**Methodology: SHAP-Informed Dynamic Factor Regimes for Portfolio Optimization**

**Author**: [Your Name]  
**Institution**: [Your Institution]

**1. Introduction**

This methodology integrates SHAP (SHapley Additive exPlanations) values into a dynamic portfolio optimization framework. The approach identifies market regimes through factor explanation patterns, constructs regime-specific portfolios, and validates performance using novel metrics. The workflow comprises four stages:

1. SHAP dynamics quantification
2. Explanation-driven regime detection
3. SHAP-informed portfolio optimization
4. Performance validation

**2. SHAP Dynamics Quantification**

Convert raw SHAP values into three time-varying metrics:

**2.1 Factor Dominance Momentum (φ)**

Measures the acceleration of a factor’s influence on ETF returns:

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φ\_t(e) = (1/τ) \* Σ\_{k=t-τ}^t I(S\_k(e,f) > 0) - (1/τ) \* Σ\_{k=t-2τ}^{t-τ} I(S\_k(e,f) > 0)

* **I(·)**: Indicator function (1 if true, 0 otherwise)
* **S\_k(e,f)**: SHAP value of factor f for ETF e at time k
* **τ**: Rolling window (e.g., 21 days)

**2.2 Explanation Stability Index (ψ)**

Quantifies consistency of factor explanations:

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ψ\_t(e) = 1 - [Var(S\_k(e,f) from k=t-τ to t) / max\_f Var(S\_k(e,f) from k=t-τ to t)]

* Ranges: [0, 1], where 1 = perfect stability

**2.3 Factor Interaction Matrix (Γ)**

Captures pairwise correlations between factor explanations:

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Γ\_t(e) = Corr(S\_t(e,f\_i), S\_t(e,f\_j)) for all f\_i, f\_j ∈ F

**3. Explanation-Driven Regime Detection**

Market regimes are states of distinct SHAP dynamics.

**3.1 Feature Engineering**

Construct a regime feature vector:

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X\_t = [φ\_t, ψ\_t, vec(Γ\_t)]

**3.2 Hybrid Clustering**

A two-step process:

1. **Dimension Reduction**:

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Z\_t = t-SNE(X\_t) ∈ ℝ²

1. **Density-Aware Clustering**:

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r\_t = argmax\_k [λ·N(Z\_t | μ\_k, Σ\_k) + (1-λ)·DTW(X\_{t-τ:t}, C\_k)]

* + **N(·)**: Gaussian likelihood
  + **DTW(·)**: Dynamic Time Warping distance
  + **λ = 0.6**: Balances spatial vs. temporal patterns

**4. SHAP-Informed Portfolio Optimization**

Construct regime-specific portfolios with explanation-aware constraints.

**4.1 Return Estimation**

Amplify returns using SHAP confidence:

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μ(r) = E[R(r)] ⊙ (1 + mean(S(r)))

**4.2 Risk Adjustment**

Modify covariance with factor interactions:

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Σ(r) = Cov(R(r)) ⊙ exp(Γ(r))

**4.3 Optimization Problem**

For regime r:

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Maximize: wᵀμ(r) - γ₁(wᵀΣ(r)w) + γ₂(Σψ(e)w\_e)

Subject to:

1. w\_e ≥ 0.05 (minimum allocation)

2. Σw\_e ∈ g\_k ≤ 0.4 (cluster diversification)

3. Σw\_e = 1 (full investment)

**5. Performance Validation**

**5.1 SHAP Regime Consistency (SRC)**

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SRC = (1/T) Σ\_{t=1}^T [Tr(S\_tS\_tᵀ) / (||S\_t||\_F ||S\_t|r||\_F)]

**5.2 Explanation Risk Premium (ERP)**

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ERP = (1/N) Σ\_{e=1}^N [E[R\_e|ψ(e) > ψ\_75] / E[R\_e|ψ(e) < ψ\_25] - 1]

**6. Implementation Framework**

**6.1 Data Pipeline**

| **Parameter** | **Value** |
| --- | --- |
| Data Period | 2005–2023 |
| ETFs | 30 DJIA constituents |
| Factors | Fama-French 5 + Momentum |
| Training Window | 252 days |
| Testing Window | 63 days |

**6.2 Workflow**

1. Compute SHAP metrics (φ, ψ, Γ)
2. Detect regimes via hybrid clustering
3. Optimize portfolios per regime
4. Validate using SRC/ERP and traditional metrics

**7. Figures and Tables**

**Figure 1: SHAP Regime Detection Workflow**

Caption: Hybrid clustering combines t-SNE and DTW to identify explanation-driven regimes.

**Table 1: Parameter Settings**

| **Parameter** | **Value** | **Description** |
| --- | --- | --- |
| τ | 21 | Rolling window |
| λ | 0.6 | Spatial-temporal weight |
| γ₁ | 0.5 | Risk aversion |
| γ₂ | 0.3 | Stability weight |

**8. Conclusion**

This methodology advances portfolio management by:

1. Defining regimes through **factor explanation patterns**
2. Incorporating SHAP stability into optimization
3. Introducing **SRC/ERP** for explainability-aware validation