THE SOLUTION OF PASSIVE ELECTRICAL NETWORKS BY MEANS OF MATHEMATICAL TREES

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The classical method of solving electrical networks is by means of determinants derived from the network, with the aid of Kirchhoff's laws. Unfortunately the expansion of the determinant of a complicated network may be a very lengthy process, and considerable care must be taken to avoid errors in sign.

A new method is described for networks not containing valves or mutual inductances, which is based on correspondences between the topological and electrical properties of networks. The diagram of an electrical network in which the branches are represented by lines is known in topology as a "graph." A set of branches on the network which connects all the nodes but does not enclose a mesh is known as a "tree" on the network. By the removal of a single branch, a tree on a network is converted into a pair of separate sub-trees which will be termed a 2-tree. Fig. 1 shows a tree on a bridge network, a 2-tree being obtained by deleting the branch *cd*

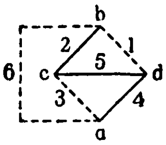

Fig. 1.—A tree (254) on a network.

The branches will be represented by numbers, and trees and 2-trees will be represented by the product of the branches of which they are composed. A set of trees T is defined as the sum of all possible trees on the network. A set of 2-trees $T_{ad.bc}$ is defined as the sum of all possible 2-trees such that one sub-tree of each 2-trees includes the nodes a and d while the other sub-tree includes the nodes b and c. For Fig. 1 $T_{ad.bc} = 42$, there being only one 2-tree which satisfies the conditions.

The difference between two sets of 2-trees such as $T_{ad.bc}$ and $T_{ac.bd}$ is termed the L-linkage $L_{ab.cd}$ between the pair of input nodes a and b and the pair of output nodes c and d. For Fig. 1 we have $L_{ab.cd} = 42 - 31$. If p is the only branch cd, we define the K-linkage by the equation $K_{ab.cd} = pL_{ab.cd}^{1}$.

It may be shown that the sum of a set of L-linkages for which the output nodes form a closed circuit is zero. This is a topological theorem analogous to Kirchhoff's law for voltages round a mesh.

It may also be shown that the sum of the K-linkages for a set of branches, such as p, meeting at a single node is zero. This is analogous to Kirchhoff's law for currents meeting at a point. The set of trees T is a generalized case of a K-linkage.

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It follows that we can set up a one-to-one correspondence between electrical and topological properties of networks. In particular, it can be shown that the ratio of a current i entering at node b and leaving at node a to the resultant voltage V of d relative to c is given by

$$\frac{i}{V} = Y_{ac.bd} = \frac{T}{L_{ab.cd}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where each branch is given the value of its admittance.

It follows that T is the nodal determinant D of the network, and $L_{ab.cd}$ is the difference between the two appropriate co-factors.

The set of trees T is evaluated with the aid of equations between networks, in which the network considered is expressed as the sum of a number of simpler networks such that the set of trees T for the network as a whole is equal to the sum of the sets of trees for the simpler networks.

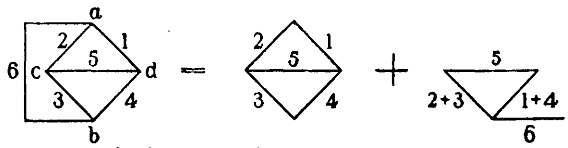

Fig. 2.—Network equation for bridge.

A few simple rules enable the network equation to be set up and the set of trees for the simpler networks to be obtained by inspection.

Fig. 2 shows the equation for the bridge network from which the nodal determinant

$$D = T = 12(3 + 4) + 34(1 + 2) + 5(1 + 2)(3 + 4) + 6[5(1 + 2 + 3 + 4) + (1 + 4)(2 + 3)] \quad . \quad (2)$$

can be obtained by inspection.

With the aid of $L_{ab.cd}$ previously obtained we have

$$Y_{ac.bd} = \frac{T}{42 - 31} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In more complicated cases $L_{ab.cd}$ can be evaluated with the aid of network equations yielding sets of 2-trees or by other means.

The method can be shown to be quicker to use than the determinant method, the advantage increasing with the complexity of the network.