Problem 1:

```
(1). for i = 1 to n {           //loop executed n times
a = a + 1;                      //loop body takes c1
 b = a + i; }                   // loop body takes c2
```
Total running time, f(n) = n(c1 + c2),
F(n) = O(n)

```
(2). for i = 1 to n {     //Loop executed n times
 for j = 1 to 2n          //Loop executed 2n times
a = a / 2; }              //Loop body takes c1
```
Total running time: f(n) = n * 2n(c1) = $2n^2$(c1),
F(n) = O(n^2)

```
(3). i = n
while (i >= 1) {        // Loop executed Log(n) times, if loop variable multiplied or divided by constant.
 a = a + 1;             // Loop body takes c1
 i = i / 2;}            // Loop body takes c2
```
Total running time: f(n) = log(n) * (c1+c2),
F(n) = O(log(n))

```
(4). i = n
while (i >= 1) {          //Loop executed Log(n) times, if loop variable multiplied or divided by constant.
 for j = 1 to n {         // Loop executed n times
a = a + 1; }              //Loop body takes c1
i = i / 2; }              // Loop body takes c2
```
Total running time: f(n) = (log(n)c2) * n(c1) = nlog(n)(c1)(c2)
F(n) = O(nlog(n))

```
(5). for i = 1 to n {                // Loop executed n times
        for j = 1 to n {             // Loop executed n times
          if j is even {             //  Loop body takes c1
               a = a + 1;            // Loop body takes c2
} } }
```
Total running time: f(n) = n * n(c1 + c2/2) = n^2 (c1+c2/2),
F(n) = O(n^2)

Problem 2:

(1). Prove the following statement using the mathematical induction method that we discussed in the class: n2 > 3n for $n \geq 4$

Base step: n = 4, LHS = 4^2 =16, RHS = 3*4 = 12, LHS > RHS

Induction step: ($n \geq 4$). Inductive hypothesis: it is true for n = k ($k \geq 4$), i.e., $k^2 > 3k$ for $k \geq 4$.

We show that it is also true for n = k + 1, i.e., $(k+1)^2 > 3(k + 1)$

LHS = $(k+1)^2 = k^2+2k+1$

> 3k + 2k + 1 (by the inductive hypothesis $k^2 >= 3k$)

5k + 1 > 3k + 3

= RHS

So, LHS > RHS.

(2). Prove the following equation using the proof by contradiction method that we discussed in the class: If a number added to itself gives itself, then the number is 0.

Let x be a number of

X + X = X

For example, x = 1

1+1 = 1 which is wrong