Date..................

Basic formula & Defination

(i) What is Expectation?

It represents the average value or the long-run average of the variable's outcomes when the experiment is reapeated many times under the same condition.

(Average)

~~The probo~~

The Expectation of random variable 'X' is $E(X)$ or μ

& It is calculated as "Sum of each possible outcome x_i of the variable" weighted by its probablity $P(X=x_i)$

That is $\boxed{E(X) = \sum_i x_i \cdot P(X=x_i)}$

① Multiply each possible outcomes by its probablity of occurrence.

② Sum up these product.

The above formula gives the average value or expected value.

understand with sort Example

Date..................

Ex:-

0 1 2 3 4 ← index

[5 , 3 , 1 , 4 , 2]

we are given an iteger array & we are appling insersion sort on the given array.

(i) for element (5), No elements in sorted portion so probablity is '0'.

(ii) for element (3), 1 Elements in sorted portion so probablity is

:- $\frac{1}{1} = 1$ $\frac{1}{i}$

(iii) for element (1), 2 Elements in sorted portion so probablity is

:- $\frac{1}{2} = 0.5$ $\frac{1}{i}$

(iv) for element (4), 3 elements in sorted portion so probablity is

:- $\frac{1}{3} = 0.33$ $\frac{1}{i}$

(v) for element (2); 4 elements in sorted portion so probablity is

:- $\frac{1}{4} = 0.25$ $\frac{1}{i}$

Date..................

~~Then~~ Expectati $\boxed{x_i = 1,2,3,4,5}$ $\boxed{\sum_{i=1}^{i-1} P_i x_i}$

(i) Second Element (3) Expectation $= 1 \times 1$

(ii) Third Element (1) Expectation $= 0.5 \times 1 + 0.5 \times 2$

(iii) fourth Element (4) Expectation $= 0.33 \times 1 + 0.33 \times 2 + 0.33 \times 3$

(iv) fifth Element (2) Expectation $= 0.25 \times 1 + 0.25 \times 2 + 0.25 \times 3$
$+ 0.25 \times 4$

Total Expected No of comparision.

$$E(T(5)) = E(x_2) + E(x_3) + E(x_4) + E(x_5)$$

$\Rightarrow 1 \times 1 + 0.5 \times 1 + 0.5 \times 2 + 0.33 \times 1 + 0.33 \times 2 + 0.33 \times 3 +$

$0.25 \times 1 + 0.25 \times 2 + 0.25 \times 3 + 0.25 \times 4$

$\Rightarrow 7$

$\boxed{E(T(5)) = 7}$

prof on Next page.

Prof of insersion sort :- Date..................

1	2	3	4	5
3	4	5	1	2
			Key	

Assume $i = i+1$
Because if we find probablty of 0^{th} element $\frac{1}{i} = \frac{1}{0} = \infty$

The key will go to j^{th} position the probablity will be P_j

$(1 \le j \le i+1)$

The Average no. of Comparition
or
~~Expectation of $(i+1)^{th}$ element will~~ be

Expectation of $(i)^{th}$ Element

$$= \sum_{j=1}^{i+1} j . P_j$$

one+ compurision extra.
Termation condition

we priviously see that every element has a prabablity $\left[\frac{1}{i}\right]$

But we takeing $\boxed{i = i+1}$

so

$$\boxed{P_1 = P_2 = P_3 = \frac{1}{i+1}}$$

Date....................

So ~~a~~ ~~no~~

Average no. of comparition of i^{th} element

$$E(i) = \sum_{j=1}^{i+1} P_i \times j$$

$$= P_i \times 1 + P_i \times 2 + P_i \times 3 \ldots P_i \times (i+1)$$

$$= \sum_{j=1}^{i+1} \frac{1}{i+1} \times j$$

$$= \frac{1}{i+1} \sum_{j=1}^{i+1} j$$

$$= \frac{1}{i+1} \left[1 + 2 + 3 \ldots i+1 \right]$$

$$= \frac{1}{\cancel{i+1}} \left(\frac{\cancel{(i+1)} \times (i+2)}{2} \right)$$

$$= \frac{i+2}{2}$$

Note it is only one element Expectation But we have N number of Elements.

$$E(T(N)) = \sum_{i=1}^{N-1} \frac{i+2}{2}$$ ← i start from 1

Spiral Teacher's Sign ....

$$\sum_{i=1}^{N-1} \frac{i}{2} + \sum_{i=1}^{N-1} 1$$

$$\frac{n(n-1)}{4} + n-1$$

$$= \frac{(n-1)(n+4)}{4}$$

$$= O(n^2)$$