Meshfree Method for Stress Driven Beams

Akhil S L, I R Praveen Krishna

Department of Aerospace Engineering,
Indian Institute of Space Science and Technology, Thiruvananthapuram, Kerala

Introduction

Low-dimensional structures with dimensions in the micro-nano range exhibit size-dependent behavior that cannot be captured by local constitutive models. This deviation occurs because local models assume material point interactions are local, whereas size-dependent behavior arises from long-range interactions. While Eringen's strain-driven nonlocal model has been widely used, it often results in ill-posed governing equations and paradoxical results for beam bending. The stress-driven nonlocal approach has emerged as a mathematically consistent and well-posed substitute. This research develops the Element-Free Galerkin (EFG) method for a stress-driven one-dimensional Bernoulli-Euler beam, utilizing its inherent nonlocal solution approximation to accurately simulate size effects.

Scope

A mems resonator ©Bhaskaran et al., ©scitime, a mems device by ©sensing-machines.

Theoretical Formulation and Methodology

Based on stress driven model, nonlocal elastic curvatures $\chi(x)$ are the output of a spatial convolution between a kernel function ϕ and the bending moment $M(t)$:

$$\chi(x) = \int_0^L \phi(x - \overline{x}, c) \frac{M(t)}{EI} dt \qquad (1)$$

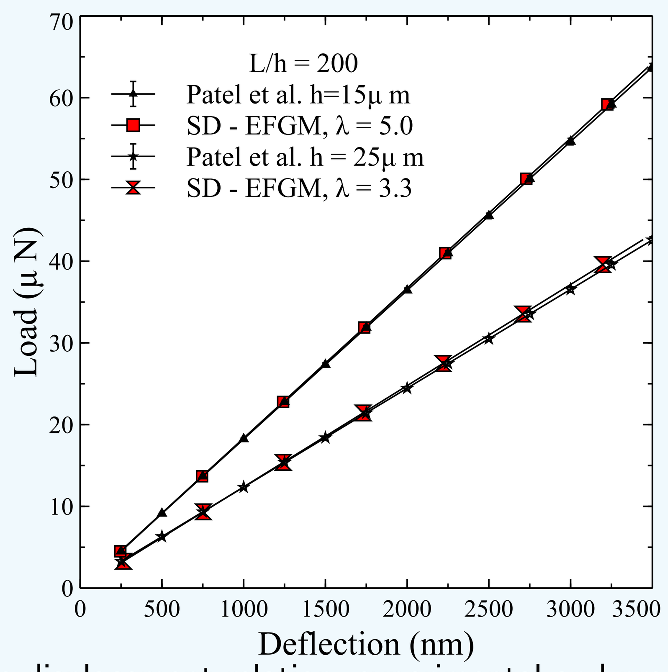
- ☐ Leads to a sixth order governing differential equation.
- ☐ EFG method rely of scattered nodes and moving least square (MLS) approximations for constructing shape functions.
- ☐ A 6th-order spline weighting function is employed to provide the higher-order continuity required for approximating bending moments and shear forces.

Numerical Implementation

- ☐ Essential and constitutive boundary conditions are enforced using a combination of Lagrange Multipliers and Scaled Transformation.
- ☐ This methodology removes the requirement for constitutive continuity conditions found in element-based formulations.
- ☐ The method results in a fully populated stiffness matrix, requiring more computational effort than traditional FEM but offers a more robust tool for nonlocal analysis.

Validation and Parametric Study

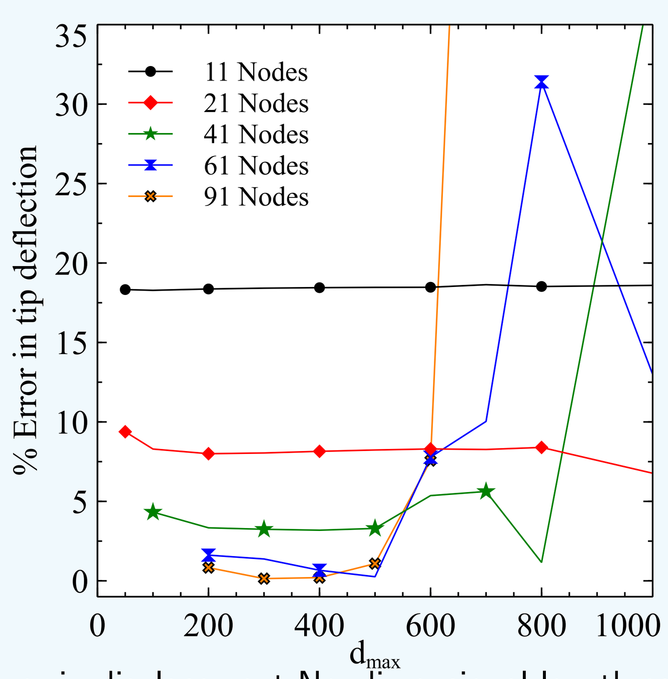
- ☐ The SD-EFGM formulation was validated against experimental results from cantilever microbeam deflection tests.
- ☐ For a beam height $h = 15\mu m$, the model matches experimental data with a nonlocal parameter $\lambda = 5.0$
- ☐ The model correctly predicts a reduction in nonlocal behavior as the beam length increases.
- ☐ Static results for simply supported, cantilever, and fixed-fixed beams under various loads align with analytical stress-driven benchmarks.



Force-displacement relation, experimental and predicted

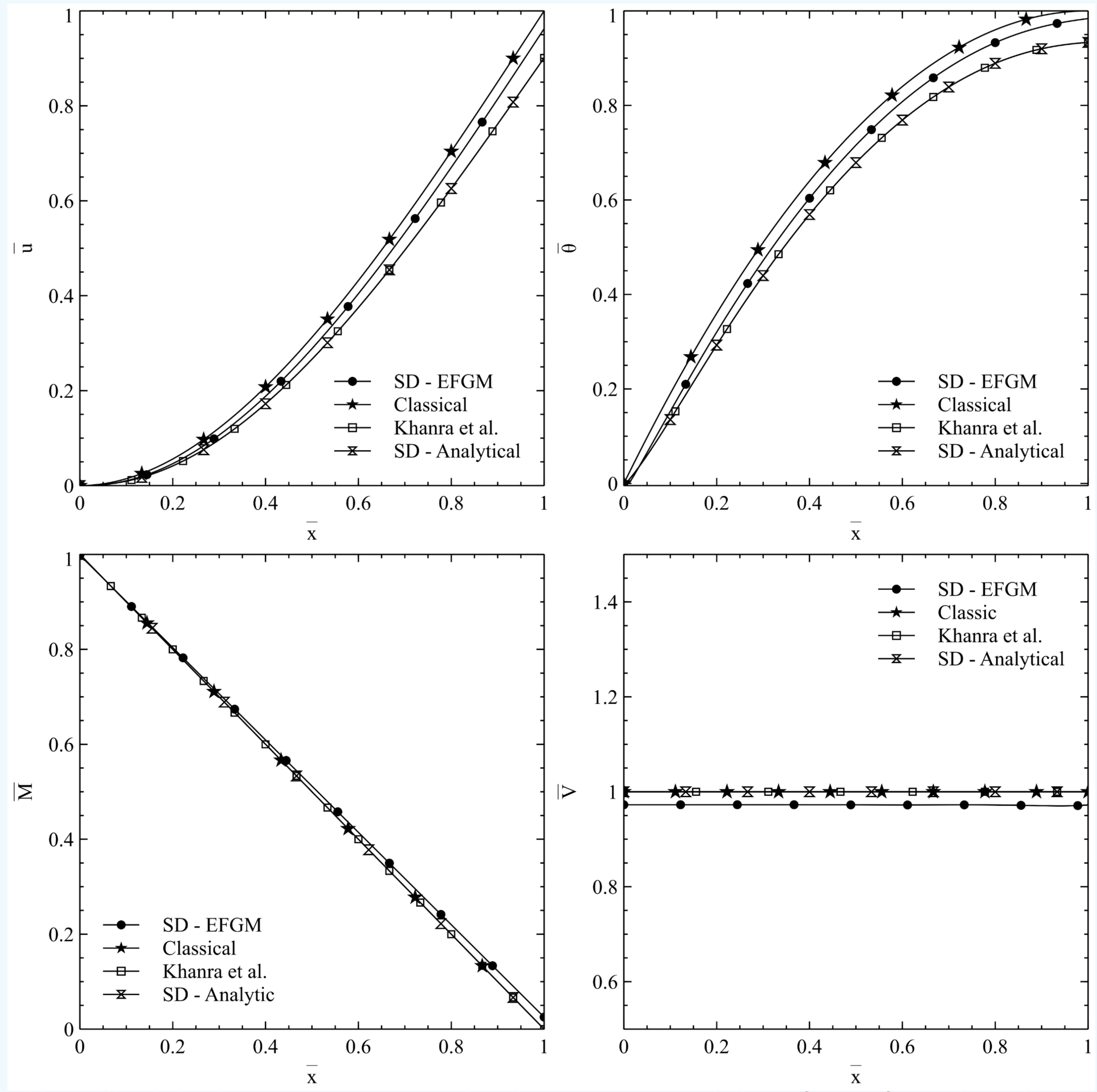
Effect of Nondimensional Length Scale

- ☐ Accuracy is heavily dependent on the local support size, scaled by the nondimensional length scale parameter, d_{max}.
- ☐ Parametric studies show that error remains high if d_{max} is below 50, with an optimal range for minimal error between 200 and 600.
- ☐ Stability requires a sufficiently large number of nodes; for example, at least 40 nodes are needed for moment balance in simply supported beams(length 25nm, width and height 1nm).
- ☐ Simulating nonlocal solids requires a support domain large enough to effectively include long range interactions.



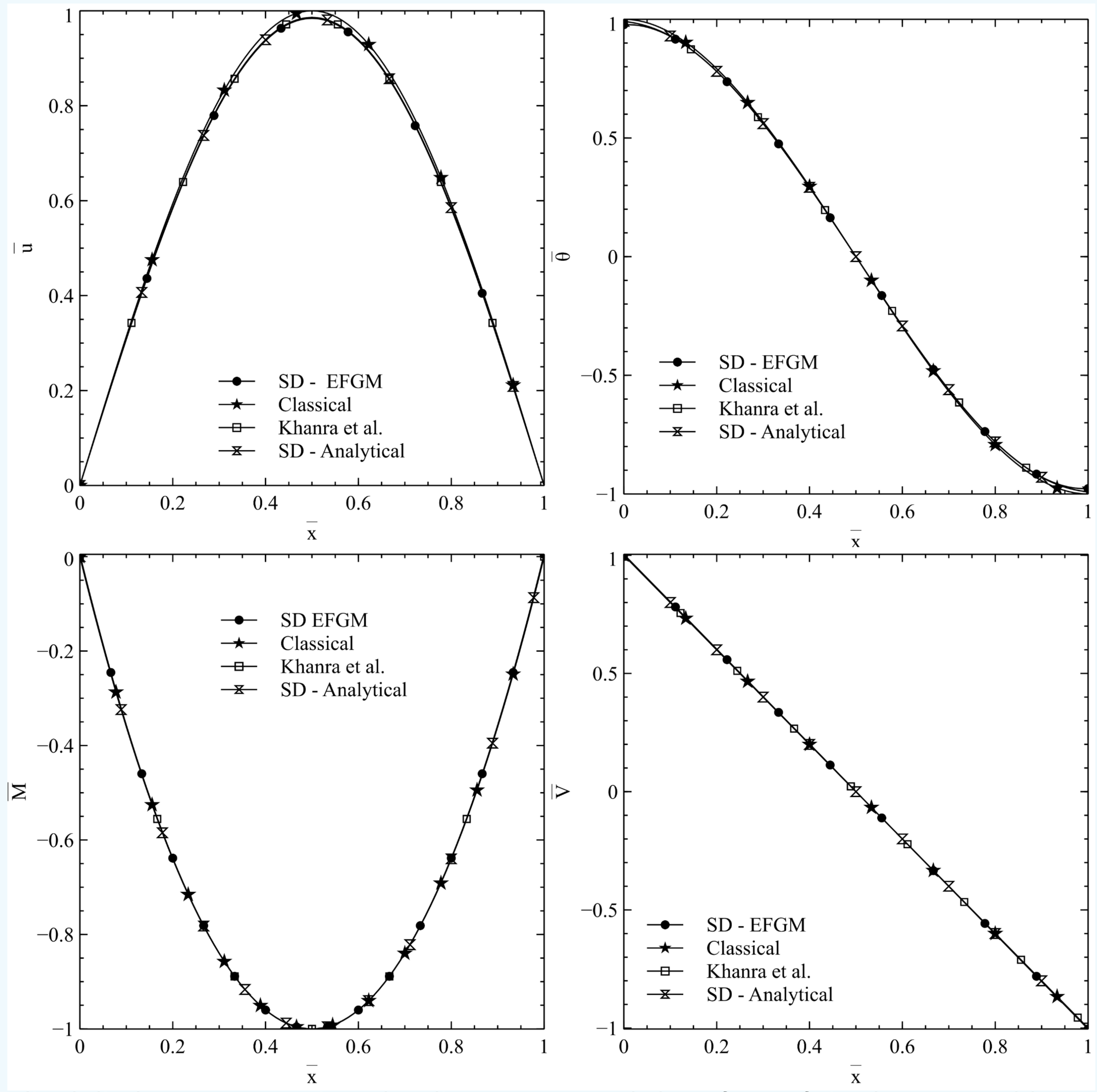
Error in displacement Nondimensional length scale

Numerical Results



Normalized displacement, rotations, bending moment and shear force of cantilever beam

Numerical Results



Normalized displacement, rotations, bending moment and shear force of simply supported beam beam

Conclusions

- ☐ SD-EFGM provides an efficient numerical scheme for the static analysis of nonlocal Bernoulli-Euler beams.
- ☐ The displacement shows a consistent stiffening effect as nonlocal parameters increase.
- ☐ Accurate nonlocal simulation requires a non-compact support domain—with an optimal nondimensional length scale (d_{max}) between 200 and 600.
- ☐ The method accurately captures displacement, slopes, bending moments, and shear forces.
- ☐ Provides a robust alternative for element based formulation and offers scalability.

References

[1] S. L. Akhil, I. R. Praveen Krishna, and M. Aswathy. Effect of non-dimensional length scale in element free Galerkin method for classical and strain driven nonlocal elasto-static problems. *Computers & Structures*, 312:107724, 2025.

[2] S. L. Akhil and I. R. Praveen Krishna. Element-Free Galerkin Method for Elastostatic Analysis of Nonlocal Stress-Driven Bernoulli–Euler Beams. *Journal of Engineering Mechanics*, 151(10):04025050, 2025.

[3] B. N. Patel and S. M. Srinivasan. Novel nickle foil micro-bend tests and the need for a relook at length scale parameter's numerical value. *Mechanics of Advanced Materials and Structures*, 29(25):3924–3933, 2022.

[4] G. Romano and R. Barretta. Nonlocal elasticity in nanobeams: the stress-driven integral model. *International Journal of Engineering Science*, 115:14–27, 2017.