undirect

degree → total link to one node

Directed

degree → incoming degree
Outgoing "

source → incoming degree 0
sink → outgoing degree = 0

~~mode~~ $= \frac{1}{N}\sum_{i=1}$

Average =

n^{th} moment

standard deviati..

Distribution

undirected

Average degree $= \frac{1}{N}\sum_{i=1}^{N} K_i$ → total degree some of all

OR

directed

Average degree $= \frac{2L}{N}$ → total link

Average degree (indegree) $= \frac{1}{N}\sum_{i=1}^{N} K_i$ → indegree count node sum

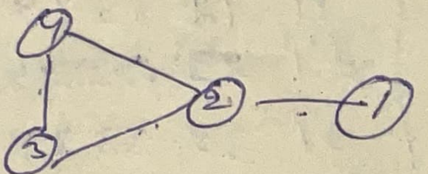
Average degree (outdegree) $= \frac{1}{N}\sum_{i=1}^{N} K_i$ → out degree count node sum

K indegree = $K^{outdegree}$

Average degree of Directed $= \langle K \rangle = \frac{L}{N}$ → total no of links

Q what is the degree of node having degree k

$$P(K) = \frac{N_K}{N}$$



Node have probability of degree 2

sum of nodes having 2 degree

$$\frac{2}{8} = \frac{1}{4}$$

8 → sum total degree of nodes of

Continuous Description $= p(K)$

$$\int_{K_1}^{K_2} p(K)\,dk$$

$$\sum_{0}^{\infty} p_K = 1 \qquad \int_{K_{min}}^{\infty} p(K)\,dK = 1$$

K_{min} is the minimal degree in the network undireded — Direded

Actor — Url

protein interaction — Phonecall

Adjacency matrix

if link → 1

no link → 0

$$\begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \end{bmatrix}$$

undirected

degree from matrix = $\dfrac{\text{count one in matrix}}{2}$

$$L = \frac{1}{2} \sum_{i=1}^{n} k_i$$

directed

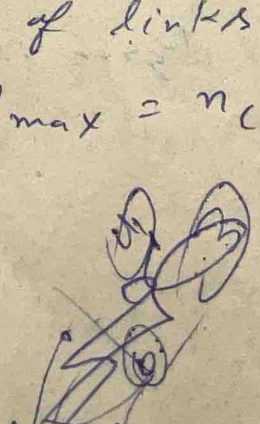
degree from matrix = count one in matrix

$$L = \sum_{i=1}^{n} k_i$$

Sparseness

Complete Graph

the max no of links a network of n nodes :- $L_{max} = {}^{n}C_2 = \dfrac{n(n-1)}{2}$



A graph with degree $L = L_{max}$ is called a complete graph

and its Average degree is $\langle K \rangle = N-1$

no of nodes

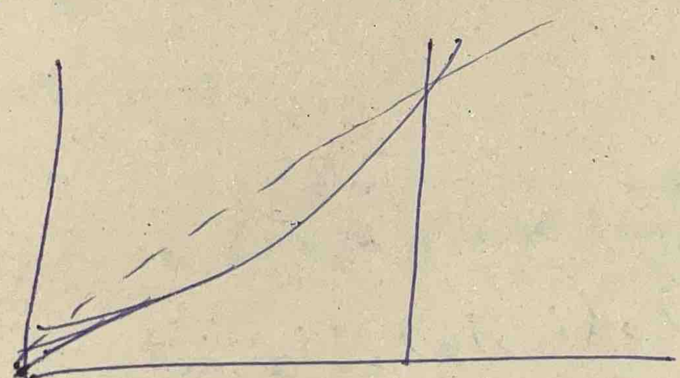
L (incomplete) $<$ L_{max} (complete)

$\langle K \rangle$ (incomplete) wr $<$ $N-1$ (complete)

Metcalfe's Law

~~Ø~~ undirected

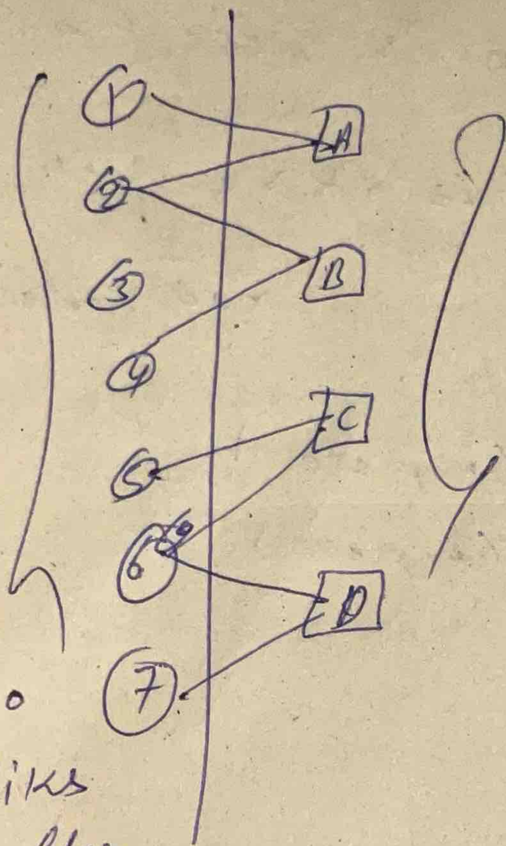
if n node comes then n^2 Links will increase



weighted and ~~~~ unweighted graph

Bipartite Network

If we devide graph in two parts so both parts have (every node) have no liks to any nodes



no liks after devide

no liks after devide

Path

$P_n = \{ i_0, i_1, i_2, ----i_n \}$

$P_n = \{ (i_0, i_1), (i_1, i_2), (i_2, i_3), --- (i_{n-1}, i_n) \}$

Distance

undirected

directed

No of paths between two nodes

we have given Adjacency matrix

undireded

we have given adjacency matrix and given go to A to B and it have n. links in between them so find no of path that have n links b/w A + B

Ans multiply matrix by n times and check Row of A and colum of B and got your ans

find distance breath first search

Network diameter and Average distance

~~[illegible]~~

1. sortest distance of every two pair in graph
2. then maximum of all of them that is our diameter

directed

$$\langle d \rangle = \frac{1}{2L_{max}} \sum_{i,j \neq i} d_{ij}$$

undirected

$$\langle d \rangle = \frac{1}{L_{max}} \sum_{i,j>i} d_{ij}$$

Shortest path

Average path length

$$= \frac{\text{Total path of A and B}}{\text{Total path b/w all nodes}}$$

Cycle

a path with the same start and end

Self-avoiding path

a path that does not intersect itself

Eulerian path

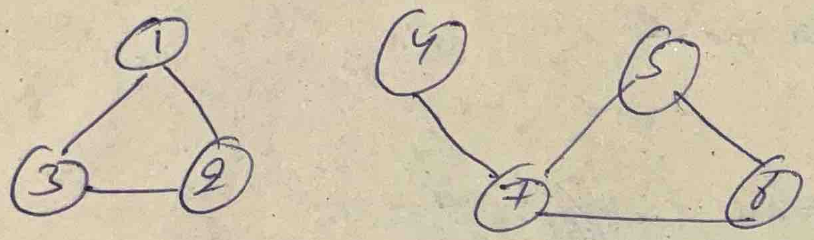
A path that traverses each link exactly once

Hamiltonian path

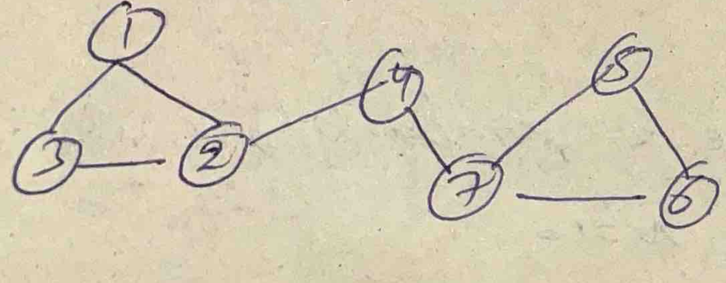
A path that visits each node exactly once

Connected and not Connected graphs

find graph is connect or not from Adjacency matrix



if you seprate matrix of one in two parts of orignal adjacency matrix so it is unconnected



else Connected

Strongly connected

directed
if we have both b/w two nodes so they are strongly Connected
else weak connected

Clustering coefficient

sum of links of neighbour of ith node

$$C_i = \frac{2e_i}{K_i(K_i-1)}$$

degree of particular nodes

$\langle C \rangle$ of Clustering coefficient

$$= \frac{2e_i}{K_i(K_i-1) \times \text{total no of nodes}}$$

Probability

Average degree = probability × nodes

$p = \frac{1}{6}$ $N = 10$

$\langle K \rangle \approx 1.5$

Q Probability having link L

$$P(L) = \binom{N_{C_2}}{L} p^L (1-p)^{\frac{N(N-1)}{2} - L}$$

L_{max}

nodes

given

$P(x) = {}^{N}C_{x}\, p^{x}(1-p)^{N-x}$ (probability)

(mean)

$\langle x \rangle = Np$

$\langle x^2 \rangle = p(1-p)N + p^2N^2$ (variance)

$\sigma_x = [p(1-p)N]^{1/2}$ (standard deviation)

Average no. of link of random graph $= p\,\frac{N(N-1)}{2}$

$\sigma^2 = p(1-p)\frac{N(N-1)}{2}$

$\langle K \rangle = \frac{2L}{N} = p(N-1)$

degree distribution

$p(k) = {}^{N-1}C_{k}\, p^{k}(1-p)^{(N-1)-k}$

$\langle k \rangle = p(N-1)$

$\sigma_k^2 = p(1-p)(N-1)$

$\frac{\sigma_k}{\langle k \rangle} = \left[\frac{1-p}{p}\frac{1}{(N-1)}\right]^{1/2} \simeq \frac{1}{(N-1)^{1/2}}$

$$P(k) = {}^{N-1}C_k \, p^k (1-p)^{(N-1)-k}$$

$${}^{N-1}C_k = \frac{(N-1)!}{k(N-1-k)!} = \frac{N-1 \times N-2 \times N-3 \cdots (N-1-k)!}{k!(N-1-k)!}$$

$$\frac{(N-1)^k \left(1 \times \frac{N-2}{N-1} \times \frac{N-3}{N-1} \cdots \frac{\cancel{(N-1-k)}!}{N-1} \right.}{k!\cancel{(N-1-k)}!}$$

$$= \frac{(N-1)^k}{k!} \quad \text{rest is} \simeq 1$$

$$(1-p)^{(N-1)-k} = 1 \cdot (1-p)^{(N-1)-k}$$

$$= \cancel{e^{\log(1-p)^{(N-1)-k}}}$$

$$= e^{\log_e (1-p)^{(N-1)-k}}$$

$$= e^{(N-1-k)\log_e(1-p)}$$

$$= e^{(N-1-k)-\left(p + \frac{p^2}{2} + \frac{p^3}{3} + \cdots \infty\right)}$$

$$= e^{-(N-1-k) \cdot p \left(1 + \frac{p}{2} + \frac{p^2}{3} + \cdots \infty\right)}$$

$$= e^{-(N-1-k) \cdot p} \qquad 1 + \frac{p}{2} + \frac{p^2}{3} + \cdots \infty \simeq 1$$

$$\langle k \rangle = p(N-1)$$

$$p = \frac{\langle k \rangle}{N-1}$$

$$= e^{-(N-1-k)\frac{\langle k \rangle}{N-1}}$$

$$= e^{-\cancel{(N-1)}\left(1-\frac{k}{N-1}\right)\frac{\langle k \rangle}{\cancel{N-1}}}$$

$$= e^{-\left(1-\frac{k}{N-1}\right)\langle k \rangle} \qquad N \simeq \infty$$

$$\text{so } \left(1 - \frac{k}{N-1}\right) \simeq 1$$

$$= e^{-\langle k \rangle}$$

Put all values

$$= \frac{(N-1)^k}{k!} \times p^k \, e^{-\langle k \rangle}$$

$$p = \frac{\langle k \rangle}{N-1}$$

$$= \frac{\cancel{(N-1)^k}}{k!} \times \frac{\langle k \rangle^k}{\cancel{(N-1)^k}} e^{-\langle k \rangle}$$

$$= \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \Rightarrow P(k) \Rightarrow e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$

$\Downarrow$

probability k^{th} degee in any random graph of from poison distribution

~~Q prob degree $k > 2000$ is 10^{-27}~~

Q $\langle k \rangle = 1000$, $N = 10^9$

$e^{-1000} \times 1000$

Distance in random graph

$\langle k \rangle^0 \Rightarrow$ Nodes at distance 0 $(d=0)$

$\langle k \rangle^1 \Rightarrow$ Nodes at distance one $(d=1)$

$\langle k \rangle^2 \Rightarrow$ " " " 2 $(d=2)$

$$N = 1 + \langle k \rangle + \langle k \rangle^2 + \cdots - \langle k \rangle^{d_{max}}$$

$$= \frac{r^n - a}{r - a}$$

$$N = \frac{\langle k \rangle^{d_{max}+1} - 1}{\langle k \rangle - 1}$$

$\langle k \rangle^{d_{max}+1}$ is so big so it have no effect when we subtract somethig from it or devide from it

$$N \simeq \langle k \rangle^{d_{max}}$$

$$\log N \simeq \log \langle k \rangle^{d_{max}}$$

$$\log N \simeq d_{max} \log \langle k \rangle$$

$$d_{max} = \frac{\log}{\log \langle k \rangle}$$

OR

$$\boxed{\langle d \rangle = \frac{\log N}{\log \langle k \rangle}}$$

$= \frac{1}{\log \langle K \rangle}$ → if graph is dense so distance b/w two nodes is min

Clustering coefficient of random graph

$C_i = \frac{2 \langle L \rangle}{K_i (K_i - 1)} = \frac{2 \times p \left(\frac{N(N-1)}{2} \right)}{K_i (K_i - 1)}$

$p = \frac{\langle K \rangle}{N}$

Average path length for random graph

$\langle d_{max} \rangle = \frac{\log N}{\log \langle K \rangle}$

api contain only contain abstract classes and interfaces not implementation.

Discrete vs Continuous formalism

Discrete

$$p_k = C k^{-\gamma}$$

$$\sum_{k=1}^{\infty} p_k = 1$$

$$C \sum_{k=1}^{\infty} k^{-\gamma} = 1$$

$$C = \frac{1}{\sum_{k=1}^{\infty} k^{-\gamma}} = \boxed{\frac{1}{\zeta(\gamma)}}$$

$$p_k = \frac{1}{\zeta \gamma} k^{-\gamma}$$

Continuous

$$p_k = C k^{-\gamma}$$

$$\int_{k_{min}}^{\infty} p(k)\, dk = 1$$

$$\int_{k_{min}}^{\infty} C k^{-\gamma}\, dk = 1$$

$$C \int_{k_{min}}^{\infty} C k^{-\gamma}\, dk = 1$$

$$C \left[\frac{k^{-\gamma+1}}{-\gamma+1} \right]_{k_{min}}^{\infty} = 1$$

$$\frac{C}{-\gamma+1} \left[k^{-\gamma+1} \right]_{k_{min}}^{\infty} = 1$$

$$\frac{C}{-\gamma+1} \left[\frac{1}{\infty} - \frac{1}{k_{min}^{\gamma-1}} \right] = 1$$

$$\cancel{-}\frac{C}{\cancel{-}(\gamma-1)} \left[\frac{1}{k_{min}^{\gamma-1}} \right] = 1 \qquad \frac{1}{\infty} = 0$$

$$c = \boxed{(\gamma-1) \times K_{min}^{\gamma-1}}$$

$$P_K = (\gamma-1) \times K_{min}^{\gamma-1} K^{-\gamma}$$

Expected maximum degree $= K_{max}$

Estimating K_{max} continuous

~~What~~ prob to have a node larger than K_{max} should not exceed the prob to have one node

i.e $\frac{1}{N}$ fraction of all node

$$\int_{K_{max}}^{\infty} P(K)\, dK \simeq \frac{1}{N}$$

$$\int_{K_{max}}^{\infty} (\gamma-1) \times K_{min}^{\gamma-1} K^{-\gamma}$$

$$(\gamma-1) K_{min}^{\gamma-1} \int_{K_{max}}^{\infty} K^{-\gamma} dK \Rightarrow K_{min}^{\gamma-1} (\gamma-1) \left[\frac{K^{-\gamma+1}}{-\gamma+1}\right]_{K_{max}}^{\infty}$$

$$= \frac{(\gamma-1) K_{min}^{\gamma-1}}{-\gamma+1} \left[K^{-\gamma+1}\right]_{K_{max}}^{\infty}$$

$$= \frac{\cancel{\gamma-1}(K_{min}^{\gamma-1})}{-(\cancel{\gamma-1})} \left[\frac{1}{\infty} - \frac{1}{K_{max}^{\gamma-1}}\right] \Rightarrow \frac{K_{min}^{\gamma-1}}{K_{max}^{\gamma-1}}$$

$$\frac{K_{main}^{\gamma-1}}{K_{max}^{\gamma-1}} \simeq \frac{1}{N}$$

$$\left[K_{max} = K_{min} N^{\frac{1}{\gamma-1}} \right]$$

Divergences in scale-free distributions

$$P(K) = CK^{-\gamma} \qquad K = [K_{min}, \infty)$$

$$P(K) = (\gamma-1) K_{min}^{\gamma-1} K^{-\gamma}$$

$$\langle K^m \rangle = \int_{K_{min}}^{\infty} K^m P(K)\, dK$$

$$\langle K^m \rangle = (\gamma-1) K_{min}^{\gamma-1} \int_{K_{min}}^{\infty} K^{m-\gamma}\, dK$$

$$= (\gamma-1) K_{min}^{\gamma-1} \left[\frac{K^{m-\gamma+1}}{m-\gamma+1} \right]_{K_{min}}^{\infty}$$

$$= \frac{(\gamma-1) K_{min}^{\gamma-1}}{m-\gamma+1} \left[K^{m-\gamma+1} \right]_{K_{min}}^{\infty}$$

$$= -\frac{(\gamma-1)}{(m-\gamma+1)} K_{min}^{m}$$

generating networks with a pre-defined pk

$$P_{ij} = \frac{K_i K_j}{2L-1}$$

Hidden parameter model

$$P(n_i, n_j) = \frac{n_i n_j}{\langle n \rangle N}$$

$$P_K = \int \frac{e^{-n} n^k}{K!} p(n)\, dn$$

$$n_j = \frac{c}{i^{\alpha}} \qquad i = 1, 2 \ldots N$$

$$P_K = \frac{1}{N} \sum_j \frac{e^{-n_j} n_j^{k}}{K!}$$

$$P_K \sim K^{-(1+\frac{1}{\alpha})}$$