

三期加测 3 函数的极限答案

每小题 5 分, 共 75 分

$$1. \quad (2023 \text{ 数三}) \quad \lim_{x \rightarrow +\infty} (\sqrt{4x^2 + x} - 2x) = \lim_{x \rightarrow +\infty} \frac{(\sqrt{4x^2 + x} - 2x)(\sqrt{4x^2 + x} + 2x)}{\sqrt{4x^2 + x} + 2x}$$

$$= \lim_{x \rightarrow +\infty} \frac{x}{\sqrt{4x^2 + x} + 2x} = \frac{1}{4}$$

$$2. \quad (2023 \text{ 数二}) \quad \lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{3}{x^2 + 3x} \right) = \lim_{x \rightarrow 0} \frac{x+3-3}{x^2 + 3x} = \lim_{x \rightarrow 0} \frac{x}{x^2 + 3x} = \lim_{x \rightarrow 0} \frac{1}{x+3} = \frac{1}{3}$$

$$3. \quad (2023 \text{ 数一}) \quad \lim_{x \rightarrow +\infty} (\sqrt{x^2 + 3x} - x) = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + 3x} - x)(\sqrt{x^2 + 3x} + x)}{\sqrt{x^2 + 3x} + x}$$

$$= \lim_{x \rightarrow +\infty} \frac{3x}{\sqrt{x^2 + 3x} + x} = \lim_{x \rightarrow +\infty} \frac{3}{\sqrt{1 + \frac{3}{x}} + 1} = \frac{3}{2}$$

$$4. \quad (2024 \text{ 数一}) \quad \text{求极限 } \lim_{x \rightarrow 3} \left(\frac{x}{x-3} - \frac{9}{x^2 - 3x} \right).$$

$$5. \quad (2024 \text{ 数三}) \quad \text{求极限 } \lim_{x \rightarrow 1} \left(\frac{1}{x^2 - x} - \frac{2}{x^2 - 1} \right).$$

1^∞ 型 (第二个重要极限) (五年高频率)

$$6. \quad (2021 \text{ 数一}) \quad \lim_{x \rightarrow \infty} \left(\frac{x+3}{x+1} \right)^x = \lim_{x \rightarrow \infty} \left(\frac{1 + \frac{3}{x}}{1 + \frac{1}{x}} \right)^x = \lim_{x \rightarrow \infty} \frac{(1 + \frac{3}{x})^x}{(1 + \frac{1}{x})^x} = \lim_{x \rightarrow \infty} \frac{(1 + \frac{3}{x})^{\frac{x}{3} \cdot 3}}{(1 + \frac{1}{x})^x} = \frac{e^3}{e} = e^2$$

$$7. \quad (2021 \text{ 数二}) \quad \text{已知 } \lim_{x \rightarrow \infty} \left(\frac{x-a}{x} \right)^x = 2, \text{ 则 } a = \underline{-\ln 2}.$$

$$8. \quad (2022 \text{ 数一}) \quad \text{设极限 } \lim_{x \rightarrow \infty} \left(1 + \frac{1}{3x} \right)^{kx} = e^2, \text{ 则 } k = \underline{6}.$$

$$9. \text{ (2022 数二)} \lim_{x \rightarrow \infty} \left(\frac{x+2}{x} \right)^{-3x} = \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x} \right)^{-3x} = \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x} \right)^{\frac{x}{2} \cdot (-6)} = e^{-6}.$$

$$10. \text{ (2022 数三)} \lim_{x \rightarrow \infty} \left(1 + \frac{1}{4x} \right)^{2x} = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{4x} \right)^{4x \cdot \frac{1}{2}} = e^{\frac{1}{2}}$$

$$11. \text{ (2023 数一)} \text{ 极限 } \lim_{x \rightarrow 0} (1+x)^{\frac{5}{x}} = e^5.$$

$$12. \text{ (2023 数三)} \text{ 极限 } \lim_{x \rightarrow \infty} \left(1 - \frac{4}{3x} \right)^x = e^{\frac{-4}{3}}.$$

$$13. \text{ (2024 数一)} \text{ 极限 } \lim_{x \rightarrow 0} \left(1 + \sin \frac{x}{2} \right)^{\frac{1}{x}} = e^{\frac{1}{2}}.$$

$$14. \text{ (2024 数二)} \text{ 极限 } \lim_{x \rightarrow 0} \left(1 + \frac{x}{2} \right)^{\frac{4}{\sin x}} = e^2.$$

$$15. \text{ (2024 数三)} \text{ 极限 } \lim_{x \rightarrow 0} (1+3x)^{\frac{a}{x}} = e, \text{ 则 } a = \underline{\frac{1}{3}}.$$

$$16. \text{ (2025 数二)} \lim_{x \rightarrow \infty} \left(\frac{x+4}{x+2} \right)^{3x-1} = \lim_{x \rightarrow \infty} \left(\frac{x+4}{x+2} \right)^{3x} \cdot \lim_{x \rightarrow \infty} \left(\frac{x+4}{x+2} \right)^{-1} = \lim_{x \rightarrow \infty} \left(\frac{x}{1+\frac{2}{x}} \right)^{3x}$$

$$= \lim_{x \rightarrow \infty} \frac{\left(1 + \frac{4}{x} \right)^{3x}}{\left(1 + \frac{2}{x} \right)^{3x}} = \frac{e^{12}}{e^6} = e^6$$

17. (2024 数二) $x \rightarrow 0$ 时, 下列函数是无穷小量的是 (C)

- A. $\cos x$ B. e^x C. $\ln(1+x)$ D. $\arcsin(1+x)$

18. (2024 数三) 当 $x \rightarrow 0$ 时, 与 x^2 为等价无穷小量的是 (A)

- A. $\sin^2 x$ B. $\sin 2x$ C. $\cos^2 x$ D. e^{2x-1}

19. (2023 数一) 当 $x \rightarrow 0$ 时, 以下函数不是无穷小量的是 (C)

- A. $\tan x$ B. $\sin 2x$ C. $\ln(1+x)$ D. $e^x + 1$

20. (2023 数三) 当 $x \rightarrow 0$ 时, 以下不是与 x 为等价无穷小量的是 (C)

- A. $\sin x$ B. $e^x - 1$ C. $1 - \cos x$ D. $\ln(1+x)$