

自己练习

1. 不定积分 $\int \sec^4 x dx = \underline{\hspace{2cm}}$.

2. 不定积分 $\int \frac{1}{x(1+3\ln x)} dx = \underline{\hspace{2cm}}$.

3. 不定积分 $\int x \cos 2x dx = \underline{\hspace{2cm}}$.

4. 不定积分 $\int x \csc^2 x dx = \underline{\hspace{2cm}}$.

5. 计算不定积分 $\int \frac{1}{1+\sqrt{x}} dx$.

6. 计算不定积分 $\int \sqrt{9-x^2} dx$.

参考答案

1. $\frac{1}{3} \tan^3 x + \tan x + C$

解析: $\int \sec^4 x dx = \int \sec^2 x \cdot \sec^2 x dx = \int \sec^2 x d(\tan x) = \int (\tan^2 x + 1) d(\tan x)$

$$= \frac{1}{3} \tan^3 x + \tan x + C$$

2. $\frac{1}{3} \ln |1+3\ln x| + C$

解析: $\int \frac{1}{x(1+3\ln x)} dx = \int \frac{1}{1+3\ln x} d(\ln x) = \frac{1}{3} \int \frac{1}{1+3\ln x} d(3\ln x)$

$$= \frac{1}{3} \int \frac{1}{1+3\ln x} d(1+3\ln x) = \frac{1}{3} \ln |1+3\ln x| + C$$

3. $\frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + C$

解析: $\int x \cos 2x dx = \frac{1}{2} \int x d(\sin 2x) = \frac{1}{2} \left(x \sin 2x - \int \sin 2x dx \right)$

$$= \frac{1}{2} x \sin 2x - \frac{1}{2} \int \sin 2x dx = \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + C$$

4. $-x \cot x + \ln |\sin x| + C$

解析: $\int x \csc^2 x dx = -\int x d(\cot x) = -\left(x \cot x - \int \cot x dx \right) = -x \cot x + \int \cot x dx$
 $= -x \cot x + \ln |\sin x| + C.$

5. 解: 令 $t = \sqrt{x}$, 则 $x = t^2, dx = 2t dt$.

$$\int \frac{1}{1+\sqrt{x}} dx = 2 \int \frac{t}{1+t} dt = 2 \int \frac{t+1-1}{1+t} dt = 2 \int \left(1 - \frac{1}{1+t} \right) dt = 2t - 2 \ln |1+t| + C$$

$$= 2\sqrt{x} - 2 \ln |1+\sqrt{x}| + C.$$

6. 解: 令 $x = 3 \sin t$, 则 $dx = 3 \cos t dt$.

$$\int \sqrt{9-x^2} dx = \int \sqrt{9-(3 \sin t)^2} \cdot 3 \cos t dt = \int \sqrt{9-9 \sin^2 t} \cdot 3 \cos t dt$$

$$= \int \sqrt{9 \cos^2 t} \cdot 3 \cos t dt = 9 \int \cos^2 t dt = 9 \int \frac{1+\cos 2t}{2} dt$$

$$= \frac{9}{2} \int (1+\cos 2t) dt = \frac{9}{2} t + \frac{9}{4} \sin 2t + C = \frac{9}{2} t + \frac{9}{2} \sin t \cos t + C$$

$$= \frac{9}{2} \arcsin \frac{x}{3} + \frac{1}{2} x \sqrt{9-x^2} + C.$$