Ch- Probability

→ Sample space: Every element of the set expressing

A sample space is called sample point.
→ {H, T} → coin   Die → {1, 2, 3, 4, 5, 6}.

→ Total no. of Sample space:
Coin → 2.
Die → 6

⇒ Mutually exclusive:
In this if any one of the given events occurs, then the others can't occur.

If A & B are mutually exclusive events then

$A \cap B = \phi$ OR $P(A \cap B) = 0$

→ Non-mutually exclusive:
In this if A & B are two events are non-mutually exclusive than both events are happen,

$A \cap B = \phi$ OR $P(A \cap B) \neq 0$

⇒ Probability:
If m is the favourable case of event A & n is total case.

$$P(A) = \frac{m}{n}$$

Date
Page No.

$P(A) + P(\bar{A}) = 1$ OR $P(A) = 1 - P(\bar{A})$

$P(\bar{A}) = 1 - P(A)$

limit = $0 \le P \le 1$

→ odds of fabourable event :-

$\Rightarrow \dfrac{\text{no. of fabourable event}}{\text{no. of unfabourable event}} = \dfrac{m}{n-m}$

→ odds against even :-

$\Rightarrow \dfrac{\text{no. of unfabourable event}}{\text{no. of fabourable event}} = \dfrac{n-m}{m}$

atleast one, either or, one of them = $\cup$ or = $\cup$

Simultaneous, this & this = $\cap$ and = $\cap$

⇒ Theorem :-

1) Additional theorant

Case - I) If events are mutually exclusive

1) If A & B are two events are mutually exclusive then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2) If A, B, C three events are mutually exclusive then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

Date
Page No.

Case-II) If events are in non-mutually exclusive then

1) If A & B two events are non-mutually exclusive then

$P(A \cup B) = P(A) + P(B)$ because $P(A \cap B)$ is zero in this case.

2) If A, B, C three events are non-mutually exclusive then

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$.

3) If A, B, C, D ---- n are n non-mutually exclusive then

$P(A \cup B \cup C, ----- \cup n) = P(A) + P(B) + P(C) ----- P(n)$

4) If A, B, C, D ---- n are n non-mutually exclusive then

$P(A) + P(B) + P(C) ---- P(n) = 1$.

⇒ A few theorems on probability:

1) $P(A \cup B) = P(A) + P(B)$ are mutually exclusive.

2) $P(A') = 1 - P(A)$.

3) $P(A \cap B') = P(A) - P(A \cap B)$

4) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

5) If A & B are two events then
P(exactly one of A, B occurs).

Date
Page No.

$\Rightarrow P[(A\cap B')\cup(A'\cap B)]$

$\Rightarrow P(A)-P(A\cap B)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cup B)-P(A\cap B)$

Also, P(exactly one of A, B occurs).

$\Rightarrow P(A\cap B')+P(A'\cap B)$

$\Rightarrow P(B')-P(A'\cap B')+P(A')-P(A'\cap B')$

$\Rightarrow P(A')+P(B')-2P(A'\cap B')$

$\Rightarrow P(A'\cup B')-P(A'\cap B')$.

6) If A & B are two events:

$P(A'\cup B')=1-P(A\cap B)$ &

$P(A'\cap B')=1-P(A\cup B)$

7) If $A_1, A_2, ---- A_n$ are n events, then

$P(A_1\cup A_2\cup ----- \cup A_n)$

$$\Rightarrow \sum_{i=1}^{n} P(A_i) - \sum_{1\le i<j\le n} P(A_i\cap A_j) + \sum_{1\le i< j\le k\le n} P(A_i\cap A_j\cap A_k) ---- + (-1)^{n-1} P(A_1\cap A_2 ----\cap A_n)$$

8) If A, B & C are three events then.

1) $P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

2) P(at least two of A, B, C occur)

$P(B\cap C)+P(C\cap A)+P(A\cap B)-2P(A\cap B\cap C)$.

Date
Page No.

3) P (exactly two of A, B, C occur)
$\Rightarrow P(B \cap C) + P(C \cap A) + P(A \cap B) - 3P(A \cap B \cap C)$

4) P(exactly one of A, B, C occur)
$P(A) + P(B) + P(C) - 2P(B \cap C) - 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

9) If $A_1, A_2, \dots A_n$ are n events then

1) $P(A_1 \cup A_2 \cup \dots \cup A_n) \leq P(A_1) + P(A_2) + \dots + P(A_n)$

2) $P(A_1 \cap A_2 \cap \dots \cap A_n) \geq 1 - P(A'_1) - P(A'_2) - \dots - P(A'_n)$

10) If $A_1, A_2, \dots A_n$ are n events then,

$P(A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n) \geq P(A_1) + P(A_2) + \dots + P(A_n) - (n-1)$

11) If A & B are two events such that $A \subseteq B$ then
$P(A) \leq P(B)$

Odds in favour & against an event: If x is the number of cases favourable to the occurance of an event A & y that for the event A', then the odds in favour of A are x:y & the odds against A are y:x.
In this case.

$P(A) = \frac{x}{x+y}$ $P(A') = \frac{y}{x+y}$

Date
Page No.

⇒ Independent event:

Two events A & B are independent if the occurance or non-occurance of A(B) does not affect the probability of occurance or non-occurance of B(A) that is if $P(B/A) = P(B)$ provided $P(A) \neq 0$

$$\frac{P(B \cap A)}{P(A)} = P(B).$$

$$P(A \cap B) = P(A)\, P(B).$$

Thus, two events A & B are independent if & only if $P(A \cap B) = P(A)\, P(B)$

$$\frac{1}{52} \times \frac{1}{52}.$$

⇒ Dependent events:

In dependent event one event occurs firstly it does affect the probability of occurance of other event

1) If A & B are two dependent event then

$$P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right)$$

OR $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$ where $P(A) \neq 0$

OR

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}.$ where $P(B) \neq 0$

DELTA Notebook

Date
Page No.

2) If A, B, C are three dependent event then,

$$P(A \cap B \cap C) = P\left(\frac{B}{A} \cap C\right) + P(C)$$

Note: If A & B are two independent event.

$$P(A \cap B) = P(A)\,P(B).$$

$$P\left(\frac{A}{B}\right) \times P(B) = P(A) \times P(B).$$

$$P\left(\frac{A}{B}\right) = P(A)$$

OR

$$P\left(\frac{B}{A}\right) = P(B)$$

Ex: $\frac{1}{51} \times \frac{1}{52}$

=> For 2 diet

Sum		times
2	→	1
3	→	2
4	→	3
5	→	4
6	→	5
7	→	6
8	→	5
9	→	4.
10	→	3
11	→	2
12	→	1

Date
Page No.

at least one = 1 − none.

=> p = probability of sucess
q = probability of failure.
r = requirement.
n = Total no. of cases.

$$x = r = {}^nC_r \; q^{n-r} \; p^r$$

at least $P(x \geq r)$

at most $P(x \leq r)$

exactly $P(x = r)$

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 15 6 1

1 7 21 35 35. 21 7. 1

=> for 3 coins:

H	Favourable case
0	1 (T, T, T)
1	3 (HTT, THT, TTH)
2	3 (HHT), THH, HTH)
3	1 (H, H, H).

$n = 2^3$

Date
Page No.

→ In case of till end throw

$$\frac{a+b}{a+2b}$$

a = Probability of A wins num. part.

b = Probability of B lose num. part.

e.g.: $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$

$$\frac{1+1}{1+2} = \frac{2}{3}$$

→ Cards:

52 — 26 black, 26 Red.

Suit card:
- 13 shade
- 13 Diamond
- 13 Club
- 13 heart

Honour card →
- 4 king
- 4 Queen
- 4 - Jack
- 4 - Ace

Face card →
- 4 king
- 4 Queen
- 4 Jack

Date
Page No.

→ Chess†

64 square → 32 white / 32 black

24 cube side है

24 square are in either side of chess board. 2-2 same colour के side है।

⇒ Multiplicative theoram:

In multiplicative theoram if A & B are two events They are dependent to each other or independent to each other.

If A & B are independent to each other then the occurance of one doesn't affect the probability of occurance of other event.

1) If A & B are two independent event then,

$$P(A \cap B) = P(A) \times P(B)$$

2) If A, B & C are three independent event then,

$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$

3) If A, B, C, ---- n are n mutually independent event then

$$P(A \cap B \cap C \ldots A_n) = P(A) \times P(B) \times P(C) \ldots P(n)$$

4) Let $A_1, A_2, \ldots A_n, A_{n+1}$ be $(n+1)$ events such that $P(A_1 \cap A_2 \cap \ldots A_n) > 0$.
Then,

$$P\left(\bigcap_{i=1}^{n+1}\right) = P(A_1)\, P(A_2/A_1)\, P(A_3/A_1 \cap A_2) \ldots \ldots P\left(\frac{A_{n+1}}{A_1} \cap \ldots \cap A_n\right).$$

⇒ Conditional probability:

If event is not independent then we use conditional probability. Since the result of event conditionally depend upon the first event.

In case conditional probability:

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \qquad P(A) \neq 0$$ where event A is already happen

OR.

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \qquad P(B) \neq 0$$ where event B is already happen

$$P(A \cap B) = \left\{\begin{matrix} P(B)\, P(A/B) \\ P(A)\, P(B/A) \end{matrix}\right\} \quad \begin{matrix} \text{if } P(B) \neq 0 \\ \text{if } P(A) \neq 0 \end{matrix}$$

Date
Page No.

→ Important note:

The function $P(\,|A)$ is a probability function & hence all theorems on probability apply to this function. For instance,

$$P(B \cup C / A) = P\left(\frac{B}{A}\right) + P\left(\frac{C}{A}\right) - P\left(\frac{B \cap C}{A}\right)$$

→

→ If we want to sit n persons in row than no. of possible cases is

$$\Rightarrow n!$$

→ If we want to sit n person in circle than no. of possible cases is

$$\Rightarrow (n-1)!$$

DELTA Notebook

Date
Page No.

→ If it said n person are sit in circle & some (1, 2, ----) are sit & some are sits b/w them then find the probability of which they are sit b/w them

$$\Rightarrow \frac{2}{n-1}$$

e.g:- Q-15 person among A & B are sit down at random at a round table. The probability that there are 4 persons b/w A & B is,

Sol:- $\frac{2}{n-1} \Rightarrow \frac{2}{15-1} \Rightarrow \frac{2}{14} \Rightarrow \frac{1}{7}$

→ In case of distribution total no. of cases:-

$$T = n^r$$

where n = Total no. of (person, things) which among you ~~what~~ want distribute things.

r = total no. things that you want to distribute

e.g:- distribute 16 items b/w peoples

$$\Rightarrow 3^{16}$$

$ABC = 3!$

$AAB = \frac{3!}{2!}$

$AAA = \frac{3!}{3!} = 1.$

Date
Page No.

⇒ Bayes' theorem:

Let S be the sample space & $E_1, E_2, ----E_n$ are the mutually exclusive event with random experiment & A is the event which is associated with $E_1, E_2, ------ E_n$ then

$$P\left(\frac{E_i}{A}\right) = \frac{P(E_i) \times P\left(\frac{E_i}{A}\right)}{\sum_{i=1}^{n} P(E_i) \times P\left(\frac{E_i}{A}\right)}$$

where $i = 1, 2, 3, ------- n$.

⇒ For 3 die

M	Favourable.
3	1
4	3
5	6
6	10
7	15
8	21
9	25
10	27
11	27
12	25
13	21
14	15
15	10
16	6
17	3
18	1

Date
Page No.

f = total no. of face marked (1, 2, 3, 4 - - - -)
n = no. of throwing.
p = total no. of sum.

coff of x^p in $\dfrac{(x^1 + x^2 + x^3 - - - - - - x^f)^n}{f^n}$

coff. of x^p in $\dfrac{x^n(1 + x^1 + x^2 + x^3 + - - - x^{f-1})^n}{f^n}$

coff. of x^{p-n} in $\dfrac{(1 + x^1 + x^2 + x^3 - - - - - x^{f-1})^n}{f^n}$

coff. of x^{p-n} in $\dfrac{\left[\dfrac{1-x^f}{1-x}\right]^n}{f^n}$ $\left(\dfrac{a}{1-r}\right)$

coff. of x^{p-n} in $\dfrac{(1-x^f)^n(1-x)^{-n}}{f^n}$

Date
Page No.

Probability Density function :

A function $f(x)$ defined over a set of real values, is called probability density function (pdf) of continuous random variable X if.

$$P(a \le x \le b) = \int_a^b f(x)\,dx$$

⇒ Properties of P.D.f:

1) A function $f(x)$ can serve as a p.d.f of a continous random variable x, if it satisfies.

a) $f(x) \ge 0, \quad -\infty < x < \infty$

b) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

2) Commulative Distribution function:

If X is a continuous random variable, then function $F(x)$ is given by

$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx; \quad -\infty < x < \infty$ is called cummulative distribution of X, where $f(x)$ is p.d.f

It may be noted that $f(x) = \frac{d}{dx} F(x)$.

2) Continuous variables & probability Density functions:

1) Mean $= \int_{-\infty}^{\infty} x\, f(x)\,dx$.

b) Geometric Mean G is

$\log G = \int_{-\infty}^{\infty} \log x \; f(x)\,dx$.

3) Harmonic mean H is:

$H^{-1} = \int_{-\infty}^{\infty} x^{-1} f(x)\,dx$

Date
Page No.

4) Median M_e is.

$$\int_{-\infty}^{M_e} f(x)\,dx = \int_{M_e}^{\infty} f(x)\,dx = \frac{1}{2}$$

5) The lower quartile Q_1 & upper Quartile Q_3 are

$$\int_{-\infty}^{Q_1} f(x)\,dx = \int_{Q_3}^{\infty} f(x)\,dx = \frac{1}{4}.$$

6) For mode, we have $f'(x) = 0$ & $f''(x) = -ve$

7) The r^{th} moment above arbitrary value is

$$\mu_r' = \int_{-\infty}^{\infty} (x-a)^r f(x)\,dx.$$

8) The rth moment about mean is

$$\mu_r = \int_{-\infty}^{\infty} (x-m)^r f(x)\,dx.$$

Standard deviation is $\sqrt{npq}$.

Binomial distribution.

$P(x=r) = {}^nC_r\, q^{n-r} p^r$

q = probability of failure, p = probability of success.

$P(x=r) = N \cdot {}^nC_r\, q^{n-r} p^r$

→ Properties:

1) If $p = q = \frac{1}{2}$ then this is called symmetrical distribution otherwise it is skew-symmetrical distribution

2) Moment about the arbitary constant:

a) $\mu_1' = np$ = mean , npq → variance.
b) $\mu_2' = npq + n^2p^2$
c) $\mu_3' = n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np$
d) $\mu_4' = n(n-1)(n-2)(n-3)p^4 + 6n(n-1)(n-2)p^3 + 7n(n-1)p^2 + np$

3) Moment about mean:

a) $\mu_1 = 0$
b) $\mu_2 = npq$
c) $\mu_3 = npq(q-p)$
d) $\mu_4 = npq\,[1 + 3(n-2)np]$

Date

Page No.

→ Constants of Binomial distribution:

1) Mean = np

2) Variance = $\sigma^2 = \mu_2 \, npq$

3) $\mu_3' = n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np$

4) $\mu_3 = npq(q-p)$

5) $\beta_1 = \dfrac{\mu_3^2}{\mu_2^2} = \dfrac{(1-2p)^2}{npq}$

6) $\beta_2 = \dfrac{\mu_4}{\mu_2^2} = 3 + \dfrac{1-6pq}{npq}$

7) $\gamma_1 = \sqrt{\beta_1} = \dfrac{1-2p}{\sqrt{npq}}$

8) $\gamma_2 = \beta_2 - 3 = \dfrac{1-6pq}{npq}$.

Note: mean > variance

Date

Page No.

Poison distribution:

$\Rightarrow \dfrac{e^{-m} \cdot m^r}{r!}$ $\quad\quad m = np = \text{mean}$

$r = 0, 1, 2, 3, \ldots$

out of $N = N \cdot \dfrac{e^{-m} \cdot m^r}{r!}$

→ Moment about the arbitary constant:

1) $\mu_1' = m = np$
2) $\mu_2' = m^2$
3) $\mu_3' = m^3 + m^2 + m$
4) $\mu_4' = m^4 + m^3 + m^2 + m$

→ Moment about the mean:

1) $\mu_1 = 0$
2) $\mu_2 = m$
3) $\mu_3 = m^2$
4) $\mu_4 = m^3$

5) $\beta_1 = \dfrac{1}{m}$

6) $\beta_2 = 3 + \dfrac{1}{m}$

7) $\gamma_1 = \dfrac{1}{\sqrt{m}}$

8) $\gamma_2 = \beta_2 - 3 = \dfrac{1}{m}$

Date
Page No.

9) $m.\sigma\ \gamma_1\ \gamma_2 = m\ \sqrt{npq}\ \frac{1}{\sqrt{m}}\ \frac{1}{m} = m.\sigma\ \sqrt{\beta_1}\ \beta_2 - 3.$

$S.D = \sqrt{npq}$ mode $= np + p > x > np - q$

Variance $= npq$

→ Mode:

Paison distribution either 0 or 1 is m when m is not a integer then mode = m
when m is integer then mode is m & m-1

If m=1 then mean deviation about mean is

$$\frac{2\sigma}{e}.$$ $np + p > x > np - q$

→ Additive property:

If X & Y are independent event.

In peison distribution mean > variance.

$e^{-1} = 0.3699$ $e^{-0.4} = 0.670$
$e^{-2} = 0.1353$
$e^{-3} = 0.0498$
$e^{-0.2} = 0.8187.$

DELTA Notebook

Normal distribution & symmetrical distribution

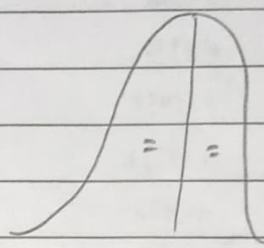
⇒ Normal distribution:

$$Z = \frac{X - m}{\sigma} \qquad m = \text{mean}$$

$\sigma =$

1) The curve is symmetrical about the mean is $X = m \; (z = 0)$

mean = median = mode.



$Q_1 = m - 0.6745\,\sigma$ ~~Q_3~~ $= 0$

$Q_3 = m + 0.6745\,\sigma$

- The point of inflaction about the mean is ⇒ $x \pm \sigma$
- The moment about the even order about mean is zero.
- The all moment about the even order about mean

$$\mu_{2n} = \sigma^2 (2n-1)\, \mu_{n-2}.$$

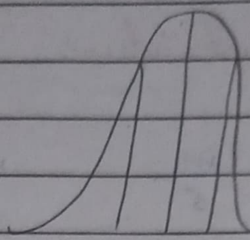
- The theortical range of distribution $(-\infty$ to $\infty)$ & practicle range is $(-3\sigma$ to $3\sigma)$ & overall range is overall 6σ.

Date

Page No.

⇒ The Area under the curve:

$z \pm 1$	68.27	
$z \pm 2$	95.45	
$z \pm 3$	99.73.	



Z	Area
0.5	0.1915
1	0.3432
1.5	0.4332
2	0.4772
2.5	0.4938
3	0.4987
3.5	0.4998
3.90	0.5
0.33	0.1293