Establishing R (or R_w) and C_d Factors for Building Seismic Provisions

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Abstract: This paper derives the basic formulas for establishing the response modification factor R and the displacement amplification factor C_d used in the National Earthquake Hazards Reduction Program (NEHRP) recommended provisions. An extension of the approach is used to derive the system modification factor R_w used in the 1988 Uniform Building Code. These terms are primarily functions of both the structural overstrength and structural ductility factors. Because of the empirical nature of the NEHRP recommended values for these factors, it is difficult to justify the relative values between R and C_d factors for most of the building systems. The structural overstrength factor, which can be determined from analytical studies, depends on structural redundancy, story drift limitations, multiple load combinations, strain hardening, participation of nonstructural elements, and other parameters. Although the ductility factor of an individual structural member can be determined experimentally, there is no general agreement within the profession of how the concept of ductility factor should be applied at the structural system level.

Introduction

In the development of seismic design provisions for building structures, the most controversial part is the development of the force reduction factors and the displacement amplification factors. The force reduction factor, expressed as either a response modification factor, R, in the National Earthquake Hazards Reduction Program (*NEHRP* 1988) or a system performance factor, R_w, in the Uniform Building Code (UBC) (1988) and the Structural Engineers Association of California (SEAOC) (*Recommended* 1988), is used to reduce the linear elastic design response spectra. A displacement amplification factor C_d is used by NEHRP to compute the expected maximum inelastic displacement from the elastic displacement induced by the design seismic forces.

Consider a typical global structural response. Fig. 1 shows that the required elastic strength, expressed in terms of base shear ratio C_{eu}, is:

$$C_{eu} = \frac{V_e}{W} \qquad (1)$$

where W = the weight of the reactive masses; and V_e = the maximum base shear that develops in the structure if it were to remain in the elastic range. Since a properly designed structure usually can provide a certain amount of ductility, a structure can be designed economically to develop an actual maximum strength of C_yW. The corresponding maximum deformation demand, expressed in terms of story drift Δ, is Δ_{max}. Since the calculation of C_yW, which corresponds to the structural mechanism or yield strength, and Δ_{max}

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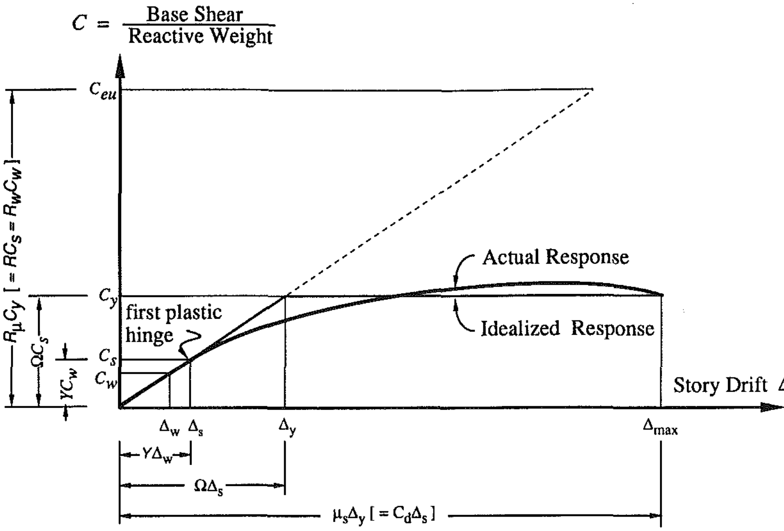

FIG. 1. General Structural Response

involves nonlinear analysis, these quantities are generally not quantified in an explicit manner.

For design purposes, NEHRP reduces the C_y level to the C_s level, which corresponds to the formation of the first plastic hinge. This level is commonly called the "first significant yield" level—a level beyond which the global structural response starts to deviate significantly from the elastic response. This force level is consistent with material codes that use a strength design approach—the American Concrete Institute's *ACI 318-89* (ACI: "Building" 1989) and the American Institute of Steel Construction's load and resistance factor design (LRFD) specification (AISC: *Load* 1986) are typical examples. To be consistent with material codes that use allowable (or working) stress design methods, such as the AISC allowable stress design (ASD) specification (AISC: *Allowable* 1989), UBC reduces further from C_s level to service load level C_w (see Fig. 1). The advantage of specifying C_s or C_w as the design level is that designers need only perform an elastic structural analysis.

The first problem associated with this type of "elastic" design procedure is that the designers do not know the true strength of the structure. If a structure's reserve strength (the so-called overstrength) beyond the design level (C_s or C_w) is significantly less than that implicitly assumed in seismic provisions, the performance of the structure is not likely to be satisfactory during severe earthquakes. The second problem is that the maximum inelastic displacements cannot be calculated from the results of an elastic analysis. The NEHRP uses the displacement amplification factor C_d and UBC uses a factor of $3R_w/8$ to predict the maximum inelastic displacement from the elastic displacement produced by the seismic design forces.

Objectives

The first objective of this paper is to formulate explicit expressions for the response modification factors R (or R_w) and the displacement amplification factors C_d; the second objective is to discuss the two most important parameters—structural overstrength factor and structural ductility factor—that are needed to calculate the R (or R_w) and C_d factors.

Definition of Relevant Terms

Structural Ductility Factor μ_s

Idealizing the actual structural response curve by the linearly elastic–perfectly plastic curve in Fig. 1, the structural ductility factor can be defined as

$$\mu_s = \frac{\Delta_{max}}{\Delta_y} \quad (2)$$

where the deformation is expressed in terms of story drift Δ.

Ductility Reduction Factor R_μ

As a result of ductility, the structure has a capacity to dissipate hysteretic energy. Because of this energy dissipation capacity, the elastic design force can be reduced to a yield strength level (C_y) by the factor R_μ:

$$R_\mu = \frac{C_{eu}}{C_y} \quad (3)$$

Note that the yield strength level refers to the structural collapse level, not the level of first significant yielding. A 5% equivalent viscous damping ratio is usually considered in the computation of this reduction factor. For a single-degree-of-freedom system, the relationship between μ_s and R_μ is well established (Newmark and Hall 1982; Riddell et al. 1989).

Overstrength Factor Ω

The reserve strength that exists between the actual structural yield level (C_y) and the NEHRP-prescribed first significant yield level (C_s) is defined in terms of the overstrength factor Ω:

$$\Omega = \frac{C_y}{C_s} \quad (4)$$

Structural overstrength results from internal force redistribution (redundancy), higher material strength than those specified in the design, strain hardening, deflection constraints on system performance, member oversize, minimum requirements according to NEHRP regarding proportioning and detailing, multiple loading combinations, effect of nonstructural elements, strain rate effect, and so on.

Allowable Stress Factor Y

This factor is used to account for the difference in the format of material codes. For allowable stress design, the corresponding design force level (C_w) in Fig. 1 is reduced from the code-specified first significant yield level (C_s) by a factor Y, that is

$$C_w = \frac{C_s}{Y} \qquad (5)$$

The value of Y is approximately in the range of 1.4–1.5. For example, the value of Y for the 1989 ASD specification (AISC: *Allowable* 1989) can be estimated as follows:

$$Y = \frac{1}{0.6 \times \frac{4}{3}} \times 1.14 = 1.4 \qquad (6)$$

where the average allowable stress of 60% of the nominal yield stress has been increased by one-third, as is permitted by the specification, and a shape factor of 1.14 is assumed for the wide-flange sections. The value of Y is equal to 1 when strength design method is used. The load factor of 1.4 for earthquake loading in *ACI 318-89* (ACI: "Building" 1989) is practically the same as the Y value in Eq. 6.

FORMULATION OF R (OR R_w) AND C_d

From Fig. 1, the total force reduction factor (or the NEHRP response modification factor R) corresponding to the strength design format can be derived as follows:

$$R = \frac{C_{eu}}{C_s} = \frac{C_{eu}}{C_y}\frac{C_y}{C_s} = R_\mu \Omega \qquad (7)$$

The total force reduction factor corresponding to the UBC allowable-stress-design format is

$$R_w = \frac{C_{eu}}{C_w} = \frac{C_{eu}}{C_y}\frac{C_y}{C_s}\frac{C_s}{C_w} = R_\mu \Omega Y \qquad (8)$$

The NEHRP displacement amplification factor C_d, which is the ratio between $\Delta_{\max}$ and Δ_s, can also be derived from Fig. 1 as follows:

$$C_d = \frac{\Delta_{\max}}{\Delta_s} = \frac{\Delta_{\max}}{\Delta_y}\frac{\Delta_y}{\Delta_s} \qquad (9a)$$

where $\Delta_{\max}/\Delta_y$ = the structural ductility factor (see Eq. 2); and Δ_y/Δ_s from Fig. 1 is

$$\frac{\Delta_y}{\Delta_s} = \frac{C_y}{C_s} = \Omega \qquad (9b)$$

Therefore Eq. 9*a* can be expressed as

$$C_d = \mu_s \Omega \qquad (10)$$

From these derivations, it is observed that both the R (or R_w) and C_d factors are functions of structural overstrength factor, structural ductility factor, and damping ratio—the effect of damping is generally included in the ductility reduction factor R_μ. Furthermore, Eqs. 7 and 8 show that it is misleading to call R or R_w the ductility reduction factor, because structural overstrength may play an equally important role to ductility in these factors.

Evaluation of the NEHRP-Recommended Response Parameters

NEHRP provides R and C_d values for different structural systems. Table 1 shows the values for steel building systems. It should be noted that these factors, which to date are empirical in nature, are based on general consensus of engineering judgment, observed structural performance in the past earthquakes, and so on (*NEHRP Recommended* 1988). Little information is available to justify the use of these values.

It is interesting to examine the ratio of R to C_d. It can be shown from Eqs. 7 and 10 that

$$\frac{R}{C_d} = \frac{R_\mu \Omega}{\mu_s \Omega} = \frac{R_\mu}{\mu_s} \quad (11)$$

Eq. 11 indicates that the ratio R/C_d for a particular structural system is a function of structural ductility factor only and is independent of the structural overstrength factor. According to a study by Newmark and Hall (1982), the ratio of R_μ/μ_s can be expressed as follows.

1. In the velocity and displacement amplification regions:

$$\frac{R_\mu}{\mu_s} = \frac{\mu_s}{\mu_s} = 1 \quad (12a)$$

TABLE 1. NEHRP-Recommended R and C_d Factors for Different Steel Structural Systems

Steel structural system (1)	R (2)	C_d (3)	R/C_d (4)
Moment resisting frame system			
SMRSF	8	5.5	1.5
OMRSF	4.5	4	1.1
Dual system			
EBF + SMRSF	8	4	2.0
CBF + SMRSF	6	5	1.2
Building frame system			
EBF	8	4	2.0
CBF	5	4.5	1.1
Bearing wall system			
CBF	4	3.5	1.1
Framed walls with shear panels	6.5	4	1.6
Inverted pendulum structures			
SMRSF	2.5	2.5	1.0
OMRSF	1.25	1.25	1.0

Note: SMRSF = special moment-resisting space frame; OMRSF = ordinary moment-resisting space frame; EBF = eccentrically braced frame; CBF = concentrically braced frame.

2. In the acceleration amplification region:

$$\frac{R_\mu}{\mu_s} = \frac{\sqrt{2\mu_s - 1}}{\mu_s}, \qquad (\mu_s \geq 1) \quad \text{(12}b\text{)}$$

Eqs. 12*a* and 12*b* indicate that the ratio R_μ/μ_s is no greater than 1. This ratio is significantly less than 1 for structures with short periods (i.e., in the acceleration region). The ratios of R/C_d for different steel structural systems are shown in Table 1. Except for inverted pendulum structures, the ratios are greater than 1. Therefore, it is difficult to justify the relative values of R and C_d used by NEHRP.

RATIONAL APPROACH TO ESTABLISH R (OR R_w) AND C_d FACTORS

Formats for evaluating R, R_w, and C_d have been presented in Eqs. 7, 8, and 10. The only way to rationalize these factors is to quantify the overstrength and structural ductility factors by analytical studies and experimental testing. More rational values for R, R_w, and C_d can then be established using Eqs. 7, 8, and 10.

Overstrength Factor Ω

The importance of overstrength in the survival of buildings during severe earthquake shaking has long been recognized (Bertero 1986; Blume 1977; Rojahn 1988). Its importance has been confirmed recently by the shaking-table testing of multistory reinforced concrete (Bertero et al. 1984; Shahrooz and Moehl 1987) and steel structures (Uang and Bertero 1986; Whittaker et al. 1989). For example, the overstrength factor observed from a six-story braced steel frame is on the order of 2.4–2.8 (Uang and Bertero 1986; Whittaker et al. 1989). Research studying the performance of buildings during the 1985 Mexico City earthquake also indicated that structural overstrength played a very important role in building survival (Osteraas and Krawinkler 1989). Its role is even more significant for buildings with short periods because ductility is ineffective in reducing the required elastic strength in this period range.

Overstrength factors can also be obtained from analytical studies. Fig. 2 shows the variation of overstrength factor for a typical interior frame of an office building located in a region of high seismic risk (Assaf 1989). The overstrength factors were obtained using inelastic static analyses. Fig. 2 shows that low-rise buildings usually have higher overstrength. A value of 2 could be used to establish R and C_d factors for special moment-resisting space frames. It should be noted that value of overstrength factor in Fig. 2 will be even higher for building structures with less than four stories, where the design is likely to be governed by gravity loads, and with more than 12 stories, where the design is more likely to be governed by stiffness (or story drift). The seismic overstrength factor will also be higher if the building is located in low seismic zones because gravity and wind loads are more likely to govern the design.

The overstrength factor shown in Fig. 2 is based on the use of the nominal material properties. Denoting this overstrength factor as Ω_o, the actual overstrength factor Ω that can be used to formulate R and C_d should consider the beneficial contribution of some other effects (F_i):

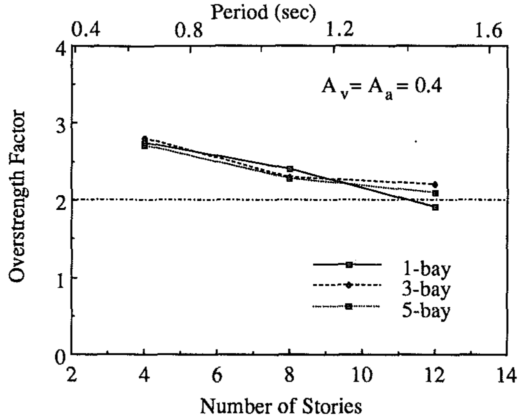

FIG. 2. Overstrength Factor versus Number of Stories for Special Moment-Resisting Steel Frames (Assaf 1989)

$$\Omega = \Omega_o F_1 F_2 \dots F_n \qquad (13)$$

For example, F_1 may be used to account for the difference between actual static yield strength and nominal static yield strength. For structural steel, a statistical study conducted by Ellingwood et al. (1980) shows that the value of F_1 may be taken as 1.05. Parameter F_2 may be used to consider the increase in yield stress as a result of the strain rate effect during an earthquake excitation. A value of 1.1, i.e., a 10% increase to account for the strain rate effect, could be used (Ellingwood et al. 1980). If only these two parameters are used to formulate the conservative value for steel frames, the overstrength factor for ductile moment frame can be taken as

$$\Omega = 2.0 \times 1.05 \times 1.1 = 2.3 \qquad (14)$$

Other parameters can also be included when reliable data is available; these parameters include nonstructural component contributions, variation of lateral force profiles, and so on.

Even though similar analytical studies can be performed to establish the level of the overstrength factor that can be used to calculate R and C_d factors for different structural systems, these factors can only be applied to structures with levels of redundancy similar to those used to develop the values of R and C_d. Current seismic design provisions in the United States do not require designers to quantify the level of overstrength, and the use of R and C_d factors does not provide a uniform level of safety against collapse, especially for structures with minimal redundancy. Notice that the current Japanese seismic design code (*Earthquake Resistant* 1988) requires that the structure's ultimate strength be checked; otherwise, the seismic forces have to be increased. It is an explicit way to ensure that at least a certain amount of structural overstrength is provided in the design process. This approach deserves more attention in the future development of the seismic provisions.

Structural Ductility Ratio μ_s

Although overstrength factors may be estimated by analytical methods, it is more reliable to establish ductility (or energy dissipation) capacities of structural members or structural systems by experimental means. The envelope of structural response shown in Fig. 1 is representative only of systems that can dissipate energy in a stable manner—special ductile moment-resisting frames and steel eccentrically braced frames, for example. For other systems that involve severe strength and stiffness deterioration, the definitions of yield displacement and maximum displacement in Eq. 1 may be incorrect.

The problems of defining structural ductility factor become even more complicated when buildings with more than one story are involved. Story drift has often been used as the deformation term to compute structural ductility factor (see Fig. 1). Newmark and Hall (1982) postulated, "The overall system ductility factor is some weighted average in general of the story ductility factors, and is defined best by considering a particular pattern of displacement corresponding to the preferred or executed mode of deformation of the structure." It is unclear what kind of response quantities should be used as the weighting function for this definition of structural ductility factor. A more rational definition of system ductility factor is urgently needed if rational values of response modification factor and displacement amplification factor are to be evaluated.

Conclusions

The main conclusions are as follows:

1. An explicit formulation of the response modification factor R and the displacement amplification factor C_d for NEHRP-type seismic provisions have been presented in Eqs. 7 and 10. The system performance factor R_w used in UBC, which is an allowable stress design code, is presented in Eq. 8. These factors are functions of structural overstrength factor Ω, structural ductility factor μ_s, and equivalent viscous damping ratio—the effect of damping is generally included in the ductility reduction factor R_μ.
2. Using a constant value of R or C_d value does not ensure the same level of safety against collapse for all structures. For buildings with little redundancy, the structural overstrength that is relied upon by the current seismic design provisions may be insufficient. There is a need to incorporate in the design process a method to quantify the overstrength of a structure; this overstrength should not be less than that assumed in establishing the R and C_d factors.
3. Values of R and C_d recommended in NEHRP are not consistent for the various structural systems. These values should be reevaluated in a more rational manner; the basic formulas derived in this paper can be used for this purpose.
4. Definition of structural ductility for multistory building structures that exhibit significant strength and stiffness deterioration is still a major obstacle for establishing R and C_d factors.

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Appendix II. Notation

The following symbols are used in this paper:

C = base shear ratio;
C_d = NEHRP displacement amplification factor;

C_{eu} = elastic base shear ratio;
C_s = NEHRP design base shear ratio;
C_w = UBC design base shear ratio;
C_y = base shear ratio at structural yield level;
F_i = modifying factor for overstrength factor;
R = NEHRP response modification factor;
R_w = UBC system modification factor;
R_μ = ductility reduction factor;
V_e = elastic base shear;
W = weight of reactive masses;
Y = allowable stress factor;
Δ = story drift;
Δ_{max} = maximum inelastic story drift;
Δ_s = NEHRP elastic story drift produced by $C_s W$;
Δ_w = UBC elastic story drift produced by $C_w W$;
Δ_y = elastic story drift at $C_y W$ force level;
μ_s = structural ductility factor; and
Ω = structural overstrength factor.