

## 第四章测试答案

一、选择题（本题 5 个小题，每小题 3 分，共 15 分）

1. D 2. D 3. D 4. D 5. B

二、填空题（本题 5 个小题，每小题 3 分，共 15 分）

1.  $f(x)dx$  2.  $\frac{2^x}{\ln 2} - \frac{3^x}{\ln 3} + C$  3.  $x \sin x + \cos x + C$  4.  $\frac{4}{7}x^{\frac{7}{4}} + C$  5.  $\frac{1}{2}e^{2x} + \frac{1}{2}$

三、解答题（本题 7 个小题，每小题 10 分，共 70 分）

1. 解： $\int \sin^2 x \cos^2 x \cdot \cos x dx = \int \sin^2 x (1 - \sin^2 x) d(\sin x) = \frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$ .

2. 解： $\int x \ln(x-2) dx = \int \ln(x-2) d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \ln(x-2) - \frac{1}{2} \int \frac{x^2 - 4 + 4}{x-2} dx$   
 $= \frac{1}{2}x^2 \ln(x-2) - \frac{1}{2} \int \left(x+2 + \frac{4}{x-2}\right) dx = \frac{1}{2}x^2 \ln(x-2) - \frac{1}{4}x^2 - x - 2 \ln|x-2| + C.$

3. 解： $\int \cos^2 x dx = \int \frac{1 + \cos 2x}{2} dx = \frac{1}{2} \left( \int dx + \int \cos 2x dx \right)$   
 $= \frac{1}{2}x + \frac{1}{4} \int \cos 2x d(2x) = \frac{x}{2} + \frac{\sin 2x \theta^0}{5\theta^0} C.$

4. 解： $\int \frac{3x^2 + x + \sin \frac{1}{x}}{x^2} dx = \int \left(3 + \frac{1}{x} + \frac{\sin \frac{1}{x}}{x^2}\right) dx = 3x + \ln|x| - \int \sin \frac{1}{x} d\frac{1}{x}$   
 $= 3x + \ln|x| + \cos \frac{1}{x} + C$

5. 解：令  $\sqrt{3x-1} = t$ ，则  $x = \frac{t^2+1}{3}$ ,  $dx = \frac{2t}{3} dt$ ,

$$\begin{aligned} \int \cos \sqrt{3x-1} dx &= \frac{2}{3} \int \cos t \cdot t dt = \frac{2}{3} \int t d \sin t \\ &= \frac{2}{3} t \sin t - \frac{2}{3} \int \sin t dt = \frac{2}{3} t \sin t + \frac{2}{3} \cos t + C = \frac{2}{3} \sqrt{3x-1} \sin \sqrt{3x-1} + \frac{2}{3} \cos \sqrt{3x-1} + C \end{aligned}$$

6. 解：令  $x = 2 \sin t$ ,  $dx = 2 \cos t dt$ ,

原式变为

$$\int \frac{2 \cdot 2 \sin t + 1}{2 \cos t} \cdot 2 \cos t dt = \int (4 \sin t + 1) dt = -4 \cos t + t + C = -2\sqrt{4-x^2} + \arcsin \frac{x}{2} + C.$$

或者  $\int \frac{2x+1}{\sqrt{4-x^2}} dx = \int \left(\frac{2x}{\sqrt{4-x^2}} + \frac{1}{\sqrt{4-x^2}}\right) dx = \int \frac{2x}{\sqrt{4-x^2}} dx + \int \frac{1}{\sqrt{4-x^2}} dx$

$$= -\int \frac{1}{\sqrt{4-x^2}} d(4-x^2) + \int \frac{1}{\sqrt{1-(\frac{x}{2})^2}} d\frac{x}{2} = -2\sqrt{4-x^2} + \arcsin \frac{x}{2} + C$$

$$7. \text{ 解: } \int \frac{x+1}{x^2-2x+5} dx = \frac{1}{2} \int \frac{2x+2}{x^2-2x+5} dx = \frac{1}{2} \int \frac{2x-2+4}{x^2-2x+5} dx$$

$$= \frac{1}{2} \int \frac{2x-2}{x^2-2x+5} dx + 2 \int \frac{1}{x^2-2x+5} dx = \frac{1}{2} \int \frac{1}{x^2-2x+5} d(x^2-2x+5) + 2 \int \frac{dx}{4+(x-1)^2}$$

$$= \frac{1}{2} \ln|x^2-2x+5| + \int \frac{1}{1+(\frac{x-1}{2})^2} d\frac{x-1}{2} = \frac{1}{2} \ln|x^2-2x+5| + \arctan \frac{x-1}{2} + C$$