

加测 16 极限提高题答案

$$(1) \text{ 解: } \lim_{n \rightarrow \infty} (\sqrt{n+2\sqrt{n}} - \sqrt{n-\sqrt{n}}) = \lim_{n \rightarrow \infty} \frac{3\sqrt{n}}{\sqrt{n+2\sqrt{n}} + \sqrt{n-\sqrt{n}}}$$

$$= \lim_{n \rightarrow \infty} \frac{3}{\sqrt{1+\frac{2}{\sqrt{n}}} + \sqrt{1-\frac{2}{\sqrt{n}}}} = \frac{\lim_{n \rightarrow \infty} 3}{\lim_{n \rightarrow \infty} \sqrt{1+\frac{2}{\sqrt{n}}} + \lim_{n \rightarrow \infty} \sqrt{1-\frac{2}{\sqrt{n}}}} = \frac{3}{2}.$$

(2) 解:

$$\lim_{x \rightarrow \infty} \frac{2x^2 + x \cos x}{x^3 - x \sin x + 1} = x^3 \lim_{x \rightarrow \infty} \frac{\frac{2}{x} + \frac{1}{x^2} \cos x}{1 - \frac{1}{x^2} \sin x + \frac{1}{x^3}} = \frac{\lim_{x \rightarrow \infty} \frac{2}{x} + \lim_{x \rightarrow \infty} \frac{1}{x^2} \cos x}{\lim_{x \rightarrow \infty} 1 - \lim_{x \rightarrow \infty} \frac{1}{x^2} \sin x + \lim_{x \rightarrow \infty} \frac{1}{x^3}} = \frac{0}{1} = 0.$$

$$(3) \lim_{x \rightarrow 0} \frac{\sin 2x}{\ln(a+x)} (\cos x - b) = 4,$$

解: 因为 $\lim_{x \rightarrow 0} \sin 2x \cdot (\cos x - b) = 0$, 所以 $\lim_{x \rightarrow 0} \ln(a+x) = 0$, 所以 $a = 1$.

$$\text{所以 } \lim_{x \rightarrow 0} \frac{\sin 2x}{\ln(a+x)} (\cos x - b) = \lim_{x \rightarrow 0} \frac{\sin 2x}{\ln(1+x)} (\cos x - b) = \lim_{x \rightarrow 0} \frac{2x}{x} (\cos x - b) = 2(1-b) = 4$$

所以 $b = -1$.

$$(4) \lim_{x \rightarrow \infty} (4x + \sqrt{ax^2 - bx - 1}) = 1, \quad a > 0, \quad \text{求 } a, b$$

$$\text{解: } \lim_{x \rightarrow \infty} (4x + \sqrt{ax^2 - bx - 1}) = \lim_{x \rightarrow \infty} \frac{(16-a)x^2 + bx + 1}{4x - \sqrt{16x^2 - bx - 1}} = 1, \quad \text{所以 } a = 16.$$

$$\text{所以 } \lim_{x \rightarrow \infty} (4x + \sqrt{ax^2 - bx - 1}) = \lim_{x \rightarrow \infty} \frac{bx + 1}{4x - \sqrt{16x^2 - bx - 1}}$$

$$= \lim_{x \rightarrow \infty} \frac{b + \frac{1}{x}}{4 + \sqrt{16 - \frac{b}{x} - \frac{1}{x^2}}} = \frac{b}{8}, \quad \text{解得 } b = 8.$$

(5) $x \rightarrow 0$ 时, $e^{x \cos x} - e^x$ 与 x^n 是同阶无穷小, 求 n

$$\text{解: 由 } e^{x \cos x} - e^x = e^x (e^{x \cos x - x} - 1) \sim x \cos x - x = x(\cos x - 1) \sim -\frac{1}{2}x^3,$$

即 $e^{x \cos x} - e^x$ 与 x^3 是同阶无穷小, 则 $n = 3$

$$(6) \lim_{x \rightarrow 0} \frac{(\sin x - \tan x)(e - e^{\cos x})}{(2x+3)\sin^5 x} = \lim_{x \rightarrow 0} \frac{(\cos x - 1)\tan x(e - e^{\cos x})}{(2x+3)x^5} = \lim_{x \rightarrow 0} \frac{-\frac{x^2}{2} \cdot x(e - e^{\cos x})}{(2x+3)x^5}$$

$$= -\frac{1}{6} \lim_{x \rightarrow 0} \frac{e - e^{\cos x}}{x^2} = -\frac{1}{6} \lim_{x \rightarrow 0} \frac{e^{\cos x} \cdot \sin x}{2x} = -\frac{1}{6} \lim_{x \rightarrow 0} \frac{e^{\cos x}}{2} = -\frac{e}{12}$$

$$(7) \quad \lim_{x \rightarrow 0} \frac{\sqrt{1+\tan x} - \sqrt{1+\sin x}}{x \ln(1+x^2)} = \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3 (\sqrt{1+\tan x} + \sqrt{1+\sin x})} = \frac{1}{2} \lim_{x \rightarrow 0} \frac{\tan x (1-\cos x)}{x^3}$$

$$= \frac{1}{2} \lim_{x \rightarrow 0} \frac{x \cdot \frac{x^2}{2}}{x^3} = \frac{1}{4}$$

$$(8) \quad \lim_{x \rightarrow 0} \frac{4 \sin x + 3x^2 \cos \frac{1}{x}}{(1+\cos x) \ln(1+3x)} = \frac{1}{2} \lim_{x \rightarrow 0} \frac{4 \sin x + 3x^2 \cos \frac{1}{x}}{3x} = \frac{1}{2} \lim_{x \rightarrow 0} \left(\frac{4 \sin x}{3x} + x \cos \frac{1}{x} \right)$$

$$= \frac{1}{2} \lim_{x \rightarrow 0} \left(\frac{4}{3} + 0 \right) = \frac{2}{3}.$$