1、求极限 $\lim\limits_{x\to\infty}\left(\dfrac{x+2}{x-1}\right)^{x}$.

2、求极限 $\lim\limits_{x\to\infty}\left(\dfrac{x-2}{x+1}\right)^{3x}$.

3、求极限 $\lim\limits_{x\to 0}\dfrac{\tan x}{5x}$.

4、求极限 $\lim\limits_{x\to 3}\dfrac{\sqrt{2x-4}-\sqrt{2}}{x-3}$.

5、求极限 $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)$.

6、求极限 $\lim\limits_{n\to\infty}\sqrt{n}\left(\sqrt{n+1}-\sqrt{n}\right)$.

7、求极限 $\lim\limits_{n\to\infty}\left[\sqrt{1+2+\cdots+n}-\sqrt{1+2+\cdots+(n-1)}\right]$.

8、求极限 $\lim\limits_{n\to\infty}\left(\dfrac{1}{4}+\dfrac{1}{4^2}+\cdots+\dfrac{1}{4^{n+1}}\right)$.

9、求 $\lim\limits_{x\to -1}\left(\dfrac{3}{x^3+1}-\dfrac{1}{x+1}\right)$.

10、 求极限 $\lim\limits_{n\to\infty}[\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{n(n+1)}]^{3n-2}$ .

# 答案

1、解：原式$=\lim\limits_{x\to\infty}(1+\dfrac{3}{x-1})^{x}=\lim\limits_{x\to\infty}e^{\frac{3x}{x-1}}=e^{3}$.

2、解：原式$=\lim\limits_{x\to\infty}(1-\dfrac{3}{x+1})^{3x}=\lim\limits_{x\to\infty}e^{\frac{9x}{x+1}}=e^{-9}$.

3、解：原式$=\lim\limits_{x\to0}\dfrac{\sin x}{\cos x\cdot 5x}=\dfrac{1}{5}\lim\limits_{x\to0}\dfrac{\sin x}{x}=\dfrac{1}{5}$.

4、解：原式$=\lim\limits_{x\to3}\dfrac{(\sqrt{2x-4}-\sqrt{2})(\sqrt{2x-4}+\sqrt{2})}{(x-3)(\sqrt{2x-4}+\sqrt{2})}=\lim\limits_{x\to3}\dfrac{2(x-3)}{2\sqrt{2}(x-3)}=\dfrac{\sqrt{2}}{2}$.

5、解：原式$=\lim\limits_{x\to\infty}\dfrac{(\sqrt{x^2+x}-\sqrt{x^2-x})(\sqrt{x^2+x}+\sqrt{x^2-x})}{\sqrt{x^2+x}+\sqrt{x^2-x}}$

$=\lim\limits_{x\to\infty}\dfrac{2x}{\sqrt{x^2+x}+\sqrt{x^2-x}}=\lim\limits_{x\to\infty}\dfrac{2}{\sqrt{1+\dfrac{1}{x}}+\sqrt{1-\dfrac{1}{x}}}=1$.

6、解：原式$=\lim\limits_{n\to\infty}\dfrac{\sqrt{n}(\sqrt{n+1}-\sqrt{n})(\sqrt{n+1}+\sqrt{n})}{\sqrt{n+1}+\sqrt{n}}$

$=\lim\limits_{n\to\infty}\dfrac{\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}=\lim\limits_{n\to\infty}\dfrac{1}{\sqrt{1+\dfrac{1}{n}}+1}=\dfrac{1}{2}$.

7、解：原式$=\lim\limits_{n\to\infty}\dfrac{1+2+\cdots+n-1-2-(n-1)}{\sqrt{1+2+\cdots+n}+\sqrt{1+2+\cdots+(n-1)}}$

$=\lim\limits_{n\to\infty}\dfrac{n}{\sqrt{\dfrac{n(n+1)}{2}}+\sqrt{\dfrac{(n-1)(1+n-1)}{2}}}$

$=\lim\limits_{n\to\infty}\dfrac{n}{\sqrt{\dfrac{n^2+n}{2}}+\sqrt{\dfrac{n^2-n}{2}}}=\dfrac{\sqrt{2}}{2}$.

8、解：原式$=\lim\limits_{n\to\infty}\dfrac{\dfrac{1}{4}(1-\dfrac{1}{4^{n+1}})}{1-\dfrac{1}{4}}=\dfrac{1}{3}$.

9、解：原式 $=\lim\limits_{x\to -1}\dfrac{3-(x^2-x+1)}{x^3+1}=-\lim\limits_{x\to -1}\dfrac{x^2-x-2}{(x+1)(x^2-x+1)}$

$$=-\lim_{x\to -1}\frac{(x+1)(x-2)}{(x+1)(x^2-x+1)}=-\lim_{x\to -1}\frac{x-2}{x^2-x+1}=1.$$

10、解：原式 $=\lim\limits_{n\to\infty}(1-\dfrac{1}{2}+\dfrac{1}{2}-\dfrac{1}{3}+\cdots+\dfrac{1}{n}-\dfrac{1}{n+1})^{3n-2}$

$$=\lim_{n\to\infty}(1-\frac{1}{n+1})^{3n-2}=\lim_{n\to\infty}e^{-\frac{3n-2}{n+1}}=e^{-3}.$$