In [2]:
```
%pip install python-igraph

import networkx as nx
import numpy as np
import matplotlib.pyplot as plt
import copy

def compute_metrics_after_deletion(G, f, deletion_strategy='random'):
    """
    Deletes a fraction f of nodes from graph G according to the specified

    Parameters:
    - G: Input graph (NetworkX Graph).
    - f: Fraction of nodes to remove (0 <= f <= 1).
    - deletion_strategy: 'random' or 'targeted' (targeted: remove highest

    Returns:
    - L: Characteristic path length (on the largest connected component).
    - S: Size of the giant component (fraction of original nodes).
    """
    G_del = G.copy()
    num_nodes = G_del.number_of_nodes()
    num_remove = int(f * num_nodes)

    if deletion_strategy == 'random':
        nodes_to_remove = np.random.choice(list(G_del.nodes()), size=num_
    elif deletion_strategy == 'targeted':
        # Sort nodes by degree (highest first) and remove the top fractio
        sorted_nodes = sorted(G_del.degree, key=lambda x: x[1], reverse=T
        nodes_to_remove = [node for node, deg in sorted_nodes[:num_remove
    else:
        raise ValueError("Unknown deletion strategy.")

    G_del.remove_nodes_from(nodes_to_remove)

    # Compute size of giant component (S)
    if len(G_del) == 0:
        S = 0
        L = np.nan
    else:
        giant_cc = max(nx.connected_components(G_del), key=len)
        S = len(giant_cc) / num_nodes
        # Compute characteristic path length on giant component
        subgraph = G_del.subgraph(giant_cc)
        if subgraph.number_of_nodes() > 1:
            L = nx.average_shortest_path_length(subgraph)
        else:
            L = 0
    return L, S

# (a) Random node deletion on a BA network
n_ba = 1000
m_ba = 3
G_ba = nx.barabasi_albert_graph(n_ba, m_ba, seed=42)
fractions = np.linspace(0, 0.8, 17)  # fractions from 0 to 0.8
L_vals = []
S_vals = []
```

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for f in fractions:
    L, S = compute_metrics_after_deletion(G_ba, f, deletion_strategy='ran
    L_vals.append(L)
    S_vals.append(S)

plt.figure(figsize=(10, 4))
plt.subplot(1, 2, 1)
plt.plot(fractions, S_vals, marker='o')
plt.xlabel('Fraction of nodes removed (f)')
plt.ylabel('Giant Component Size (S)')
plt.title('Random Deletion: Giant Component Size')
plt.grid(True)

plt.subplot(1, 2, 2)
plt.plot(fractions, L_vals, marker='s', color='r')
plt.xlabel('Fraction of nodes removed (f)')
plt.ylabel('Characteristic Path Length (L)')
plt.title('Random Deletion: Characteristic Path Length')
plt.grid(True)
plt.tight_layout()
plt.show()
# Completing part (b) by explicitly plotting characteristic path length (

import networkx as nx
import numpy as np
import matplotlib.pyplot as plt

# Network parameters
n_net = 1000
m_ba = 3
p_er = 0.01

# Create ER and BA networks
G_er = nx.erdos_renyi_graph(n_net, p_er, seed=1)
G_ba = nx.barabasi_albert_graph(n_net, m_ba, seed=1)

# Node deletion strategies
strategies = ['random', 'targeted']
fractions = np.linspace(0, 0.8, 17)

results_L = {'ER': {s: [] for s in strategies}, 'BA': {s: [] for s in str

def compute_metrics(G, f, strategy):
    G_temp = G.copy()
    num_remove = int(f * len(G_temp))
    if strategy == 'random':
        nodes_to_remove = np.random.choice(list(G_temp.nodes()), num_remo
    elif strategy == 'targeted':
        sorted_nodes = sorted(G_temp.degree, key=lambda x: x[1], reverse=
        nodes_to_remove = [node for node, _ in sorted_nodes[:num_remove]]
    G_temp.remove_nodes_from(nodes_to_remove)
    if len(G_temp) == 0:
        return np.nan
    giant_cc = max(nx.connected_components(G_temp), key=len)
    if len(giant_cc) > 1:
        return nx.average_shortest_path_length(G_temp.subgraph(giant_cc))
    return 0

# Computing characteristic path length L explicitly
for f in fractions:
```

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    for net_name, G in zip(['ER', 'BA'], [G_er, G_ba]):
        for strategy in strategies:
            L = compute_metrics(G, f, strategy)
            results_L[net_name][strategy].append(L)

# Plotting Characteristic Path Length (L)
plt.figure(figsize=(10, 5))
for net_name in ['ER', 'BA']:
    for strategy in strategies:
        plt.plot(fractions, results_L[net_name][strategy], marker='o', li
                 label=f"{net_name} - {strategy.capitalize()}")
plt.xlabel('Fraction of Nodes Removed (f)')
plt.ylabel('Characteristic Path Length (L)')
plt.title('Characteristic Path Length under Random vs Targeted Node Delet
plt.legend()
plt.grid(True)
plt.tight_layout()
plt.show()
# Let's load your provided edge data, construct the graph without using N
# and analyze its scale-free nature before proceeding with deletion analy

import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import igraph as ig

# Assume you've saved your provided edges into a file at 'C:\Users\om\Des
file_path = "edges.txt"

# Load edges from the provided txt file
edges_df = pd.read_csv(file_path, sep=" ", header=None, names=["source",

# Build the graph using igraph
g_real = ig.Graph.TupleList(edges_df.itertuples(index=False), directed=Fa

# Basic analysis to confirm scale-free nature
degrees = g_real.degree()
plt.hist(degrees, bins=30, log=True)
plt.xlabel('Degree')
plt.ylabel('Frequency (log-scale)')
plt.title('Degree Distribution (Check Scale-Free)')
plt.grid(True)
plt.show()

# Display basic properties
num_nodes = g_real.vcount()
num_edges = g_real.ecount()
avg_degree = np.mean(degrees)
print(f"Real-world Graph loaded successfully:\nNodes: {num_nodes}, Edges:

# (d) Comments:
print("Comments on Observations:")
print("\nDataset is: https://snap.stanford.edu/data/ego-Facebook.html\n")
print("""The response seen in the networks is consistent with Albert et a
With random deletion the size of the giant component (S) of the network d
until a major proportion of nodes is eliminated. This is as a result of t

But in targeted deletion, scale-free networks disintegrate quickly when h
fragments into isolated components. In contrast Erdős–Rényi (ER) networks
```
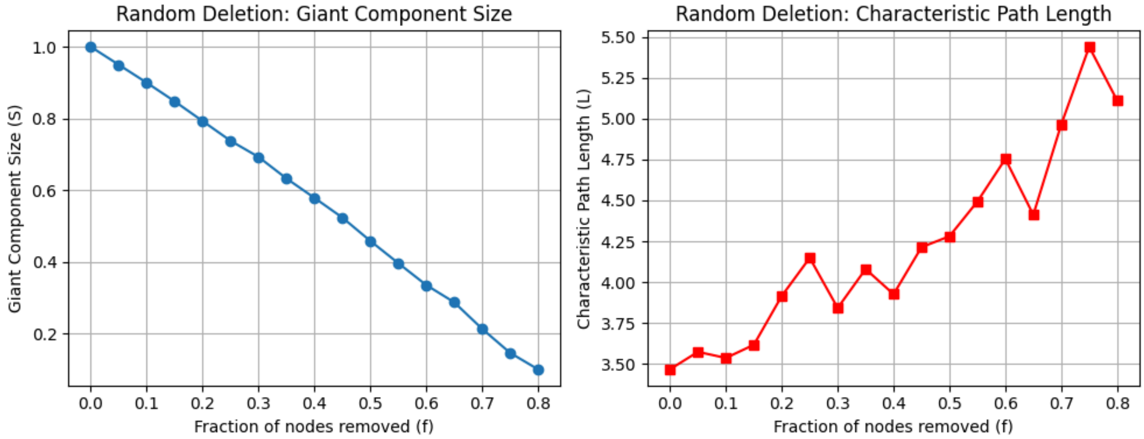
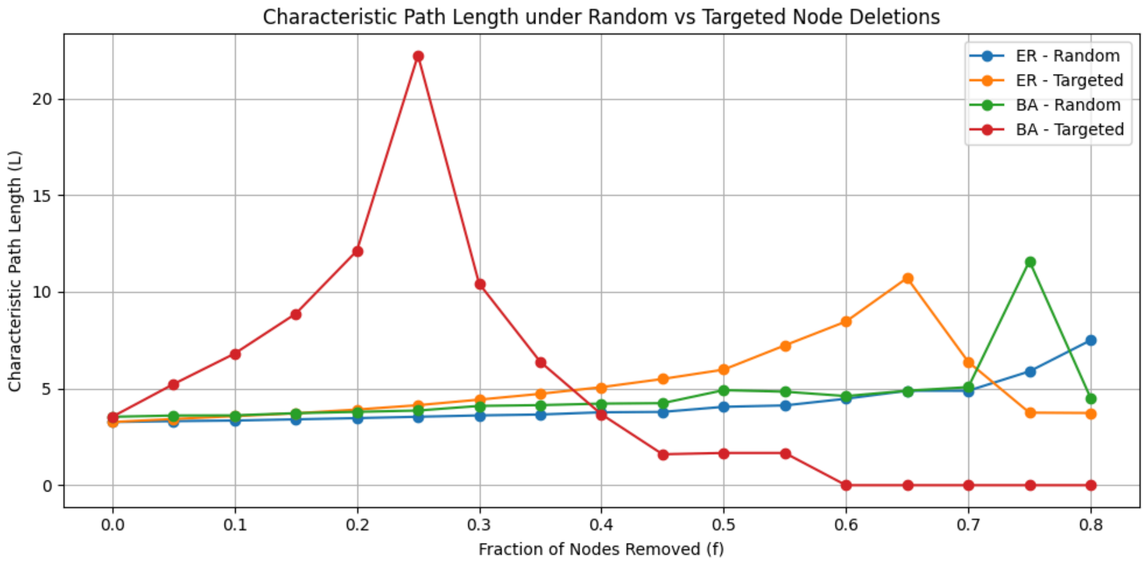
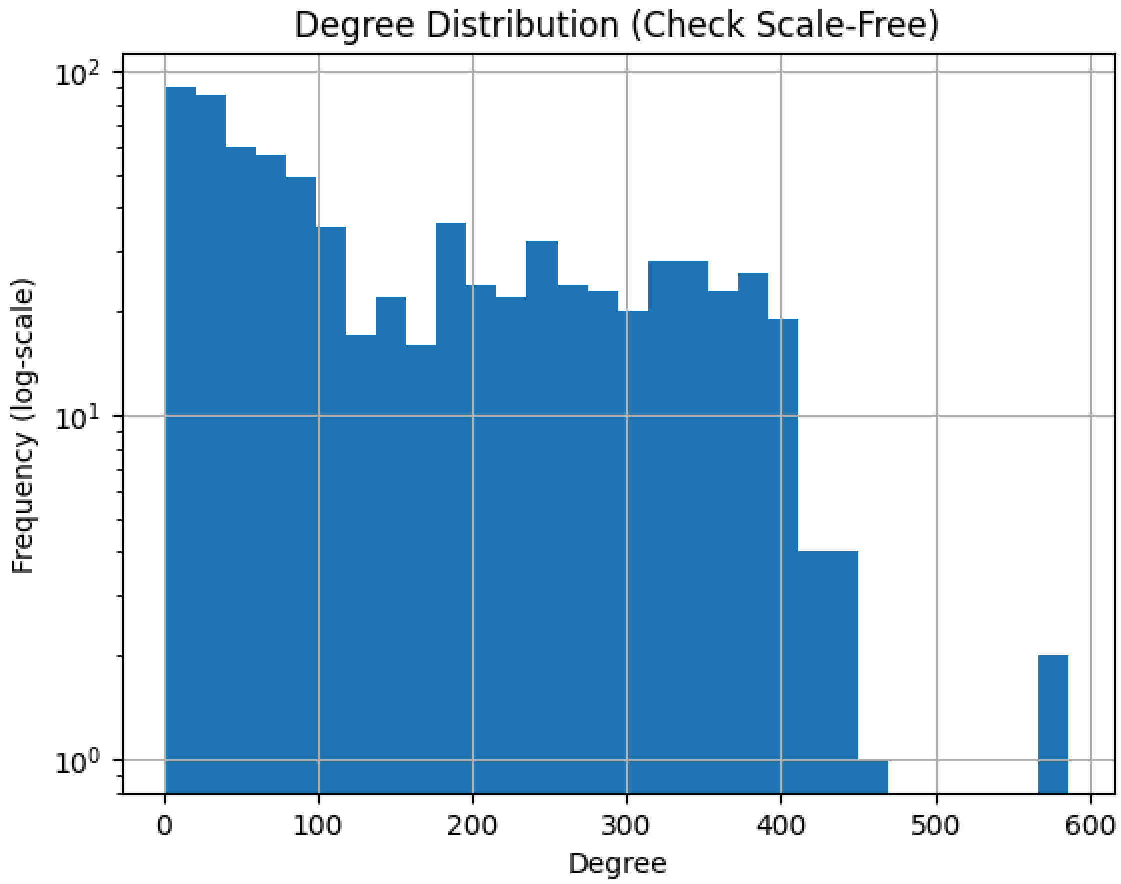

```
The results highlight that although scale-free networks have efficient co
in real-world systems like communication systems and power grids.""")
```

```
Requirement already satisfied: python-igraph in /home/akash/miniconda3/l
ib/python3.11/site-packages (0.11.8)
Requirement already satisfied: igraph==0.11.8 in /home/akash/miniconda3/
lib/python3.11/site-packages (from python-igraph) (0.11.8)
Requirement already satisfied: texttable>=1.6.2 in /home/akash/miniconda
3/lib/python3.11/site-packages (from igraph==0.11.8->python-igraph) (1.
7.0)
Note: you may need to restart the kernel to use updated packages.
```








```
Real-world Graph loaded successfully:
Nodes: 747, Edges: 60050, Average Degree: 160.78
Comments on Observations:

Dataset is: https://snap.stanford.edu/data/ego-Facebook.html

The response seen in the networks is consistent with Albert et al.(2000)
which shows scale-free networks to be robust to random failures but extr
emely vulnerable to attacks.
With random deletion the size of the giant component (S) of the network
declines gradually and the characteristic path length (L) rises slowly m
eaning that connectivity is preserved
until a major proportion of nodes is eliminated. This is as a result of
the existence of many low-degree nodes that sustain overall network stru
cture.

But in targeted deletion, scale-free networks disintegrate quickly when
high-degree hubs are deleted. S decreases steeply and L first increases
but then decreases as the network
fragments into isolated components. In contrast Erdős–Rényi (ER) network
s however degrade more uniformly since they do not contain dominant hub
s.
The results highlight that although scale-free networks have efficient c
onnectivity distribution, they are attack-prone structurally and thus hu
b protection is essential
in real-world systems like communication systems and power grids.
```