

极限竞赛题

(每小题 5 分, 共 100 分, 考试时间 120 分钟)

(1—18 题每小题 4 分, 19—23 每小题 7 分, 共 100 分)

$$1. \lim_{x \rightarrow \infty} \frac{3x^2 - 2x \sin x}{2x + x \sin x}$$

$$2. \lim_{x \rightarrow \infty} \frac{5x^2 - 3}{2x + 1} \sin \frac{2}{x}$$

$$3. \lim_{x \rightarrow 0} \frac{\sin(4x)}{\sqrt{x+2} - \sqrt{2}}$$

$$4. \lim_{x \rightarrow 2} \frac{\sin(x^2 - 4)}{x^2 + x - 6}$$

$$5. \lim_{x \rightarrow 1} \frac{3x^3 - 2x^2 - 1}{\arcsin(x^2 - 1)}$$

$$6. \lim_{x \rightarrow 0} \frac{x - xe^x}{\sin^2 x}$$

$$7. \lim_{x \rightarrow 0} \frac{\sqrt{1+5x} - \sqrt{1-3x}}{x^2 + 2x}$$

$$8. \lim_{x \rightarrow 0} \frac{\sqrt{1+\sin x} - \sqrt{1+\tan x}}{x(1-\cos x)}$$

$$9. \lim_{x \rightarrow 0} \frac{e - e^{\cos x}}{\sqrt[3]{x^2 + 1} - 1}$$

$$10. \lim_{x \rightarrow +\infty} \frac{\sqrt{4x^2 + x - 1} + x + 3}{\sqrt{x^2 + \sin x}}$$

$$11. \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x \ln(1 + \sin^2 x)}$$

$$12. \lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$$

$$13. \lim_{x \rightarrow +\infty} \ln\left(1 + \frac{1}{x}\right) \cdot \ln x$$

$$14. \lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{e^x - 1} \right)$$

$$15. \lim_{x \rightarrow +\infty} \left(3x - \sqrt{ax^2 - x + 1} \right) = \frac{1}{6}, \text{ 求 } a.$$

$$16. \lim_{x \rightarrow +\infty} x(\ln(x-2) - \ln(x+1))$$

$$17. \lim_{x \rightarrow 0} \cot x \left(\frac{1}{\sin x} - \frac{1}{x} \right)$$

$$18. \lim_{x \rightarrow 0} (x + e^x)^{\frac{1}{x}}$$

19. 已知 $f(x) = 2^x$, 求 $\lim_{n \rightarrow \infty} \frac{1}{n^2} \ln[f(1) \cdot f(2) \cdots f(n)]$

20. $\lim_{x \rightarrow 0} \frac{\sin 2x}{\ln(a+x)} (\cos x - b) = 4$, 求 a, b

21. $\lim_{x \rightarrow -\infty} (4x + \sqrt{ax^2 - bx - 1}) = 1$, $a > 0$, 求 a, b

22. 已知极限 $\lim_{x \rightarrow +\infty} (\frac{x^2}{x+1} - x - a) = 2$, 求 a .

23. 若 $\lim_{x \rightarrow 1} f(x)$ 存在, 且 $f(x) = x^3 + \frac{2x^2+1}{x+1} + 2\lim_{x \rightarrow 1} f(x)$, 求 $f(x)$