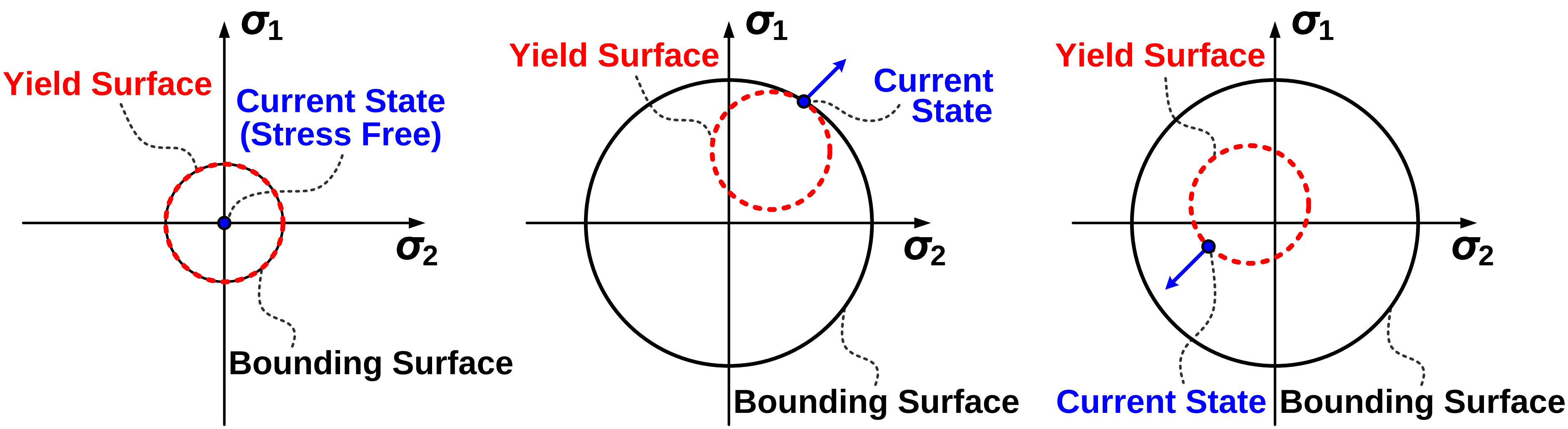
Hardening Material

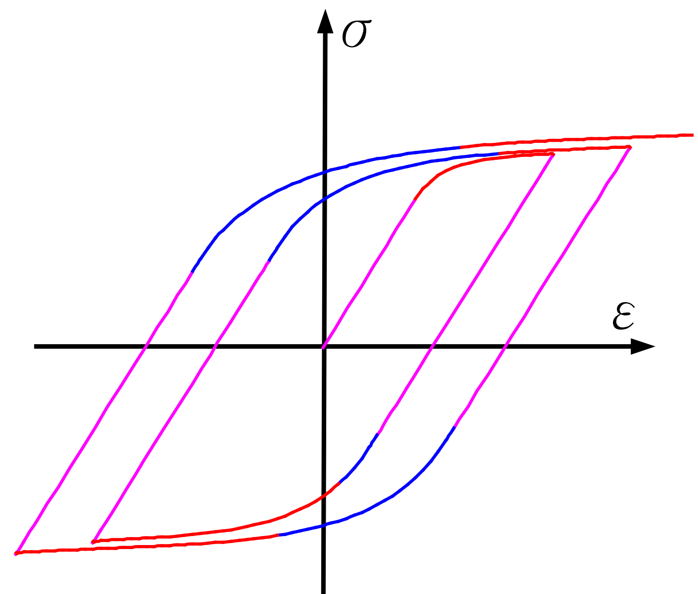
Command: Material, Hardening (Manual: F03)

Two-Surface Model



Bounding Surface
(Isotropic Hardening)

Yield Surface
(Kinematic Hardening)



The kinematic and isotropic hardening characteristics of the steel members under cyclic loadings can be well simulated using the two-surface plasticity model [23]. The essential features of the two-surface model can be revealed in a 2-dimentional space as in Figure 13. A bounding surface exists in the stress space in addition to the yield surface. The elastic region is represented by the interior of the yield surface. The bounding surface is identical to the yield surface prior to the first yielding (Figure 13(a)). The size of the yield surface remains constant and the yield surface is constrained to always move within the bounding surface. When the stress state reaches the yield surface and touches the bounding surface, the bounding surface can expand and isotropic-hardening is allowed (Figure 13(b)). When the stress state is on the yield surface before touching the bounding surface, the kinematic-hardening rule is adopted (Figure 13(c)). Dafalias and Popov [23] suggested that the plastic modulus E_P is a function of two parameters, the distance δ from the stress state under consideration to the corresponding bound, and the value of δ at the initiation of yielding for each loading process, denoted by δ_{ini}.

In PISA3D [22], the two-surface hardening material model consists of an elastic part and a plastic part. These two parts are connected in series. The stiffness of the stress-strain relationships is dealt with using the following equations:

$$E_T^{-1} = E_E^{-1} + E_P^{-1}, \quad F_T = F_E + F_P \tag{9}$$

where E_T and E_E are the total and elastic moduli, respectively; F_T, F_E, and F_P are the total, elastic, and plastic flexibilities, respectively. When isotropic hardening takes place, the plastic flexibility of F_{Pi} is calculated from:

$$F_{Pi} = \frac{F_E}{C_1 + (1 - C_1)e^{\left(-\frac{BS}{YS}C_2\right)}} \tag{10}$$

where BS/YS is the proportion of the bounding surface to the yield surface. C_1 is the reduction factor of the initial modulus. Equations (9) and (10) demonstrate that F_{Pi} finally approaches F_E/C_1, and E_T approaches $E_E C_1$ when C_1 is small enough. C_2 is the modulus reduction rate coefficient. The larger the value of C_2, the faster F_{Pi} approaches F_E/C_1. When the stress state goes into kinematic hardening the plastic flexibility of F_{Pk} is computed from:

$$F_{Pk} = \frac{F_{Pi}}{1 + C_3\left(\frac{\delta}{\delta_{ini} - \delta}\right)^{C_4}} \tag{11}$$

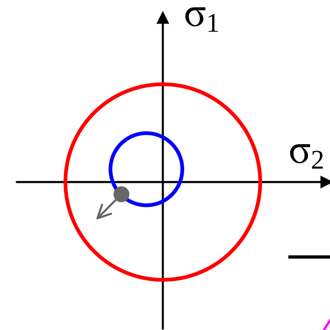
Equation (11) satisfies the fact that F_{Pk} is between F_{Pi} (when δ=0) and 0 (when δ=δ_{ini}). C_3 and C_4 control the modulus variation in the kinematic hardening state. The C_1, C_2, C_3, and C_4 parameters and the BS/YS ratio have been incorporated into the hardening material model as user-specified variables in PISA3D.

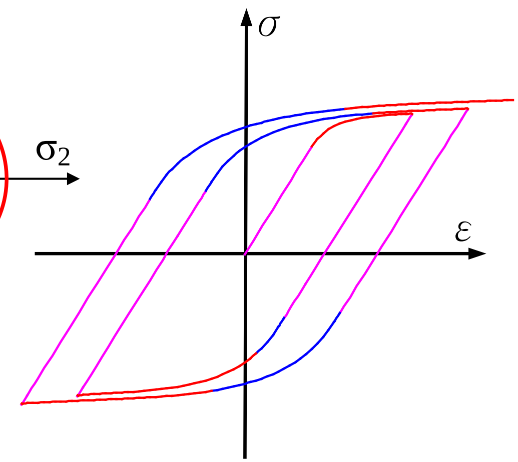
Hardening Material

			C_1^+	C_2^+	C_1^-	C_2^-	C_3	C_4	
E_i	f_{yp}	f_{yc}	H_{iso1+}	H_{iso2+}	H_{iso1-}	H_{iso2-}	H_{kin1}	H_{kin2}	BS/YS

$$F_T = F_E + F_P$$

F_T: total flexibility
F_E: elastic flexibility
F_P: plastic flexibility





Isotropic hardening:

$$F_{Pi} = \frac{F_E}{\left[C_1 + \left(1 - C_1\right) e^{\left(-\frac{BS}{YS} C_2\right)}\right]}$$

***BS/YS* ↑, $F_{Pi} \rightarrow F_E/C_1$,**

$E_T = E_E{}^* C_1/(1+C_1)$,

If C_1 is small, $E_T \approx E_E C_1$

Kinematic hardening:

$$F_{Pk} = \frac{F_{Pi}}{\left[1 + C_3 \left(\frac{\delta}{\delta_{ini} - \delta}\right)^{C_4}\right]}$$

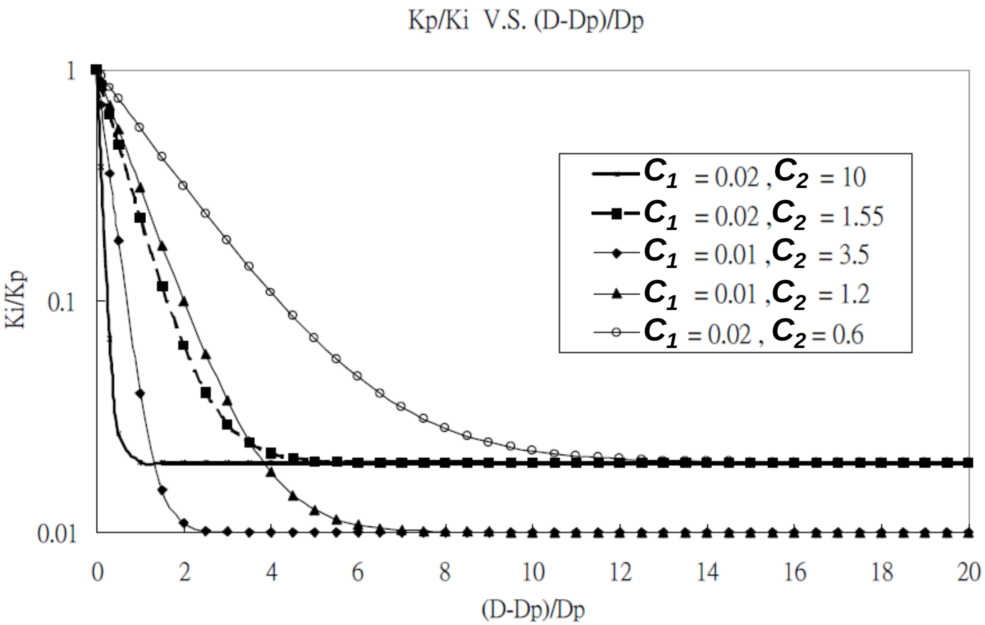
When the material begins kinematic hardening:
$\delta = \delta_{ini}$, $F_{Pk} = 0$

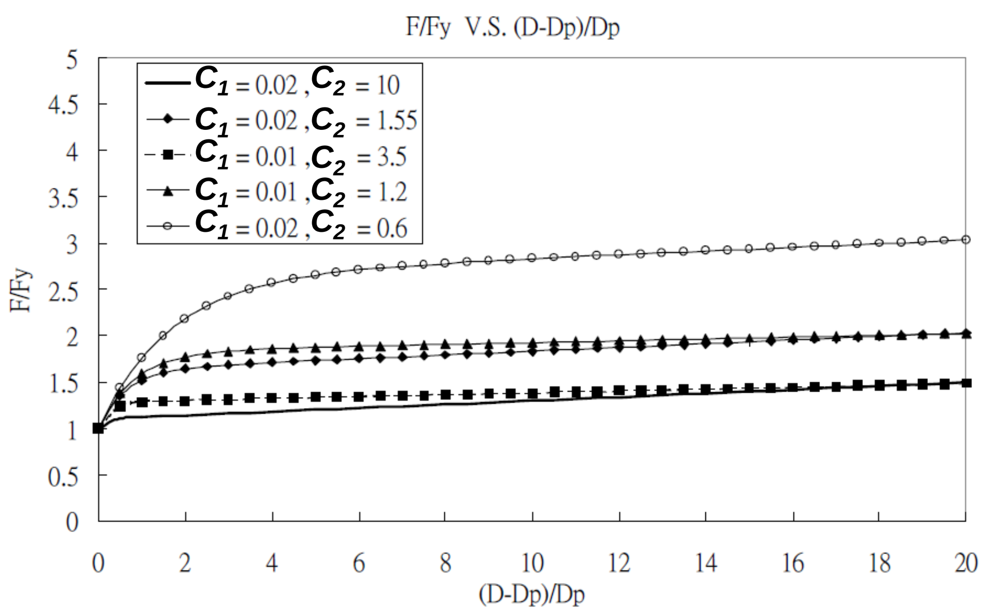
When the material begins isotropic hardening:
$\delta = 0$, $F_{Pk} = F_{Pi}$

δ: the distance of the stress state under consideration from the corresponding bound

δ_{ini}: the value of δ at the initiation of yielding for each loading process

Hardening Material





Isotropic hardening:

$$F_{Pi} = \frac{F_E}{\left[C_1 + (1 - C_1) e^{\left(-\frac{BS}{YS} C_2\right)} \right]}$$

BS/YS ↑,
$e^{[-BS*C_2/YS]} \rightarrow 0$, $F_{Pi} \rightarrow F_E/C_1$,

代入$F_T=F_E+F_P$再取倒數,可得：

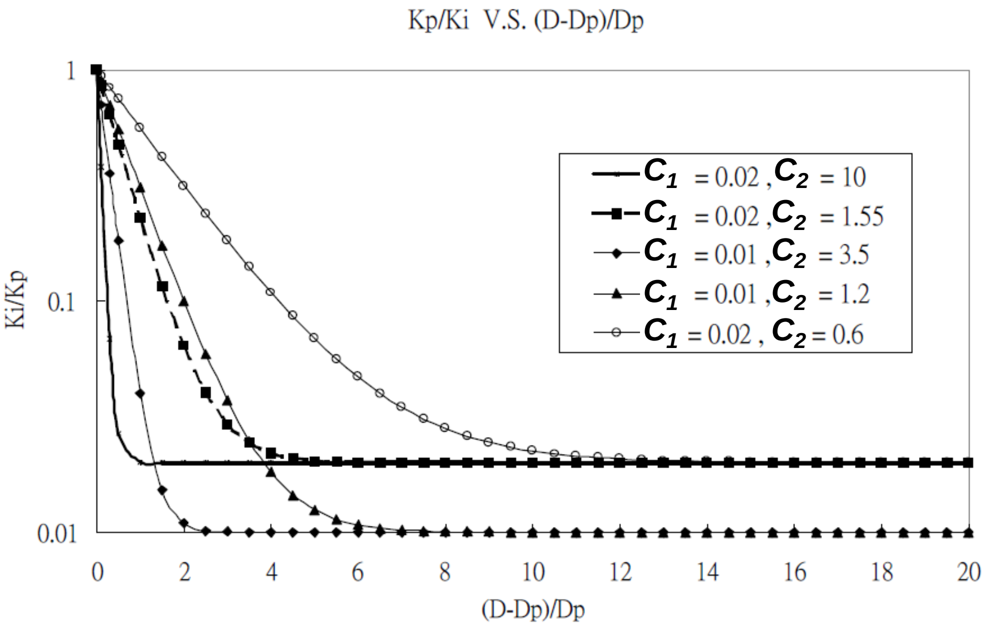
$E_T = E_E * C_1/(1+C_1)$

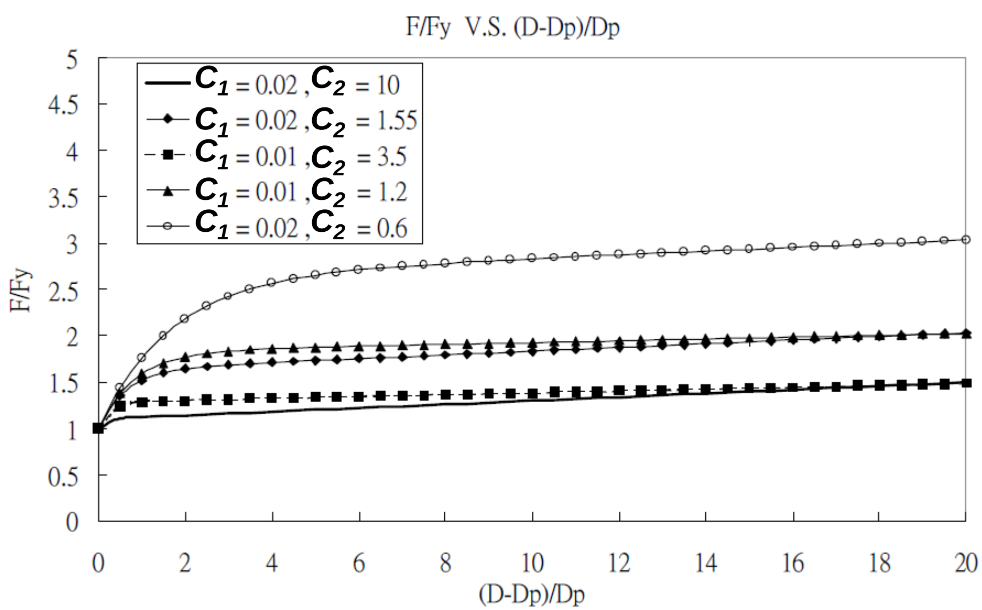
當C_1夠小時, $E_T \approx E_E C_1$

(C_1: 初始斜率折減係數)

C_2為斜率折減速率係數，C_2愈大時F_{Pi}愈快趨近於F_E/C_1

Hardening Material





Isotropic hardening:

$$F_{Pi} = \frac{F_E}{\left[C_1 + (1 - C_1) e^{\left(-\frac{BS}{YS} C_2 \right)} \right]}$$

***BS/YS* ↑,**

$e^{[-BS*C_2/YS]} \rightarrow 0$, $F_{Pi} \rightarrow F_E/C_1$,

Substitute $F_T = F_E + F_P$, take reciprocal, then:

$E_T = E_E * C_1/(1+C_1)$

when C_1 *is very small*, $E_T \approx E_E C_1$

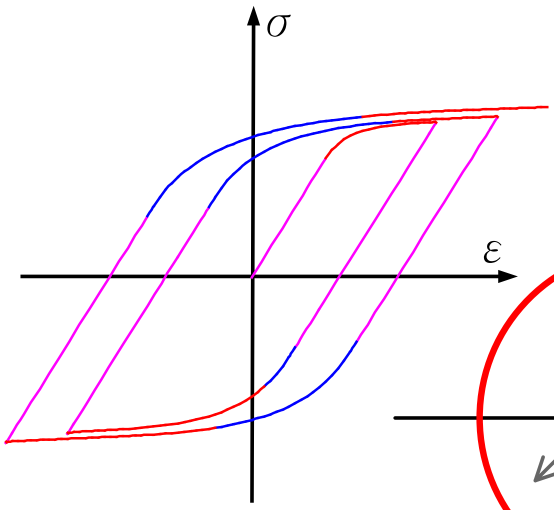
(C_1: reduction factor of the initial stiffness)

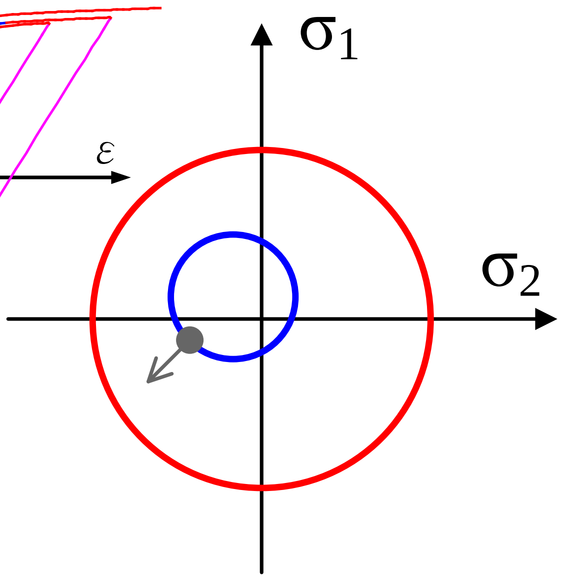
C_2: reduction rate coefficient, the larger the C_2 , the F_{pi} approaches F_E/C_1 faster

Hardening Material

Kinematic hardening:

$$F_{Pk} = \frac{F_{Pi}}{\left[1 + C_3\left(\frac{\delta}{\delta_{ini} - \delta}\right)^{C_4}\right]}$$





***δ*:** the distance of the stress state under consideration from the corresponding bound

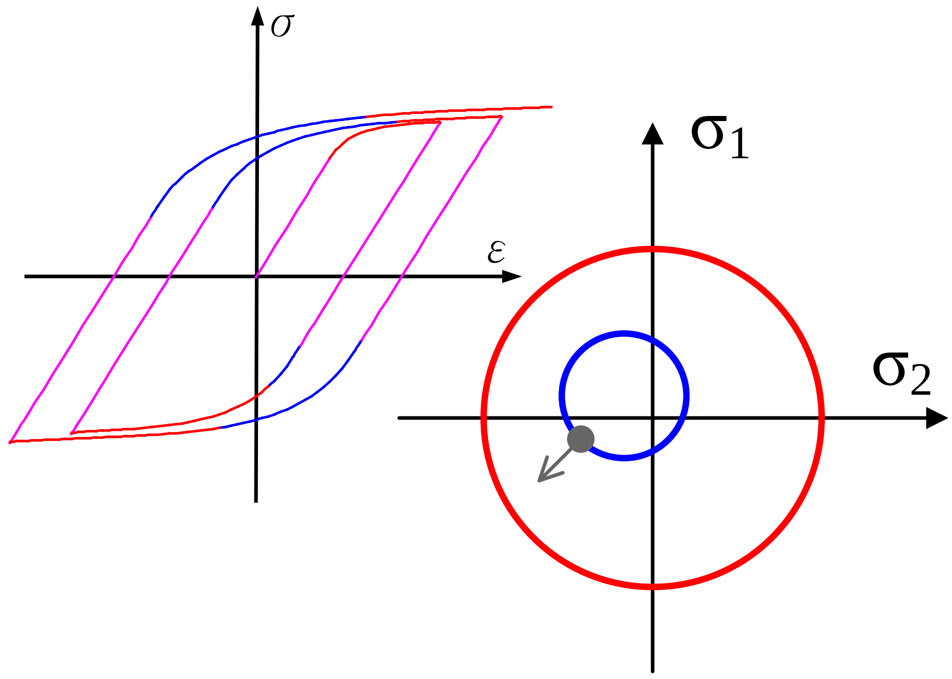
δ_{ini}*:** the value of ***δ at the initiation of yielding for each loading process

走動硬化時，塑性區之柔度F_{Pk}介於F_{pi} ($\delta = 0, E_{Pk} = E_{Pi}$)與0 ($\delta = \delta_{ini}$, $E_{Pk} = E_E$)之間，C_3與C_4為斜率折減速率係數，兩者愈大則F_{Pk}愈慢趨近於F_{Pi}

Hardening Material

Kinematic hardening:

$$F_{Pk} = \frac{F_{Pi}}{\left[1 + C_3\left(\frac{\delta}{\delta_{ini} - \delta}\right)^{C_4}\right]}$$



***δ*:** the distance of the stress state under consideration from the corresponding bound

δ

δ_{ini}: the value of ***δ*** at the initiation of yielding for each loading process

When it is kinematic hardening, plastic flexibility F_{Pk} is between F_{pi} ($\delta = 0, E_{Pk} = E_{Pi}$) and 0 ($\delta = \delta_{ini}, E_{Pk} = E_E$), C_3 and C_4 factors control the rate of stiffness reduction, the larger the factors, F_{Pk} converges to F_{pi} slower.

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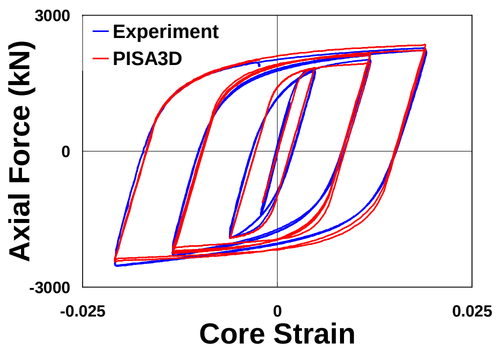
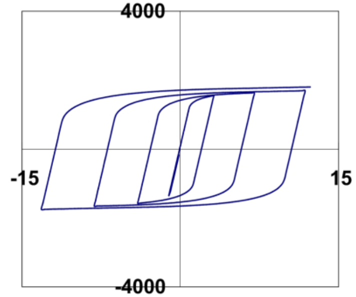
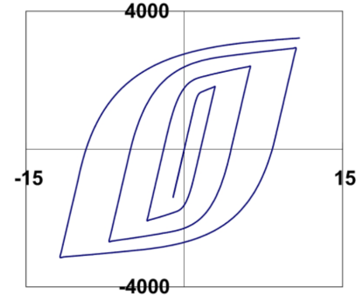
Command: Material, Hardening (Manual: F03)

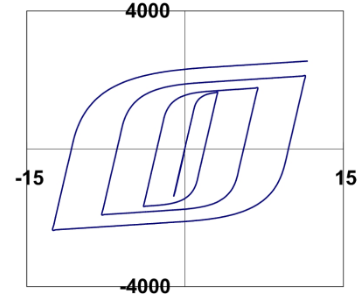




(a)



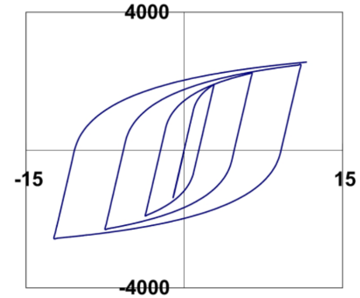
(b)



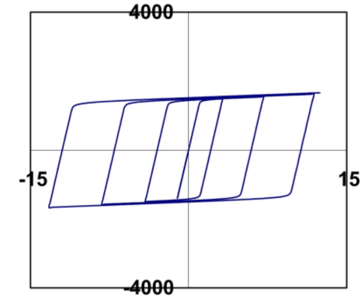
(c)

$$F_{Pi} = \frac{F_E}{\left[C_1 + (1 - C_1) e^{\left(-\frac{BS}{YS} C_2 \right)} \right]}$$

$$F_{Pk} = \frac{F_{Pi}}{\left[1 + C_3 \left(\frac{\delta}{\delta_{ini} - \delta} \right)^{C_4} \right]}$$



(d)



(e)

				C_1^+	C_2^+	C_1^-	C_2^-	C_3	C_4	
Case	E_i	f_{yp}	f_{yc}	H_{iso1+}	H_{iso2+}	H_{iso1-}	H_{iso2-}	H_{kin1}	H_{kin2}	BS/YS
(a)	1320	1250	-1250	0.005	5.0	0.005	5.0	1.0	24	1.3
(b)	1320	1250	-1250	0.005	1.7	0.005	1.7	1.0	24	1.3
(c)	1320	1250	-1250	0.015	5.0	0.015	5.0	1.0	24	1.3
(d)	1320	1250	-1250	0.005	5.0	0.005	5.0	1.0	24	2.3
(e)	1320	1250	-1250	0.005	5.0	0.005	5.0	5.0	24	1.3