



Meshfree Method for Stress Driven Beams

Akhil S L, I R Praveen Krishna



Department of Aerospace Engineering,
Indian Institute of Space Science and Technology, Thiruvananthapuram, Kerala

Introduction

Low-dimensional structures with dimensions in the micro-nano range exhibit size-dependent behavior that cannot be captured by local constitutive models. This deviation occurs because local models assume material point interactions are local, whereas size-dependent behavior arises from long-range interactions. While Eringen's strain-driven nonlocal model has been widely used, it often results in ill-posed governing equations and paradoxical results for beam bending. The stress-driven nonlocal approach has emerged as a mathematically consistent and well-posed substitute. This research develops the Element-Free Galerkin (EFG) method for a stress-driven one-dimensional Bernoulli-Euler beam, utilizing its inherent nonlocal solution approximation to accurately simulate size effects.

Scope



A MEMS resonator (©Bhaskaran et al.), images from ©scitime, a MEMS device by ©sensing-machines.

Theoretical Formulation and Methodology

Based on stress driven model, nonlocal elastic curvatures $\chi(x)$ are the output of a spatial convolution between a kernel function ϕ and the bending moment $M(x)$:

$$\chi(x) = \int_0^L \phi(x - \bar{x}, c) \frac{M(\bar{x})}{EI} d\bar{x} \quad (1)$$

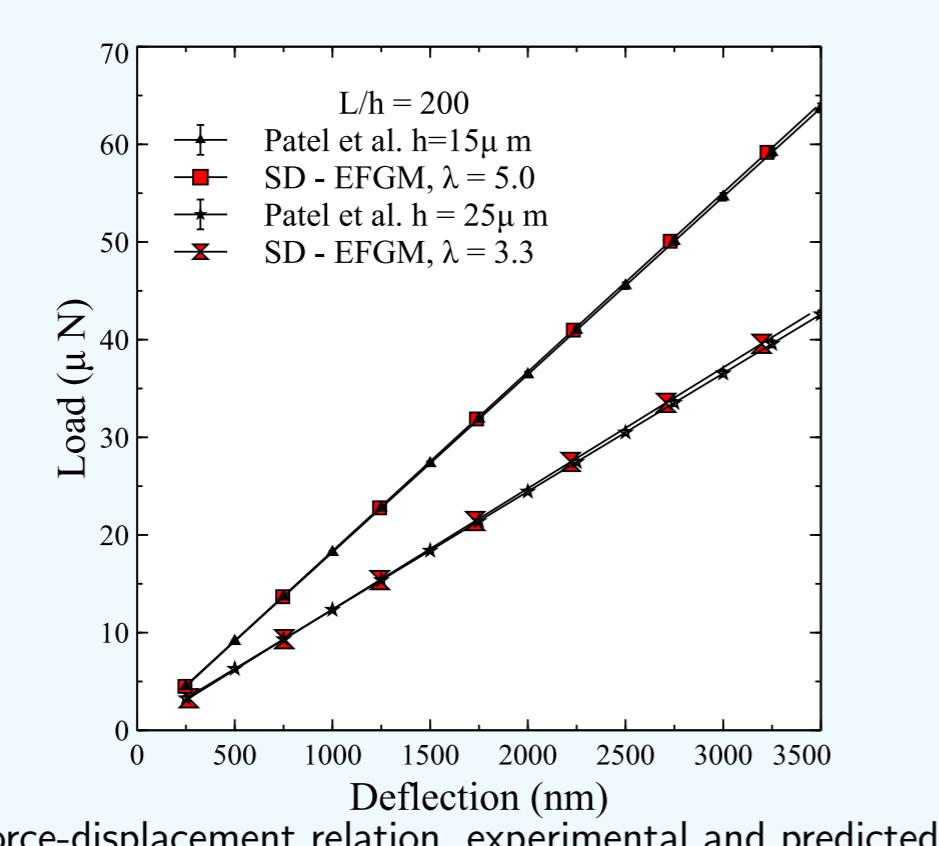
- Leads to a sixth order governing differential equation.
- EFG method rely of scattered nodes and moving least squares (MLS) approximations for constructing shape functions.
- A 6th-order spline weighting function is employed to provide the higher-order continuity required for approximating bending moments and shear forces.

Numerical Implementation

- Essential and constitutive boundary conditions are enforced using a combination of Lagrange multipliers and scaled transformation.
- This methodology removes the requirement for constitutive continuity conditions found in element-based formulations.
- The method results in a fully populated stiffness matrix, requiring more computational effort than traditional FEM but it offers a more robust tool for nonlocal analysis.

Validation and Parametric Study

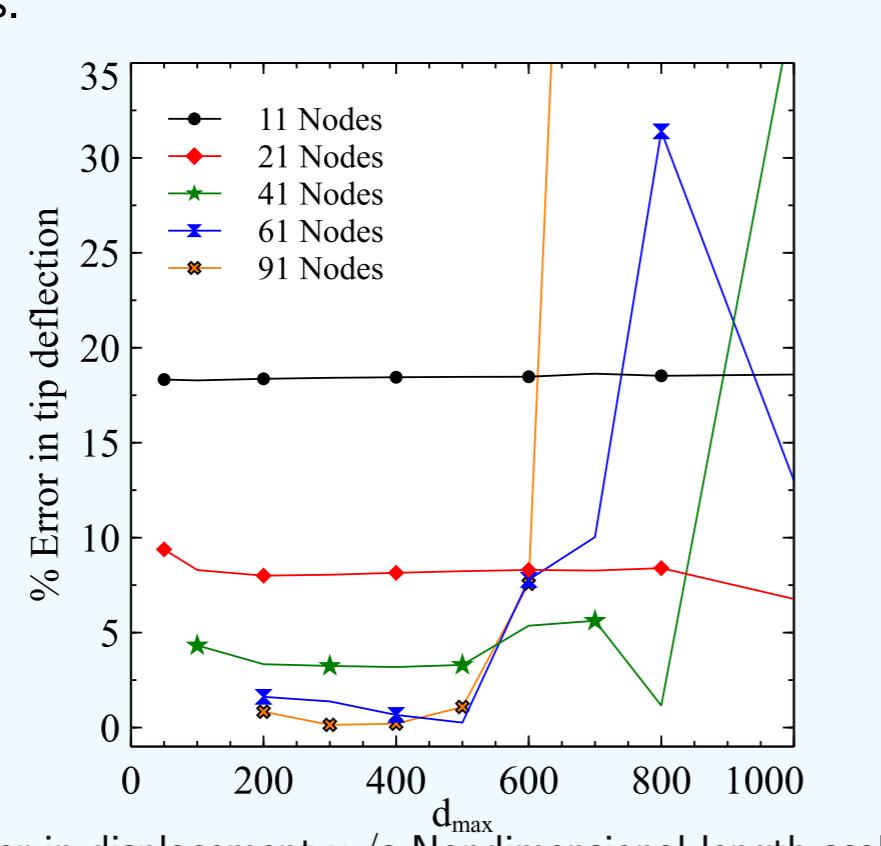
- The SD-EFGM formulation was validated against experimental results from cantilever micro-beam deflection tests.
- For a beam height $h = 15\mu m$, the model matches experimental data with a nonlocal parameter, $\lambda = 5.0$
- The model correctly predicts a reduction in nonlocal behavior as the beam length increases.
- Static results for simply supported, cantilever, and fixed-fixed beams under various loads align with analytical stress-driven benchmarks.



Force-displacement relation, experimental and predicted

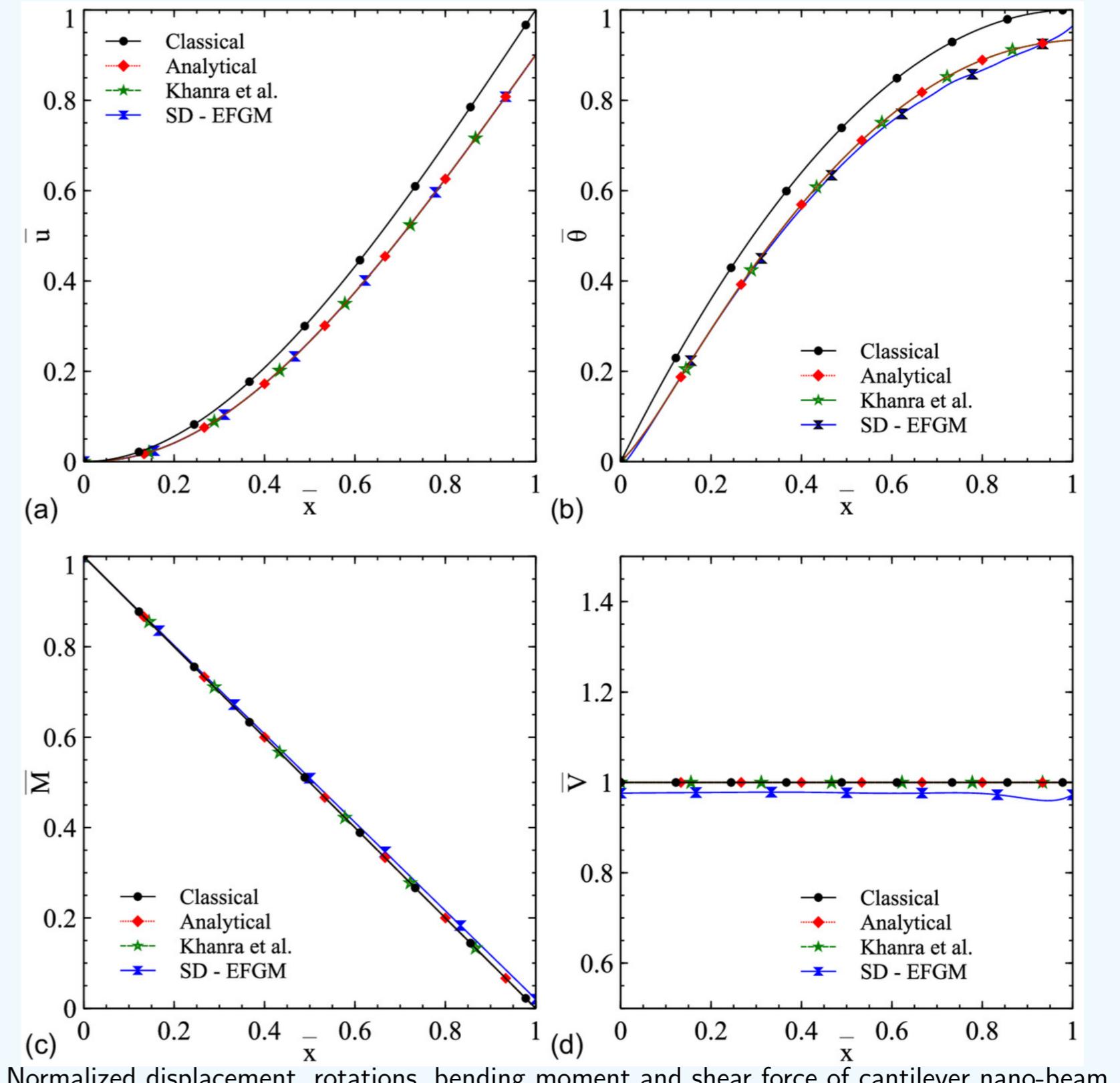
Effect of Nondimensional Length Scale

- Accuracy is heavily dependent on the local support size, scaled by the nondimensional length scale parameter, d_{max} .
- Parametric studies show that error remains high if d_{max} is below 50, with an optimal range for minimal error between 200 and 600.
- Stability requires a sufficiently large number of nodes; for example, at least 40 nodes are needed for moment balance in simply supported beams (length 25nm, width and height 1nm).
- Simulating nonlocal solids requires a support domain large enough to effectively include long range interactions.



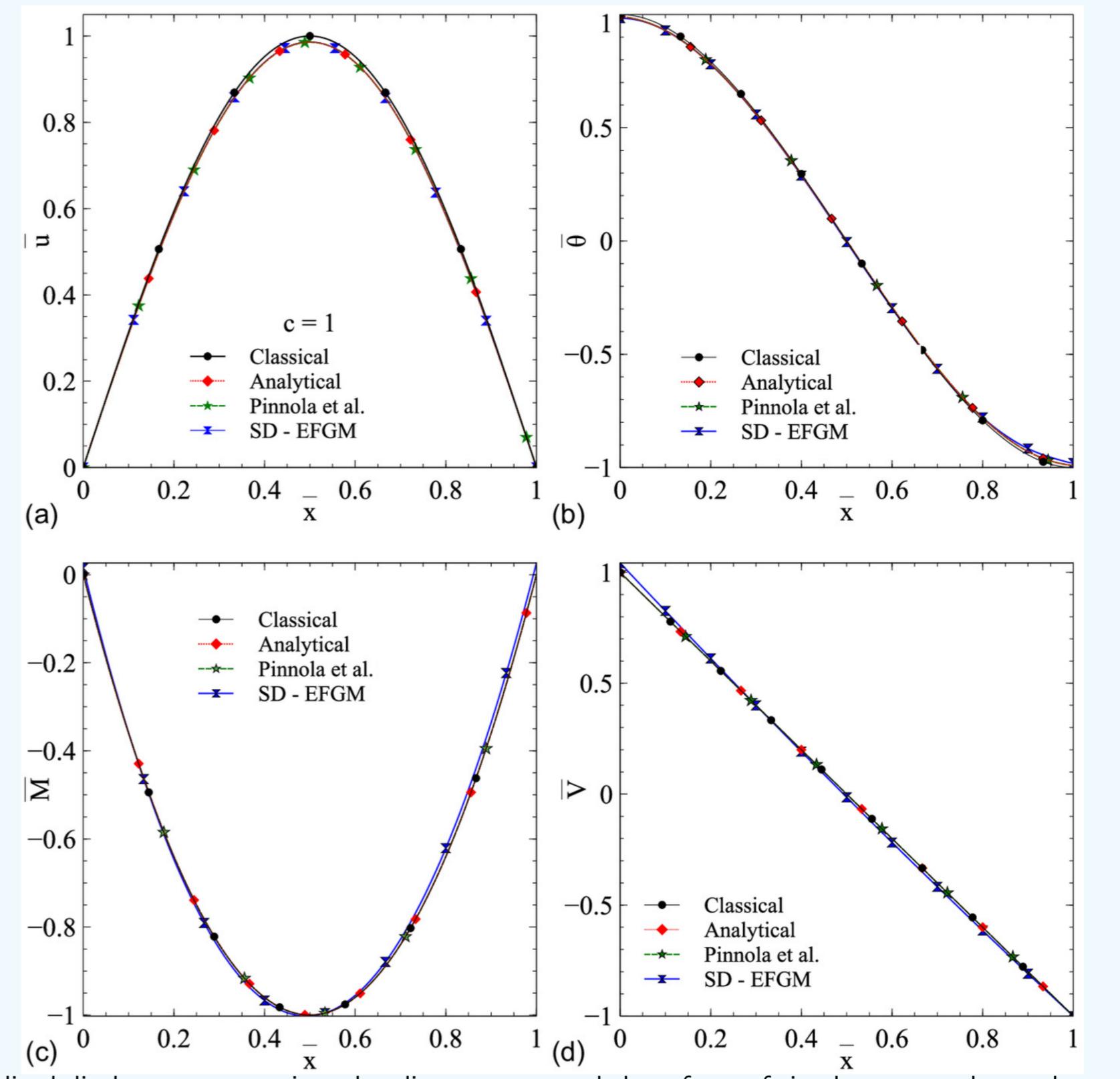
Error in displacement v/s Nondimensional length scale

Numerical Results



Normalized displacement, rotations, bending moment and shear force of cantilever nano-beam

Numerical Results



Normalized displacement, rotations, bending moment and shear force of simply supported nano-beam

Conclusions

- SD-EFGM provides an efficient numerical scheme for the static analysis of nonlocal Bernoulli-Euler beams.
- The displacement shows a consistent stiffening effect as nonlocal parameters increase.
- Accurate nonlocal simulation requires a non-compact support domain—with an optimal nondimensional length scale (d_{max}) between 200 and 600.
- The method accurately captures displacement, slopes, bending moments, and shear forces.
- Provides a robust alternative for element based formulation and offers scalability.

References

- [1] S. L. Akhil, I. R. Praveen Krishna, and M. Aswathy. Effect of non-dimensional length scale in element free Galerkin method for classical and strain driven nonlocal elasto-static problems. *Computers & Structures*, 312:107724, 2025.
- [2] F. P. Pinnola, M. S. Vaccaro, R. Barretta, and F. M. de Sciarra. Finite element method for stress-driven nonlocal beams. *Engineering Analysis with Boundary Elements*, 134:22–34, 2022.
- [3] B. N. Patel and S. M. Srinivasan. Novel nickel foil micro-bend tests and the need for a relook at length scale parameter's numerical value. *Mechanics of Advanced Materials and Structures*, 29(25):3924–3933, 2022.
- [4] G. Romano and R. Barretta. Nonlocal elasticity in nanobeams: the stress-driven integral model. *International Journal of Engineering Science*, 115:14–27, 2017.