Name: ______________________________________

Student ID: ___________________________________

National Taiwan University	**Finite Element Method**
Department of Civil Engineering	Instructor: C.-S. CHEN

Midterm Examination, April 15, 2014
Open Printed Books and Notes, **Time: 180 minutes**
Return the Exam Sheet (試卷隨答案卷繳回)

Part I (25%): Conceptual Problems: answer the following questions **briefly**

1. (12%) (a) Explain the major differences/features between the classical Galerkin method and finite element method. (b) Explain the advantages gained by these features in the finite element method.

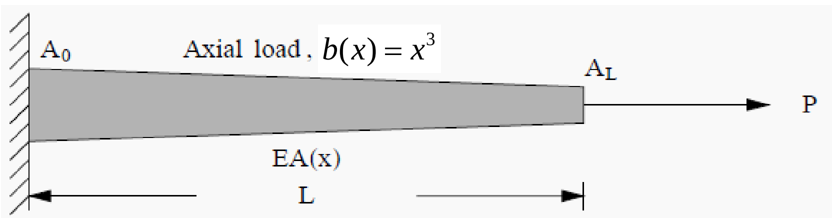
2. (5%) Write down the form of a complete polynomial up to the cubic order for 2D (x, y).

3. (8%) Consider a tapered bar fixed at one end and subjected to a static point load at the other end as shown below. The bar is also subjected to a varying body load $b(x) = x^3$. The Young's modulus is constant and the area varies linearly from A_0 to A_L where

$$A(x) = \frac{(L + (-1 + r)x)A_0}{L} \qquad \text{in which} \qquad r = \frac{A_L}{A_0}$$

(a) If we use the cubic element to mesh the domain, what is the minimum number of Gauss points we need to numerically integrate the element stiffness matrix $\mathbf{K}^e$ exactly?

(b) If we use the quadratic element to mesh the domain, what is the minimum number of Gauss points we need to numerically integrate the element external force matrix $\mathbf{f}^e$ exactly?

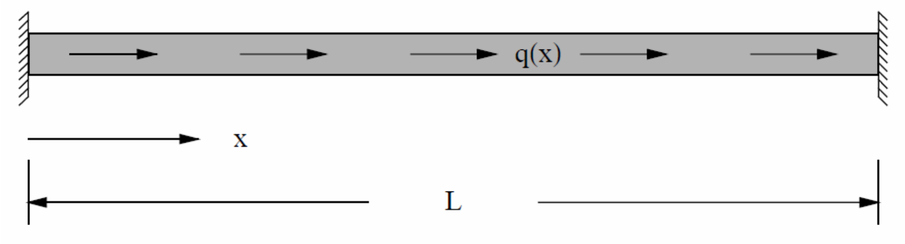


Part II (75%): Derivation Problems: give detailed derivations of the following questions

4. (20%) Consider a uniform bar with a constant Young's modulus and area. The bar is fixed at both ends and subjected to an axial load $q(x) = cx^2$ where c is a constant as shown below.

Assume a quadratic polynomial solution $u(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2$.

(a) (10%) Write down the total potential energy Π in terms of unknown polynomial coefficients.
(b) (10%) Obtain the solution using the Rayleigh–Ritz method.



5. (20%) An engineering analysis problem is formulated in terms of the following ordinary differential equation:

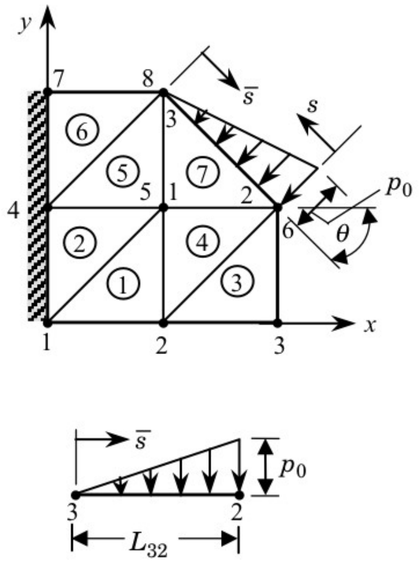
$$-\frac{d^2u}{dx^2} = x^2, \quad 0 < x < 1.$$
$$u(0) = u(1) = 0.$$

(a) (10%) Develop the corresponding weak form from the strong form.

(b) (10%) Obtain the solution to the weak form by using trial solution and weight function of a cubic form:

$$u(x) = x(1-x)(\alpha_0 + \alpha_1 x)$$
$$w(x) = x(1-x)(\beta_0 + \beta_1 x)$$

6. (15%) Determine **analytically** the element external force matrix (that is, equivalent nodal forces) $\mathbf{f}^e = \begin{bmatrix} f_{x1}^e \\ f_{y1}^e \\ f_{x2}^e \\ f_{y2}^e \\ f_{x3}^e \\ f_{y3}^e \end{bmatrix}$ in the global Cartesian coordinate for the following. Assume the thickness for the problem is t.



7. (20%) Theory of linear elastic fracture mechanics tells us that stresses near a crack tip are inversely proportional to $\sqrt{x}$, where x is the distance from a crack tip. It happens that we can accomplish this by simply placing the mid-side nodes of a quadratic element to their quarter points. A 1D element shown in following figure is the best to illustrate the singular behavior.

   (a) (6%) Derive the mapping relationship between the physical coordinate (Cartesian space) x and the parametric coordinate ξ.

   (b) (6%) Let us first consider a standard 1D quadratic element with a mid-side node ($\alpha = \frac{1}{2}$). Show the expected linear expression for the strain $\frac{du}{dx}$ using the isoparametric formulation.

   (c) (8%) Now if we move the mid-side node at the Cartesian space only to the quarter point ($\alpha = \frac{1}{4}$). Show that we now have a desirable singular expression for the strain $\frac{du}{dx}$ using the isoparametric formulation.