Given

$$A = \begin{bmatrix} N & \sum_t x^t & \sum_t (x^t)^2 & \cdots & \sum_t (x^t)^k \\ \sum_t x^t & \sum_t (x^t)^2 & \sum_t (x^t)^3 & \cdots & \sum_t (x^t)^{k+1} \\ \vdots \\ \sum_t (x^t)^k & \sum_t (x^t)^{k+1} & & \cdots & \sum_t (x^t)^{2k} \end{bmatrix}, \quad w = \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_k \end{bmatrix}, \quad y = \begin{bmatrix} \sum_t r^t \\ \sum_t r^t x^t \\ \vdots \\ \sum_t r^t (x^t)^k \end{bmatrix}$$

Now,

$$D = \begin{bmatrix} 1 & x^1 & (x^1)^2 & \cdots & (x^1)^k \\ 1 & x^2 & (x^2)^2 & \cdots & (x^2)^k \\ \vdots \\ 1 & x^N & (x^N)^2 & \cdots & (x^N)^k \end{bmatrix}, \quad r = \begin{bmatrix} r^1 \\ r^2 \\ \vdots \\ r^N \end{bmatrix}$$

To prove: $D^T D = A$ and $y = D^T r$

Then,

$$D^T D = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x^1 & x^2 & & x^N \\ (x^1)^2 & (x^2)^2 & & (x^N)^2 \\ \vdots & \vdots & & \vdots \\ (x^1)^k & (x^2)^k & \cdots & (x^N)^k \end{bmatrix} \begin{bmatrix} 1 & x^1 & (x^1)^2 & \cdots & (x^1)^k \\ 1 & x^2 & (x^2)^2 & \cdots & (x^2)^k \\ \vdots \\ 1 & x^N & (x^N)^2 & \cdots & (x^N)^k \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+\cdots+1 & x^1+x^2+\cdots+x^N & (x^1)^2+(x^2)^2+\cdots(x^N)^2 & \cdots & (x^1)^k+(x^2)^k+\cdots+(x^N)^k \\ x^1+x^2+\cdots+x^N & (x^1)^2+(x^2)^2+\cdots+(x^N)^2 & \cdots & & (x^1)^{k+1}+(x^2)^{k+1}+\cdots+(x^N)^{k+1} \\ \vdots \\ (x^1)^k+(x^2)^k+\cdots+(x^N)^k & \cdots & & & (x^1)^{2k}+(x^2)^{2k}+\cdots+(x^N)^{2k} \end{bmatrix}$$

$= A$ Hence, proved.

And,

$$D^T r = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x^1 & x^2 & & x^N \\ (x^1)^2 & (x^2)^2 & & (x^N)^2 \\ \vdots & \vdots & & \vdots \\ (x^1)^k & (x^2)^k & \cdots & (x^N)^k \end{bmatrix} \begin{bmatrix} r^1 \\ r^2 \\ \vdots \\ r^N \end{bmatrix}$$

$$= \begin{bmatrix} r^1+r^2+\cdots+r^N \\ x^1 r^1 + x^2 r^2 + \cdots + x^N r^N \\ \vdots \\ (x^1)^k r^1 + (x^2)^k r^2 + \cdots + (x^N)^k r^N \end{bmatrix} = \begin{bmatrix} \sum_t r^t \\ \sum_t r^t x^t \\ \vdots \\ \sum_t r^t (x^t)^k \end{bmatrix}$$

$= y$ Hence, proved.