Problems in Math and Computer Science

Compiled and to be solved by Chad Estioco

Contents

Chapter 1

2015.1 General Logic

1. Prove that
$$\exists x \forall y P(x, y) \leftrightarrow \forall y \exists x P(x, y)$$
(Curiously, the "reverse" of this implication is not true. That is,
$$\forall y \exists x P(x, y) \leftrightarrow \exists x \forall y P(x, y)$$
How do you reconcile this?)

2. Prove the following inference rules
$$\frac{P \to Q, Q \to R}{P \to R}$$
$$\frac{P \to Q, \neg Q}{\neg P}$$
$$\frac{\neg P \to \neg Q}{Q \to P}$$

3. Is the following also true? Prove or disprove.
$$\frac{\neg P \to \neg Q}{P \to Q}$$

4. Regarding the Horses proof in section 3.2.6 [MCS-LEHMAN], why is it wrong to claim that the mistake in the proof is with $P(n)$ being false for $n \geq 2$ and that it assumes something false, namely, $P(n)$ in order to prove $P(n+1)$?

Chapter 2

2015.2 Number Theory

1. Given integers, n, a, and b, determine whether there exists integers s and t such that $n = sa + tb$.

Chapter 3

Programming Problems

A collection of programming problems I've thought up (ala Programming Praxis).

Away Goals

In some major football championships, each match-up sees teams play in home and in away conditions. Should there be a tie after two games, the winner is decided by the *away goals* rule: whoever scored the most goals while playing in away territory wins that leg of the championships.

Given the results of the first leg of a match-up and the projection for the second leg, determine the *minimum* goals the *first leg loser* needs in order to win the fixture.

Input

Each test case is made up of three lines: the first two indicate the score for each team in the first leg while the last one will indicate the projected score of the first-leg winner in the second leg. The format will be

```
TN1 S1
TN2 S2
PS
```

Where `TN1` and `TN2` are the team names and `S1` and `S2` are the scores in the first leg. The team names will contain no spaces and the scores (including the projected score) are guaranteed to be integers. The scores will be no more than 10.

Output

The output for each test case is a single integer indicating the number of goals the first-leg losing team needs to score in order to win the fixture.

Example Input

```
FCBarcelona 3
FCBayern 0
2
Juventus 2
RealMadrid 1
1
ParisSaintGermain 1
FCBarcelona 3
2
```

Example Output

6
3
4

Chapter 4

Miscellaneous Problems

(I.e., problems that were compiled before and left unsolved)

1. Let $J(n)$ be the Josephus Numbers[CA-KNUTH], and 2^m be the largest power of 2 not exceeding n. Prove that $J(5 \cdot 2^m) = 2^{m+1} - 1$. What can you conjecture about $J(x2^m)$ if x is any arbitrary integer?

2. **Number Theory?** $2^m - 2$ is a multiple of 3 when m is an odd but no when m is even.

3. **Statistics** A soft drink machine is regulated so that it dispenses an average of 200ml per cup. If the amount of drink dispensed is normally distributed with a standard deviation equal to 15ml.

   (a) What fraction of the cups will contain more that 224ml?

   (b) What is the probability that a cup contains between 191ml and 209ml?

   (c) How many cups will likely overflow if 230ml cups are used for the next 1000 drinks?

   (d) Below what value do we get the smallest 2.5% of the drinks?

4. Prove: Let,

$$I = \sum \text{length of paths from root to internal node}$$

$$E = \sum \text{length of paths from root to external node}$$

$$n = \text{number of internal nodes}$$

   Then, $E = I + 2n$

5. What is the significance of the relationship between Pascal's triangle and combinations? That is, why does adding the two numbers above an entry in Pascal's triangle give the number of combinations?

6. Give a strong proof of the following statement: If n is composite, no item in the nth row of Pascal's triangle divides n.

7. **Statistics** Prove: The positive and negative distance from the mean cancel each other out.

Bibliography

[CA-KNUTH] Knuth, D. *Concrete Mathematics*

[MCS-LEHMAN] Lehman E. *Mathematics for Computer Science*