Module 3

Historical return

- Historical returns are often associated with the past performance of a security or index, such as the Nifty 50.
- Analysts review historical return data when trying to predict future returns or to estimate how a security might react to a particular situation, such as a drop in consumer spending. Historical returns can also be useful when estimating where future points of data may fall in terms of standard deviations.

Historical return

- Historical returns are often associated with the past performance of a security or index, such as the S&P 500.
- Investors study historical return data when trying to forecast future returns or to estimate how a security might react in a situation.
- Calculating the historical return is done by subtracting the most recent price from the oldest price and divide the result by the oldest price.

Historical return

- Analyzing historical data can provide insight into how a security or market has reacted to a variety of different variables, from regular economic cycles
- to sudden, exogenous world events.
- Investors looking to interpret historical returns should bear in mind that past results do not necessarily predict future returns. The older the historical return data, the less likely it'll be successful at forecasting returns in the future.

Historical return

- Calculating or measuring the historical return of an asset or investment is relatively straightforward.
- Subtract the most recent price from the oldest price in the data set and divide the result by the oldest price. We can move the decimal two places to the right to convert the result into a percentage.
- For example, let's say we want to calculate the return of the S&P 500 for 2019. We start with the following data:
- 3,756= the S&P 500 closing price on December 31, 2020
- 4,766 = the S&P 500 closing price on December 31, 2021
- 4,766 - 3,756 = 1,010
- 1,010/3,756 = .269 or 27%*

Historical return

- The historical returns are often analyzed for trends or patterns that may align with current financial and economic conditions
- Technical analysts believe potential market outcomes may follow past patterns. Hence, there is a hidden value available from the study of historical return trends. However, technical analysis is more often applied to short-term price movements of those assets that frequently fluctuate in price, such as commodities.

Historical return

- From there, investors can plan their asset allocation, meaning what types of holdings to invest in, and develop a risk management strategy in case the price of the market or asset moves adversely. In short, historical returns analysis might not predict future price movements, but it can help investors be more informed and better prepared for what the future holds.

Historical risk

- There are five principal risk measures, and each measure provides a unique way to assess the risk present in investments that are under consideration.

Historical risk

- There are five principal risk measures, and each measure provides a unique way to assess the risk present in investments that are under consideration. The five measures include the
- alpha,
- beta,
- R-squared
- standard deviation
- Sharpe ratio

Alpha

- Alpha measures risk relative to the market or a selected benchmark index. For example, if the Nifty 50 has been deemed the benchmark for a particular fund, the activity of the fund would be compared to that experienced by the selected index.
- If the fund outperforms the benchmark, it is said to have a positive alpha. If the fund falls below the performance of the benchmark, it is considered to have a negative alpha.

BETA

- Beta measures the volatility or systemic risk of a fund in comparison to the market or the selected benchmark index.
- A beta of one indicates the fund is expected to move in conjunction with the benchmark. Betas below one are considered less volatile than the benchmark, while those over one are considered more volatile than the benchmark.

R-Squared (coefficient of determination)

- R-Squared measures the percentage of an investment's movement attributable to movements in its benchmark index.
- R-squared (R^2) is a statistical measure that represents the proportion of the variance for a dependent variable that's explained by an independent variable in a regression model.
- Whereas correlation explains the strength of the relationship between an independent and a dependent variable, R-squared explains the extent to which the variance of one variable explains the variance of the second variable. So, if the R^2 of a model is 0.50, then approximately half of the observed variation can be explained by the model's inputs.

R-Squared vs. Beta

- Beta and R-squared are two related, but different, measures of correlation. Beta is a measure of relative riskiness. A mutual fund with a high R-squared correlates highly with a benchmark. If the beta is also high, it may produce higher returns than the benchmark, particularly in bull markets.
- R-squared measures how closely each change in the price of an asset is correlated to a benchmark.
- Beta measures how large those price changes are relative to a benchmark. Used together, R-squared and beta can give investors a thorough picture of the performance of asset managers. A beta of exactly 1.0 means that the risk (volatility) of the asset is identical to that of its benchmark.
- Essentially, R-squared is a statistical analysis technique for the practical use and trustworthiness of betas of securities.

Calculation of β by Co-variance Method

$$\beta_x = \frac{Cov_{xm}}{\sigma_m^{\ 2}}$$

Calculation of β by Correlation method

$$\beta_x = Cor_{xm} \times \frac{\sigma_x}{\sigma_m}$$

INTRODUCTION

- A portfolio is a bundle or a combination of individual assets or securities.
- Portfolio theory provides a normative approach to investors to make decisions to invest their wealth in assets or securities under risk.
- Extend the portfolio theory to derive a framework for valuing risky assets. This framework is referred to as the capital asset pricing model (CAPM). An alternative model for the valuation of risky assets is the arbitrage pricing theory (APT).
- The return of a portfolio is equal to the weighted average of the returns of individual assets (or securities).

PORTFOLIO RETURN: TWO-ASSET CASE

- The return of a portfolio is equal to the weighted average of the returns of individual assets (or securities) in the portfolio with weights being equal to the proportion of investment value in each asset.
- We can use the following equation to calculate the expected rate of return of individual asset:

$$E(R_x) = (R_1 \times P_1) + (R_2 \times P)_2 + (R_3 \times P_3) + \ldots\ldots + (R_n \times P_n)$$

$$E(R_x) = \sum_{i=1}^{n} R_i P_i$$

Expected Rate of Return: Example

- Suppose you have an opportunity of investing your wealt either in asset X or asset Y. The possible outcomes of tw assets in different states of economy are as follows:

Possible Outcomes of two Assets, *X and Y*

State of Economy	*Probability*	*Return (%)*	
		X	*Y*
A	0.10	−8	14
B	0.20	10	−4
C	0.40	8	6
D	0.20	5	15
E	0.10	−4	20

The expected rate of return of X is the sum of the product of outcomes and their respective probability. That is:

$$E(R_x) = (-8 \times 0.1) + (10 \times 0.2) + (8 \times 0.4) + (5 \times 0.2) + (-\ 4 \times 0.1) = 5\%$$

Similarly, the expected rate of return of Y is:

$$E(R_y) = (14 \times 0.1) + (-\ 4 \times 0.2) + (6 \times 0.4) + (15 \times 0.2) + \ (20 \times 0.1) = 8\%$$

PORTFOLIO RISK: TWO-ASSET CASE

7

- Risk of individual assets is measured by their variance or standard deviation.

- We can use variance or standard deviation to measure the risk of the portfolio of assets as well.

- The risk of portfolio would be less than the risk of individual securities, and that the risk of a security should be judged by its contribution to the portfolio risk.

Measuring Portfolio Risk for Two Assets

- The portfolio variance or standard deviation depends on the co-movement of returns on two assets.
- Covariance of returns on two assets measures their co-movement.
- Three steps are involved in the calculation of covariance between two assets:

Determining the expected returns on assets.

Determining the deviation of possible returns from the expected return for each asset.

Determining the sum of the product of each deviation of returns of two assets and respective probability.

The CAPM Theory

- Markowitz, William Sharpe, John Lintner and Jan Mossin provided the basic structure for the CAPM model.
- It is a model of linear general equilibrium return.
- The required rate return of an asset is having a linear relationship with asset's beta value *i.e.,* undiversifiable or systematic risk.

Assumptions

- An individual seller or buyer cannot affect the price of a stock.
- Investors make their decisions only on the basis of the expected returns, standard deviations and covariances of all pairs of securities.
- Investors are assumed to have homogenous expectations during the decision-making period.
- The investor can lend or borrow any amount of funds at the riskless rate of interest.
- There is no transaction cost.
- There is no personal income tax.
- Unlimited quantum of short sales, is allowed.

Lending and Borrowing

- It is assumed that the investor could borrow or lend any amount of money at riskless rate of interest.

  R_p = Portfolio return

  X_f = The proportion of funds invested in risk free assets

  $1 - X_f$ = The proportion of funds invested in risky assets

  R_f = Risk free rate of return

  R_m = Return on risky assets

- The expected return on the combination of risky and risk free combination is

$$R_p = R_f X_f + R_m(1 - X_f)$$

Risk Free Rate = 7% p.a.

Rate of Return in the Market = 15% p.a.

Average Rate of Return on Security 'X' = 17.69% p.a.

Beta of Security 'X' = 1.2 times.

You are required to analyse the situation using Capital Asset Pricing Model and determine the alpha for Security 'X'.

Decide whether it is worth investing in Security 'X'

Rate of Return	
Rate of Return in the Market Less: Risk Free Rate	15% 7%
Risk Premium	8%

An investor who is ready to invest in the market portfolio would expect 15% p.a. returns, i.e. 7% as risk free rate and 8% as premium for taking the risk prevailing in the market.

Security 'X' has a beta of 1.2 that means the risk associated with Security 'X' is 20% more than the risk prevailing in the market. Therefore, the investor will definitely expect a higher reward for such high risk i.e. by expecting 20% more premium. Therefore, the expected risk premium for the investor equals to 8 + 20% = 9.6%.

∴ *Expected Rate of Return as per Capital Asset Pricing Model*
= 7% + 9.6% = 16.6%

$$E(R_x) = 7 + (15 - 7) \times 1.20 = 16.6\%$$

The rate of return, on an average, observed on Security 'X' is higher than the expected rate of return based on Capital Asset Pricing Model. Therefore, it is advisable to invest into such security. Alpha of Security 'X' is the difference between Average rate of return of Security 'X' and the expected rate of return on Security 'X' as per Capital Asset Pricing Model.

$$\alpha_x = R_x - E(R_x)$$

$$\alpha_x = 17.69 - 16.60 = 1.09$$

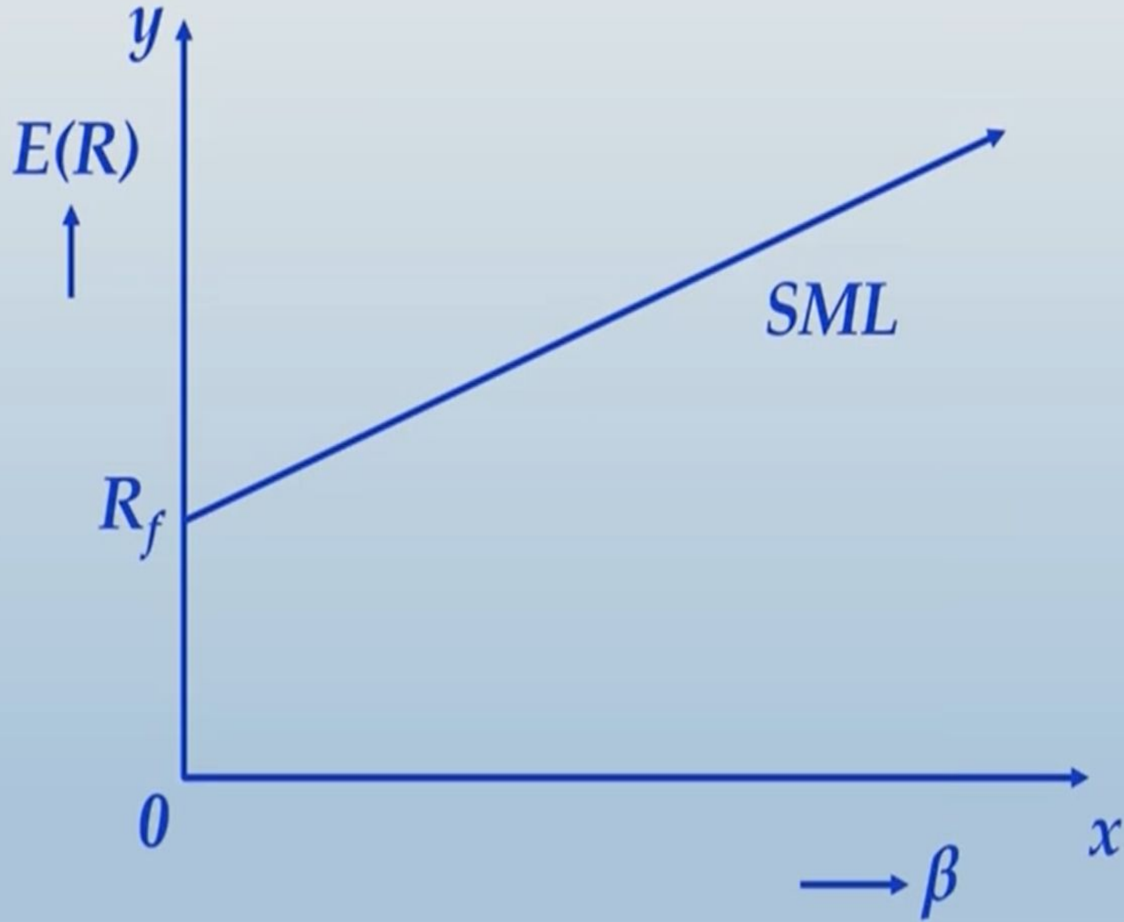
α of Security 'X' is positive indicating that there is a possibility of getting rate of return higher than expected as per Capital Asset Pricing Model. Therefore, it is worth investing in Security 'X'.

Security Market Line (SML)

*When the expected rate of return as per **CAPM** is presented graphically;*

*It will be a straight line **sloping upward.***

*Such line is known as **"Security Market Line"***



SML sloping upward
Indicates that:
The higher the β
The higher will be E(R)

The higher the Risk
the higher should be
the Returns

Standard Deviation

- Standard deviation is a method of measuring data dispersion in regards to the mean value of the dataset and provides a measurement regarding an investment's volatility.
- As it relates to investments, the standard deviation measures how much return on investment is deviating from the expected normal or average returns.

Sharpe Ratio

- **The Sharpe ratio measures performance as adjusted by the associated risks**. This is done by removing the rate of return on a risk-free investment, such as a G-sec , from the experienced rate of return.
- This is then divided by the associated investment's standard deviation and serves as an indicator of whether an investment's return is due to wise investing or due to the assumption of excess risk.

Sharpe Ratio

- the Sharpe ratio divides a portfolio's excess returns by a measure of its volatility to assess risk-adjusted performance
- Excess returns are those above an industry benchmark or the risk-free rate of return
- The calculation may be based on historical returns or forecasts
- A higher Sharpe ratio is better when comparing similar portfolios.

Sharpe ratio

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$

where:

R_p = return of portfolio

R_f = risk-free rate

σ_p = standard deviation of the portfolio's excess return

Consider the following example:

R_m = 25%

R_f = 10%

σ_m = 6%

Determine Lambda

$$\lambda = \frac{R_m - R_f}{\sigma_m}$$

$$\lambda = \frac{25 - 10}{6}$$

$$\lambda = 2.5$$

The above computed λ indicates:

- *For each percent of market risk,*
- *In the form of standard deviation,*
- *The risk premium should be 2.5%*

Limitation of Sharpe ratio

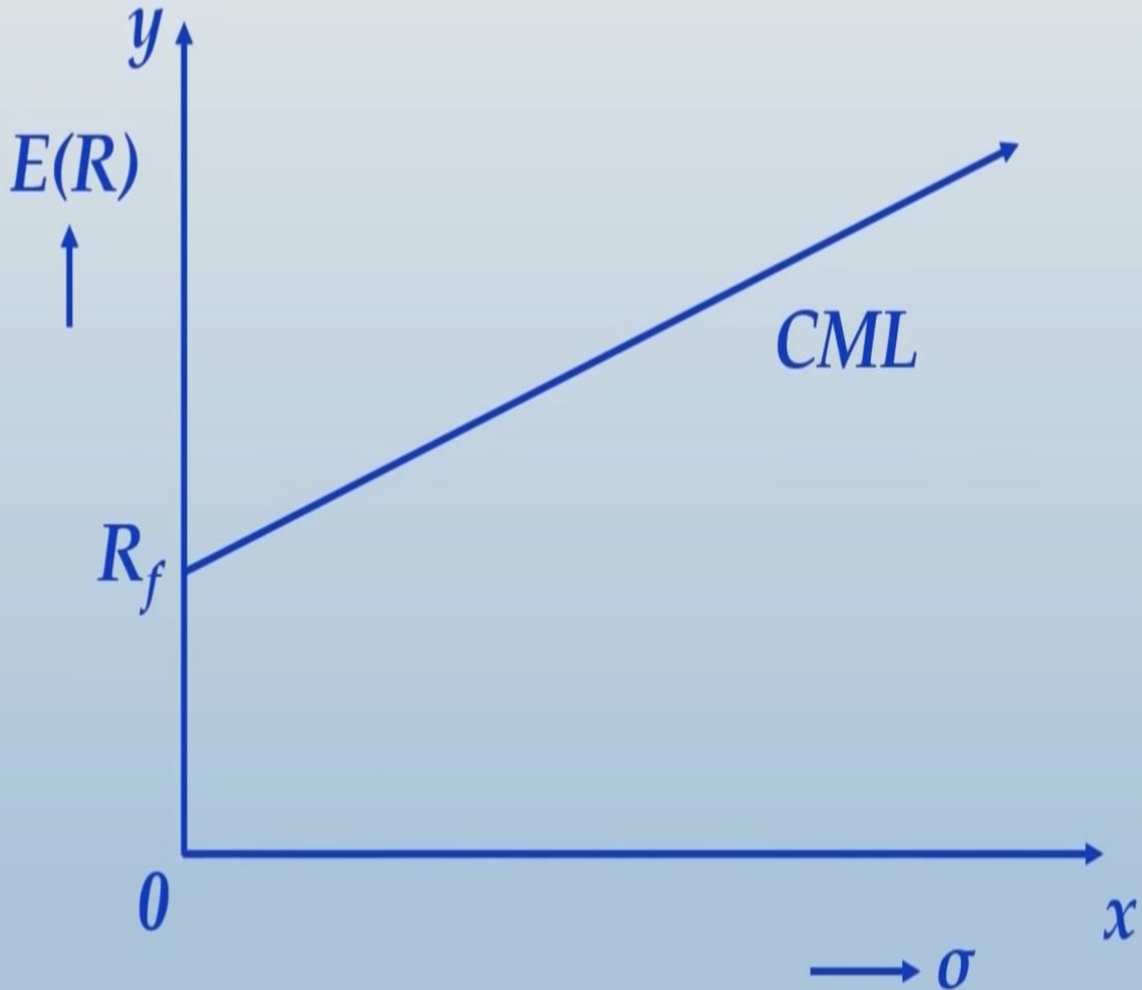
- The Sharpe ratio can be manipulated by portfolio managers seeking to boost their apparent risk-adjusted returns history. This can be done by lengthening the return measurement intervals, which results in a lower estimate of volatility. For example, the standard deviation (volatility) of annual returns is generally lower than that of monthly returns, which are in turn less volatile than daily returns. Financial analysts typically consider the volatility of monthly returns when using the Sharpe ratio.

Capital Market Line (CML)

- *When the expected rate of return is related to absolute risk of the stock, i.e. with the standard deviation,*
- *Presented graphically,*
- *It will result in a straight line sloping upward.*
- *Such line is known as "Capital Market Line"*

$$E(R) = R_f + (R_m - R_f)\frac{\sigma_x}{\sigma_m}$$

Or, $E(R) = R_f + \lambda.\sigma_x$



CML sloping upward
Indicates that:
The higher the σ
The higher will be E(R)

The higher the Risk the higher should be the Returns

Consider the following data:

$R_m = 25\%$

$R_f = 10\%$

$\sigma_m = 6\%$

$\sigma_x = 4\%$

Determine the expected rate of return using CML.

Expected rate of return as per CML

$$E(R) = R_f + (R_m - R_f)\frac{\sigma_x}{\sigma_m} \qquad E(R) = 10 + (25 - 10)\ \frac{4}{6}$$

$$E(R) = 10 + 15 \times \frac{4}{6} \qquad E(R) = 20\%$$

Alternatively:

$$\lambda = \frac{R_m - R_f}{\sigma_m} \qquad E(R) = R_f + \lambda.\sigma_x$$

$$E(R) = 10 + (2.5 \times 4)$$

$$\lambda = \frac{25 - 10}{6} \qquad E(R) = 20\%$$

$$\lambda = 2.5$$

Expected Rate of Return as per SML:

$$E\ (R_x) = R_f + (R_m - R_f)\beta_x$$

$$\beta_x = \frac{\sigma_x}{\sigma_m} \times Cor_{xm}$$

$$E\ (R_x) = R_f + (R_m - R_f).\frac{\sigma_x}{\sigma_m} \times Cor_{xm}$$

Expected rate of return as per CML:

$$E(R) = R_f + (R_m - R_f)\frac{\sigma_x}{\sigma_m}$$

Note:

On comparing CML with SML, it is observed that:

- *CML is based on Standard Deviation (σ), whereas SML is based on Beta (β).*
- *CML considers absolute risk of Stock and Market (based on Standard Deviations), whereas SML considers relative risk of Stock with that of Market.*
- *In CML, correlation between the security and the market is not considered, whereas SML considers correlation between the security and the market. (SML is based on CAPM).*
- *If the correlation between security and market is perfectly positive, (i.e. $Cor_{xm} = 1.00$), the conclusions as per CML and SML will be the same.*

Arbitrage

- Arbitrage is a process of earning profit by taking advantage of differential pricing for the same asset.
- The process generates riskless profit.
- In the security market, it is of selling security at a high price and the simultaneous purchase of the same security.

The Assumptions

- ❖ The investors have homogenous expectations.
- ❖ The investors are risk averse and utility maximisers.
- ❖ Perfect competition prevails in the market and there is no transaction cost.

Arbitrage Portfolio

- According to the APT theory, an investor tries to find out the possibility to increase returns from his portfolio without increasing the funds in the portfolio.
- He also likes to keep the risk at the same level.
- If X indicates the change in proportion,

$$\Delta X_A + \Delta X_B + \Delta X_C = 0$$

Factor Sensitivity

- The factor sensitivity indicates the responsiveness of a security's return to a particular factor. The sensitiveness of the securities to any factor is the weighted average of the sensitivities of the securities.
- Weights are the changes made in the proportion. For example b_A, b_B and b_C are the sensitivities, in an arbitrage portfolio the sensitivities become zero.

$$b_A \, \Delta X_A + b_B \, \Delta X_B + b_C \, \Delta X_C = 0$$

The APT Model

- According to Stephen Ross, returns of the securities are influenced by a number of macro economic factors.
- The macro economic factors are: growth rate of industrial production, rate of inflation, spread between long term and short term interest rates and spread between low grade and high grade bonds. The arbitrage theory is represented by the equation:

$$R_i = \lambda_0 + \lambda_1\, bi_1 + \lambda_2\, bi_2 \ldots + \lambda_j\, bi_j$$

where R_i = average expected return

λ_1 = sensitivity of return to bi_1

bi_1 = the beta co-efficient relevant to the particular factor

Arbitrage Pricing Equation

In a single factor model, the linear relationship between the return R_i and sensitivity b_i can be given in the following form

$$R_i = \lambda_o + \lambda_i b_i$$

- R_i = return from stock A
- λ_o = riskless rate of return
- b_i = the sensitivity related to the factor
- λ_i = slope of the arbitrage pricing line

APT and CAPM

- The simplest form of APT model is consistent with the simple form of the CAPM model.
- APT is more general and less restrictive than CAPM.
- The APT model takes into account of the impact of numerous factors on the security.
- The market portfolio is well defined conceptually. In APT model, factors are not well specified.
- There is a lack of consistency in the measurements of the APT model.
- The influences of the factors are not independent of each other.

What is expected return

- Expected return is the anticipated profit or loss an investor can predict for a specific investment based on historical rates of return (RoR).
- **Expected return = (Return A x probability A) + (Return B x probability B)**

How to calculate expected return

- Determine the probability of each return that might occur
- Determine the expected return for each possible return
- Multiply each expected return by its corresponding percentage (weight)
- Add each of the products together to find the weighted average expected return

What is a portfolio

Portfolio means a set or combination of different financial assets (or securities) held at a point of time for the purpose of investment and not for consumption purposes. These financial assets (or securities) may include shares, debentures, bonds, government securities, etc. Different portfolios are designed according to the investor's risk tolerance, time frame and investment objectives.

Purpose of Porfolio

Diversification means that a portfolio must contain large number of securities across different firms and industry lines which could be easily managed. It is strategy of holding more than one security type in a portfolio for minimizing risk. Generally, a portfolio of 15-20 securities is considered as manageable. This helps in reducing the non- systematic risk of the portfolio as poor performance of one security can be offset by the better performance of another. *This is called not keeping all eggs in one basket.*

Negative correlation means that a change in one variable is inversely related to the changes in other variable. The notion of negative correlation denotes that, as far as possible, the securities included in a portfolio should be negatively correlated. It is hedging strategy for risk. The securities with negative correlation also serve the purpose of diversification. This helps in

- Reduces the impact of market volatility
• Reduces the time spent in monitoring the portfolio
• Helps seek advantage of different investment instruments
• Helps achieve long-term investment plans
• Helps avail of benefit of compounding of interest
• Helps keep the capital safe
• Lets you shuffle amongst investments
• Offers peace of mind

Amount to invest: $100,000

OPTION A	OPTION B	OPTION C
Google	1. Google	1. Google
	2. Microsoft	2. Deutsche Bank
	3. Apple	3. American Airlines
	4. Intel	4. Target

Option A
- Single investment
- No Diversification

Option B
- Portfolio
- Not well diversified
- High correlation

Option C
- Portfolio
- Well diversified
- Low correlation

Expected return of a portfolio

- The expected return on a portfolio is nothing but simply the weighted average of the expected returns of the individual sesecurities in a given portfolio

- The risk of Individual security is estimated through Deviation or variance. The portfolio risk is calculated using the
- risk of individual assets,
- Weight of asset in portfolio
- Either correlaton between the asset or covariance between the portfolio returns

Example 31.1: Suppose a portfolio consists of three securities A, B and C with expected returns of 12 percent, 15 percent and 18 percent respectively. The proportions of total investment in these securities are 0.30, 0.40 and 0.30 respectively. Find the expected return on the portfolio.

Here,

W_A=0.30	$E(R_A)$= 12%
W_B=0.40	$E(R_B)$= 15%
W_C=0.30	$E(R_C)$=18%

$$\boxed{E(R_P)= [\{W_A * E(R_A)\} + \{W_B * E(R_B)\} + \{W_C * E(R_C)\}]}$$

= (0.30*0.12) + (0.40* 0.15) + (0.30*0.18)

=0.15

=15%

Therefore, the expected rate of return of the given portfolio of three securities is 15%.

Sates of the economy	Probability	Return A	Return B
Good	45%	15%	16%
Average	40%	8%	9%
Bad	15%	-11%	-13%

Expected Return

Return A:

$$0.45*0.15+0.4*0.08+0.15*-0.11$$

$$= 0.083 = 8.3\%$$

Sates of the economy	Probability	Return A	Return B
Good	45%	15%	16%
Average	40%	8%	9%
Bad	15%	-11%	-13%

Expected Return

Return B:

$$0.45*0.16+0.4*0.09+0.15*-0.13$$

$$= 0.0885 = 8.85\%$$

Sates of the economy	Probability	Return A	Return B
Good	45%	15%	16%
Average	40%	8%	9%
Bad	15%	-11%	-13%

Standard deviation:

$$\text{Sum}\sqrt{\text{Probability}*(\text{Return} - \text{Expected Return})^2}$$

Sates of the economy	Probability	Return A	Return B
Good	45%	15%	16%
Average	40%	8%	9%
Bad	15%	-11%	-13%

SD=0.45(0.15-0.083)^2+0.40(0.08-0.083)^2+0.15(-0.11-0.083)^2

$$SD=\sqrt{0.07611}$$

SD=0.087 or 8.7%

Standard deviation:

$\sqrt{\text{Probability*(Return –Expected Return)^2}}$

Sates of the economy	Probability	Return A	Return B
Good	45%	15%	16%
Average	40%	8%	9%
Bad	15%	-11%	-13%

SD=0.45(0.16-0.0885)^2+0.40(0.09-0.0885)^2+0.15(-0.13-0.0885)^2

SD=√0.0946

SD=0.097 or 9.7%

Expected Portfolio Return (Mean): $\hat{r}_P$

$$\hat{r}_P = \sum w_i \hat{r}_i$$

$$= w_1\hat{r}_1 + w_2\hat{r}_2 + \ldots + w_N\hat{r}_N$$

- For the 3-asset portfolio:

$$\hat{r}_P = w_A\hat{r}_A + w_B\hat{r}_B + w_C\hat{r}_C$$

$$= 0.5(0.23) + 0.3(0.13) + 0.2(0.10) = 0.174 \; or \; 17.4\%$$

Example

As an investor, you're considering three different investment options. You determine the performance of each investment option over the past five years and find the following performance results:

Investment A	10%	25%	-4%	6%	15%
Investment B	5%	12%	8%	3%	7%
Investment C	20%	7%	18%	-10%	4%

Assuming the probability of each return scenario occurring again is equal, the probability of each return occurring is 20%. You calculate the expected return of each investment as follows:

Investment A Expected Return	(10 x .20) + (25 x .20) + (-4 x .20) + (6 x .20) + (15 x .20)	Equals 10.4%
Investment B Expected Return	(5 x .20) + (12 x .20) + (8 x .20) + (3 x .20) + (7 x .20)	Equals 7%
Investment C Expected Return	(20 x .20) + (7 x .20) + (18 x .20) + (-10 x .20) + (4 x .20)	Equals 7.8%

Using the same scenario from the example above, you decide to create a portfolio and invest in all three investments. You decide to allocate 50% of your portfolio in Investment A, 30% of your portfolio in Investment B and 20% of your portfolio in Investment C. You calculate the expected return of your entire portfolio as follows:

Investment A	Weight of investment - 50%	Expected return - 10.4%
Investment B	Weight of investment - 30%	Expected return - 7%
Investment C	Weight of investment - 20%	Expected return - 7.8%

Portfolio Expected Return	(50 x .104) + (30 x .07) + (20 x .078)	Equals 8.86%	-4%	6%	15%

Portfolio Risk

- Measures of portfolio risk:
  1. Variance of portfolio (σ^2_P)
  2. Standard deviation of portfolio (σ_P)

- Factors affecting portfolio risk:
  1. Individual security risk: σ^2_i
  2. Investment proportion: w_i
  3. Degree of diversification between stocks, measured by:
     - Covariance (σ_{ij}) or
     - Correlation coefficient (ρ_{ij})

Portfolio Selection

1. Individual Security risk: σ_i
2. Investment %
3. Portfolio diversification

Stock A
High Risk

Stock B
High Risk

Stock C
Low Risk

Stock D
Low Risk

Covariance

- Covariance is a statistical tool used to determine the relationship between the movements of two random variables.
- When two stocks tend to move together, they are seen as having a positive covariance; when they move inversely, the covariance is negative.
- Covariance is different from the correlation coefficient, a measure of the strength of a correlative relationship.
- Covariance is an important tool in modern portfolio theory for determining what securities to put in a portfolio.
- Risk and volatility can be reduced in a portfolio by pairing assets that have a negative covariance.

Formula for covariance

$$\text{Covariance} = \sum \frac{(\text{Ret}_{abc} - \text{Avg}_{abc}) \times (\text{Ret}_{xyz} - \text{Avg}_{xyz})}{\text{Sample Size} - 1}$$

where:

$\text{Ret}_{abc} = \text{Day's return for ABC stock}$

$\text{Avg}_{abc} = \text{ABC's average return over the period}$

$\text{Ret}_{xyz} = \text{Day's return for XYZ stock}$

$\text{Avg}_{xyz} = \text{XYZ's average return over the period}$

$\text{Sample Size} = \text{Number of days sampled}$

Positive Covariance

- A positive covariance between two variables indicates that these variables tend to be higher or lower at the same time. In other words, a positive covariance between stock one and two is where stock one is higher than average at the same points that stock two is higher than average, and vice versa. When charted on a two-dimensional graph, the data points will tend to slope upwards.

Negative Covariance

- When the calculated covariance is less than negative, this indicates that the two variables have an inverse relationship. In other words, a stock one value lower than average tends to be paired with a stock two value greater than average, and vice versa.
-

Measures of Association

Historical data:

Time	Return r_A	Return r_B
1	0.50	0.25
2	0.40	0.15
3	-0.07	0.32
4	0.09	-0.20

Additional Portfolio Data

Note that…

- Sample mean of Stock A: $\bar{r}_A = 0.23$
- Sample mean of Stock B: $\bar{r}_B = 0.13$

- Sample standard deviation of Stock A: $s_A = 0.2655$
- Sample standard deviation of Stock B: $s_B = 0.2308$

Time	r_A	r_B	Cov
1	0.50	0.25	0.0324
2	0.40	0.15	0.0034
3	-0.07	0.32	-0.0570
4	0.09	-0.20	0.0462
			0.0250

Sample Covariance: Cov(A,B)

$$Cov(A,B) = \frac{\sum (r_A - \bar{r}_A)(r_B - \bar{r}_B)}{n-1}$$

For securities A, and B:

$$Cov(A,B) = \frac{(r_{A1} - \bar{r}_A)(r_{B1} - \bar{r}_B) + (r_{A2} - \bar{r}_A)(r_{B2} - \bar{r}_B) + (r_{A3} - \bar{r}_A)(r_{B3} - \bar{r}_B) + (r_{A4} - \bar{r}_A)(r_{B4} - \bar{r}_B)}{4-1}$$

Substituting,

$$Cov(A,B) = \frac{(0.5 - 0.23)(0.25 - 0.13) + (0.4 - 0.23)(0.15 - 0.13) + (-0.07 - 0.23)(0.32 - 0.13) + (0.09 - 0.23)(-0.2 - 0.13)}{4-1}$$

$$= \frac{0.0250}{3} = 0.00833$$

DrawBack of Covariance

One drawback of the covariance measure is that its magnitude does not tell us much about the strength of the co-movement. Correlation is a standardized measure of co-movement.

Correlation between Assets

- Investing wealth in more than one security reduces portfolio risk.
- This is attributed to diversification effect.
- However, the extent of the benefits of portfolio diversification depends on the correlation between returns on securities.
- When correlation coefficient of the returns on individual securities is perfectly positive then there is no advantage of diversification. The weighted standard deviation of returns on individual securities is equal to the standard deviation of the portfolio.
- Diversification always reduces risk provided the correlation coefficient is less than 1.

Co-relation

The coefficient of correlation is another measure to reflect the degree of co- movement between two variables. The correlation between two securities depends upon the covariance between two securities and the standard deviation of each security.

$$\boxed{r_{xy} = \frac{Cov_{xy}}{\sigma_x \, \sigma_y}}$$

Where,
r_{xy}= Coefficient of correlation between x and y
Cov_{xy} = Covariance between x and y
σ_x= Standard deviation of x
σ_y= Standard deviation of y
The value of coefficient of correlation between two variables lies between -1 and +1. The correlation coefficient of -1 between two securities implies a perfect negative correlation between the securities. A zero correlation coefficient means that the securities are independent of each other and +1 implies that there exists a perfect positive correlation among the securities.

PORTFOLIO RISK-RETURN ANALYSIS: TWO-ASSET CASE

- Perfectly positive correlation (+1.0);
- Perfectly negative correlation (–1.0);
- No correlation (0.0),
- Positive correlation (0.5), and
- Negative correlation (– 0.25).

Special situations

Sample Correlation Coefficient: $r_{A,B}$

$$r = \frac{Cov(A,B)}{(s_A)(s_B)} = \frac{0.00833}{(0.2655)(0.2308)} = 0.14, \quad r = -1 \le r \le 1$$

Covariance	Correlation
It is a measure to show the extent to which given two random variables change with respect to each other.	It is a measure used to describe how strongly the given two random variables are related to each other.
It is a measure of correlation.	It is defined as the scaled form of covariance.
The value of covariance lies between -∞ and +∞.	The value of correlation lies between -1 and +1.
It indicates the direction of the linear relationship between the given two variables.	It measures the direction and strength of the linear relationship between the given two variables.

Calculation of portfolio risk

FORMULA

In a two-asset (A and B) portfolio , the portfolio risk is defined as

$$\sigma\rho = \sqrt{W_A^2\sigma_A^2 + W_B^2\sigma_B^2 + 2w_A w_B \cdot \rho_{AB}\sigma_A\sigma_B}$$

σ_A and σ_B = standard deviations of assets A and B, respectively

w_A and w_B= weights, or fractions, of total funds invested in assets A and B

ρ_{AB} = the correlation coefficient between assets A and B

Q From the following data compute Beta?

SD (j) = 12%; SD(market) =9% ; Corelation (j,m) = +0.72

Ans= 0.96