

If investors are risk averse they will buy portfolio at point P

An efficient portfolio is one that has the highest expected returns for a given level of risk. The efficient frontier is the frontier formed by the set of efficient portfolios!



Point M is the optimum risk portfolio. The optimum risky portfolio is the market portfolio of all risky assets where each asset is held in proportion to its market value.

It is the best portfolio since it dominates all the other portfolios.

$$R_f = \text{Slope of Capital line}$$

$$\text{Slope of CML} = \frac{E(C_m) - R_f}{\sigma_m}$$

Slope of CML is also called reward to variability portfolio.

$$\text{eg} \quad \text{Portfolio return} = 14$$

$$SD = 3.6\%$$

$$R_f = 5\%$$

$$\therefore \text{Slope of CML} = \left[\frac{11.0 - 5.0}{3.6} \right] = 1.67$$

The slope of CML describes the best price of a given level of risk in equilibrium. Therefore the expected return on a portfolio on CML is defined by the following equation

$$E(R_p) = R_f + \left[\frac{E(R_m - R_f)}{\sigma_m} \right] \sigma_p$$

\downarrow
Expected Return

of portfolio

along the

Capital market line

Q. Risk free rate of interest = 8%.
 Market portfolio ~~(σ)~~ = 18%
 $\sigma_D = 6\%$

If an investor desires to earn an expected rate of return of 15%, in what combination should he hold the market portfolio and risk free security.

~~Solution~~
 If the investor invest w in the market portfolio, his investment in risk free security will be $1-w$ and his expected rate of return from a portfolio consisting of the risk-free security and market portfolio will be:

$$E(R_p) = 18 \times w + (1-w) \times 8$$

$$15 = 18 \times w + (1-w) \times 8$$

$$10w = 15 - 8$$

$$w = (15 - 8) / 10 = 0.7 \times 8$$

The portfolio risk will be

$$\sigma_p = w \sigma_m = 0.7(6) = 4.2\%$$

By combining the risk free security (lending) with the market portfolio, the investor can reduce his risk (from 6% to 4.2%) but his return will be less (15% instead 18%).

OR

$$E(R_p) = 8\% + \left[\frac{(18\% - 8\%)}{6\%} \right] 4.2\% = 8\% + 1.667 \\ = 8\% + 7\% = 15\%$$

The relevant measure of assets risk is its covariance with the market portfolio of risky assets.

The SECURITY MARKET LINE

Exemplifies the relationship between an asset's risk and its required rate of return.

Assumption of CAPM \rightarrow Relationship between risk and expected rate of return on risky security

1) Market efficiency \rightarrow Share price reflect all available info.

2) Risk aversion and mean-variance optimization
investor are risk averse and prefer highest return for a given level of risk.

3) Heterogeneous Expectations about risk and return of security.

4) Investor investment decision is based on

single time period.

g) All investors can lend and borrow at risk-free rate of interest.

Since unsystematic risk can be mostly eliminated without any cost there is no price paid for it. Therefore it will have no influence on the return of individual securities. Market will pay premium only for systematic risk since it is not diversifiable.

Capital Asset Pricing Model provides a framework to determine the required rate of return on an asset and indicated the relationship between return and risk of the asset.

Under CAPM Risky individual security = volatility of securities return vis-à-vis the return of the market portfolio.

This is Systematic risk = covariance of individual security to risk security.

$$E(R_j) = R_f + \frac{(E(R_m) - R_f)}{\sigma_m^2} \text{Cov}_{pm} \quad (\text{Cov}_{pm})$$

Riskfree rate σ_m^2 Risk premium

$$\text{Securities } \beta = \frac{\text{Cov}_{pm}}{\sigma_m^2}$$

Q Following is the data given for
Divine home company
How much is the total Return
as per APY model.

Factor	Beta	Expected value.	Actual value
GNP	1.95	6.00	6.50
Inflation	0.85	5.00	5.75
Interest Rate	1.20	7.00	8.0
Stock mkt index	2.50	9.50	11.50
Industrial Profit	2.20	9.00	10.00

$$E(R) = R_f + \beta_1 (R_{F_1} - R_f) + \beta_2 (R_{F_2} - R_f) + \\ \dots + \beta_n (R_{F_n} - R_f)$$

$$E(R) = 9 + 1.95(6.5 - 6) + 0.85(5.75 - 5) + \\ 1.20(8 - 7) + 2.5(11.5 - 9.5) + \\ 2.20(10 - 9) \\ = 9 + 10 = 19\%$$

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A investor holds two equity shares X and Y in equal proportion with the following risk and return characteristics

$$E(R_X) = 24\%$$

$$E(R_Y) = 19\%$$

$$\sigma_X = 28\% \quad \sigma_Y = 23\%$$

$$\text{Solution} = E(R_p) = 24(0.5) + 19(0.5) \\ = 12 + 9.5 = 21.5\%$$

and portfolio risk

$$\sigma_p = \sqrt{\sigma_p^2}$$

$$\sigma_p^2 = (28)^2(0.5)^2 + (23)^2(0.5)^2 + 2(0.5)(28)(23) \\ (0.6) \\ = 196 + 132.25 + 193 \cdot 2 = 521.45$$

$$\sigma_p = \sqrt{521.45} = 22.84\%$$

If the investor desires portfolio SD = 15%,
 the correlation coefficient will be as

$$(15)^2 = (28)^2(0.5)^2 + (23)^2(0.5)^2 \\ + 2(0.5)(0.5)(28)(23) \text{ Corr}_{XY} \\ 225 = 196 + 132.25 + 322 \text{ Corr}_{XY}$$

$$\text{Corr}_{XY} = \frac{-103.25}{322} = -0.321$$

A portfolio consists of three securities P, Q and R with the following parameters.

	P	Q	R	Cor
Expected Return (%)	25	22	20	
S.D (%)	30	26	24	

Correlation

PQ	-0.50
PR	+0.40
QR	+0.60

Solution

The portfolio return

$$E(R_p) = (25)(V_3) + 22(V_3) + 20(V_3) = 22.33\%$$

$$\begin{aligned}\sigma_p^2 &= (30)^2(V_3)^2 + (26)^2(V_3)^2 + (24)^2(V_3)^2 \\ &\quad + 2(V_3)(V_3)(-0.5)(30)(26) \\ &\quad + 2(V_3)(V_3)(0.4)(26)(24) \\ &\quad + 2(V_3)(V_3)(0.6)(30)(24)\end{aligned}$$

$$\begin{aligned}&= 100 + 75.11 + 64 - 86.67 + 55.47 \\ &\quad + 96 = 303.91\end{aligned}$$

$$\sigma_p = \sqrt{303.91} = 17.43\%$$

Q From the following data Compute beta
of security J

$$\sigma_j = 12\%$$

$$\sigma_m = 9\%$$

$$\text{Cor}_{jm} = +0.75$$

$$\beta = \frac{\sigma_j \sigma_m \text{Cor}_{jm}}{\sigma_m^2} = \frac{12 \times 9 \times 0.75}{9^2}$$

$$= \frac{77.76}{81} = 0.96$$

Q Calculate the expected rate of return
for security P from the following
information.

$$R_f = 10\% \quad R_m = 18\% \quad \beta_i = 1.35$$

Solution

$$E(R_i) = R_f + (R_m - R_f)\beta$$

$$= 10\% + (18\% - 10\%) 1.35$$

$$= 10\% + 10.8\% = 20.8\%$$

Expected dividend receivable by year end is Rs 45. Expected market price of Equity share by year end is Rs 300. The expected rate of return by the shareholder is 25%. P.a. Determine the current market price of equity share.

$$P_0 = \frac{D_1 + P_1}{1 + k_e}$$

D_1 = Expected dividend receivable by year end = 45

P_1 = Expected Price of Equity Share by year end = 300

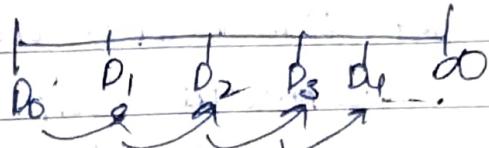
k_e = Expected Rate of Return by the shareholder = 0.25

P_0 = Current Market Price of equity share.

$$P_0 = \frac{45 + 300}{1 + 0.25} = 276$$

Gordon Model = $P_0 = \frac{D_1}{k_e - g}$

Growing perpetuity



$D_0 = 10$

$g = 10\%$

$D_1 = 10 + 10\% = 11$

$D_5 = D_0 \cdot (1+g)^5$

Future value = Present value $\cdot (1+i)^{-n}$

$D_1 = D_0 \cdot (1+g)^n$

$P_0 = \frac{D_1}{k_e - g} \text{ or } \frac{D_0 \cdot (1+g)^n}{k_e - g}$

Growing perpetuity $\Rightarrow \frac{A}{1-g} \Leftarrow \frac{D_1}{k_e - g}$

Growth rate = $b \times r \rightarrow \text{ROE}$
 \downarrow
Retention ratio

$1 - \text{Payout ratio} = \text{Retention ratio}$

$k_e = \frac{D_1}{P_0} + g$ \leftarrow Capital a.

Overall
Required
rate of return
or
cost of equity

Q Suppose you expect Med Pharma to pay dividends of Re 0.44 per share and trade for Rs 33 per share at the end of the year. If investments with equivalent risk to Med Pharma's stock have an expected return 8.5%. what is the most you would pay today for Med Pharma's stock? What dividend yield and capital gain rate would expect at this price.

$$P_0 = \frac{Div_1 + P_1}{1 + r_E} = \frac{0.44 + 33.0}{1.085} = 30.82$$

Dividend Yield $Div_1/P_0 = 0.44/30.82 = 1.43\%$

The expected gain is $33.00 - 30.82 = 2.18$

Capital gain rate of $2.18/30.82 = 7.07\%$.

Med Pharma's Expected total return is

$$1.43\% + 7.07\% = 8.5\%$$

which is equal to equity cost of capital.

Q Consolidated Nijmal Inc is a regulated utility company that services the New Delhi area. Suppose Consolidated Nijmal Inc. plans to pay 2.86 Rs per share in dividends in the coming year. If its equity cost of capital is 7.5% and dividends are expected to grow by 1.5% per year in the future. Estimate the value of Consolidated Nijmal Inc's Stock.

If dividends are expected to grow perpetually @ 1.5% per year

$$\therefore P_0 = \frac{\text{Div}_1}{r_e - g} = \frac{\text{Rs } 2.86}{0.075 - 0.015}$$

or
 r_e

$$\gamma_E = \frac{Div_1}{P_0} + g = \frac{P_0 - Div}{\gamma_E - g}$$

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Q Fancy sporting goods expects to have earnings per share = Rs 6 in the coming year. The firm decides to payout all its earnings as dividend. With these expectation of no growth, Fancy's current share price = 60Rs.

Suppose Fancy could set its dividend payout rate to 75% for the foreseeable future and use the retained earning to open new stores. The return on its investment in these stores is expected to be 12%. Assuming its equity cost of capital is unchanged, what effect would this new policy have on Fancy's stock price.

currency $EPS = \frac{Div}{P_0}$

share price = 60

dividend payout = $6/60 = 10\%$

if growth is not expected

$$\gamma_E = \frac{Div}{P_0} + g = 10\% + 0\% = 10\%$$

To justify Fancy's stock price under its current policy, the expected return of other stocks in the market with the equivalent risk must be 10%.

New Policy \rightarrow Dividend payout = 75%

then Div, = EPS \times 75% = $6 \times 75\% = 4.50$

For growth Now firm will retain
15% of its earning.

growth = Retention Rate \times Return on

New investment
= 15% \times 12% = 3%

$$P_0 = \frac{4.50}{0.10 - 0.03} = 64.29$$

Thus fancy's share price should rise
from Rs 60 to 64.29.

Tomatina Fry Limited has just invented a potato chip that looks and taste like a French fry. Given the phenomenal market response of this product Tomatina Fry is repositioning all of its earnings to expand its operations. Earnings were \$2 per share this past year and are expected to grow at a rate of 20% per year until the end of year 4. At that point other companies are likely to bring out competitive products. Analysts project that at the end of year 4, Tomatina Fry will cut investment and begin paying 60% of its earnings as dividends and its growth will slow to a long-run rate of 4%. If Tomatina Fry's equity cost of capital is 8%, what is the value of share today.

	Year 0	1	2	3	4	5	6
Earnings							
1) EPS Growth Rate	-	20%	20%	20%	20%	4%	4%
2) EPS	2	2.40	2.88	3.46	4.15	4.31	4.49
Dividends							
3) Dividend Payout	-	60%	60%	60%	60%	60%	60%
	-	-	-	-	-	2.49	2.59

From the year 4 onward, dividends will grow $\rightarrow 4\%$ per year.

$$P_0 = \frac{\text{Div}_1}{r_E - g} = \frac{2.49}{0.08 - 0.04} = 62.25$$

$$P_0 = \frac{\text{Div}_1}{1+r_E} + \frac{\text{Div}_2}{(1+r_E)^2} + \frac{\text{Div}_3}{(1+r_E)^3} + \frac{P_2}{(1+r_E)^3}$$

The return of ABC company at present is 21%. This is assumed to continue for the next five years and after that it is assumed to have growth rate of 10 percent indefinitely. The dividend paid for the year 2011-12 is 3.2. The required rate of return is 20% and the present price is 57. What is the estimated price according to two stage model.

$$P_0 = ?$$

$$g = \text{for 5 years} = 21\% \text{ or } 0.21$$

$$D_0 = \text{Rs } 3.2$$

$$k_e = 20\% = 0.20$$

$$\approx \frac{3.2(1+0.21)}{(1+0.2)^1} + \frac{(3.2(1.21))^2}{(1+0.2)^2} + \dots + \frac{3.2(1.21)^5}{(1+0.2)^5}$$

$$= \frac{3.872}{1.2} + \frac{4.6857}{1.44} + \frac{5.669}{1.728} + \frac{6.8595}{2.0736} + \frac{8.3}{2.4883}$$

$$= 3.2267 + 3.2535 + 3.2807 + 3.3080$$

$$+ 3.3356$$

$$= 16.4045$$

Step 2

dividend in 5th year is 0.3
 Div growth rate for indefinite period
 $\equiv 10\%$

∴ we need to find terminal
 value of ~~indefinite~~ sixth year

$$P_6 = \frac{D_5(1+g)}{K_e - g}$$

$$= \frac{0.3(1+0.10)}{0.20 - 0.10}$$

$$= \frac{0.3(1+0.10)}{0.20 - 0.10}$$

$$= \frac{0.3(1+0.10)}{0.20 - 0.10}$$

Now present value of P_6

$$= \frac{0.3(1+0.10)}{0.20 - 0.10} \times \frac{1}{(1+0.2)^5}$$

$$= \frac{91.8 \times 1}{2.4883} \leq \text{Rs } 36.69$$

$$P_0 = 16.4045 + 36.69$$

$$P_0 = 53.09$$

O肚 has bought the Everest Company stock that has paid Rs 3.00 as dividend per share during the last financial year. He anticipates two situations either 5% decline in the dividend or 5% growth in the dividend in the next year. His anticipated return is 20%.

Fix the price you both the situation.

(a) 5% growth

$$P_0 = \frac{D_1}{k-g}$$

$$\frac{3(1.05)}{0.20 - 0.05} = \text{Rs } 21$$

(b) 5% decline

$$P_0 = \frac{D_1}{k-g}$$

$$= \frac{3(1 - 0.05)}{0.20 - (-0.05)} = \text{Rs } 11.4$$

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A Vigilant Company stock is currently selling at Rs 25 per share. The stock is expected to pay Rs 1 as dividend per share at the end of the next year. It is reliably estimated that the stock will be available for ~~as~~ as Rs 29 at the end of one year.

solution

$$P_0 = \frac{D_1}{1+r} + \frac{P_1}{1+r}$$

$$\frac{1}{1.2} + \frac{29}{1.2} = 0.833 + 24.16$$

$$P_0 = 25$$

Since the estimated price and the actual price are equal, the investor could buy it.

b) $P_0 = \frac{D_1}{1+r} + \frac{P_1}{1+r}$

$$25 = \frac{1}{1.15} + \frac{P_1}{1+r}$$

$$25 = 0.87 + \frac{P_1}{1.15}$$

$$25 = 0.87 + \frac{P_1}{1.15}$$

$$25 - 0.87 = \frac{P_1}{1.15}$$

$$P_1 = 24.13 \times 1.15$$

$$P_1 = 27.75$$

The value of the stock at the end of the period should be Rs. 27.75 if the required rate of return is 15%.

Q. Fashions Ltd operates a large ready made garment system in the textile industry. Assume that its common stock can be purchased in the beginning of 2009 at Rs 40. The dividend per share would be Rs 2 for the next 3 years. It is estimated that at the end of 2012, the stock will be sold for Rs 55. What is the rate of return for fashions stock?

$$P_0 = \frac{D_1}{(1+\alpha)^1} + \frac{D_2}{(1+\alpha)^2} + \frac{D_3}{(1+\alpha)^3} + \frac{P_1}{(1+\alpha)^3}$$

$$40 = \frac{2}{(1+\alpha)} + \frac{2}{(1+\alpha)^2} + \frac{2}{(1+\alpha)^3} + \frac{55}{(1+\alpha)^3}$$

This requires trial and errors procedure.

Let us try 15% return

$$= \frac{2}{1-15} + \frac{2}{1-3225} + \frac{2}{1-5209} + \frac{55}{1-5209}$$

$$= 1.739 + 1.512 + 1.3150 + 86.162$$

$$= 40.728$$

\therefore the rate of return is 15%.