

第四章测试答案

一、选择题 (本题 5 个小题, 每小题 3 分, 共 15 分)

1. D 2. D 3. D 4. D 5. B

二、填空题 (本题 5 个小题, 每小题 3 分, 共 15 分)

1. $f(x)dx$ 2. $\frac{2^x}{\ln 2} - \frac{3^x}{\ln 3} + C$ 3. $x \sin x + \cos x + C$ 4. $\frac{4}{7}x^{\frac{7}{4}} + C$ 5. $\frac{1}{2}e^{2x} + \frac{1}{2}$

三、解答题 (本题 7 个小题, 每小题 10 分, 共 70 分)

1. 解: $\int \sin^2 x \cos^2 x \cdot \cos x dx = \int \sin^2 x (1 - \sin^2 x) d(\sin x) = \frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$.

2. 解: $\int x \ln(x-2) dx = \int \ln(x-2) d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \ln(x-2) - \frac{1}{2} \int \frac{x^2 - 4 + 4}{x-2} dx$
 $= \frac{1}{2}x^2 \ln(x-2) - \frac{1}{2} \int \left(x+2 + \frac{4}{x-2}\right) dx = \frac{1}{2}x^2 \ln(x-2) - \frac{1}{4}x^2 - x - 2 \ln|x-2| + C.$

3. 解: $\int \cos^2 x dx = \int \frac{1 + \cos 2x}{2} dx = \frac{1}{2} \left(\int dx + \int \cos 2x dx \right)$
 $= \frac{1}{2}x + \frac{1}{4} \int \cos 2x d(2x) = \frac{x}{2} + \frac{\sin 2x}{4} + C.$

4. 解: $\int \frac{3x^2 + x + \sin \frac{1}{x}}{x^2} dx = \int \left(3 + \frac{1}{x} + \frac{\sin \frac{1}{x}}{x^2}\right) dx = 3x + \ln|x| - \int \sin \frac{1}{x} d\frac{1}{x}$
 $= 3x + \ln|x| + \cos \frac{1}{x} + C$

5. 解: 令 $\sqrt{3x-1} = t$, 则 $x = \frac{t^2+1}{3}, dx = \frac{2t}{3} dt$,

$$\int \cos \sqrt{3x-1} dx = \frac{2}{3} \int \cos t \cdot t dt = \frac{2}{3} \int t d \sin t$$
$$= \frac{2}{3} t \sin t - \frac{2}{3} \int \sin t dt = \frac{2}{3} t \sin t + \frac{2}{3} \cos t + C = \frac{2}{3} \sqrt{3x-1} \sin \sqrt{3x-1} + \frac{2}{3} \cos \sqrt{3x-1} + C$$

6. 解: 令 $x = 2 \sin t, dx = 2 \cos t dt$,

原式变为

$$\int \frac{2 \cdot 2 \sin t + 1}{2 \cos t} \cdot 2 \cos t dt = \int (4 \sin t + 1) dt = -4 \cos t + t + C = -2\sqrt{4-x^2} + \arcsin \frac{x}{2} + C.$$

或者 $\int \frac{2x+1}{\sqrt{4-x^2}} dx = \int \left(\frac{2x}{\sqrt{4-x^2}} + \frac{1}{\sqrt{4-x^2}} \right) dx = \int \frac{2x}{\sqrt{4-x^2}} dx + \int \frac{1}{\sqrt{4-x^2}} dx$

$$= -\int \frac{1}{\sqrt{4-x^2}} d(4-x^2) + \int \frac{1}{\sqrt{1-(\frac{x}{2})^2}} d\frac{x}{2} = -2\sqrt{4-x^2} + \arcsin \frac{x}{2} + C$$

$$\begin{aligned} 7. \text{ 解: } \int \frac{x+1}{x^2-2x+5} dx &= \frac{1}{2} \int \frac{2x+2}{x^2-2x+5} dx = \frac{1}{2} \int \frac{2x-2+4}{x^2-2x+5} dx \\ &= \frac{1}{2} \int \frac{2x-2}{x^2-2x+5} dx + 2 \int \frac{1}{x^2-2x+5} dx = \frac{1}{2} \int \frac{1}{x^2-2x+5} d(x^2-2x+5) + 2 \int \frac{dx}{4+(x-1)^2} \\ &= \frac{1}{2} \ln|x^2-2x+5| + \int \frac{1}{1+(\frac{x-1}{2})^2} d\frac{x-1}{2} = \frac{1}{2} \ln|x^2-2x+5| + \arctan \frac{x-1}{2} + C \end{aligned}$$