Date: / /
Page No.

Protection against Market risk

↳ Investor should study the Price behaviour of Stock SD, β

Protection against Interest Rate Risk

↳ Holding the Investment to maturity
↳ Buying T-Bill and Bonds of Short maturity so that money can be reinvested at Interest Rate

Protection against Inflation

↳ Bond with high Interest Rate 13-15%.
↳ Diversification of portfolio

Protection against Business and financial Risk

- Analysing strength and weakness of the industry to which company belongs.

Ex-ante Risk Measurement

$$Avg = \bar{X} = \frac{\Sigma x}{N}$$

$$SD\ (\sigma) = \sqrt{\frac{\Sigma x - \bar{x}}{N-1}}$$

The Arithmatic mean of Expected Return of companies may ~~vary~~ be Same but the returns may vary.

Company A				Company B		
r_i	P_i	$P_i r_i$		r_i	P_i	$P_i r_i$
6	0.10	0.6		4	0.1	0.4
7	0.25	1.75		6	0.2	1.2
8	0.30	2.4		8	0.4	3.2
9	0.25	2.25		10	0.2	2.0
10	0.10	1.00		12	0.1	1.2

$\sum E(r) = 8.00$ $\quad\quad$ $\sum E(r) = 8.00$

Companies A's return vary from 6% to 10%.
B " " " 4% to 12%.

∴ to find out variation SD technique is applied.

$$\sigma = \sqrt{\sum_{i=1}^{N} P[r_i - E(r)]^2}$$

$$\text{Variance} = \sigma^2 = \sum_{i=1}^{N} P[r_i - E(r)]^2$$

Hence $\sigma = \sqrt{\text{variance}}$

Company A

$r_i - E(r)$	$[r_i - E(r)]^2$	$P_i[r_i - E(r)]^2$
−2	4	0.4
−1	1	0.25
0	0	0
+1	1	0.25
+2	4	0.40
		1.30

Date : / /
Page No.

$$\sigma = \sqrt{\sum_{i=1}^{N} P[r_i - E(r)]^2}$$

$$\sigma = \sqrt{1.30} = 1.14$$

Company B

$r_i - E(r)$	$r_i - E(r)^2$	$P_i[r_i - E(r)]^2$
-4	16	1.6
-2	4	0.8
0	0	0
$+2$	4	0.8
$+4$	16	1.6
		4.8

$$\sigma = \sqrt{\sum_{i=1}^{N} P[r_i - E(r)^2}$$

$$\sigma = \sqrt{4.80} = 2.19.$$

Company A's Expected return is stable compared to B's

Characteristic Regression line

$$R_i = \alpha_i + \beta_i R_m + e_i$$

R_i = Return of the i^{th} stock
α_i = Intercept
β = Slope of the i^{th} stock
R_m = Return of the market index
e_i = The error term.

$$\text{Return} = \frac{\text{Today's Price} - \text{Yesterday's Price}}{\text{Yesterday's Price}} \times 100$$

Let us consider the daily prices of Shaji Auto Stock and the index for the ~~Computation of Beta. Usually beta values a~~ Period of 5 October 2012 to 16 Oct 2012. Calculat β

INDEX (X)	Shaji Auto (Y)
904.95	597.80
845.75	570.80
874.25	582.95
847.95	559.85
849.10	554.60
835.80	545.10
816.75	519.15
843.55	560.70
835.55	560.95
839.50	597.40

$$\beta = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$$

$$\alpha = \bar{y} - \beta \bar{x}$$

To calculate the Beta, the return have to be calculated

Index Return(x)	x^2	Shaji Auto(y)	y^2	xy
-6.54	42.77	-4.52	20.43	29.56
3.37	11.36	2.13	4.54	7.18
-3.01	9.06	-3.96	15.68	11.92
0.14	0.02	-0.94	00.88	-0.13
-1.57	2.46	-1.71	9.92	2.68
-2.28	5.20	-4.76	22.66	10.85
3.28	10.76	-8.00	64.00	26.24
-0.95	0.90	0.04	0.00	0.04
0.47	0.22	6.50	42.25	3.06
-7.09	82.75	0.78	173.36	91.32

$$\beta = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$$

$$= \frac{9 \times 91.32 - (-7.09)(0.78)}{9 \times 82.75 - (-7.09)^2}$$

$$\beta = 1.19$$

Date : / /
Page No.

$\alpha = \bar{y} - \beta\bar{x}$

$\bar{Y} = \frac{0.78}{9} = 0.086$

$\bar{X} = \frac{-7.09}{9} = -0.79$

$= 0.086 - (1.19 \times -0.79)$

$\alpha = 1.02$

α = indicate that Stock Return is independen of market return
+ve α is a healthy sign

Q

Probability	Security A	Security B
0.5	4	0
0.4	2	3
0.1	0	3

Security A Return $= 4 \times 0.5 + 2 \times 0.4 + 0 \times 0.1$
$= 2 + 0.8 = 2.8$

Security B Return $= 0 \times 0.5 + 3 \times 0.4 + 3 \times 0.1$
$0 + 1.2 + 0.3 = 1.5$

A's return is high

Risk

$\sigma = \sqrt{\sum_{i=1}^{n} P_i [r_i - E(r)]^2}$

Date : / /
Page No.

Security A

r_1	P_1	$(r - E(r)]^2$	$P_i[r_i - E(r)]^2$
4	0.5	$[4-2.8]^2 = 1.44$	0.720
2	0.4	$[2-2.8)^2 = 0.64$	0.256
0	0.1	$[0-2.8]^2 = 7.84$	0.784
			1.76

Security B

r_1	P_1	$[r_1 - E(r)]^2$	$P_i[r_1 - E(r)]]^2$
4	0.5	$[0-1.5]^2 = 2.25$	1.125
2	0.4	$[3-1.5]^2 = 2.25$	0.9
0	0.1	$[3-1.5]^2 = 2.25$	0.225
			2.25

Security A
$\sigma = \sqrt{1.76} = 1.33$

Security B
$\sigma = \sqrt{2.25} = 1.5$

Date: / /
Page No.

Q. Return in Risk free asset = 12.5%.
~~Investment~~ Return in Risky asset = 20%.
if 50% invested in Risky asset
50% invested in Risk free then
what is the return of the
portfolio

$$R_p = R_f X_f + R_m(1 - X_f)$$

$$= 12.5 \times 0.5 + 20(1 - 0.5)$$

$$= 6.25 + 10$$

$$= 16.25\%$$

(i) if there is 0 Investment in risk free asset and 100% in risky

$$R_p = R_f X_f + R_m(1 - X_f)$$

$$0 + 20\%$$

$$= 20\%$$

(ii) If we assume that variance of Risky asset is 15 then Variance of portfolio will depend on variance of Risky ~~free~~ asset.

i.e

Proportion of Risky asset	Portfolio Risk
0.5	7.5
1.0	15.0
1.5	22.5

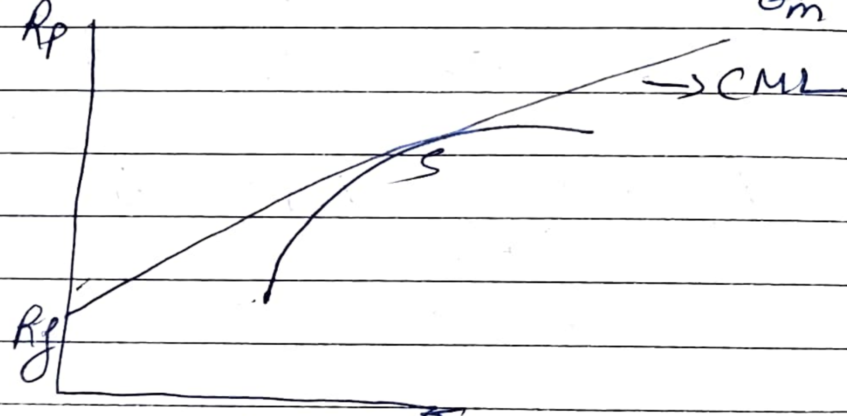
Risk premium of a portfolio = $(R_m - R_f)$.

Capital Market line

$$E(R_p) = R_f + \left(\frac{R_m - R_f}{\sigma_m}\right) \times \sigma_p$$

For a portfolio on the Capital market line, the expected rate of return in excess of the risk-free rate is in proportion to the SD of the market portfolio.

$$\text{Price of the Risk} = \text{Slope of line} = \frac{\text{Risk premium}}{\sigma_m}$$



Capital Market Line

$\therefore$ Expected Return = Price of time + (Price of Risk × Amount of Risk)

CML only shows effecient portfolio whose return is more than Expected return.. For the measure of inefficient portfolio and Individual securities Security market line (SML) formula is used.

SD = Systematic + unsystematic Risk

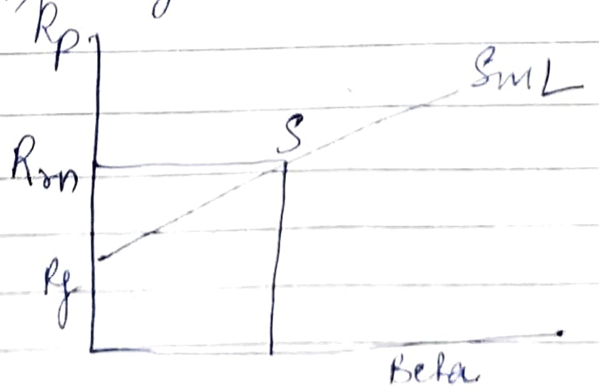
β = Systematic risk

The variance of a portfolio is equal to the weighted sum of the covariances of individual securities in a portfolio. If we add a security the market portfolio, its marginal contribution to the variance of the market is the covariance between security's return and market portfolio return. Covariance can be standardized by dividing it by the SD of a market portfolio (Cov_{im}/σ_m). Then the expected return of the security is given by the equation.

$$R_i - R_f = \frac{R_m - R_f}{\sigma_m} Cov_{im}/\sigma_m$$

$$= R_i - R_f = \frac{Cov_{im}}{\sigma_m^2} [R_m - R_f)$$

$$E(R_i) = R_f + \beta_i [E(R_m) - R_f)$$



Date : / /
Page No.

Q. Assume that the Risk-free rate of return is 7%. The market portfolio has an Expected return of 14%. and SD of return = 25%. Under Equilibrium Conditions as described by CAPM what would be the expected return for a portfolio having no unsystematic risk and 20%. SD of the return.

$$R_i = R_f + \left[\frac{R_m - R_f}{\sigma_m}\right]\sigma_i$$

$$= 0.07 + \left(\frac{0.14 - 0.07}{0.25}\right) \times 0.2$$

$$= 0.126$$

The portfolio return is 12.6%.

Q. Assume you are a portfolio manager. Based on the following details, determine the securities that are overpriced and those that are underpriced in terms of SML

Security	Actual Return	β	σ
A	0.33	1.7	0.50
B	0.13	1.4	0.35
C	0.26	1.1	0.40
D	0.12	0.95	0.24
E	0.21	1.05	0.28
F	0.14	0.70	0.18
Nifty index	0.13	1.00	0.20
T-Bills	0.09	0	0.00

Solution

$R_i = R_f + \beta_i (R_m - R_f)$

A security return

$R_i = 0.09 + 1.7 (0.13 - 0.09)$

$= 0.158$

Security	Actual return	Estimated Return	Remark
A	0.33	0.158	underprice
B	0.13	0.146	overprice
C	0.26	0.134	underpriced
D	0.12	0.128	overpriced
E	0.21	0.132	underpriced
F	0.15	0.118	underpriced

Date: / /
Page No.

Q// Assume that a portfolio is constructed by using equal portions of the six stocks listed in above Q and find out Expected return of the portfolio.

Solution

$$E(R) = \sum_{i=1}^{N} X_i R_i$$

$$= 1/6 \times 0.33 + 1/6 \times 0.13 + 1/6 \times 0.26 + 1/6 \times 0.12 + 1/6 \times 0.21 + 1/6 \times 0.15$$

$$= 0.2$$

Q// Estimate the stock return by using CAPM model and arbitrage model. The particulars are as follows.

(a) The expected return of the market is 15% and equity $\beta = 1.2$, $R_f = 8\%$.

Solution

$$R_i = R_f + \beta_i (R_m - R_f)$$
$$= 0.08 + 1.2(0.15 - 0.08)$$
$$= 0.08 + 0.084$$
$$R_i = 0.164$$
$$= 16.4\%$$

Date :
Page No.

h_0 = Risk free Rate of Return

b)

Factor	Market price of risk (%)	Sensitivity index
inflation	6	1.1
Industrial prod	~~3~~ 2	0.8
Risk premium	3	1.0
Interest Rate	4	-0.9

APT Model

$$= h_0 + h_{11} b_{i1} + h_2 b_{i2} + h_3 b_{i3} + h_4 b_{i4}$$

Since h_0 is not given in the APT equilibrium model with a risk free asset, it can be found out using the formula.

$$R_i = h_0 + h_i 1 \Rightarrow R_i - h_0 = h_i$$

$$R_i = R_f + (b_{i1} h_1 + b_{i2} h_2 + b_{i3} h_3 + b_{i4} h_4)(R_m - R_f)$$

$$= \cancel{0.08}$$

$$= 0.08 + (0.06 \times 1.1 + 0.02 \times 0.8 + 0.03 \times 1.0 + 0.04 \times -0.9)(0.15 - 0.08)$$

$$= 0.08 + (0.066 + 0.016 + 0.03 - 0.036) 0.07$$

$$= 0.08 + 0.005$$

$$R_i = 0.085$$

The Rate of return differs in CAPM and APT model because the variables used are entirely different. The return is low according to the APT model because the interest rate have a negative effect on return.

Date :

Page No.

Direct Method

β = Covariance between market Return and security return to the market return variance

$$B_j = \frac{Cov_{jm}}{\sigma_m^2}$$

$$\text{or Corelation}_{j,m} \times \frac{\sigma_j}{\sigma_m}$$

Q. Shows the percentage returns on the market represented by the BSE Sensex and the Share of the Laya Infotech Limited for recents five years

Year	Market Return	Laya infotech
1	18.60	23.46
2	-16.50	-36.180
3	63.83	54.64
4	-20.65	-7.29
5	-17.87	-12.95
	5.48	3 95

1). Calculate the avg return of market and Laya

2.) Calculate deviation of returns on market from avg return

3) Calculate deviations of returns on Laya share from avg return.

Date :
Page No.

4) Multiply deviation of market returns and deviations of Laya return. Take the sum and divide by ~~to~~ 5

$$Cov_{m,l} = \frac{4666.30}{5} = 933.26$$

5 Calculate the squared deviations of the Market return. Take the sum and divide by 5 to calculate the ~~sum of~~ variance of market return.

$$\sigma_m^2 = \frac{5288.23}{5} = 1,057.65$$

6. Divide the covariance of market and Laya by market variance to get beta.

$$B_j = \frac{Cov_{j,m}}{\sigma_m^2} = \frac{933.26}{1,057.65} = 0.88$$

7. The intercept term is given by the following formula:

$$\alpha_j = \bar{r}_j - B_j \times \bar{r}_m$$

$\bar{r}_j \to$ avg return on investment J

$r_m \to$ avg market return

$\alpha = 3.95 - 0.88 \times 5.48 = -0.89$

Date : / /
Page No.

Thus the characteristic line of Jaya infotech is

$$\boxed{r_i = -0.89 + 0.88\, r_m}$$

Portfolio Theory. Assumption

1) Assume that investors are risk averse i.e. they well diversify their portfolio. So they have expected return of the portfolio

2) Assume that returns of the asset are normally distributed.
i.e. the mean and variance analysis is the foundation of portfolio decision.

When we extend the portfolio Theory to derive a framework for valuing risk asset. This framework is reffered to as Capital asset pricing model (CAPM) and alternative model for valuation of Risky asset is Arbitrage pricing model.

Date: / /
Page No.

Expected Rate of Return of Individual security

$E(R_x) = \sum_{i=1}^{n} P_i R_i$ — P_i → Probability of that return; R_i → ith return

Expected Rate of Return of a portfolio

= weight of security X × expected Return on security
+ weight of security Y × expected return on security Y

$E(R_p) = W \times E(R_x) + (1-W) \times E(R_y)$

Returns on individual assets fluctuate more than portfolio return.

Portfolio Risk

Example - 1

Economic Condition	Prob	Returns % A	Returns % B
~~Good~~			
Good	0.5	40	0
Bad	0.5	0	40

Assuming that the investor invest in both the assets equally.

$E(R_A) = 0.5 \times 40 + 0.5 \times 0 = 20\%$

$\sigma_A^2 = 0.5(40-20)^2 + 0.5(0-20)^2 = 400$

$\sigma_A = \sqrt{400} = 20\%$

$\sigma_A \rightarrow$ SD of A
$\sigma_A^2 \rightarrow$ variance of A

(ii) $E(R_B) = 0.5 \times 0 + 0.5 \times 40 = 20\%$.
$\sigma_B^2 = 0.5(0-20)^2 + 0.5(40-20)^2 = 400$
$\sigma_B = \sqrt{400} = 20\%$.

Both investments A and B have the same expected rate of return 20%. and same variance. Thus they are equally profitable and equally risky.

$\therefore$ if portfolio of A and B are constructed

$$E(R_p) = 0.5 \times 20 + 0.5 \times 20 = 20\%.$$

This return is the same as the expected return from individual securities, But without any risk. Why? if the economic conditions are good, the A would yeild 40%. return and B 0 and the portfolio return will be

$$E(R_p) = 0.5 \times 40 + 0.5 \times 0 = 20\%.$$

Thus, by investing equal amounts in A and B, rather than the entire amount only in A and B, the investor is able to eliminate risk altogether.

Measuring Portfolio Risk for two assets

1) variance of portfolio
2) SD of "

Covariance = comovement.

Calculation

1) Determine Expected return
2) Determine the deviation of possible returns from the Expected return for each asset
3) Determine the sum of the product of each deviation of return of two assets and respective probability.

$$\text{Covariance}_{xy} = \sum_{i=1}^{n} [R_x - E(R_x)][R_y - E(R_y)] \times P_i$$

In case of variance and covariance squared deviations are used so
∴ the direction is Explained but the number cannot be Explained.

Date :
Page No.

Correlation is a measure of the linear relationship between two variables

$$\text{Covariance}_{XY} = \underbrace{\sigma_x}_{\text{variability in } x} \times \underbrace{\sigma_y}_{\text{variability in } y} \times \underbrace{\text{Corelation}_{XY}}_{\text{association between 2}}$$

$$\therefore \text{Corelation}_{xy} = \frac{\text{Cov}_{xy}}{\sigma_x \sigma_y}$$

Corelation coefficient ranges between -1 and +1

Variance and SD of Two asset Portfolio

$$\text{Variance} \rightarrow \sigma_p^2 = \sigma_x^2 w + \sigma_y^2 w + 2 w_x w_y \text{Cov}_{xy}$$

$$= \sigma_x^2 w + \sigma_y^2 w^2 + 2 w_x w_y (\sigma_x \sigma_y \text{Cor}_{xy})$$

$$SD \rightarrow \sqrt{\sigma_x^2 w_x^2 + \sigma_y^2 w_y^2 + 2 w_x w_y \sigma_x \sigma_y \text{Cor}_{xy}}$$

if σ_p (SD) = 2.45%
and Expected return is 6.5%
then the security can vary by
3.96% (I) 6.5 − 2.54
(II) 6.5 + 2.54

Minimum variance portfolio or optimum Portfolio

$$w^* = \frac{\sigma_y^2 - Cov_{xy}}{\sigma_x^2 + \sigma_y^2 - 2Cov_{xy}}$$

w^* = is the optimum proportion in security x and $1 - w^*$ in y

Portfolio Risk Depends upon the Correlation between Assets

if The SD of portfolio of x and y is 2.54%
Then the portfolio ~~weighted~~ SD is not the weighted average of Individual Securities
Thus the SD of portfolio is considerably lower than the weighted SD of these individual securities.
This is diversification effect.

However, the extent of the benefits of portfolio diversification depends on the correlation between returns on securities.

A negative correlation can cause significant reduction in portfolio risk.

Date:
Page No.

//

Q. Securities M and N are equally risky But they have different expected Returns:

	M	N
Expected return %	16.00	24.00
weight	0.50	0.50
SD %	20.00	20.00

what is the portfolio risk if (variance)

(a) $Cor_{mn} = +1.0$
(b) $Cor_{mn} = -1.$
(c) $Cor_{mn} = 0$
(d) $Cor_{mn} = +0.10$
(e) $Cor_{mn} = -0.10$

Ans. (a) $\longrightarrow \sigma_p = \sqrt{\sigma_x^2 w_x^2 + \sigma_y^2 w_y^2 + 2 w_x w_y \sigma_x \sigma_y} = \sigma_x w_x + \sigma_y w_y$

SD of portfolio $= 20 \times 0.5 + 20 \times 0.5 = 20.0\%$.

(b) $\rightarrow \sigma_p = \sqrt{20^2 \times 0.5^2 + 20^2 \times 0.5^2 + 2 \times 0.5 \times 0.5 \times 20 \times 20 \times -10}$

$= \sqrt{100 + 100 - 200} = 0.0\%$

(c) $\sigma_p = \sqrt{\sigma_x^2 w_x^2 + \sigma_2^2 w_y^2}$

$\sigma_p = \sqrt{20^2 + 0.5^2 + 20^2 \times 0.5^2} = \sqrt{200}$
$= 14.14\%$

Date :
Page No.

(d) $\sigma_p^2 = \sqrt{20^2 + 0.5^2 + 20^2 + 0.5^2 + 2 \times 0.5 \times 0.5 \times 20 \times 20 \times 0.10}$

$= \sqrt{100 + 100 + 20} = \sqrt{220}$

$= 14.83\%$

(e) $\sigma_p^2 = \sqrt{20^2 \times 0.5^2 + 20^2 \times 0.5^2 + 2 \times 0.5 \times 0.5 \times 20 \times 20 \times -0.10}$

$= \sqrt{100 + 100 - 20} = \sqrt{180}$

$= 13.42\%$

Formula For β

Date : / /
Page No.

Calculation of β By Co-variance Method

$$\boxed{\beta_x = \frac{Cov_{xm}}{\sigma_m^2}}$$

Calculation of β by correlation Method

$$\boxed{\beta_x = Cor_{xm} \times \frac{\sigma_x}{\sigma_m}}$$

High β Stocks, due to their large volatility will be more ~~pr~~ unpredictable
Low β stocks show relatively small volatility and more predictable.

β measures → Systematic Risk

Statistical formula for β

$$\boxed{\beta = \frac{n\Sigma(xy) - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2}}$$

$$\boxed{\alpha = \frac{\Sigma y - \beta \Sigma x}{n}}$$

Date : / /
Page No.

ii) Dividend Rate = 25%
Dividend is applied on Par value of share
Suppose if Par value = Rs 10
then 25% of 10 = 2.50 Rs
Total dividend = 2.50 × No. of shares

Suppose at the end of the year the share price has increased then the Capital gain from share is (SP − BP) × No. of Shares

Total Return = Div + Capital gain.

Rate of Return = $\frac{Div}{P_0} + \frac{P_1 - P_0}{P_0}$

$$= \frac{Div + (P_1 - P_0)}{P_0}$$

direct
β

Date
Page No.

Calculate the β of Hauck Corporation from the following data.

Year	Price	Dividend	Market Index	Market dividend	Riskless rate
2005	25-27	1	100-105	3.05%	6.00%
2006	27-29	1	105-110	3.00%	6.00%
2007	29-32	1.50	110-120	2.95%	5.95%
2008	32-33	1.50	120-125	2.80%	5.90%

Capital gain → dividend

2005 $25-27 = (2+1)/25 = 0.12$

2006 $27-29 = (2+1)/27 = 0.111$

2007 $29-32 = (3+1.5)/29 = 0.155$

2008 $32-33 = (1+1.5)/32 = 0.078$

Riskfree — y (Security return)

2005 $0.12 - 0.06 = 0.06$

2006 $0.111 - 0.06 = 0.5111$

2007 $0.155 - 0.595 = 0.95672$

2008 $0.078 - 0.059 = 0.019125$

R_m (market return) — x

2005 $100-105 = (5/100 + 0.305) - 0.06 = 0.0205$

2006 $105-110 = (5/105 + 0.03) - 0.06 = 0.017619$

2007 $110-120 = (10/110 + .0295) - 0.0595 = 0.060909$

2008 $120-125 = (5/120 + .028) - 0.059 = 0.010667$

Σxy	Σx	Σy	Σx^2
$(0.0205)(0.06)$	0.0205	0.06	$(0.0205)^2$
$(0.017619)(0.05111)$	0.017619	0.05111	$(0.017619)^2$
$(0.060909)(0.09567)$	0.060909	0.09567	$(0.060909)^2$
$(0.010667)(0.019125)$	0.010667	0.019125	$(0.010667)^2$
$= 0.0081618$	0.109695	0.225905	0.00455437

$$\beta = \frac{n\Sigma x - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = 1.27$$

Date: / /
Page No.

Q A portfolio is formed as follows

Stock	Amount Invested	β	$\sigma(R)$
Childs Corporation	12000	1.25	25%
Jermyn Co.	13000	1.20	22

Riskless Rate $= 7\%$

Expected return of market $= 14\%$

Covariance between the two stock $= 0.0385$

Find Expected return and SD of the portfolio

Total weight $= 12000 + 13000 = 25$

Ans weight of $w_1 = 12/25 = 0.48$

$w_2 = 1 - 0.48 = 0.52$

The Expected returns are

$E(R_1) = 0.07 + 1.25(0.14 - 0.07) = 0.1575$

$E(R_2) = 0.07 + 1.2(0.07) = 0.1540$

(i) portfolio Return $= E(R_p) = 0.48(0.1575) + 0.52(0.154) = 15.568\%$

$\therefore 0.15568 \times 25000 = 3892$ Rs

(ii) Corelation Coefficient $= \dfrac{0.0385}{(0.25)(0.22)} = 0.7$

(iii) SD of portfolio $(\sigma(R_p)) \rightarrow \sqrt{0.48^2 \, 0.25^2 + 0.52^2 \, 0.22^2 + 2(0.48)(0.52)(0.25)(0.22)(0.7)}$

$= 21.61\%$

$$\sigma(R_p) = 0.2161(25000) = \underline{5403 Rs}$$

Date: / /
Page No.

How to Calculate Variance and SD.

1) Calculate ~~the~~ avg Rate of Return

$$\bar{R} = \frac{1}{n} \sum_{n=1}^{n} R_t$$

2) Calculate the deviations of individual Rate of return from avg and square it

$$(R_i - \bar{R})^2$$

~~3)~~

3) variance $= \frac{1}{n-1} \sum_{t=1}^{n} (R - \bar{R})^2$

4) SD $= \sqrt{\text{variance}}$

Date : / /
Page No.

Q The following table gives dividend and share price data for Hind Manufacturing Company.

Year	Div per share	Closing share price
1999	2.50	12.25
2000	2.50	14.20
2001	2.50	17.50
2002	3	16.75
2003	3	18.45
2004	3.25	22.25
2005	3.50	23.50
2006	3.50	27.75
2007	3.50	25.50
2008	3.75	27.95
2009	3.75	31.80

You are required to calculate
(i) the annual rate of return
(ii) the expected rate of return
(iii) the variance
(iv) The SD of returns

Date:
Page No

Solution

(i) Annual rate of return

Year	Div.	Closing price	Annual Rate of Return
1999	2.50	12.25	
2000	2.50	14.20	2.50 + (14.20 − 12.25)/12.25 = 36.33
2001	2.50	17.50	2.50 + (17.50 − 14.20)/14.20 = 40.85
2002	3.00	16.75	3.00 + (16.75 − 17.50)/17.50 = 12.86
2003	3.00	18.45	3 + (18.45 − 16.75)/16.75 = 28.06
2004	3.25	22.25	3.25 + (22.25 − 18.45)/18.45 = 38.21
2005	3.50	23.50	3.50 + (23.50 − 22.25)/22.25 = 21.35
2006	3.50	27.75	3.50 + (27.75 − 23.50)/23.50 = 32.98
2007	3.50	25.50	3.50 + (25.50 − 27.75)/27.75 = 4.50
2008	3.75	27.95	3.75 + (27.95 − 25.50)/25.50 = 24.31
2009	3.75	31.30	3.75 + (31.30 − 27.95)/27.95 = 25.40

(ii) Average Rate of Return

(36.33 + 40.85 + 12.86 + 28.06 + 38.21 + 21.35 + ~~28~~ 32.98 + 4.50 + 24.31 + 25.40)/10

= 26.48%

Date : / /
Page No.

(iii) variance

=

	Annual Return	$(R_i - \bar{R})$	$(R_i - \bar{R})^2$
1999	36.33	9.84	96.86
2000	40.85	14.36	206.22
2001	12.86	−13.63	185.71
2002	28.06	1.57	2.48
2003	38.21	11.73	137.51
2004	21.35	−5.14	26.38
2005	32.98	6.49	42.17
2006	4.50	−21.98	483.13
2007	24.31	−2.17	4.71
2008	25.40	−1.08	1.17
	264.85		1186.36

$\bar{R}$ Avg = $\frac{264.85}{10}$ = 26.48

variance = 1186.36/(10 − 1) = 131.82

SD = $\sqrt{131.82}$ = 11.48

Date: / /
Page No.

Q// Calculate the Expected Return, variance and Standard deviation

$$E(R) = R_1 \times P_1 + R_2 \times P_2 + \ldots R_n \times P_n = \sum_{i=1}^{n} R_i P_n$$

Return %	Probability	
-20	0.05	$(-20-11)^2$
-10	0.65	$(-10-11)^2$
-5	0.10	$(-5-11)^2$
5	0.10	$(5-11)^2$
10	0.15	$(10-11)^2$
18	0.25	$(18-11)^2$
20	0.25	$(20-11)^2$
30	0.05	$(30-11)^2$

$E(R) \rightarrow 11$

$$E(R) = (-20 \times 0.05) + (-10 \times 0.05) + (-5 \times 0.10) + (5 \times 0.10) + (10 \times 0.15) + (18 \times 0.25) + (20 \times 0.25) + (30 \times 0.05)$$
$$= 11$$

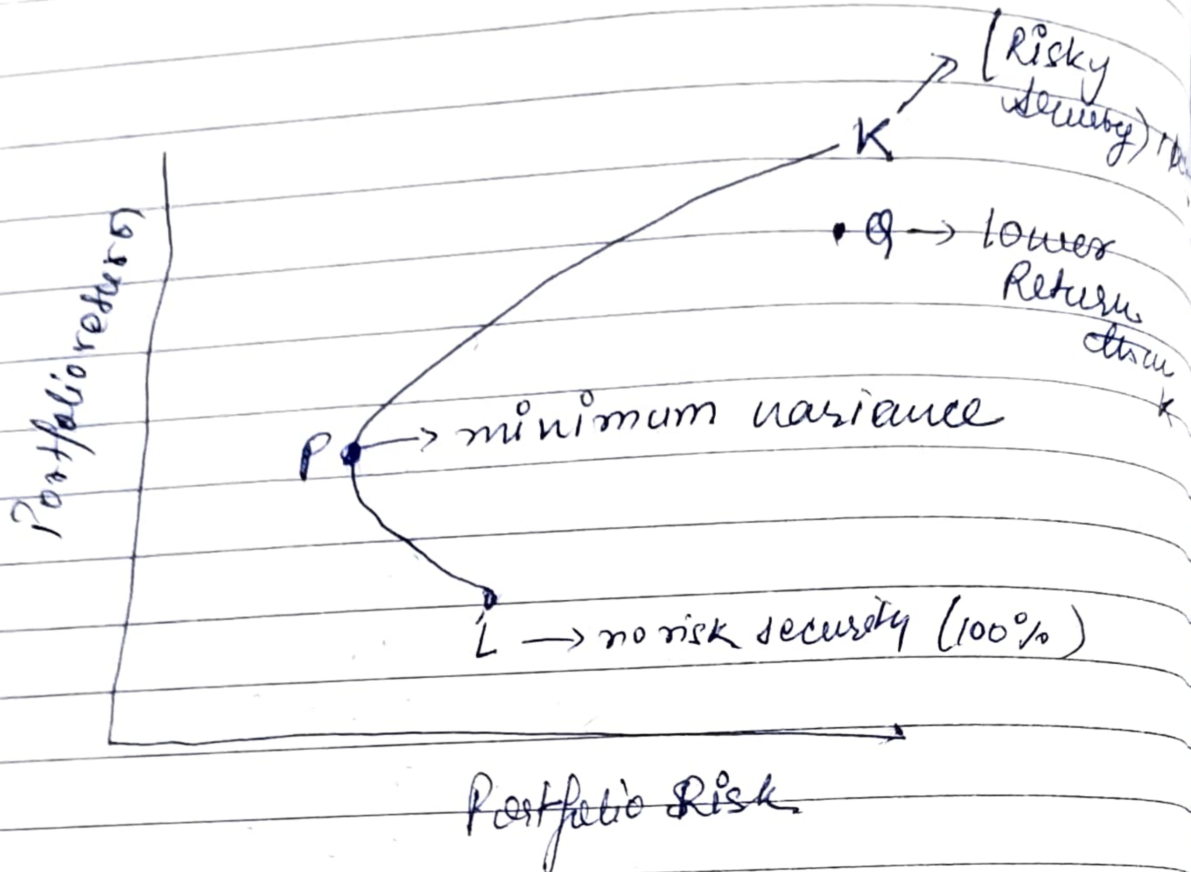
variance of Return

$$\sigma^2 = [R_1 - E(R)]^2 \times P_1 + [R_2 - E(R)]^2 \times P_2 + [R_n - E(R)]^2 \times P_n$$

$$= (-20-11)^2 \times 0.05 + (-10-11)^2 \times 0.05 + (-5-11)^2 \times 0.10 + (5-11)^2 \times 0.10 + (10-11)^2 \times 0.15 + (18-11)^2\, 0.25 + (20-11)^2 \times 0.25 + (30-11)^2 \times 0.05 = 150$$

Page No.

$\sigma = \sqrt{150} = \underline{\underline{12.25}}$



(I) When corelation is 0 then minimum portfolio $W_x = \dfrac{\sigma_y^2}{\sigma_x^2 + \sigma_y^2}$

(II) Minimum variance portfolio when corelation is +ve but less than 1

$$W_x = \frac{\sigma_y^2 - \sigma_x \sigma_y \, Cor_{xy}}{\sigma_x^2 + \sigma_y^2 - 2\sigma_x \sigma_y \, Cor_{xy}}$$