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Evaluation of Strength Reduction Factors for Earthquake-Resistant Design

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Strength reduction factors which permit estimation of inelastic strength demands from elastic strength demands are evaluated. Results from various investigations of strength reduction factors carried out over the last 30 years are reviewed, and their results are presented in a common format which facilitates their comparison. The main parameters that affect the magnitude of strength reductions are discussed. The evaluation of the results indicates that strength reductions are primarily influenced by the maximum tolerable displacement ductility demand, the period of the system and the soil conditions at the site. Simplified expressions of strength reduction factors to estimate inelastic design spectra as functions of these primary-influencing parameters are presented.

INTRODUCTION

Design lateral strengths prescribed in earthquake-resistant design provisions are typically lower and in some cases much lower than the lateral strength required to maintain a structure in the elastic range in the event of severe earthquake ground motions. Strength reductions from the elastic strength demand are commonly accounted for through the use of reduction factors. While reduction factors prescribed in seismic codes are intended to account for damping, energy dissipation capacity as well as for overstrength, the level of reduction specified in seismic codes is primarily based on observation of the performance of different structural systems in previous strong earthquakes. Several researchers have expressed their concern about the lack of rationality in the reduction factors currently specified in building codes [1-6]. Furthermore, the improvement of reduction factors has been identified as a way to improve the reliability of present earthquake-resistant design provisions [7,8].

Reductions in forces produced by the hysteretic energy dissipation capacity of a structure (i.e., reduction in forces due to nonlinear hysteretic behavior) are expressed by strength reduction factors or by their reciprocals (typically referred to as deamplification factors). Thus, the assessment of the minimum lateral strength capacity that will result in an adequate control of inelastic deformations (i.e., damage) during strong earthquake ground motions requires a good estimation of the strength reduction factors.

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Strength reduction factors have been the topic of several investigations over the last 30 years. However, so far many of the findings of these investigations have not been incorporated into building codes. The objectives of this paper are: (i) to review previous studies as well as recently-published investigations on strength reduction factors; (ii) to present their results in a common format which allows for straight forward comparison; and (iii) to discuss the implications of these results for earthquake-resistant design.

STRENGTH REDUCTION FACTOR SPECTRA

An adequate design is accomplished when a structure is dimensioned and detailed in such a way that the local (story and member) ductility demands are smaller than their corresponding capacities. Thus, during the preliminary design of a structure, there is a need to estimate the lateral strength (lateral load capacity) of the structure that is required in order to limit the global (structure) displacement ductility demand to a certain pre-determined value which results in the adequate control of local ductility demands.

The level of inelastic deformation experienced by the system under a given ground motion is typically given by the displacement ductility ratio, μ, which is defined as the ratio of maximum absolute relative displacement to its yield displacement

$$m = \frac{max.\ |u(t)|}{u_y} \tag{1}$$

The time history of the response of a nonlinear single-degree-of-freedom (SDOF) system to earthquake ground motions is given by the solution of the following differential equation

$$m\,\ddot{u}(t) + c\,\dot{u}(t) + F(t) = -\,m\,\ddot{u}_g(t) \tag{2}$$

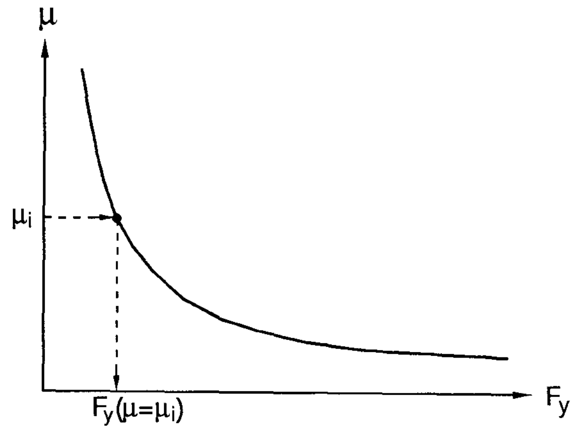


Figure 1. Variation of displacement ductility demand with changes in the lateral strength of the system

where m, c, and $F(t)$ are the mass, damping coefficient, and restoring force of the system, respectively; $u(t)$ is the relative displacement; $\ddot{u}_g(t)$ is the ground displacement; and the dot over a quantity represents its derivative with respect to time. The initial period of the system is given by,

$$T = 2\pi\left[\frac{m}{k}\right]^{1/2} = 2\pi\left[\frac{m\,u_y}{F_y}\right]^{1/2} \tag{3}$$

where k is the initial stiffness of the system; and F_y is the system's yield strength, respectively.

The strength reduction factor (i.e., the reduction in strength demand due to nonlinear hysteretic behavior), R_μ, is defined as the ratio of the elastic strength demand to the inelastic strength demand,

$$R_\mu = \frac{F_y(\mu = 1)}{F_y(\mu = \mu_i)} \tag{4}$$

where $F_y(\mu = 1)$ is the lateral yielding strength required to avoid yielding in the system under a given ground motion and $F_y(\mu = \mu_i)$ is the lateral yielding strength required to maintain the displacement ductility ratio demand, μ, less than or equal to a pre-determined target ductility ratio, μ_i when subjected to the same ground motion.

In general, for structures responding inelastically during earthquake ground motions, inelastic deformations increase as the lateral yielding strength of the structure decreases (or as the design reduction factor increases), as shown schematically in Fig. 1.

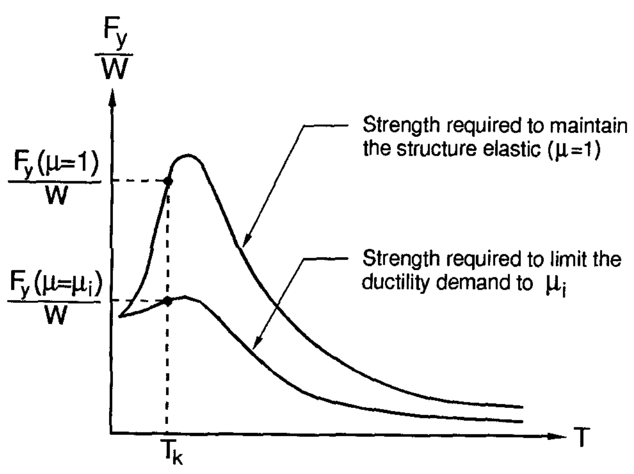


Figure 2. Linear and constant ductility nonlinear response spectra

For a given ground motion and a maximum tolerable displacement ductility demand μ_i, the problem is to compute the minimum lateral strength capacity $F_y(\mu = \mu_i)$ that has to be supplied to the structure in order to avoid ductility ratio demands larger than μ_i. As shown in Fig. 2, lateral strengths $F_y(\mu = 1)$ and $F_y(\mu = \mu_i)$, if normalized by the weight of the system, correspond to ordinates of a linear elastic response spectrum and a constant displacement ductility ratio nonlinear response spectrum, respectively.

For design purposes, R_μ corresponds to the maximum reduction in strength that is consistent with limiting the displacement ductility ratio demand to the pre-determined target ductility μ_i in a structure that will have a lateral strength equal to the design lateral strength. An additional strength reduction can be considered in the design of a structure to take into account the fact that structures usually have a lateral strength higher than the design strength. For a more detailed discussion on strength reductions due to overstrength the reader is referred to Refs. 8, 9 and 10.

For a given ground motion, computation of $F_y(\mu = \mu_i)$ involves iteration, for each period and for each target ductility, of the lateral strength F_y using Eq. 2 until the computed ductility demand μ is, within a certain tolerance, the same as the target ductility μ_i. For a given ground acceleration time history, $\ddot{u}_g(t)$, a R_μ spectrum can be constructed by plotting the strength reduction factors (computed with Eq. 4) of a family of SDOF systems (with different periods of vibrations) undergoing different levels of inelastic deformation, μ_i, when subjected to $\ddot{u}_g(t)$.

REVIEW OF PREVIOUS STUDIES

The R_μ spectrum is a function both of the characteristics of the system (damping and hysteretic behavior) and the earthquake ground motion. In this section, previous studies that have investigated strength reduction factors are reviewed and proposed expressions to estimate R_μ are presented in a common format in order to facilitate the comparison among them.

Newmark and Hall [11].- Based on elastic and inelastic response spectra of the NS component of the El Centro, California earthquake of May 18, 1940, as well as on previous studies of the response on simple systems to pulse-type excitations and two other recorded ground motions [12, 13], the authors observed that: (i) in the low-frequency and medium-frequency spectral regions, an elastic and an inelastic system have approximately the same maximum displacement; (ii) in the extremely high-frequency region, an elastic and an inelastic system have the same force; and (iii) in the moderately high-frequency region, the principle of conservation of energies can be used by which the monotonic load-deformation diagram of the elastic system up to the maximum deformation is the same as that of an elastic-perfectly plastic system subjected to the same excitation. These observations resulted in the recommendation of a procedure to construct inelastic spectra from the elastic spectra. The procedure consisted of the reduction of the elastic spectra by different factors for each spectral region. The strength reduction factors, R_μ, consistent with the Newmark and Hall procedure are given by:

$$For\ 0 \le T < \frac{T_1}{10} \qquad R_\mu = 1 \tag{5}$$

$$For\ \frac{T_1}{10} \le T < \frac{T_1}{4} \qquad R_\mu = \sqrt{2\mu - 1}\left[\frac{T_1}{4T}\right]^{2.513 \log\left[\frac{1}{\sqrt{2\mu - 1}}\right]} \tag{6}$$

$$For\ \frac{T_1}{4} \le T < T_1' \qquad R_\mu = \sqrt{2\mu - 1} \tag{7}$$

$$For\ T_1' \le T < T_1 \qquad R_\mu = \frac{T\mu}{T_1} \tag{8}$$

$$For\ T_1 \le T < T_2 \qquad R_\mu = \mu \tag{9}$$

$$For\ T_1 \le T < 10.0\ s \qquad R_\mu = \mu \tag{10}$$

where the limiting periods T_1 , T'_1 and T_2 are given by

$$T_1 = 2\pi \frac{\phi_{ev} V}{\phi_{ea} A} \tag{11}$$

$$T'_1 = T_1 \frac{\mu}{\sqrt{2\mu - 1}} \tag{12}$$

$$T_2 = 2\pi \frac{\phi_{ed} D}{\phi_{ev} V} \tag{13}$$

where A, V and D are the maximum ground acceleration, maximum ground velocity and maximum ground displacement, respectively; ϕ_{ea} , ϕ_{ev} and ϕ_{ed} are amplification factors that, applied to the maximum ground motion parameters give the ordinates of the elastic design spectrum in the acceleration, velocity and displacement spectral regions, respectively. Although Eqs. 9 and 10 yield the same R_μ, they are included here to be consistent with the spectral regions defined in the original study by Newmark and Hall (Ref. 11).

The strength reduction factors are characterized by constant reduction factors with the exception of two transition regions, where a nonlinear (for $T_1 / 10 \le T < T_1 / 4$) and a linear (for $T_1' \le T < T_1$) reduction factor are specified. The reduction factors in these transition regions result in piece-wise linear inelastic spectra when plotted on logarithmic paper.

Strength reduction factors computed using recommended average values for V/A and D/V^2 ratios (i.e., 48 in/sec/g and 6, respectively) combined with the use of recommended amplification factors ϕ_{ea} , ϕ_{ev} and ϕ_{ed} for a damping ratio of β = 5% (i.e., 2.6, 1.9 and 1.4, respectively, resulting in T_1 = 0.57 sec.) are shown in Fig. 3a.

Lai and Biggs [14].- Design inelastic response spectra were proposed based on mean inelastic spectra computed for 20 artificial ground motions whose elastic response spectra were compatible with the Newmark-Hall elastic design spectrum. Analyses were made for 50 natural periods equally spaced between 0.1 second and 10 seconds on a logarithmic scale.

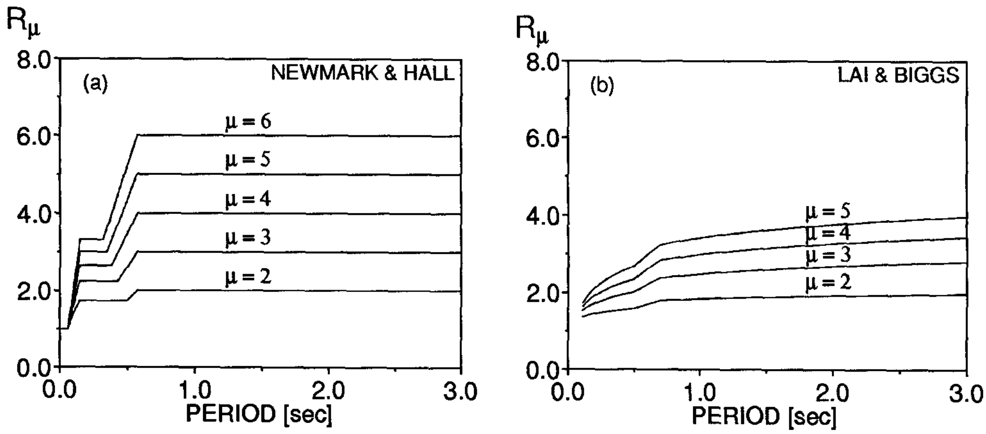


Figure 3. Strength reduction factors proposed by: (a) Newmark & Hall; and(b) Lai & Biggs

Two levels of damping and four ductility ratios were considered. The study was limited to elasto-plastic systems.

The proposed inelastic response spectra consisted of the reduction of the elastic spectra by means of deamplification factors consisting of linear segments (when plotted on semi-logarithmic paper) for each of three spectral regions. The strength reduction factors corresponding to the proposed deamplification factors are given by the following equation:

$$R_\mu = \alpha + \beta(\log T) \tag{14}$$

where coefficients α and β depend on the displacement ductility ratio and the spectral region (period range) as shown in Table 1.

The strength reduction factors consistent with the deamplification proposed by Lai and Biggs are plotted in Fig. 3b. As mentioned before, these strength reduction factors are piece-wise linear when plotted on semi-logarithmic paper (i.e., plotting R_μ vs. $\log T$).

PERIOD RANGE	COEFFICIENT	μ = 2	μ = 3	μ = 4	μ = 5
0.1 ≤ T < 0.5 s	α	1.6791	2.2296	2.6587	3.1107
	β	0.3291	0.7296	1.0587	1.4307
0.5 ≤ T < 0.7 s	α	2.0332	2.7722	3.3700	3.8336
	β	1.5055	2.5320	3.4217	3.8323
0.7 ≤ T < 4.0 s	α	1.8409	2.4823	2.9853	3.4180
	β	0.2642	0.6605	0.9380	1.1493

Table 1. Coefficients to compute strength reduction factors proposed by Lai & Biggs.

Riddell and Newmark [15].- An improved set of deamplification factors was based on a statistical analysis of inelastic response spectra for elasto-plastic systems with 2, 5, and 10 percent damping, and for bilinear and stiffness degrading systems with 5 percent damping and for ductility values from 1 to 10. The study concluded that peak responses of elasto-plastic, bilinear and stiffness degrading systems are very similar, and that the use of an elastic-plastic spectrum for inelastic analysis is generally conservative. The study, the first to consider a statistical analysis of inelastic spectra of recorded ground motions, considered ten earthquake ground motions recorded on rock and alluvium sites.

Similarly to the previous study by Newmark [11], the proposed inelastic spectra are computed with the use of deamplification factors which depend on displacement ductility and the spectral region. However, in contrast to the Newmark-Hall deamplification factors, in this study the proposed deamplification factors depend also on the damping ratio, β. The strength reduction factors, R_μ, consistent with the Riddell and Newmark procedure are given by:

$$\text{For } 0 \leq T < 0.0303\,s \qquad R_\mu = 1 \tag{15}$$

$$\text{For } 0.0303\,s \leq T < 0.125\,s \qquad R_\mu = (p_a\,\mu - q_a)^{r_a}\left[\frac{1}{8T}\right]^{1.625\,\log\left[(p_a\,\mu - q_a)^{-r_a}\right]} \tag{16}$$

$$\text{For } 0.125\,s \leq T < T_1' \qquad R_\mu = (p_a\,\mu - q_a)^{r_a} \tag{17}$$

$$\text{For } T_1' \leq T < T_1 \qquad R_\mu = \frac{T}{T_1}(p_v\,\mu - q_v)^{r_v} \tag{18}$$

$$\text{For } T_1 \leq T < T_2' \qquad R_\mu = (p_v\,\mu - q_v)^{r_v} \tag{19}$$

$$\text{For } T_2' \leq T < T_2 \qquad R_\mu = \frac{T}{T_2\,p_d\,\mu^{-r_d}} \qquad 1.5 \leq \mu < 10 \tag{20}$$

$$\text{For } T_2 \leq T < 10.0\,s \qquad R_\mu = \frac{1}{p_d\,\mu^{-r_d}} \qquad 1.5 \leq \mu < 10 \tag{21}$$

where the parameters p_a, q_a, r_a, p_v, q_v, r_v, p_d and r_d are given by

$$p_a = q_a + 1 \qquad q_a = 3.0\,\beta^{-0.3} \qquad r_a = 0.48\,\beta^{-0.08} \qquad 2 \leq \beta \leq 10 \tag{22}$$

$$p_v = q_v + 1 \qquad q_v = 2.7\,\beta^{-0.4} \qquad r_v = 0.66\,\beta^{-0.04} \qquad 2 \leq \beta \leq 10 \tag{23}$$

$$p_d = 0.87\beta^{-0.055} \qquad r_d = 1.07 \qquad 2 \leq \beta \leq 10 \tag{24}$$

and the limiting periods T_1, T'_1 T_2 and T_2' are given by

$$T_1 = 2\pi\frac{\phi_{ev}\,V}{\phi_{ea}\,A} \tag{25}$$

$$T'_1 = T_1\frac{(p_a\,\mu - q_a)^{r_a}}{(p_v\,\mu - q_v)^{r_v}} \tag{26}$$

$$T_2 = 2\pi\frac{\phi_{ed}\,D}{\phi_{ev}\,V} \tag{27}$$

$$T'_2 = T_2\,p_d\,\mu^{-r_d}(p_v\,\mu - q_v)^{r_v} \tag{28}$$

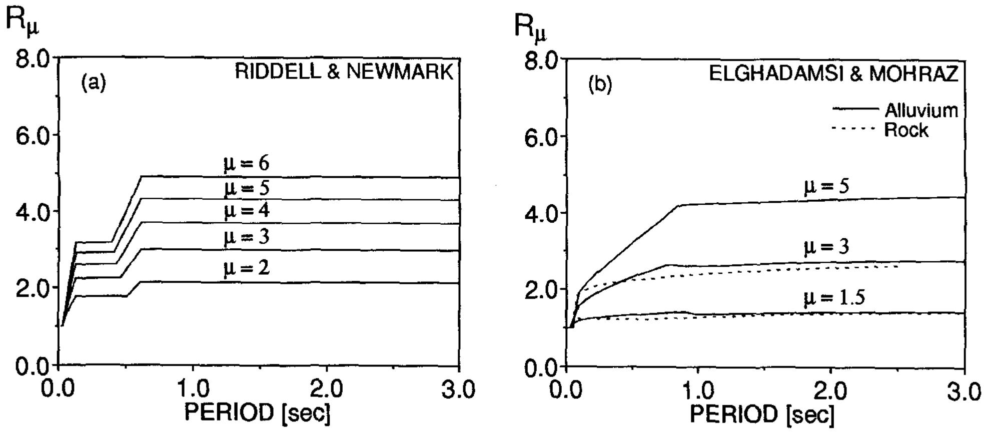


Figure 4. Strength reduction factors proposed by: (a) Riddell & Newmark; and (b) Elghadamsi & Mohraz

where *A*, *V* and *D* are the maximum ground acceleration, maximum ground velocity and maximum ground displacement, respectively; ϕ_{ea}, ϕ_{ev} and ϕ_{ed} are amplification factors that, applied to the maximum ground motion parameters give ordinates of the elastic design spectrum in the acceleration, velocity and displacement spectral regions, respectively.

Strength reduction factors computed using recommended average values for V/A and D/V^2 ratios (i.e., 48 in/sec/g and 6, respectively) combined with the use of recommended amplification factors ϕ_{ea}, ϕ_{ev} and ϕ_{ed} for a damping ratio of β = 5% (i.e., 2.77, 2.15 and 2.10, respectively) are shown in Fig. 4a.

Elghadamsi and Mohraz [16].- To the best of the authors' knowledge this is the first study that considered the effect of soil conditions on deamplification factors. The study considered inelastic response spectra computed for SDOF systems with elasto-plastic behavior when subjected to 50 horizontal ground motions recorded on alluvium and 26 horizontal components of ground motions recorded on rock. This study concluded that deamplification factors are not significantly influenced by soil conditions and that their effects stems primarily from their effects on elastic response spectra.

Based on statistical analysis a new approach was proposed to estimate inelastic response spectra. In the procedure suggested in this study, inelastic yield spectra are obtained by interpolation from two smoothed limiting spectra. The two limiting spectra correspond to an elastic spectrum and an inelastic spectrum for a yield level of 0.05 in. Using interpolated inelastic spectra, deamplification factors were computed for rock and alluvium sites. Strength reduction factors corresponding to the deamplification factors proposed by Elghadamsi and Mohraz are shown in Fig. 4b.

Riddell, Hidalgo and Cruz [17].- This study was based on inelastic spectra computed for four sets of earthquake records computed for SDOF systems with an elasto-plastic hysteretic behavior and with 5 percent damping. Simplified strength reduction factors were proposed based on approximate mean strength reduction factors. The mean strength reduction factors

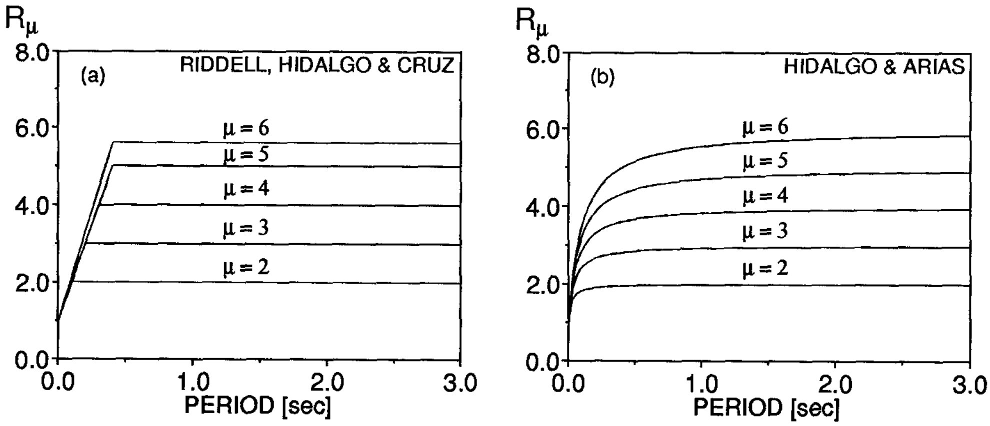


Figure 5. Strength reduction factors proposed by: (a) Riddell, Hidalgo & Cruz; and (b) Hidalgo & Arias

are only an approximation of the exact mean reduction factors because they were computed as the ratio of mean elastic to inelastic strength demand spectra and not as the mean of the ratios of elastic to inelastic strength demand spectra.

The strength reduction factors proposed in this study consist of two linear segments given by

$$\text{For} \quad 0 \leq T \leq T^* \qquad R_\mu = 1 + \frac{R^* - 1}{T^*} T \tag{29}$$

$$\text{For} \quad T \geq T^* \qquad R_\mu = R^* \tag{30}$$

where the value of T^* was proposed to vary between 0.1 and 0.4 seconds for ductility ratios between 2 and 10 and the value of R^* was proposed to be equal to μ for $2 \leq \mu \leq 5$ and smaller than μ for $5 \leq \mu \leq 10$ as follows

PARAMETER	μ = 2	μ = 3	μ = 4	μ = 5	μ = 6	μ = 7	μ = 8
R^*	2.0	3.0	4.0	5.0	5.6	6.2	6.8
T^*	0.1	0.2	0.3	0.4	0.4	0.4	0.4

Strength reduction factors computed with the above equations are shown in Fig. 5a.

Arias and Hidalgo [18].- Based on the approximate mean strength reduction factors computed by Riddell, Hidalgo and Cruz [17], this study proposed an expression to compute strength reduction factors consisting of a nonlinear curve that is applicable in the whole period range of interest. The proposed expression is given by

$$R_\mu = 1 + \frac{T}{k\,T_o + \dfrac{T}{\mu - 1}} \tag{31}$$

where the factor $k\ T_O$ was reported to vary for different groups of ground motions. For the draft of the Chilean code this study recommended a value of $k = 0.1$.

Strength reduction factors computed with the above equation assuming $T_O = 0.2$ are shown in Fig. 5b.

Nassar and Krawinkler [19].- This study considered the response of SDOF nonlinear systems when subjected to 15 ground motions recorded in the Western United States. The records used were obtained at alluvium and rock sites. The influence of site conditions, however, was not explicitly considered. The sensitivity of mean strength reduction factors to the epicentral distance as well as structural system parameters such as natural period, yield level, strain-hardening ratio and the type of inelastic material behavior (i.e. bilinear versus stiffness degrading) was examined. The study concluded that epicentral distance and stiffness degradation have a negligible influence on strength reduction factors.

Based on mean strength reduction factors the following expression was proposed to estimate strength reduction factors:

$$R_\mu = \left[c\,(\mu - 1) + 1 \right]^{1/c} \tag{32}$$

where

$$c\,(T\,,\,\alpha) = \frac{T^a}{1 + T^a} + \frac{b}{T} \tag{33}$$

where α is the post-yield stiffness as percentage of the initial stiffness of the system, and the parameters a and b are given by

α	a	b
0.00	1.00	0.42
0.02	1.00	0.37
0.10	0.80	0.29

Strength reduction factors computed with the above equations are shown in Fig. 6a.

Vidic, Fajfar and Fischinger [20].- Based on mean strength reduction factors computed for 20 ground motions recorded on western United States and in the 1979 Montenegro, Yugoslavia, earthquake a simplified expressions were proposed to estimate strength reduction factors.

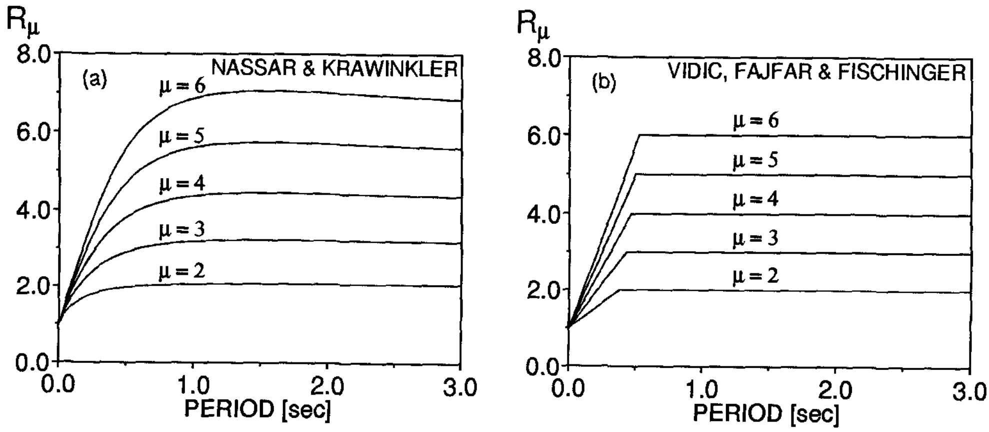


Figure 6. Strength reduction factors proposed by: (a) Nassar & Krawinkler; and (b) Vidic, Fajfar & Fischinger

The study considered SDOF systems with bilinear and stiffness degrading (Q-model) hysteretic behavior and viscous damping proportional to the mass and to the instantaneous stiffness of the system.

The simplified expressions consist of two linear segments. In the first segment which corresponds to the short-period region, R_μ increases linearly with increasing period from $R_\mu = 1$ to a value that it is equal or nearly equal to the ductility factor. In the second segment, the strength reduction factor maintains a constant value. The details of the proposed relations depend on the hysteretic behavior and damping of the system. For systems with a Q-model hysteretic behavior and 5% mass-proportional damping the following expressions were proposed:

$$\text{For } T \leq T_o \qquad R_\mu = (\mu - 1)\frac{T}{T_o} + 1.0 \tag{34}$$

$$\text{For } T > T_o \qquad R_\mu = \mu \tag{35}$$

where T_o is given by

$$T_o = 0.65\,\mu^{0.3}\,T_1 \tag{36}$$

$$T_1 = 2\pi\frac{\phi_{ev}\,V}{\phi_{ea}\,A} \tag{37}$$

Strength reduction factors computed using mean amplification factors ϕ_{ea} and ϕ_{ev} of the 20 ground motions considered in the study (i.e., $\phi_{ea} = 2.5$ and $\phi_{ev} = 2.0$) are shown in Fig. 6b.

Miranda [21].- In this study, an effort was made to consider a relatively large number of recorded ground motions in order to study the effects of the variability of the characteristics of recorded ground motions on strength reduction factors. In order to study the influence of local site conditions on strength reduction factors, a group of 124 ground motions recorded on a wide range of soil conditions during various earthquakes was considered. Based on the local sitte conditions at the recordin station, ground motions were classified into three groups: ground motions recorded on rock; ground motions recorded on alluvium; and ground motions recorded on very soft soil deposits characterized by low shear wave velocities.

Strength reduction factors were computed for 5% damped bilinear SDOF systems undergoing displacement ductility ratios between 2 and 6. Afterwards, mean strength reduction factors were computed for each soil group. In addition to the influence of soil conditions, the investigation considered the influence of magnitude and epicentral distance on strength reduction factors. The study concluded that while soil conditions may influence significantly the reduction factors (particularly for soft soil sites), magnitude and epicentral distance have a negligible effect on mean strength reduction factors.

Based on mean strength reduction factors, the following simplified expressions were proposed to estimate the reduction factors:

$$R_\mu = \frac{\mu - 1}{\Phi} + 1 \geq 1 \tag{38}$$

where Φ is a function of μ, T and the soil conditions at the site, and is given by

For rock sites
$$\Phi = 1 + \frac{1}{10\,T - \mu T} - \frac{1}{2T}\,exp\left[-\frac{3}{2}\left(ln\,T - \frac{3}{5}\right)^2\right] \tag{39}$$

For alluvium sites
$$\Phi = 1 + \frac{1}{12\,T - \mu T} - \frac{2}{5T}\,exp\left[-2\left(ln\,T - \frac{1}{5}\right)^2\right] \tag{40}$$

For soft soil sites
$$\Phi = 1 + \frac{T_g}{3\,T} - \frac{3\,T_g}{4\,T}\,exp\left[-3\left(ln\,\frac{T}{T_g} - \frac{1}{4}\right)^2\right] \tag{41}$$

where T_g is the predominant period of the ground motion, defined as the period at which the maximum relative velocity of a 5% damped linear elastic system is maximum throughout the whole period range.

Due to the important variations in mean R_μ with changes in the T/T_g ratio, combined with uncertainties in the estimation of the T/T_g ratio, Eq. 41 was not based on mean strength reduction factors but on modified strength reduction factor spectra which consider a ±10% error in the estimation of the T/T_g ratio. Strength reduction factors computed with Eqs. 38 to 41 are shown in Fig. 7.

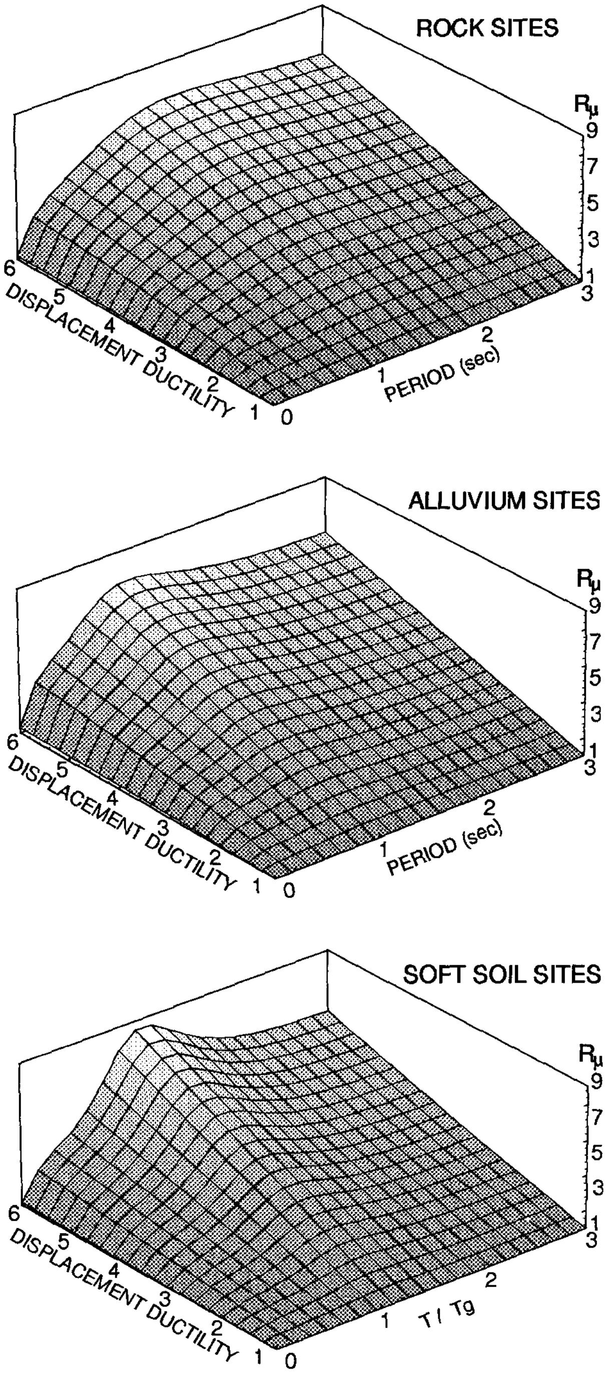


Figure 7. Strength reduction factors proposed by Miranda for rock, alluvium,and soft soil sites.

Other studies.- The effects of various types of stiffness and strength degradation on the inelastic response of SDOF systems with 5% damping were studied by Al-Sulaimani and Roessett [22]. Using nine recorded ground motions, the study concluded that the strength reduction factors for systems which exhibit stiffness degradation with softening and pinching should be smaller than those used for elasto-plastic systems.

Using a stochastic procedure, Peng et al. [23] computed deamplification factors for elasto-plastic systems with various levels of damping when subjected to accelerograms with different durations recorded on rock and recorded on alluvium. The study concluded that for a given displacement ductility ratio a longer duration of strong motion results in a greater deamplification or a smaller strength reduction factor, and that damping and soil conditions do not affect deamplification factors to a significant degree.

Takada et al. [24] used three types of stick models to study the strength reduction factors in MDOF systems. Based on artificially-generated ground motions and a Monte-Carlo simulation the following period-independent expression was proposed

$$R_\mu = \varepsilon \sqrt{2\mu - 1} \tag{43}$$

where ε is an adjustment factor representing the degree of deviation of the R_μ vs. μ relationship from the equal energy expression $\sqrt{2\mu - 1}$. Results from the Monte Carlo simulation indicate that median values of ε varied between 1.05 and 1.34.

EVALUATION OF STRENGTH REDUCTION FACTORS

Main factors influencing strength reduction factors.- The strength reduction factor depends not only on the characteristics of the system, but also on the ground motion input (i.e., the ground acceleration time history). For a given ground motion, R_μ is a function of the period of vibration T of the structure, the damping, the type of hysteretic behavior and the level of inelastic deformation (i.e., the displacement ductility ratio). Studies reviewed in the previous section agree that for a given acceleration time history the strength reduction factor is primarily influenced by the period of vibration and the level of inelastic deformation, and to a much lesser degree by the damping and the hysteretic behavior of the system (under the assumption that there is no significant strength deterioration). Thus, the strength reduction factor can be expressed as

$$R_\mu = R_\mu (T, \mu_i) \tag{43}$$

From the definition of R_μ (Eq. 4), it is clear that for any ground motion, regardless of the period of the structure, for systems behaving elastically ($\mu_i = 1$) the strength reduction factor must satisfy the following equation

$$R_\mu = R_\mu (T, \mu_i = 1) = 1 \tag{44}$$

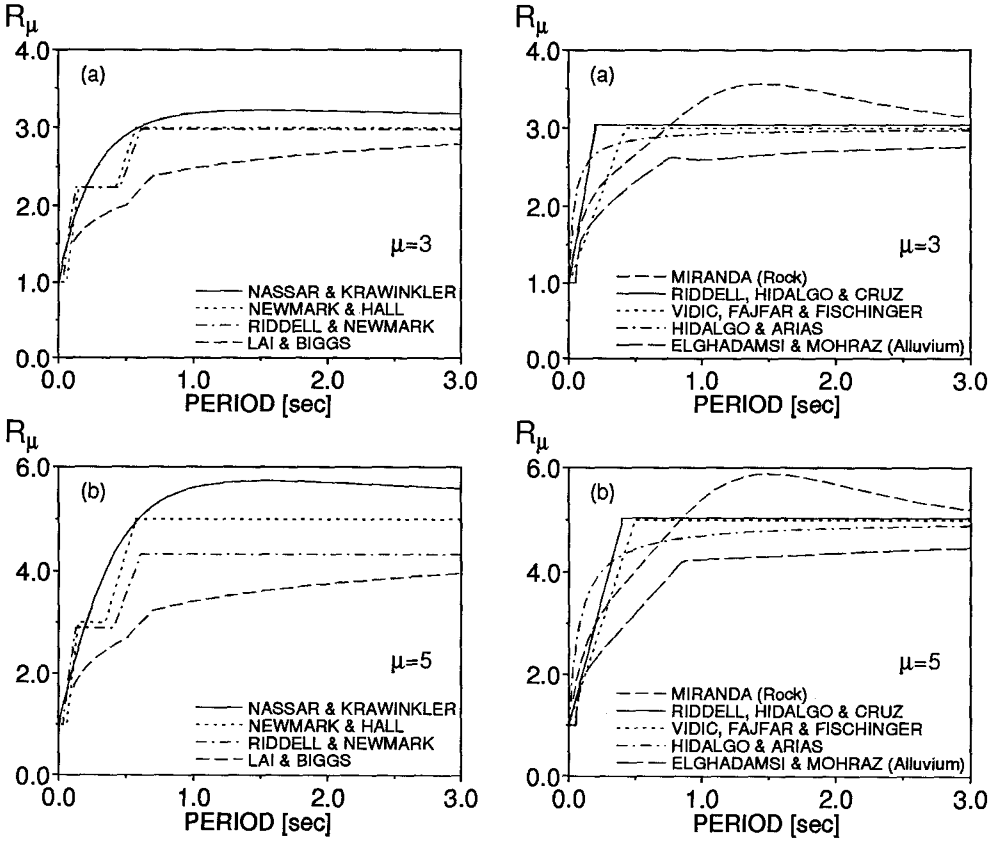


Figure 8. Comparison of strength reduction factors proposed in various studies for (a) $\mu = 3$, and for (b) $\mu = 5$

For very rigid systems whose yield displacement tends to zero (i.e., $u_y \to 0$, $T \to 0$), even a very small reduction in lateral strength in from the lateral strength required to keep the system in the elastic range results in large inelastic deformations. Thus, for any ground motion the inelastic strength demand in these systems is the same as the elastic strength demand, and therefore the strength reduction factor must satisfy the following equation

$$R_\mu = R_\mu\left(T \to 0 \, , \, \mu_i\right) \to 1 \tag{45}$$

With the exception of the strength reduction factor proposed by Takada et al. which is period-independent all other proposed expressions to estimate R_μ satisfy this equation.

For very flexible systems (i.e., $T \to \infty$), regardless of the strength of the system, the maximum relative displacement tends toward the maximum ground displacement. Therefore, for any ground acceleration time history the inelastic strength demand is equal to the elastic strength demand divided by the displacement ductility ratio and the strength reduction factor for these systems must satisfy the following equation

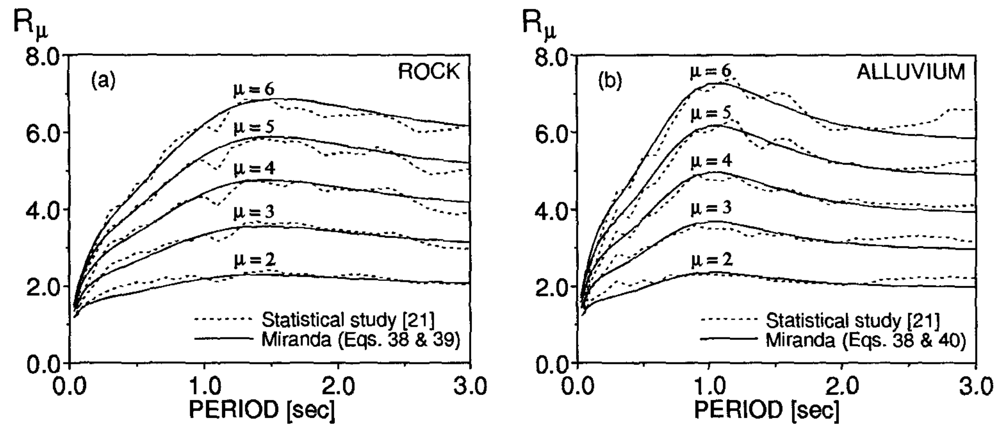


Figure 9. Comparison of mean strength reduction factors for (a) rock and (b) alluvium with those computed using equations 38-40.

$$R_\mu = R_\mu\left(T \to \infty,\, \mu_i\right) \to \mu \tag{46}$$

This equation is not satisfied by the expressions proposed in the following studies: Lai & Biggs [14], Riddell et al. [17] (for μ larger than 5), and Takada et al. [24].

A comparison of strength reduction factors proposed by the various investigations for displacement ductility ratios equal to 3 and 5 is shown in Fig. 8. Although all proposed reduction factors follow a similar trend, some significant differences exist between them. As shown in this figure, for $\mu = 5$ the smallest reduction factors are those proposed by Lai & Biggs, which for certain periods are 40% and 38% smaller than those proposed by Nassar & Krawinkler and those proposed by Miranda, respectively. For periods smaller than 0.3 s and $\mu = 5$ the largest reductions are those proposed by Hidalgo & Arias which can be up to 44% larger than those proposed by Riddell, Hidalgo & Cruz (despite being based on the same statistical study). Furthermore, only expressions proposed by Nassar & Krawinkler and those proposed by Miranda recognize the fact that for a certain range of periods mean strength reduction factors are larger than the target ductility.

Since the strength reduction factor is a function of the ground motion, for a given system undergoing a ductility demand, μ_i, the reduction, R_μ, will be different for different ground motions. While statistical studies indicate that epicentral distance has a very small influence on mean strength reduction factors [19, 21], soil conditions at the site can have an important effect on R_μ, particularly for very soft soils [10, 21]. A comparison of mean strength reduction factors computed for a relatively large number of ground motions recorded on rock sites and on alluvium sites with those computed with the simplified expressions proposed by Miranda (Eqs. 38-40) is shown in Fig. 9. It can be seen that the proposed equations are a good approximation of average strength reduction factors.

Using mean strength reduction factors computed using a different set of ground motions (consisting of 15 acceleration time histories recorded on rock and on alluvium) Nassar & Krawinkler proposed Eqs. 32 and 33 to estimate R_μ for structures built on firm sites (either rock or alluvium sites).

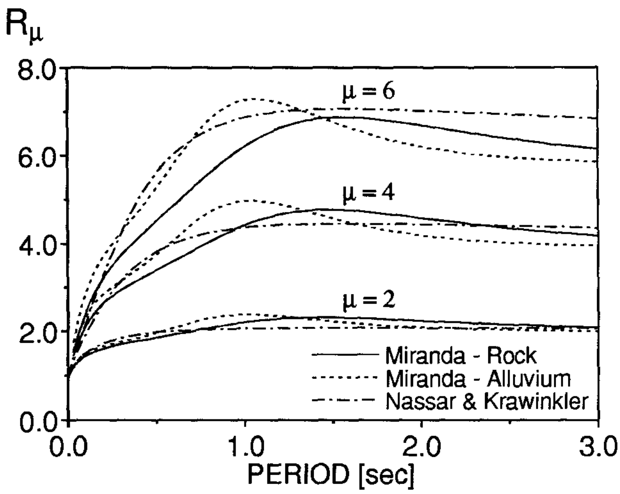


Figure 10. Comparison of strength reduction factors proposed by Miranda for rock and alluvium sites with those porposed by Nassar and Krawinkler

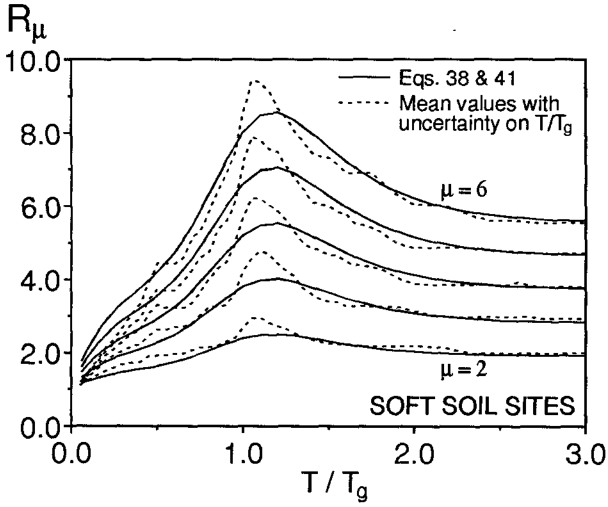


Figure 11. Comparison of mean strength reduction factors of soft soil sites with uncertainty in the estimation of the T/T_g ratio with those propose by Miranda

A comparison of these strength reduction factors to those proposed by Miranda can be seen in Fig. 10. It can be seen that the differences among the three expressions are relatively small and that, in general, these differences increase with increasing displacement ductility ratio. For periods smaller than about 1.0 s strength reduction factors computed for systems on alluvium sites are larger than those for systems on rock sites, while for systems with periods larger than 1.5 s the opposite is true.

As shown in Ref. 21, strength reduction factors for systems on soft soils are characterized by values much larger than the target ductility for systems with periods close to the predominant period of the ground motion. Mean strength reduction factors (modified to account for a ±10% error in the estimation of the T/T_g ratio) and those computed using Eqs. 38 and 41 are shown in Fig. 11.

In general, the use of strength reduction factors derived for firm sites if applied to soft soil sites will be unconservative for systems with periods shorter than about 2/3 of the predominant period and conservative for systems with periods close to the predominant period.

Implementations of period-dependent strength reduction factors.- With the exception of the study by Takada et al., all investigations reviewed herein recommend the use of period-dependent strength reduction factors. Variations in R_μ with changes in period of vibration are not incorporated in current seismic provisions for building structures in the U.S. The permissible level of strength reduction is only based on the type of structural system. Strength reduction factors recommended by the 1988 National Earthquake Hazard Reduction Program [25] (response modification factor, R) and by the 1991 Uniform Building Code [26] (system performance factor, R_W, divided by 1.4 to increase the reduction associated with allowable design strength to approximately that associated with the first significant yield strength) for ductile systems are shown in Fig. 12. Mean strength reduction factors computed for systems undergoing displacement ductility ratios of 2 and 4 when subjected to ground motions recorded on rock and on alluvium (taken from Ref. 21) are also shown in the figure. It can be seen that strength reduction factors specified by U.S. building codes are much larger than the strength reductions due to nonlinear hysteretic behavior for system undergoing displacement ductility ratios equal to four. Thus, structures designed according to these seismic provisions need to have significant amounts of overstrength in order to avoid excessive inelastic deformations (for most structures, global ductility ratios in excess of four are associated with very large local ductility demands). Furthermore, this figures indicates that these design provisions may not provide the intended uniform degree of protection for buildings with different periods of vibration.

For a given target ductility, the strength-reduction factor can exibit great variations from one ground motion to another. For the design of a structure this means that the lateral strength capacity required to avoid displacement ductility demands larger than a given limit can have important variations from one ground motion to another. Thus, this means that in some cases using mean strength-reduction factors may not result in the desired level of conservativism. In those cases it is necessary to use strength-reduction factors associated with smaller levels of probability of exceedance (for example those associated with mean minus one standard deviation). The reader is referred to Ref. 21 for a discussion on the dispersion on R_μ for different levels of ductility.

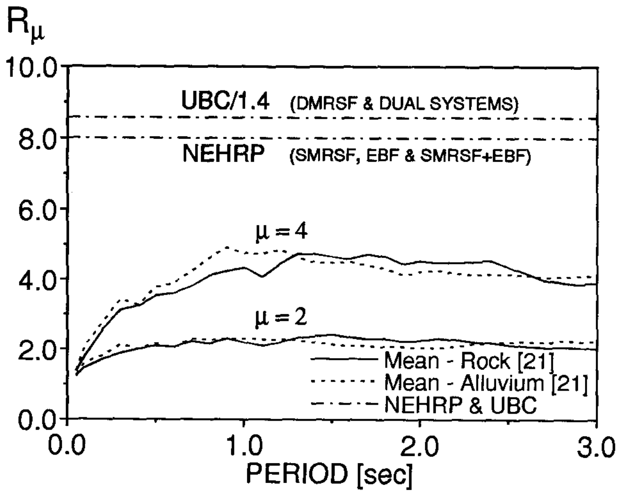


Figure 12. Comparison of strength reduction factors recommended by UBC and NEHRP for ductile systems with those from statistical analysis

One of the few seismic codes in the U.S. that specifies period-dependent strength reduction factors is the 1990 Bridge Design Specifications [27] used by the California Department of Transportation (Caltrans), which for ductile single- and multi-column bent highway bridges recommends the use of a so called adjustment factor for ductility and risk assessment, Z, which is constant for bridges with periods shorter than 0.6 s and linearly decreases for bridges with periods between 0.6 and 3.0 s. Figure 13 compares results from statistical analyses of systems undergoing displacement ductility demands of 2 and 4 when subjected to rock and alluvium ground motions with period-dependent strength reductions recommended by Caltrans seismic provisions. It can be seen that the variation of Z with changes in period is opposite to that computed for nonlinear systems under earthquake excitation; that is, the Caltrans Z factors decrease with increasing period while, in general, average strength reductions due to nonlinear behavior increase with increasing period.

The first earthquake-resistant seismic code to explicitly prescribe period-dependent strength reduction factors which account for smaller reductions in the short period range was the 1976 Mexico City Building Code [28], which included a bilinear R_μ spectrum like that computed using Eqs. 34 and 35 with three different values of T_O specified as a function of the soil condition as reflected in the microzonation of the city. The use of a bilinear R_μ spectrum, similar to that used in the Mexico City Code, has also been recommended in the CIRSOC 103 Argentine Code [29] and for highway bridges in New Zealand [30, 31] (using a $T_O = 0.7$ s). More recently, bilinear expressions for R_μ (with $T_O = 0.5$ s) were suggested [6] to improve the period-independent reduction factors of the 1990 edition of the National Building Code of Canada. Period-dependent R_μ factors computed using Eq. 31 have been proposed [18] for the new version of the Chilean seismic Code.

The strength reduction factors shown in Figs. 3 to 11 are based on statistical studies for SDOF systems. The extrapolation of these results to MDOF structures requires the knowledge of the relationship between local (story) ductility demands and the global (structure) ductility demands. This relationship is a function of both, the distribution of inelastic deformations

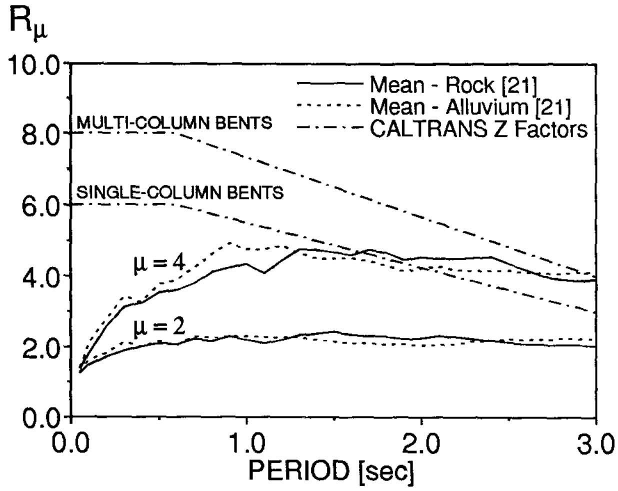


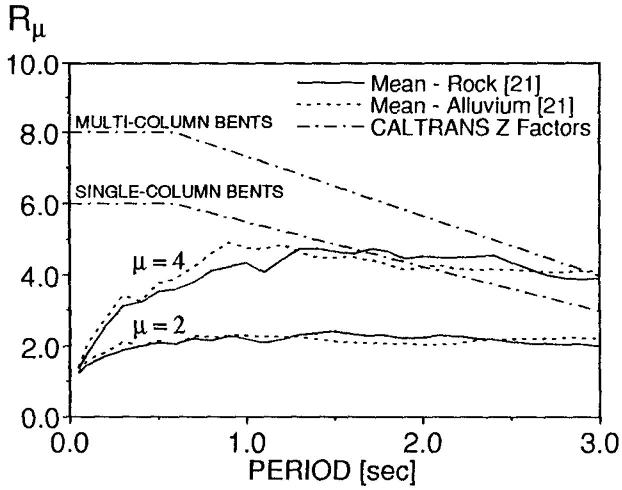
Figure 13. Comparison of strength reduction factors recommended by Caltrans with those from statistical analysis

within the structure (i.e., collapse mechanism) and the ground motion. For many types of structures, a good approximation of this relationship can be obtained with nonlinear static analyses [9, 10, 32].

CONCLUSIONS

The main objective of this study was to review investigations on reduction factors which allow the estimation of lateral strengths required to control the level of inelastic deformations during strong earthquake ground motions. To attain this objective, strength reduction factors proposed by several investigations have been presented in a common format. The following conclusions can be drawn from this study:

1. Except for the idealy infinitely rigid structures allowing nonlinear hysteretic behavior to take place in the event of severe ground motions results in important reductions in design lateral strengths. The magnitude of these strength reductions is primarily a function of the maximum tolerable displacement ductility demand, the period of the system, and the soil conditions at the site. Other factors that may affect the strength reduction factor but to a much lesser degree are the type of hysteretic behavior and damping of the structure, as well as the distance to the epicenter of the earthquake.
2. Depending on the period of vibration strength reduction factors for systems on alluvium can be slightly higher or lower than those of systems on rock. With the exception of relatively long periods (larger than two and one half times the predominant period of the ground motion) strength reduction factors for systems on soft soil sites are significantly different from those of systems on either rock or alluvium sites. For short-period structures (those with periods smaller than two thirds the predominant period of the ground motion), the strength reduction factor is significantly smaller than that corresponding to systems with the same period on either rock or alluvium sites. Thus, the use of strength reduction factors derived for firm sites, if used for short-period structures



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on soft soil sites, can lead to displacement ductility demands considerably larger than those originally intended in its design.

3. The use of ductility-, period- and site-dependent strength reduction factors like those presented in this paper, together with estimates of the overstrength of the structure and the relationship between global and ductility demands, can lead to a more rational and transparent seismic design approach than the approach currently used in seismic codes in the United States.

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