श्रीराम श्रीराम (1)

Sem-3 MST Prep | Maths

Two coins are tossed

Domain: all the possible outcomes → $S = \{HHH, TTT, HHT, TTH, HTT, THH, HTH, THT\}$

Range: all the Desirable outcomes → HHT, HTH, TH, H

Sample Space (X): Asked outcomes. $X \to$ 2 heads

$$P(X) = {}^nC_r (p)^{n-r} (p)^r$$

Q → A coin is tossed 2 times;

$x = 0, 1, 2$

$$P(X)_0 = {}^2C_0 \left(\frac{1}{2}\right)^{(2-0)} \left(\frac{1}{2}\right)^0 = {}^2C_0 \left(\frac{1}{2}\right)^2 = \frac{2!}{2!} \times \frac{1}{4} = \frac{1}{4}$$

$$P(X)_1 = {}^2C_1 \left(\frac{1}{2}\right)^{(2-1)} \left(\frac{1}{2}\right)^1 = 2 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$$

$$P(X) = {}^2C_2 \left(\frac{1}{2}\right)^{(2-2)} \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Domain f^n = $P(X \le 0)$ $= P(X=0)$ $= \frac{1}{4}$

, $P(X \le 1)$, $P(X=0) + P(X=1)$ $= \frac{1}{4} + \frac{\cancel{2}1}{2} = \frac{1+2}{4} = \frac{3}{4}$

, $P(X \le 2)$

$P(X=0) + P(X=1) + P(X=2)$

$= \frac{3}{4} + \frac{1}{4} = 1$

all of that can be seen in another way too.

next page.

② 10 times करेंगे 2^{10} आएगा sample space.

tossed one no. of times

Coin tossed → ① $S = \{H, T\}$
→ two outcomes

$2^{②}$ → tossed ; $S = \{HH, HT, TH, TT\}$

X (No. of Head)	0	1	2
P(X)	1/4	2/4	1/4

random variable

DRV | CRV

mass | density

p.m.f ① $P(x) \geq 0$, $\sum P(x) = 1$

$f(x)$ p.d.f ① $f(x) \geq 0$ ⓘⓘ $\int_{-\infty}^{\infty} f(x) = 1$

Binomial, Poisson

Exponential, Normal, Uniform

Q 4 bad oranges are mixed with 16 good oranges. Find the probability distribution of the no of bad oranges (2 drawn)

~~probability distribution~~

X (No. of bad oranges)	0	1	2	3	4
P	$\frac{16C_2}{20C_2}$	$\frac{16C_1 \times 4C_1}{20C_2}$	$\frac{4C_2}{20C_2}$	0	0

$\boxed{16G \quad 4B} \to 2$

dis cut. / commulative distri. fn.

$$f(x) = \begin{cases} \frac{16C_2}{20C_2} & x \leq 0 \\ \frac{16C_2}{20C_2} + \frac{16C_1 \times 4C_1}{20C_2} & x \leq 1 \\ \frac{16C_2}{20C_2} + \frac{16C_1 \times 4C_1}{20C_2} + \frac{4C_2}{20C_2} & x \leq 2 \end{cases}$$

Conditional Probability

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

Q:- If x is a continuous r.v. with the following p.d.f

$$f(x) = \begin{cases} \alpha(2x - x^2) & 0 < x < 2 \\ 0 & o/w \end{cases}$$

Find (i) α (ii) $P(X > 1)$

Soln by defn of p.d.f $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{0} \cancel{f(x)}\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{\infty} f(x)\,\cancel{dx} = 1$$

(value not given) value not given

$$= \int_{0}^{2} \alpha(2x - x^2) = 1$$

$$= \int_{0}^{2} 2x\alpha - x^2\alpha = \left[\frac{\cancel{2}x^3\alpha}{\cancel{2}} - \frac{x^3}{3}\alpha\right]_0^2 = 1$$

$$4\alpha - \frac{8\alpha}{3} - [0] = 1$$

$$12\alpha - 8\alpha = 3$$

$$4\alpha = 3$$

$$\alpha = \frac{3}{4}$$

(ii) $P(X \geq 1)$

$$= \int_{1}^{\infty} f(x)\,dx$$

$$= \int_{1}^{2} f(x)\,dx.$$

$$= \int_{1}^{2} \frac{3}{4}(2x - x^2)\,dx.$$

$$= \frac{1}{2}.$$

(4)

Probability density function is for continuous r.v & distribution f^n is for discrete random. v

Note: rel^n betw distribution f^n & density f^n:

$$\frac{d}{dx} F(x) = f(x)$$

Q: Probability density f^n of r.v X

$$f(x) = \begin{cases} x & 0 \le x < 1 \\ 2-x & 1 < x < 2 \\ 0 & o/w \end{cases}$$

(i) → Find $P(x \geq 1.5)$

(ii) → Find cummulative distr f^n

$$P(x \geq 1.5) = \int_{1.5}^{2} f(x)\,dx = \int_{1.5}^{2} 2-x = \left| 2x - \frac{x^2}{2} \right|_{1.5}^{2}$$

$$= 4 - 3 - \left[2 - \frac{9}{4}\left(\frac{1}{2}\right) \right] = 1 - \left(2 - \frac{9}{8}\right)$$

$$= 1 - \left(\frac{7}{8}\right) = \frac{1}{8}$$

(ii) Cummulative distr f^n

$x \le 0$

$$P(X \le x) = \int_{-\infty}^{x} f(x)\,dx = 0$$

$x < 1$

$$P(X \le x) = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} f(x)\,dx = \frac{x^2}{2}$$

$1 < x < 2, \quad x \le 2$

$\int_{-\infty}^{0} f(x)dx + \int_0^1 f x\,dx + \int_1^x f(x)dx = \left|\frac{x^2}{2}\right|_0^1 + \int_1^x (2-x)$

$= \frac{1}{2} + \left[\frac{x}{2} - \frac{1}{2}\right] = \frac{x}{2} \quad \frac{1}{2} + \frac{x^2}{2} - \frac{1}{2} = \frac{x^2}{2}$

$\frac{1}{2} + \left|2x - \frac{x^2}{2}\right|_1^x \rightarrow 2x - \frac{x^2}{2} - \left[2 - \frac{1}{2}\right] = 2x - \frac{x^2}{2} - \left[\frac{3}{2}\right]$

$= 2x - \frac{x^2}{2} - 1$

$x \ge 2$

$\int_{-\infty}^{0} f(x)dx + \int_0^1 f x\,dx + \int_1^2 f(x)dx + \int_2^x f(x)dx$

$= \int_0^1 x\,dx + \int_1^2 (2-x)\,dx = \left|\frac{x^2}{2}\right|_0^1 + \left|2x - \frac{x^2}{2}\right|_1^2$

$= \frac{1}{2} + 4 - 2 - \left[2 - \frac{1}{2}\right] = \frac{1}{2} + 2 - \left[\frac{3}{2}\right]$

$= 1$

$$F_x = \begin{cases} 0 & -\infty < x < 0 \\ x^2/2 & x \le 1 \\ 2x - x^2/2 - 1 & x \le 2 \\ 1 & 2 < x < \infty \end{cases}$$

~~$x \le 0$~~ ~~$x \le 1$~~ ~~$x \le 2$~~

अगर इनका df करेंगे तो P(x)

(distribution fn आ जाएगा)

Expected value & Mean एक ही है।

Mathematical Expectation

$$E(\phi(x))$$

D.R.V: $\sum_x \phi(x) f(x)$

C.R.V: $\int_{-\infty}^{\infty} \phi(x) f(x)\, dx$

if $\phi(x) = x$

$\bar{x}$ = Mean $= E(x)$

$\xrightarrow{D.R.V}$ $E(x) = \sum_x x\, P(x)$

$\xrightarrow{C.R.V}$ $E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$

$E(\bar{x}) = \bar{x}$

Variance $= E(x-\bar{x})^2$

$= E(x^2) - [E(x)]^2$

Q Find the mean & variance of the probability distribution Given by the following table!

continuous random variable

x	1	2	3	4	5
$P(x)$	0.2	0.35	0.25	0.15	0.05

$$E(x) = \sum x P(x) = \underbrace{0.2}_{1\times 0.2} + \underbrace{0.7}_{2\times 0.35} + \underbrace{0.75}_{0.25\times 3} + \underbrace{0.6}_{0.6\times 4} + \underbrace{0.25}_{5\times 0.05}$$

Mean $= 2.5$

$E(x^2) = \Sigma x^2 p = 1\times0.2 + (2)^2\times0.35 + (3)^2 + 0.25 + (4)^2\times0$
$\times 25\times0.05$
$= 0.2 + 1.4 + 2.25 + 2.4 + 12.5 = 7.5$

$Var(x) = \sigma^2 x = E(x^2) - (E(x))^2$
$= 7.5 - (2.5)^2 = 7.5 - 6.25$
$= 1.25$

Q 13 cards are drawn simultaneously from a pack of 52 cards. If ace count 1 & face card 10 & other according to their denomination, find expectation of total score in 13 cards

Discrete random variable

X	1	2	3	4	5	6	7	8	9	10	10	10
P	$\frac{1}{13} = 4/52$	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13

$E(x) = \frac{1}{13}(1+2+3+4+5+6+7+8+9+4(10))$

$= \frac{1}{13}\left(\frac{9}{2}(1+9) + 30\right) = \frac{1}{13}(45+40)$

$= \frac{1}{13}[85]$

Q A c.r.v X has density fn given by $f(x) =$ $\begin{cases} 2e^{-2x} & x>0 \\ 0 & o/w \end{cases}$

Find expected value & variance of X.

Sol $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_0^{\infty} x f(x)\,dx = \int_0^{\infty} x^{1}(2e^{-2x})\,dx$

$= 2\int_0^{\infty} x e^{-2x}\,dx$

$= 2\left(\frac{\Gamma 2}{2^2}\right) = \frac{1}{4}$

$\int_0^{\infty} x^{n-1} e^{-ax}\,dx = \frac{\Gamma n}{a^n}$

gamma fn

⑦

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^{\infty} x^2 (2e^{-2x})\,dx.$$

$$= 2\frac{\Gamma 3}{2^3} = 2\frac{(2\cdot 1)}{8} = \frac{1}{2}$$

variance $= \sigma^2_x = E[x^2] - [E(x)]^2$

Binomial Distribution

i) All the trials are independent.
ii) No of n trials is finite (बहुत ज्यादा वाले को फिक्स करो)
iii) The probability p of success is same of each trial.

$$P(x) = {}^nC_x\, p^x\, q^{n-x}$$

n = trials
p → probability of success
& q = " " failure
x → conditionals

$p+q = 1$

Moment generating f^n of Binomial distribution

$$M_x(t) = (pe^t + q)^n$$

Characteristic f^n:-

$$\phi_x(t) = E(e^{ixt})$$
$$= (pe^{it} + q)^n$$

Probability dist^n f^n

$Z_x(t) = (zP+q)^n$

Mean $= E(x) = np$

Variance $= \sigma^2 x = E(x)^2 - [E(x)]^2$

$= ~~npq~~\ nPq$ or npq

Q The probability that man aged 60 will live upto 70 is 0.65 out of 10 men, now aged 60, find probability

(i) At least 7 men will live upto 70

(ii) Exactly 9 will live upto 70.

(iii) At most 9 will live upto 70.

Sol- $n=10$, $P=0.65$, $Q=1-p=0.35$

$P(x) = {}^nC_x\, p^x\, q^{n-x}$

$= {}^{10}C_x (0.65)^x\, ~~q~~ (0.35)^{(10-x)}$

(i) $P(x\geq 7) = P(7)+P(8)+P(9)+P(10)$

$= {}^{10}C_7 (0.65)^7 (0.35)^3 + {}^{10}C_8 (0.65)^8 (0.35)^2 + {}^{10}C_9 (0.65)^9 (0.$

$+ {}^{10}C_{10} (0.65)^{10}$.

(ii) ~~$1-{}^{10}C_{10}(0.65)^{10}(0.35)^0$~~ $P(9) = {}^{10}C_9 (0.65)^9 (0.35)$

(iii) not more than 9 (1-9) $P(x\leq 9) = 1-P(x>9)$

$= 1-P[10] = 1-{}^{10}C_{10}(0.65)^{10}$

Q Out of 800 families with 5 children each, how many families would be expected to have

(i) 3 boys (ii) 5 girls (iii) Either 2 or 3 boys

(iv) at least 2 girls

Sol^n $N=800$, $n=5$, $P(B)=\frac{1}{2}$, $P(\bar{B})=\frac{1}{2}=q$

$P(x) = {}^nC_x\, p^x\, q^{n-x}$

$= {}^5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x} = {}^5C_x \left(\frac{1}{2}\right)^5$

(i) $P(3) = {}^5C_3 \left(\frac{1}{2}\right)^5 = \frac{5}{16}$

No. of famili $= 800 \times 5/16 = 250$.

(10)

(ii) 5 girls → mean 0 boys → $^5C_0\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

No of families $= \frac{1}{32} \times 800 = 25$

(iii) Either 2 or 3 boys $= P(2) + P(3) = {^5C_2}\left(\frac{1}{32}\right) + {^5C_3}\left(\frac{1}{32}\right) = \frac{20}{32}$

$= \frac{20}{32} \times 800 = 500$

(iv) At least 2 girls

2	3	4	5	G
3	2	1	0	B

$= 1 - [P(5) + P(4)] = 1 - \left({^5C_5}\left[\frac{1}{32}\right] + {^5C_4}\frac{1}{32}\right)$

$1 - \left(\frac{1}{32} + \frac{5}{32}\right) = 1 - \frac{6}{32} = \frac{26}{32} \times 800$

$= 650$ families

25(25+1)
625+25
650

Q 4 coins are tossed 100 times & following were obtained. Fit a binomial distribution for data & calculate theoretical frequency.

No of Head x	frequencies f	xf	$P(x) =$
0	5	0	P(0)
1	29	29	P(1)
2	36	72	P(2)
3	25	75	P(3)
4	5	20	P(4)
		$\Sigma xf = 196$	

$^4C_x (0.49)^x (0.5)^{4-x}$

$\bar{X} = \frac{\Sigma xf}{\Sigma f} = \frac{196}{100} = 1.96$

$n = 4$

$np = 4p$

$4p = 1.96$

$p = 0.49$

$q = 0.5$

$f(P(x))$
7
26
37
24
46
100

Poisson Distribution

A Discrete Random variable x which has the following probability mass fn:-

$$\boxed{P(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \ldots \infty}$$

when n is 8000 big & probability of distribution is very less.

Poisson Distribution is a limiting case of Binomial distribution under following condition:-

→ $n \to \infty$

→ $p \to 0$

→ $np = \lambda$ (finite)

Mean $= E(x) = \lambda$ Variance $= \lambda = \sigma^2$

$\therefore E(x^2) = \lambda^2 + \lambda$ $E(x^2) - [E(x)]^2$

Q In which distribution, the mean & variance come out to be same?

Ans:- Poisson distribution

MGF; $M_x(t) = \cancel{e^{\lambda}(e^t} \; e^{\lambda(e^t-1)}$

Characteristic f^n $= e^{\lambda(e^{it}-1)}$

PGF $= e^{\lambda(z-1)}$

Mean & Variance by M.G.F

$E(x) = $ Mean $= \lambda$

Variance $= \lambda$

Q Given that 2% of the fuses manufactured by a firm are defective, find probability that a box containing 200 fuses has

(i) At least 1 defective fuses

(ii) 3 or more defective fuses

(iii) No defective fuses

Sol:- $n = 200$, $p = 0.02$, $\lambda = np = 200 \times 0.02 = 4$

$P(x) = \cancel{\lambda^x / e^{-\lambda}}$ $\dfrac{\lambda^x e^{-\lambda}}{x!}$

(12).

(1) $P(x \geq 1) = 1 - P(x < 1) = 1 - P(0) = 1 - \frac{4^0 e^{-4}}{0!} = 1 - e^{-4}$

(2) $P(x \geq 3) = 1 - P(x < 3)$

$= 1 - (P(0) + P(1) + P(2))$

$= 1 - 13e^{-4}$

(3) $P(x = 0) = \frac{4^0 e^{-4}}{0!} = e^{-4}$

Q* If probability of a bad reaction from a certain injection is 0.01, find the chance that out of 200 individuals, more than two will get bad reaction.

Sol: $p = 0.01$, $n = 200$, $np = \cancel{2} = \lambda$

$\frac{\lambda^x e^{-\lambda}}{x!} = \frac{(\lambda)^x e^{-\lambda}}{x!}$

$P(x > 2) = 1 - P(x \leq 2)$

$= 1 - [P(0) + P[1] + P[2])$

$= 1 - \left[\frac{(2)^0 e^{-2}}{0!} + \frac{(2)^1 e^{-2}}{1!} + \frac{(2)^2 e^{-2}}{2!}\right]$

$= 1 - \left[2e^{-2} + 2e^{-2} + \frac{e^{-2}}{2}\right]$

$= 1 - \left[2e^{-2} + \frac{e^{-2}}{2}\right] = 1 - 5e^{-2}$

$= \frac{2 + 3e^{-2}}{2}$

Fitting of Poisson distribution

Q A skilled typist, on routine work kept a record of mistake made per day during 300 working days.

mistake/day	0	1	2	3	4	5	6
No. of days	143	90	42	12	9	3	1

x Mistake/day	f	$P(x) = \frac{(0.89)^x e^{-0.89}}{x!}$
0	143	$P(0) = (0.89)^0 e^{-0}$
1	90	
2	42	
3	12	
4	9	
5	3	
6	1	
	$\Sigma f = 300$	

$\bar{x} = \frac{\Sigma xf}{\Sigma f}$

$\lambda = \frac{267}{300}$

$= 0.89$

Geometric distribution

It is the distribution of the no. of trials needed to get the first success in repeated Bernoulli trials.

Q In a large population of adults, 30% have received CPR training. If adults from this population are randomly selected, what is the probability that the 6th person sampled is the first that has received CPR training?

OR lets try diff question.

(14)

Q What is the probability of getting the I "6" on the 4^{th} roll using a six-sided die?

I	II	III	IV	
Fail,	Fail	Fail	success	← rolls
5/6	5/6	5/6	1/6	$= \frac{5^3}{6^4} = \frac{125}{1296} = 0.09645 \simeq 9.6\%$ chance

Sol^n

$S = \{1, 2, 3, 4, 5, 6\}$

So lets see its formula

$$P(X = x) = q^{x-1} \cdot p$$

p → probability of getting a successful event

q → fail

$q = (1 - p)$

getting to that question,

$$P(X = 4) = \left(\frac{5}{6}\right)^{4-1} \cdot \left(\frac{1}{6}\right) = 0.09645$$

Q 15% of all cars passing along a certain road are blue. What is the P(x) that the 7^{th} car will be the first blue car that you see passing along this road?

Ans $p = \frac{15}{100} = \frac{3}{25}$: $\frac{3}{25}$ $q = 1 - \frac{3}{25} = \frac{22}{25}$

$q = t$, $x = 7$

$$P(X=7) = (q)^{x-1} (p) = \left(\frac{22}{25}\right)^6 \left(\frac{3}{25}\right) = 0.055 \simeq 5.7\% \text{ chance}$$

Q 4% of the population in a small town work as a teacher

(a) $P(X = 10)$ (that every 10th person is a teacher).

(b) Calculate the mean, variance & standard deviation.

$x = 10$, $p = 0.04$, $q = 0.96$

(a) $= (0.96)^9 (0.04) = 0.027 \simeq 2.7\%$ chance.

Mean $= \frac{1}{p} = \frac{1}{0.04} = 25$

Variance; $\sigma^2 \Rightarrow \left(\frac{1}{p}\right)\left(\frac{1}{p} - 1\right) = (25)(25-1) = 625 - 25 = 600$

$\boxed{S.D = \sqrt{\text{Variance}} = \sqrt{600} = 24.49}$

If we want to find, $P(X \leq x)$, then there is a formula, that's called cumulative geometric distribution for. ~~(If P(X > x) then~~ $= (1-q^x)$

so for $P(X > x) \to 1 - (1-q^x) = q^x$

If you come across a ques. that how many times do you expect to test until you find the first defective one?
In this case, you will have to find the Mean $= \frac{1}{p}$