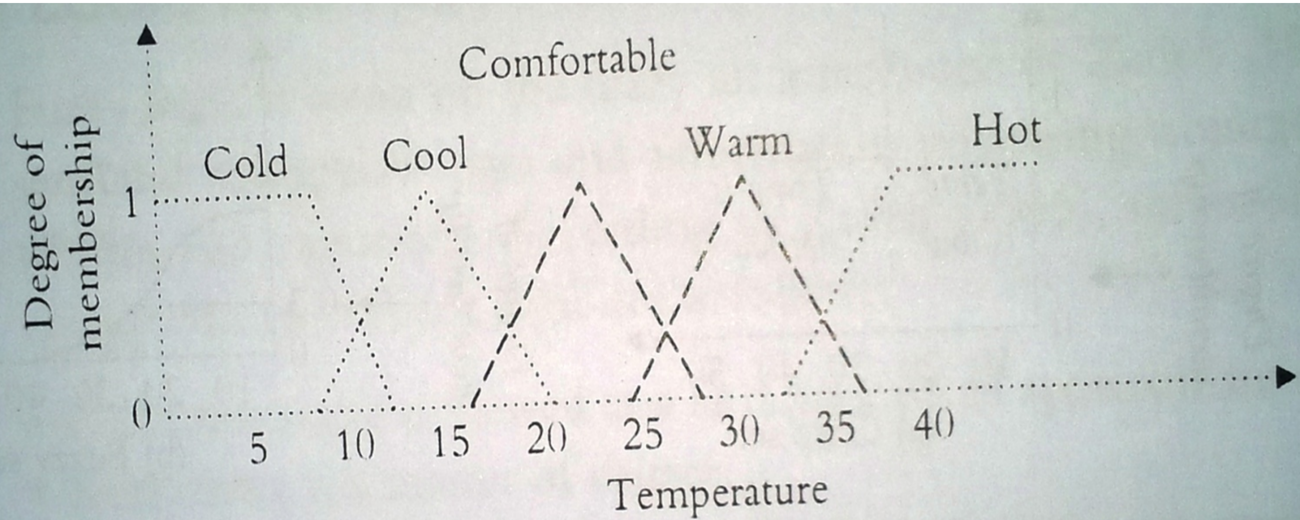
MEMBERSHIP FUNCTIONS

Membership Function

- The curve representing the mathematical function is a membership function that determines the degree of belonging of member x to the fuzzy set T.
- Maps elements of a fuzzy set to real numbered values in the interval 0 to 1.

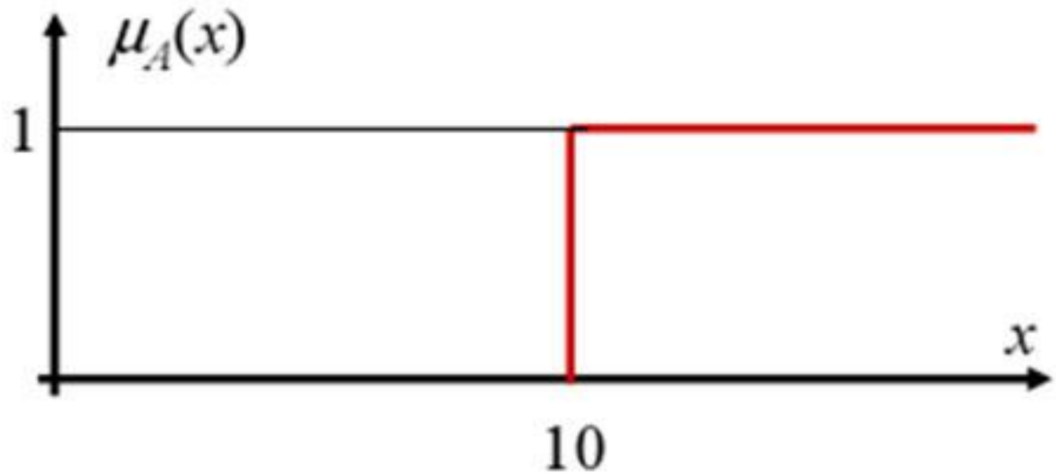
FIGURE 5.4
Fuzzy membership functions for a typical air conditioner control



CRISP MEMBERSHIP FUNCTIONS

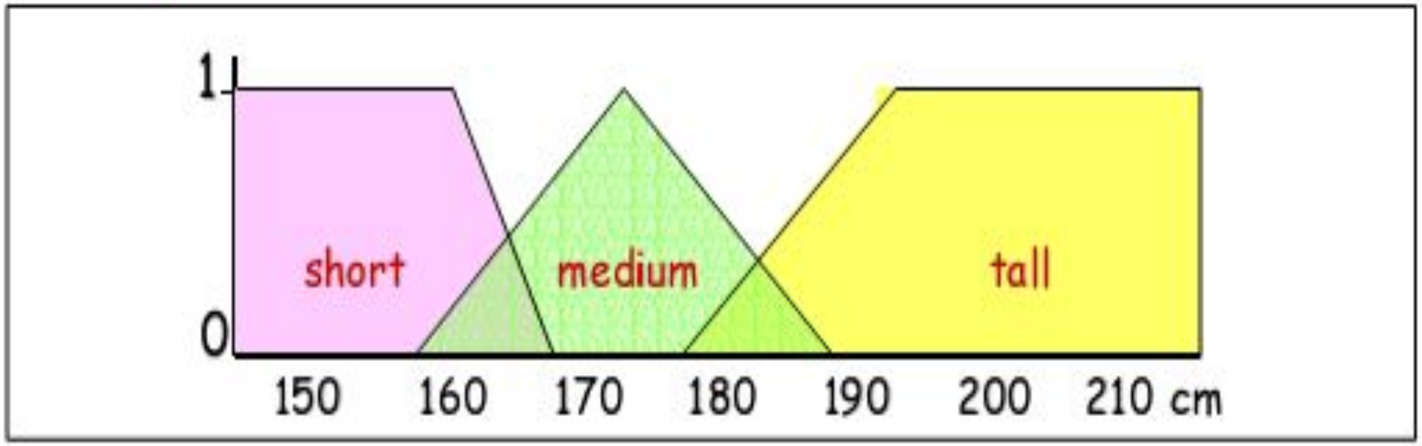
- Crisp membership functions (μ) are either one or zero.
- Consider the example: Numbers greater than 10. The membership curve for the set A is given by

$A = \{x \mid x > 10\}$



REPRESENTING A DOMAIN IN FUZZY LOGIC

Fuzzy sets (men's height):



FUZZY MEMBERSHIP FUNCTIONS

- Categorization of element x into a set A

  described through a membership function $\mu_A(x)$
- Formally, given a fuzzy set A of universe X

  $\mu_A(x)$: $X \rightarrow [0,1]$, where

  - $\mu_A(x) = 1$ if x is totally in A
  - $\mu_A(x) = 0$ if x is totally not in A
  - $0 < \mu_A(x) < 1$ if x is partially in A

  $\mu_{Tall}(200) = 1$

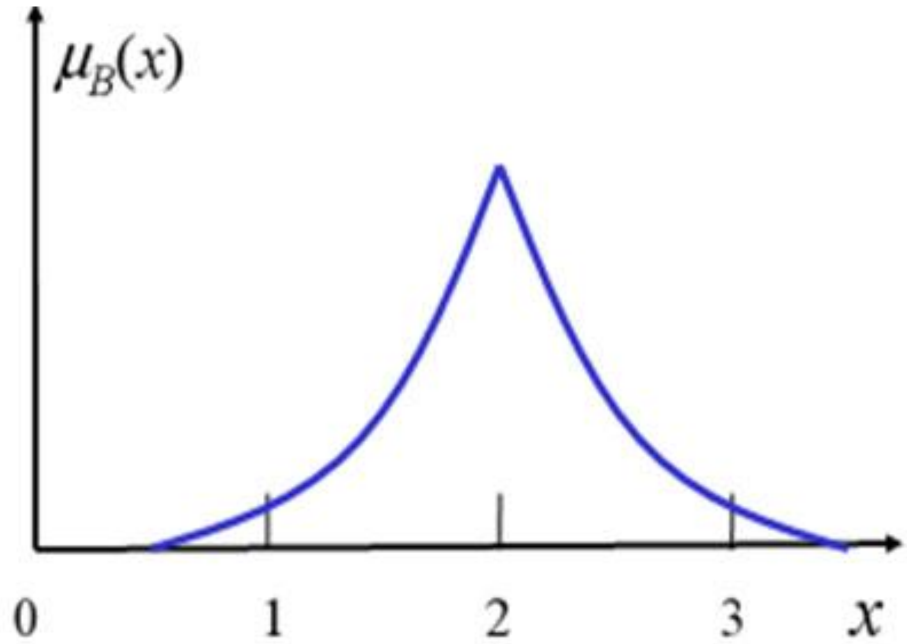
  $\mu_{Tall}(160) = 0$

  $0 < \mu_{Tall}(180) < 1$
- (Discrete) Fuzzy set A is represented as:

  $A = \{\mu_A(x_1)/x_1, \mu_A(x_2)/x_2, \ldots, \mu_A(x_n)/x_n\}$

The set B of numbers approaching 2 can be represented by the membership function

$$\mu_B(x) = e^{-|x-2|}$$



FUZZINESS vs PROBABILITY

- When first exposed to fuzzy logic, humans associate membership functions with density functions.
- This is not so, since:
  - Probability density is an abstraction from empirical frequency.

    ⇒ an aggregate property.
    - how often events occur in different ways.
    - ways that are quite crisp and mutually exclusive after occurrence.
  - Fuzzy relations, by contrast, are properties of single events that are always there, and not different from occurrence to occurrence.

Linguistic Variable

A variable whose values are words or sentences in natural language.

Example 1: Temperature is linguistic variable if it takes values hot, cool, warm, comfortable etc.

Example 2: the values of the fuzzy variable *height* could be *tall, very tall, very very tall, somewhat tall, not very tall, tall but not very tall, quite tall, more or less tall.*

LINGUISTIC VARIABLE

- A linguistic variable associates words or sentences with a measure of belief functions, also called membership function.
- The set of values that it can take is called term set.
- Each value in the set is a fuzzy variable defined over a base variable.
- The base variable defines the Universe of discourse for all the fuzzy variables in the term set.

A linguistic variable is a quintuple [X, $T(X)$, U, G, M] where

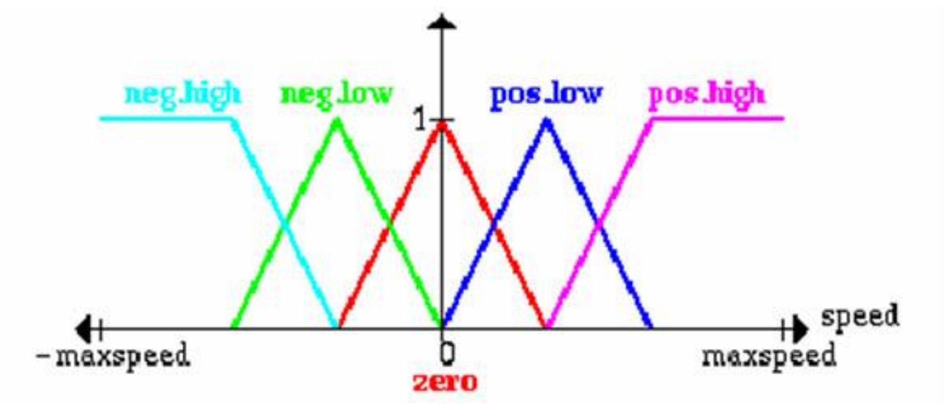
- X is the name of the variable,
- $T(X)$ is the term set, i.e. the set of names of linguistic values of X,
- U is the universe of discourse,
- G is the grammar to generate the names and
- M is a set of semantic rules for associating each X with its meaning.

LINGUISTIC VARIABLE

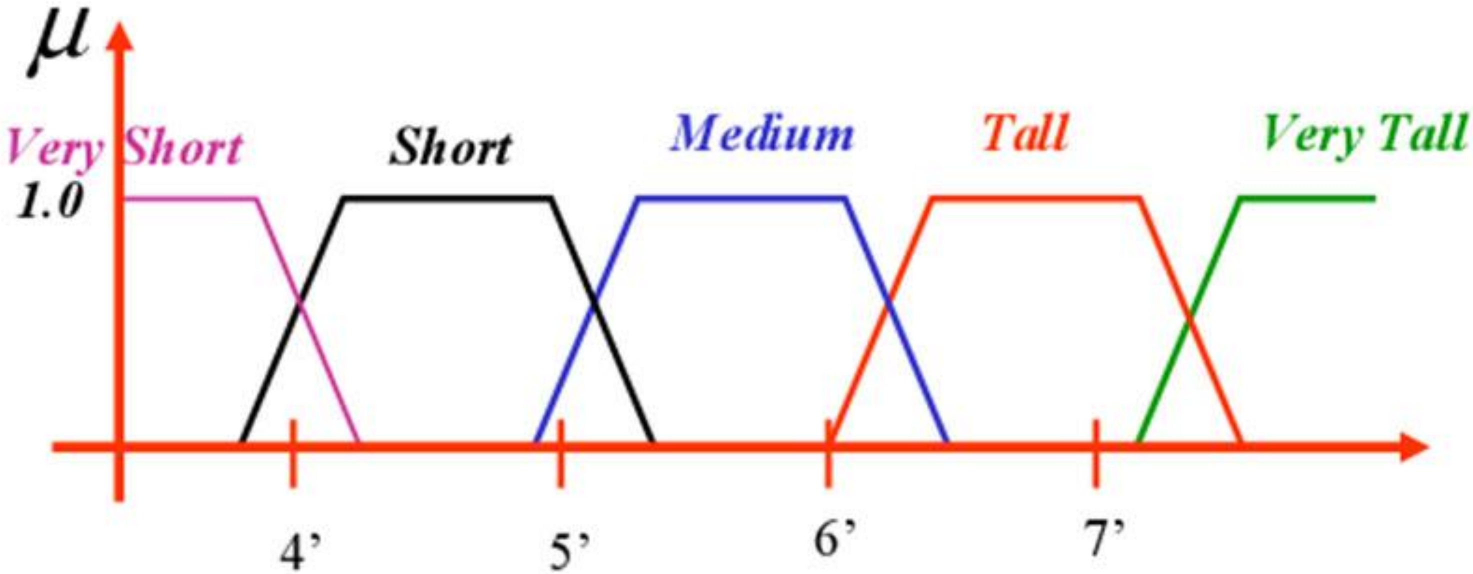
- Let x be a linguistic variable with the label “speed”.
- Terms of x, which are fuzzy sets, could be “positive low”, “negative high” from the term set T:

    T = {PostiveHigh, PositiveLow, NegativeLow, NegativeHigh, Zero}

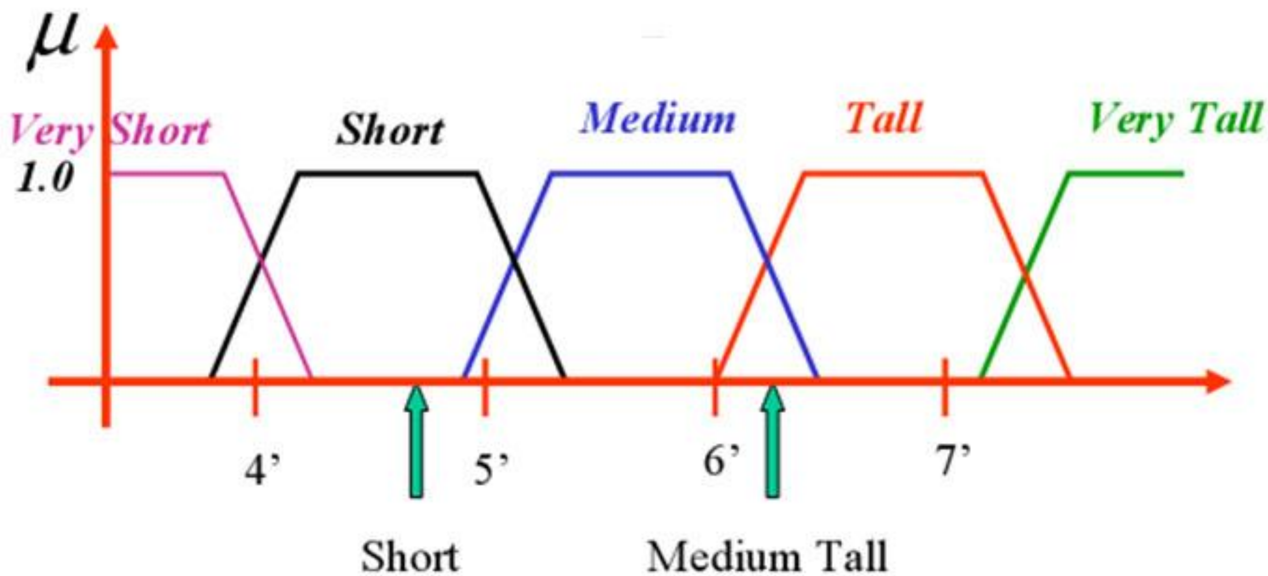
- Each term is a fuzzy variable defined on the base variable which might be the scale of all relevant velocities.



MEMBERSHIP FUNCTIONS



$$\mu = [\mu_{vs}, \mu_{s}, \mu_{m}, \mu_{t}, \mu_{vt}]$$



$$\mu = [0,\ 1,\ 0,\ 0,\ 0]$$

$$\mu = [0,\ 0,\ 0.5,\ 0.5,\ 0]$$

FEATURES OF MEMBERSHIP FUNCTIONS

core(A) is set of all points x in X such that $\{(x \mid \mu_A(x) = 1\}$

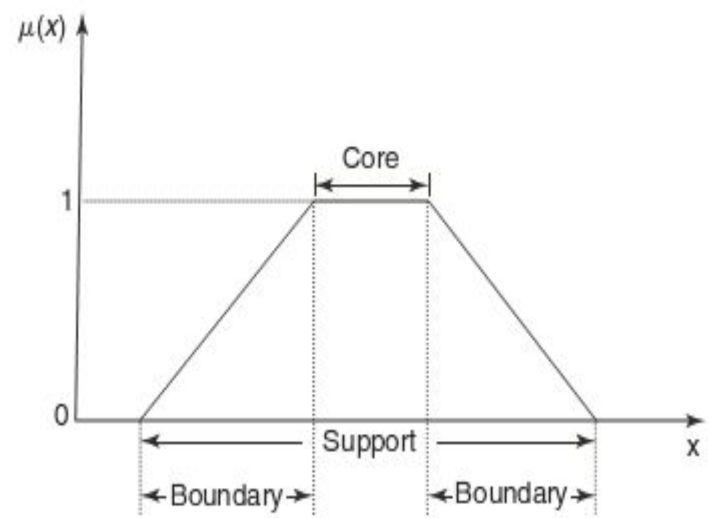
Support(A) is set of all points x in X such that $\{(x \mid \mu_A(x) > 0\}$

Boundary (A) is set of all points x in X such that $\{(x \mid 0 < \mu_A(x) < 1\}$

Core of a fuzzy set may be an empty set.

Fuzzy set whose support is a single point in X with $\mu_A(x) = 1$ is called fuzzy singleton

- CORE: $\mu_{\underset{\sim}{A}}(x) = 1$
- SUPPORT: $\mu_{\underset{\sim}{A}}(x) > 0$
- BOUNDARY: $0 < \mu_{\underset{\sim}{A}}(x) < 1$



Normal and subnormal fuzzy set

- Normal fuzzy set : whose membership function has atleast one element in the universe whose membership value is unity.
- Prototypical element : the element for which the membership is equal to 1.
- Subnormal fuzzy set: whose membership function has no element in the universe whose membership value is unity.

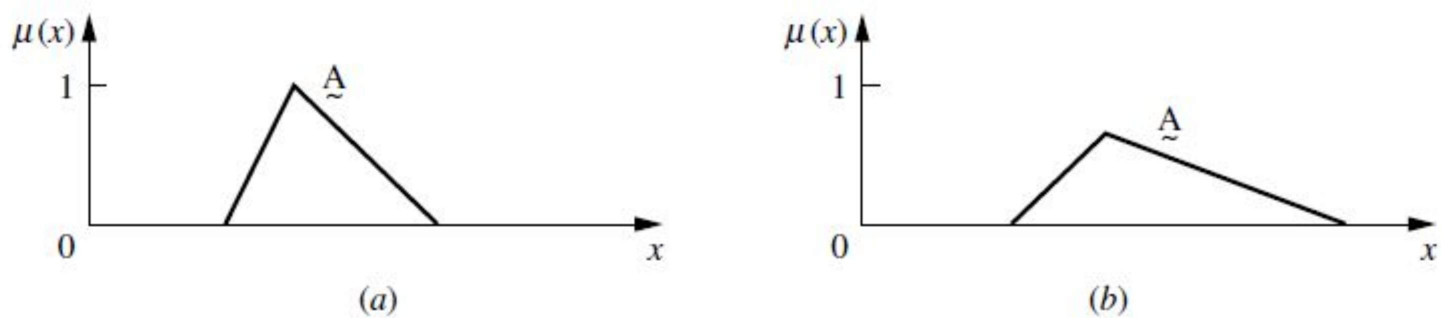


FIGURE 4.2
Fuzzy sets that are normal (a) and subnormal (b).

Convex and non convex fuzzy set

- Convex fuzzy set has a membership function whose membership values are strictly monotonically increasing or strictly monotonically decreasing or strictly monotonically increasing than strictly monotonically decreasing with increasing values for the elements in the universe.

for any elements x, y, and z in a fuzzy set $\underset{\sim}{A}$, the relation $x < y < z$ implies

$$\mu_{\underset{\sim}{A}}(y) \geq \min[\mu_{\underset{\sim}{A}}(x), \mu_{\underset{\sim}{A}}(z)] \tag{4.1}$$

Nonconvex fuzzy set: possessing the characteristics opposite to the convex fuzzy set.

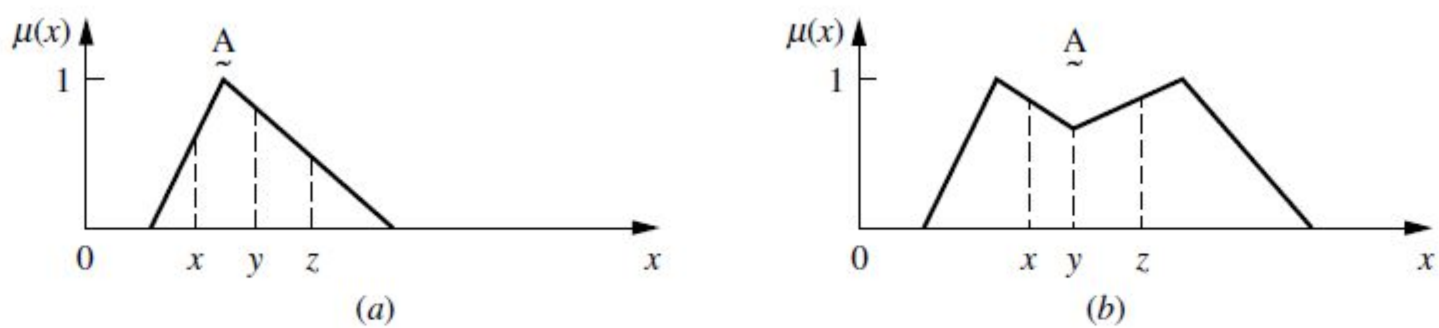


FIGURE 4.3
Convex, normal fuzzy set (a) and nonconvex, normal fuzzy set (b).

Intersection of two convex fuzzy sets

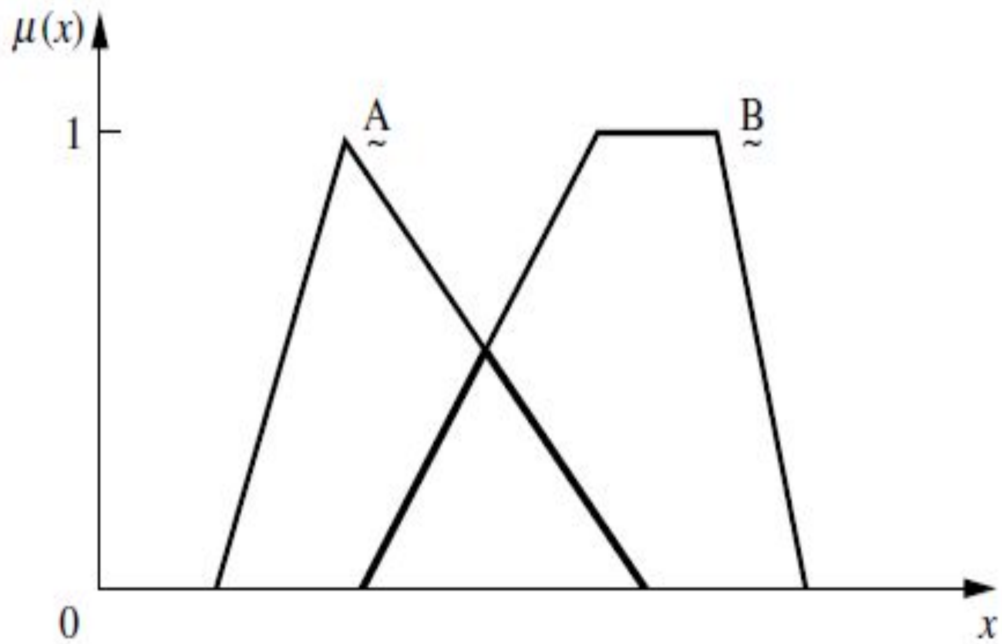


FIGURE 4.4
The intersection of two convex fuzzy sets produces a convex fuzzy set.

FEATURES OF MEMBERSHIP FUNCTIONS

- Crossover point of a fuzzy set *A* is a point x in X such that $\{(x \mid \mu_A(x) = 0.5\}$
- There can be more than one crossover point in a fuzzy set.
- Height of fuzzy set : maximum value of the membership function.
- For a normal fuzzy set, height =1
- For a subnormal fuzzy set, height < 1

Fuzzification

The process of transforming crisp(bivalued) input values into linguistic values is called fuzzification

Steps of Fuzzification:

Step 1: Input values are translated into linguistic concepts, which are represented by fuzzy set.

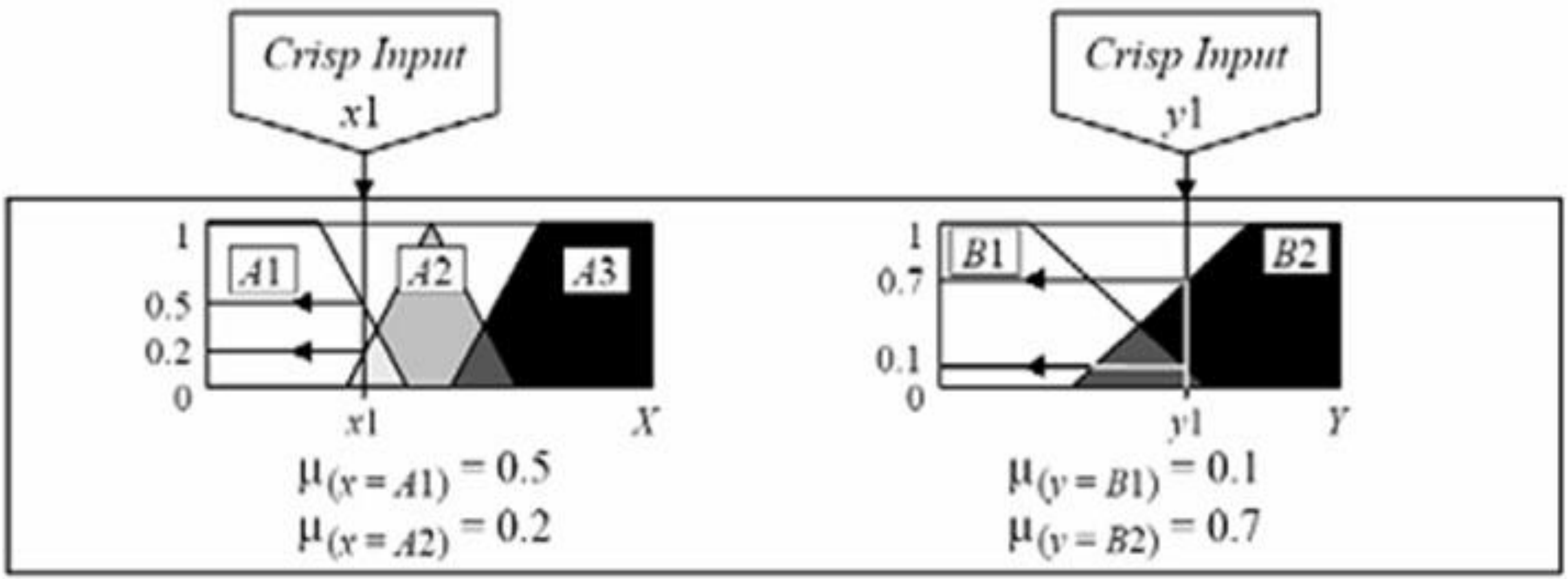
Step 2: Membership functions are applied to the measurements, and the degree of membership is determined

FUZZIFICATION

- Fuzzifier converts a crisp input into a fuzzy variable.
- Definition of the membership functions must
  - reflects the designer's knowledge
  - provides smooth transition between member and nonmembers of a fuzzy set
  - simple to calculate
- Typical shapes of the membership function are Gaussian, trapezoidal and triangular.

- Use crisp inputs from the user.

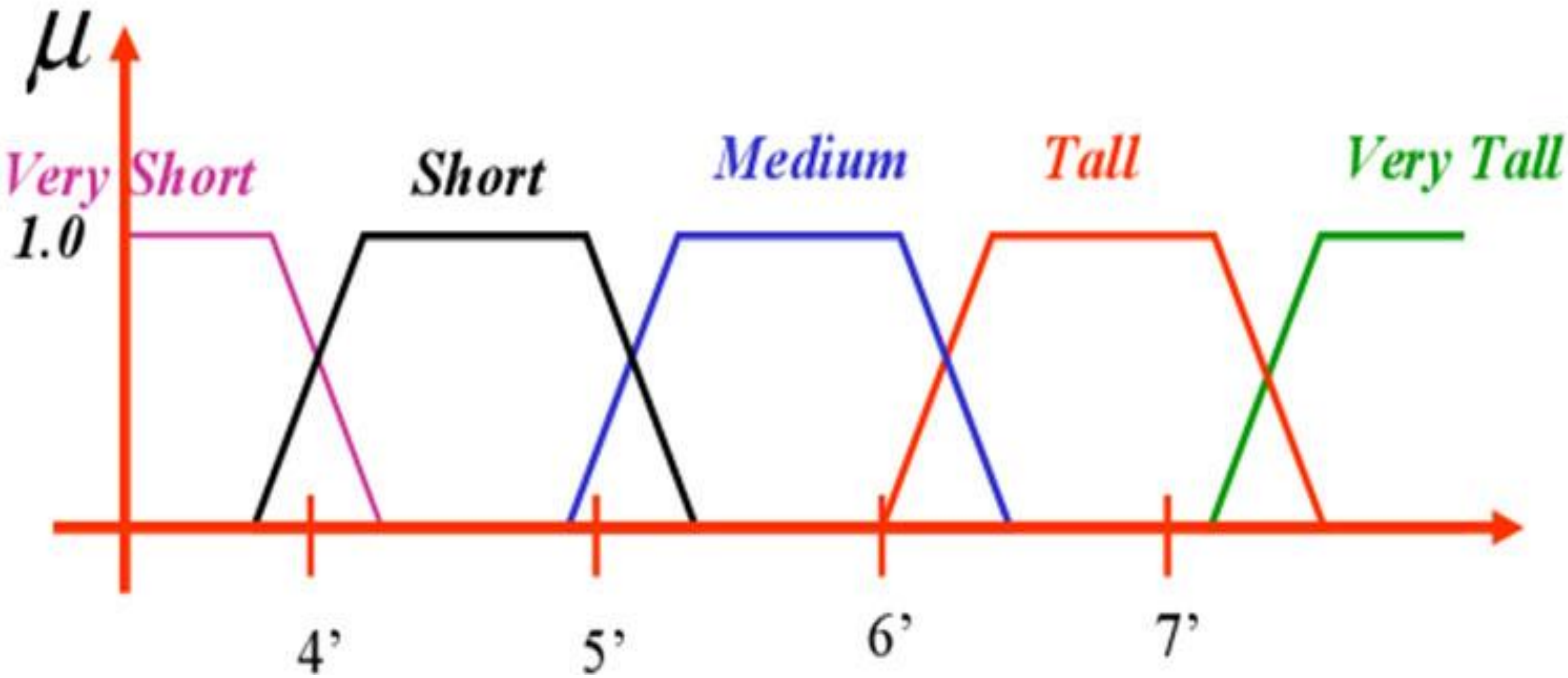
- Determine membership values for all the relevant classes (i.e., in right Universe of Discourse).



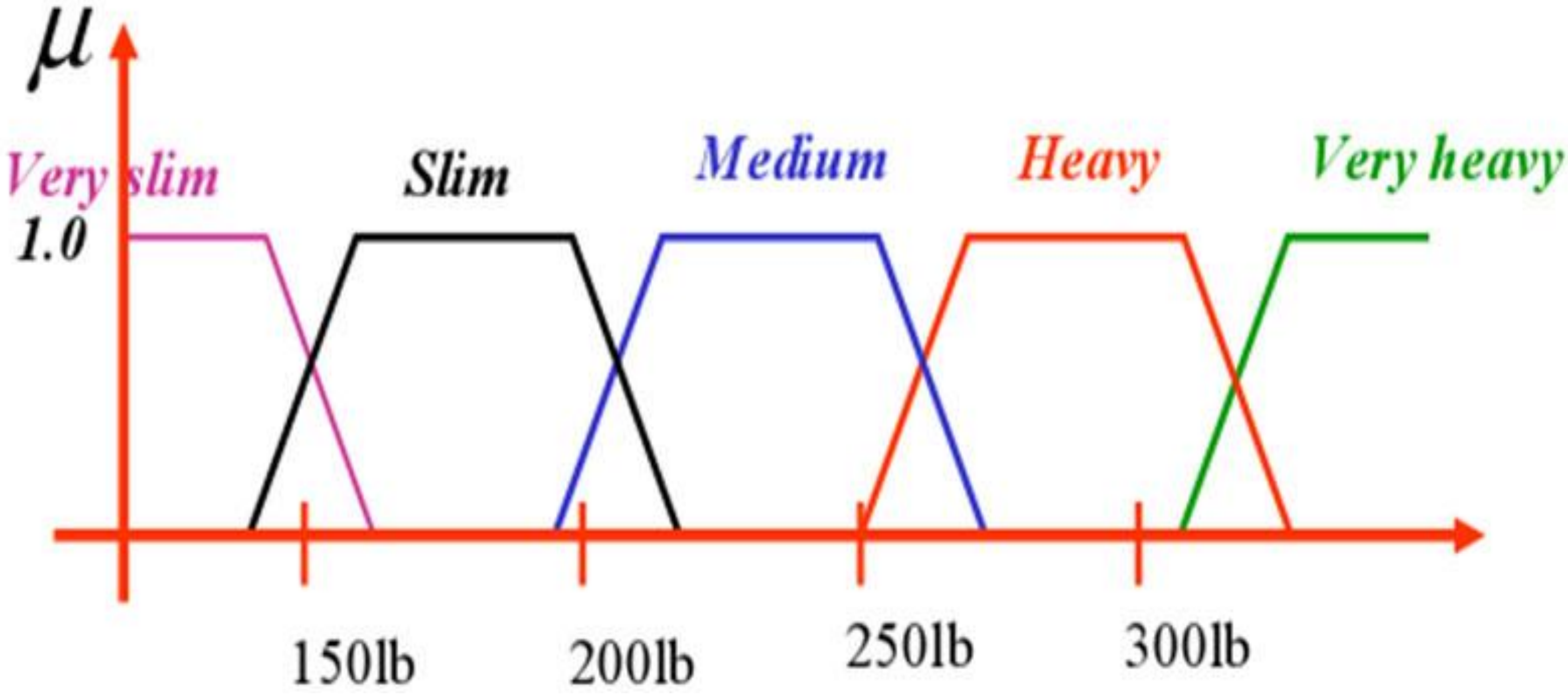
EXAMPLE - FUZZIFICATION

- Assume we want to evaluate the health of a person based on his height and weight.
- The input variables are the crisp numbers of the person's height and weight.
- Fuzzification is a process by which the numbers are changes into linguistic words

FUZZIFICATION OF HEIGHT



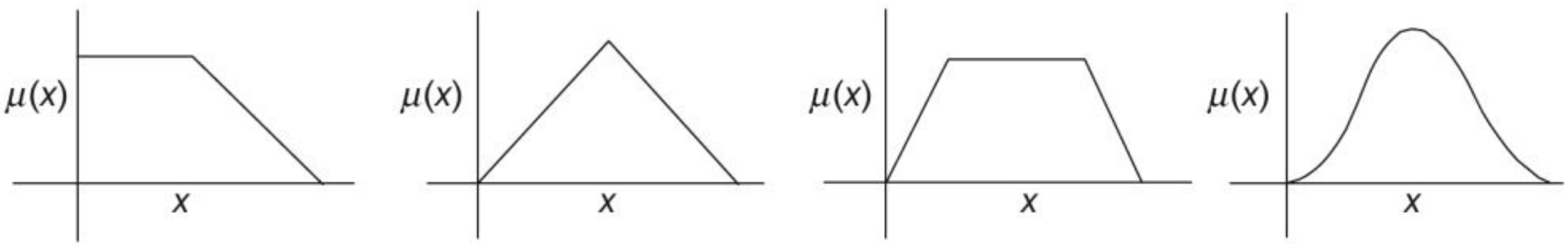
FUZZIFICATION OF WEIGHT



REPRESENTING FUZZY LOGIC IN CONTINUS DOMAIN

- Membership functions for fuzzy logic can be described through continuous functions.

- Different shapes of membership functions exist for example: triangular, trapezoidal, curved etc.



REPRESENTING FUZZY LOGIC IN CONTINUS DOMAIN

Example

Consider the set of people in the following age groups

0–10	40–50
10–20	50–60
20–30	60–70
30–40	70 and above

The fuzzy sets "young", "middle-aged", and "old" are represented by the membership function graphs as illustrated