

## 二期 21 不定积分加测 2

分部积分法——五年内数一考了 2 次（21 和 22 计算）、数二考了 2 次（22 和 24 计算）、数三考了 2 次（22 和 23 计算）

$$1. \text{ 【解析】} \int \frac{\ln(1+x^2)}{x^2} dx = \int \ln(1+x^2) d\left(-\frac{1}{x}\right) = -\frac{\ln(1+x^2)}{x} + \int \frac{1}{x} d\ln(1+x^2)$$

$$= -\frac{\ln(1+x^2)}{x} + 2 \int \frac{1}{1+x^2} dx = -\frac{\ln(1+x^2)}{x} + 2 \arctan x + C$$

$$2. \text{ 【解析】} \int (2x \ln x + \sin x) dx = \int \ln x dx^2 - \cos x = x^2 \ln x - \int x^2 d \ln x - \cos x$$

$$= x^2 \ln x - \int x dx - \cos x = x^2 \ln x - \frac{x^2}{2} - \cos x + C$$

3. 【解析】

$$\int (x \cos x + \sec^2 x) dx = \int x d \sin x + \tan x = x \sin x - \int \sin x dx + \tan x = x \sin x + \cos x + \tan x + C$$

$$4. \text{ 【解析】} \int (2x \ln x + \frac{1}{x^2}) dx = \int \ln x dx^2 - \frac{1}{x} + C = x^2 \ln x - \int x^2 d \ln x - \frac{1}{x} + C$$

$$= x^2 \ln x - \int x dx - \frac{1}{x} + C = x^2 \ln x - \frac{x^2}{2} - \frac{1}{x} + C$$

$$5. \text{ 【解析】} \int (\arctan x + 2^x) dx = \int \arctan x dx + \int 2^x dx = x \arctan x - \int \frac{x}{1+x^2} dx + \frac{2^x}{\ln 2}$$

$$= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} d(1+x^2) + \frac{2^x}{\ln 2} = x \arctan x - \frac{1}{2} \ln(1+x^2) + \frac{2^x}{\ln 2} + C$$

$$6. \text{ 【解析】} \int (x e^{3x} + \cos x) dx = \int x d \frac{1}{3} e^{3x} + \sin x = \frac{1}{3} x e^{3x} - \frac{1}{3} \int e^{3x} dx + \sin x$$

$$= \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + \sin x + C$$

7. 【解析】

$$\int \left( \frac{1}{x^2} + x \cos x \right) dx = -\frac{1}{x} + \int x d \sin x = -\frac{1}{x} + x \sin x - \int \sin x dx = -\frac{1}{x} + x \sin x + \cos x + C$$

其他

1. 【解析】

$$\int \frac{1}{x^2 + 10x + 26} dx = \int \frac{1}{1+(x+5)^2} dx = \int \frac{1}{1+(x+5)^2} d(x+5) = \arctan(x+5) + C$$

$$2. \text{ 【解析】} \int \frac{2x^3 - x + 3}{x^2} dx = \int \left(2x - \frac{1}{x} + \frac{3}{x^2}\right) dx = x^2 - \ln|x| - \frac{3}{x} + C$$

$$3. \text{ 【解析】} \int (2\tan^2 x - x + 5) dx = 2 \int (\sec^2 x - 1) dx - \frac{x^2}{2} + 5x = 2 \tan x - 2x - \frac{x^2}{2} + 5x + C \\ = 2 \tan x + 3x - \frac{x^2}{2} + C$$

4. 【解析】

$$\int \frac{3x^2 + 6x + 1}{3x^2 + 1} dx = \int \frac{3x^2 + 1}{3x^2 + 1} dx + \int \frac{6x}{3x^2 + 1} dx = x + \int \frac{1}{1 + 3x^2} d(1 + 3x^2) = x + \ln|1 + 3x^2| + C$$