Master of Computer Applications
MCAE 309: Automata Theory (Year of Admission: 2023)
Unique Paper Code: 223422310
MCAE 310: Automata Theory (Year of Admission: 2022)
Unique Paper Code: 223401303
Semester III
December-2024

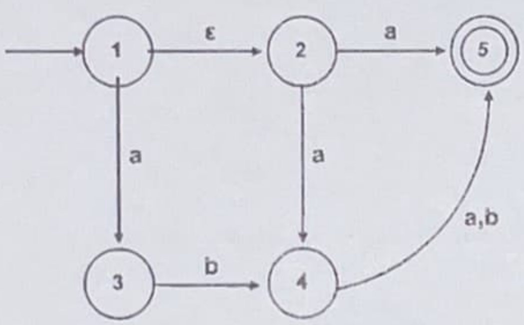
Duration: Three Hours **Max. Marks: 70**

Note: All questions carry 05 marks. Notations have their usual meaning. Assume $\Sigma = \{a, b\}$ as the underlying alphabet unless mentioned otherwise.

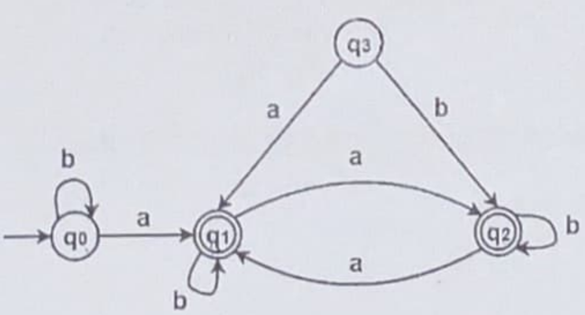
1. For the language $L = \{w \in \Sigma^*: \text{w has no two consecutive a's}\}$, construct the following:
   i. Regular expression
   ii. Finite Automaton (FA)

   Also, provide unambiguous context-free grammar (CFG) for the above FA.

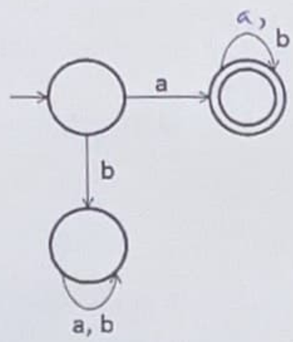
2. Construct a deterministic finite automaton for the following $NFA - \epsilon$:



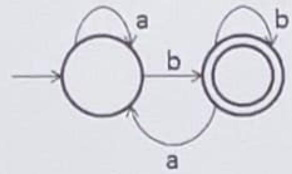
3. Construct a minimum state finite automaton equivalent to the following finite automaton:



4. For languages L_1 and L_2, described by the following finite automata (FA), construct the FA that defines $L_1 . L_2$.

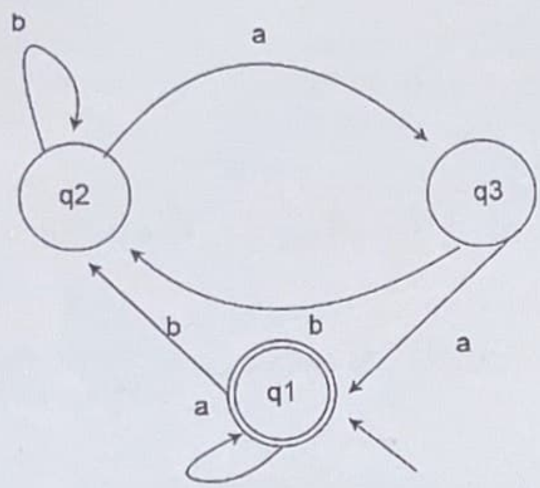


FA₁ (L₁)



FA₂ (L₂)

5. Find the regular expression for the following finite automaton (q_1 is an initial state):



6. Use pumping lemma to check whether the language $L = \{ww \mid w \in \{a,b\}^*\}$ is context-free or not.

7. Convert the following context-free grammar (CFG) into Chomsky Normal Form (CNF).

$$E \to E + T \mid T * F \mid (E) \mid a \mid b \mid Ga \mid Gb$$
$$T \to T * F \mid (E) \mid a \mid b \mid Ga \mid Gb$$
$$F \to (E) \mid a \mid b \mid Ga \mid Gb$$
$$F \to a \mid b \mid Ga \mid Gb$$

8. Consider the following context-free grammar (CFG):

$$S \to ASB \mid \epsilon$$
$$A \to aAS \mid a$$
$$B \to SbS \mid A \mid bb$$
$$C \to B$$

a) Eliminate ϵ-productions in the above CFG.
b) Eliminate any unit productions in the resulting grammar.
c) Eliminate any useless symbols in the resulting grammar.

9. For the languages L_1 and L_2 described by the regular expressions $(\mathbf{ab}^*)^*$ and $a(\mathbf{a}+\mathbf{b})^*$, respectively, construct the DFA that defines $L_1 \cap L_2$.

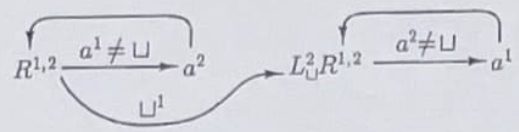
10. Use CYK algorithm to check the membership of the string $w = ababa$ in the language generated by the following grammar:

$$\begin{aligned} S &\to AB \mid BC \\ A &\to BA \mid a \\ B &\to CC \mid b \\ C &\to AB \mid a \end{aligned}$$

11. Is it possible to construct a deterministic pushdown automaton for the language $L = \{ww^R : w \in \{a, b\}^*\}$? If not, construct the pushdown automaton (PDA) that accepts L. Thereafter, trace the string "aaabbb" through the constructed PDA.

12. Construct the Turing Machine (TM) that semi-decides the language $L: \{a^n b^n c^n : n \geq 0\}$

13. Assuming the content of the first and second tapes is $\triangleright\underline{\sqcup}ab$ and $\triangleright\underline{\sqcup}$, respectively, describe the operation of the following 2-tape Turing machine:



14. Consider the Turing machine $M = (K, \Sigma, \delta, s, \{h\})$, where $K = \{s, q, h\}, \Sigma = \{\sqcup, \triangleright, a\}$, and δ is given as follows:

state,	symbol	δ
s	a	$(q, \sqcup)$
s	$\sqcup$	$(h, \sqcup)$
s	$\triangleright$	$(s, \rightarrow)$
q	a	(s, a)
q	$\sqcup$	$(s, \rightarrow)$
q	$\triangleright$	$(q, \rightarrow)$

Give the encoded representation of M.