M.Sc. Computer Science
MCSC-101: Design and Analysis of Algorithms
Unique Paper Code: 223411101
Semester I
Year of Admission: 2022

Time: Three Hours **Max. Marks: 70**

***All questions are compulsory.**

1. Consider the following algorithm to insert x in an array A containing n elements in ascending order so that the resulting array is sorted. We assume that A has sufficient space to accommodate x.

```
i = n
while i ≥ 1 do
    if A[i] > x then
        A[i+1] = A[i]        /* shift the element to right
        i - -
    end
    else break
end
A[i+1] = x
return
```

i = 5

7+8+8+8+8 +9

   a. Define the Loop -Invariant H(r): when the control reaches the while statement for the rth time ______________________

   b. Prove the Loop Invariant using Mathematical Induction.

(2+4)

2.

   a. Consider the interval scheduling problem, with processing time(p_i) and deadlines(d_i), to minimize lateness. Give an example to show that picking jobs in increasing order of (d_i-p_i) will not work.

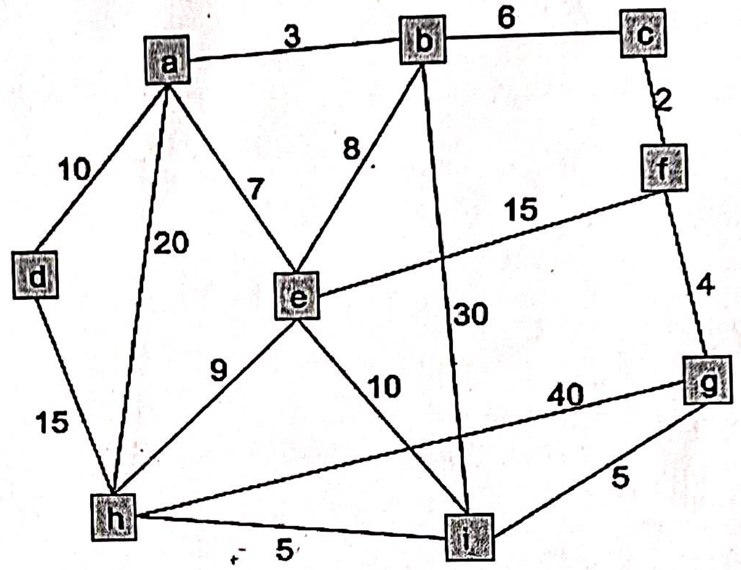
   b. Consider a situation where a telephone company wants to set up connections between 'n' houses in the society. The company devises a plan to efficiently connect all the houses at minimum cost and also ensure that every household gets the connection. Assume that the cost to connect two houses is determined by the distance between them.

      i. Model the problem using graphs?

ii. What plan the company must have used to efficiently connect all the houses at minimum cost ensuring that every household gets the connection.

iii. Apply the above plan in part (b) on the following example. Show all the steps. What is the total cost paid by the company?

(10)



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(2+2+2+4)

3. Suppose your friend is in the road ministry and she along with her team constructed a road network in a state to connect n cities. Her senior officials want her to give a simple algorithm to check how robust the road network is, that is, what is the minimum number of roads that if blocked disconnects one set of cities in the state from the remaining set of cities. Help your friend by designing a simple algorithm that gives the correct solution with high probability and runs in no more than $O(n^4 \log n)$ time.

(4)

4. Give an $O(\log n)$ divide and conquer algorithm for finding x^n. Justify the complexity.

(5)

5.

a. Consider a parallel algorithm for sorting an array of 'n' elements using comparison based sorting. The algorithm runs in 'n' steps such that step 1 has 'n' instructions that can be executed in parallel and all the remaining steps have 'log n' instructions. Suppose the algorithm uses 'n' processors. Is the algorithm optimal? If not, can you make it optimal? Justify your answer.

(4)

b. Consider an algorithm using a collision CRCW model and 10 processors. Following table represents the 10 processors, the memory location each processor will write to and the data that will be written to the respective memory locations.

Also, let the old data in memory locations m1, m2, m3, m4, m5, m6 and m7 be 100, 40, 170, 20, 90, 12 and 38 respectively.

Explain briefly, how the above task will be accomplished if we have 4 processors. Show all the steps.

Processor	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10
Memory location	m3	m7	m6	m5	m5	m2	m4	m3	m2	m1
Data	5	2	1	9	5	7	3	8	0	1

(4+4)

6.

a. Consider the following input: 3, 9, 2, 6, 0, 5, 8, 1. What is the probability of comparing 2 and 3 in the randomized quicksort algorithm? 2

b. Consider the randomized select algorithm. "For every input, there exists a random choice of pivots for which the algorithm takes $O(n^2)$ time. Comment. What happens if the input is in sorted order?

(2 + 2)

7. A subsequence of a string is a sequence that is generated by deleting some characters (possibly 0) from the string without altering the order of the remaining characters. For example, "abc", "abg", "bdf", "aeg", '"acefg", etc are subsequences of the string "abcdefg". Given two strings, S1 and S2, give a DP solution to find the length of the longest subsequence present in both of the strings. For example, if S1 = "AGGTAB", S2 = "GXTXAYB" then the longest subsequence which is present in both strings is "GTAB". If S1 = "ABCDGH", S2 = "AEDFHR" then the longest subsequence which is present in both strings is "ADH".

c

a. Define opt(i,j).
b. What is opt(i, j) if S1[i] is equal to S2[j]?
c. What is opt(i, j) if S1[i] is not equal to S2[j]?
d. Write the final recurrence for opt(i, j).
e. What is the running time of your algorithm?

4-1+1

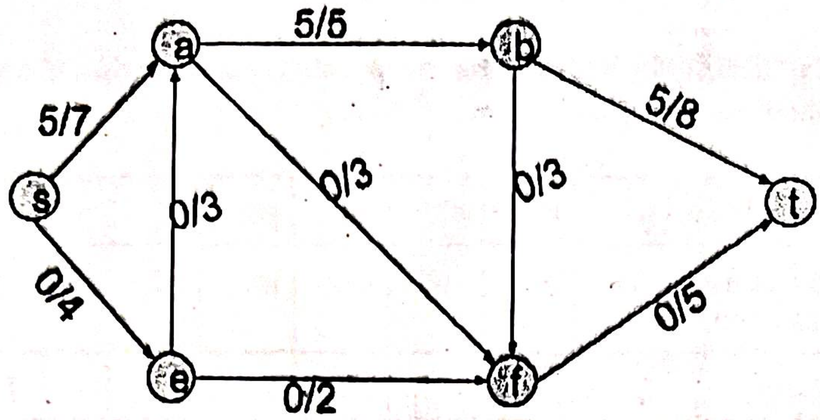
(6)

$opt(i,j) = 1 + opt(i-1, j-1)$

8.

a. Consider the following s-t flow in a network. Numbers ~~after~~ before "/" show the flow and the numbers after before "/" show the original capacities. Augment the given flow to compute the maximum flow using Ford and Fulkerson Algorithm.

b. Find the minimum cut in the network using the algorithm discussed in the class.



c. Given a graph G = (V, E), and matchings M and M' with |M| > |M'|. Let Q = (M / M') U (M' / M). Let G' = (V, Q) be a graph. Comment on the following statements.

i. Degree of each vertex in G' is at least 2.
ii. G' either consists of all cycles or all paths.
iii. G' contains at least one path with more edges from M than from M'.
iv. G' contains at least one cycle with more edges from M than from M'.

(5 + 2 + 4)

-3 +19 52 6

2+1+2

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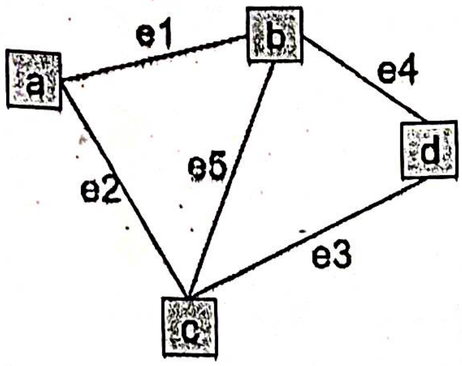
9. X In the Set Cover problem, Given a set of elements U = {1, 2, ..., n} and a collection S = $\{S_1, S_2, \ldots, S_m\}$ of sets where each S_i is a subset of U. The set cover problem is to identify the smallest sub-collection of S whose union equals the universe. For example, consider the universe U = {1, 2, 3, 4, 5} and the collection of sets S = { {1, 2, 3}, {2, 4}, {3, 4}, {4, 5} }. The solution is { {1, 2, 3}, {4, 5} }.

Consider the following reduction from vertex cover to set cover: let G(V, E) be the instance of vertex cover. For every edge e in E, add an element to U. For every vertex v in V, construct a subset S_v such that elements in S_v are the edges incident to v.

a. Construct an instance of set cover for the following vertex cover instance.



b. Show that if there exists a vertex cover of size k in G then there is a set cover of size k.

c. Show that if there exists a set cover of size k then there is a vertex cover of size k.

(2+2+2)

10.

a. Write the decision version of the interval partitioning problem. Show that the problem is in NP.

b. Consider the Traveling Salesman (TS) Problem on graph G. Show the following:

   i. The Cost of a Minimum Spanning Tree in G <= Optimal Cost of the TS tour.

   ii. Let V' be a subset of V. Show that there exists a matching M on the vertex set V' of the graph such that cost(M) <= Optimal Cost of the TS tour/2.

   iii. Number of odd degree vertices in an MST 'T' is even.

(4 + 2 + 2 + 2)