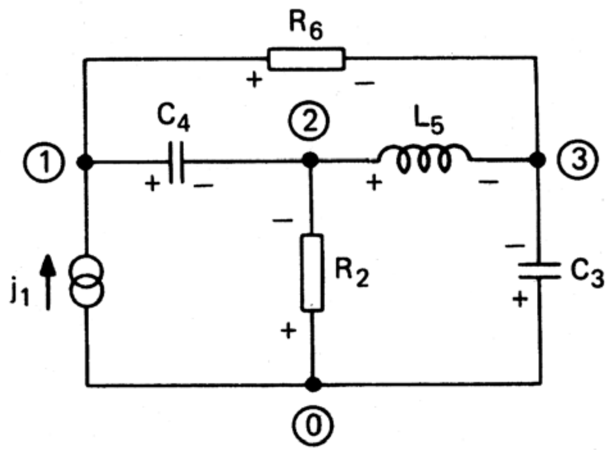
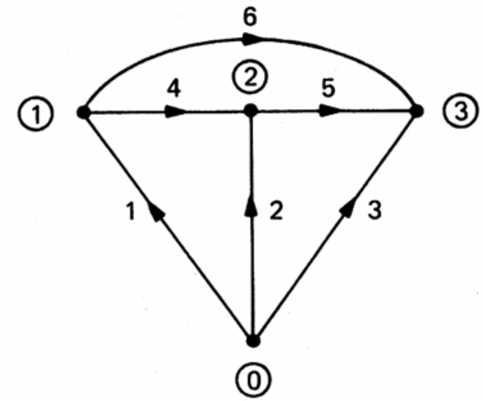
Richiami sui metodi generali di formulazione delle equazioni di una rete elettrica

- Matrice incidenza
- Relazioni costitutive
- Metodo del Tableau
- Metodo ai nodi modificato
- Metodo a due grafi

Matrice Incidenza

Esempio





rami →

nodi ↓

$$\mathbf{A} = \begin{array}{cc} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix} \end{array}$$

Matrice Incidenza

$$\text{tensioni di ramo} \qquad \text{correnti di ramo} \qquad \text{tensioni di nodo}$$

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_b \end{bmatrix} \qquad \mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_b \end{bmatrix} \qquad \mathbf{v}_n = \begin{bmatrix} v_{10} \\ v_{20} \\ \vdots \\ v_{n,0} \end{bmatrix}$$

$$KCL \quad \rightarrow \quad \mathbf{Ai} = \mathbf{0}$$

$$KVL \quad \rightarrow \quad \mathbf{A}^t \mathbf{v}_n = \mathbf{v}$$

Matrice Incidenza

Nell'esempio precedente:

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_{10} \\ v_{20} \\ v_{30} \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} \quad \rightarrow \quad \begin{cases} -v_{10} = v_1 \\ -v_{20} = v_2 \\ -v_{30} = v_3 \\ v_{10} - v_{20} = v_4 \\ v_{20} - v_{30} = v_5 \\ v_{10} - v_{30} = v_6 \end{cases}$$

$$sostituendo\, le\; prime\, 3\, eq.\, nelle\; altre:$$

$$\begin{cases} -v_1 + v_2 = v_4 \\ -v_2 + v_3 = v_5 \\ -v_1 + v_3 = v_6 \end{cases} \qquad (b-n\;\; eq.,\;\; n = num.\, nodi\; ind.)$$

Matrice Incidenza

Proprietà

$$Rank\left(\mathbf{A}\right) = n$$

$$\mathbf{A}^t \mathbf{v}_n = \mathbf{v} \qquad \rightarrow \qquad \mathbf{v}^t = \mathbf{v}_n^t \mathbf{A} \qquad \rightarrow \qquad \mathbf{v}^t \mathbf{i} = \mathbf{v}_n^t \mathbf{A} \mathbf{i}$$

$$\Rightarrow$$

$$\mathbf{v}^t \mathbf{i} = \mathbf{0} \qquad \textit{Teorema di Tellegen}$$

$$\text{in regime sinusoidale:}$$

$$\mathbf{V}^t \mathbf{I}^* = \mathbf{0} \qquad \textit{Teorema di Boucherot}$$

Relazioni Costitutive

$$\begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{K}_2 \end{bmatrix} \mathbf{V}_b + \begin{bmatrix} \mathbf{K}_1 \\ \mathbf{Z}_2 \end{bmatrix} \mathbf{I}_b = \begin{bmatrix} \mathbf{W}_{b_1} \\ \mathbf{W}_{b_2} \end{bmatrix}$$

$$\mathbf{Y}_b \mathbf{V}_b + \mathbf{Z}_b \mathbf{I}_b = \mathbf{W}_b$$

Element	Constitutive Equation	Value of Y_b	Value of Z_b	Value of W_b
Resistor	$V_b - R_b I_b = 0$	1	$-R_b$	0
Conductor	$G_b V_b - I_b = 0$	G_b	-1	0
Capacitor	$sC_b V_b - I_b = C_b V_0$	sC_b	-1	$C_b V_0$
Inductor	$V_b - sL_b I_b = -L_b I_0$	1	$-sL_b$	$-L_b I_0$
Voltage source	$V_b = E_b$	1	0	E_b
Current source	$I_b = J_b$	0	1	J_b

Formulazione Generale: Tableau

$$\mathbf{V}_b - \mathbf{A}^t\mathbf{V}_n = \mathbf{0} \qquad \text{KVL}$$

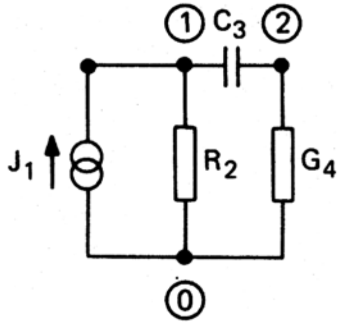
$$\mathbf{Y}_b\mathbf{V}_b + \mathbf{Z}_b\mathbf{I}_b = \mathbf{W}_b \qquad \text{R.C.}$$

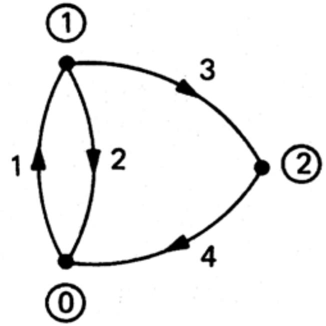
$$\mathbf{A}\mathbf{I}_b = \mathbf{0} \qquad \text{KCL}$$

$$\begin{array}{c} \\ b \\ b \\ n \end{array}
\begin{array}{c} \begin{array}{ccc} b & b & n \end{array} \\
\begin{bmatrix} \mathbf{1} & \mathbf{0} & -\mathbf{A}^t \\ \mathbf{Y}_b & \mathbf{Z}_b & \mathbf{0} \\ \mathbf{0} & \mathbf{A} & \mathbf{0} \end{bmatrix} \end{array}
\begin{array}{c} \\ \begin{bmatrix} \mathbf{V}_b \\ \mathbf{I}_b \\ \mathbf{V}_n \end{bmatrix} \end{array}
\begin{array}{c} \\ = \end{array}
\begin{array}{c} \\ \begin{bmatrix} \mathbf{0} \\ \mathbf{W}_b \\ \mathbf{0} \end{bmatrix} \end{array}$$

$$\mathbf{TX} = \mathbf{W}$$

Esempio

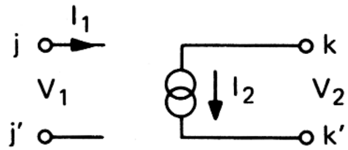
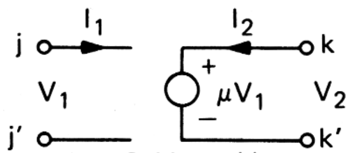
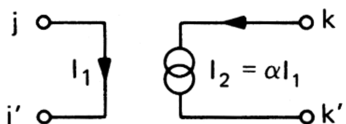
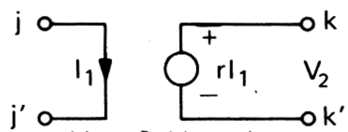
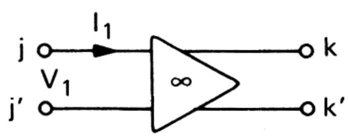




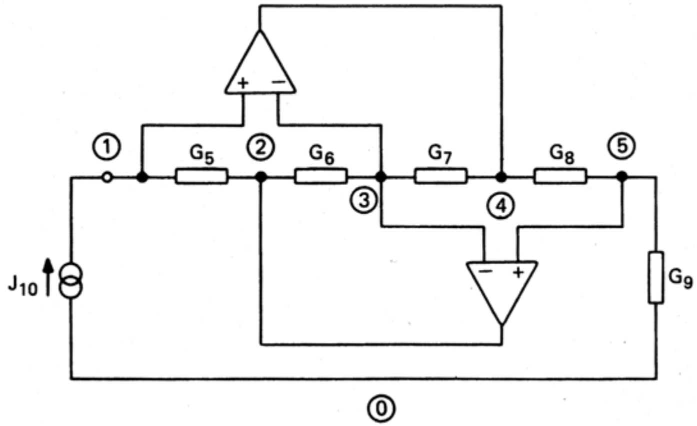
$$
\begin{array}{c c}
 & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\mathbf{A} = & \begin{bmatrix} -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}
\end{array}
$$

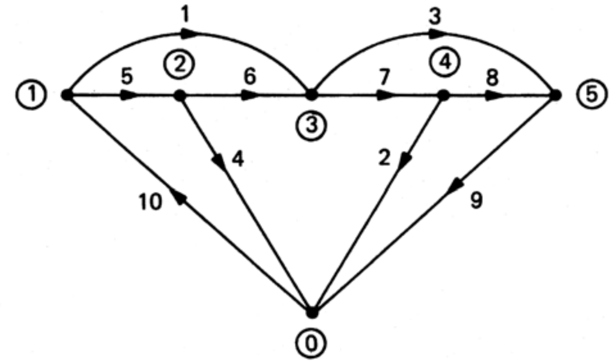
$$
\left[\begin{array}{cccc|cccc|cc}
1 & & & & & & & & 1 & 0 \\
 & 1 & & & & & \mathbf{0} & & -1 & 0 \\
 & & 1 & & & & & & -1 & 1 \\
 & & & 1 & & & & & 0 & -1 \\
\hline
0 & & & & 1 & & & & & \\
 & 1 & & & & -R_2 & & & \mathbf{0} & \\
 & & sC_3 & & & & -1 & & & \\
 & & & G_4 & & & & -1 & & \\
\hline
 & & & & -1 & 1 & 1 & 0 & & \\
 & \mathbf{0} & & & 0 & 0 & -1 & 1 & \mathbf{0} &
\end{array}\right]
\left[\begin{array}{c}
V_1 \\ V_2 \\ V_3 \\ V_4 \\ \hline I_1 \\ I_2 \\ I_3 \\ I_4 \\ \hline V_{n1} \\ V_{n2}
\end{array}\right]
=
\left[\begin{array}{c}
\\ \mathbf{0} \\ \\ \\ \hline J_1 \\ 0 \\ 0 \\ 0 \\ \hline \mathbf{0} \\ \\
\end{array}\right]
$$

Relazioni Costitutive: Componenti a due porte

ELEMENT	SYMBOL	CONSTITUTIVE EQUATIONS
VCT	 $I_1 = 0; I_2 = gV_1$	$\begin{bmatrix} 0 & 0 \\ g & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
VVT	 $I_1 = 0; V_2 = \mu V_1$	$\begin{bmatrix} 0 & 0 \\ \mu & -1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
CCT	 $V_1 = 0; I_2 = \alpha I_1$	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \alpha & -1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
CVT	 $V_1 = 0; V_2 = rI_1$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ r & 0 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
OPAMP	 $V_1 = 0; I_1 = 0$	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Esempio





$$
\begin{array}{cc}
 & \begin{array}{cccccccccc|cccccccccc|ccccc} 1&2&3&4&5&6&7&8&9&10&1&2&3&4&5&6&7&8&9&10&1&2&3&4&5 \end{array} \\
\begin{array}{c} 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\ 1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\ 1\\2\\3\\4\\5 \end{array} &
\left[\begin{array}{cccccccccc|cccccccccc|ccccc}
1&&&&&&&&& & &&&&&&&&& & -1&&1&& \\
&1&&&&&&&& & &&&&&&&&& & &&&-1& \\
&&1&&&&&&& & &&&&&&&&& & &&-1&&1 \\
&&&1&&&&&& & &&&&&&&&& & &-1&&& \\
&&&&1&&&&& & &&&&&&&&& & -1&1&&& \\
&&&&&1&&&& & &&&&&&&&& & &-1&1&& \\
&&&&&&1&&& & &&&&&&&&& & &&-1&1& \\
&&&&&&&1&& & &&&&&&&&& & &&&-1&1 \\
&&&&&&&&1& & &&&&&&&&& & &&&&-1 \\
&&&&&&&&&1 & &&&&&&&&& & 1&&&& \\
\hline
1&&&&&&&&& & &&&&&&&&& & &&&& \\
&&&&&&&&& & 1&&&&&&&&& & &&&& \\
&&1&&&&&&& & &&&&&&&&& & &&&& \\
&&&&&&&&& & &&1&&&&&&& & &&&& \\
&&&&G_5&&&&& & &&&&-1&&&&& & &&&& \\
&&&&&G_6&&&& & &&&&&-1&&&& & &&&& \\
&&&&&&G_7&&& & &&&&&&-1&&& & &&&& \\
&&&&&&&G_8&& & &&&&&&&-1&& & &&&& \\
&&&&&&&&G_9& & &&&&&&&&-1& & &&&& \\
&&&&&&&&& & &&&&&&&&&1 & &&&& \\
\hline
&&&&&&&&& & 1&&&&1&&&&&-1 & &&&& \\
&&&&&&&&& & &&&1&-1&1&&&& & &&&& \\
&&&&&&&&& & -1&&1&&&-1&1&&& & &&&& \\
&&&&&&&&& & &1&&&&&-1&1&& & &&&& \\
&&&&&&&&& & &&-1&&&&&-1&1& & &&&&
\end{array}\right]
\end{array}
\left[\begin{array}{c} V_1\\V_2\\V_3\\V_4\\V_5\\V_6\\V_7\\V_8\\V_9\\V_{10}\\ \hline I_1\\I_2\\I_3\\I_4\\I_5\\I_6\\I_7\\I_8\\I_9\\I_{10}\\ \hline V_{n_1}\\V_{n_2}\\V_{n_3}\\V_{n_4}\\V_{n_5} \end{array}\right]
=
\left[\begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ J_{10}\\ \\ \\ \\ \\ \\ \end{array}\right]
$$

dim: 25x25

Metodo ai Nodi

Eliminazione tensioni di ramo:

$$\mathbf{V}_b = \mathbf{A}^t\mathbf{V}_n$$

$$\mathbf{Y}_b\mathbf{V}_b + \mathbf{Z}_b\mathbf{I}_b = \mathbf{W}_b \longrightarrow \begin{aligned}\mathbf{Y}_b\mathbf{A}^t\mathbf{V}_n + \mathbf{Z}_b\mathbf{I}_b &= \mathbf{W}_b \\ \mathbf{A}\mathbf{I}_b &= \mathbf{0}.\end{aligned} \qquad \begin{bmatrix}\mathbf{Y}_b\mathbf{A}^t & \mathbf{Z}_b \\ \mathbf{0} & \mathbf{A}\end{bmatrix}\begin{bmatrix}\mathbf{V}_n \\ \mathbf{I}_b\end{bmatrix} = \begin{bmatrix}\mathbf{W}_b \\ \mathbf{0}\end{bmatrix}$$

$$\mathbf{A}\mathbf{I}_b = \mathbf{0}$$

Ipotesi: tutti componenti con rapp. ammettenza
solo gen. indip. di corrente

$$\begin{aligned}\mathbf{I}_b &= \mathbf{Y}_b\mathbf{V}_b + \mathbf{J}_b \\ \mathbf{V}_b &= \mathbf{A}^t\mathbf{V}_n\end{aligned} \longrightarrow \mathbf{I}_b = \mathbf{Y}_b\mathbf{A}^t\mathbf{V}_n + \mathbf{J}_b$$

sostituendo in $\mathbf{A}\mathbf{I}_b = \mathbf{0}$

$$\mathbf{A}(\mathbf{Y}_b\mathbf{A}^t\mathbf{V}_n + \mathbf{J}_b) = 0$$

cioè:

$$\mathbf{A}\mathbf{Y}_b\mathbf{A}^t\mathbf{V}_n = -\mathbf{A}\mathbf{J}_b \longrightarrow \mathbf{Y}\mathbf{V}_n = \mathbf{J}_n$$

Formulazione ai nodi
valida per componenti con rapp. ammettenza
e gen. ind. di corrente

Metodo ai Nodi Modificato

KCL

$$[\mathbf{A}_1 \quad \mathbf{A}_2 \quad \mathbf{A}_3]\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\\ \mathbf{J}\end{bmatrix} = \mathbf{0} \qquad \mathbf{A}_1\mathbf{I}_1 + \mathbf{A}_2\mathbf{I}_2 = -\mathbf{A}_3\mathbf{J}$$

1. $\mathbf{I}_1$ correnti di ramo dei componenti che hanno rapp. ammettenza
2. $\mathbf{I}_2$ correnti di ramo dei componenti che non hanno rapp. ammettenza
   + correnti dei generatori di tensione
3. $\mathbf{J}$ correnti dei gen. ind. di corrente

KVL

$$\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\\ \mathbf{V}_J\end{bmatrix} = \begin{bmatrix}\mathbf{A}_1^t\\ \mathbf{A}_2^t\\ \mathbf{A}_3^t\end{bmatrix}\mathbf{V}_n \qquad \begin{aligned}\mathbf{V}_1 &= \mathbf{A}_1^t\mathbf{V}_n\\ \mathbf{V}_2 &= \mathbf{A}_2^t\mathbf{V}_n\\ \mathbf{V}_J &= \mathbf{A}_3^t\mathbf{V}_n\end{aligned}$$

Rel. Cost.

$$\mathbf{Y}_1\mathbf{V}_1 = \mathbf{I}_1.$$

$$\mathbf{Y}_2\mathbf{V}_2 + \mathbf{Z}_2\mathbf{I}_2 = \mathbf{W}_2$$

Metodo ai Nodi Modificato

$$\mathbf{A}_1\mathbf{Y}_1\mathbf{A}_1^t\mathbf{V}_n + \mathbf{A}_2\mathbf{I}_2 = -\mathbf{A}_3\mathbf{J}$$

$$\mathbf{Y}_2\mathbf{A}_2^t\mathbf{V}_n + \mathbf{Z}_2\mathbf{I}_2 = \mathbf{W}_2.$$

$$\begin{bmatrix} \mathbf{A}_1\mathbf{Y}_1\mathbf{A}_1^t & \mathbf{A}_2 \\ \mathbf{Y}_2\mathbf{A}_2^t & \mathbf{Z}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_n \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} -\mathbf{A}_3\mathbf{J} \\ \mathbf{W}_2 \end{bmatrix}$$

$$\mathbf{A}_1\mathbf{Y}_1\mathbf{A}_1^t = \mathbf{Y}_{n1}$$

$$-\mathbf{A}_3\mathbf{J} = \mathbf{J}_n.$$

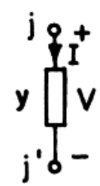
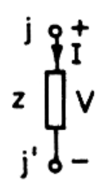
tensioni di nodo

correnti addizionali

$$\begin{array}{lcl} \textbf{KCL} & \begin{bmatrix} \mathbf{Y}_{n1} & \mathbf{A}_2 \\ \mathbf{Y}_2\mathbf{A}_2^t & \mathbf{Z}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_n \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{J}_n \\ \mathbf{W}_2 \end{bmatrix} & \text{gen. corrente ai nodi} \\ \text{equazioni addizionali} & & \text{influenza dei gen. tensione} \end{array}$$

MNA per ispezione
(componente per componente, a partire da una *netlist*)

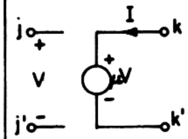
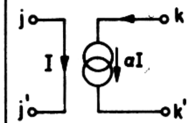
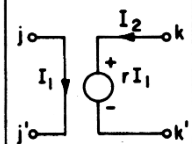
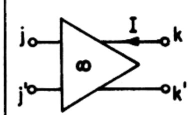
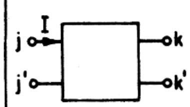
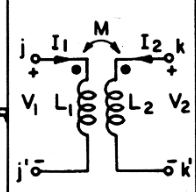
- **Tx** = **w**
- **T** = **G** + s**C**
- Si contano i nodi (n) e si dimensiona **T** ad n x n e **w** ad n
- Si esamina la netlist componente per componente:
  - Componenti con rapp. Ammettenza vengono inseriti in **T** con le regole note (matrice ammettenza ai nodi).
  - Per i componenti con rapp. Impedenza, si aumenta l'ordine di **T** e **w** (si aggiunge a **T** una riga ed una colonna) e si aggiunge un'incognita I.

	$\begin{array}{c c} & \begin{array}{cc} V_j & V_{j'} \end{array} \\ \begin{array}{c} j \\ j' \end{array} & \begin{bmatrix} y & -y \\ -y & y \end{bmatrix} \end{array}$	$I_j = y(V_j - V_{j'})$ $I_{j'} = -y(V_j - V_{j'})$
	$\begin{array}{c c} & \begin{array}{ccc} V_j & V_{j'} & I \end{array} \\ \begin{array}{c} j \\ j' \\ m+1 \end{array} & \left[\begin{array}{cc:c} & & 1 \\ & & -1 \\ \hdashline 1 & -1 & -z \end{array}\right] \end{array}$	$V_j - V_{j'} - zI = 0$ $I_j = -I_{j'} = I$

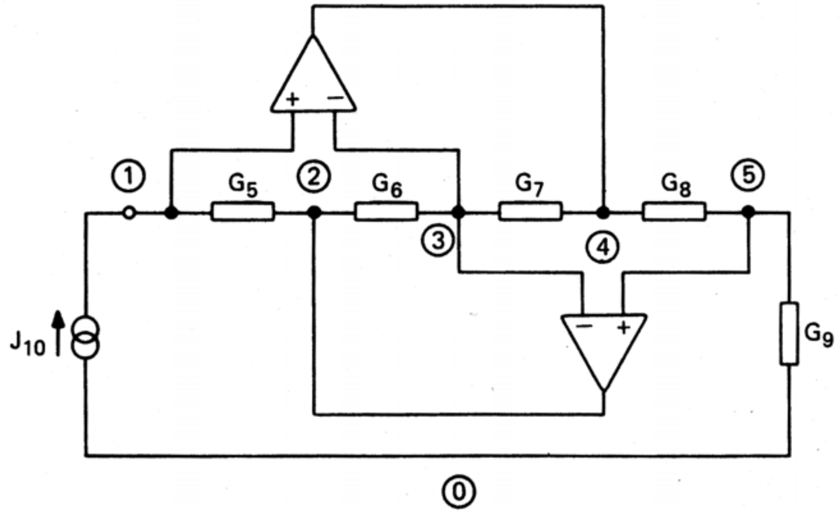
MNA per ispezione

ELEMENT	SYMBOL	MATRIX	EQUATIONS
CURRENT SOURCE		$\begin{array}{c} j \\ j' \end{array}\begin{bmatrix} -J \\ J \end{bmatrix}$ SOURCE VECTOR	$I_j = J$ $I_{j'} = -J$
VOLTAGE SOURCE		$\begin{array}{c} \\ j \\ j' \\ m+1 \end{array}\begin{array}{c} \begin{array}{ccc} V_j & V_{j'} & I \end{array} \\ \begin{bmatrix} & & 1 \\ & & -1 \\ 1 & -1 & \end{bmatrix} \end{array}$ SOURCE VECTOR $\begin{bmatrix} \\ \\ E \end{bmatrix}$	$V_j - V_{j'} = E$ $I_j = I$ $I_{j'} = -I$
OPEN CIRCUIT		——	$V = V_j - V_{j'}$
SHORT CIRCUIT		$\begin{array}{c} \\ j \\ j' \\ m+1 \end{array}\begin{array}{c} \begin{array}{ccc} V_j & V_{j'} & I \end{array} \\ \begin{bmatrix} & & 1 \\ & & -1 \\ 1 & -1 & \end{bmatrix} \end{array}$	$V_j - V_{j'} = 0$ $I_j = I$ $I_{j'} = -I$
ADMITTANCE		$\begin{array}{c} \\ j \\ j' \end{array}\begin{array}{c} \begin{array}{cc} V_j & V_{j'} \end{array} \\ \begin{bmatrix} y & -y \\ -y & y \end{bmatrix} \end{array}$	$I_j = y(V_j - V_{j'})$ $I_{j'} = -y(V_j - V_{j'})$
IMPEDANCE		$\begin{array}{c} \\ j \\ j' \\ m+1 \end{array}\begin{array}{c} \begin{array}{ccc} V_j & V_{j'} & I \end{array} \\ \begin{bmatrix} & & 1 \\ & & -1 \\ 1 & -1 & -z \end{bmatrix} \end{array}$	$V_j - V_{j'} - zI = 0$ $I_j = -I_{j'} = I$
NULLATOR		$\begin{array}{c} \\ \\ m+1 \end{array}\begin{array}{c} \begin{array}{cc} V_j & V_{j'} \end{array} \\ \begin{bmatrix} & \\ 1 & -1 \end{bmatrix} \end{array}$	$V_j - V_{j'} = 0$ $I_j = I_{j'} = 0$
NORATOR		$\begin{array}{c} \\ j \\ j' \end{array}\begin{array}{c} \begin{array}{cc} & I \end{array} \\ \begin{bmatrix} & 1 \\ & -1 \end{bmatrix} \end{array}$	V, I ARE ARBITRARY
VCT		$\begin{array}{c} \\ k \\ k' \end{array}\begin{array}{c} \begin{array}{cc} V_j & V_{j'} \end{array} \\ \begin{bmatrix} g & -g \\ -g & g \end{bmatrix} \end{array}$	$I_j = 0$ $I_{j'} = 0$ $I_k = g(V_j - V_{j'})$ $I_{k'} = -g(V_j - V_{j'})$

MNA per ispezione

ELEMENT	SYMBOL	MATRIX	EQUATIONS
VVT		$\begin{array}{c\|c} & \begin{matrix} V_j & V_{j'} & V_k & V_{k'} & I \end{matrix} \\ \begin{matrix} j \\ j' \\ k \\ k' \\ m+1 \end{matrix} & \left[\begin{array}{cccc\|c} & & & & \\ & & & & \\ & & & & 1 \\ & & & & -1 \\ \hline -\mu & \mu & 1 & -1 & \end{array}\right] \end{array}$	$-\mu V_j + \mu V_{j'} + V_k - V_{k'} = 0$ $I_k = I$ $I_{k'} = -I$
CCT		$\begin{array}{c\|c} & \begin{matrix} V_j & V_{j'} & V_k & V_{k'} & I \end{matrix} \\ \begin{matrix} j \\ j' \\ k \\ k' \\ m+1 \end{matrix} & \left[\begin{array}{cccc\|c} & & & & 1 \\ & & & & -1 \\ & & & & \alpha \\ & & & & -\alpha \\ \hline 1 & -1 & & & \end{array}\right] \end{array}$	$V_j - V_{j'} = 0$ $I_j = -I_{j'} = I$ $I_k = -I_{k'} = \alpha I$
CVT		$\begin{array}{c\|c} & \begin{matrix} V_j & V_{j'} & V_k & V_{k'} & I_1 & I_2 \end{matrix} \\ \begin{matrix} j \\ j' \\ k \\ k' \\ m+1 \\ m+2 \end{matrix} & \left[\begin{array}{cccc\|c\|c} & & & & 1 & \\ & & & & -1 & \\ & & & & & 1 \\ & & & & & -1 \\ \hline 1 & -1 & & & & \\ \hline & & 1 & -1 & -r & \end{array}\right] \end{array}$	$V_j - V_{j'} = 0$ $V_k - V_{k'} - rI_1 = 0$ $I_j = -I_{j'} = I_1$ $I_k = -I_{k'} = I_2$
OPERATIONAL AMPLIFIER		$\begin{array}{c\|c} & \begin{matrix} V_j & V_{j'} & V_k & V_{k'} & I \end{matrix} \\ \begin{matrix} j \\ j' \\ k \\ k' \\ m+1 \end{matrix} & \left[\begin{array}{cccc\|c} & & & & \\ & & & & \\ & & & & 1 \\ & & & & -1 \\ \hline 1 & -1 & & & \end{array}\right] \end{array}$	$V_j - V_{j'} = 0$ $I_k = -I_{k'} = I$
CONVERTOR		$\begin{array}{c\|c} & \begin{matrix} V_j & V_{j'} & V_k & V_{k'} & I \end{matrix} \\ \begin{matrix} j \\ j' \\ k \\ k' \\ m+1 \end{matrix} & \left[\begin{array}{cccc\|c} & & & & 1 \\ & & & & -1 \\ & & & & -K_2 \\ & & & & K_2 \\ \hline 1 & -1 & -K_1 & K_1 & \end{array}\right] \end{array}$	$V_j - V_{j'} - K_1 V_k + K_1 V_{k'} = 0$ $I_j = -I_{j'} = I$ $I_k = -I_{k'} = -K_2 I$ FOR IDEAL TRANSFORMER $K_1 = K_2 = n$
TRANSFORMER		$\begin{array}{c\|c} & \begin{matrix} V_j & V_{j'} & V_k & V_{k'} & I_1 & I_2 \end{matrix} \\ \begin{matrix} j \\ j' \\ k \\ k' \\ m+1 \\ m+2 \end{matrix} & \left[\begin{array}{cccc\|c\|c} & & & & 1 & \\ & & & & -1 & \\ & & & & & 1 \\ & & & & & -1 \\ \hline 1 & -1 & & & -sL_1 & -sM \\ \hline & & 1 & -1 & -sM & -sL_2 \end{array}\right] \end{array}$	$V_j - V_{j'} - sL_1 I_1 - sM I_2 = 0$ $V_k - V_{k'} - sM I_1 - sL_2 I_2 = 0$ $I_j = -I_{j'} = I_1$ $I_k = -I_{k'} = I_2$

Esempio




```
NETLIST
J10 1 0 1
G5  1 2 0.1
G6  2 3 0.1
G7  3 4 0.1
G8  4 5 0.1
G9  5 0 0.1
OP1 4 0 1 3
OP2 2 0 5 3
```

$$
\left[\begin{array}{ccccc|cc}
G_5 & -G_5 & 0 & 0 & 0 & 0 & 0 \\
-G_5 & G_5+G_6 & -G_6 & 0 & 0 & 0 & 1 \\
0 & -G_6 & G_6+G_7 & -G_7 & 0 & 0 & 0 \\
0 & 0 & -G_7 & G_7+G_8 & -G_8 & 1 & 0 \\
0 & 0 & 0 & -G_8 & G_8+G_9 & 0 & 0 \\
\hline
1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & 0 & 0
\end{array}\right]
\left[\begin{array}{c}
V_{n_1} \\ V_{n_2} \\ V_{n_3} \\ V_{n_4} \\ V_{n_5} \\ \hline I_{OP1} \\ I_{OP2}
\end{array}\right]
=
\left[\begin{array}{c}
J_{10} \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0
\end{array}\right]
$$

dim: 7x7

Metodo dei due grafi

Si usano grafi separati per correnti e tensioni

Grafo delle correnti:

1. Se la corrente nel ramo considerato non è presente nelle relazioni costitutive (e non interessa come uscita), i nodi corrispondenti collassano in un solo nodo
2. Se la corrente nel ramo considerato è nulla (per le r.c.), il ramo è cancellato

Grafo delle tensioni:

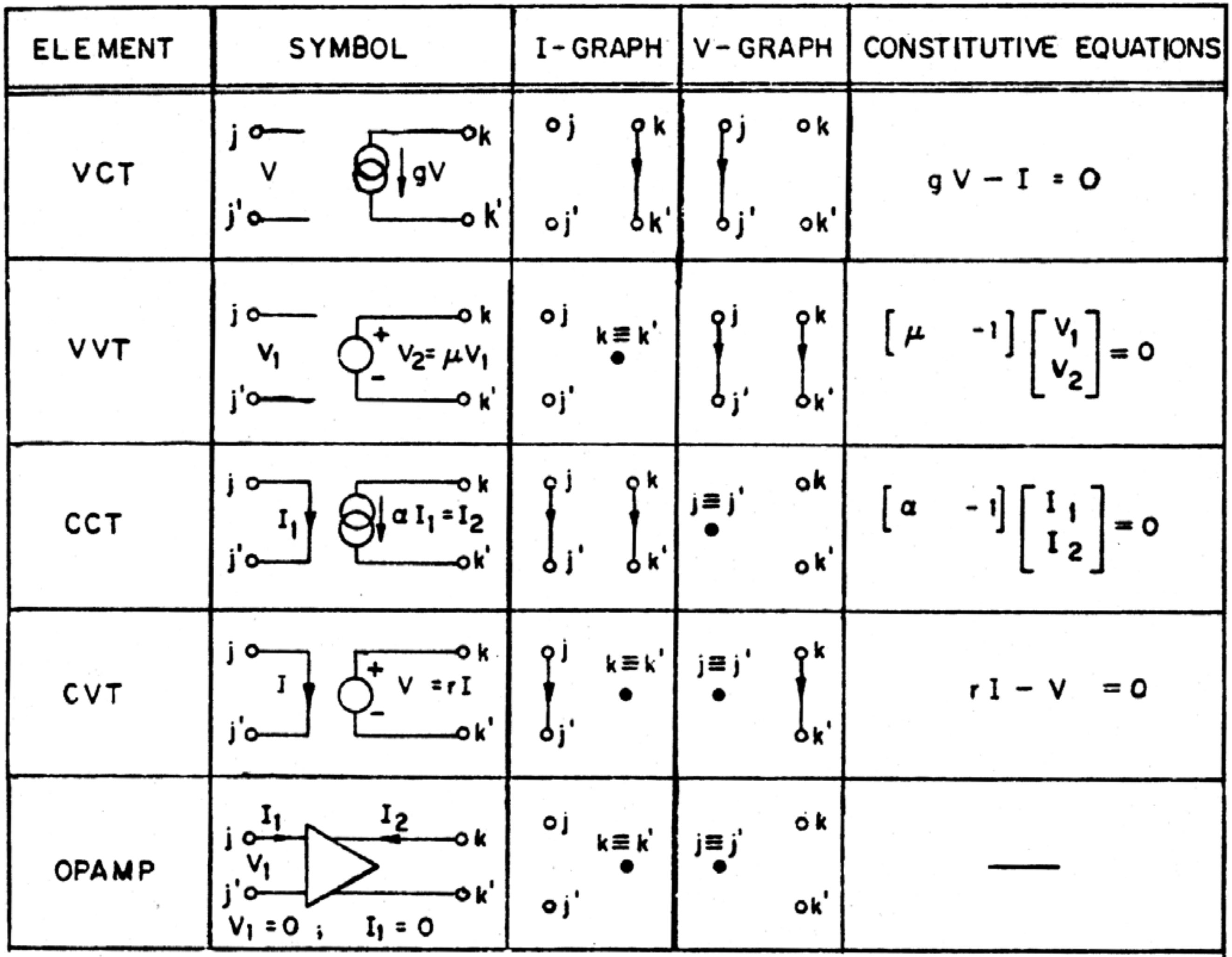
1. Se la tensione sul ramo considerato non è presente nelle relazioni costitutive (e non interessa come uscita), il ramo è cancellato
2. Se la tensione sul ramo considerato è nulla (per le r.c.), i nodi corrispondenti collassano in un solo nodo.

Metodo dei due grafi

ELEMENT	SYMBOL	I - GRAPH	V - GRAPH	CONSTITUTIVE EQUATIONS
CURRENT SOURCE	j, J, j'	j, j'	j, j'	I = J
VOLTAGE SOURCE	j, +, E, −, j'	j ≡ j	j, j'	V = E
OPEN CIRCUIT	j, j'	j, j'	j, j'	—
SHORT CIRCUIT	j, j'	j, j'	j ≡ j'	—
ADMITTANCE	j, y, j'	j, j'	j, j'	yV − I = 0
IMPEDANCE	j, z, j'	j, j'	j, j'	−V + zI = 0
NULLATOR	j, j'	j, j'	j ≡ j'	—
NORATOR		j ≡ j'	j, j'	—

rami di controllo di gen. controllati

Metodo dei due grafi



ELEMENT	SYMBOL	I-GRAPH	V-GRAPH	CONSTITUTIVE EQUATIONS
VCT	j, k, V, gV, j', k'	j, k, j', k'	j, k, j', k'	$gV - I = 0$
VVT	j, k, V_1, $+$, $-$, $V_2 = \mu V_1$, j', k'	j, $k \equiv k'$, j'	j, k, j', k'	$\begin{bmatrix} \mu & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0$
CCT	j, k, I_1, $\alpha I_1 = I_2$, j', k'	j, k, j', k'	$j \equiv j'$, k, k'	$\begin{bmatrix} \alpha & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = 0$
CVT	j, k, I, $+$, $-$, $V = rI$, j', k'	j, $k \equiv k'$, j'	$j \equiv j'$, k, k'	$rI - V = 0$
OPAMP	I_1, I_2, j, k, V_1, j', k', $V_1 = 0$; $I_1 = 0$	j, $k \equiv k'$, j'	$j \equiv j'$, k, k'	—

Tableau a due grafi

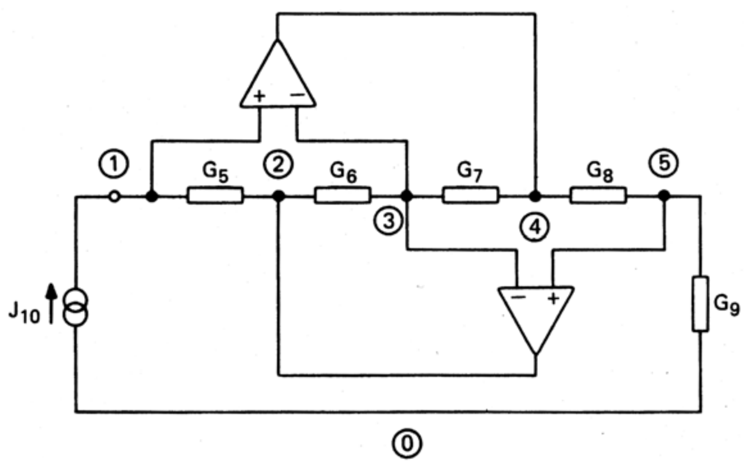
$$\mathbf{V}_b = \mathbf{A}_v^t \mathbf{V}_n$$

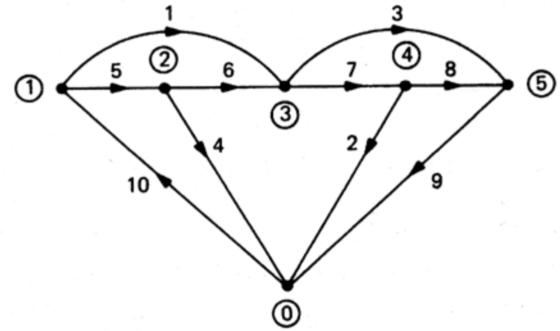
$$\mathbf{Y}_b \mathbf{V}_b + \mathbf{Z}_b \mathbf{I}_b = \mathbf{W}_b$$

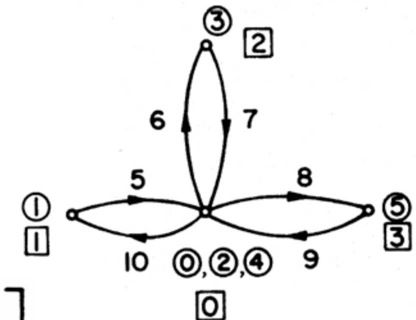
$$\mathbf{A}_i \mathbf{I}_b = \mathbf{0}$$

$$\begin{array}{r|ccc|}
 & \text{tensioni di ramo} & \text{correnti di ramo} & \text{nodi nel grafo } \mathbf{V} \\
\text{tensioni di ramo} & \mathbf{1} & \mathbf{0} & -\mathbf{A}_v^t \\
\text{relazioni costitutive} & \mathbf{Y}_b & \mathbf{Z}_b & \mathbf{0} \\
\text{nodi nel grafo } \mathbf{I} & \mathbf{0} & \mathbf{A}_i & \mathbf{0}
\end{array}
\begin{bmatrix} \mathbf{V}_b \\ \mathbf{I}_b \\ \mathbf{V}_n \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{W}_b \\ \mathbf{0} \end{bmatrix}$$

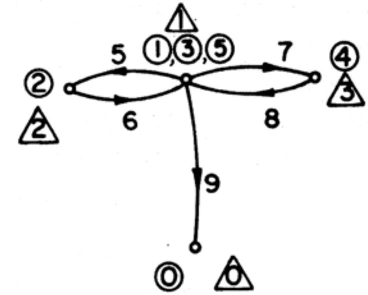
Esempio







I - GRAPH



V - GRAPH

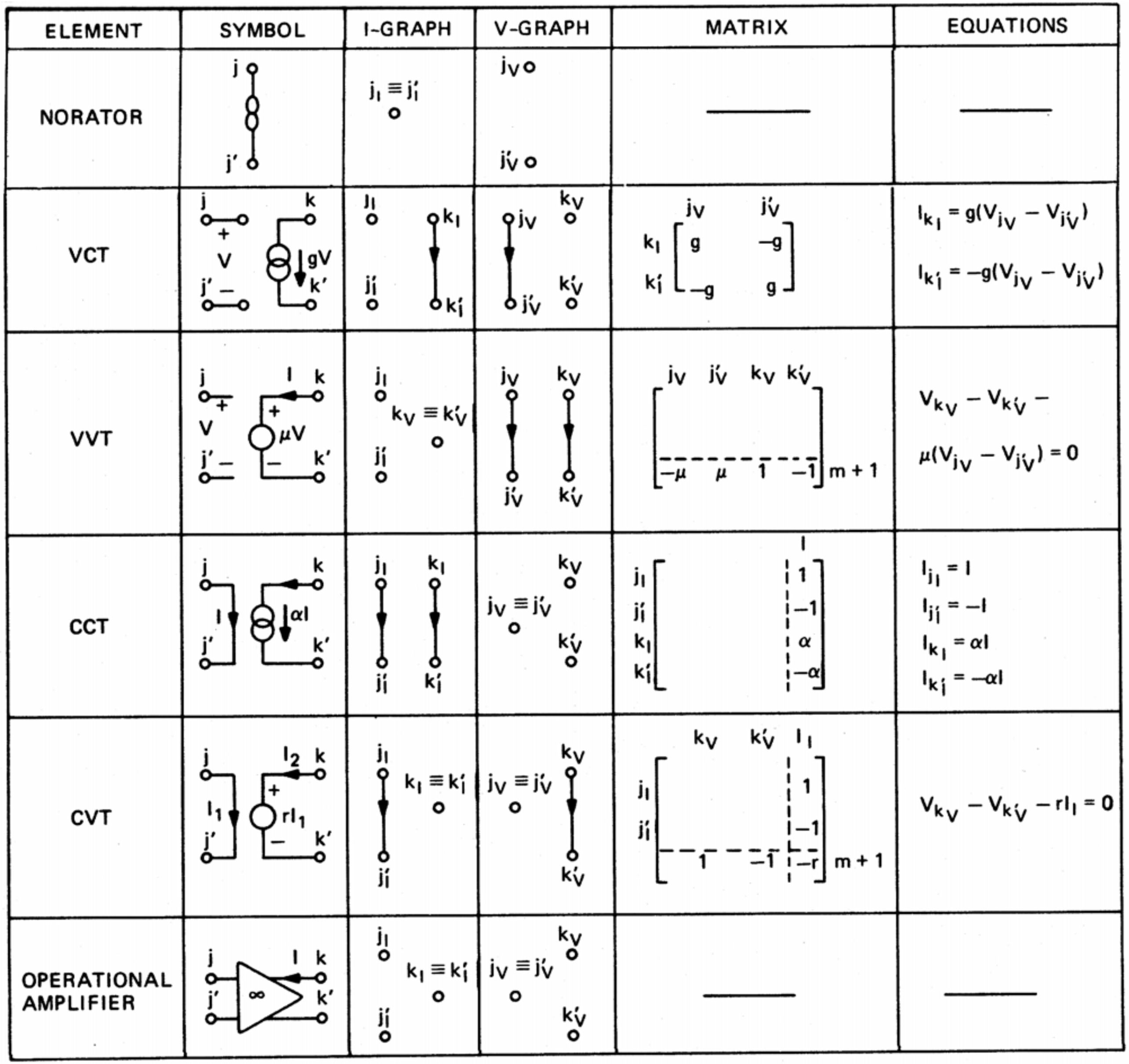
$$
\left[\begin{array}{ccccc|cccccc|ccc}
1 & & & & & & & & & & & -1 & 1 & \\
 & 1 & & & & & & & & & & 1 & -1 & \\
 & & 1 & & & & & & & & & -1 & & 1 \\
 & & & 1 & & & & & & & & 1 & & -1 \\
 & & & & 1 & & & & & & & -1 & & \\
\hline
G_5 & & & & & -1 & & & & & & & & \\
 & G_6 & & & & & -1 & & & & & & & \\
 & & G_7 & & & & & -1 & & & & & & \\
 & & & G_8 & & & & & -1 & & & & & \\
 & & & & G_9 & & & & & -1 & & & & \\
 & & & & & & & & & & 1 & & & \\
\hline
 & & & & & 1 & & & & & -1 & & & \\
 & & & & & & -1 & 1 & & & & & & \\
 & & & & & & & & -1 & 1 & & & &
\end{array}\right]
\left[\begin{array}{c}
V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ \hline I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \\ I_{10} \\ \hline V_{\triangle 1} \\ V_{\triangle 2} \\ V_{\triangle 3}
\end{array}\right]
=
\left[\begin{array}{c}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ J_{10} \\ \hline 0 \\ 0 \\ 0
\end{array}\right]
$$

dim=14x14

MNA a due grafi

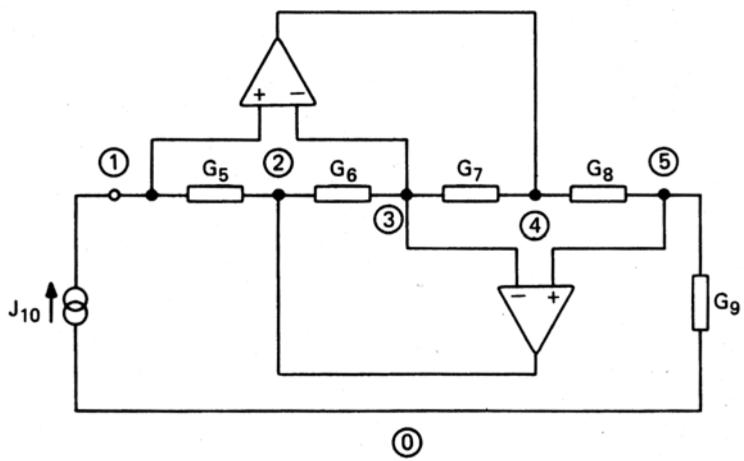
ELEMENT	SYMBOL	I-GRAPH	V-GRAPH	MATRIX	EQUATIONS
CURRENT SOURCE	j J j'	j_I j'_I	j_V j'_V	$\begin{array}{c} j_I \\ j'_I \end{array}\begin{bmatrix} -J \\ +J \end{bmatrix}$ SOURCE VECTOR	$I_{j_I} = J$ $I_{j'_I} = -J$
VOLTAGE SOURCE	j I + E − j'	$j_I \equiv j'_I$	j_V j'_V	$\begin{array}{cc} & \begin{array}{cc} j_V & j'_V \end{array} \\ m+1 & \begin{bmatrix} & \\ 1 & -1 \end{bmatrix} \end{array}$ SOURCE VECTOR $\begin{bmatrix} \\ E \end{bmatrix}$	$V_{j_V} - V_{j'_V} = E$
OPEN CIRCUIT	j + V j' −	j_I j'_I	j_V j'_V	——	$V = V_{j_V} - V_{j'_V}$
SHORT CIRCUIT	j I j'	j_I j'_I	$j_V \equiv j'_V$	$\begin{array}{cc} & I \\ \begin{array}{c} j_I \\ j'_I \end{array} & \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{array}$	I IS ARBITRARY
ADMITTANCE	j + I y V j' −	j_I j'_I	j_V j'_V	$\begin{array}{cc} & \begin{array}{cc} j_V & j'_V \end{array} \\ \begin{array}{c} j_I \\ j'_I \end{array} & \begin{bmatrix} y & -y \\ -y & y \end{bmatrix} \end{array}$	$I_{j_I} = y(V_{j_V} - V_{j'_V})$ $I_{j'_I} = -y(V_{j_V} - V_{j'_V})$
IMPEDANCE	j + I z V j' −	j_I j'_I	j_V j'_V	$\begin{array}{cc} & \begin{array}{ccc} j_V & j'_V & I \end{array} \\ \begin{array}{c} j_I \\ j'_I \\ m+1 \end{array} & \begin{bmatrix} & & 1 \\ & & -1 \\ 1 & -1 & -z \end{bmatrix} \end{array}$	$V_{j_V} - V_{j'_V} - zI = 0$ $I_{j_I} = -I_{j'_I} = I$
NULLATOR	j j'	j_I j'_I	$j_V \equiv j'_V$	——	——

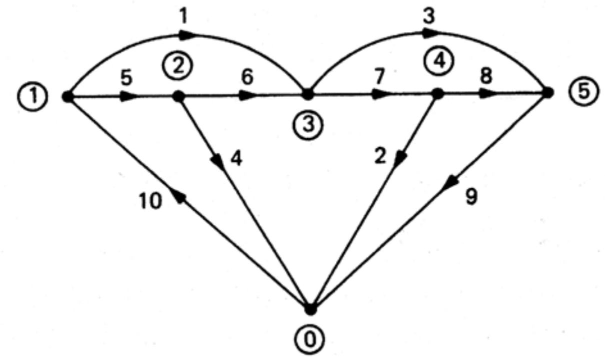
MNA a due grafi

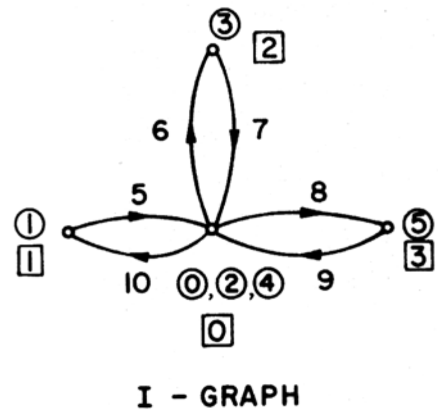


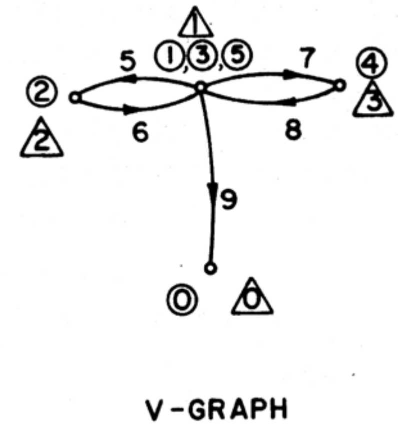
ELEMENT	SYMBOL	I-GRAPH	V-GRAPH	MATRIX	EQUATIONS
NORATOR	j, j′	$j_I \equiv j'_I$	j_V, j'_V	——	——
VCT	j, k, +, V, gV, j′ −, k′	j_I, k_I, j'_I, k'_I	j_V, k_V, j'_V, k'_V	$\begin{array}{c} \\ k_I \\ k'_I \end{array} \begin{array}{cc} j_V & j'_V \\ \hline g & -g \\ -g & g \end{array}$	$I_{k_I} = g(V_{j_V} - V_{j'_V})$ $I_{k'_I} = -g(V_{j_V} - V_{j'_V})$
VVT	j, I, k, +, V, +, μV, j′ −, −, k′	j_I, $k_V \equiv k'_V$, j'_I	j_V, k_V, j'_V, k'_V	$\begin{array}{ccccc} j_V & j'_V & k_V & k'_V & \\ \hline -\mu & \mu & 1 & -1 & m+1 \end{array}$	$V_{k_V} - V_{k'_V} - \mu(V_{j_V} - V_{j'_V}) = 0$
CCT	j, k, I, αI, j′, k′	j_I, k_I, j'_I, k'_I	$j_V \equiv j'_V$, k_V, k'_V	$\begin{array}{c\|c} & I \\ \hline j_I & 1 \\ j'_I & -1 \\ k_I & \alpha \\ k'_I & -\alpha \end{array}$	$I_{j_I} = I$ $I_{j'_I} = -I$ $I_{k_I} = \alpha I$ $I_{k'_I} = -\alpha I$
CVT	j, I_2, k, +, I_1, rI_1, j′, −, k′	j_I, $k_I \equiv k'_I$, j'_I	$j_V \equiv j'_V$, k_V, k'_V	$\begin{array}{c\|ccc} & k_V & k'_V & I_1 \\ \hline j_I & & & 1 \\ j'_I & & & -1 \\ m+1 & 1 & -1 & -r \end{array}$	$V_{k_V} - V_{k'_V} - rI_1 = 0$
OPERATIONAL AMPLIFIER	j, I, k, j′, ∞, k′	j_I, $k_I \equiv k'_I$, j'_I	$j_V \equiv j'_V$, k_V, k'_V	——	——

Esempio









$$
\begin{array}{c}
 & V\text{-graph nodes} \\
 & \begin{array}{ccc} \triangle_1 & \triangle_2 & \triangle_3 \end{array} \\
I\text{-graph nodes} \begin{array}{c} \boxed{1} \\ \boxed{2} \\ \boxed{3} \end{array} &
\begin{bmatrix} G_5 & -G_5 & 0 \\ G_6 + G_7 & -G_6 & -G_7 \\ G_8 + G_9 & 0 & -G_8 \end{bmatrix}
\begin{bmatrix} V_{\triangle_1} \\ V_{\triangle_2} \\ V_{\triangle_3} \end{bmatrix}
=
\begin{bmatrix} J_{10} \\ 0 \\ 0 \end{bmatrix}
\end{array}
$$

dim=3x3