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Valuing Bonds

First launched in 2003

Terms

1) Bond certificate
2) Maturity date or term of Bond
3) Coupon Rat

Q → Rs 1000 Bond
10% Coupon Rate and Semi annual payments

CPN = 1000 × 10%/2 = 50 Rs per Six mo
= Coupon Rate × Face value / Number of Coupon Payments

Amount of each Coupon payment = Coupon Rate × Face value / No. of Coupon Payment per year.

Yield to Maturity → of a a bond is the discount Rate that sets the present value of the promised Bond payments equal to the current market price of the Bond.

$$PV = \frac{FV}{(1+YTM_n)^n}$$

∴ YTM of zero Coupond Bond

$$YTM_n = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$$

YTM = IRR of Bond and holding it to maturity.

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Q1 Suppose the following zero-coupon Bond are trading at the prices shown below per Rs 100 Face value. Determine the corresponding Spot interest rates that determine the zero-coupon yield Curve.

Maturity	1 year	2 year	3 year	4 years
Price	Rs 96.62	Rs 92.45	Rs 87.68	Rs 83.06

$r_1 = YTM_1 = (100/96.62) - 1 = 3.50\%$

$r_2 = YTM_2 = (100/92.45)^{1/2} - 1 = 4.00\%$

$r_3 = YTM_3 = (100/87.68)^{1/3} - 1 = 4.50\%$

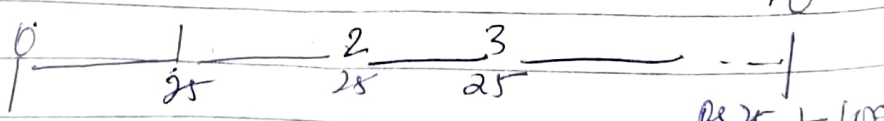
$r_4 = YTM_4 = (100/83.06)^{1/4} - 1 = 4.75\%$

Q The US Treasury has just issued a five-year Rs 1000 Bond with a 5% coupon rate and semiannual coupons. What cash flows will you receive if you hold this Bond until maturity.

FV = 1000

Payment = Semiannual

$\therefore$ = 1000 X 5%/2 = Rs 25



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Now if the Bond is currently trading you a price 957.35 what is the Bonds Yield to Maturity?

$$P = CPN \times \frac{1}{y}\left(1 - \frac{1}{(1+y)^N}\right) + \frac{FV}{(1+y)^N}$$

y = ~~Coupon Rate~~ yeild

$$957.35 = 25 \times \frac{1}{y}\left(1 - \frac{1}{(1+y)^{10}}\right) + \frac{1000}{(1+y)^{10}}$$

$y = 3\%$. Because Bond Pays premium semiannually this yeild is for 6 months.

We can solve it by trial and errors.

→ ∴ Annual Payment Rate (APR) = 6%

Q2 Consider 3 30 year Bonds with annual coupon payments.

Bond 1 → 10%

Bond 2 → 5%

Bond 3 → 3%.

if YTM of each Bond is 5% what is the price of each Bond per Rs 100 face value? Which bond trade at premium, which trades at a discount and which trades at par?

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Compute the price of each bond

(i) $10 \times \frac{1}{(0.05)} \times \left(1 - \frac{1}{(1.05)^{30}}\right) + \frac{100}{(1.05)^{30}} = 176.86$ → at premium

(ii) $5 \times \frac{1}{(0.05)} \times \left(1 - \frac{1}{(1.05)^{30}}\right) + \frac{100}{(1.05)^{30}} = 100$ → at par

(iii) $3 \times \frac{1}{(0.05)} \times \left(1 - \frac{1}{(1.05)^{30}}\right) + \frac{100}{(1.05)^{30}} = \text{Rs}69.26$ ↓ at discount.

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Time and Bond prices

Q. Suppose you purchase a 30 year Zero Coupon Bond with a YTM = 5%
FV = 100

$$\therefore PV = \frac{100}{(1.05)^{30}} = ₹23.14$$

Now lets consider the price of this Bond five year later when it has 25 year remaining for maturity

$$PV(25 \text{ year to maturity}) = \frac{100}{(1.05)^{25}} = ₹29.53$$

Note the Bond price is higher, and hence the Discount from its face value is smaller, when there is less time to maturity. The discount shrinks because the yield has not changed but there is less time until face value will be received.
If you purchased the Bond for Rs 23.14 and then sold it after 5 years at Rs 29.53, the IRR of your investment would be

$$\left(\frac{29.53}{23.14}\right)^{1/5} - 1 = 5\%$$

YTM = IRR

IRR = discount Rate that equates the PV of the Bond's remaining Cash flows to its current price

∴ Effect of time on the price of coupon

Q4 Suppose 30 year Bond
10% - Coupon Rate (annual)
FV = 100 Rs

what is the initial price of this Bond if it has 5% ~~years~~ YTM.

$$P = 10 \times \frac{1}{0.05}\left(1 - \frac{1}{(1.05)^{30}}\right) + \frac{100}{(1.05)^{30}}$$

$$= Rs\ 176.86$$

Price of the Bond immediately before and after the first coupon yeild.

(i) $P = \underset{\text{first coupon}}{10} + 10 \times \frac{1}{0.05}\left(\frac{1-0}{(1.05)^{29}}\right) + \frac{100}{(1.05)^{29}} = 185.71$

(ii) P (just after first coupon)

$$= 10 \times \frac{1}{0.05}\left(1 - \frac{1}{(1.05)^{29}}\right) + \frac{100}{(1.05)^{29}} = 175.71$$

As each coupon is paid the price of a Bond drops by the amount of coupon. (assuming YTM remains constant)

(i) When Bond is trading at premium)

the price drop when a coupon is paid will be larger than the price increase between coupons; so the Bond's premium will tend to decline as time passes.

(ii) If Bond is trading at discount

The price increase between coupons will exceed the drop when coupon is paid, so Bond price will rise and its discount will decline as the time passes.

Ultimately the price of all Bonds approach the Bonds face value when the Bond matures and there last Coupon is paid.

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~~if the yield to maturity remains 5%.~~

Q Consider a 30 year zero Coupon Bond with YTM = 5%. FV = 100 find PV of Bond
Now interest rate changes and rise so that Investor demand 6% YTM in this Bond

$$P_1 = \frac{100}{1.05^{30}} = \text{Rs } 23.14$$

$$P_2 = \frac{100}{(1.06)^{30}} = \text{Rs } 17.41$$

Q Consider a 15-year zero coupon Bond and a 30 year coupon Bond with 10% annual coupons. By what % will the price of each Bond changes if its yield to Maturity increase from 5% to 6%.

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	15 year zero Coupon	30 year 10% Coupon
YTM		
5%	$\frac{100}{1.05^{15}} = Rs\ 48.10$	$\frac{10 \times 1}{0.05}\left(1 - \frac{1}{1.05^{30}}\right) + \frac{100}{1.05^{30}}$ $= Rs\ 176.86$
6%	$\frac{100}{1.06^{15}} = Rs\ 41.73$	$\frac{10 \times 1}{0.06}\left(1 - \frac{1}{1.06^{30}}\right) + \frac{100}{1.06^{30}}$ $= Rs\ 155.06$

Price of zero Coupon Bond changes by $(41.73 - 48.10)/48.10$
$= -13.2\%$

Price of 10% Coupon Bon
$(155.06 - 176.86)/176.86 = -12.3\%$

Bond prices are subject to the effect of Both the passage of time and changes in Interest rate.
Bond price converge to its Bond FV due to time effect

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Q MP Ltd issued Bond on 1/1/2010
Par (FV) = 1000
Coupon = 10% RA
Maturity = 31/12/2025
Coupon payment = Semi annually
on 30 June & 31 Dec

You purchased outstanding Bond
1/3/2018
when the going interest rate was
12%

(1) YTM of Bond as on 1/1/2018