Negative Binomial

Negative Binomial is the generalised case of Geometric distribution. It is the distribution of the no. of trials needed to get the r^{th} success.

$r=2,\ r=12$ ..

It's like, What is the probability that it will take us X no. of trials to reach r success

$X \to$ trials

$r \to$ total success

$x-r \to$ failure

$\overline{f}\ \overline{s}\ \overline{s}\ \overline{f}\ \overline{s} \mid \underline{s} \to$ fixed

$\longleftarrow x-1 \longrightarrow$

$p =$ probability of success

$q =$ " " failure $=(1-p)$

$$f(x) = \binom{(x-1)}{(r-1)} (1-p)^{x-r} (p)^{r}$$

$\hookrightarrow {}^{x-1}P_{r-1}$

$X \geq r$

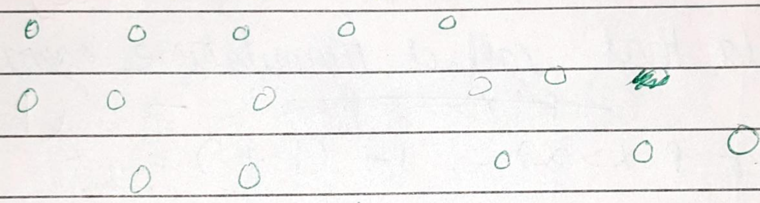
succ

Q Your sis is selling cookies & going around house to house until she sells all 5 boxes. What is the probability that she ~~finds~~ finishes selling them by the 8^{th} house? Given that any house chose to buy the box, probability is 30%.

$p = 0.3 \quad , r = 5 \quad , x?$

possibilities are;

$H_1 \quad H_2 \quad H_3 \quad H_4 \quad H_5 \quad H_6 \quad H_7 \quad H_8$



like these,

min

$x = 5, x = 6, x = 7, x = 8$

$p(5) + p(6) + p(7) + p(8)$

$x = 5$

$$p(5) = \binom{5-1}{5-1}(1-0.3)^{5-5}(0.3)^5$$

$$x = 6 = \binom{6-1}{5-1}(1-0.3)^{6-5}(0.3)^5$$

⋮

$$x = 8 \quad \binom{8-1}{5-1}(1-0.3)^{8-5}(0.3)^5$$

add them up

Exponential Distribution

A continuous random variable x, which has the following P.d.f.

$$\boxed{f(x) = \lambda e^{-\lambda x}} \qquad \boxed{\begin{array}{c}\lambda > 0 \\ 0 < x < \infty\end{array}}$$

Mean & variance of Exponential distribution

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx = \int_{0}^{\infty} x \lambda e^{-\lambda x} dx$$

$$\boxed{E(x) = \text{Mean} = \frac{1}{\lambda}} \qquad \boxed{\text{var}(x) = \frac{1}{\lambda^2}}$$

$$\boxed{\text{M.G.F} = \frac{\lambda}{t-\lambda}} \qquad \boxed{\text{characteristic } f^n = \frac{\lambda}{it - \lambda}}$$

Q The length of Telephonic Conversation is an exponential variable with mean 3 min. Find probability that ca

(i) ends less than 3 min

(ii) takes between 3 to 5 min.

Sol^n: $\frac{1}{\lambda} = 3$, $\lambda = \frac{1}{3}$, $F(x) = \lambda e^{-\lambda x} = \frac{1}{3} e^{-x/3}$

(i) $P(x<3) = \int_0^3 f(x) dx = \frac{1}{3}\int_0^3 e^{-x/3} dx = \left(-\left|\frac{\frac{1}{3} e^{-x/3}}{1/3}\right|_0^3\right)$

$= e^{-3/3} - e^{-0} \stackrel{o}{=} -(e^{-1} - 1)$

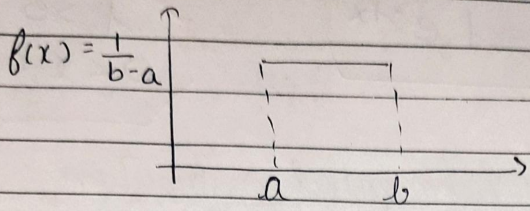
(ii) $P(3<x<5) = \int_3^5 f(x) dx = \int_3^5 \frac{1}{3} e^{-x/3} dx = -\left(e^{-5/3} - e^{-1}\right)$

$a^2+b^2+2ab-4ab = (a-b)^2$
$(a+b)^2-4ab = (a-b)^2$

Rectangular or Uniform Distribution

$$f(x) = \begin{cases} K & a < x < b \\ 0 & o/w \end{cases} \qquad K = \frac{1}{b-a}$$

$a < b$.



$$\boxed{\text{Mean} = E(x) = \frac{a+b}{2}} \qquad \boxed{\text{Variance}(x) = \frac{(a-b)^2}{12}}$$

$$MGF = M_x(t) = E(e^{xt}) \quad \boxed{= \frac{e^{bt}-e^{at}}{t(b-a)}}$$

Q. If x is uniformly distributed with mean 1 & variance 4/3, find $P(x<0)$

Sol^n: $\frac{a+b}{2} = 1$, $\quad \frac{(a-b)^2}{12} = \frac{4}{3}$ $\quad \rightarrow (a-b)^2 = 16$

$(a+b)^2 = 4$, $\quad (a-b)^2 = 16$

$(a+b)^2 - 4ab = 16$

$(a+b)^2 - (a+b)^2 + 4ab = -12$

$ab = -3$, $\quad b = \frac{-3}{a}$

$a - \frac{3}{a} = 2$

$a^2 - 3 = 2a$

$a^2 - 2a - 3 = 0$

$a^2 - 3a + a - 3 = 0$

$a(a-3) + 1(a-3) = 0$

$a = 3, \; a = -1$

$a = 3, b = 1$ $\quad$ $a = -1, b = 3$ $\quad$ - bcz a gives lower limit

$$f(x) = \begin{cases} \frac{1}{b-a} & , a < x < b \\ 0 & o/w \end{cases}$$

$$= \begin{cases} \frac{1}{3-(-1)} & -1 < x < 3 \\ & -1 < x < 3 \end{cases}$$

$f(x) = \begin{cases} \frac{1}{4} \\ 0 \end{cases}$ $\quad -1 < x < 3$ o/w)

$$f(x) = \begin{cases} 1/4 & -1 < x < 3 \\ 0 & o/w \end{cases}$$

$$P(x<0) = \int_{-\infty}^{0} f(x)\,dx = \int_{-1}^{0} \frac{1}{4}\,dx = \frac{1}{4}(x)\Big|_{-1}^{0} = \frac{1}{4}$$