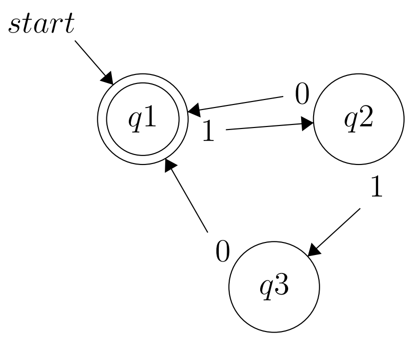
CSC236, A2

Youngsin Ryoo

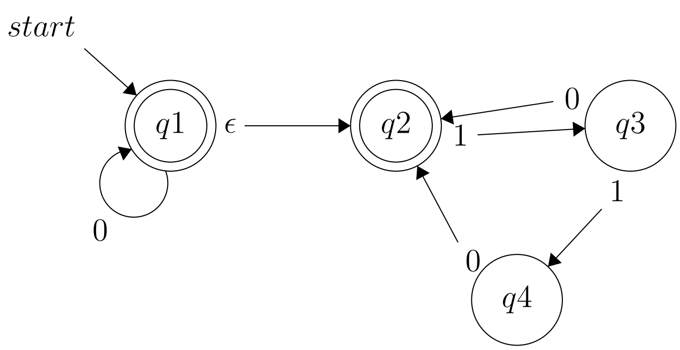
Q1

(a) i.



Text

ii.



(b) i. $(b + \epsilon)(ab)^*(a + \epsilon)$

ii. $a^*ba^*ba^* + a^*ba^*ba^*ba^*$

Q2

First, by the given fact, we have:

$$
\begin{aligned}
&\textit{1. } R^* \subseteq (R+S)^* && \text{(by the given fact)}\\
&\textit{2. } S^* \subseteq (R+S)^* && \text{(by the given fact)}
\end{aligned}
$$

Then, by the given fact, we have:

$$
\begin{aligned}
R^*S^* &\subseteq (R+S)^*(R+S)^* && \text{(by the given fact)}\\
&= (R+S)^* && \text{(by the property of the kleene star)}\\
(R^*S^*)^* &\subseteq ((R+S)^*)^* && \text{(by the given fact)}\\
(R^*S^*)^* &\subseteq (R+S)^* && \text{(by the given fact)}
\end{aligned}
$$

Secondly, we also have:

$$
\begin{aligned}
\textit{1. } (R^*)^* &\subseteq (R^*+S^*)^* && \text{(by the given fact)}\\
R^* &\subseteq (R^*+S^*)^* && \text{(by the given fact)}\\
\\
\textit{2. } (S^*)^* &\subseteq (R^*+S^*)^* && \text{(by the given fact)}\\
S^* &\subseteq (R^*+S^*)^* && \text{(by the given fact)}
\end{aligned}
$$

Then, we have:

$$
\begin{aligned}
R^*S^* &\subseteq (R^*+S^*)^*(R^*+S^*)^* && \text{(by the given fact)}\\
&= (R^*+S^*)^* && \text{(by the property of the kleene star)}\\
(R^*S^*)^* &\subseteq ((R^*+S^*)^*)^* && \text{(by the given fact)}\\
(R^*S^*)^* &\subseteq (R^*+S^*)^* && \text{(by the given fact)}
\end{aligned}
$$

Now, note that $R+S \subseteq R^*S^*$ since $R \subseteq R^*$ and $S \subseteq S^*$.
Also we know that $R^*+S^* \subseteq R^*S^*$ by the property of union and concatenation operations.

$$
\begin{aligned}
&\textit{1. } R+S \subseteq R^*S^*\\
&(R+S)^* \subseteq (R^*S^*)^* && \text{(by the given fact)}\\
\\
&\textit{2. } R^*+S^* \subseteq R^*S^*\\
&(R^*+S^*)^* \subseteq (R^*S^*)^* && \text{(by the given fact)}
\end{aligned}
$$

Altogether, we have:

$$
\begin{aligned}
(R^*S^*)^* &\subseteq (R+S)^*\\
(R^*S^*)^* &\subseteq (R^*+S^*)^*\\
(R+S)^* &\subseteq (R^*S^*)^*\\
(R^*+S^*)^* &\subseteq (R^*S^*)^*
\end{aligned}
$$

We have $(R^*S^*)^* \subseteq (R+S)^*, (R^*+S^*)^* \subseteq (R^*S^*)^*$. This means that $(R+S)^*$ and $(R^*+S^*)^*$ are equivalent. Therefore, $(R+S)^* \equiv (R^*+S^*)^*$. ■

Q3

The answer is yes. $Rev(L)$ is regular.
Let there be the NFA, M_1, for L.
Then, $\mathcal{L}(M_1) = L$.
Let $M_1 =< Q_1, \Sigma, \delta_1, \mathcal{S}_1, \mathcal{F}_1 >$
Let M_2 be the NFA for $Rev(L)$.
Then, $\mathcal{L}(M_2) = Rev(L)$.
Let q be a new initial state for M_2.
Then, the new set of states for $Rev(L)$ should include this new initial state q that is linked with an epsilon to the next state.
Since $Rev(L)$ is reversing the original string in L, the alphabet of symbols used will still be the same for $Rev(L)$.
Let $w \in \Sigma^*$. $w \in Rev(L)$ iff $w^R \in L$.
Let $q', p' \in Q_1$. In L, a string w will be read from a state q' to a state p'.
In other words, $p' \in \delta_1^*(q', w)$.
Then, for $Rev(L)$, we have $q' \in \delta_r^*(p', w)$ since all the paths should be reversed.
Again, since $Rev(L)$ is reversing the original string in L, all the initial states should now become the accepting states. Finally, we have the formal definition for $Rev(L)$.
That is, $M_2 =< Q_1 \cup \{q\}, \Sigma, \delta_r, q, \mathcal{S}_1 >$.

Q4

Assume for a contradiction that L is a regular.
Then L must satisfy PLC. That is, there exists $p \in \mathbb{N}^+$ such that $\forall w \in L$, if $|w| \geq p$, then $\exists x, y, z$ such that the conditions, $w = xyz, |y| \geq 1$, $|xy| \leq p$, and $\forall i \geq 0, xy^iz \in L$ hold.
Let $w = a^{(p+1)}b^p$.
Then $w \in L$ and $|w| \geq p$.
Assume w satisfies the first three conditions, $w = xyz, |y| \geq 1$, and $|xy| \leq p$.
That is, $w = xyz$ such that $|y| \geq 1$ and $|xy| \leq p$.
Since w starts with $(p+1)$ a's and $|xy| \leq p$, x and y must only contain a's.
Let $x = a^s$, $y = a^t$, and $z = a^rb^p$, where $s + t + r = p + 1$ and $t \geq 1$ (by $|y| \geq 1$).
Consider when $i = 0$.
Then $xy^0z = xz = a^sa^rb^p$ and $s + r \leq p$ because $s + r = p - t + 1$ and $t \geq 1$.
This means that for $i = 0$, $xy^iz \notin L$.
Since the length of a's is not greater than the length of b's, $xy^0z \notin L$ and L does not satisfy PLC.
This is a contradiction. So L is non-regular. $\blacksquare$