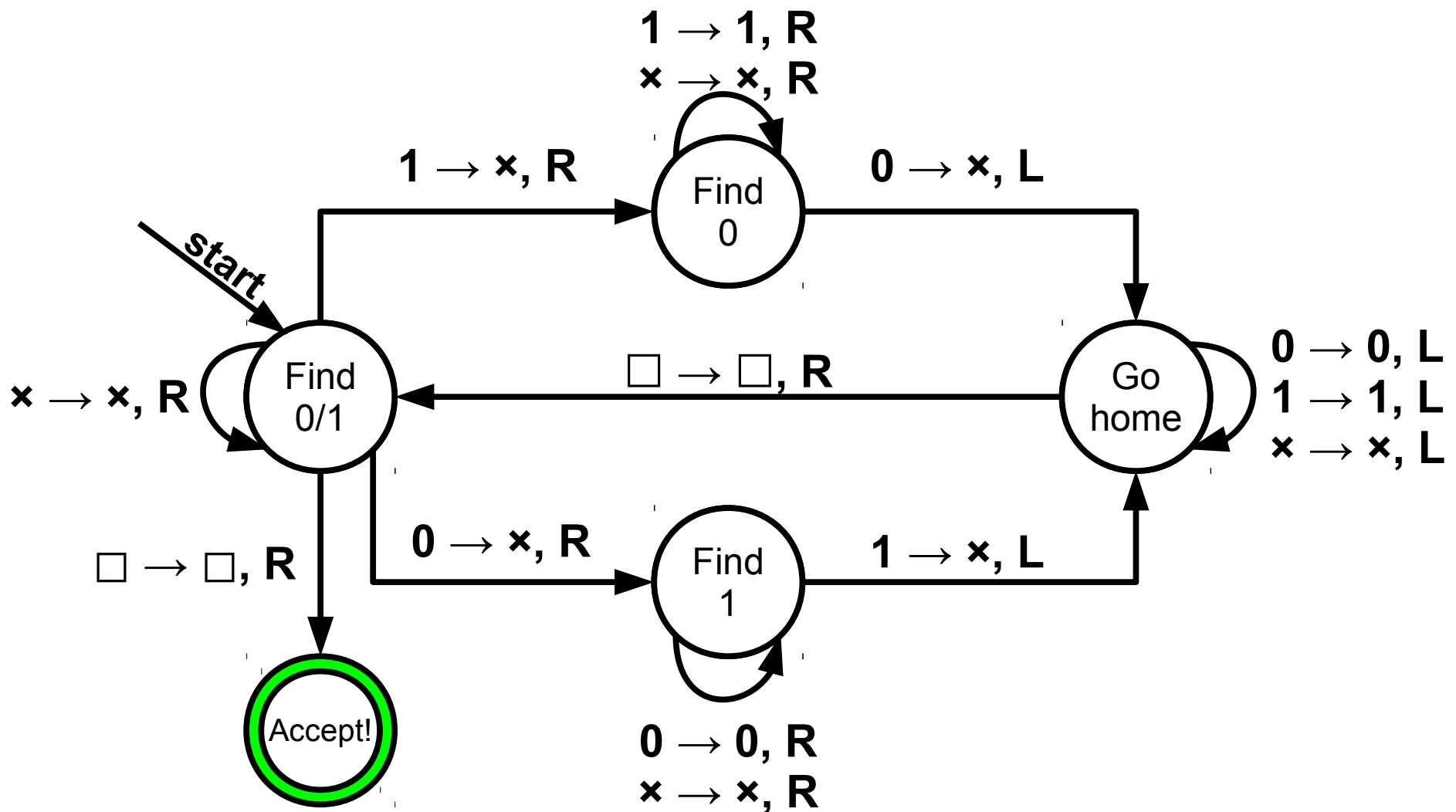




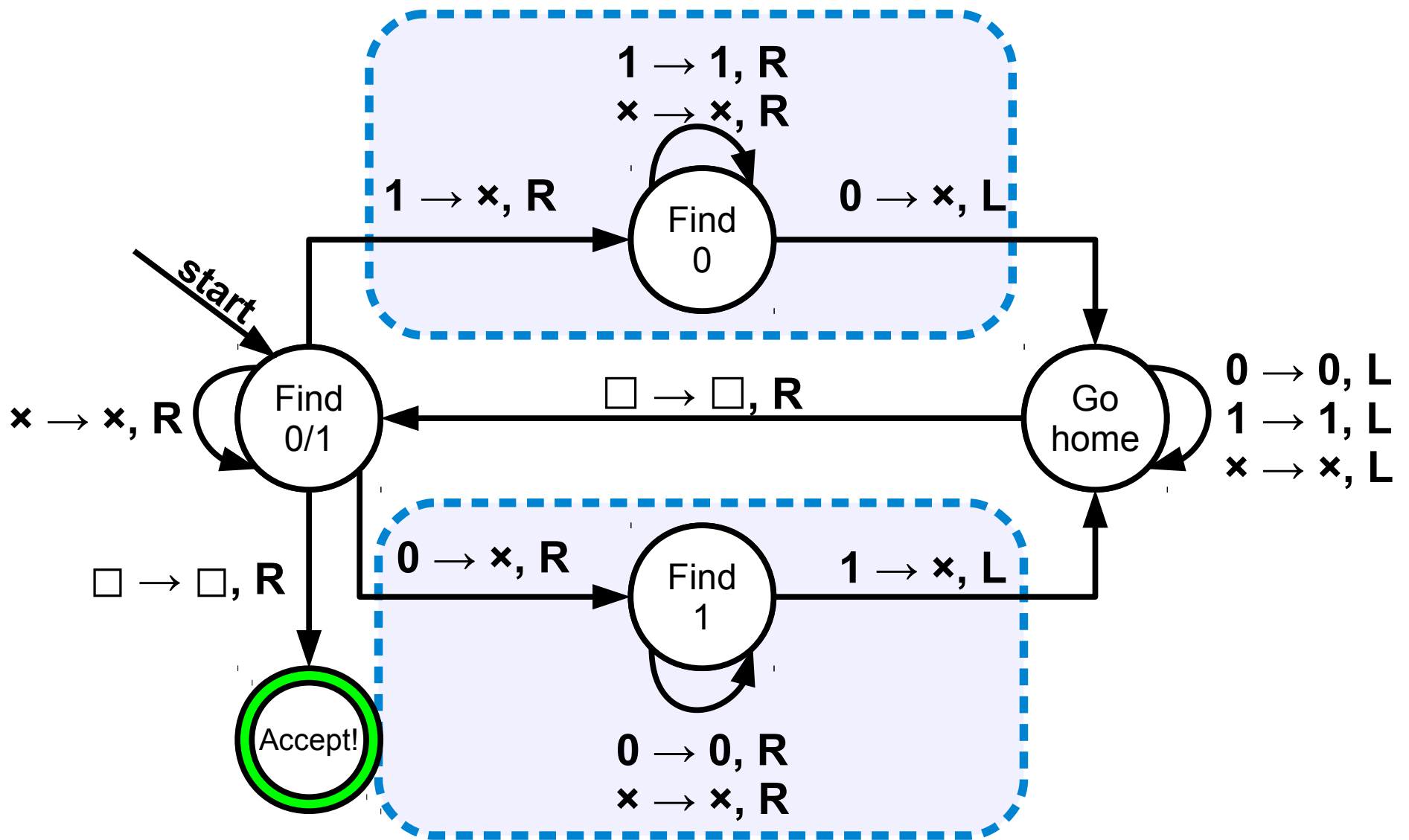








Going forward, we'll ignore the missing transitions and pretend they implicitly reject.



# Constant Storage

- Sometimes, a TM needs to remember some additional information that can't be put on the tape.
- In this case, you can use similar techniques from DFAs and introduce extra states into the TM's finite-state control.
- The finite-state control can only remember one of finitely many things, but that might be all that you need!



Time-Out for Announcements!

# Problem Set Seven

- Problem Set Seven is due on Friday.
- Have questions? Please feel free to stop by office hours or to ask questions on Piazza.

# Second Midterm Exam

- The second midterm exam is next **Monday, February 29** from 7PM – 10PM, location TBA.
- Topic coverage:
  - Focus is on PS4 – PS6 and lectures 09 – 16.
  - Topics from PS7 and from lecture 17 onward will not be tested... yet. ☺
  - **Major topics:** strict orders, graphs, the pigeonhole principle, induction, finite automata, regular expressions, regular languages, closure properties.
- Policies and procedures same as the first midterm:
  - Three hours, four questions.
  - Closed-computer, closed-book, and limited-note. You can have a double-sided 8.5" × 11" sheet of paper with you when you take the exam.

# Preparing for the Exam

- We have just released four sets of extra practice problems (EPP4 – EPP7) online. Solutions will go out on Wednesday.
- We will be holding a practice midterm on **Wednesday** from **7PM - 10PM** in **Bishop Auditorium**.
- By popular demand, we've put together *two* practice exams – one that we'll give out at the practice exam and one that will be posted online on Wednesday.
- We've also released a set of challenge problems, and this week's CS103A class will be a cumulative review.
- That's (by my reckoning) four sets of practice problems, two practice exams, one set of challenge problems, and one set of CS103A problems. If that's not enough, let me know and I'll see what I can do.

Stanford Women  
In Computer Science

# CASUAL CS DINNER

{w}

Wednesday Feb. 24 from 5:30-7:30pm at the WCC  
RSVP at [goo.gl/forms/U2JY1e9CSu](https://goo.gl/forms/U2JY1e9CSu)

Come have dinner with CS students and faculty.  
Everyone is welcome, especially students  
just starting out in CS!

Your Questions

“What's your favorite (and lesser-known) place on campus?”

There (used to be? still is?) a fire escape on one of the inner quad buildings that lets you climb up to the third floor. You get a great view and it's nice and quiet. It's a great place to study or cuddle and watch a movie.

# “What keeps you motivated?”

I never cease to be amazed by the world. There's always so much to learn and discover.

Which reminds me – I haven't yet asked you the Three Key Questions yet!



# Three Questions

- What is something that you now know that, at the start of the quarter, you knew you didn't know?
- What is something that you now know that, at the start of the quarter, you *didn't* know you didn't know?
- What is something that you *don't* know that, at the start of the quarter, you didn't know you didn't know?

Back to CS103!

# Another TM Design

- Consider the following language over  $\Sigma = \{0, 1\}$ :

$$L = \{0^n 1^m \mid n, m \in \mathbb{N} \text{ and } m \text{ is a multiple of } n \}$$

- Is this language regular?
- How might we design a TM for this language?

# An Observation

- We can recursively describe when one number  $m$  is a multiple of  $n$ :
  - If  $m = 0$ , then  $m$  is a multiple of  $n$ .
  - Otherwise,  $m$  is a multiple of  $n$  iff  $m - n$  is a multiple of  $n$ .
- **Idea:** Repeatedly subtract  $n$  from  $m$  until  $m$  becomes zero (good!) or drops below zero (bad!)

# The Challenge



...				0	0	1	1	1	1	1	1		...
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# One Solution



...				0	0	1	1	1	1	1	1		...
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# One Solution



...				×	0	1	1	1	1	1	1		...
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# One Solution



...				×	0	1	1	1	1	1	1		...
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# One Solution



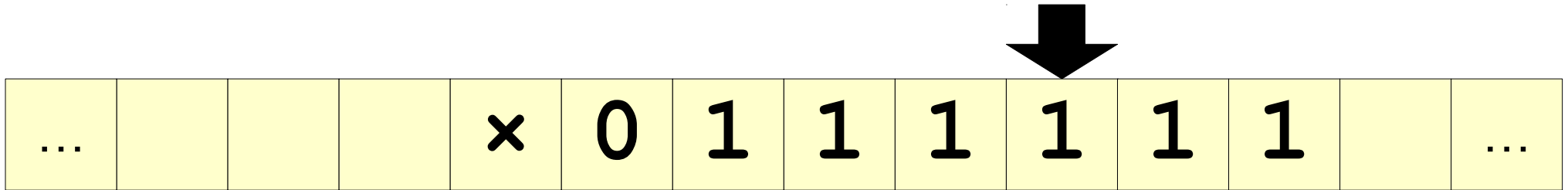
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# One Solution

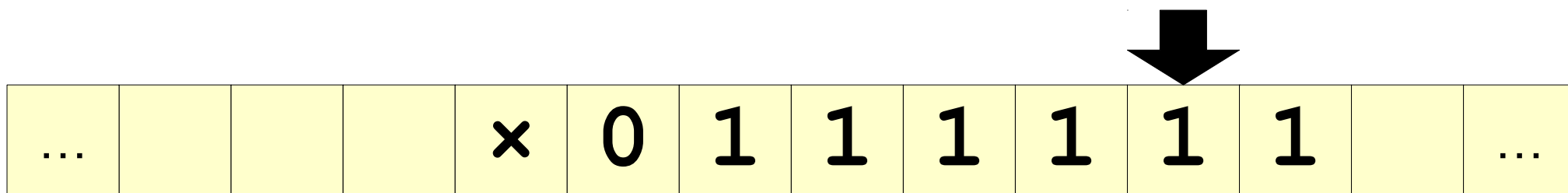


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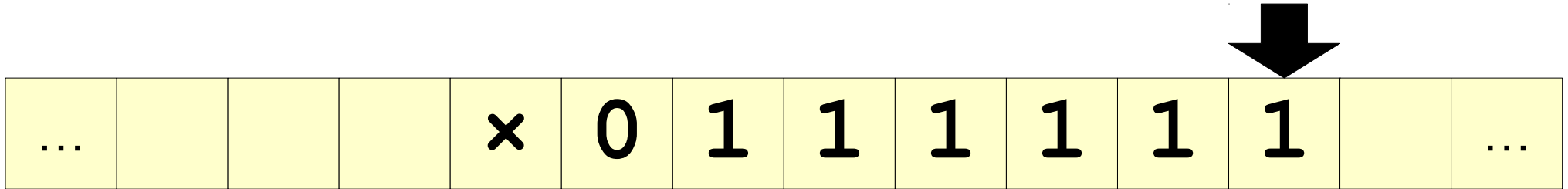
# One Solution



# One Solution



# One Solution

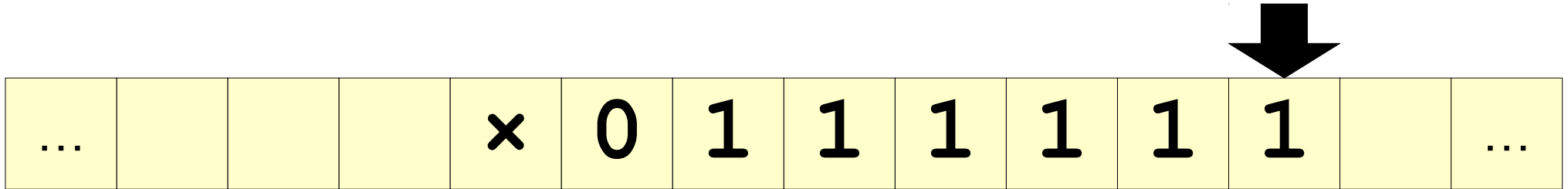


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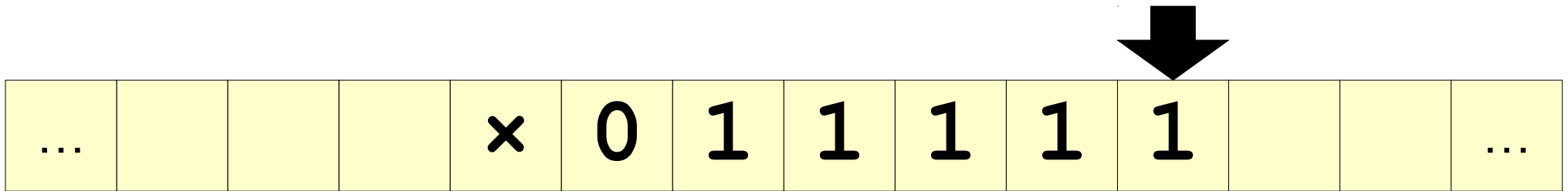


...				×	0	1	1	1	1	1	1		...
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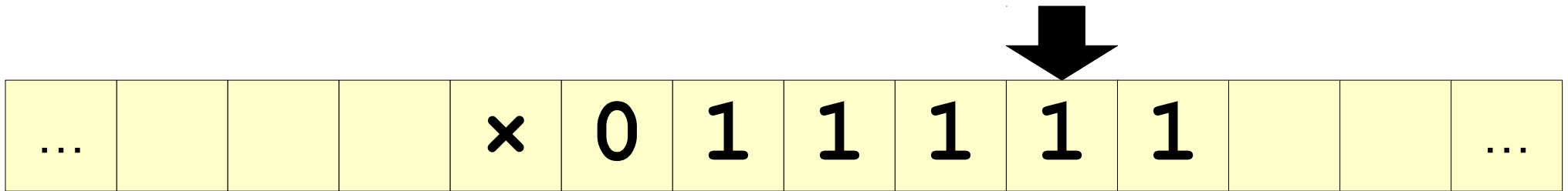


# One Solution





# One Solution



# One Solution



...				×	0	1	1	1	1	1			...
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# One Solution



...				×	0	1	1	1	1	1			...
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# One Solution



...				×	0	1	1	1	1	1			...
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# One Solution



...				×	0	1	1	1	1	1			...
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# One Solution



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# One Solution



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# One Solution



...				×	0	1	1	1	1	1			...
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# One Solution



...				×	0	1	1	1	1	1			...
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# One Solution



...				x	x	1	1	1	1	1			...
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# One Solution



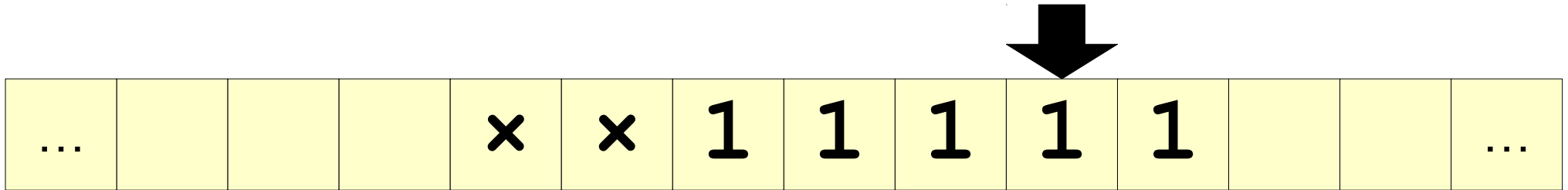
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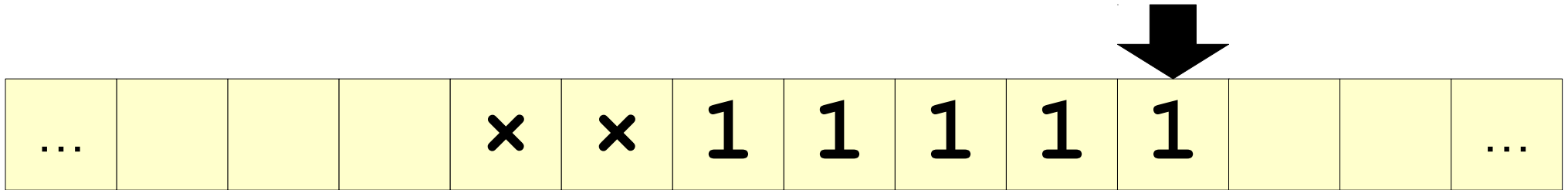


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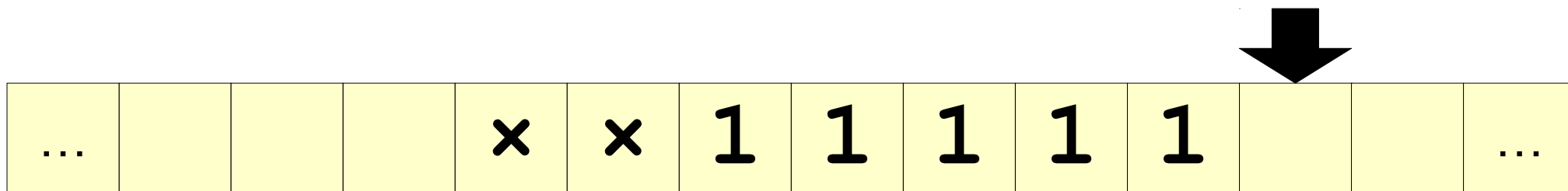
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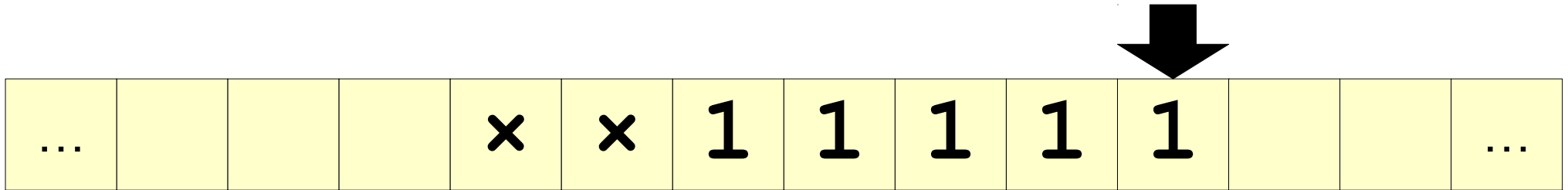
# One Solution



# One Solution

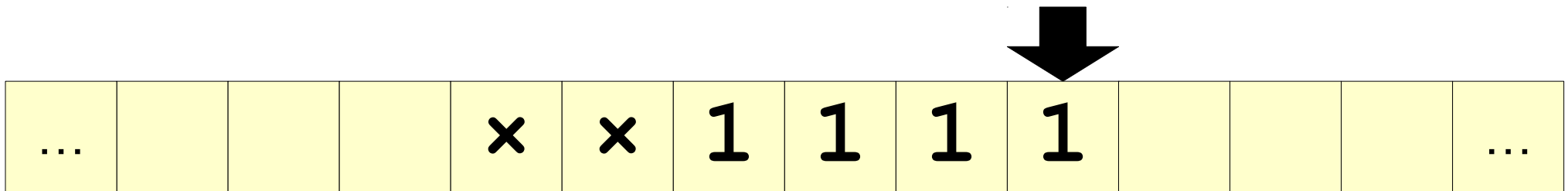


# One Solution





# One Solution



# One Solution



...				x	x	1	1	1	1				...
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# One Solution



...				x	x	1	1	1	1				...
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# One Solution



...				x	x	1	1	1	1				...
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# One Solution



...				x	x	1	1	1	1				...
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# One Solution



...				x	x	1	1	1	1				...
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# One Solution



...				x	x	1	1	1	1				...
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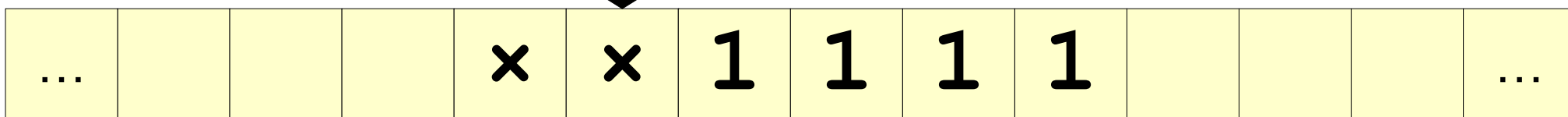
# One Solution



...				x	x	1	1	1	1				...
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# One Solution

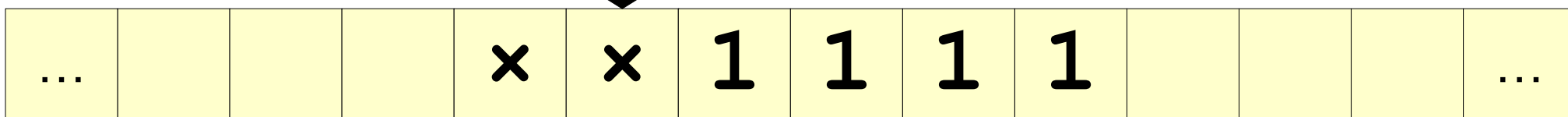


# One Solution



...				x	x	1	1	1	1				...
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# One Solution



# One Solution



...				x	0	1	1	1	1				...
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# One Solution

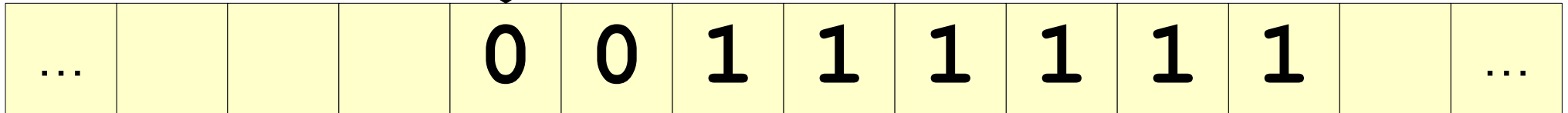
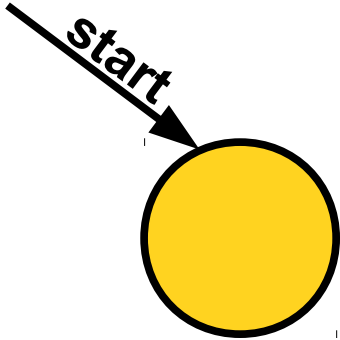


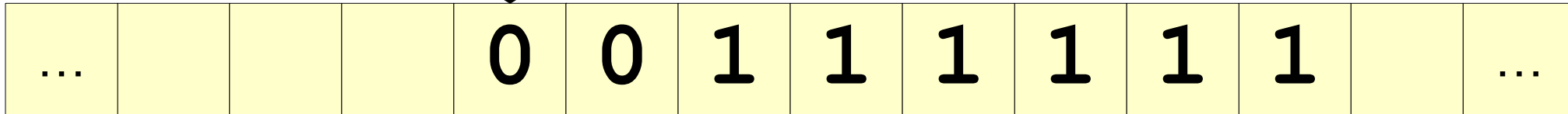
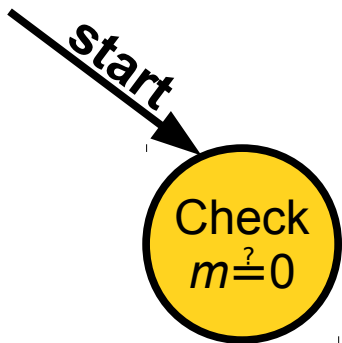
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# One Solution

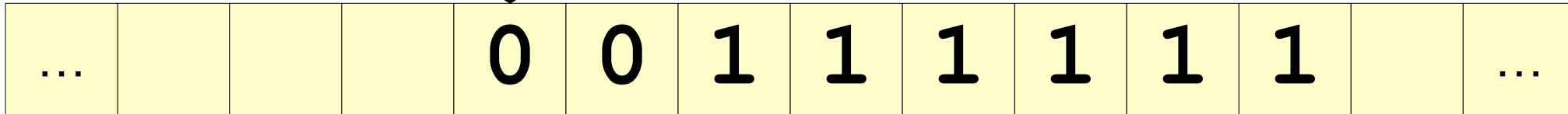
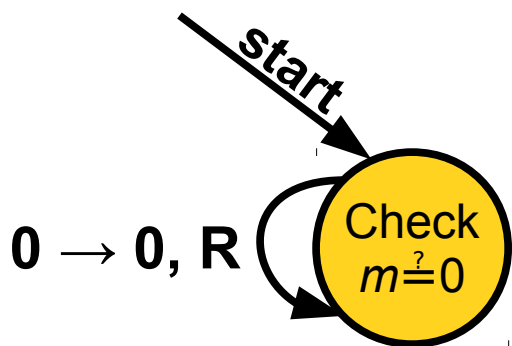


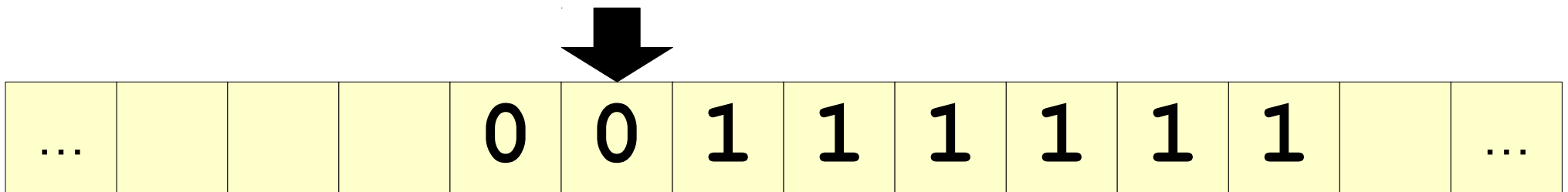
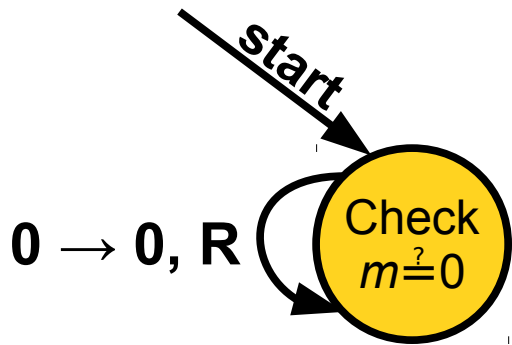
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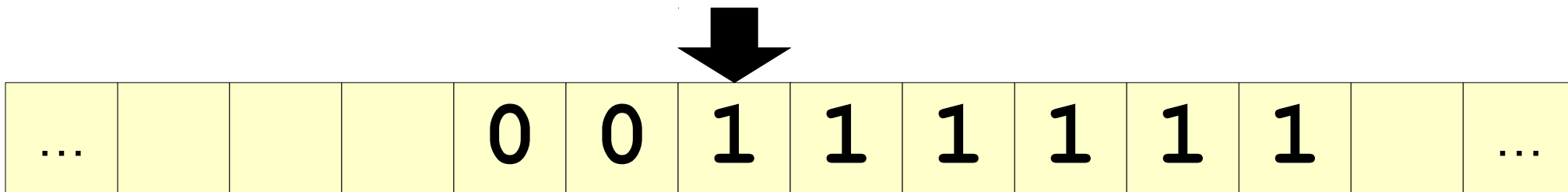
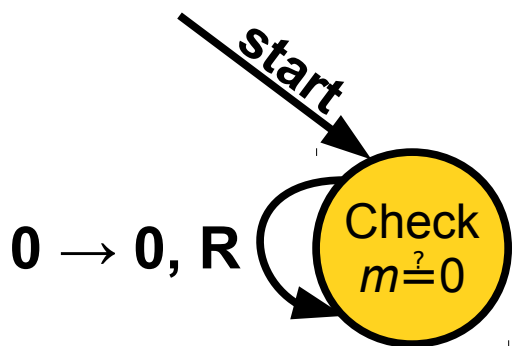


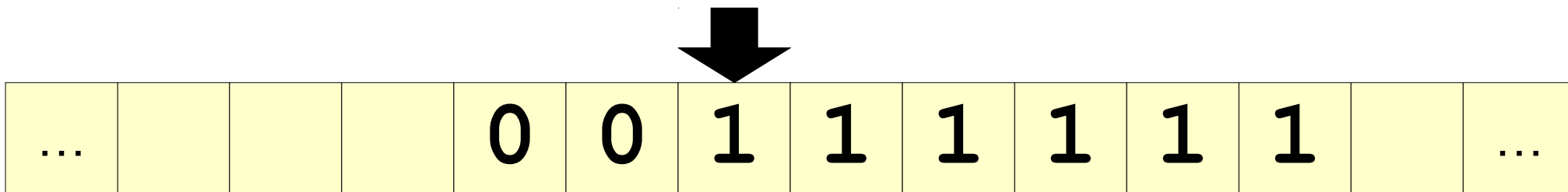
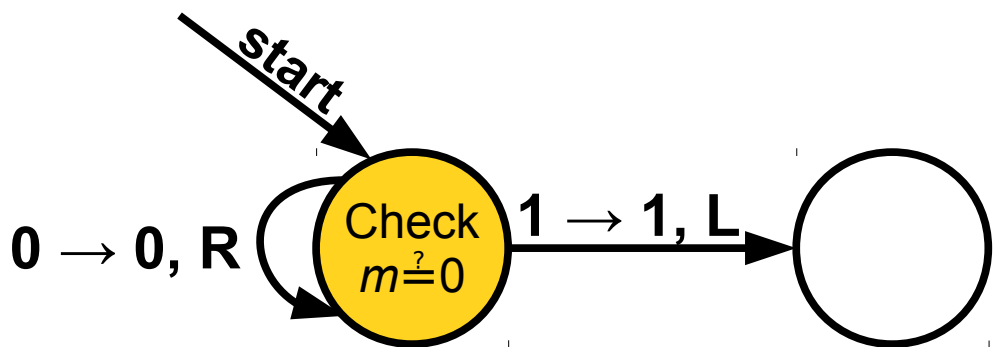


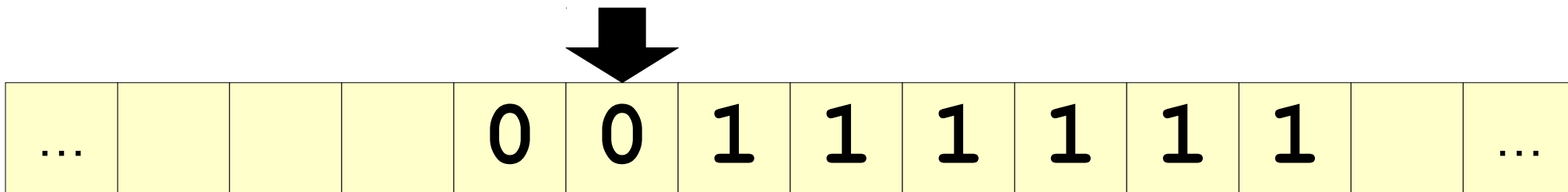
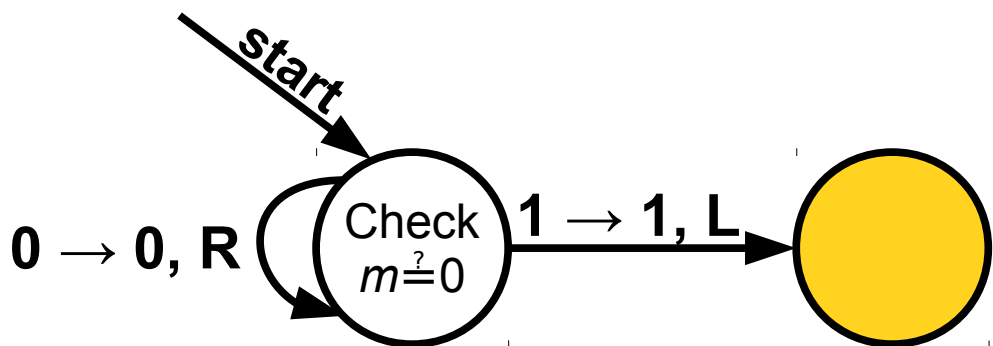


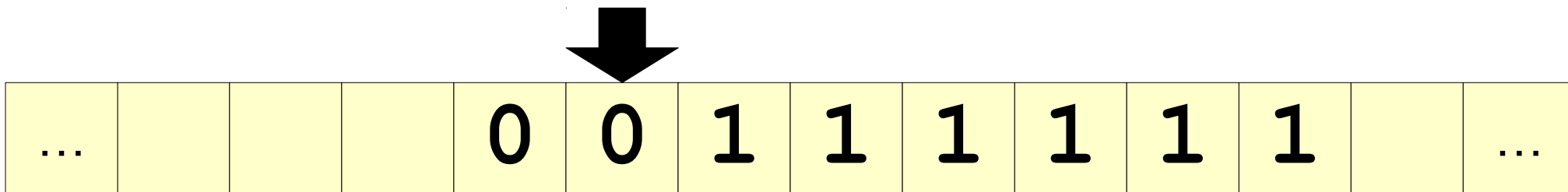
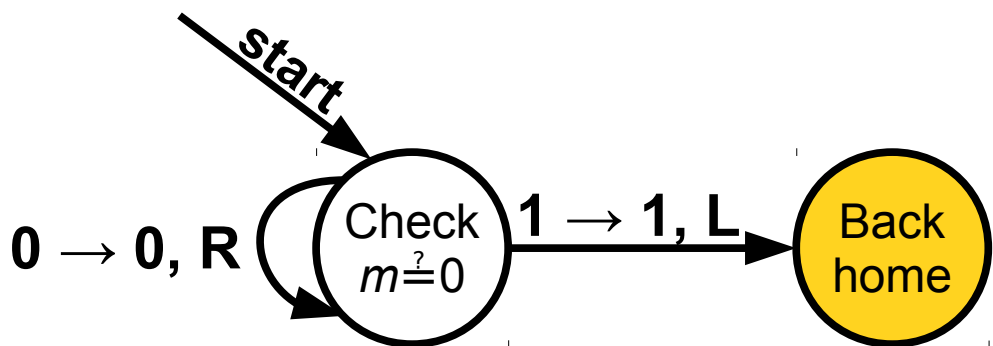


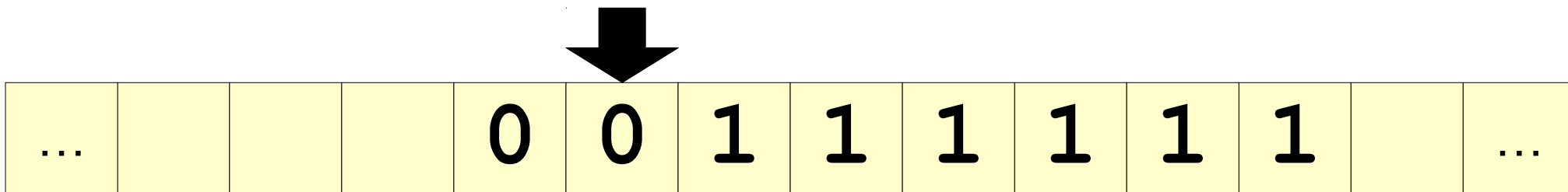
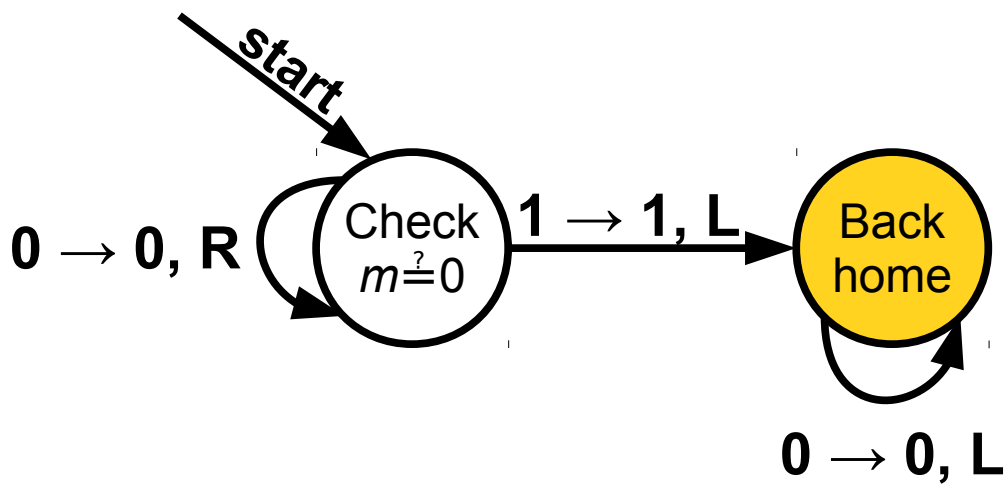


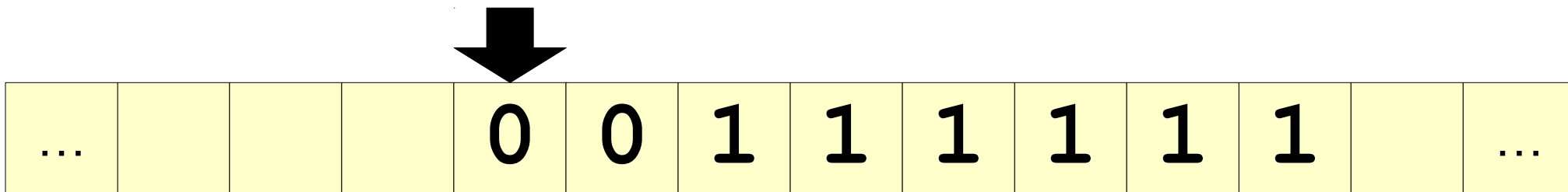
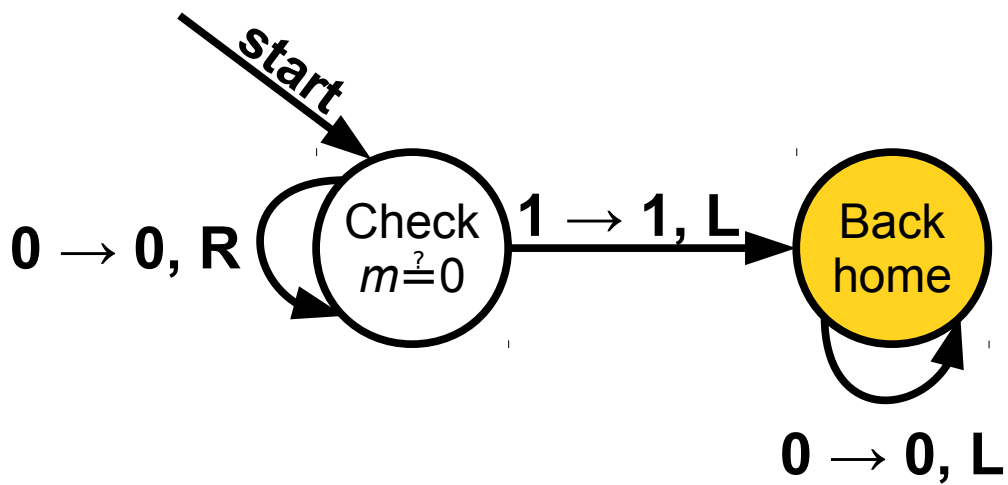




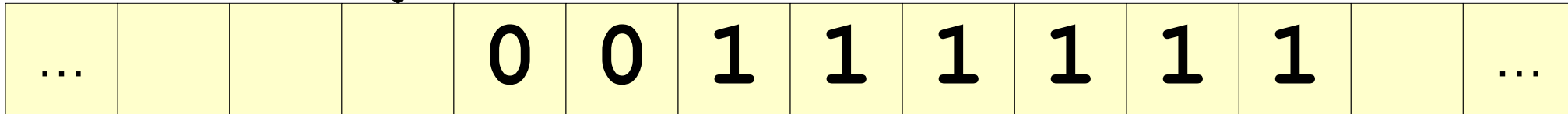
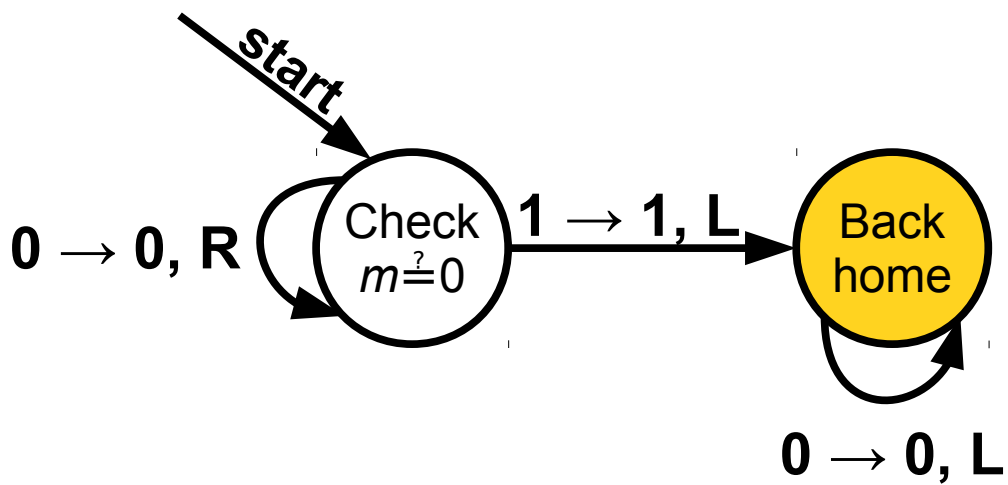


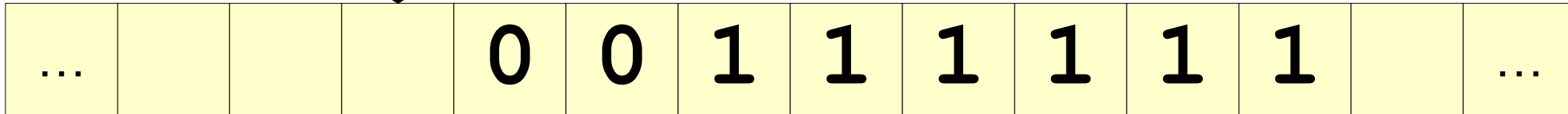
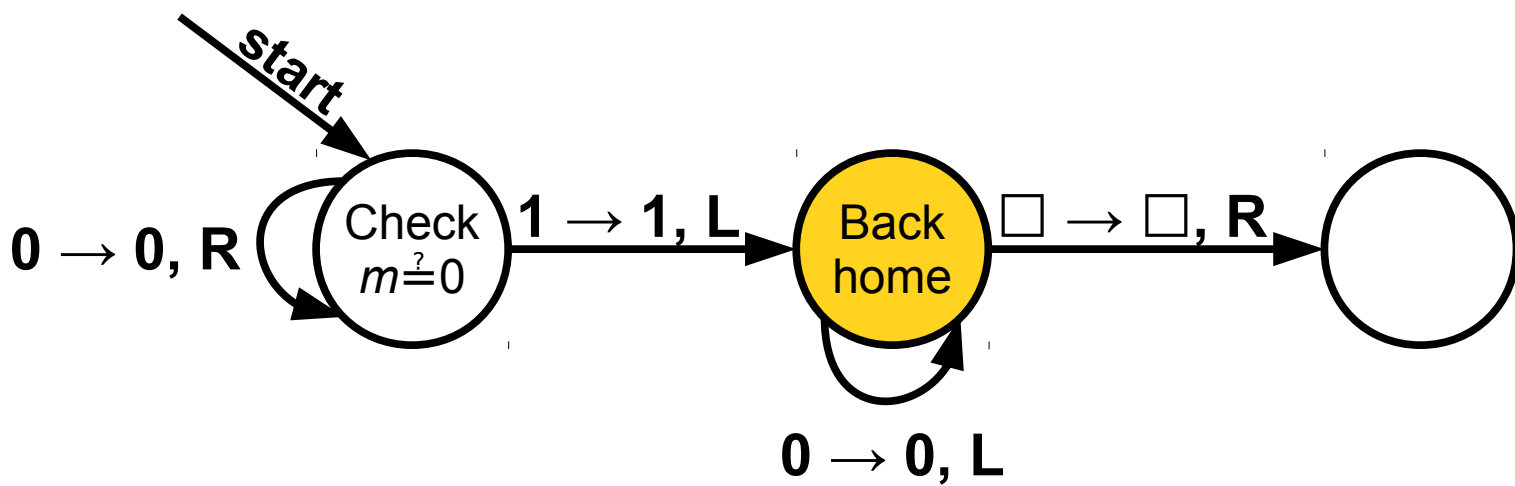


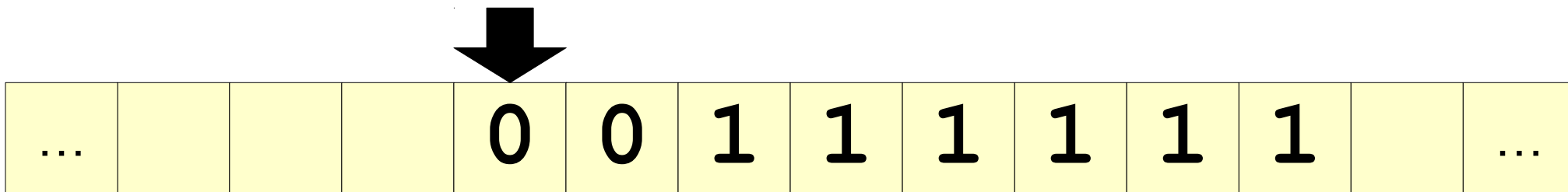
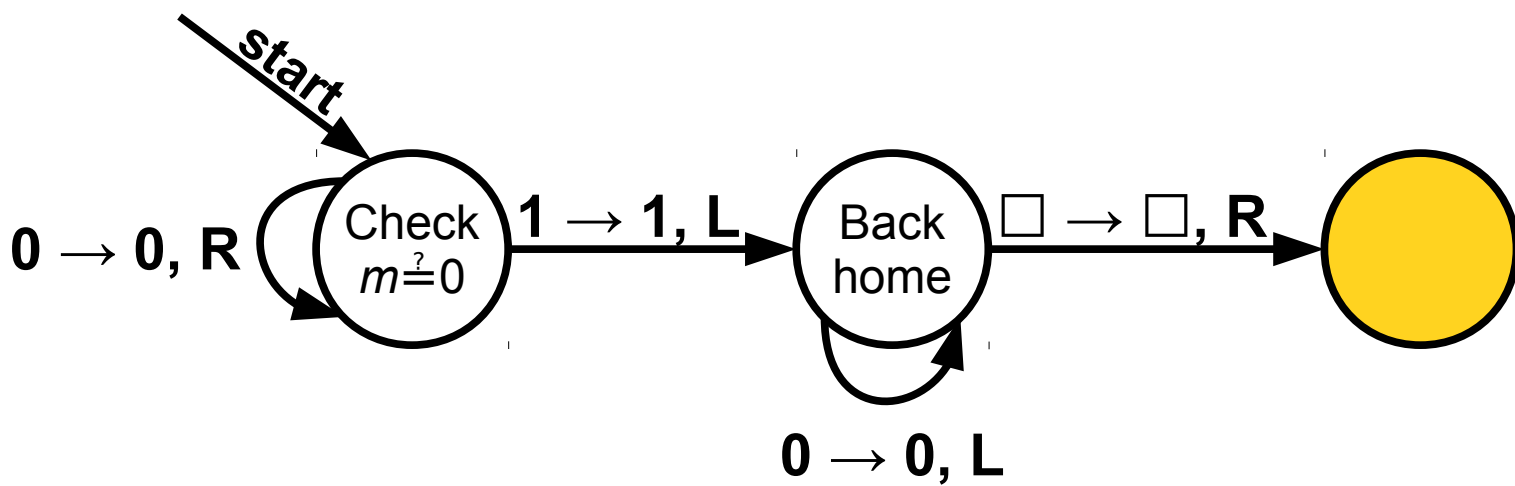


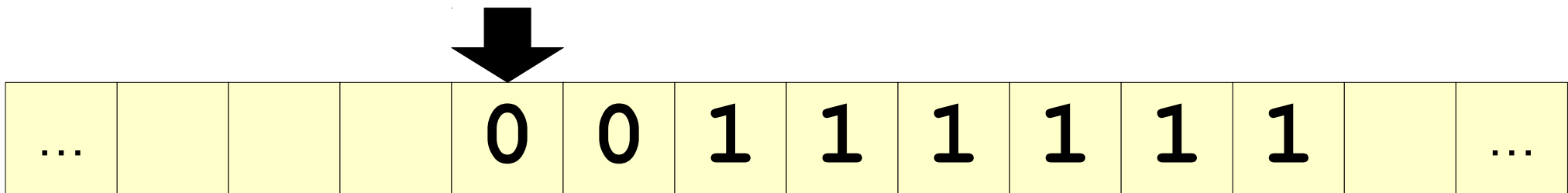
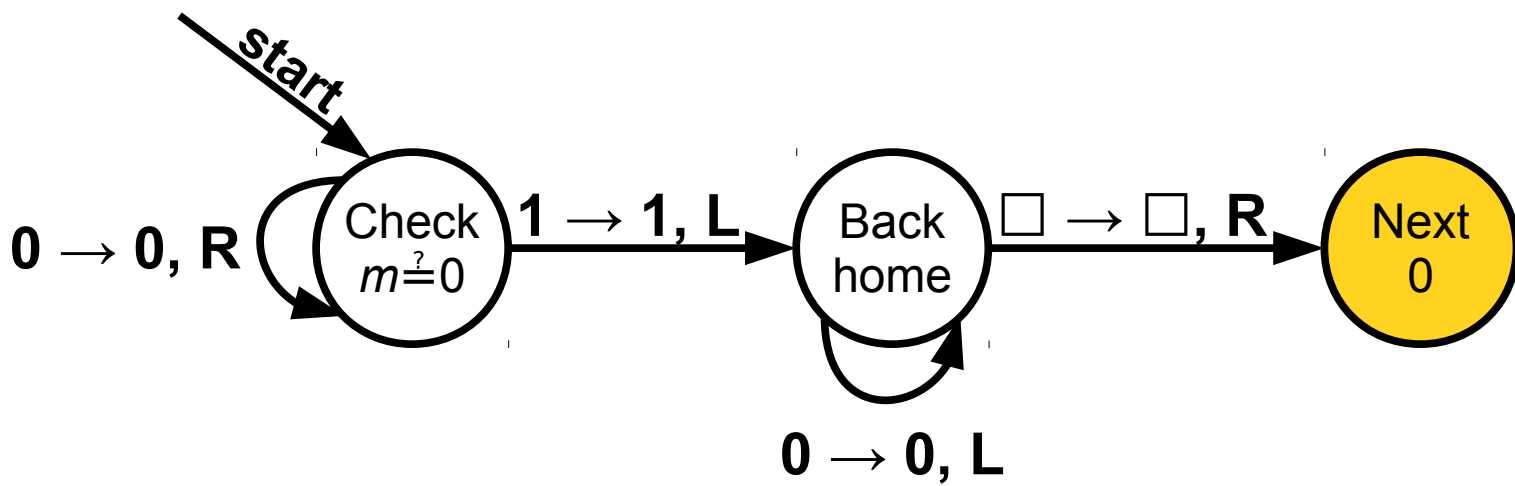


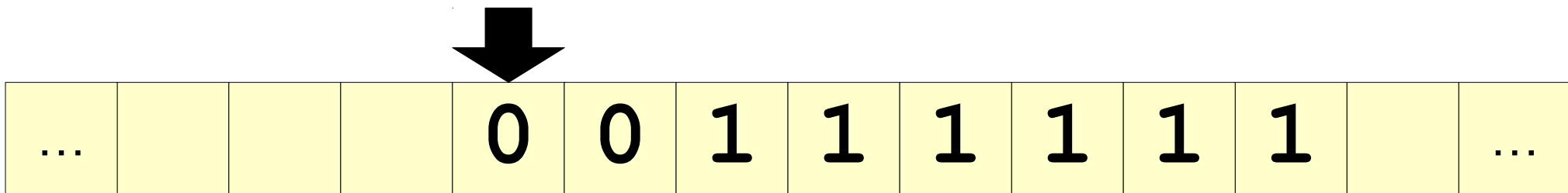
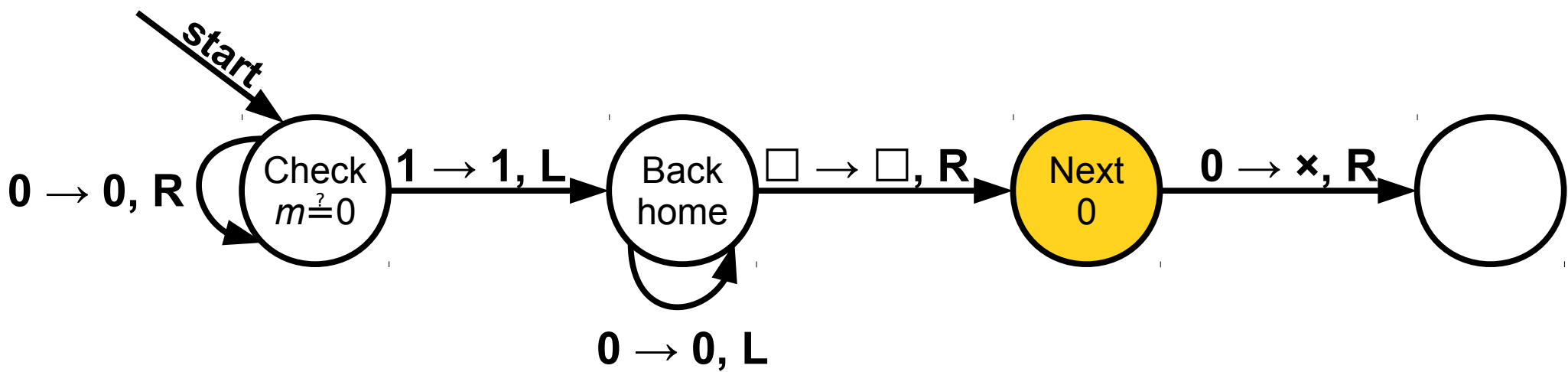


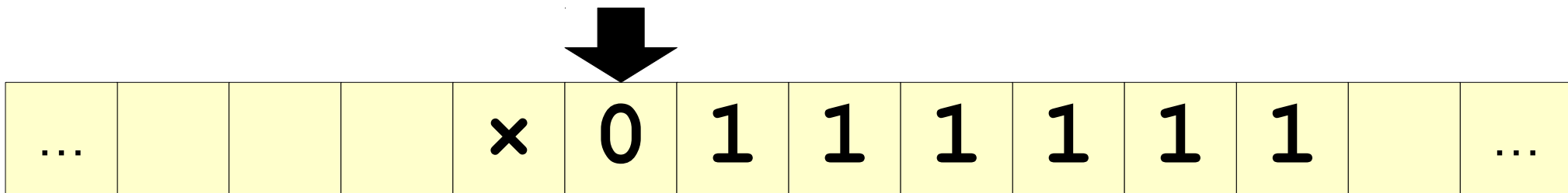
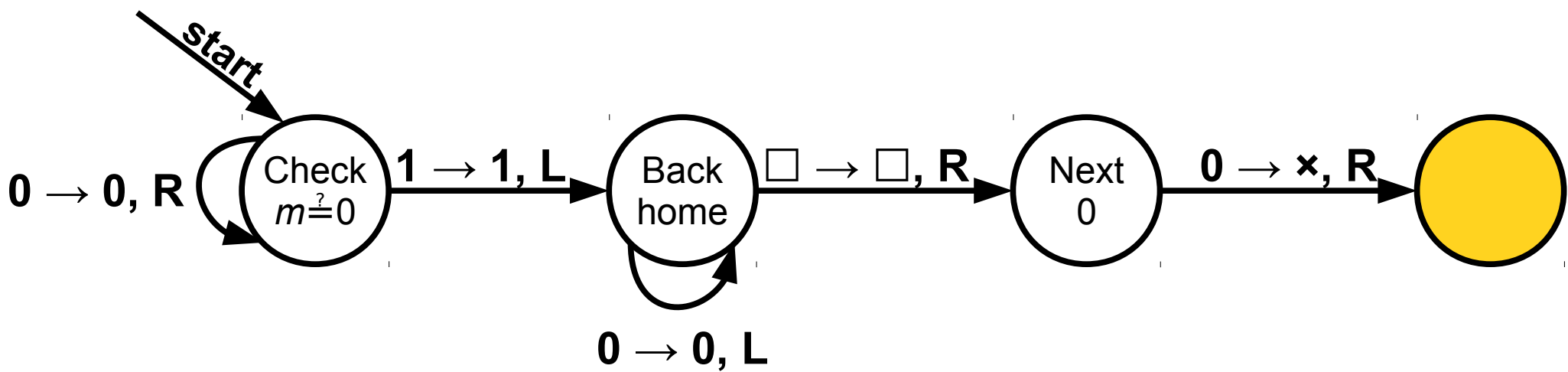


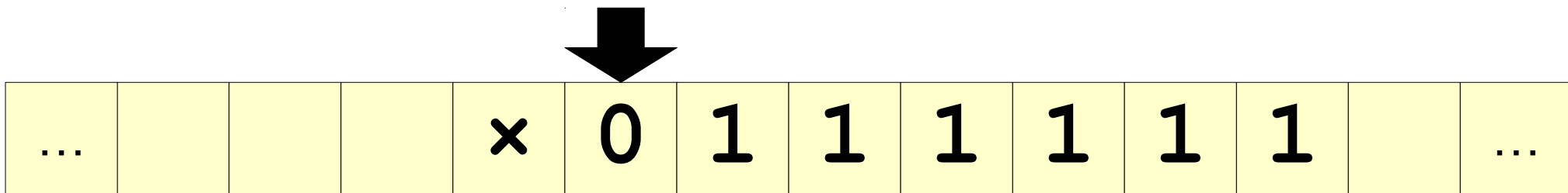
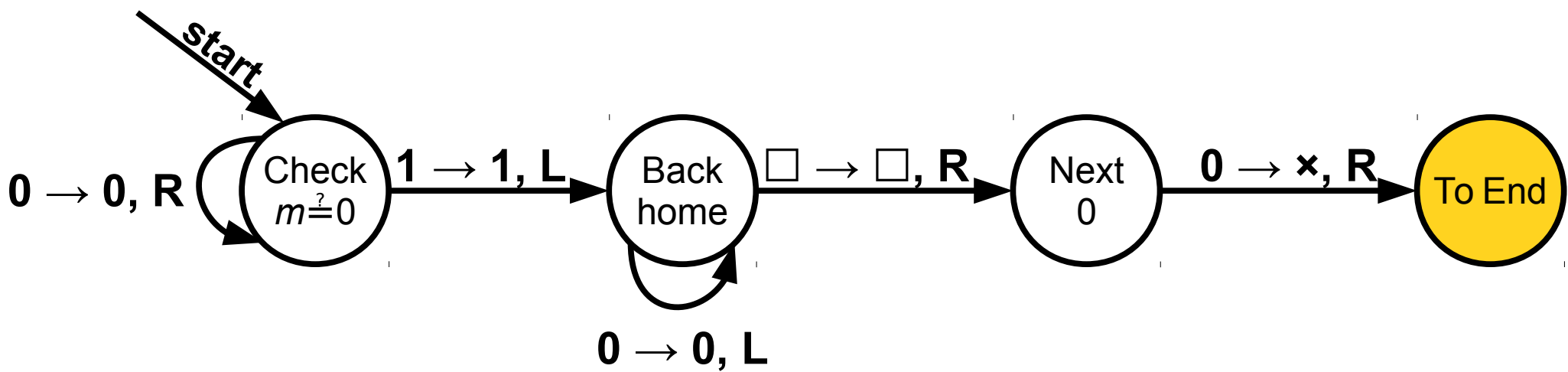


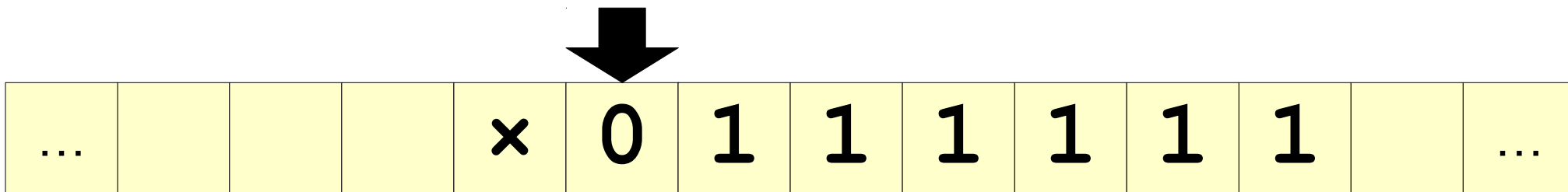
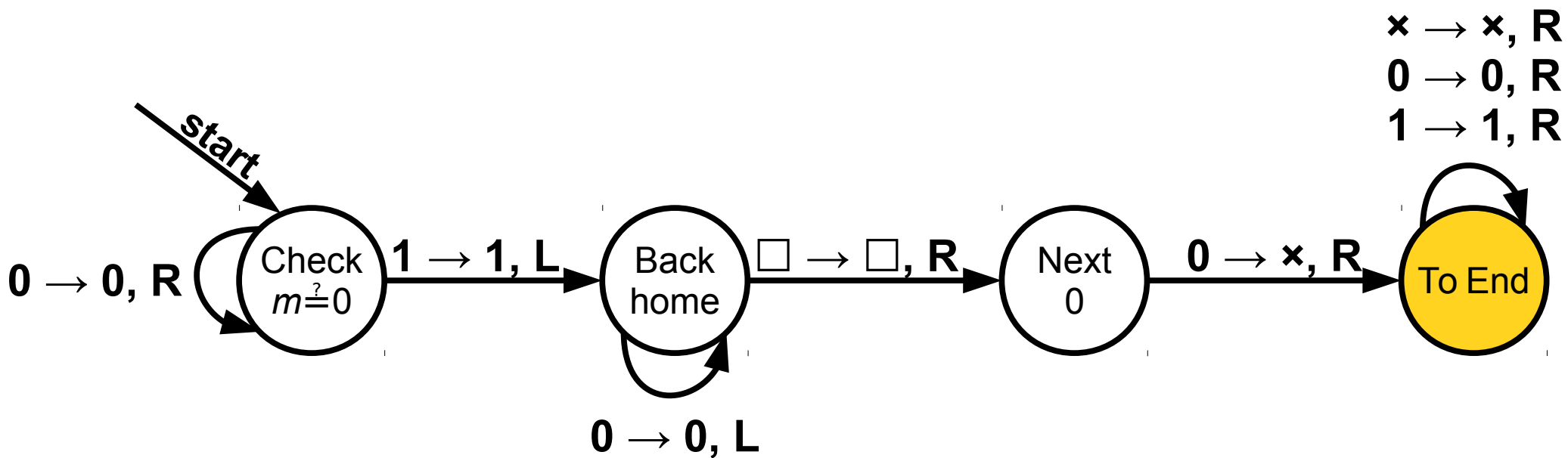




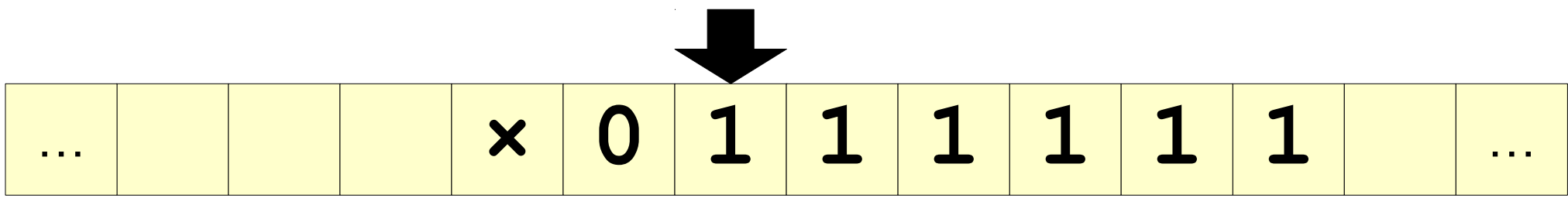
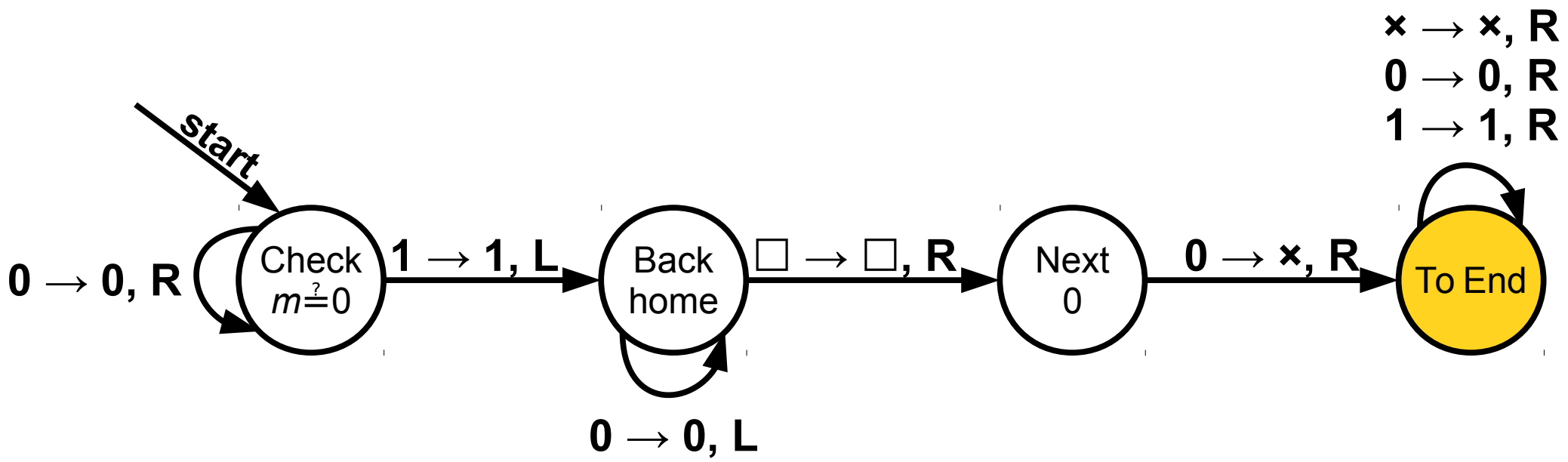


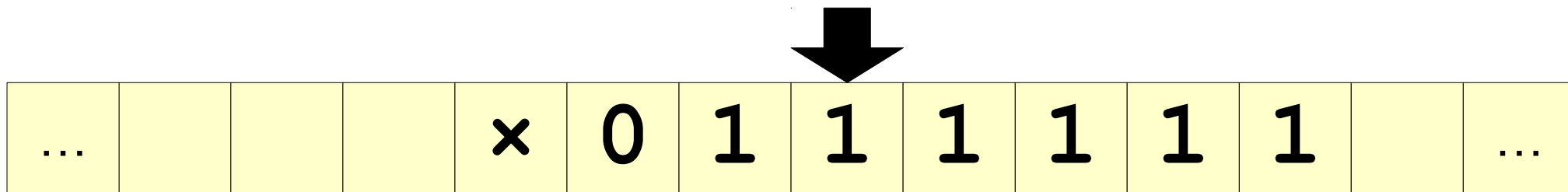
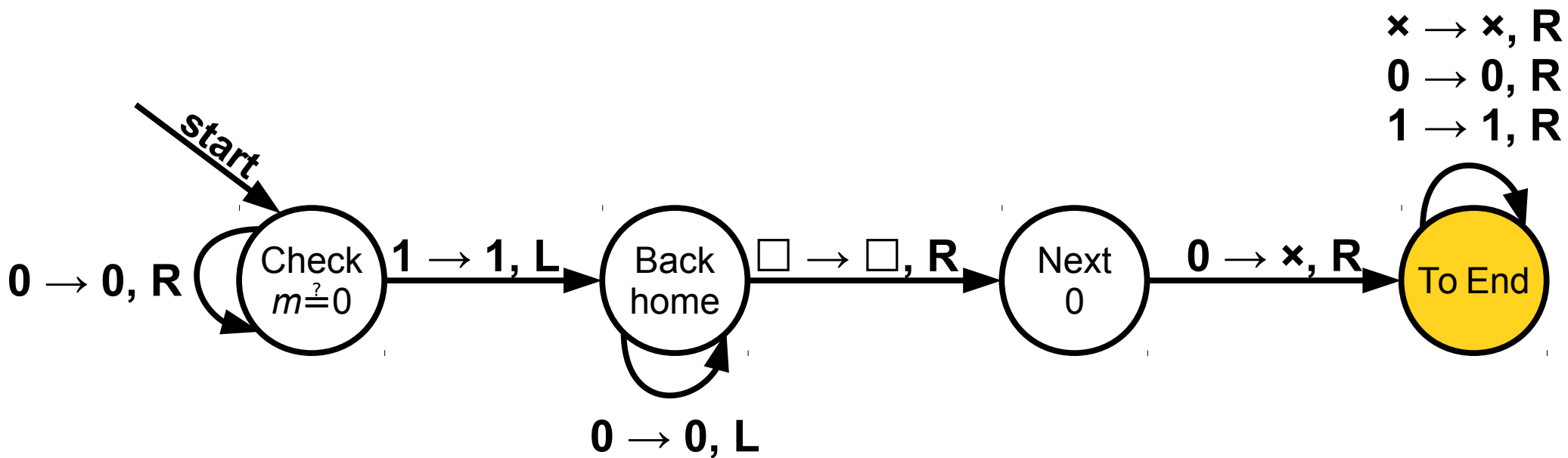


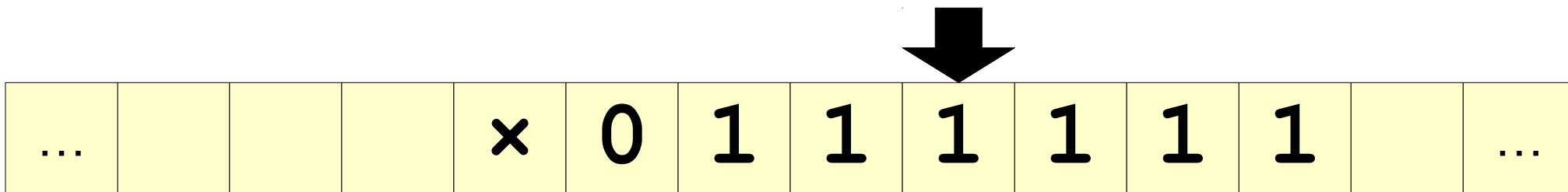
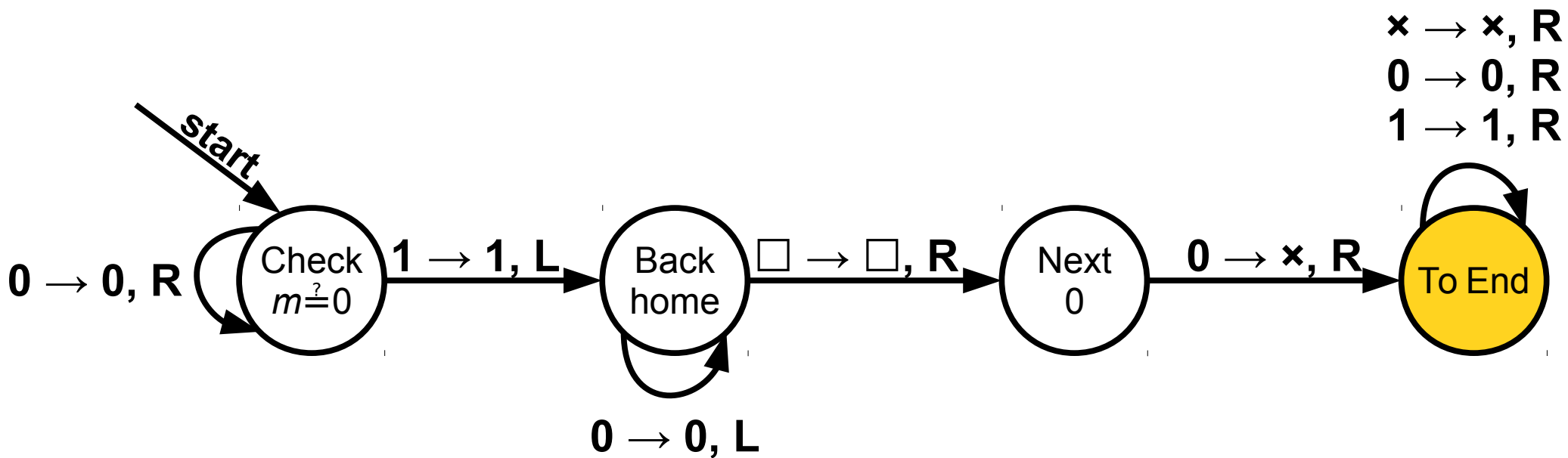


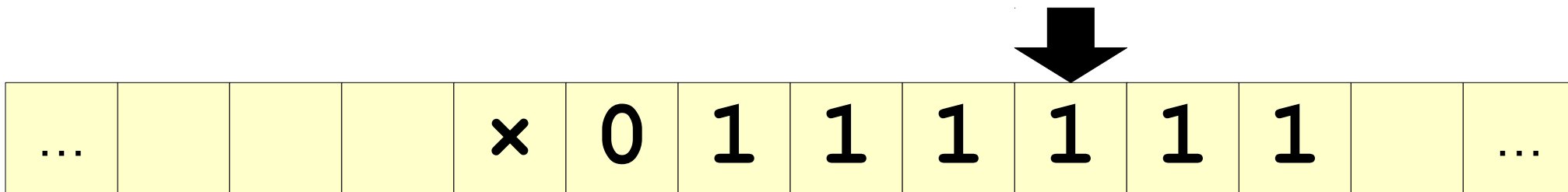
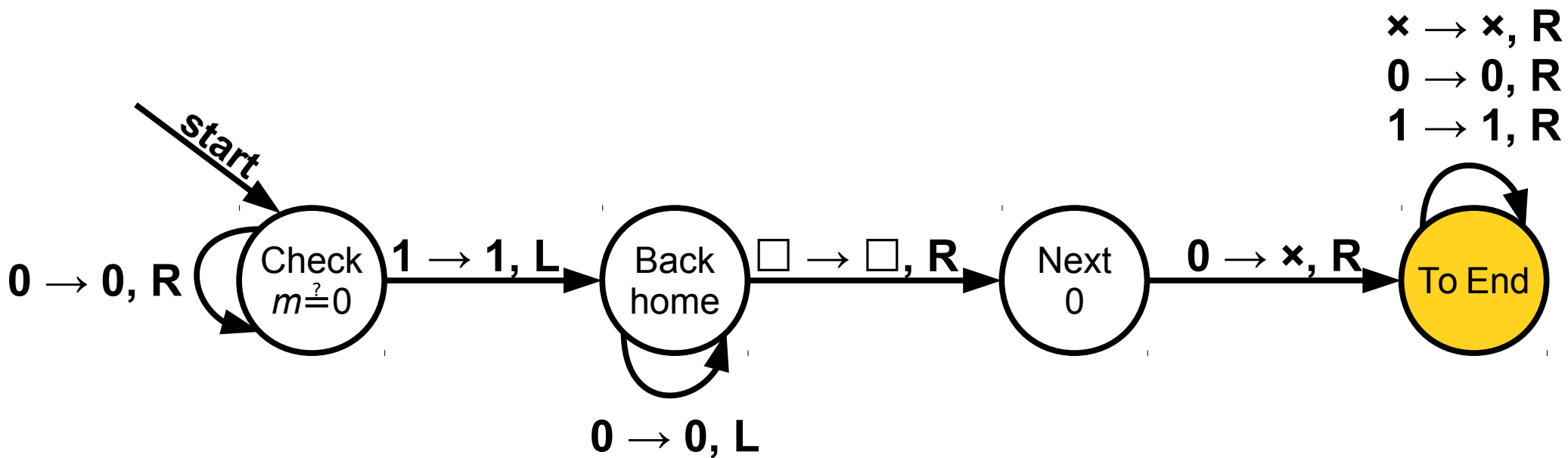


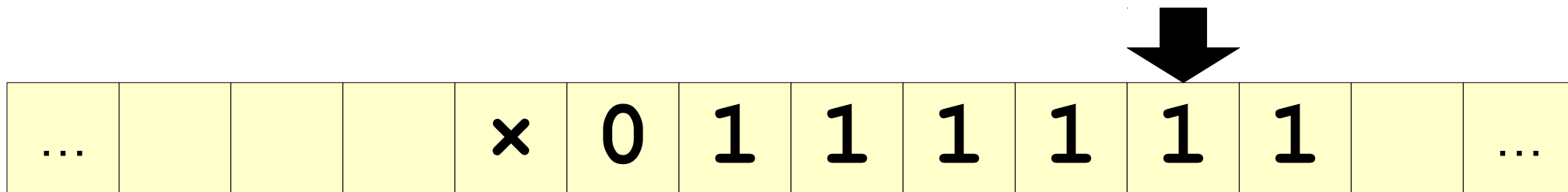
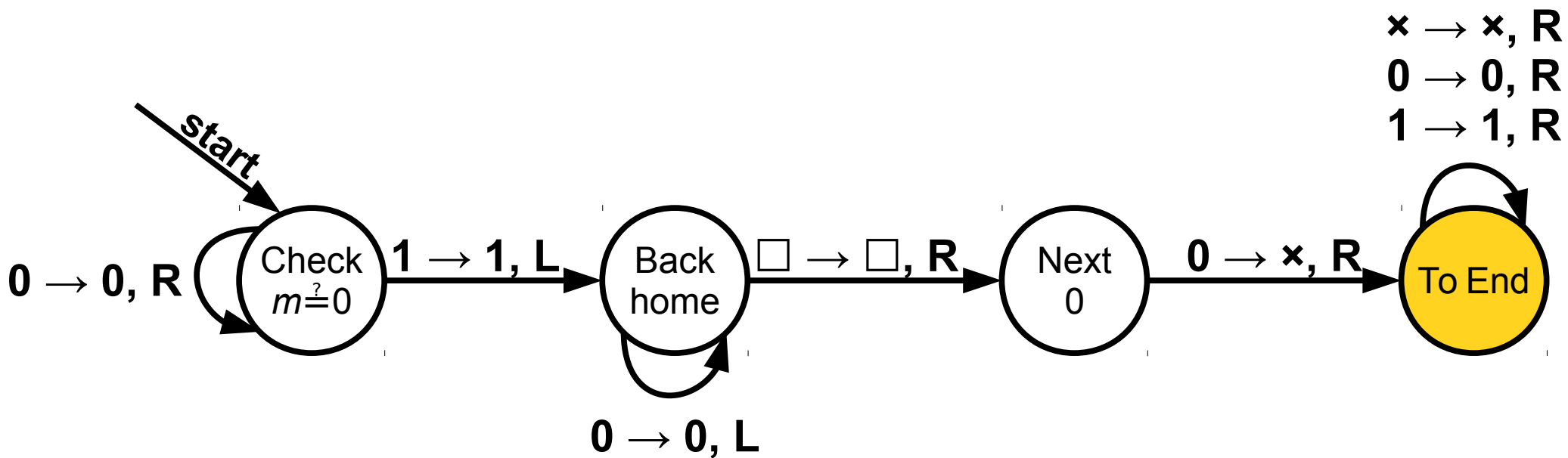


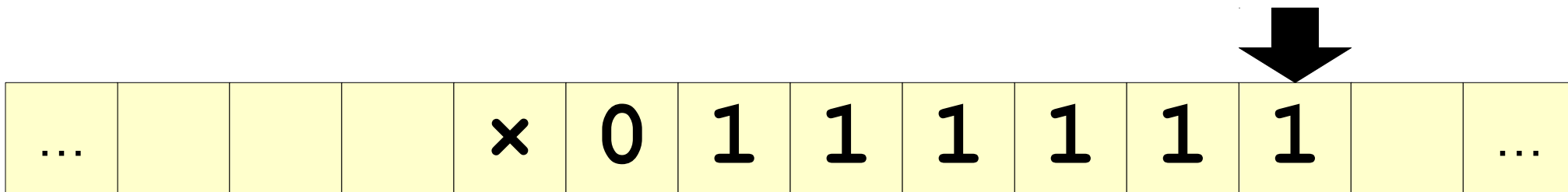
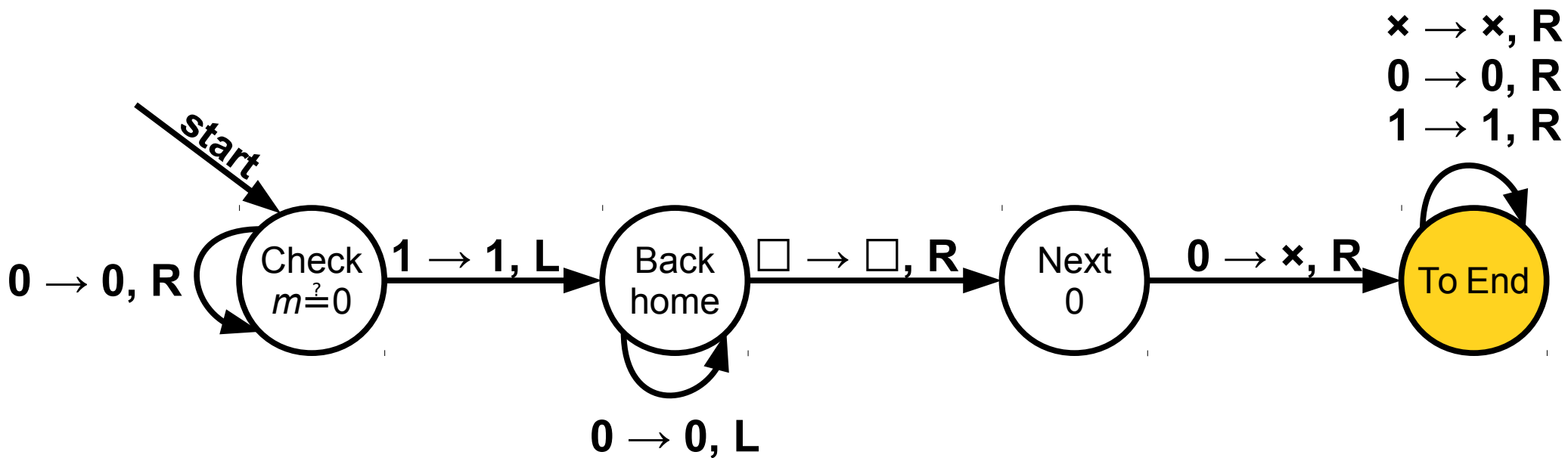


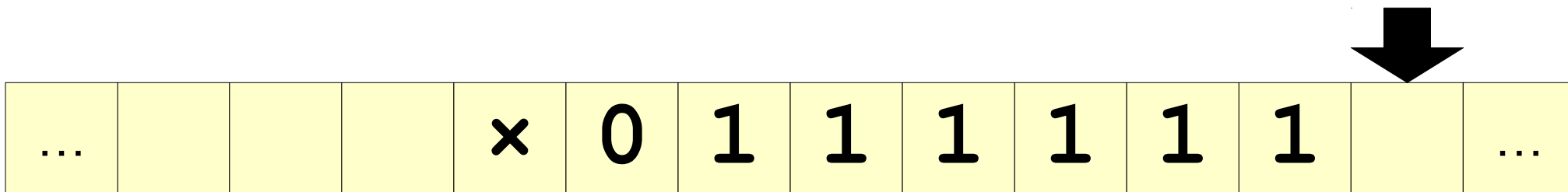
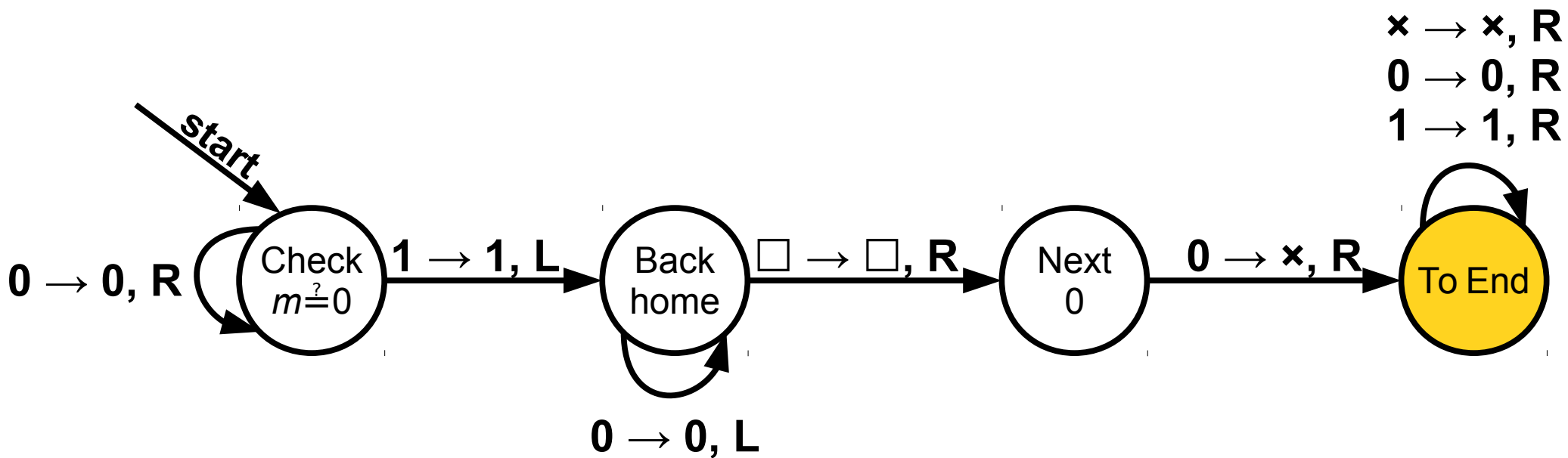


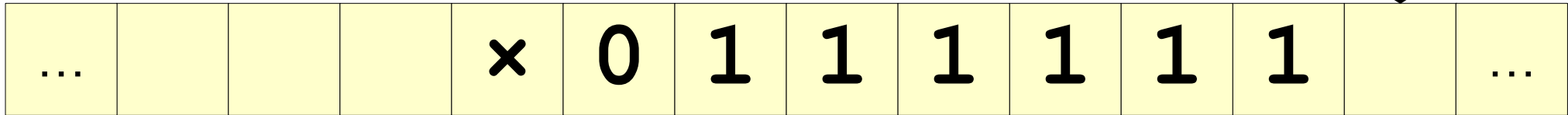
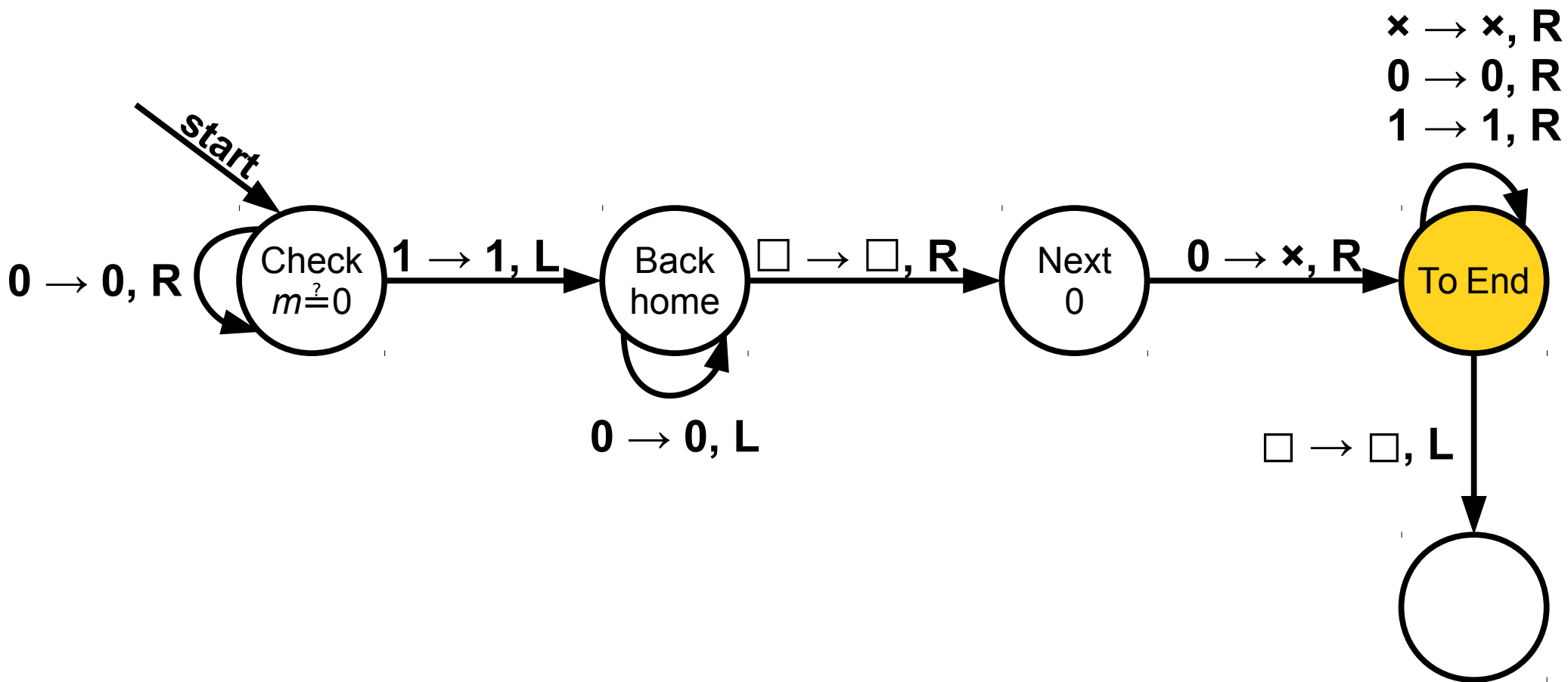




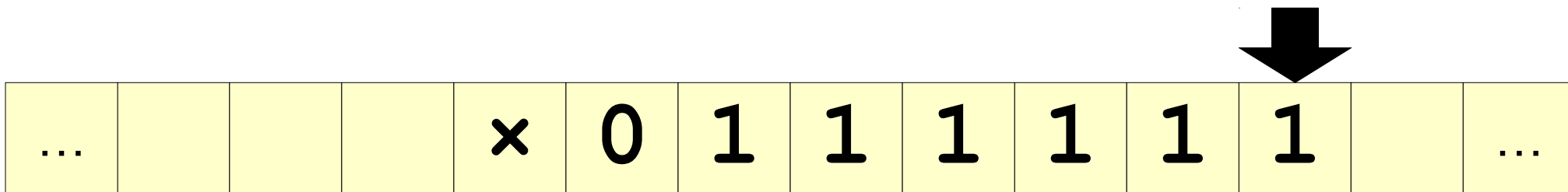
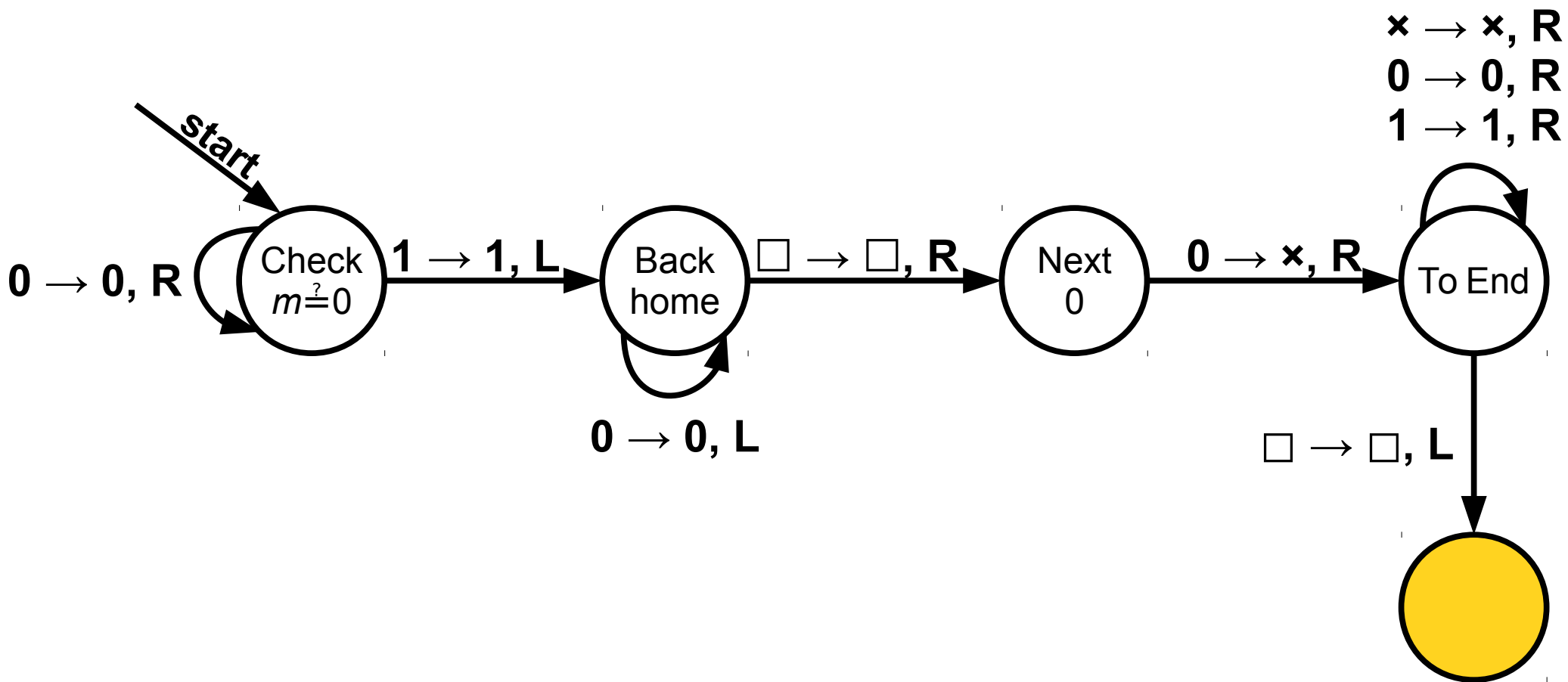


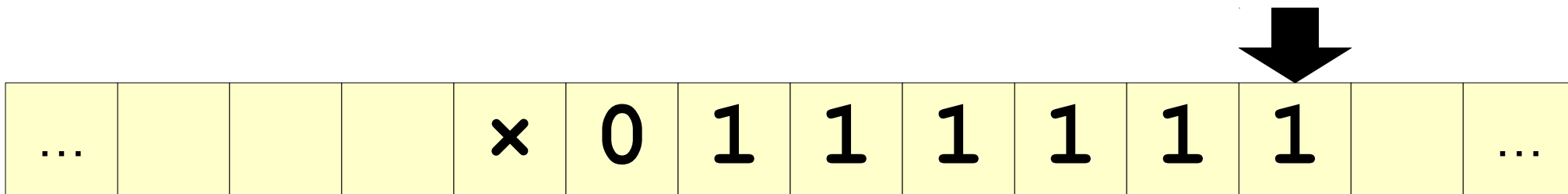
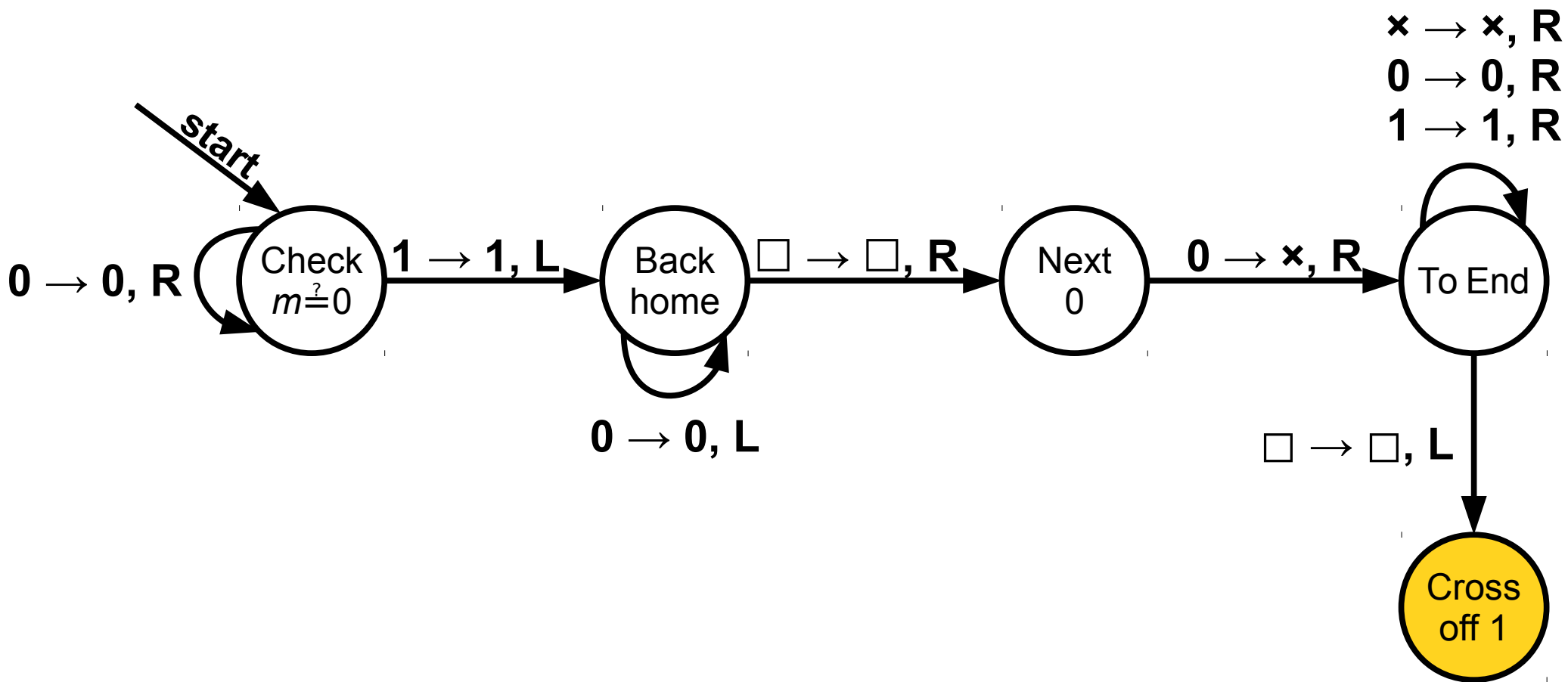




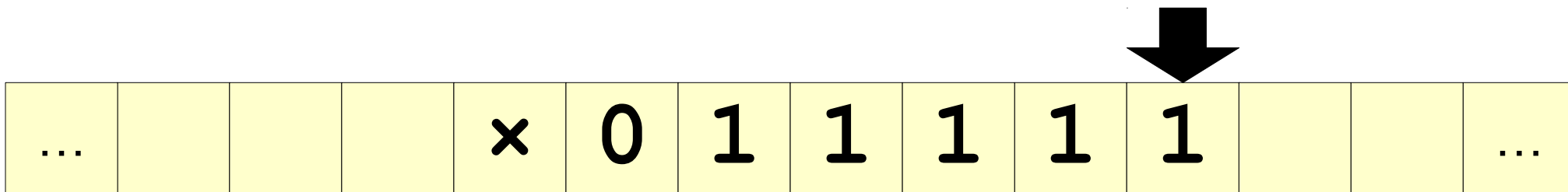
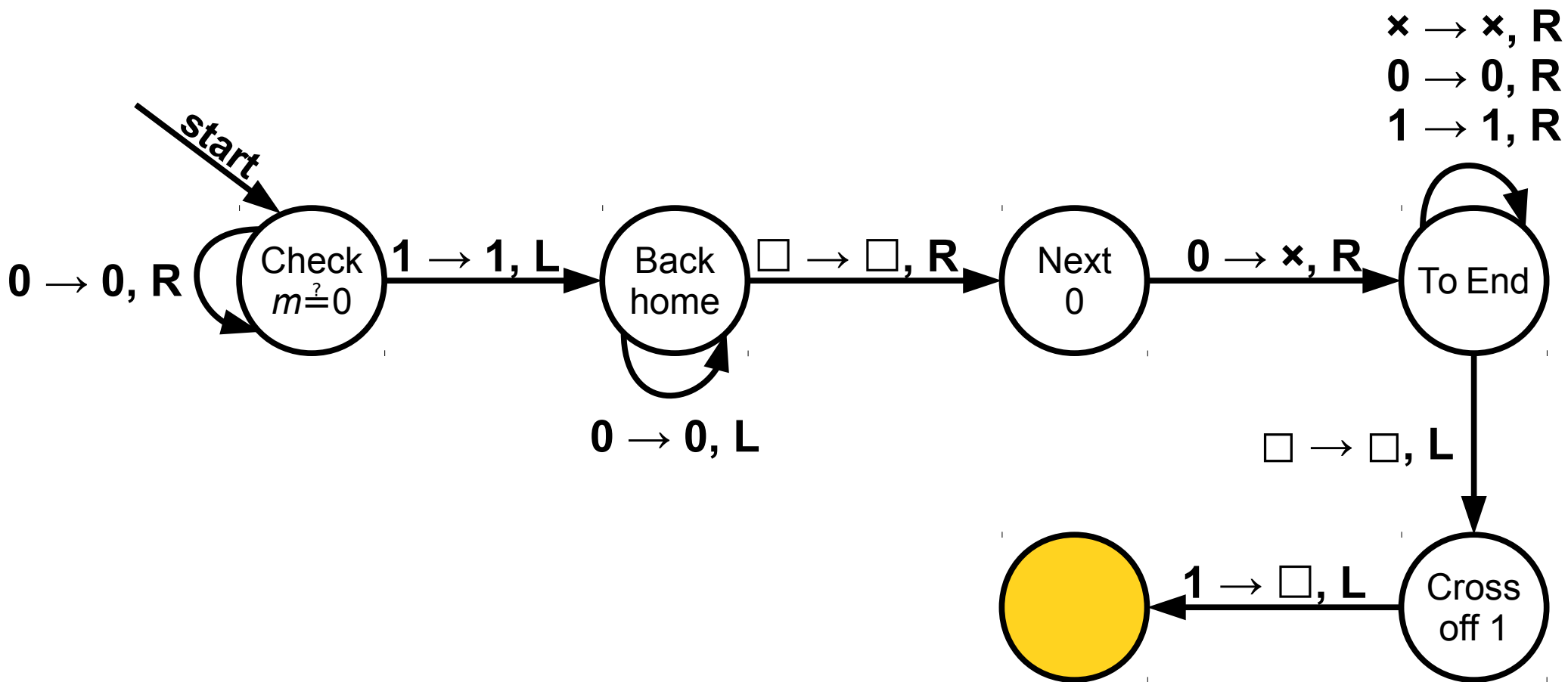


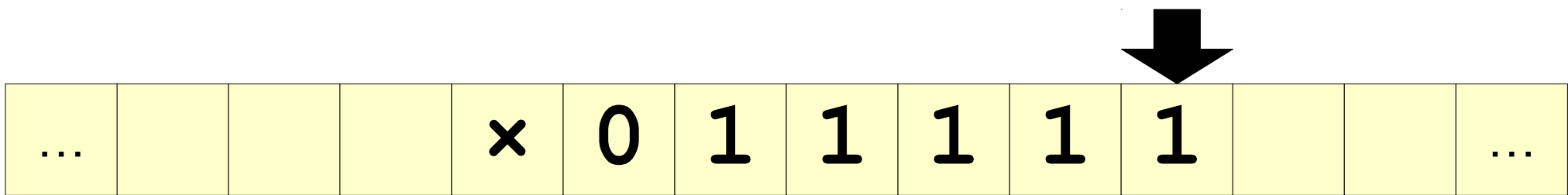
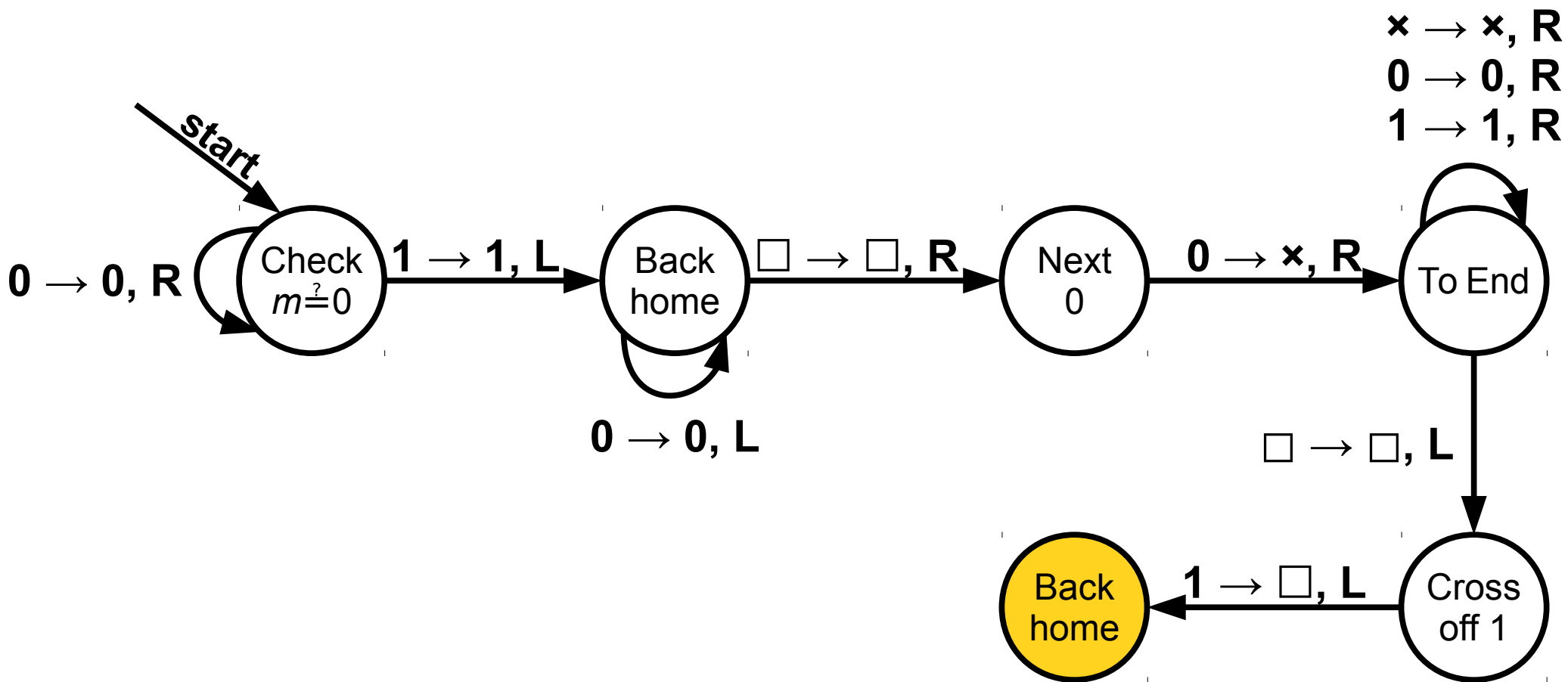


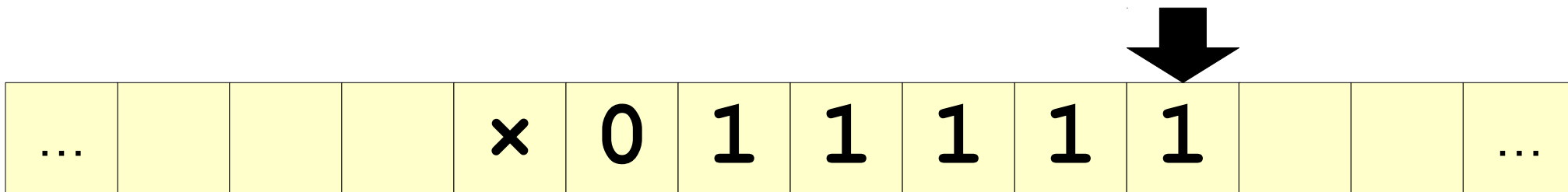
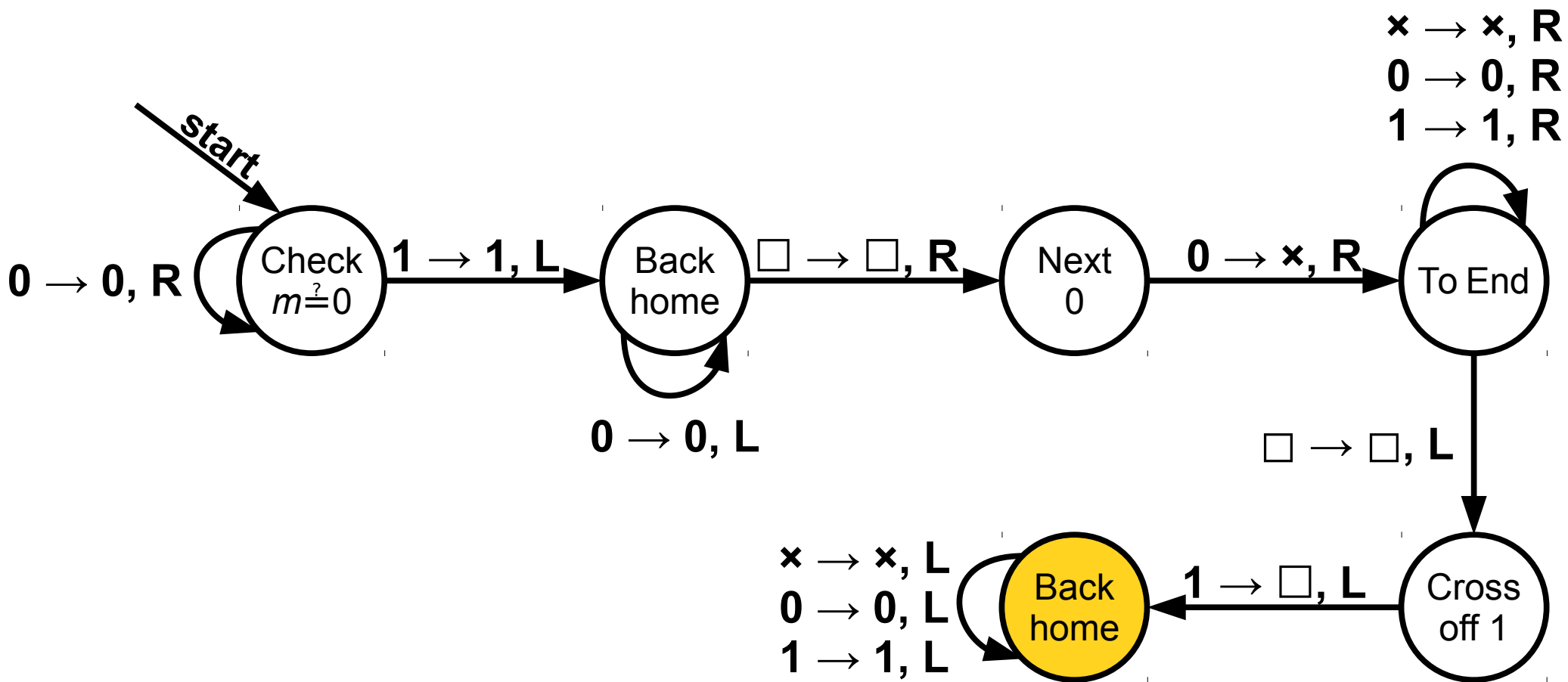


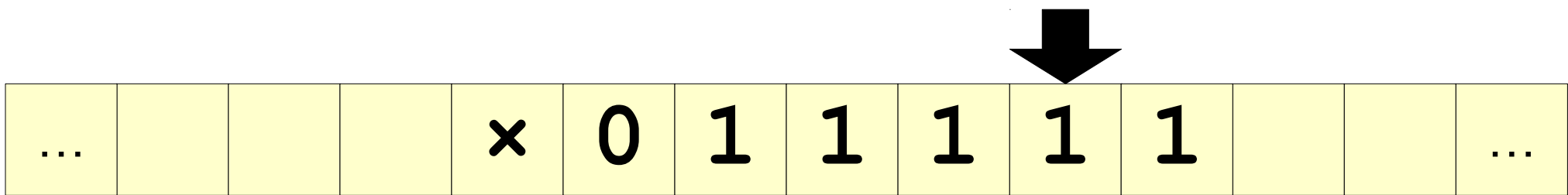
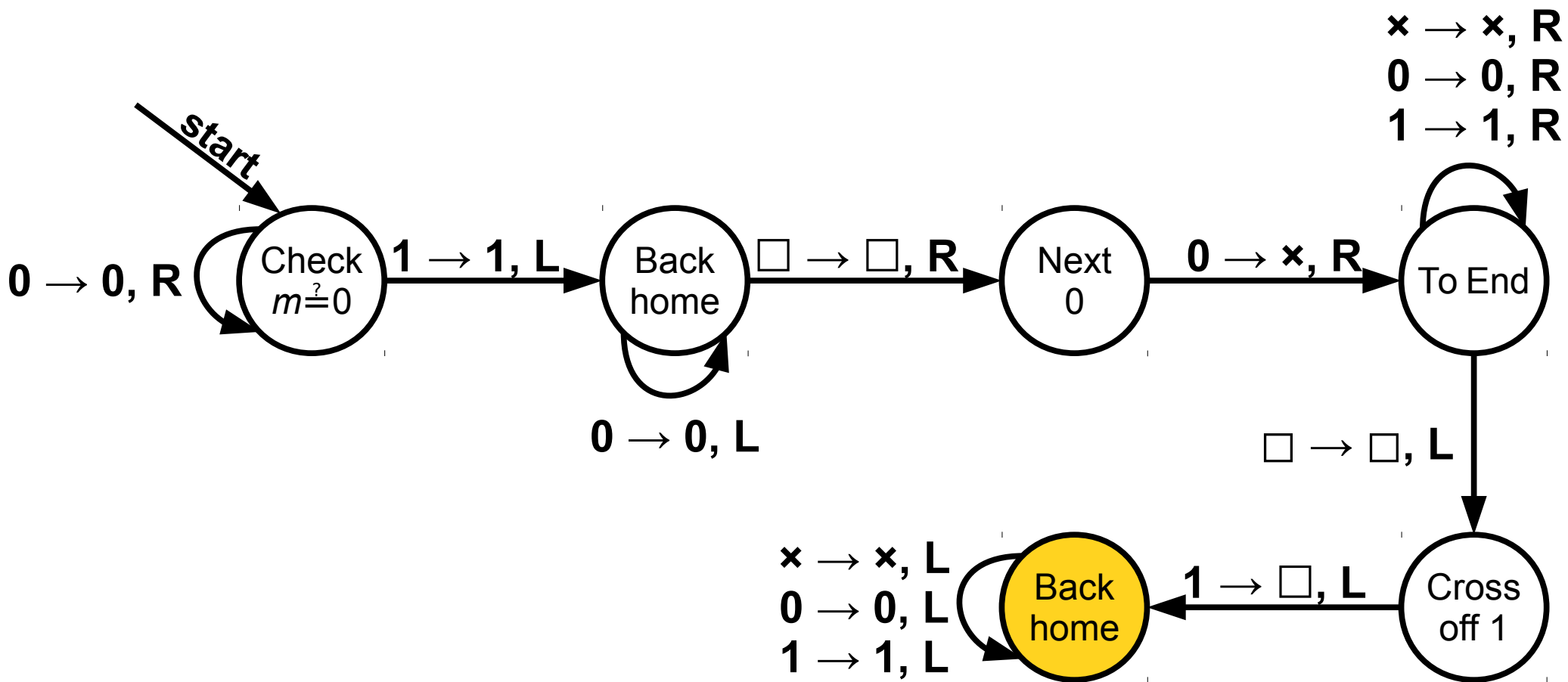


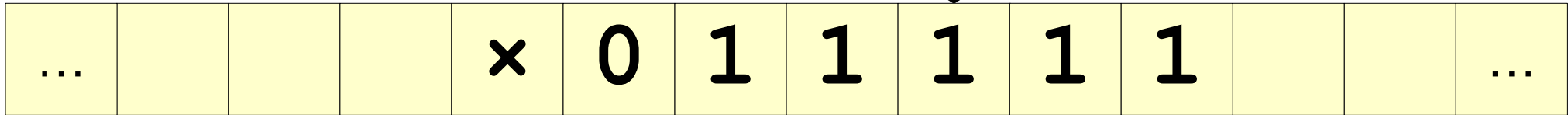
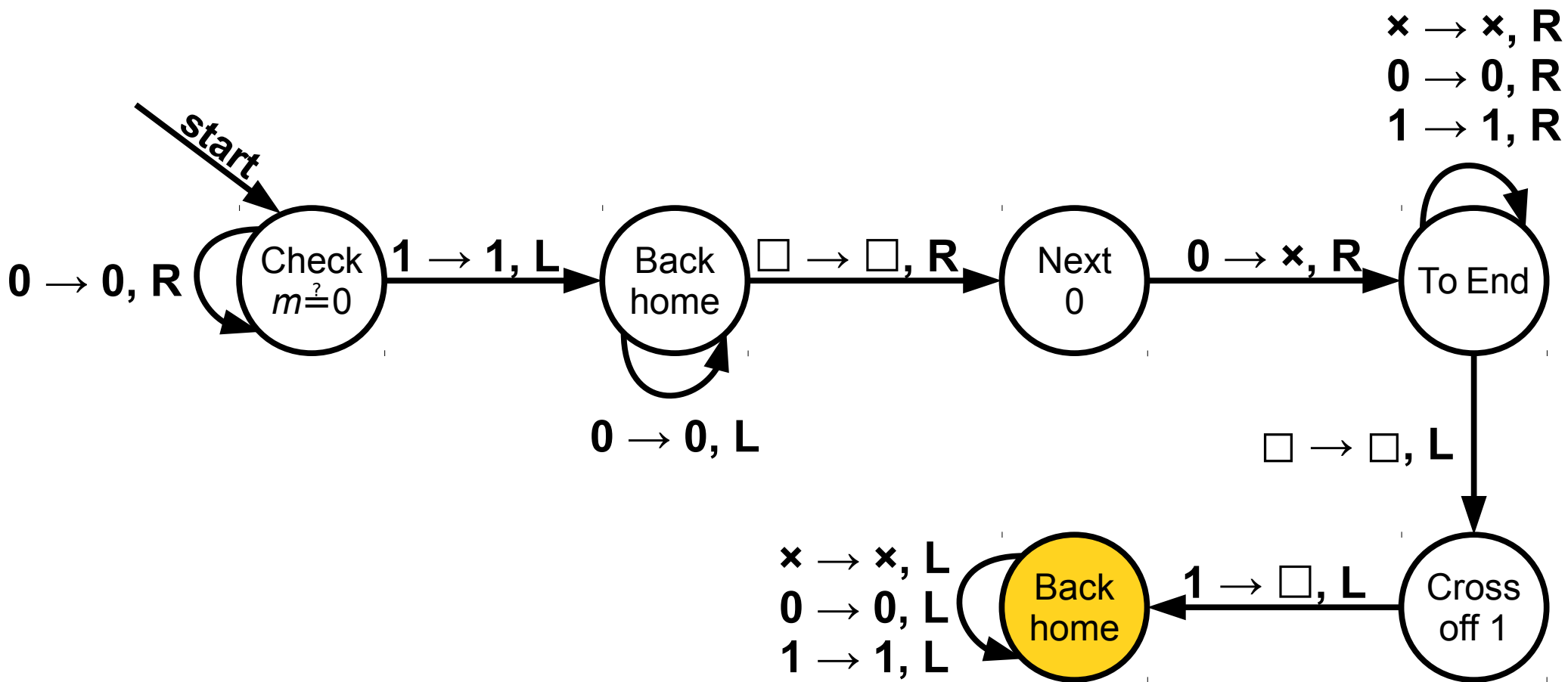




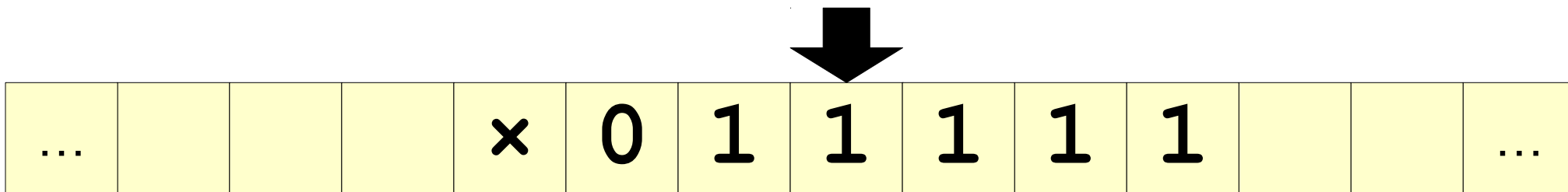
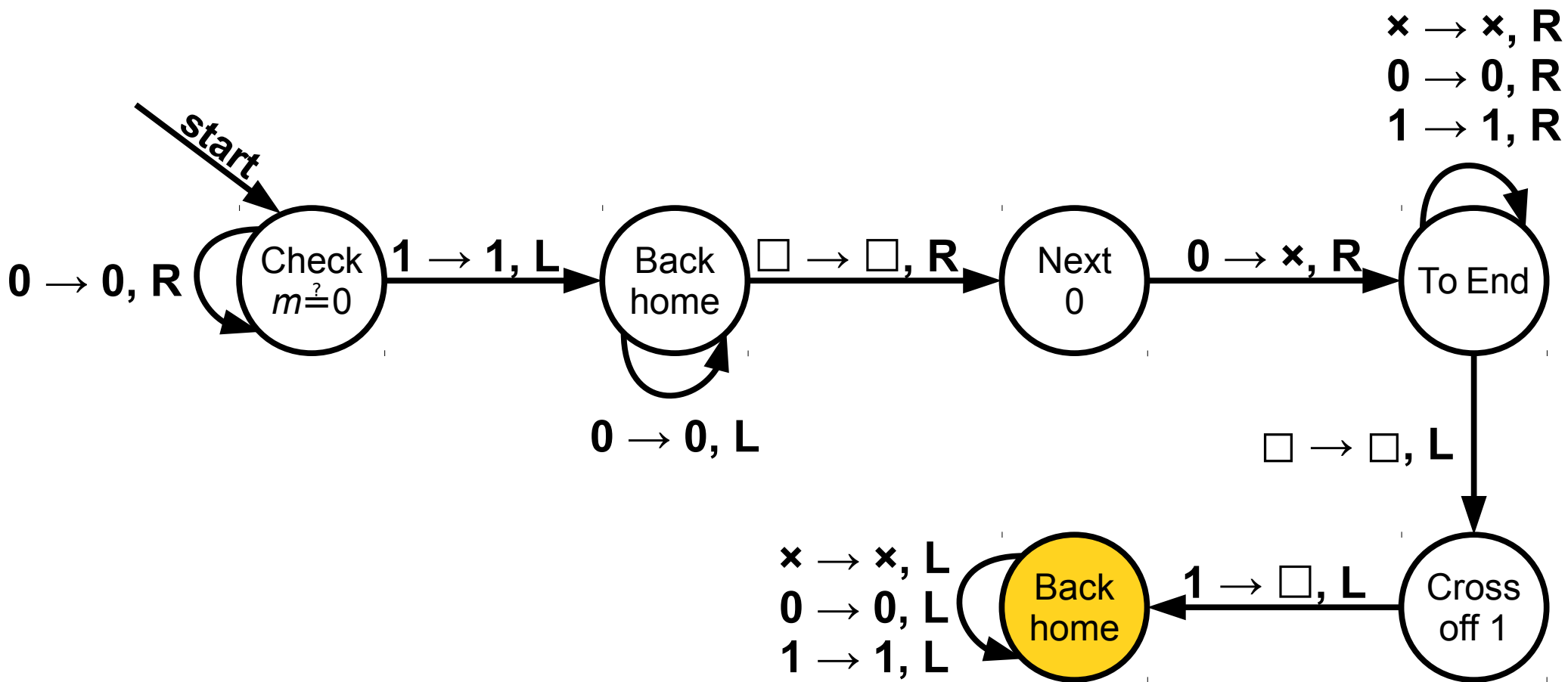


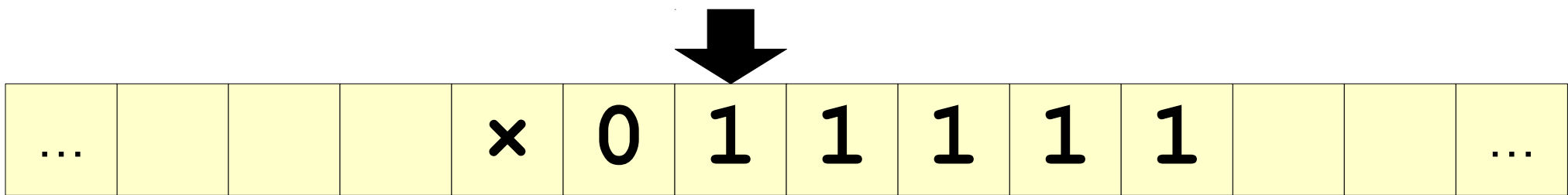
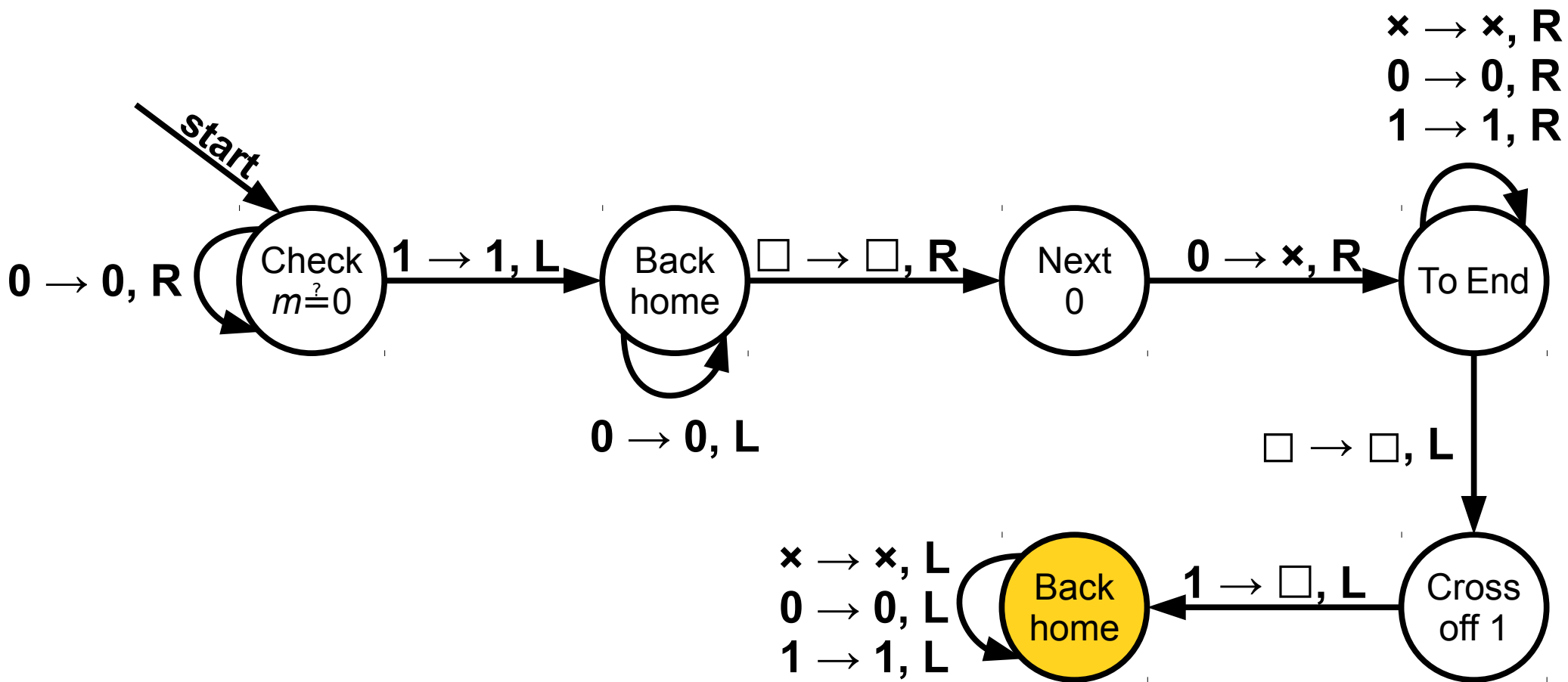


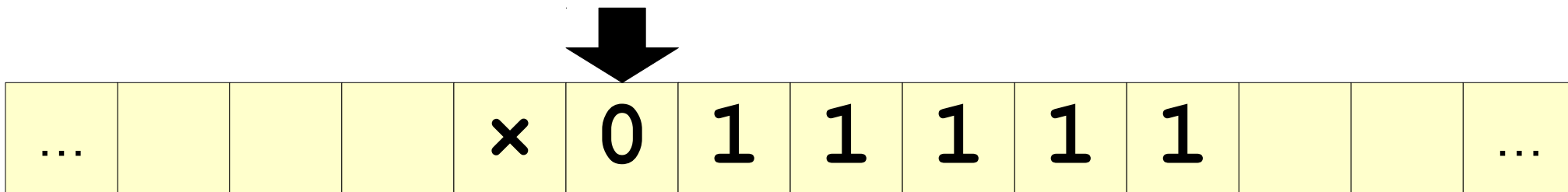
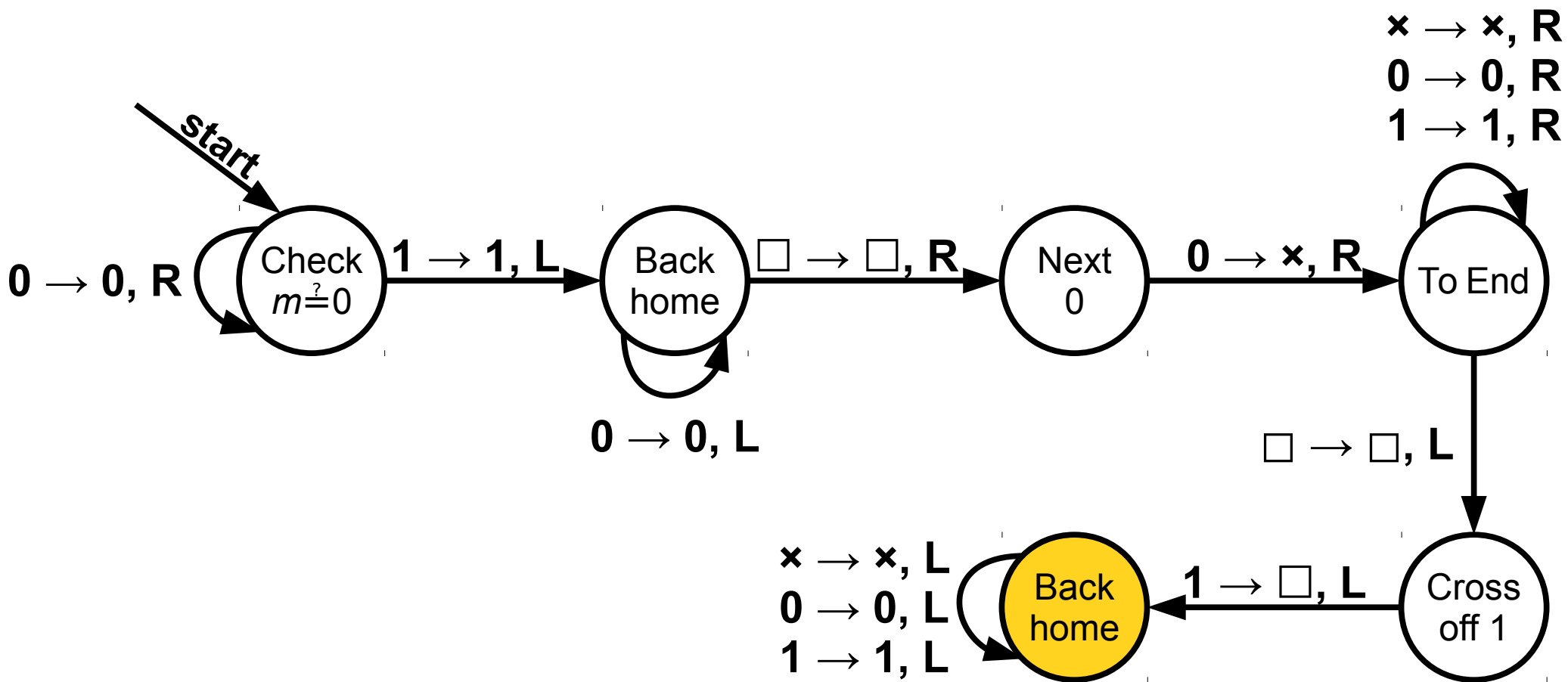


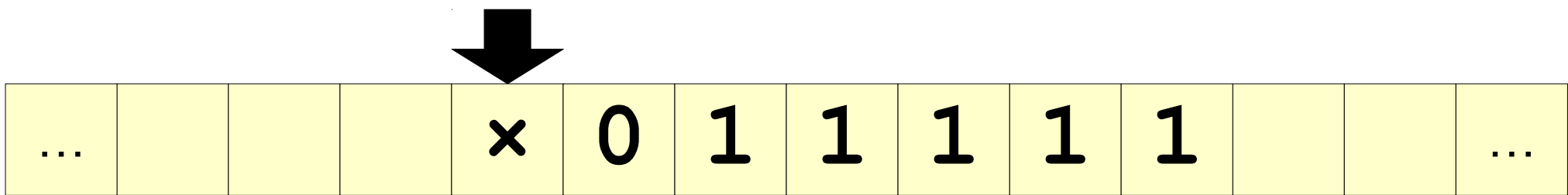
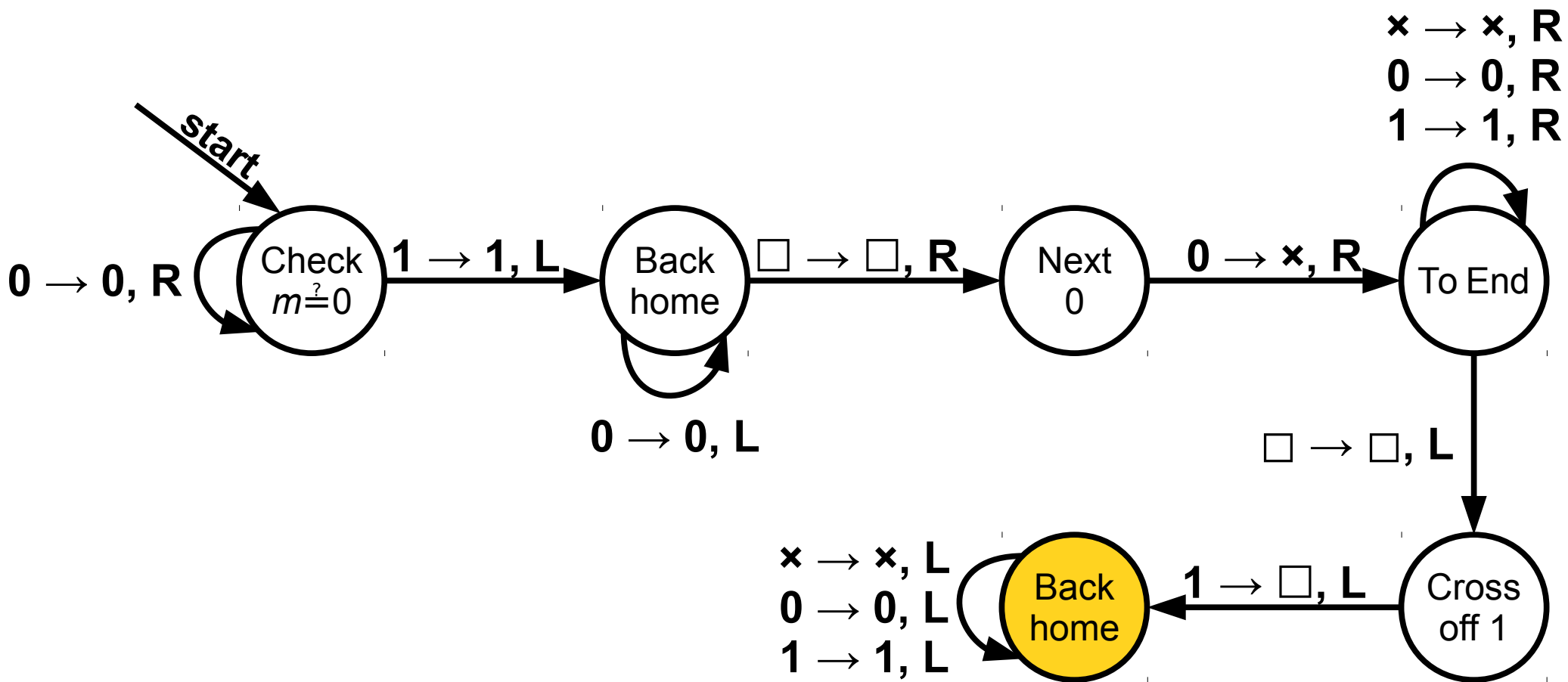


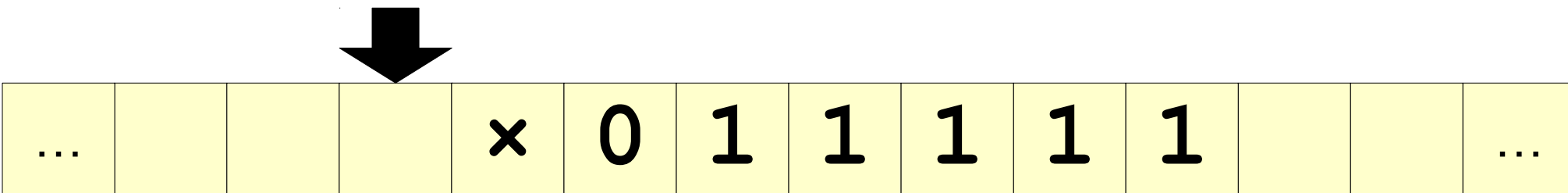
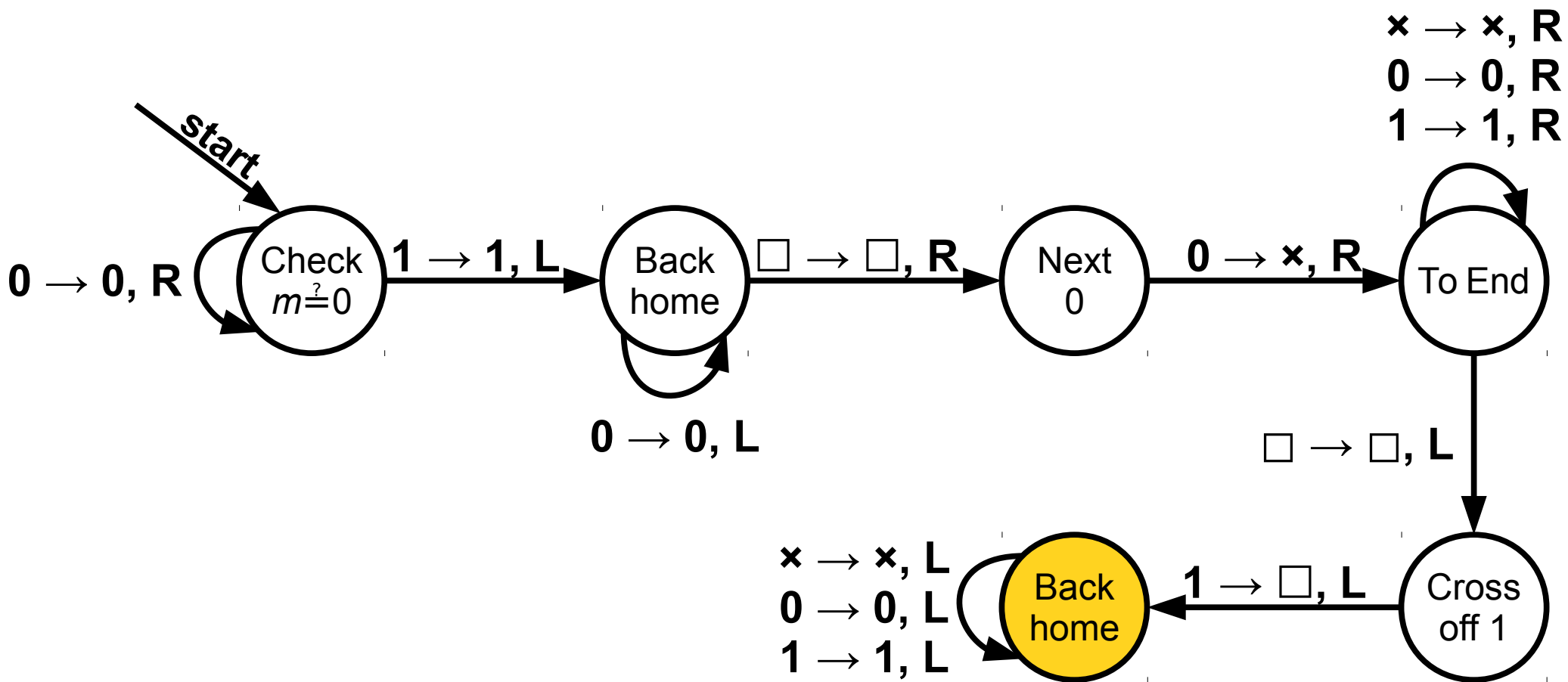


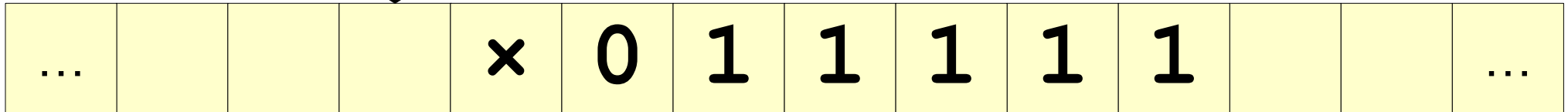


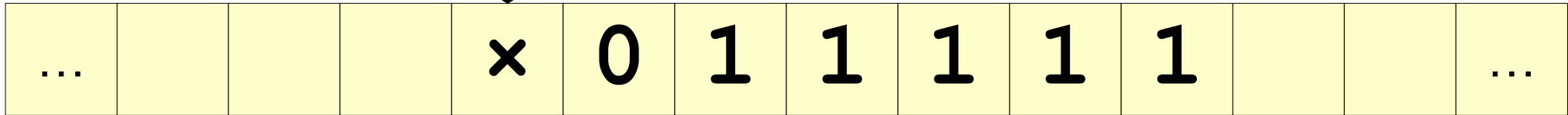
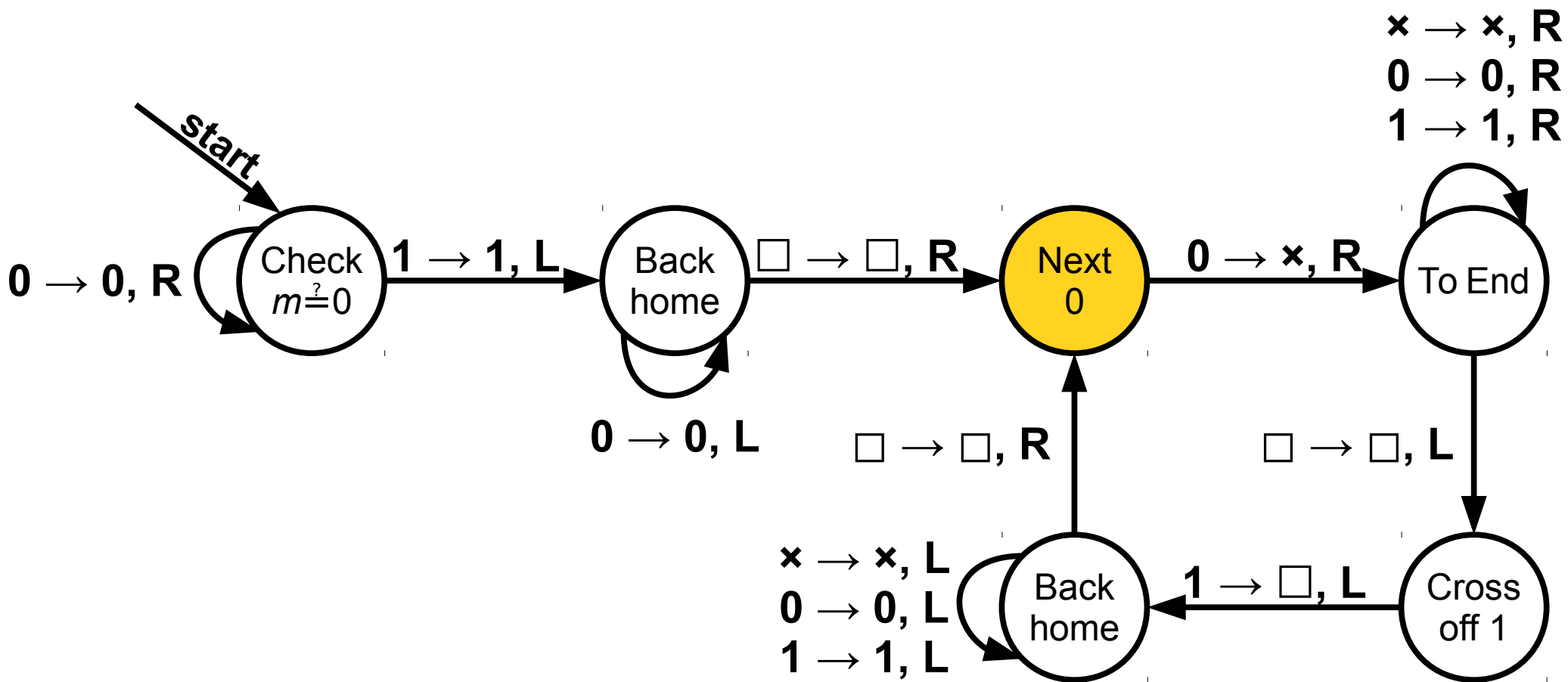


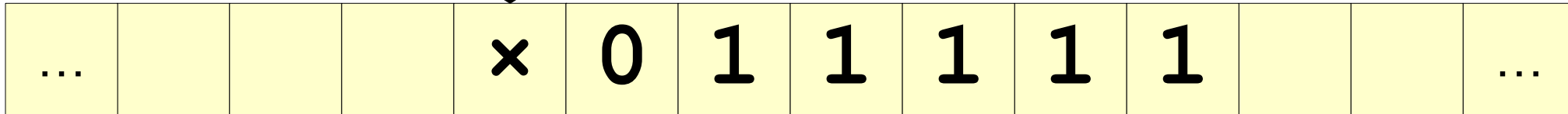
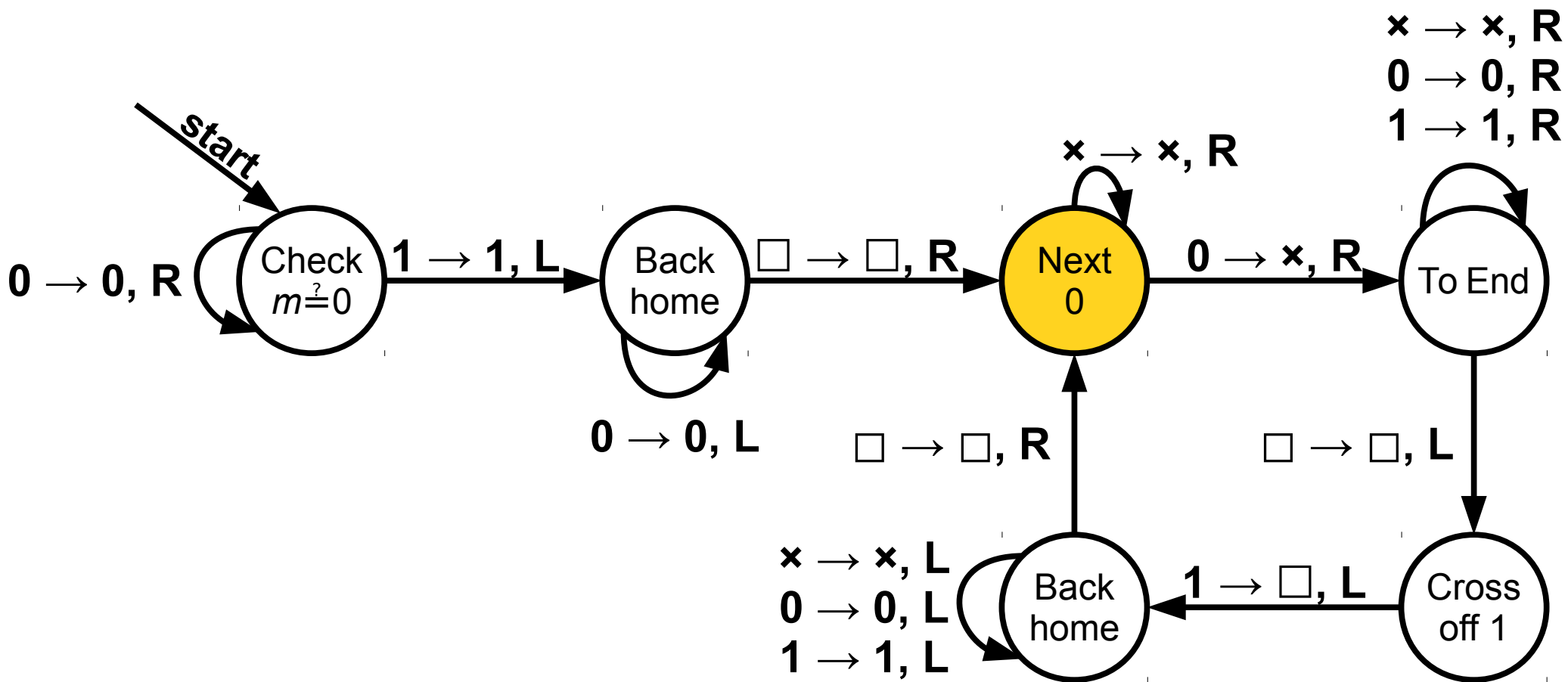




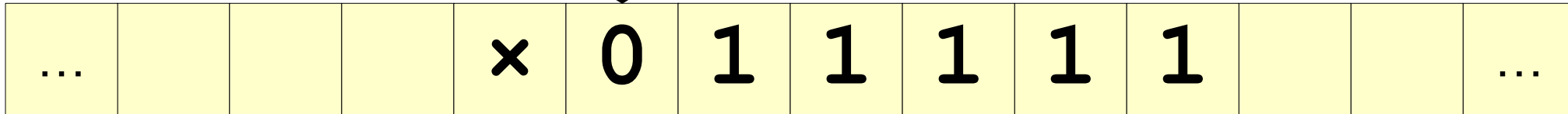
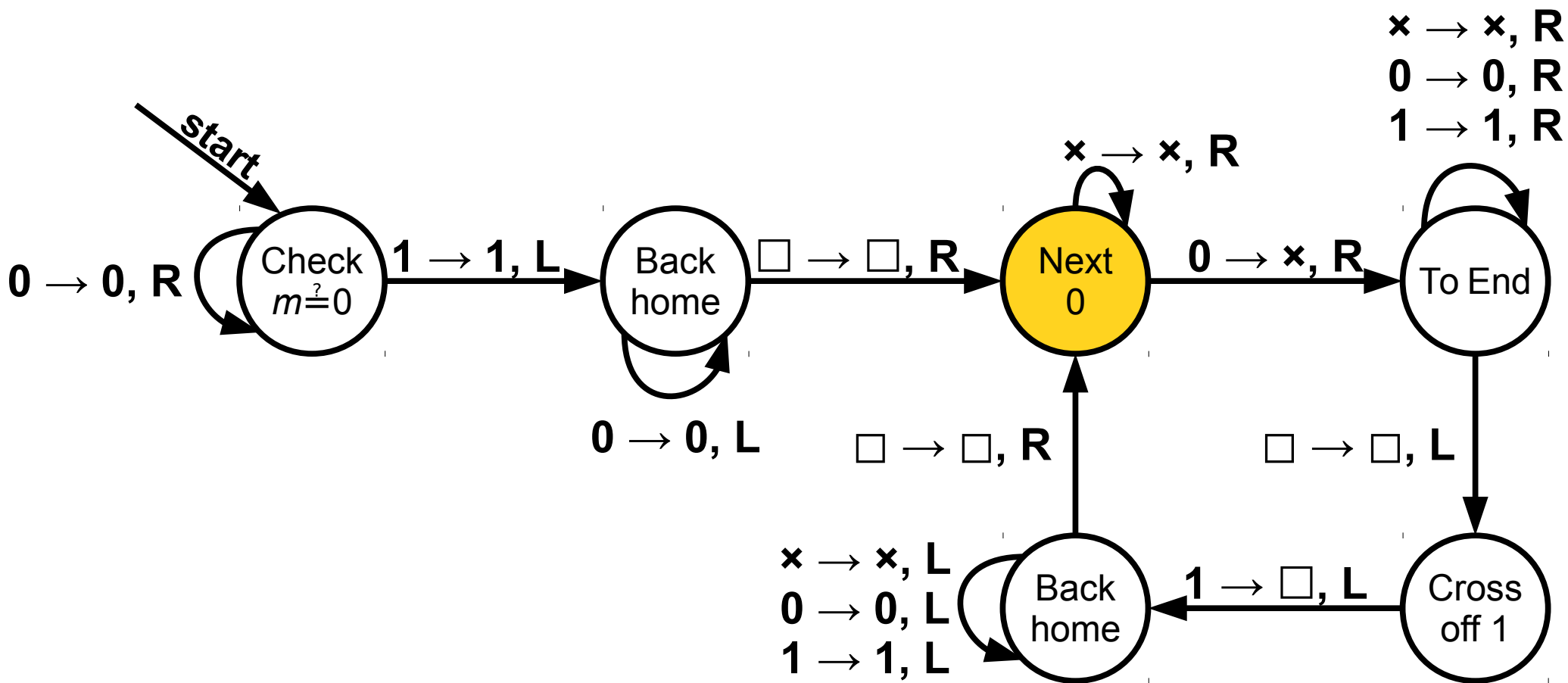


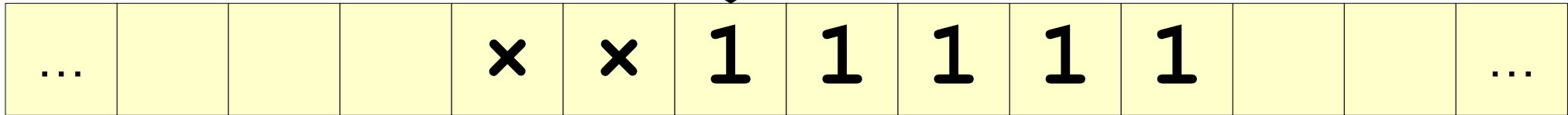
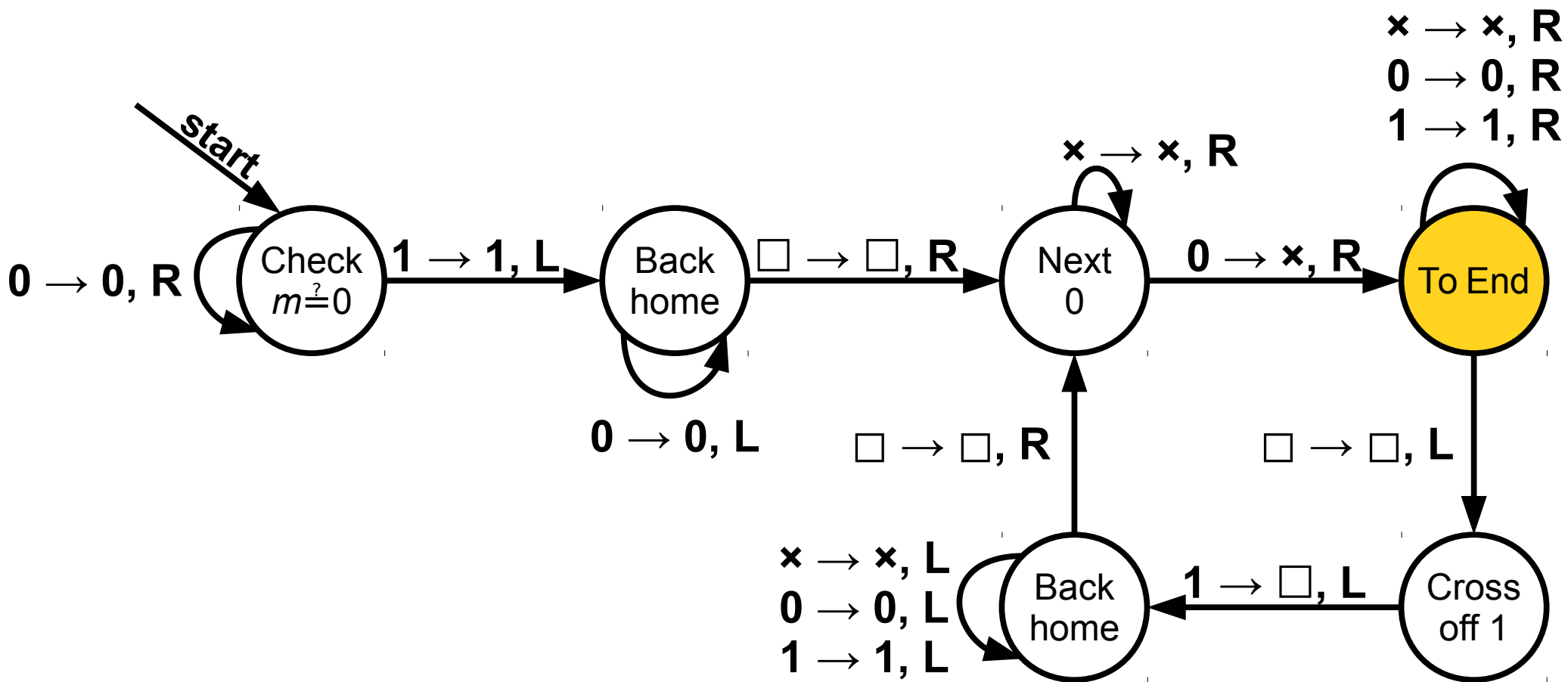


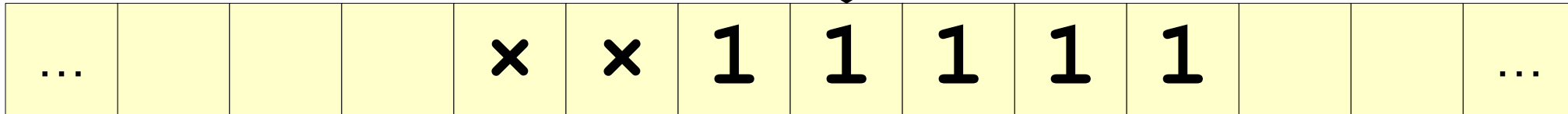
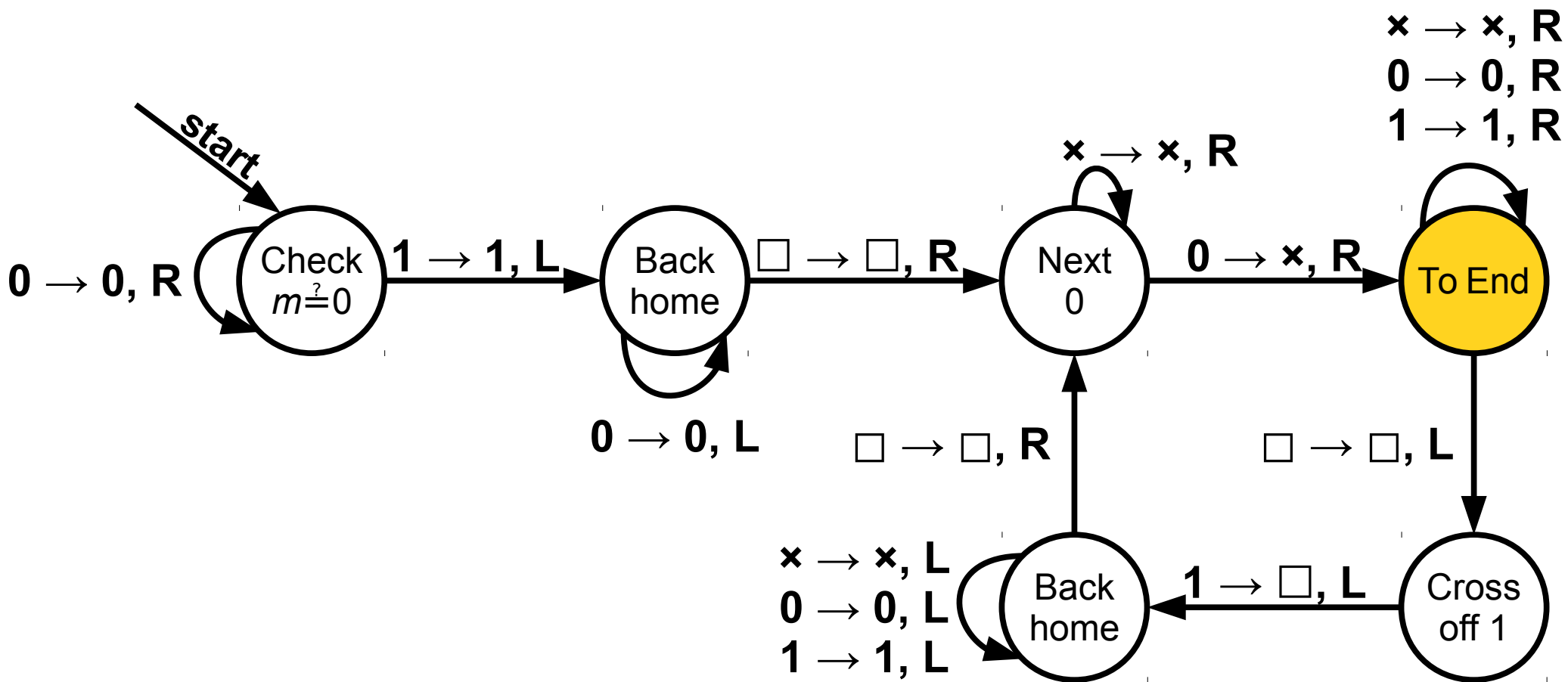


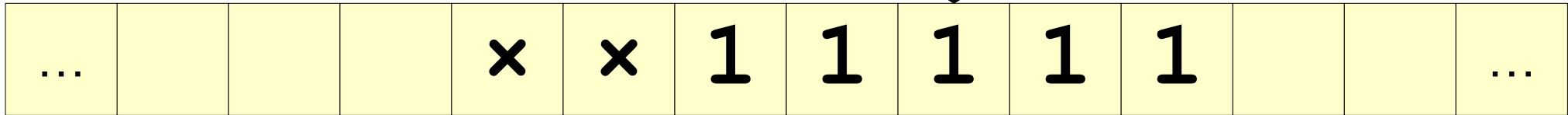
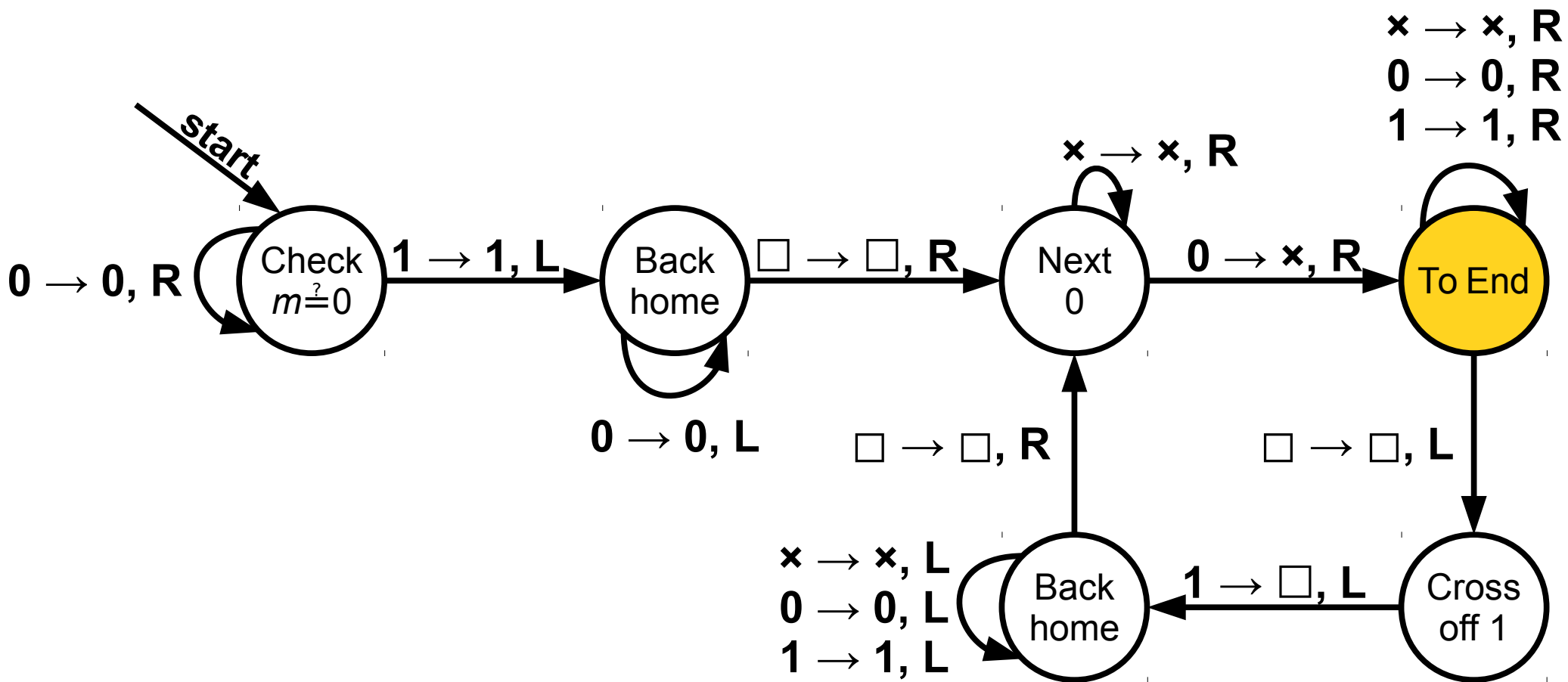


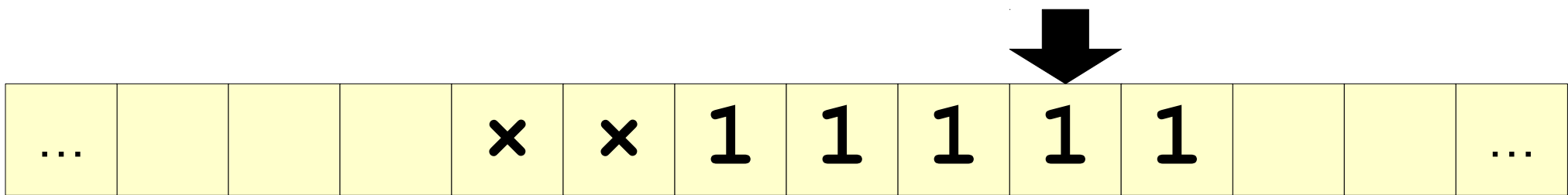
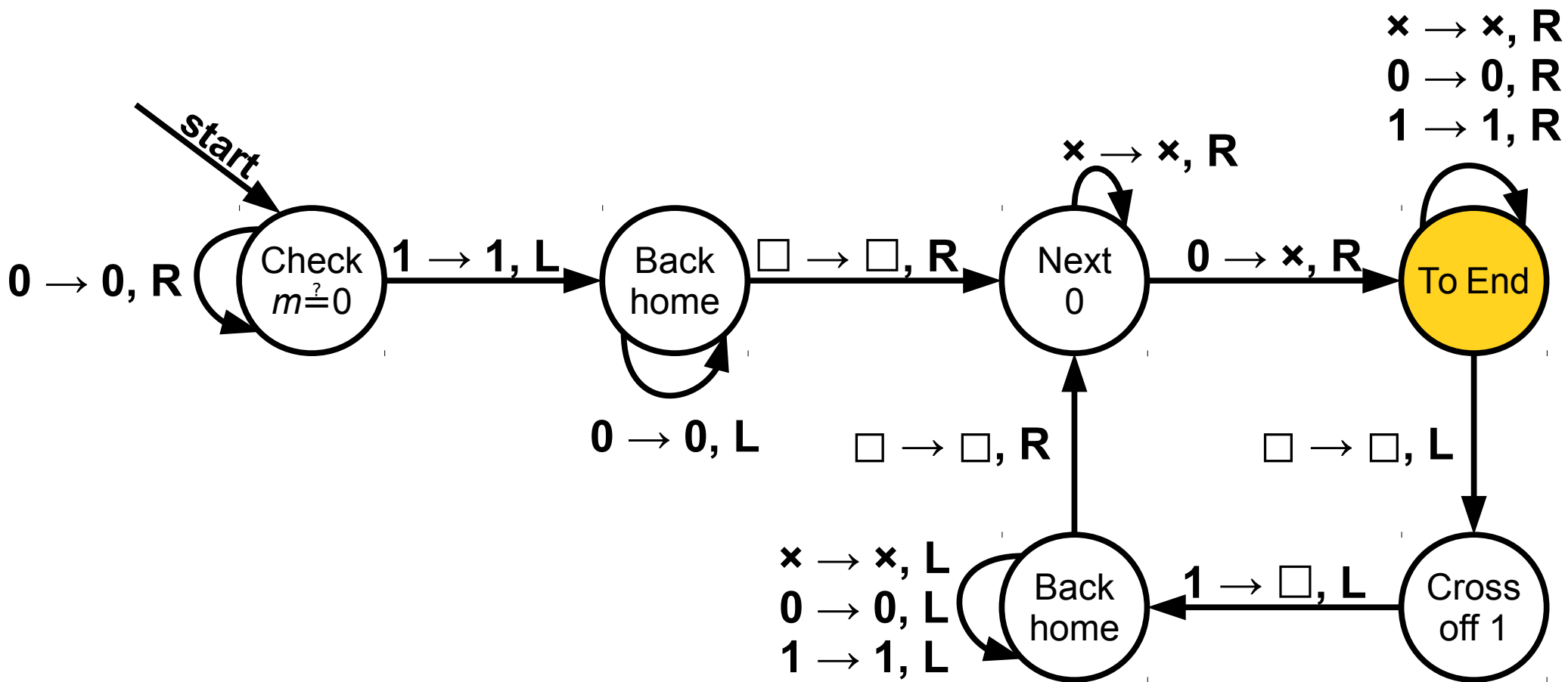


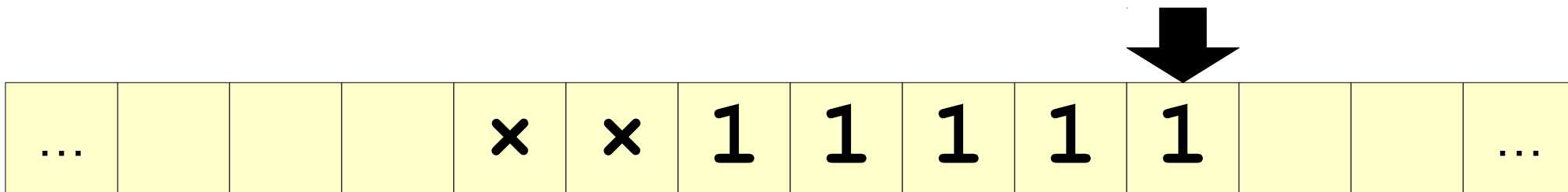
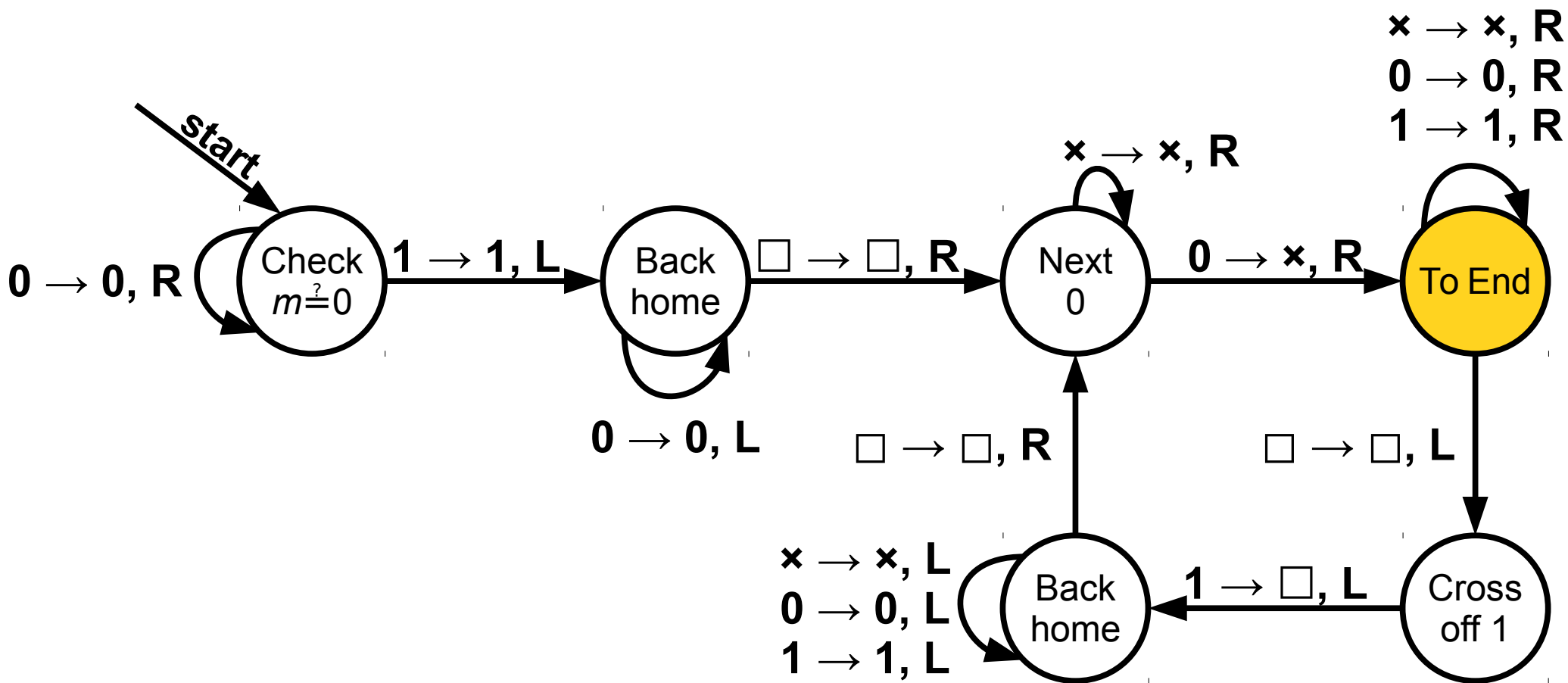




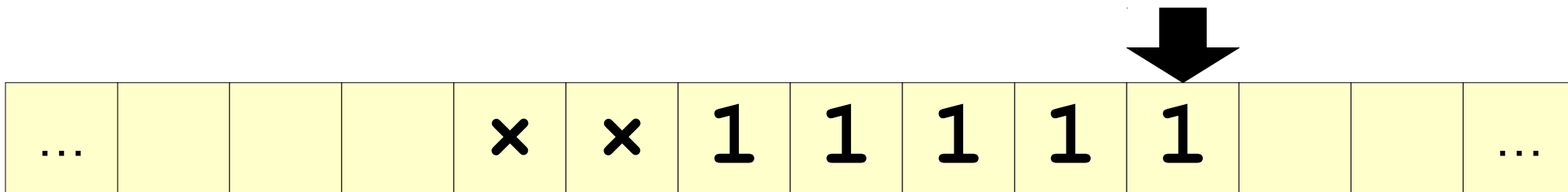
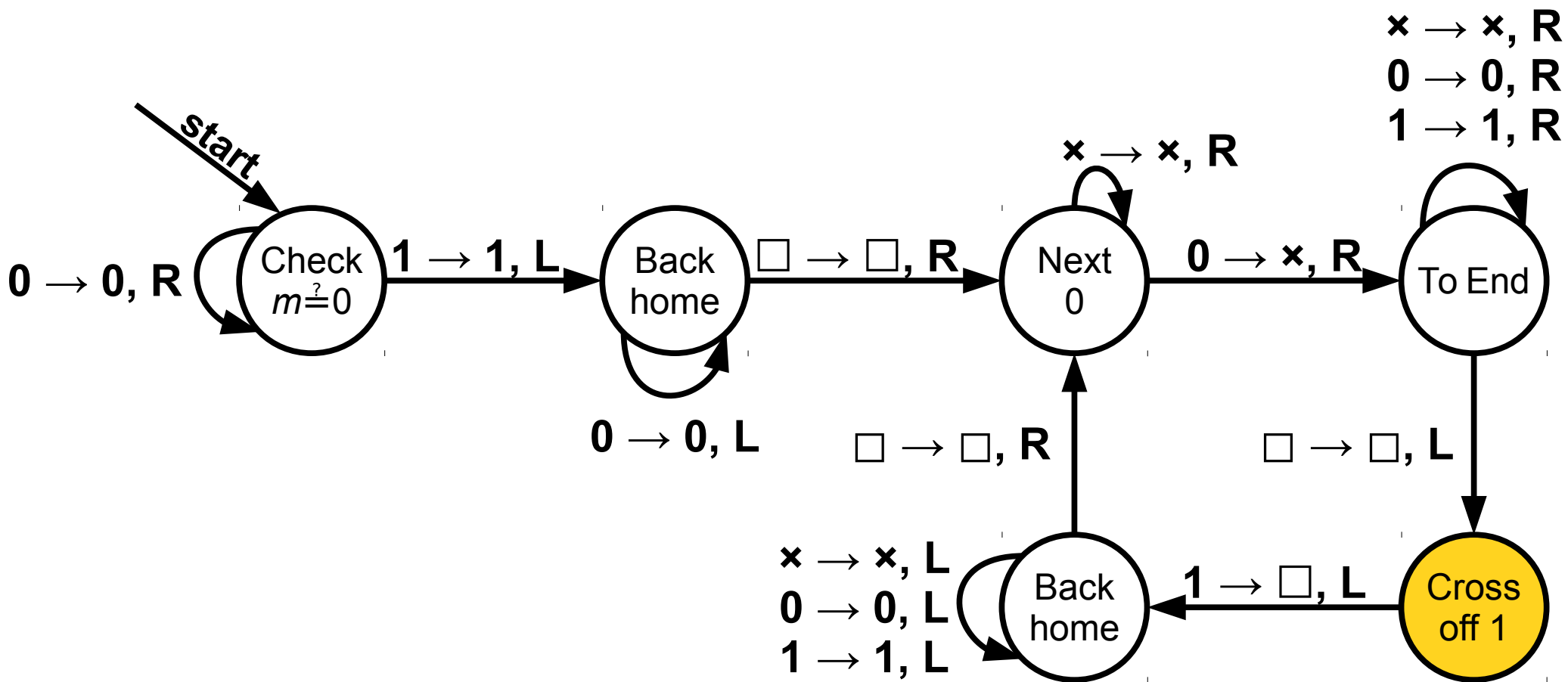




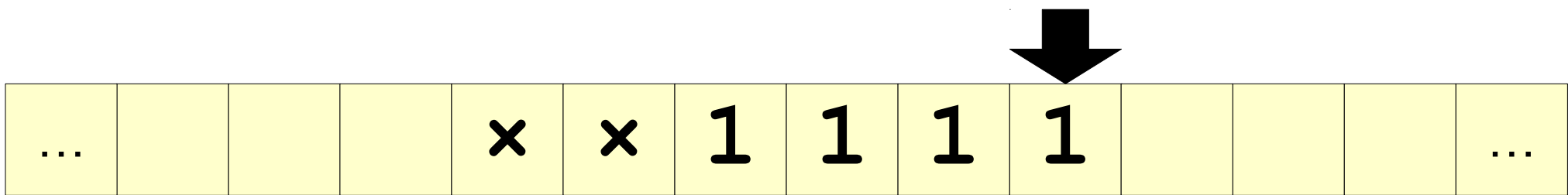
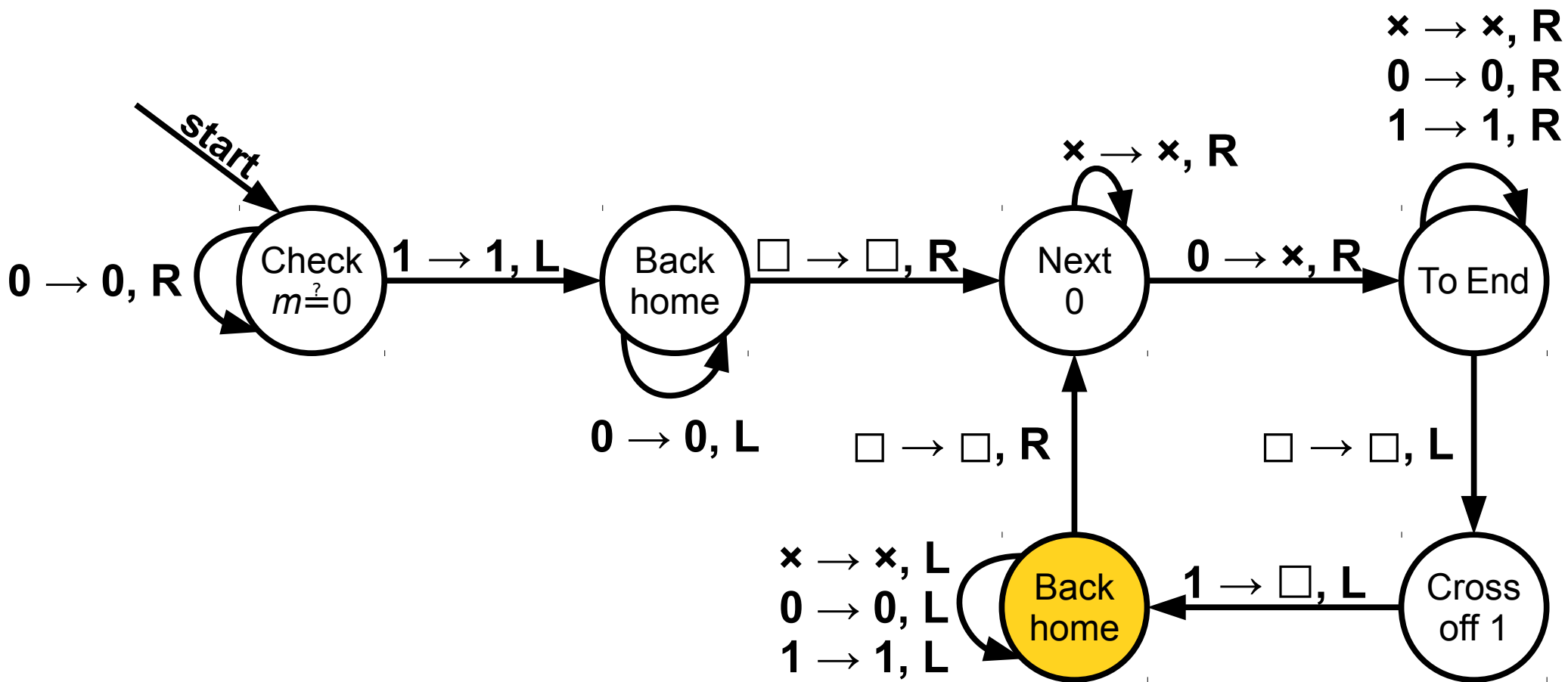


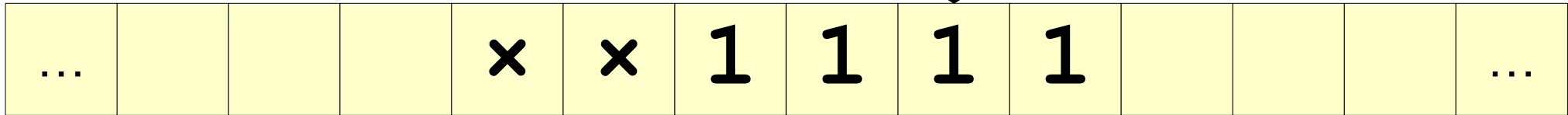
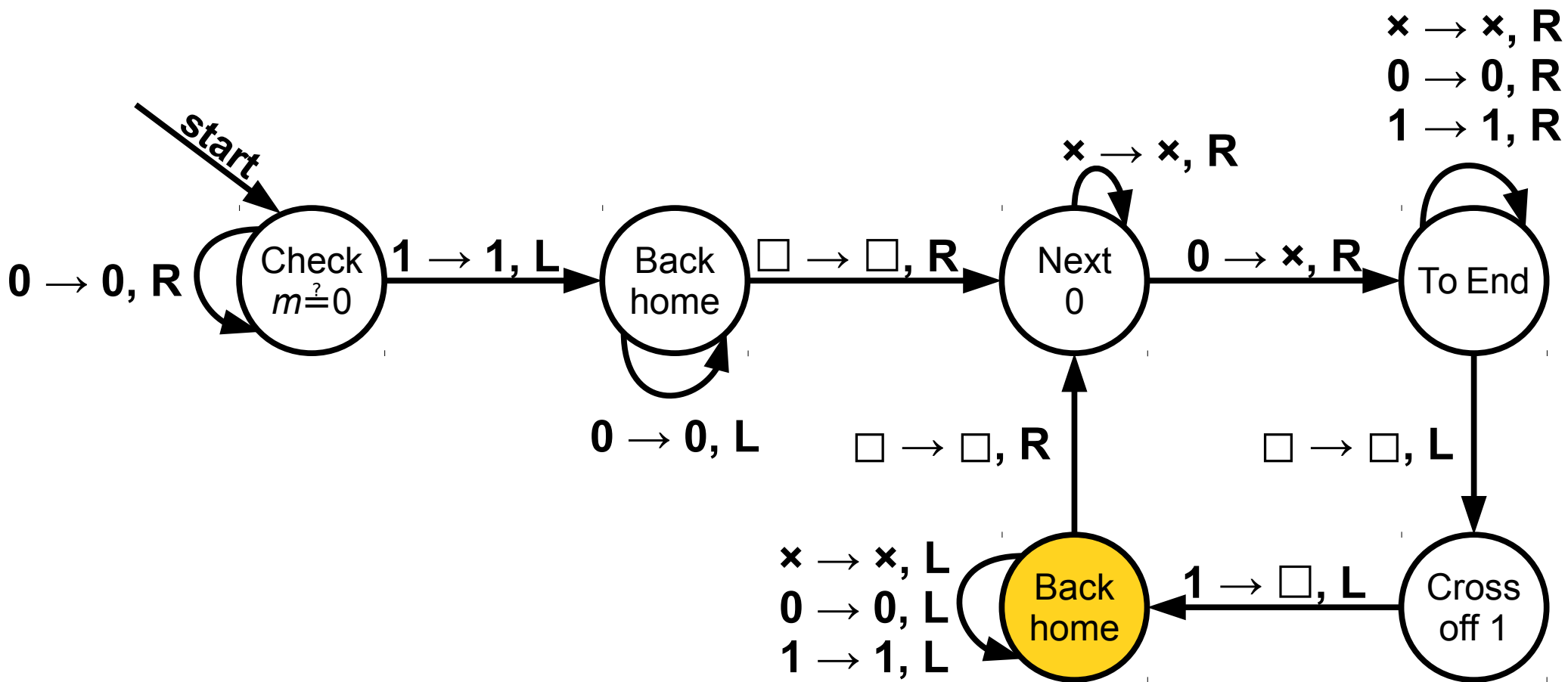


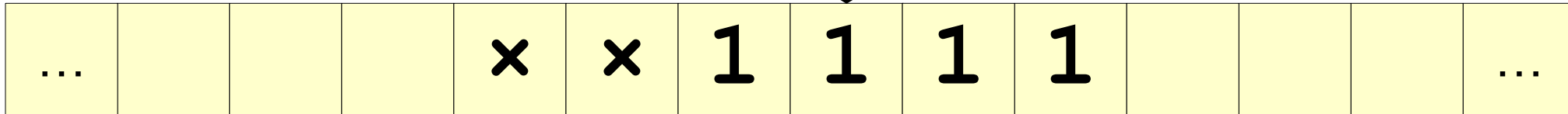
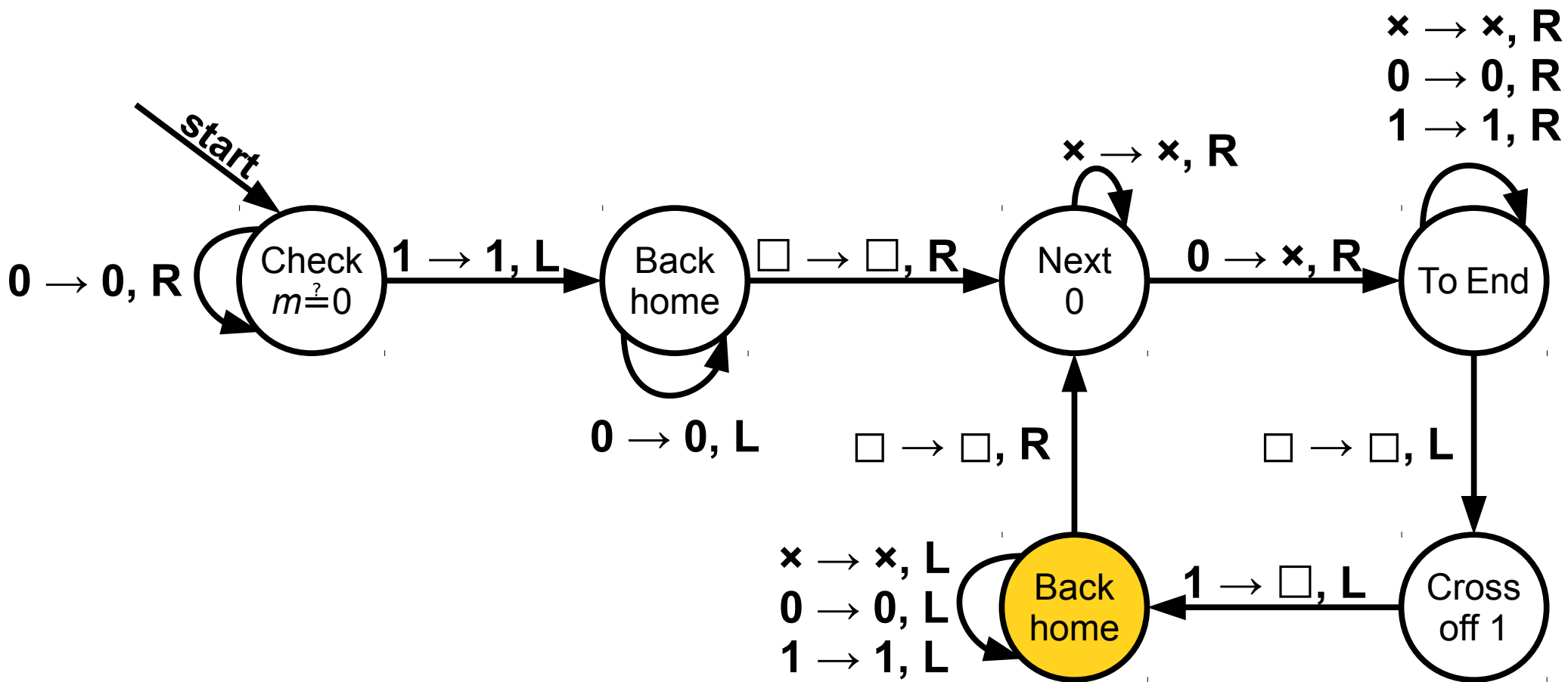


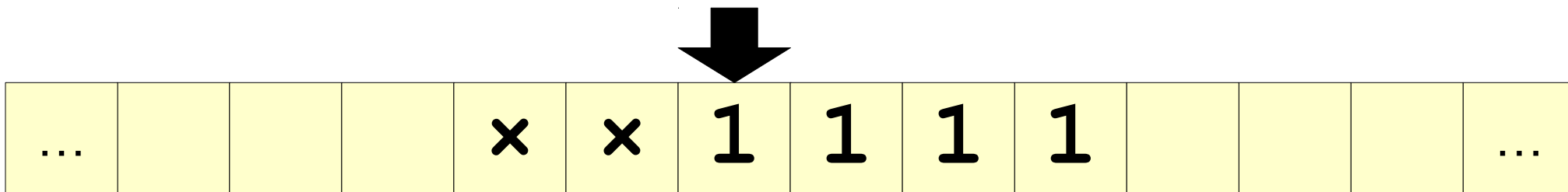
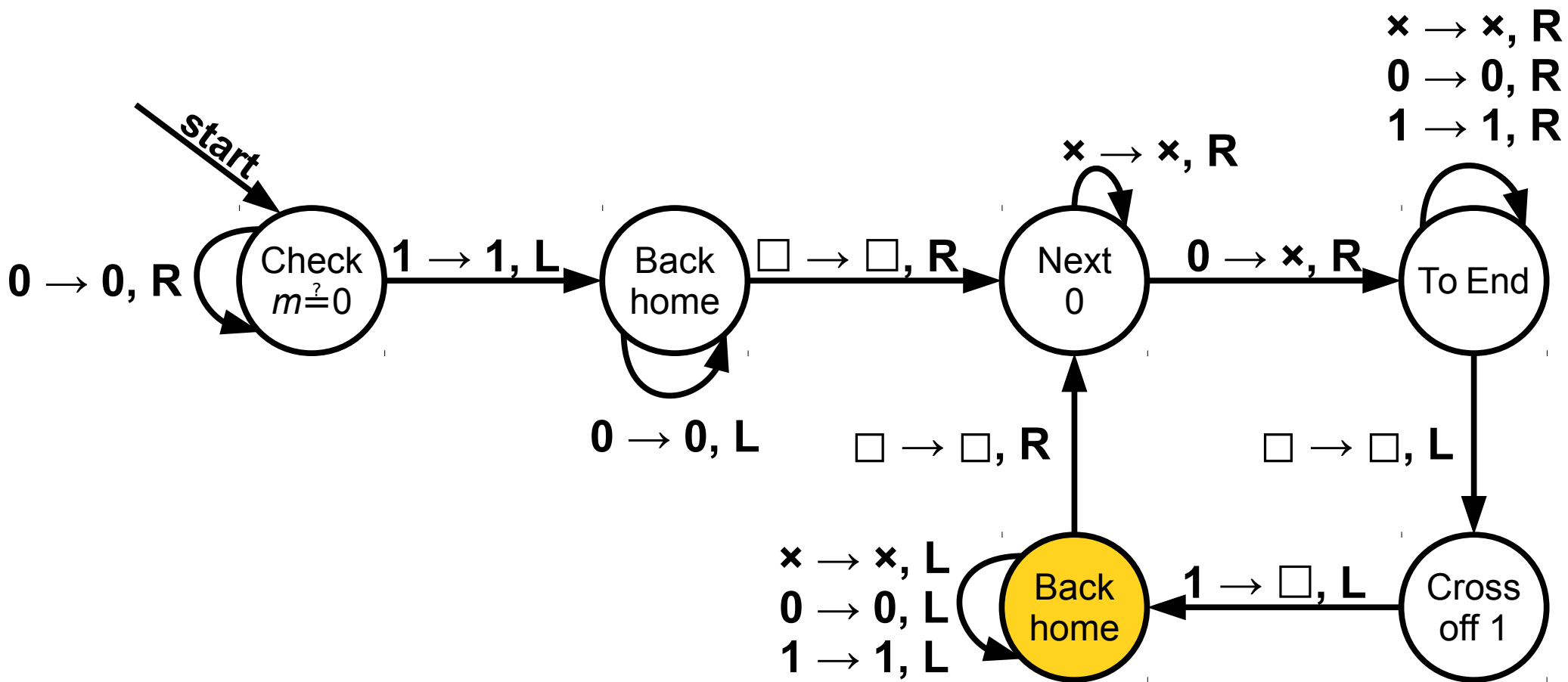


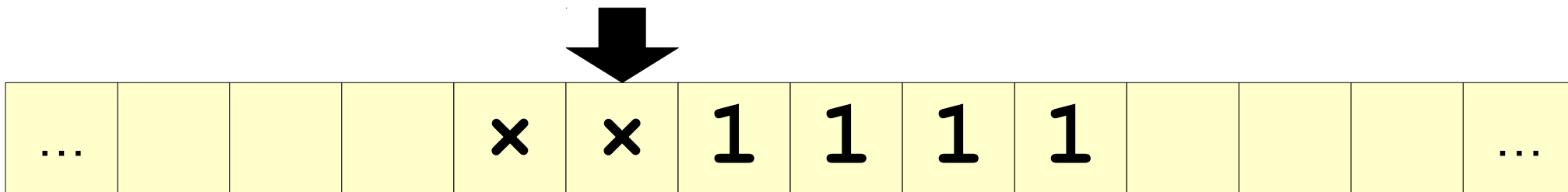
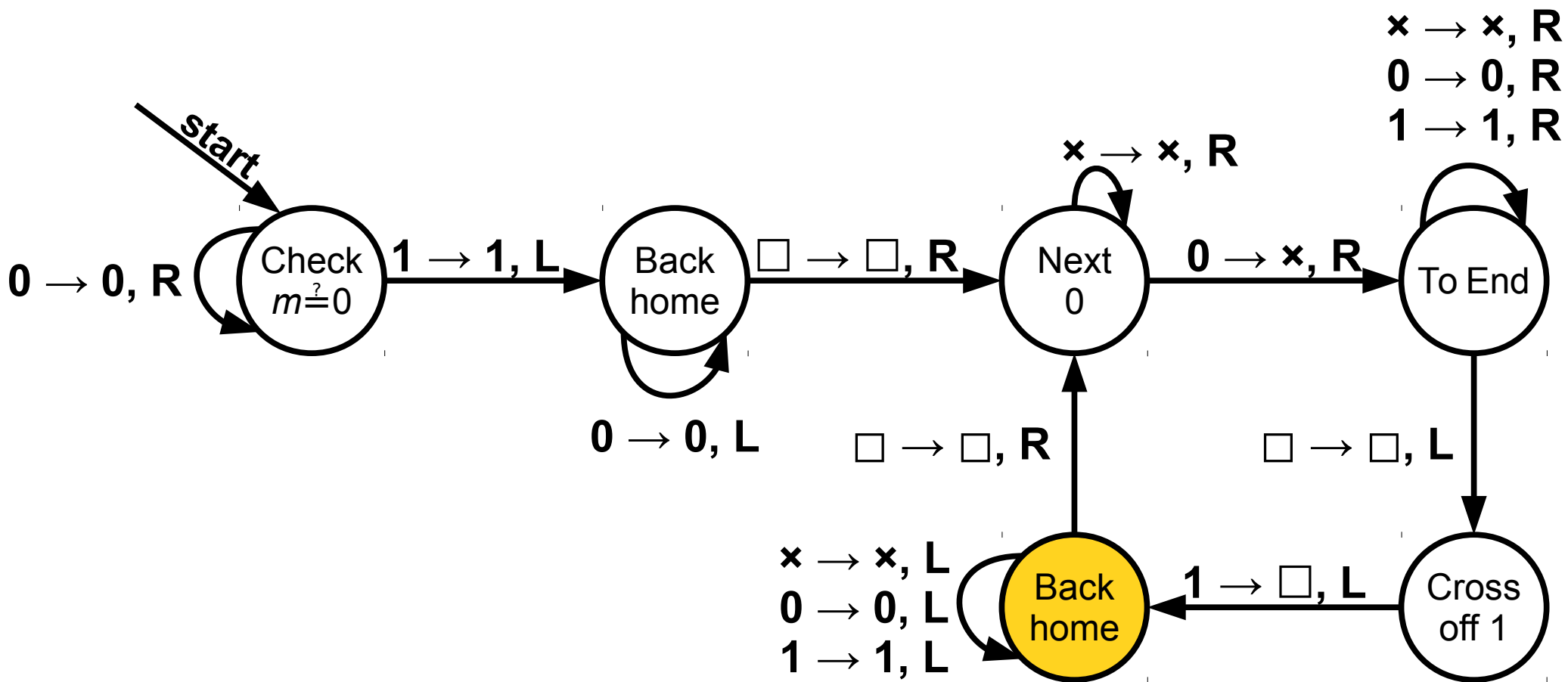


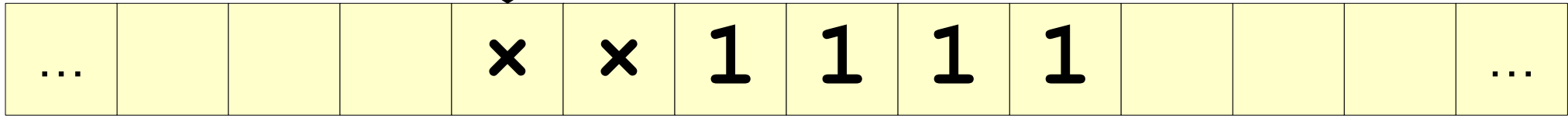
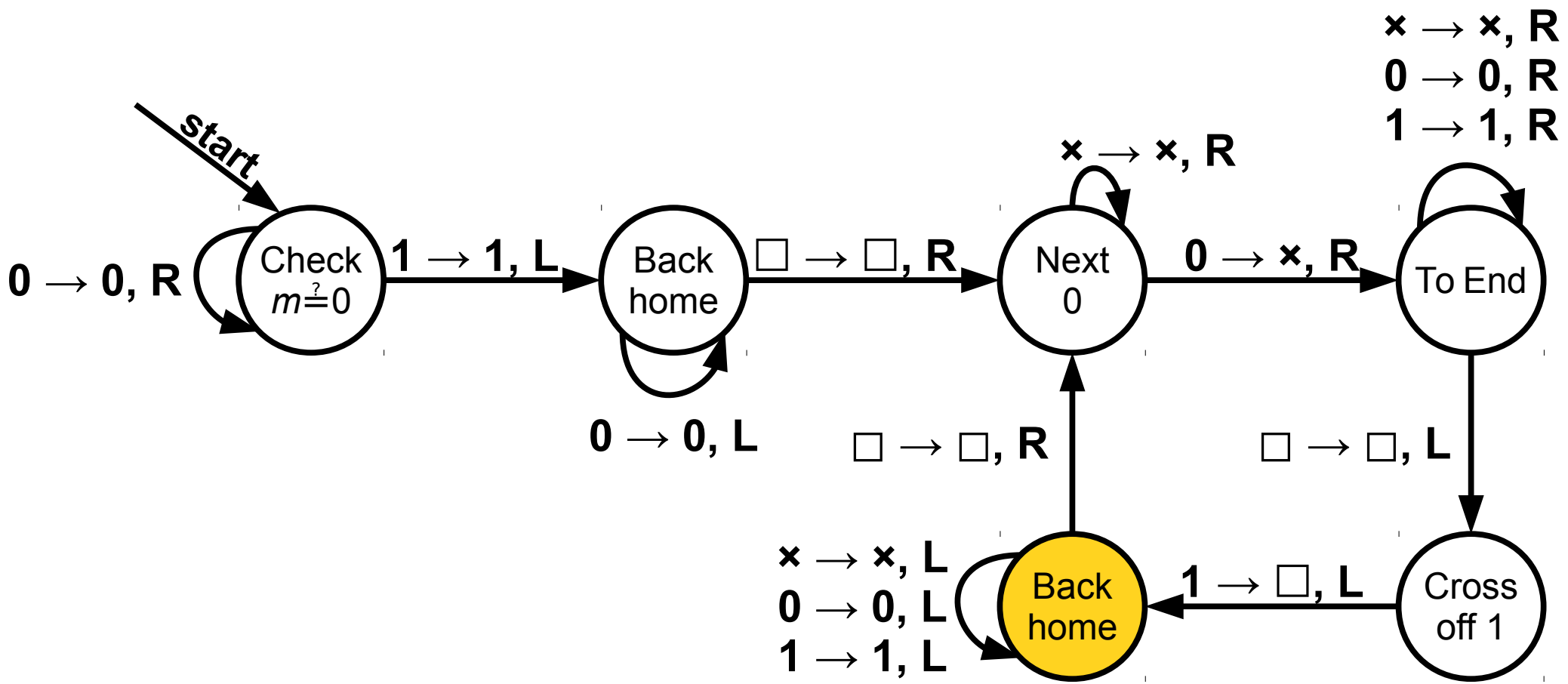


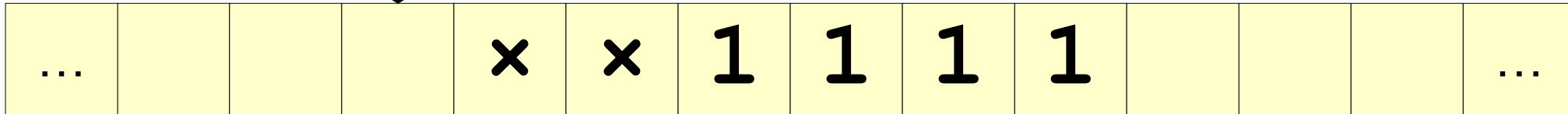
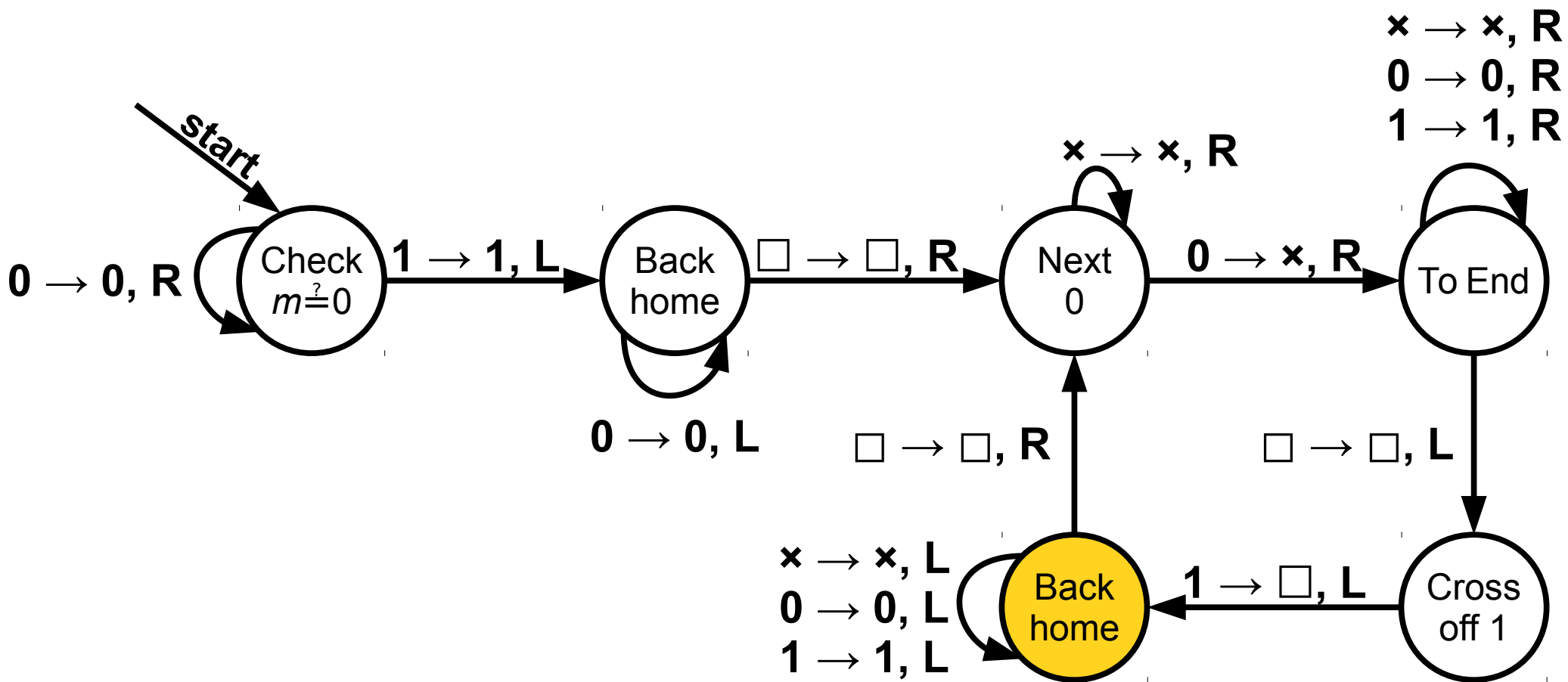


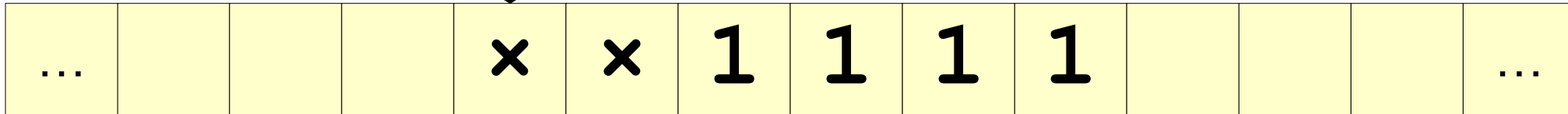
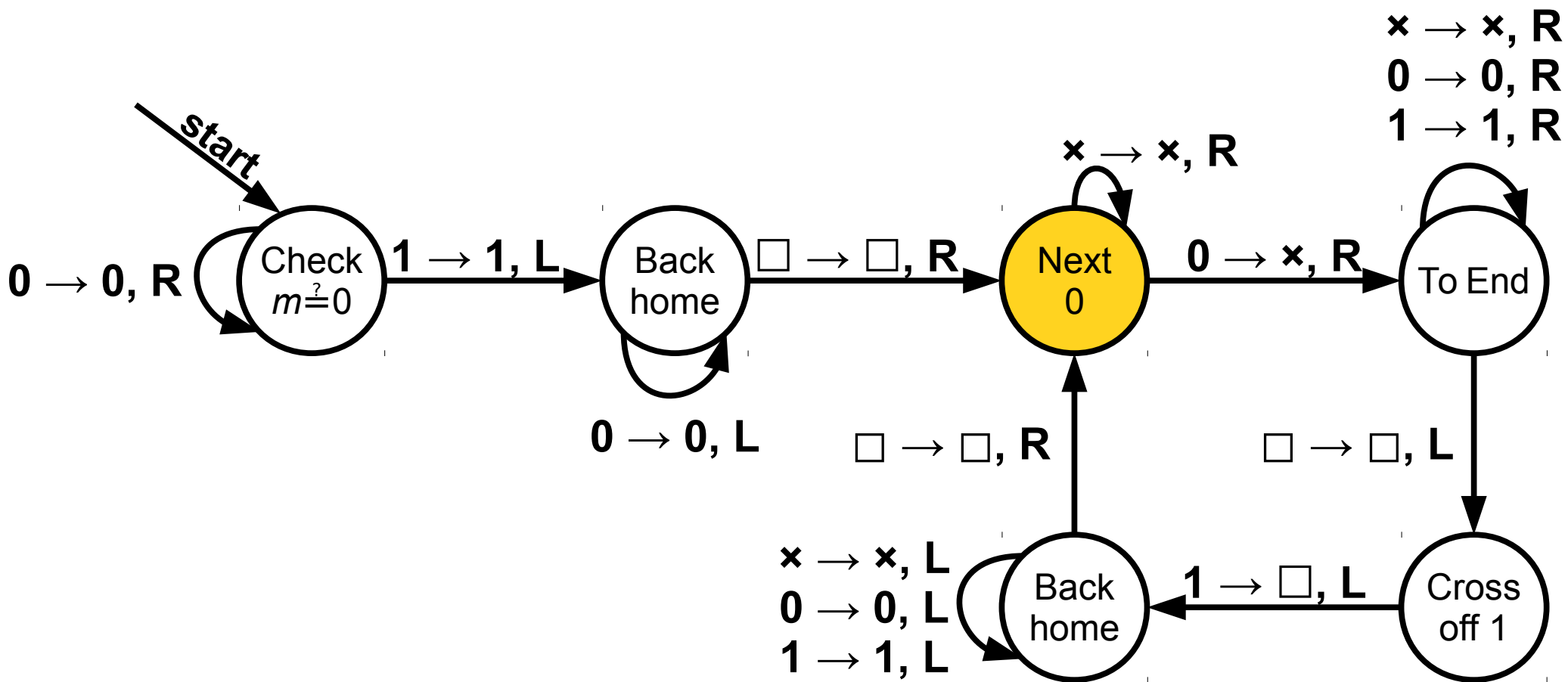




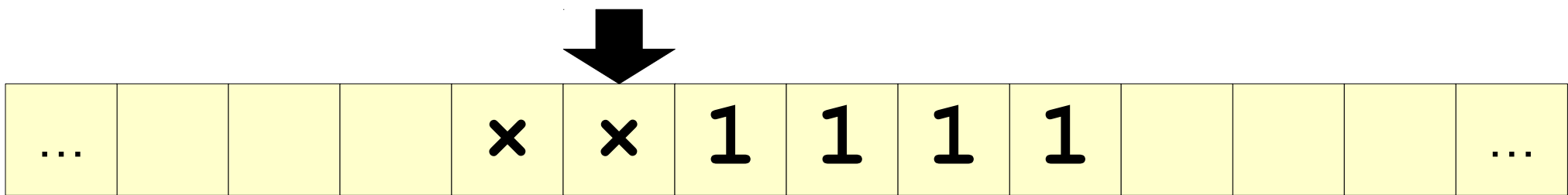
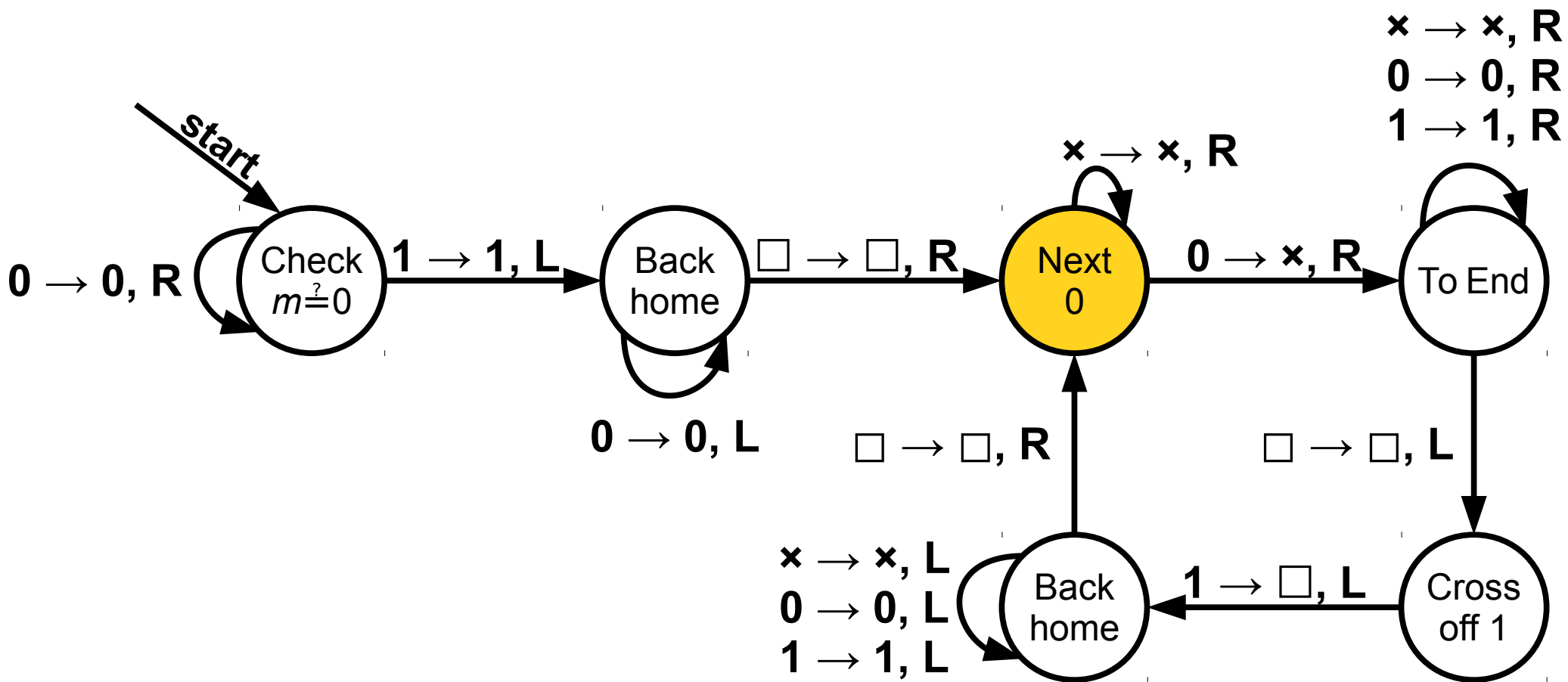


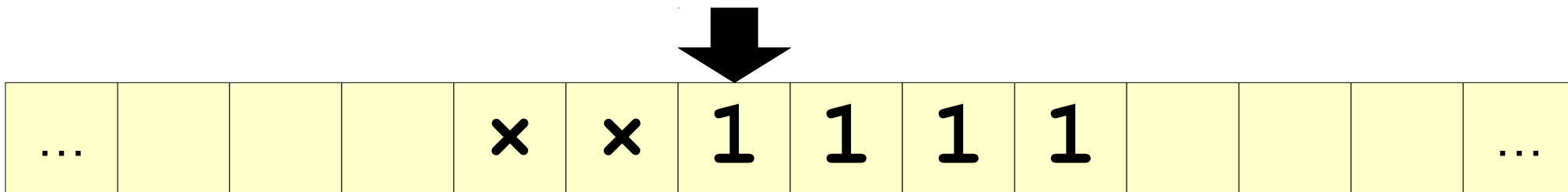
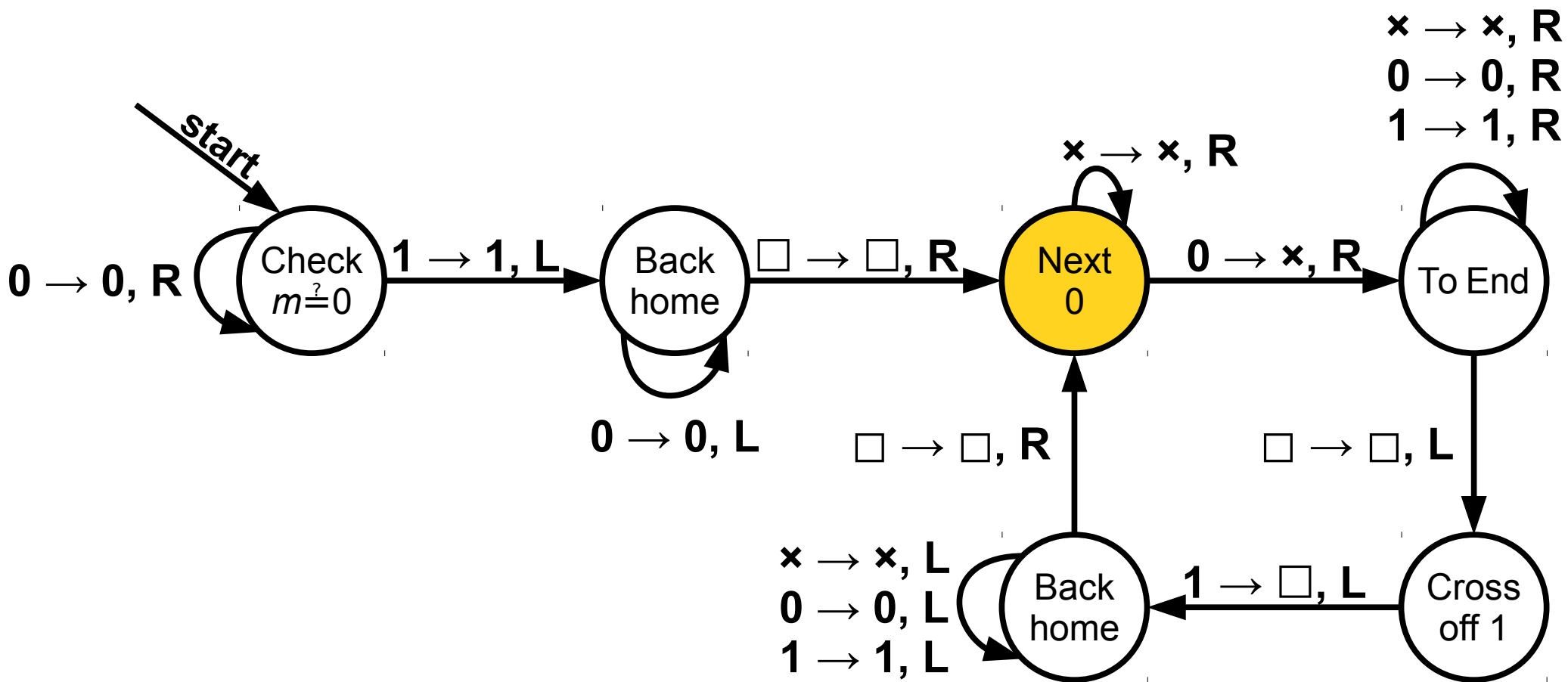


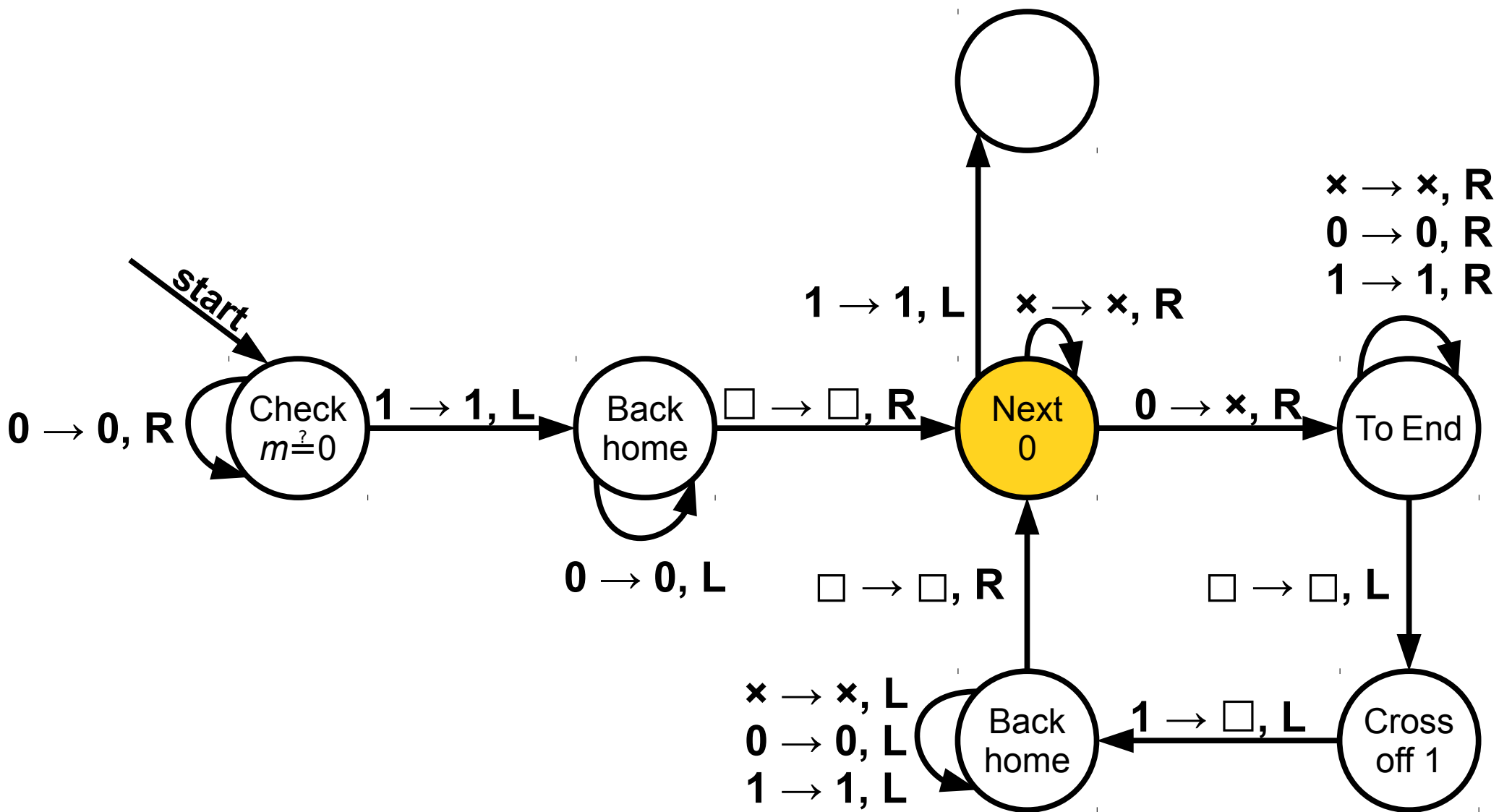


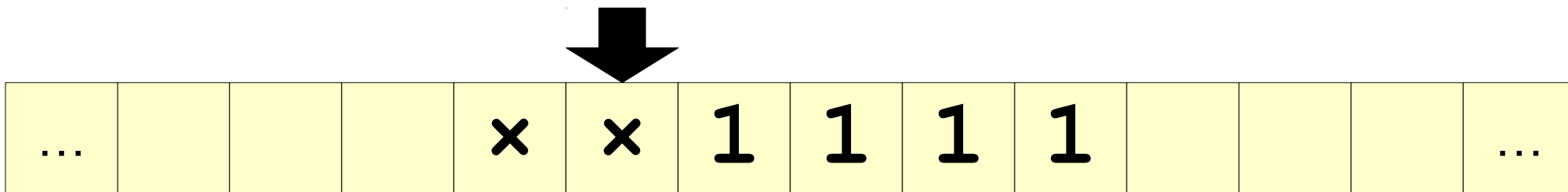
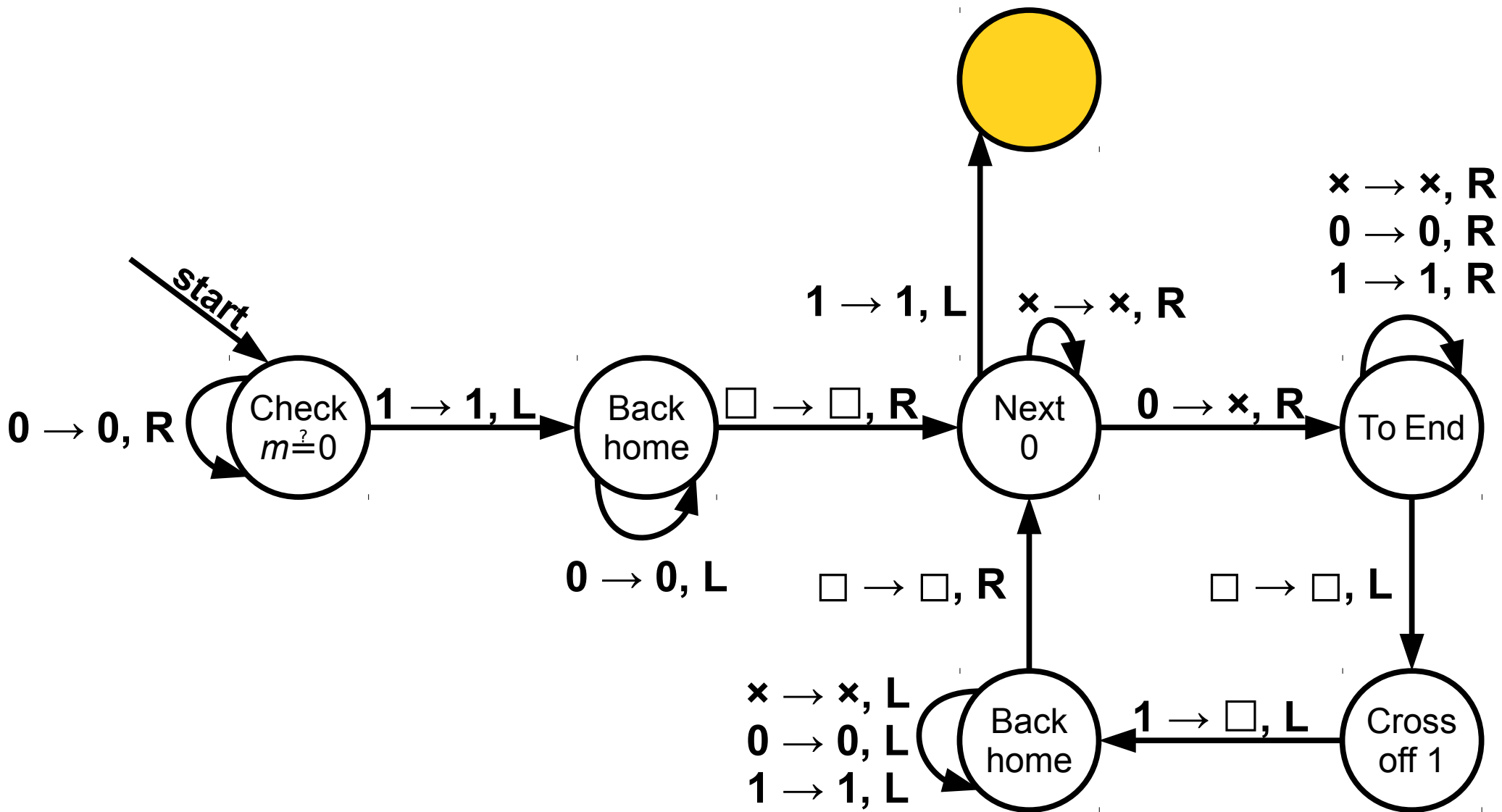


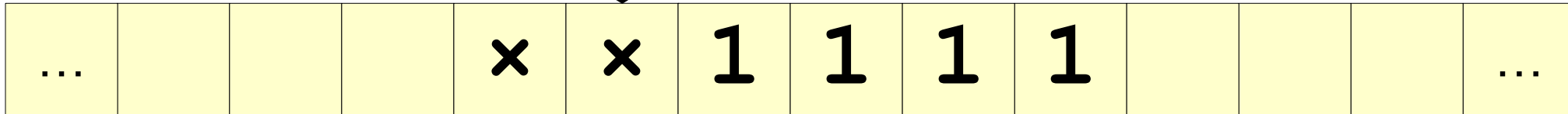
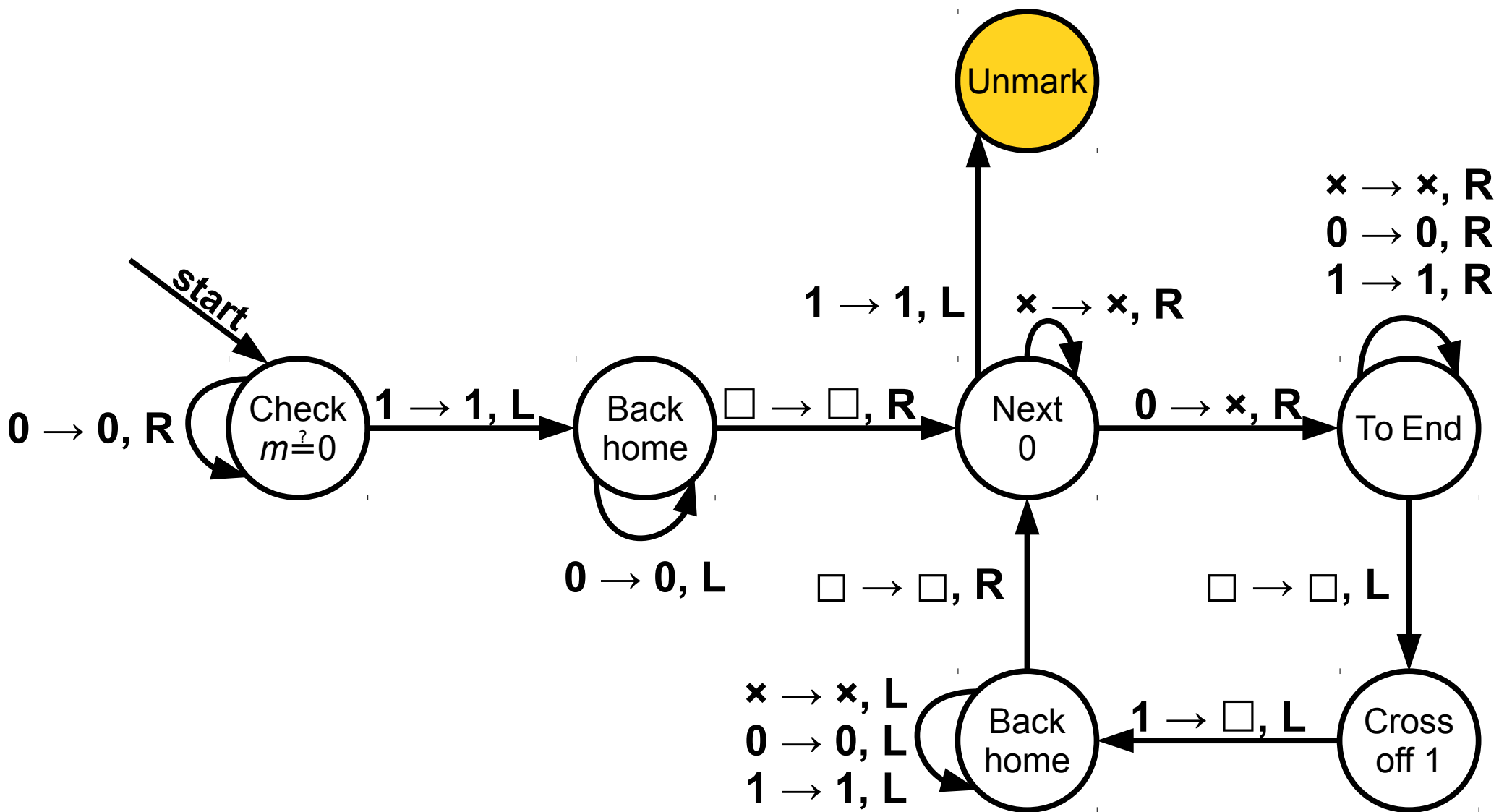


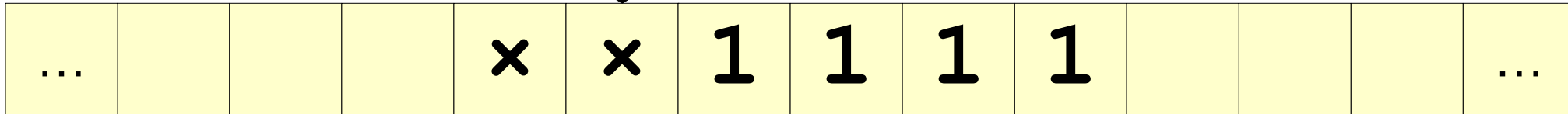
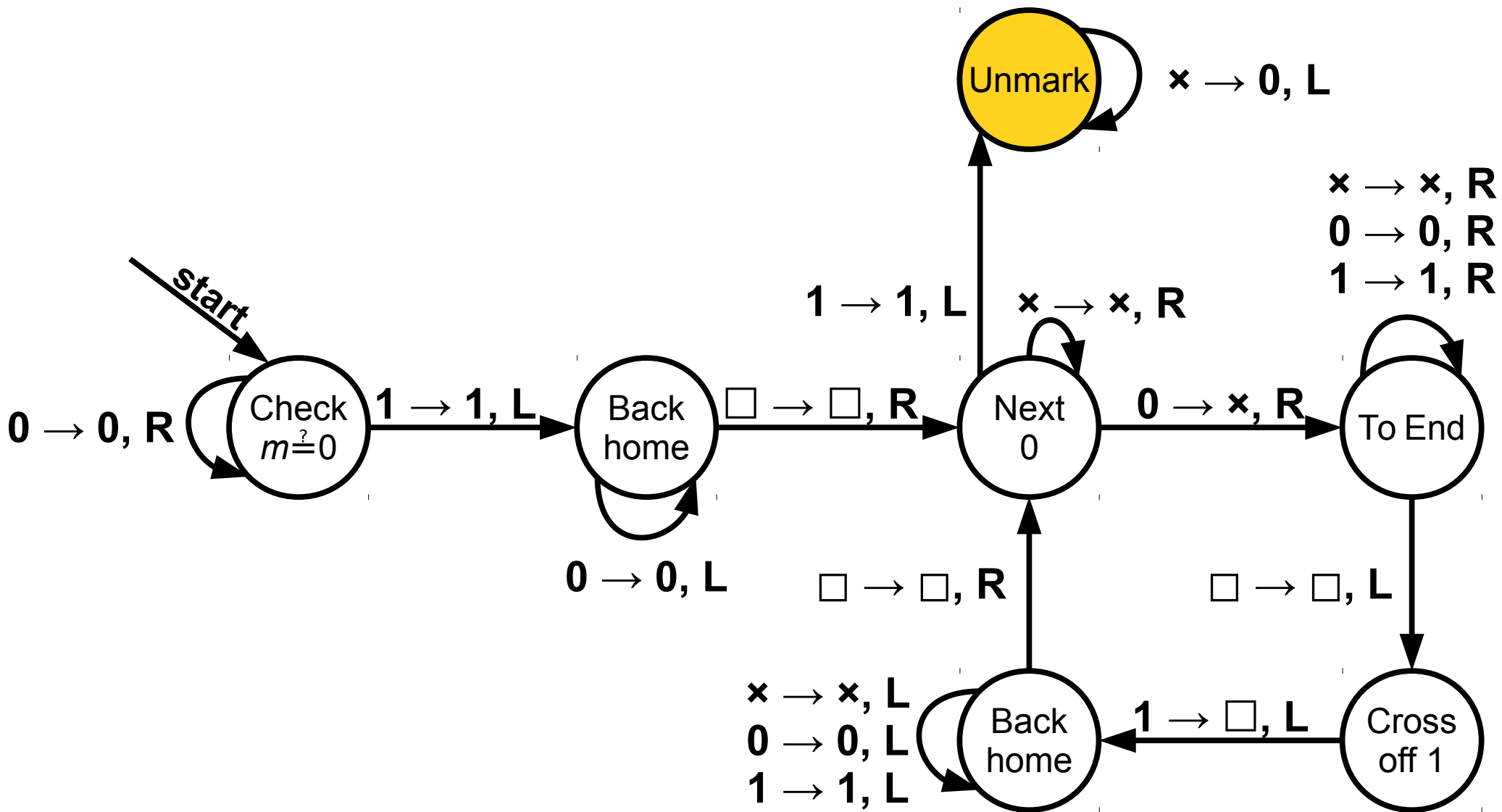


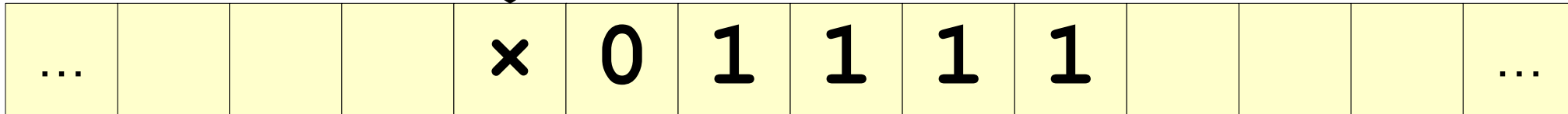
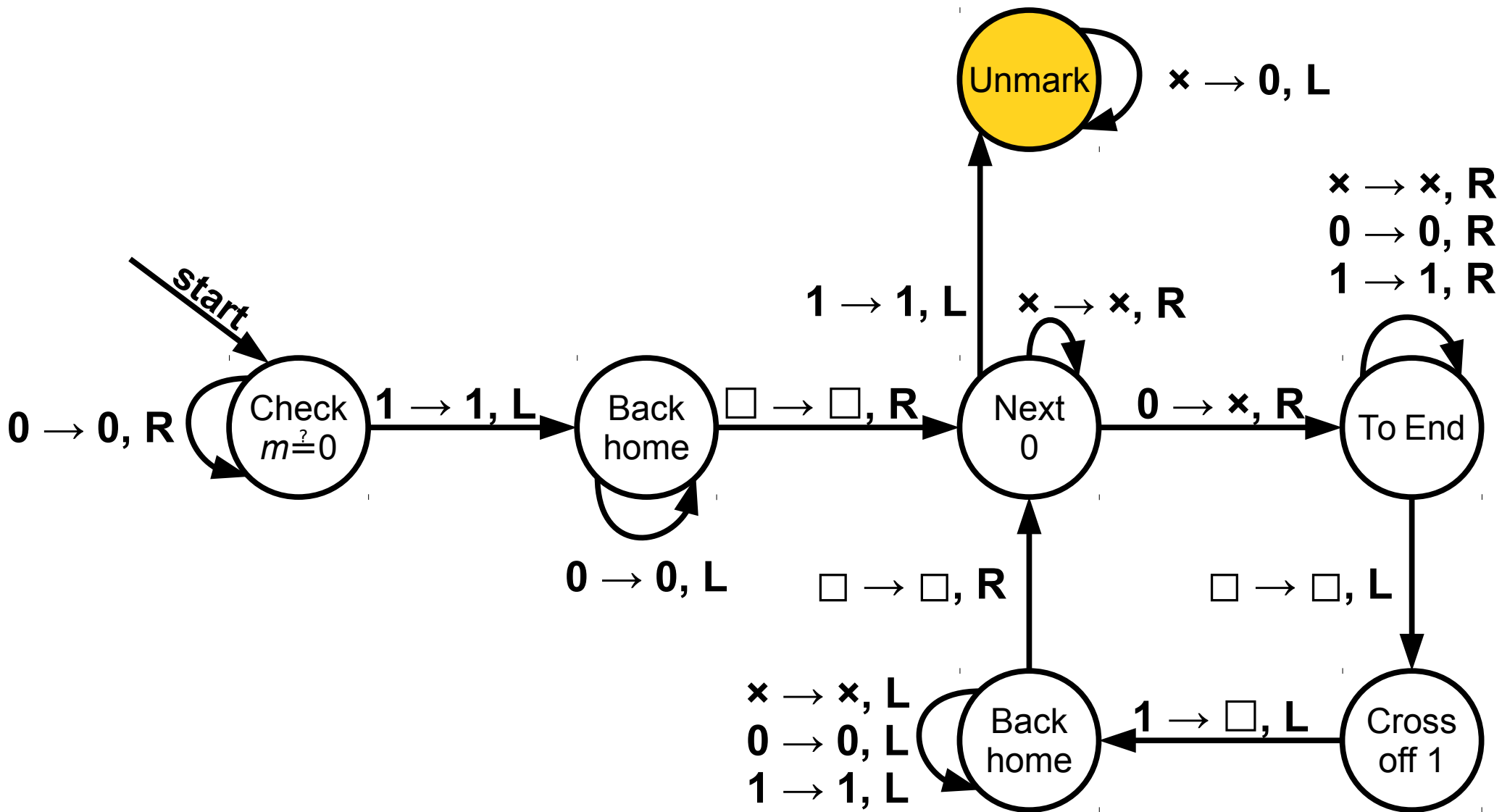


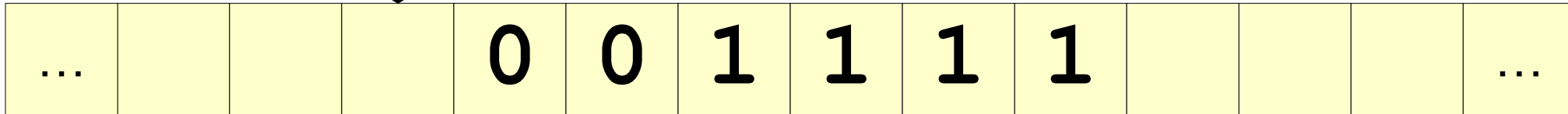
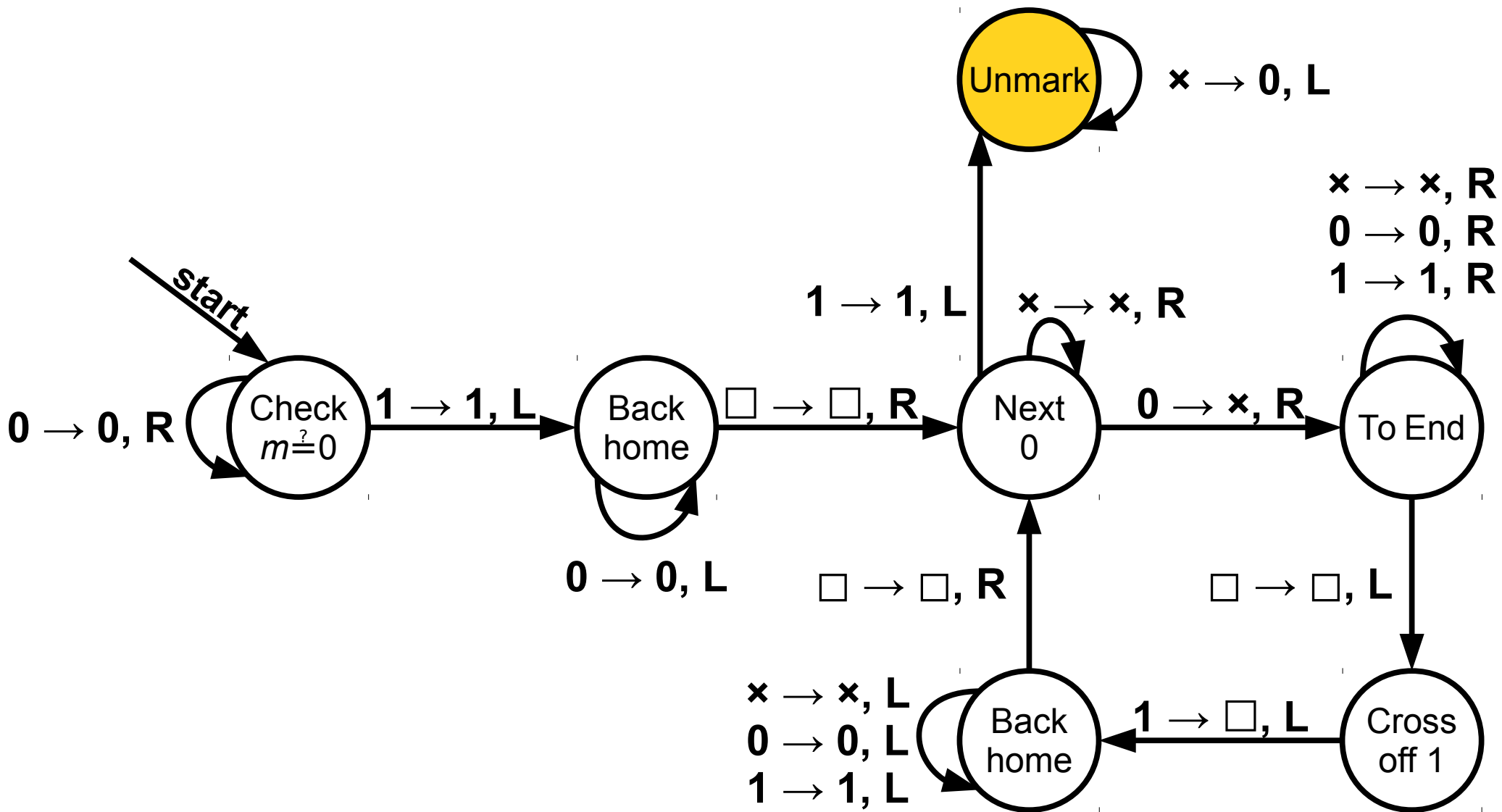




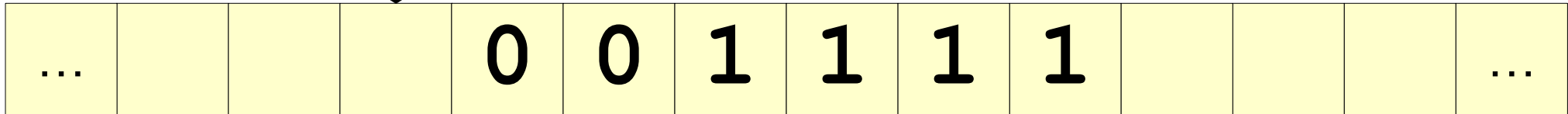
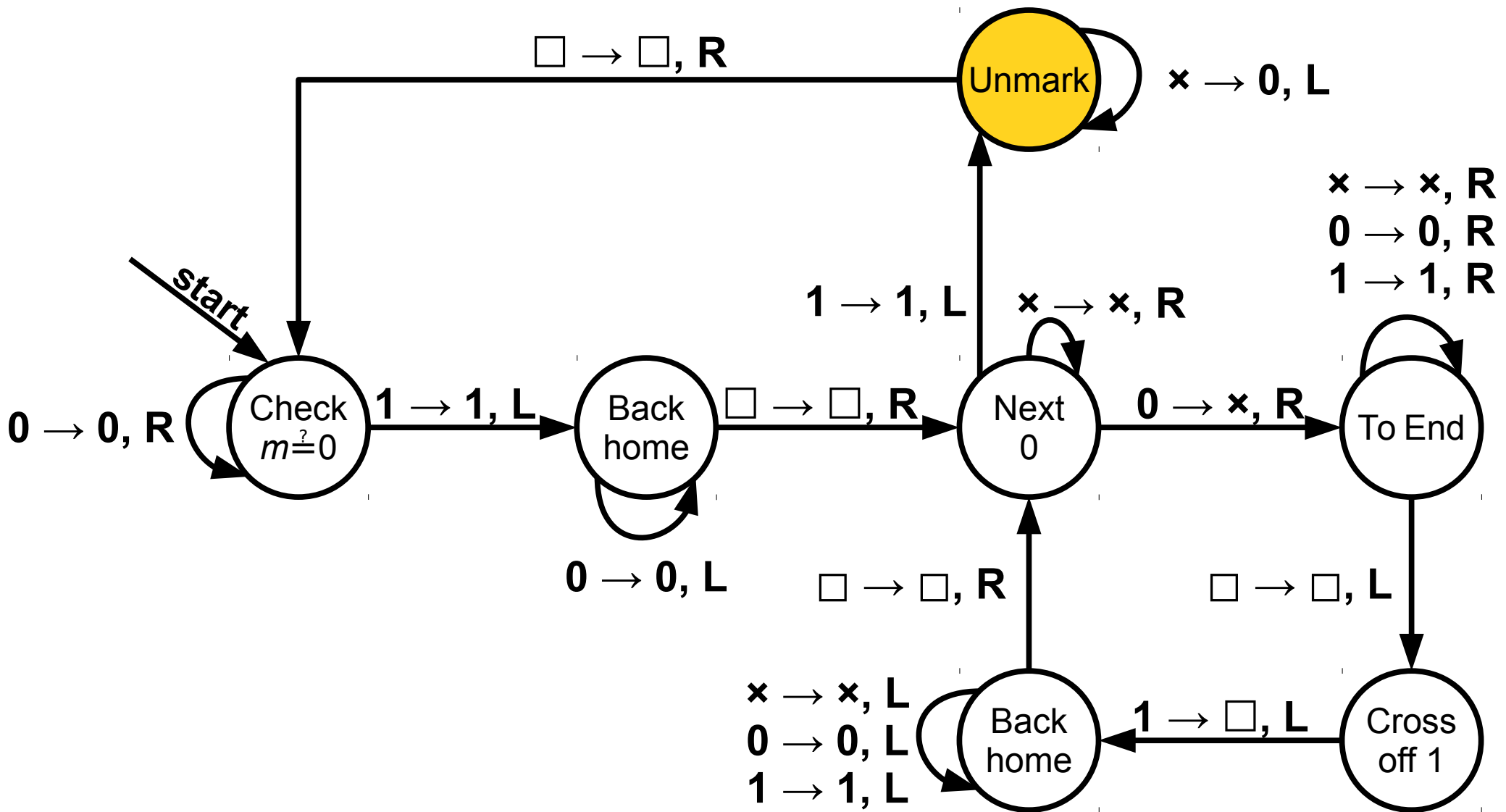


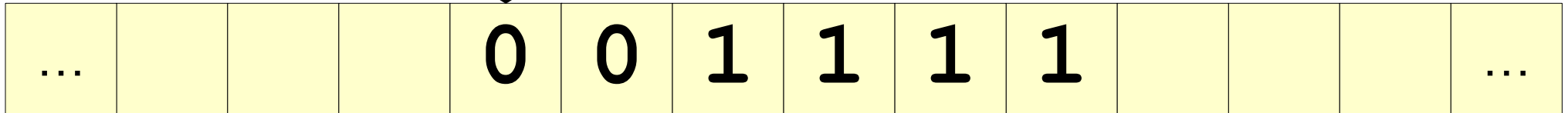
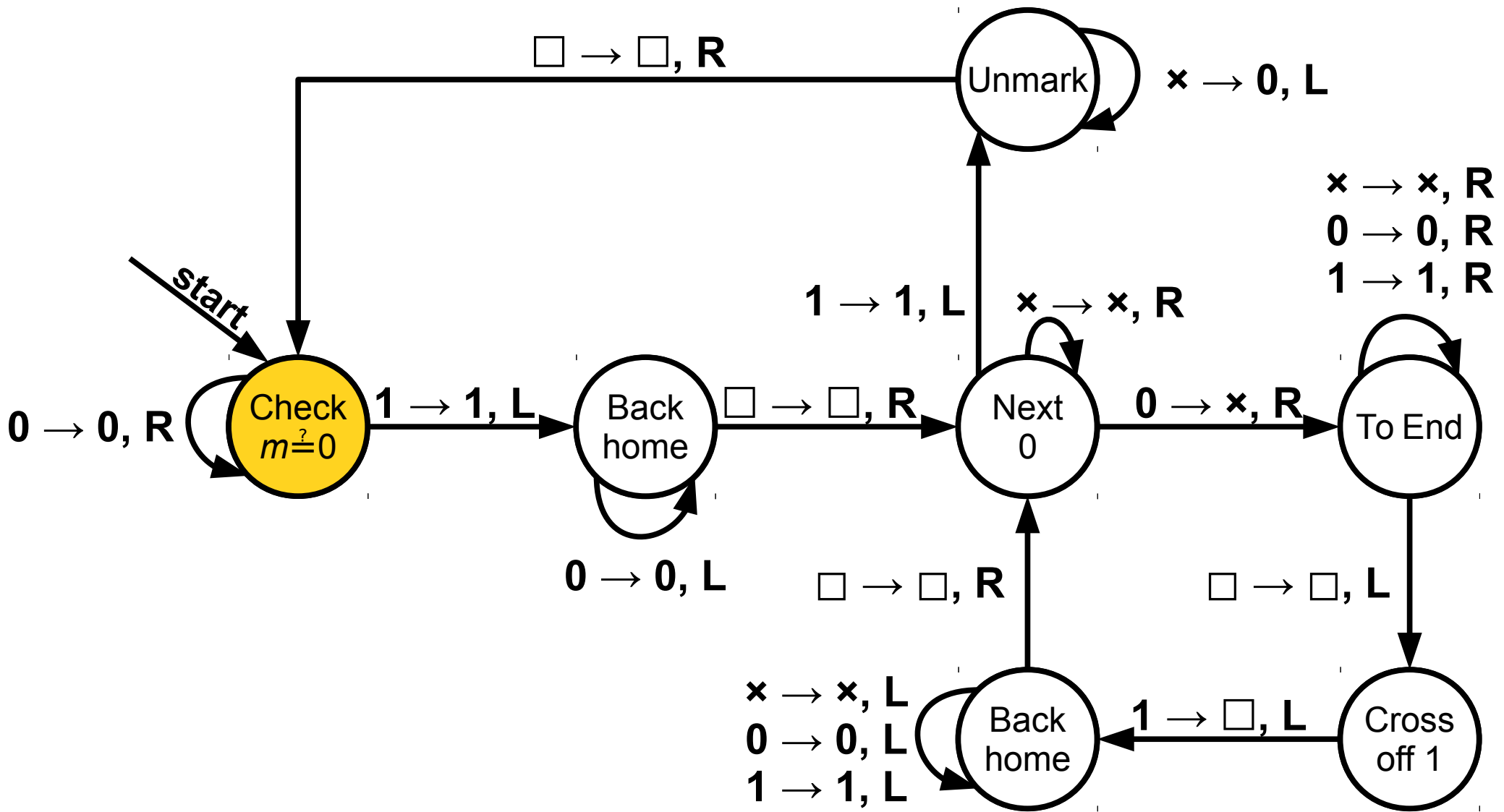


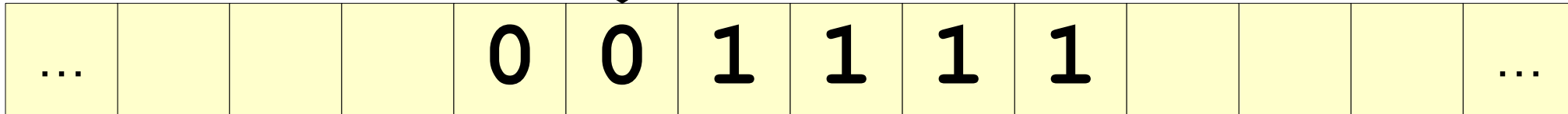
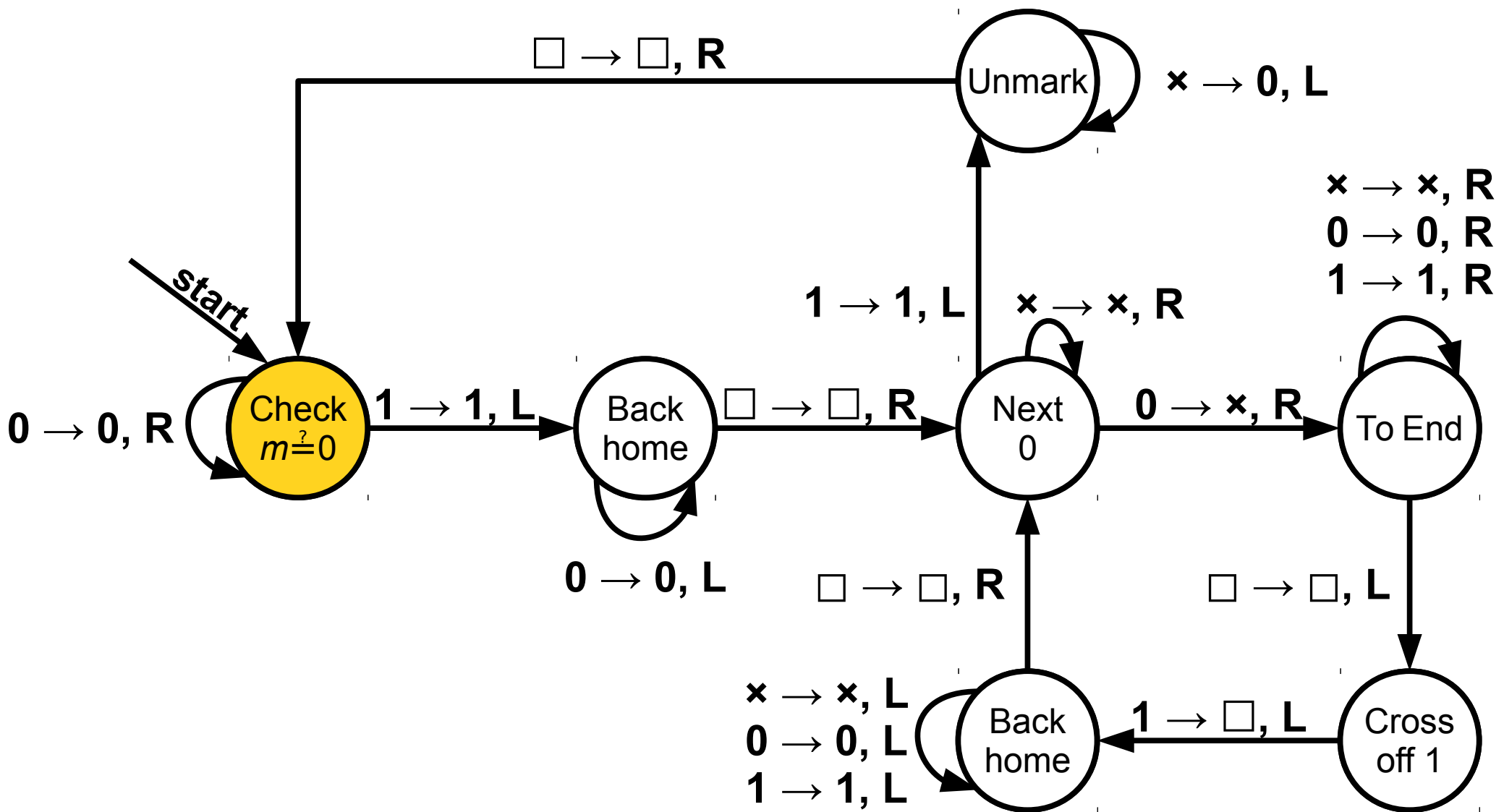


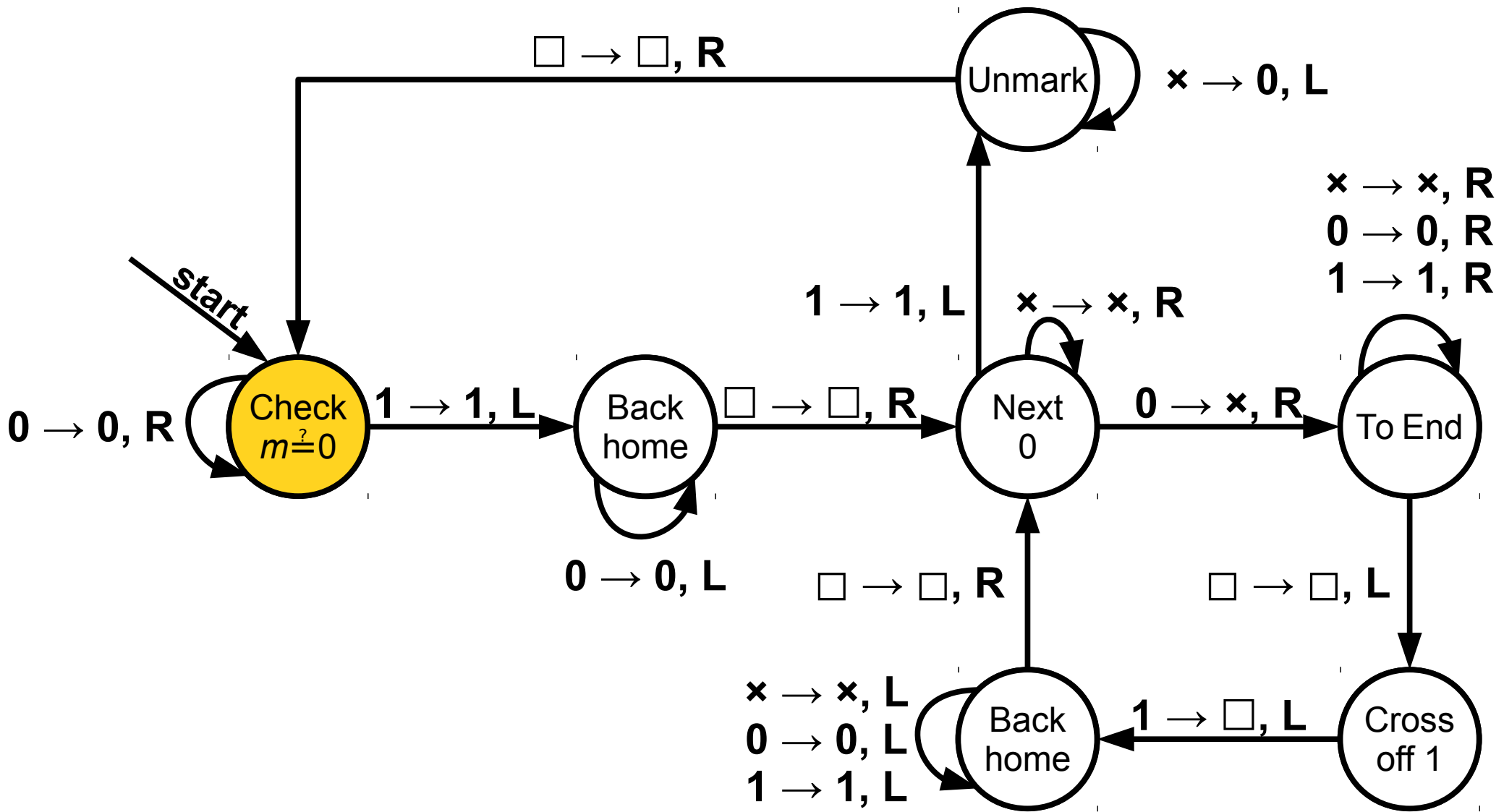


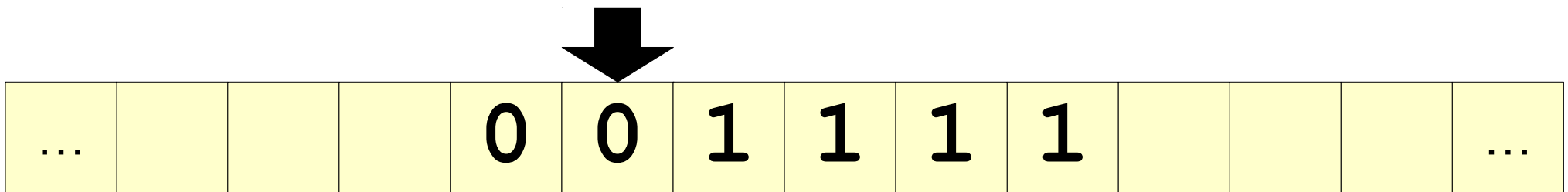
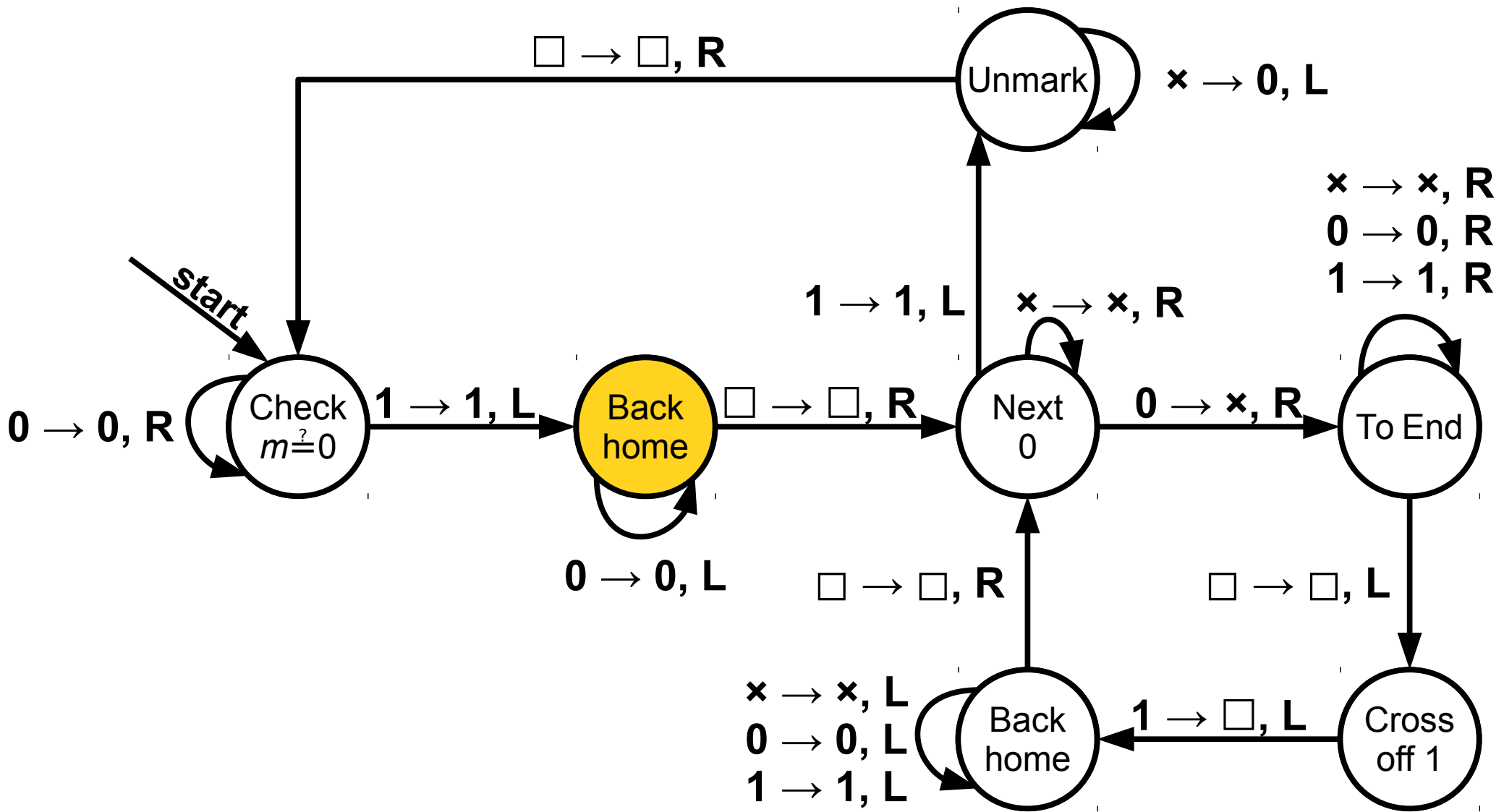


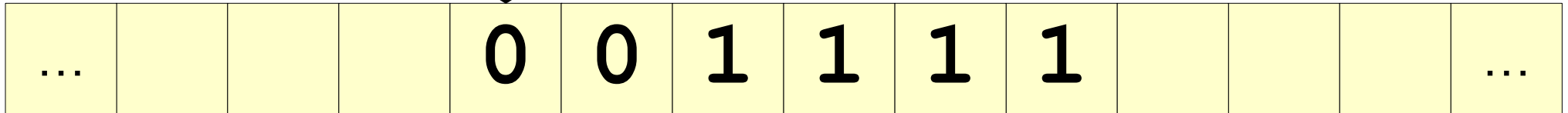
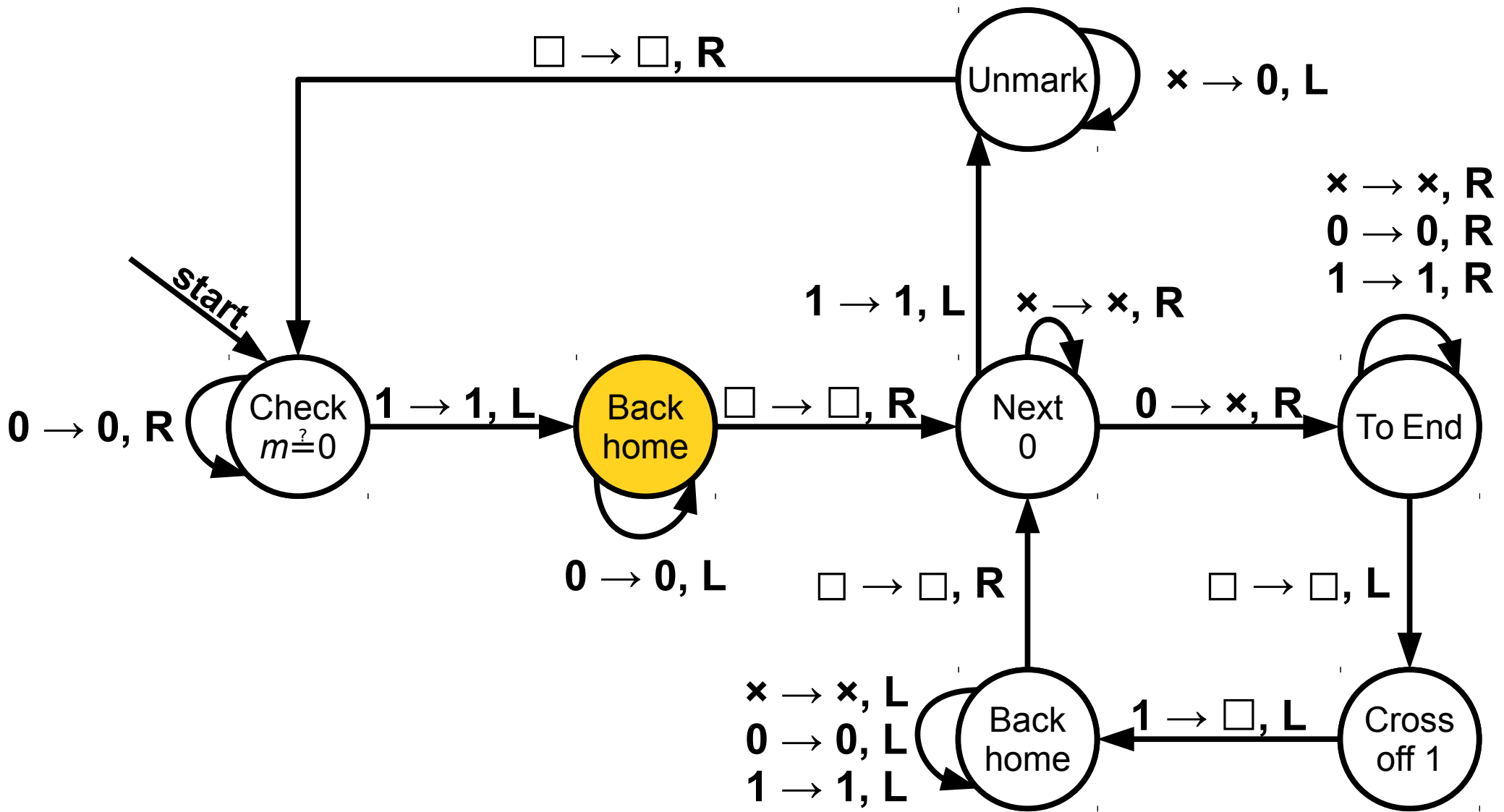


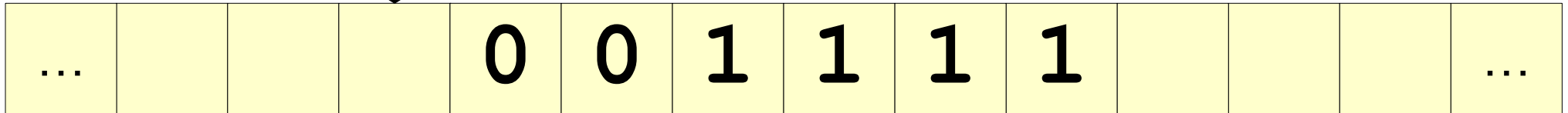
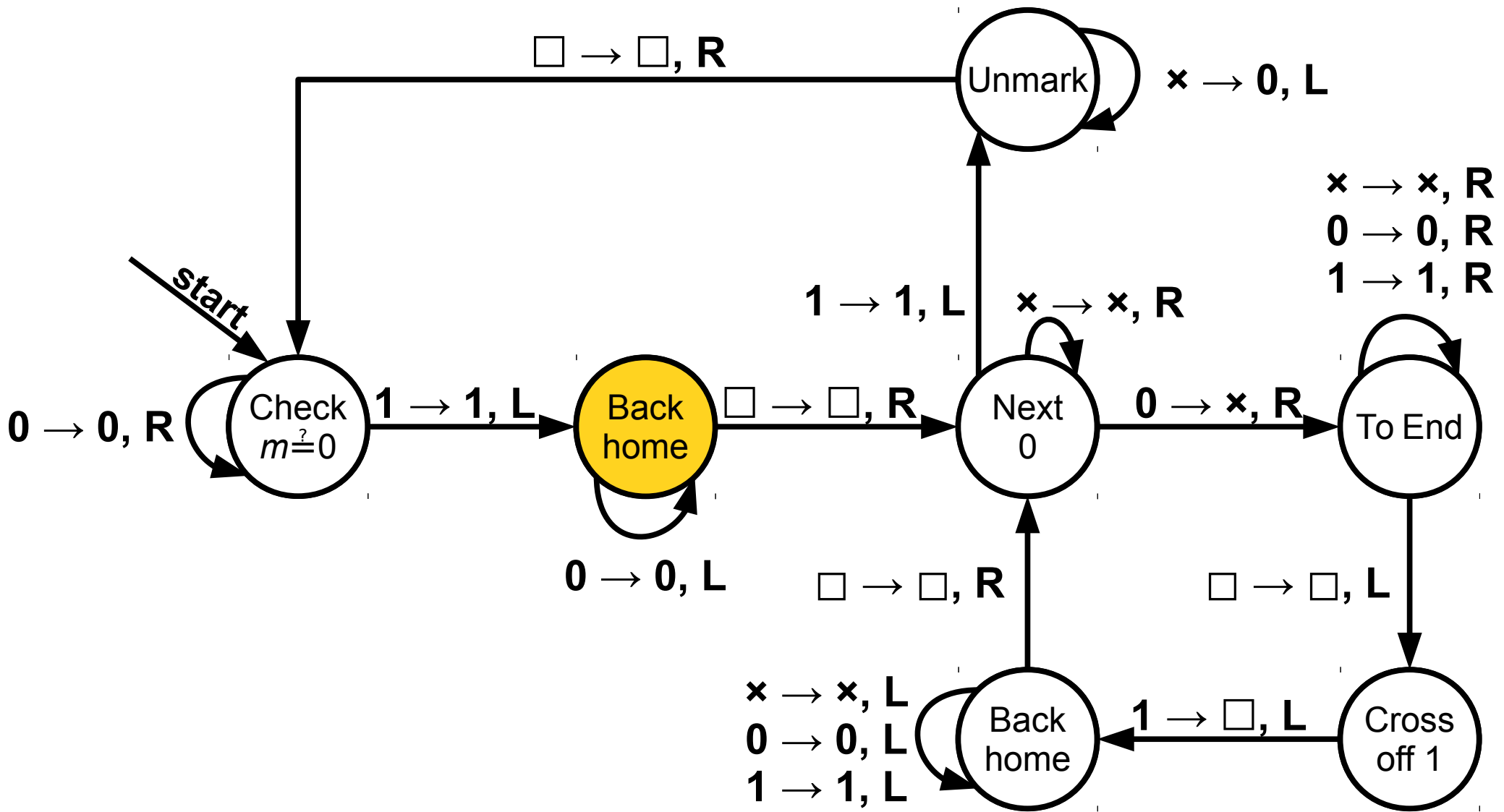


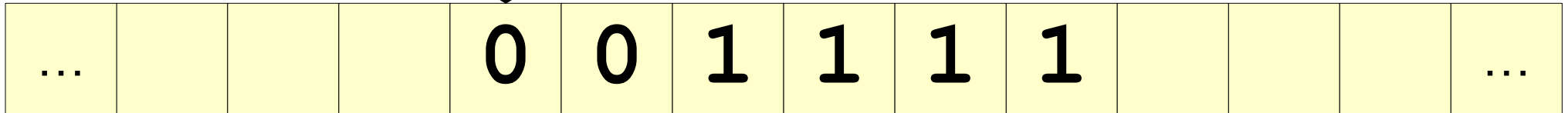
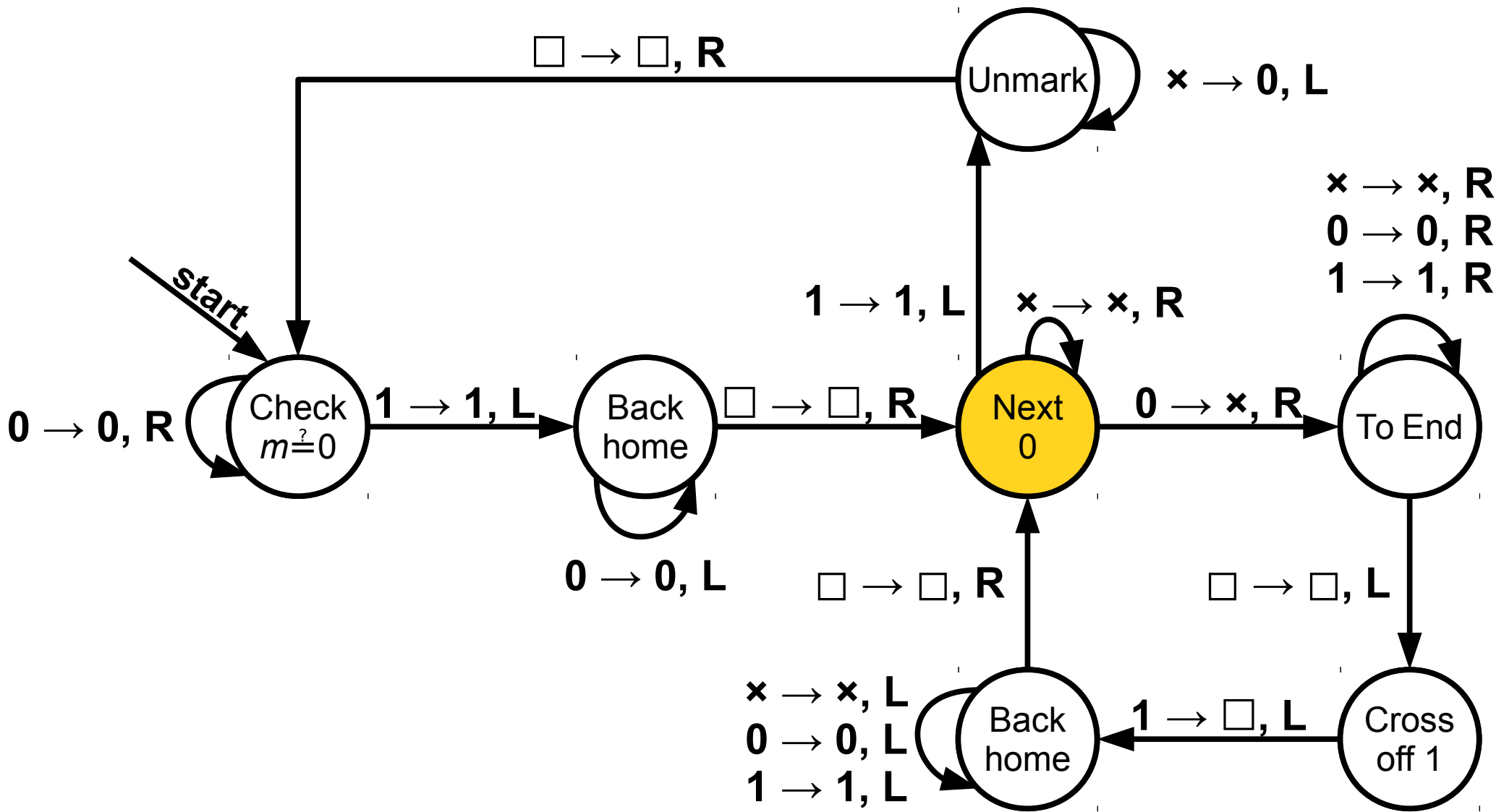




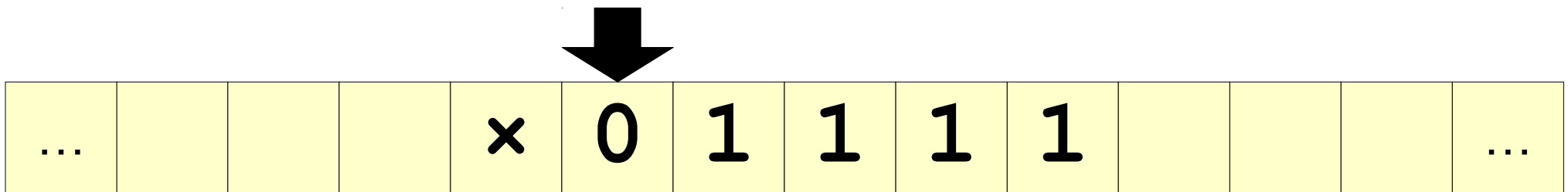
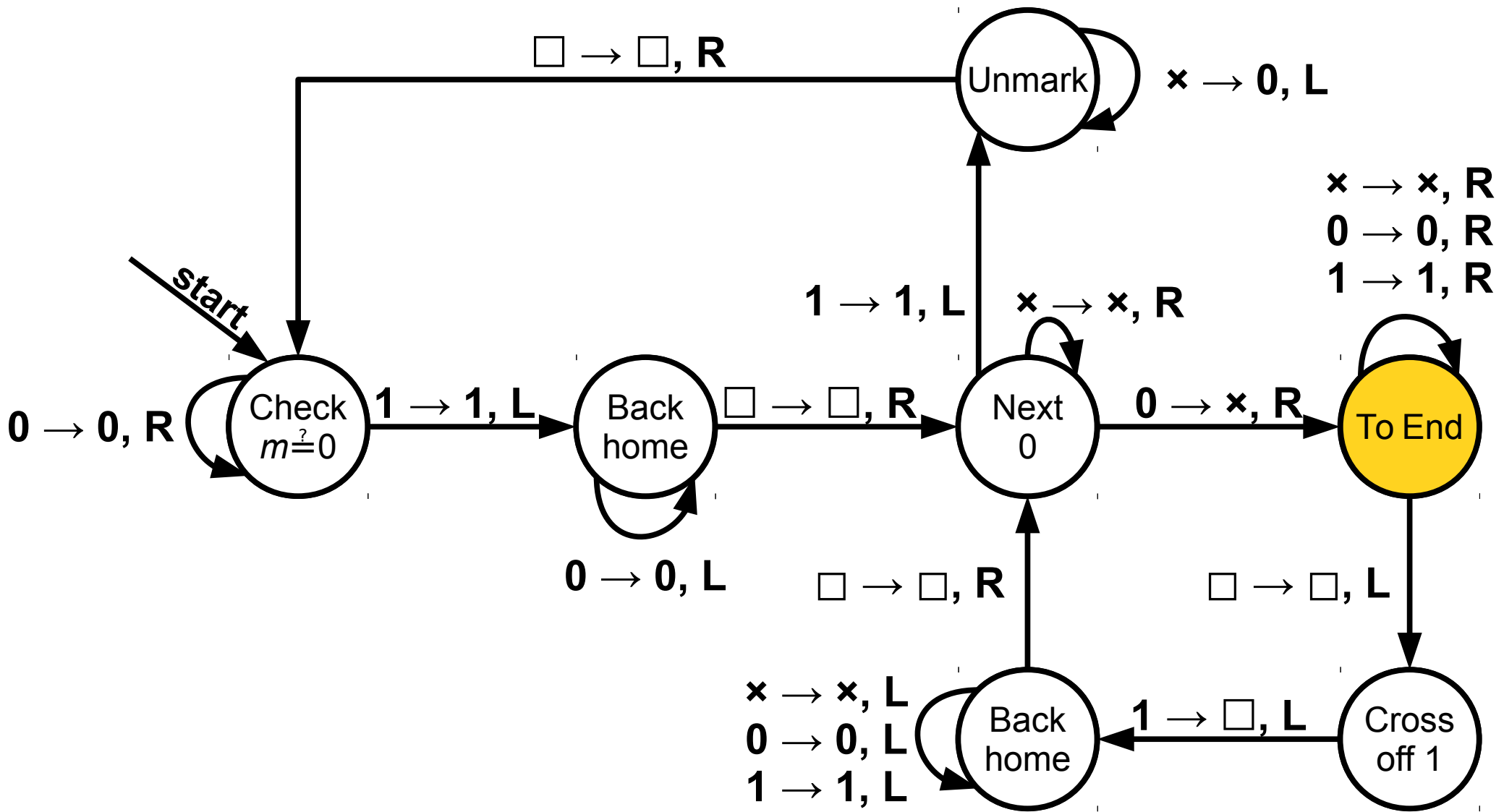


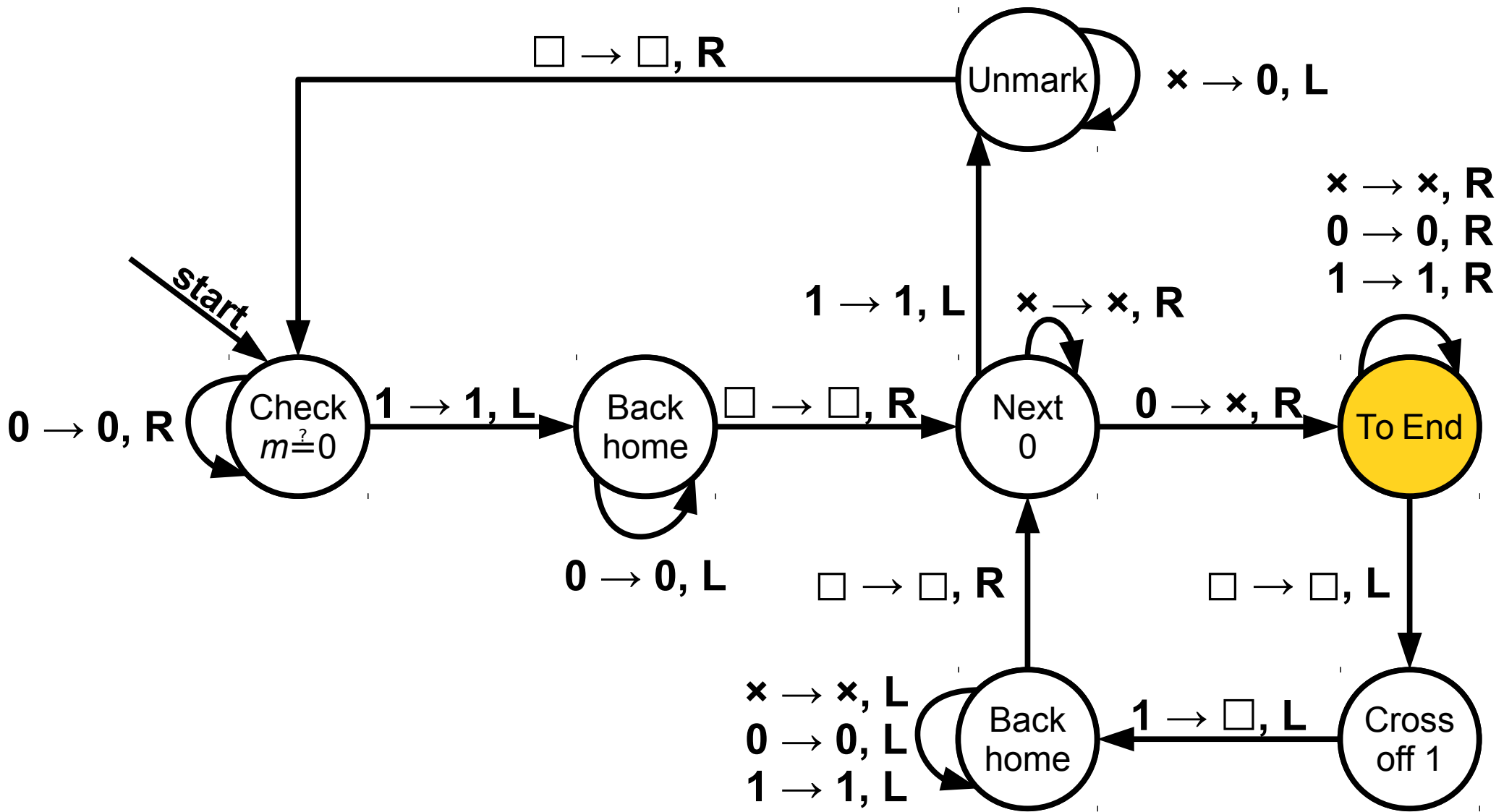


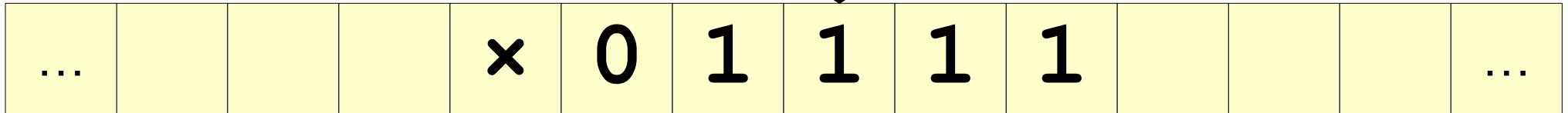
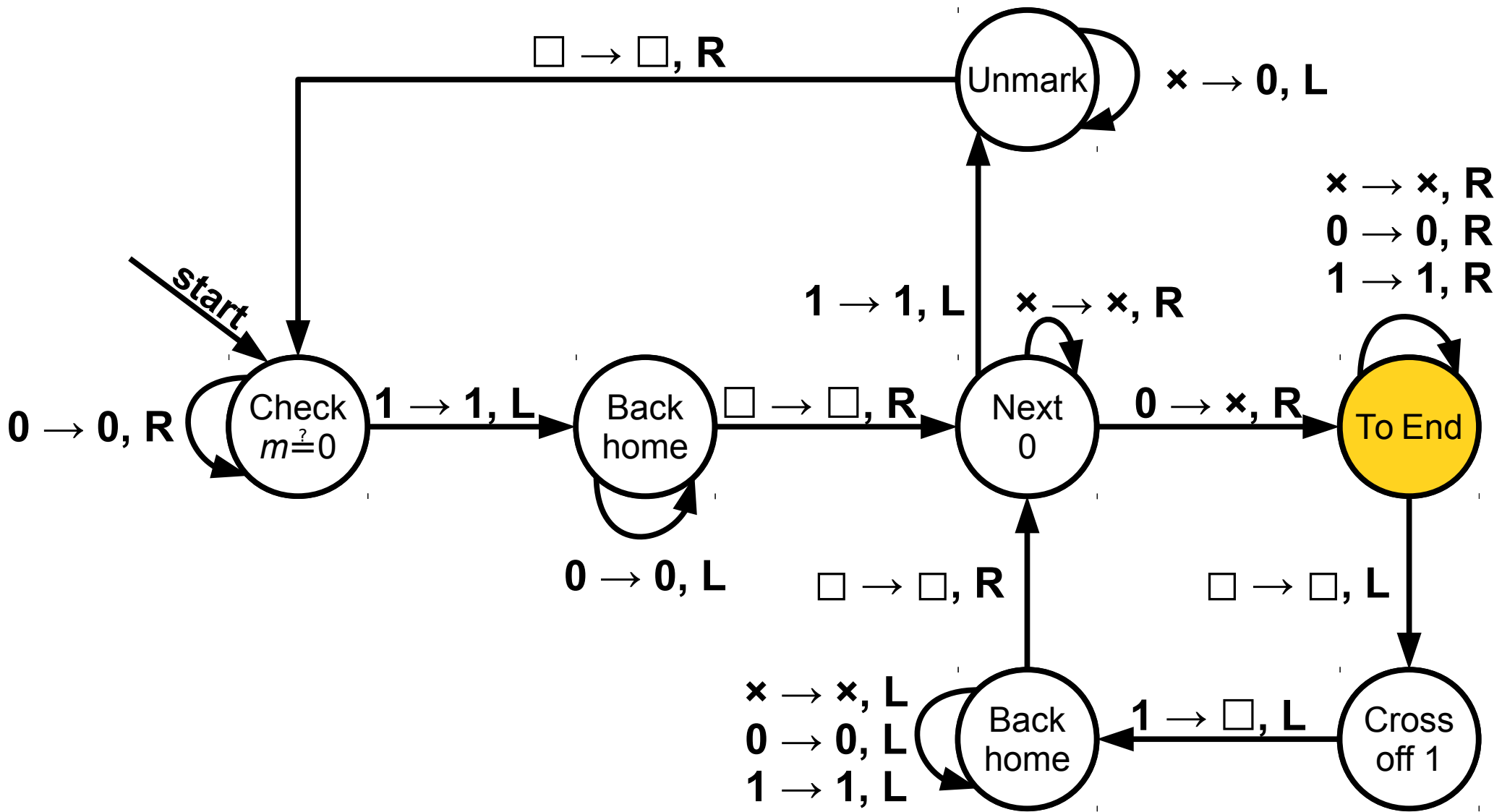


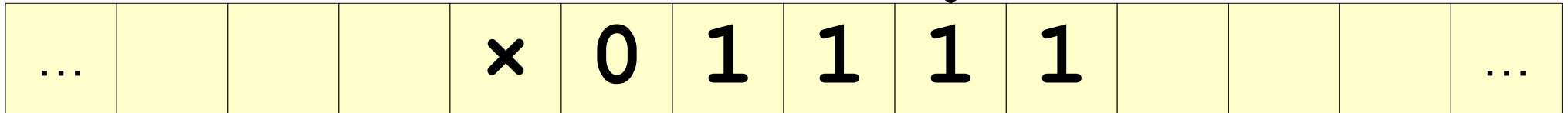
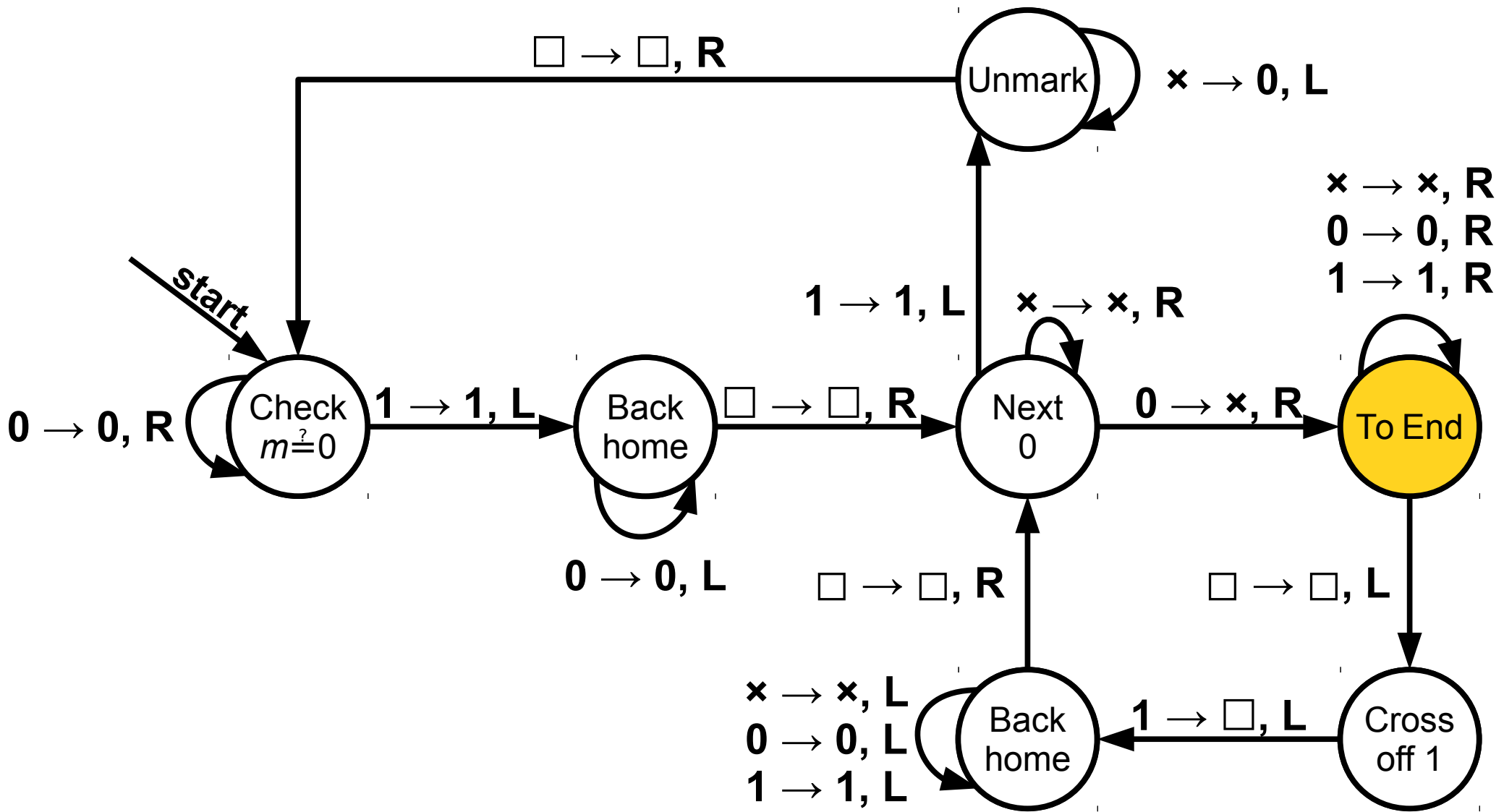


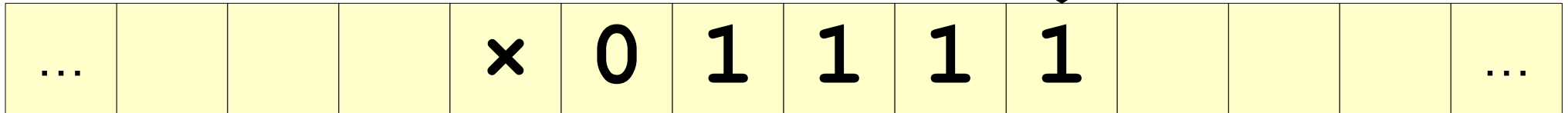
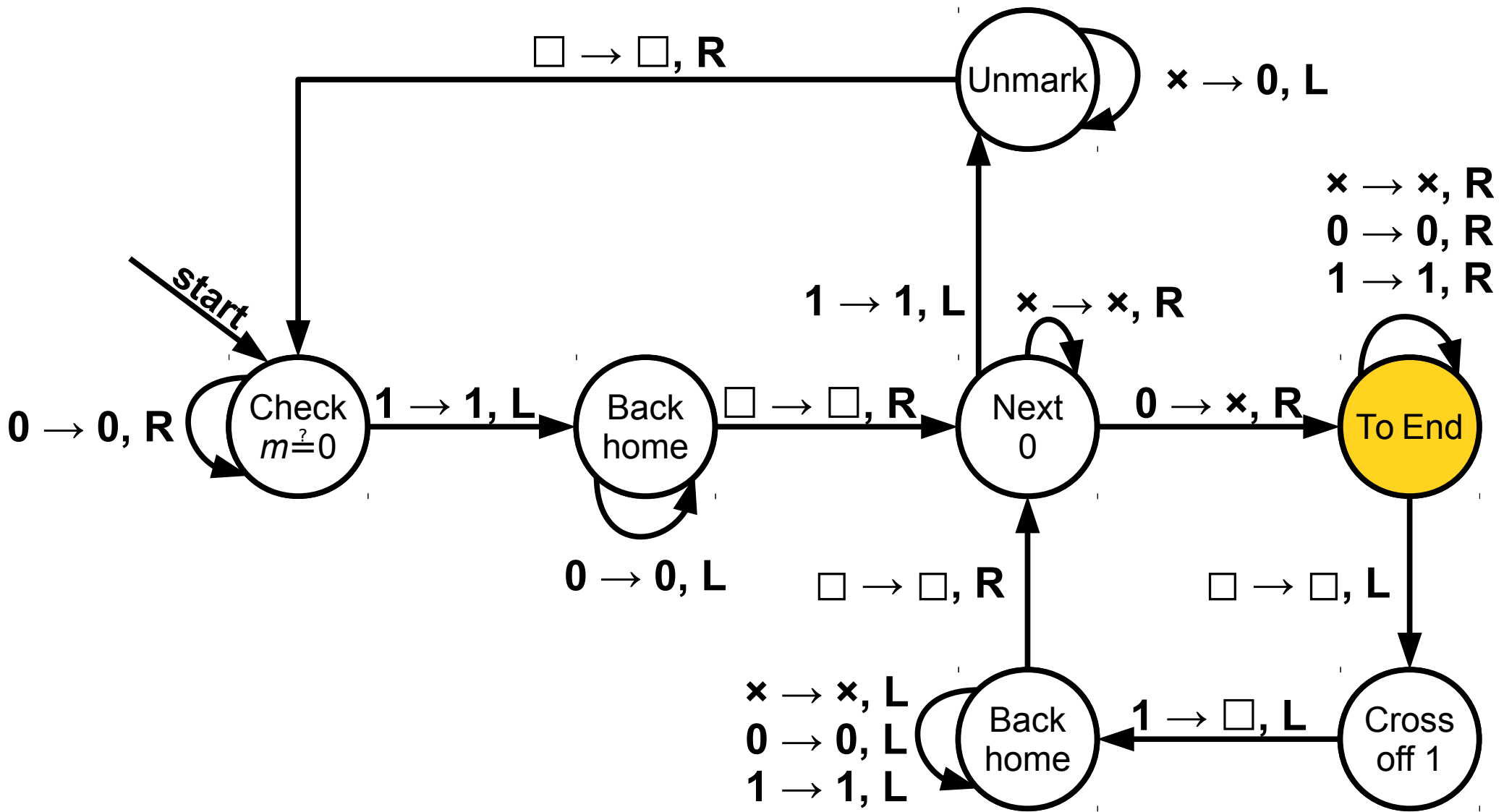


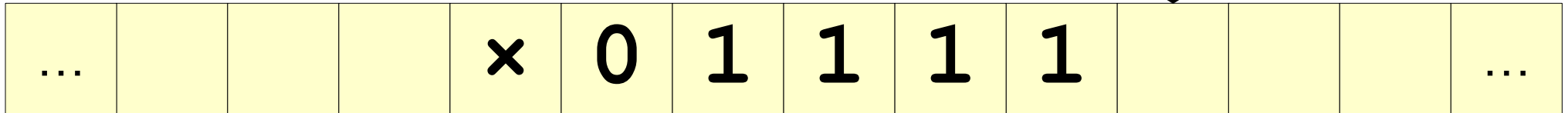
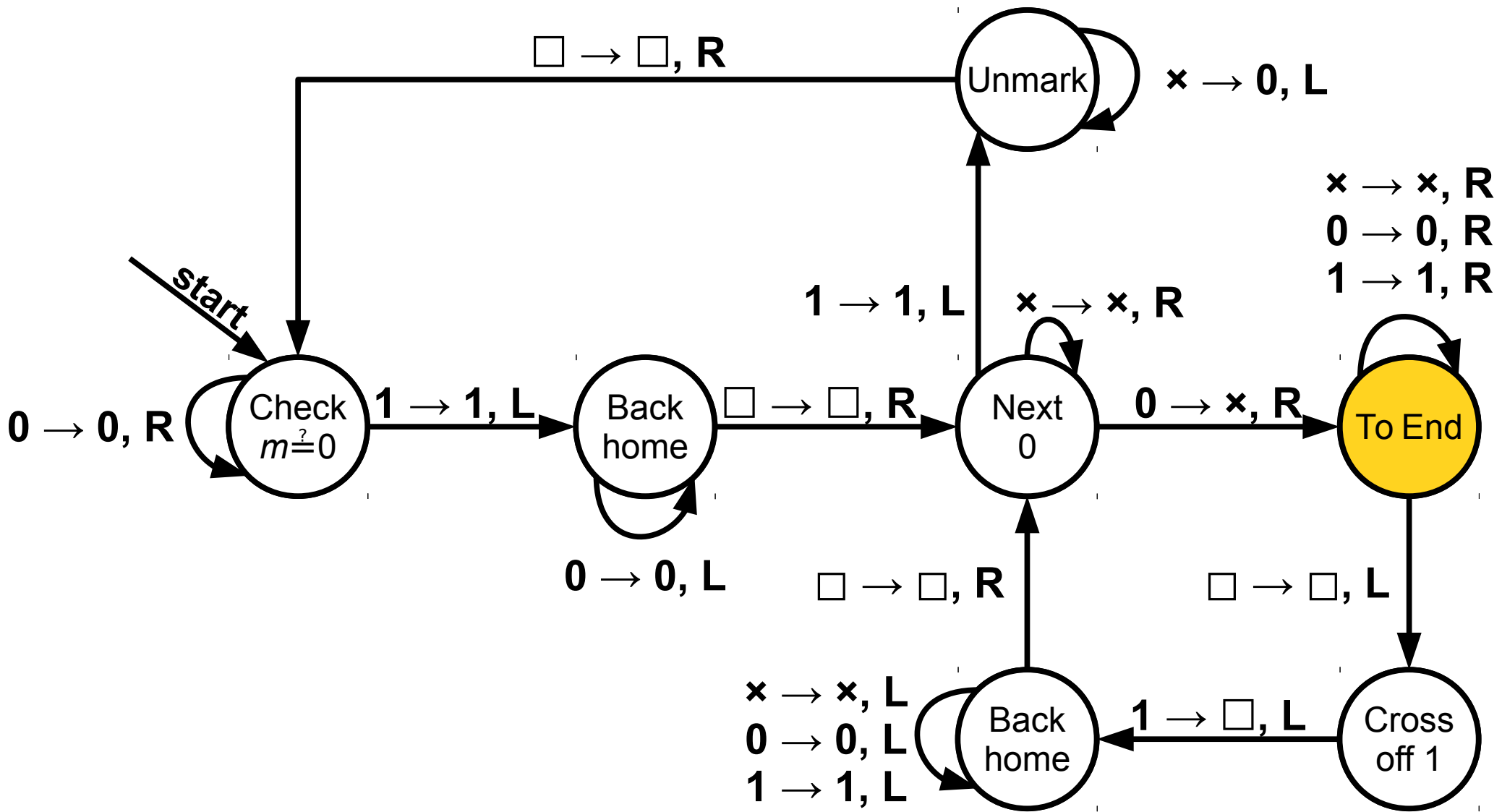


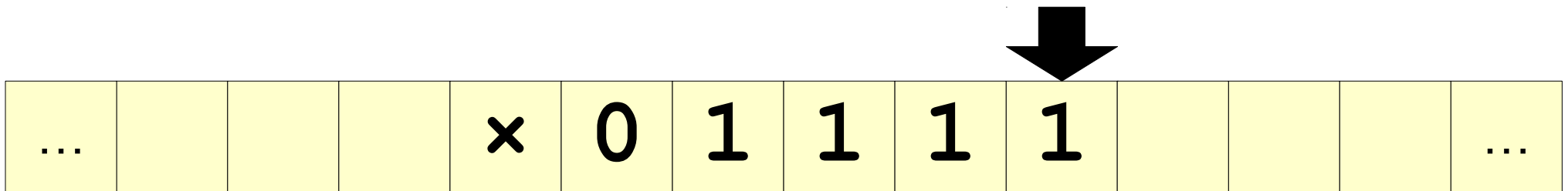
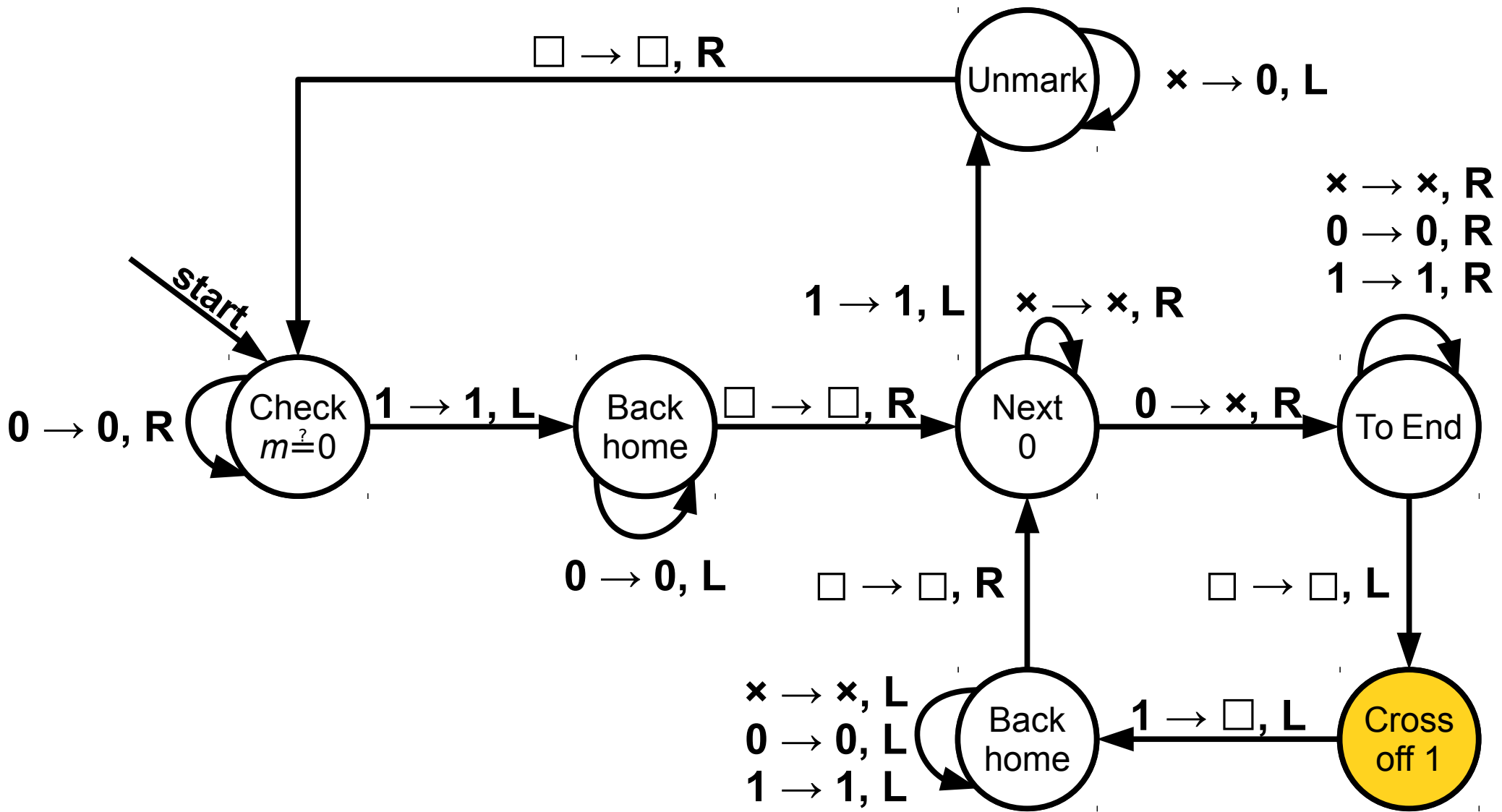


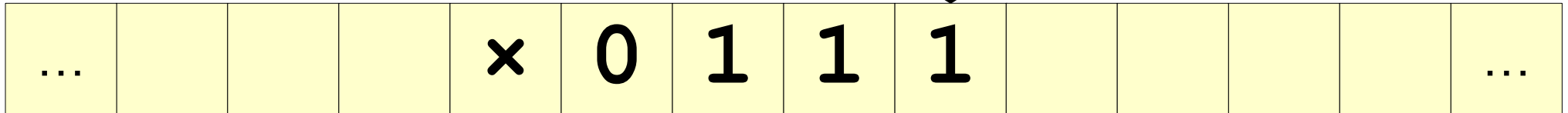
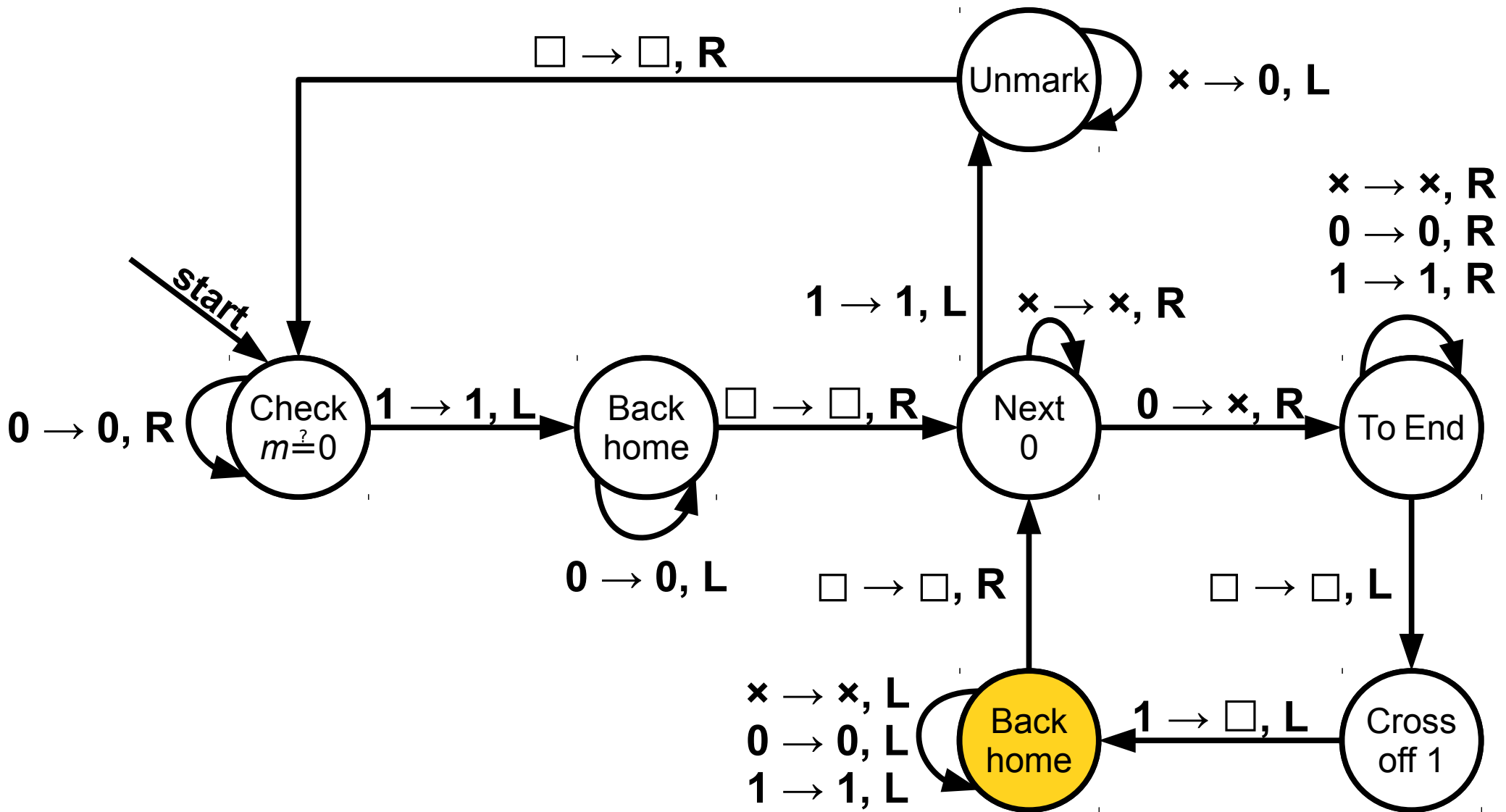




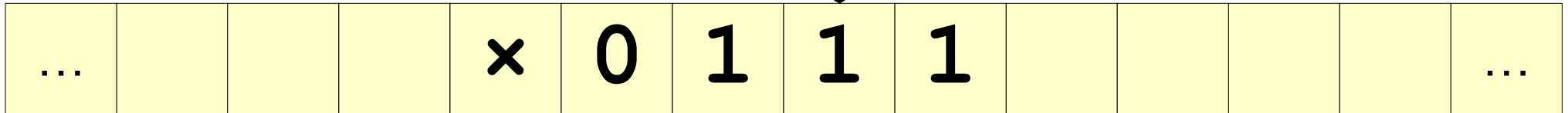
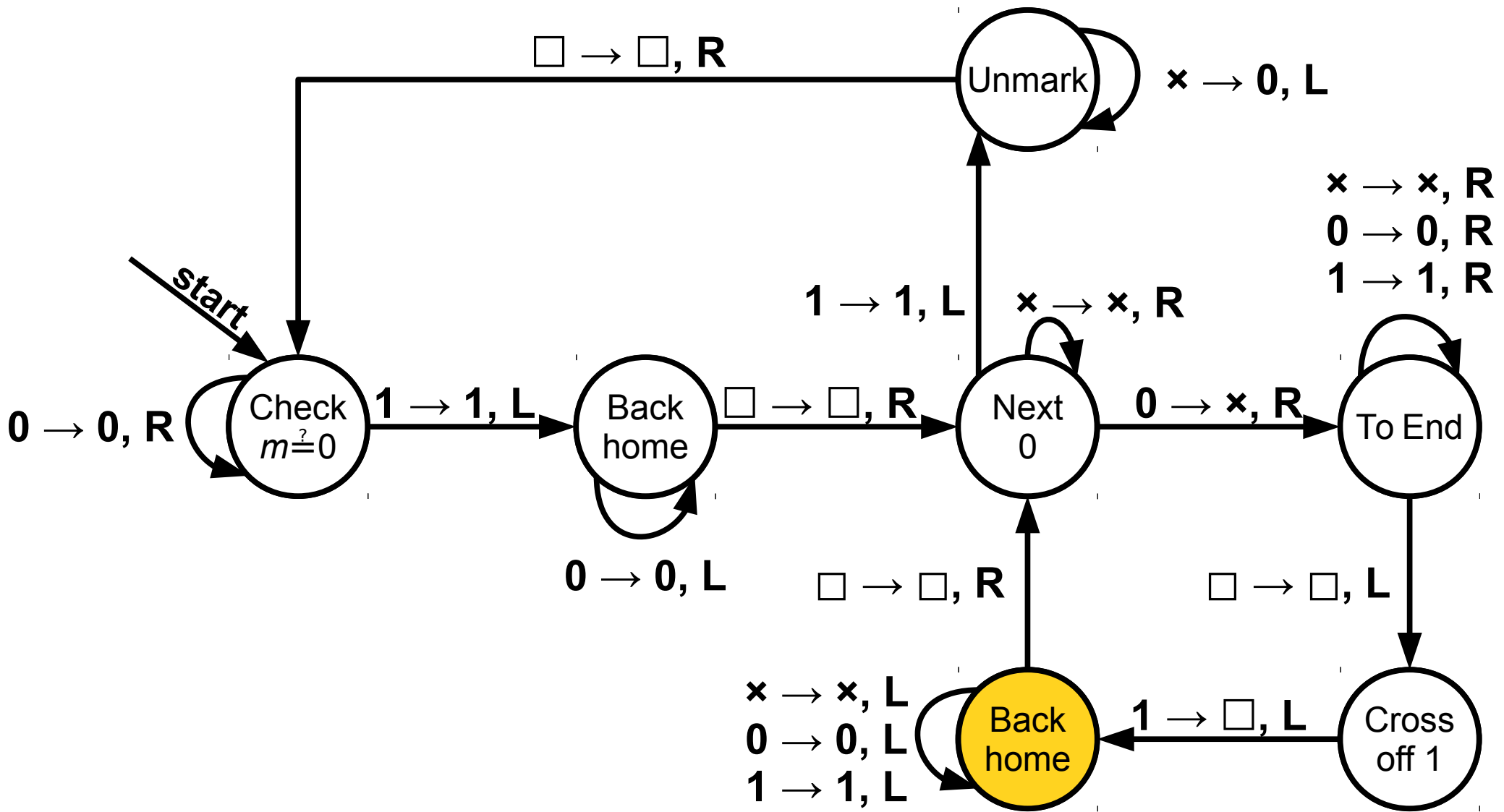


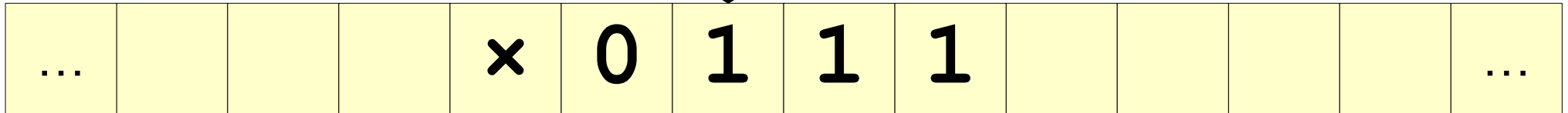
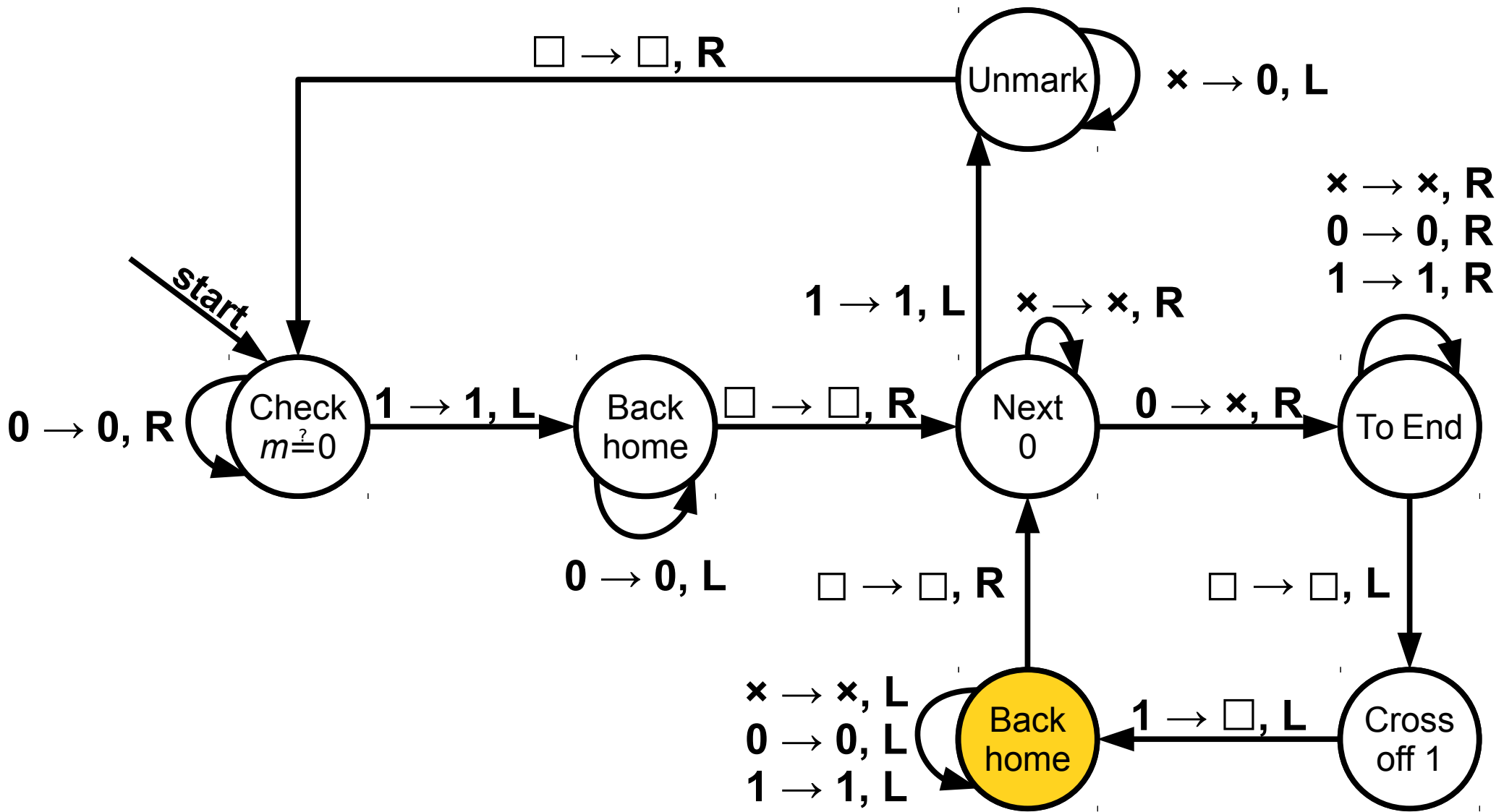


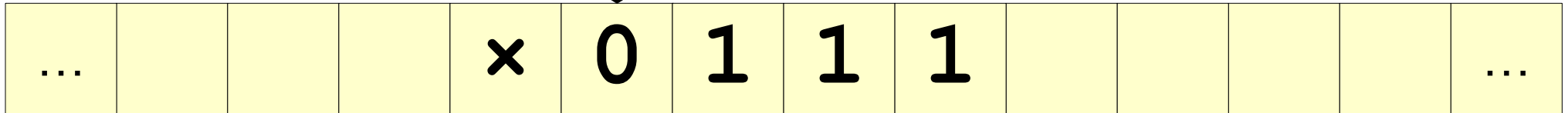
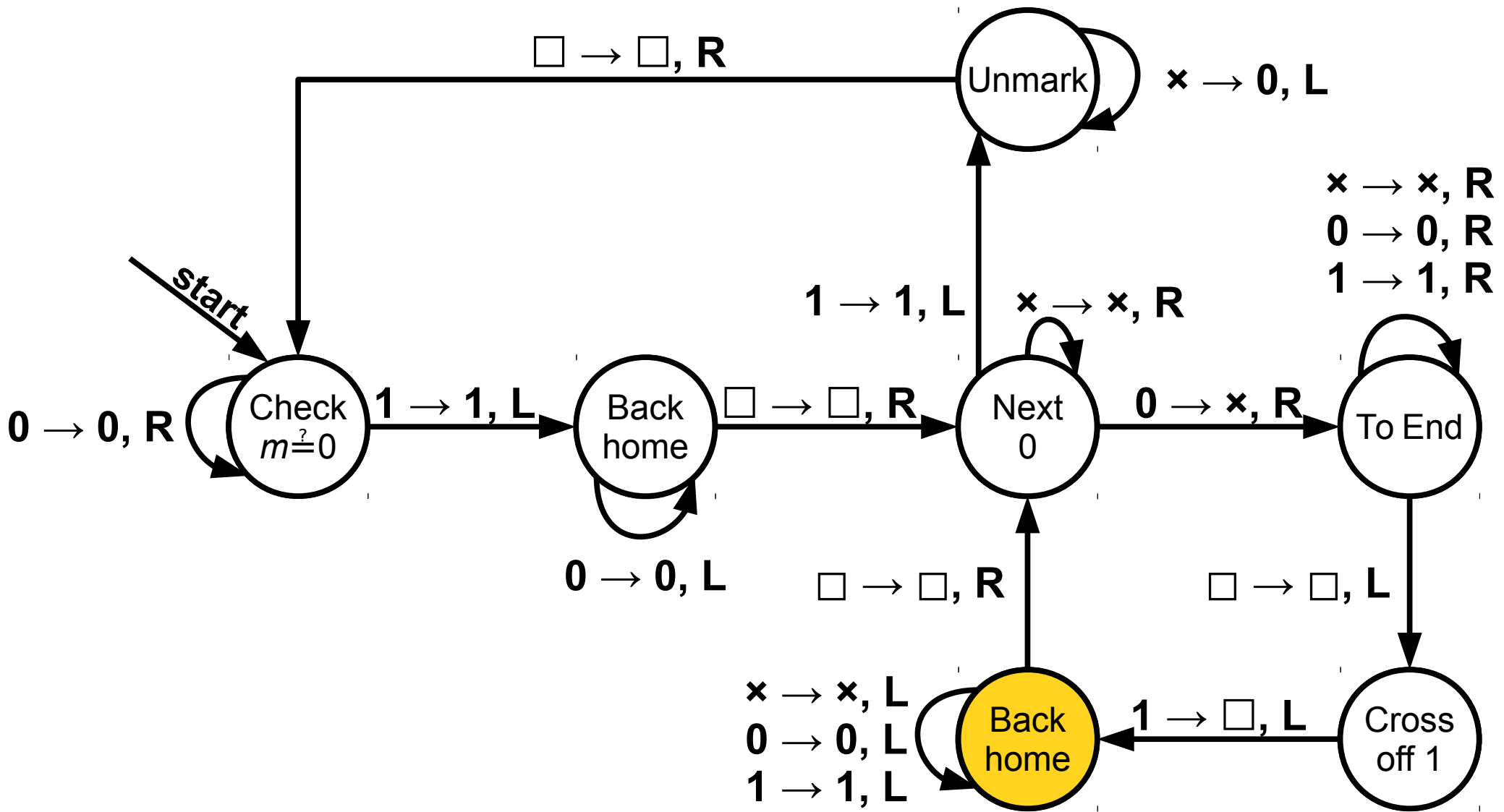


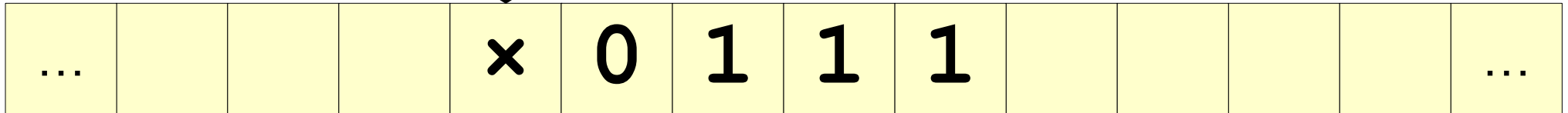
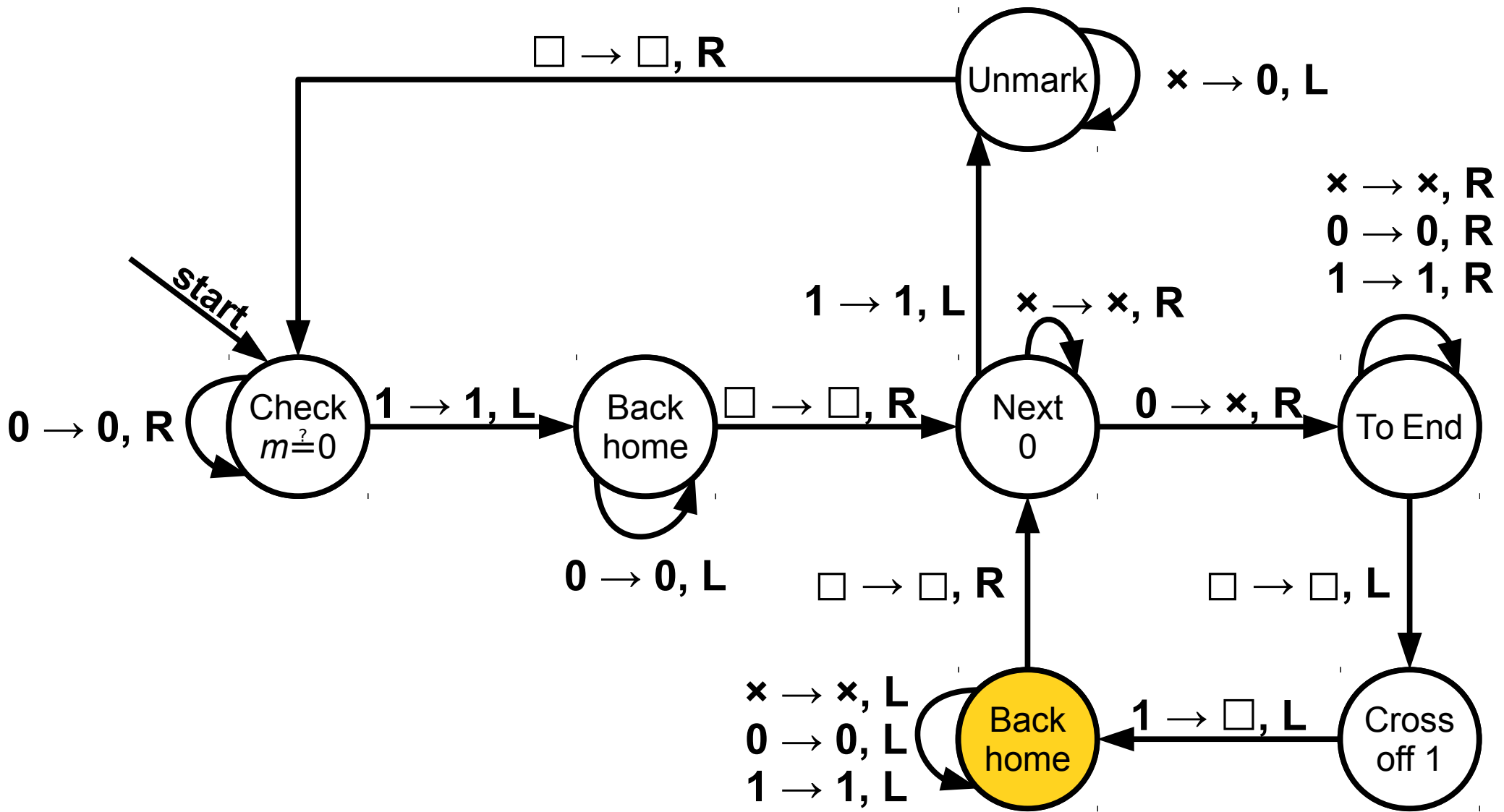


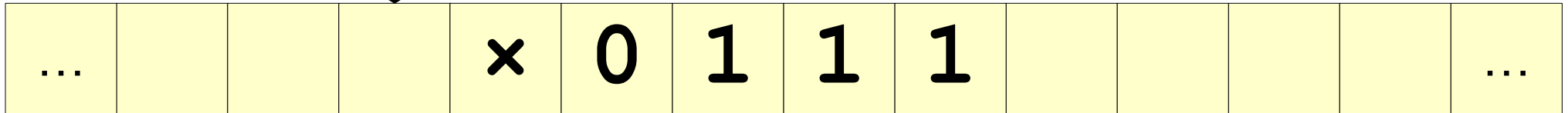
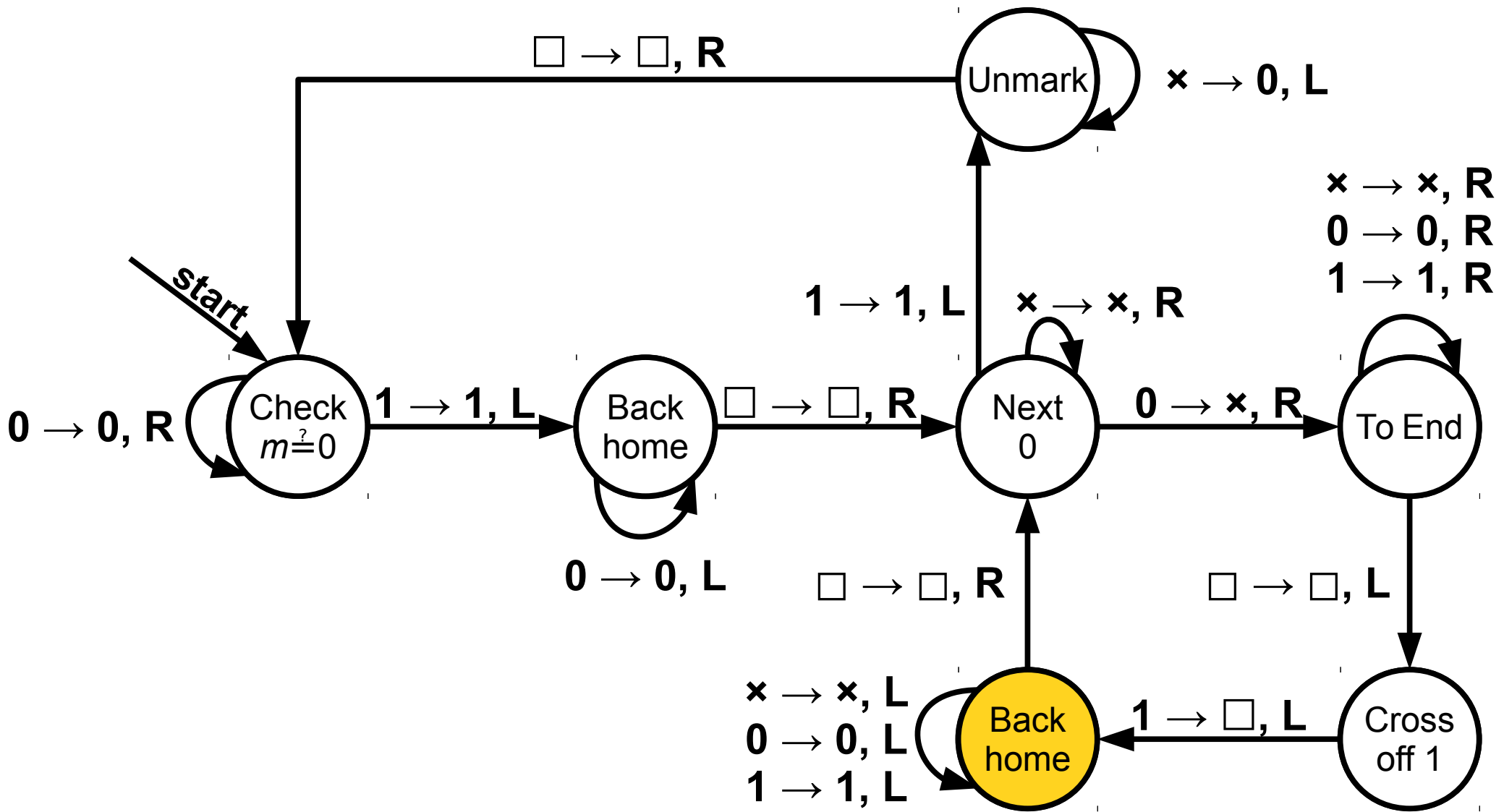


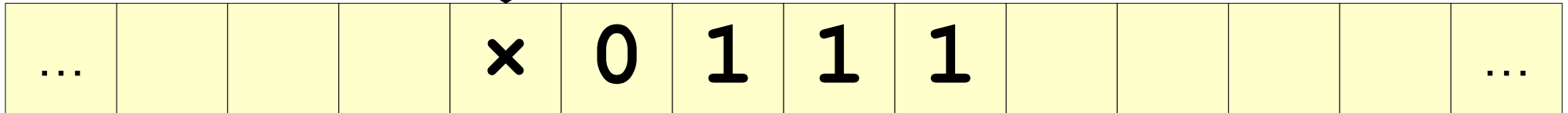
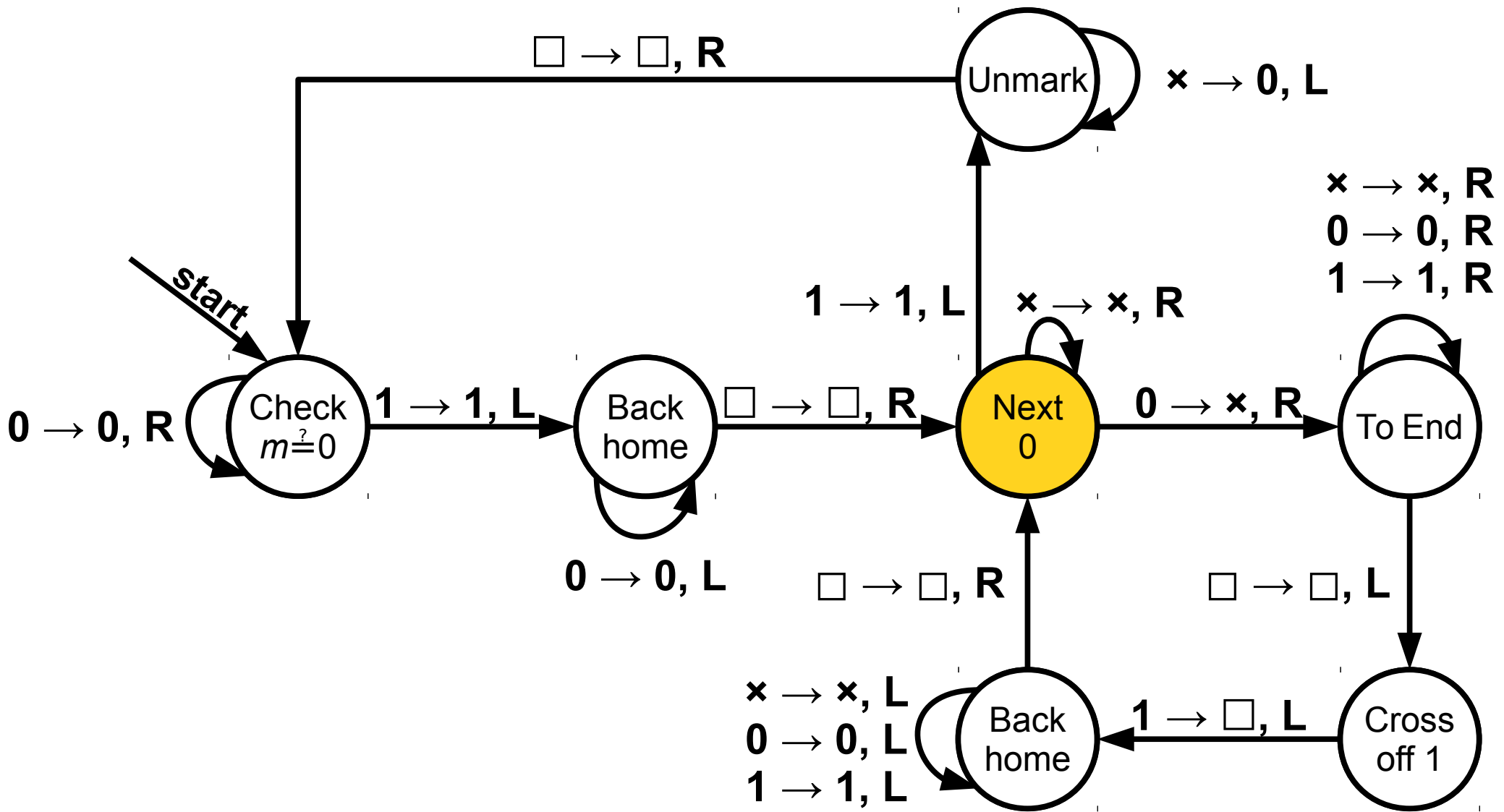


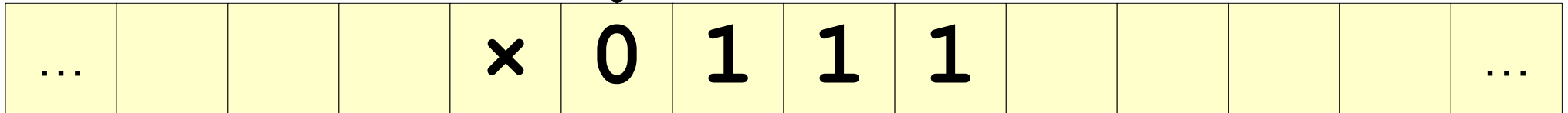
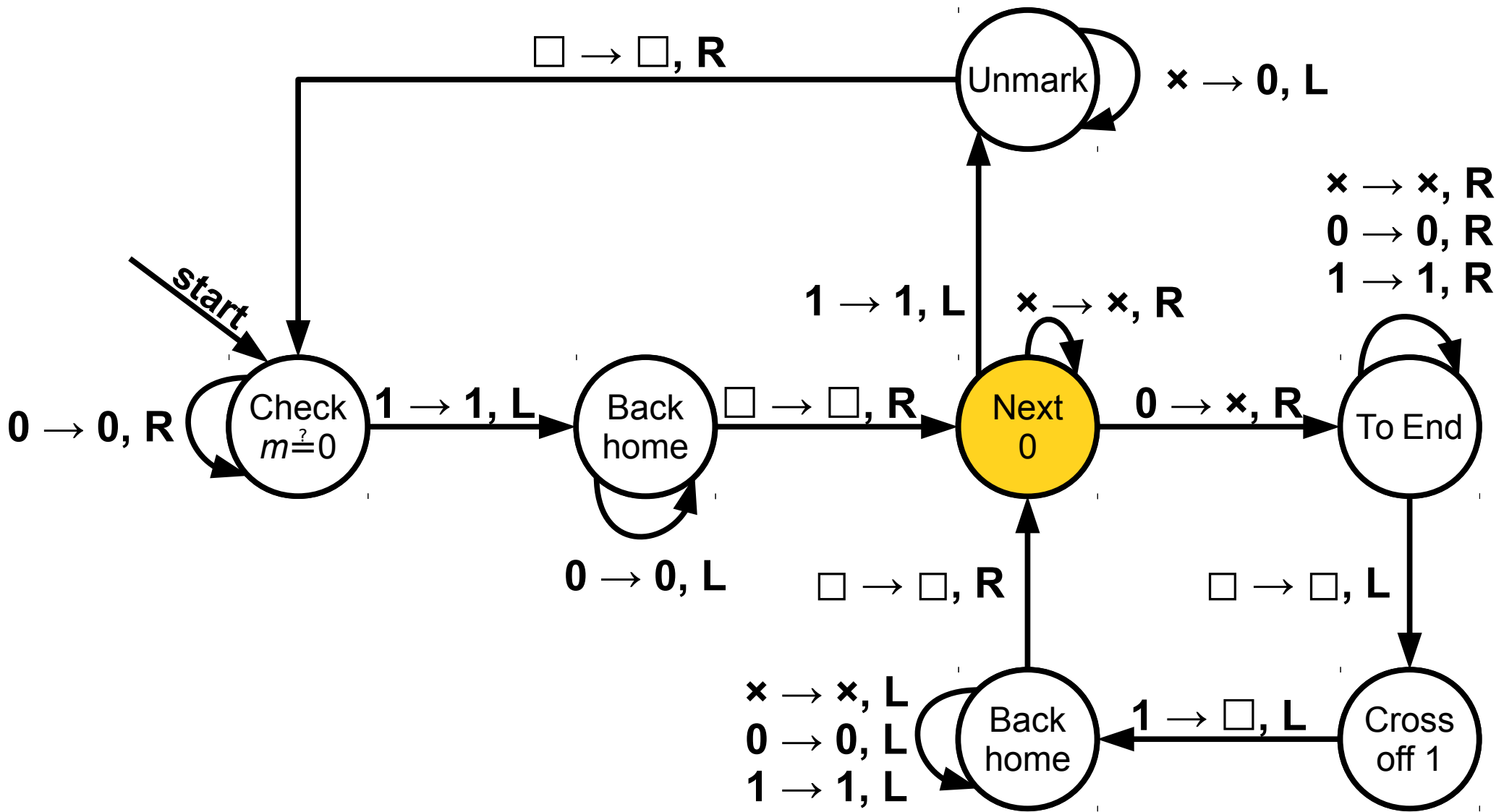


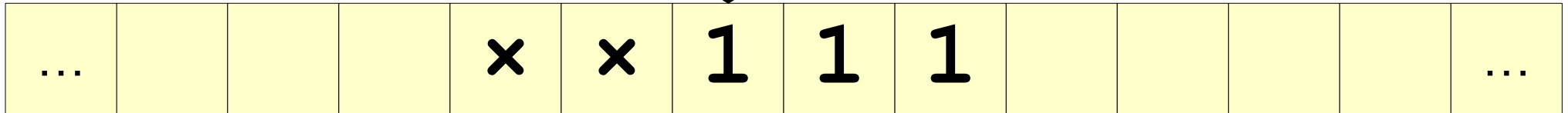
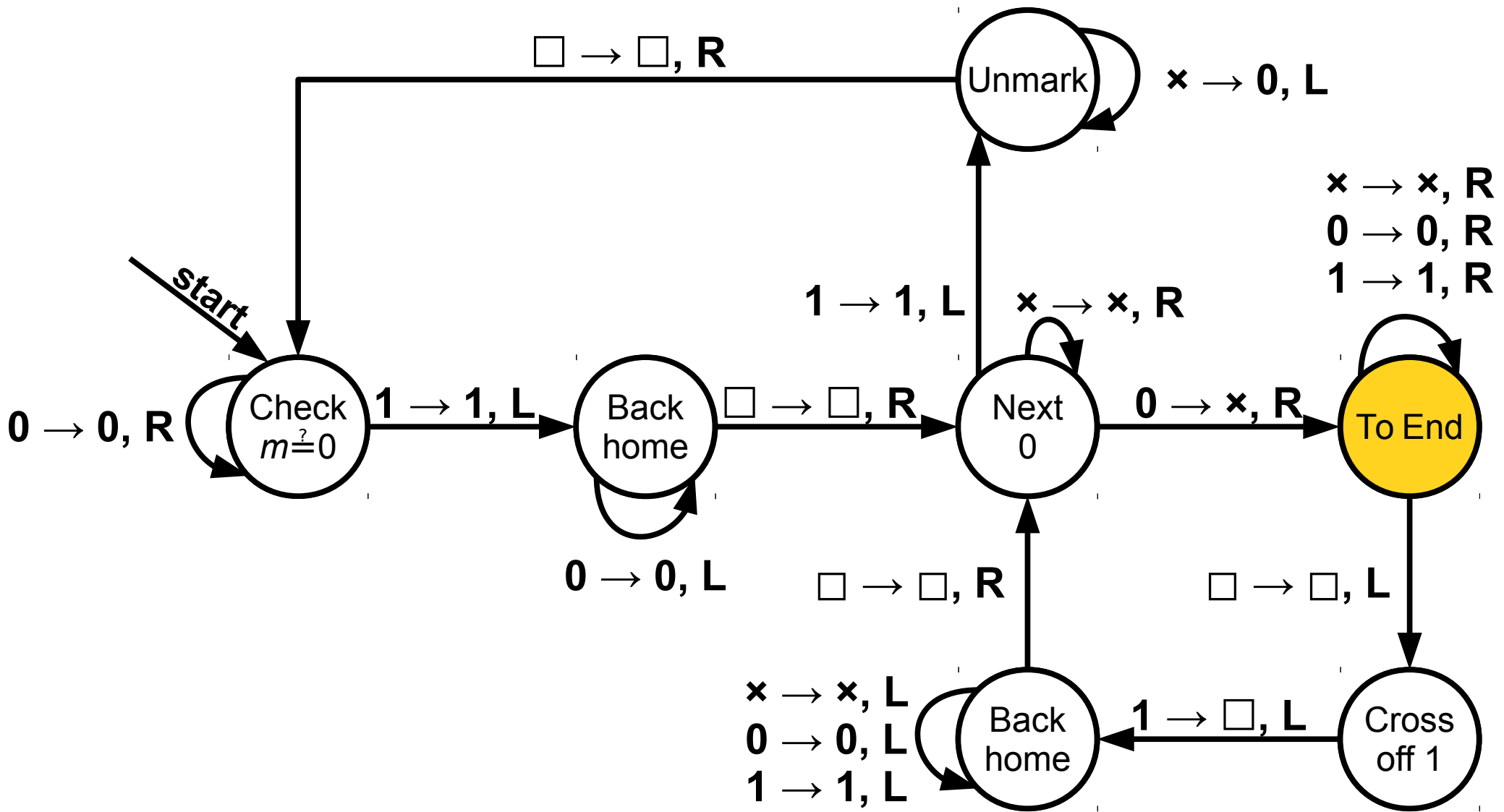




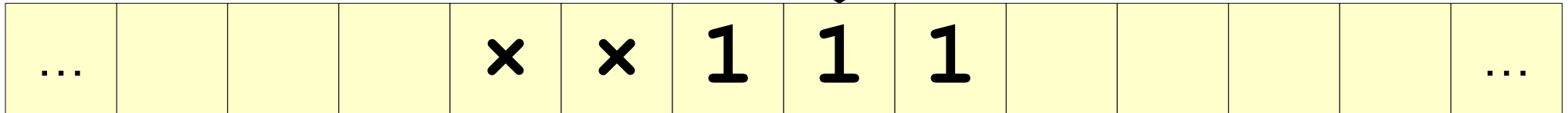
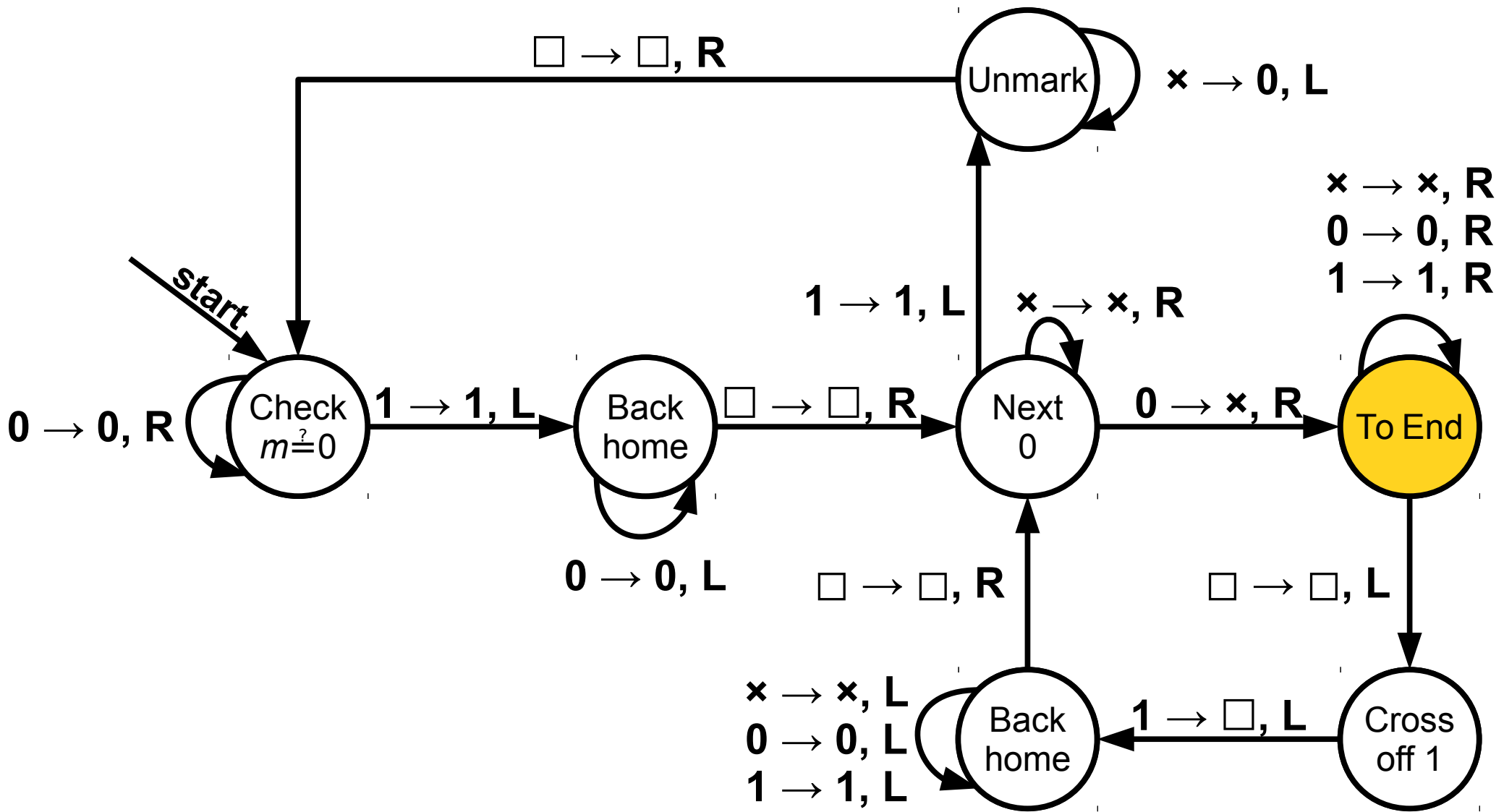


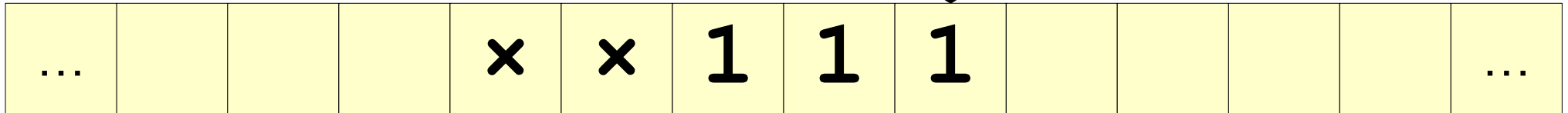
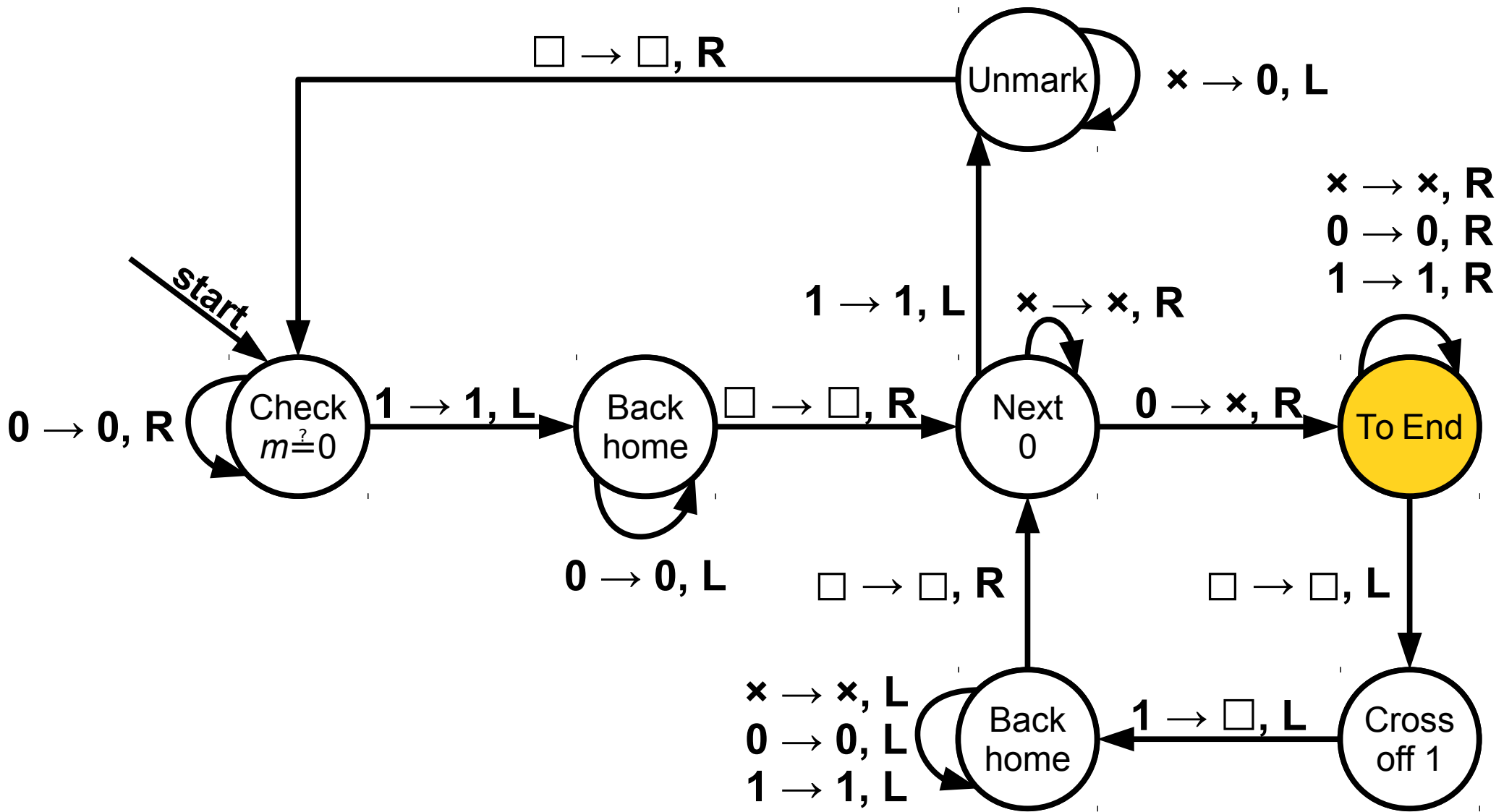


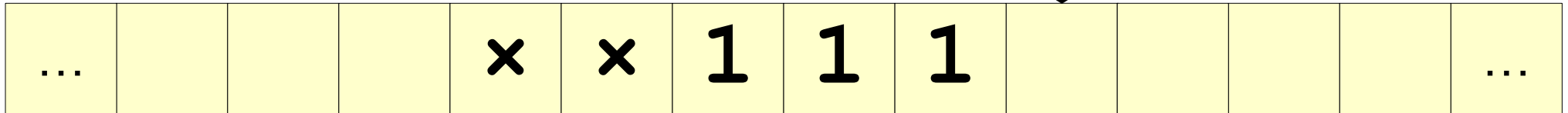
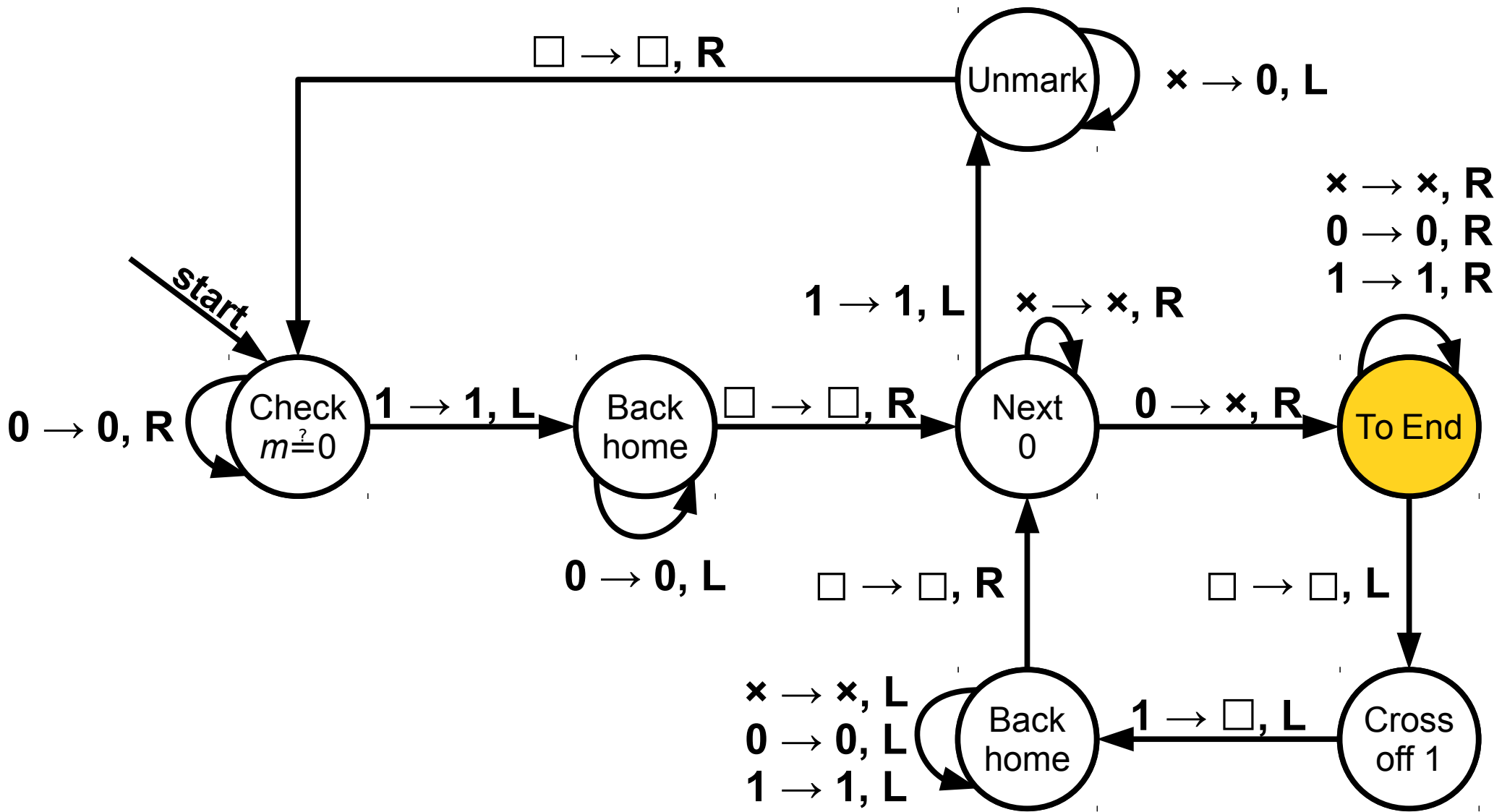


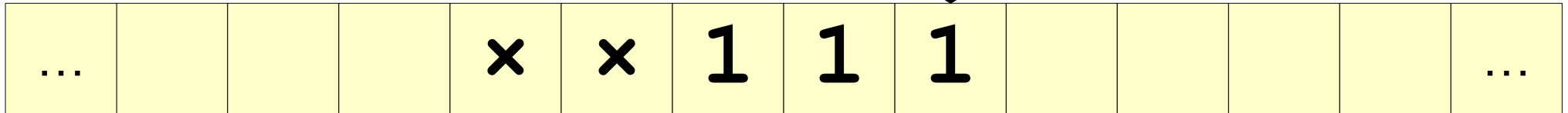
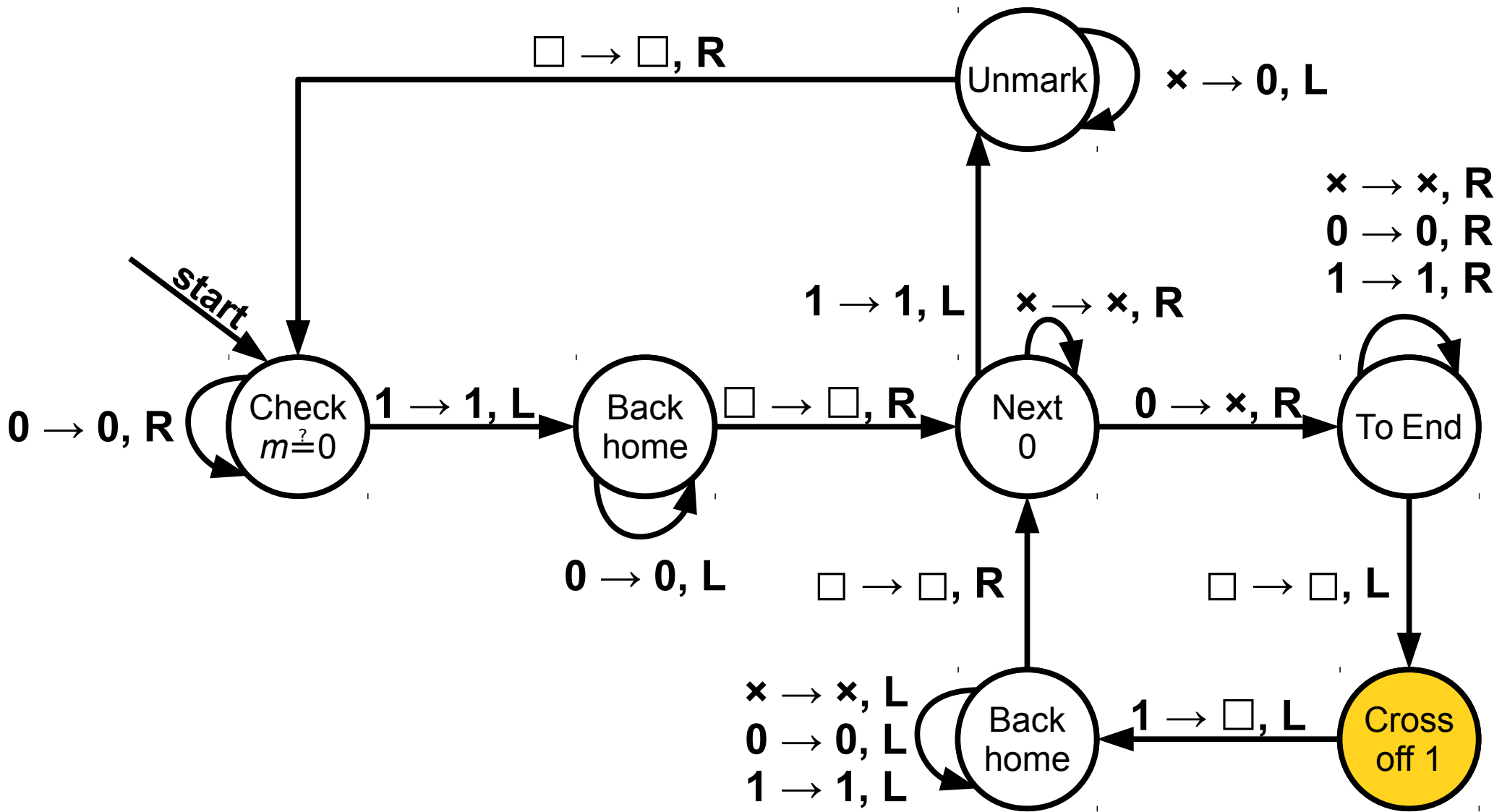


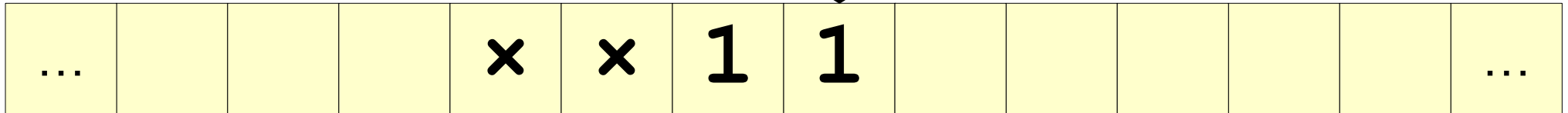
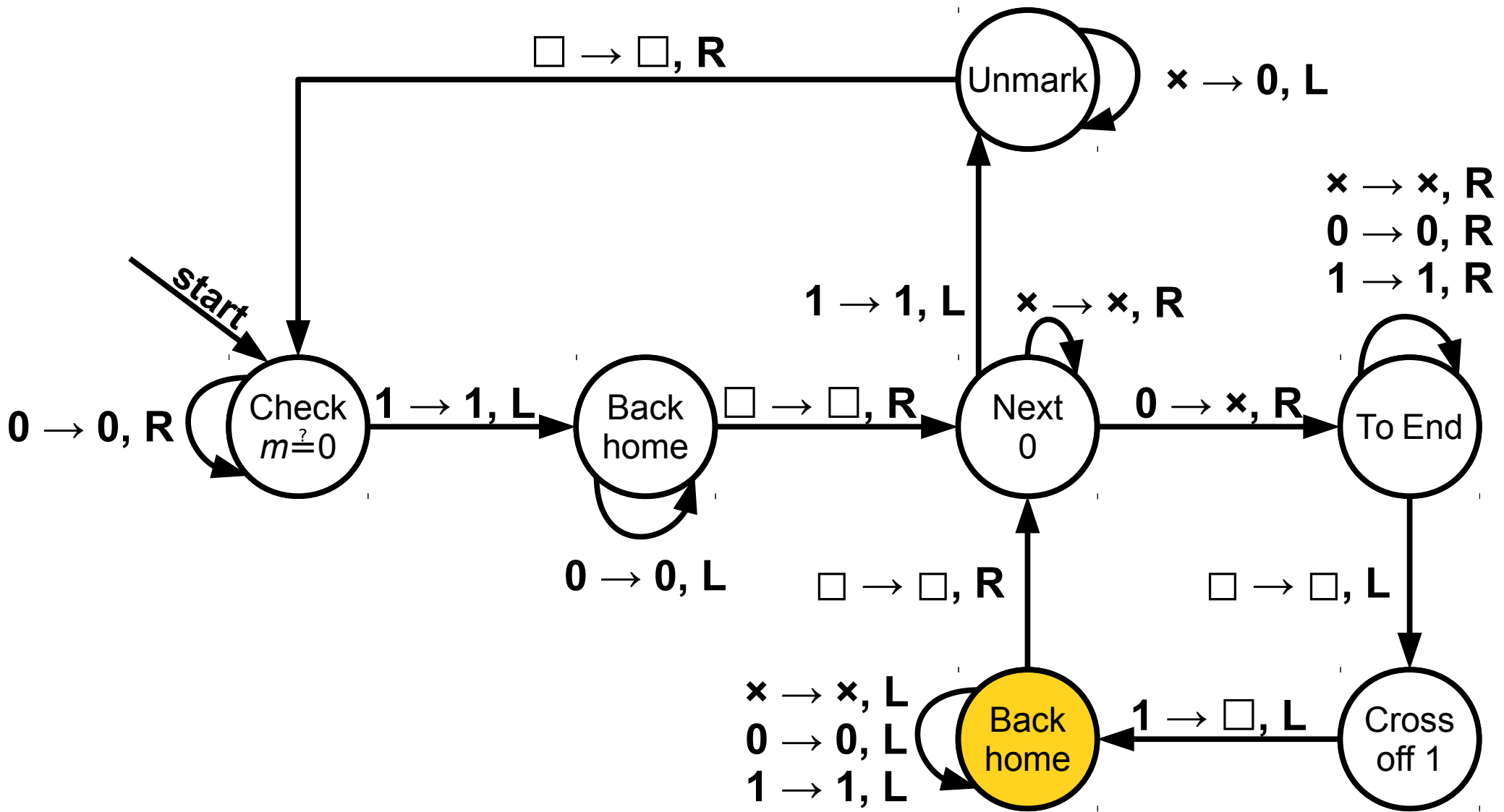


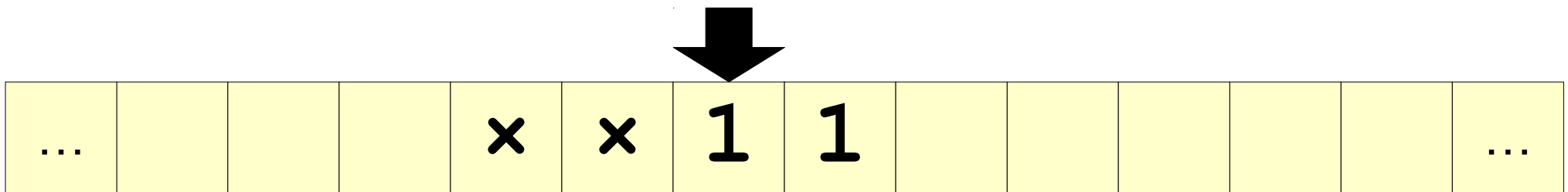
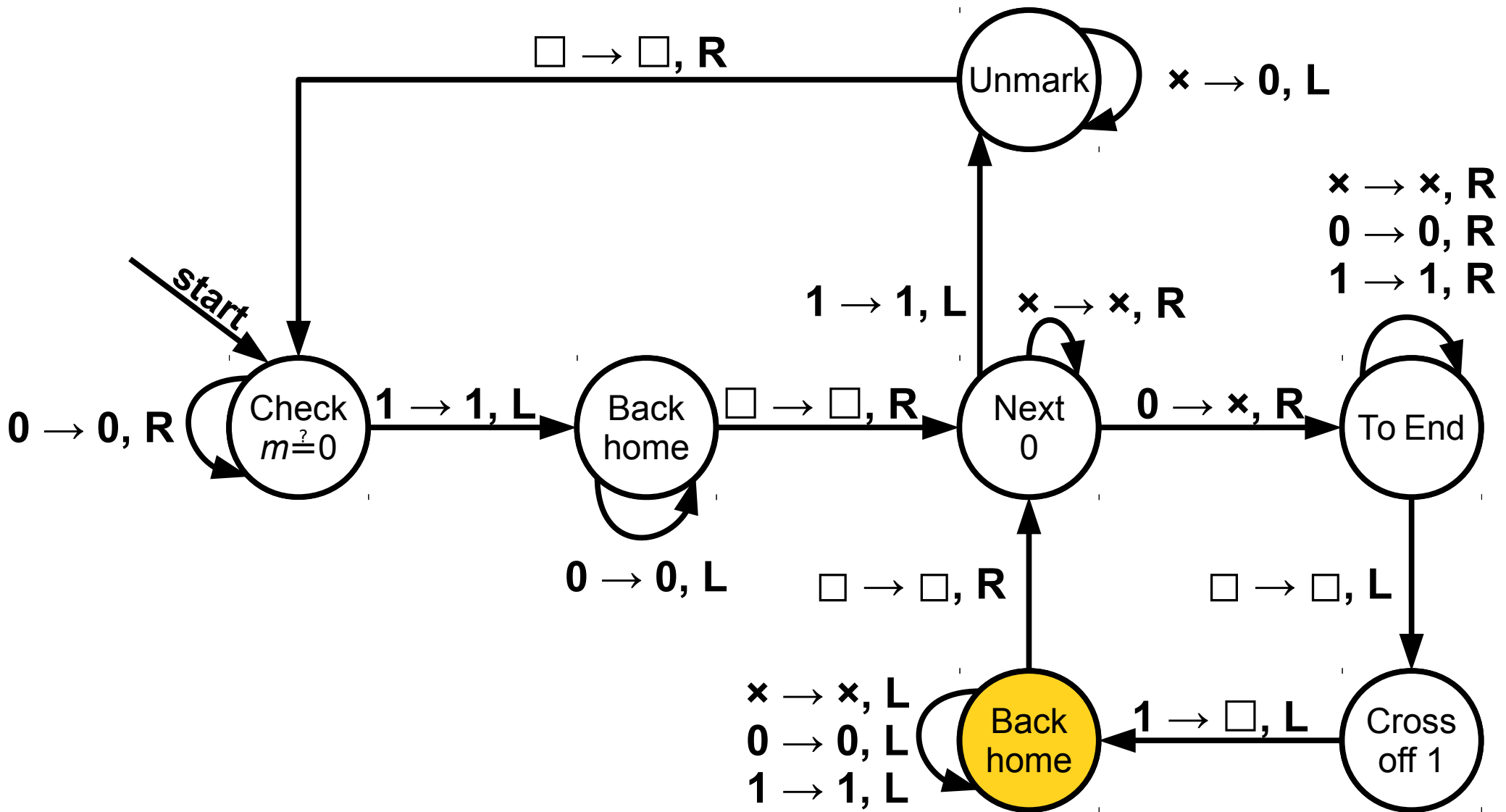


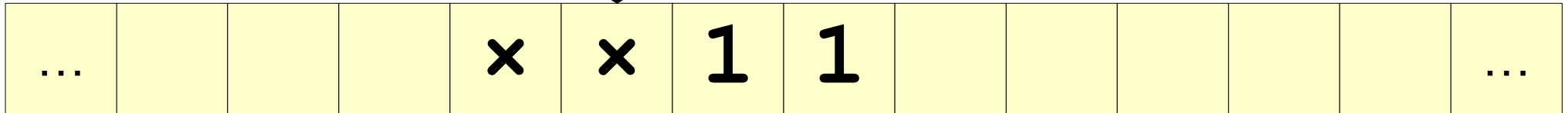
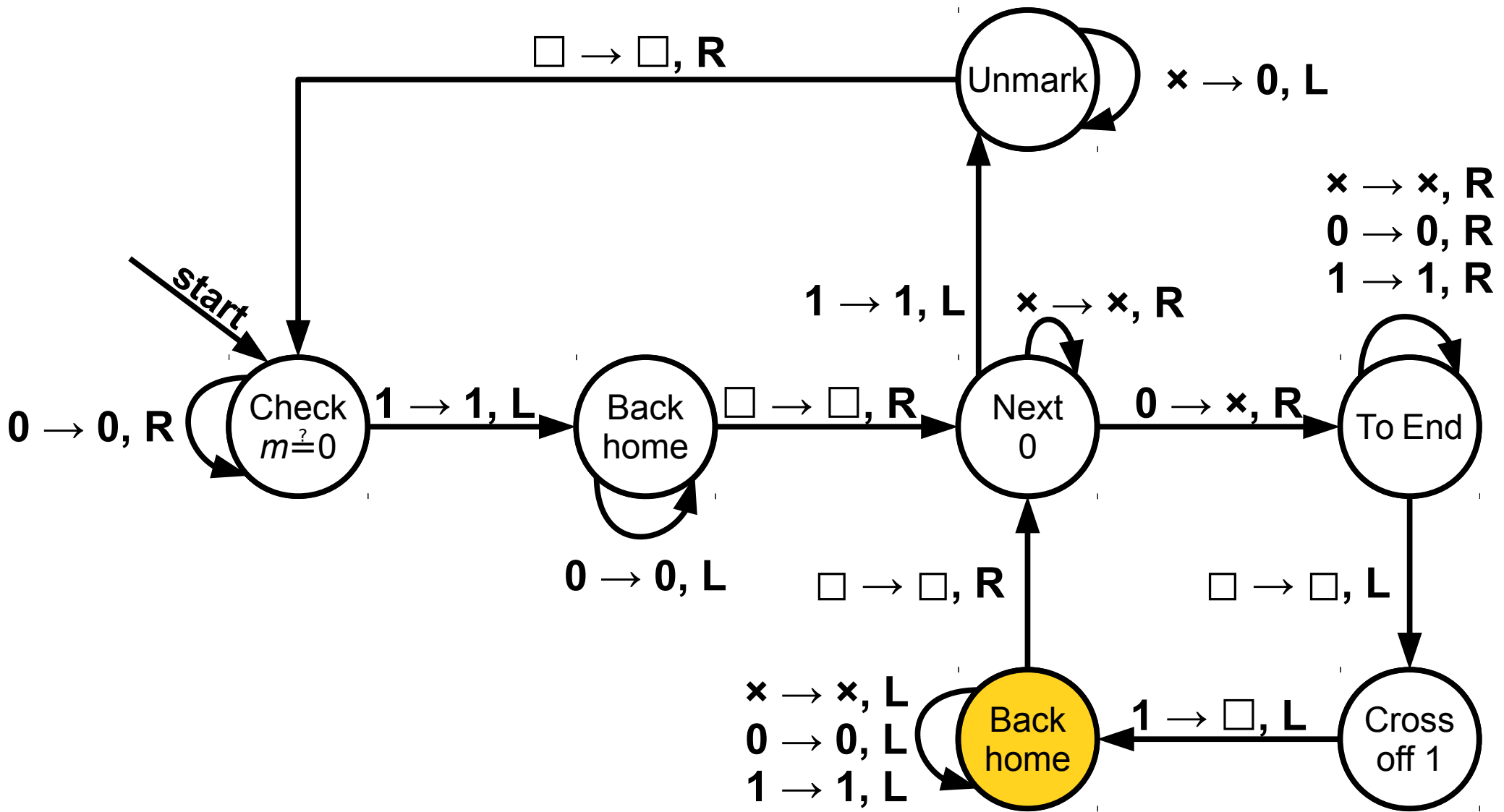


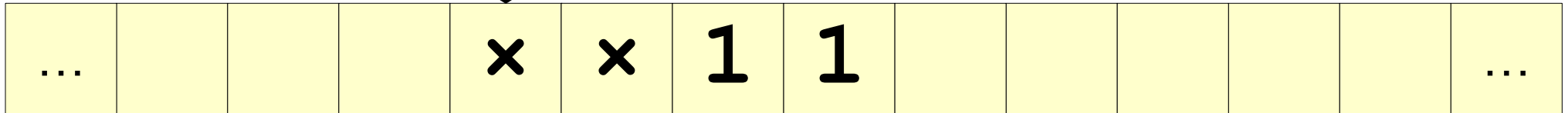
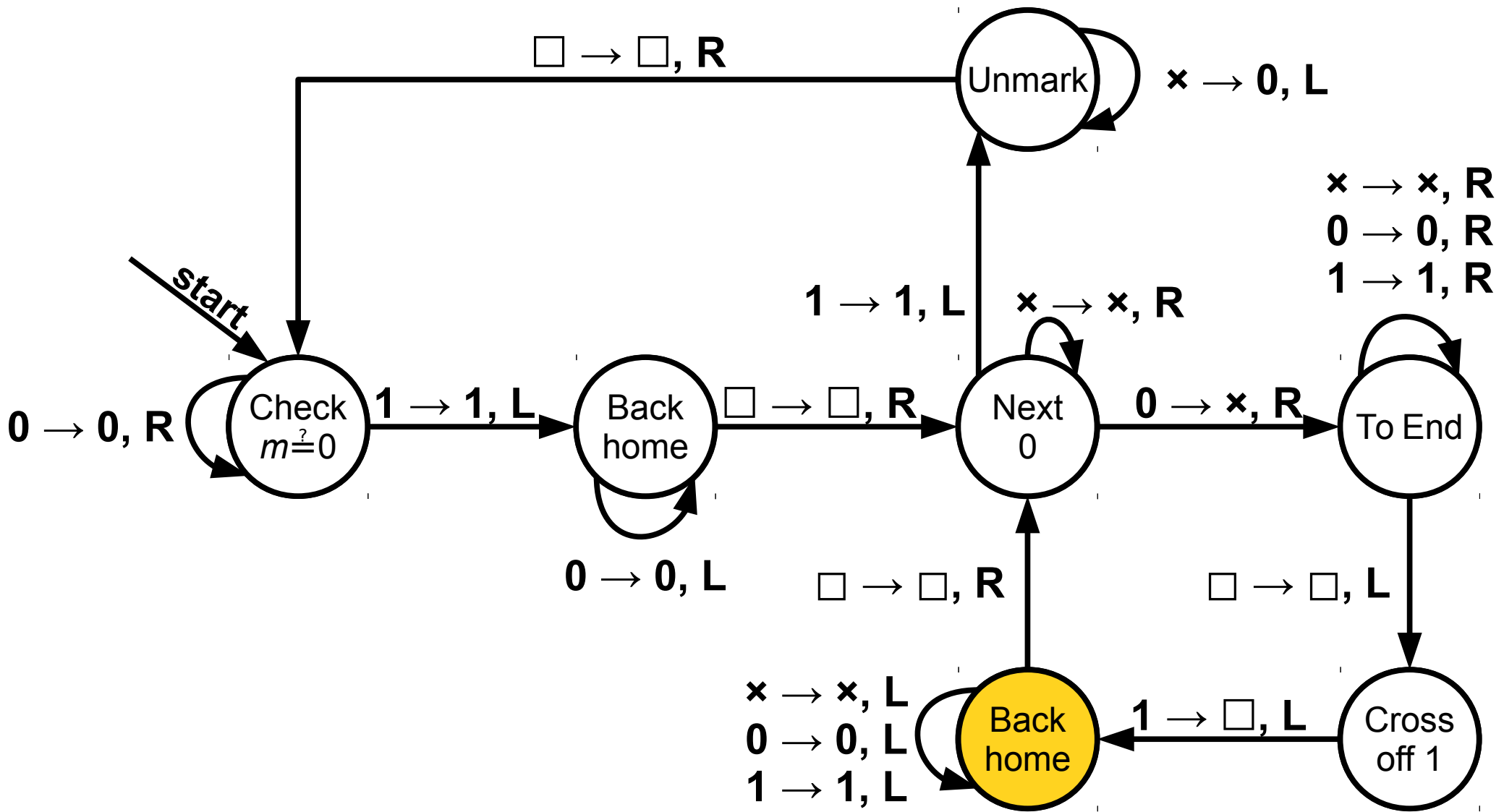




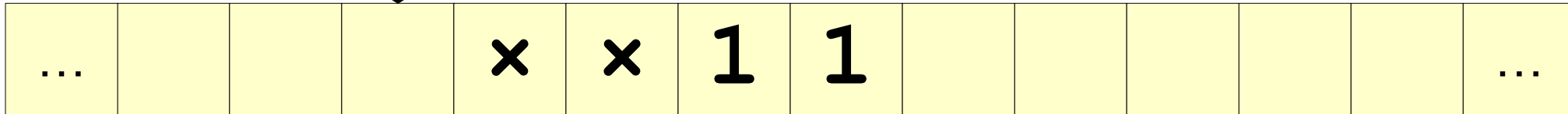
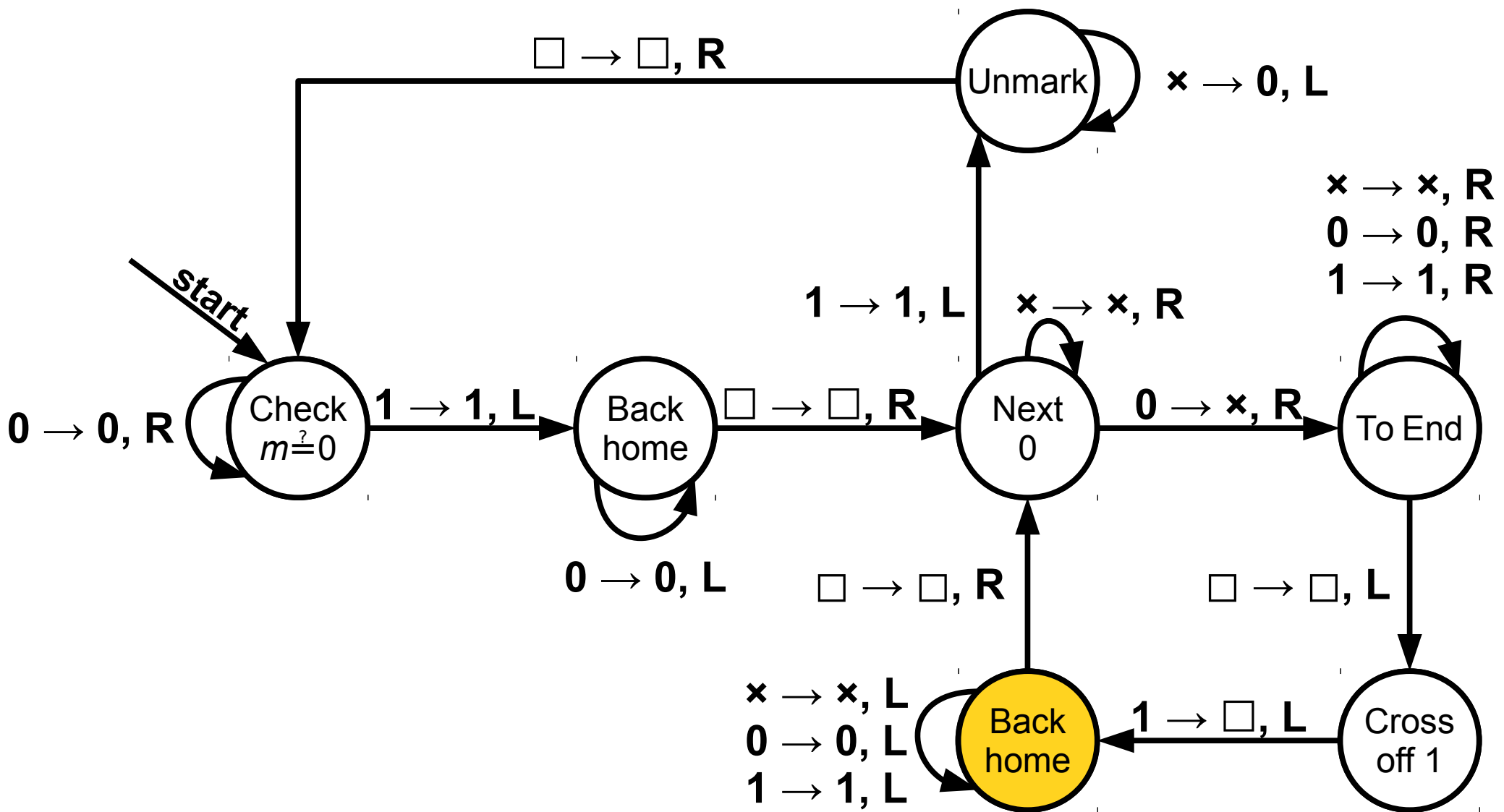


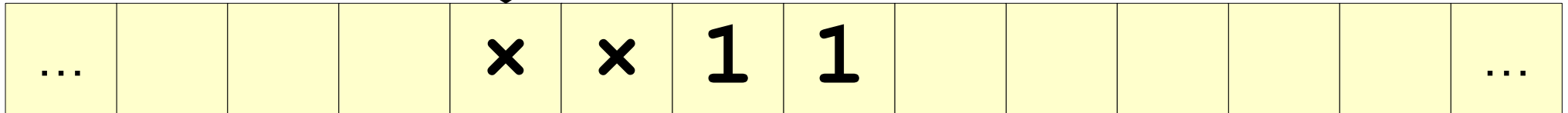
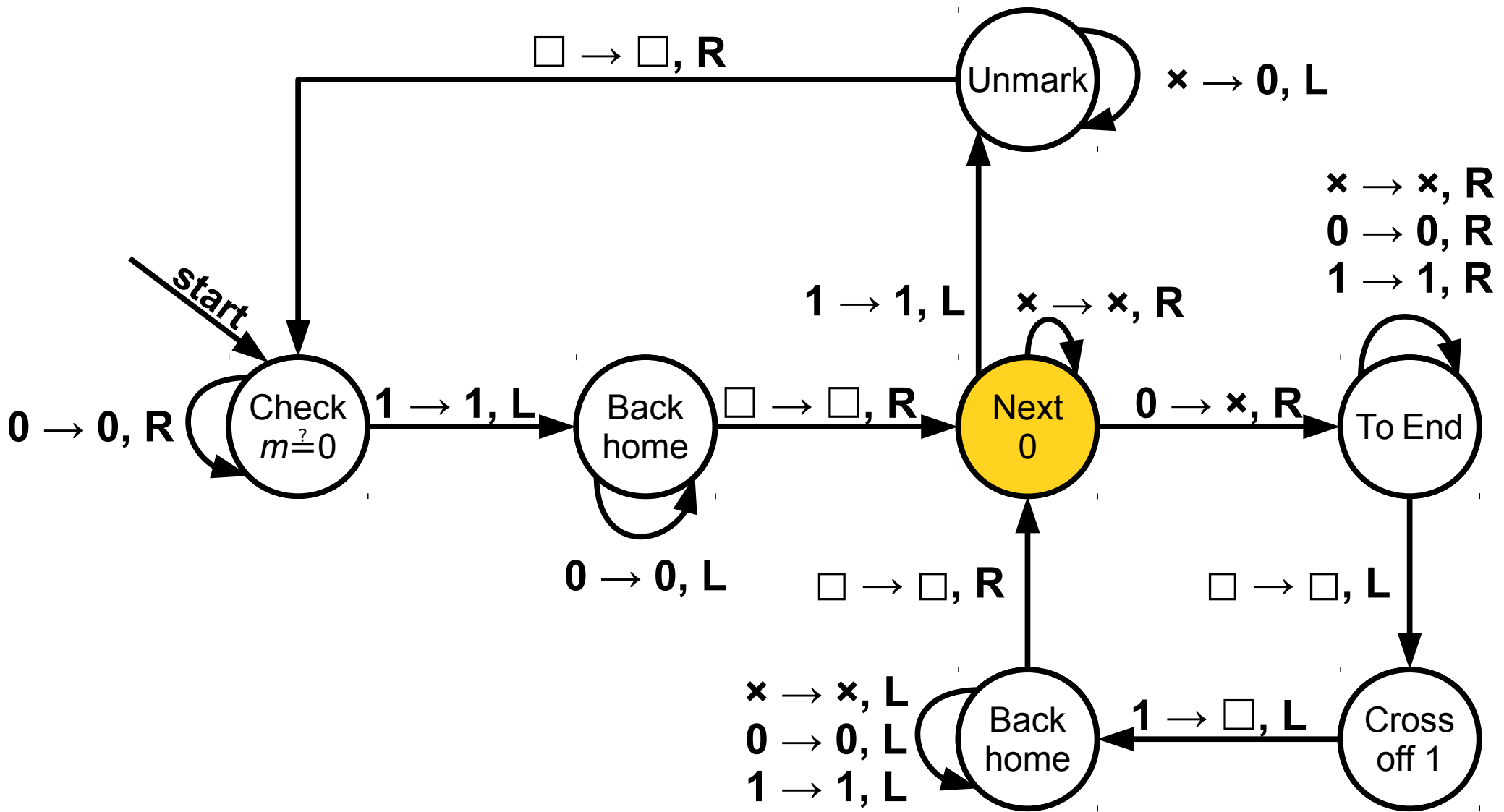


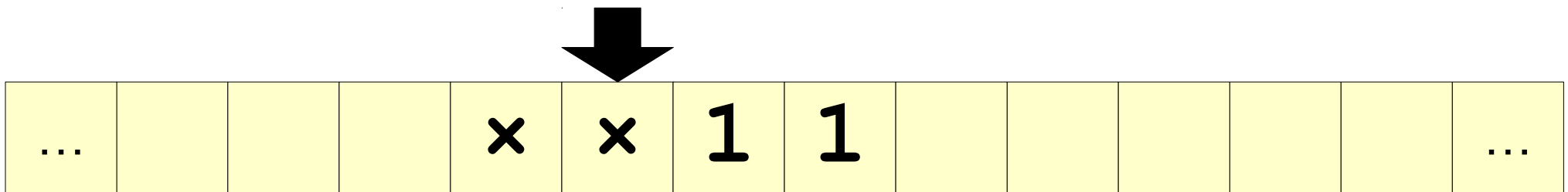
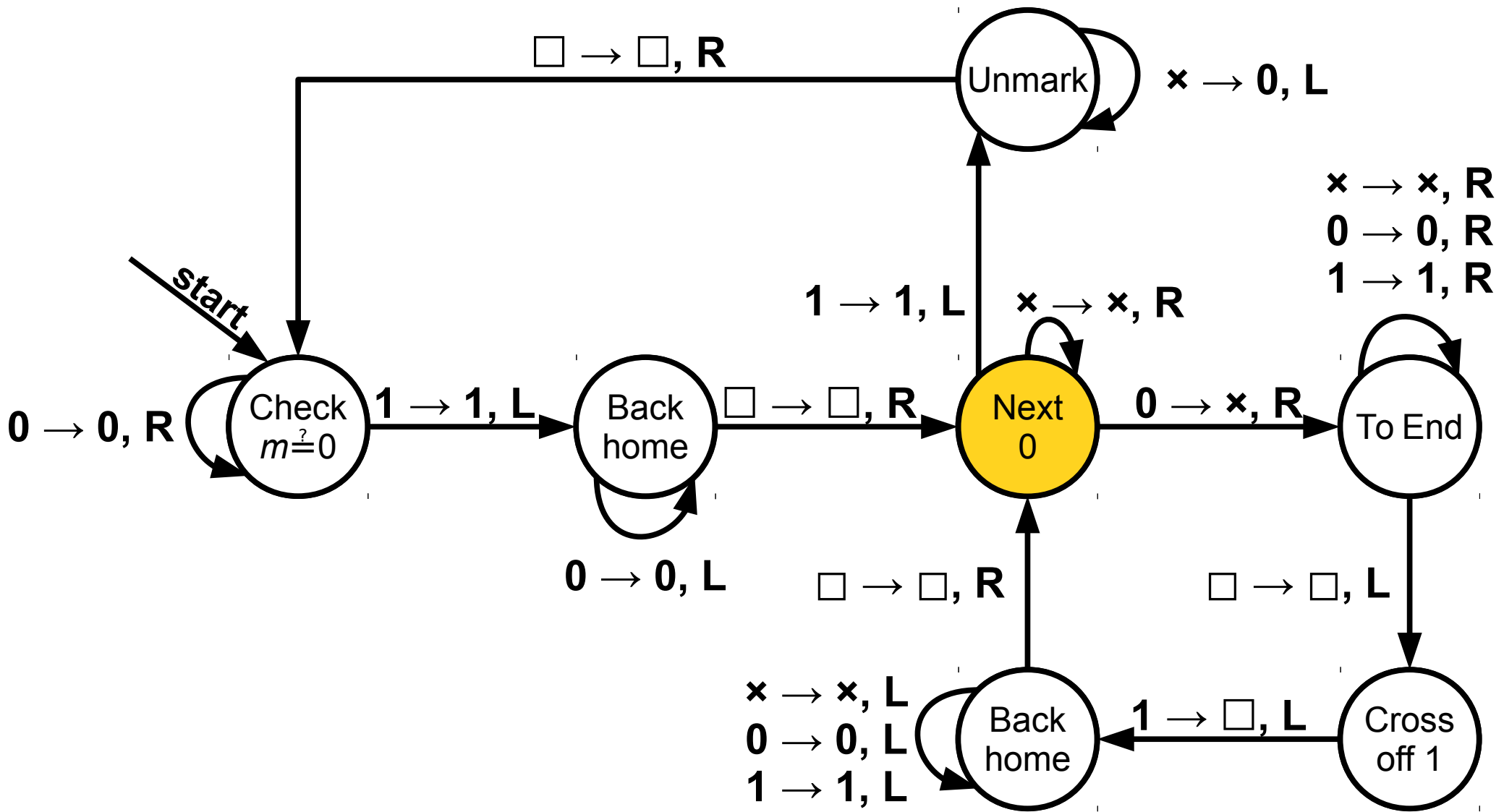


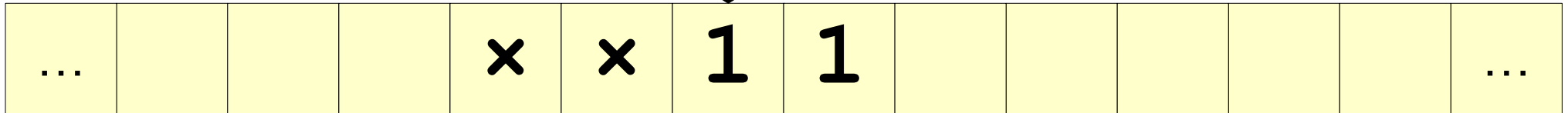
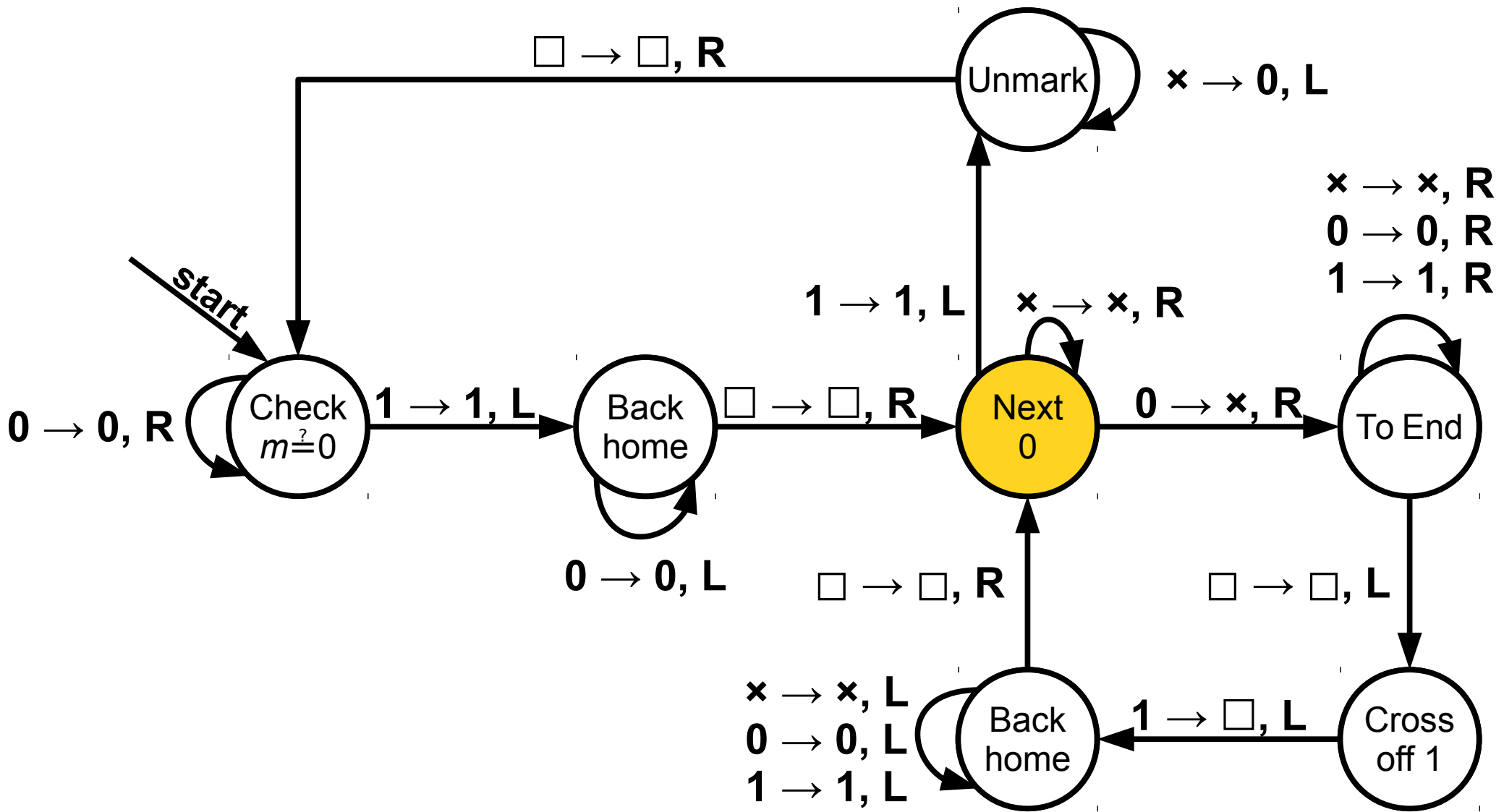


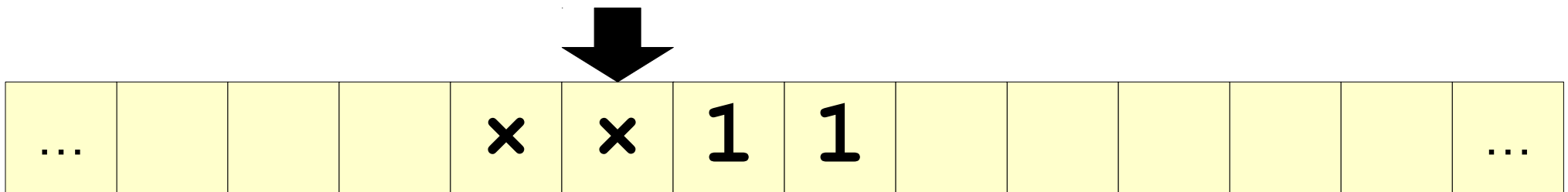
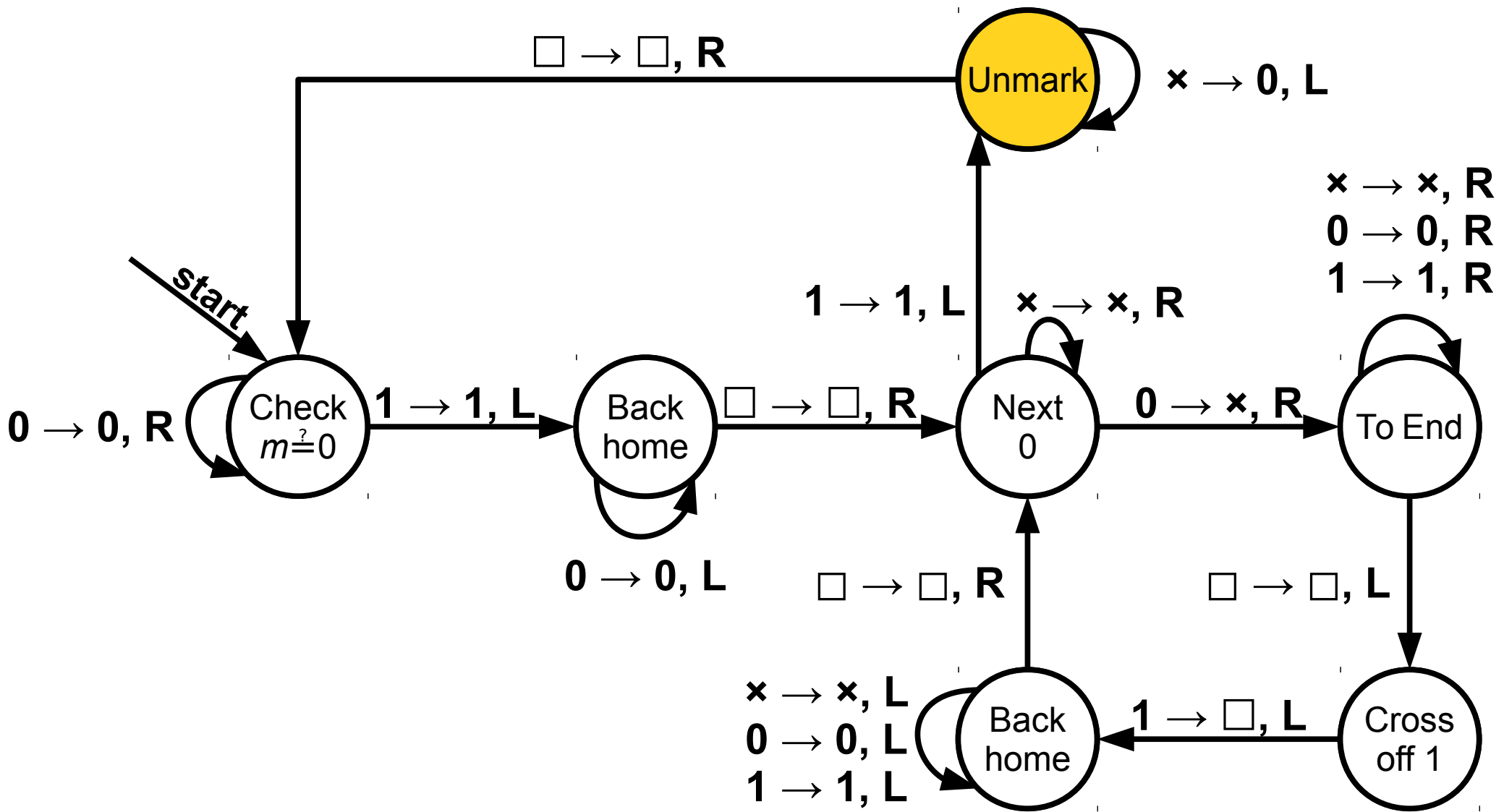


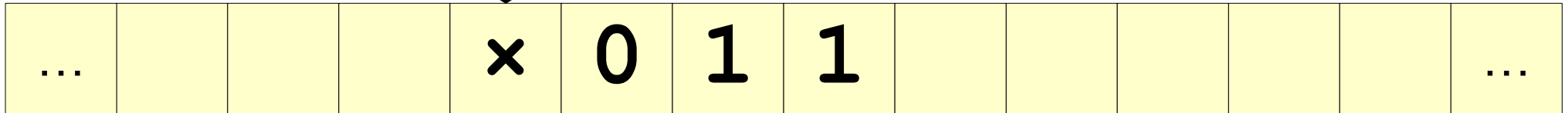
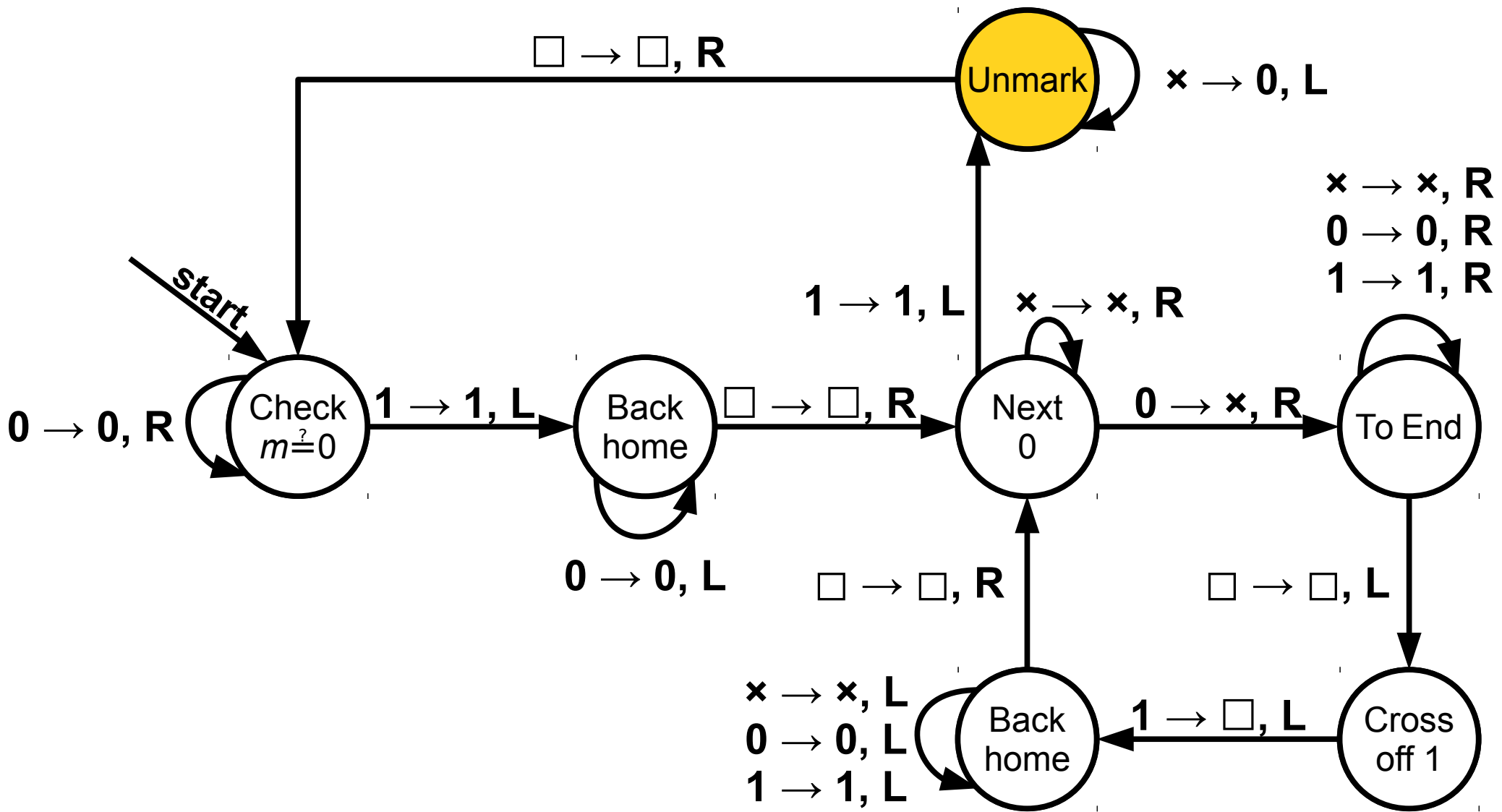


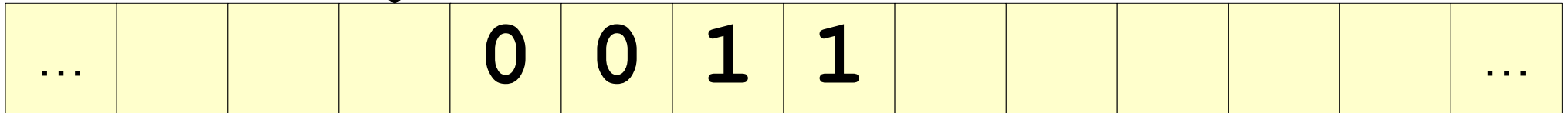
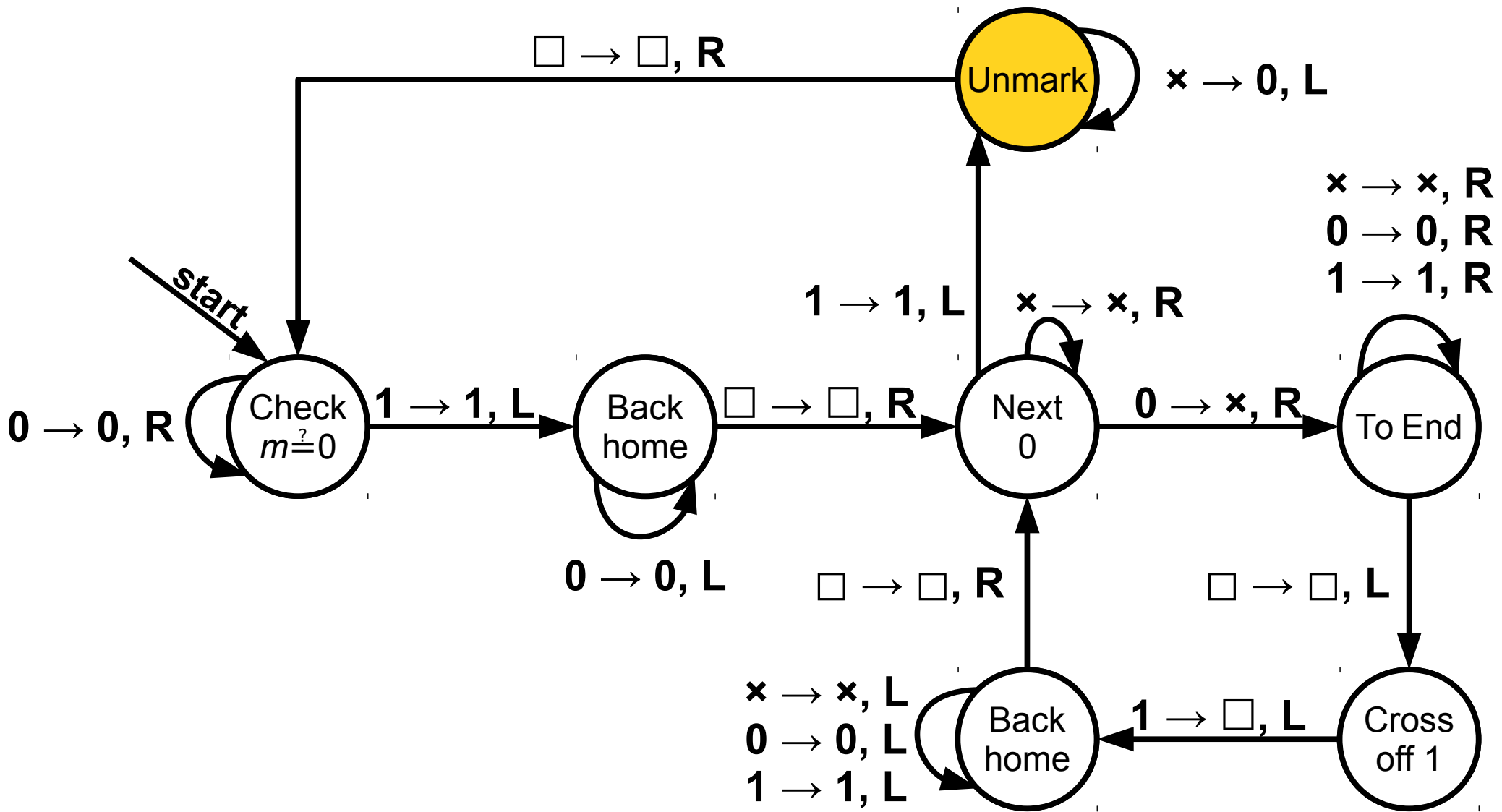


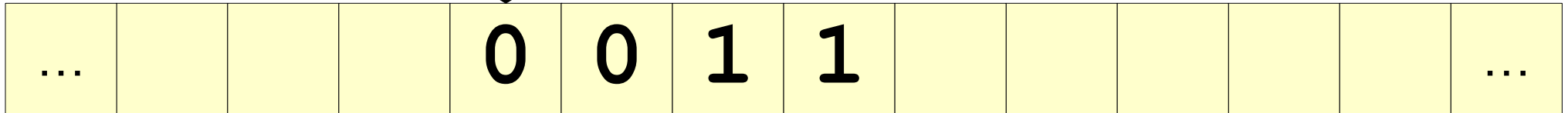
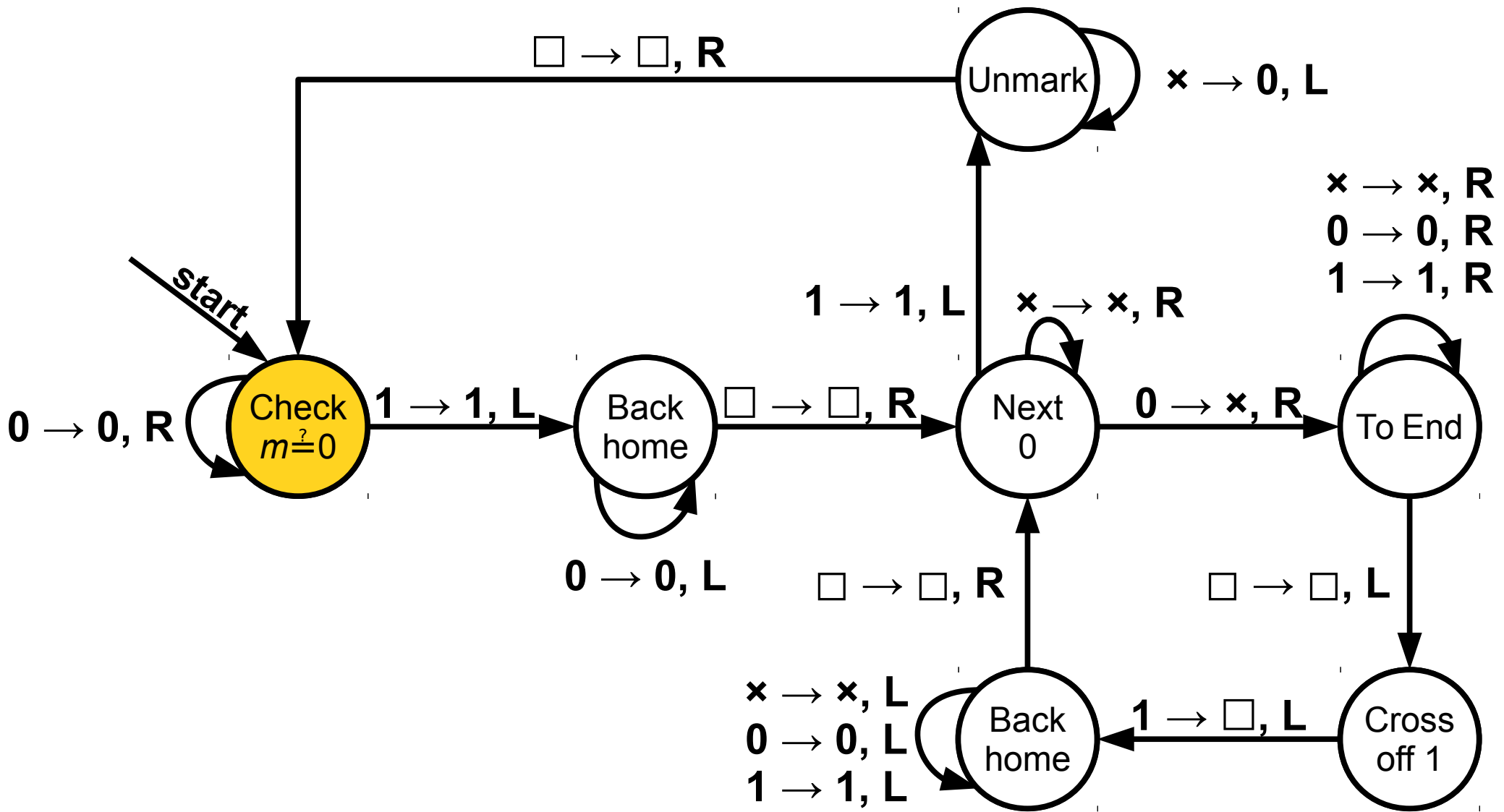




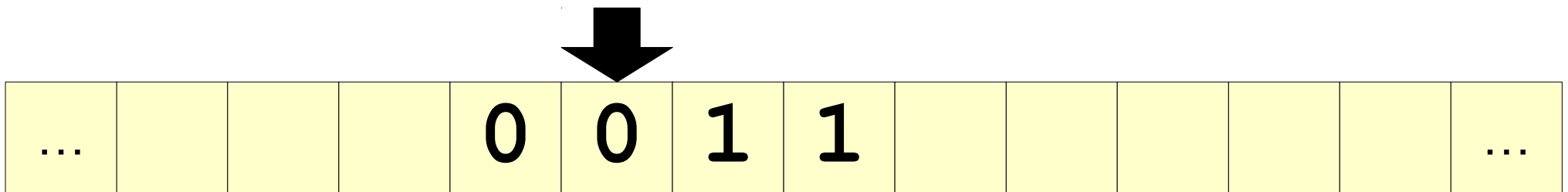
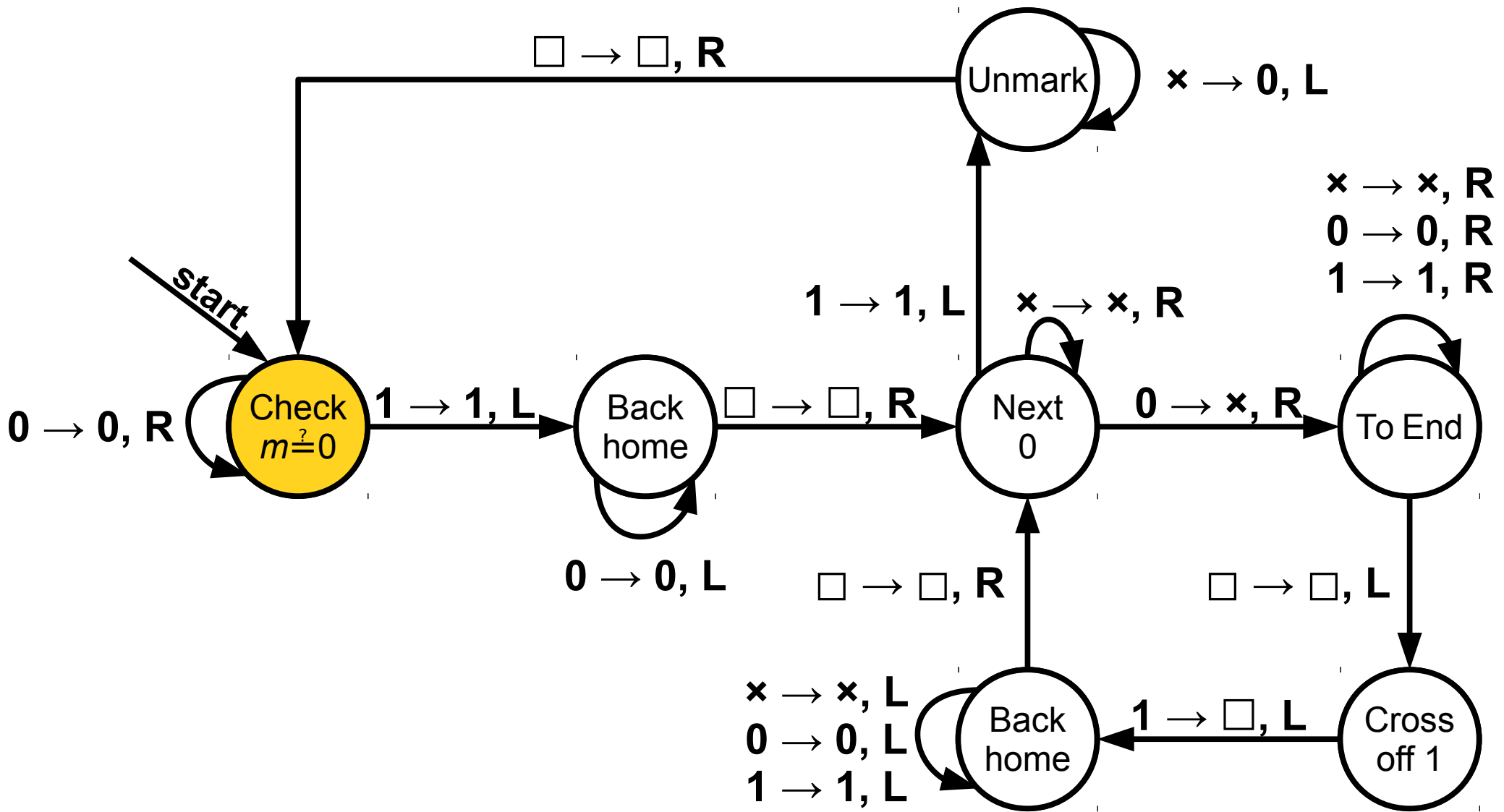


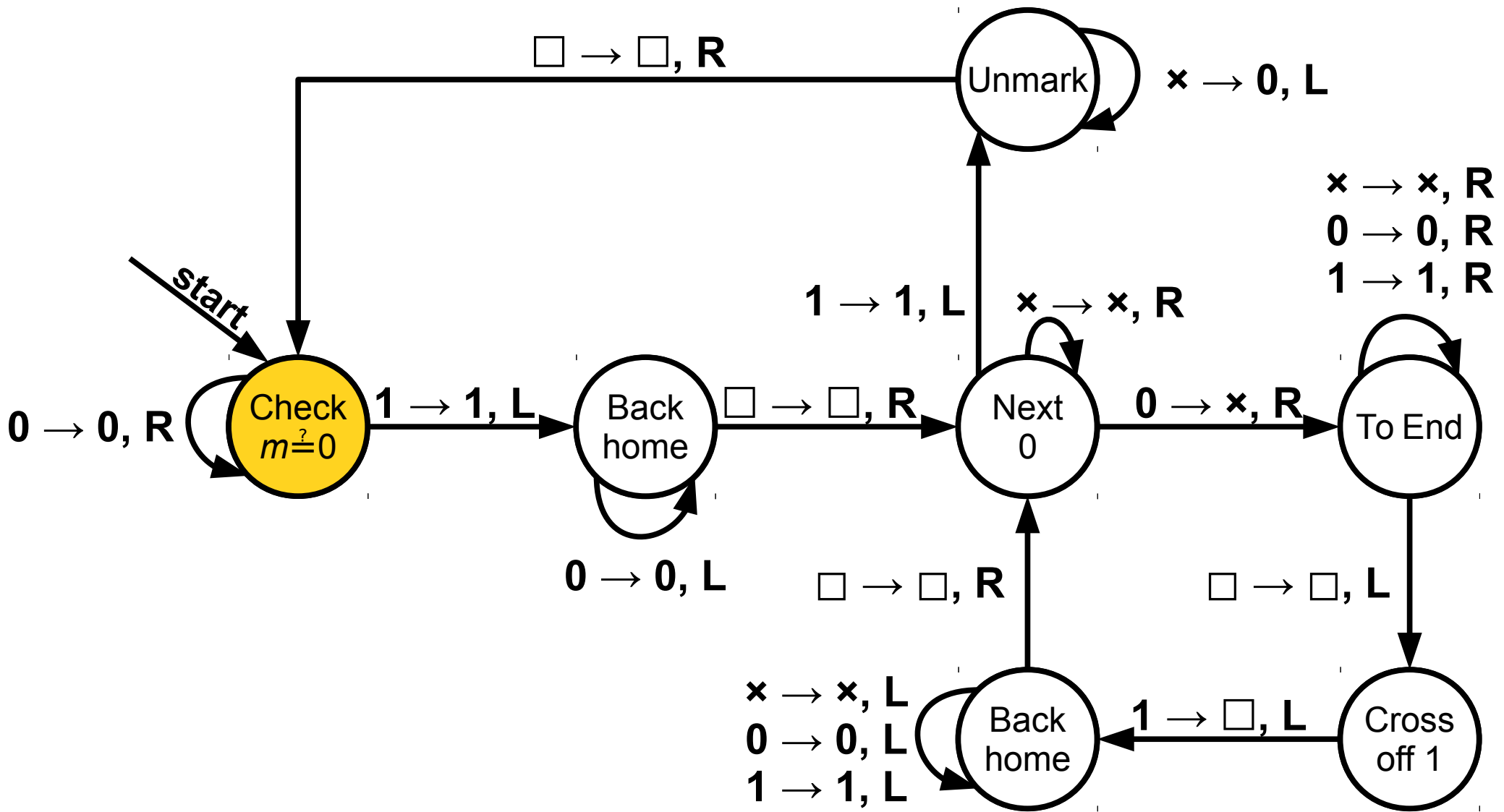


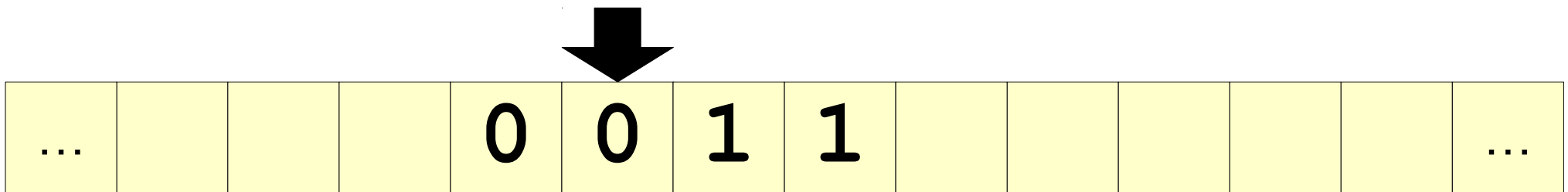
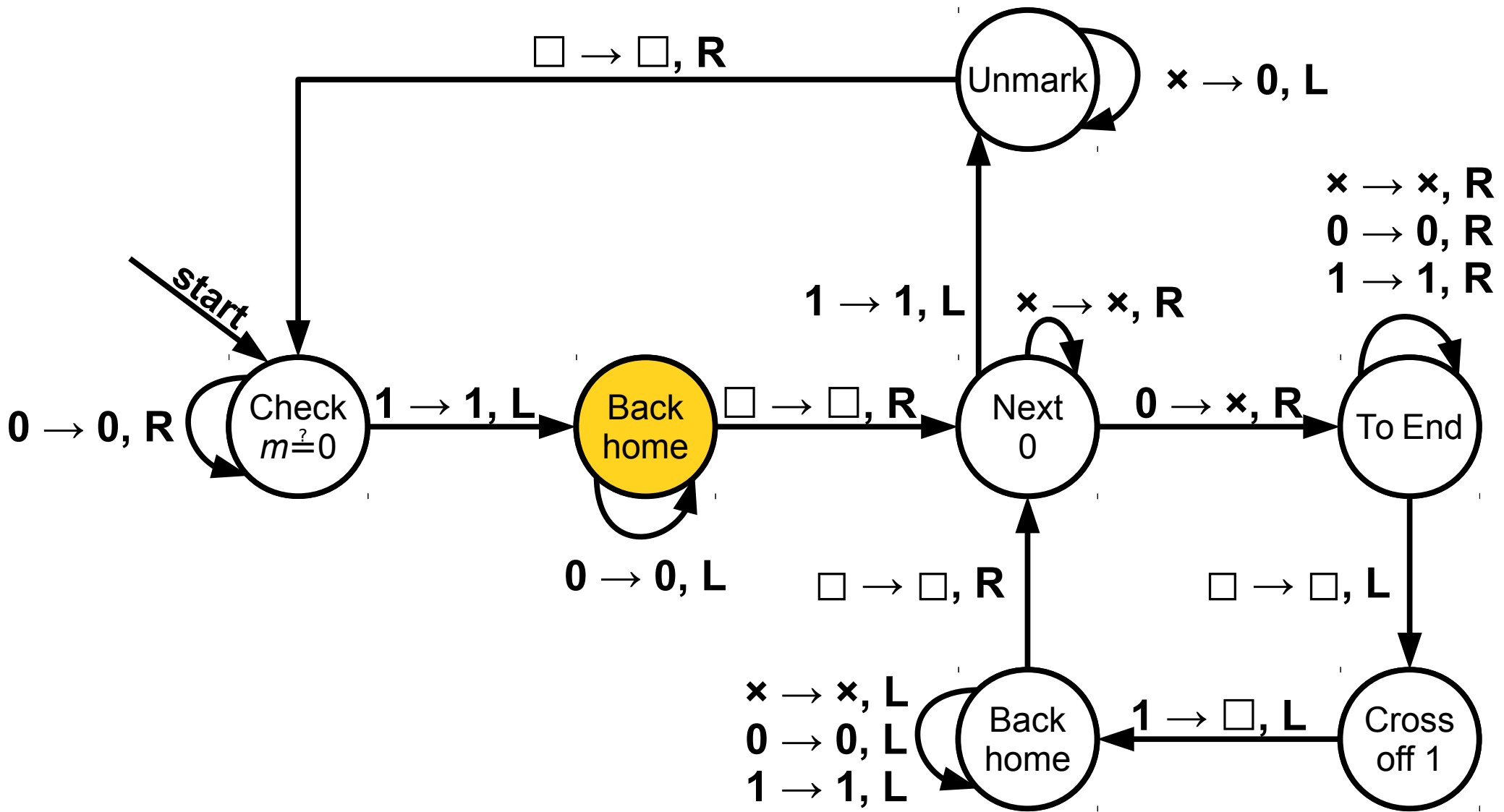


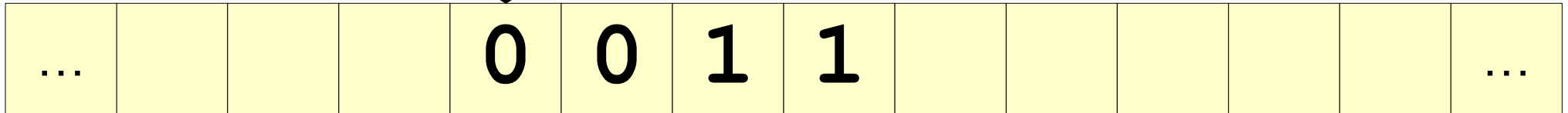
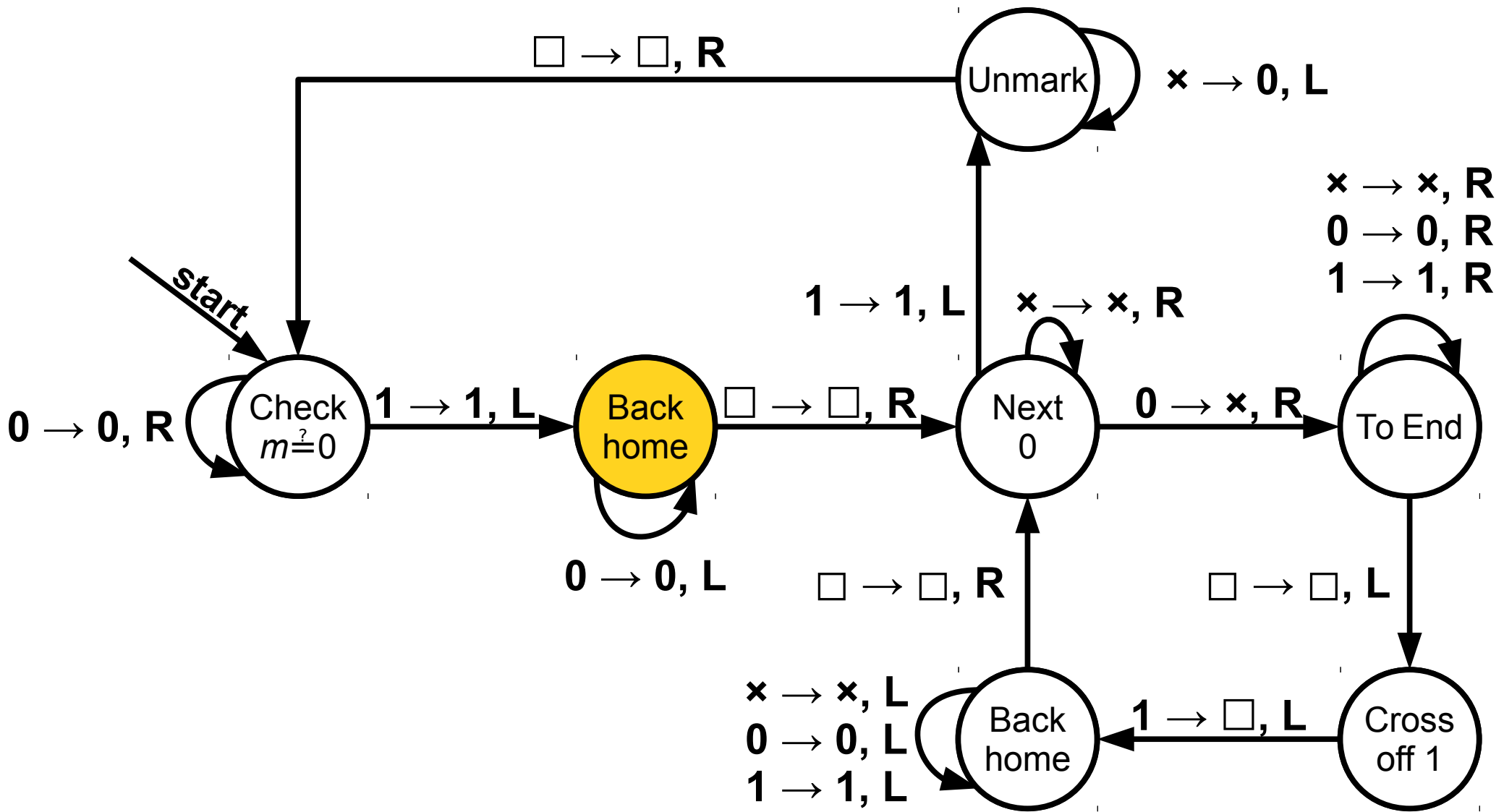


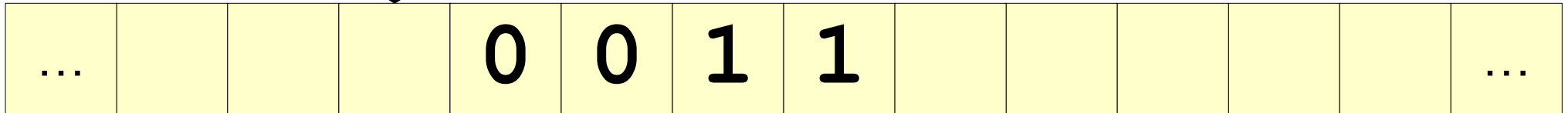
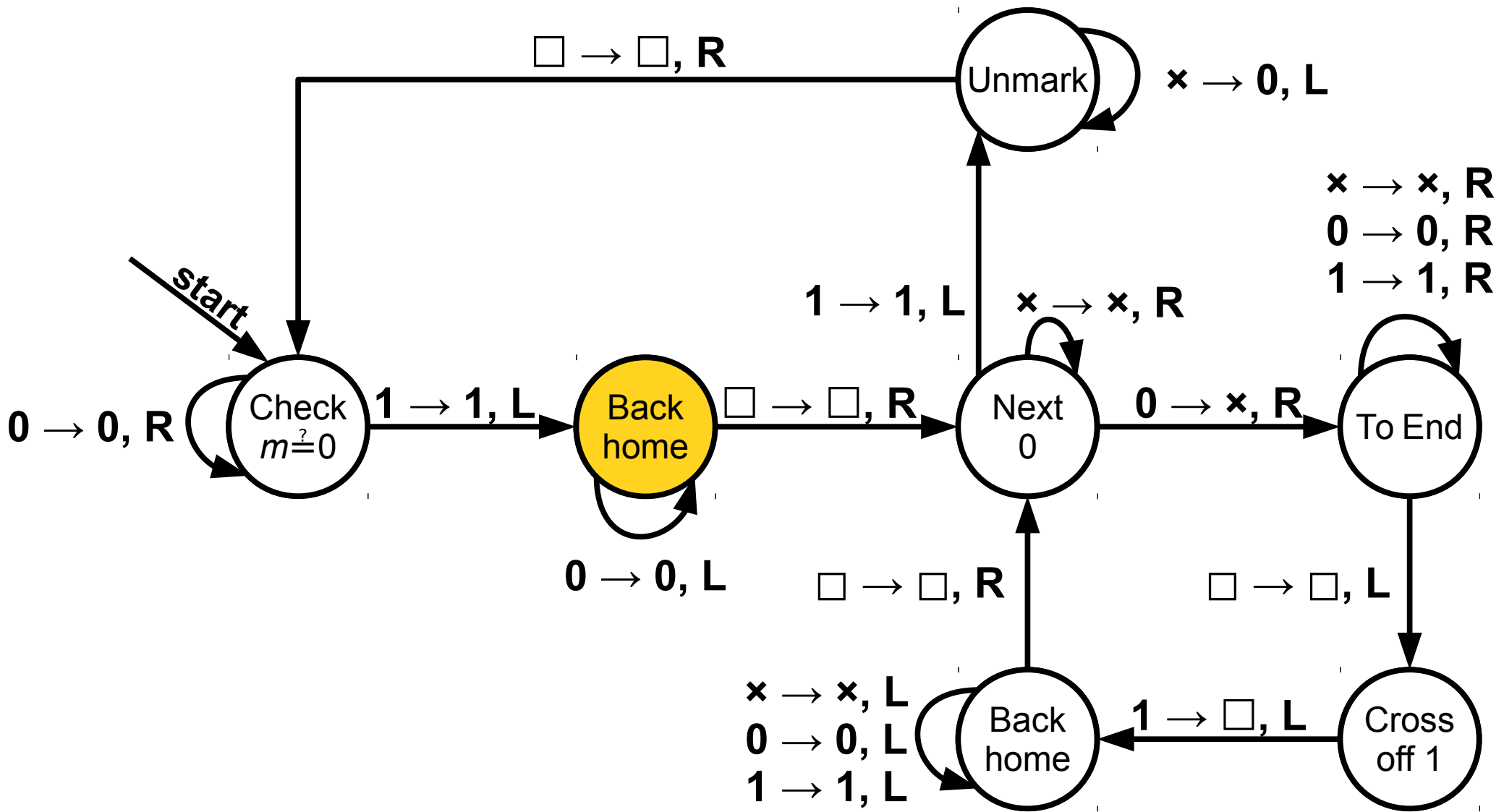


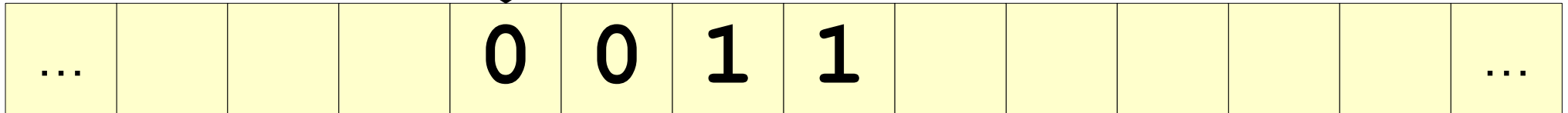
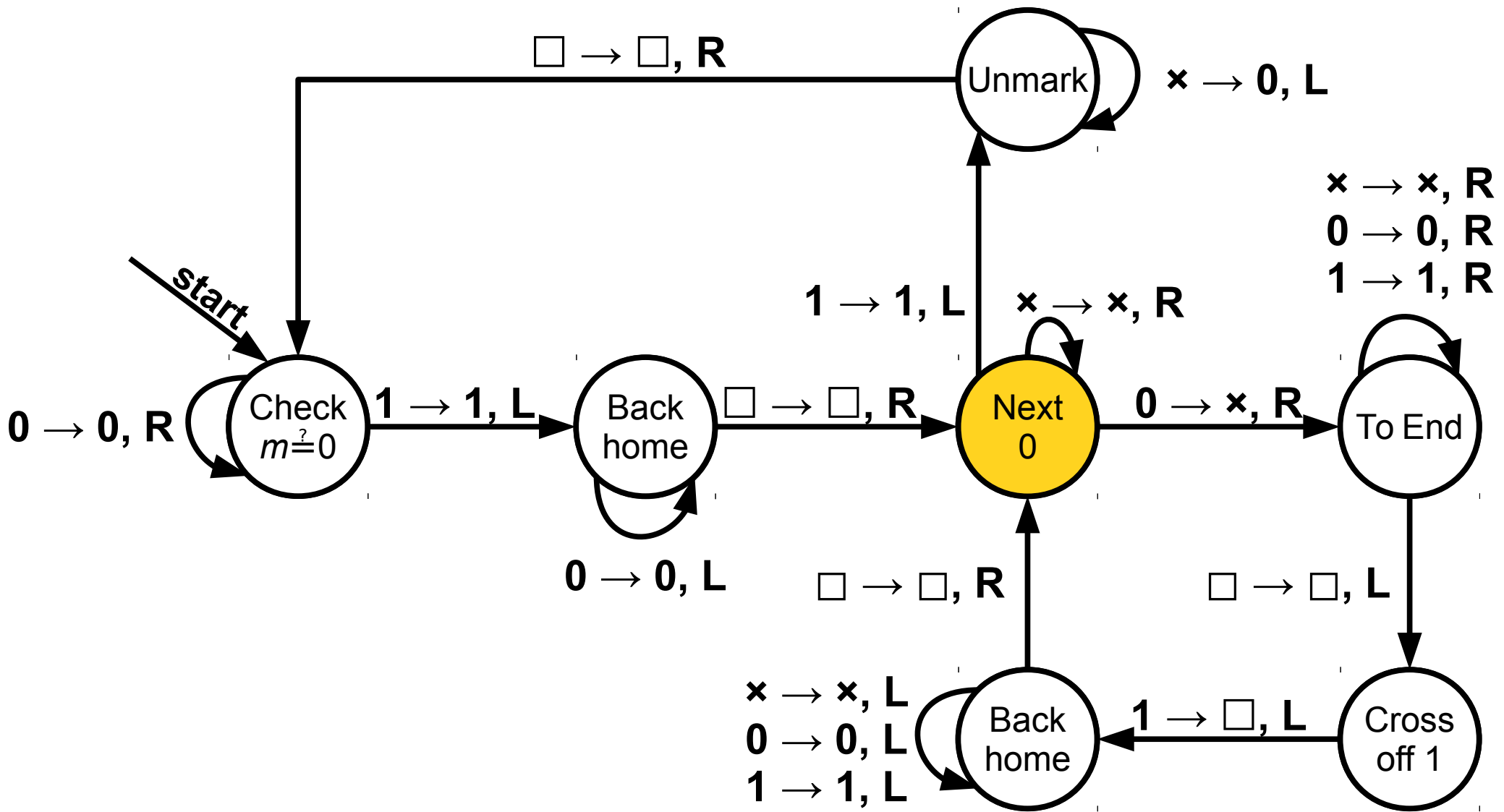


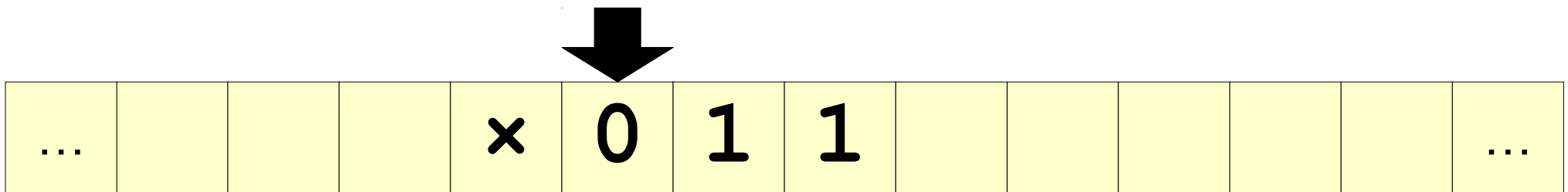
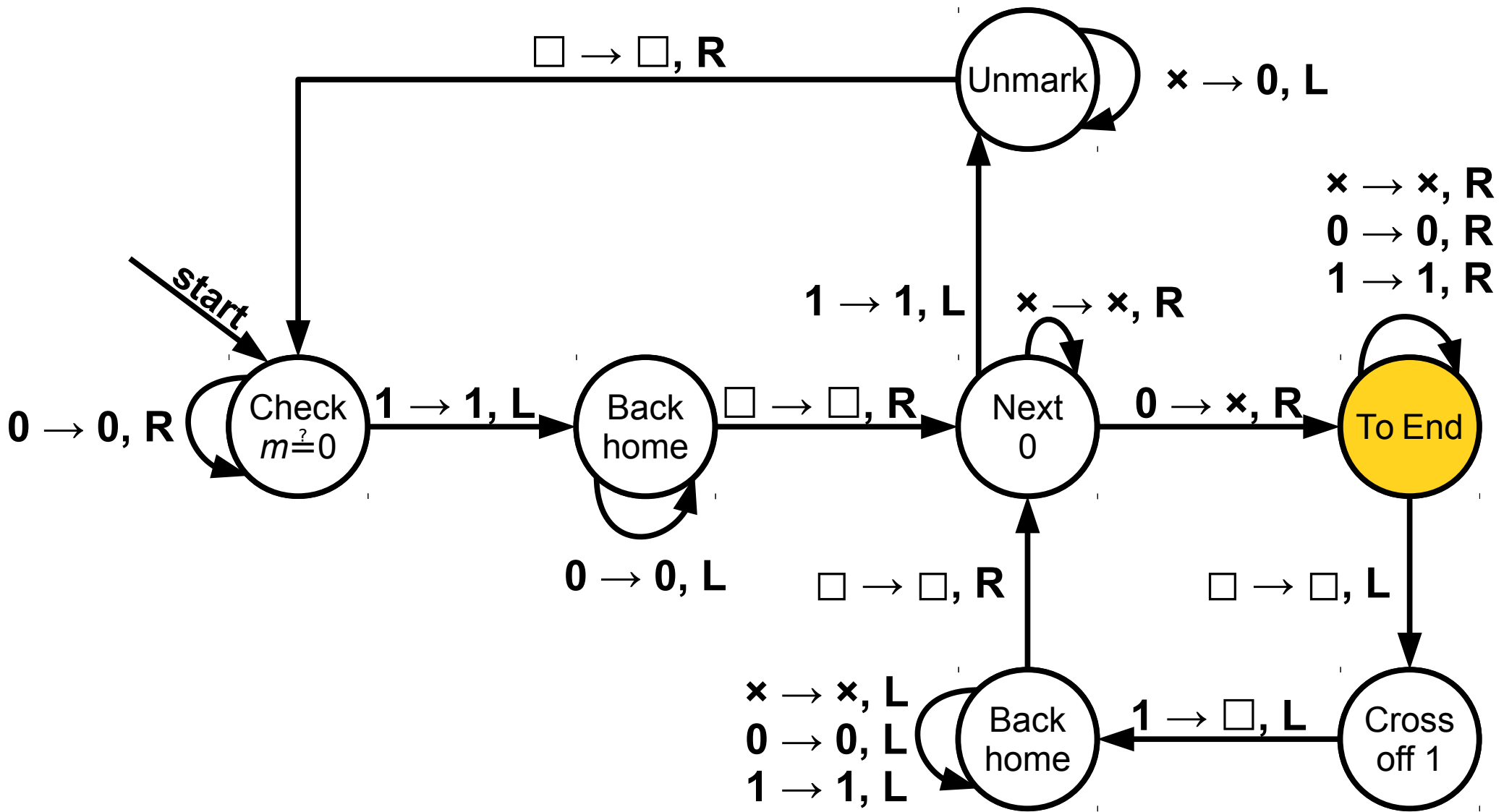


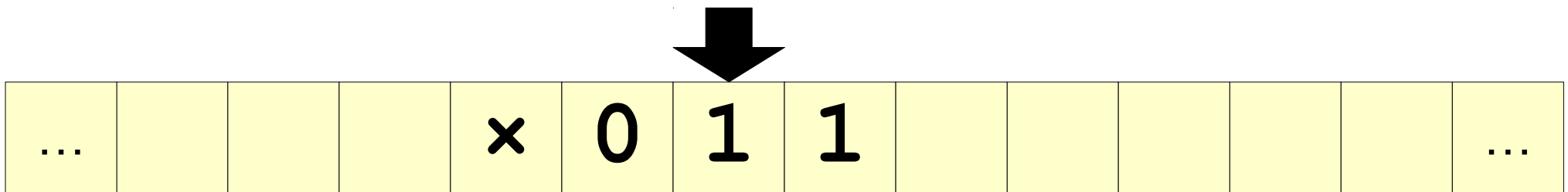
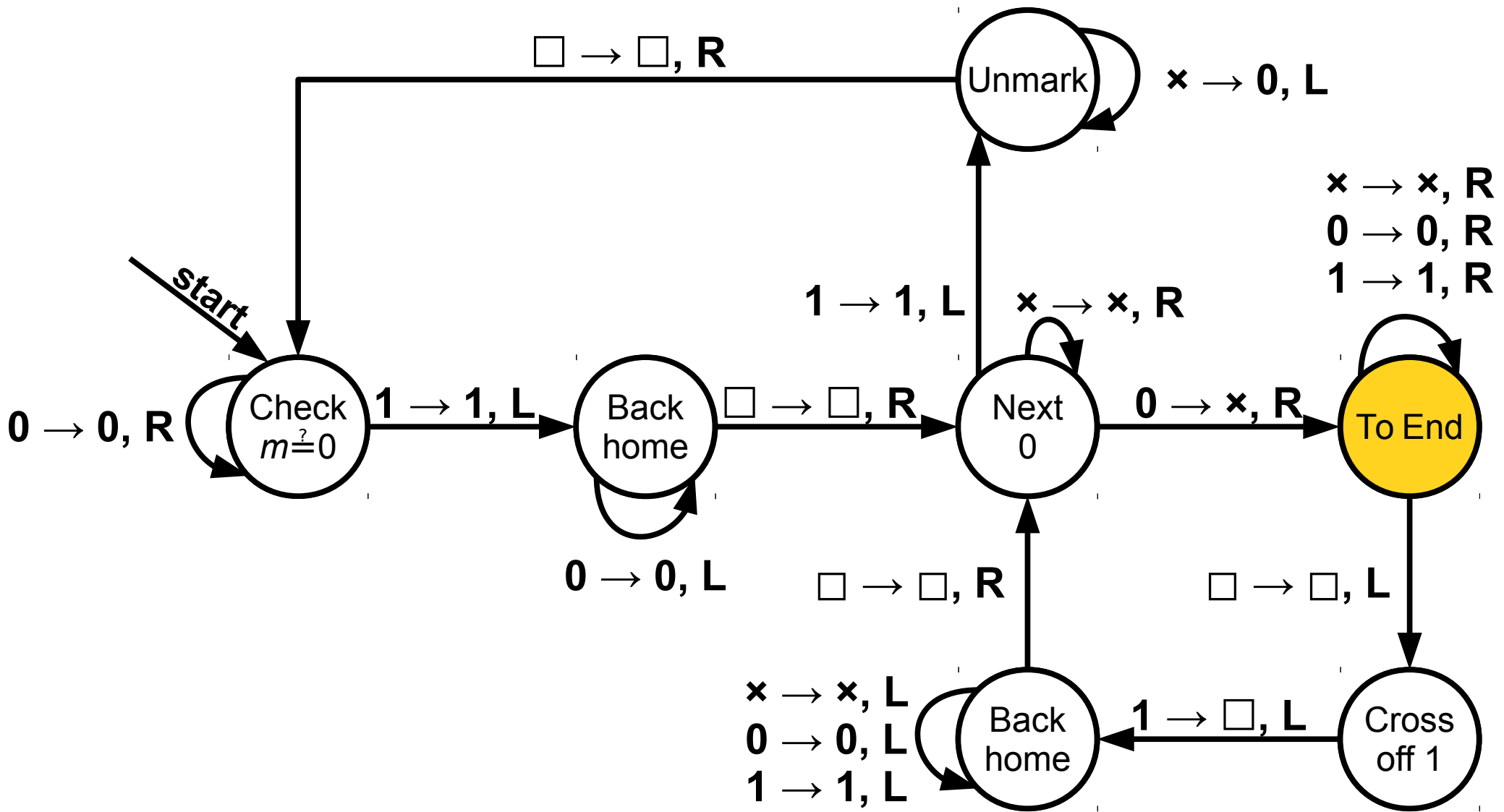




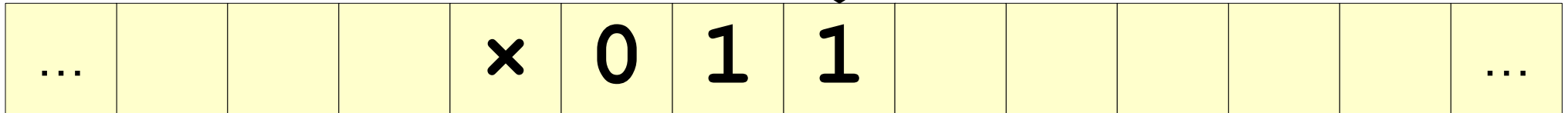
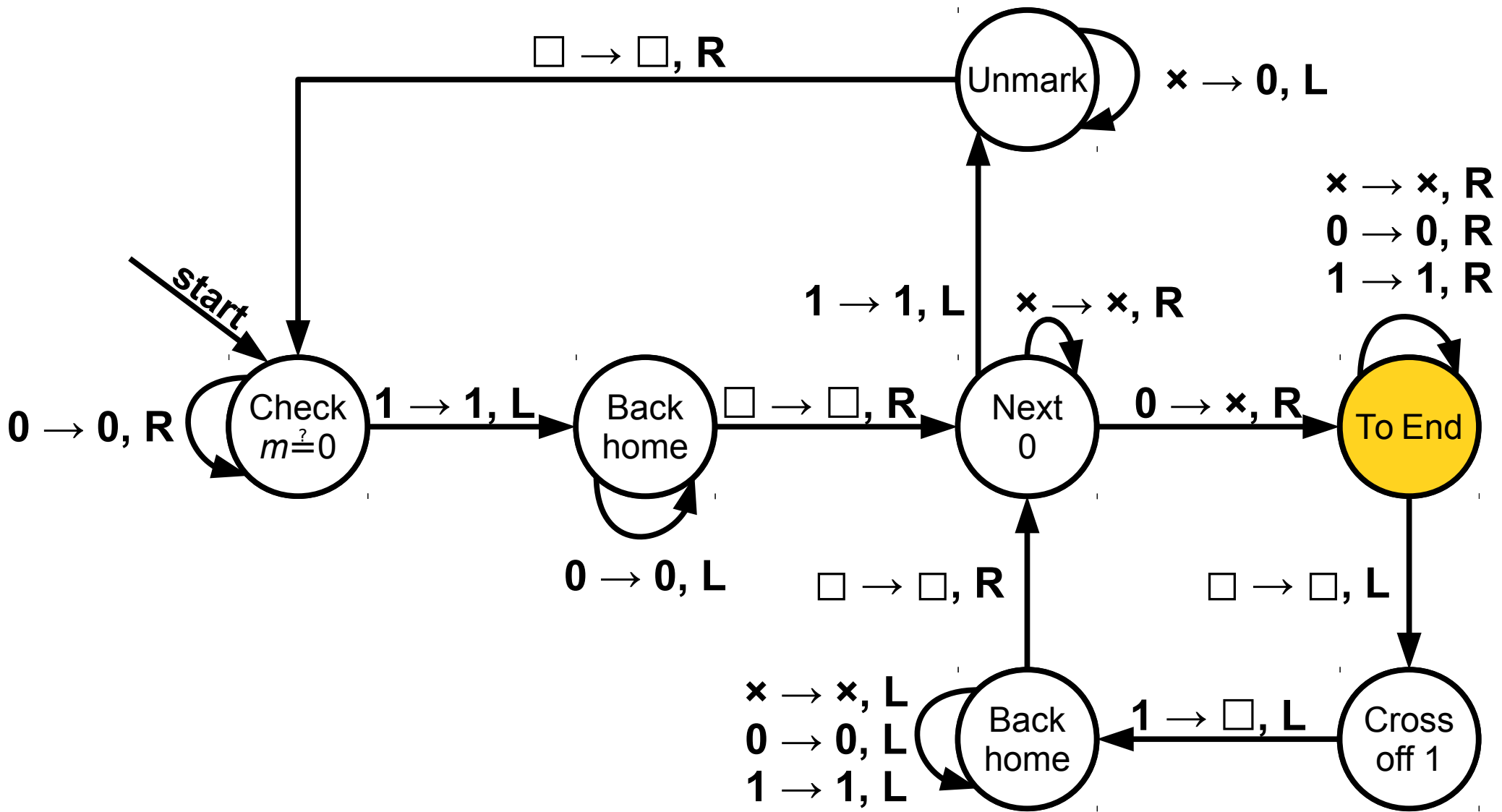


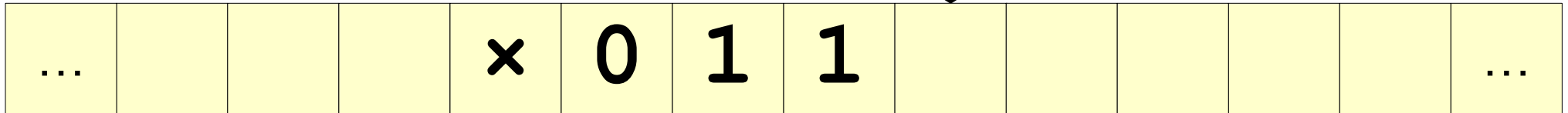
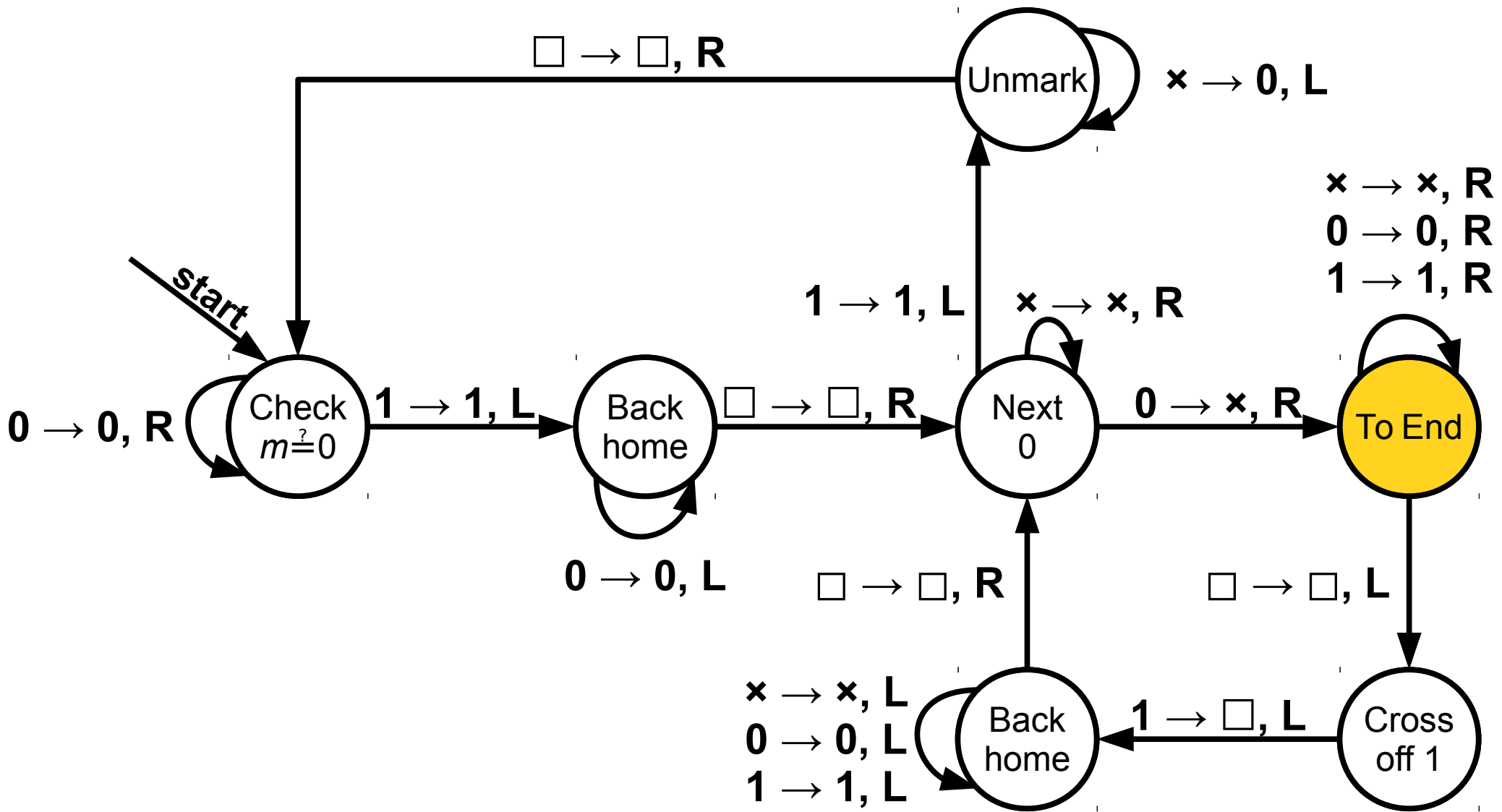


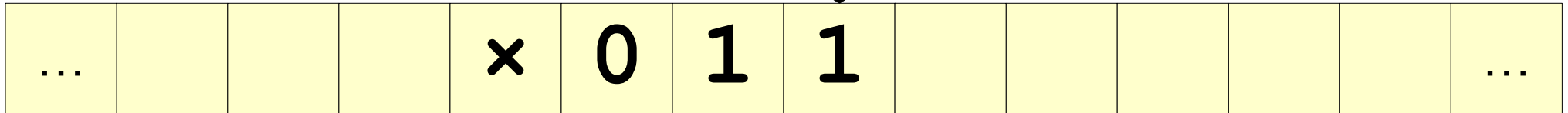
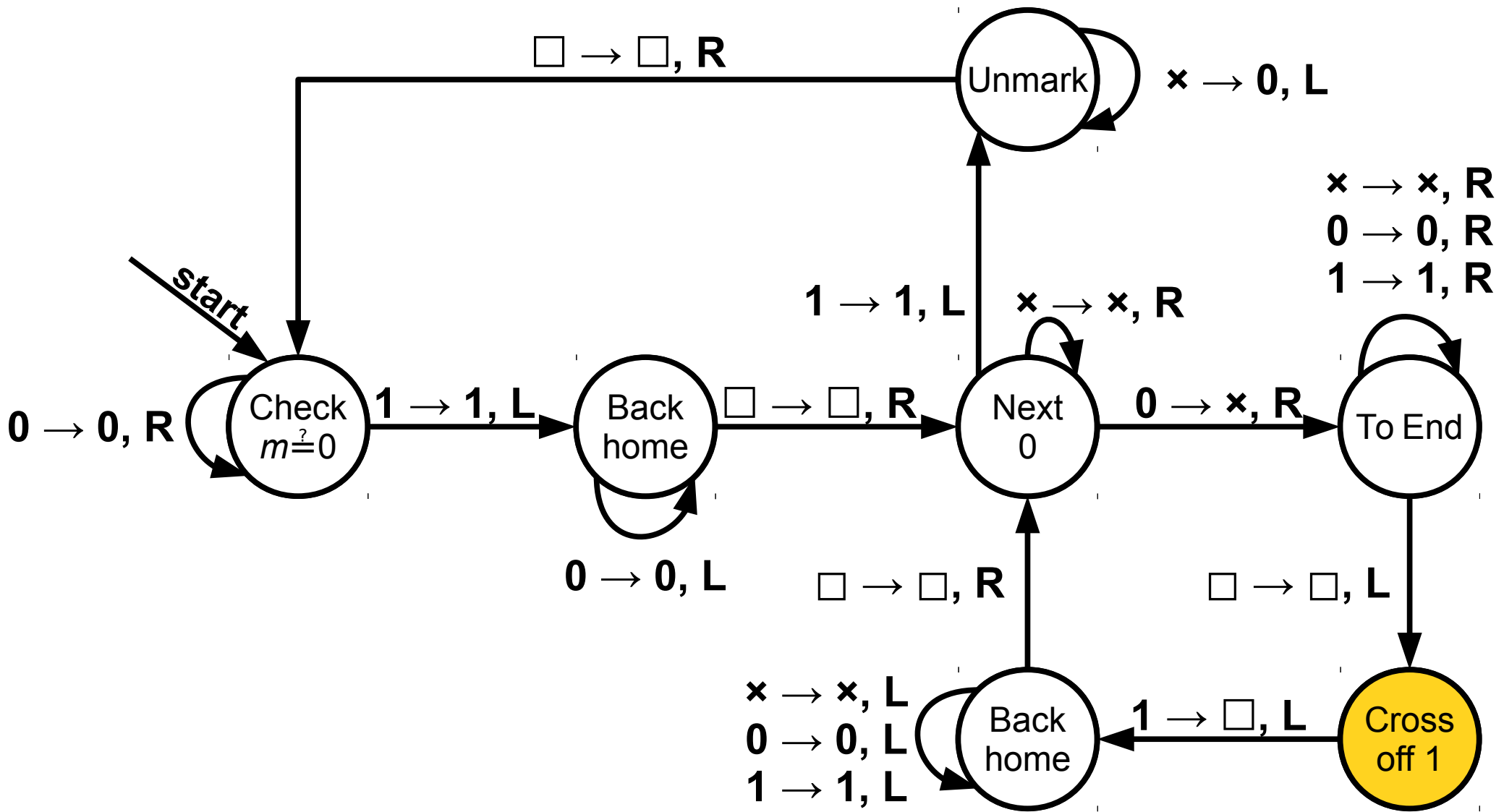


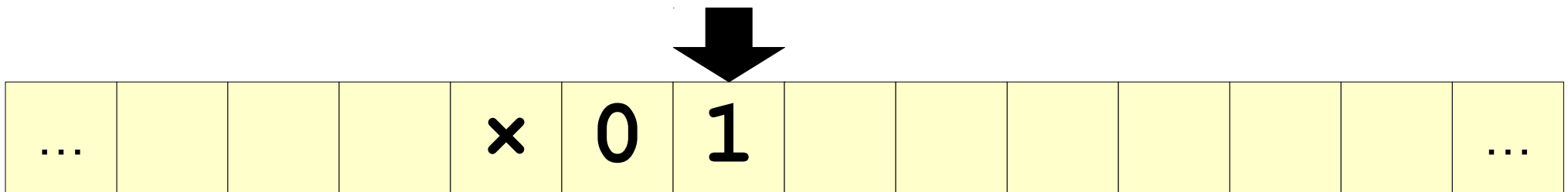
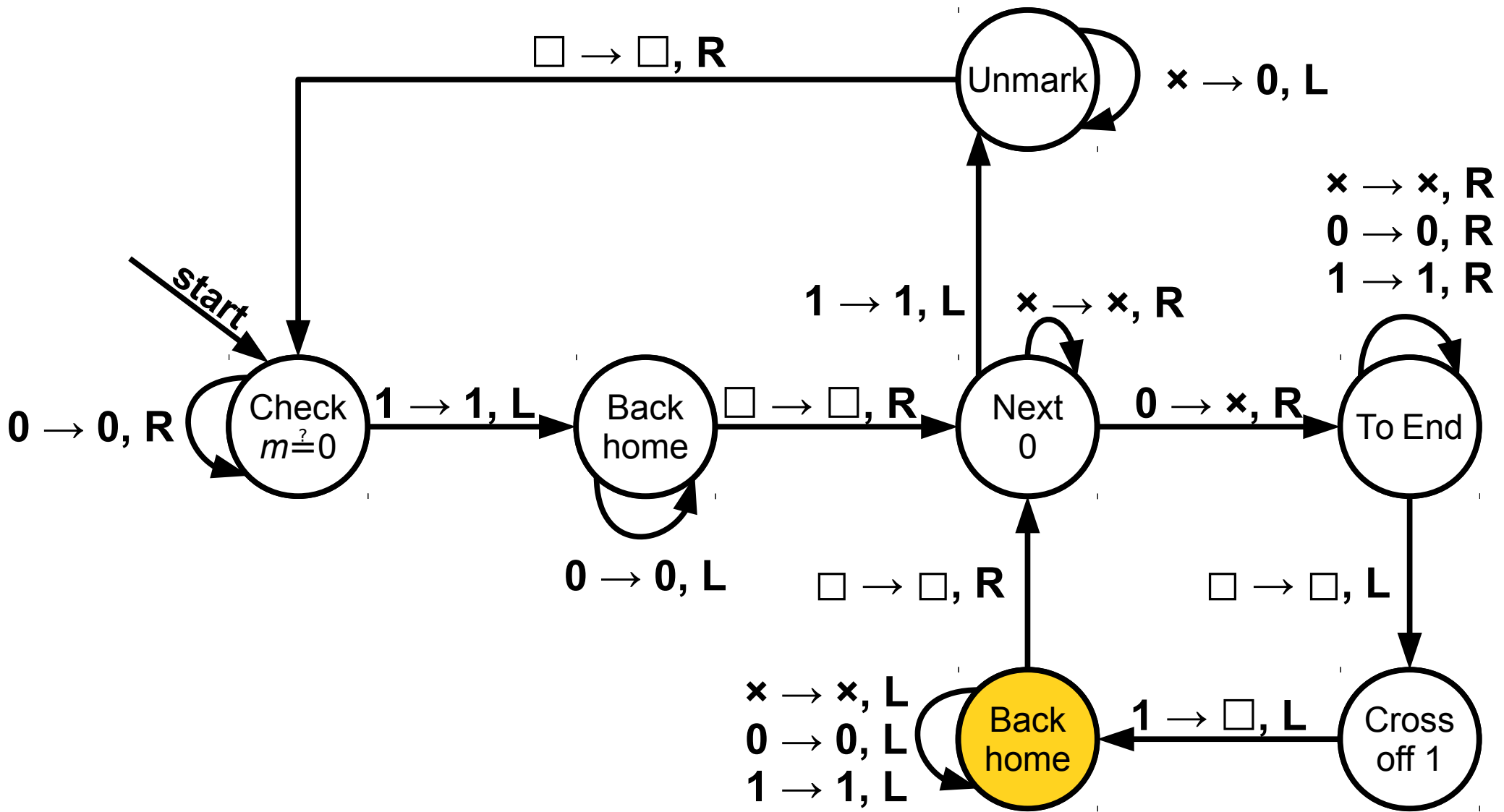






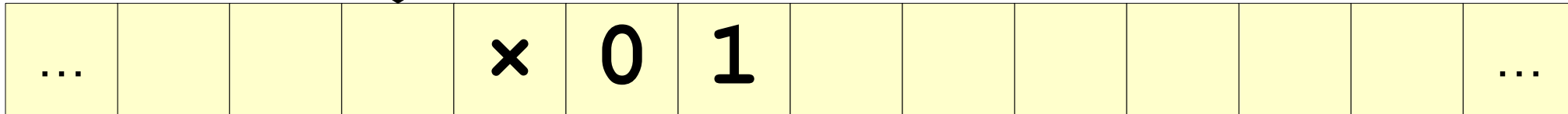
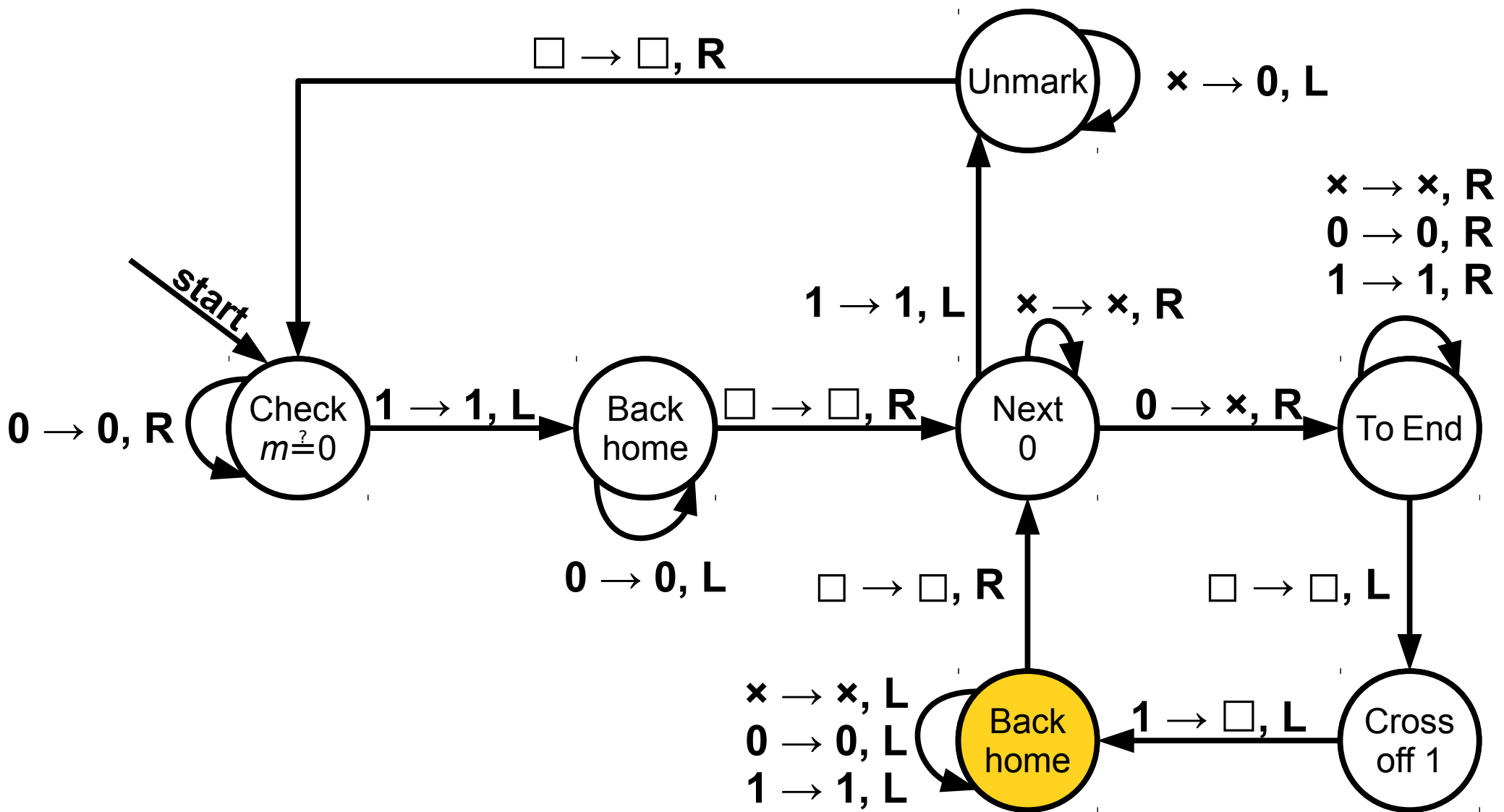


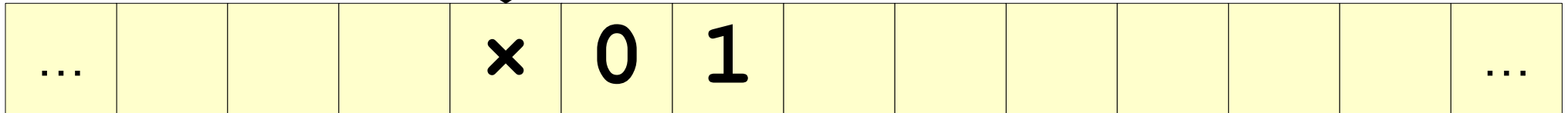
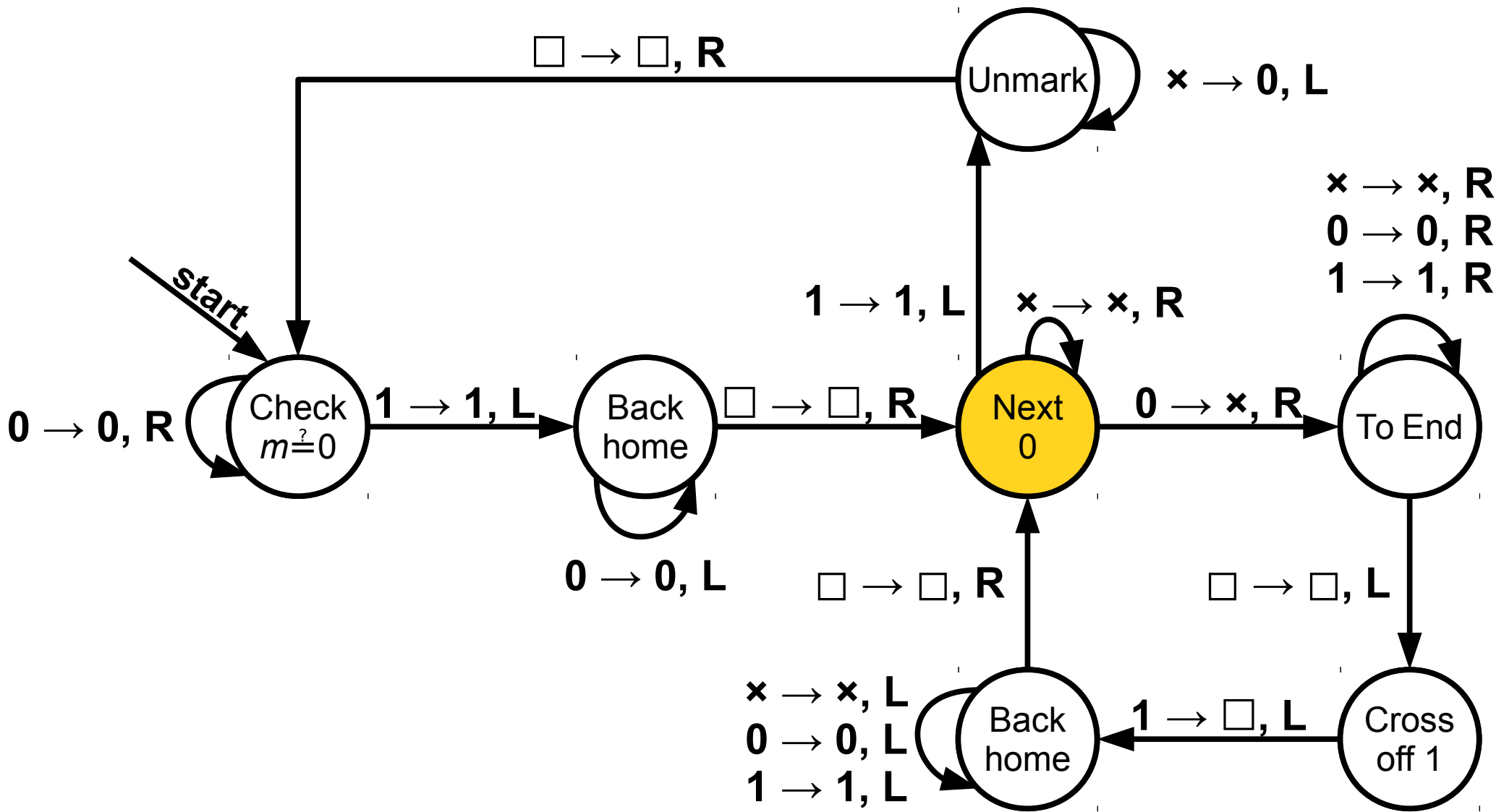






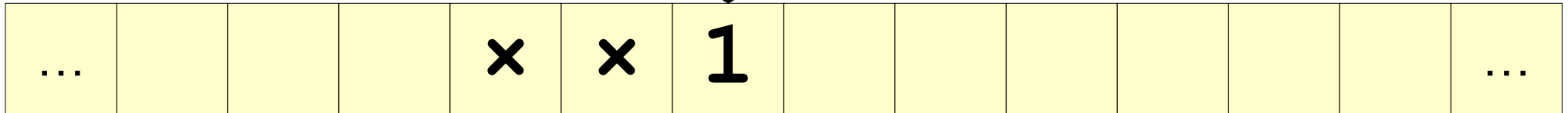
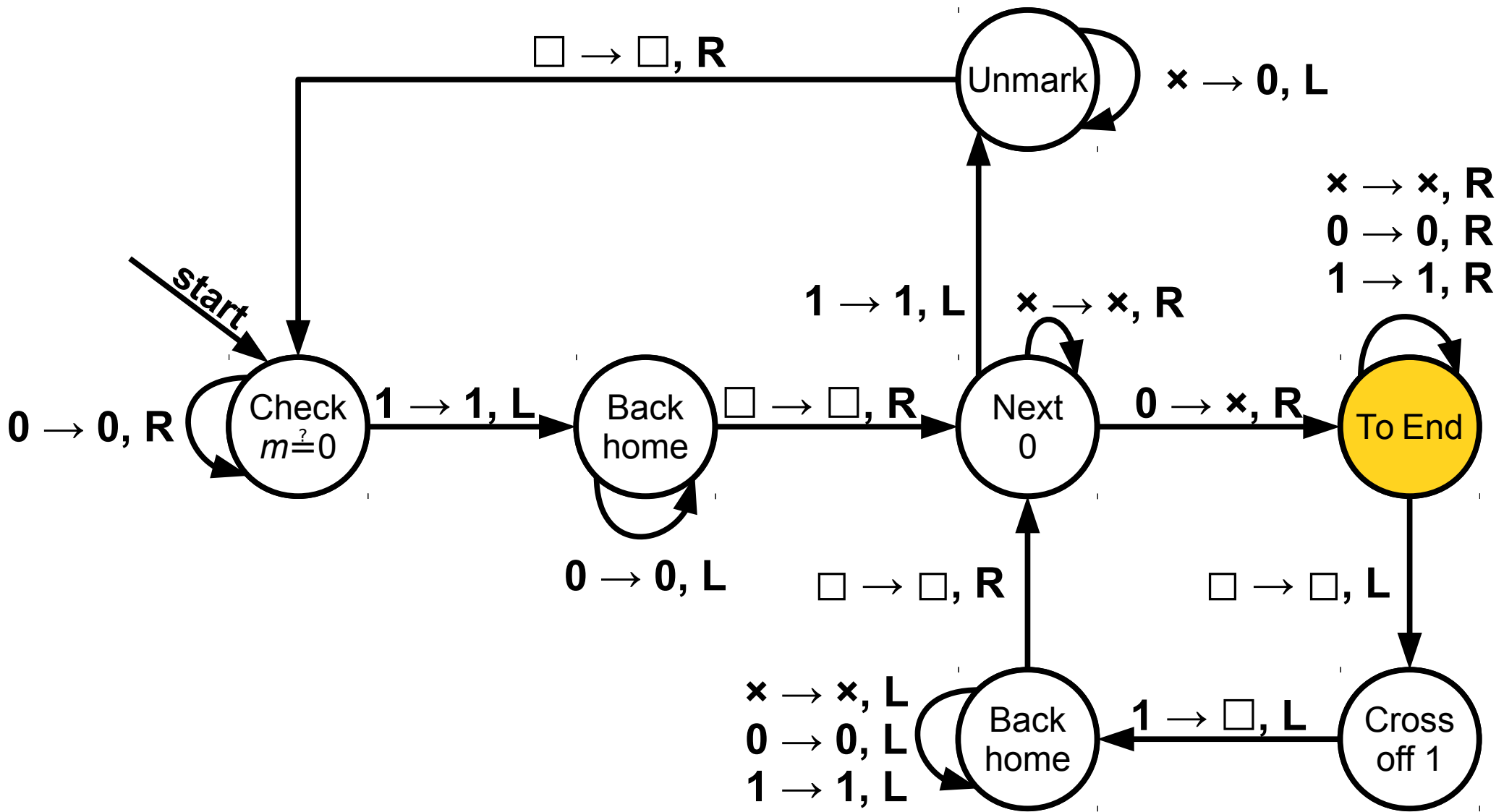


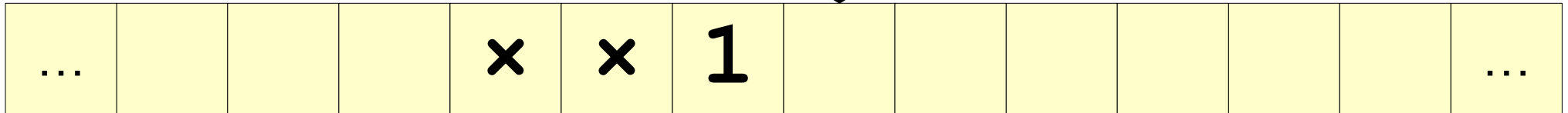
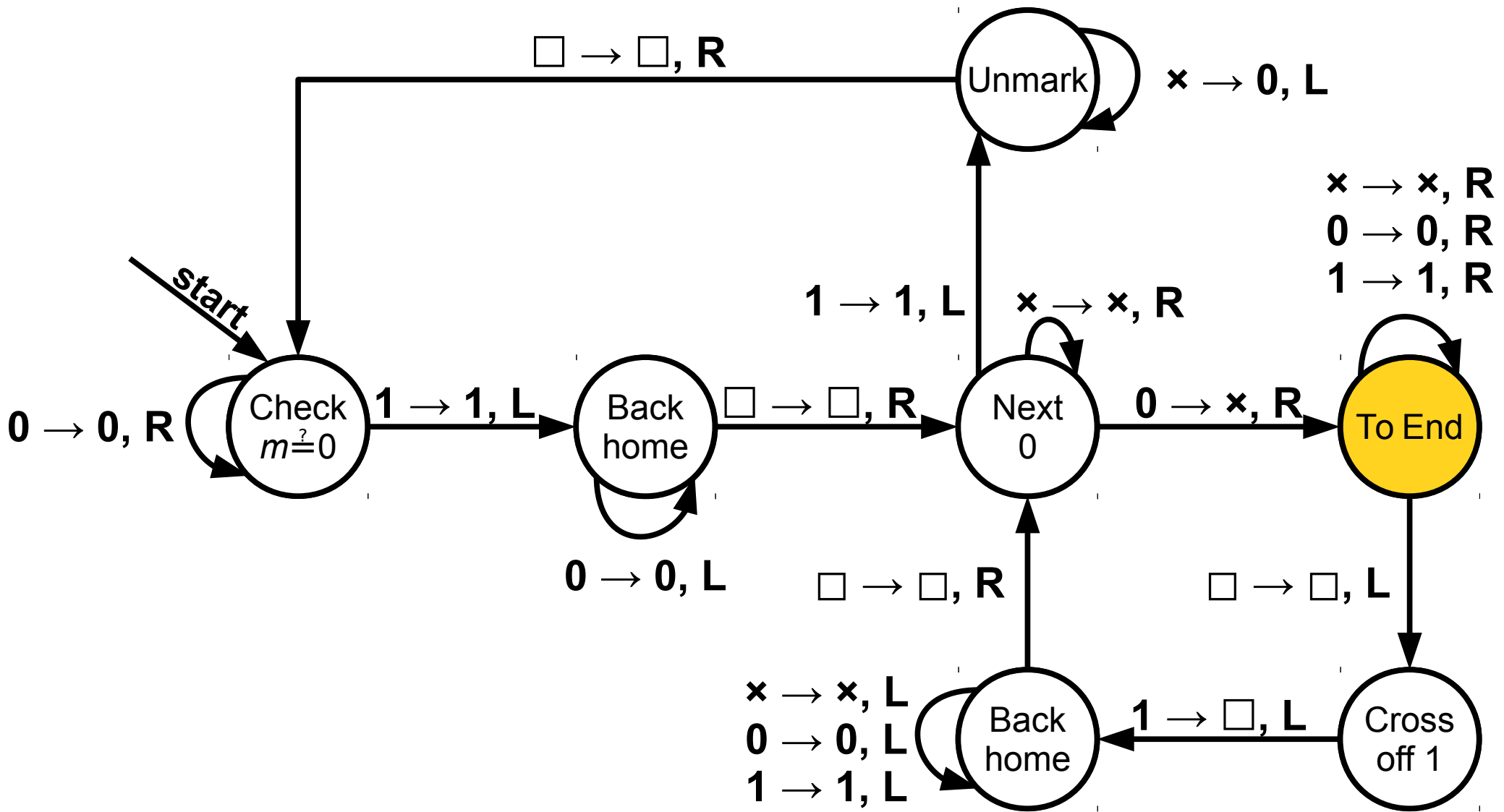


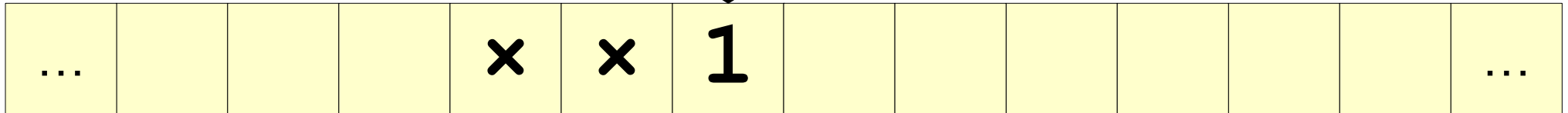
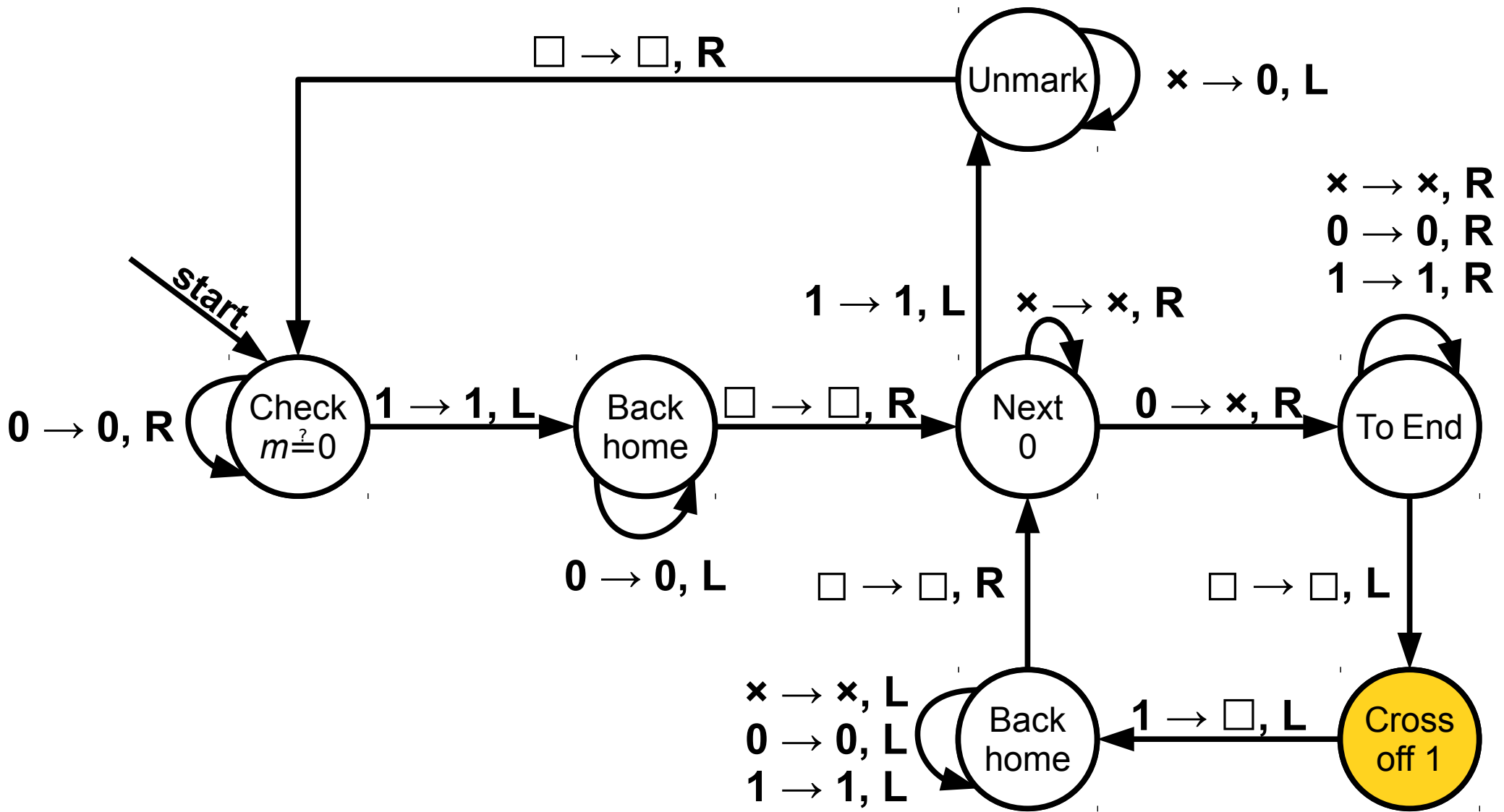




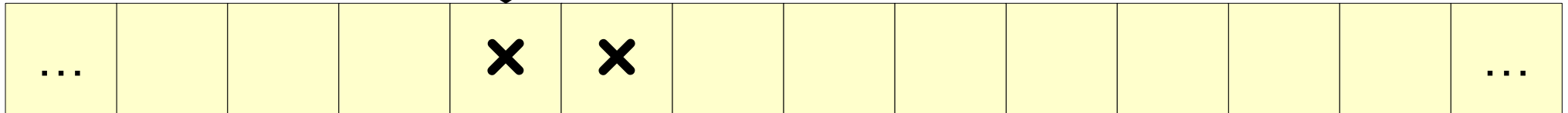
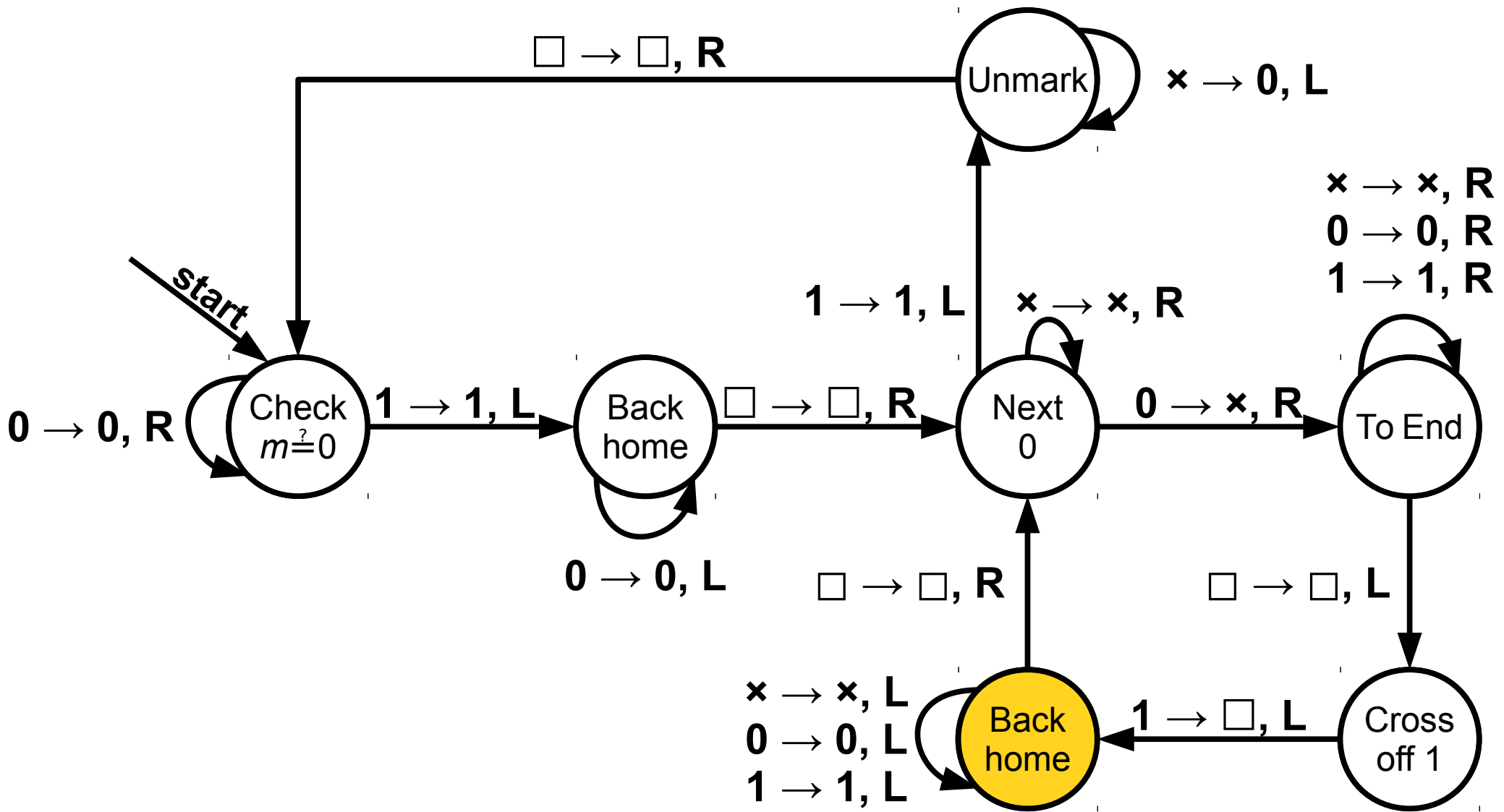


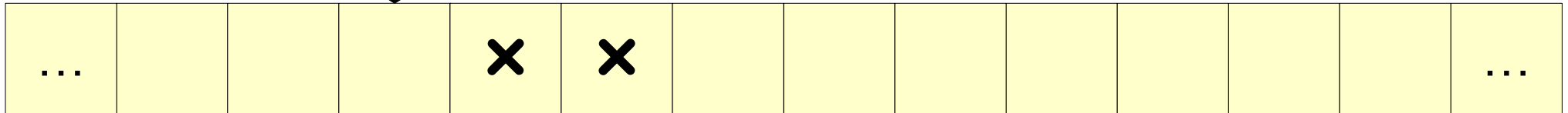
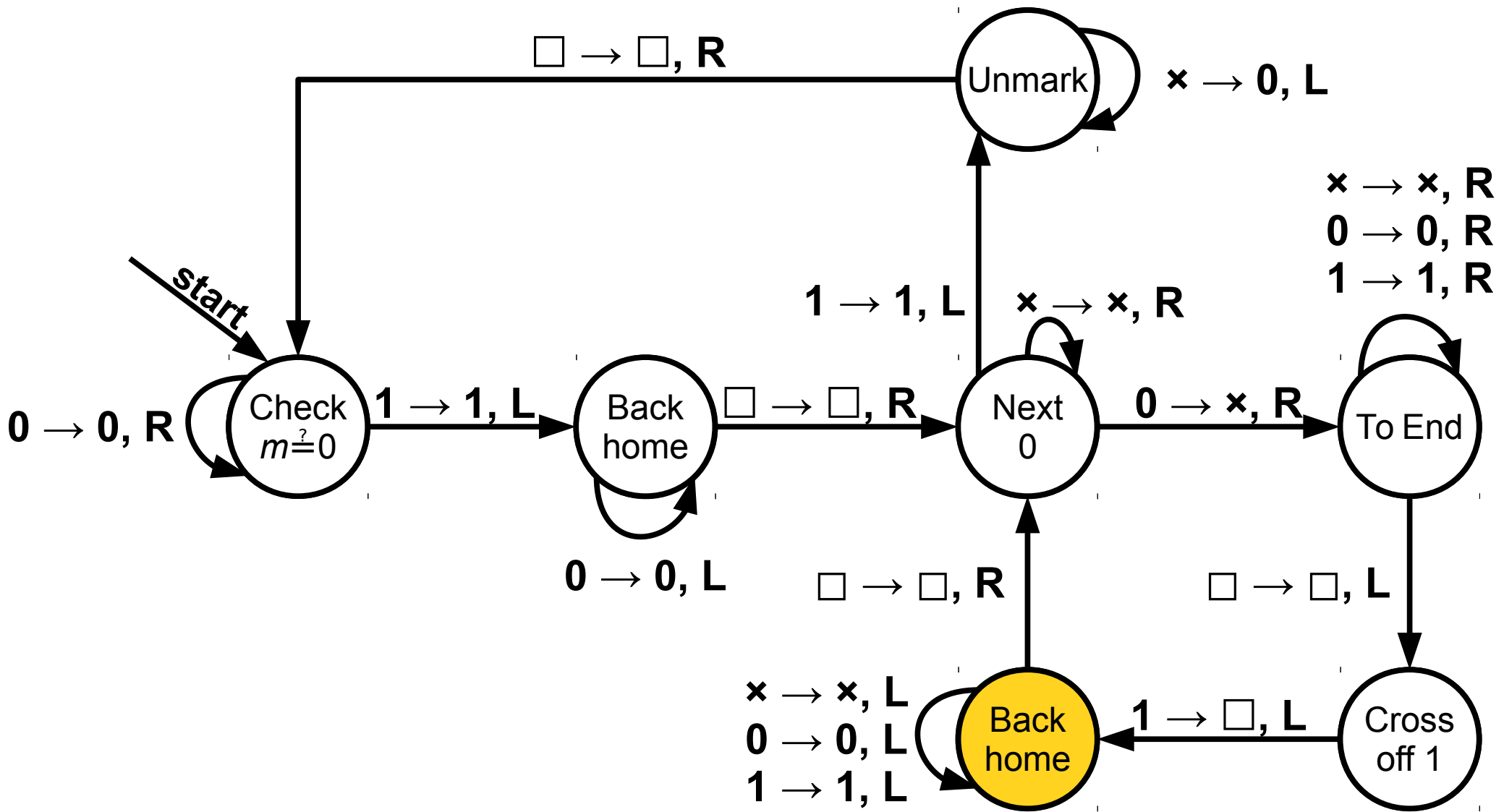


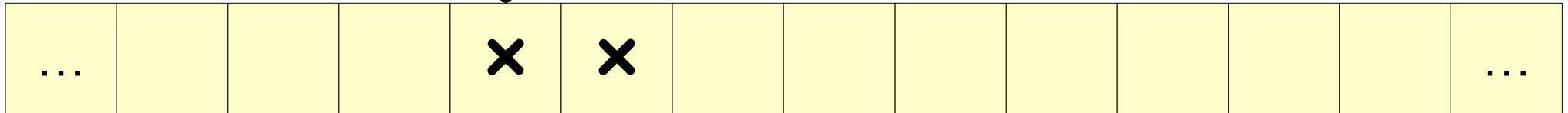
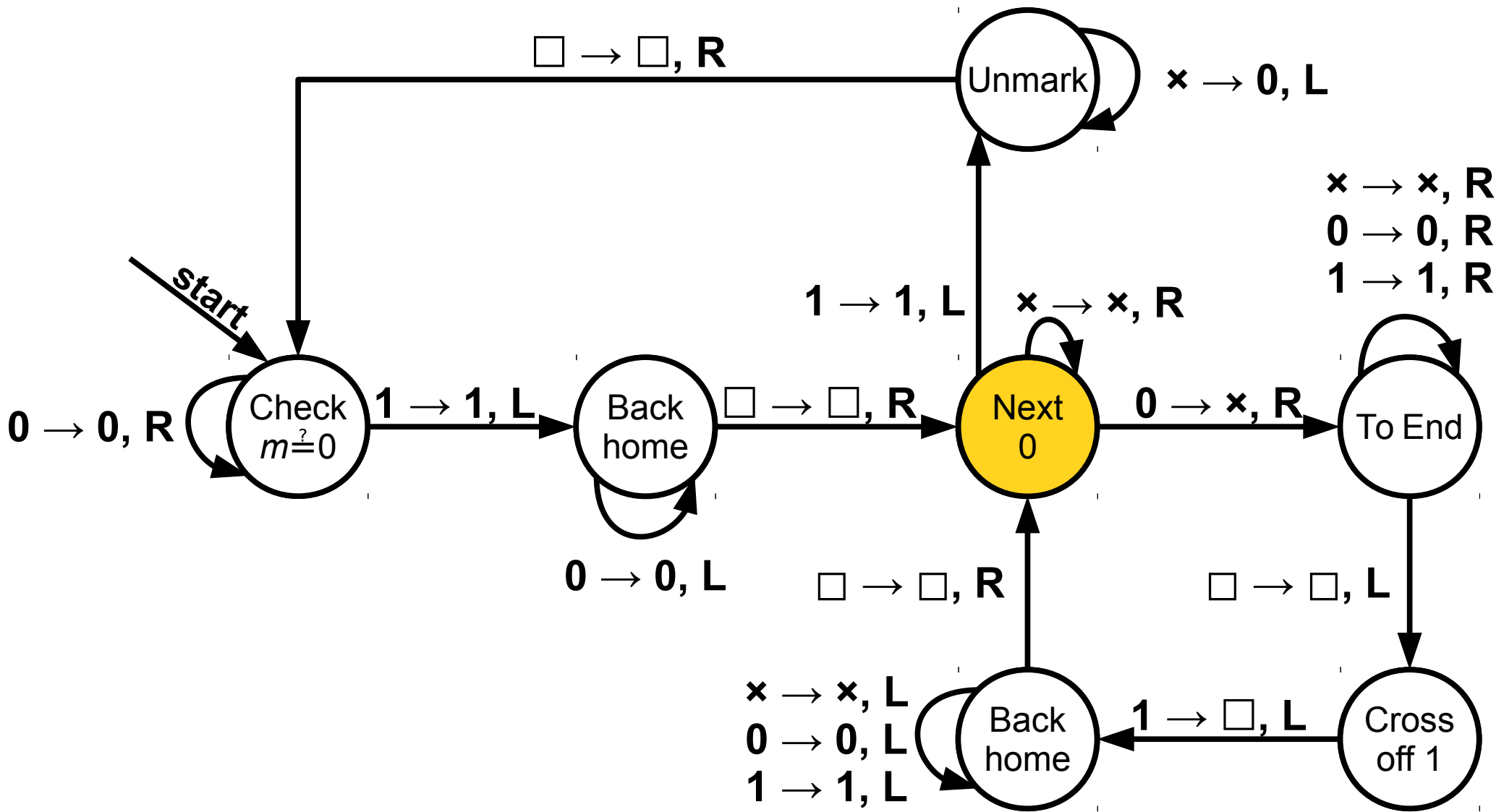




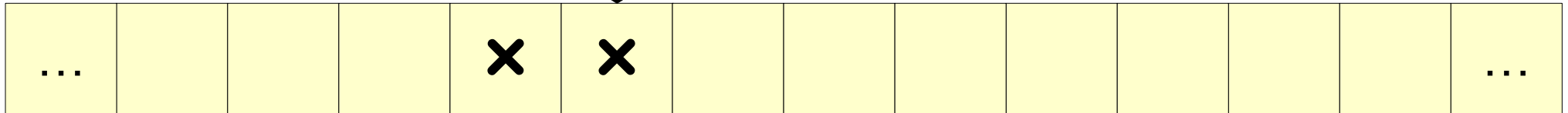
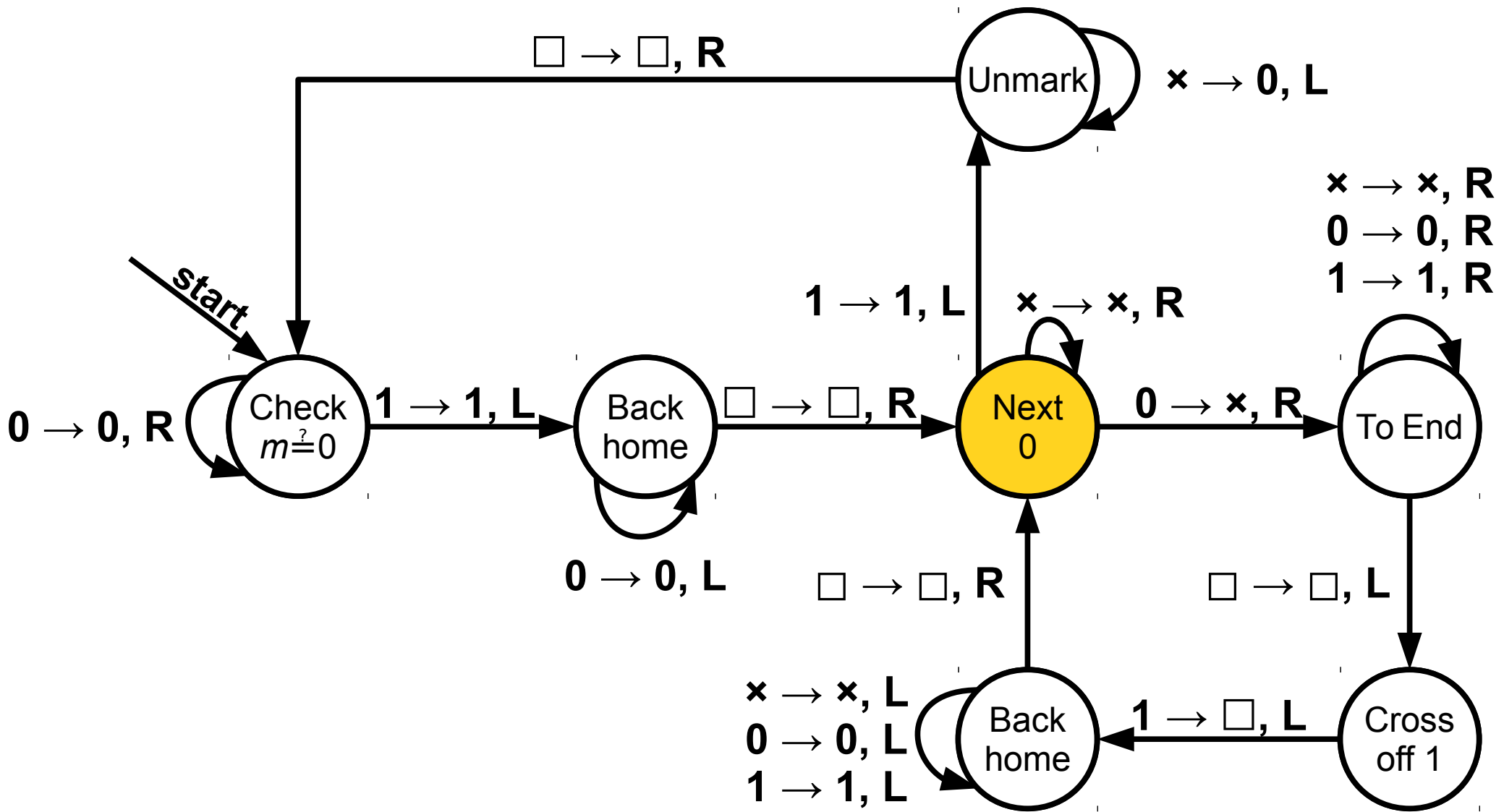


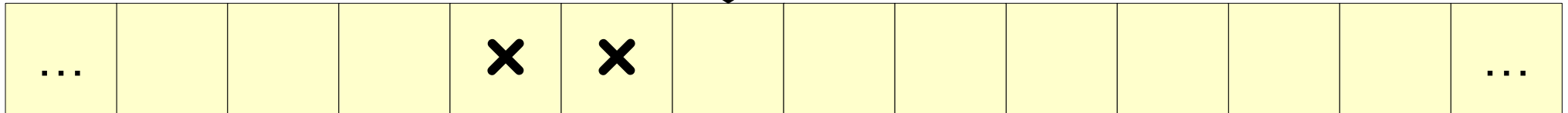
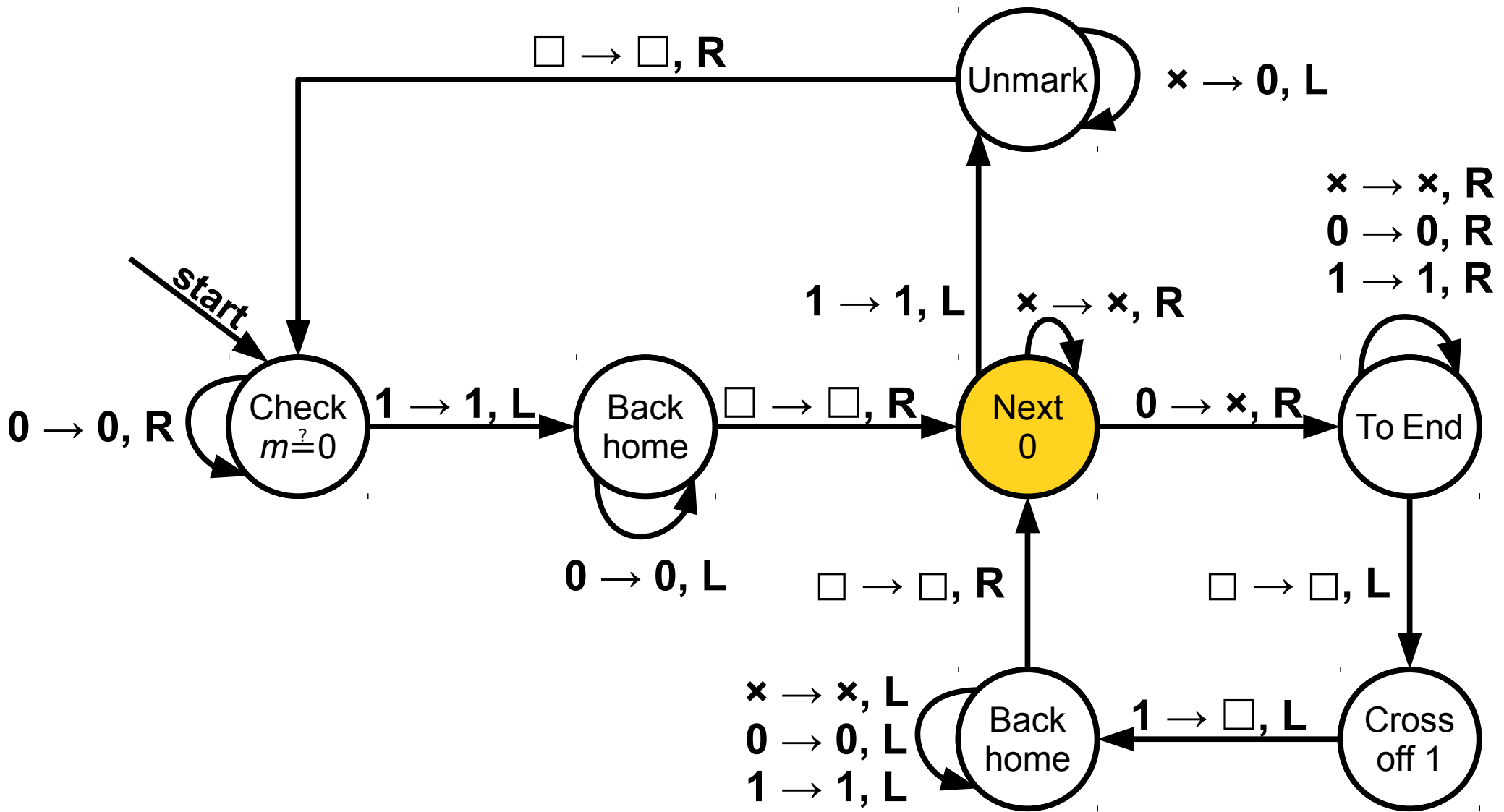


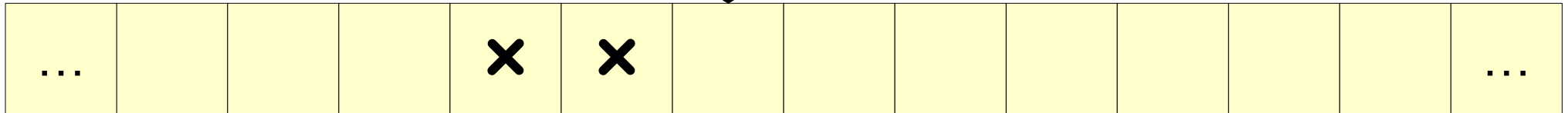
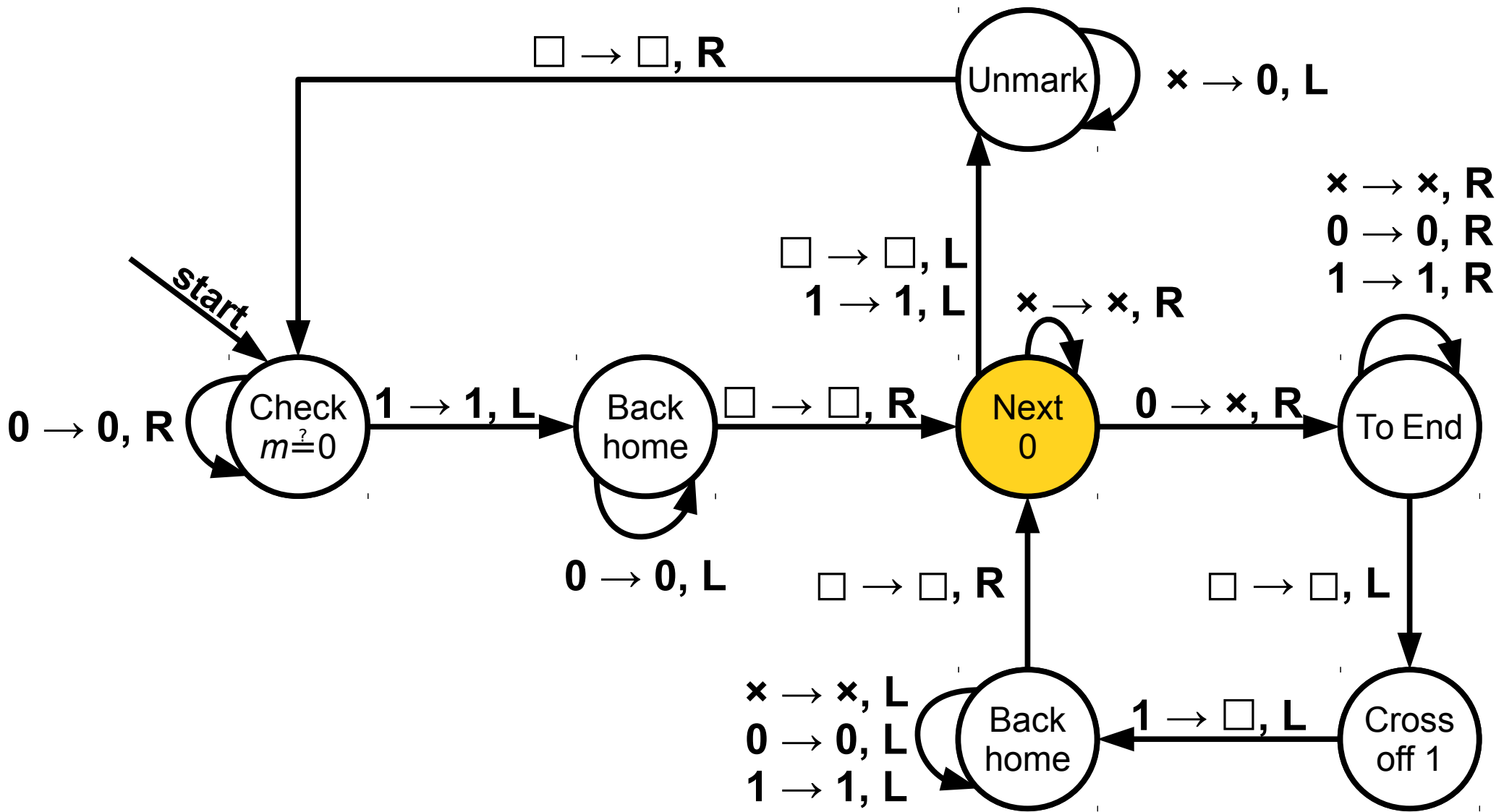


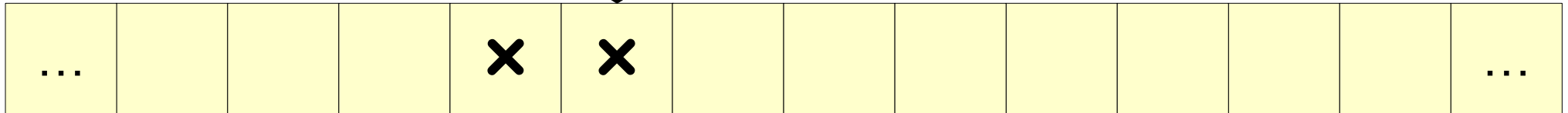
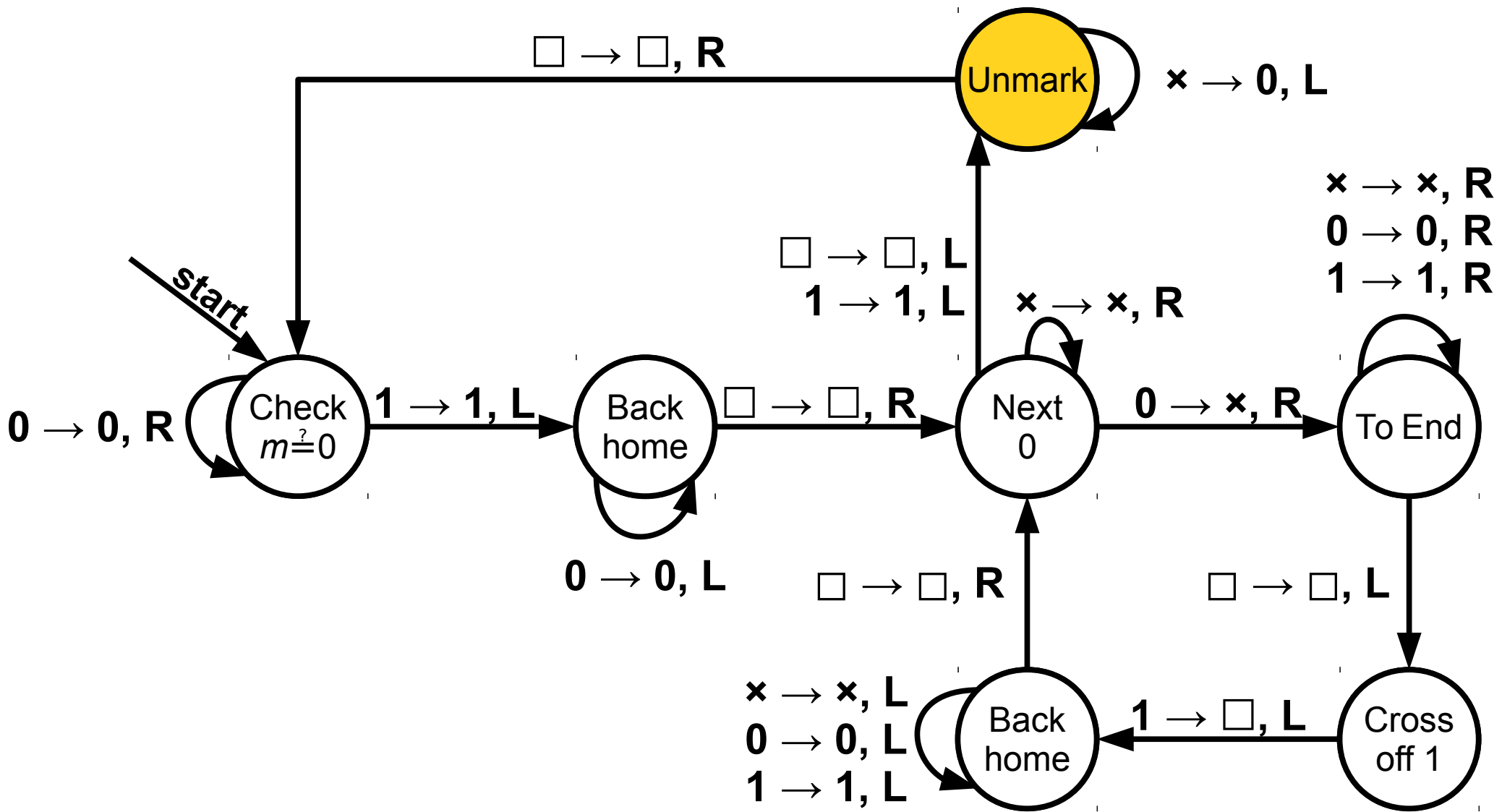




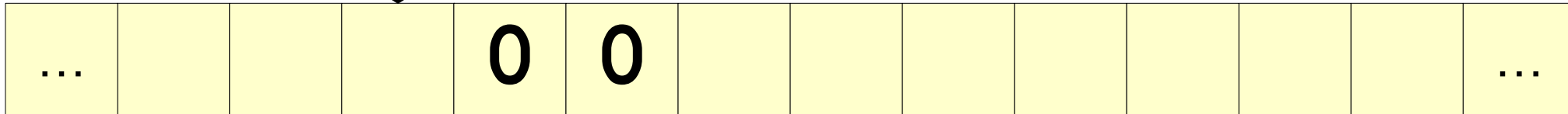
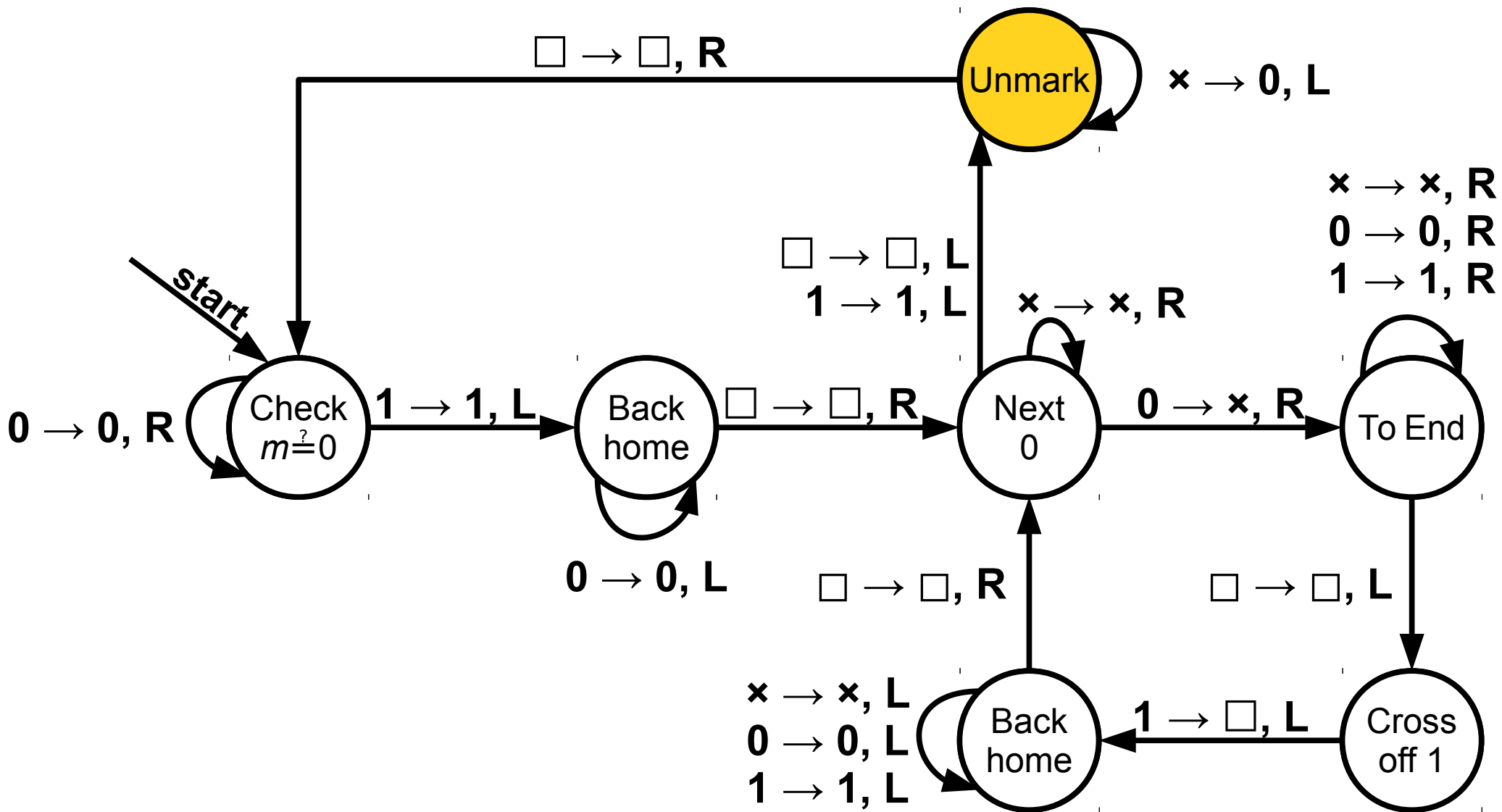




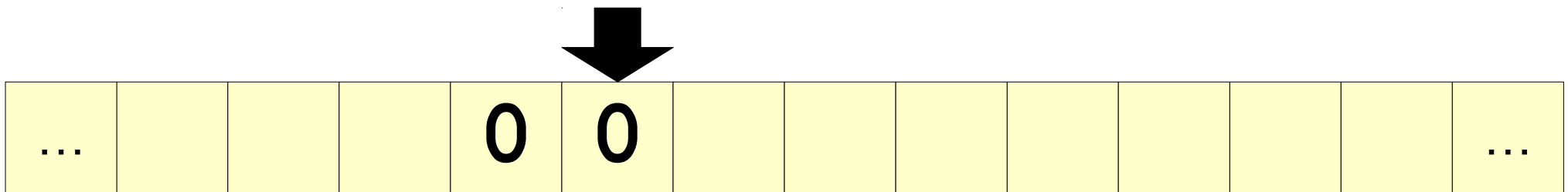
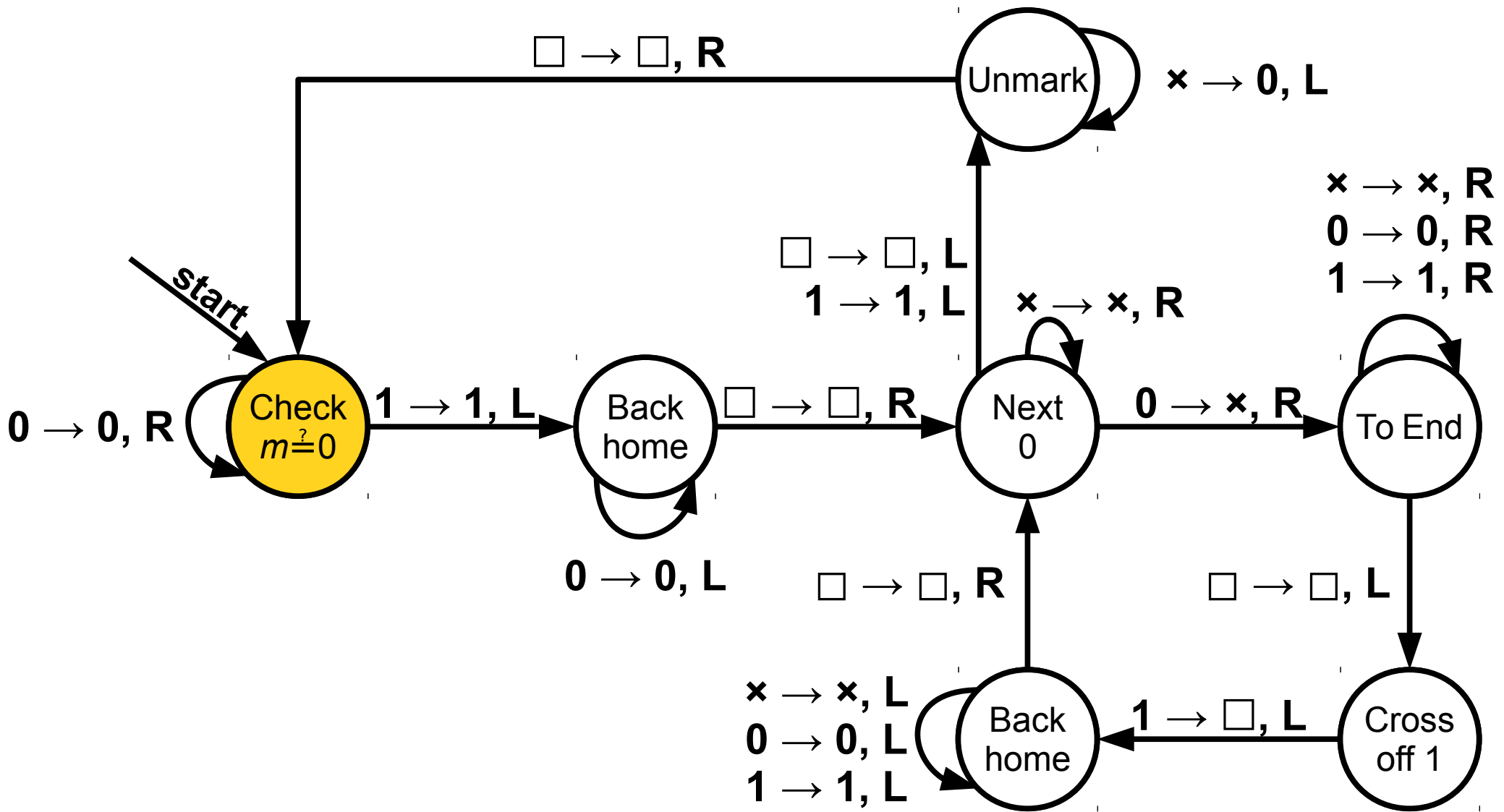












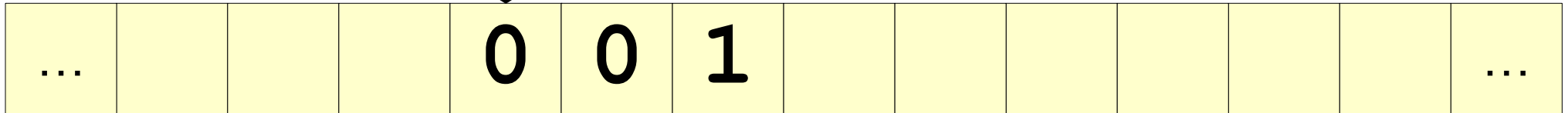
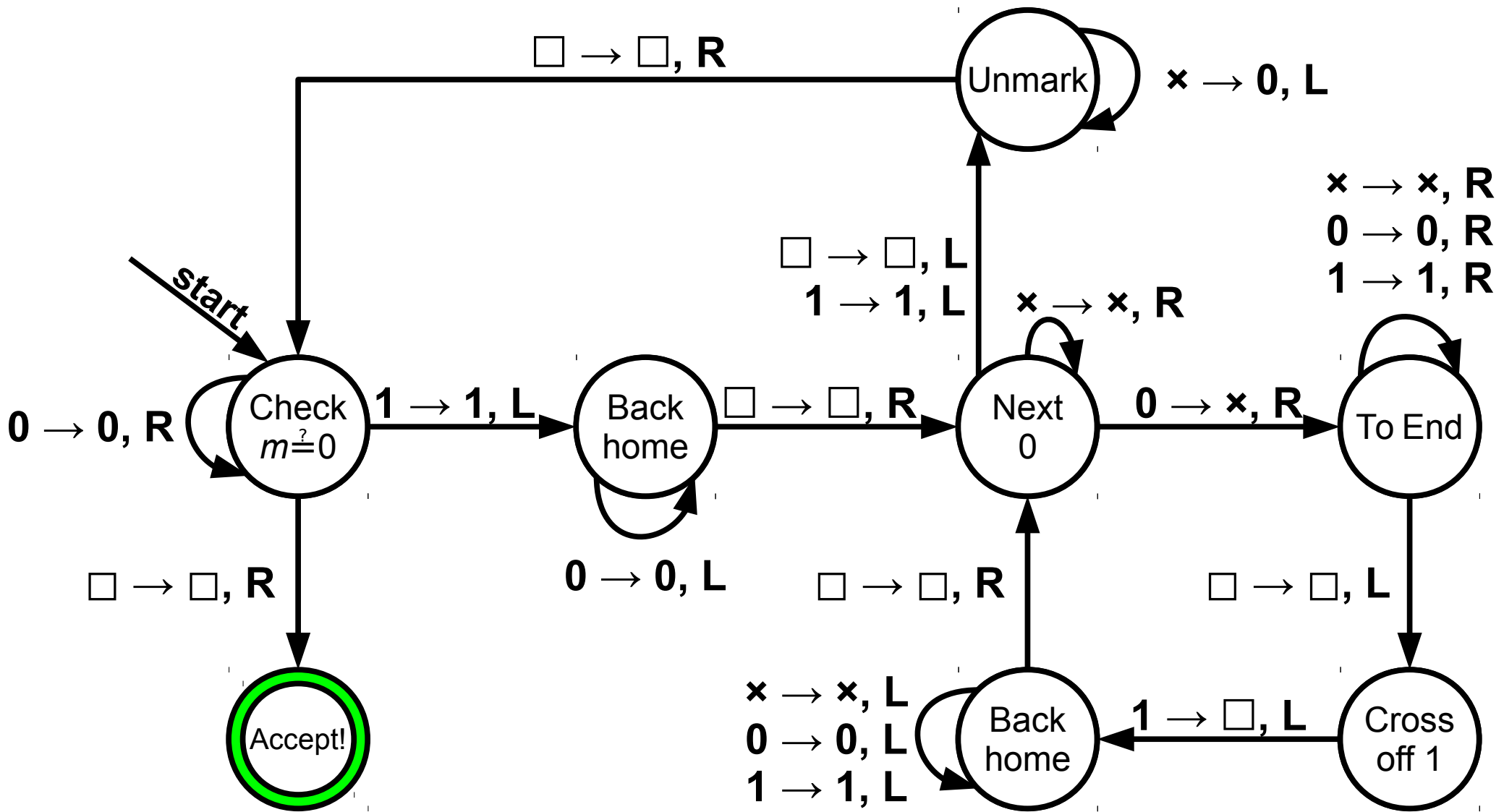


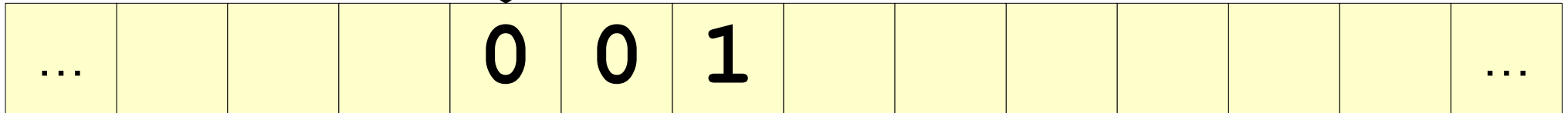
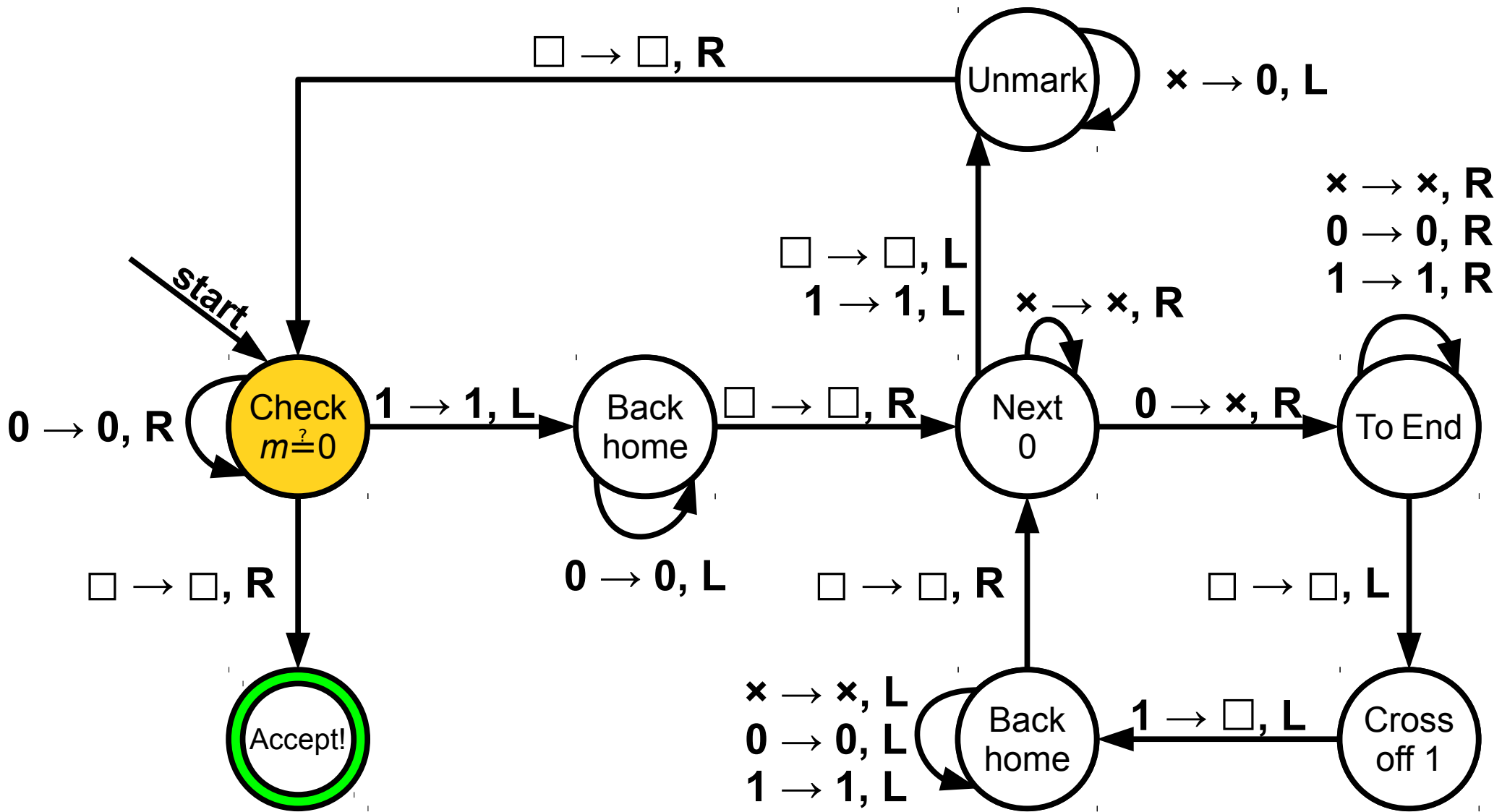




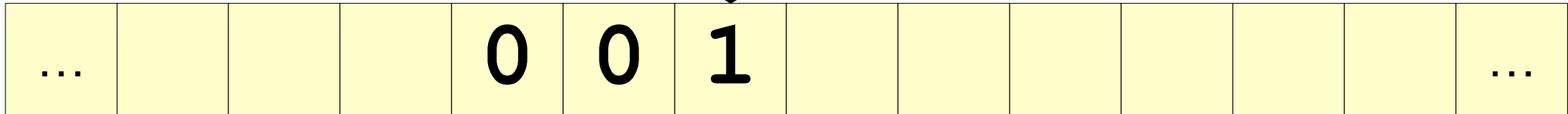
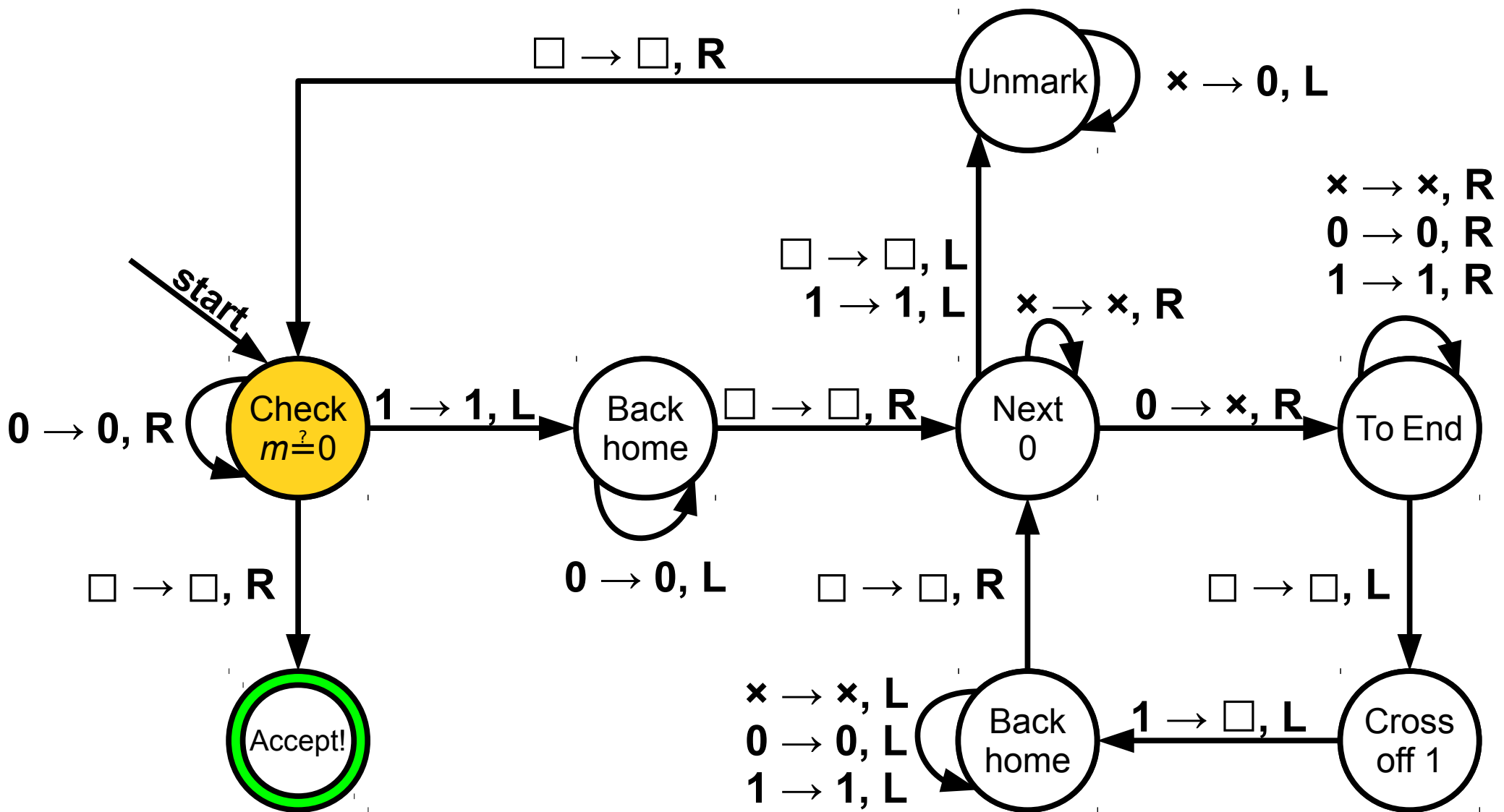






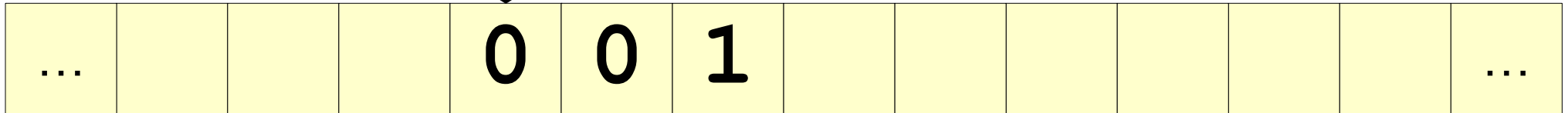
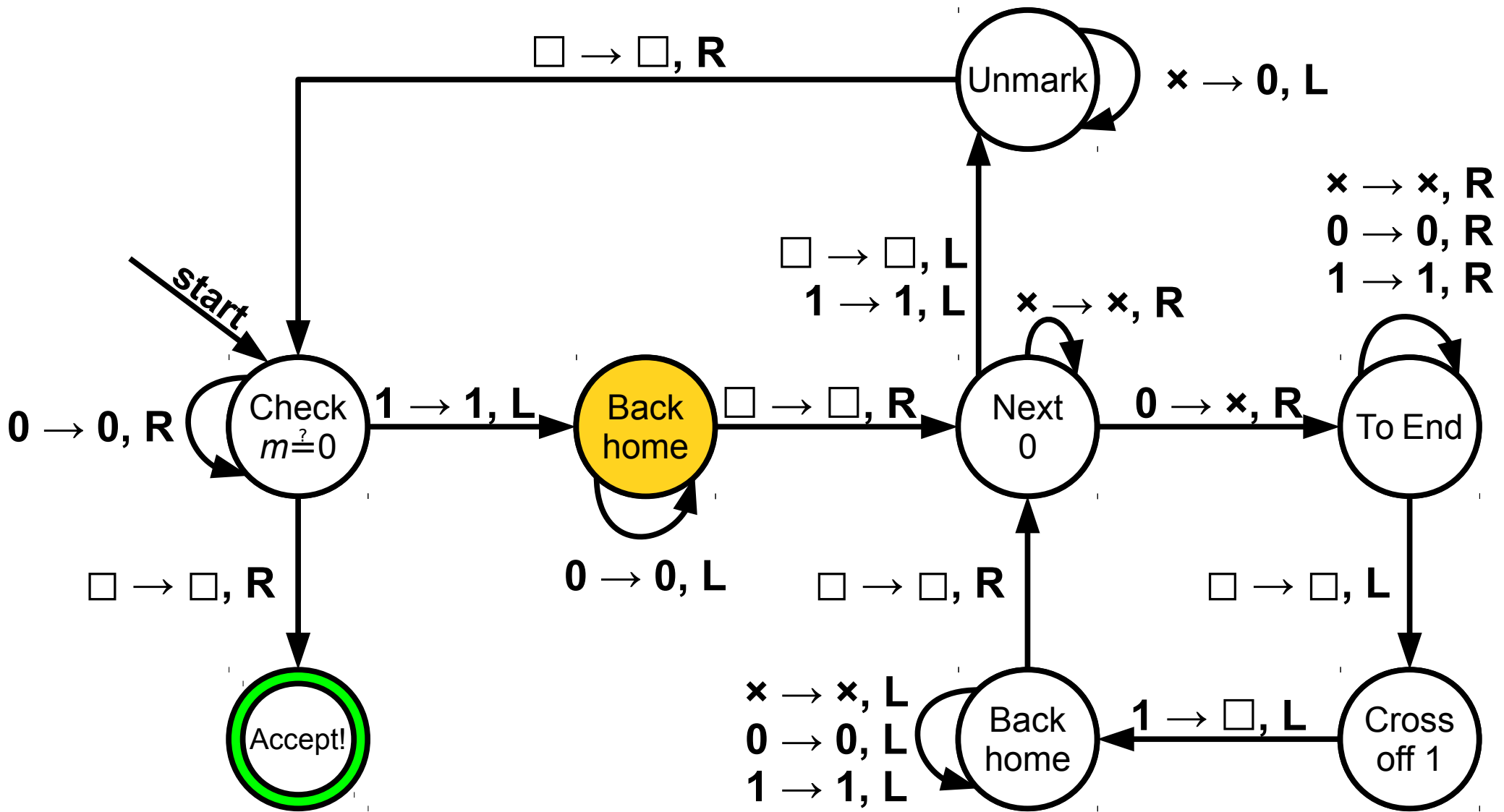


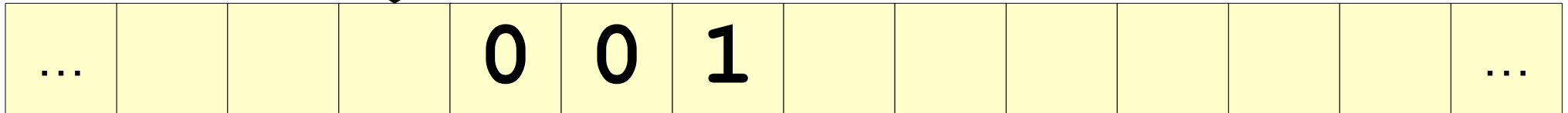
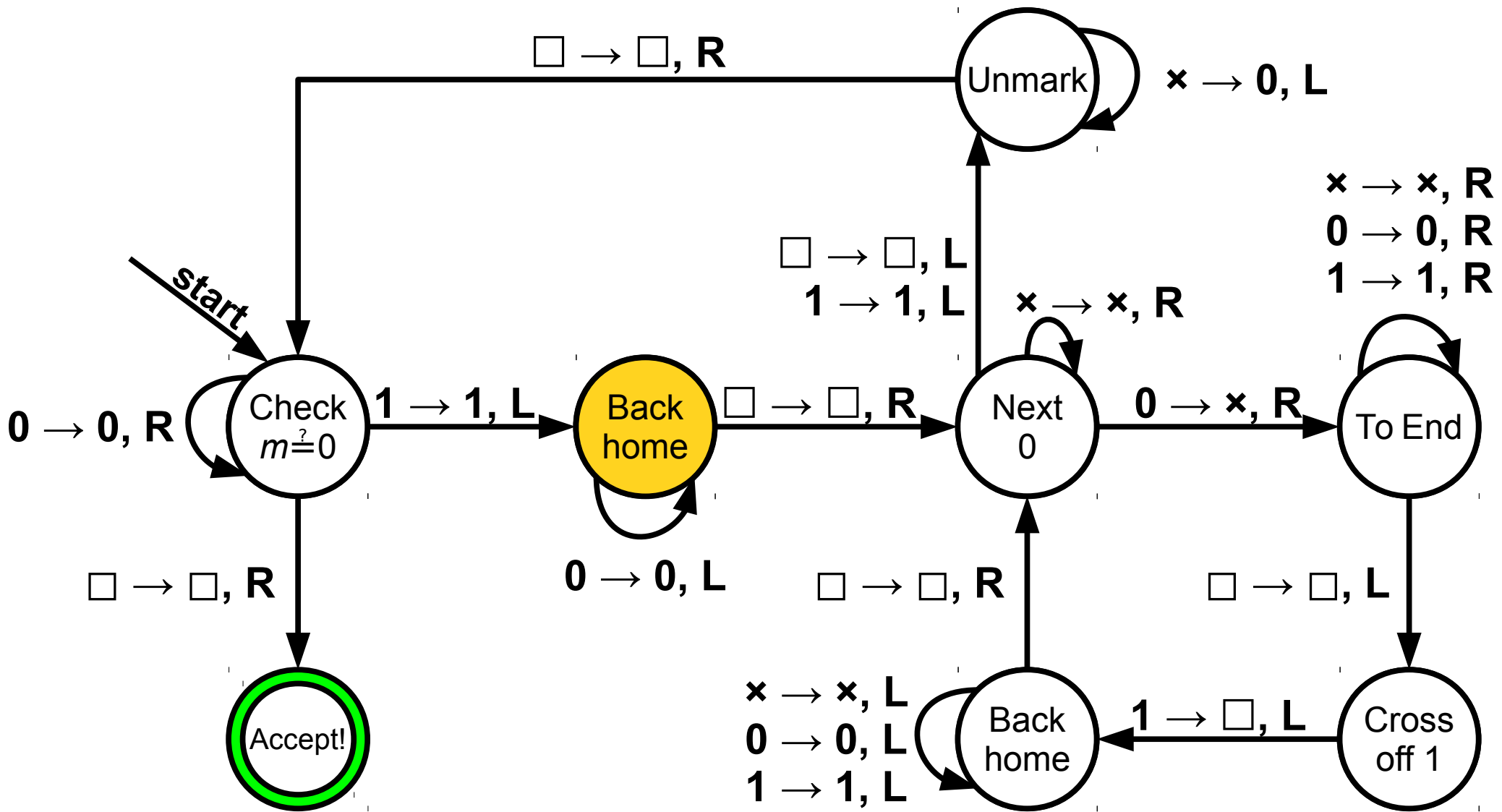






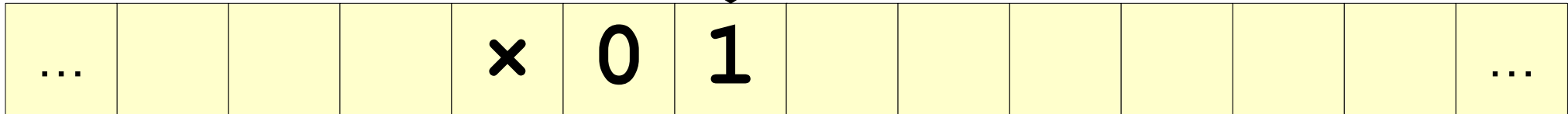
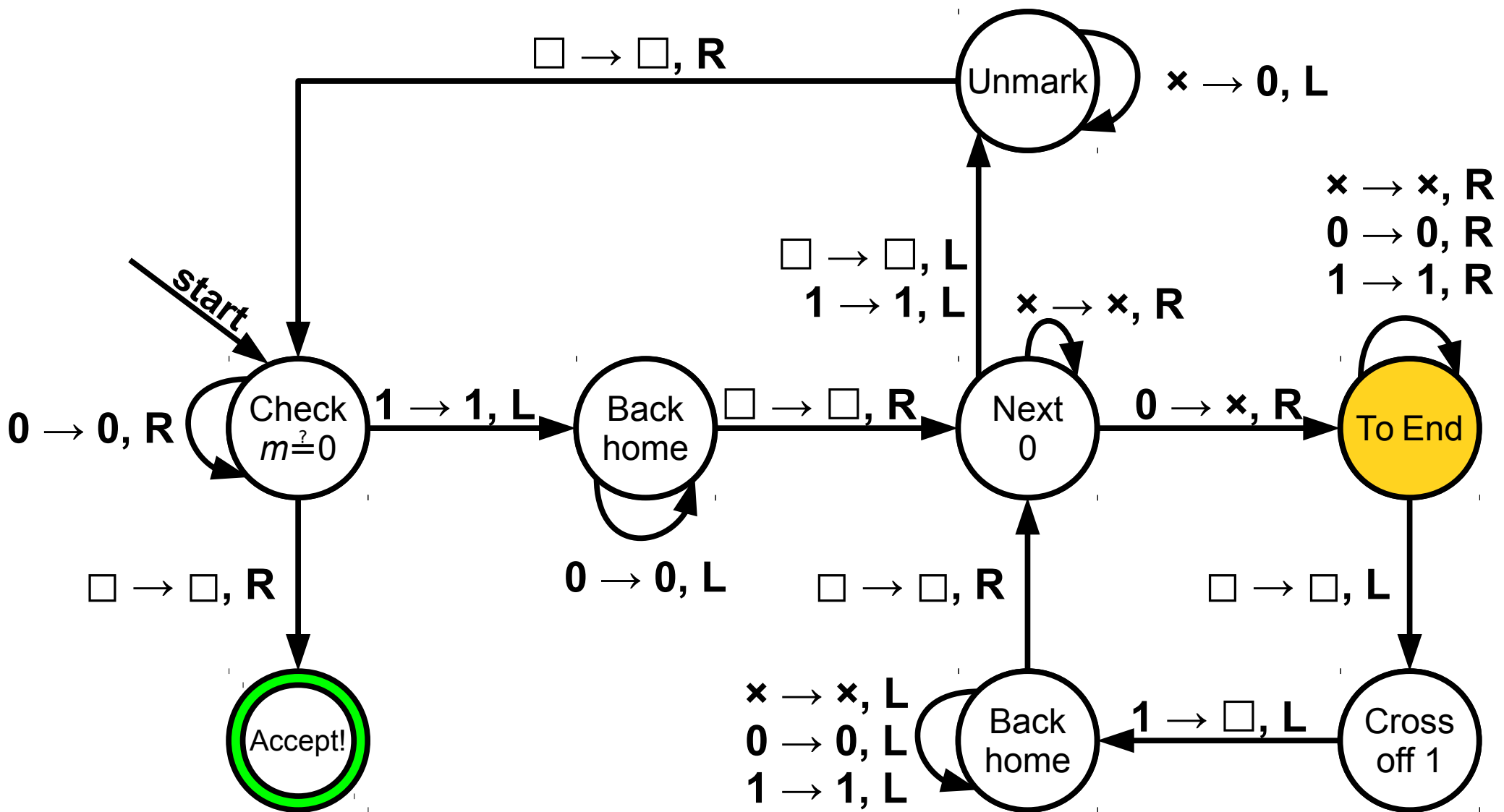


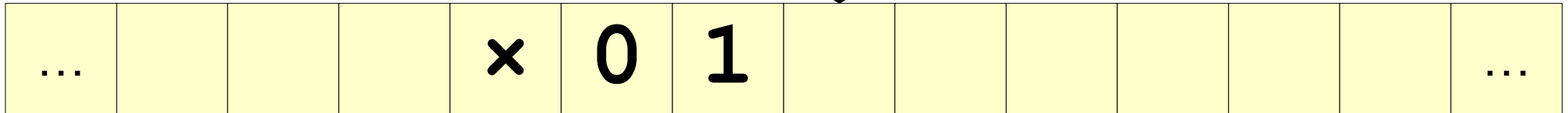
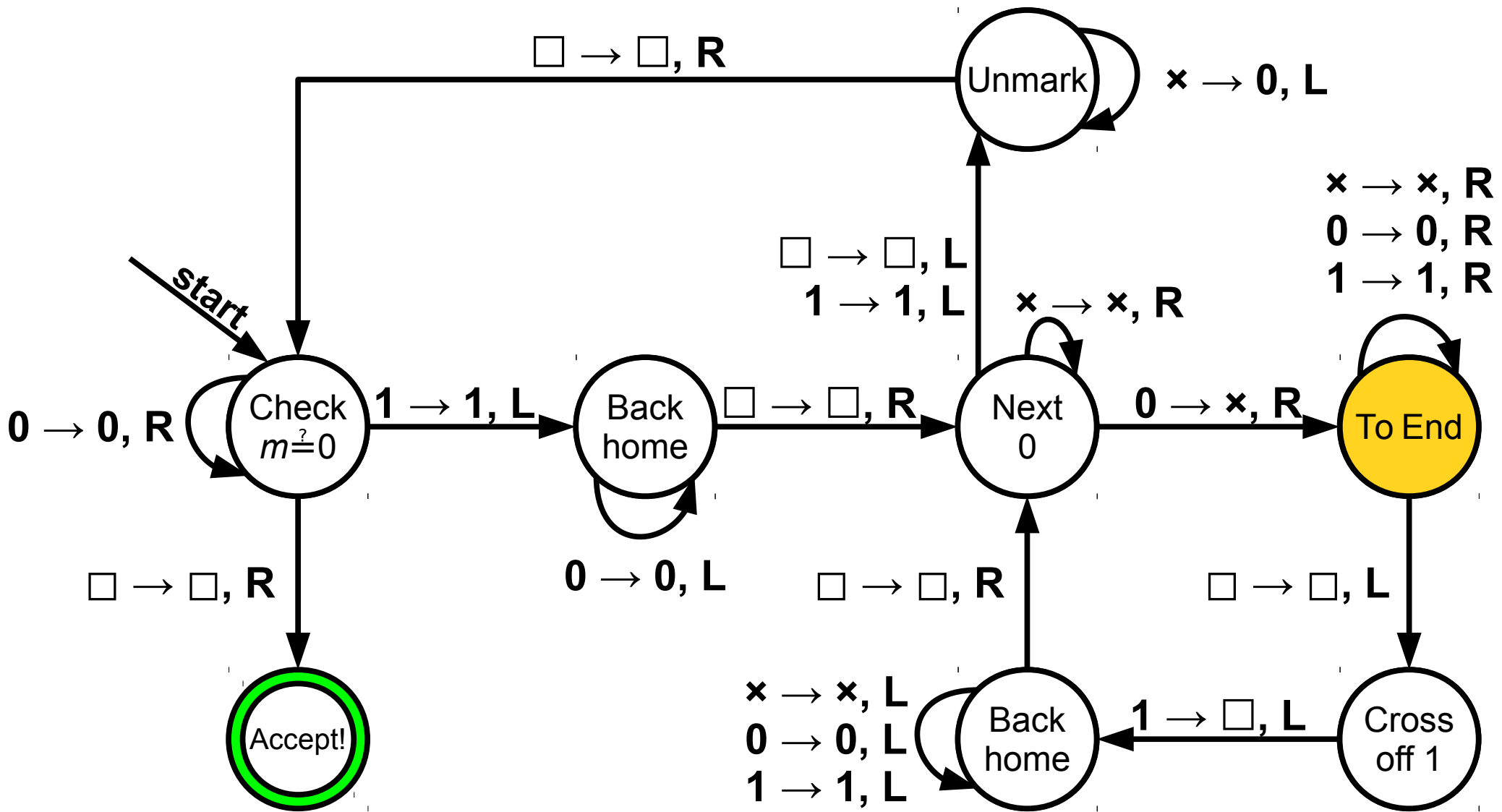


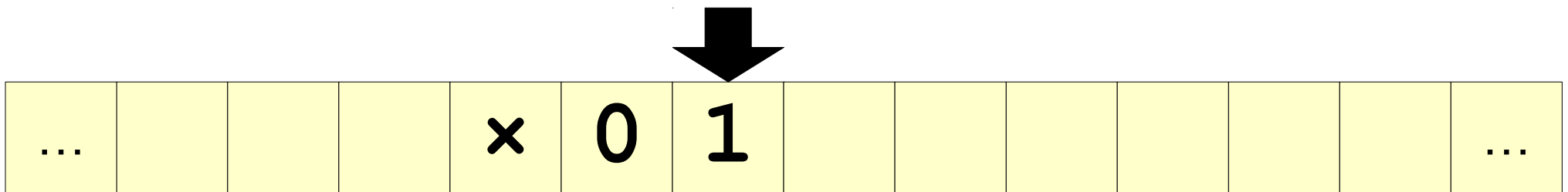
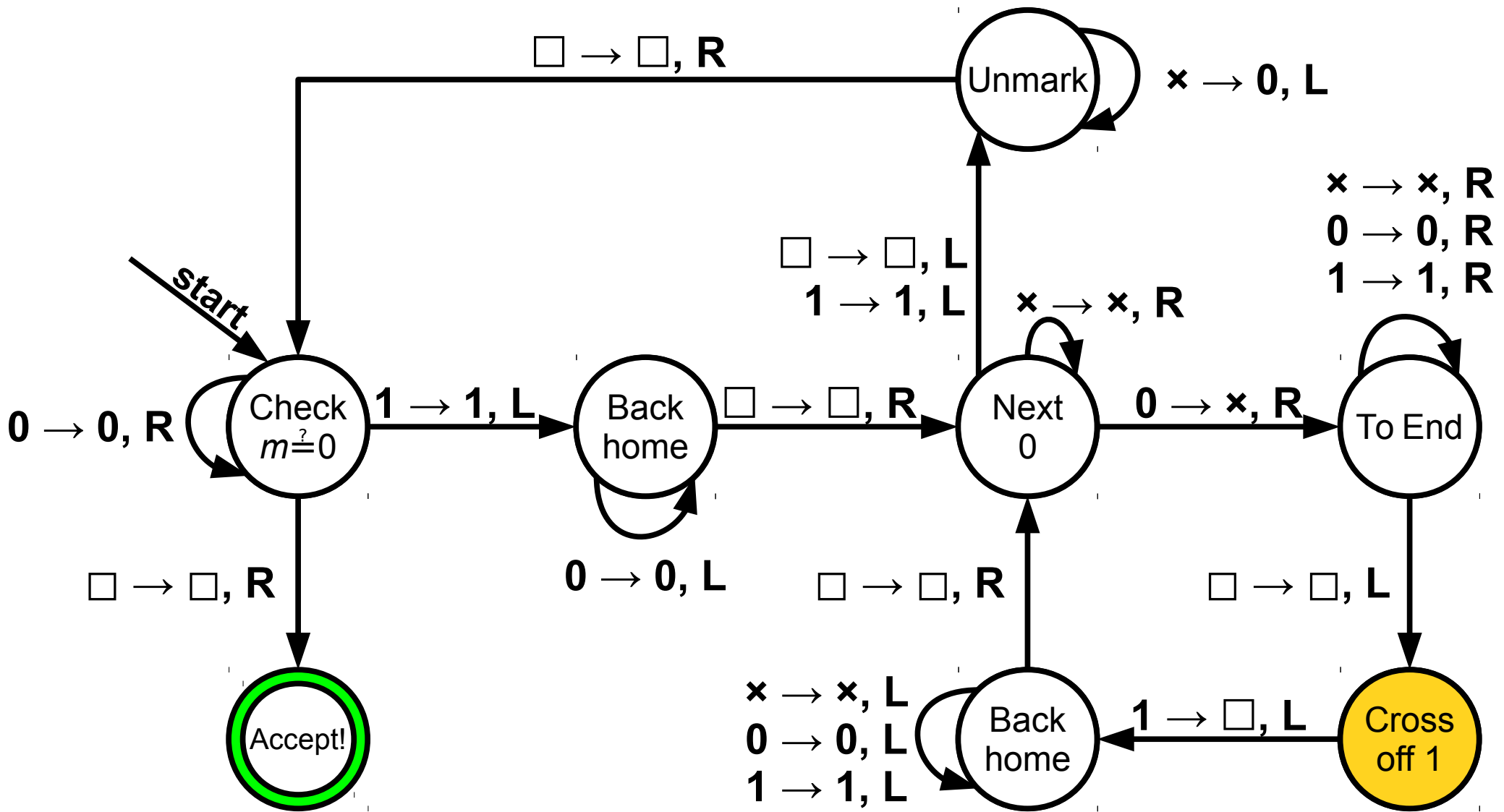




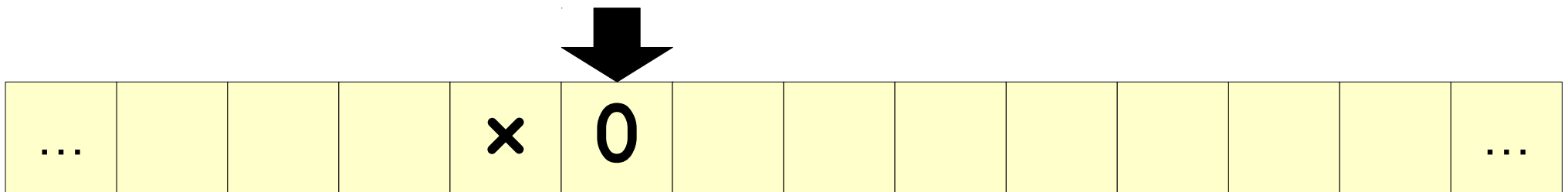
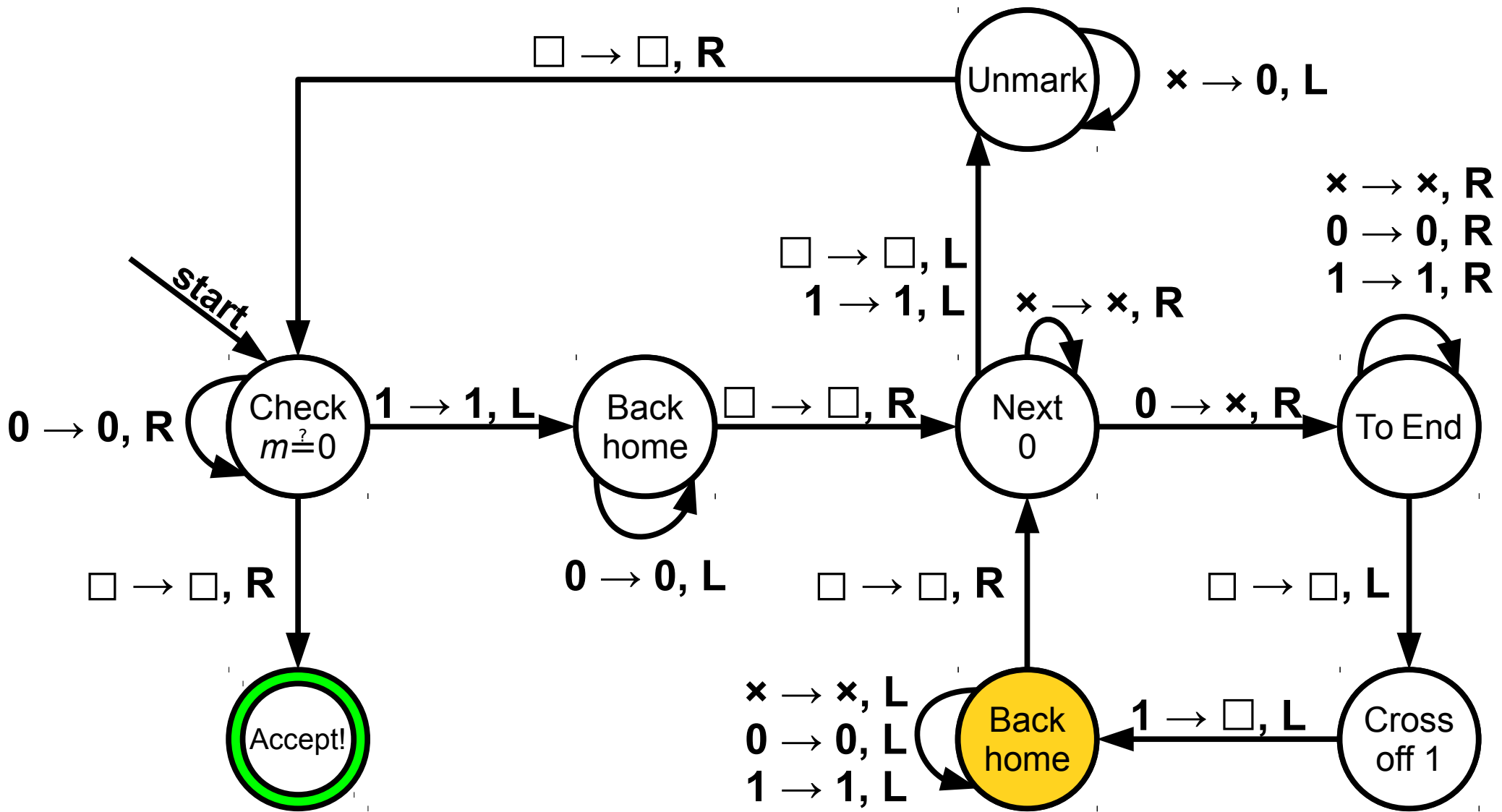














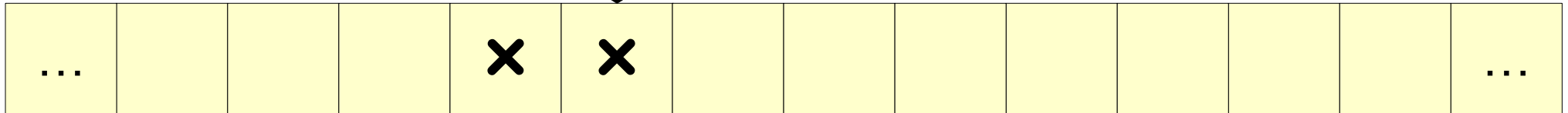
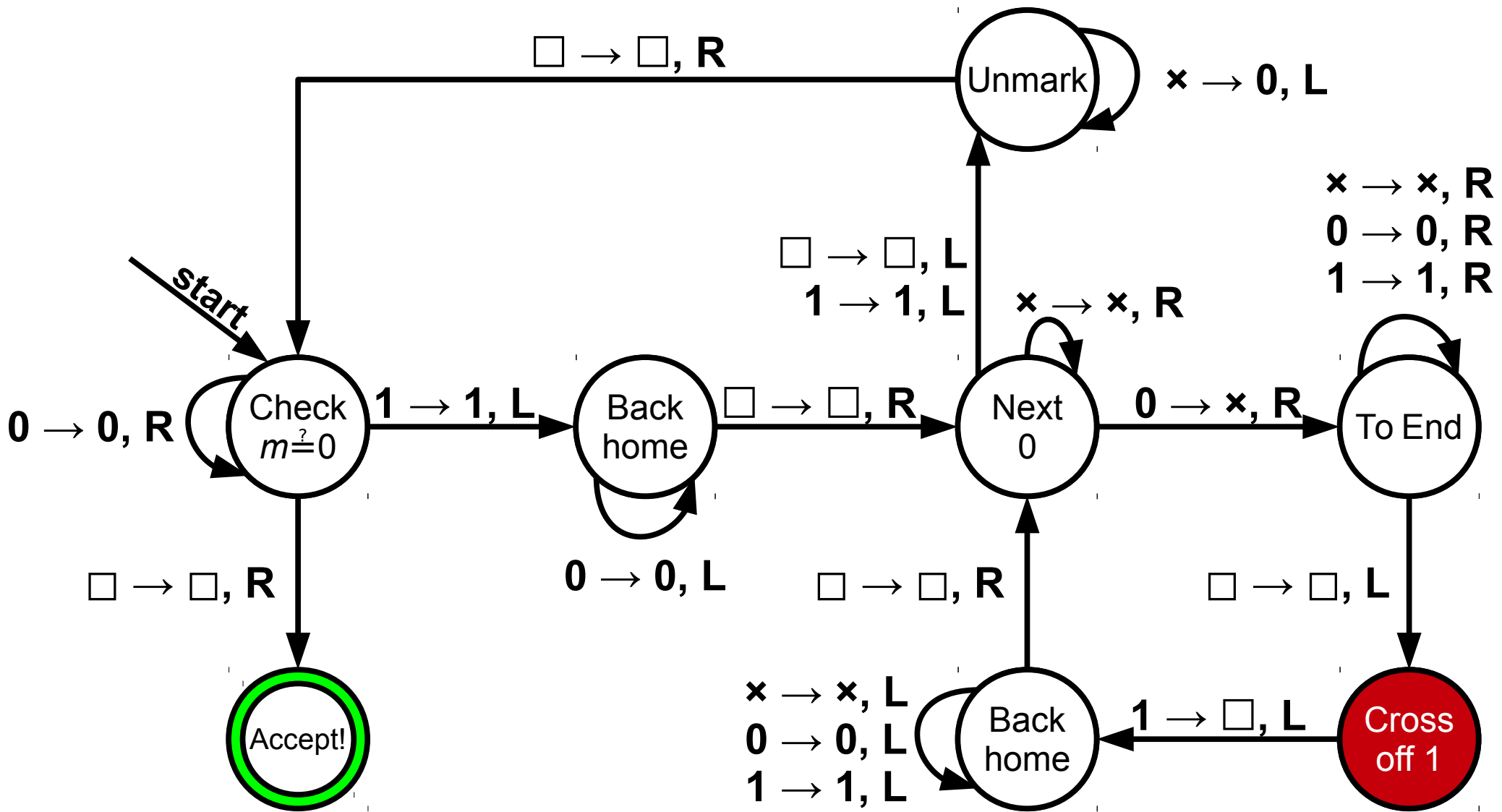




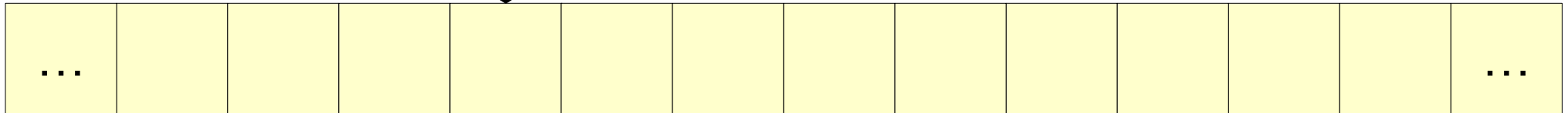
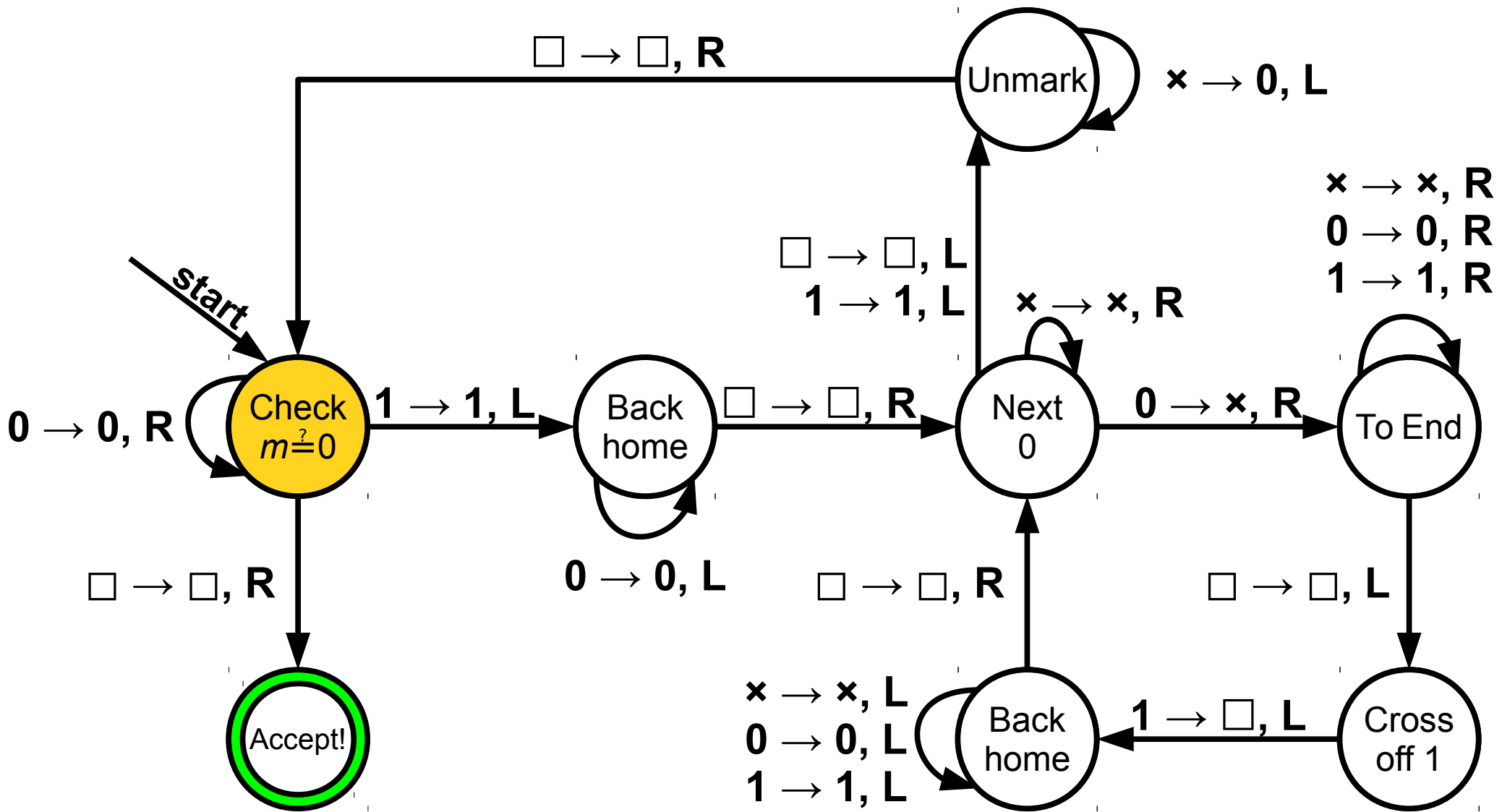








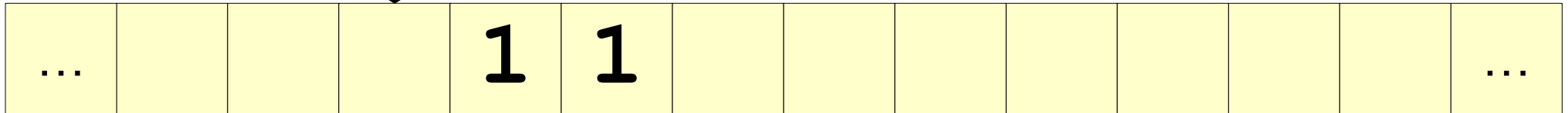
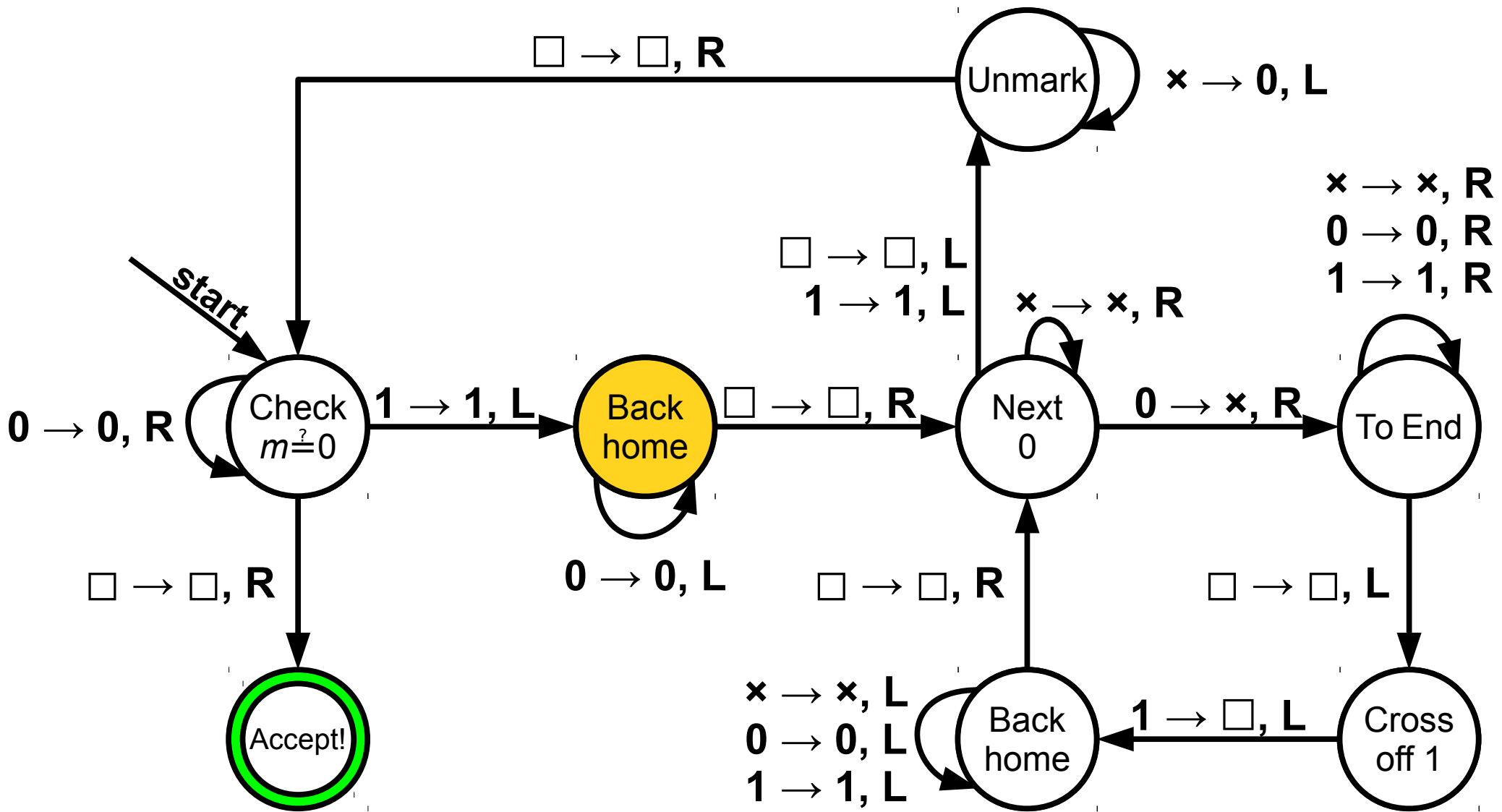






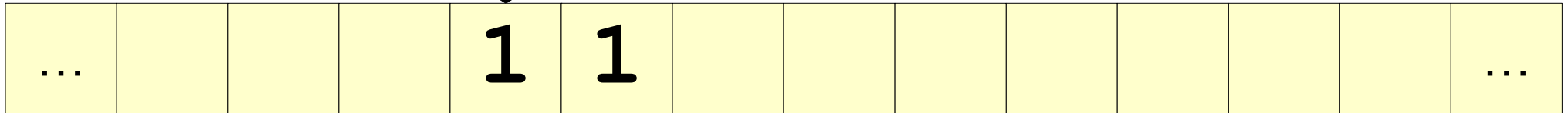
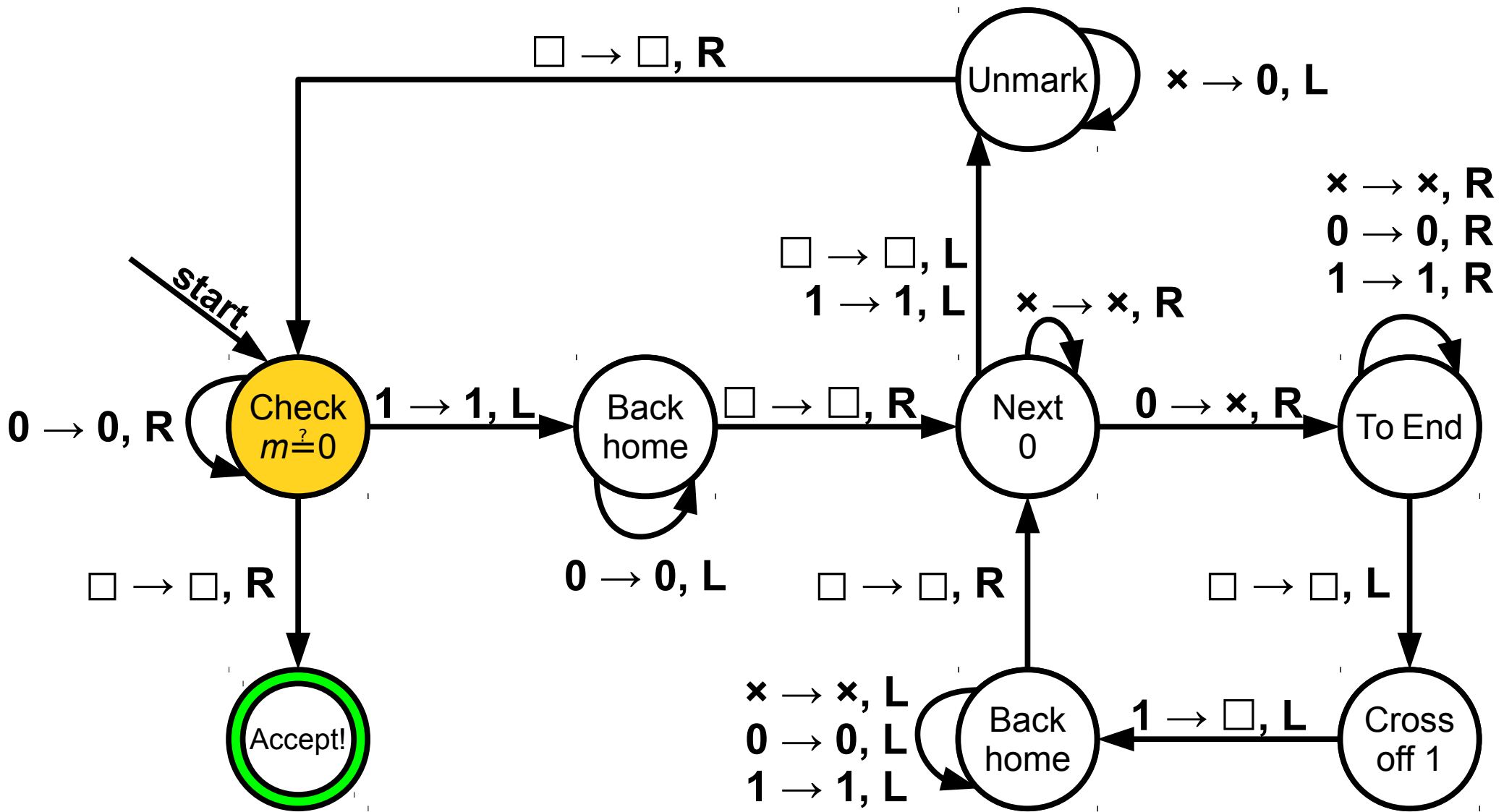








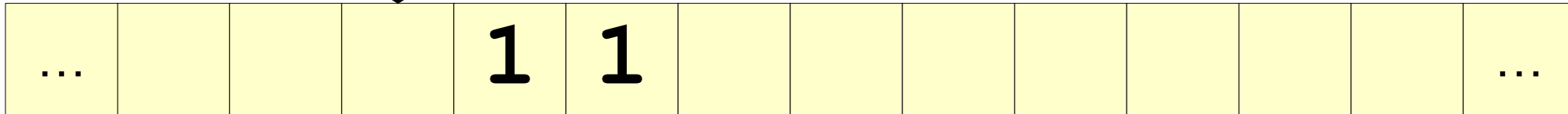
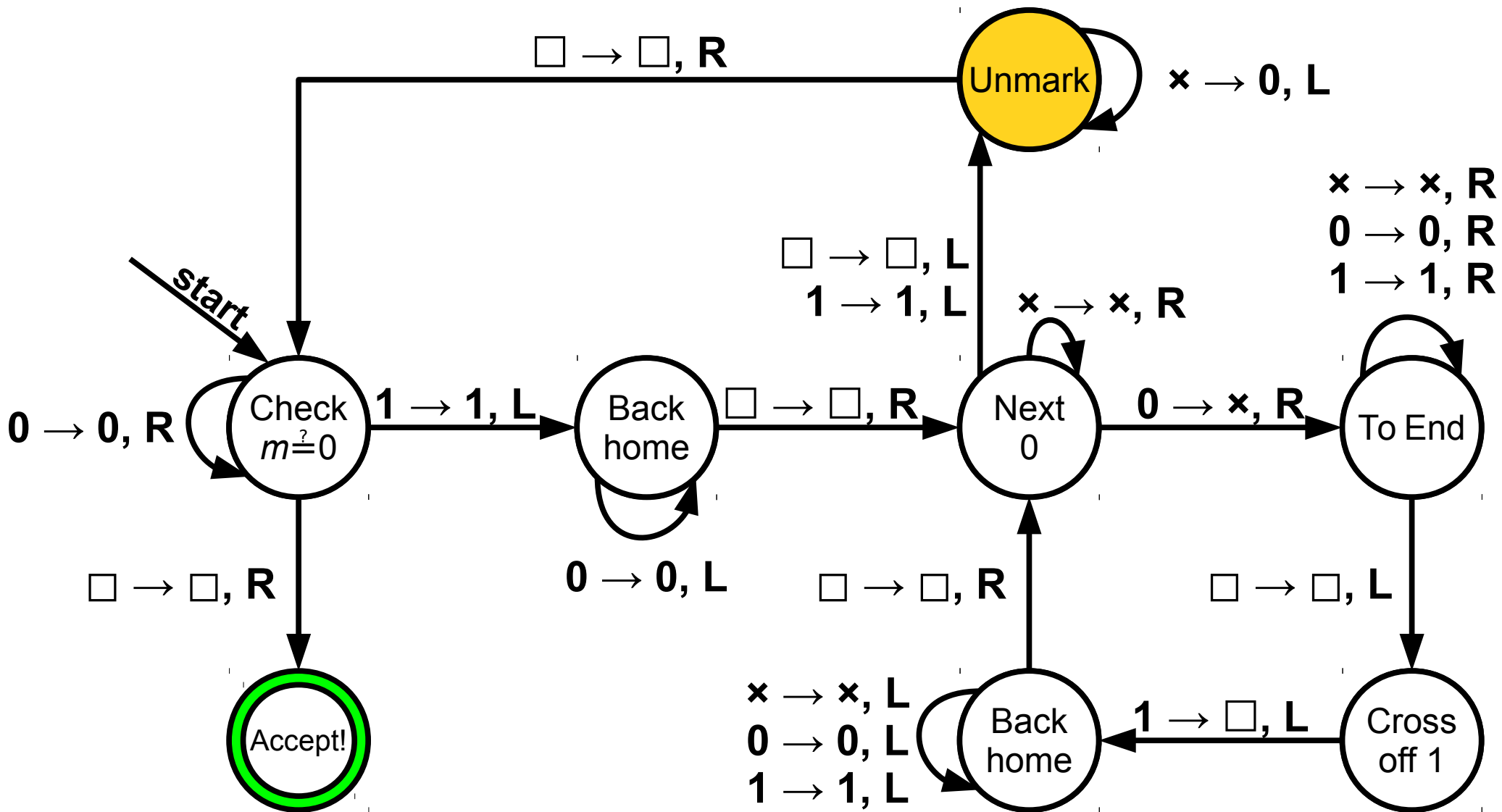


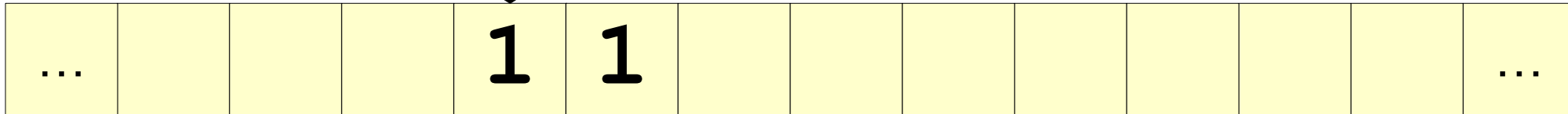
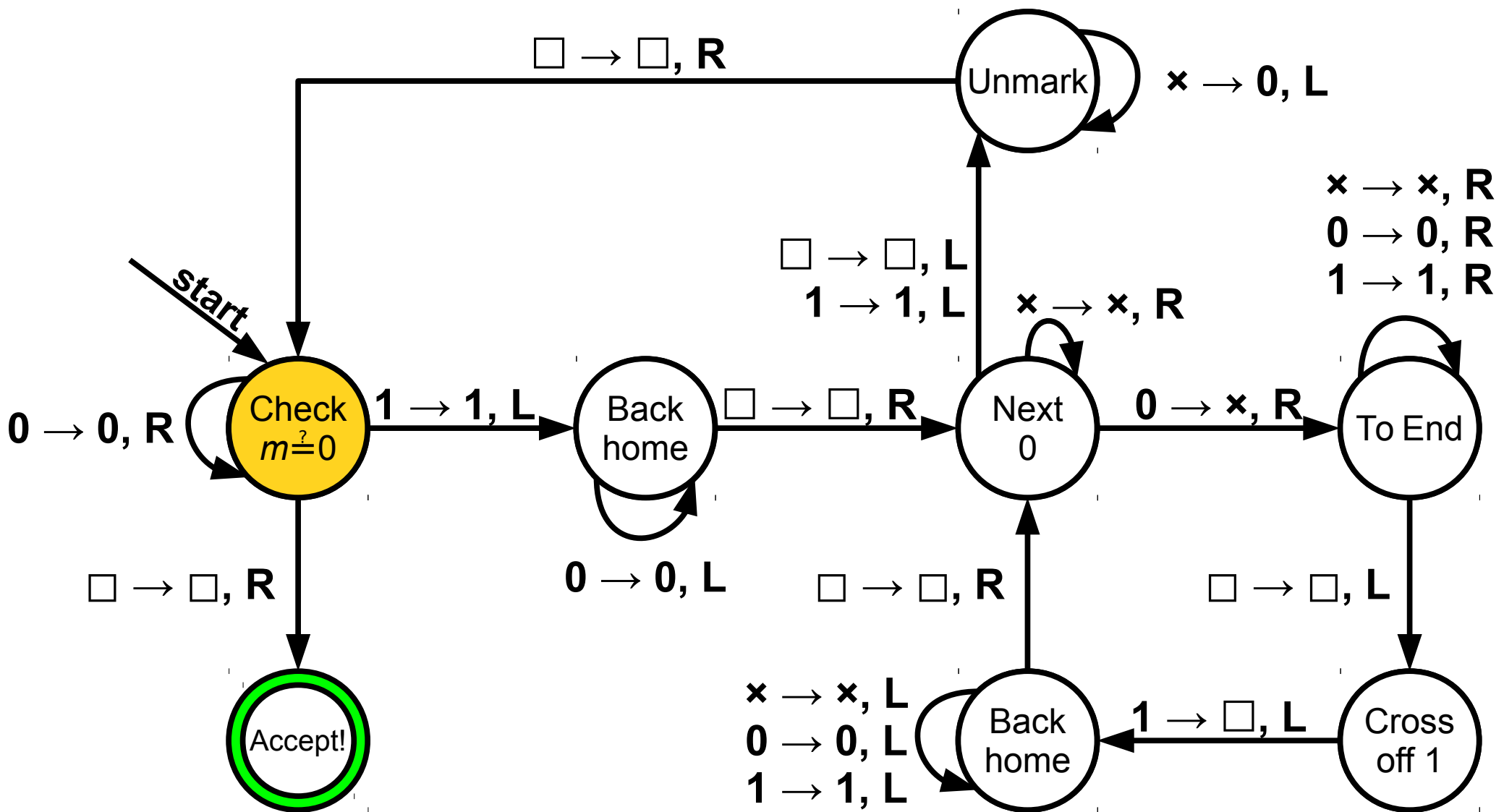






















# Concepts from Today

- Turing machines are a generalization of finite automata equipped with an infinite tape.
- It's often helpful to think recursively when designing Turing machines.
- It's often helpful to introduce new symbols into the tape alphabet.
- Watch for edge cases that might lead to infinite loops – though we'll say more about that later on.

# Next Time

- **TM Subroutines**
  - Combining multiple TMs together!
- **The Church-Turing Thesis**
  - Just how powerful *are* Turing machines?
- **Objects and Encodings**
  - Computing on graphs, images, etc.