Problems in Math and Computer Science

Compiled and to be solved by Chad Estioco

Contents

Chapter 1

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1. Prove that

$$\exists x \forall y P(x, y) \leftrightarrow \forall y \exists x P(x, y) \tag{1.1}$$

Chapter 2

Miscellaneous Problems

(I.e., problems that were compiled before and left unsolved)

1. Let $J(n)$ be the Josephus Numbers[CA-KNUTH], and 2^m be the largest power of 2 not exceeding n. Prove that $J(5 \cdot 2^m) = 2^{m+1} - 1$. What can you conjecture about $J(x2^m)$ if x is any arbitrary integer?

2. **Number Theory?** $2^m - 2$ is a multiple of 3 when m is an odd but no when m is even.

3. **Statistics** A soft drink machine is regulated so that it dispenses an average of 200ml per cup. If the amount of drink dispensed is normally distributed with a standard deviation equal to 15ml.

   (a) What fraction of the cups will contain more that 224ml?

   (b) What is the probability that a cup contains between 191ml and 209ml?

   (c) How many cups will likely overflow if 230ml cups are used for the next 1000 drinks?

   (d) Below what value do we get the smallest 2.5% of the drinks?

Bibliography

[CA-KNUTH] Knuth, D. *Concrete Mathematics*

[MCS-LEHMAN] Lehman E. *Mathematics for Computer Science*