

1. 三角函数公式

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}, \quad \sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x}. \quad (\text{切割化弦})$$

$$\sin^2 x + \cos^2 x = 1, \quad 1 + \tan^2 x = \sec^2 x, \quad 1 + \cot^2 x = \csc^2 x,$$

$$\sin 2x = 2 \sin x \cos x.$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x.$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}. \quad \cos^2 x = \frac{1 + \cos 2x}{2}. \quad (\text{降幂公式})$$

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}. \quad \cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}. \quad (\text{降幂公式})$$

2. $\arcsin x$ 的定义域为 $[-1, 1]$; 值域为 $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\arccos x$ 的定义域为 $[-1, 1]$; 值域为 $[0, \pi]$

$\arctan x$ 的定义域为 $(-\infty, +\infty)$; 值域为 $(-\frac{\pi}{2}, \frac{\pi}{2})$

$\operatorname{arccot} x$ 的定义域为 $(-\infty, +\infty)$; 值域为 $(0, \pi)$

3. 第一重要极限 $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, 推广形式 $\lim_{\otimes \rightarrow 0} \frac{\sin \otimes}{\otimes} = 1$.

第二重要极限 $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$, $\lim_{x \rightarrow \infty} (1+\frac{1}{x})^x = e$, 推广形式 $\lim_{\otimes \rightarrow 0} (1+\otimes)^{\frac{1}{\otimes}} = e$.

4. 等差数列求和公式 $S_n = \frac{(a_1 + a_n)n}{2}$.

等比数列求和公式 $S_n = \frac{a_1(1-q^n)}{1-q}$, $q \neq 1$.

5. 当 $x \rightarrow 0$ 时, $\sin x \sim x$, $\arcsin x \sim x$, $\tan x \sim x$,

$$\arctan x \sim x, \quad 1 - \cos x \sim \frac{x^2}{2}, \quad \cos x - 1 \sim -\frac{x^2}{2}.$$

$$\sqrt[4]{1+x} - 1 \sim \frac{x}{2}, \quad \sqrt[4]{1+x} - 1 \sim \frac{x}{4}, \quad 1 - \sqrt{1+x} \sim -\frac{x}{2}, \quad 1 - \sqrt[n]{1+x} \sim -\frac{x}{n}$$

$$e^x - 1 \sim x, \quad 1 - e^x \sim -x, \quad \ln(1+x) \sim x.$$

当 $\varphi(x) \rightarrow 0$ 时, $\sin \varphi(x) \sim \varphi(x)$, $\arcsin \varphi(x) \sim \varphi(x)$, $\tan \varphi(x) \sim \varphi(x)$,

$$\arctan \varphi(x) \sim \varphi(x), \quad 1 - \cos \varphi(x) \sim \frac{[\varphi(x)]^2}{2}, \quad \cos \varphi(x) - 1 \sim -\frac{[\varphi(x)]^2}{2}.$$

$$\begin{aligned} \sqrt{1+\varphi(x)} - 1 &\sim \frac{1}{2}\varphi(x) , \quad \sqrt[4]{1+\varphi(x)} - 1 \sim \frac{1}{4}\varphi(x) , \quad 1 - \sqrt{1+\varphi(x)} \sim -\frac{1}{2}\varphi(x) , \\ \sqrt[n]{1+\varphi(x)} - 1 &\sim \frac{1}{n}\varphi(x) , \quad e^{\varphi(x)} - 1 \sim \varphi(x) , \quad 1 - e^{\varphi(x)} \sim -\varphi(x) , \quad \ln(1+\varphi(x)) \sim \varphi(x) . \end{aligned}$$

6. 函数 $f(x)$ 在 $x = x_0$ 处的导数定义式为

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} .$$

$$\text{推广式 } f'(x_0) = \lim_{\otimes \rightarrow 0} \frac{f(x_0 + \otimes) - f(x_0)}{\otimes}$$

$$7. \quad \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + m\Delta x) - f(x_0 + n\Delta x)}{k\Delta x} = \frac{(m-n)}{k} f'(x_0)$$

8. 求导法则：设 $u = u(x), v = v(x)$ 均在点 x 处可导，则

$$(1) \quad [u(x) \pm v(x)]' = u'(x) \pm v'(x) . \quad (2) \quad [u(x) \cdot v(x)]' = u'(x)v(x) + u(x)v'(x) .$$

$$(3) \quad \left[\frac{u(x)}{v(x)} \right]' = \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)} . \quad (4) \quad [cu(x)]' = c[u(x)]' .$$

9. 基本初等函数求导公式

$$(C)' = 0 , \quad (x^k)' = kx^{k-1} , \quad (\log_a x)' = \frac{1}{x \ln a} , \quad (\ln x)' = \frac{1}{x}$$

$$(a^x)' = a^x \ln a , \quad (e^x)' = e^x , \quad (\sin x)' = \cos x , \quad (\cos x)' = -\sin x ,$$

$$(\tan x)' = \sec^2 x , \quad (\cot x)' = -\csc^2 x , \quad (\sec x)' = \sec x \tan x , \quad (\csc x)' = -\csc x \cot x ,$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} , \quad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}} ,$$

$$(\arctan x)' = \frac{1}{1+x^2} , \quad (\operatorname{arccot} x)' = -\frac{1}{1+x^2} . \quad (\sqrt{x})' = \frac{1}{2\sqrt{x}} , \quad \left(\frac{1}{x}\right)' = -\frac{1}{x^2} .$$

10. 复合函数求导链式法则

$$([u(x)]^k)' = ku(x)^{k-1} \cdot u'(x) , \quad (\log_a u(x))' = \frac{1}{u(x) \ln a} \cdot u'(x) , \quad (\ln u(x))' = \frac{1}{u(x)} \cdot u'(x)$$

$$(a^{u(x)})' = a^{u(x)} \ln a \cdot u'(x) , \quad (e^{u(x)})' = a^{u(x)} \cdot u'(x) , \quad (\sin u(x))' = \cos u(x) \cdot u'(x) ,$$

$$(\cos u(x))' = -\sin u(x) \cdot u'(x) , \quad (\tan u(x))' = \sec^2 u(x) \cdot u'(x) ,$$

$$(\cot u(x))' = -\csc^2 u(x) \cdot u'(x),$$

$$(\sec u(x))' = \sec u(x) \tan u(x) \cdot u'(x), \quad (\csc u(x))' = -\csc u(x) \cot u(x) \cdot u'(x),$$

$$(\arcsin u(x))' = \frac{1}{\sqrt{1-u^2(x)}} \cdot u'(x),$$

$$(\arccos u(x))' = -\frac{1}{\sqrt{1-u^2(x)}} \cdot u'(x), \quad (\arctan u(x))' = \frac{1}{1+u^2(x)} \cdot u'(x),$$

$$(\operatorname{arccot} u(x))' = -\frac{1}{1+u^2(x)} \cdot u'(x),$$

$$(\sqrt{u(x)})' = \frac{1}{2\sqrt{u(x)}} \cdot u'(x), \quad \left(\frac{1}{u(x)}\right)' = -\frac{1}{u^2(x)} \cdot u'(x).$$