$S = \{ HH, \boxed{HT}, TH, TT \}$

$\frac{2}{4} = \frac{1}{2}$ ✓

RB   RW   WR   BR

$\frac{1}{3} \times \frac{1}{2}$ $\frac{1}{3} \times \frac{1}{2}$ $\frac{1}{3} \times \frac{1}{2}$ $\frac{1}{3} \times \frac{1}{2}$

$\frac{3}{4}$ $\frac{1}{4}$

H T

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ ✓

$\frac{4}{6} = \frac{2}{3}$ ✓

1 R
1 B
1 W

$\frac{2}{3C_2} = \frac{2}{3}$ ✓   5   7   2

8

(1,1) 1.

$\frac{4}{36}$ ✓ $\frac{6}{36}$ ✓ $\frac{5}{36}$ ✓

1 2 3 4 5 6

7

AB, C, D, E

AB ✓
BA ✓

$^5P_2 = \frac{5!}{3!} = 20$

2

4 R
2 G

6

$\frac{8}{15} = \frac{4 \times 2}{15} = \frac{^4C_1 \times {^2C_1}}{^6C_2}$

RG

P(A∪B)   Girl or Brown hair   11

Q — $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$

Ques ①
$$\frac{{}^{16}C_2}{{}^{52}C_2} = \frac{16 \times 15}{52 \times 51}$$

②

$P(A) + P(B) + P(C) + P(D) = 1$

$4P(D) + 2P(D) + 2P(D) + P(D) = 1$

$9P(D) = 1$

$P(D) = 1/9$

$P(C) = 2P(D) = 2/9$ $\quad P(A) = \frac{4}{9}$

$P(\bar{A}) = 5/9$

② 3 freshm
4 Sen
5 junio
2 sen

${}^3C_1 \times {}^4C_1 \times {}^5C_1 \times {}^2C_1$

$3 \times 4 \times 5 \times 2$

$= 120$

③ 4 math
3 chem
2 phy
1 bio

$4! \times (4! \times 3! \times 2! \times 1!)$

④ $\frac{\cancel{{}^7C_5 + {}^6C_5}}{{}^8C_5}$

$\cancel{{}^8C_3 \times {}^5C_2}$

${}^2C_1 \times \cancel{{}^7C_4} \times {}^6C_3$

${}^6C_5 + {}^2C_1 \times {}^6C_4$

① 9 → 3, 4, 2

$^9C_3 \times {}^6C_4 \times {}^2C_2$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$P(A/B) \cdot P(B) = P(A \cap B)$

$P(A) \cdot P(B) = P(A \cap B)$

②

16 R, x B	4 R, 6 B
II	I

$P(A/B)$

$P(R)\ \frac{16C_1 + 4C_1}{16+x\ C_1 \times {}^{10}C_1} = 0.44$

$\frac{16+4}{} = \frac{20}{}$

$\frac{16}{16+x} \times \frac{4}{10} + \frac{x}{16+x} \times \frac{6}{10} = 0.44$

$\boxed{x = 4}$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

③ $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$P(A \cap B) = P(A/B) \cdot P(B)$

$= P(A) \cdot P(B)$

$\{ P(A/B) = P(A)$

④ 3 R

2 B

E1 : 2nd marbel red

E2 : 1st marb - red

$P(E1/E2) = \frac{P(E_1 \cap E_2)}{P(E_2)}$

$\frac{3C_2 / 5C_2}{3C_1 \times 2C_1}$

$= \frac{3C_2/5C_2}{\cancel{5C_2}}$

$= \frac{3}{\cdot}$

$$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{{}^3C_2 / {}^5C_2}{{}^3C_1 \times {}^2C_1 / {}^5C_2}$$

$$= \frac{3}{6} = \frac{1}{2}$$

13

2. E_1: X has ex— two big

E_2: Y h—— One big

52

4   48

$$P\left(\frac{E_2}{E_1}\right) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{\overbrace{{}^4C_2 \times {}^{48}C_{11}}^{x} \times \overbrace{{}^2C_1 \times {}^{37}C_{12}}^{y}}{{}^{52}C_{13} \times {}^{39}C_{13}}$$

52

4   48

$$\frac{{}^4C_2 \times {}^{48}C_{11}}{{}^{52}C_{13}}$$

3. $$= \frac{{}^5C_1}{{}^{12}C_1} \cdot \frac{{}^6C_1}{{}^{13}C_1} \times \frac{{}^7C_1}{{}^{14}C_1}$$

$\boxed{\begin{matrix} 5 & W \\ 7 & B \end{matrix}}$

4. $P(A/B) = 4/5$

$P(A/C) = \frac{1}{7}$

$P(B) = \frac{1}{2}$

$P(C) = \frac{1}{2}$

B

C

$P(C) = P(C) \cdot P(A/C)$

$= \frac{1}{2} \times \frac{1}{7}$

$$P(A/B) = \frac{P(A\cap B)}{P(B)} \Rightarrow P(A/B)\cdot P(B) = P(A\cap B)$$

$$P(B/A) = \frac{P(B\cap A)}{P(A)} = P(B/A)\cdot P(A) = P(A\cap B)$$

$$\therefore P(A/B)\,P(B) = P(B/A)\cdot P(A)$$

$$\boxed{P(A/B) = \frac{P(B/A)\cdot P(A)}{\boxed{P(B)}}}$$

① $\frac{2}{3}$ $\frac{4}{1}$ $\frac{2}{6}$ $\frac{3}{1}$ $\frac{2}{4}$

$\boxed{P(R)} = \frac{1}{3}\times\frac{2}{5} + \frac{1}{5}\times\frac{4}{5} + \frac{1}{5}\times\frac{2}{6}$

② $P(G) = \frac{60}{100} = .6$

$P(B) = .4$

		LH	
40%	B	20%	
60%	G	30%	

$P(\cancel{A/E})$

$$P(L) = P(G)\cdot P(L/G) + P(B)\cdot P(L/B)$$

$= \cancel{\frac{60}{100}} \times \cancel{\frac{30\times 60}{60}} + \cancel{\frac{40}{100}}\times$

$= .6\times .3 + .4\times .2 =$

③ $P(K) = P$

$P(G) = 1-P$

$P(C/G) = \frac{1}{m}$

$$P(K/C) = \frac{P(C/K)\cdot P(K)}{P(C)}$$

$$= \frac{1\times P}{P(C/K)\cdot P(K) + P(C/G)\cdot P(G)}$$

$$P + \frac{1}{m}\times(1-P)$$

$$\frac{mp}{pm+1-p} = \frac{p}{p+\frac{1-p}{m}}$$

Qn.

$P(P/D) = .99$ | $P(D) = 0.1$

$P(P/\bar{D}) = 0.5$ | $P(\bar{D}) = 0.9$

$$P(D/P) = \frac{P(P/D)\cdot P(D)}{P(P)}$$

$$= \frac{.99 \times 0.1}{P(P/D)\cdot P(D) + P(P/\bar{D})\cdot P(\bar{D})}$$

$$= \frac{.99 \times 0.1}{.99 \times 0.1 + 0.5 \times 0.9}$$

②

$P(TZ) = .4$

$P(TO) = .6$

$P(C/TZ) = 0.95$ | $0.05 = P(O/TZ)$

$P(C/TO) = 0.90$

$$P(O_m) = P(O/TZ) \times P(TZ) + P(O/TO)\cdot P(TO)$$

$$= 0.05 \times 0.4 + 0.90 \times .6$$

$P(T0/0) = \frac{P(0/T0) \cdot P(T0)}{P(0)}$

$= \frac{0.90 \times .6}{0.56}$

(1) X 0, 1, 2, 3

X : 5 उछाला

(3)

X : 5

Discrete R.V. (int)

Continuos R.V. (float)

HH

HT, TH

$P(2) = P(X=2)$

pmf

Expectation

0 var

$\mu = E[X] = \sum_i x_i P(x_i)$

$var(X) = E[(X-\mu)^2]$

$\frac{(x_1 + x_2 \ldots)}{n}$

$\boxed{\mu = E[X] = \sum_i x_i P(x_i)}$

$\boxed{var(X) = E[(X-\mu)^2] = \sum (x_i-\mu)^2 \cdot P(x_i)}$

$E[f(x)]$

$= \sum f(x_i) P(x_i)$

$X : 15, 16, 17, \ldots 20,$

$1, 2, 3, \ldots, 20$

$$P(X=15) = \frac{{}^{14}C_4}{{}^{20}C_5}$$

$$P(X=16) = \frac{{}^{15}C_4}{{}^{20}C_5}$$

$\vdots$

$$P(X=i) = \frac{{}^{i-1}C_4}{{}^{20}C_5}$$

$P(X \geq 15) =$

Q.

3 ← 3 W, 3 R, 5 B

11

$$\boxed{Var(X) = E[X^2] - (E[X])^2}$$

Q. $P(S) = 0.20 = p$

$P(\bar{S}) = 0.80 = q$

$\boxed{n = 12}$

${}^{n}C_r \, q^{n-r} \, p^r$

$r = 0 \quad {}^{12}C_0 (0.8)^{12} \times (0.20)^0$

$1 \times (0.8)^{12}$

$r = 2$ 72 person / hour

$$\frac{(3.6)^4 \times e^{-3.6}}{4!}$$

$$\boxed{P(x) = \frac{\lambda^x e^{-\lambda}}{x!}}$$

$\Downarrow$

$P \to$ 4 per / 3 mint

x

60 min → 72

1 min → $\frac{72}{60}$

3 min → $\frac{72 \times 3}{60_{20}} = \frac{36}{10} = \boxed{3.6}$ λ

6

3 min → 2 D.

~~9 min~~ → 6

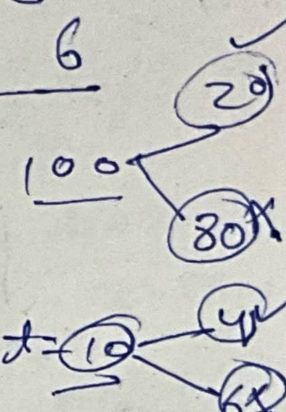
$\boxed{\lambda = 6}$ $x \ge 5$

$= 1 - (P \le 4)$

$1 - \sum_{x=0}^{4} e$

$^{n}C_r \, q^{n-r} \, p^r$

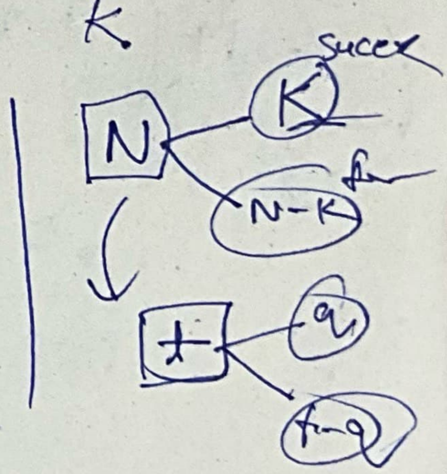
$$\frac{{}^{20}C_4 \times {}^{80}C_6}{{}^{100}C_{10}}$$



$P(X=x) = {}^{x-1}C_{r-1} \, q^{x-r} \, p^r$

PMF(

$P(X=q) = {}^{K}C_q \times {}^{N-K}C_{t-q}$



$$P(X=q) = \frac{{}^{K}C_q \; {}^{N-K}C_{t-q}}{{}^{N}C_t}$$

Random Graph

① →

② graph theor-

D1 →

Digm $\langle k \rangle = \frac{2L}{N}$

$\langle k \rangle = \frac{L}{N}$

$\frac{N(N-1)}{2} p$

$^nC_2$

$\langle x \rangle = E[x]$

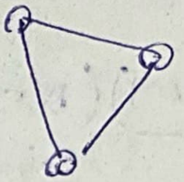
NP

$\langle x^2 \rangle = E[x^2]$

$^nC_r \, q^{n-r} \, p^r$

$\cancel{2} \times \frac{N(N-1)}{\cancel{2}} \times P$

$P(N-1)$



$\sigma_x = \sqrt{E[x^2] - (E[x])^2}$

NPq

$\frac{\sigma_k}{\langle k \rangle}$

$P(1-P)N + \cancel{P^2N^2} - \cancel{N^2P^2}$

$$\frac{p^{1/2}(1-p)^{1/2}(N-1)^{1/2}}{P(N-1)}$$

$P(K) =$

$$\boxed{{}^{N}C_r \, p^r q^{n-r}}$$

0.01

100000

99998

$$ {}^{100000}C_2 \, (0.01)^2 (0.99)^{99998} $$

Poission

$${}^{N-1}C_K = \frac{(N-1)!}{K!\,(N-1-K)!}$$

${}^{7}C_2$

$\frac{\lfloor 7}{\lfloor 2 \lfloor 5}$

$$\frac{(N-1)(N-1-1)(N-1-2)\ldots(N-1-(K+1))\,\cancel{(N-1-K}}{\lfloor K}$$

$$\frac{(N-1)^K\left[1 \times \left(1-\frac{1}{N-1}\right)\left(1-\frac{2}{N-1}\right)\cdots\left(1-\frac{K+1}{N-1}\right)\right]}{\lfloor K}$$

$$\frac{(N-1)^K}{\lfloor K}$$

$(1-p)^{(N-1)-K}$

0.01

$e^{\log_e (1-p)^{N-1-K}}$

$e^{N-1-K(\log(1-p))}$

$e^{(N-1-K)}$

$\log(1-p) = -\left(p + \frac{p^2}{2} + \frac{p^3}{3} \cdots \infty\right)$

$e^{(N-1-K)(-p)\left(1+\frac{p}{2}+\frac{p^2}{3} \cdots \infty\right)}$

$e^{-p(N-1-K)}$

$e^{-\frac{\langle k\rangle}{N-1}(N-1-K)}$

$e^{-\langle k\rangle\left(1-\frac{K}{N-1}\right)}$

$e^{-\langle k\rangle}$

$\frac{\cancel{(N-1)^K}}{\lfloor K} \frac{\langle K\rangle^K}{\cancel{(N-1)^K}} \times e^{-\langle K\rangle}$

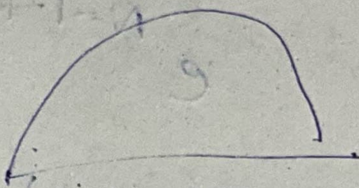
$\frac{e^{-\langle K\rangle} \langle K\rangle^K}{\lfloor K}$

$$\int_{49.5}^{50.5} \left(1.5 - 6(x-50.2)^2\right) \cdot dx$$

$$\left[1.5x - 2(x-50.2)^3\right]_{49.5}^{50.5}$$

120 140

$$\int_{-\infty}^{\infty} f(x) = 1$$



$$\cancel{\int_{-\infty}^{49.5} f(x)\,dx} + \int_{49.5}^{50.5} f(x)\,dx + \cancel{\int_{50.5}^{+\infty} f(x)\,dx}$$

$$0 + 1 + 0$$

$$\int_{49.8}^{50.1} = 0.43$$

$\frac{3}{1}$

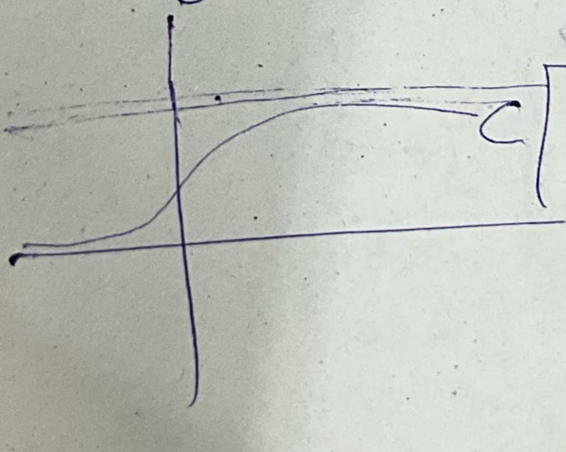
| 0 | 1 | 2 | 9 |

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx + \int_{3}^{+\infty} f(x)\,dx$$

$$0 + \int_0^3 cx^2\,dx + 0$$

e^x

$$+\, c\left[\frac{x^3}{3}\right]_0^3$$



$$c\left[\frac{27}{3} - 0\right] = 1$$

$$c = 1 \times \frac{3}{27} = \frac{1}{9}$$

$$\int_a^b \quad + \int_b^c$$

$\le \quad <$

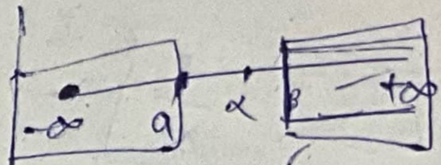
$\ge \quad >$

$120 \le X \le 140$

$20 \qquad \frac{1}{20}$

$$f(x) = \begin{cases} \frac{1}{20} & \text{for } 120 \le x \le 140 \\ 0 & \text{else} \end{cases}$$

$$f(x) = \frac{10}{20} = 0.50$$



$$\mu = \frac{x_1 + x_2 + \dots}{n}$$

Uniform Random Variable:—

$$F(a) = \begin{cases} 0 & \boxed{a \le \alpha} \\ \frac{a-\alpha}{\beta-\alpha} & \alpha < a < \beta \\ \boxed{1} & \text{else} \end{cases}$$

$$F(a) = P(X \le a) = \int_{-\infty}^{a} f(x)\,dx$$

$$\boxed{E[X] = \frac{\alpha+\beta}{2}}$$

$$F(a) = \int_0^a \lambda e^{-\lambda x} dx$$ $a \geq 0$

$\dfrac{\lambda^x \cdot e^{-\lambda}}{x!}$

$f(x)$

$$= \lambda \int_0^a e^{-\lambda x} dx$$

$$= \lambda \left[\frac{-e^{-\lambda x}}{\lambda}\right]_0^a$$

$$= -e^{-\lambda a} + e^0$$

$$= -e^{-\lambda a} + 1 = \boxed{1 - e^{-\lambda a}}$$

$\boxed{E[X] = \frac{1}{\lambda}}$

$$\int x \cdot y \, dx = x \int y \, dx - \int \frac{dw}{dx} \int y \, dx \, dx$$

$f(x) = x \cdot (1)$

UV

$E[x] = \boxed{\dfrac{x_1 + x_2 + x_3 + \dots}{n}}$

ILATE

$\boxed{\cancel{U \, dv - v \, du}}$

$E[x] = \dfrac{\Sigma x}{n}$

$$\int xy \, dx \quad x \int y - \left[\frac{dy}{dx}\right.$$

$$= x \int y \, dx - \int \left(\frac{d}{dx}(x) \int y \, dx\right) dx$$

Gamma R.V.

$$f(x) = \begin{cases} \dfrac{\lambda e^{-\lambda x} (\lambda x)^{\alpha - 1}}{\gamma(\alpha)} & \text{if } x \geq 0 \\ 0 & x < 0 \end{cases}$$

$\lambda > 0, \quad \alpha > 0$

$$\gamma(\alpha) = \int_0^\infty e^{-x} x^{\alpha-1} \, dx$$

Normal R.V. :-

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

Joint Distribution :-

Continuous case.

$$P(a \leq X \leq b, \ c \leq Y \leq d) = \int_c^d \int_a^b f(x, y) \, dx \, dy$$

$$f(x, y) = \begin{cases} 2e^{-x} e^{-2y}, & 0 < x < \infty, 0 < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

$$P\{X > 1, Y < 1\} = e^{-1}(1 - e^{-2})$$

Discrete Case

$$P(X = x, Y = y) = f(x, y)$$

4-6

Q-1 $\frac{16}{52} \times \frac{15}{51}$

$^{6}C_5 + {^2C_1} \times {^6C_4}$

2 $P(A)+B+C+D=1$

$4D+2D+2D+D$

$9D=1$

$\boxed{D=\frac{1}{9}}$

$^9C_3 \times {^6C_4} \times {^2C_2}$

8 → 6, 2

$P(C)=\frac{2}{9}$

$P(\bar{A})=1-\frac{4}{9}=$

$=5/9$

H T

HH HT TH TT

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

$\frac{4}{8}=\frac{1}{2}$

3

4 R	16 R
6 B	x B

$P(A/B)=\frac{P(A\cap B)}{P(B)}$

$\frac{4}{10}\times\frac{16}{16+x}+\frac{6}{10}\times\frac{x}{16+x}=0.44$

$\boxed{x=4}$

6 W
7 B

$\frac{5}{12}\times\frac{6}{13}\times\frac{7}{14}$

$P(A/B)=4/5$

$P(A/C)=1/7$

$\frac{1}{2}\times\frac{1}{7}$

$P(K)=P$

$P(\bar{K})=1-P$

$P(C/K)=\frac{1}{m}$ $P(C/K)=1$

$P(K/C)=\frac{P(C/K)\cdot P(K)}{P(C)}=\frac{1\times P}{\frac{m+1}{m}}=\frac{mP}{m+1}$

$P(C)=P(C/K)+P(C/\bar{K})$

$=1+\frac{1}{m}$

(1) ~~P(K)=~~

$P(K) = p$

$P(G) = 1-p$

$P(C/G) = \frac{1}{m}$

$P(C/K) = 1$

$P(C) =$

$P(C/K)\cdot P(K) + P(C/G)\cdot P(G)$

$p + \frac{1}{m}\cdot(1-p)$

$\frac{mp+1-p}{m}$

$P(K/C) = \frac{P(C/K)\cdot P(K)}{P(C)} = \frac{1\times pm}{mp+1-p}$

(2)

$P(P/D) = .99$

$P(P/\bar{D}) = .05 \times \frac{9}{10}$

D $\bar{D}$

.01 ~~.999~~

$\frac{1}{10}$ $\frac{9}{10}$

$P(P) = P(P/D) + P(P/\bar{D})$

$P(D/P) = \frac{P(P/D)\times P(D)}{P(P)} = \frac{.99\times\frac{1}{1000}}{.99\times\frac{1}{1000} + .05\times\frac{9}{10}}$

(3) $P(R/Z) = .95$

$P(R/O) = .90$

$P(O) = P(O/TO)\cdot P(TO) + P(O/TZ)\cdot P(TZ)$

$= .90\times .6 + .05\times .4$

$E[X] = \sum_i x_i\, P(x_i)$

$$\mu = E[X] = \sum_i x_i P(x_i)$$

$$E[g(X)] = \sum_i g(x_i) P(X = x_i)$$

Expectation

$$\text{Var}(X) = \sigma^2 = E[(X-\mu)^2] = \sum_i (x_i - \mu)^2 P(X = x_i)$$

S.D.

$$\sigma = \sqrt{\sigma^2}$$

Theorem

ⓐ $E[X+Y] = E[X] + E[Y]$

ⓑ $E[aX] = a\,E[X]$

ⓒ $E[aX+bY] = a\,E[X] + b\,E[Y]$

$$\text{Var}(X) = \sigma^2 = E[(X-\mu)^2] = \boxed{E[X^2] - (E[X])^2}$$

Discrete R.V.

$\cancel{E[X] = x_1}$

uniform discrete r.v. :—

① $E[X] = \dfrac{x_1 + x_2 + \dots + x_n}{n}$

② $\text{Var}[X] = \dfrac{x_1^2 + x_2^2 + \dots + x_n^2}{n} - \left(\dfrac{x_1 + x_2 + \dots + x_n}{n}\right)^2$

Bernoulli r.v. :-

$$I_A = \begin{cases} 1 & \text{if A happens} \\ 0 & \text{otherwise} \end{cases}$$

$E[X] = p$

$Var[I_A] = E[I_A^2] - (E[I_A])^2$

$= p - p^2 = \boxed{p(1-p)}$

Binomial r.v. :-

PMF $\quad P(X=i) = {}^nC_i \, p^i \, (1-p)^{n-i} \qquad \left\{ {}^nC_r \, q^{n-r} \, p^r \right.$

$E[X] = np$

$Var(X) = np(1-p)$

Poisson Distribution

PMF $\quad P(x) = \dfrac{\lambda^x e^{-\lambda}}{x!} \qquad e = 2.71$

$E[X] = \lambda$

$Var(X) = \lambda$

Geometric Distribution

$f(x) = \frac{1}{p}$

$E[X] = \frac{1}{p}$

Negative Bionomial

pmf $\Rightarrow$ $P(X=r) = {}^{x-1}C_{r-1}\, p^{r} (1-p)^{x-r}$

$E[X] = \frac{r}{p}$

$var[X] = \frac{r(1-p)}{p^2}$

Hypergeometric distribution

pmf $P(X=q) = \frac{{}^{K}C_{q} \times {}^{N-k}C_{t-q}}{{}^{N}C_{t}}$

$E[X] = \frac{tK}{N}$

$Var[X] = \frac{tk}{N}\left(\frac{N-k}{N}\right)\left(\frac{N-t}{N-1}\right)$