CHAPTER 8

BOND RETURN AND VALUATION

CHAPTER QUERY

Mr Ratan is not in favour of equity investments because of the volatility of equity prices. He believes debt instruments are safer than equity stocks. Mr Shah, a stockbroker, tells him that debt instruments also have risk features. He informs Mr Ratan that maturity period and duration are different concepts. Mr Ratan is confused. As a passive investor, he is worried. He wants to know what the risks associated with bonds are. What is meant by duration? What strategy should he adopt?

CHAPTER OBJECTIVES

- To comprehend the risks associated with bonds
- To calculate the yield to maturity
- To assess the duration of a bond
- To choose a suitable investment strategy

The answer to Mr Ratan's questions is that interest rate risk and default risks are the most significant risks in a bond portfolio. Duration is different from maturity period. This chapter will discuss the concept of duration. For a passive investor, a passive buy-and-hold strategy or a quasi-passive strategy is most appropriate. For this, he must select a technique that will suit him. Debt instruments are the cherished conduit for an investor's money. An assured return and high interest rate are responsible for the preference for bonds over equities.

Rising bank lending rates made companies search for cheap sources of funds outside the banking system. This gave an impetus to the corporate bond market. In August 2011, state-owned Rural Electrification (REC) Corp. (REC) Ltd raised between ₹3,500 crore and ₹4,200 crore by selling bonds that will mature in three tranches of three years, five years, and 10 years at 9.43 per cent, 9.45 per cent, and 9.48 per cent, respectively. Infrastructure Development Finance, a company formed by the Indian government to lend to energy and road projects, issued a 10-year tax-exempt bond in 2011, and raised ₹2,930 crore (US$640 million). This was followed by Indian Overseas Bank's successful issue of a 15-year bond. Financial institutions, banks, and corporate bodies offer attractive bonds like retirement bonds, education bonds, deep discount bonds, encash bonds, money multiplier bonds and index bonds. In this chapter, the risk, yield to maturity and bond duration, besides bond portfolio management immunization, are discussed in detail.

BOND BASICS

A bond is a contract that requires the borrower to pay an interest income to the lender. It resembles the promissory note and is issued by governments and/or corporations. The par value of the bond indicates the face value of the bond, i.e., the value stated on the bond paper. Generally, the face values of bonds are ₹1,000, ₹2,000, ₹5,000 and the like. Most bonds offer fixed interest payments till their maturity. This specific rate of interest is known as the coupon rate. Coupons are paid quarterly, semiannually and annually. At the end of the maturity period, the value is repaid.

BOND RISK

Generally, stocks are considered to be risky but bonds are not. This is not absolutely correct. Bonds do carry risks, but the nature and types of risks may be different. The risks are related to the interest rate, default, marketability, and callability.

Interest rate risk

Variability in the return from debt instruments to investors is caused by the changes in the market interest rate. This is known as interest rate risk. Changes in interest rates affect bonds more directly than they affect equity. There is a relationship between the coupon rate and market interest rate. If the market interest rate moves up, the price of the bond declines and vice versa. For example, if one holds a 14.5 per cent bond and the market interest rate falls from 14 per cent to 13 per cent, the bond value would be higher. In contrast, if the market interest rate goes up to 15 per cent, the price would decline to offer the buyer a yield that is proximate to the market interest rate.

Default risk

The failure to pay the agreed value of the debt instrument by the issuer in full, on time, or both is known as default risk. Treasury bills and bonds issued by the central government are devoid of this risk. The same cannot be assured of bonds/debentures issued by corporate bodies. Default risk can arise because of macroeconomic factors or firm-specific factors. The macroeconomic factors affect the overall system. In the case of the CRB Capital Market, the bankruptcy had more to do with firm-specific factors such as inefficient management, rather than macroeconomic factors.

One of the steps taken to avoid default risk is to have rating agencies rate the capacity of a company to serve the debt. Regulators such as RBI and SEBI use credit rating to determine the eligibility of the fixed income instruments. The Credit Rating Information Services of India Limited (CRISIL), the Investment Information and Credit Rating Agency of India Limited (ICRA), and the Credit Analysis and Research Limited (CARE) rate bonds and other fixed income securities. In the international bond market, Moody's investor service and Standard and Poor's ratings are well known.

Marketability risk

Variations in return caused by difficulties in selling the bonds quickly without having to make a substantial price concession is known as marketability risk. This risk is different from the market risk that affects all securities in the market in that it is a specific risk. The marketability or liquidity of a particular bond depends on the corporate entity that issues it. There is the possibility of a particular company's bond becoming illiquid owing to the downgrading of the bond's rating by the rating agencies. The managerial inefficiencies and fall in profits may create a fear in the minds of investors, and they may not be willing to buy such bonds in the secondary market. Sometimes, a particular instrument of a company whose other instruments enjoy good liquidity may be illiquid. If an investor has to sell such illiquid investments, he may be forced to sell it at a high discount. For example, the bonds/debentures issued by Reliance Industries enjoy high liquidity, but the same may not be true of the debentures issued by the smaller companies. Thus, liquidity of the particular bond or debenture depends on the corporate image.

Callability risk

The uncertainty created for the investor's return by the issuer's ability to call the bond at any time is known as callability risk. Debt instruments used to carry a call option. The call option provides the issuer the right to call back the instruments by redeeming them. This facility provides a way out for the issuer if the market interest rate declines. The issuer can call the bond with high interest rate and again raise funds at a lower interest rate. Since the bond or debenture can be called at any time, there is uncertainty regarding the maturity period. This feature of the bond may depress the price of the bond and the uncertainty element attached to callable bonds makes the investors ask for higher yields.

TIME VALUE CONCEPT

The time value concept of money is that a rupee received today is more valuable than a rupee received tomorrow. An investor will postpone current consumption only if he can add to future consumption opportunities through investment. Individuals generally prefer current consumption to future consumption. If there is inflation in the economy, a rupee today will represent more purchasing power than a rupee at a future date.

Interest is the rent paid to the owners to part with their money. The interest that the borrower pays to the lender causes the money to have a future value that is different from its present value. The time value of money makes the rupee invested today grow more than a rupee in the future. To quantify this concept mathematically, compounding and discounting principles are used. The one-period future time value of money is given by the equation:

Future value = Present value (1 + interest rate). If ₹100 are put in a savings bank account for one year, the future value of money will be:

$$\begin{aligned}\text{Future value} &= ₹100\,(1.0+6\%)\\ &= 100 \times 1.06 = ₹106\end{aligned}$$

If the deposited money is allowed to cumulate for more than one time period, the period exponent is added to the previous equation.

$$\text{Future value} = (\text{Present value})\,(1+\text{ interest rate})^t$$

Where t is the number of periods the deposited money accumulates interest. Suppose ₹100 is put for two years at a 6 per cent rate of interest, the money will grow to be ₹112.36.

$$\begin{aligned}\text{Future value} &= \text{Present value}\,(1 + \text{interest rate})^2\\ &= 100\,(1+.06)^2\\ &= 100\,(1.1236)\\ &= 112.36\end{aligned}$$

To find out the values in a simple manner, the compound sum of ₹1 at the end of a period FVIF1,/K,n and compound sum of an annuity of ₹1 per period FVIFA, tables are given in Appendix 1.

PRESENT VALUE

The present value of money can be found simply by reversing the earlier equation.

$$\text{Present value} \times (1 + \text{interest rate}) = \text{Future value}$$

$$\text{Present value} = \frac{\text{Future value}}{1 + \text{interest rate}}$$

Here, the discounting principle is used. Today's worth of ₹100 to be received after a year at 10 per cent interest would be:

$$\text{Present value} = \frac{\text{Future value}}{1 + \text{interest rate}}$$

$$= \frac{100}{1 + 0.10} = \frac{100}{1.1} = ₹90.90$$

Multiple periods of present value equation take into account the multiple periods.

$$\text{Present value} = \frac{\text{Future value}}{1 + (\text{interest rate})^t}$$

To make the calculation easier, the present value of ₹1: PVIF $= 1/(1+K)^t$ and present value of annuity of ₹1 per period:

$$\text{PVIFA} = \left[\sum_{i=1}^{n} \frac{1}{\{1+K\}^t}\right] \text{ are given in the appendix.}$$

BOND RETURN

When an investor buys a bond and sells it after holding it for a while, the rate of return in that holding period is called the holding period return.

Holding period return

This return is calculated as follows:

$$\text{Holding period return} = \frac{\text{Price gain or loss during the holding period + Coupon interest rate, if any}}{\text{Price at the beginning of the holding period}}$$

The holding period rate of return is also called the one-period rate of return. This return can be calculated daily, monthly or annually. If the fall in the bond price is greater than the coupon payment, the holding period return will be negative.

Example 8.1

(a) An investor purchases a bond at ₹900 with ₹100 as coupon payment and sells it at ₹1,000. What is his holding period return?
(b) If the bond is sold for ₹750 after receiving ₹100 as coupon payment, what is the holding period return?

Solution

(a)

$$\text{Holding period return} = \frac{\text{Price gain} + \text{Coupon payment}}{\text{Purchase price}}$$

$$= \frac{100 + 100}{900} = \frac{200}{900} = 0.2222$$

$$\text{Holding period return} = 22.22\%$$

(b)

$$\text{Holding period return} = \frac{\text{Gain or loss} + \text{Coupon payment}}{\text{Purchase price}}$$

$$= \frac{-150 + 100}{900} = \frac{-50}{900} = -0.0555$$

$$\text{Holding period return} = -5.5\%$$

Current yield

The current yield is the coupon payment as a percentage of current market price.

$$\text{Current yield} = \frac{\text{Annual coupon payment}}{\text{Current market price}}$$

With this measure, investors can know the rate of cash flow from their investments every year. The current yield differs from the coupon rate, since the market price differs from the face value of the bond. When the bond's face value and the market price are the same, the coupon rate and the current yield would be the same. For example, when the coupon payment is 8 per cent for a ₹100 bond with the same market price, the current yield is 8 per cent. If the current market price is ₹80 then, the current yield would be 10 per cent.

Yield to maturity

The concept of yield to maturity (YTM) is one of the most widely used tools in bond investment management. Arithmetically, YTM is the single discount factor that makes the present value of future cash flows from a bond equal to the current price of the bond. Intuitively, YTM is the rate of return that an investor can expect to earn if the bond is held till maturity.

The YTM is calculated based on certain assumptions:

- There should not be any default. The coupon and principal amount should be paid as per schedule.
- The investor has to hold the bond till maturity.
- All coupon payments should be reinvested immediately at the same interest rate as the YTM of the bond.

Understanding this is crucial for better investment decisions. For example, if a 11 per cent coupon paying bond with four years to mature has a YTM of, say, 13 per cent it would be realized only if two conditions are met: (1) the bond is held till maturity (for four years), and (2) the interest received from the bond is reinvested for the rest of the period at 13 per cent. Otherwise, the actual or realized rate of return to the investor will be different from the expected return.

In the above example, if coupon receipts are reinvested at, say, 10 per cent for the rest of the period then the realized rate of return will be less than the YTM. Conversely, if the coupon receipts are reinvested at 14 per cent, the realized rate of return will be higher than the YTM.

Any difference in the reinvestment rate will cause a difference between the actual return and the YTM. In this sense, the YTM is only a measure of yield. It cannot be regarded as a measure of return from a coupon-paying bond.

The YTM concept has a slightly different meaning for zero coupon bonds (ZCB), popularly known as deep discount bonds (DDB). ZCBs do not carry any coupon but are issued at a price discounted to the face value. On maturity, these bonds are redeemed at face value. Since these bonds do not have any coupon payments during the life of the bond, the question of reinvestment of coupon payments does not arise. There is no reinvestment risk for ZCBs.

To find out the YTM, the present value technique is adopted. The formula is as follows:

$$\text{Present value} = \frac{\text{Coupon}_1}{(1+y)^1} + \frac{\text{Coupon}_2}{(1+y)^2} + \ldots + \frac{(\text{Coupon}_n + \text{face value})}{(1+y)^n}$$

Y = Yield to maturity.

Example 8.2

A four-year bond with a 7 per cent coupon rate and maturity value of ₹1,000 is currently selling at ₹905. What is its yield to maturity?

Solution

Yield to maturity can be found out by using trial-and-error method. Let us try 10 per cent as yield to maturity.

Cash flow	*PV for 10%*	*PV of CF*
70	0.9091	63.64
70	0.8264	57.85
70	0.7513	52.59
1070	0.6830	730.82
		₹904.90

The yield to maturity is 10 per cent.

The approximate YTM can also be calculated using the following formula:

$$Y = \frac{C + (P \text{ or } D / \text{years to maturity})}{(P_o + F)/2}$$

Y = Yield to maturity
C = Coupon interest
P or D = Premium or discount
P_o = Present value
F = Face value

In the above example,

$$= \frac{70 + (95/4)}{(905 + 1000)/2} = \frac{93.75}{952.5} = 0.098$$

$$Y = 9.8\%$$

Yield to maturity is 9.8 per cent.

Using the formula, the value of the bond can be assessed and buying decisions can be made. By knowing the expected YTM, the present price of the bond can be found. The difference between the actual price and calculated price indicates whether the bond is underpriced or overpriced. When the bond's prevailing price is lower than the calculated price, it is considered as being underpriced. Taking the previous example, the expected YTM is 10 per cent at the price of ₹905. If the market price is ₹850, it is underpriced. The simple way to calculate is

$$NPV = \frac{\text{Coupon}}{(1+y)^t} + \frac{P_m}{(1+y)^t}$$

NPV = Net present value
P_m = Market price

Example 8.3

A ₹100 par value bond bearing a coupon rate of 11 per cent matures after five years. The expected yield to maturity is 15 per cent. The present market price is ₹82. Can the investor buy it?

Solution

$$NPV = \frac{\text{Coupon}}{(1+y)^t} + \frac{P_m}{(1+y)^t}$$

$$= 11 \times 3.3522 + 100 \times 0.4972$$
$$= 36.87 + 49.72 = 86.59$$
$$= 86.59 - 82 = 4.59$$

The net present value is higher, i.e., the market value is lower than the calculated value and hence the bond is underpriced; the investor can buy the bond.

BOND VALUE THEOREMS

The value of the bonds depends upon three factors, namely, the coupon rate, years to maturity, and the expected yield to maturity or the required rate of return. On the basis of this, bond value theorems have evolved.

Theorem 1

If the market price of the bond increases, the yield would decline and vice versa.

Factors	*Bond A*	*Bond B*
Par value	₹1,000	₹1,000
Coupon rate	10%	10%
Maturity period	2 years	2 years
Market price	₹874.75	₹1035.66
Yield	18%	8%

Even though the bonds A and B are of the same maturity and bear the same coupon rate, the difference in the market price leads to a difference in yields. The bond with a low price has high yield because with less money more return is earned.

Theorem 2

If the bond's yield remains the same over its life, the discount or premium depends on the maturity period.

Factors	*Bond A*	*Bond B*
Par value	₹1,000	₹1,000
Coupon rate	10%	10%
Yield	15%	15%
Maturity period	2 years	3 years
Market price	₹918.71	₹885.86
Discount	₹81.29	₹114.14

This means, A bond with a short-term to maturity sells at a lower discount than B bond with a long-term to maturity.

Theorem 3

If a bond's yield remains constant over its life, the discount or premium amount will decrease at an increasing rate as its life gets shorter. Consider a bond with a face value of ₹1,000, and maturity period of five years with 10 per cent yield to maturity. The calculated values are given below.

Years to maturity	*Present value*
5	620.9
4	683.0
3	751.3
2	826.4
1	909.1

The above example shows that the discount rate declines at a lower when the bond approaches maturity. The same point is illustrated in Figure 8.1.

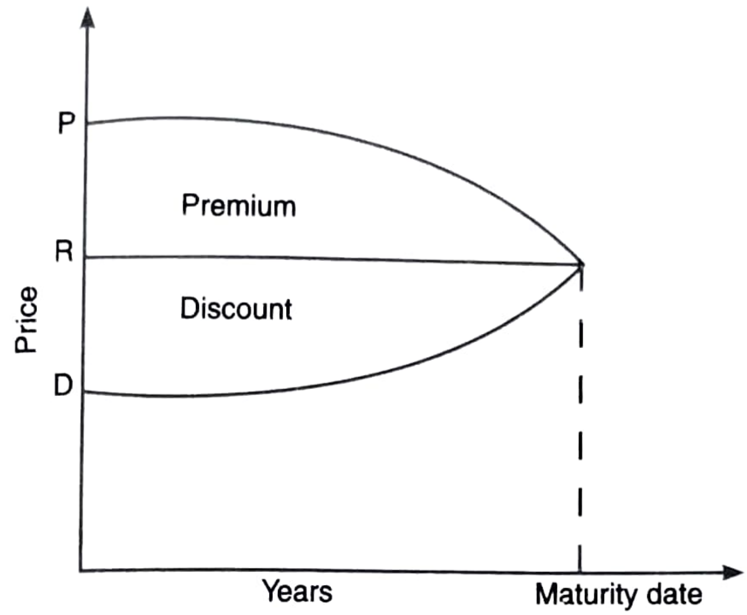

Figure 8.1 Bond's price changes during its life

Theorem 4

A rise in the bond's price for a decline in the bond's yield is greater than the fall in the bond's price for a rise in the yield. Take a bond with a 10 per cent coupon rate, maturity period of five years, and a face value of ₹1,000. If the yield declines by 2 per cent, that is, to 8 per cent, then the bond price will be ₹1079.87.

= 100 (PVIFA 8%, 5 years) + 1000 (PVIF 8%, 5 years)
= 100 × 3.9927 + 1000 × 0.6806
= ₹1079.87

If the yield increases by 2 per cent, the bond price will be ₹927.88.
= 100 (PVIFA 12%, 5 years) + 1000 (PVIF 12%, 5 years)
= 100 × 3.6048 + 1000 × 0.5674
= ₹927.88

Now the fall in yield has resulted in a rise of ₹79.86 but the rise in yield caused a variation of ₹72.22 in the price.

Theorem 5

The change in the price will be less for a percentage change in the bond's yield if its coupon rate is higher. This is explained by the following example:

Factors	*Bond A*	*Bond B*
Coupon rate	10%	8%
Yield	8%	8%
Maturity period	3 years	3 years
Price	₹105.15	₹100
Face value	₹100	₹100
Yield increase	1%	1%
Price after yield increases	₹102.53	₹97.47
Percentage change in price	2.4%	2.53%

CONVEXITY

A bond's price and yield are inversely related. The rise in bond prices would cause a fall in yield and vice versa. This has been shown in Theorem 1. According to Theorem 4, the relationship is not linear. The quantum increase in the bond's price for a given decline in the yield is higher than the decline in bond's price for a similar amount of increase in the bond's yield. It is not linear in nature. This relationship is often referred to as convexity. The convexity concept is applicable to all types of bonds. The degree of convexity differs from bond to bond depending upon the size of the bond, the years to maturity, and the current market price. Figure 8.2 shows the convexity in yield.

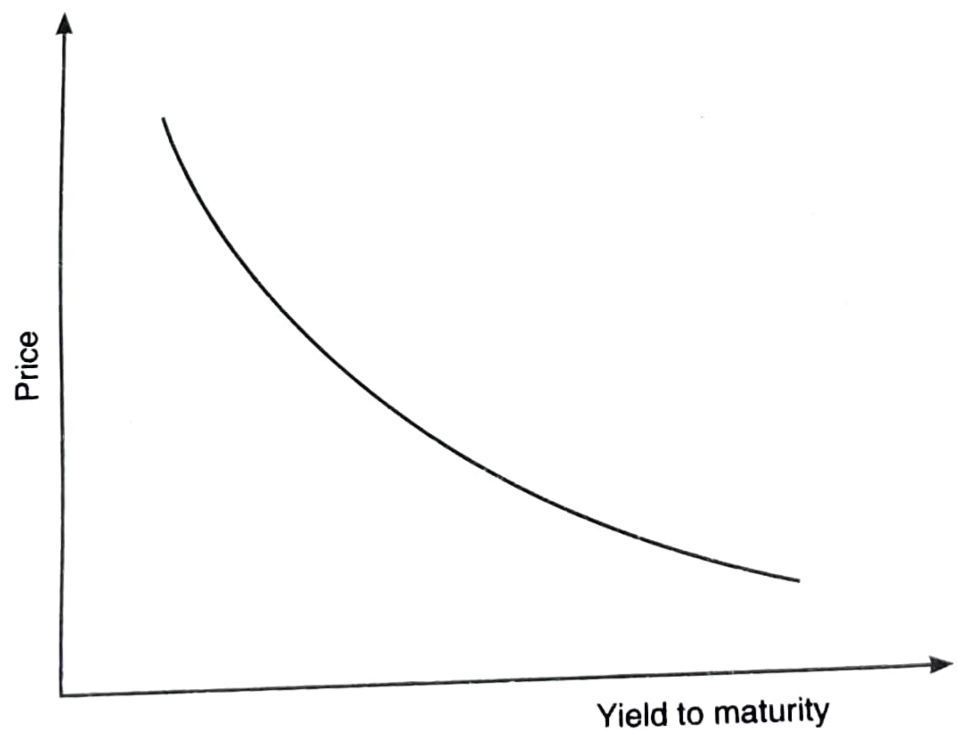

Figure 8.2 Bond convexity

THE TERM STRUCTURE OF INTEREST RATES (YIELD CURVE)

The bond portfolio manager is often concerned with two aspects of interest rates—the interest rate level and the term structure of interest rates. The relationship between the yield and time or years to maturity is called the term structure. The term structure is also known as yield curve. In analysing the effect of maturity on yield and all other influences are held constant. Usually, pure discount instruments are selected to eliminate the effect of coupon payments. The selected bonds should not have early redemption features. The maturity dates are different, but the risks, tax liabilities, and redemption possibilities are similar.

The general perception is that the curve will be upward moving up to a point and then become flat. This is shown in Figure 8.3.

Figure 8.3 Rising yield curve

There are at least three competing theories that attempt to explain the term structure of interest rates—the expectation theory, the liquidity preference theory and the preferred habitat or segment theory.

Expectation theory

This theory was developed by J R Hicks (1939), F Lutz (1940), and B Malkiel (1966). According to the expectation theory, the shape of the curve can be explained by the expectations of investors about future interest rates. If the short-term rates are expected to be relatively low in the future, then the long-term rate will be below the short-term rate. There are three reasons for investors to anticipate a fall in the interest rate.

1. Anticipation of the fall in the inflation rate and reduction in the inflation premium
2. Anticipation of a balanced budget or cut in the fiscal deficit
3. Anticipation of recession in the economy, and a fall in the demand for funds by private companies

The long-term rates will exceed the current short-term rates, if there is an expectation that the market rates will be higher in the future. Thus, the yield curve depends upon the expectations of the investors. This is illustrated in Figure 8.4.

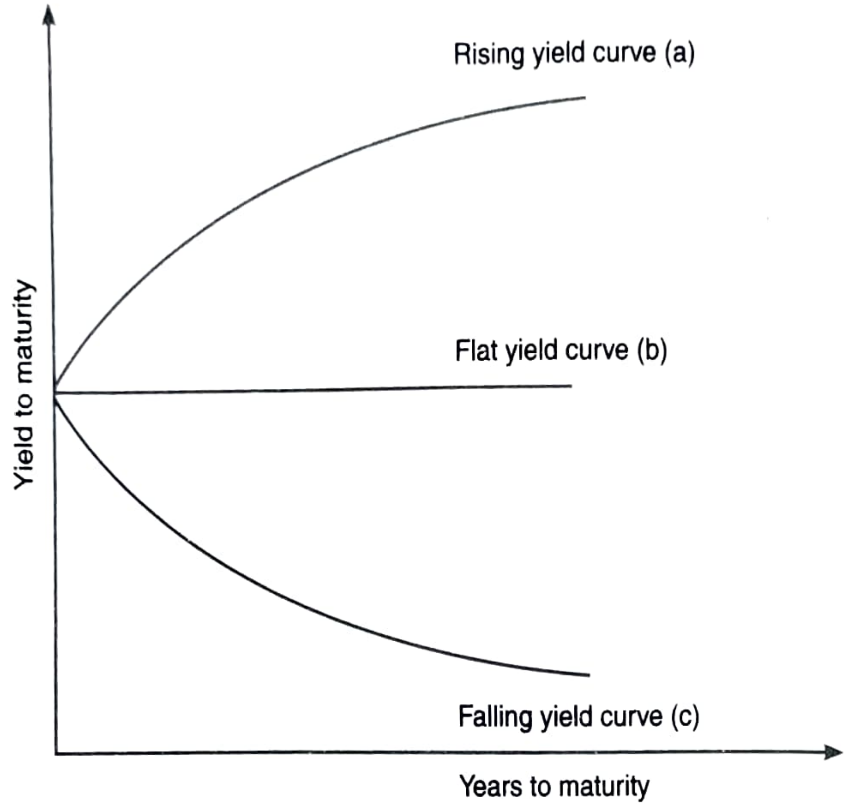

Figure 8.4 Yield curve

A rising yield curve (a) indicates investors' expectations of a continuous rise in interest rates. A flat yield curve (b) means that investors expect the interest rate to remain constant. A declining yield curve (c) shows that investors expect the interest rate to decline.

Liquidity preference theory

Keynes' liquidity preference theory as advocated by J R Hicks (1939) accepts that expectations influence the shape of the yield curve. In a world of uncertainty, it would be more desirable for investors to invest in short-term bonds than in long-term ones because of their greater liquidity. If no premium exists for holding long-term instruments, investors would prefer to hold short-term bonds to minimize the possible variation in the nominal value of their portfolio.

The exponents of the liquidity preference theory believe that investors prefer the short term rather than the long term. Hence, they have to be motivated to buy the long-term bonds or lengthen the investment horizon. The bond issuing corporation or contributor pays a premium to motivate the investors to buy. This liquidity premium theory asserts that over time the forward rates are actually higher than the projected interest rate.

Segmentation theory

Critics of the expectations theory, such as J Culbertson (1957), and F V Modigliani and R Sutch (1966) pointed out that liquidity preferences cannot be the main consideration for all classes of investors. In their view, insurance companies, pension funds, and even retired persons prefer long-term rather than short-term securities to avoid possible fluctuations in the interest rate. This can be explained in detail.

Life insurance companies offer insurance policies that do not require any payment for a long time. For example, an insurance policy issued to a 25-year-old may involve another 20 or more years before the company has to make a payment. Premium payments are fixed by the expected future rate of interest. If the insurance company invests the funds in a long-term bond, the interest the bond would earn is certain and if the earned interest rate is higher than the promised interest rate, the company stands to gain, and its risk is also reduced. If it invests in one-year bonds, the risk of reinvestment is there and if there is a fall in the market interest rate, the insurance company stands to lose, and it would be difficult for the company to meet its obligation. This leads insurance companies to prefer long-term bonds to short-term ones.

On the other hand, commercial banks and corporates may prefer liquidity to meet their short-term requirements and therefore, they prefer short-term issues. The supply of and demand for funds are segmented in sub-markets because of the preferred habits of individuals. Thus, the yield is determined by the demand for and supply of funds.

Riding the Yield Curve

When the long-term coupon rates are higher than the short term rates, the yield curve has an upward sloping shape. The bond portfolio manager tries to exploit this to his advantage and tries to increase the yield by purchasing the long-term bonds. This strategy is known as riding the yield curve. When the long-term bond approaches to maturity, the interest rate may come closer to that of the short-term bond but there would be capital gain. The bond portfolio manager may maintain the long-term bond to utilize the capital gains as the bond moves to maturity date and 'ride down the yield curve' to the lower interest rate, which will be appropriate when it becomes a shorter term bond. Riding the yield curve will succeed only if the market interest rate does not rise. Sometimes, the market interest rate may increase or the short-term end of the yield curve may slope upwards causing capital losses to the bond portfolio manager. To manage the situation efficiently, he should be continuously watchful about the shape of the yield curve and the shifts that occur in the market interest rates.

Duration

Duration measures the time structure of a bond and the bond's interest rate risk. The time structure of investment in bonds is expressed in two ways. The common way is to state how many years an investor has to wait until the bond matures and the principal money is paid back. This is known as asset time to maturity or its years to maturity. The other way is to measure the average time taken for all interest coupons and the principal to be recovered. This is called Macaulay's duration. Duration is defined as the weighted average of periods to maturity, with the weights being present values of the cash flow in each period. The formula for duration is,

$$D=\frac{\frac{C_1}{(1+r)}}{P_o}+\frac{\frac{C_2}{(1+r)^2}}{P_o}2+\ldots+\frac{\frac{C_t}{(1+r)^t}}{P_o}\times t$$

This can be summarized as

$$D=\sum_{t=1}^{T}\frac{P_v(C_t)}{P_o}\times t$$

D = Duration
C = Cash flow
r = Current yield to maturity
t = Number of years
$P_v(C_t)$ = Present value of the cash flow
P_o = Sum of the present values of cash flows

Example 8.4

Calculate the duration for bond A and bond B with 7 per cent and 8 per cent coupons having a maturity period of four years. The face value is ₹1,000. Both the bonds currently yield 6 per cent.

Solution

$$D = \frac{\frac{C_1}{(1+r)}}{P_o}1 + \frac{\frac{C_2}{(1+r)^2}}{P_o}2 + \frac{\frac{C_3}{(1+r)^3}}{P_o}3 + \frac{\frac{C_4}{(1+r)^4}}{P_o}4$$

C_4 includes the principal repayment.

The following relates to bond A with a 7 per cent coupon rate:

Year	*Cash flow* C_t	$\frac{1}{(1+r)^t}$	$P_v \times C_t$	$\frac{\frac{C_t}{(1+r)^t}}{P_o}$	$\frac{\frac{C}{(1+r)^t}}{P_o} \times t$
1.	70	0.943	66.01	0.0638	0.0638
2.	70	0.890	62.30	0.0602	0.1204
3.	70	0.8396	58.77	0.0568	0.1704
4.	1070	0.7921	847.55	0.8191	3.2764
			P_0 = ₹1034.63		D = 3.6310

The following relates to bond B with an 8 per cent coupon rate:

Year	*Cash flow* C_t	$\frac{1}{(1+r)^t}$	$P_v \times C_t$	$\frac{\frac{C_t}{(1+r)^t}}{P_o}$	$\frac{\frac{C_t}{(1+r)^t}}{P_o} \times t$
1	80	0.943	75.44	0.0706	0.0706
2	80	0.890	71.200	0.0666	0.1332
3	80	0.8396	67.168	0.0628	0.1884
4	1080	0.7921	855.468	0.8000	3.2000
			P_0 = ₹1069.276		D = 3.5922

	Bond A	*Bond B*
Face value	₹1,000.00	₹1,000.00
Coupon rate	7%	8%
Years to maturity	4.0	4.0
Macaulay's duration	3.631 years	3.592 years

From this example it is clear that the bond with higher coupon payments has a shorter duration compared to the bond with a lower coupon rate.

As a general rule,

- The higher the coupon rate, the lower the duration and the less volatile the bond price.
- The longer the term to maturity, the longer the duration and the more volatile the bond.
- The higher the yield to maturity, the lower the bond duration and bond volatility, and vice versa.
- In a zero coupon bond, the bond's term to maturity and duration are the same. The zero coupon bond involves only one balloon payment to repay the principal and interest on the maturity date.

Importance of duration

The concept of duration is important because it provides a more meaningful measure of the length of a bond, helps evolve immunization strategies for portfolio management, and measures the sensitivity of the bond price to changes in the interest rate.

Duration and price changes

The price of a bond changes according to the interest rate. A bond's price changes are commonly called bond volatility. Duration analysis helps to determine the changes in bond price as the yield to maturity changes. The relationship between the duration of a bond and its price volatility as the market interest rate changes is given by the following formula

$$\text{Percentage change in price} = \frac{-\text{MD}[\Delta \text{BP}]}{100}$$

MD = Modified duration
BP = Basis point is 0.01 of 1 per cent (1 per cent = 100)
Δ = change in interest rate

$$\text{Modified duration MD} = \frac{D}{1+\frac{R}{P}}$$

where,
D = Duration
R = Market yield
P = Interest payment per year (usually two)

Immunization

Immunization is a technique that allows the bond portfolio holder to be relatively certain about the promised stream of cash flows. The bond interest rate risk arises from changes in the market interest rate. The market rate affects the coupon rate and the price of the bond. In the immunization process, the coupon rate risk and the price risk can be made to offset each other. Whenever there is an increase in the market interest rate, the price of the bonds falls. At the same time, the newly issued bonds offer higher interest rates. The coupon can be reinvested in the bonds offering higher interest rate and losses that occur due to the fall in the price of the bond can be offset and the portfolio is said to be immunized.

The process The bond portfolio manager or investor has to calculate the duration of the promised outflow of the funds and invest in a portfolio where the bonds have identical durations. The bond portfolio duration is the weighted average of the durations of the individual bonds in the portfolio. For example, if an investor has invested an equal amount of money in three bonds, namely A, B and C with durations of two, three, and four years respectively, then the bond portfolio duration is

$$D = 1/3\times\ 2 + 1/3\times 3 + 4\times 1/3$$
$$= 0.66 + 1 + 1.33$$
$$D = 2.99 \text{ (or) three years}$$

The bond manager can offset the interest rate and price risks by matching the outflow duration with cash inflow duration from bond investment. The portfolio of money to be invested between the different types of bonds also can be found by using the following equation:

Investment outflow = (X_1 × duration of bond 1) + (X_2 × duration of bond 2)

where X_1, X_2 refer to the proportions of investment in bond 1 and 2.

Example 8.5

Abisekh has ₹50,000 to make a one-time investment. His son has entered higher secondary school and he needs his money back after two years for his son's education. As Abisekh's outflow is a one-time outflow, the duration is simply two years. Now he has a choice of two types of bonds.

- Bond A, which has a coupon rate of 7 per cent, a maturity period of four years, and a current yield of 10 per cent. The current price is ₹904.90.
- Bond B, which has a coupon rate of 6 per cent, a maturity period of one year, and a current yield of 10 per cent. The current price is ₹963.64.

The two bonds pose two types of risks for him. He can invest all his money in bond B with the aim of reinvesting the proceeds from the maturing bonds in another issue of one-year period. If the interest rate declines in the market during the next year, he must reinvest his money in low-yielding bonds and may incur a loss. Now, he has to face the reinvestment risk.

On the other hand, if he invests his money in bond A, this too involves a certain amount of risk. He cannot hold it till maturity because he needs the money after two years and has to sell it earlier. If there is a rise in the market interest rate, then the price of the bond will fall and vice versa. If a rise in interest rate is assumed, the investor has to incur a loss.

Solution

Abisekh can solve the problem by investing part of money in one-year bonds and a part in four-year bonds. He should know how much to invest in each of these bonds. This can be calculated by solving the following equation:

$$(X_1 \times D_1) + (X_2 \times D_2) = 2$$

Where,

X_1 = the proportion of investment in bond A
X_2 = the proportion of investment in bond B
D_1 = duration of bond A
D_2 = duration of bond B

While the duration of the one-year bond is 1 year only as it makes a one-time payment, the duration of bond 2 is calculated as follows:

$$D=\left[\sum_{t=1}^{T}\frac{P_v(C_t)}{P_o}\times t\right]$$

Year	Cash flow C_t	Present value factor 10%	$P_v(C_t)$	$\frac{P_v(C_t)}{P_o}$	$\frac{P_v(C_t)}{P_o}\times t$
1	70	0.9091	63.64	0.0703	0.0703
2	70	0.8264	57.85	0.0639	0.1278
3	70	0.7513	52.59	0.0581	0.1743
4	1070	0.6830	730.81	0.8076	3.2305
			$P_o =$ 904.89		D = 3.6029

Applying the formula,

$(X_1 \times 1) + (X_2 \times 3.6030) = 2$

Since X_1 can be written as $(1 - X_2)$

$[(1 - X_2)1] + [X_2 \times 3.603] = 2$

or $1 - X_2 + 3.6030\, X_2 = 2$

or $X_2 = 0.3842$

or $X_1 = 0.6158$

Abisekh should put 61.58 per cent of his investible funds in the one-year bond and 38.42 per cent in the four-year bond.

For investing in both the bonds he needs ₹41,322.31 = ₹50,000 × $\frac{1}{(1.10)^2}$ to have a fully immunized bond portfolio. The money to be invested is,

One-year bond = 41,322.31 × X_1 = 41,322.31 × 0.6158
= ₹25,446.28

Four-year bond = 41322.31 × 0.3842 = ₹ 15,876.03

From here we can find out how many bonds he can buy.

Since the 1-year bond price is ₹963.64, he can buy,

$$\frac{25446.28}{963.64}=26.4$$

Approximately 26 bonds,

With the four-year bond price being ₹904.89, he can buy

$$\frac{15876.03}{904.89}=17.54$$

Approximately 18 bonds.

Theoretically, the rise in the market interest is offset by the reinvestment of matured bonds at a higher rate of interest. In practice, this is not so simple because of the following reasons:

- Immunization and duration are based on the assumption that the change in the interest rate will occur before payments are received from both bonds. This may not always be true. The shift may occur after receiving the cash flows.
- Another assumption is that the bonds have same yield. This may also not be applicable. The yield may vary according to the period of maturity.
- It is assumed that the shift in interest rates will affect all bonds equally. Many a time, the shift in interest rates affects different bonds differently.
- The whole analysis is based on the belief that there will not be any call risk or default risk. Evidence shows that bond investment is not free from call risk or default risk.