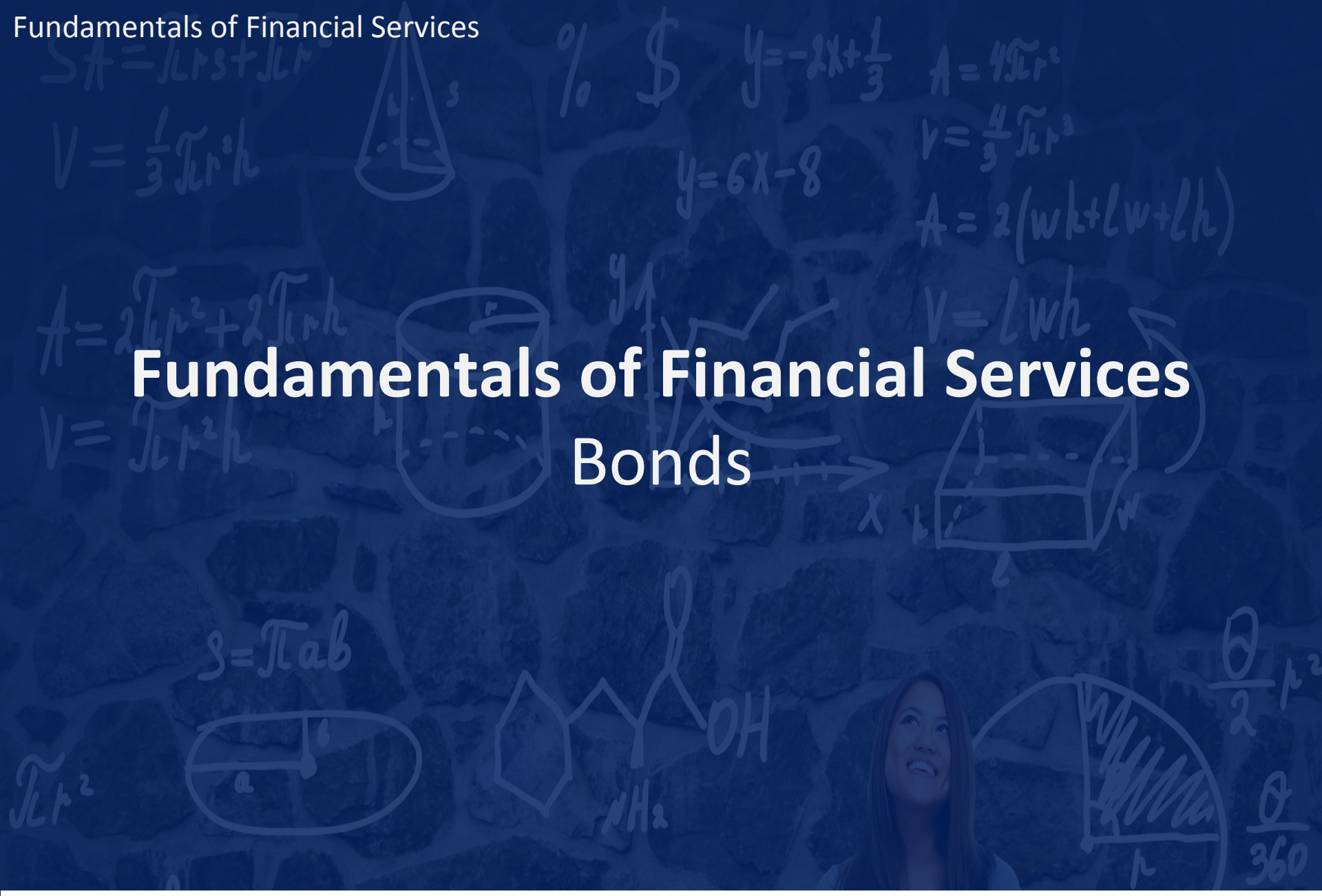
Fundamentals of Financial Services

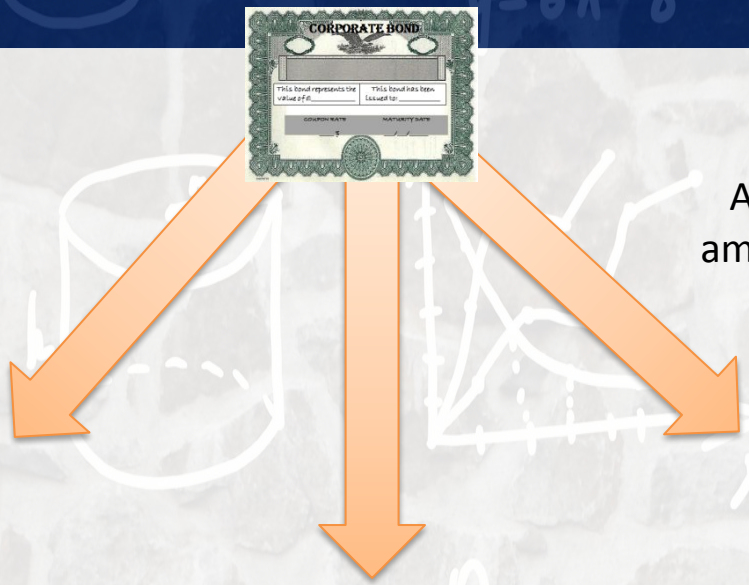
Bonds



What is a BOND?

A debt instrument whereby an investor lends money to an entity (such as a company or a government) that borrows the funds for a defined period of time at a fixed interest rate.

Bond Features



Repayment Date

When the money will be returned. Also known as the redemption or maturity date. This is when the bond will be redeemed.

Nominal Value

Also known as the face value. It is the amount that is written on the face of the bond certificate.

Tradeable

Bonds can be sold before they reach their repayment date

Interest Rate & Frequency

Percentage paid as interest and how often it is paid. More typically termed as the **COUPON** on a bond.

- Face value
- Coupon Rate
- Coupon payment
- Issue price
- Market Price
- Maturity date
- Redemption price
- Intrinsic value

- Zero coupon bond
- Deep discount bond
- Callable and puttable bond
- Annuity bond
- Bond strips

Risk associated with bonds

1)Interest rate risk

- For fixed-income securities, as interest rates rise security prices fall (and vice versa). This is because when interest rates increase, the opportunity cost of holding those bonds increases – that is, the cost of missing out on an even better investment is greater. The rates earned on bonds therefore have less appeal as rates rise, so if a bond paying a fixed rate of 5% is trading at its par value of $1,000 when prevailing interest rates are also at 6%, it becomes far less attractive to earn that same 5% when rates elsewhere start to rise to say 6% or 7%.

Risk associated with bonds

2) Default Risk

Default risk is the risk that a lender takes on in the chance that a borrower will be unable to make the required payments on their debt obligation. Lenders and investors are exposed to default risk in virtually all forms of credit extensions. A higher level of default risk leads to a higher required return, and in turn, a higher interest rate.

Risk associated with bonds

3) Marketability Risk

Variation in returns caused by difficulties in selling the bonds quickly without having to make a substantial price concession is known as marketability risk. Marketability of a bond depends upon the corporate entity that issues it . There is possibility of a particular company's bond becoming illiquid Owing to the downgrading of bonds rating by rating agencies. If an investor has to sell such illiquid investments, he may be forced to sell it at high discount.

Risk associated with bonds

4)Callability risk

- A callable bond is one that can be redeemed prior to its maturity date. The bond has an embedded option that is similar to a call option, giving the issuer the right to call the bond before it matures. When interest rates drop in the market, bond issuers seek to take advantage of the lower rates by redeeming the outstanding bonds and reissuing at a lower financing rate.

Callability risk

- A callable bond is issued with a coupon rate of 5% and has a maturity of 10 years. The call protection period is four years, which means the issuer cannot call the bonds for the first four years of the bond's life regardless of how interest rates change. After the call protection period ends, bondholders are exposed to the risk that the bonds may be paid off if interest rates drop below 5%.

Definition of Systematic Risk

- The term ‘systematic risk’, we mean the variation in the returns on securities, arising due to macroeconomic factors of business such as social, political or economic factors. Such fluctuations are related to the changes in the return of the entire market. Systematic risk is caused by the
- changes in government policy,
- the act of nature such as natural disaster,
- changes in the nation’s economy,
- international economic components. The risk may result in the fall of the value of investments over a period. It is divided into three categories, that are explained as under:

Definition of Systematic Risk

- **Interest risk**: Risk caused by the fluctuation in the rate or interest from time to time and affects interest-bearing securities like bonds and debentures.
- **Inflation risk**: Alternatively known as purchasing power risk as it adversely affects the purchasing power of an individual. Such risk arises due to a rise in the cost of production, the rise in wages, etc.
- **Market risk**: The risk influences the prices of a share, i.e. the prices will rise or fall consistently over a period along with other shares of the market.

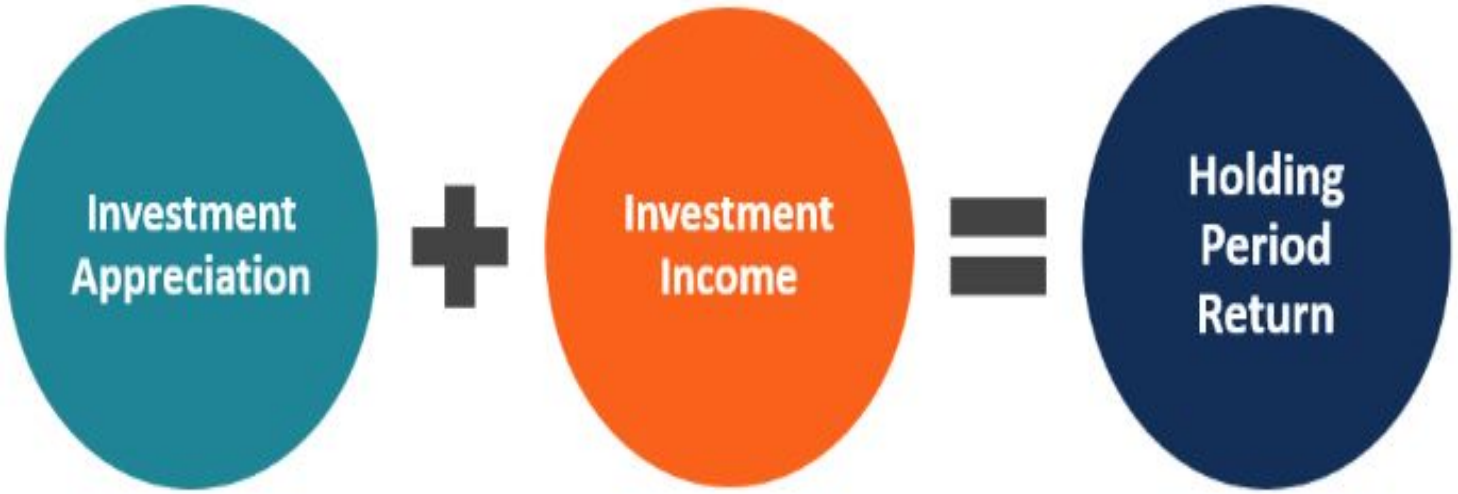
Definition of Unsystematic Risk

- The risk arising due to the fluctuations in returns of a company's security due to the micro-economic factors,
- i.e. factors existing in the organization, is known as unsystematic risk.
- The factors that cause such risk relates to a particular security of a company or industry so influences a particular organization only. The risk can be avoided by the organization if necessary actions are taken in this regard. It has been divided into two category business risk and financial risk, explained as under:

- **Business risk:** Risk inherent to the securities, is the company may or may not perform well. The risk when a company performs below average is known as a business risk. There are some factors that cause business risks like changes in government policies, the rise in competition, change in consumer taste and preferences, development of substitute products, technological changes, etc.
- **Financial risk**: Alternatively known as leveraged risk. When there is a change in the capital structure of the company, it amounts to a financial risk. The debt – equity ratio is the expression of such risk.

BASIS FOR COMPARISON	SYSTEMATIC RISK	UNSYSTEMATIC RISK
Meaning	Systematic risk refers to the hazard which is associated with the market or market segment as a whole.	Unsystematic risk refers to the risk associated with a particular security, company or industry.
Nature	Uncontrollable	Controllable
Factors	External factors	Internal factors
Affects	Large number of securities in the market.	Only particular company.
Types	Interest risk, market risk and purchasing power risk.	Business risk and financial risk
Protection	Asset allocation	Portfolio diversification

Holding Period Return



Holding Period Return

- It is a return in which an investor buys a bond and liquidates it in the market after holding it for a definite period of time.
- The formula for calculating holding period of return is as follows:

$$\frac{\text{Price gain} + \text{Coupon payment}}{\text{Purchase price}}$$

- It can be calculated on a daily, monthly or annual basis.

question

- An investor purchases a bond of Rs 900 with 100 as coupon payment and sells it at Rs 1000.

What is the holding period return.

$$\frac{100+100}{900} = 200/900 = 0.2222*100 = 22.22\%$$

Question

- If a bond is sold for Rs 750 after receiving Rs 100 as a coupon payment , What is the holding period return?

$$-\frac{150+100}{900} = -50/900 = -0.0555 * 100 = -5.5\%$$

Bond Yields

YIELD: This is just another word for “return” and it is expressed as an ANNUAL PERCENTAGE

COUPON: This is the INTEREST RATE paid on the FACE/NOMINAL value of the bond

YIELD and COUPON are not the same thing.

They are only the same when the bond is bought/sold at its FACE/NOMINAL value.

Otherwise, generally they are different.

Their relationship can be explained as follows:

Bond Prices

Bond prices respond to changes in INTEREST RATES.

- This is because, for their yield (return) to be attractive to investors it must remain competitive (when compared to the return available on alternative investment instruments).
- If the bank increases the rate of interest, then alternative investments may appear to have a more competitive return.
- In response to this the bond price decreases to make the bond appear cheaper to purchase, and the yield offered needs to increase.

The Current Yield

- It is a measure through which the investors can easily figure out the rate of cash flow on the investments made by them every year.
- It is calculated as:

$$\frac{\text{Annual Coupon Payment}}{\text{current market price}}$$

Current Yield

$$\text{Current Yield} = \frac{\text{Annual Coupon}}{\text{Current Bond Price}}$$

For instance, if a corporate bond with a $1,000 face value (FV) and an $80 annual coupon payment is trading at $970, then the implied yield is 8.25%.

- Current Yield = $80 Annual Coupon ÷ $970 Bond Price = 8.25%

In practice, the calculation of the metric is simple and convenient, yet its utility tends to be limited in scope.

The current yield is most often relevant only if the market price deviates from its par value.

- **Bond Price < Par Value** → "Discount" Bond
- **Bond Price = Par Value** → "Par" Bond
- **Bond Price > Par Value** → "Premium" Bond

Since the price of a bond adjusts based on the prevailing macro conditions and news surrounding the underlying issuer, bonds can be purchased at discounts or premiums relative to par.

- If a bond is trading below its face value (FV), the current yield is *higher* than the coupon rate.
- If a bond is trading at its face value (FV) – i.e. "at par" – the current yield is *equivalent* to the coupon rate.
- If a bond is trading in excess of its face value, the current yield is *lower* than the coupon rate.

Question

assume three bonds were each issued at a face (par) value of Rs1,000 with an annual coupon rate of 6%.

- Face Value of Bond (FV) = Rs1,000
- Coupon Rate (%) = 6.00%

Since the annual coupon depends on the bond's original face value (FV), the coupon can be calculated by multiplying the coupon rate by the FV of the bond.

- Annual Coupon = 6.00% × Rs1,000
- Annual Coupon = Rs60

Question

However, the current market prices of the bonds are all different, with the bonds trading below par, at par, and above par, respectively:

- Discount Bond = Rs950
- Par Bond = Rs1,000
- Premium Bond = Rs1,050

For each bond, the current yield is equal to the annual coupon divided by the bond's face value (FV).

- Discount Bond = Rs60 ÷ Rs950 = 6.32%
- Par Bond = Rs60 ÷ Rs1,000 = 6.00%
- Premium Bond = Rs60 ÷ Rs1,050 = 5.71%

Yield to maturity

- Yield to maturity (YTM) is the total return anticipated on a bond if the bond is held until it matures. Yield to maturity is considered a long-term bond yield but is expressed as an annual rate.
- In other words, it is the internal rate of return (IRR) of an investment in a bond if the investor holds the bond until maturity, with all payments made as scheduled and reinvested at the same rate.

Yield to maturity

- The YTM is the estimated annual rate of return that a bond is expected to earn until reaching maturity, with three notable assumptions:
- **Assumption #1 →** The return assumes the bond investor held onto the debt instrument until the maturity date.
- **Assumption #2 →** All the required interest payments and principal repayment were made on schedule.
- **Assumption #3 →** The coupon payments were reinvested at the same rate as the yield-to-maturity (YTM).

Yield to Maturity

- It is the single discount factor that makes the present value of future cashflows from a bond equivalent to the current price of the bond.
- The following assumptions are used to calculate yield to maturity:
  - There should not be any default.
  - The interest payments are reinvested at yield to maturity.
  - The investor has to hold the bond till its maturity.
  - It is calculated as:

$$\text{Present value} = \frac{\text{Coupon}_1}{(1+y)^1} + \frac{\text{Coupon}_2}{(1+y)^2} + \ldots + \frac{(\text{Coupon}_n + \text{Face value})}{(1+y)^n}$$

Yield to Maturity (YTM)

- Assume that there is a bond on the market priced at $850 and that the bond comes with a face value of $1,000 (a fairly common face value for bonds). On this bond, yearly coupons are $150. The coupon rate for the bond is 15% and the bond will reach maturity in 7 years.

- An important distinction between a bond's YTM and its coupon rate is the YTM fluctuates over time based on the prevailing interest rate environment, whereas the coupon rate is fixed.
- The relationship between the yield to maturity and coupon rate (and current yield) are as follows.
- YTM < Coupon Rate and Current Yield → The bond is being sold at a "premium" to its par value.
- YTM > Coupon Rate and Current Yield → The bond is being sold at a "discount" to its par value.
- YTM = Coupon Rate and Current Yield → The bond is said to be "trading at par".

Question

- A four year bond with a 7 per cent coupon rate and maturity value of Rs 1000 is currently selling at Rs 905. What is its yield to maturity.

Cash flow	PV for 10%	PV of CF
70	0.9091	63.64
70	0.8264	57.85
70	0.7513	52.59
1070	0.6830	730.82
		904.90

$$Price = \frac{CF_1}{(1+YTM)^1} + \frac{CF_2}{(1+YTM)^2} + \frac{CF_3}{(1+YTM)^3} + \cdots + \frac{CF_n}{(1+YTM)^n}$$

Face Value of bond = ₹ 1,000. Redeemable after 4 years @ ₹ 1,200; Coupon Rate = 8% p.a. Determine the yield to maturity, if the present market price of the bond is ₹ 1,050.

Determining the intrinsic value of bond using desired rate of return as 10% p.a.

Years	Cash Flows	PV Factors @ 10%	PV of CF
1 – 4	80	3.1699	253.59
4	1,200	0.6830	819.60
		Intrinsic Value	1,073.19

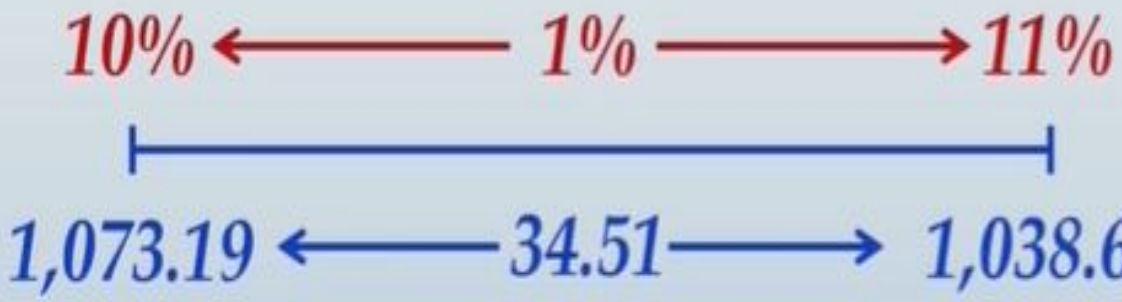
The market price of the bond, i.e. ₹1,050 is lower than the Intrinsic Value determined above.

Determining the intrinsic value of bond using desired rate of return as 11% p.a.

Years	Cash Flows	PV Factors @ 11%	PV of CF
1 – 4	80	3.10245	248.20
4	1,200	0.65873	790.48
		Intrinsic Value	1,038.68

The market price of the bond, i.e. ₹1,050 is higher than the Intrinsic Value determined above.
Therefore, YTM of the bond must be between 10% & 11%.

Using interpolation for determining YTM:



$1{,}073.19 - 1050 = 23.19$

$YTM = 10\% + ?$

$1\% \longrightarrow 34.51$

$? \longrightarrow 23.19$

$$? = \frac{23.19}{34.51} \times 1 = 0.672\%$$

$YTM = 10\% + 0.672\%$

$YTM = 10.672\%$ (Approx)

RELATIONSHIP OF THE BOND PRICE AND YIELD

The yield and bond price have an important but **inverse relationship**.

- **When the bond price is lower than the face value,** the bond yield is higher than the coupon rate. When the bond price is higher than the face value, the bond yield is lower than the coupon rate.
- So, the bond yield calculation depends on the price of the bond and the coupon rate of the bond. If the bond price falls, the yield rises, and if the bond price rises, the yield falls. Let us understand why this is the case:

RELATIONSHIP OF THE BOND PRICE AND YIELD

- 1. When interest rates fall, it causes a fall in the value of the related investments. However, bonds that have been issued will not be affected in such a way. They will keep paying the same coupon rate as issued from the beginning, which will now be at a higher rate than the prevailing interest rate. This higher coupon rate makes these bonds attractive to investors willing to buy these bonds at a premium.
- 2. Conversely, when interest rates rise, newer bonds will pay investors better interest rates than existing bonds. Here, the older bonds are less attractive and will drop their prices as compensation and sell at a discounted price

The relationship with the Current Price and Face value of the Bond

Market price = Face value	Coupon = Current yield = Yield to maturity
Market price < Face value	Coupon < Current yield < Yield to maturity
Market price > Face value	Coupon > Current yield > Yield to maturity

RELATIONSHIP OF THE BOND PRICE AND YIELD

- There is a 10-year bond with a price of Rs 5000 and a coupon amount of Rs 200. The yield on this bond is calculated as per the formula below
- ● Yield = interest on bond / market price of the bond x 100
  ● So, yield = (200/5000) x 100% = 4%

RELATIONSHIP OF THE BOND PRICE AND YIELD

- Suppose the price of the bond increases from Rs 5000 to Rs 5500 due to strong investor demand. So, the bond now trades at a price of 10% above the issue price. However, the coupon amount remains the same at Rs 200.
- Now the yield changes to (200/5500) x 100% = 3.64%
- So, the bond price has gone up, which causes the yield on the bond to decrease

RELATIONSHIP OF THE BOND PRICE AND YIELD

- Now suppose the price on the same bond considered above decreases.
- ● Initial bond price = Rs 5000
  ● Coupon = Rs 200
  ● Bond price falls to Rs 4300
  ● Coupon remains Rs 200
  ● Now yield is (200/4300) x 100% = 4.65%
- Due to the inverse relationship between bond price and bond yield, the yield has now gone up. You can also invest in short-term mutual funds for similar benefits

Clean vs. Dirty Prices

- Clean price: quoted price
- Dirty price: price actually paid = quoted price plus accrued interest
- Clean Price = Dirty Price − Accrued Interest
- Suppose ABC makes coupon payments on March 1 and Sept. 1 each year, and you buy ABC's bonds on April 1. Let's assume ABC follows a 30/360 day-count convention. That means that even though there are 31 days in March, you'd use 30 for "D," or the number of days since the last coupon payment. You'd use 180 (i.e., 360 divided by two) for "T," or days between coupon payments, even though the coupon is semiannual and there are 365 days in a year.

Pricing styles of Bonds

- Prevailing Market Price of any Bond can be:
  - Clean Price
  - Dirty Price
- Clean Price is the price of a bond excluding the accrued interest
- Dirty Price is the price of a bond including any interest that has accrued since the most recent coupon payment

Bank A enter into a Repo for 14 days with Bank B in 10% Government of India Bonds 2018 @ 5.65% for ₹ 8 crore. Assuming that clean price be ₹ 99.42 and initial Margin be 2% and days of accrued interest be 262 days.

You are required to determine

1. Dirty Price
2. Repayment at maturity.

(Consider 360 days in a year).

$$\text{Dirty Price} = \text{Clean Price} + \text{Interest Accrued}$$

$$= 99.42 + \left[100 \times \frac{10}{100} \times \frac{262}{360}\right] = ₹106.70$$

First Leg (Start Proceed)

$$= \text{Nominal Value} \times \frac{\text{Dirty Price}}{100} \times \frac{100 - \text{Initial Margin}}{100}$$

$$= ₹8,00,00,000 \times \frac{106.70}{100} \times \frac{100 - 2}{100} = ₹8,36,52,800$$

Second Leg (Repayment at Maturity)

$$= \text{Start Proceed} \times \left(1 + \text{Repo Rate} \times \frac{\text{No. of Days}}{360}\right)$$

$$= ₹8,36,52,800 \times \left(1 + 0.0565 \times \frac{14}{360}\right)$$

$$= ₹8,38,36,604$$

Term Structure of Interest Rates

The relationship between yield and maturity The term structure of interest rates, commonly known as the **yield curve**, analysis **effect of maturity on yield**

Yield curve depicts the interest rates of similar quality bonds at different maturities.

The term structure of interest rates reflects expectations of market participants about future changes in interest rates and their assessment of monetary policy conditions.

Term Structure of Interest Rates

Upward sloping—long-term yields are higher than short-term yields. This is considered to be the "normal" slope of the yield curve and signals that the economy is in an expansionary mode.

Downward sloping—short-term yields are higher than long-term yields. Dubbed as an "inverted" yield curve and signifies that the economy is in, or about to enter, a recessive period.

Flat—very little variation between short and long-term yields. This signals that the market is unsure about the future direction of the economy.

A bond's with different maturity period also affects its interest rate (yield), and the relationship among interest rates(yield) on bonds with different terms to maturity is called the term structure of interest rates.

The curve which shows term structure of interest rates is called Yield Curve. In other words, the curve which shows different yields corresponding to different maturity period like 6month, 1 year, 2 year etc. for a similar debt contract called Yield Curve.

Yield curve is commonly used for the treasury bonds or government bonds for various maturities.

Term Structure of Interest Rates

Analysing the effect of maturity on yield

Keeping other features constant

Eliminate effect of coupon payment

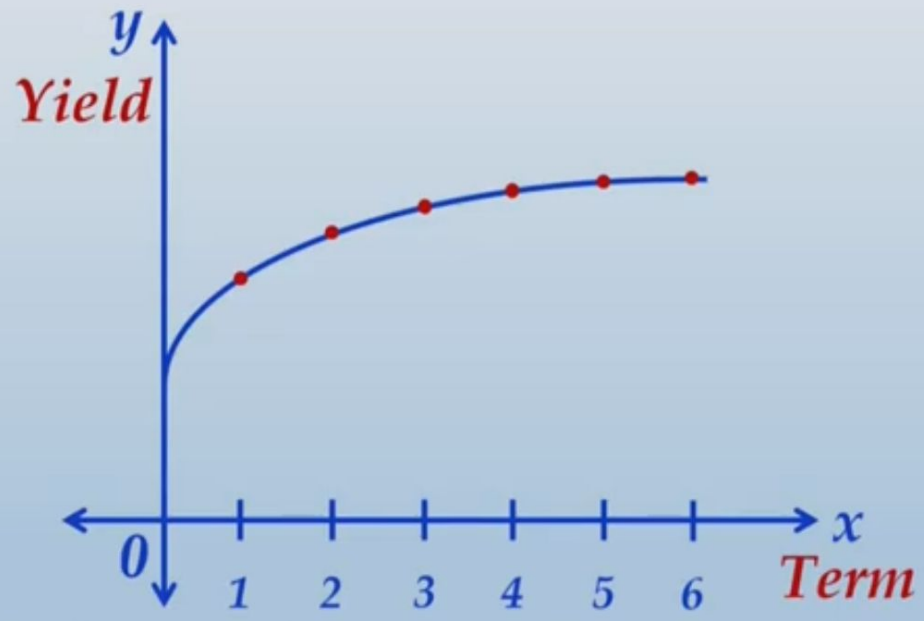
No early redemption feature

Risk , tax liability and redemption possibilities are similar

Term Structure of Interest Rates

- Term structure is the relationship between time to maturity and yields, all else equal
- Term structure is also known as yield curve
- It is important to recognize that we pull out the effect of default risk, different coupons, etc.
- Yield curve – graphical representation of the term structure
  - Normal – upward-sloping; long-term yields are higher than short-term yields
  - Inverted – downward-sloping; long-term yields are lower than short-term yields

Yield Curve



The "Longer" the term for maturity,
The "Higher" is the Yield Rate of the bond.

The "Yield Curve" is a curve, showing several yields or interest rates across different maturities say, 1 year, 2 years, 10 years, etc., for a similar debt contract.

The curve shows the relation between the interest rate and the time to maturity, known as the "term", of the debt.

The Treasury Bonds or Government Bonds for various maturities are commonly plotted on a graph which is called "Yield Curve".

The mathematical descriptions of this relation are often called the term structure of interest rates.

Expectation Theory

The expectations theory states that the shape of the yield curve represents the investor's anticipation regarding the future interest rates.

Investors anticipate fall in interest rates

Anticipation of fall in inflation

Anticipation of a cut in fiscal deficit

Anticipation of recession in economy

The long term interest rates will exceed the current short term rates, If there is an expectation that the market rates will be higher in future.

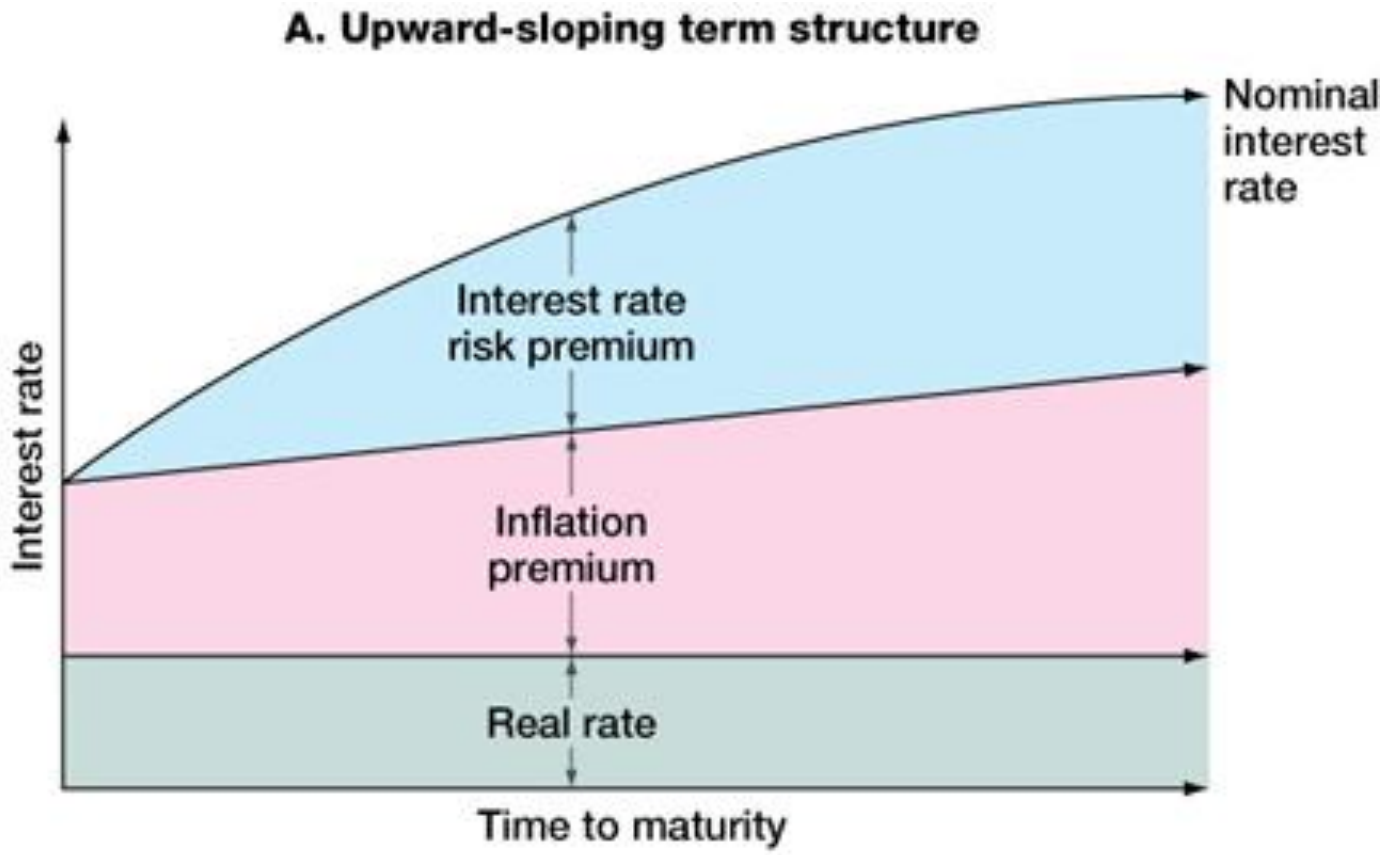
The liquidity preference theory

The liquidity preference theory states that investors prefer short-term bonds as they are more liquid than long-term bonds. If no premium exists for holding a long term instrument investor will prefer to hold a short term bond.

Investors need to be motivated to buy a long term bond by paying a premium.

This theory states that the yield curve will be upward sloping as the long-term bonds will provide extra interest to attract investors to hold the bonds for a long period.

Figure 7.6 – Upward-Sloping Yield Curve



Advantage of term structure

Indicator of the overall health of the economy – An upward sloping and steep curve indicates good economic health while inverted, flat, and humped curves indicate a slowdown.

Knowing how interest rates might change in the future, investors are able to make informed decisions.

Sensitivity of bond

Interest rate sensitivity is a measure of how much the price of a bond will fluctuate due changes in interest rate.

Securities that are more sensitive have greater price fluctuations than those with less sensitivity.

This type of sensitivity must be taken into account when selecting a bond.

Fixed-income securities and interest rates are inversely correlated. Therefore, as interest rates rise, prices of the Bond tend to fall and vice versa.

$$Sensitivity = \frac{\sum n.PV}{1 + i}$$

Question

Face value = ₹ 1,000, Coupon Rate 9% p.a.
Remaining Maturity = 6 years
Redemption price = ₹ 1,100
Desired yield rate = 9% p.a.

Determine the following:

1. Intrinsic value of the bond
2. Sensitivity of the bond

Year (n)	Cash Flow	PV F @ 9%	PV of CF	n.PV
1				
2				
3				
4				
5				
6				
6				

Year (n)	Cash Flow	PV F @ 9%	PV of CF	n.PV
1	90	0.9174		
2	90	0.8417		
3	90	0.7722		
4	90	0.7084		
5	90	0.6499		
6	90	0.5963		
6	1100	0.5963		

Year (n)	Cash Flow	PV F @ 9%	PV of CF	n.PV
1	90	0.9174	82.57	
2	90	0.8417	75.75	
3	90	0.7722	69.50	
4	90	0.7084	63.76	
5	90	0.6499	58.49	
6	90	0.5963	53.66	
6	1100	0.5963	655.93	

Year (n)	Cash Flow	PV F @ 9%	PV of CF	n.PV
1	90	0.9174	82.57	82.57
2	90	0.8417	75.75	151.50
3	90	0.7722	69.50	208.50
4	90	0.7084	63.76	255.04
5	90	0.6499	58.49	292.45
6	90	0.5963	53.66	321.96
6	1100	0.5963	655.93	3,935.58
			1,059.66	5,247.60

Year (n)	Cash Flow	PV F @ 9%	PV of CF	n.PV
1	90	0.9174	82.57	82.57
2	90	0.8417	75.75	151.50
3	90	0.7722	69.50	208.50
4	90	0.7084	63.76	255.04
5	90	0.6499	58.49	292.45
6	90	0.5963	53.66	321.96
6	1100	0.5963	655.93	3,935.58
			1,059.66	5,247.60

Sensitivity

$$= \frac{\Sigma\, n.PV}{1+i}$$

$$= \frac{5,247.60}{1.09}$$

$$= 4,812.48$$

Intrinsic Value = ₹1,059.66

Use the data given in Question 8A and determine the intrinsic value of bond if the rate of return desired by the investor increases from 9% to 10% p.a. Show how does sensitivity, as determined in Question 8A can be applied in this regard.

Using Sensitivity

Intrinsic Value @ 9% = ₹1,059.66

Sensitivity @ 9% = 4,812.48

Increase in desired rate of return from 9% to 10% p.a.

Increase in desired rate of return = 1% p.a.

Approximate decrease in Intrinsic Value = 4,812.48 X 1%

Approximate decrease in Intrinsic Value = 48.12

Intrinsic Value @ 10% using sensitivity = ₹1,059.66 – 48.12

Determining the intrinsic value of bond when rate of return desired by the investor increases from 9% to 10% p.a.

Years	Cash Flows	PV Factors @ 10%	PV of CF
1 - 6	90	4.3553	391.97
6	1,100	0.5645	620.95
		Intrinsic Value	1,012.92

Duration of Bond

- Duration is the time period, for which holder of a bond must wait to receive PV of cash flows.
- It is expressed in terms of period (generally years)
- If duration of a 3 year 6% (HY) bond with face value of Rs.1000 is 2.77 years, it means that total amount to be received over 3 year of Rs.1180 (1000 + 180), will be equal to receiving Rs.1180, at the end of 2.77 years as a one time (bullet) payment.
- Bonds having higher duration, experience greater increase in value when market interest rate declines and greater losses, when market interest rate rise (and vice versa).

Duration of a bond

- Duration measures the time structure of a bond and bond's interest rate risk . The time structure of investment in bond is expressed in two ways. The common way is to state how many years an investor has to wait until the bond matures and principal money is paid back.
- Duration measures how much a bond's price is likely to change given a corresponding change in interest rates.

Duration of a bond

- Duration is a way of gauging how sensitive your bond or bond fund is to changes in interest rates—also commonly known as interest-rate risk. The metric is measured in years.
- Essentially, for every 1% change in interest rates, a bond's price will change in the opposite direction by 1% for every year of duration.
- All other things being equal, a bond with a duration of five years will be more sensitive to changes in interest rates than a bond with a duration of three years, but not as sensitive to changes as a bond with a duration of 10 years.
- average lifetime of a debt securities stream of future payments

Macaulay's Duration

The Macaulay's duration is the weighted average term to maturity of the cash flows from a bond.

The weight of each cash flow is determined by dividing the present value of the cash flow by the price.

$$\text{Macaulay's Duration} = \frac{\sum n.PV}{\text{Intrinsic Value}}$$

Volatility of modified duration

$$\text{Modified Duration (Volatility)} = \frac{\text{Sensitivity}}{\text{Intrinsic Value}}$$

Or,

$$\text{Modified Duration (Volatility)} = \frac{\text{Macaulay's Duration}}{1 + i}$$

Use the data given in Question 8A and determine the following:

1. Macaulay's Duration
2. Modified Duration or Volatility

$$Macaulay's\ Duration = \frac{\Sigma\, n.PV}{Intrinsic\ Value}$$

$$= \frac{5{,}247.60}{1{,}059.66} = 4.952\ years$$

$$Modified\ Duration\ (Volatility) = \frac{Sensitivity}{Intrinsic\ Value} = \frac{4{,}812.48}{1{,}059.66} = 4.54$$

$$Or, \quad = \frac{Macaulay's\ Duration}{1 + i} = \frac{4.952}{1.09} = 4.54$$

Bond Duration

Bond duration measures the sensitivity of the bonds price to changes in market interest rates.

if a bond duration =3 then for every 1% increase in interest rate bond price will decrease by 3%

*Larger the coupon rate lower the duration

*The longer the time to maturity the higher the duration

Macaulay duration

- The Macaulay duration and the modified duration are chiefly used to calculate the duration of bonds.
- The Macaulay duration calculates the weighted average time before a bondholder would receive the bond's cash flows. Conversely,
- the modified duration measures the price sensitivity of a bond when there is a change in the yield to maturity.
- Larger the coupon rate lower the duration

Macaulay duration

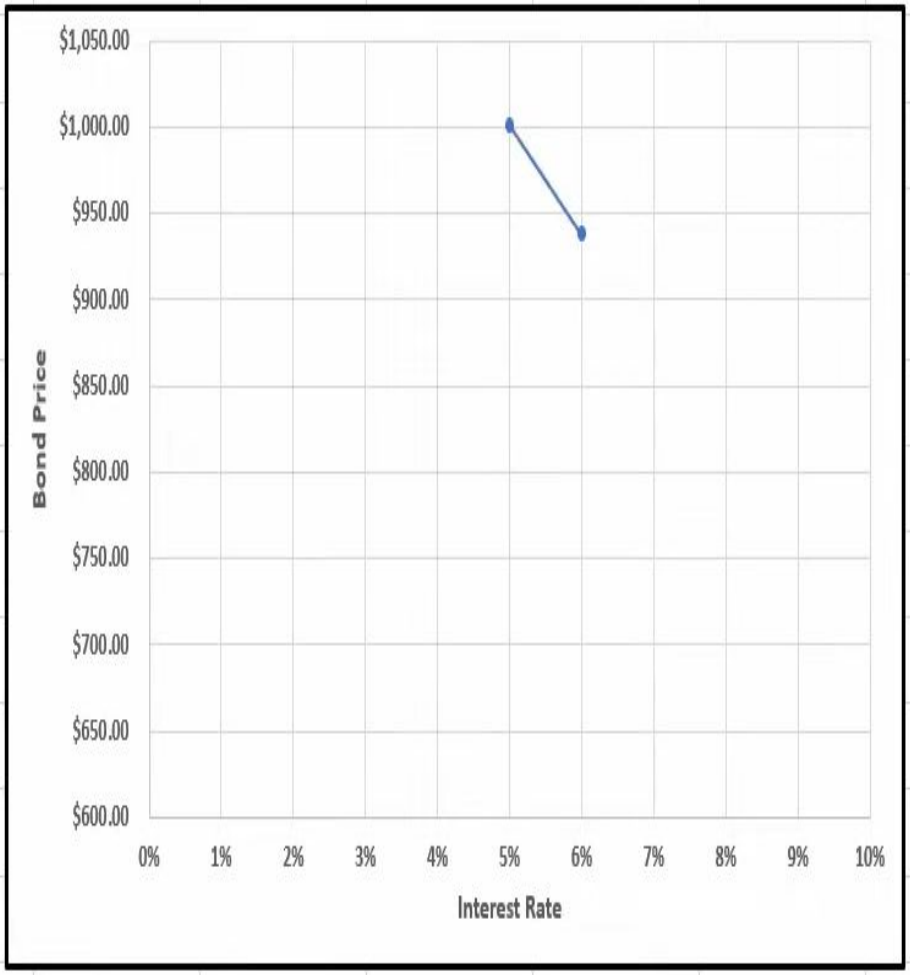
- The Macaulay duration is calculated by multiplying the time period by the periodic coupon payment and dividing the resulting value by 1 plus the periodic yield raised to the time to maturity. Next, the value is calculated for each period and added together. Then, the resulting value is added to the total number of periods multiplied by the par value, divided by 1, plus the periodic yield raised to the total number of periods. Then the value is divided by the current bond price.

Inputs	
Time to Maturity (Years)	8
Yield to Maturity	6%
Coupon Rate	5%

Period	Cash Flow	Present Value	Weight		Period*Weight
1	$ 50	$ 47.17	5.03%		0.05
2	$ 50	$ 44.50	4.74%		0.09
3	$ 50	$ 41.98	4.48%		0.13
4	$ 50	$ 39.60	4.22%		0.17
5	$ 50	$ 37.36	3.98%		0.20
6	$ 50	$ 35.25	3.76%		0.23
7	$ 50	$ 33.25	3.55%		0.25
8	$ 1,050	$ 658.78	70.24%		5.62
		$ 937.90	100.00%	Macaulay Duration	6.74

Period*Weight
0.05
0.09
0.13
0.17
0.20
0.23
0.25
5.62
6.74

Yield to Maturity	Bond Price
2%	
3%	
4%	
5%	$ 1,001.12
6%	$ 937.90
7%	
8%	
9%	



Period*Weight
0.05
0.09
0.13
0.17
0.20
0.23
0.25
5.62
6.74

Yield to Maturity	Bond Price
2%	$1,190.78
3%	$1,127.56
4%	$1,064.34
5%	$1,001.12
6%	$ 937.90
7%	$ 874.68
8%	$ 811.46
9%	$ 748.25

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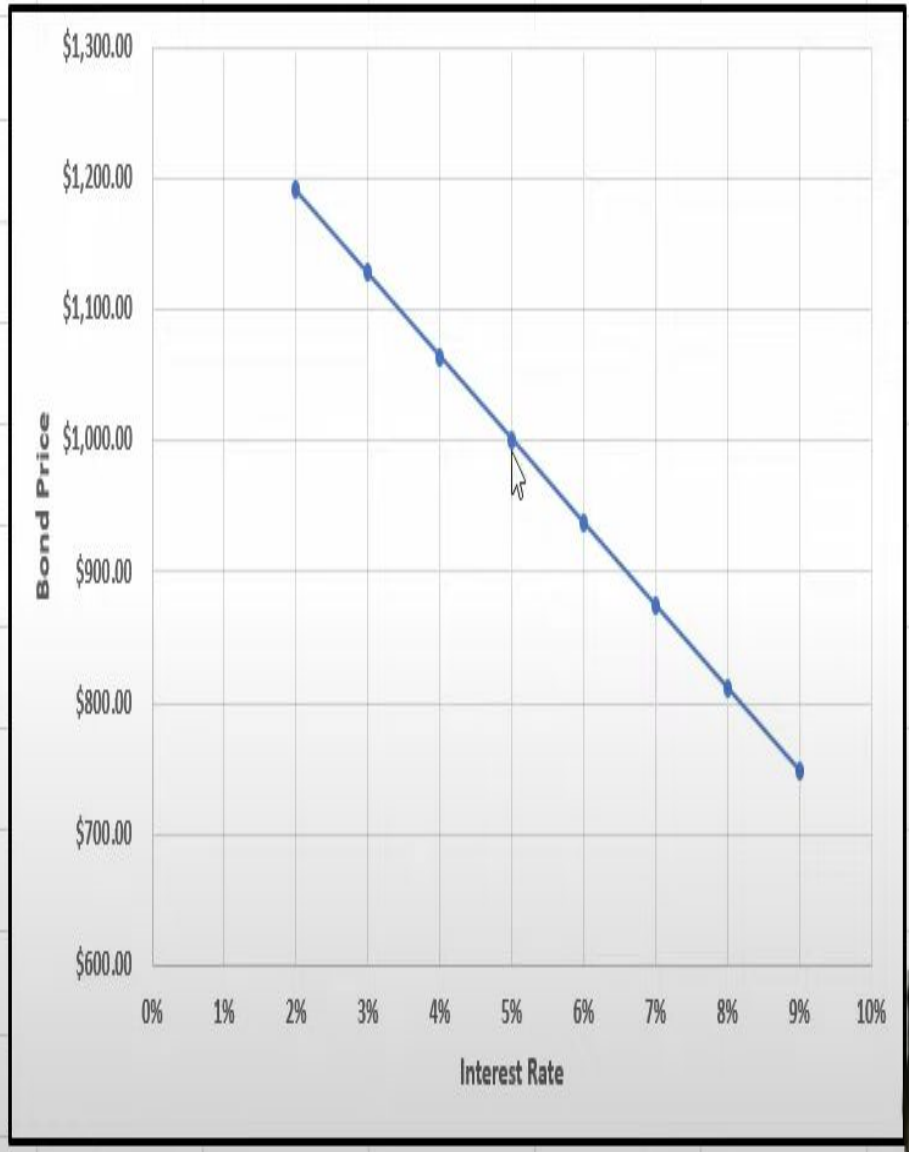


modified duration

- The modified duration is an adjusted version of the Macaulay duration, which accounts for changing yield to maturities. The formula for the modified duration is the value of the Macaulay duration divided by 1, plus the yield to maturity, divided by the number of coupon periods per year. The modified duration determines the changes in a bond's duration and price for each percentage change in the yield to maturity

Rules

1. Duration of Coupon paying bond is always less than its Maturity

2. Lesser the coupon, More the duration

3. Zero Coupon Bond Duration is equal to its maturity

4. Longer Bond's Maturity, longer its duration

5. When interest rate rises, YTM falls, so does Duration

Convexity of the Bond

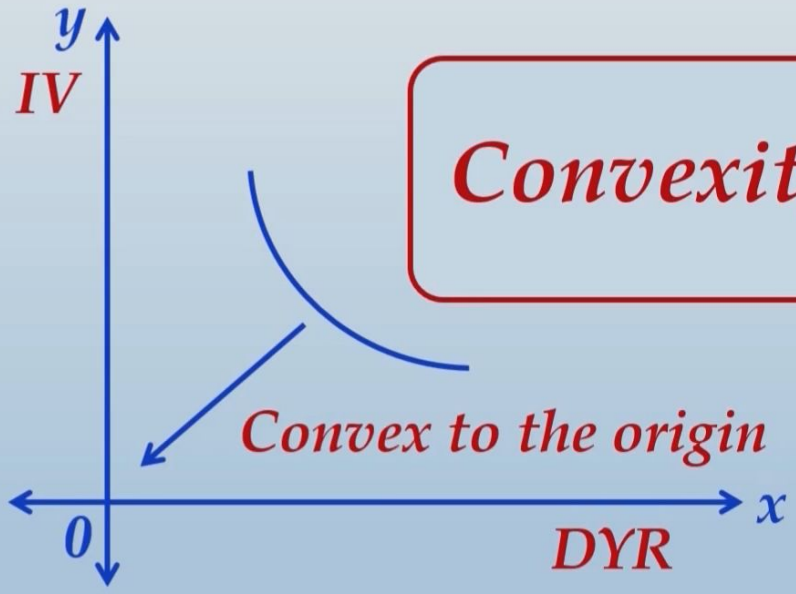
It is known that when interest rate expected by the investor changes, the value of the bond will also change.

The desired yield rate and bond value are inversely related to each other.

Therefore, when desired yield rate increases the bond value declines and vice-versa.

When this inter-relationship is presented graphically, the outcome will not be a straight line but a curve which will be convex to the origin.

Measuring the Bond Convexity

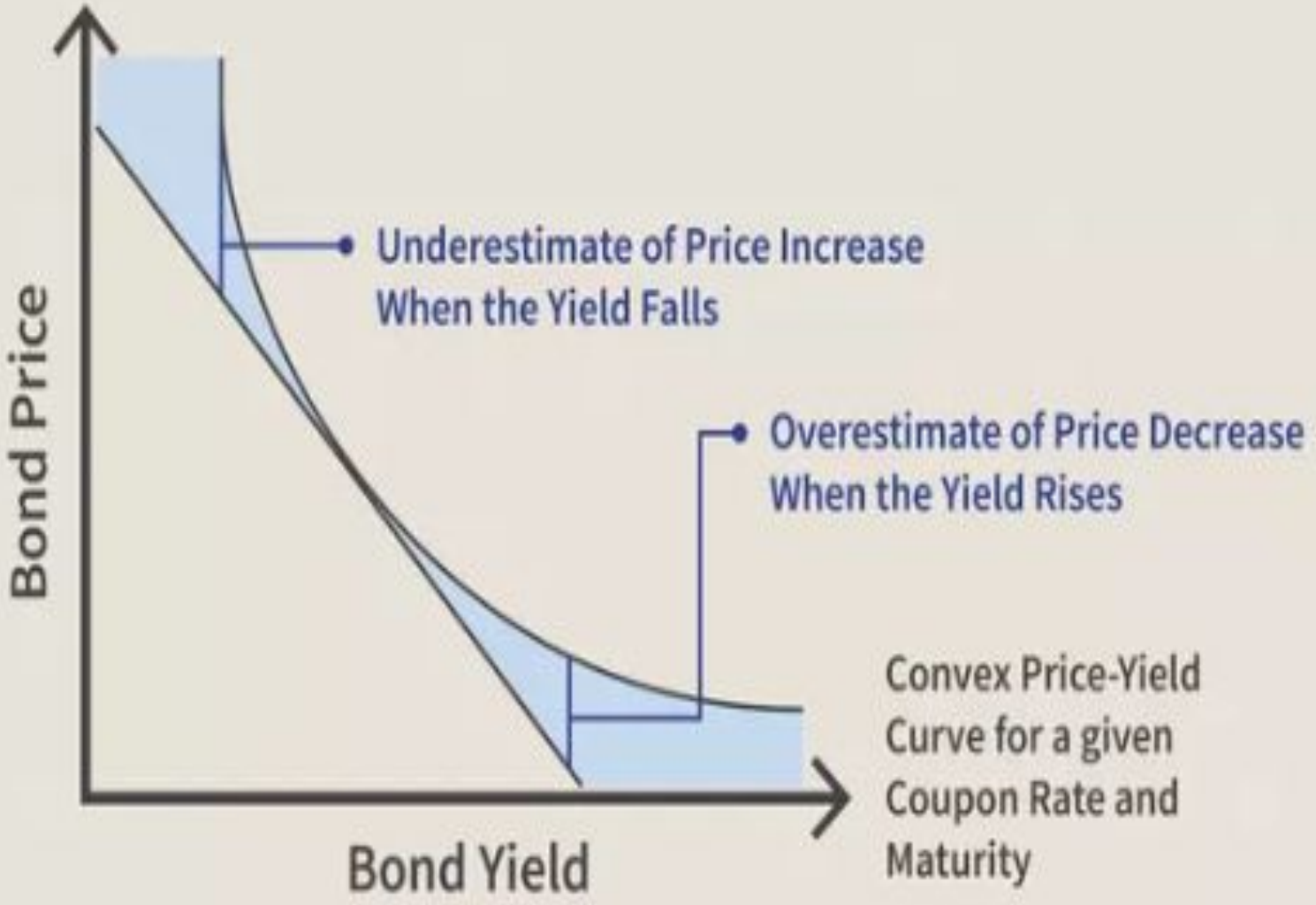


$$\text{Convexity} = \frac{\sum n(n+1)PV}{V(1+i)^2}$$

Measuring convexity is measuring the rate of change in the slope of this curve.

Convexity

- Unfortunately, duration has limitations when used as a measure of interest rate sensitivity. While the statistic calculates a linear relationship between price and yield changes in bonds, in reality, the relationship between the changes in price and yield is convex.
- In the image below, the curved line represents the change in prices, given a change in yields. The straight line, tangent to the curve, represents the estimated change in price, via the duration statistic. The shaded area reveals the difference between the duration estimate and the actual price movement. As indicated, the larger the change in interest rates, the larger the error in estimating the price change of the bond.


Bond Price
Underestimate of Price Increase
When the Yield Falls
Overestimate of Price Decrease
When the Yield Rises
Convex Price-Yield
Curve for a given
Coupon Rate and
Maturity
Bond Yield

- Convexity, a measure of the curvature of the changes in the price of a bond, in relation to changes in interest rates, addresses this error, by measuring the change in duration, as interest rates fluctuate.

- Bond prices do not change linearly, rather the relationship between bond prices and interest rates is convex.
- Convexity is a measure of the curvature of the price change w.r.t. interest rate changes, or the second derivative of the price function w.r.t. relevant interest rates.
- Convexity is also a measure of the spread of future cash flows.
- Duration gives the discounted mean term; convexity is used to calculate the discounted standard deviation of return.

immunization

Process of Bond Immunization

- Here coupon rate risk and the price risk can be made to offset each other.
- Whenever there is an increase in the market interest rate ,the prices of the bonds fall. At the same time the newly issued bonds offer higher interest rate.
- The coupon can be re-invested issued bonds offer higher int rate & losses that occur due to the fall in the price of bond can be offset &the portfolio is said to be immunized.

Investment in bonds cannot be considered completely risk free. Such investment can have the following two types of risk:

1. *Interest Rate Risk*
2. *Reinvestment Risk*

Immunization of bond is a process of creating a bond portfolio through which the interest rate risk and reinvestment risk can be either eliminated or minimized.

Note:

Immunization cannot be achieved through single bond and creating a bond portfolio is necessary for immunization.

Question 24

A company requires to immunize its bond portfolio for a targeted period of 3 years. For this purpose the company has decided to invest ₹ 10,00,000 at present and has identified two bonds A and B.

Particulars	Bond A	Bond B
Face Value	1,000	1,000
Market Price	986.5	1,035
Duration	5 years	2 years

You are required to determine the following:

1. Amount of money to be invested in each of the two bonds.
2. Number of units of each bond to be purchased.

Step 1: *Identify the target period of investment i.e. 3 years.*

Step 2: *Identify bonds for constructing a bond portfolio such that at least one bond should have its duration less than target period and at least one bond should have its duration greater than the target period. "Bond A" & "Bond B"*

Step 3: *Determine Optimum Weights.*

The optimum weights should be determined in such a way that the duration of bond portfolio should be target period of 3 years.

Let w_A and w_B be the weights of Bond A and Bond B respectively.

We know that $w_A + w_B = 1$

$\therefore w_B = 1 - w_A$

Duration of portfolio will be:

$$D_P = D_A.w_A + D_B.w_B$$

$$\therefore 3 = 5 \times w_A + 2(1 - w_A)$$

$$\therefore 3 = 5 \times w_A + 2 - 2w_A$$

$$\therefore 3 - 2 = 5 \times w_A - 2w_A$$

$$\therefore 1 = 3w_A$$

$$\therefore w_A = \frac{1}{3}$$

$\therefore w_B = 1 - w_A$

$= 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore$ *Total amount invested in Bond A:*

$= 10,00,000 \times \frac{1}{3} = ₹3,33,333$

& Total amount invested in Bond B:

$= 10,00,000 \times \frac{2}{3} = ₹6,66,667$

Determining the number of bonds to be purchased:

$$\text{Bond A:} \quad \frac{₹3,33,333}{₹986.50} = 337.89 \text{ units or } 338 \text{ units}$$

$$\text{Bond B:} \quad \frac{₹6,66,667}{₹1,035} = 644.12 \text{ units or } 644 \text{ units}$$

- Abisekh has 50000 Rs to make one time investment . His son has entered higher secondary school and he needs his money back after 2 years for his son's education . As abisekh's outflow is one time outflow, the duration is simply 2 years. Now he has a choice of two types of bonds . FV of bond 1000rs

1) Bond A , Which has a coupon rate of 7 percent , a maturity period of four years , and a current yield of 10 percent . The current Price is 904.90.
2) Bond B , Which has a coupon rate of 6 per cent , a maturity period of one year , and current yield of 10- percent . The current price is 963.64

year	Cash Flow	PVF @ 10 %	Intrinsic value	N.PV	
1	70	0.9091	63.64	63.64	
2	70	0.8264	57.85		
3	70	0.7513	52.59		
4	1070	0.6830	730.81		
			904.89		