

ANSWERS

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1) The Konigsburg bridge graph has no Euler circuit. As we shall see later this is due to the fact that it has a vertex with an odd degree (in fact, ALL vertices have odd degree).

2) Only B has an Euler Circuit. Again, all other graphs have a vertex of odd degree.

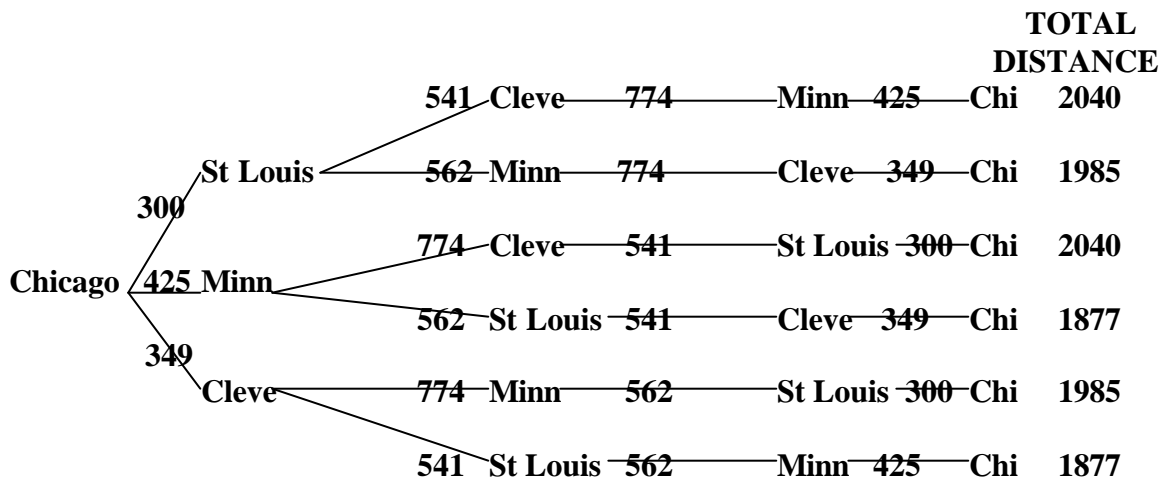
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4) A graph will have an Euler Path which is not an Euler Circuit, if it has exactly 2 vertices of odd degree. The Eulerian Path must begin at one of the vertices of odd degree and end at the other one.

5) Graphs A and D have Euler Paths since they have exactly 2 vertices of odd degree.

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1) We start at Chicago but it doesn't matter as these are circuits so the total distance doesn't depend on the starting point.



Using a tree diagram all possible routes are constructed with the total mileage (the method of *Brute Force*). Clearly the shortest distance is 1877 miles and occurs for Chicago-Minneapolis-St. Louis-Cleveland-Chicago and Chicago-Cleveland-St. Louis-Minneapolis-Chicago which is just the first one traveled in the opposite direction (so there are really only 3 “distinct” possibilities).

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2) a) 24 b) 120 c) 3,628,800 d) 2.43290×10^{18}

3) a) $9.33262154 \times 10^{157}$

The TI89 can actually compute $100!$ but the TI83 gives an overflow error. There are many ways around this problem. One is to compute $60! = 8.320987113 \times 10^{81}$ (actually the limit of the TI83 is $69!$) and then do $100 \cdot 99 \cdot 98 \cdot \dots \cdot 62 \cdot 61 = {}_{100}P_{40} = 1.121576253 \times 10^{76}$. Multiplying $8.320987113 \cdot 1.121576253$ and rounding to 8 places yields the desired result above.

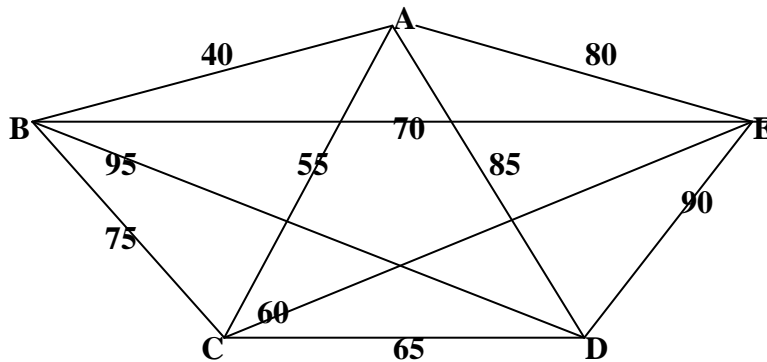
b) $1.555436923 \times 10^{147}$ c) $2.592394872 \times 10^{145}$ d) $1.08016453 \times 10^{144}$

e) $1.543092186 \times 10^{143}$ f) $5.14364062 \times 10^{141}$ (using 30 days in a month)

g) $4.286367183 \times 10^{140}$ h) $4.286367183 \times 10^{138}$ i) $4.286367183 \times 10^{137}$

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4)



Starting at A: A—40—B---70---E---60---C---65---D---85---A for a total of 320

Starting at B: B---40---A---55---C---60---E---90---D---95---B for a total of 340

Starting at C: C---55---A---40---B---70---E---90---D---65---C for a total of 320

Starting at D: D---65---C---55---A---40---B---70---E---90---D for a total of 320

Starting at E: E---60---C---55---A---40---B---95---D---90---E for a total of 340

Clearly, in the case of the Nearest Neighbor Algorithm, the starting point makes a difference but the best value is 320.

Note that the circuit C-A-B-E-D-C is the “same” as D-C-A-B-E-D.

Brute Force shows that the best possible value is also 320.

Note: While I have tried to check all the answers above, errors can occur. If you discover any, I would greatly appreciate it if you would notify me.