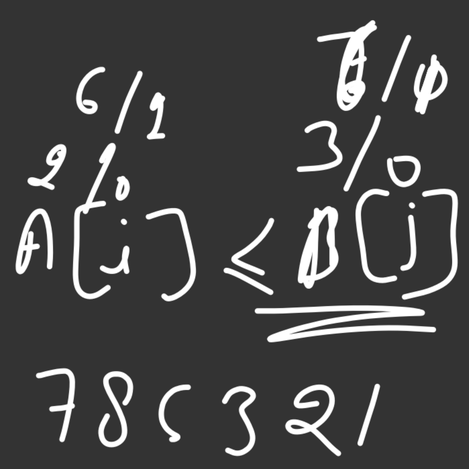
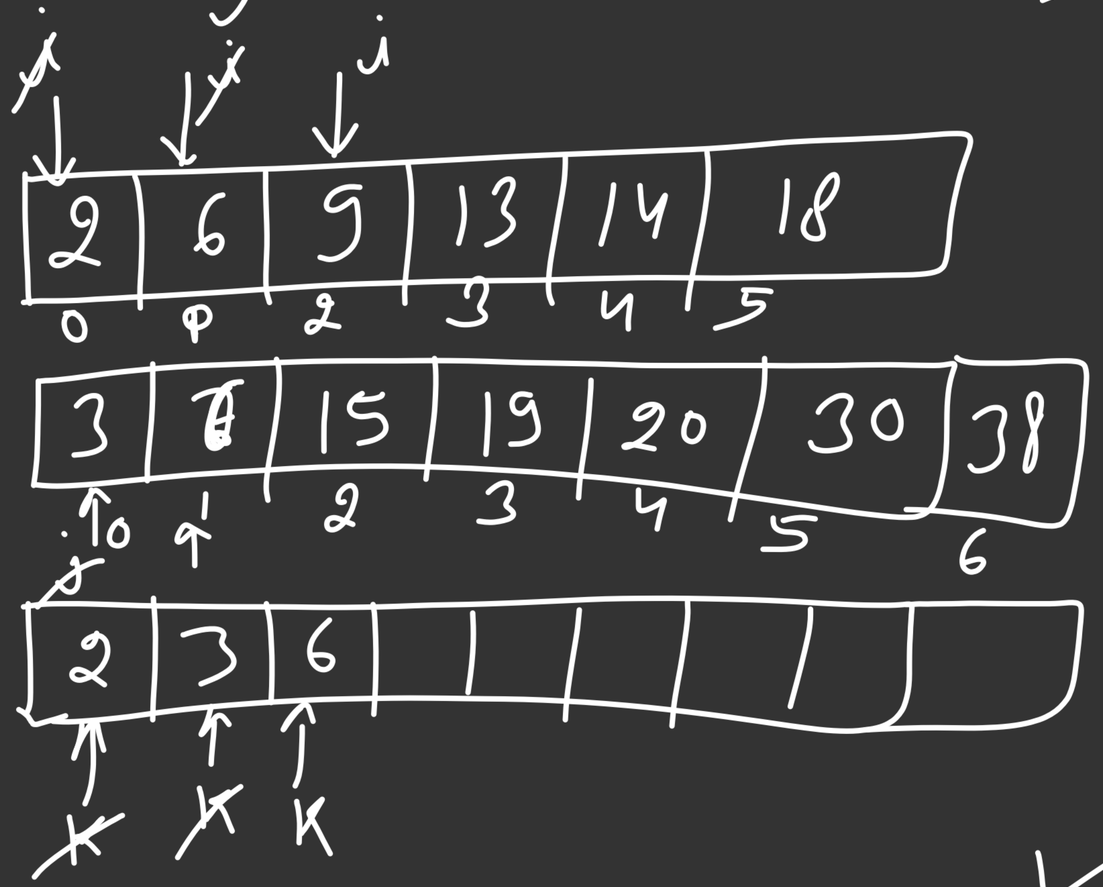
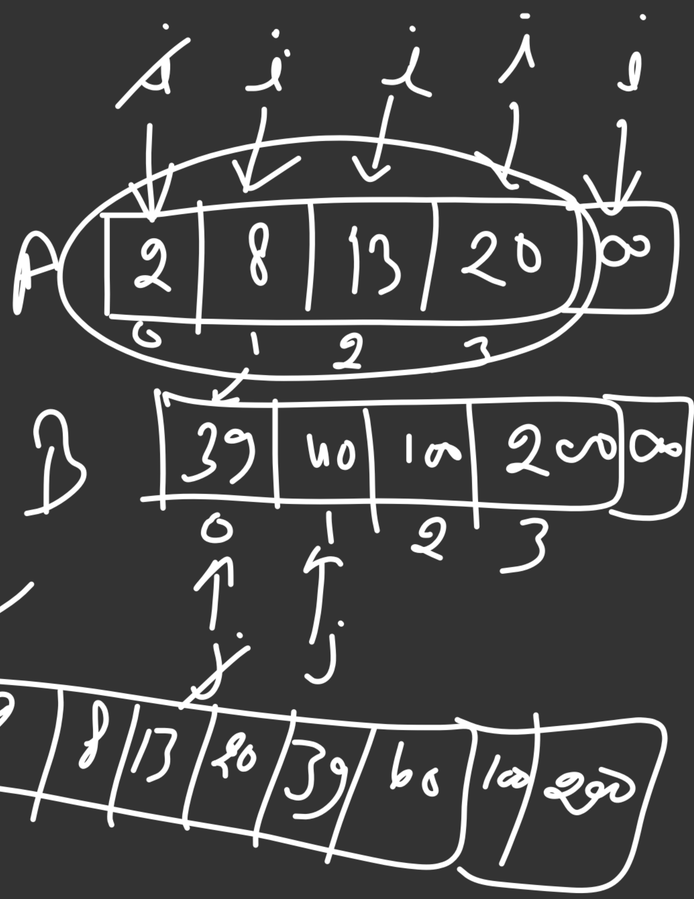
Date
14/12/22

§ Data Structures §

merging::







Sorting::

Instance

Algorithm :: Step by Step Solution of algorithmic problem.

Analyze the Algorithm :: multiple solutions.

- Time Complexity → Time taken by Algorithm
  - CPU time ✗ (depends on Hardware)
  - No. of lines ✗ (Hardware)
  - No. of Steps / Asymptotic Notations ✓
- Space Complexity → memory required

$f(n) \le c \cdot \frac{g(n)}{n}$ → Very large.

$3n+2$

$f(n) = 3n+2$ → Linear function

```
int i=0                          → Constant time = 1
for(i=0; i<n; i++)               ] n
    Pf("Hi");
}
```

$f(n) = 2n^2 + 3n + 5$

```
for (i=1; i<=n; i++)
{
    pf("Hi")
}
```

n times.

```
for (i=1; i<=n; i=i*2)
{
    pf("Hi");
}
```

$n = 2^k$

$k = \log_2 n$

i	Print (Hi)
$1 = 2^0$	1
$2 = 2^1$	2
$4 = 2^2$	3
$8 = 2^3$	4
⋮	
$n = 2^k$	$k+1 \Rightarrow \log_2 n + 1$

$O(\log n)$

```
for(i=n; i>1; i=i/2)
{
   Pf("Hi")
}
```

$\log_2 n$

Rate of growth

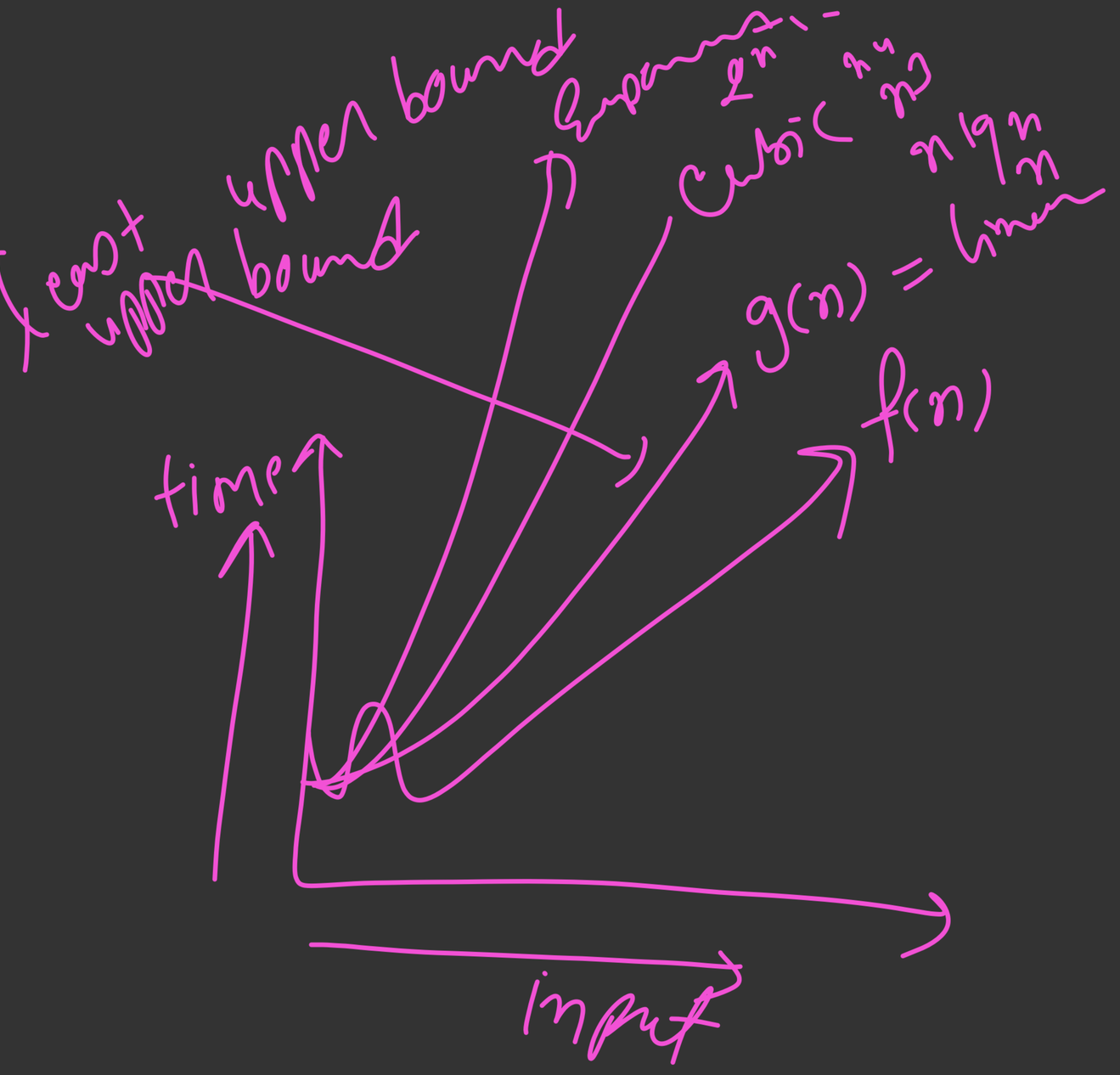
$f(n) \leq c \cdot g(n)$

$f(n) = 3n + 2$

$g(n) = n$

$f(n) \leq c \cdot g(n)$

$3n + 2 \leq c \cdot n$

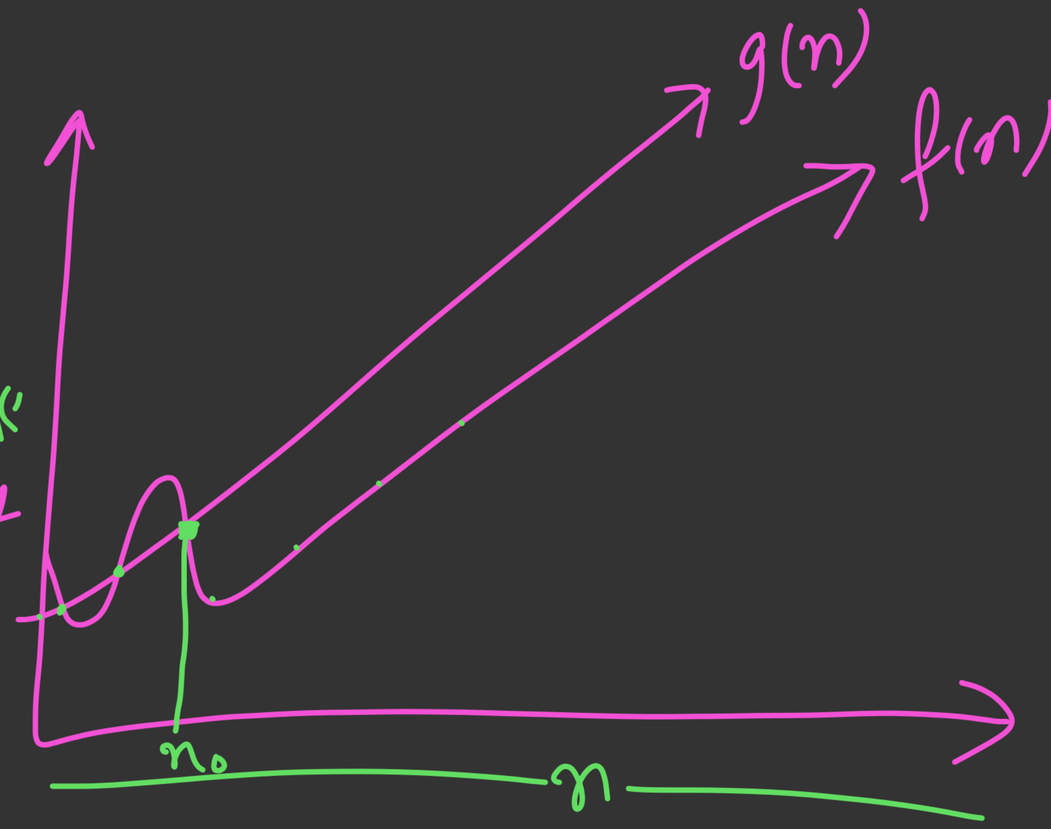


Big oh (O) notation: (worst Case)

$f(n) \le c \cdot g(n)$

$f(n) = O(g(n))$

$O(g(n)) = \{$ there exists a one time function $f(n)$ such that $f(n) \le c \cdot g(n)$ $\}$



$c > 0$, $n \ge n_0$, $n_0 \ge 1$.

$f(n) = 3n + 2$ $\quad c = 9$ $\quad n_0 = 9$. $\quad g(n) = 9n$

$f(n) \le c \cdot g(n)$ $\quad c = 5$, $n_0 = 9$.

$3n + 2 \le c \cdot n$

$3n + 2 \le 5 \cdot n$

$n \ge n_0$

$n \ge 1$

$f(n) = 2n^2 + 3n + 4$ find $f(n)$, c, & n_0

Add g c , $n_0 = 1$

$f(n) = 2n^2 - n + 5$, find $g(n)$, c, n_0

$n_0 = 1$, $c = 6$ $\quad n_0 = 2$, $c = 1$ $\quad \geq 2$

$2n^2 - n + 5 \leq 6n^2$

$2*4 - 2 - 5 \leq 4$

$2*100 - 10 - 5 \leq 100 \times 1$

$200 - 15 \leq 100$

$185 \leq 100$ ✗

$n = \sum_{i=0}^{100} \sum_{i=50}^{n/2} \sum_{i=0}^{10} 4$

Assignment

```
for(i=0; i<=n/2; i++)
```

Best Case

Big omega (Ω) notation:

$3n+2 > n / \lg n / \lg\lg n / \ldots / 1$

greatest lower bound

$f(n) = 3n+2$

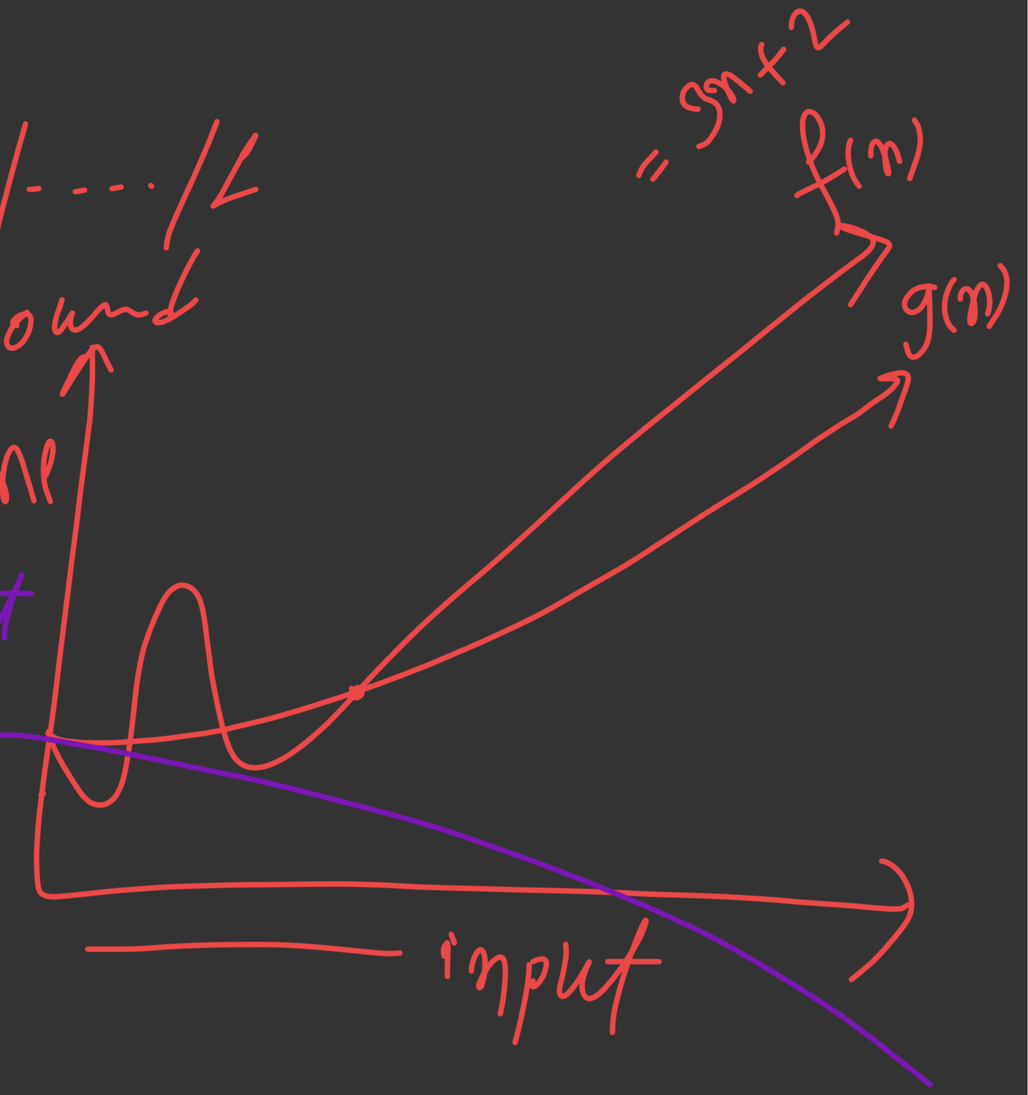
$f(n) \geq c \cdot g(n)$

$3n+2 \geq c \cdot n$

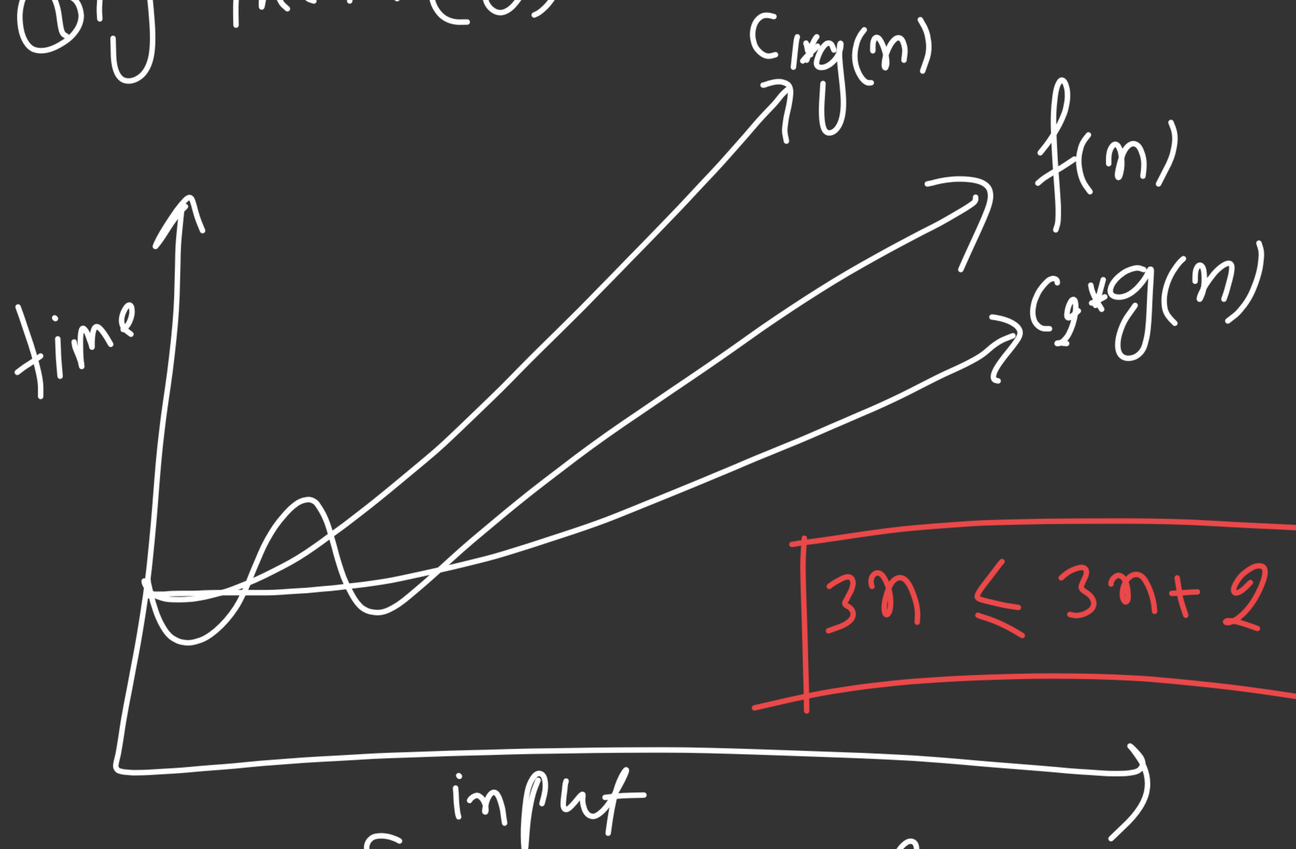
$c = 3, n_0 = 1$

$3n+2 \geq 3n$

$\Omega(g(n)) = \{ f(n)$ such that $f(n) \geq c \cdot g(n)$ where $c > 0$ & $n_0 \geq 1$



Big theta (Θ) notation...



$$3n \leq 3n+2 \leq 5n$$

$$\Theta(g(n)) = \begin{cases} \text{there exist a function} \\ f(n) \text{ such that } C_2 g(n) \leq f(n) \leq C_1 g(n) \end{cases}$$

$f(n) = 3n+2$.

$C_1 g(n) \leq 3n+2 \leq C_2 g(n)$

$C_1 n \leq 3n+2 \leq C_2 n$

C_1 C_2 & n_0

5 3 1

A

2	8	16	12	5	4	3

$n = 7$

Key = 2

Best Case = $O(1)$

Worst Case = $O(n)$

Average Case = $O(n/2)$

Amortized Analysis

$f(n) = n^{\sqrt{n}}$ $g(n) = n^{\lg n}$

$f(n) = O(g(n))$

$\sqrt{n} \lg n$ $\lg n * \lg n$

$\frac{1}{2} \lg n > \lg \lg n$