Simulazione in continua

• Determinazione del punto di lavoro

circuiti con dispositivi a semiconduttore, non lineari

• Analisi a regime con eccitazioni costanti

Si opera su un circuito senza memoria (resistivo),

in genere, non lineare.

• E' usata anche nella simulazione della risposta temporale di circuiti dinamici

metodi di discretizzazione

• MNA (o Tableau) —> sistema algebrico non lineare

Algoritmo di Newton-Raphson

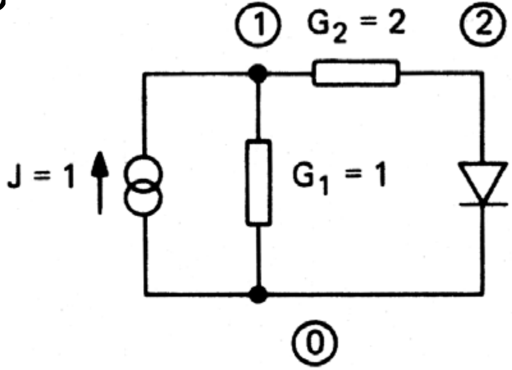
Caso Scalare:

$$f(x) = 0$$

iterazione:

$$x^{k+1} = x^k + \Delta x^k = x^k - f(x^k)/f'(x^k)$$

Esempio



$$i_D = e^{40 v_D} - 1$$

equazioni ai nodi:

$$3v_1 - 2v_2 = 1$$

$$-2v_1 + 2v_2 + (e^{40v_2} - 1) = 0$$

$$v_1 = \frac{1}{3} + \frac{2}{3} v_2$$

$$f(v_2) = \frac{2}{3} v_2 + e^{40v_2} - \frac{5}{3} = 0$$

$$f'(v_2) = \frac{2}{3} + 40e^{40 v_2}$$

$$v_2^0 = 0.1 \text{ V}$$

iterazioni:

k	v_2^k	Δv_2^{k-1}
1	$0.75740D - 01$	$-0.24260D - 01$
2	$0.52712D - 01$	$-0.23029D - 01$
3	$0.32705D - 01$	$-0.20007D - 01$
4	$0.18883D - 01$	$-0.13822D - 01$
5	$0.13356D - 01$	$-0.55267D - 02$
6	$0.12654D - 01$	$-0.70199D - 03$
7	$0.12644D - 01$	$-0.99424D - 05$
8	$0.12644D - 01$	$-0.19579D - 08$

Algoritmo di Newton-Raphson

$$\mathbf{f}(\mathbf{x}) = \mathbf{0}$$

$$\begin{cases} f_1(x_1, x_2, \ldots, x_n) = 0 \\ f_2(x_1, x_2, \ldots, x_n) = 0 \\ \vdots \\ f_n(x_1, x_2, \ldots, x_n) = 0. \end{cases}$$

espansione in serie di Taylor (1° ordine, linearizzazione):

$$\begin{cases} f_1(\mathbf{x}^*) = f_1(\mathbf{x}) + \dfrac{\partial f_1}{\partial x_1}(x_1^* - x_1) + \dfrac{\partial f_1}{\partial x_2}(x_2^* - x_2) + \cdots \\ \qquad + \dfrac{\partial f_1}{\partial x_n}(x_n^* - x_n) + \cdots \\ \vdots \\ f_n(\mathbf{x}^*) = f_n(\mathbf{x}) + \dfrac{\partial f_n}{\partial x_1}(x_1^* - x_1) + \dfrac{\partial f_n}{\partial x_2}(x_2^* - x_2) + \cdots \\ \qquad + \dfrac{\partial f_n}{\partial x_n}(x_n^* - x_n) + \cdots. \end{cases}$$

$$\mathbf{f}(\mathbf{x}^*) \approx \mathbf{f}(\mathbf{x}) + \mathbf{M}(\mathbf{x}^* - \mathbf{x})$$

Algoritmo di Newton-Raphson

$$\mathbf{f}(\mathbf{x}^*) \approx \mathbf{f}(\mathbf{x}) + \mathbf{M}(\mathbf{x}^* - \mathbf{x}) \longrightarrow \mathbf{f}(\mathbf{x}^k) + \mathbf{M}(\mathbf{x}^{k+1} - \mathbf{x}^k) =$$

$$\mathbf{x}^{k+1} = \mathbf{x}^k - \mathbf{M}^{-1}\mathbf{f}(\mathbf{x}^k)$$

$$\mathbf{M} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}_{|\mathbf{x}}$$

Matrice
Jacobiana

Algoritmo di Newton-Raphson

$$\mathbf{x}^{k+1} = \mathbf{x}^k - \mathbf{M}^{-1}\mathbf{f}(\mathbf{x}^k)$$

$$\Big\downarrow \quad \Delta\mathbf{x}^k = \mathbf{x}^{k+1} - \mathbf{x}^k$$

$$\mathbf{M}\Delta\mathbf{x}^k = -\mathbf{f}(\mathbf{x}^k)$$

$$\Big\downarrow$$

sistema lineare
fattorizzazione LU

$$\mathbf{x}^{k+1} = \mathbf{x}^k + \Delta\mathbf{x}^k$$

Algoritmo di Newton-Raphson

per ottenere, ad ogni iterazione:

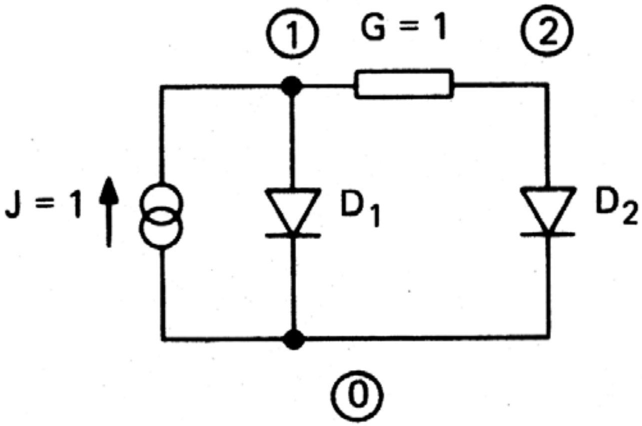
$$\| \mathbf{f}(\mathbf{x}^{k+1}) \| \leqslant \| \mathbf{f}(\mathbf{x}^{k}) \|$$

si usa:

$$\mathbf{x}^{k+1} = \mathbf{x}^{k} + t^{k} \Delta \mathbf{x}^{k}$$

$$0 < t^{k} \leqslant 1$$

Esempio



$$i_D = e^{40 v_D} - 1,$$

stima iniziale:

$$v_1^0 = v_2^0 = 0.1$$

equazioni ai nodi:

$$i_{D1} + G(v_1 - v_2) = j$$

$$G(-v_1 + v_2) + i_{D2} = 0$$

$$\longrightarrow$$

$$f_1(v_1, v_2) = e^{40 v_1} + v_1 - v_2 - 2 = 0$$

$$f_2(v_1, v_2) = -v_1 + v_2 + e^{40 v_2} - 1 = 0$$

Jacobiano: $$\mathbf{M} = \begin{bmatrix} 40e^{40v_1} + 1 & -1 \\ -1 & 40e^{40v_2} + 1 \end{bmatrix}$$

Esempio

prima iterazione:

$$f_1 = 52.59815$$

$$f_2 = 53.59815$$

$$\mathbf{M} = \begin{bmatrix} 2184.926 & -1 \\ -1 & 2184.926 \end{bmatrix}$$

$$\Delta v_1^0 = -0.0240844$$

$$\Delta v_2^0 = -0.0245419$$

$$v_1^1 = 0.0759156$$

$$v_2^1 = 0.07545810$$

k	Δv_1^k	Δv_2^k	v_1^{k+1}	v_2^{k+1}
0	−0.02408	−0.02454	0.07592	0.07546
1	−0.02260	−0.02378	0.05331	0.05168
2	−0.01909	−0.02182	0.03423	0.02986
3	−0.01234	−0.01736	0.02188	0.01250
4	−0.00432	−0.00962	0.01757	0.00289
5	−0.00044	−0.00236	0.01712	0.00053
6	−0.00001	−0.00012	0.01712	0.00041
7	−0.00000	−0.00000	0.01712	0.00041

Formulazione Nodale

supponiamo:
solo gen. indipendenti di corrente
nonlinearità costitutite da resistori controllati in tensione:

$$i_b = g(v_b)$$

KVL: $\mathbf{v}_b = \mathbf{A}^t \mathbf{v}_n$

KCL: $\mathbf{A}\mathbf{i}_b = -\mathbf{A}_J \mathbf{j}_b = \mathbf{j}_n$

$$\downarrow$$

$$\mathbf{A}\mathbf{g}(\mathbf{v}_b) = \mathbf{j}_n$$

$$\downarrow$$

$$\mathbf{A}\mathbf{g}(\mathbf{A}^t \mathbf{v}_n) = \mathbf{j}_n$$

Formulazione Nodale

Newton-Raphson:

$$\mathbf{f}(\mathbf{v}_n) \equiv \mathbf{A}\mathbf{g}(\mathbf{A}^t\mathbf{v}_n) - \mathbf{j}_n = \mathbf{0}.$$

$$\mathbf{M} = \frac{\partial \mathbf{f}}{\partial \mathbf{v}_n} = \mathbf{A}\,\frac{\partial \mathbf{g}}{\partial \mathbf{v}_b}\,\frac{\partial \mathbf{v}_b}{\partial \mathbf{v}_n}.$$

$$\frac{\partial \mathbf{g}}{\partial \mathbf{v}_b} = \mathbf{G}_b \qquad \frac{\partial \mathbf{v}_b}{\partial \mathbf{v}_n} = \mathbf{A}^t$$

$$\downarrow$$

$$\mathbf{M} = \mathbf{A}\mathbf{G}_b\mathbf{A}^t$$

Tableau (MNA)

conduttanza nonlineare tra i nodi *k* ed *l*:

$$i_b = g(v_b).$$

equazioni ai nodi:

$$k: \qquad \cdots + g(v_k - v_l) \cdots = 0$$

$$l: \qquad \cdots - g(v_k - v_l) \cdots = 0.$$

Jacobiano:

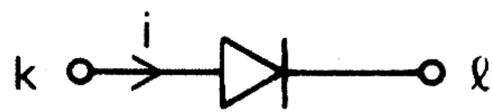
$$\begin{array}{ll} \text{variable} \longrightarrow & \begin{array}{ccccc} \qquad & v_k & \qquad & v_l & \qquad \end{array} \\ \begin{array}{l} \text{row } k \\ \text{row } l \end{array} & \begin{bmatrix} \cdots & \partial g/\partial v_b & \cdots & -\partial g/\partial v_b & \cdots \\ \cdots & -\partial g/\partial v_b & \cdots & \partial g/\partial v_b & \cdots \end{bmatrix} \end{array}$$

Lo Jacobiano ha esattamente la stessa forma della matrice nodale delle conduttanze.
Le conduttanze lineari restano invariate, al posto delle conduttanze nonlineari c'è la derivata della r.c., calcolata alla tensione dell'iterazione.

Tableau (MNA)

$$
\begin{array}{c c}
 & \begin{array}{cc} k & \ell \end{array} \\
\begin{array}{c} k \\ \ell \end{array} & \left[\begin{array}{cc} \partial h/\partial v & -\partial h/\partial v \\ -\partial h/\partial v & \partial h/\partial v \end{array}\right]
\end{array}
\qquad
\begin{array}{cc}
\left[\begin{array}{c} h(v) \\ -h(v) \end{array}\right] & \begin{array}{c} k \\ \ell \end{array}
\end{array}
$$

Jacobian　　　　　　　　Right-hand side



$i = h(v)$; $v = v_u - v_e$

Tableau

Equazioni del Tableau:

$$\mathbf{v}_b - \mathbf{A}^t\mathbf{v}_n = \mathbf{0}$$

$$\mathbf{p}(\mathbf{v}_b, \mathbf{i}_b) = \mathbf{w}$$

$$\mathbf{A}\mathbf{i}_b = \mathbf{0}.$$

Formulazione per Newton-Raphson:

$$\mathbf{f}(\mathbf{x}) \equiv \begin{bmatrix} \mathbf{v}_b - \mathbf{A}^t\mathbf{v}_n \\ \mathbf{p}(\mathbf{v}_b, \mathbf{i}_b) - \mathbf{w} \\ \mathbf{A}\mathbf{i}_b \end{bmatrix} = \mathbf{0}$$

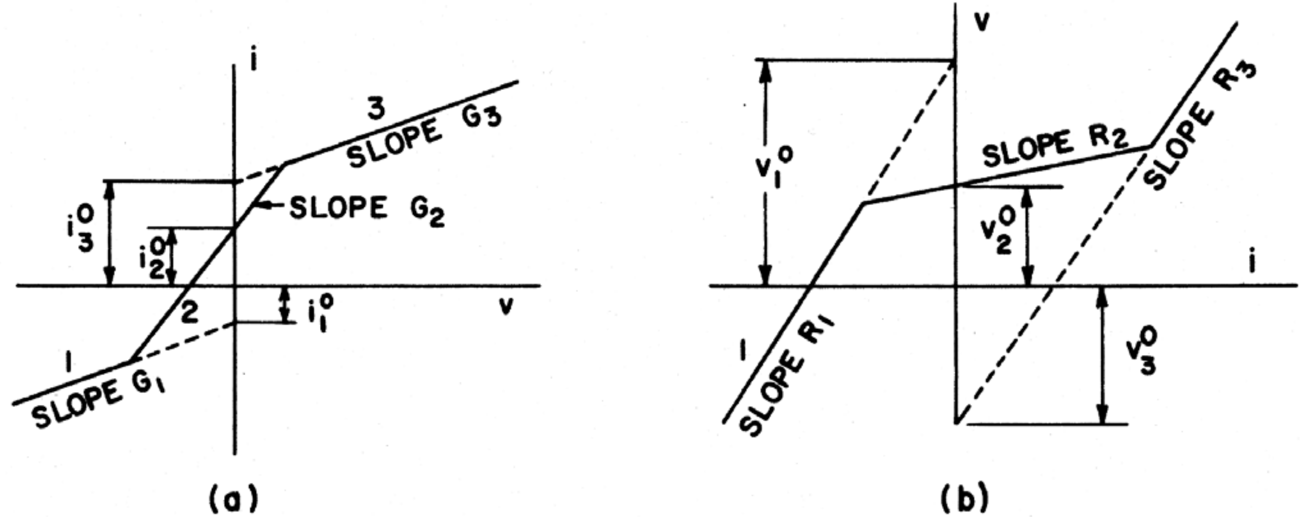
$$\mathbf{M}^k \Delta\mathbf{x}^k = -\mathbf{f}(\mathbf{x}^k)$$

Jacobiano alla k-esima iterazione:

$$\mathbf{M} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & -\mathbf{A}^t \\ \mathbf{G}^k & \mathbf{R}^k & \mathbf{0} \\ \mathbf{0} & \mathbf{A} & \mathbf{0} \end{bmatrix} \longrightarrow \Delta\mathbf{x}^k = \begin{bmatrix} \Delta\mathbf{v}_b \\ \Delta\mathbf{i}_b \\ \Delta\mathbf{v}_n \end{bmatrix}.$$

dove: $\mathbf{G}^k = \left.\dfrac{\partial\mathbf{p}}{\partial\mathbf{v}_b}\right|_{\mathbf{x}^k}; \quad \mathbf{R}^k = \left.\dfrac{\partial\mathbf{p}}{\partial\mathbf{i}_b}\right|_{\mathbf{x}^k}$

Tecniche PWL



$$v = v_l^0 + R_l i$$

$$i = i_l^0 + G_l v$$

$\longrightarrow$ $\mathbf{G}_l \mathbf{v}_b + \mathbf{R}_l \mathbf{i}_b = \mathbf{w}_l.$

tableau

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & -\mathbf{A}^t \\ \mathbf{G}_l & \mathbf{R}_l & \mathbf{0} \\ \mathbf{0} & \mathbf{A} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v}_b \\ \mathbf{i}_b \\ \mathbf{v}_n \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{w}_l \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{w} \\ \mathbf{0} \end{bmatrix}$$

$$\mathbf{T}_l \mathbf{x}_l = \mathbf{w}_l + \mathbf{w}.$$

Algoritmo di Katzenelson

$$\mathbf{T}_l \mathbf{x}_l = \mathbf{w}_l + \mathbf{w}.$$

alla k-esima iterazione:

$$\mathbf{f}^k = \mathbf{T}_l^k \mathbf{x}^k - \mathbf{w}_l^k - \mathbf{w}$$

Jacobiano:

$$\mathbf{M}^k = \mathbf{T}^k$$

Newton-Raphson:

$$\mathbf{T}_l^k \Delta\mathbf{x}^k = -\mathbf{f}^k$$

$$\hat{\mathbf{x}}^{k+1} = \mathbf{x}^k + \Delta\mathbf{x}^k$$

Se nessun elemento cambia regione, $\hat{\mathbf{x}}^{k+1}$ è la soluzione.

Altrimenti:

$$\mathbf{x}^{k+1} = \mathbf{x}^k + t^k \Delta\mathbf{x}^k$$

Algoritmo di Katzenelson

$$\mathbf{x}^{k+1} = \mathbf{x}^k + t^k \Delta \mathbf{x}^k$$

Katzenelson: si sceglie t^k in modo che il punto di lavoro
di un solo elemento vada a finire sul confine della regione attuale.

$$t_i^k = \frac{x_{li} - x_i^k}{\Delta x_i}, \qquad \text{se } \Delta x_i > 0$$

$$t_i^k = \frac{x_i^k - x_{(l-1)i}}{\Delta x_i}, \qquad \text{se } \Delta x_i < 0$$

$$t^k = \min_i \, (t_i^k)$$

Algoritmo di Katzenelson

sul confine, due equazioni, per due regioni contigue, sono valide:

$$\mathbf{f}^{k+1} = \mathbf{T}_l^{k+1}\mathbf{x}^{k+1} - \mathbf{w}_l^{k+1} - \mathbf{w}$$

$$= \mathbf{T}_l^{k}\mathbf{x}^{k+1} - \mathbf{w}_l^{k} - \mathbf{w}.$$

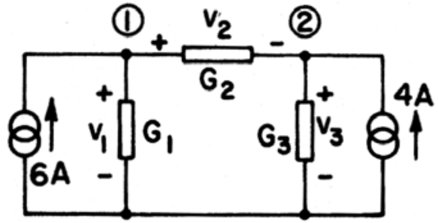
cioè:

$$\mathbf{f}^{k+1} = (\mathbf{T}_l^{k}\mathbf{x}^{k} - \mathbf{w}_l^{k} - \mathbf{w}) + t^k\mathbf{T}_l^{k}\Delta\mathbf{x}^{k}$$

$$= \mathbf{f}^{k} - t^k\mathbf{f}^{k}$$

$$= (1 - t^k)\mathbf{f}^{k}$$

Esempio

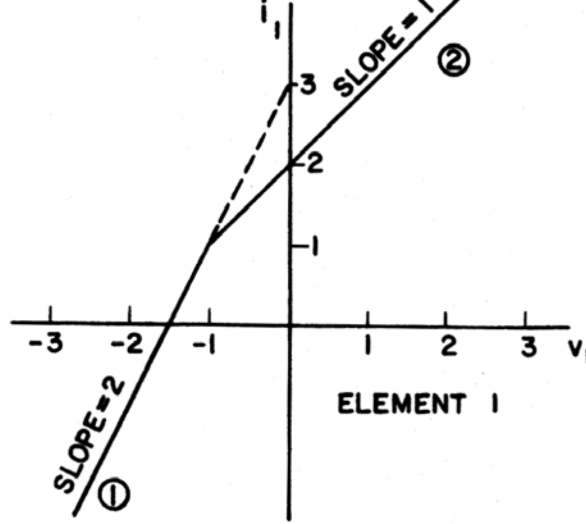


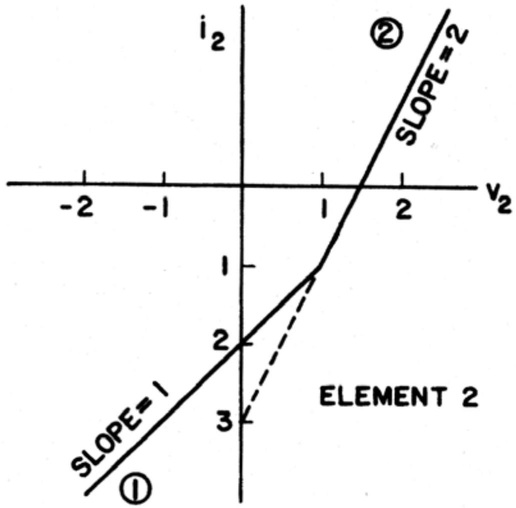
tensioni di nodo iniziali :

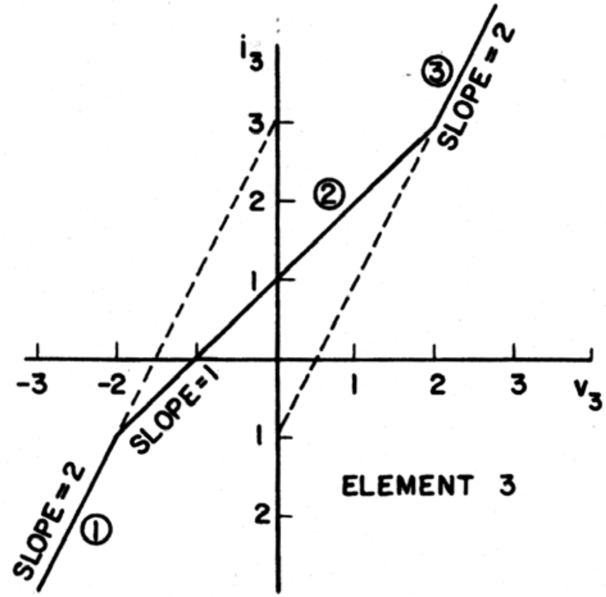
$v_{n1} = -3 \quad v_{n2} = -3$

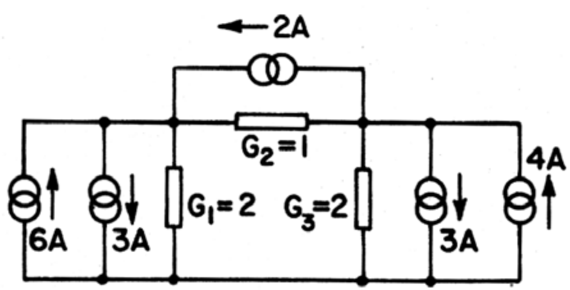
tensioni di ramo :

$v_1 = v_3 = -3 \quad v_2 = 0$









stato iniziale

Esempio

$$\mathbf{f}^0 = \mathbf{T}_l^0 \mathbf{x}^0 - \mathbf{w}_l^0 - \mathbf{w}.$$

$$\mathbf{f}^0 = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} -3 \\ -3 \end{bmatrix} - \begin{bmatrix} -1 \\ -5 \end{bmatrix} - \begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} -11 \\ -5 \end{bmatrix}$$

$$\mathbf{T}_l^k \Delta \mathbf{x}^k = -\mathbf{f}^k.$$

$$\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} \Delta v_{n1}^0 \\ \Delta v_{n2}^0 \end{bmatrix} = \begin{bmatrix} 11 \\ 5 \end{bmatrix} \longrightarrow \begin{bmatrix} \Delta v_{n1}^0 \\ \Delta v_{n2}^0 \end{bmatrix} = \begin{bmatrix} \frac{19}{4} \\ \frac{13}{4} \end{bmatrix}$$

$$\begin{bmatrix} \hat{v}_{n1}^1 \\ \hat{v}_{n2}^1 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \end{bmatrix} + \begin{bmatrix} \frac{19}{4} \\ \frac{13}{4} \end{bmatrix} = \begin{bmatrix} \frac{7}{4} \\ \frac{1}{4} \end{bmatrix}$$

$$\begin{bmatrix} \hat{v}_1^1 \\ \hat{v}_2^1 \\ \hat{v}_3^1 \end{bmatrix} = \begin{bmatrix} \frac{7}{4} \\ \frac{6}{4} \\ \frac{1}{4} \end{bmatrix} \longleftarrow \text{tutti gli elementi cambiano regione}$$

Esempio

$$t_1^0 = \frac{-1 - (-3)}{\frac{19}{4}} = \frac{8}{19}$$

$$t_2^0 = \frac{1 - 0}{\frac{6}{4}} = \frac{4}{6} \longrightarrow t^0 = \min_i (t_i^0) = \tfrac{4}{13}$$

$$t_3^0 = \frac{-2 - (-3)}{\frac{13}{4}} = \frac{4}{13}$$

tensioni di nodo:

$$\begin{bmatrix} v_{n1}^1 \\ v_{n2}^1 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \end{bmatrix} + \tfrac{4}{13} \begin{bmatrix} \frac{19}{4} \\ \frac{13}{4} \end{bmatrix} = \begin{bmatrix} -\frac{20}{13} \\ -\frac{26}{13} \end{bmatrix}$$

$$\mathbf{f}^1 = (1 - \tfrac{4}{13}) \begin{bmatrix} -11 \\ -5 \end{bmatrix} = \begin{bmatrix} -\frac{99}{13} \\ -\frac{45}{13} \end{bmatrix}$$

tensioni sui componenti:

$$\begin{bmatrix} v_1^1 \\ v_2^1 \\ v_3^1 \end{bmatrix} = \begin{bmatrix} -\frac{20}{13} \\ \frac{6}{13} \\ -\frac{26}{13} \end{bmatrix}$$

fine della 1° iterazione

siamo in una nuova regione, con: $G_1 = 2 \quad G_2 = 1 \quad G_3 = 1$

$\downarrow$

Nuova iterazione.