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Identification of Modal Combinations for Nonlinear Static Analysis of Building Structures

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Abstract: *An essential requisite in performance-based seismic design is the estimation of inelastic deformation demands in structural members. An increasingly popular analytical method to establish these demand values is a "pushover" analysis in which a model of the building structure is subjected to an invariant distribution of lateral forces. Although such an approach takes into consideration the redistribution of forces following yielding of sections, it does not incorporate the effects of varying dynamic characteristics during the inelastic response. Simple modal combination schemes are investigated in this article to indirectly account for higher mode effects. Because the modes that contribute to deformations may be different from the modes that contribute to forces, it is necessary to identify unique modal combinations that provide reliable estimates of both force and deformation demands. The proposed procedure is applied to typical moment frame buildings to assess the effectiveness of the methodology. It is shown that the envelope of demands obtained from a series of nonlinear static analysis using the proposed modal-combination-based lateral load patterns results in better estimation of inter-story drift, a critical parameter in seismic evaluation and design.*

1 BACKGROUND AND INTRODUCTION

Modern seismic design is a force-based procedure. However, there is general consensus both in the engineering and the research community that force-based design procedures do not account for force redistribution in elements following yielding, and they do not take into consideration the influence of changing dynamic characteristics of the system, particularly those arising from the contribution of higher modes. Because structural damage is directly related to local deformations, a more rational approach for seismic evaluation should be based on inelastic displacements rather than elastic forces and several articles on the subject of displacement-based seismic design can be found in the literature (Bertero et al., 1991; Nassar and Krawinkler, 1991; Moehle, 1992; Priestley and Calvi, 1997; Aschhiem and Black, 2000; Chopra and Goel, 2002).

Advances in displacement-based seismic design have, in many ways, contributed to the progress of performance-based design. The emergence of FEMA-356 (2000) has now laid the foundation for the development of future performance-based seismic codes. Inherent in FEMA-356 is the assumption that a nonlinear static analysis is more reliable than a linear static procedure, and that a regular building with a dominant first mode response in the elastic state is generally not influenced by higher modes. A nonlinear static procedure (NSP) incorporates nonlinear material characteristics in representing the force–displacement response of the structure. A mathematical model of the building, that includes all significant lateral force-resisting elements, is subjected to a monotonically increasing "invariant" lateral force pattern until a predetermined target displacement is reached or the building is on the verge of incipient collapse. The stiffness of elements is revised as yielding occurs. The capacity curve of the structure is determined in terms of base shear versus control node displacement. The internal forces and deformations in the structural elements are then evaluated. FEMA-356 recommends using at least two lateral load patterns that approximately bound the likely distribution of the inertia forces: a uniform load pattern or a modal load pattern. The modal load pattern is an approximate representation of the inertial forces in the elastic range, whereas the uniform load pattern represents their likely distribution

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in the inelastic range assuming that the structure has formed a soft story. Using this basic procedure, numerous methods have been developed in an effort to minimize the errors and approximations of using a static analysis to reproduce a response similar to a dynamic analysis while maintaining its simplicity. Obviously, a static procedure cannot account for inertia effects, damping, ground motion characteristics, and effects of system degradation.

1.1 Overview of current pushover procedures

The term "pushover" analysis of structures is a modern variation of the well-known "collapse" analysis that is based on classical plastic analysis of frame structures. However, unlike classical plastic analysis where ultimate strength (typically for gravity loads) is of main interest, pushover analysis aims at characterizing the lateral strength as well as local deformations in the structure. The concept gained prominence after its introduction in the *capacity spectrum method* (CSM) by Freeman (1978). CSM uses a pushover analysis to establish the base shear versus control node displacement and then converts these quantities into spectral acceleration and spectral displacement that is plotted in acceleration-displacement response spectrum (ADRS) format. Sasaki et al. (1998) extend the basic CSM approach to account for higher modes. Load patterns based on higher modes are used to generate a series of pushover curves. The following relationships provide the conversion to ADRS format:

$$S_{a,n} = \alpha_n (V_n / W) \text{ expressed in units of } g \tag{1}$$

$$S_{d,n} = \frac{\Delta_{c,n}}{\beta_n \Phi_{c,n}} \tag{2}$$

$$\alpha_n = \frac{\sum_{i=1}^{N} w_i \Phi_{i,n}^2}{\left(\sum_{i=1}^{N} w_i \Phi_{i,n}\right)^2} \tag{3}$$

$$\beta_n = \frac{\sum_{i=1}^{N} w_i \Phi_{i,n}}{\sum_{i=1}^{N} w_i \Phi_{i,n}^2} \tag{4}$$

where

$S_{a,n}$ is the spectral acceleration for mode n,
$S_{d,n}$ is the spectral displacement for mode n,
V_n is the base shear for mode n,
W is the seismic weight of building,
w_i is the seismic weight of floor at level i,
$\Phi_{i,n}$ is the modal amplitude at level i for mode n, and
$\Delta_{c,n}$,$\Phi_{c,n}$ are displacement and modal amplitude of control node for mode n, respectively.

The next step in CSM is to convert the standard response spectrum (which represents the demand side of the equation) into ADRS format as well, thereby permitting a comparison of demand versus capacity. A contentious issue in CSM is the representation of damping when developing the demand curve to account for inelastic effects. When the structure responds inelastically under the action of seismic forces, the input energy is dissipated by viscous damping and yielding of the structure. The net effect of yielding is to efficiently increase the overall damping of the system. This total damping is termed equivalent viscous damping. The determination of the damping due to yielding of the system is an iterative process because it involves computation of energy dissipated by damping and the strain energy stored in the system.

The *displacement coefficient method* (DCM) has been adopted by NEHRP in their prestandard for seismic rehabilitation of buildings (FEMA-356, 2000) as the preferred method to determine the expected maximum displacement (or target displacement) for the nonlinear static analysis procedure. The target displacement (δ_t) is computed by modifying the spectral displacement (Equation (2)) of an equivalent SDOF system as follows:

$$\delta_t = C_o C_1 C_2 C_3 S_a \frac{T_e^2}{4\pi^2} g \tag{5}$$

The factors C_o, C_1, C_2, and C_3 are modification factors that account for spectral displacement, inelasticity, hysteresis shape, and P-Δ effects, respectively. T_e is the effective fundamental period computed at a specified secant stiffness of the system and T_i is the elastic fundamental period. Because the nonlinear static response of the structure is extremely sensitive to choice of load pattern, FEMA-356 recommends using at least two load patterns that approximately bound the distribution of the inertia forces along the building height in a seismic event. The first is a profile based on lateral forces that are proportional to the total mass at each level called the uniform pattern. The second pattern can be a triangular one that is dependent on the fundamental period of the structure in the direction under consideration or a pattern resulting from a modal combination using SRSS or CQC modal response combination.

The inability of regular pushover methods to identify possible failure mechanisms due to higher mode effects in structures with large periods has led engineers to look for other methods that minimize this shortcoming. The multi-mode pushover (MMP) described earlier (Sasaki et al., 1998) is an attempt to introduce higher modes by considering multiple pushover curves derived from different modal force patterns. The *adaptive pushover method* (APM) developed by Gupta and Kunnath (2000) uses a load pattern that changes depending on the instantaneous dynamic properties of the system and considers as many modes as necessary to capture significant higher-mode effects. An elastic response spectrum for the site-specific ground motion to be used is established so that the modal forces can be determined for various steps. Story forces, as described in Equation (6), are then computed following an eigenvalue analysis of the model.

$$F_{ij} = \Gamma_j \phi_{ij} W_i S_a(j) \tag{6}$$

The modal participation factor for the jth mode is $\Gamma_j = \sum_{i=1}^{i=N} m_i \phi_{ij}$, ϕ_{ij} is the mass-normalized ($\Phi^T M \Phi = 1$) mode shape value at level i for mode j, m_i is the mass of story level i, and N is the number of stories. A static analysis is then performed using story forces corresponding to each mode independently, resulting in push-and-pull forces at different levels. The incremental element forces, deformations, and story drifts are then computed by SRSS combination of the respective modal quantities for the current step. The accumulated member forces are compared with their respective yield values at every step. If element yielding is detected, the member and global stiffness matrices are updated and a new eigenvalue analysis is carried out. This process continues until the specified target displacement is reached.

A number of alternative methods for pushover analysis have also been investigated. These include methods where deformation levels and/or the stiffness state determine the load pattern. Fajfar and Fischinger (1988) suggested using story forces proportional to the deflected shape of the structure. Eberhard and Sozen (1993) used force patterns based on mode shapes derived from the secant stiffness at each load step and were able to demonstrate the effectiveness of the method for shear-wall structures. An enhanced capacity-spectrum-based approach, wherein demands and capacities are estimated at the story level as opposed to overall base shear versus top story displacement, was investigated by Bracci et al. (1997). Other variations of these approaches can be found in published articles by Saiidi and Sozen (1981), Qi and Moehle (1991), Biddah et al. (1995), and Kilar and Fajfar (1997).

An important fact that emerges from the above discussion is that there exists the need to predict expected deformations across the height of the structure using some reliable analytical procedure.

Given the fact that a pushover method is a static method which does not require the selection of ground motions and the modeling effort in building a computer model of a building is significantly less than for a time–history method, engineers are more comfortable with pushover procedures than nonlinear time–history methods. Given this appeal of pushover procedures, it is reasonable to assume that pushover analyses will become commonplace in seismic evaluation compared to time–history methods.

1.2 Unresolved issues in pushover methods

Pushover methods have been the subject of numerous studies as it found its way into performance-based evaluation documents such as FEMA-356 and ATC-40. However, the ability of a static procedure to predict dynamic response raises several significant questions that must be addressed before they can be used reliably in performance-based seismic evaluation. Kunnath and John (2000) identify several inconsistencies in the different pushover procedures currently used to estimate seismic demands. Iwan (1999) raises questions about the validity of pushover methods for pulse-like near-fault ground motions. The same study also suggests that restrictions need to be placed on the use of equivalent damping in CSMs. The most significant issue with a pushover analysis using an inverted triangular or uniform lateral load pattern is that it fails to account for certain critical higher mode contributions thereby underestimating drift demands in the mid and upper stories.

Kunnath and Gupta (2000) investigated the different lateral load patterns recommended in FEMA-356. In their study, a typical eight-story reinforced concrete office building was first subjected to a strong recorded ground motion. Details of the building and the ground motion used are described in Section 3 of the article. The resulting nonlinear time–history response was considered to be the benchmark solution. The same building model was then analyzed using a pushover analysis with all three lateral load patterns recommended in FEMA-356. The target displacement for the pushover analyses was the maximum computed roof displacement in the time–history evaluation. Figure 1 shows the peak

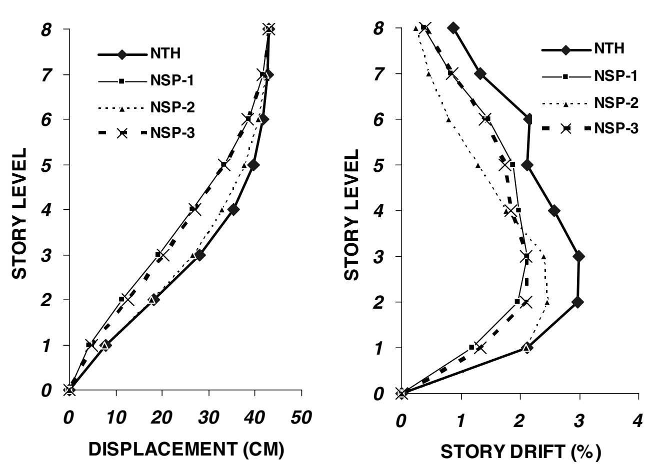

Fig. 1. Peak displacement and drift profile using different analytical methods (NTH: nonlinear time–history analysis; NSP-1: pushover analysis using inverted triangular load; NSP-2: uniform load; NSP-3: lateral loads derived from modal combination).

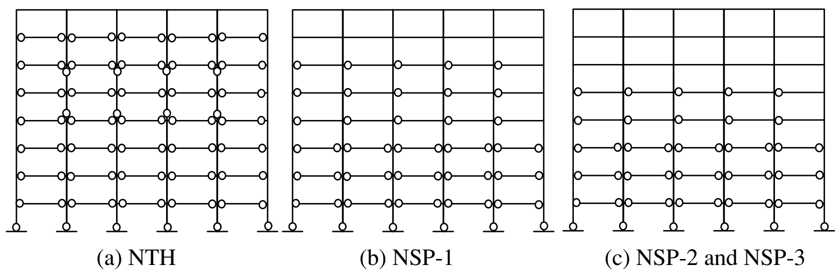

Fig. 2. Plastic hinge locations predicted using various analysis methods (circles represent plastic hinges).

displacement profile and the peak inter-story drift profile obtained with the different analyses. In this case, the peak displacements are generally well represented by the uniform load pattern (NSP-2). The remaining two patterns underestimate the displacements at almost all levels. The plot of peak inter-story drift, on the other hand, clearly highlights the inability of "all" nonlinear static methods to predict this critical deformation parameter. The importance of the inter-story drift demands in the upper stories is further demonstrated in Figure 2, which shows the predicted plastic hinges in the frame. The potential for the formation of a story mechanism in the upper stories can be overlooked with traditional pushover techniques.

An important consideration in evaluating a pushover method, therefore, is its ability to predict inter-story drifts rather than roof displacements. Consequently, the concept of a roof ductility factor is not meaningful in the design or assessment of structures because the controlling failure mechanism may be a local story mechanism. Based on these and similar findings from related studies, it appears that lateral load patterns such as those specified in FEMA-356 are adequate only for structures with a first-mode dominant response. Further, current FEMA guidelines to establish the limits of a pushover procedure are also flawed because they are based on a modal approximation in the undamaged, elastic state of the structure.

There is growing evidence that FEMA-356 prescribed pushover analyses are becoming commonplace in seismic evaluation. The introduction of nonlinear static analysis options in popular computer programs such as SAP2000 (Computers and Structures, Inc., 1998) has resulted in widespread use of pushover analyses without adequate attention being paid to the results obtained and their implications in design.

2 METHOD OF MODAL COMBINATIONS

Previous attempts to incorporate higher mode effects into a NSP have generally been based on the premise that the effects of each mode can be uncoupled. Modal demands are then combined to study the overall response of a building structure. An interesting alternative to combining modal contributions was used by Matsumori et al. (1999), wherein the combination is performed by numerically adding and subtracting the contribution of different modes. To better estimate maximum earthquake demands, they used two patterns of story shear distributions: the sum of two modal story shears and the difference of two modal story shears. In their study, they considered only the first and second modes. Their article suggests that the pushover analysis was carried out till the first mode displacement was equal to the maximum earthquake response and that separate pushover analyses were carried out for each mode. The application of the methodology on two building models produced good correlation with demands resulting from nonlinear time–history analyses.

2.1 Rationale for the methodology

The central idea behind the proposed modal combination scheme is best illustrated through an example. Returning to the example of the eight-story frame structure considered in Section 1.2, the difficulties in using static procedures to replace dynamic processes becomes obvious when examining a snapshot of the time–history response of inter-story drift for the building. Figure 3 displays the variation in inter-story drift for approximately

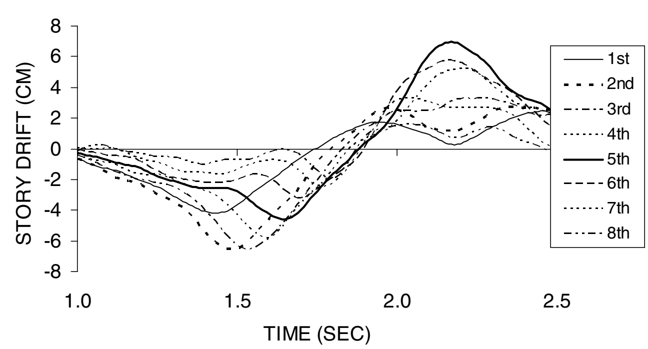

Fig. 3. Contributions of higher modes to inter-story drift response.

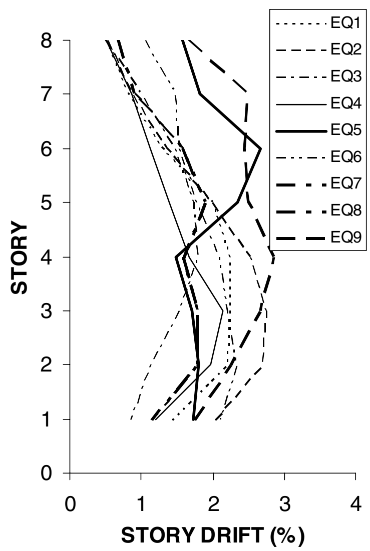

Fig. 4. Effect of ground motion characteristics on system response.

one full cycle of response. The peak inter-story drift in the first half-cycle occurs between the 2nd and 3rd levels, whereas in the second half cycle the peak response is observed between the 4th and 5th levels.

Another aspect of the dynamic behavior of structures is the influence of ground motion characteristics on the structural response. The different modes of the same eight-story structure subjected to different earthquakes respond differently; hence the modal demands at each story level vary from one earthquake to another. Figure 4, which presents the maximum inter-story deformation for nine different earthquakes, demonstrates further the significance of this statement. The records were scaled such that the spectral value at the fundamental period was the same in all cases. The peak inter-story drift at each story level is not necessarily caused by the same earthquake. For example, the demands imposed by the earthquake denoted by EQ5 on the upper levels are much higher than the demands imposed on the lower levels when compared to the remaining records. Although this points to the general need to include multiple ground motions when estimating seismic demands, a similar analogy can be made when assessing inelastic static methods which do not incorporate essential characteristics of the loading.

These observations indicate that two aspects of the dynamic response that must somehow be incorporated into a static procedure are: (a) consideration of multiple modes in estimating the lateral force pattern to be used in a pushover analysis and (b) consideration of ground motion characteristics in establishing these forces. More importantly, the findings also suggest that the modes should not be considered independently but in some appropriate combination, which reasonably represents the significant contributing modes to the final response.

2.2 Conceptual development

A clearer understanding of the process of modal combinations is afforded by reviewing methods of response spectrum analysis. The idea of separating the time-dependent component of the forcing function in the dynamic equation of motion is discussed in Clough and Penzien (1993). The method presented here is based on the procedure described in Chopra (2001). The governing equation of motion of a multi-degree-of-freedom system is given by

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\mathbf{m}\iota\ddot{u}_g \tag{7}$$

where $\mathbf{m}$, $\mathbf{c}$, $\mathbf{k}$ are mass, damping, and stiffness matrices, respectively, $\ddot{\mathbf{u}}$, $\dot{\mathbf{u}}$, $\mathbf{u}$ are acceleration, velocity, and displacement vectors, respectively, ι is a vector with unit values, and $\ddot{u}_g$ is the ground acceleration.

Applying the modal transformation $\mathbf{u} = \mathbf{\Phi q}$, Equation (7) can be recast into the following form:

$$\ddot{q}_n + 2\zeta_n\omega_n\dot{q}_n + \omega_n^2 q_n = -\Gamma_n\ddot{u}_g \tag{8}$$

where q is the generalized coordinate representing the modal amplitude, ς is the damping ratio, ω is the natural circular frequency, n is the mode number and

$$\Gamma_n = \frac{\mathbf{\Phi}^{\mathrm{T}}\mathbf{m}\iota}{\mathbf{\Phi}^{\mathrm{T}}\mathbf{m}\mathbf{\Phi}} \tag{9}$$

which is independent of the time-varying loading function. A more informative way of expressing the right hand side of Equation (7) is to consider independent modal contributions as presented by Chopra (2001):

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\sum_{n=1}^{N}\mathbf{R}_n\ddot{u}_g \tag{10}$$

Comparing Equation (10) to Equation (7) and following through the modal transformation that results in Equation (8) it can be shown that

$$\mathbf{R} = \sum \mathbf{R}_n = \Gamma_n\mathbf{m}\Phi_n \tag{11}$$

Each term in the above expansion contains the modal contribution of the respective mode. Another way of visualizing Equation (11) is to consider the load vector on the right-hand side of Equation (7) as follows:

$$\mathbf{m}\iota\ddot{u}_g = \mathbf{R}f(t) \tag{12}$$

where $\mathbf{R}$ is a load distribution vector. For a general loading function $\mathbf{p(t)} = \mathbf{r}f(t)$, the vector $\mathbf{r}$ represents a displacement transformation vector resulting from a unit support displacement. For earthquake loading, this simply becomes a vector with unit values. The external loading can obviously vary as a function of time both in terms of amplitude and spatial distribution. The objective of deriving an expression of the form given by Equation (12) is to separate the spatial distribution from the time-varying amplitude function.

The next step is to introduce features of the earthquake loading. Because the procedure being developed is a static one, the most appropriate form of earthquake loading that can be considered is a response spectrum. The spatial distribution of lateral forces to be used in conjunction with a pushover analysis is approximated in terms of the peak modal contributions, as follows:

$$f_n = \Gamma_n \mathbf{m} \Phi_n S_a(\zeta_n, T_n) \tag{13}$$

where S_a is the spectral acceleration for the given earthquake loading at a frequency corresponding to the period, T and damping ratio, ζ for mode n.

The modal forces computed using Equation (13) will represent the contributions to mode n only. In the approach proposed by Chopra and Goel (2002), it is assumed that the inelastic response can also be approximated by modal superposition because the nth mode is expected to be dominant, even for inelastic systems. The advantage with such an approach is that an inelastic response spectrum may then be used to estimate the peak inelastic displacements for each mode. The validity of the procedure, though illustrated for a single building and a single earthquake, needs to be demonstrated for a variety of structural configurations and varying ground motion characteristics.

The modal combination procedure involves identifying appropriate modes to include in the analysis and the manner in which the combination will be carried out. In general, the spatial variation of the applied forces will be computed from the following expression:

$$F_j = \sum_{m=n1,n2}^{nn} \alpha_m \Gamma_m \mathbf{m} \Phi_m S_a(\zeta_m, T_m) \tag{14}$$

where F_j is the lateral force to be applied at story level *j; m* represents the mode number, and $n1$, $n2$, and nn the initial mode, mode interval (step), and final modes to be considered in the summation. The factor α_m is a modification factor, which can be used to control the relative effects of each mode being included in the combination. A default value of positive or negative unity can be assigned to this factor though the response may be sensitive to this parameter if the mass participation of the mode is small but the spectral acceleration demand is significant for higher modes. The summation in Equation (14) can include as many modes as necessary to adequately represent critical modes and their contribution to the response. If the first three modes were being combined, the following combinations would be used:

$$\begin{aligned} F_j = {} & \alpha_1 \Gamma_1 \mathbf{m} \Phi_1 S_a(\zeta_1, T_1) \pm \alpha_2 \Gamma_2 \mathbf{m} \Phi_2 S_a(\zeta_2, T_2) \\ & \pm \alpha_3 \Gamma_3 \mathbf{m} \Phi_3 S_a(\zeta_3, T_3) \end{aligned} \tag{15}$$

The procedure, therefore, requires multiple pushover analyses, wherein a range of modal load patterns are applied. In each case, the pattern itself is invariant. To arrive at estimates of deformation and force demands, it is necessary to consider peak demands at each story level and then establish an envelope of demand values for use in performance-based evaluation.

Two questions that obviously arise from the above formulations are: precisely what modes should be included in the combination and how are the modification factors to be assigned? Preliminary studies carried out on structures ranging in height from 4 to 20 stories indicate that the number of modes to be included is a function of the height of the structure. While a single (fundamental) mode is adequate for low-rise structures, more modes need to be included for taller structures. No more than three modes were necessary to obtain a conservative envelope of the demand values in all cases considered. Sample results and pertinent observations from an ongoing study are reported here to illustrate the methodology and to highlight features of the procedure that will enhance our understanding of pushover analyses in general and modal combination techniques in particular.

3 STRUCTURAL MODELS AND GROUND MOTIONS

3.1 Building details

An eight-story and a 16-story reinforced concrete frame building were analyzed to investigate important features of the proposed modal combination procedure. The plan view for both buildings is shown in Figure 5. The building

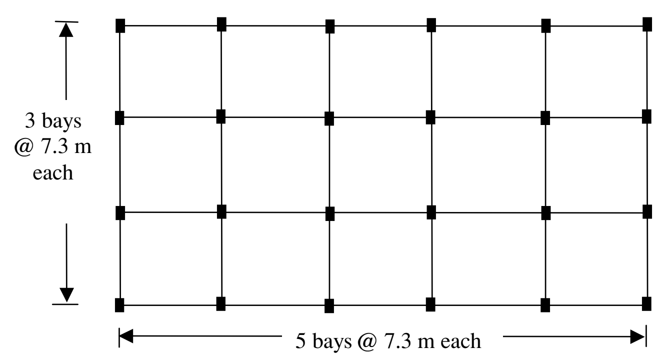

Fig. 5. Typical plan of prototype structure used in evaluation study.

Table 1
Details of members for various frames used in validation study.

No. of stories	*Levels*	*Column details*			*Beam details*			
		Size[1]	f'_c	*Steel*[2]	*Size*[1]	f'_c	*Top steel*[2]	*Bottom steel*[2]
8	5–8	45 × 60	27.6	12−#8	45 × 60	27.6	4−#8	2−#8
	1–4	45 × 75	34.5	14−#9	45 × 60	27.6	3−#9+2−#8	3−#8
16	13–16	60 × 45	27.6	10−#8	45 × 60	27.6	2−#8+3−#7	2−#7+1−#6
	9–12	60 × 45	27.6	12−#9	45 × 60	27.6	3−#9+2−#8	3−#8
	5–8	60 × 30	34.5	18−#9	45 × 60	27.6	5−#9	3−#9
	1–4	60 × 30	41.3	18−#10	45 × 60	27.6	2−#10+3−#9	3−#8+2−#7

[1]All dimensions are in cm; concrete stress in MPa.
[2]Number of bars/diameter: #6 = 19 mm; #7 = 22 mm; #8 = 25 mm; #9 = 29 mm; #10 = 32 mm.

is composed of three bays in the transverse direction and five in the longitudinal direction. A story height of 3.7 m was assumed for all levels. The seismic design of the buildings was based on the requirements of UBC (1988) with the following values: $R_W = 12$, seismic zone factor, $Z = 0.4$, and importance factor, $I = 1.0$. A live load of 2.4 kPa was assumed to act on each floor. Normal weight concrete with a *specified* compressive strength (f'_c) as shown in Table 1 was used in the design. The yield stress for all reinforcement was assumed as 414 MPa. Fixed-base conditions were assumed in the analysis and P-Δ effects were ignored. The latter assumption may be unreasonable for the 16-story building, however, the main purpose of this investigation is to study modal combinations for use in static analysis and P-Δ effects are expected to influence failure modes rather than the spatial distribution of lateral forces. The buildings were designed to satisfy the drift criterion and the strong column-weak beam criteria required by UBC. Table 1 presents member details for each building. Only a typical longitudinal frame was considered for detailed evaluation in the present study. The same set of frames was also utilized in an earlier study (Gupta and Kunnath, 2000), where the reader may find additional details on modeling assumptions. Table 2 displays essential dynamic properties for both frames which were computed at the initial elastic state of the system based on effective stiffness values. All analyses reported in this article were carried out using an updated version of IDASS (Kunnath, 1995).

3.2 Seismic loading

The pushover analyses presented in Section 4 of the article are compared to benchmark solutions obtained from nonlinear time–history analyses of the target buildings. Each building was subjected to a magnitude-scaled S48W component of the Rinaldi receiving station free field motion recorded during the 1994 Northridge earthquake. The original record used in the analysis was taken from the SAC database (Somerville et al., 1997) and is classified as an event with a 10% probability of exceedance in 50 years. The accelerogram and spectra of the free field motion as used in the study are displayed in Figure 6. The objective of selecting this record was to identify an earthquake that would induce higher mode response in the system and produce deformations in the inelastic range of each system.

4 EVALUATION OF THE METHODOLOGY

4.1 Analysis of building I

An eigenvalue analysis of the eight-story frame building is conducted in the initial state of the structure based on effective stiffness values described in Section 3.1. The eigenvalue problem is solved using a condensed stiffness matrix in which only the lateral floor degrees-of-freedom (dofs) are retained, hence reducing it to a problem with only eight dofs. Rather than examining the

Table 2
Dynamic properties of buildings

Building	*Mode 1*		*Mode 2*		*Mode 3*		*Mode 4*	
	T_1	*% mass*	T_2	*% mass*	T_3	*% mass*	T_4	*% mass*
8-story	1.75	81.8	0.60	10.6	0.34	3.5	0.23	2.0
16-story	2.82	77.9	1.01	10.7	0.58	4.0	0.40	2.2

Note: Building periods, T_i (seconds).

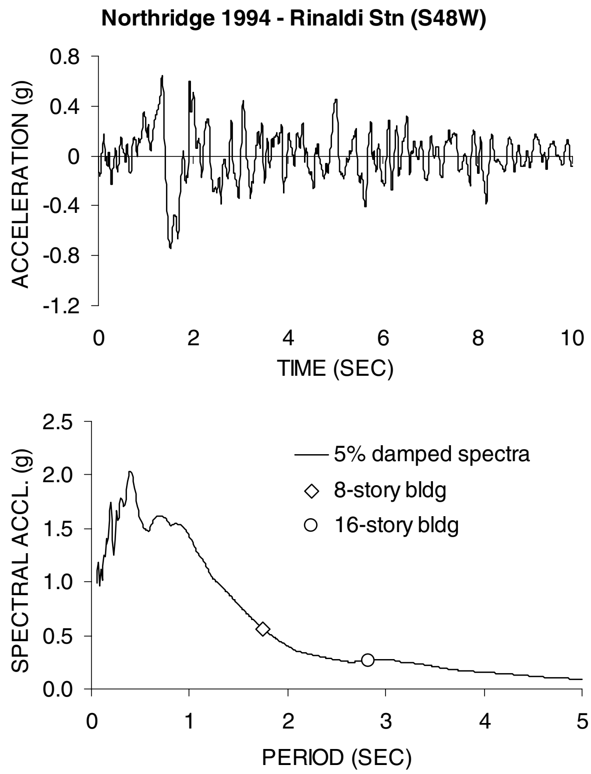

Fig. 6. Time history and spectrum of ground motion used in benchmark analysis.

resulting eigenvectors, the relative modal contributions (which is equivalent to the relative modal mass contribution) were compared. It was determined that the modal participation in the first two modes was 55.7 and 21.0%, respectively, thus contributing to almost 80% of the total response. Addition of the third mode would include about 90% of the modal contributions to the response. If Equation (13) is used to estimate story forces to be used in the lateral load analysis, the resulting spatial distribution is displayed in Figure 7. Only the first two modes are shown. Next, the proposed modal combination scheme is used. For the eight-story structure under consideration, only the first two modes are used. A closer observation of the modal forces shown in Figure 7 will reveal that a modal sum of the first two modes will amplify the demands at the lower levels and that a modal difference will amplify the demands at the top. Intuitively, therefore, the envelope of the two demands will provide a conservative estimate of the inter-story drift demands across the entire building height. Without resorting to the use of modification factors, the following combinations are utilized:

$$F_j = \Gamma_1 \mathbf{m} \Phi_1 S_a(\zeta_1, T_1) \pm \Gamma_2 \mathbf{m} \Phi_2 S_a(\zeta_2, T_2) \qquad (16)$$

The resulting modal forces to be used in the pushover analyses are shown in Figure 8.

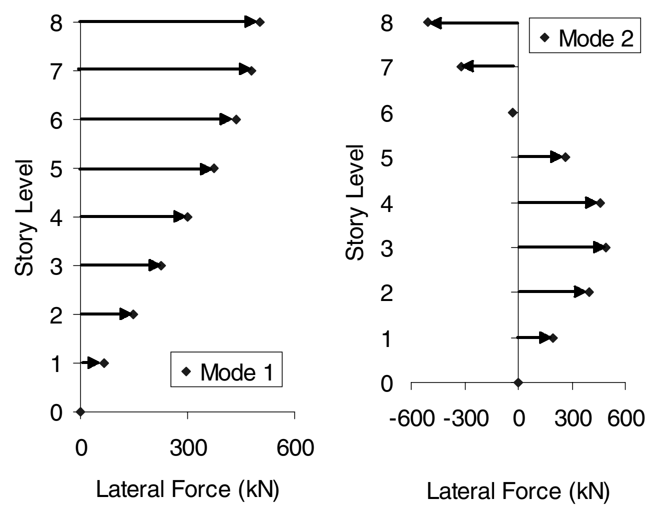

Fig. 7. Spatial distribution of lateral forces considering each mode independently.

Pushover analyses were carried out using the two modal patterns shown in Figure 7 independently. Next, the modal combinations shown in Figure 8 were used as the applied lateral forces. In each case, the lateral loads were incrementally applied (as a small fraction of the loads shown) until the roof displacement was the same as that obtained in the nonlinear time–history analysis. The resulting displacement profiles are shown in Figure 9. The displacement amplification at the top story levels is obvious in the second mode response for the first case and in the modal difference combination for the latter. The main problem with the responses obtained from the independent modal pushovers is the need to develop a procedure to combine the individual responses.

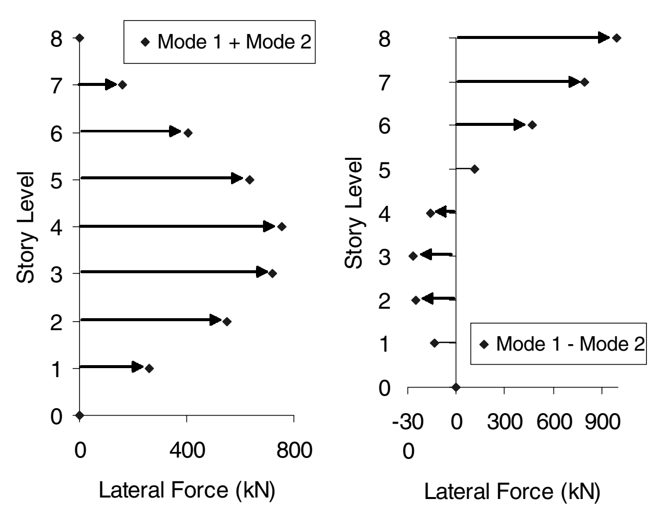

Fig. 8. Spatial distribution of lateral forces using modal combination.

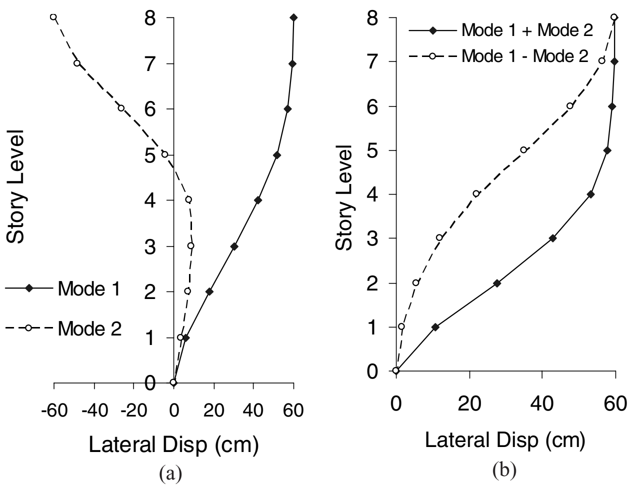

Fig. 9. Displacement profile obtained with (a) individual modal pushover analyses, (b) displacement profile obtained with modal combinations.

Figure 10 displays the peak drift envelopes for each case. When the envelope of the two independent modal pushovers is considered, the drifts at the upper levels are grossly overestimated and the drifts at the lower levels are somewhat underestimated. If the results of the independent analyses are combined using ratios based on modal contributions, then the drifts at the lower levels would be underestimated. The envelope based on the proposed modal combination scheme, however, provides a more reasonable estimate of the peak inter-story drift at all levels compared to nonlinear time–history analysis.

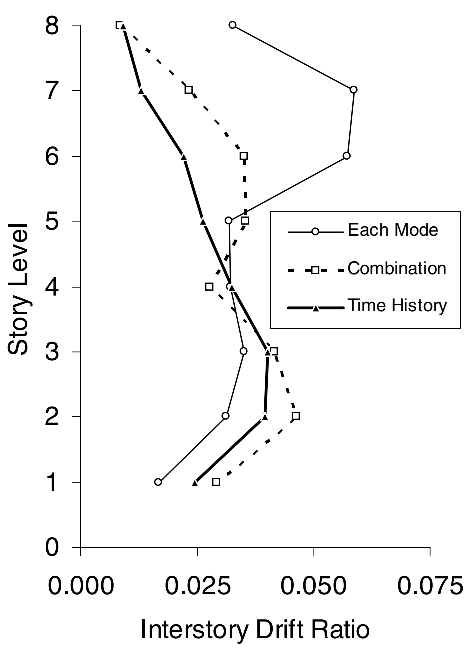

Fig. 10. Envelope of peak drift estimates for different approaches.

4.2 Analysis of building II

The previous example considered a building of medium height where only two modes were adequate to obtain an acceptable envelope of the displacement and drift responses. Next, the modal combination procedure is applied to a taller structure. As before, only two modes will be considered in developing the lateral forces. The computed modal forces will be applied independently to obtain individual modal responses followed by a series of lateral load analyses using the proposed combination schemes. The benchmark solution against which the pushover procedures will be compared is a nonlinear time–history analysis. The earthquake record described in Section 3.2 is used to establish the target displacements for the nonlinear static analyses. Figure 11 displays both the individual modal forces and the combined forces to be used in separate pushover analyses.

The resulting displacements are shown in Figure 12. The problem with using independent modes in a pushover procedure is defining an appropriate target displacement. Hence, two options are used when using lateral forces computed from Mode 2 contributions: the first involves pushing the structure until the roof displacement matches the time–history value (Mode 2a), and the second option considers any story level reaching this target value (Mode 2b). The different displacement profiles provide a preview of the inter-story drifts for the different pushover methods. The independent first mode displacements and the modal combination consisting of the sum of the first two modes both provide a good estimate of floor displacements.

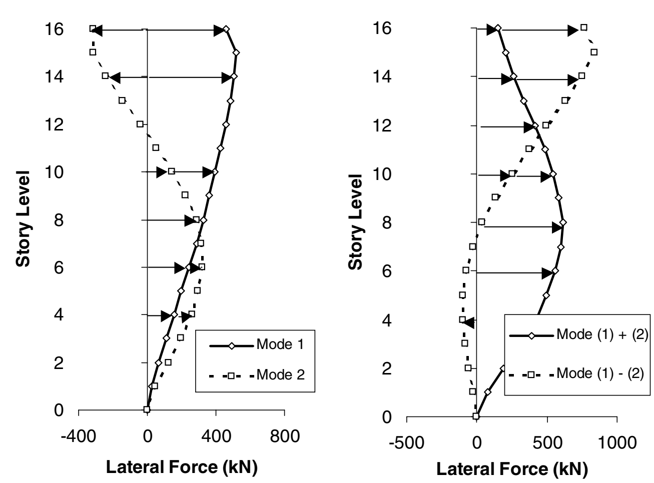

Fig. 11. Lateral forces used in pushover procedures.

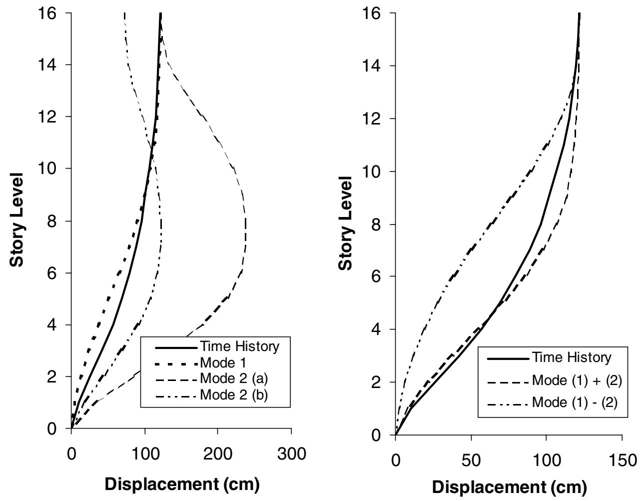

Fig. 12. Comparison of displacement profile using different methods.

Displacement estimates resulting from the pushover analysis using second mode forces alone overestimate displacements at the lower levels and underestimate them at upper levels. Results of the modal combination consisting of the difference of modes generally underestimate the floor displacements at all levels. A comparison of the maximum inter-story drifts using the different approaches (Figure 13) identifies several shortcomings in all methods.

The use of independent modes to estimate lateral forces can make the task of interpreting the response a challenging task. Drifts are grossly overestimated at most levels. Even if the modal contribution of the second mode is incorporated when estimating the response, the resulting drifts will provide a poor measure of the actual values. The proposed modal combination procedure provides a more reasonable estimate of the drifts though the estimates at the upper levels require improvement. Recall that only the first two modal forces were used in establishing the pushover forces. It is clear that additional modes must be included or that modification factors may be needed to better approximate the contribution of higher modes. Rather than using random combinations that improve the drift estimates, a systematic approach needs to be developed so as to identify features of the different modes that are critical to the response. The basis for such a method is explored in the next section.

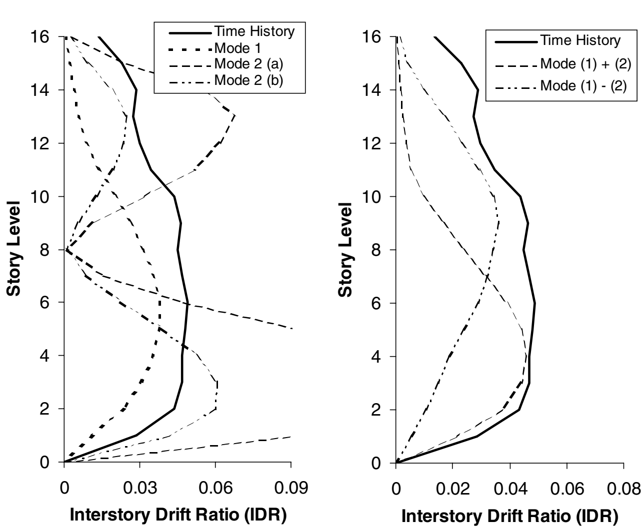

Fig. 13. Comparison of inter-story drift ratios.

4.2.1 Advanced combination schemes. The most critical aspect of structural response that needs to be understood if static procedures are to be used to predict a dynamic process is the mechanics of inter-story drift and the factors that influence its magnitude. The next two figures attempt to examine some of the features of inter-story drift. Figure 14 presents the variation of inter-story drift at each level throughout the time–history response for the eight-story structure. Figure 15 does the same for the 16-story building. The peak drifts in the lower levels occur at approximately the same time during the same cycle of response. This cycle also leads to the first inelastic excursion in the system. Higher modes do not seem to play a significant role early in the response. The drift ratios at the upper levels happen later in the response and suggest the role of inelastic behavior in modifying the modal contributions of the higher modes. A similar phenomenon is observed in the response of the 16-story building (Figure 15). This implies that a modal combination resulting from an eigenvalue analysis of the structure in the elastic state is probably valid only for predicting the drifts at the lower levels. To properly incorporate higher mode effects, the distribution of modal forces in the inelastic state of the system is needed. To facilitate a better understanding of the variation of modal contributions, eigenvalue solutions were obtained at several discrete steps throughout the time–history analysis for both buildings. The duration of the record used in the time–history analyses is 12 seconds (which represents the strong motion component only). To generate Figure 16, an eigenvalue analysis was carried out at intervals of 0.2 seconds providing snapshots of the modal period shifts at 60 discrete points. The findings (Figure 16) are quite illuminating. Only the first six modes of vibration are analyzed. The overall system characteristics can be defined in two states: an initial elastic response and a post-yield behavior. The spikes in the modal periods represent plastic behavior at relatively large deformations (similar to the post-yield behavior in a pushover analysis).

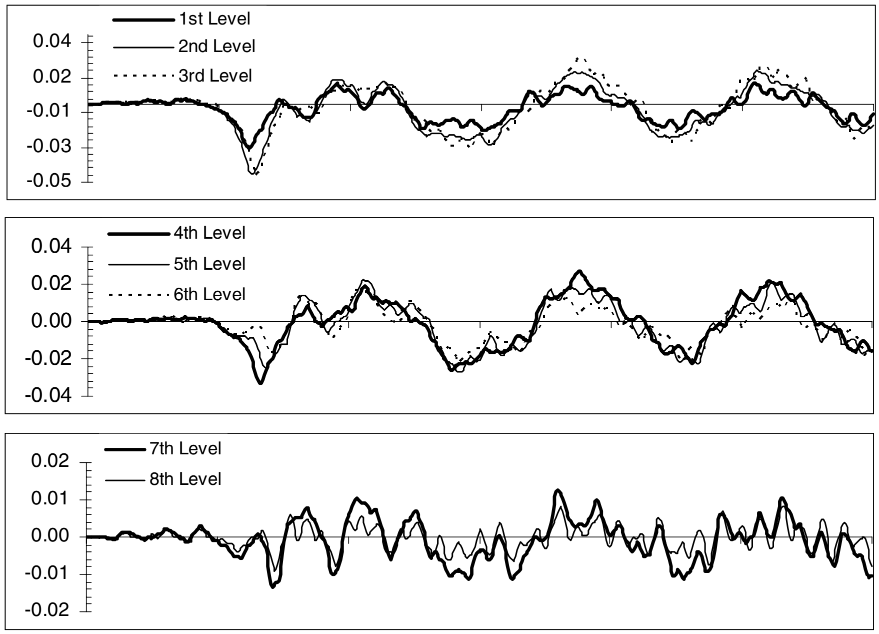

Fig. 14. Variation of inter-story drift ratio during earthquake response.

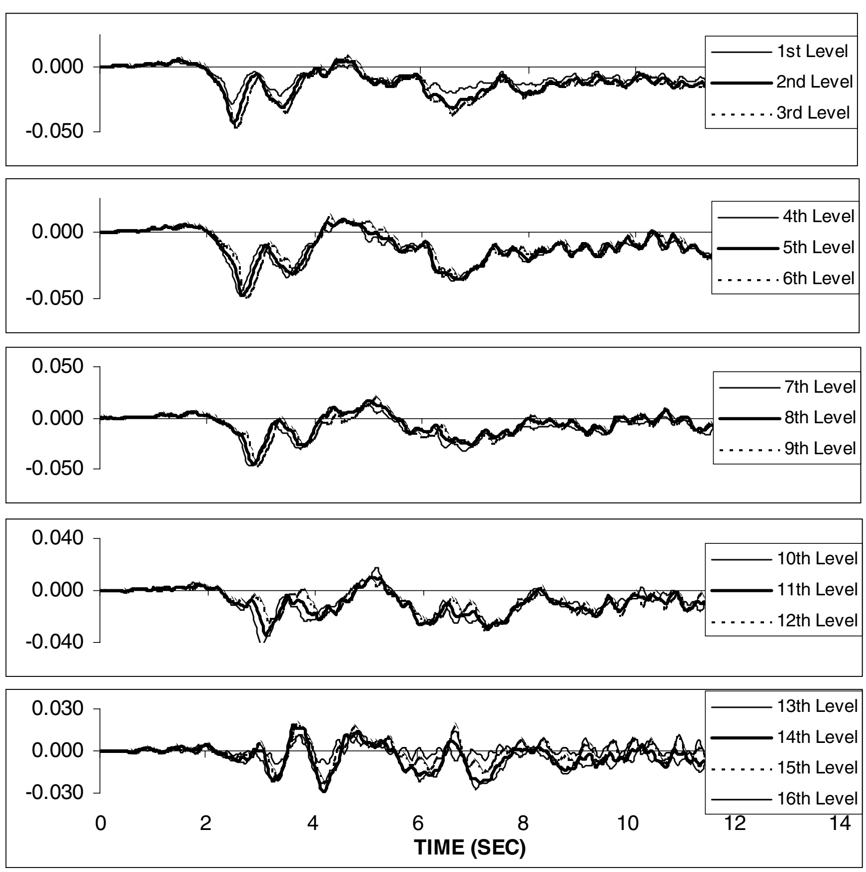

Fig. 15. Variation of inter-story drift ratio for 16-story building.

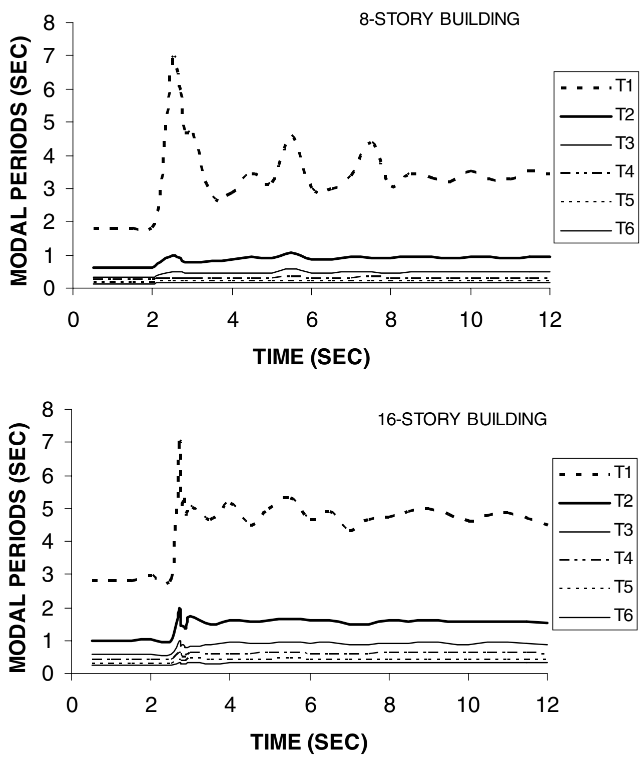

Fig. 16. Shifts in the first six modal periods during earthquake response.

However, earthquake response is characterized by cyclic action, which returns the system to a more stable behavior. Clearly, the variation in dynamic characteristics is a function of the material and hysteretic models used in the analysis. A more complex model with multiple paths will result in significantly more variations. In the present study, the multi-parameter hysteretic model (Figure 17) in IDASS was used. This model uses several control parameters to establish the rules under which inelastic loading reversals take place. For example, α, which can be expressed as a function of the deformation, controls the amount of stiffness loss; ϕ and χ control the initiation and degree of pinching; and the slope **s** and the change in expected peak strength (M to M^*) controls the softening due to system deterioration. For the analysis of the buildings in this study, the following parameters were used: $\alpha = 0.5$ (nominal degradation); $\Phi = 0$; $\chi = 1.0$ (no pinching); and an energy-based strength deterioration parameter $\beta = 0.01$ (see IDASS manual, Kunnath, 1995). Hence, the inelastic behavior model used in the present analysis is reasonably complex and the frequency variation in the different modes shown in Figure 16 is representative of the changes in the dynamic characteristics of the building as it responds inelastically. It must be pointed out, however, that eigenvalue samples were taken only at discrete time intervals of 0.25 seconds.

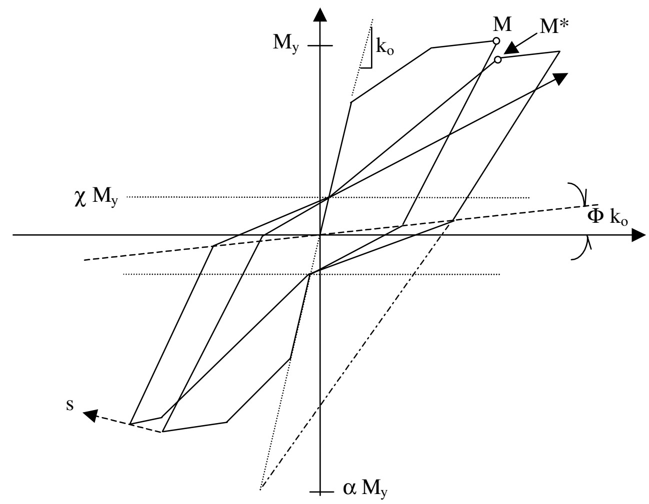

Fig. 17. Hysteresis model used in nonlinear time–history analysis.

Figure 16 suggests that a snapshot of the lateral force distribution at two stages of the analysis should be adequate to capture the participation of the different modes. An additional factor to keep in mind is the earthquake demands at these modal periods. The fundamental (first mode) period experiences a significant change and a corresponding decrease in seismic demands, whereas the higher modal periods are not altered to the same extent. This leads to the conclusion that when considering higher

Table 3
Modal combinations used to develop drift profile shown in Figure 18

Story level	*Time history IDR*	*Modal combination envelope value*	*Modal combination case*
16	0.0138	0.032	α_1M1 + M2
15	0.0229	0.044	α_1M1 + M2
14	0.0288	0.049	α_1M1 + M2
13	0.0272	0.049	α_1M1 – M2
12	0.0298	0.047	α_1M1 − M2
11	0.0346	0.042	α_1M1 − M2
10	0.0438	0.034	M1 − M2
9	0.0464	0.043	α_1M1 + α_2M2 + α_3M3
8	0.0450	0.048	α_1M1 + α_2M2 + α_3M3
7	0.0469	0.044	α_1M1 + α_2M2 + α_3M3
6	0.0488	0.039	α_1M1 − α_2M2 + α_3M3
5	0.0481	0.047	α_1M1 − α_2M2 + α_3M3
4	0.0467	0.049	α_1M1 − α_2M2 + α_3M3
3	0.0469	0.049	α_1M1 – M3
2	0.0435	0.049	α_1M1 + α_2M2 − α_3M3
1	0.0286	0.034	α_1M1 + α_2M2 − α_3M3

Note: M1 = Mode 1; M2 = Mode 2; M3 = Mode 3; $\alpha_1 = 0.08$; $\alpha_2 = 0.46$; and $\alpha_3 = 0.46$.

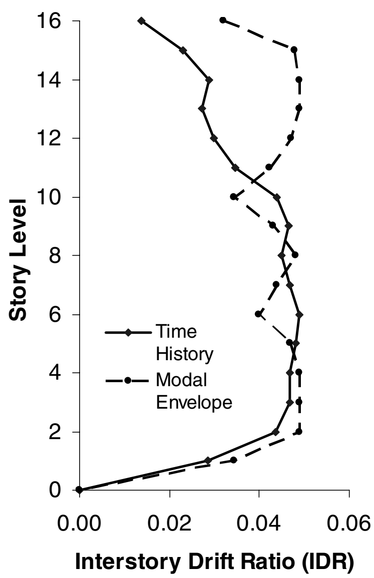

Fig. 18. Improved simulation of inter-story drift using combinations in first three modes.

modes, the combination should either exclude the first mode or its contribution should be reduced through the use of the modification factor with $\alpha_m < 1$.

To illustrate this concept, consider once again the 16-story structure. Several pushover analyses of the structural model were carried out using the following combinations:

- Mode 1 $\pm$ Mode 2 $\rightarrow$ 2 runs
- Mode 2 $\pm$ Mode 3 $\rightarrow$ 2 runs
- $\alpha_1 \times$ Mode 1 $\pm$ Mode 2 or ($\pm$ Mode 3) $\rightarrow$ 4 runs
- $\alpha_1 \times$ Mode 1 $\pm \alpha_2 \times$ Mode 2 $\pm \alpha_3 \times$ Mode 3 $\rightarrow$ 4 runs.

The modification factors were obtained by dividing the spectral acceleration demands (Figure 6) at the respective modal periods in the initial elastic state of the system. Though the relative contributions between the modes seem unaffected (on the average) in the elastic and inelastic states, the actual demands may be different. Table 3 provides a listing of the different modal combinations that provided the peak story drift for a given level. The modification factors used are also given in the table footnote. The resulting envelope of drift demands from each of the above runs is compared in Figure 18 with time–history estimates. A significant improvement in the predictions is observed.

5 CONCLUDING REMARKS

The procedure described in this article is based on ongoing research that seeks to examine the validity and reliability of static procedures to reproduce dynamic response information. One of the main concepts that this article attempts to convey is that static procedures are limited in their ability to reproduce dynamic behavior. Hence, there are no absolute alternatives to nonlinear time–history analyses. However, given the state of practice in seismic engineering, there is reason to believe that pushover procedures will become commonplace and that there is a growing misconception that pushover procedures are accurate and superior to traditional force-based design. As long as static methods continue to be used in engineering practice, there exists the need to improve upon current methodologies. The issues identified in this article and the alternatives offered are meant to contribute to the ongoing effort to enhance pushover analyses for performance-based seismic evaluation of structures. The proposed methodology avoids the complexity of adaptive methods by using an invariant load pattern. The proposed approach does not require a special computer program and can be used in any existing nonlinear software because the modal forces can be computed separately and supplied as the input lateral forces. Compared to the Chopra–Goel procedure, the proposed method requires significantly less effort. However, no comparative studies between different methods have been conducted.

The spectral accelerations to be used in the expressions presented in Equations (14) and (15) are based on estimates of the modal periods at a given state of the structure. As indicated in the previous section, one set of modal periods can be established at the initial elastic state of the system and additional sets may be determined at other inelastic states during the pushover procedure. Because higher modes become more dominant as the inelasticity in the system increases, it is reasonable to consider modes and spectral values of the higher modes at different states. Because the objective of a pushover analysis is to avoid resorting to time–history procedures, the question of identifying inelastic modes becomes a central issue. One alternative is to sample the eigenvectors at different states of the pushover procedure and to use response spectrum techniques outlined in Section 2.2 to combine modal contributions. There are obviously numerous other alternatives to identifying and combining modal load patterns, however, a rational basis for arriving at the combination scheme is needed if the procedure is to find application in routine seismic evaluation studies. Finally, it is necessary to point out that the proposed method also provides a good representation of story shears across the height of the building. However, the focus of this study has been on deformations rather than forces. Because peak inter-story drifts are proportional to shears, it is not surprising that the effectiveness of the method to predict inter-story shear is comparable to the accuracy in predicting peak drifts.

ACKNOWLEDGMENTS

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