National Taiwan University **Finite Element Method**

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Homework 6, 04/25/2018 Due: 05/02/2018

A4 professional format, collecting at the BEGINNING of class (09:09 am)

(late submission within 24 hours: score*0.9; late submission before post of solution: score*0.8

(the solution will be posted usually within a week))

Total 90%

1. **(30%)** The plane stress and plane strain constitutive matrix $\mathbf{D}$ can be deduced from the isotropic Hooke's law in 3D. Use the 3D constitutive matrix given in the note and derive the constitutive matrix $\mathbf{D}$ for the plane stress and plane strain problems.

2. **(20%)** A state of plane stress is one where the out-of-plane components of the stresses are zero, $\sigma_{zz}=0,\ \sigma_{zx}=0,\ \sigma_{zy}=0$. Show that for this case if

$$\sigma_{xx}=\frac{\partial^2\phi}{\partial y^2},\quad \sigma_{yy}=\frac{\partial^2\phi}{\partial x^2},\quad \sigma_{xy}=-\frac{\partial^2\phi}{\partial x\partial y}$$

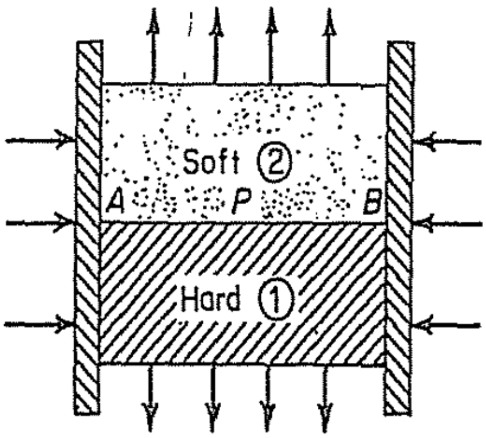
where $\phi(x,y)$ is the Airy stress function, an unknown function to be determined, then the static equilibrium equations are satisfied identically in the absence of body forces.

3. **(20%)** The components of the stress at a certain point of a continuous medium are given by

$$\begin{bmatrix}\sigma_{xx} & \sigma_{xy} & \sigma_{xz}\\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz}\\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz}\end{bmatrix}=\begin{bmatrix}200 & 400 & 300\\ 400 & 0 & 0\\ 300 & 0 & -100\end{bmatrix}\text{psi}$$

Determine the stress vector **t** and its normal and tangential components at the point on the plane $\phi(x,y)\Rightarrow x+2y+2z=\text{constant}$, passing through the point.

4. **(20%)** Consider a thin body composed of a hard material (high elastic modulus) joined to a soft material (low elastic modulus), as shown in the Figure below. Let the body be compressed between two walls. Both the soft material and the hard material will be stressed. At a point P on the interface AB, show that $\sigma_{xx}^{(1)}\neq\sigma_{xx}^{(2)}$, $\sigma_{yy}^{(1)}=\sigma_{yy}^{(2)}$ and $\sigma_{xy}^{(1)}=\sigma_{xy}^{(2)}$.


Soft 2
A
P
B
Hard 1