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Midterm Examination, April 27, 2017
Open Printed Books and Notes, **Time: 180 minutes**
Return the Exam Sheet (試卷隨答案卷繳回)

Total 100%

Part I (20%): Conceptual Problems: answer the following questions **briefly**

1. (4%) What is the inter-element continuity needed for a weak form for a second-order differential equation?
2. (4%) Write down the form of a complete polynomial up to the quadratic order for 3D (x, y, z).
3. (4%) For a viscous fluid, the velocities at various depths are measured as follows:

Velocity (m/s)	3.75	4	6.75
Depth (m)	0	1	1.4

Use the Lagrange interpolation to obtain an expression for fluid velocity as a function of depth. You don't need to expand the expression.

4. (8%)

(a) (4%) What are the requirements for the admissible solutions and admissible weight functions when developing a weak form for a strong form in the classical Galerkin method?

(b) (4%) Consider the differential equation $\frac{d^2u}{dx^2}+4u=12$ in the range $0<x<1$, with essential boundary conditions $u(0)=3$ and $u(1)=1$. Write down an admissible polynomial solution with only <u>one</u> parameter α left to be solved.

Part II (80%): Derivation Problems: give detailed derivations of the following questions

5. (16%) An engineering analysis problem is formulated in terms of the following ordinary differential equation:

$$2x\frac{d^2u}{dx^2}+3\frac{du}{dx}=4, \quad 1<x<2.$$

$$\frac{du(1)}{dx}=1; \; u(2)-2=0$$

$u(2)=2$
$w(2)=0$

(a) (8%) Develop the corresponding weak form from the strong form.

(b) (8%) Use trial solution and weight function of a linear polynomial $u(x)=\alpha_0+\alpha_1 x$ and $w(x)=\beta_0+\beta_1 x$ and obtain the approximate solution using the Galerkin method.

6. (24%) Consider problem of finding axial displacement of a truncated solid cone made of concrete, hanging under its own weight and subjected to a downward load $F=100$ kN at the tip, as illustrated in Figure 6 below. Let the length $L=5$ m, the diameter at the top is $d_0=1$ m and changes linearly to $d_L=\frac{1}{4}$ m at the bottom. The concrete weighs 24 kN/m^3 and its modulus is $E=2\times10^7$ kN/m^2.

   (a) (5%) State the strong form (governing equation and boundary conditions).

   (b) (10%) Construct the element stiffness matrix and element external force matrix using two linear elements equally distributed as shown below. The linear element needs to take the linear cross section into consideration. Assemble these two elements to obtain the global stiffness matrix and global external force matrix.

   (c) (5%) Find the nodal displacements.

   (d) (4%) Find the element nodal forces and draw the free body diagram at node 2. Is the equilibrium condition satisfied at this node? Comment your findings.

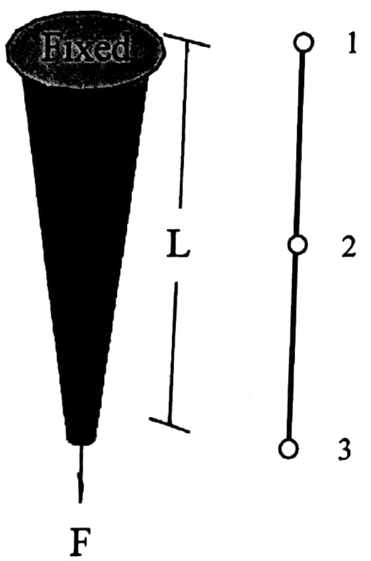

Figure 6

7. (15%) Consider a single T3 element shown in Figure 7 (unit in meters). The element is subjected to a linearly varying pressure acting on the normal direction along the edge and a body force from gravity in the negative y direction. Let the acceleration of gravity $g=9.8$ m/s^2, the density $\rho=1200$ Kg/m^3, the element thickness $t=0.12$ m, the maximum of the pressure $p_0=1000$ kN/m^2. Calculate the external force matrix (i.e., equivalent nodal

forces) for the element.

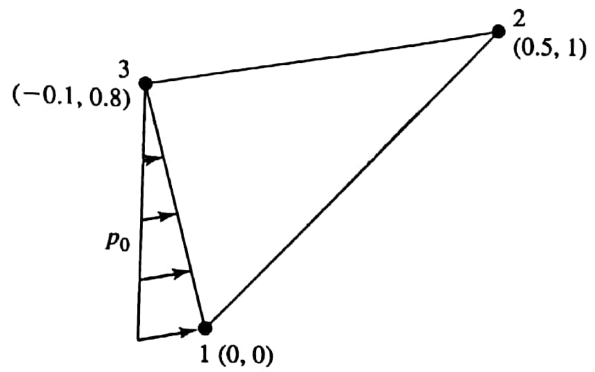

Figure 7 (unit in meter)

~~8~~. (15%) Consider a plane strain rectangular element as shown in Figure 8 (unit in cm). The nodal displacements are given below:

$$u_{x1} = 0 \text{ cm} \quad u_{x2} = 0.005 \text{ cm} \quad u_{x3} = 0.0025 \text{ cm} \quad u_{x4} = 0 \text{ cm}$$
$$u_{y1} = 0 \text{ cm} \quad u_{y2} = 0.0025 \text{ cm} \quad u_{y3} = -0.0025 \text{ cm} \quad u_{y4} = 0 \text{ cm}$$

Let the elastic modulus $E = 210$ GPa and the Poisson ratio $\nu = 0.3$. Calculate the element strains and stresses at the centroid of the element and at the node 3.

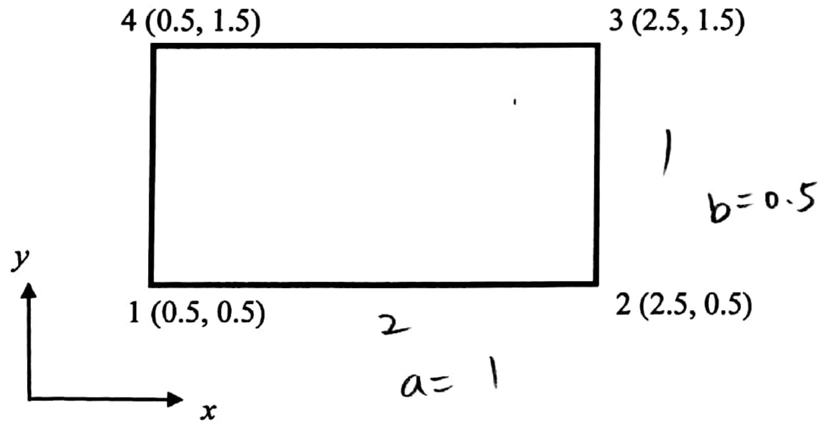

Figure 8 (unit in cm)

9. (10%) A 1D infinite element is created by using a conventional shape function to describe variation of the field quantity (e.g., displacement) but describe element geometry by placing one side of element at infinity as illustrated in Figure 9. Establish the mapping relationship between the parametric coordinate (ξ) to the physical coordinate (r) where $r = x - x_0$.

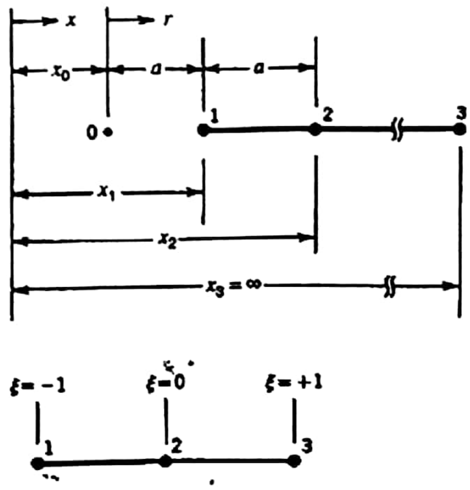

Figure 9