Concept space & hypothesis space

F1 → A, B → 2   2 × 2 = 4
F2 → X, Y → 2

Instance Space :- (A, X), (A, Y), (B, X), (B, Y) → 4

F1 → A, B, ?, ϕ → 4   4 × 4 = 16
F2 → X, Y, ?, ϕ → 4

Hypothesis Space : (A, X), (A, Y), (A, ?), (A, ϕ), (B, X), (B, Y), (B, ?), (B, ϕ), (?, X), (?, Y), (?, ?), (?, ϕ), (ϕ, X), (ϕ, Y), (ϕ, ?), (ϕ, ϕ) — 16

Note :- In instance space we don't have any ϕ but in Hypothesis space we have ϕ i.e it is actually not possible
So we take only one ϕ and remove others

F1 → A, B, ? → 3   3 × 3 = 9 + 1 (ϕ) = 10
F2 → X, Y, ? → 3

Semantically Distinct Hypothesis : (A, X), (A, Y), (A, ?), (B, X), (B, X), (B, ?), (?, X), (?, Y), (?, ?), (ϕ, ϕ) — 10

SKY :- 3
AirTemp :- 2
Humidity :- 2
Wind :- 2
Water :- 2
Forecast :- 2

Possible distinct instances :- 3 × 2 × 2 × 2 × 2 × 2 = 96

SKY :- 3, ?, ϕ
AirTemp :- 2, ?, ϕ
Humidity :- 2, ?, ϕ
Wind :- 2, ?, ϕ
Water :- 2, ?, ϕ
Forecast :- 2, ?, ϕ

Hypothesis Space :- $5 \times 4 \times 4 \times 4 \times 4 \times 4 = 5120$

SKY :- 3, ?
AirTemp :- 2, ?
Humidity :- 2, ?
Wind :- 2, ?
Water :- 2, ?
Forecast :- 2, ?

Semantically Distinct Hypothesis : $(4 \times 3 \times 3 \times 3 \times 3 \times 3) + 1 = 973$

FIND-S Algorithm

Finding a maximally Specific Hypothesis

Steps:- 1. Initialize h to the most specific hypothesis in H

$$h_0 = \langle \phi, \phi, \phi, \phi, \phi, \phi \rangle$$

2. For the +ve traning instance x
   - For each attribute constraint a_i in h
     if the Constraint a_i is satisfied by x
     Then do nothing
     else replace a_i in h by the next more general Constraint that is satisfied by x

If we have -ve instance ignore them

eg:-

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	Enjoy
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Same	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

$$h_0 = \langle \phi, \phi, \phi, \phi, \phi, \phi \rangle$$ Compare

a_1 a_2 a_3 a_4 a_5 a_6

x_1 = < Sunny, Warm, Normal, Strong, Warm, Same >

h1 = < Sunny, warm, Normal, Strong, Warm, Same >

h1 = < Sunny , warm, Normal , Strong, Warm, Same >

x2 = < Sunny , warm, Hight , Strong, Warm, Same >

h2 = < Sunny, Warm, ? , Strong , Warm, Same >

x3 = < Rainy , Cold, High, Strong , Warm, Change >

this -ve instance so ignore it

h2 = < Sunny, Warm, ? , Strong , Warm, Same >

x4 = < Sunny, warm , High, Strong, Cool, Change >

h3 = < Sunny, warm , ? , Strong, ? , ? >

Output :- h3 = < Sunny, warm , ? , Strong, ? , ? >

this is Maximally Specific Hypothesis

Candidate Elimination Algorithm

here we have one generic and Specific boundary

generic :- < ?, ?, ?, ?, ?, ? >

Specific :- < ϕ, ϕ, ϕ, ϕ, ϕ, ϕ >

Steps :- 1 :- If d is positive example
Remove from G any hypothesis
h inconsistent with d

For each hypothesis s in S not consistent with d:
Remove s from S
Add to S all minimal generalizations of s
consistent with d

If d is negative example

Remove from S any hypothesis h inconsistent with d
For each hypothesis g in G not consistent with d
Remove g from G
Add to G all minimal specializations of
g consistent with d

Steps :- 1. firstly we take one generic and specific hypothesis

generic :- < ?, ?, ?, ?, ?, ? >

Specific :- < ϕ, ϕ, ϕ, ϕ, ϕ, ϕ >

2. Then check example if it is negative go to specific boundary and compare it with that if it is matches

3. If it is +ve example than go to generic boundary and compare it with that if that is consistent that we retain it as it is otherwise generic boundary is same as previous will try to write all hypothesis which is consistent with all the training example till now.
the go to specific boundary and compare it with our example if it is consistent than retain it as it is

4. If it is -ve example than go to specific boundary and check it is consistent or not if it is consistent than retain as it is otherwise make next general hypothesis (that not same replace? to them) if we have all ϕ then replace it with our example
Then we go to generic boundary and compare it with our example if it is consistent then remain it as it otherwise will we try to write all hypothesis which is consistent with all the training example till now.

eg:-

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	Enjoy
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Same	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

S0 <ϕ, ϕ, ϕ, ϕ, ϕ, ϕ>

S1 <Sunny, warm, Normal, Strong, warm, Same>

S2 S3 <Sunny, warm, ?, Strong, warm, Same>

S4 <Sunny, warm, ?, Strong, ?, ?>

G4 <Sunny, ?, ?, ?, ?, ?> < ?, warm, ?, ?, ?, ?>

G3 <Sunny, ?, ?, ?, ?, ?> <?, warm, ?, ?, ?, ?> <?, ?, Normal, ?, ?, ?> <?, ?, ?, ?, Cool, ?>
<?, ?, ?, ?, ?, change>

Note :- when it is inconsistent then Replace every ? one by one with opposite value of current example position (if our current example is -ve)

G0 G1 G2 <?, ?, ?, ?, ?, ?>

S4 <Sunny, warm, ?, Strong, ?, ?>

<Sunny, ?, ?, Strong, ?, ?> <Sunny, warm, ?, ?, ?, ?> <?, warm, ?, Strong, ?, ?>

G4 <Sunny, ?, ?, ?, ?, ?> < ?, warm, ?, ?, ?, ?>

Example	Eye	Nose	Head	Fcolor	Hair	Smile
1	Round	Triangle	Round	Purple	Yes	Yes
2	Square	Square	Square	Green	Yes	No
3	Square	Tringle	Round	Yellow	Yes	Yes
4	Round	Tringle	Round	Green	No	No
5	Square	Square	Round	Yellow	Yes	Yes

S0 < ϕ, ϕ, ϕ, ϕ, ϕ >

S1 S2 < Round, Triangle, Round, Purple, Yes >

S3 S4 < ?, Triangle, Round, ?, Yes)

S5 < ?, ?, Round, ?, Yes)

G5 (?, ?, Round, ?, Yes)

G4 (Square, Triangle, ?, ?, ?) (?, Triangle, Square, ?, ?)
(?, Triangle, ?, Yellow, ?) (?, Triangle, ?, Purple, ?) (?, Triangle, ?, ?, Yes)
(Square, ?, Round, ?, ?) (?, Square, Round, ?, ?)
(?, ?, Round, Yellow, ?) (?, ?, Round, Purple, ?) (?, ?, Round, ?, Yes)

G3 (?, Triangle, ?, ?, ?) (?, ?, Round, ?, ?)

G2 (Round, ?, ?, ?, ?) (?, Triangle, ?, ?, ?) (?, ?, Round, ?, ?)
(?, ?, ?, Purple, ?)

G0 G1 (?, ?, ?, ?, ?)

Output :- (?, ?, Round, ?, Yes)

Naive Bayes Classifier

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

P(PlayTennis = Yes) = 9/14 = 0.64
P(PlayTennis = No) = 5/14 = 0.36

outlook	Y	N
sunny	2/9	3/5
overcast	4/9	0
rain	3/9	2/5

outlook	Y	N
sunny	0.22	0.6
overcast	0.44	0
rain	0.33	0.4

Humidity	Y	N
high	3/9	4/5
normal	6/9	1/5

Humidity	Y	N
high	0.33	0.8
normal	0.66	0.2

Temprerature	Y	N
hot	2/9	2/5
mild	4/9	2/5
cool	3/9	1/5

Temprerature	Y	N
hot	0.22	0.4
mild	0.44	0.4
cool	0.33	0.2

Windy	Y	N
Strong	3/9	3/5
Weak	6/9	2/5

Windy	Y	N
Strong	0.33	0.6
Weak	0.66	0.4

New Instance :- <Sunny, Cool, high, strong>

For Yes :- $(0.22 \times 0.33 \times 0.33 \times 0.33) \times 0.64 = 5.05$

For No :- $(0.6 \times 0.2 \times 0.8 \times 0.6) \times 0.34 = 0.020$

we have to normalize these probabilities because these are equivalent probabilites because we remove the deminatar term

$$Yes = \frac{Yes}{Yes + No} \qquad No = \frac{No}{Yes + No}$$

$$Yes = \frac{5.05}{5.07} = 0.996 \qquad No = \frac{0.020}{5.07} = 3.944$$

So No is greater so Clasify it as NO

Gaussian Naive Bayes Algorithm

Person	Height(ft)	Weight(lbs)	footsize(inch)
Male	6.00	180	12
Male	5.92	190	11
Male	5.58	170	12
Male	5.92	165	10
Female	5.00	100	6
Female	5.50	150	8
Female	5.42	130	7
Female	5.75	150	9

P(Male) :- $4/8 = 0.5$
P(Female) :- $4/8 = 0.5$

Male :-

Mean(Height) = $\frac{6+5.92+5.58+5.92}{4} = 5.855$

Variance (Height) = $\frac{\Sigma (x_i - \bar{x})^2}{n-1}$

$$= \frac{(6-5.855)^2+(5.92-5.855)^2+(5.58-5.855)^2+(5.92-5.855)^2}{4-1}$$

$= 0.035033$

Mean (weight) :- 176.25

Variance (weight) :- 122.92

Mean(foot size) :- 11.25

Variance(foot size) :- 0.91667

Female :-

Mean(height) :- 5.4175

Variance (height) :- 0.097225

Mean (weight) :- 132.5

Variance (weight) :- 558.33

Mean(foot size) :- 7.5

Variance (foot size) :- 1.6667

$$\text{Posterior (Male)} = \frac{P(M) * P(H/M) * P(W/M) * P(FS/M)}{\text{Evidence}}$$

$$\text{Posterior (Female)} = \frac{P(F) * P(H/F) * P(W/F) * P(FS/F)}{\text{Evidence}}$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

New Instance is :-

Height	weight	foot size	Person
6	130	8	?

$$(e(-(x-\mu)^2 \div (2\sigma^2))) \div (\sqrt{(2\pi\sigma^2)})$$

$$P(H/M) = \frac{1}{\sqrt{2 * 3.142 * 0.035033}} * e^{\frac{-(6-5.855)^2}{2 * 0.035033}} = 1.5789$$

$$P(W/M) = 5.9881e^{-6} = 5.9855$$

$$P(FS/M) = 1.3112e^{-3} = 1.3065$$

$$P(H/F) = 2.2346e^{-1} = 0.0200$$

$$P(W/F) = 1.6789e^{-2} = 0.0167$$

$$P(FS/F) = 2.8669e^{-1} = 0.157$$

$$\text{Posterior (Male)} = 0.5 * 1.5789 * 5.9881e^{-6} * 1.3112e^{-3} = 6.1984e^{-9}$$

$$\text{Posterior (Female)} = 0.5 * 2.2346e^{-1} * 1.6789e^{-2} * 2.8669e^{-1} = 5.377e^{-4}$$

here Posterior (Female) is more than Posterior (Male) so new example is classified as female in this case

SVM

x1	x2	class
2	2	-1
4	5	+1
7	4	+1

$N = 3$

$\vec{x}_1 = (2,2)$, $\vec{x}_2 = (4,5)$, $\vec{x}_3 = (7,4)$

$y_1 = -1$, $y_2 = 1$, $y_3 = 1$

hyperplane eq^n $\Rightarrow f(\vec{x}) = \vec{\omega} \cdot \vec{x} + b$

$\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$

subject to conditions

$$\sum_{i=1}^{N} \alpha_i y_i = -\alpha_1 + \alpha_2 + \alpha_3 = 0$$

$$\alpha_1 > 0, \alpha_2 > 0, \alpha_3 > 0$$

find value of α by maximize $\phi(\vec{\alpha})$

$$\phi(\vec{\alpha}) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1, j=1}^{N} \alpha_i \alpha_j y_i y_j (\vec{x}_i \cdot \vec{x}_j)$$

$$= \sum_{i=1}^{3} \alpha_i - \frac{1}{2} \sum_{i=1, j=1}^{N} \alpha_i \alpha_j y_i y_j (\vec{x}_i \cdot \vec{x}_j)$$

$(\vec{x}_1 \cdot \vec{x}_1) = (2,2)(2,2) \Rightarrow (2\times2) + (2\times2) = 8$

$(\vec{x}_1 \cdot x_2) = (2,2)(4,5) \Rightarrow (2\times4) + (2\times5) = 18$

$(\vec{x}_1 \cdot \vec{x}_1) = 08 \qquad (\vec{x}_1 \cdot \vec{x}_2) = 18 \qquad (\vec{x}_1 \cdot \vec{x}_3) = 22$

$(\vec{x}_2 \cdot \vec{x}_1) = 18 \qquad (\vec{x}_2 \cdot \vec{x}_2) = 41 \qquad (\vec{x}_2 \cdot \vec{x}_3) = 48$

$(\vec{x}_3 \cdot \vec{x}_1) = 22 \qquad (\vec{x}_3 \cdot \vec{x}_2) = 48 \qquad (\vec{x}_3 \cdot \vec{x}_3) = 65$

$$\phi(\vec{\alpha}) = \alpha_1 + \alpha_2 + \alpha_3 - \frac{1}{2}\Big[8\alpha_1^2 - 18\alpha_1\alpha_2 - 22\alpha_1\alpha_3 - 18\alpha_1\alpha_2 + 41\alpha_2^2 + 68\alpha_2\alpha_3 - 22\alpha_1\alpha_3 + 68\alpha_2\alpha_3 + 65\alpha_3^2\Big]$$

$$\phi(\vec{\alpha}) = (\alpha_1 + \alpha_2 + \alpha_3) - \frac{1}{2}\Big[8\alpha_1^2 + 41\alpha_2^2 + 65\alpha_3^2 - 36\alpha_1\alpha_2 - 44\alpha_1\alpha_3 + 96\alpha_2\alpha_3\Big]$$

Put $\alpha_1 = \alpha_2 + \alpha_3$

$$2(\alpha_2+\alpha_3) - \frac{1}{2}\Big[8(\alpha_2+\alpha_3)^2 + 41\alpha_2^2 - 36(\alpha_2+\alpha_3)\alpha_2 - 44(\alpha_2+\alpha_3)\alpha_3 + 96\alpha_2\alpha_3 + 65\alpha_3^2\Big]$$

$$2(\alpha_2+\alpha_3) - \frac{1}{2}\Big(8(\alpha_2^2 + \alpha_3^2 + 2\alpha_2\alpha_3) + 41\alpha_2^2 - 36\alpha_2^2 - 36\alpha_2\alpha_3 - 44\alpha_2\alpha_3 - 44\alpha_3^2 + 96\alpha_2\alpha_3 + 65\alpha_3^2\Big)$$

$$2(\alpha_2+\alpha_3) - \frac{1}{2}\Big[8\alpha_2^2 + 8\alpha_3^2 + 16\alpha_2\alpha_3 + 41\alpha_2^2 - 36\alpha_2^2 - 36\alpha_2\alpha_3 - 44\alpha_2\alpha_3 - 44\alpha_3^2 + 96\alpha_2\alpha_3 + 65\alpha_3^2\Big]$$

$$2\alpha_2 + 2\alpha_3 - \frac{1}{2}\Big[13\alpha_2^2 + 29\alpha_3^2 + 32\alpha_2\alpha_3\Big]$$

$$2\alpha_2 + 2\alpha_3 - \frac{13}{2}\alpha_2^2 - \frac{29}{2}\alpha_3^2 - 16\alpha_2\alpha_3\Big]$$

For $\phi(\vec{\alpha})$ to be maximum we must have

$$\frac{\partial \phi}{\partial \alpha_2} = 0 \qquad \frac{\partial \phi}{\partial x_3} = 0$$

That is

$$2-13\alpha_2-16\alpha_3=0 \qquad 2-16\alpha_2-29\alpha_3=0$$

$$\alpha_2=\frac{26}{121} \qquad \alpha_3=\frac{-6}{121} \qquad \alpha_1=\frac{20}{121}$$

Now have to calculate the weight vector

$$\vec{\omega}=\sum_{i=1}^{N}\alpha_i y_i \vec{x}_i$$

$$=\frac{20}{121}(-1)(2,2)+\frac{26}{121}(+1)(4,5)-\frac{6}{121}(+1)(7,4)$$

$$-\frac{40}{121}+\frac{104}{121}-\frac{42}{121}\ ,\ -\frac{40}{121}+\frac{130}{121}-\frac{24}{121}$$

$$=\left(\frac{2}{11},\frac{6}{11}\right)$$

Now we have to calculate the baise

$$b=\frac{1}{2}\Big(\underset{\underset{\text{take +ve examples}}{i:y_i=+1}}{\min}(\vec{\omega}\cdot\vec{x}_i)+\underset{\underset{\text{take -ve examples}}{i:y_i=-1}}{\max}(\vec{\omega}\cdot\vec{x}_i)\Big)$$

$$=\frac{1}{2}\left(\min\{(\vec{\omega}\cdot\vec{x}_2),(\vec{\omega}\cdot\vec{x}_3)\}+\max\{(\vec{\omega}\cdot\vec{x}_i)\}\right)$$

$$=\frac{1}{2}\left(\min\left[\left(\frac{2}{11},\frac{6}{11}\right)(4,5),\left(\frac{2}{11},\frac{6}{11}\right)(7,4)\right]+\max\left[\left(\frac{2}{11},\frac{6}{11}\right)(2,2)\right]\right)$$

$$=\frac{1}{2}\left(\min\left\{\frac{38}{11},\frac{38}{11}\right\}+\max\left\{\frac{16}{11}\right\}\right)$$

$$=\frac{1}{2}\left(\frac{38}{11}+\frac{16}{11}\right) \Rightarrow \frac{27}{11}$$

$b = \frac{27}{11}$

The SVM classifier function

$f(\vec{x}) = \vec{\omega} \cdot \vec{x} - b$

where

$\vec{x} = (x_1, x_2)$

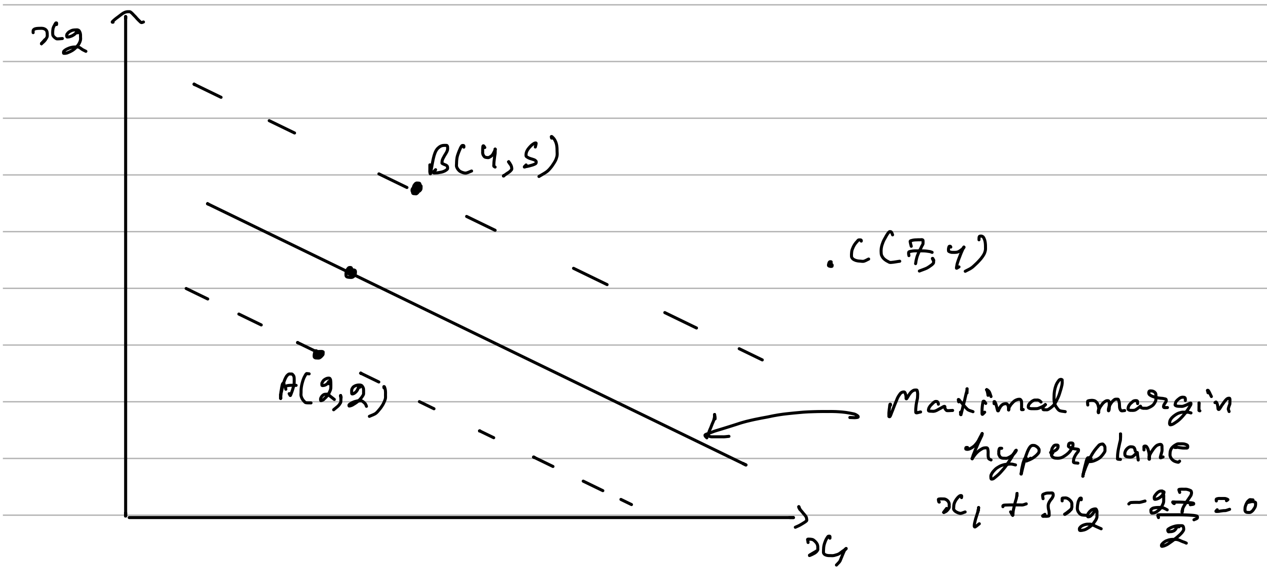
$f(\vec{x}) = \frac{2}{11} x_1 + \frac{6}{11} x_2 - \frac{27}{11}$

$f(\vec{x}) = 0$

$\frac{2}{11} x_1 + \frac{6}{11} x_2 - \frac{27}{11} = 0$ Multiply by $\frac{11}{2}$

$x_1 + 3x_2 - \frac{27}{2} = 0$

Note:- we have α_3 is -ve so we can't take x_3 as support vector



Mid point of A & B is

$A(2,2)$, $B(4,5)$ $\Rightarrow$ $\left(\frac{2+4}{2}, \frac{2+5}{2}\right) = (3, 3.5)$

Put these in eqⁿ

$3 + 3 \times 3.5 - \frac{27}{2} = 0$ satisfy

x_1	x_2	Class
2	1	+1
4	3	-1

$N = 2$

$\vec{x}_1 = (2, 1)$, $\vec{x}_2 = (4, 3)$

$y_1 = +1$ $y_2 = -1$

$\vec{\alpha} = (\alpha_1, \alpha_2)$

Subject to the conditions

$\alpha_1 - \alpha_2 = 0 \Rightarrow \alpha_1 = \alpha_2$

$\alpha_1 \geq 0$, $\alpha_2 \geq 0$

find value of α by maximize $\phi(\vec{\alpha})$

$$\phi(\vec{\alpha}) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j (\vec{x}_i \cdot \vec{x}_j)$$

$$= (\alpha_1 + \alpha_2) - \frac{1}{2} [\alpha_1 \alpha_1 y_1 y_1 (\vec{x}_1 \cdot \vec{x}_1) + \alpha_1 \alpha_2 y_1 y_2 (\vec{x}_1 \cdot \vec{x}_2)$$

$$+ \alpha_2 \alpha_1 y_2 y_1 (\vec{x}_2 \cdot \vec{x}_1) + \alpha_2 \alpha_2 y_2 y_2 (\vec{x}_2 \cdot \vec{x}_2)]$$

$$= (\alpha_1 + \alpha_2) - \frac{1}{2} [\alpha_1^2 (+1)(+1)(2 \times 2 + 1 \times 1) + \alpha_1 \alpha_2 (+1)(-1)$$

$$(2 \times 4 + 1 \times 3) + \alpha_2 \alpha_1 (-1)(+1)(4 \times 2 + 3 \times 1)] + \alpha_2^2 (-1)(-1)$$

$$(4 \times 4 + 3 \times 3)]$$

$$= (\alpha_1 + \alpha_2) - \frac{1}{2} [5\alpha_1^2 - 22\alpha_1 \alpha_2 + 25\alpha_2^2]$$

$\alpha_1 = \alpha_2$

$\phi(\vec{\alpha}) = 2\alpha_1 - 4\alpha_1^2$

For ϕ to be maximum we have

$$\frac{d\phi}{d\alpha_1} = 2 - 8\alpha_1 = 0$$

$$\alpha_1 = \frac{1}{4} \qquad \alpha_2 = \frac{1}{4}$$

Calculate the weight vector

$$\vec{\omega} = \sum_{i=1}^{N} \alpha_i y_i \vec{x}_i$$

$$= \alpha_1 y_1 x_1 + \alpha_2 y_2 \vec{x}_2$$

$$= \frac{1}{4}(+1)(2,1) + \frac{1}{4}(-1)(4,3)$$

$$= \frac{1}{4}(-2,-2)$$

$$\omega = \left(\frac{-1}{2}, \frac{-1}{2}\right)$$

Now calculate baise

$$b = \frac{1}{2}\left(\min_{i:y_i=+1}(\vec{\omega}\cdot\vec{x}_i) + \max_{i:y_i=-1}(\vec{\omega}\cdot\vec{x}_i)\right)$$

$$= \frac{1}{2}\left((\vec{\omega}\cdot\vec{x}_1) + (\vec{\omega}\cdot\vec{x}_2)\right)$$

$$= \frac{1}{2}\left(\left(\frac{-1}{2}\times 2 - \frac{1}{2}\times 1\right) + \left(\frac{-1}{2}\times 4 - \frac{1}{2}\times 3\right)\right)$$

$$= \frac{1}{2}\left(\frac{-10}{2}\right) = \frac{-5}{2}$$

$b = -\frac{5}{2}$

The SVM classifier function

$f(\vec{x}) = \vec{\omega} \cdot \vec{x} - b$

$= \left(-\frac{1}{2}, -\frac{1}{2}\right) \cdot (x_1, x_2) - \left(-\frac{5}{2}\right)$

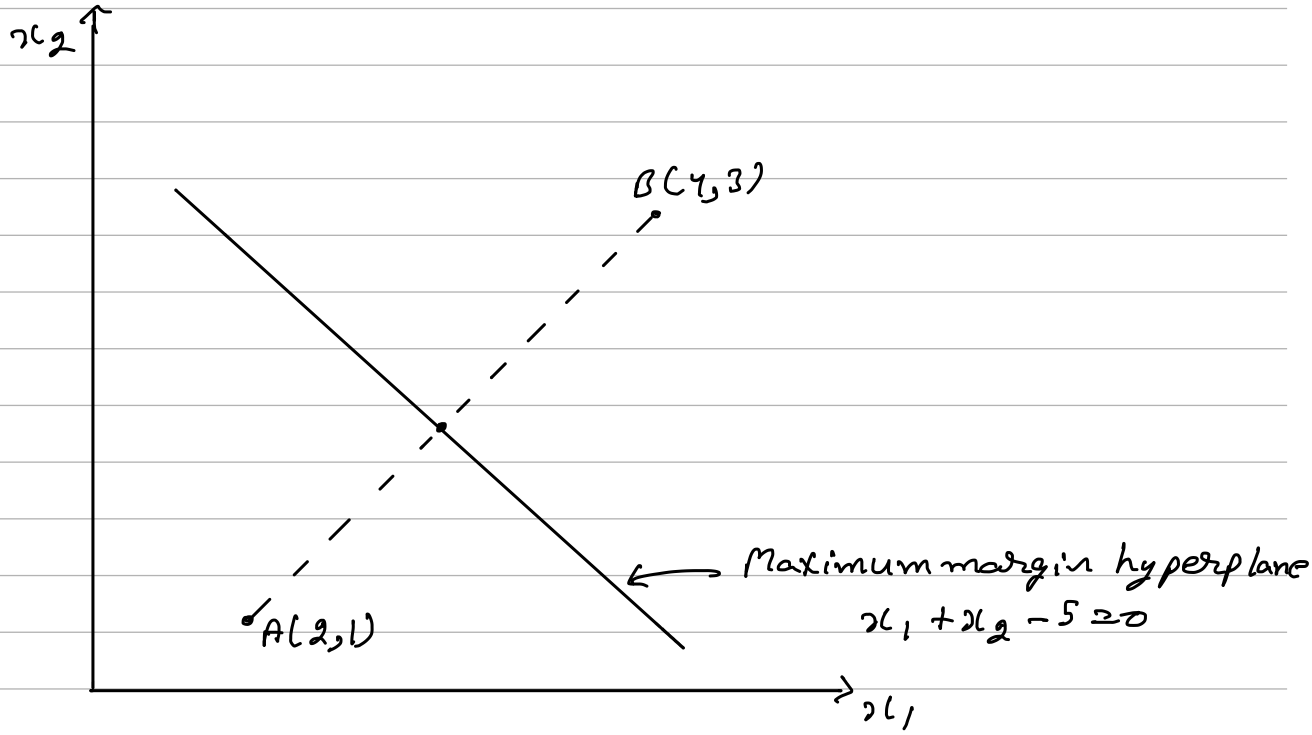
$= -\frac{1}{2} x_1 - \frac{1}{2} x_2 + \frac{5}{2}$ // Remember Here when Multiplying

$f(x) = -\frac{1}{2}(x_1 + x_2 - 5)$

$f(x) = 0$

$-\frac{1}{2}(x_1 + x_2 - 5) = 0$

$x_1 + x_2 - 5 = 0$



Mid point of A & B is

$A(2,1)\ B(4,3) \Rightarrow \left(\frac{2+4}{2}, \frac{1+3}{2}\right) \Rightarrow (3,2)$

put these in eqn

$3+2-5=0$ satisfy

Kernal Trick

In this we transform data to one dimension to other

Kernal Trick for 2^{nd} degree Polynomial Mapping

$$\Rightarrow \phi(x, y) = (x^2, \sqrt{2}xy, y^2)$$

$$\phi(a)^T \cdot \phi(b) = \begin{pmatrix} a_1^2 \\ \sqrt{2}a_1a_2 \\ a_2^2 \end{pmatrix}^T \begin{pmatrix} b_1^2 \\ \sqrt{2}b_1b_2 \\ b_2^2 \end{pmatrix} = a_1^2b_1^2 + 2a_1b_1a_2b_2 + a_2^2b_2^2$$

$$= (a_1b_1 + a_2b_2)^2 = \left(\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}^T \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}\right)^2 = (a^T \cdot b)^2$$

$$= (a^T \cdot b)^2$$

Types of Kernal in SVM

Linear Kernal

$K(x, y) = x^T \cdot y$ $\quad$ x & y are 2 vector

eg:- $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\quad y = (2, 3)$ $\quad c = 1$

$$\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix}^T \cdot (2, 3)\right) \Rightarrow ((1, 2)\ (2, 3)) = (1\times2) + (2\times3)$$
$$= 8$$

Polynomial Kernal

$$K(x, y) = (x^T y)^2$$

if $q=2$ then it is called quadratic Kernal

eg:- $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $y = (2, 3)$

$$\left(\begin{pmatrix} 1 \\ 2 \end{pmatrix}^T \cdot (2,3)\right)^2 \Rightarrow \left((1,2)(2,3)\right)^2$$

$$\left((1+2)(2+3)\right)^2 \Rightarrow 8^2 = 64$$

For Inhomogenous Kernal

$$K(x,y) = (c + x^T y)^q$$

eg:- $x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $y = (2, 3)$ $c = 1$

$$= \left(1 + \begin{pmatrix} 1 \\ 2 \end{pmatrix}^T (2,3)\right)^2 = (1+8)^2 = 9^2 = 81$$

Gaussian Kernal ☆☆

$$K(x,y) = e^{\frac{-(x-y)^2}{2\sigma^2}}$$

if y is small than it is similar to linear SVM

y is large than it influenced by more SVM

eg:- $x = (1,2)$ $y = (2,3)$ $\sigma = 1$

distance $\Rightarrow (1-2)^2 + (2-3)^2 = 2$

if $\sigma = 1$, then $K(x, y) = e^{\left(-\frac{2}{2}\right)} = e^{-1} = 0.3679$

Sigmoid kernal

$$K(x_i, y_i) = \tanh(k x_i y_i - \sigma)$$

How two select Best Hyperplane

Let we have 2 hyperplane classifier

$5 + 2x_1 + 5x_2$, $5 + 20x_1 + 50x_2$

firstly we have find Distance Error $= \sqrt{a_1^2 + a_2^2}$

Distance Error 1 $= \sqrt{2^2 + 5^2} = 5.39$

Distance Error 2 $= \sqrt{20^2 + 50^2} = 53.85$

For classifier $\Rightarrow \frac{2}{||w||}$ → length of particular distance

For first Classifier $= \frac{2}{5.39} = 0.37$

For 2nd Classifier $= \frac{2}{53.85} = 0.037$

Here 1st classifier give max value so we select 1st hyperplane