

第四章测试答案

一、选择题 (本题 5 个小题, 每小题 3 分, 共 15 分)

1. D 2. D 3. D 4. D 5. B

二、填空题 (本题 5 个小题, 每小题 3 分, 共 15 分)

$$1. \int f(x)dx \quad 2. \frac{2^x}{\ln 2} - \frac{3^x}{\ln 3} + C \quad 3. x \sin x + \cos x + C \quad 4. \frac{4}{7} x^{\frac{7}{4}} + C \quad 5. \frac{1}{2} e^{2x} + \frac{1}{2}$$

三、解答题 (本题 7 个小题, 每小题 10 分, 共 70 分)

$$1. \text{解: } \int \sin^2 x \cos^2 x \cdot \cos x dx = \int \sin^2 x (1 - \sin^2 x) d(\sin x) = \frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C .$$

$$\begin{aligned} 2. \text{解: } \int x^2 \ln(x-2) dx &= \int \ln(x-2) d\left(\frac{1}{3} x^3\right) = \frac{1}{3} x^3 \ln(x-2) - \frac{1}{3} \int \frac{x^3 - 8 + 8}{x-2} dx \\ &= \frac{1}{3} x^3 \ln(x-2) - \frac{1}{3} \int \left(x^2 + 2x + 4 + \frac{8}{x-2} \right) dx \\ &= \frac{1}{3} x^3 \ln(x-2) - \frac{1}{9} x^3 - \frac{1}{3} x^2 - \frac{4}{3} x - \frac{8}{3} \ln|x-2| + C \end{aligned}$$

$$\begin{aligned} 3. \text{解: } \int \cos^2 x dx &= \int \frac{1 + \cos 2x}{2} dx = \frac{1}{2} \left(\int dx + \int \cos 2x dx \right) \\ &= \frac{1}{2} x + \frac{1}{4} \int \cos 2x d(2x) = \frac{x}{2} + \frac{\sin 2x \theta^0}{5\theta^0} C. \end{aligned}$$

$$\begin{aligned} 4. \text{解: } \int \frac{3x^2 + x + \sin \frac{1}{x}}{x^2} dx &= \int \left(3 + \frac{1}{x} + \frac{\sin \frac{1}{x}}{x^2} \right) dx = 3x + \ln|x| - \int \sin \frac{1}{x} d \frac{1}{x} \\ &= 3x + \ln|x| + \cos \frac{1}{x} + C \end{aligned}$$

$$5. \text{解: 令 } \sqrt{3x-1} = t, \text{ 则 } x = \frac{t^2+1}{3}, dx = \frac{2t}{3} dt,$$

$$\begin{aligned} \int \cos \sqrt{3x-1} dx &= \frac{2}{3} \int \cos t \cdot t dt = \frac{2}{3} \int t d \sin t \\ &= \frac{2}{3} t \sin t - \frac{2}{3} \int \sin t dt = \frac{2}{3} t \sin t + \frac{2}{3} \cos t + C = \frac{2}{3} \sqrt{3x-1} \sin \sqrt{3x-1} + \frac{2}{3} \cos \sqrt{3x-1} + C \end{aligned}$$

$$6. \text{解: 令 } x = 2 \sin t, dx = 2 \cos t dt,$$

原式变为

$$\int \frac{2 \cdot 2 \sin t + 1}{2 \cos t} \cdot 2 \cos t dt = \int (4 \sin t + 1) dt = -4 \cos t + t + C = -2\sqrt{4-x^2} + \arcsin \frac{x}{2} + C.$$

$$\text{或者 } \int \frac{2x+1}{\sqrt{4-x^2}} dx = \int \left(\frac{2x}{\sqrt{4-x^2}} + \frac{1}{\sqrt{4-x^2}} \right) dx = \int \frac{2x}{\sqrt{4-x^2}} dx + \int \frac{1}{\sqrt{4-x^2}} dx$$

$$= -\int \frac{1}{\sqrt{4-x^2}} d(4-x^2) + \int \frac{1}{\sqrt{1-(\frac{x}{2})^2}} d\frac{x}{2} = -2\sqrt{4-x^2} + \arcsin \frac{x}{2} + C$$

$$7. \text{ 解: } \int \frac{x+1}{x^2-2x+5} dx = \frac{1}{2} \int \frac{2x+2}{x^2-2x+5} dx = \frac{1}{2} \int \frac{2x-2+4}{x^2-2x+5} dx$$

$$= \frac{1}{2} \int \frac{2x-2}{x^2-2x+5} dx + 2 \int \frac{1}{x^2-2x+5} dx = \frac{1}{2} \int \frac{1}{x^2-2x+5} d(x^2-2x+5) + 2 \int \frac{dx}{4+(x-1)^2}$$

$$= \frac{1}{2} \ln|x^2-2x+5| + \int \frac{1}{1+(\frac{x-1}{2})^2} d\frac{x-1}{2} = \frac{1}{2} \ln|x^2-2x+5| + \arctan \frac{x-1}{2} + C$$