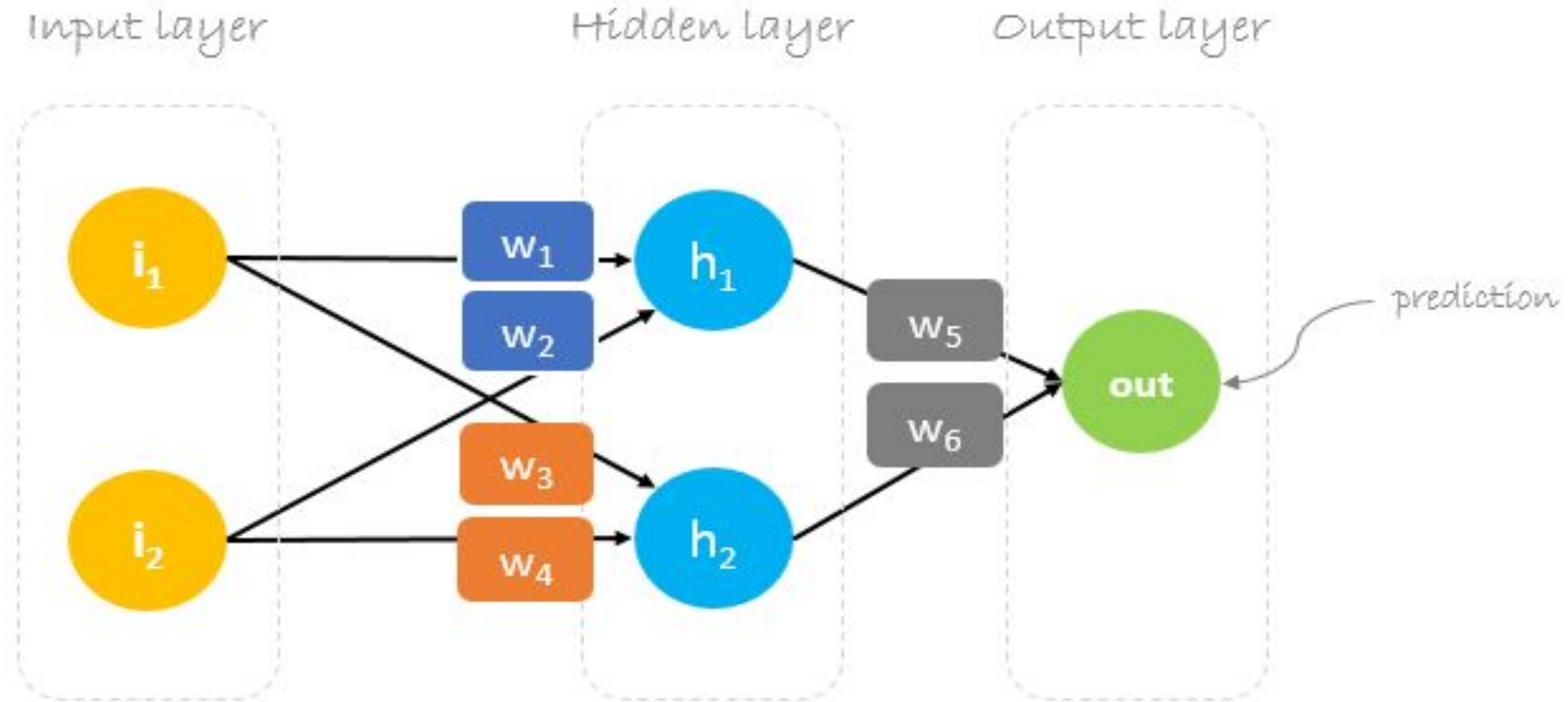


Lecture

- **Backpropagation Step by Step**

Backpropagation Step by Step



Initial Weights

- Initial weights are following:

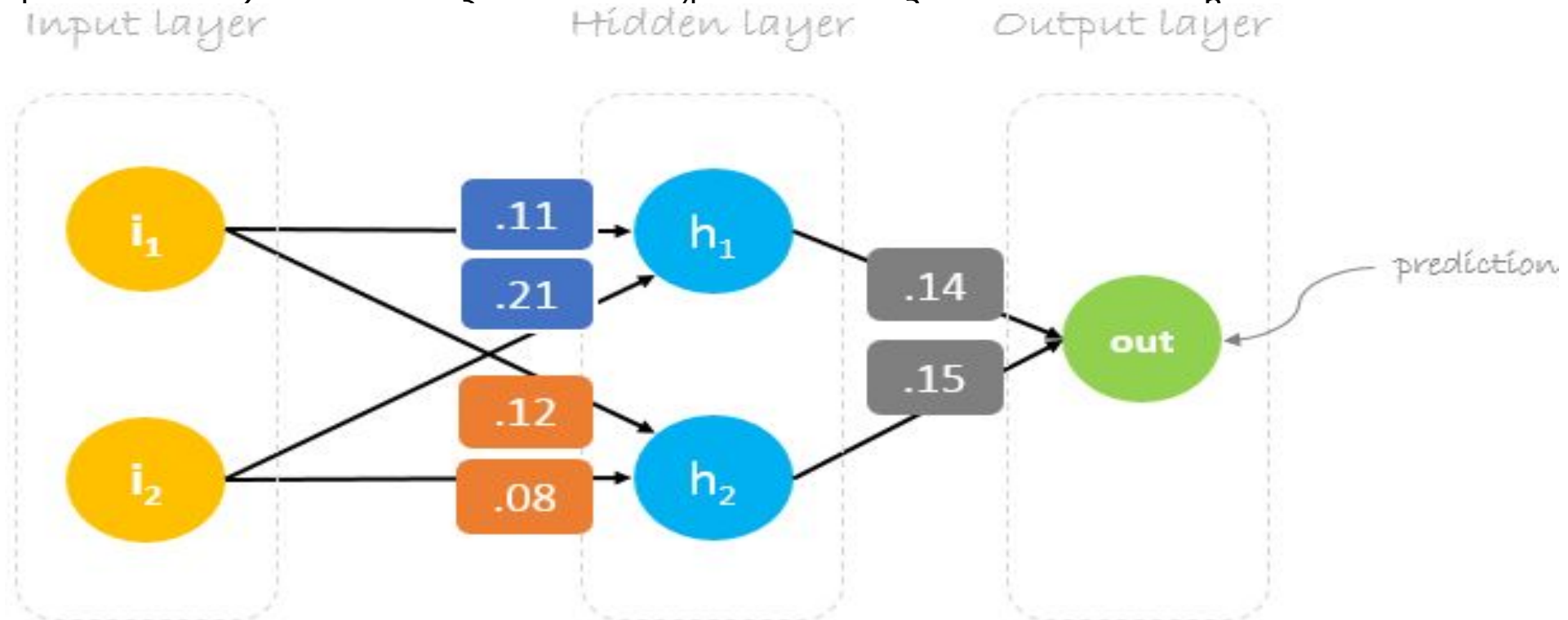
$$w_1 = 0.11, w_2 = 0.21, w_3 = 0.12, w_4 = 0.08, w_5 = 0.14 \text{ and } w_6 = 0.15$$

LAYOUT?

Initial Weights

- Initial weights are following:

$w_1 = 0.11$, $w_3 = 0.21$, $w_2 = 0.12$, $w_4 = 0.08$, $w_5 = 0.14$ and $w_6 = 0.15$



Dataset

- Dataset has one sample with two inputs and one output.



- Single sample is as following inputs=[2, 3] and output=[1].

LAYOUT?

Dataset

- Dataset has one sample with two inputs and one output.



- Single sample is as following inputs=[2, 3] and output=[1].



Forward Pass

- Use given weights and inputs to predict the output. Inputs are multiplied by weights; the results are then passed forward to next layer.

LAYOUT

?

Forward Pass

What are the values in hidden layer?

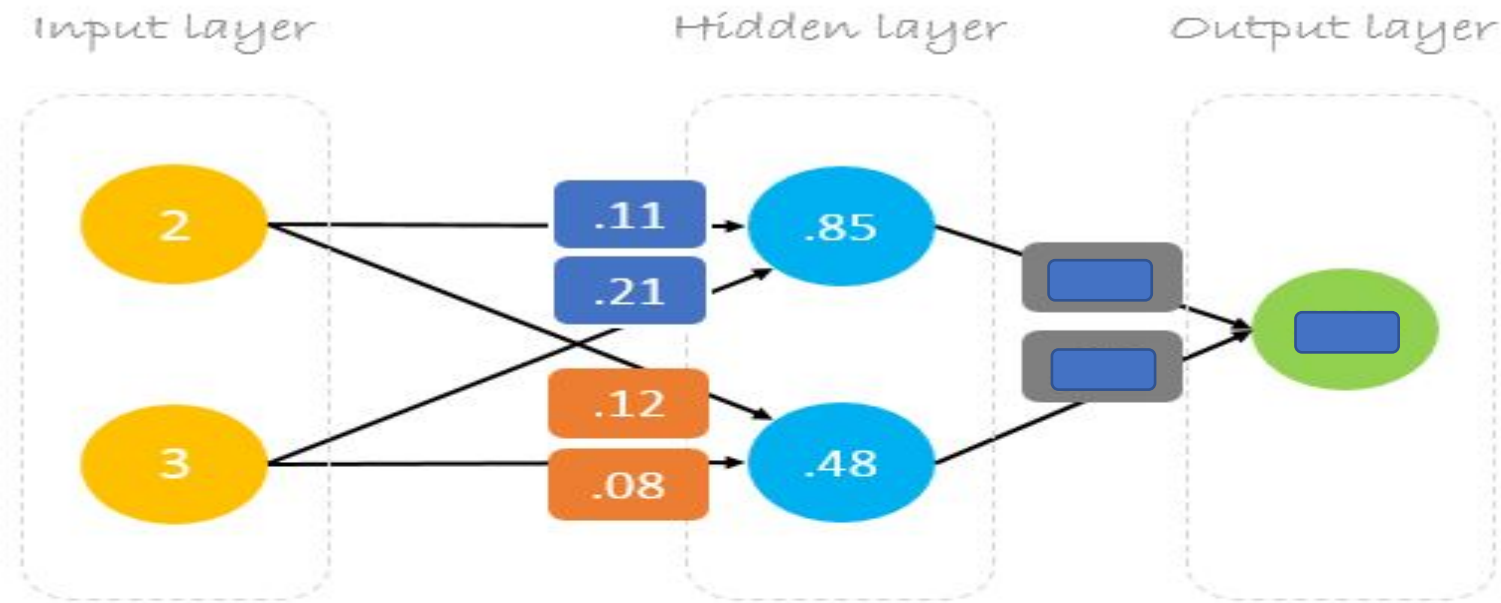
Compute them.

Forward Pass

What are the values in hidden layer?

(0.85, 0.48)

Forward Pass



$$\begin{bmatrix} 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0.11 & 0.12 \\ 0.21 & 0.08 \end{bmatrix} = \begin{bmatrix} 0.85 & 0.48 \end{bmatrix}$$

Forward Pass

What is the value in output layer?

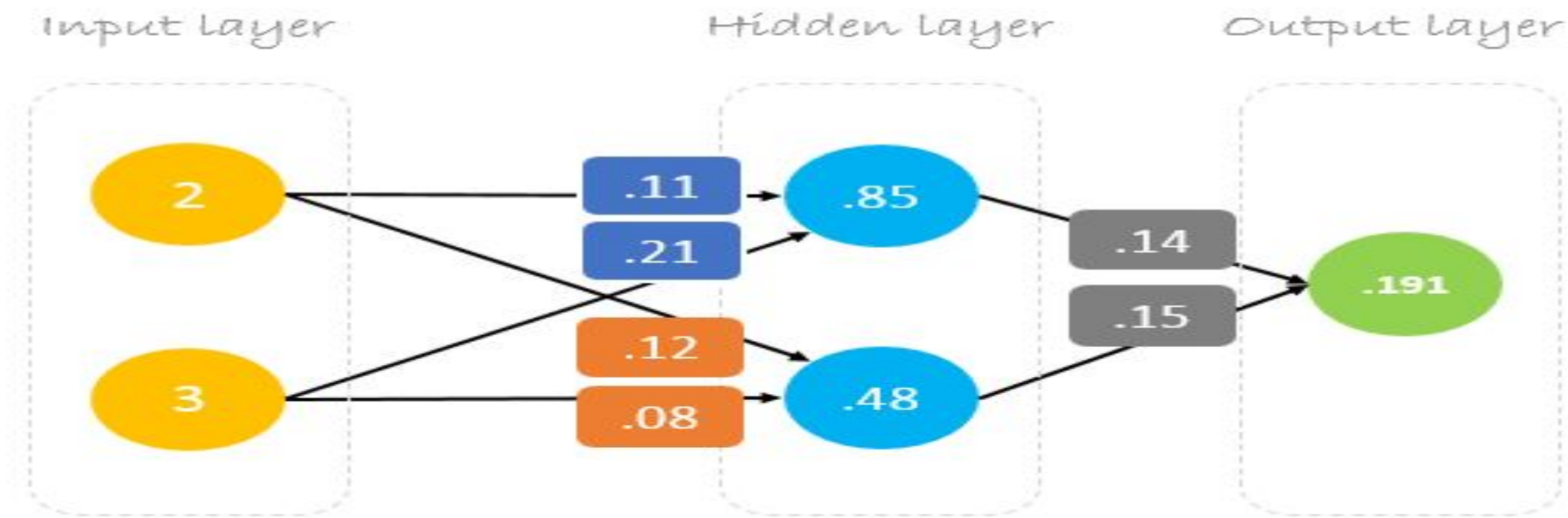
Compute the value.

Forward Pass

What is the values in output layer?

(0.191)

Forward Pass



$$\begin{bmatrix} 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0.11 & 0.12 \\ 0.21 & 0.08 \end{bmatrix} = \begin{bmatrix} 0.85 & 0.48 \end{bmatrix} \cdot \begin{bmatrix} 0.14 \\ 0.15 \end{bmatrix} = \begin{bmatrix} 0.191 \end{bmatrix}$$

$$2 \times .11 + 3 \times .21 = .85$$

$$2 \times .12 + 3 \times .08 = .48$$

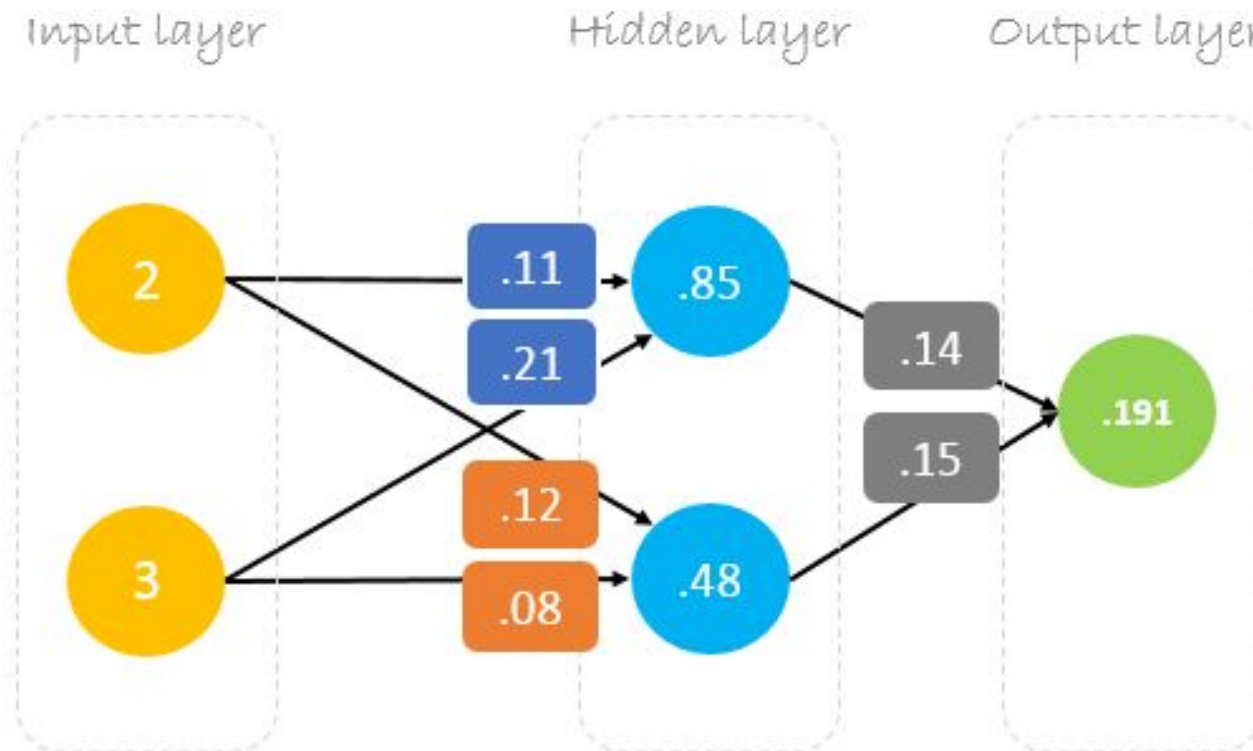
$$.85 \times .14 + .48 \times .15 = .191$$

Matrix multiplication

Details

Forward Pass

- Use given weights and inputs to predict the output. Inputs are multiplied by weights; the results are then passed forward to next layer.



Calculate Error

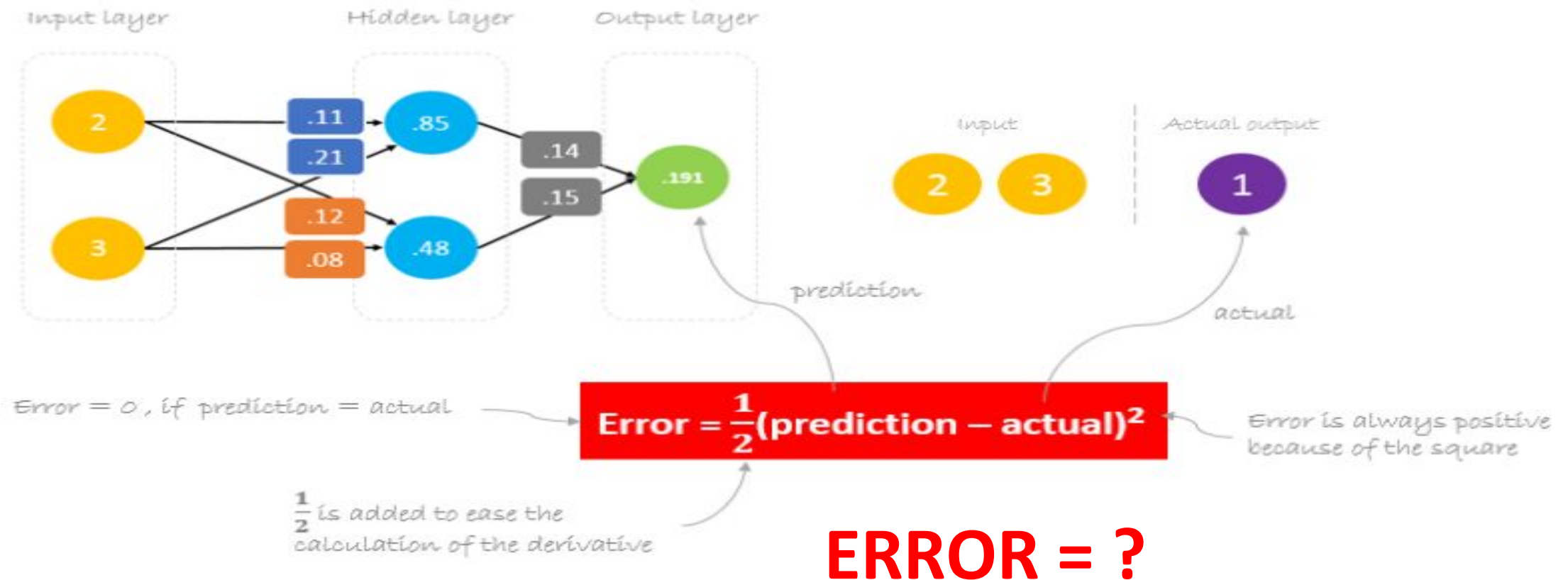
- Network output, or **prediction (0.191)**, is not even close to **actual output (1)**. We can calculate the difference or the error.

LAYOUT

?

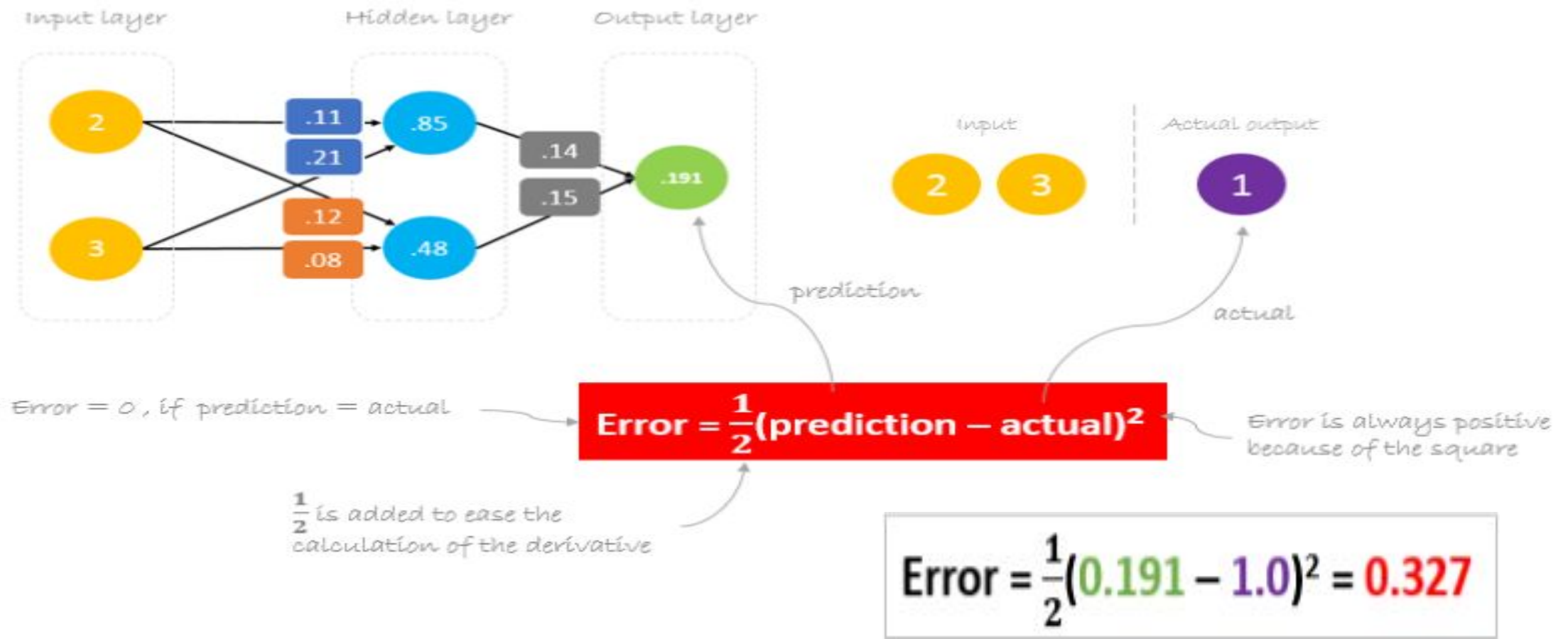
Calculate Error

- Network output, or **prediction (0.191)**, is not even close to **actual output (1)**. We can calculate the difference or the error as following.



Calculate Error

- Network output, or **prediction (0.191)**, is not even close to **actual output (1)**. We can calculate the difference or the error as following.



Reducing Error

- Our main goal of the training is to reduce the **error** or the difference between **prediction** and **actual output**.
- Since **actual output** is constant, “not changing”, the only way to reduce the error is to change **prediction** value.
- The question now is, how to change **prediction** value?

Reducing Error

- Our main goal of the training is to reduce the **error** or the difference between **prediction** and **actual output**.
- Since **actual output** is constant, “not changing”, the only way to reduce the error is to change **prediction** value. The question now is, how to change **prediction** value?
- By decomposing **prediction** into its basic elements we can find that **weights** are the variable elements affecting **prediction** value.
- In other words, in order to change **prediction** value, we need to **change weights values**.

Reducing Error

prediction = out



prediction = (?) w_5 + (?) w_6



prediction = (?) w_5 + (?) w_6



to change *prediction* value,
we need to change *weights*

Reducing Error

prediction = out



prediction = (?) w_5 + (?) w_6




prediction = (?) w_5 + (?) w_6

to change **prediction** value,
we need to change **weights**

Reducing Error

$$\text{prediction} = \text{out}$$



$$\text{prediction} = (h_1) w_5 + (?) w_6$$


$$\text{prediction} = (\text{?}) w_5 + (\text{?}) w_6$$

to change *prediction* value,
we need to change *weights*

Reducing Error

$$\text{prediction} = \text{out}$$


$$\text{prediction} = \frac{(h_1) w_5 + (h_2) w_6}{}$$


$$\text{prediction} = (\text{?}) w_5 + (\text{?}) w_6$$

to change *prediction* value,
we need to change *weights*

Reducing Error

prediction = out



prediction = $(h_1) w_5 + (h_2) w_6$



prediction = () w_5 + () w_6

to change *prediction* value,
we need to change *weights*

Reducing Error

$$\text{prediction} = \text{out}$$



$$\text{prediction} = (h_1) w_5 + (h_2) w_6$$



$$\text{prediction} = (i_1 w_1 + i_2 w_2) w_5 + (\text{?}) w_6$$

to change *prediction* value,
we need to change *weights*

Reducing Error

$$\text{prediction} = \text{out}$$



$$\text{prediction} = (h_1) w_5 + (h_2) w_6$$



$$\text{prediction} = (i_1 w_1 + i_2 w_2) w_5 + (i_1 w_3 + i_2 w_4) w_6$$

to change *prediction* value,
we need to change *weights*

Reducing Error

$$\text{prediction} = \text{out}$$



$$\text{prediction} = (h_1) w_5 + (h_2) w_6$$



$$\begin{aligned} h_1 &= i_1 w_1 + i_2 w_2 \\ h_2 &= i_1 w_3 + i_2 w_4 \end{aligned}$$

$$\text{prediction} = (i_1 w_1 + i_2 w_2) w_5 + (i_1 w_3 + i_2 w_4) w_6$$

to change *prediction* value,
we need to change *weights*

Reducing Error

$$\text{prediction} = \text{out}$$



$$\text{prediction} = (h_1) w_5 + (h_2) w_6$$



$$\begin{aligned} h_1 &= i_1 w_1 + i_2 w_2 \\ h_2 &= i_1 w_3 + i_2 w_4 \end{aligned}$$

$$\text{prediction} = (i_1 w_1 + i_2 w_2) w_5 + (i_1 w_3 + i_2 w_4) w_6$$

to change *prediction* value,
we need to change *weights*

The question now is **how to change\update the weights value so that the error is reduced?**

The answer is **Backpropagation!**

Backpropagation

- **Backpropagation**, short for “backward propagation of errors”, is a mechanism used to update the **weights** using gradient descent.
- It calculates the gradient of the error function with respect to the neural network’s weights.
- The calculation proceeds backwards through the network.
- **Gradient descent** is an iterative optimization algorithm for finding the minimum of a function; in our case we want to minimize the error function.

Gradient Descent

- **Gradient descent** is an iterative optimization algorithm for finding the minimum of a function; in our case we want to minimize the error function.
- To find a local minimum of a function using gradient descent, one takes steps proportional to the negative of the gradient of the function at the current point.

Update Weight

$$\begin{array}{c} \text{Old weight} \quad \quad \quad \text{Derivative of Error} \\ \quad \downarrow \quad \quad \quad \downarrow \\ *W_x = W_x - \text{a} \left(\frac{\partial \text{Error}}{\partial W_x} \right) \\ \uparrow \quad \quad \quad \uparrow \\ \text{New weight} \quad \quad \text{Learning rate} \end{array}$$

- For example, to update w_6 , we take the current w_6 and subtract the partial derivative of **error** function with respect to w_6 . Optionally, we multiply the derivative of the **error** function by a selected number to make sure that the new updated **weight** is minimizing the error function; this number is called ***learning rate***.

Local Minima

- Training is essentially minimizing the mean square error function.
 - ▶ Key problem is avoiding local minima
 - ▶ Traditional techniques for avoiding local minima:
 - Simulated annealing: Perturb the weights in progressively smaller amounts
 - Genetic algorithms: Use the weights as chromosomes, Apply natural selection, mating, and mutations to these chromosomes

REFERENCES

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- Tom M. Mitchell, *Machine Learning*, McGrawHill Publications, Indian Edition, 2017

Tutorial:

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- https://www.bing.com/images/search?view=detailV2&ccid=wQD8qKaG&id=07A6E6CB5195A27FD4E601CAB57551B787D0D817&thid=OIP.wQD8qKaGR_lf902l5hx18wHaD4&mediaurl=https://cdn-images-1.medium.com/max/1600/1*_M4bZyūwaGby6KMiYVYXvg.jpeg&exph=840&expw=1600&q=Multilayer+Neural+Network&simid=608048841338979193&ck=3093FFEA6AD1D032F7E34A5AF059AE83&selectedIndex=0&FORM=IRPRST&ajaxhist=0
- <https://hmkcode.com/ai/backpropagation-step-by-step/>