2/3/21

Curvillinear Regression

There are two types of regression
1. Linear Regression
2. Curvilinear Regression.

1. Linear Regression → A curve of the form $Y = a + bX$ is called linear regression of y on X where b is the regression coeff. & a ~~b~~ is intercept on Y-axis.

2. Curvilinear → A ~~cur~~ curve of the form $Y = a + b_1 X + b_2 X^2$ is called curvilinear regression.

Date
Page No.

To calculate or to find the regression equations, we need certain equations that are known as normal equations.

1. To fit a linear regression

$$Y = a + bX \quad — (0)$$

The normal eq.^{n} will be

constants of eq^n

$$\Sigma Y = \Sigma a + b\Sigma X$$
$$= na + b\Sigma X \quad — (1)$$

Multiply by ΣX — (0)

$$\Sigma XY = a\Sigma X + b\Sigma X^2 \quad — (2)$$

2. To fit a curvilinear Regression

$$Y = a + b_1 X + b_2 X^2 \quad — ☆$$

The normal eq^n are

$$\Sigma Y = na + b_1 \Sigma X + b_2 \Sigma X^2 \quad — (1)$$

Multiply ☆ with ΣX

$$\Sigma XY = a\Sigma X + b_1 \Sigma X^2 + b_2 \Sigma X^3 \quad — (2)$$

Multiply ☆ with ΣX^2

$$\Sigma X^2 Y = a\Sigma X^2 + b_1 \Sigma X^3 + b_2 \Sigma X^4 \quad — (3)$$

On solving normal eq^n find the value of constants.
After finding the value of constants, put back in given eq^n

(3) $y = ab^x$

$\log y = \underline{\log a} + x \underline{\log b}$

$Y = \log y = A + BX$

$Y = A + Bx$, where

$Y = \log y$
$A = \log a$
$B = \log b$

Then the normal eq^n are

$$\Sigma Y = nA + b\Sigma X$$

$$\Sigma XY = A\Sigma X + b\Sigma X^2$$

Q. For 10 randomly selected observations, the following data were recorded.

Sol^n

X	y	X^2	XY	X^3	X^2y	X^4
1	2	1	2	1	2	1
1	7	1	7	1	7	1
2	7	4	14	8	28	16
2	10	4	20	8	40	16
3	8	9	24	27	72	81
3	12	9	36	27	108	81
4	10	16	40	64	160	256
5	14	25	70	125	350	625
6	11	36	66	216	396	1296
7	14	49	98	343	686	2401
34	95	154	377	820	1849	4774

Determine the coefficients of regression & regression eq^n using
$$Y = a + b_1X + b_2X^2$$

$\Rightarrow$ Given, $Y = a + b_1X + b_2X^2$
The normal eq^n are

$$\Sigma Y = na + b_1\Sigma X + b_2\Sigma X^2 \quad -(1)$$

$$\Sigma XY = a\Sigma X + b_1\Sigma X^2 + b_2\Sigma X^3 \quad -(2)$$

$$\Sigma X^2Y = a\Sigma X^2 + b_1\Sigma X^3 + b_2\Sigma X^4 \quad -(3)$$

$\therefore$ 1,2,3 becomes

$$95 = 10a + 34b_1 + 154b_2$$
$$377 = 34a + 154b_1 + 820b_2$$
$$1849 = 154a + 820b + 4774b_2$$

Date
Page No.

$$\begin{bmatrix} 10 & 34 & 154 \\ 34 & 154 & 820 \\ 154 & 820 & 4474 \end{bmatrix} \begin{bmatrix} a \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 95 \\ 377 \\ 1849 \end{bmatrix}$$

Make use of row operation to convert into upper Δ matrix

~~$R_2 \to 5R_2 - 17R_1$~~

$$\begin{bmatrix} 10 & 34 & 154 \\ & & \\ & & \end{bmatrix} \begin{bmatrix} a \\ b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$R_2 \to 5R_2 - 17R_1$

$R_3 \to 10R_3 - 154R_1$

On solving we will get

$a = 1.80$

$b_1 = 3.48$

$b_2 = -0.27$

Thus the eq becomes

$$Y = a + b_1 X + b_2 X^2$$
$$= 1.80 + 3.48 X + (-0.27) X^2$$

Ans.

Q5 Fit an exponential curve of the form ~~x~~ $Y = ab^x$ to the foll. data

X	Y	$u = \log Y$	X^2	Xu
1	1	0.0000	1	0
2	1.2	0.0792	4	0.1584
3	1.8	0.2553	9	0.7659
4	2.5	0.3979	16	1.5916
5	3.6	0.5563	25	2.7815
6	4.7	0.6721	36	4.0326
7	6.6	0.8195	49	5.7365
8	9.1	0.9590	64	7.6720

$\Rightarrow$ To fit exponential curves, we need $Y = ab^x$

$$\log Y = \log a + X \log b$$

Put $\log Y = u$
$\log a = A$
$\log b = B$

$$u = A + BX$$

Then the normal eq^n are

$$\Sigma u = nA + B\Sigma X$$

$$\Sigma Xu = A\Sigma X + B\Sigma X^2$$