CSC165H1, Problem Set 1

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Question 1

(a)

$$\exists t_1 \in T, \forall f \in F, (\neg BelongsTo(t_1, f) \wedge (\forall t_2 \in T, \neg BelongsTo(t_2, f) \Rightarrow t_2 = t_1))$$

(b)

$$\exists f \in F, \forall t \in T, BelongsTo(t, f) \Rightarrow Oak(t)$$

(c)

$$\exists f \in F, \forall t \in T, Pine(t) \Rightarrow BelongsTo(t, f)$$

(d)

$$\forall f \in F, \forall t_1, t_2 \in T, (Pine(t_1) \wedge Oak(t_2)) \Rightarrow (\neg BelongsTo(t_1, f) \vee \neg BelongsTo(t_2, f))$$

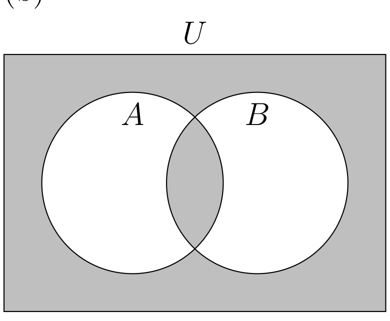
(e) Let G = $\mathcal{P}(T)$, where $\mathcal{P}(T)$ is the power set of T, i.e., $\mathcal{P}(T) = \{S | S \subseteq T\}$ is a set of all subsets of T. (reference: Course Notes page 15.)

$$\forall g \in G, \exists f \in F, f = g$$

Question 2

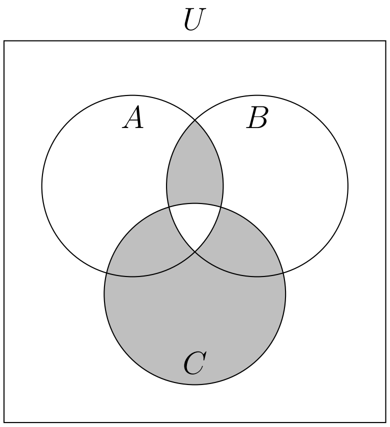
(a) $A \cap B^{\complement}$

(b)



(c) $(A \cup B \cup C) \setminus ((A \cap B) \cup (B \cap C) \cup (A \cap C) \setminus (A \cap B \cap C))$

(d)



Question 3

(a) Let $S = \{-1, 1\}$, Let $T = \{1, -1\}$
Define predicate $P(x) : x > 0$
Define predicate $Q(x, y) : |x|y > 0$

The first statement in the question is True if for all $x \in S$, there exists a $y \in T$ such that $(P(x) \vee Q(x, y)) \wedge (\neg P(x) \vee \neg Q(x, y))$ is True:
Choose $y = -x$.
When $x = 1$, $y = -1$. Therefore, $P(x)$ is True, and $Q(x, y)$ is False, $(P(x) \vee Q(x, y)) \wedge (\neg P(x) \vee \neg Q(x, y))$ is True.
When $x = -1$, $y = 1$. Therefore, $P(x)$ is False, and $Q(x, y)$ is True, $(P(x) \vee Q(x, y)) \wedge (\neg P(x) \vee \neg Q(x, y))$ is True.
Therefore, the first statement is True for the definitions provided.

The second statement is False if $(P(x) \vee Q(x, y)) \wedge (\neg P(x) \vee \neg Q(x, y))$ is False for any $y \in T$ and a chosen $x \in S$.
If y is chosen to be 1, let $x = 1$. Therefore, $P(x)$ is True and so is $Q(x, y)$. Therefore, $(\neg P(x) \vee \neg Q(x, y))$ is False, and therefore $(P(x) \vee Q(x, y)) \wedge (\neg P(x) \vee \neg Q(x, y))$ is False.
If y is chosen to be -1, let $x = -1$. Therefore, $P(x)$ is False and so is $Q(x, y)$. Therefore, $(P(x) \vee Q(x, y))$ is False, and therefore $(P(x) \vee Q(x, y)) \wedge (\neg P(x) \vee \neg Q(x, y))$ is False.
Therefore, there does not exist a $y \in T$ such that for all $x \in S$, $(P(x) \vee Q(x, y)) \wedge (\neg P(x) \vee \neg Q(x, y))$ is True. This means that the second statement is False for the given definitions of S, T, P, and Q.

(b) If we assume the second statement to be True, then there must exist a $y \in T$ such that for all $x \in S$, $(P(x) \vee Q(x,y)) \wedge (\neg P(x) \vee \neg Q(x,y))$ is True. Take this y value to be y_0. Now in the first statement, for any x values, we can choose y to be y_0 and $(P(x) \vee Q(x,y)) \wedge (\neg P(x) \vee \neg Q(x,y))$ will be True. Therefore, it is impossible for the first statement to be False while the second statement is True, as the second statement being True implies the first statement being True.

Question 4

(a)

$$\exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, (n \leq n_0) \vee Even(n)$$

(b)

$$Even(0) \wedge (\forall x \in \mathbb{N}, Even(x) \Rightarrow Even(x+2))$$

(c)

$$\forall n_0 \in \mathbb{N}, \exists n \in \mathbb{N}, n > n_0 \wedge Even(n) \wedge (n = \sum_{i=1}^{n-1} i \cdot \mathbb{I}(i|n))$$

(d)

$$\exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, ((n > n_0) \wedge (n = \sum_{i=1}^{n-1} i \cdot \mathbb{I}(i|n)) \Rightarrow \neg Even(n))$$