Meshfree Method for Stress Driven Beams

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Introduction

Low-dimensional structures with dimensions in the micro-nano range exhibit size-dependent behavior that cannot be captured by local constitutive models. This deviation occurs because local models assume material point interactions are local, whereas size-dependent behavior arises from long-range interactions. While Eringen's strain-driven nonlocal model has been widely used, it often results in ill-posed governing equations and paradoxical results for beam bending. The stress-driven nonlocal approach has emerged as a mathematically consistent and well-posed substitute. This research develops the Element-Free Galerkin (EFG) method for a stress-driven one-dimensional Bernoulli-Euler beam, utilizing its inherent nonlocal solution approximation to accurately simulate size effects.

Scope

A MEMS resonator (©Bhaskaran et al.), images from ©scitime, a MEMS device by ©sensing-machines.

Theoretical Formulation and Methodology

Based on stress driven model, nonlocal elastic curvatures $\chi(x)$ are the output of a spatial convolution between a kernel function ϕ and the bending moment $M(x)$:

$$\chi(x) = \int_0^L \phi(x - \overline{x}, c)\frac{M(x)}{EI}dx \tag{1}$$

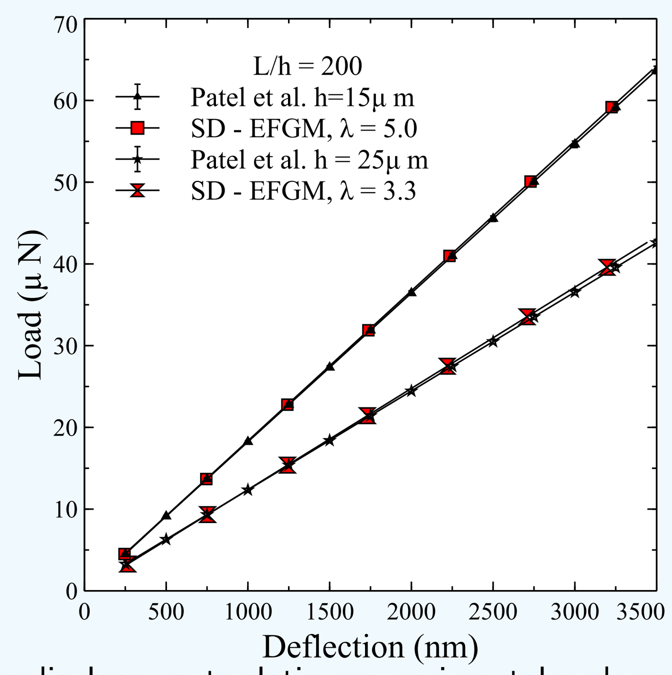
- ☐ Leads to a sixth order governing differential equation.
- ☐ EFG method rely of scattered nodes and moving least squares (MLS) approximations for constructing shape functions.
- ☐ A 6th-order spline weighting function is employed to provide the higher-order continuity required for approximating bending moments and shear forces.

Numerical Implementation

- ☐ Essential and constitutive boundary conditions are enforced using a combination of Lagrange multipliers and scaled transformation.
- ☐ This methodology removes the requirement for constitutive continuity conditions found in element-based formulations.
- ☐ The method results in a fully populated stiffness matrix, requiring more computational effort than traditional FEM but it offers a more robust tool for nonlocal analysis.

Validation and Parametric Study

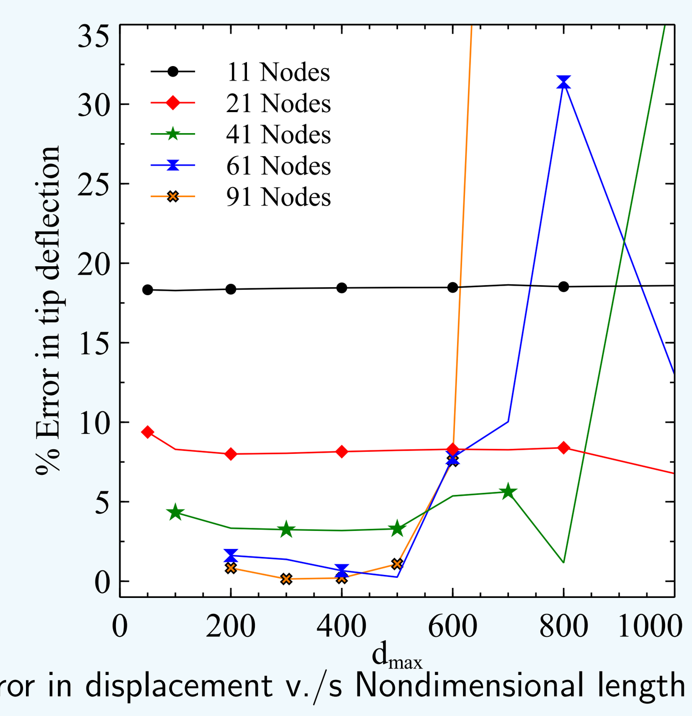
- ☐ The SD-EFGM formulation was validated against experimental results from cantilever micro-beam deflection tests.
- ☐ For a beam height $h = 15\mu m$, the model matches experimental data with a nonlocal parameter, $\lambda = 5.0$
- ☐ The model correctly predicts a reduction in nonlocal behavior as the beam length increases.
- ☐ Static results for simply supported, cantilever, and fixed-fixed beams under various loads align with analytical stress-driven benchmarks.



Force-displacement relation, experimental and predicted

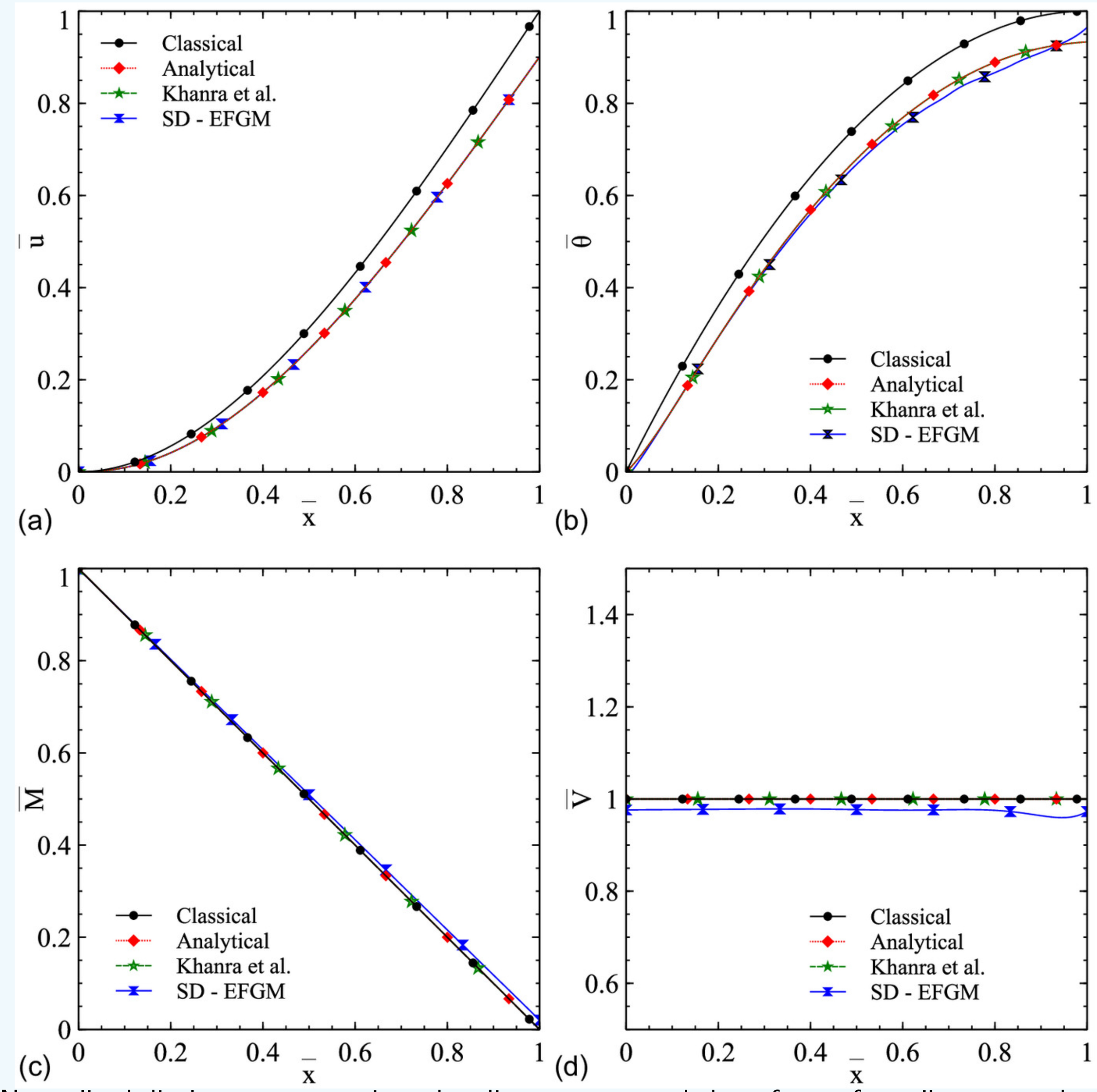
Effect of Nondimensional Length Scale

- ☐ Accuracy is heavily dependent on the local support size, scaled by the nondimensional length scale parameter, d_{max}.
- ☐ Parametric studies show that error remains high if d_{max} is below 50, with an optimal range for minimal error between 200 and 600.
- ☐ Stability requires a sufficiently large number of nodes; for example, at least 40 nodes are needed for moment balance in simply supported beams(length 25nm, width and height 1nm).
- ☐ Simulating nonlocal solids requires a support domain large enough to effectively include long range interactions.



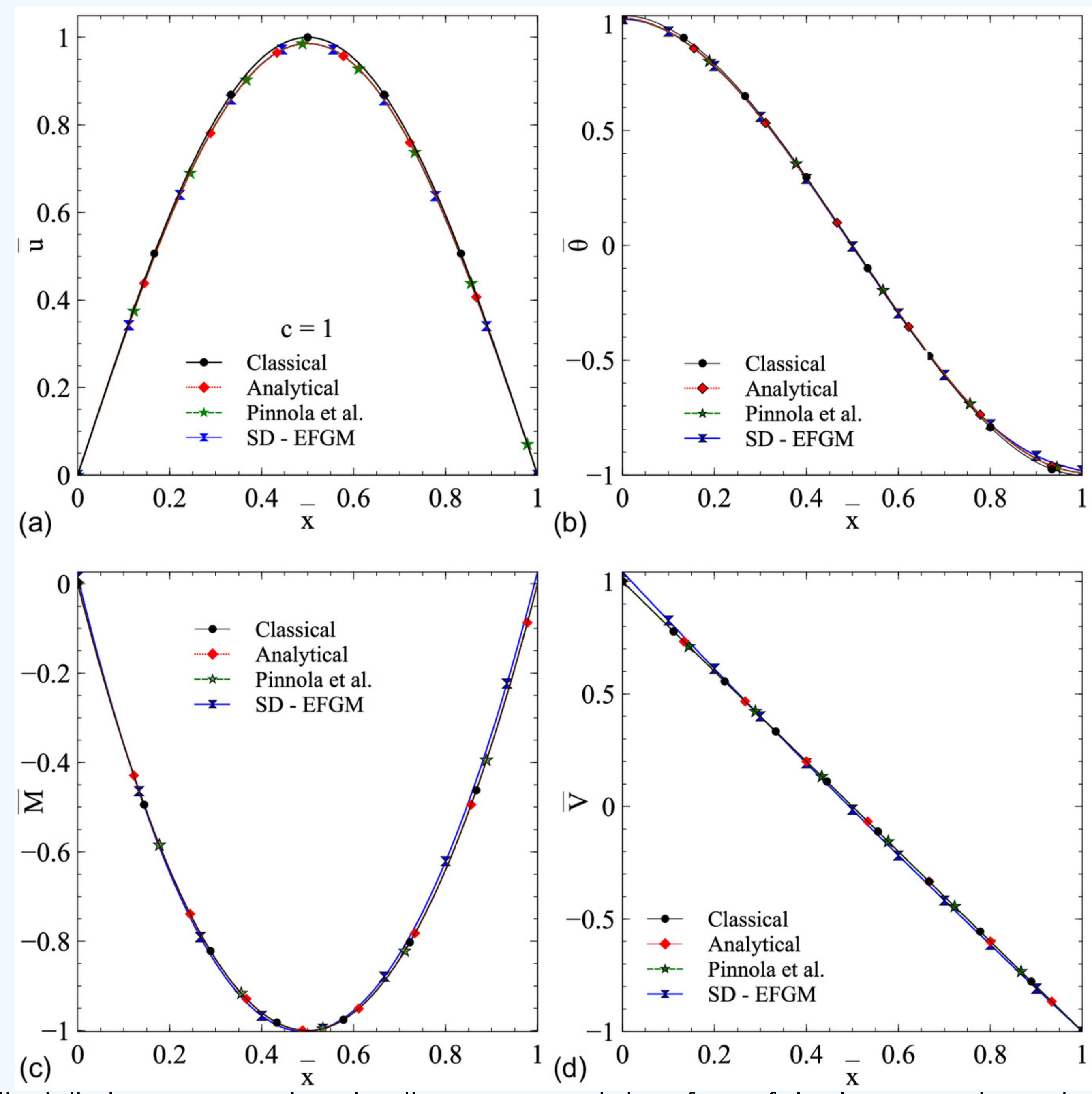
Error in displacement v./s Nondimensional length scale

Numerical Results



Normalized displacement, rotations, bending moment and shear force of cantilever nano-beam

Numerical Results



Normalized displacement, rotations, bending moment and shear force of simply supported nano-beam beam

Conclusions

- ☐ SD-EFGM provides an efficient numerical scheme for the static analysis of nonlocal Bernoulli-Euler beams.
- ☐ The displacement shows a consistent stiffening effect as nonlocal parameters increase.
- ☐ Accurate nonlocal simulation requires a non-compact support domain—with an optimal nondimensional length scale (d_{max}) between 200 and 600.
- ☐ The method accurately captures displacement, slopes, bending moments, and shear forces.
- ☐ Provides a robust alternative for element based formulation and offers scalability.

References

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