
An Introduction and Tutorial to Bmad and Tao

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1 A Guide for the Perplexed



This tutorial is a guide to introduce the reader to the *Bmad* toolkit for relativistic charged-particle and X-Ray simulations. Additionally, this tutorial serves as an introduction to *Tao* which is a simulation program based upon the *Bmad* toolkit.

It is assumed that you know something about accelerator physics. For example, it is assumed that you know something about beta functions, dispersion and closed orbits. It is also assumed that you are familiar with the basics of Unix and how to use a command line interface.

1.1 Overview: What is Bmad? What is Tao?

Bmad

Bmad is an open-source software library (aka toolkit) for simulating charged particles and X-rays. *Bmad* is not a program itself but is used by programs for doing calculations. The advantage of *Bmad* over a stand-alone simulation program is that when new types of simulations need to be developed, *Bmad* can be used to cut down on the time needed to develop such programs with the added benefit that the number of programming errors will be reduced.

Note: Only the first letter of *Bmad* is capitalized. the rest is lower case.

Over the years, *Bmad* has been used for a wide range of charged-particle and X-ray simulations. This includes:

Lattice design	X-ray simulations
Spin tracking	Wakefields and HOMs
Beam breakup (BBU) simulations in ERLs	Touschek Simulations
Intra-beam scattering (IBS) simulations	Dark current tracking
Coherent Synchrotron Radiation (CSR)	Frequency map analysis

Tao

The disadvantage of *Bmad* is that, as a toolkit, one cannot perform any calculations without first developing a program. To get around this, the *Tao* program was developed. *Tao* is a general purpose simulation program, based upon *Bmad*. *Tao* can be used to view lattices, do Twiss and orbit calculations, nonlinear optimization on lattices, etc., etc. Additionally, *Tao*'s object oriented design makes it relatively easy to extend it. For example, it can be used for orbit flattening in an online machine control system.

1.2 How to use this Tutorial!

This tutorial is meant to be “hands on”. That is, you should be **using Tao** as you read through the chapters. Additionally, it is **expected** (nay, **necessary**) that you **consult** the **Bmad** and **Tao** manuals for information on topics that are only partially covered in this tutorial (and in fact, the exercises at the ends of the tutorial chapters are meant to motivate further reading of the manuals).

2 Orientation

The *Bmad* web site is at:

www.classe.cornell.edu/bmad/

2.1 Prerequisites

Bmad and *Tao* are generally used with Linux or Mac OS-X. Historically, while *Bmad* could be built under Windows, running *Tao* was problematical due to graphics issues. As present, these graphics issues have been resolved.

Except when using a Python interface, *Tao* is accessed through the command line. Therefore you will need to run a terminal program to be able to run *Tao*.

2.2 Bmad Slack Workspace

There is a Slack workspace for *Bmad*. This workspace is a valuable resource if you have questions about anything related to *Bmad* and *Tao*. Sign up is at:

join.slack.com/t/bmad-simulation/shared_invite/zt-flwsmsc3-ITpqJyhRKNwWkZSA6b4LUw

This link allows you to sign up if you have a recognized email domain. If you do not, please contact a *Bmad* maintainer (click the “Help” button in the *Bmad* web site) and an invitation can be issued.

2.3 Minimum Bmad/Tao Version Compatible with Tutorial

This tutorial can be used with any version of *Bmad* and *Tao* dated July 12, 2020 or after (§2). For versions of *Bmad* and *Tao* before this, some difficulties may be encountered.

2.4 Tao Executable Installation

If you have a local *Bmad* Guru, check with your Guru to see if there is a *Tao* executable that you can use which will save you the trouble of installing one.

Installation instructions are posted on the *Bmad* web site. There are two possible routes. One possibility is to use **Conda-forge** (a Python package manager) to download a *Tao* executable. The other is to compile a *Bmad* Release from GitHub (§2.5).

Even if you do use **Conda**, you should also download (but you do not need to compile) a Release (§2.5) since a Release contains many useful lattice examples.

If you compile a **Release**, a useful check that everything is setup is the **accinfo** command which shows the environment variables that have been setup by the compile scripts.

```
> accinfo

DIST_DEBUG=/Users/dcs16/Bmad/bmad_dist_2020_0712-0/debug/bin
DIST_OS=Darwin
DIST_BUILD=TRUE
DIST_F90=gfortran
DIST_UTIL=/Users/dcs16/Bmad/bmad_dist_2020_0712-0/util
DIST_BASE_DIR=/Users/dcs16/Bmad/bmad_dist_2020_0712-0
... etc...
ACC_EXE=/Users/dcs16/Bmad/bmad_dist_2020_0712-0/production/bin
ACC_CMAKE_VERSION=2.8.5
ACC_DEBUG=/Users/dcs16/Bmad/bmad_dist_2020_0712-0/debug/bin
ACC_BUILD_SYSTEM=/Users/dcs16/Bmad/bmad_dist_2020_0712-0/build_system
ACC_PLOT_DISPLAY_TYPE=X
... etc...
```

One common mistake, if you have multiple Releases on your system, is having the environment variables pointing to a Release other than the one you want to use. The `accinfo` command is a quick way to tell if there is a problem.

2.5 Releases

A (GitHub) **Release** is a set of files, including *Bmad* and *Tao* source files, which are used to build the *Bmad*, the *Tao* program, and various other simulation programs. A **CLASSE Release** is like a GitHub **Release** except that it is created on the Linux computer system at CLASSE (Cornell's Laboratory for Accelerator-based Sciences and Education). More information can be obtained from the *Bmad* web site. For the purpose of this tutorial, it is assumed that you are dealing with a GitHub **Release**.

The directories in a Release are:

INSTALL	fftw	lattice	regression_tests
PGPLOT	fgsl	lux	sim_utils
bmad	bmad-doc	forest	mad_tpsa
tao	bsim	gnu_utilities_src	open_spacecharge
util	build_system	gsl	openmpi
util_programs	cpp_bmad_interface	hdf5	plplot
xraylib	examples	lapack	xsif

For an explanation of what directories contain what, see:

<https://wiki.classe.cornell.edu/ACC/ACL/OffsiteDoc#DistDirs>

3 Resources

More information is readily available at the *Bmad* web site:

www.classe.cornell.edu/bmad/

Links to the most up-to-date *Bmad* and *Tao* manuals can be found there as well as manuals for other programs and instructions for downloading and setup. The manuals are at:

www.classe.cornell.edu/bmad/bmad-manual.pdf

www.classe.cornell.edu/bmad/tao-manual.pdf

This tutorial, along with associated lattice files and other examples may be obtained from [GitHub](#) ([§2](#))

```
\url{github.com/bmad-sim/bmad-doc}
```

To keep in touch with the latest *Bmad* developments, there are two mailing lists for *Bmad*. The **bmad-I** mailing list is used to send information on *Bmad* developments. The **bmad-devel-I** mailing list is for programmers. The volume of e-mail is small – Typically less than one a week. Instructions on how to sign up can be obtained from the *Bmad* web page.

3.1 Bmad Directories

A quick introduction to the most important directories in a **Release**:

bmad

The **bmad** directory holds the code for the *Bmad* library.

bsim

The **bsim** directory holds the code for some *Bmad* based programs for simulating synchrotron radiation (programs: **synrad** and **synrad3d**), **dynamic_aperture** (program: **dynamic_aperture**), intra-beam scattering (programs: **ibs_linac** and **ibs_ring**), etc.

debug

The **debug** directory is like the **production** directory except that the executables in the **debug/bin** directory have been compiled with the **debug** flag. These executables can be used with a debugger but, since they typically run much slower than their counterparts in **production/bin**, these executables should not be used for normal running.

examples

The **examples** directory holds example programs.

production

The **production** directory is created when the code in a **Release** is compiled. It contains various libraries, modules, and other files. In particular, the **production/bin** directory contains executable files for *Tao* and other *Bmad* based simulation programs. Note: Many directories will themselves contain a **production** directory. For example, there will be a **bmad/production** and a **tao/production** directory. Do not confuse these directories with the top level **production** directory. The second tier production directories do not contain anything of interest.

tao

The **tao** directory holds the code for the *Tao* program as well as example input files in the **bmad-doc/tao_examples/** directory.

util

The **util** directory holds various build scripts.

util_programs

The **util_program** directory holds the code for various utility programs including programs for converting between *Bmad* lattice files to other formats (MAD, SAD, etc.).

4 Bmad Based Programs

Below is a partial list of simulation and helper programs that are based upon *Bmad*. All of these programs are included in any **Release**. The program executable will be in the **production/bin** directory (§3.1).

Documentation for some of the program can be found at:

www.classe.cornell.edu/bmad/other_manuals.html

Example input files, along with documentation source files and other information, can generally be found in the directory that holds the source code as noted below.

Note: Before running any program, the appropriate environment variables must be setup. [The setup for running programs and the setup for compiling *Bmad* based programs is one and the same.] Ask your local *Bmad* guru about this or consult the *Bmad* web page (§3) for directions.

bmad_to_mad_and_sad

The **bmad_to_mad_and_sad** program converts *Bmad* lattice format files to **MAD8**, **MADX** and **SAD** format.

bbu

The **bbu** program simulates the beam breakup instability in Energy Recovery Linacs (ERLs). The source code is in the directory **bsim/bbu**.

dynamic_aperture

The **dynamic_aperture** program finds the dynamic aperture through tracking. The source code is in the **bsim/dynamic_aperture** directory.

ibs_linac

The **ibs_linac** program simulates the effect of intra-beam scattering (ibs) for beams in a Linac. The source code is in the **bsim/ibs_linac** directory.

ibs_ring

The **ibs_linac** program simulates the effect of intra-beam scattering (ibs) for beams in a ring. The source code is in the **bsim/ibs_linac** directory.

long_term_tracking

The **long_term_tracking_program** is for long term tracking of a particle or beam possibly including tracking of the spin.

lux

The **lux** program simulates X-ray beams from generation through to experimental end stations. The source code is in the **lux** directory.

mad8_to_bmad.py, madx_to_bmad.py

These python programs will convert **MAD8** and **MADX** lattice files to to *Bmad* format. The scripts are in the **util_program/mad_to_bmad** directory

synrad

The **synrad** program computes the power deposited on the inside of a vacuum chamber wall due to synchrotron radiation from a particle beam. The calculation is essentially two dimensional but the vertical emittance is used for calculating power densities along the centerline. Crotch geometries can be handled as well as off axis beam orbits. The source code is in the **bsim/synrad** and **bsim/code_synrad** directories.

synrad3d

The **synrad3d** program tracks, in three dimensions, photons generated from a beam within the vacuum chamber. Reflections at the chamber wall is included. Source code is in the **bsim/synrad3d** and **bsim/code_synrad3d** directories.

tao

Tao is a general purpose simulation program described in §1.1. The source code can be found in the **tao** directory. Documentation is available from the web (§3).

tune_scan

The **tune_scan** program produces data for plotting resonances in the transverse (Q_a, Q_b) tune plane. This is done by adjusting the lattice tunes Q_a and Q_b to some values and then tracking a beam of particles while recording various parameters like the beam size. This is done repeatedly on a grid of points in the (Q_a, Q_b) tune plane.

5 Starting Tao

5.1 Preliminaries

To find out if everything is properly setup. Issue the command

```
> which tao
```

The response should be the location of the *Tao* executable which will look something like:

```
/Users/dcs16/bmad/bmad_dist/production/bin/tao
```

If the **which** command does not find an executable, consult your local *Bmad* Guru. One common problem is not initializing the *Bmad* environment variables (which is the same setup used when programs are to be compiled).

To see if X11 windowing is properly setup, run the **xeyes** program (just type “**xeyes**” on the command line) and a pair of eyes should pop up. Note: **xeyes** is not automatically installed on all systems so if you get an error message about “command not found” you will either need to install **xeyes** or just skip this test completely.

5.2 Tao Startup

To run, *Tao* needs as input a *Bmad* lattice file that describes the machine to be simulated. Lattice files used in this tutorial are available for download from the web. Go to the *Bmad* web site (§3), Follow a link to either the *Bmad* or *Tao* manual page, and there should be a further link to example lattice files. Alternatively, the lattice files are available in any **Release** (§2) in the directory:

```
bmad-doc/tutorial_bmad_tao/lattice_files
```

And each lattice shown in this tutorial lists the lattice file name on the first line.

Note: Other lattice examples can be obtained from the directory:

```
bmad-doc/lattices
```

As an example of how to run *Tao*, the lattice file shown in section §6 will be used. This lattice file is named **simple.bmad**. Make sure that the directory from which you are running *Tao* does not have a file called **tao.init** since this file will affect things (more on that later). Copy **simple.bmad** to your working directory and run *Tao* with the command:

```
> tao -lat simple.bmad
```

(alternatively just supply the full path name to **simple.bmad**.) *Tao* should open a window for plotting as shown in Figure 1. If this window is too large for your screen, you can adjust the size of the plotting window by using the **-geometry** option at startup. Example:

```
> tao -lat simple.bmad -geom 400x400
```

Consult the *Tao* manual for a list of command line arguments or start *Tao* with:

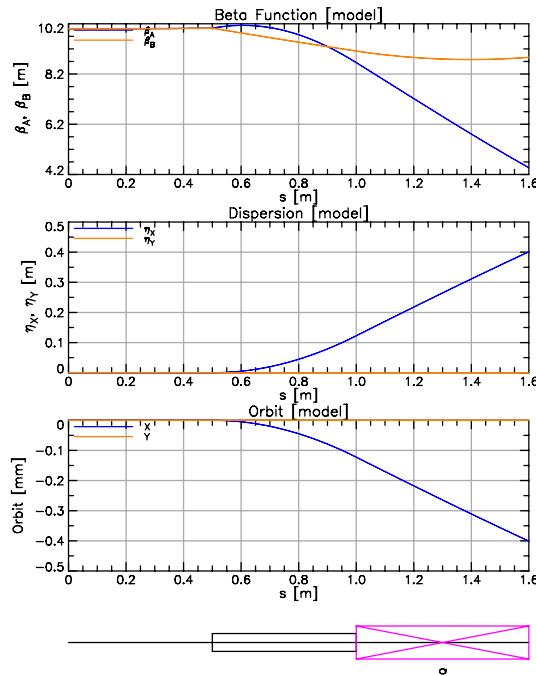


Figure 1: Initial graphics when *Tao* is run with the **simple.bmad** lattice file.

```
> tao -help
```

After initialization, there should be a “**Tao>**” prompt where you can type *Tao* commands.

5.3 Tao: Online Help

When *Tao* is running, to get a list of *Tao* commands, use the **help** command:

```
Tao> help
```

```
Type ‘‘help <command>’’ for help on an individual command
Available commands:
```

alias	read
call	restore
change	reinitialize
clip	run_optimizer
continue	scale
derivative	set
end_file	show
exit	single_mode
flatten	spawn
... etc...	

Note: For brevity's sake, "... etc..." is used when the output has been truncated. Also the output shown is sometimes modified to fit the printed page.

Tao commands are documented in the “**Tao Commands**” (§11) chapter of the *Tao* manual. When running *Tao*, this same documentation can be displayed using the “**help <command>**” command where **<command>** is the name of a command. Example:

```
Tao> help set
```

```
The "set" command is used to set values for data, variables, etc.  
Subcommands are:
```

```
  set beam_init {n@}<component> = <value>  
  set particle_start {n@}<coordinate> = <value>  
  set bmad_com <component> = <value>  
  set space_charge_com <component> = <value>  
  set curve <curve> <component> = <value>  
  set data <data_name>|<component> = <value>  
  set default <parameter> = <value>  
  set element <element_list> <attribute> = <value>  
  set floor_plan <component> = <value>  
  set geodesic_lm <component> = <value>  
... etc...
```

```
When running \tao, to see documentation on any of the subcommands, use the  
help set <subcommand> command. For example, help set element  
will show information on the set element subcommand.  
... etc...
```

Two commands, **set** and **show**, are complicated enough so that they have “subcommands”. The output of “**help set**” and “**help show**” will give you a list of the **set** and **show** subcommands. Once you have a subcommand list, You can then type, for example, “**help set curve**” for help on the **set curve** subcommand.

5.4 Tao Initialization Files

Besides lattice files, *Tao* uses *Tao* specific initialization files for doing such things as configuring plotting and setting optimization parameters. These initialization files are discussed in the **Tao Initialization** chapter of the *Tao* manual.

Besides the initialization files supplied for this tutorial, example *Tao* initialization files can be found in a **Release** in the directory:

```
bmad-doc/tao_examples
```

Tao initialization can be split among several initialization files but there is always one main initialization file that will reference secondary initialization files if needed. The default name for the main initialization is **tao.init**. The main initialization file can be specified using the **-init** option when starting *Tao*.

Tao initialization files use **namelist** input. The general syntax is:

```
&namelist-name
  parameter1_name = value1
  parameter2_name = value2
  ... etc...
/
```

The namelist starts with **&namelist-name** where **namelist-name** is the name of the namelist. The namelist ends with the slash “/” character. In between, there is a set of lines that set appropriate parameter values. Example:

```
&tao_design_lattice
  n_universes = 1
  design_lattice(1)%file = "lat.bmad"
/
```

A detailed discussion of namelist syntax is given in the **Namelist Syntax** (§10.2) section of the **Tao Initialization** chapter of the *Tao* manual.

A **Command file** (§5.5) can be executed automatically at startup. If a command file is used at startup, it is also considered to be an initialization file. The initialization command file, if there is one, will be executed at the end of initialization. By default, a file called **tao.startup**, if it exists in the current directory, is used as the initialization command file. The name of the startup command file may be specified either in the main initialization file or on the command line using the **-startup** option. See section §10.3 for an example.

5.5 Tao Command Files

A **command file** is a file that contain a set of *Tao* commands. A command file can be executed automatically at startup. See Sec. §5.4 for more details. To execute a command file from the *Tao* command line, use the **call** command. Example:

```
Tao> call my_cmd_file -no_calc
```

This calls a command file called **my_cmd_file**. To speed up processing, the **-no_calc** optional argument is used here to suppresses computations during the running of the command file.

Command files can call other command files. If using relative path names, *Tao* will look for the second command file relative to the directory of the first command file.

Parameters may be passed to a command file. For example the command

```
Tao> call my_cmd_file abc def
```

will pass the argument “abc” and “def” to the command file. In the command file itself, the parameters can be referred to using “[1]” for the first argument (here “abc”), “[2]” for the second argument (here “def”), etc. Thus in the command file there might be the line:

```
show ele [[2]]
```

which, when executed, would be translated to

```
show ele def
```

Loops can be put constructed in a command file. For example, in a command file the following will print information on the first ten elements in the lattice:

```
do ix = 1, 10
    show ele [[ix]]
enddo
```

If an **end_file** command is encountered in a command file, everything below this command is ignored. Also a **pause** command can be used to temporarily stop a command file. See the **Command File and Aliases** section (§2.6) of the *Tao* manual for more details.

5.6 Exercises [Answers in section §25.1]

- 5.1 Create an initialization file named **tao.init** (which is the default name for the main initialization file) with a **tao_design_lattice** namelist that specifies **simple.bmad** as the lattice file. With this, you should be able to start *Tao* without the **-lat** option.
- 5.2 Create a command file that runs the command **help <argument1>** where **<argument1>** is the (first) argument when the command file is run using the "call" command. For example, if the command file is called "doit", the command "call doit flatten" should run the command "help flatten". See the section "Command Files and Aliases" (§2.6) in the *Tao* manual.
- 5.3 In your **tao.init** initialization file from the first exercise, put in a **tao_start** namelist (See §10.3 "Beginning Initialization" in the *Tao* manual) that sets the **startup_file** parameter of this namelist to the name of the command file from the second exercise. What happens when you start *Tao*?

6 Introduction to Bmad Lattices

The basis of any *Bmad* based simulation is a lattice file. The following is a simple example:

```
! Lattice file: simple.bmad
beginning[beta_a] = 10.    ! m a-mode beta function
beginning[beta_b] = 10.    ! m b-mode beta function
beginning[e_tot] = 10e6   ! eV Or can set beginning[p0c]

parameter[geometry] = open           ! Or closed
parameter[particle] = electron      ! Reference particle.

d: drift, L = 0.5
b: sbend, L = 0.5, g = 1, e1 = 0.1, dg = 0.001 ! g = 1/design_radius
q: quadrupole, L = 0.6, k1 = 0.23

lat: line = (d, b, q)           ! List of lattice elements
use, lat                         ! Line used to construct the lattice
```

Note: Part I of the *Bmad* manual covers lattice syntax so please refer to that for information that is not covered here.

Some comments on the above lattice:

beginning[...], parameter[...]

Global parameters (parameters of the lattice that are not associated with any one given lattice element) can be set using constructs like **beginning[...]**, **parameter[...]**, **particle_start[...]**, etc. See the **Lattice File Global Parameters** chapter in the *Bmad* manual for a complete list of parameters that can be set.

parameter[geometry]

The **parameter[geometry]** parameter sets whether the geometry of the lattice is considered **open** (like a 1-pass accelerating linac) or **closed** (like a storage ring). If the lattice is closed, the closed orbit is used as the reference orbit and the computed beta functions correspond to the periodic solution. If the lattice is **open**, the orbit and beta functions are computed using the beginning orbit and beta functions as set in the lattice. The default geometry is **open** if the lattice contains an **Icavity** (linac accelerating RF cavity) element and is **closed** if no **Icavity** is present.

beginning[beta_a], beginning[beta_b]

The **beginning[beta_a]** and **beginning[beta_b]** parameters set the beta functions at beginning of the lattice. The beginning beta functions will be only used by *Bmad* if the lattice geometry is set to **open**. Note: Some programs will use the labels **beta_x** and **beta_y** for the beta functions. This is inaccurate since the beta functions are associated with the normal modes of oscillation of the beam and, if there is horizontal/vertical coupling, the normal modes will not correspond to purely horizontal and purely vertical motion. By convention, in the limit of no coupling, the **a**-mode will correspond to horizontal oscillations and the **b**-mode will correspond to vertical oscillations.

beginning[e_tot]

The **beginning[e_tot]** parameter sets the reference total energy at the beginning of the lattice. Alternatively, **beginning[p0c]** can be used to set the reference momentum at the beginning of the lattice.

parameter[particle]

The **parameter[particle]** parameter sets the reference particle. Besides fundamental particles, ions can be used. For example, the reference particle could be set to **#12C+3** which is triply charged carbon-12. The default reference particle is a positron.

d: drift, ..., b: sbend, ..., q: quad, ...

The lattice element named **d** is a drift element. That is, a field free region. The element named **b** is a dipole bend and the element named **q** is a quadrupole element. See the **Elements** chapter in the *Bmad* manual for more details.

lat: line = (...)

A line consists of an ordered list of elements that the beam goes through. **lines** are used to define the ordered list. In this instance, a line called **lat** contains the elements **d**, **b**, and **q** in that order. See the “**Beam lines and Replacement Lists**” chapter of the *Bmad* manual for more details.

use, lat

The **use** statement in a lattice file identifies the particular line used to construct the lattice. This is needed since a lattice file may define multiple lines and lines can contain sub-lines, etc. In some cases, a **use** statement may reference have more than one line. More on that later.

6.1 Exercises [Answers in Section §25.2]

- 6.1 Element **b** is an **sbend**. What exactly is an **sbend**? And what is the meaning of the **g** and **dg** parameters of element **b**? Hint: Information on **sbend** elements can be found in the **Bends: Rbend and Sbend** section §4.5 of the **Elements** chapter of the *Bmad* manual.
- 6.2 Make a simple closed geometry “FODO” lattice with a single cell: [drift, quad1, drift, quad2]. Set both drifts to have the same length and both quadrupole to have the same length (does not have to be the same as the drift length). Set the quadrupole **k1** value of **quad2** to have the opposite sign as **quad1** and adjust the **k1** values so that the machine is stable.
- 6.3 Make a FODO lattice with ten of these cells. [Hint: Make the “**lat**” line a subline of another line and use a repetition count as explained in section §7.2 “Beam Lines and Lattice Expansion” of the *Bmad* manual.]

7 Tao Show Command

The **show** command of *Tao* is used to display information on anything from the makeup of lattice elements to particle positions within a tracked particle beam. This chapter gives an introduction to using the **show** command.

Start *Tao* as explained in section §5.2 with the lattice file **simple.bmad**. The **show** command has a large set of subcommands. To see the list of subcommands, use the **help show** command:

```
Tao> help show
The "show" command is used to display information.
Format:
  show {-append <file_name>} {-noprint} {-no_err_out} <subcommand>
  show {-write <file_name>} {-noprint} {-no_err_out} <subcommand>

"<subcommand>" may be one of:
  alias           ! Show aliases .
  beam ...        ! Show beam info .
  branch ...      ! Show lattice branch info .
  building_wall   ! Show building wall info .
  ... etc...
  wall ...        ! Show vacuum chamber wall info .
  wave            ! Show wave analysis info .
```

The "show" command has "-append" and "-write" optional arguments which can be used to write the results to a file. The "show -append" command will ...

For example, to see information on using the **show branch** subcommand, use the command **help show branch**.

The output from the **show** command may be piped to a file or appended to an existing file using the **-append** or **-write** switches which must appear before **<subcommand>** on the command line. For example:

```
Tao> show -write abc.dat lattice
```

This will dump lattice information to a file named **abc.dat**.

7.1 Showing a List of Elements in the Lattice

To show the elements in the lattice use the **show lattice** subcommand:

```
Tao> show lat
# Values shown are for the Exit End of each Element:
# Index    name      key          s      l     beta     phi ...
#                   a          a ...
#               0 BEGINNING Beginning_Ele    0.000    ---   10.00   0.000 ...
#               1 D          Drift       0.500    0.500  10.03   0.050 ...
#               2 B          Sbend      1.000    0.500   7.87   0.104 ...
#               3 Q          Quadrupole 1.600    0.600   3.50   0.217 ...
```

```

4 END      Marker          1.600  0.000  3.50  0.217 ...
# Index   name      key           s      l    beta    phi ...
#
# Values shown are for the Exit End of each Element:

```

The **s** column shows the longitudinal position from the beginning of the lattice (§14.1) and the **l** column shows the length of the elements.

Comparing the output of the **show lattice** command to the **simple.bmad** lattice file, notice that *Bmad* adds two extra elements to the lattice. A zero length beginning element called **BEGINNING** and a zero length marker element at the end called **END**.

Each element has an index number associated with it. the **BEGINNING** element has index zero, the next element has index 1, etc. Generally an element can be referred to in a command by its name or its index.

The “**show lat**” command, like many other commands, has optional parameters to customize the table of information printed:

```

Tao> help show lat

Syntax:
  show lattice {-0undef} {-all} {-attribute <attrib>} {-base}
            {-blank_replacement <string>} {-branch <name_or_index>}
            {-custom <file_name>} {-design} {-floor_coords} {-lords} {-middle}
            {-no_label_lines} {-no_tail_lines} {-no_slaves} {-orbit}
            {-remove_line_if_zero <column #>} {-s <s1>:<s2>} {-tracking_elements}
            {<element_list>}

Show a table of Twiss and orbit data, etc. at the specified element locations.
The default is to show the parameters at the exit end of the elements.
... etc...

```

For example, the **-attribute** switch makes it easy to make a custom table of element attributes:

```

Tao> show lat -no_tail_lines -attrib b1_gradient
Values shown are for the Exit End of each Element:
Index   name      key           s      l    b1
                                         gradient
0 BEGINNING Beginning_Ele     0.000    ---    ---
1 D        Drift          0.500  0.500    ---
2 B        Sbend          1.000  0.500  0.0000E+00
3 Q        Quadrupole     1.600  0.600 -7.6620E-03
4 END      Marker         1.600  0.000    ---

```

When using the **-attribute** switch the first five columns are fixed and additional columns are specified by each instance of **-attribute** appearing on the command line. In this example there is one additional column showing the **b1_gradient** element attribute. The string — is printed for elements that do not have this attribute (The string used when an element does not have a particular attribute may be changed using the appropriate **show lattice** switches).

7.2 To Show the Attributes of a Lattice Element

Use the **show element** command to show the attributes of a particular lattice elements. Example:

```
Tao> show ele b ! Or "show ele 2" since element B has index 2.
```

```
Element #          2
Element Name: B
Key: Sbend
Sub Key: SBend
S_start, S:    0.500000,      1.000000
Ref_time:   3.340005E-09

Attribute values [Only non-zero/non-default values shown]:
 1  L           = 5.0000000E-01 m
 6  G           = 1.0000000E+00 1/m
 7  DG          = 1.0000000E-03 1/m
 8  RHO         = 1.0000000E+00 m
10  FRINGE_TYPE = Basic_Bend (7)
11  FRINGE_AT  = Both_Ends (3)
12  HIGHER_ORDER_FRINGE_TYPE = None (1)
13  SPIN_FRINGE_ON = T (1)
14  EXACT_MULTIPOLES = Off (1)
19  E1           = 1.0000000E-01 rad
29  L_SAGITTA   = 3.1087578E-02 m
... etc...

Twiss at end of element:
      A           B           Cbar     ...
Beta (m)    8.65422245  9.11594461 | 0.00000000 ...
Alpha       3.56155250  0.86569936 | 0.00000000 ...
Gamma (1/m) 1.58126929  0.19190939 | Gamma_c = ...
Phi (rad)   0.10144612  0.10228316 | X        ...
Eta (m)    0.12252488  0.00000000 | 0.12252488 ...
Etap        0.47990496  0.00000000 | 0.47990496 ...

Orbit: Electron State: Alive
      Position[mm] Momentum[mrad]      Spin | ...
X:      -0.12240995  -0.47942554 | t_particle [sec]: ...
Y:      0.00000000  0.00000000 | t_part-t_ref [sec]: ...
Z:      0.02055389  0.00000000 | (t_ref-t_part)*Vel ...
```

Note: By default, only non-zero attributes are shown. Use the **-all** option to see all the attributes.

7.3 Using Wild Cards in Element Names

Wild card characters can be used with element names. The wild card characters are:

```
"*" -- Matches to any number of characters (including zero).  
 "%" -- Matches to any single character.
```

The general syntax is:

```
<name_with_wild_card_characters>           ! or  
<element_type>::<name_with_wild_card_characters>
```

where **<element_type>** is the type of element (**drift**, **quadrupole**, etc.). [Actually the syntax is a bit more complicated than this. See the “**Matching to Lattice Element Names and Other Attributes**” section (§3.6) in the *Bmad* manual.]

In the examples below a more complicated lattice is used. Start *Tao* with:

```
> tao -lat <path-to bmad-doc>/tao_examples/cesr/bmad_L9A18A000_-_MOVEREC.lat
```

For example, to show all elements whose name begins with "Q" use the **show element** command:

```
Tao> show ele q*  
  
     870  Q00W                                2.163  
      5  Q00W\CLEO_SOL                      1.755  
      6  Q00W#1                             2.163  
     10  Q01W                                3.874  
     14  Q02W                                5.932  
... etc...  
Number of Matches: 110
```

Or to show all sbend elements the command is:

```
Tao> show ele sbend::*  
  
     22  B03W                                11.949  
     24  B03AW                               13.970  
     30  B04W                                18.653  
     38  B05W                                23.698  
     46  B06W                                28.743  
... etc...  
Number of Matches: 84
```

Element names with wild cards can also be used with the **show lattice** command. For example:

```
Tao> show lat q%1*  
  
# Values shown are for the Exit End of each Element:  
# Index   name    key          s      l     beta      phi ...  
#                                     a      a
```

```

10  Q01W Quadrupole      3.874  0.950  66.94  0.173
100 Q11W Quadrupole     67.129  0.600   4.57  1.100
178 Q21W Quadrupole    149.739  0.600   7.35  2.145
266 Q31W Quadrupole    235.539  0.600   5.30  3.195
... etc...
# Index  name   key           s      l     beta   phi ...
#                               a      a
# Values shown are for the Exit End of each Element:

```

Note: When **rbend** (rectangular bend) elements are read in, internally they are converted to **sbend** (sector bend) elements. Thus, a search for **sbend** elements will include all **rbend** elements.

7.4 Exercises [Answers in Section §25.3]

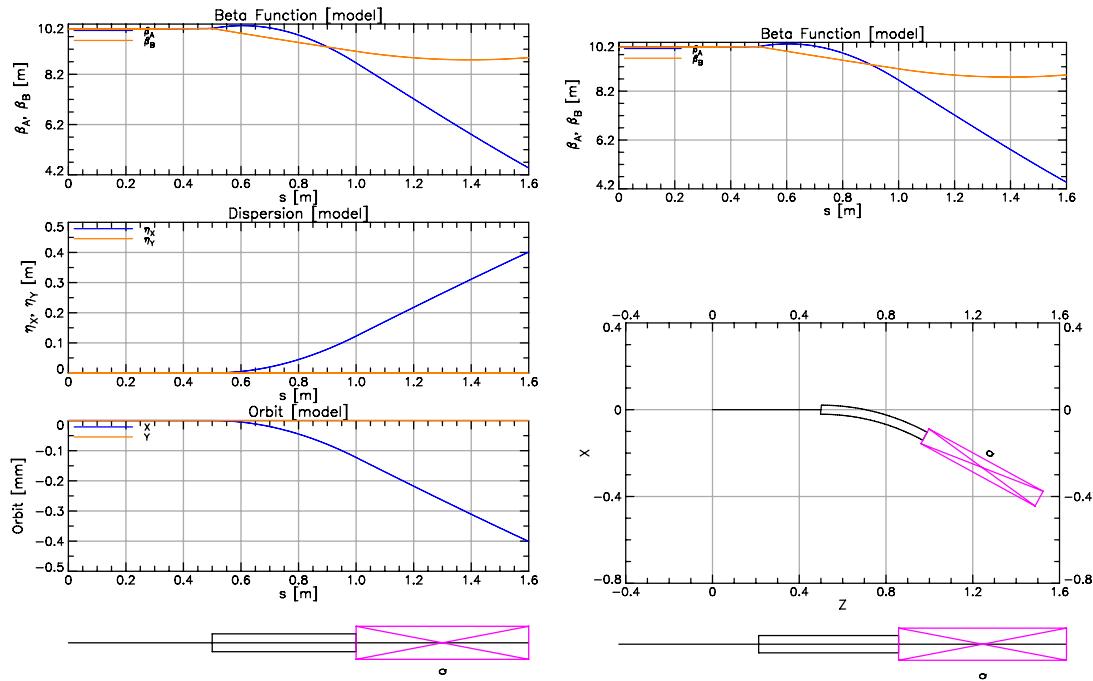
- 7.1 Lattice elements have string attributes named **type**, **alias** and **descrip**. These parameters can be used for sorting elements. Modify any lattice so that, say, elements have a non-blank **alias**. Open *Tao* with this lattice and use the **show element** command to, say, search for all elements whose **alias** attribute begins with the letter “z”. Hint: See the “**Matching to Lattice Element Names and Other Attributes**” section (§3.6) in the *Bmad* manual.
- 7.2 Start *Tao* with a lattice with, say, multiple elements named **q** and determine how to show the second element in the lattice with the name **q**. Hint: See the “**Matching to Lattice Element Names and Other Attributes**” section (§3.6) in the *Bmad* manual.
- 7.3 Explore using other **show** commands. For example, what does **show universe** show? How about **show global**?
- 7.4 What is the command to show a list of prior commands that you have typed in?
- 7.5 After looking at the documentation for “**show lattice**”, construct a “**show lattice**” command with the appropriate arguments so that there is a column showing the value of the quadrupole “**k1**” for the elements.
- 7.6 Construct a “**show lattice**” command with the appropriate arguments so that only rows for **sbend** elements are shown.
- 7.7 Look at the output of “**show ele -all q**” and “**show ele q**” and observe the differences.

8 Introduction to Plotting in Tao

First: Start *Tao* as explained in section §5.2 with the lattice file **simple.bmad**.

The default is to have three plots as shown in Figure 2a: beta, dispersion, and orbit, along with what is called a **lat_layout** plot situated at the bottom of the window that graphically shows the lattice elements as a function of longitudinal position. Note: If you do not want *Tao* to display the plot window, use the **-noplots** option on the command line when you start *Tao*.

Plotting is described in the **Plotting** chapter in the *Tao* manual and the setup of custom plots is described in the **Initializing Plotting** (§10.13) section of the **Tao Initialization** chapter.



(a) Initial graphics when *Tao* is run with the `simple.bmad` lattice file.

(b) Graphics after a `place r22 floor_plan` command.

Figure 2: Example *Tao* graphics.

8.1 Nomenclature

A given “**plot**” has a number of “**graphs**” associated with it. A **graph** consists of horizontal and vertical axes (which may or may not be displayed) along with a number of “**curves**”. A **curve** is a set of (x, y) points. These curve points can be plotted by drawing a symbol at each point or the points can be connected together using line segments. For the curves in Figure 2a, *Tao* calculates enough points per curve so that the line drawn to connect the points looks smooth. Some graphs, like the **lat_layout** do not have any associated curves.

In Figure 2a, all four plots have exactly one associated graph. The **lat_layout** graph does not display its axes and has no associated curves. The other graphs each have two associated curves.

8.2 Displaying a Plot

To change the plots that are being displayed, you have to tell *Tao* what plot to draw and where to draw it. The **show plot -templates** command prints the list of plots that can be displayed:

Tao> show plot -templates	
Templates:	
Plot	Description
alpha	Twiss alpha function
b_div_curl	Magnetic Field Divergence and Curl.
b_field	Magnetic Field Along Orbit
beta	Twiss beta function
bunch_sigma_xy	Bunch transverse sigmas
bunch_R1_R2	Bunch phase space plot.
cbar	Cbar coupling matrix
dbeta	Chromatic normalized beta beat
... etc...	

The output of **show plot -templates** shows plot “**templates**”. A plot template specifies the parameters needed to draw the plot: what is to be plotted, the number of associated graphs, the x and y-axis scales, colors to be used for the curves, etc. *Tao* defines a set of default templates and custom ones can be defined by constructing the appropriate initialization file. Directions for constructing templates are given in the **Initializing Plotting** (§10.13) section of the **Tao Initialization** chapter of the *Tao* manual and an example of how to construct a template plot is given in Section §11.6 below.

To see where in the plot templates can be placed in the plotting window, use the **show plot** command without any additional arguments:

Tao> show plot		Location on Page				
Plot Region	<-->	Plot	x1	x2	y1	y2
-----	-----	-----	-----	-----	-----	-----

```

layout          <--> lat_layout      0.00  1.00  0.00  0.15
r11           <-->                  0.00  1.00  0.15  1.00
r12           <-->                  0.00  1.00  0.58  1.00
r22           <-->                  0.00  1.00  0.15  0.58
r13           <--> beta            0.00  1.00  0.72  1.00
r23           <--> dispersion       0.00  1.00  0.43  0.72
r33           <--> orbit           0.00  1.00  0.15  0.43
r14           <-->                  0.00  1.00  0.79  1.00
... etc...

```

The output of the command shows a list of plot “**regions**” along with what plot (if any) is associated with a given region. A plot region is a rectangular box within the plot window into which a plot can be placed as illustrated in Figure 3. With the present example, there are four regions that have a plot. For example, The **r13** region has a **beta** plot.

The position of a region within the plotting window is determined by four numbers as shown in Figure 3. **x1** and **x2** determine the horizontal position with a value of 0.0 corresponding to the left border edge (which is a distance **x1b** from the window edge) and 1.0 corresponding to the right border edge (which is a distance **x2b** from the window edge). Similarly, the **y1** and **y2** numbers determine the vertical position with 0.0 corresponding to the bottom border edge and 1.0 corresponds to the top border edge.

Values for **x1**, **x2**, **y1**, and **y2** are given in the right most 4 columns of the output of the **show plot** command. In the present case, for example, the **r13** region has **x1** = 0 and **x2** = 1 so the plot occupies the full horizontal width of the page. See the **Initializing Plotting** (§10.13) section of the **Tao Initialization** chapter of the *Tao* manual for more details.

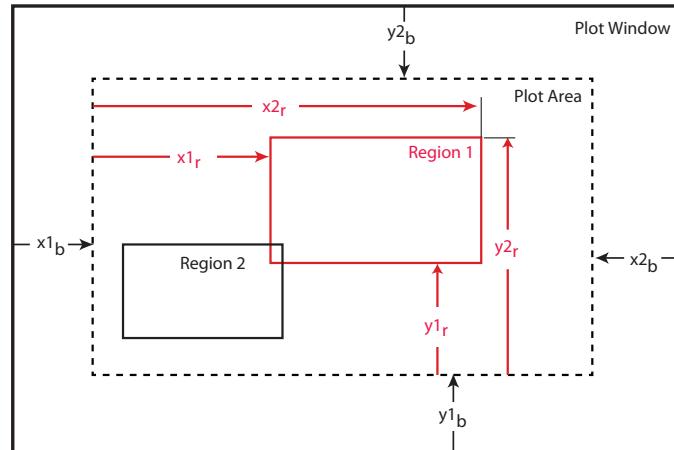


Figure 3: The plot window is divided up into a number of rectangular **regions** into which a plot **template** can be placed. Regions can overlap but if a plot is placed in a given region, plots in any other region that overlap are cleared. The **border**, within which regions are placed, is displaced from the edge of the window by distances **x1b**, **x2b**, **y1b**, and **y2b**.

The **place** command is used to place a **template** plot into a plot **region**. Example:

```
Tao> place r22 floor_plan
```

This places a **floor_plan** plot in the **r22** region (the **floor_plan** plot draws a bird's eye view of the machine). The result is shown in Figure 2b. Plots associated with regions that overlap the region that is used in the **place** command are erased.

Example place commands:

```
Tao> place r22 none      ! Clear a region
Tao> place * none       ! Clear all regions
```

Note: Plots of the same type can be placed in multiple regions. For example, there could be multiple orbit plots displayed with each plot, say, having differing x-axis scaling.

8.3 Scaling Plots

Plots can be scaled vertically using the **scale** command:

```
Tao> scale beta 0 20   ! Set y_min = 0, y_max = 20.
Tao> scale r13 0 20    ! Same as above if the beta plot is in the r13 region.
Tao> scale beta        ! Tao will calculate nice bounds.
Tao> scale all         ! "all" = all plots.
Tao> scale              ! Same as "scale all".
```

The **x_scale** command can be used to scale the horizontal axis and the **xy_scale** command can be used to simultaneously scale the x and y axis (used for **floor_plan** plots).

```
Tao> x_scale beta 0.8 1.0   ! Scale horizontal axis
Tao> xy_scale floor_plan   ! Combined scale and x_scale.
```

8.4 Getting Information on a Plot

To see the parameters associated with a given plot use the command “**show plot**” with the name or region of the plot. Example:

```
Tao> show plot beta  ! or "show plot r13" is equivalent.

Plot:  beta
Region:  r13
Visible          = T
Location [x1,x2,y1,y2] = .000  1.000  .717  1.000
x_axis_type      = s
... etc...
x%draw_label     = T
x%draw_numbers   = T
autoscale_x      = F
```

```

autoscale_y      = F
autoscale_gang_x = T
autoscale_gang_y = T
n_curve_pts      = -1
Graphs:
    g

```

The output shows that the **beta** plot is associated with the region **r13**. Also the output shows that there is one associated graph called “**g**”. This graph can be referred to as “**beta.g**” or “**r13.g**”. To see parameters of the **beta** template plot, use a “**T::**” prefix. In this case the the command would be “**show plot T::beta**”.

To display information on a graph use the command **show graph**. Example:

```

Tao> show graph beta

Region.Graph: r13.g
Plot.Graph:   beta.g
type          = data
title         = Beta Function
title_suffix  = [model]
component     = model
margin        = 0.15   0.06   0.12   0.12 %BOX
scale_margin  = 0.00   0.00   0.00   0.00 %GRAPH
... etc...
y%max        = 1.02000000E+01
y%min        = 4.20000000E+00
y%major_div  = 3
y%major_div_nominal = 4

Curves:
    a
    b

```

Here the “**show graph beta**” command works since there is only one graph associated with the beta plot. The output shows that the graph has two associated curves called **a** and **b**. The **a** curve can be referred to as “**beta.g.a**” or **r13.g.a**” with similar names for the **b** curve.

To display information on curves use the “**show curve**” command. using the “**-line**” option with this command will display the set of points that are used to draw the curve:

```

Tao> show curve -line beta
Region.Graph.Curve: r13.g.a
                    r13.g.b
Plot.Graph.Curve:   beta.g.a
                    beta.g.b
data_source       = lat
... etc...
draw_symbol_index = F
smooth_line_calc = T
line%width        = 2

```

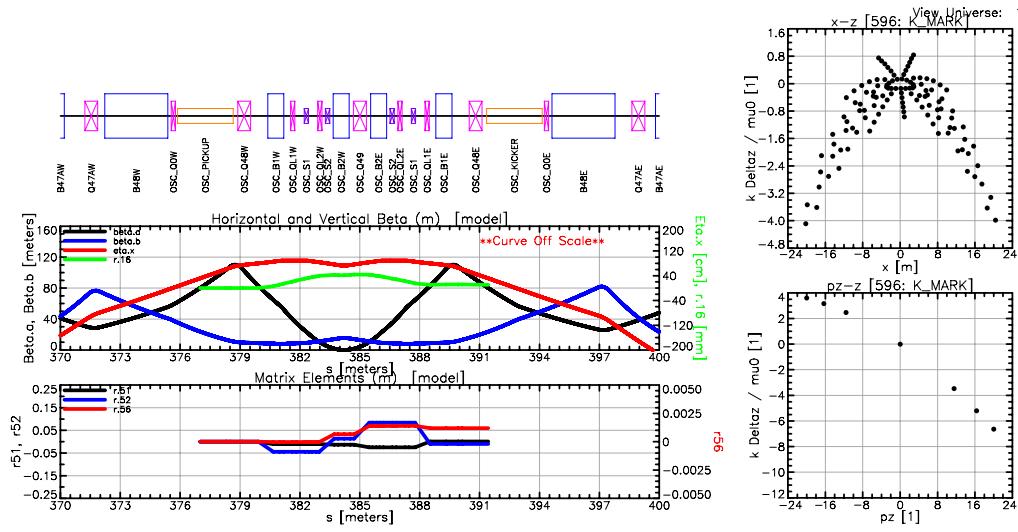


Figure 4: Example of what can be done with custom plotting.

```

line%color      = 4  Blue
line%pattern    = 1  solid
... etc.....

# Smooth line points:
# index          x-axis           a             b
1   0.000000E+00  1.000000E+01  1.000000E+01
2   4.000000E-03  1.000000E+01  1.000000E+01
3   8.000000E-03  1.000001E+01  1.000001E+01
4   1.200000E-02  1.000001E+01  1.000001E+01
... etc....

```

8.5 Changing Plot Parameters

The `set` command can be used to change plot parameters. The various subcommands that affect plotting are

Set command	Corresponding Show cmd	Notes
<code>set plot</code>	<code>show plot</code>	Set plot parameters
<code>set graph</code>	<code>show graph</code>	Set graph parameters
<code>set curve</code>	<code>show curve</code>	Set curve parameters
<code>set plot_page</code>	<code>show plot -plot_page</code>	Set global plot parameters.
<code>set lat_layout</code>	<code>show plot -lat_layout</code>	Set lat_layout shapes
<code>set floor_plan</code>	<code>show plot -floor_plan</code>	Set floor_plan shapes

8.6 Custom Plotting

The default template plots that *Tao* defines are sufficient for many purposes but at times you may want to define your own. Custom plotting is out of the scope of this tutorial but the reader is referred to the section on **Initializing Plotting** (§10.13) in the **Initializing Tao** chapter of the *Tao* manual. Figure 4 shows an example of what can be done.

8.7 Exercises [Answers in Section §25.4]

- 8.1 Try placing other plot templates onto the plot page, such as **b_field**.
- 8.2 Use the **set** command to set the **draw_symbols** curve logical for the curves in the **beta** plot to True. What does the **beta** plot look like now?
- 8.3 Start from Figure 2b and enable the drawing of the beam orbit in the **floor_plan** plot. [Hint: First read the “Drawing a Floor Plan” subsection in the “Initializing Plotting” section of the “Tao Initialization” chapter.] The beam orbit deviates from the reference trajectory through the center of the elements due to the non-zero **dg** in the bend. To make the trajectory plot more interesting, give the beam an initial energy offset. [Hint: Either put a **particle_start** command in the lattice file (see the *Bmad* manual) or use the **set particle_start** command in *Tao*.]
- 8.4 What is the command to scale the x-axis for all the plots with one command? Use this to scale the x-axis for all plots to be the range [0.5, 1.5].

9 Model, Design and Base Lattices in Tao

When *Tao* runs, *Tao* instantiates three lattices (Technically, *Tao* instantiates three lattices per **universe**. See §12):

Design Lattice

The **design** lattice corresponds to the lattice read in from the lattice description file(s). Parameters in this lattice are never varied.

Model Lattice

Initially, when *Tao* is started, The **model** lattice is the same as the **design** lattice. Parameters in the **model** lattice are allowed to vary. That is, all commands to vary lattice parameters vary parameters of the **model** lattice.

Base Lattice

The **base** lattice is a reference lattice used so that changes in the **Model** lattice may be easily viewed. The **Design** lattice can also be used as a reference lattice but since the parameters of the **design** lattice are fixed, this is not always desirable. The parameters of the **base** lattice are set by setting the parameters of the **base** lattice equal to the present state of the **model** lattice.

9.1 Changing Model Parameters

To see the difference between the **model** and **design** lattices, start *Tao* as explained in section §5.2 with the lattice file **simple.bmad**.

Now issue the following commands:

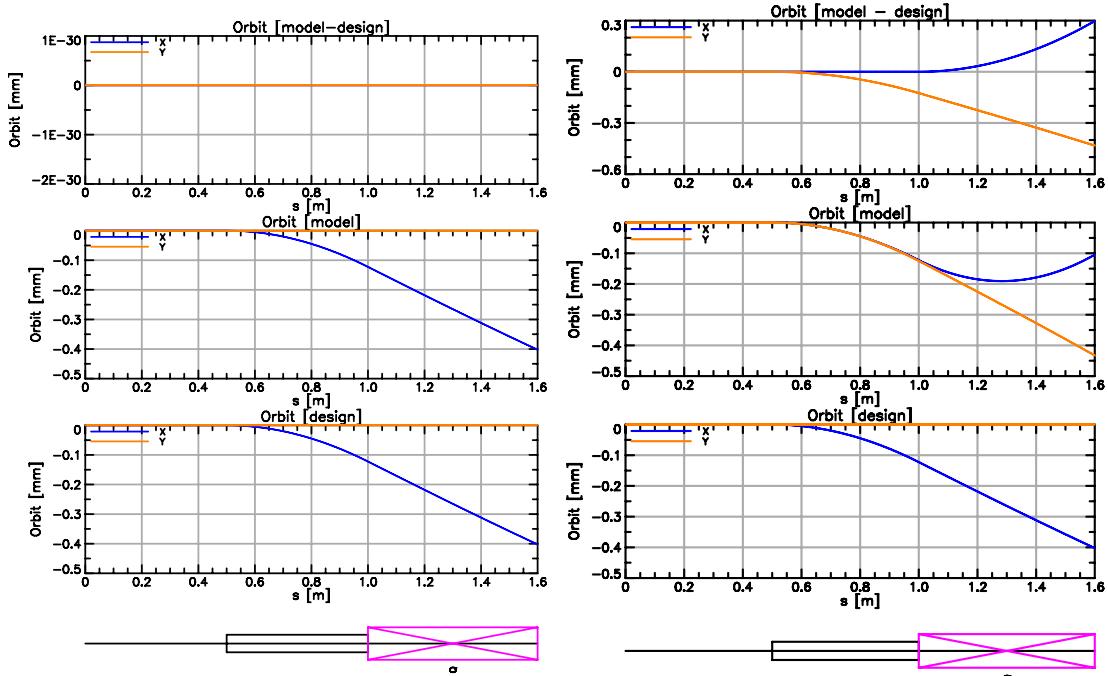
```
Tao> place r23 orbit
Tao> place r13 orbit
Tao> set plot r33 component = design           ! Bottom plot
Tao> set plot r13 component = model - design ! Plot difference orbit
Tao> scale
```

The “**set plot <plot_name> component = ...**” command sets where the data to be plotted comes from. The result is shown in Figure 5a. The bottom plot shows the **design** lattice orbit, the middle plot shows the **model** lattice orbit and the top plot shows the difference in orbits between **model** and **design**. Since the two lattices are the same when *Tao* is started, the difference orbit is zero.

Now change the **model** lattice using the following commands:

```
Tao> change element b vkick -0.0005 ! Changes by a given delta
Tao> set element q hkick = 0.001      ! Another way of changing a parameter.
Tao> scale
```

The **change** command changes real numbers by a given delta. The **set** command sets a parameter to a specific value. Unlike the **change** command, the **set** command can also be used with



(a) Initially, the **model** lattice and the **design** lattices are the same.

(b) The **set** and **change** commands will modify **model** lattice parameters.

Figure 5

integer, string and logical parameters. The result is shown in Figure 5b. Since now the **model** lattice is not the same as the **design** lattice, the difference orbit is non-zero.

9.2 Using the Base Lattice

The **base** lattice is used to view changes when the desired reference lattice does not correspond to the **design** lattice.

Continuing from the previous section, issue the following commands:

```
Tao> set lattice base = model ! Set the Base lattice = Model lattice.
Tao> set plot r33 component = model - base
Tao> set ele q vkick = 5e-4
Tao> scale
```

The **set lattice** command sets the **base** lattice equal to the **model** lattice. The third command varies the **model** lattice. The result is shown in Figure 6. The bottom plot of the orbit difference between **model** and **base** is not the same as the orbit difference between **model** and **design**.

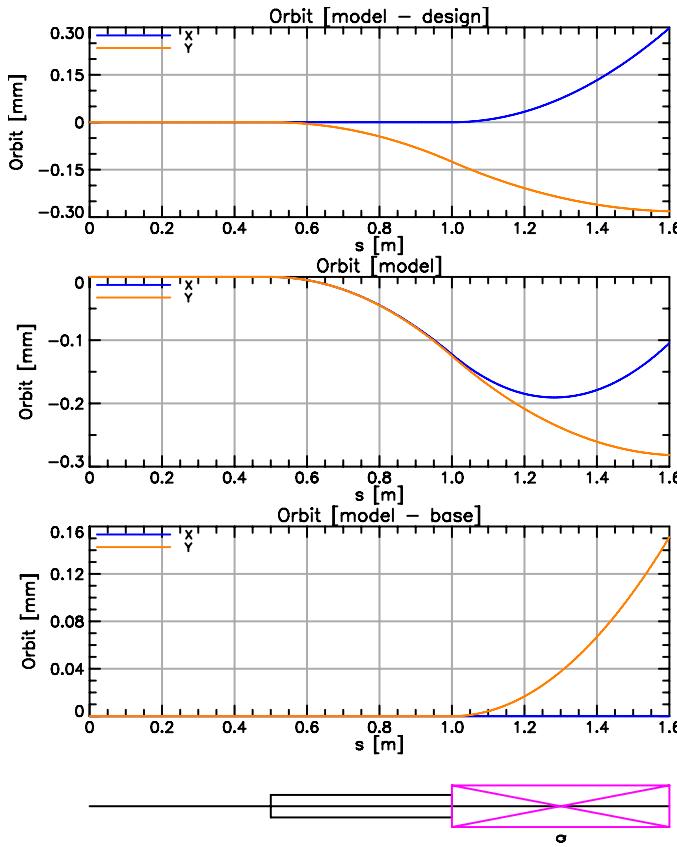


Figure 6: The **base** lattice is used to view changes when the reference lattice configuration does not correspond to the **design** lattice.

9.3 Exercises [Answers in Section §25.5]

- 9.1 Modify the middle graph in Fig. 6 so that the vertical orbit curve becomes the horizontal orbit of the **design** lattice.
- 9.2 To save on typing, alias commands may be defined. Define an alias command called **setit** so that typing “**setit 1e-6**” is equivalent to “**set ele q vkick = 1e-6**”. Hint: Look in the manual or type “**help alias**” to get more information on setting and using alias commands. Note: Do not confuse **Tao** alias commands with the alias string component of a lattice element.

10 Optimization with Tao

10.1 What is Optimization?

“Optimization” is the process of varying (model) lattice parameters to create a lattice with a certain set of properties as close to a “desired” state as possible. Optimization is covered in detail in the “Optimization: Lattice Correction and Design” chapter in the *Tao* manual.

Optimization problems generally fall into one of two categories. One category involves “lattice design” where lattice parameters are varied to achieve some set of ideal properties. For example, varying sextupole magnet strengths in order to give maximum dynamic aperture.

The other category of optimization problems involves “lattice correction”. Problems in this category involve matching the **model** lattice to actual measured data. For example, orbit flattening involves varying steering in the **model** lattice so that the orbit calculated from the **model** lattice matches as close as possible the measured orbit. The steering strengths in the **model** lattice can then be used to calculate what changes are needed to correct the orbit as discussed section §11.1.

An example of lattice design is given below. An example of lattice correction is given in Chapter §11.

10.2 Optimization Overview and the Merit Function

Optimization involves “**data**” and “**variables**”. **Data** are the parameters to be optimized. For example, orbit positions when flattening an orbit or the value of beta at the interaction point when designing a lattice. **Variables** are what is to be varied which can be steering strengths, magnet positions, etc.

Each datum has a set of associated parameters. For example, each datum has “**model**” and “**design**” values which is the value of the datum as calculated from the **model** and **design** lattices. Each datum also has a “**measured**” value which is set by the User. This value can be from an actual measurement which is typical when doing lattice correction or may be the desired value of the datum which is typical when doing lattice design. This is further discussed in section §10.4.

Like datums, each variable has a number of associated parameters. For example, each variable has a “**model**” value which controls the corresponding value or values (a variable can control multiple parameters simultaneously) in the **model** lattice. There are also “**low_lim**” and “**high_lim**” values that can be set by the user that are used to keep the variable **model** value within a given range. This is further discussed in section §10.5.

Optimization involves minimizing one or more “objectives” or “merit functions”. *Tao* itself implements “single objective” optimization. For “multiple objective” optimization, there is a separate program called **moga** that can be used (§4). The general form of the merit function **M** in *Tao* is

$$\mathcal{M} \equiv \sum_i w_i [\delta D_i]^2 + \sum_j w_j [\delta V_j]^2 \quad (1)$$

where the first sum is a sum over the **data** and the second sum is a sum over the **variables**. The

w_i and w_j are weights specified by the user and the δD_i and δV_j are functions of the data and variables. The merit function is discussed in depth in Chapter §8 in the *Tao* manual.

The form of the δD_i can be different for each datum and similarly the form of δV_j can be different for each variable and will be illustrated below in the example optimization.

There are several different optimizers that can be used with *Tao*. The one optimizer that is good for finding global merit function minima is the **de** (differential evolution) optimizer. All of the others are good for finding local minima.

10.3 Lattice Design Example: Initialization

An example of lattice design is given in the following section. For a lattice correction example, see chapter §11.

The files used to illustrate lattice design optimization are in the directory

```
bmad-doc/tutorial_bmad_tao/lattice_files/lattice_optimization
```

Copy these files to your working directory. Here the main initialization file is named **tao.init**. Since this is the default name for initialization files (§5.4), and since the **tao.init** file contains the name of the lattice file, *Tao* can be started without the **-lat** option (§5.2).

There are four files here:

```
lat.bmad      ! Lattice file .
tao.init      ! Primary Tao initialization file. Includes plotting setup.
setup.tao     ! Command file run at startup.
optimized_var.out ! Output optimized values
```

The file **tao.init** is the main initialization file (§5.4). The file is divided into three parts. The first part sets some general parameters while the next two sections setup data §10.4 and variable §10.5 lists.

```
&tao_start
    startup_file = "setup.tao"
/

&tao_design_lattice
    n_universes = 1
    design_lattice(1)%file = "lat.bmad"
/

&tao_plot_page
    plot_page%size = 500, 400
    place(1) = "layout", "lat_layout"
    place(2) = "r12", "beta"
    place(3) = "r22", "key"
/
```

Namelist format is used as explained in Section §5.4. A single **universe** will be used (§12) and the lattice file name for the universe is “**lat.bmad**”. The command file “**setup.tao**” will be run after all other initialization is complete.

The **tao_plot_page** namelist sets some plotting parameters. The setting of **plot_page%size** overrides the default size of the plot window and the **place(1)**, **place(2)**, and **place(3)** settings define initial placement of plots (§8.2).

The startup command file **setup.tao** defines an **alias** commands:

```
alias opt run lm
```

This defines the command “**opt**” to be equivalent to “**run lm**”. The **run** command starts optimization and the “**lm**” option specifies the Levenburg-Marquardt optimizer which is a good optimizer for finding a local minimum. See the **Tao Commands** chapter in the *Tao* manual for more details.

10.4 Data in Tao

In order to optimize, you must tell Tao what **data** will contribute to the merit function. A detailed description on how to do this is given in the “**Data**” chapter of the *Tao* manual.

The data that is used in the present example is defined in the middle section of the **tao.init** file:

```
&tao_d2_data
  d2_data%name = "twiss"
  n_d1_data = 2
/
&tao_d1_data
  ix_d1_data = 1
  d1_data%name = "a"
  datum(1) = "beta.a"      "" "" "END"    "target"   12.0   1e1
  datum(2) = "alpha.a"     "" "" "END"    "target"   -0.4   1e2
/
&tao_d1_data
  ix_d1_data = 2
  d1_data%name = "b"
  datum(1) = "beta.b"      "" "" "END"    "target"   12.0   1e1
  datum(2) = "alpha.b"     "" "" "END"    "target"   -0.4   1e2
/
```

In general, the **data** is grouped into a three level tree as illustrated in Figure 7. Nodes at the highest level are instances of what is called **d2_data** structures. For example, a **d2_data** structure may be setup to hold **orbit** data. In the present case, there is a single **d2_data** structure named “**twiss**”.

A **d2_data** structure will hold an array of one or more **d1_data** structures. For example, a **orbit d2_data** structure may hold “x” and “y” **d1_data** structures which represent horizontal and vertical orbit data. In the present case, the **twiss d2_data** structure has two **d1_data_structures**

named “a” and “b” representing the two transverse normal modes of oscillation. The syntax to refer to a particular **d1_data** structure is:

```
d2-data-name.d1-data-name
```

So with the above example, the a structure would be referred to as **twiss.a**.

A **d1_data** structure will hold an array of one or more **datum** structures. For example, the **x d1_data** structure contained in an **orbit d2_data** structure may be setup with an array of datums, one for each beam position monitor in the machine, with each datum representing a horizontal orbit measurement at a specified BPM. In this case, both **a** and **b d1_data** structures hold two datums, one representing β and α Twiss values. To refer to an individual **datum** use the syntax:

```
d2-data-name.d1-data-name[datum-index]
```

where **datum-index** is the index for the datum. So with the above example, the first datum in **twiss.a**, which is the a-mode β , would be referred to as **twiss.a[1]**.

An individual **datum** is structure that has a number of components. With the present **tao.init** file, seven components of each datum are set. These components are, in order:

data_type	! Type of data: "orbit.x", etc.
ele_ref_name	! Name of reference lattice element
ele_start_name	! Name of starting lattice element when there is a range
ele_name	! Name of the lattice element where datum is evaluated.
merit_type	! Type of constraint: "target", "max", "min", etc.
meas	! Measured datum value.
weight	! Weight for the merit function term

Thus for the **twiss.b[2]** datum which is set on the line:

```
datum(2) = "alpha.b"    "" "" "END"    "target" -0.4   1e2
```

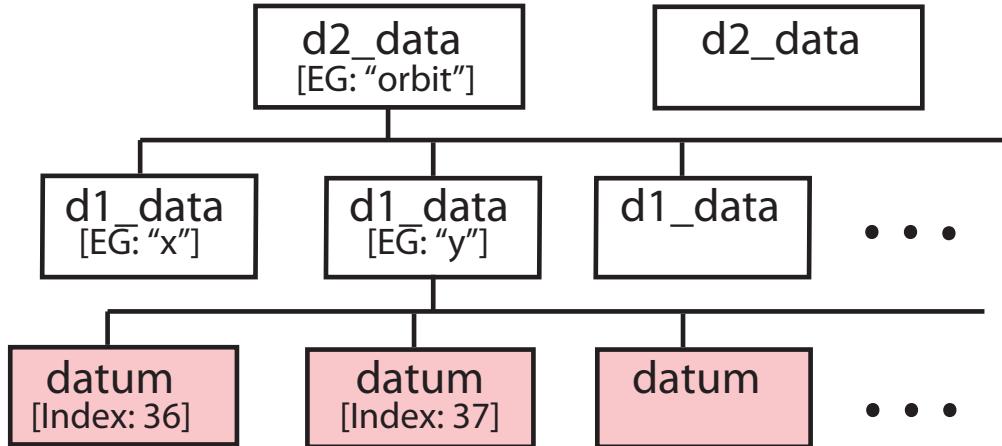


Figure 7: Data is grouped into a three level tree. A **d2_data** structure holds a set of **d1_data** structures. A **d1_data** structure holds an array of **datums**.

The **data_type** component is set to “**alpha.b**” [For a list of data types that *Tao* recognizes, see the **Tao Data Types** section (§6.9) of the **Data** chapter of the *Tao* manual.], the **ele_ref_name** and **ele_start_name** are set to the blank string. These parameters are not used in the present example.

The **ele_name** component for **twiss.b[2]** is set to “**END**” which is where the datum is to be evaluated. Since the **merit_type** is set to **target**, this means that δD in Eq. (1) is evaluated using the equation

$$\delta D = \text{model} - \text{meas}$$

where **model** is the value as calculated from the **model** lattice and **meas** is the “**measured**” value as set on the datum line. For **twiss.b[2]**, **meas** is set to -0.4. Also the weight *w* for **twiss.b[2]** is set to 100. Thus if the model b-mode alpha function is, say, 1.0 at element **END**, then this datum would contribute $100 * (1.0 - 0.4)^2$ to the merit function.

Start *Tao* (remember, no **-lat** argument needed). The **d2_data** structures can be shown with the command **show data**:

```
Tao> show data
```

Name	Using for Optimization
twiss.a[1:2]	Using: 1:2
twiss.b[1:2]	Using: 1:2

To see a list of datums for an individual **d1_data** structure append the **d1_data** name after **show data**. For example:

```
Tao> show data twiss.b
Data name: twiss.b
...
| Useit
... Ele Meas Model Design | Opt Plot
1 beta.b <target> ... END 1.200E+01 9.292E+00 9.292E+00 T F
2 alpha.b <target> ... END -4.000E-01 -4.427E-01 -4.427E-01 T F
... Ele Meas Model Design | Opt Plot
... | Useit
```

To see the parameters of an individual datum append the datum name after **show data**. For example:

```
Tao> show data twiss.a[2]
%ele_name      = END
... etc...
%data_type     = alpha.a
... etc...
%model         = -4.42701763E-01
%design        = -4.42701763E-01
... etc...
%good_model    = T
%good_design   = T
```

```
%good_base      = T
%good_meas     = T
%good_ref      = F
%good_user     = T
%good_opt      = T
%good_plot     = F
%useit_plot    = F
%useit_opt     = T
```

The **useit_opt** logical indicates whether the datum will be used when the merit function is evaluated. For example, if the **meas** value has not been set, *Tao* will set **good_meas** to False and this will cause *Tao* to set **useit_opt** to False. The user also has control as to whether a datum will be used in optimization and this is controlled by setting the **good_user** component. The commands that control this are **use**, **veto** and **restore**. Example:

```
Tao> veto data twiss.a
twiss.a[1:2]                                Using:
twiss.b[1:2]                                Using: 1:2
```

In this example the two **twiss.a** datums have been vetoed. It can be checked that the **twiss.a** datums now have their **good_user** components set to False.

Notice that it does not matter to the optimization process how data is divided into **d1_data** and **d2_data** groups. It is only a matter of convenience to the user. Also a given **d1_data** group of data does not have to contain data of a single type. Thus the **twiss.a** datums include both **beta** and **alpha** type data.

10.5 Variables in Tao

In order to optimize, you must tell *Tao* what **variables** you want to vary to minimize the merit function. A detailed description on how to construct **variables** is given in the “Variables” chapter of the *Tao* manual.

The variables that are used in the present example are defined in the bottom section of the **tao.init** file:

```
&tao_var
v1_var%name = "quad"
search_for_lat_els = "Quad::*"
default_step = 1e-4
default_attribute = "k1"
default_merit_type = "limit"
default_low_lim = -50
default_high_lim = 50
default_weight = 1
ix_min_var = 1
default_key_delta = 1e-2
default_key_bound = T
/
```

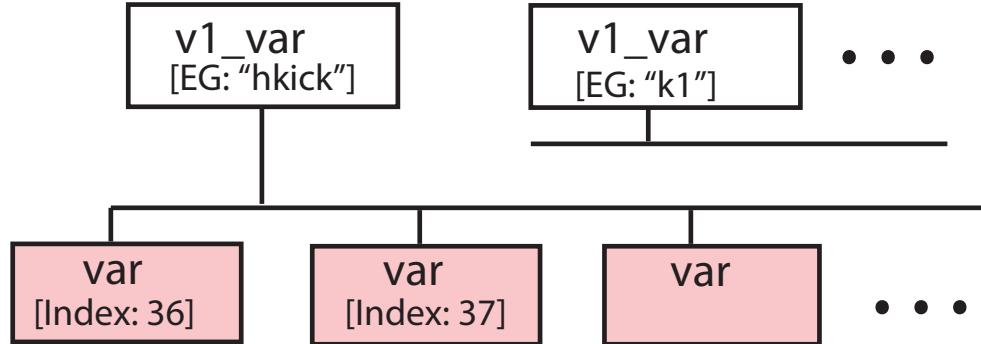


Figure 8: Variables are grouped into a two level tree. A **v1_var** structure holds an array of **variables**. Illustrated is a **var_var** structure holding an array of variables with each variable controlling the **hkick** attribute of a particular lattice element.

Just like **data**, **variables** are grouped into a two level tree as illustrated in Fig. 8. The top level nodes of the tree are called **v1_var** structures. In this example a **v1_var** structure is defined called **quad** which controls the **k1** attribute of all element whose name matches **quad::***. This will match to all quadrupoles. In this case, the lattice has 6 quadrupoles named **Q1** through **Q6**. Thus there will be an array of 6 **variables** associated with the **quad v1_var** structure.

To refer to an individual variable use the syntax:

```
v1-var-name [var-index]
```

where **var-index** is the index of the variable. For example, the first variable in the **quad** structure is **quad[1]**.

The parameters like **default_step** in the above namelist establish a default value for the **step** attribute of each variable that is created for the **quad** structure. The **step** attribute is used by *Tao* to calculate derivatives that are used by some of the optimizers. Essentially, to calculate derivatives, *Tao* varies the variable by \pm **step** and looks at the changes in the **data**. Like many attributes associated with **optimization** it is important that the **step** attribute be set properly. To small a setting and round-off error can throw off the derivative calculation. On the other hand, if the value of **step** is too large, nonlinearities can throw off the calculation.

The **weight** of a variable sets the value of w_j in Eq. (1). Since the **merit_type** in this case is **limit**, the δV used in the merit function is:

$$\delta V = \begin{cases} \text{model} - \text{high_lim} & \text{model} > \text{high_lim} \\ \text{model} - \text{low_lim} & \text{model} < \text{low_lim} \\ 0 & \text{Otherwise} \end{cases} \quad (2)$$

That is, the contribution to the merit function will be zero if the value of the variable is between **low_lim** and **high_lim** which in this case is -50 and 50.

Running *Tao*, the **v1_var** structures can be shown with the command **show variable**:

```
Tao> show var
      Name
quad[1:6]
```

Using for Optimization
1:6

To see a list of individual variables of a given **v1_var** structure, append the **v1_var** name to the **show variable** command:

```
Tao> sho var quad
Variable name: quad
```

Index	Controlled Attribs(s)	Meas	Model	Design	Useit_opt
1	Q1[K1]	8.6924-311	0.0000E+00	0.0000E+00	F
2	Q2[K1]	8.6924-311	0.0000E+00	0.0000E+00	F
3	Q3[K1]	8.6924-311	0.0000E+00	0.0000E+00	F
4	Q4[K1]	8.6924-311	0.0000E+00	0.0000E+00	F
5	Q5[K1]	8.6924-311	0.0000E+00	0.0000E+00	F
6	Q6[K1]	8.6924-311	0.0000E+00	0.0000E+00	F

To see the parameters of an individual variable, append the variable name to the **show var** command. Example:

```
Tao> sho var quad[2]
%ele_name      = Q2
%attrib_name   = K1
... etc...
%exists        = T
%good_var      = T
%good_user     = T
%good_opt      = T
%useit_opt     = T
... etc...
```

The **useit_opt** logical indicates whether the variable will be used in the optimization process. The user has some control as to whether a variable will be used in optimization and this is controlled by setting the **good_user** component. The commands that control this are **use**, **veto** and **restore**. Example:

```
Tao> veto var quad[4]
quad[1:6]
```

Using: 1:3 5:6

Variable properties can also be changed within Tao. For example,

```
Tao> set var quad[1:4] low_lim = -1
```

will set the lower limit for the first four quads.

Notice that, like data, it does not matter to the optimization process how variables are divided groups. It is only a matter of convenience to the user. Also a given **v1_var** instance, the array of associated **variables** does not all have to be of a single type.

If you want to have one *Tao variable* control a set of parameters, construct an **overlay** or **group** element (§13) and then have the *Tao variable* control the overlay or group. For example, the following overlay gangs the **k1** parameters of elements **Q1** and **Q3** together:

```
ps1: overlay = {Q1, Q3}, var = {k1}, k1 = 0.8
```

10.6 Key Bindings

Tao has two modes for entering commands. In **single mode**, each keystroke represents a command. That is, with a few exceptions, the user does not have to press the carriage control key to signal the end of a command. This is to be contrasted with **line mode**, which you have been using up to now, where *Tao* waits until the return key is depressed to execute a command. **single mode** is useful for quickly varying parameters to see how they affect a lattice but the number of commands in single mode is limited. **Single mode** is covered in detail in the “Single Mode” chapter in the *Tao* manual.

The main purpose of **single mode** is to associate certain keyboard keys with certain variables so that the pressing of these keys will change their associated model value of the variable. This is called a **key binding** and is illustrated in Figure 9.

Start *Tao* using the example optimization files (§10.3) or use the **reinit tao** command to reinitialize *Tao*. The plot window should look like Figure 10a. The **key_table** plot in the middle shows what variables (§10.5) have been bound to what keyboard keys. In this instance the **quad[1]** variable is bound so that pressing the “1” key will change **quad[1]** by **+delta**, and pressing the “q” key will change **quad[1]** by **-delta**. Pressing the shift key when pressing the “1” or “q” keys will change **quad[1]** by **+10×delta** and **-10×delta** respectively. Similarly, the **quad[2]** variable is bound to the **2** and **w** keys, etc.

Single mode and key bindings are useful for getting a feel for how variables affect the lattice. Get into single mode by issuing the **single_mode** command. Play around with varying variables. To get out of single mode press capital Z.

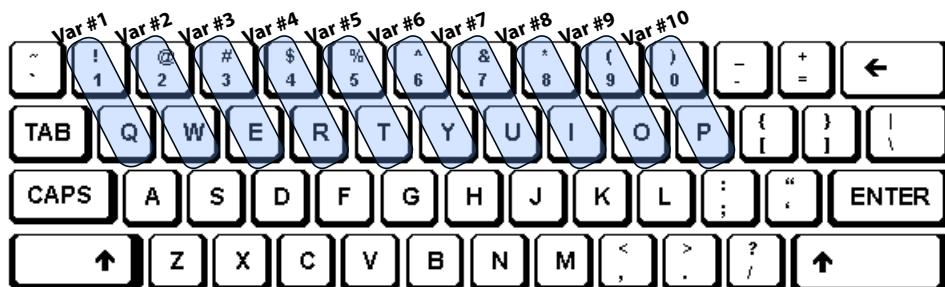


Figure 9: Ten pairs of keys on the keyboard are bound to ten variables so that pressing a key of a given pair will either increment or decrement the associated variable. The first key pair bound to variable number 1 are the 1 and Q keys, etc.

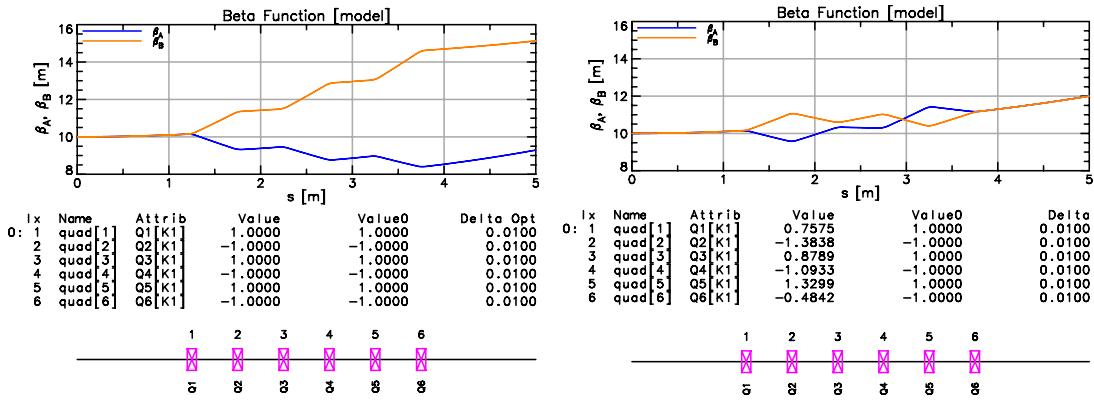


Figure 10

10.7 Running an optimization

Start *Tao* using the example optimization files (§10.3) or use the **reinit tao** command to reinitialize *Tao*. The plot window should look like Figure 10a.

To see what data and what variables are being used in the optimization, use the **show data** and **show variables** commands as illustrated above or use the **show optimizer** command:

```
Tao> show opti
Data Used:
  twiss.a[1:2]                                     Using: 1:2
  twiss.b[1:2]                                     Using: 1:2
Variables Used:
  quad[1:6]                                         Using: 1:6

optimizer:          lm
Global optimization parameters (use "set global" to change):
  %de_lm_step_ratio      =  1.00000000E+00
  %de_var_to_population_factor =  5.00000000E+00
  %lm_opt_deriv_reinit   = -1.00000000E+00
  %lmdif_eps             =  9.99999996E-13
  %merit_stop_value      = -1.00000000E+00
  %svd_cutoff             =  9.99999975E-06
  ... etc...
```

There are many parameters associated with optimization and it is important to carefully consider what values these parameters have in order to be able to have a successful optimization.

To see what the biggest contributions to the merit function are use the **show top10** command:

```
Tao> show top
```

```

Constraints          ...   Ele/S    Target    Value    Merit
twiss.b[1]  beta.b <target>  ...   END      1.200E+01  1.513E+01  9.82E+01
twiss.a[1]  beta.a <target>  ...   END      1.200E+01  9.292E+00  7.33E+01
twiss.b[2]  alpha.b <target> ...   END     -4.000E-01 -2.527E-01  2.17E+00
twiss.a[2]  alpha.a <target> ...   END     -4.000E-01 -4.427E-01  1.82E-01
quad[6]     Q6[K1]           ...   3.80    -5.000E+01 -1.000E+00  0.00E+00
quad[5]     Q5[K1]           ...   3.30    5.000E+01  1.000E+00  0.00E+00
... etc...

figure of merit: 1.738388E+02

List of non-zero contributors to the Merit Function:
Name                  Merit      Sigma [= sqrt(Chi^2/N)]
twiss.b              1.0034E+02  2.2179E+00
twiss.a              7.3499E+01  1.9149E+00

```

This shows, among other things, that the value of the merit function is 173.8 and that the largest contributor to the merit function is **twiss.b[1]** which has a contribution of 98.2 which is over 50%.

To run, say, the **lm** optimizer use the **run lm** command. In this case, this command has been aliased to the **opt** command by the **setup.tao** command file that was run at initialization (§10.3):

```

Tao> opt
Optimizing with: lm
Type ‘.’ to stop the optimizer before it’s finished.
[INFO] tao_dmodel_dvar_calc:
      Remaking dModel_dVar derivative matrix.
      This may take a while...

Cycle      Merit    A_lambda
 1      5.4532E-01  1.00E-04
 2      8.7067E-02  1.00E-05
... etc...
 19      3.5034E-04  1.00E-22
 20      2.9333E-04  1.00E-23
Written: var1.out
 21      2.9333E-04  0.00E+00

```

The optimization has managed to reduce the merit function from to 2.9E-4 or about six orders of magnitude. Further reductions in the merit function can be had by running the optimizer repeatedly. At the end of optimization, *Tao* creates a file **var1.out** which contains the optimized variable values:

```

Q1[K1] = 7.76217267682967E-01
Q2[K1] = -1.40330956873957E+00
Q3[K1] = 8.78773428675319E-01
Q4[K1] = -1.07943627873167E+00
Q5[K1] = 1.29072113653010E+00
Q6[K1] = -4.57469896955973E-01
... etc...

```

This file should be virtually identical to the **optimized_var.out** file (§10.3). The format of this file conforms to *Bmad* lattice file syntax so this file can be used to create a lattice with the optimized values. One way to form a lattice with optimized values is to create a new lattice file that calls the original lattice and **var1.out**. That is, the new file would look like:

```
call, file = lat.bmad  
call, file = var1.out
```

Note: At any time to print in *Bmad* format the variable values used in the optimization use the command:

```
show var -bmad -good
```

To save directly to a file, add the write option:

```
show -write solution.bmad var -bmad -good
```

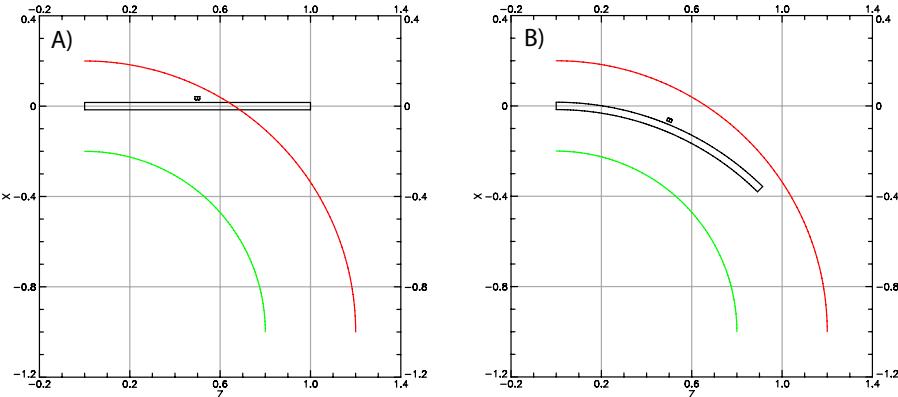


Figure 11: Optimization to keep a machine within existing building walls. A) Initial setup. The end of the machine (a 1 meter long dipole) is outside the walls (red and green semi circles). B) After optimization the machine is safely inside the walls.

10.8 Exercises

- 10.1 Start with the files in `bmad-doc/tutorial_bmad_tao/lattice_files/lattice_optimization`. Change all the quadrupole strengths to zero in `lat.bmad` and change the alpha Twiss `meas` targets to -1. Now run with the `opt` command and verify that the optimizer does not find a good solution! Why is this? The problem is that the good solutions (and there is more than one) are outside of the local minimum that the optimizer is stuck in. Note: To easily reset the lattice use the command:

```
set lattice model = design
```

- 10.2 Start with the situation in Exercise 10.1 and find a good solution by first using `single mode` to vary the quadrupole strengths to find an approximate solution. Then run the `lm` optimizer to polish the results. This is a general strategy, often there is no single method that will work so a combination of methods is what is needed.

- 10.3 Start with the situation in Exercise 10.1 and find a good solution by first using the `de` optimizer which can find global minimum. Then run the `lm` optimizer to polish the results. Warning: Success here depends upon finding the right `de` parameter settings to use. This will take some thought and experimentation so successful completion of this exercise will not be quick.

- 10.4 This exercise shows how to do an optimization with a constraint that the machine stay within existing building walls. If you get stuck, a working example can be viewed in the directory:

```
bmad-doc/tutorial_bmad_tao/lattice_files/building_wall_optimization
```

- (a) Construct a lattice with a single element which is a 1 meter long bend with zero bend angle.

-
- (b) Setup a *Tao* input file that defines two building wall sections as shown in Figure 11A. The sections are two circular arcs of radius 0.8 meters and 1.2 meters.
 - (c) Setup two datums: `wall[1]` and `wall[2]`. The first datum constrains the end of the lattice to be to the inside of the outside (left) wall with a 0.1 meter clearance. The second datum constrains the end of the lattice to be to the outside of the inside (right) wall with a 0.1 meter clearance.
 - (d) Setup a variable to vary either the `g` or `angle` component of the bend.
 - (e) Run the `lm` optimizer to produce Figure 11B. Voila! The machine is within the walls with the desired clearance.
 - (f) When you startup *Tao* and the dipole is unbent, you should get the warning:

```
[WARNING] tao_init:  
    DATUM EXISTS BUT CANNOT COMPUTE A MODEL VALUE: wall[2]  
    INVALID SINCE: No wall section found in the transverse plane  
                    of the evaluation point.
```

Explain why you are getting this warning. Also explain why this warning goes away when the dipole gets bent enough. [Hint: Read carefully the description of how the datum value is computed.] Note that not being initially able to compute a model value does not hinder the optimization.

11 Lattice Correction (Including Orbit Correction)

The previous chapter (§10.1) discussed optimization and, in particular, how optimization could be used for “lattice design” type problems. This chapter discusses “lattice correction” type optimization. In particular, this chapter shows how to calculate corrections to correct the Twiss parameters using measured orbits. It is assumed that the reader is familiar with the material in the previous chapter.

Side note: With an actual machine, calculating corrections is only one step in the correction process. Also needed is the ability to take data and the ability to load corrections. The object oriented nature of *Tao*’s design makes it relatively straightforward to implement machine communication within *Tao* itself. This allows *Tao* to act as an “online model” for the machine control system. Programming with *Bmad* and *Tao* is outside of the scope of this tutorial and the reader is referred to the appropriate sections of the *Bmad* and *Tao* manuals.

11.1 Correcting the Orbit

Before discussing Twiss parameter correction, consider the simpler case of correcting the orbit. The **design** lattice will represent the state we want the machine to be in. Typically, the orbit in the **design** lattice will be zero but that is not a necessary condition. The analysis starts with an orbit measurement. By varying steerings in the **model** lattice, the orbit, as calculated in the **model**, can be fit to the measured orbit.¹ If the fit is good, the orbit is corrected in the machine by changing the actual steering strengths K_{actual} by an amount dK given by

$$dK_i = K_i^{(design)} - K_i^{(model)} \quad (3)$$

where i denotes the i th steering, and $K_i^{(design)}$ and $K_i^{(model)}$ are the **design** and **model** values for the steering strengths respectively. Typically, the **design** steering strengths are zero but that is not necessary for the analysis.

Eq. (3) is derived using the following logic: Once a fit to the measured data has been made, the **model** represents the actual state of the machine. On the other hand, the desired state is represented by the **design** lattice. Thus the difference $K_i^{(design)} - K_i^{(model)}$ represents “desired - actual” so the final state of the steering magnets after correction will be

```
Final_State = Initial_State + Change
            = Actual_State + (Desired_State - Actual_State)
            = Desired_State
```

There are a few points that should be kept in mind here. First, it does not matter to the correction whether the deviations of the orbit from the ideal are caused by steering strength errors or other errors such as dipole rolls or quadrupole offsets. To the extent that the measured orbit can be well fit determines the extent to which the orbit can be corrected. For example, if an unwanted

¹Typically, the measured orbit in a machine with a closed geometry (EG storage ring) will be the closed orbit while the orbit in an open geometry machine (EG Linac) will be the orbit as measured with the beam starting from some initial position. In the open geometry case, varying the initial position in the model lattice along with steerings may be necessary to fit the orbit.

kick is generated by some element at a spot that is far from any correctors, it will not be possible to fit the measured orbit well and it will not be possible to make a good correction. If, on the other hand, an unwanted kick is generated next to one corrector, the measured orbit can be well fit and the **model** lattice will have a strength change from the **design** for that one corrector. Varying that one corrector can then cancel out the unwanted kick.

Another point is that the correction algorithm will work with varying any set of parameters as long as the variation in the parameters affect the **model** data. Thus an analysis can be made using dipole rolls and/or quadrupole offsets as variables or any combination thereof. If the fit is good, rolling the dipoles and moving the quadrupoles will correct the orbit. With *Tao*, the User has complete freedom to vary any parameters in the fitting process.

A third point is that the fitting process is independent of the strengths of the parameters in the **design** lattice. That is, the fit involves the actual machine state independent of what the desired state is. It is not until the values needed for the correction are computed that the parameter strengths in the **design** lattice come into play.

Typically, at the start of the fit, the **model** lattice is, by default, equal to the **design** lattice but this is not necessary. Generally, the actual machine state is near enough to the **design** machine state so that the machine will behave roughly linearly with the variation in the parameters (typically, if the machine parameters are far from the design values, it will not be possible to store beam to take a measurement in the first place). This means that there will be only one minimum merit function state so that the parameter values (steering strengths in this case) at the end of the fit are independent of the starting state. To put this in other terms, the User generally does not have to worry about the initial state of the **model** at the start of a fit. This is in contrast to the lattice design example in chapter §10. With lattice design, there are typically many local minima and it can take days of work to find a good operating point. With lattice correction, on the other hand, the near linear nature of the problem means that finding a solution in a machine with hundreds of correctors and hundreds of BPM readings can be done in seconds.

11.2 Twiss Parameters Correction

As mentioned in the previous section, with *Tao*, the User has complete freedom to fit to any arbitrary set of lattice parameters. *Tao* also gives the User complete freedom to optimize using any arbitrary collection of data. In particular, as discussed below, by varying quadrupole strengths to fit a multiple orbit data sets taken with multiple steering strength settings allows correction of the Twiss parameters.

One way to correct Twiss parameters is to vary, one-by-one, the steerings in the machine and for each varied steering, take before and after orbits. By fitting the measured orbit differences using the **model** quadrupole strengths, the strengths of the quadrupoles can be determined. With this, the difference between the **model** quadrupole strengths and the **design** strengths gives the needed quadrupole corrections. See Eq. (3) where, in this case, the K 's represent quadrupole strengths. In practice the steering strengths will also be included in the variable list. This correction scheme is similar to the Orbit Response Matrix (ORM) approach in that both uses multiple orbits to correct the Twiss parameters.

It should be mentioned that even without doing the correction step, the fitting of the data in itself gives values for the optics in the current state of the machine.

11.3 Twiss Parameter Correction Example Input Files

The example files used to illustrate correction of the Twiss parameters are in the directory

```
bmad-doc/tutorial_bmad_tao/lattice_files/twiss_correction
```

Copy these files to your working directory. There are six files:

```
create_data.tao      ! For creating "measured" data.  
lat.bmad            ! Lattice file.  
tao.init             ! Primary Tao initialization file.  
tao_plot.init        ! Plot setup initialization file.  
startup.tao          ! Command file run at startup.  
set_meas.tao         ! Set measured data.
```

The lattice being used here has one horizontal steering named **H1** and one vertical steering named **V2**. Two universes (§12) will be needed to store the measured data. The first universe will hold the measured data from changing **H1** and the second universe will hold the measured data from changing from **V2**.

The file **tao.init** is the main initialization file (§5.4) and is similar to the **tao.init** file in Chapter §10.3. One difference is the the **tao.init** file here initializes *Tao* with two universes instead of one:

```
&tao_design_lattice  
n_universes = 2  
design_lattice(1)%file = "lat.bmad"  
design_lattice(2)%file = "lat.bmad"  
/
```

Here both universes use the same lattice.

The **tao.init** file lists the secondary initialization files in the **tao_start** namelist:

```
&tao_start  
  startup_file = "startup.tao"  
  plot_file = "tao_plot.init"  
/
```

11.4 Data Setup

Data (§10.4) for the optimization is setup in the file **tao.init** file. There is just one **d2_data** block called **orbit**:

```
&tao_d2_data  
  d2_data%name = "orbit"
```

```

universe = "*"
n_d1_data = 2
default_weight = 1e8
/
&tao_d1_data
ix_d1_data = 1
d1_data%name = "x"
search_for_lat_eles = "det*"
/
&tao_d1_data
ix_d1_data = 2
d1_data%name = "y"
search_for_lat_eles = "det*"
/

```

There are two **d1_data** arrays for the horizontal and vertical orbits. A datum will be set up for each element whose name begins with “det”. That is, at the four detectors in the lattice **det1** through **det4**. The **universe = *** setting results in an **orbit** data block being setup one for each universe.

11.5 Variable Setup

The variables (§10.5) setup is in the file **tao.init**. There are three **v1_var** blocks — for quadrupole strengths and tilts, along with steering strengths. By varying quadrupole tilts, the coupling can be corrected. [Typically, coupling — that is, coupling between horizontal and vertical orbits due to skew fields — is corrected by adjusting the strength of skew quadrupoles. Here, to simplify things somewhat, the same quadrupoles are used for both beta and coupling correction.]

The variable blocks are defined in **tao_var** namelists. The quadrupole strength block is

```

&tao_var
v1_var%name = "quad_k1"
search_for_lat_eles = "Quad:::*"
default_step = 1e-4
default_attribute = "k1"
default_merit_type = "target"
default_weight = 1
ix_min_var = 1
default_key_delta = 1e-2
default_key_bound = T
/

```

The **search_for_lat_eles** setting will searches for quadrupole elements in all universes. All quadrupoles of a given name in all universes will be bound together with a single variable. In this case there are two quadrupoles named **q1** and **q2** in each universe so the **quad_k1** variable array, which has the name **quad_k1[1]**, will have two elements. One element will control the **k1**

attribute of the **q1** element in universe 1 and the the **k1** attribute of the **q1** element in universe 2. Similarly the other variable, **quad_k1[2]** will control the **k1** attribute of the **q2** elements.

When a variable controls multiple elements, the controlled parameters (the **k1s** of the two **q1** quads in this case) can have different values as long as the **model** value of the variable is not touched. When the **model** value is set, either during the optimization or with a **set** or **change** command, all the controlled parameters will be set to the same value as the **model** value of the variable. This setup mirrors the orbit measurement conditions were the quadrupole strengths were the same for all measurements.

The **merit_type** is set to **target** which means that the contribution to the merit function in Eq. (1) will be

$$\delta V = \text{model} - \text{measured} \quad (4)$$

Here **measured** is the value of the quadrupole strength as “measured” when the orbit was obtained. Typically, magnet strengths are “measured” using the known calibration between current going through the field windings or, if it is a permanent magnet, the measured value is taken to be the field that was measured on the bench before installation.

The reason why the merit function is setup to have a contribution from the variables is to avoid near-degeneracies where, if there where no δV terms, inaccuracies in the orbit measurement coupled with magnets that have a similar effect on the orbit results in the optimization minimum being a state where the near-degenerate magnets have large unphysical strengths but the **model** orbit will look reasonable since the effect of the near-degenerate magnets nearly cancel one another. The trick is to adjust the weights for the δV terms to be large enough to keep the magnet strengths reasonable but not too large that the fit of the orbit is degraded due to inaccuracies in the variable measurement value.

The second variable block defines variables that control quadrupole tilts. These variables are setup similar to the quadrupole strength variables.

The third variable block holding the steering strengths is setup with

```
&tao_var
  v1_var%name = "steering"
  default_step = 1e-4
  default_merit_type = "target"
  default_weight = 1
  default_key_delta = 1e-2
  default_key_bound = T
  var(1:2)%universe = 1,           2
  var(1:2)%ele_name = "h1",       "v2"
  var(1:2)%attribute = "hkick",   "vkick"
/
```

Here there are two variables **steering[1]** and **steering[2]**. **steering[1]** controls the **hkick** attribute of element **h1** in universe 1 and **steering[2]** controls the **vkick** attribute of element **v2** in universe 2.

11.6 Plotting Setup

For details of how to setup custom plots, see the **Initializing Plotting** (§10.13) section of the **Tao Initialization** chapter of the *Tao* manual. Here the plotting setup is done in the `tao_plot.init` file. This file defines a plot template (§8.2) called `data_orbit` in a `tao_template_plot` structure:

```
&tao_template_plot
  plot%name = "data_orbit" ! Name of template plot.
  plot%x_axis_type = "s" ! Graph x-axes are longitudinal s-coordinate.
  plot%autoscale_gang_y = F ! So scale command scales the graphs separately.
  plot%n_graph = 2           ! There are 2 associated graphs.
/
```

This template plot has two associated graphs defined in two `tao_template_graph` namelists below the `tao_template_plot` namelist. The first graph's namelist looks like:

```
&tao_template_graph
  graph_index = 1           ! First graph.
  graph%name = "x"          ! Graph name is "data_orbit.x"
  graph%box = 1, 1, 1, 2    ! Graph is in lower half of the plot region.
  graph%title = "X Orbit [mm]" ! Graph title
  graph%y%label = "X (mm)"   ! Graph y-axis label.
  ... etc. ...
```

This graph is named “x” so the fully qualified name is “`data_orbit.x`”. There are three curves associated with this graph and are defined further down in this namelist. The parameters of the first curve are:

```
curve(1)%name = "lat"        ! 1st curve name is "data_orbit.x.lat"
curve(1)%data_source = "lat" ! Data is drawn from lattice calculations...
curve(1)%data_type = "orbit.x" ! ... In this case the horizontal orbit.
curve(1)%draw_symbols = F    ! Do not draw any symbols.
curve(1)%draw_line = T       ! But do draw a orbit curve.
curve(1)%line%color = "blue"
```

This curve draws the horizontal orbit (`data_type = "orbit.x"`) as a curve (`draw_line = T`) with values calculated from the lattice (`data_source = "lat"`). The default, since the `component` of the curve is not set, is values will be extracted from the `model` lattice.

The second curve of the graph is defined in the namelist by:

```
curve(2)%name = "dat"        ! 2nd curve name is "data_orbit.x.dat"
curve(2)%data_source = "data" ! Data is drawn from a data block...
curve(2)%data_type = "orbit.x" ! ... In particular, the "orbit.x" data block.
curve(2)%draw_symbols = T    ! Draw symbols at the data points.
curve(2)%draw_line = F       ! Do not draw lines between the data points.
curve(2)%symbol%height = 25
```

This curve draws a symbol at each data point of the `orbit.x` data block. This data block defines datums at each detector element. The default, since the `component` of the curve is not set, is

values will be extracted from the **model** lattice just like the first curve. The result is that symbols are drawn on top of the line drawn by the first curve.

The third curve of the graph is defined in the namelist by:

```
curve(3)%name = "meas"  
curve(3)%component = "meas"  
curve(3)%data_source = "data"  
curve(3)%data_type = "orbit.x"  
curve(3)%draw_symbols = T  
curve(3)%draw_line = F  
curve(3)%symbol%height = 15
```

This is similar to the second curve except that the values for drawing the symbols comes from the “**meas**” component of the **orbit.x** data block.

The second graph associated with the **data_orbit** graph is similar to the first except the data is drawn from the vertical orbit instead of the horizontal orbit.

Besides setting up the **data_orbit** template plot, the **tao_plot.init** file places plots using the **tao_plot_page** namelist

```
&tao_plot_page  
  place(1) = "r12",    "data_orbit"  
  place(2) = "r22",    "data_orbit"  
  place(3) = "layout" "lat_layout"  
/
```

The plot page will have 3 plots: Two **data_orbit** plots and one **lat_layout** plot. In the **startup.tao** command file that is run at startup (since the **startup_file** is set to this file in the **tao_start** namelist in **tao.init**), there is:

```
set graph r12 ix_universe = 1  
set graph r22 ix_universe = 2
```

This sets one of the **data_orbit** plot to use the data from universe 1 and the other **data_orbit** plot to use the data from universe 2.

11.7 Measured Data and Variables Setup

The “measured” data and variable values are set by the command file **set_meas.tao** which is called by the **startup.tao** file:

```
set dat 1@orbit.x|meas = [2.9627E-03, 3.1590E-03, 2.3705E-03, 2.1821E-03]  
set dat 1@orbit.y|meas = [-1.1841E-03, -1.0370E-03, -1.3810E-03, -1.5749E-03]  
set dat 2@orbit.x|meas = [-1.0481E-03, -1.1265E-03, -9.6521E-04, -8.8252E-04]  
set dat 2@orbit.y|meas = [7.6870E-04, 6.6229E-04, 8.6742E-04, 9.6448E-04]  
  
set var steering[1:2]|meas = [1e-4, 1e-4]  
set var quad_k1[1:2]|meas = [1.0, -1.1]  
set var quad_tilt[1:2]|meas = [0, 0]
```

Since the lattice used in this example does not represent an actual machine, the “measured” orbit data had to be generated and this was done using the file the script `create_data.tao`:

```
set var quad_k1[1]|model = 0.9
set var quad_k1[2]|model = -1.05
set var quad_tilt[1]|model = 0.1
set var quad_tilt[2]|model = -0.05
set var steering[1]|model = 1.2e-4
set var steering[2]|model = 0.9e-4
show value 1@orbit.x|model + 5e-5*ran_gauss() -form es12.4 ! Add 50um noise
show value 1@orbit.y|model + 5e-5*ran_gauss() -form es12.4
show value 2@orbit.x|model + 5e-5*ran_gauss() -form es12.4
show value 2@orbit.y|model + 5e-5*ran_gauss() -form es12.4
```

This file represents the state of the machine at the time the orbit measurements are made. The results of running this script was, with a little bit of reformatting, been put in the file `set_meas.tao`. The “measured” values for the variables in `set_meas.tao` is somewhat different from the “true” values `create_data.tao` to simulate inaccuracies inherent in measuring these variables. Additionally, 50 μm Gaussian noise has been added to the orbit data.

11.8 Running the Optimization

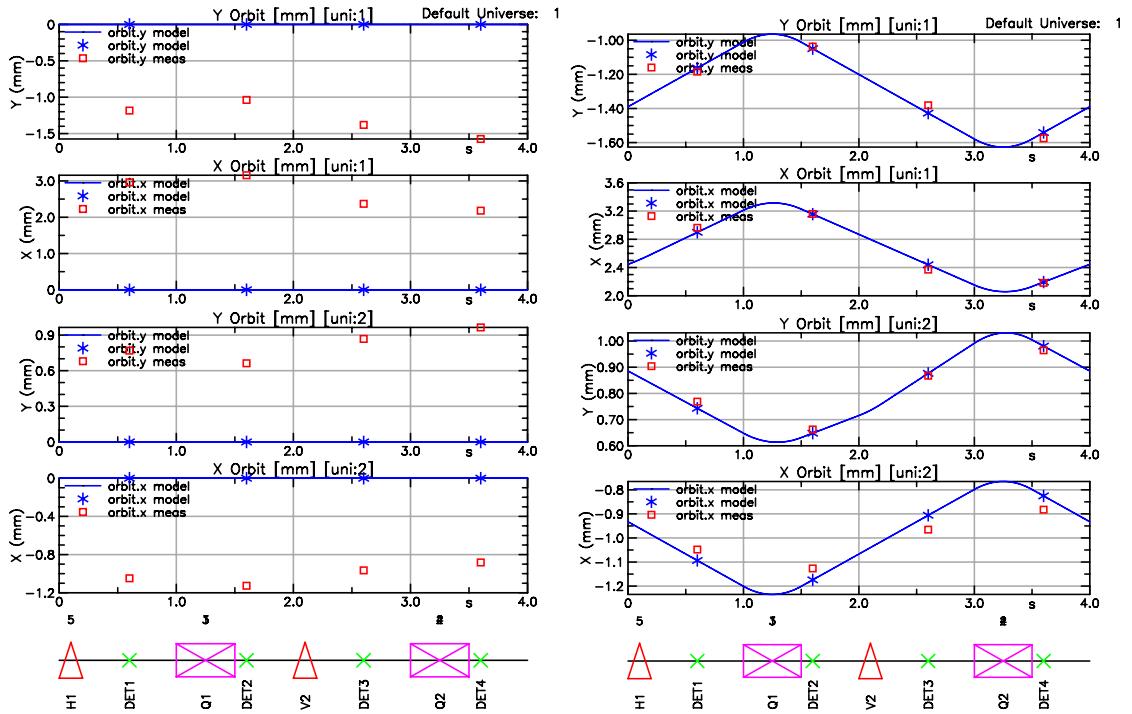
At startup, the resulting plot window is shown in Fig. 12a. It is no surprise that the `model` orbit, being zero, does not well fit the `measured` data. Running the optimization (§10.7) with the `run` command and after doing a `scale` command produces Fig. 12b. Now the `measured` orbit data is fairly well fit.

Even though the optimization can fit the orbit fairly well, the `model` values for the variable parameters are a bit off from the “true” values (that is, the values when the “measurements” were made) which are the values as set in the `create_data.tao` file (§11.7):

Tao> show var *	Variable	Slave Parameters	Meas	Model	Design
	quad_k1[1]	[1:2]@Q1[K1]	1.0000E+00	9.8011E-01	1.0000E+00
	quad_k1[2]	[1:2]@Q2[K1]	-1.1000E+00	-1.1711E+00	-1.1000E+00
	quad_tilt[1]	[1:2]@Q1[TILT]	0.0000E+00	8.8747E-02	0.0000E+00
	quad_tilt[2]	[1:2]@Q2[TILT]	0.0000E+00	-7.2506E-02	0.0000E+00
	steering[1]	1@H1[HKICK]	1.0000E-04	1.5353E-04	0.0000E+00
	steering[2]	2@V2[VKICK]	1.0000E-04	1.1896E-04	0.0000E+00

This is due to a combination of the errors that were introduced and some degeneracies among the variables. Even though there is a significant differences between here between true and `model` values, a correction using Eq. (3) does a good job of correcting the beta function. To simulate a correction after the fit was done, the following can be done

```
Tao> run ! Run the optimizer
Tao> set lattice base = model ! Save the model fit values in the base lattice
Tao> call create_data.tao ! This will load the "true" values
Tao> set var *|model = *|model + *|design - *|base ! Load the correction.
```



(a) Plot window at startup. The **measured** orbit data points (red) are far from the **model** orbit which corresponds to the zero orbit.

(b) Plot window after optimization showing the **measured** orbit points is fairly well fit by the **model** orbit.

Figure 12

The result is shown in Fig. 13. Fig. 13A shows the **design** beta function. Fig. 13B shows the shows the “true” beta functions that existed at the time of the measurements. Finally, Fig. 13C shows the beta functions after correction. Fig. 13C is close to Fig. 13A showing that the lattice is fairly well corrected.

11.9 Exercises [Answers in Section §25.6]

- 11.1 With orbit correction as discussed in section (§11.1), it was explicitly assumed that the **design** orbit was the desired orbit one wanted. But what if this is not the case? Suppose that the desired orbit is one that has been empirically determined to minimize, say, the beam size, or, say, maximize the lifetime. How should orbit correction proceed in this case where the “golden” orbit is not the **design** orbit?
- 11.2 One important component in the optimization process is the setting of the merit function weights for the datums and variables. This can frequently be done empirically by varying the weights and while observing how well the data is fitted and how well behaved the

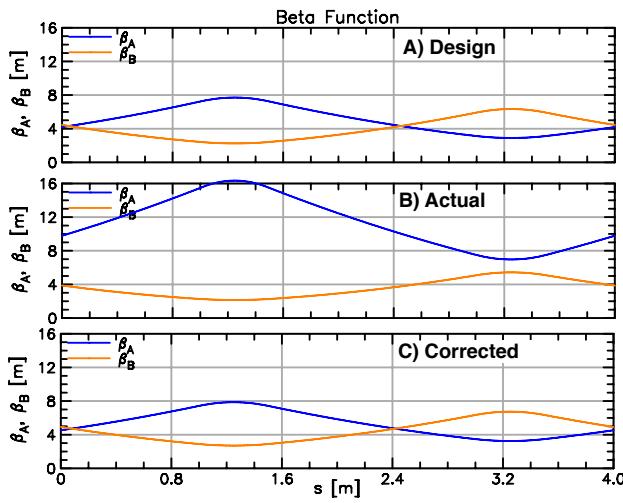


Figure 13: Beta plots showing the Twiss parameter correction. A) Design Beta functions. B) Beta functions before correction. C) Beta function after a correction. The corrected Beta function is near the design which is what is wanted.

variable values are. Try varying weights in the example in this chapter to see what happens to the fit.

12 Multiple Universes

Tao has a concept called a **universe**. A **universe** consists of **model**, **design**, and **base lattice** as explained in Chapter §9. *Tao* can be initialized with multiple universes. This can be useful in a number of situations. For example, the simultaneous analysis of multiple data sets to correct the machine optics as discussed in Chapter §11. Universes are discussed in the *Tao* manual in the chapter titled “Overall Organization and Structure”.

12.1 Multiple Universe Example

The example files used to illustrate multiple universes are in the directory

```
bmad-doc/tutorial_bmad_tao/lattice_files/multiple_universes
```

Copy these files to your working directory. There are four files here:

```
tao.init      ! Tao initialization file.  
setup.tao     ! Command file run at startup.  
lat1.bmad     ! Lattice file for universe 1.  
lat2.bmad     ! Lattice file for universe 2.
```

Here the main initialization file is named **tao.init**. The **tao.init** file contains a **tao_design_lattice** namelist which specifies that there will be two universes:

```
&tao_design_lattice  
  n_universes = 2  
  design_lattice(1)%file = "lat1.bmad"  
  design_lattice(2)%file = "lat2.bmad"  
/
```

The lattice file **lat1.bmad** will be used for universe 1 and **lat2.bmad** will be used for universe 2.

The **tao.init** file also contains a **tao_start** namelist that specifies the command file to run at startup:

```
&tao_start  
  startup_file = "setup.tao"  
/
```

Since “tao.init” is the default name for the main initialization file, and since this file sets the lattice file names, *Tao* can be started using just the “tao” command without any arguments.

12.2 Using Tao With Multiple Universes

With the example files copied to your working directory, start *Tao* with the “tao” command. *Tao* starts up with the default set of orbit, beta, and dispersion plots as shown in Figure 14a. For each plot curve, there is an associated universe where the data used to construct the curve is

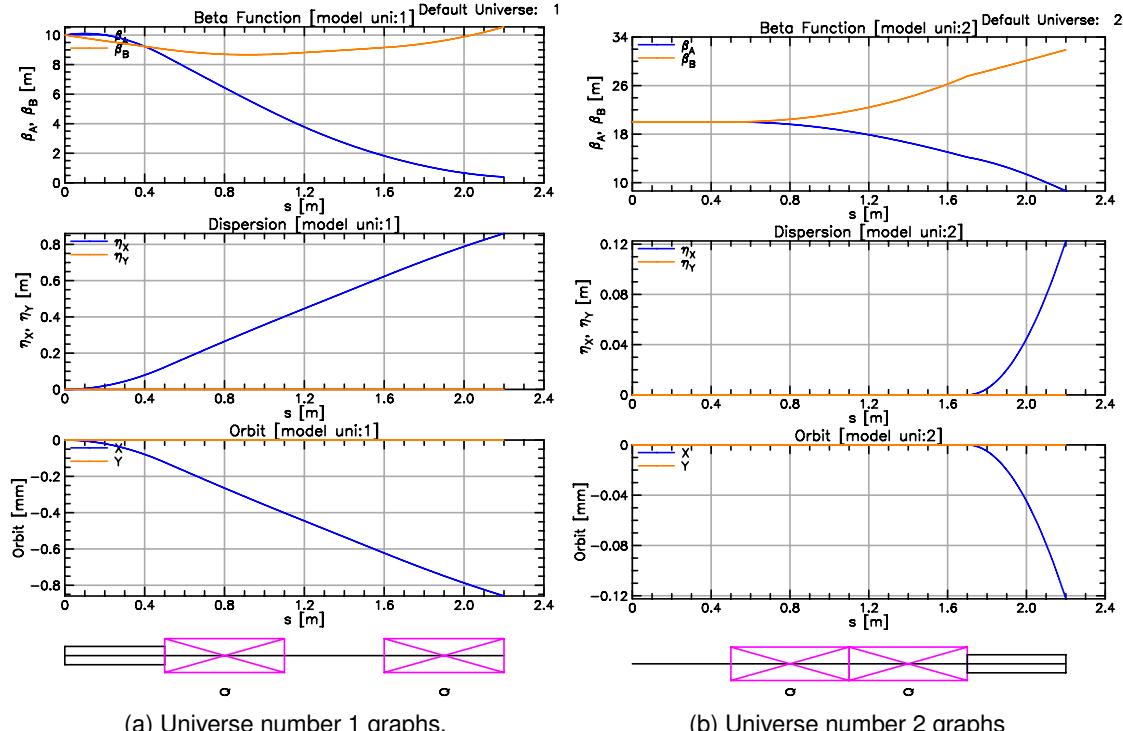


Figure 14: Two Universe Example

obtained. To see what the associated universe for, say, the horizontal orbit curve is, use the `show curve` command:

```
Tao> show curve orbit
Region.Graph.Curve: r33.g.x
                    r33.g.y
Plot.Graph.Curve:   orbit.g.x
                    orbit.g.y
data_source        = "lat"
data_index         = ""
...
ix_bunch          = 0
ix_universe        = -1
...
```

[Note: In this example, since “orbit” is a plot name and not a curve name, the parameters shown are for the first associated curve which is `orbit.g.x` and which is what is wanted.] The `ix_universe` index for the curve is set to -1 which indicates that what will be plotted is what is called the “**default**” universe. The default universe is printed in the upper right hand corner of Figure 14a which shows the default universe to be universe #1. Additionally, the titles above each graph in Figure 14a show that all the curves of all the graphs are associated with universe

#1.

To print the default universe use the **show global** command:

```
Tao> show global
Tao Global parameters [Note: To print optimizer globals use: "show optimizer"]
  %beam_timer_on          = F
  %bunch_to_plot          = 1
  ...
Tao Parameters:
  Universe index range:      = 1  2
  default_universe:          = 1  ! Set using: "set default universe = ..."
  default_branch:            = 0  ! Set using: "set default branch = ..."
  ...
```

To set the default universe use the command **set default universe**. In this example, to save on typing, an alias command called **view** has been set in the **setup.tao** file that changes the default universe and scales all the plots:

```
alias view set default universe = [[1]]; scale
```

So using the command

```
Tao> view 2
```

will produce Figure 14b.

Besides plotting, the default universe also plays a role when using some *Tao* commands. For example, the **show lattice** command will show the default universe unless the optional **-universe** switch is used override the default. Example:

```
Tao> show lat -uni 2    ! Show lattice elements from universe 2
```

Another example is the **set element** command which sets lattice element parameters. Here the **@** prefix can be used to specify the universe

```
Tao> set ele 2@b dg = 0.01  ! Apply to elements B in universe 2
Tao> set ele *@b dg = 0.02  ! Apply to all elements B in all universes
```

12.3 Exercises [Answers in Section §25.7]

- 12.1 For the beta plot, use the **set** command to set **ix_universe** equal to **1** for the **a**-mode curve and **ix_universe** equal to **2** for the **b**-mode curve so that the beta plot will always look the same independent of the setting of the default universe. Verify this by changing the default universe.

13 Control Elements

13.1 Overview

Control elements are elements that control the parameters of other elements. There are three types of control elements: **groups**, **overlays**, and **girders**. **Groups** and **overlays** are convenient to do such things as simulate control room "knobs". For example a power supply that powers a chain of magnets. **Girder** elements (§13.6) are used for simulating lattice element support structures.

Note: **Group**, **overlay**, and **girder** elements are known as “**minor lords**” since they only control a subset of an element’s attributes. The other type of **lord** elements, **multipass lords** (§17) and **superposition lords** (§16) are called “**major lords**”.

13.2 Example Lattice

The lattice used to illustrate control elements is named **control.bmad**:

```
! Lattice File: control.bmad
beginning[beta_a] = 10
beginning[beta_b] = 10

parameter[particle] = muon
parameter[p0c] = 1e9
parameter[geometry] = open

q: quadrupole, l = 1
b: sbend, l = 1
ll: line = (q, b)
use, ll

ov1: overlay = {q[k1]: a+b^2, b[g]: 0.1*a+tan(b)}, var = {a, b}, a = 0.02
ov2: overlay = {q[k1]: 0.7, q[x_offset]: 0.1*hh}, var = {hh}, hh = 0.01
gr1: group = {b[k1]: 0.4*sqrt(z)}, var = {z}
```

See the **group** (§4.22) and **overlay** (§4.37) sections of the **Elements** chapter of the *Bmad* manual.

Notes:

- The overlay **ov1** controls two parameters: The **k1** attribute of element **q** (denoted **q[k1]**) and the **g** attribute of element **b** (denoted **b[g]**).
- Overlay **ov1** has two variables called **a** and **b** that are used to control the two attributes.
- The formulas that overlay **ov1** uses to calculate the values of the two controlled attributes are **a+b²** for **q[k1]** and **0.1*a+tan(b)** for **b[g]**.
- Since overlay **ov2** also controls **q[k1]**, the value of **q[k1]** is the sum of the contributions of **ov1** and **ov2**.

-
- The given "formula" for the control of **q[k1]** by **ov2** is just a constant: 0.7. This is a shorthand notation and the actual formula used is **0.7*hh**. Note: When this shorthand notation is used, only one control variable (in this case **hh**) may be used by the overlay.
 - The initial values for control variables may be set when defining the control element. For example, **hh** of **ov2** is set to 0.2. Control variables default to a value of zero.

13.3 Control Element Organization in the Lattice

Start *Tao* as explained in section §5.2 with the lattice file **control.bmad**. To see the elements in the lattice use the **show lat** command:

```
Tao> show lat

      Values at End of Element:
Index   name      key          s       l     beta    phi ...
                                         a       a ...
0     BEGINNING Beginning_Ele  0.000    ---  10.00  0.000 ...
1       Q        Quadrupole  1.000  1.000  9.83  0.101 ...
2       B        Sbend       2.000  1.000  9.60  0.204 ...
3     END        Marker      2.000  0.000  9.60  0.204 ...
Lord Elements:
4     OV1        Overlay    2.000    ---  9.60  0.204 ...
5     OV2        Overlay    1.000    ---  9.83  0.101 ...
6     GR1        Group      2.000    ---  9.60  0.204 ...
Index   name      key          s       l     beta    phi ...
                                         a       a ...
      Values at End of Element:
```

The list of lattice elements is divided up into two sections:

- The "**tracking**" part of the lattice are the elements to be tracked through. Here the tracking part of the lattice contains elements with index 1 through 3 (the **beginning** element with index 0 is not tracked through and is present to hold Initial parameters like the Twiss parameters).
- The "**lord**" section of the lattice are where the lord elements reside. Here the lord section contains elements with index 4 through 6.
- **Group** and **overlay** elements get assigned a longitudinal s-position based upon the s-position of the last slave element. This does not affect any calculations and is done since it can be useful information when using the **show lat** and other **show** commands.

13.4 Overlay Control

To see how things are controlled, use the **show element** command. Examining lord **ov1** shows:

```
Tao> show ele 4      ! Or: show ele ov1

Element #          4
Element Name: OV1
Key: Overlay
... etc...
Slave_Status: Free
Lord_Status: Overlay_Lord
Control Variables:
  1  A           =  2.000000E-02
  2  B           =  0.000000E+00
Slaves: [Attrib_Val = Expression_Val summed over all controlling overlays.]
Index   Ele_Name   Attribute Attrib_Value   Expression_Val   Expression
  1     Q         K1        2.7000E-02    2.0000E-02   a+b^2
  2     B         G        2.0000E-03    2.0000E-03   0.1*a+tan(b)
... etc...
```

- All lattice elements have a **slave_status** which shows what type of slave the element is and a **lord_status** which shows what type of lord the element is. **overlay** elements automatically have a **lord_status** of **overlay_lord**. In this case, **ov1** has a **slave_status** of **free** since there are no other lord elements that control parameters of **ov1**. In general, **overlay** and **group** lords may control parameters of other lords as well as non-lords.
- When an element parameter is controlled by one or more overlays, the value of that element parameter is the sum of the values for each overlay. Thus in the above example, the contribution to **q[k1]** due to overlay **ov1** is 0.02 ($= a + b^2$) as shown in the “**Expression_Val**” column above. There is also a contribution of 0.007 ($= 0.7 \cdot hh$) due to overlay **ov2** making the value of **q[k1]** equal to 0.027 as shown in the “**Attrib_Value**” column above.

Examining the **q** slave element shows that indeed the **k1** attribute has a value of 0.027:

```
Tao> show ele q
Element #          1
Element Name: Q
... etc...
  1  L           =  1.0000000E+00  m
  4  K1          =  2.7000000E-02  1/m^2
... etc...
Slave_Status: Minor_Slave
Controller Lord(s):
Index   Name       Attribute   Lord_Type   Expression
  4  OV1         K1          Overlay      a+b^2
  5  OV2         K1          Overlay      0.7*hh
  5  OV2         X_OFFSET   Overlay      0.1*hh
```

```
Lord_status: Not_a_Lord  
... etc...
```

The **slave_status** of element **q** is set to **minor_slave** to show that it is controlled by one or more minor lords. The **lord_status** of **q** is **not_a_lord** indicating that it does not control anything (“tracking elements”, that is elements in the tracking part of the lattice, never control other elements).

Since the value of an attribute that is controlled by overlays depends directly on the overlay variable values, the attribute may not be directly changed. For example, trying to change **q[k1]** directly will result in an error:

```
Tao> set ele q k1 = 0.02  
[ERROR | 2017-AUG-26 13:29:26] attribute_free:  
    THE ATTRIBUTE: K1  
    OF THE ELEMENT: Q  
    IS NOT FREE TO VARY SINCE:  
    IT IS CONTROLLED BY THE OVERLAY: OV1
```

13.5 Group Control

Overlay elements use what is called “**absolute**” control since the value of a controlled parameter is determined directly by the settings of the overlay variables that the controlled parameter is slaved to. On the other hand, **group** elements use what is called “**relative**” control which is different from absolute control in two respects:

- Only changes in group variable values affect controlled parameters.
- With group control, a controlled parameter may be varied directly.

Looking at an example will make this clear. Starting from the **control.bmad** lattice, consider the effect of changing the **z** variable of the group **gr1** to 0.01.

```
Tao> set ele gr1 z = 0.01  
  
Tao> show ele gr1  
Element #          6  
Element Name: GR1  
Key: Group  
  
... etc...  
  
Slave_Status: Free  
Lord_Status: Group_Lord  
Control Variables:  
    1   Z   =  1.0000000E-02           OLD_Z   =  1.0000000E-02  
Slaves:  
    Index   Ele_Name   Attribute   Attrib_Value   Expression_Val   Expression  
          2       B         K1           4.0000E-02        4.0000E-02      0.4*sqrt(z)
```

For **group** elements, *Bmad* keeps track of what is called the “old” value of a variable. The name of the old value is the variable name with a “**old_**” prefix. In this case the old value of **z** is given the name “**old_z**”. Before the **set ele gr1** command was executed, the value of **z** and **old_z** is zero. When the above **set ele gr1** command is executed, the value of **z** becomes 0.01. *Bmad* detects that **z** and **old_z** are different and updates **b[k1]** using the following procedure:

1. Evaluates the formula for **b[k1]** using **z** and **old_z** and takes the difference. In this case the difference is $0.4*\sqrt{z} - 0.4*\sqrt{\text{old}_z} = 0.04$
2. Changes the value of **b[k1]** by the difference (0.04). Since the old value of **b[k1]** was zero. The new value of **b[k1]** is 0.04.
3. Sets the value of **old_z** equal to **z**.

A consequence of the last step in the above procedure means that the **show ele** command will always show that **old_z** and **z** are equal.

Now consider the effect of the following commands:

```
Tao> reinit tao
Tao> set ele gr1 z = 0.01
Tao> set ele b k1 = 0.02
Tao> set ele gr1 z = 0.04
```

The result is:

```
Tao> show ele gr1
... etc...
Control Variables:
  1  Z   =  4.0000000E-02          OLD_Z   =  4.0000000E-02
Slaves:
  Index   Ele_Name   Attribute   Attrib_Value   Expression_Val   Expression
    2       B         K1           6.0000E-02      8.0000E-02      0.4*sqrt(z)
```

1. The “**reinit tao**” command resets *Tao* to its initial state.
2. The “**set ele gr1 z = 0.01**” command acts as explained above.
3. The “**set ele b**” command sets the value of **b[k1]** to 0.02. This is independent of the state of element **gr1**.
4. The “**set ele gr1 z = 0.04**” command sets the value of **gr1[z]** to 0.04 which causes the value of **b[k1]** to increase by 0.04 (= 0.08 - 0.04) from 0.02 to a value of 0.06.

What is a group element useful for? Example: Consider the situation where you want to control the chromaticity (change in tune with particle energy) of a ring by varying sextupole strengths. To change the chromaticity by 1 unit you want to change the sextupole strengths by some amount that you compute. Here you don’t care about the value of the sextupole strengths per se, you only want to vary the sextupole strengths by a certain delta. So the sextupole “knob” can be simulated using a **group** controller which may look like:

```
raw_xqune_1 : group ={SEX_08W:-.6415E-03*k2,...}, var = {k2}
```

Note: In this case, since the parameter to be controlled for the **sex_08w** element was not specified, the parameter is taken to be the same as the variable of the controller. **k2** in this case.

Notes:

- Group and overlay elements can control other group and overlay elements.
- A given element parameter may only be controlled by a set of group elements or a set of overlay elements but may not be controlled by both group and overlay elements since this would create an ambiguous situation as to how to evaluate the parameter.

13.6 Girders

A third type of controller is the **girder** element which can be used to simulate support structures like an I-beam that supports a number of magnets or an optical table supporting an optical setup. This is discussed further in Section §14.5. Also see the *Bmad* manual for more details.

13.7 Exercises [Answers in Section §25.8]

- 13.1 The function that a controller uses to control a slave attribute may be specified using an arithmetical expression as in the above examples, or may be specified by a list of “**knot**” points with spline interpolation used to evaluate the function in between points. As an exercise, setup a controller that uses knot points that mimics the action of **ov2** at least over some limited interval. Hint: Look at the documentation for **overlay** or **group** elements in the **elements** chapter of the *Bmad* manual.
- 13.2 **Group** controllers are good for varying the longitudinal position of elements. Starting with the file **simple.bmad** add a group controller that varies the s-position of the upstream edge of element **B** while keeping the length of the entire lattice constant (hint: The lengths of both **B** and **D** must change in tandem). This situation occurs frequently enough that there is a shortcut attribute called **start_edge** that can be used instead of directly varying the lengths of elements. See the documentation on **group** elements (§4.22) in the **Elements** chapter of the *Bmad* manual for more details.
- 13.3 Modify the lattice file **simple.bmad** to include a **girder** element supporting elements **B** and **Q**. Use the **show ele** command to verify that indeed the girder is supporting these two elements.

14 Machine Coordinates and Patch Elements

14.1 Coordinate Systems

As explained in the **Coordinates** chapter of the *Bmad* manual, *bmad* uses three coordinate systems to describe the positioning of lattice elements as shown in Figure 15:

Global Coordinates

The **(X, Y, Z) global** (also called **floor**) coordinate system (notice that capital letters are used) is independent of the accelerator machine and is "attached" to the building the accelerator is in. Typically, the **Y**-axis is taken to be pointing vertically up and **(X, Z)** is the horizontal plane.

Local Coordinates

The **global** coordinate system is not convenient for describing where particles are as they move through the lattice. For this, shown in red, there is the **local** (also called **laboratory**, also called **reference**) curvilinear coordinate system. Laboratory coordinates are also used to describe the nominal (that is, without any "misalignments") position of the lattice elements. The laboratory coordinates start with a curved line often called the "**reference trajectory**". The distance along the reference trajectory is denoted by s . At each point along the reference trajectory, there is a Cartesian **(x, y, z)** coordinate system with the origin at the reference trajectory point. The **z**-axis is always tangent to the reference trajectory and the **x** and **y**-axes always transverse to the reference trajectory.

Element Body Coordinates

Elements can be shifted ("misaligned") from their nominal position. To describe things like electric and magnetic fields or apertures (which can depend upon the elements actual position), **element body** coordinates are used. The **element body** coordinates are the coordinates attached to the physical element. Without any "misalignments", the **element** coordinates correspond to the **laboratory** coordinates.

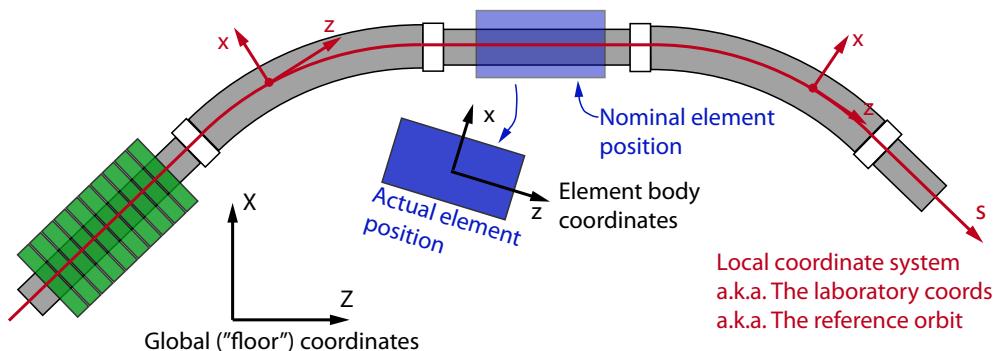


Figure 15: The three coordinate systems used to describe lattice element positioning: **Global**, **reference**, and **element body** coordinates.

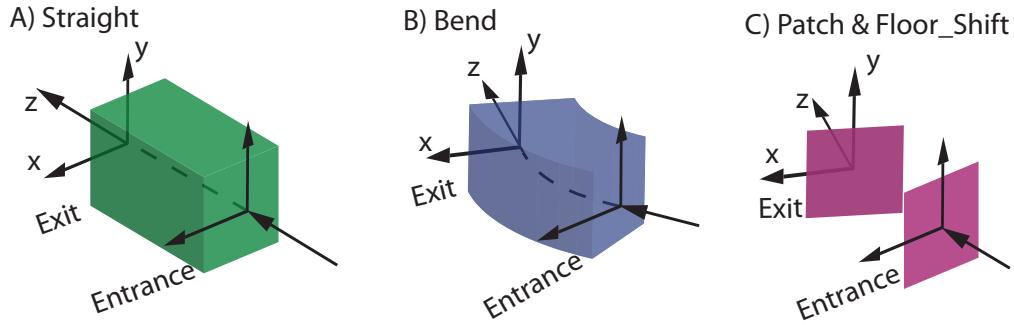


Figure 16: Lattice element geometry types: **Straight**, **bend**, and **patch**. All elements have an **entrance** coordinate system and an **exit** coordinate system.

14.2 Element Geometry Types

All lattice elements have an “entrance” end and an “exit” end. Normally a particle will enter the element at the **entrance** end and exit at the **exit** end but it is possible to simulate particles going backwards or have lattice elements that are reversed longitudinally.

Lattice elements in *Bmad* have one of four geometry types. Three of them will be discussed here and are shown in Figure 16. [The fourth geometry type, used for X-ray simulations, is used with **mirror**, **multilayer_mirror**, and **crystal** elements.] These three types are called **straight**, **bend** and **patch** based upon how the (x, y, z) laboratory coordinates transform as a function of the longitudinal s position from the **entrance** end of the element to the **exit** end.

Straight Geometry

The straight geometry shown in Figure 16A is used with elements like **drifts** and **quadrupoles**.

The orientation of the (x, y, z) coordinates is independent of s and the reference trajectory is a straight line so that the z -axis at the **exit** end is co-linear with the **entrance** z -axis.

Bend Geometry

The bend geometry shown in Figure 16B is used with **sbend** and **rbend** dipole elements. With this geometry, the reference trajectory is a semi-circle. The (x, y, z) coordinates rotate about an axis which is perpendicular to the z -axis. The default is to have the rotation axis parallel to the y -axis which keeps the orientation of the y -axis independent of s .

Patch Geometry

The patch geometry shown in Figure 16C is used with **patch** and **floor_shift** elements. With the **patch** geometry, the exit coordinates can be arbitrarily positioned with respect to the entrance coordinates. See section §14.6. The reference trajectory within the patch region is undefined since there is no good way to define it.

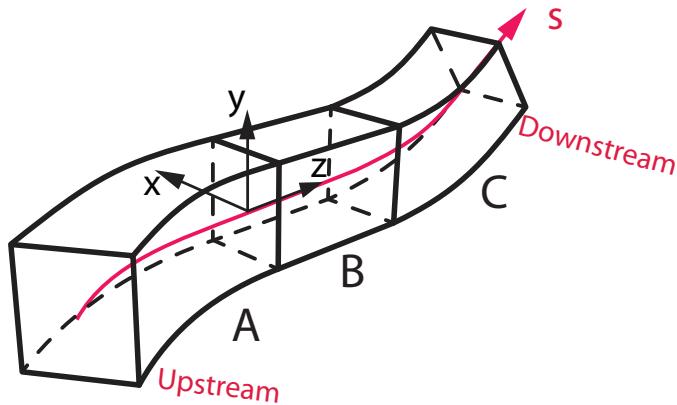


Figure 17: The **local** coordinate system is constructed by taking the ordered list of lattice elements and connecting the **exit** frame of one element to the **entrance** frame of the next.

14.3 Local Coordinate System Construction

The **local** coordinate system is constructed by taking the ordered list of lattice elements and connecting the **(x, y, z)** exit frame of one element to the **(x, y, z)** entrance frame of the next (just like LEGO blocks). Given a line constructed as:

```
lat: line = (A, B, C)
```

The result could look as shown in Figure 17. The reference trajectory is shown in red.

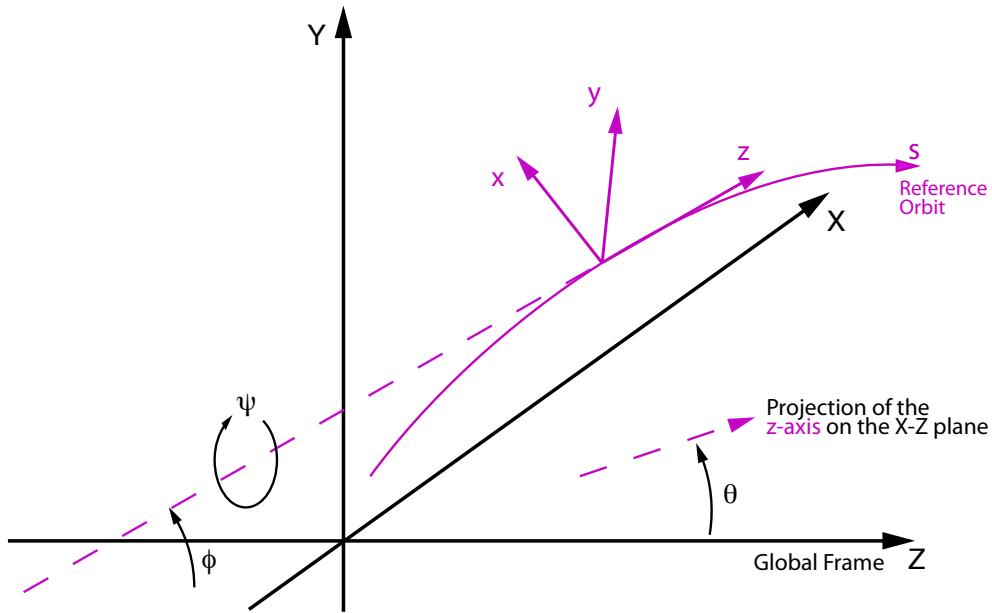


Figure 18: The global coordinate system.

14.4 Laboratory Coordinates Relative to Global Coordinates

For any given s -position on the reference orbit, the **local** coordinate system is described with respect to the **global** coordinate system by 6 parameters as shown in Figure 18:

- (X, Y, Z) global position
- θ azimuth angle in the (X, Z) plane.
- ϕ elevation angle
- ψ roll angle.

Notes:

- The default is for the beginning of the lattice ($s = 0$) is to have the local (x, y, z) coordinate system aligned with the global (X, Y, Z) coordinate system with θ, ϕ and ψ all being zero.
- For a machine that lies in the horizontal plane, the $\phi(s)$ and $\psi(s)$ angles are zero for all s .

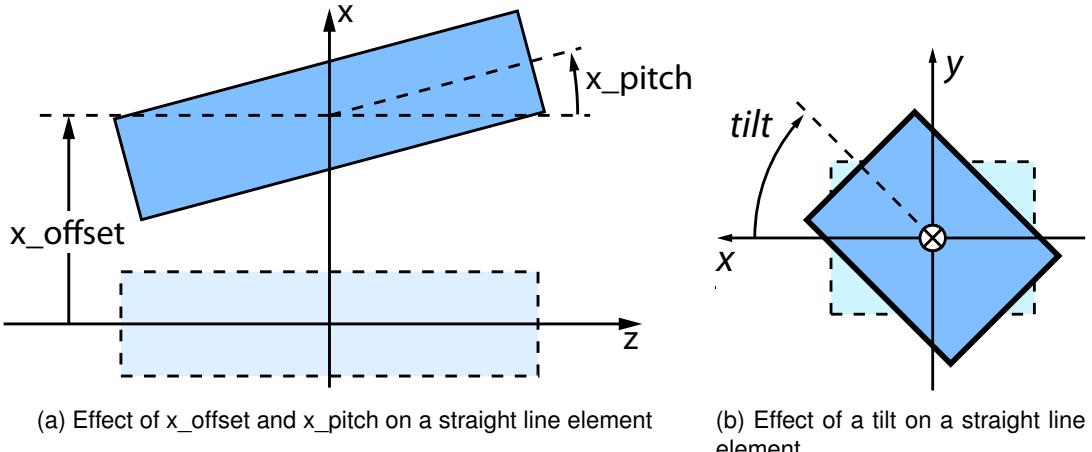


Figure 19

14.5 Element Misalignments

Once the **reference** coordinate system is established, the position of any physical element can be shifted (“misaligned”). [Note: **Patch** and **floor_shift** elements cannot be misaligned.] For straight elements, the element attributes that determine any misalignment are:

x_offset, y_offset, z_offset

The `x_offset`, `y_offset`, and `z_offset` attributes offset the element in the `x`, `y`, and `z` directions respectively. See Figure 19a.

x_pitch, y_pitch

The `x_pitch` and `y_pitch` attributes rotate the element. A `x_pitch` of $\pi/2$ would rotate the element around the `+y`-axis so that the body `+z`-axis is aligned with the local `+x`-axis. Similarly, a `y_pitch` of $\pi/2$ would rotate the element around the `-x`-axis so that the body `+z`-axis is aligned with the local `+y`-axis. See Figure 19a.

tilt

A `tilt` rotates the element around the `+z`-axis as shown in Figure 19b

Note: The above only applies to straight elements. Patch like elements are explained below. For a discussion of misalignments for bend type elements see the *Bmad* manual.

Example:

```
! Lattice File: misalign.bmad
beginning[beta_a] = 10.    ! m a-mode beta function
beginning[beta_b] = 10.    ! m b-mode beta function
beginning[e_tot] = 10e6   ! eV
parameter[geometry] = open ! or closed
q: quadrupole, L = 1, x_offset = 0.1, x_pitch = 0.04
```

```
lat: line = (q)      ! List of lattice elements
use, lat           ! Line used to construct the lattice
```

Start *Tao* as explained in section §5.2 with the lattice file **misalign.bmad**. The misalignment can be viewed using the **-floor** option with the **show element** command:

```
Tao> show ele q -floor

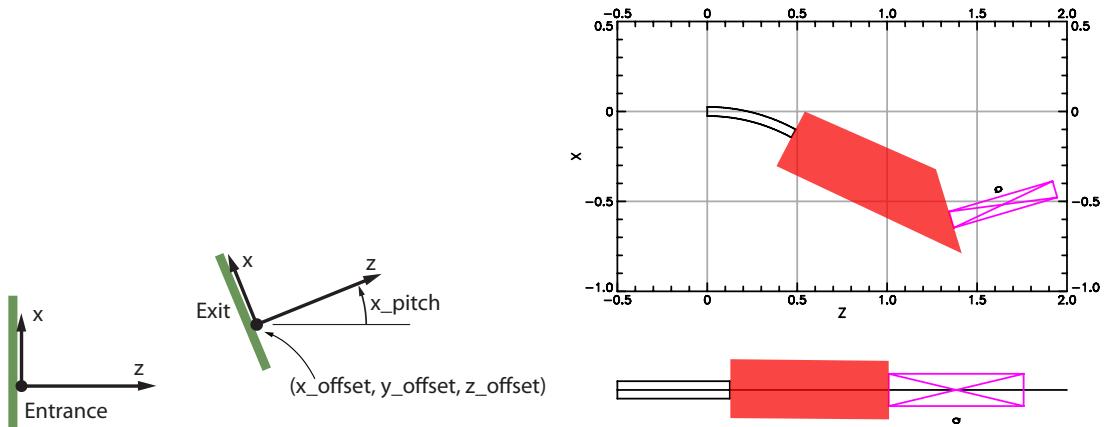
Element #          1
Element Name: Q
... etc...

Attribute values [Only non-zero/non-default values shown]:
  1  L            =  1.0000E+00 m
 13 SPIN_FRINGE_ON =  T (1)
 31 L_HARD_EDGE   =  1.0000E+00 m
 34 X_PITCH       =  4.0000E-02      55 X_PITCH_TOT   =  4.0000E-02
 36 X_OFFSET      =  1.0000E-01 m    57 X_OFFSET_TOT  =  1.0000E-01 m
... etc...

Global Floor Coords at End of Element:
      X          Y          Z      Theta
Reference  0.00000  0.00000  1.00000  0.00000 ... ! Without misalignments
Actual     0.11999  0.00000  0.99960  0.04000 ... ! With misalignments
... etc...
```

In the “**Global Floor Coords**” section, the **Reference** row shows the nominal position of the **exit** end of the element without misalignments. [Due to space constraints the **phi** and **psi** columns are not shown. They are zero in this case.] The **Actual** row shows the position of the physical element at the **exit** end.

Associated with each misalignment attribute there is a corresponding attribute with a “**_tot**” suffix. The difference is that an attribute like **x_offset** is the misalignment with respect to any **girder** (§13.6) that may be supporting it while the corresponding **x_offset_tot** is the total misalignment of the lattice element with respect to the element’s nominal position. Another difference is that misalignments attributes are set by the user while the corresponding **_tot** attributes are calculated by *Bmad*. If there is no **girder** support, the **_tot** attributes will be the same as the misalignment attributes as it is in this case so **x_pitch** is equal to **x_pitch_tot**, etc.



(a) The body coordinates at the exit end of a **patch** is set by the element attributes **x_offset**, **y_offset**, **z_offset**, **x_pitch**, **y_pitch**, and **tilt**.

(b) Lattice with a patch element. The patch element is the coordinates patch element in a lattice. [Note: The default is not to draw **patch** elements in a **floor_plan** plot.]

Figure 20

14.6 Patch Elements

Patch elements are used to shift the reference orbit. As a consequence, the nominal placements of all elements downstream of the patch are affected. This is useful in simulating things like injection or extraction lines where the patch is used to reorient the reference orbit so that it follows the injection or extraction line.

For **patch** elements the same six parameters that are used to misalign straight line elements are, for a **patch**, used to set the placement of the **exit** frame relative to the **entrance** frame. The transformation from entrance coordinates to exit coordinate is:

1. Initially the exit coordinates coincide with the entrance coordinates.
2. The origin of the exit coordinates is translated by $(x_{\text{offset}}, y_{\text{offset}}, z_{\text{offset}})$
3. The **x_pitch** and **y_pitch** rotations (in radians) are applied. The **x_pitch** rotation rotates the $+z$ axis towards the $+x$ axis (rotation around the $+y$ axis). The **y_pitch** rotation rotates the $+z$ axis towards the $+y$ axis (rotation around the $-x$ axis).
4. The **tilt** rotation (in radians) rotates the exit coordinates around the exit coordinate's $+z$ axis.

This transformation is illustrated in Figure 20a. The transformation from **patch entrance** to **exit** coordinates is the same transformation from laboratory coordinates at the center of a straight element to the element body coordinates at the center of the misaligned element.

Example:

```
! Lattice File: patch.bmad
beginning[beta_a] = 10.    ! m a-mode beta function
beginning[beta_b] = 10.    ! m b-mode beta function
beginning[e_tot] = 10e6   ! eV
parameter[geometry] = open ! or closed

b: sbend, L = 0.5, g = 1   ! g = 1 / bending_radius
p: patch, z_offset = 1, x_pitch = pi/4
q: quadrupole, L = 0.6, k1 = 0.23

lat: line = (b, p, q)     ! List of lattice elements
use, lat                  ! Line used to construct the lattice
```

Start *Tao* as explained in section §5.2 with the lattice file **patch.bmad**. Create a **floor_plan** with the command **place r11 floor**. The result is shown in Figure 20b except that, by default, *Tao* does not draw a patch element so, in the figure, the patch has been drawn in by hand. The global coordinates of the nominal positions of the elements can be seen by using the **show lat -floor** command:

```
Tao> show lat -floor

      Values at End of Element:
Ix  name      key      s          X          Y          Z          Theta ...
0  BEGINNING Beginning_Ele  0.000  0.0000  0.0000  0.0000  0.0000 ...
1  B          Sbend     0.500 -0.1224  0.0000  0.4794 -0.5000 ...
2  P          Patch     1.207 -0.6018  0.0000  1.3570  0.2854 ...
3  Q          Quadrupole 1.807 -0.4329  0.0000  1.9327  0.2854 ...
4  END        Marker    1.807 -0.4329  0.0000  1.9327  0.2854 ...
```

A **patch** represents a field free space so a particle traveling through a patch propagate as in a drift. The difference is that in a **patch** there is a coordinate transformation from entrance coordinates to exit coordinates.

14.7 Exercises [Answers in Section §25.9]

- 14.1 Modify the lattice file **simple.bmad** to include a **girder** element supporting elements **B** and **Q**. Misalign the **girder** and verify that a supported element will have **_tot** attributes different from the misalignment attributes.
- 14.2 Create a lattice with elements **drift**, followed by a **mirror**, followed by a **drift**. Give the **mirror** a finite **graze_angle** and verify that the laboratory coordinate after the **mirror** are rotated by twice the **graze_angle** with respect to the coordinates before the **mirror** so that a photon traveling on the zero-orbit before the **mirror** will stay on the zero-orbit after the **mirror**.
- 14.3 Using lattice **simple.bmad**, calculate by hand the floor coordinates at the exit end of element **Q** and compare this with the coordinates as calculated by **Bmad**.

15 Particle Phase Space Coordinates

The previous chapter showed how to describe the placement of lattice elements. This chapter covers how to describe particle trajectories.

15.1 Particle Phase Space Coordinates

Figure 21a shows the reference orbit. As explained in the previous chapter, the reference coordinates (x, y, z) associated with a given point a distance s along the reference point has the coordinate origin at the given point and the z -axis tangent to the reference orbit.

A particle being simulated has its own trajectory as shown in the figure. Given a particle at some point on its trajectory (blue dot in Figure 21a), there is a point at position s on the reference orbit such that the z coordinate of the particle is zero in the (x, y, z) coordinate frame associated with the reference orbit point.

With this, the particle's position and momentum \mathbf{P} can be described using the coordinates:

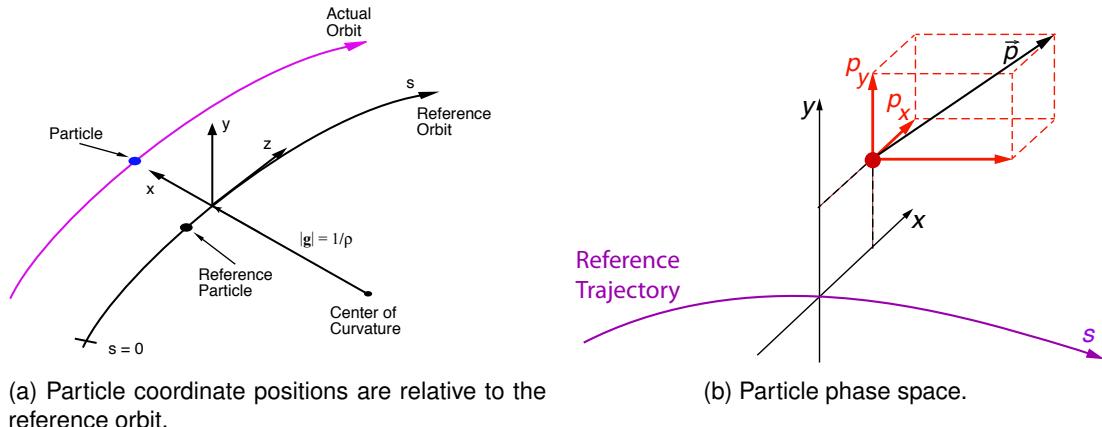
$$(x(s), y(s), P_x(s), P_y(s), P_z(s), t(s))$$

where $t(s)$ is the time that the particle is at the given point. From now on, to simplify the notation, the s dependence will be dropped.

For tracking purposes, canonical phase space coordinates are used with the convention that upper case \mathbf{P} denotes (unnormalized) momentum (Figure 21b) and lower case \mathbf{p} denotes phase space momentum. The phase space coordinates are denoted

$$(x, p_x, y, p_y, z, p_z)$$

where



(a) Particle coordinate positions are relative to the reference orbit.

(b) Particle phase space.

Figure 21

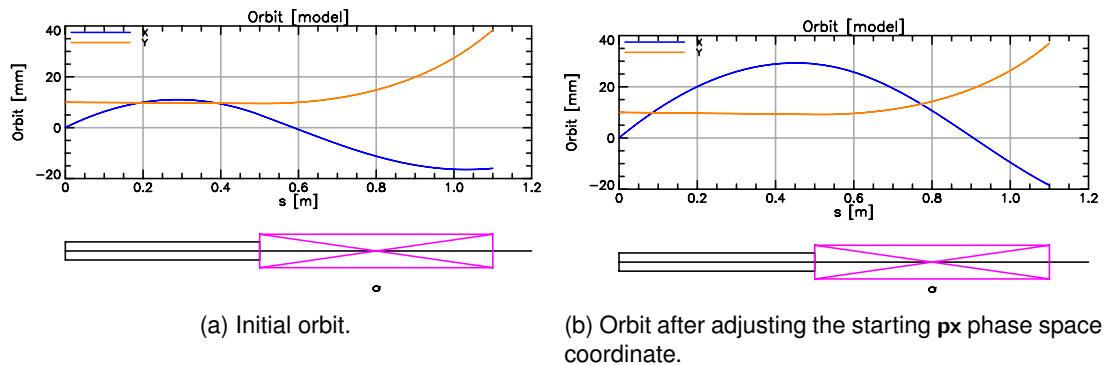


Figure 22

```

px = Px / P0
py = Py / P0
pz = (P - P0) / P0
z = c * β * (t_ref - t)

```

with

- $\mathbf{P}0$ is the reference momentum. See section §15.3.
- $c * \beta$ is the velocity of the particle,
- $t_{\text{ref}}(s)$ is the time the reference particle reaches the point s . The reference particle is a fictitious particle that can be imagined to be traveling on the reference orbit. Frequently, this reference particle is thought of as describing the center of a bunch of particles.

Notes:

- Do not confuse the canonical z coordinate with the z coordinate of the particle in the (x, y, z) coordinate frame. By construction, The latter is always zero.
- For a bunch of particles at a given s position, in general, the particles will have differing time t .
- If the reference particle has the same β value as a particle, canonical z will be the longitudinal distance the particle is with respect to the reference particle. Positive z indicates that the particle is in front of the reference particle and vice versa.

15.2 Example

Example lattice:

```

! Lattice File: orbit.bmad
beginning[beta_a] = 10. ! m a-mode beta function
beginning[beta_b] = 10. ! m b-mode beta function
beginning[e_tot] = 10e6 ! eV
parameter[geometry] = open ! or closed
bmad_com[spin_tracking_on] = T

particle_start[y] = 0.01
particle_start[px] = 0.06
particle_start[pz] = -0.2
particle_start[spin_x] = 1

b: sbend, L = 0.5, g = 1 ! g = 1 / bending_radius
q: quadrupole, L = 0.6, k1 = 10

lat: line = (b, q) ! List of lattice elements
use, lat ! Line used to construct the lattice

```

Start *Tao* as explained in section §5.2 with the lattice file **orbit.bmad**. Here spin tracking is turned on (**bmad_com[spin_tracking_on] = T**) and a non-zero initial orbit is set using **particle_start** parameters. The resulting orbit is shown in Figure 22a.

The initial phase space coordinates can now be varied using the **change** or **set** commands. For example:

```

Tao> change particle_start px 0.04
      Old          New      Old-Design      New-Design      Delta
      0.060000    0.100000    0.000000    0.040000    0.040000

```

The result is shown in Figure 22b.

View Orbits with **show lattice** command

```

Tao> show lat -spin -orbit
      Values at End of Element:
      Index   name       key           ...           orbit   ...           spin
                           ...           x   ...           x
      0  BEGINNING Beginning_Ele ...  0.000000E+00 ...  0.000000E+00
      1    B            Sbend        ...  5.027722E-03 ... -1.414516E-01
      2    Q            Quadrupole  ... -1.599068E-02 ... -7.350543E-02
      3  END           Marker     ... -1.599068E-02 ... -7.350543E-02

```

or the **show element** command

```

Tao> show ele 1
      Element #             1
      Element Name: B
      ... etc...
      Orbit: Positron State: Alive
              Position[mm] Momentum[mrad]      Spin   |
      X:      5.02772161   -44.34051580  -0.14145163 | Particle [sec]: ...

```

Y:	9.52710654	-1.27804602	-0.00171549	Part-Ref [sec]:	...
Z:	-4.78648430	-200.00000000	0.98994368	(Ref-Part)*Vel [m]:	...

15.3 Reference Energy and the Lcavity and RFcavity Elements

Every lattice element has a reference particle, a reference energy called **E_tot**, and a reference momentum named **p0c** (in eV). The three are interrelated so knowing the reference particle and either the reference momentum or energy, the third quantity can be calculated. The reference momentum is used for normalization of the phase space momentum (§15.1) as well as normalized strength parameters. For example, for a quadrupole element, the **b1_gradient** parameter (units: Tesla/m) can be used to specify the linear field gradient and the **k1** parameter (units: 1/m²) is the normalized field gradient normalized using the value of the reference momentum as discussed in the “Magnetostatic Multipole Fields” section of the “Electromagnetic Fields” chapter of the *Bmad* manual.

For lattices with an **open** geometry (§18), the reference energy/momentum at the beginning of the lattice is what is set in the lattice file. The reference energy/momentum for a downstream element inherits the reference energy/momentum of the element just upstream. The exception is for **Lcavity** elements, which represent an RF cavity, where the reference energy/momentum at the exit end of the **Lcavity** is set so that a particle entering the cavity with zero phase space coordinates leaves with zero phase space coordinates and in particular phase space **pz** will be zero at the exit end. **Lcavity** elements also have **p0c_start** and **E_tot_start** parameters that are set to the reference energy of the upstream element. Notice that with **rfcavity** elements, which also represent an RF cavity, the reference energy/momentum is the same as the upstream element. Example:

```
! Lattice File: cavity.bmad
beginning[beta_a] = 10. ! m a-mode beta function
beginning[beta_b] = 10. ! m b-mode beta function
beginning[p0c] = 1e8 ! eV

parameter[geometry] = open      ! or closed
parameter[particle] = He+

q1: quad, l = 0.1, k1 = 0.14
q2: quad, l = 0.1, b1_gradient = parameter[p0c] * q1[k1] / c_light
lc: lcavity, l = 1, voltage = 10e8, rf_frequency = 1e9
rf: rfcavity, l = 1, voltage = 10e8, phi0 = 0.25

lat: line = (q1, q2, lc, q1, q2, rf)
use, lat
```

Notes:

- For a **Lcavity** $\phi_0 = 0$ corresponds to peak acceleration.
- For an **rfcavity** $\phi_0 = 0.25$ corresponds to peak acceleration.

Start *Tao* as explained in section §5.2 with the lattice file **cavity.bmad**. Examining the **Icavity** element shows:

```
> tao -lat cavity.bmad

Tao> show ele 3
Element #           3
Element Name: LC
Key: Lcavity
... etc...
 51   POC_START    = 1.000000E+08 eV      BETA_START    = 0.02681151
 52   E_TOT_START  = 3.729740E+09 eV      DELTA_E        = 1.000000E+09 eV
 53   POC          = 2.910237E+09 eV      BETA          = 0.61530588
 54   E_TOT        = 4.729740E+09 eV      GAMMA         = 1.268571E+00
... etc...
Orbit: He+  State: Alive
      Position[mm] Momentum[mrad]      Spin | 
      X: 0.00000000 0.00000000 | Particle [sec]: ...
      Y: 0.00000000 0.00000000 | Part-Ref [sec]: ...
      Z: -0.00000000 0.00000000 | (Ref-Part)*Vel [m]: ...
```

The reference energy at the start of the element, **E_tot_start**, is not the same as the reference energy at the end of the element **E_tot**. The particle orbit, which started out with zero phase space coordinates (there were no **particle_start** statements to give a non-zero starting orbit), still has zero phase space coordinates at the end of the **Icavity** element.

Compare this to the **rfcavity** element:

```
Tao> show ele 6
Element #           6
Element Name: RF
Key: RFcavity
... etc...
 53   POC          = 2.9102374E+09 eV      BETA          = 0.615305883
 54   E_TOT        = 4.7297409E+09 eV      GAMMA         = 1.2685712E+00
... etc...
Orbit: He+  State: Alive
      Position[mm] Momentum[mrad]      Spin | 
      X: 0.00000000 0.00000000 | Particle [sec]: ...
      Y: 0.00000000 0.00000000 | Part-Ref [sec]: ...
      Z: 140.37425587 494.97867675 | (Ref-Part)*Vel [m]: ...
```

Here there is no **E_tot_start** parameter since the ending reference energy is always equal to the starting one. Here, the **pz** coordinate at the end of the element is nonzero.

15.4 Exercises [Answers in Section §25.10]

- 15.1 Modify the **Icavity** in the **cavity.bmad** to have a small length (so the transit time is small), and set the beginning momentum small enough so the relativistic beta is significantly than

one. Starting the particle with a finite z , calculate the ending z after the cavity and verify that the change in z is consistent with the equation for z given in §15.1.

- 15.2 **Lcavity** elements have an attribute **phi0_err** which varies the RF phase that a particle sees but does not change the reference energy. Add a finite **phi0_err** to the **cavity** element and verify that the reference energy does not change but that the phase space **pz** of the particle (which is the normalized momentum deviation from the reference §15.1) does change. With the **cavity.bmad** lattice, there is a range of values for **phi0_err** where the cavity will decelerate the particle enough so that the particle will turn around and not make it through the cavity. Approximately what is this range?

16 Superposition

Superposition is used when elements overlap spatially. In such a case, *Bmad* creates “**slave**” elements that will be tracked through and “**lord**” elements that represent the individual elements. Some examples will make this clear. Note: Superposition is discussed in the “Superposition and Multipass” chapter in the *Bmad* manual.

16.1 Example 1

Superposition works by defining a **line** as done in any lattice file and then defining an element that will be “superimposed” on top of the line. To superimpose an element you need to specify where the element is placed. To do this, a reference position is specified and the superimposed element is placed at that position shifted by a specified offset as illustrated in Figure 23. The lattice file **superimpose1.bmad** illustrates how superposition is done.

```
! Lattice File: superimpose1.bmad

beginning[beta_a] = 10.    ! m a-mode beta function
beginning[beta_b] = 10.    ! m b-mode beta function
beginning[e_tot] = 10e6   ! eV Or can set p0c
parameter[geometry] = open      ! or closed

q: quadrupole, L = 1, k1 = 0.2
d: drift, L = 1

m1: marker, superimpose, ref = q, ref_origin = beginning, offset = 0.3
m2: marker, superimpose, ref = q, ref_origin = end, offset = 0.4

lat: line = (q, d)    ! List of lattice elements
use, lat              ! Line used to construct the lattice
```

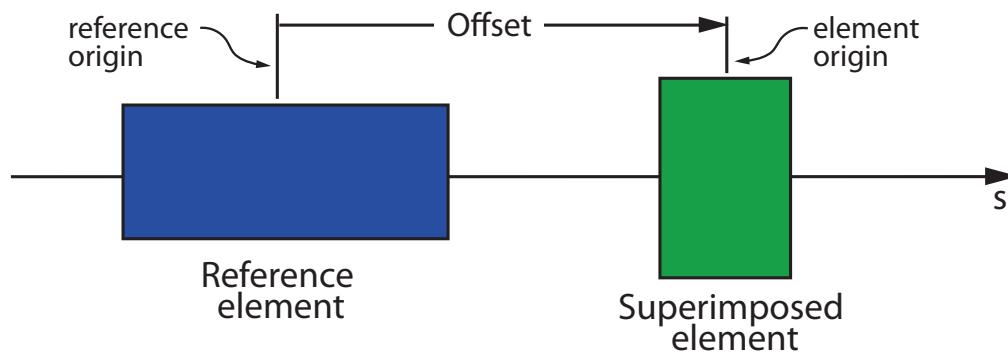


Figure 23: The placement of superimposed elements is determined by an offset from a reference element.

Here two marker elements named **m1** and **m2** are superimposed on the lattice. The offsets for **m1** and **m2** are 0.3 meters and 0.4 meters respectively. The reference position is determined by the reference element, which is specified by the **ref** attribute, and the **ref_origin** attribute which specifies where on the reference element the reference position is. In this example, the reference point for **m1** is the **beginning** (upstream) end of element **q** and the reference point for **m2** is the (downstream) **end** of element **q**. The element origin is similarly defined using the **ele_origin** attribute. In this case, since marker elements have zero length, the setting of **ele_origin** is immaterial.

Start *Tao* as explained in section §5.2 with the lattice file **superimpose1.bmad**. The lattice looks like:

```
Tao> show lat
      Values at End of Element:
Index   name      key          s       l     beta    phi    eta ...
                                         a       a       a ...
0 BEGINNING Beginning_Ele    0.000    ---  10.00  0.000  0.00 ...
1 Q#1        Quadrupole    0.300    0.300  9.83  0.030  0.00 ...
2 M1         Marker        0.300    0.000  9.83  0.030  0.00 ...
3 Q#2        Quadrupole    1.000    0.700  8.22  0.107  0.00 ...
4 D#1        Drift         1.400    0.400  6.97  0.160  0.00 ...
5 M2         Marker        1.400    0.000  6.97  0.160  0.00 ...
6 D#2        Drift         2.000    0.600  5.37  0.258  0.00 ...
7 END         Marker        2.000    0.000  5.37  0.258  0.00 ...
Lord Elements:
8 Q          Quadrupole    1.000    1.000  8.22  0.107  0.00 ...
Index   name      key          s       l     beta    phi    eta ...
                                         a       a       a ...
                                         Values at End of Element:
```

The quadrupole **Q** has been split by marker **M1** and so has become a **super_lord** element:

```
Tao> show ele Q
Element #                      8
Element Name: Q
Key: Quadrupole
... etc...
Slave_status: Free
Lord_status: Super_Lord
Slaves:
Index   Name      Type
1      Q#1      Quadrupole
3      Q#2      Quadrupole
```

The two **super_slaves** of **Q**, elements **Q#1** and **Q#2**, will be used when tracking a particle through the lattice.

If parameters of element **Q** are modified, Bmad bookkeeping routines will automatically update the **super_slaves**. Thus if the **k1** attribute of **Q** is modified:

Tao> change ele Q k1 0.11	Old	New	Old-Design	New-Design	Delta
	0.200000	0.310000	0.110000	0.110000	0.110000
					Q

The change will be reflected in the slave elements:

```
Tao> show ele 1
Element #           1
Element Name: Q#1
Key: Quadrupole

Attribute values [Only non-zero/non-default values shown]:
  1   L             =  3.000000E-01 m
  4   K1            =  3.100000E-01 1/m^2
... etc...

Slave_status: Super_Slave
Associated Super_Lord(s):
  Index   Name          Type
    8     Q            Quadrupole
Lord_status: Not_a_Lord
```

Parameter values of the super_slave elements are determined by the super_lord and may not be directly set:

```
Tao> change ele Q#1 k1 0.01

[ERROR | 2017-AUG-28 22:38:36] tao_change_ele:
  ATTRIBUTE NOT FREE TO VARY. NOTHING DONE
```

Notes:

- The default value for **ref_origin** and **ele_origin**, if not present, is **center** — the center of the element.
- The default reference element if **ref** is not present is the zero length **beginning** element at the beginning of the lattice.
- With closed lattices (§18), a superimposed element may "wrap" around so that part of the superimposed element is at the end of the lattice and part of the element is at the beginning of the lattice. See the example in Section §18.
- No super_lord element is made when a drift element is split. Thus in the above example, there is no **D** super_lord and the two elements **D#1** and **D#1** are *not* **super_slaves**. Drifts are the only type of element where, if split, a super_lord element is not created. This is done to simplify the lattice. If you don't want this behavior, use a **pipe** in place of a **drift**.

16.2 Example 2

The second superposition example involves superposition of an element with finite length:

```
! Lattice File: superimpose2.bmad
beginning[beta_a] = 10. ! m a-mode beta function
beginning[beta_b] = 10. ! m b-mode beta function
beginning[e_tot] = 10e6 ! eV Or can set p0c
parameter[geometry] = open ! or closed

Q: quad, l = 4
D: drift, l = 12
S: solenoid, l = 8, superimpose, ref = Q, ele_origin = beginning
M: marker, superimpose, ref = S, offset = 1

lat: line = (Q, D)
use, lat
```

The superimposes a solenoid on top of a quadrupole and a drift. Start *Tao* as explained in section §5.2 with the lattice file **superimpose2.bmad**.

```
Tao> show lat
      Values at End of Element:
Index   name     key           s       l     beta    phi    eta ...
          a       a       a ...
0 BEGINNING Beginning_Ele 0.000    --- 10.00  0.000  0.00 ...
1 Q#1      Quadrupole   2.000   2.000 10.40  0.197  0.00 ...
2 Q\S     Sol_Quad     4.000   2.000 11.60  0.381  0.00 ...
3 S#1      Solenoid    7.000   3.000 14.90  0.611  0.00 ...
4 M       Marker       7.000   0.000 14.90  0.611  0.00 ...
5 S#2      Solenoid   10.000   3.000 20.00  0.785  0.00 ...
6 D#1      Drift       16.000   6.000 35.60  1.012  0.00 ...
7 END      Marker     16.000   0.000 35.60  1.012  0.00 ...
Lord Elements:
8 Q       Quadrupole   4.000   4.000 11.60  0.381  0.00 ...
9 S       Solenoid    10.000   8.000 20.00  0.785  0.00 ...
Index   name     key           s       l     beta    phi    eta ...
          a       a       a ...
      Values at End of Element:
```

The **Q\S** super_slave element has both quadrupole **Q** and solenoid **S** elements as super_lords. This makes **Q\S** a **sol_quad** or combination solenoid and quadrupole element.

```

Tao> show ele q\s
Element #           2
Element Name: Q\S
Key: Sol_Quad
... etc...

Slave_status: Super_Slave
Associated Super_Lord(s):
Index   Name          Type
  8     Q            Quadrupole
  9     S            Solenoid
Lord_status: Not_a_Lord

```

Notes:

- This superposition works since *Bmad* has a **sol_quad** element which is a combination solenoid/quadrupole. On the other hand, *Bmad* does not have a combination quadrupole/-sextupole element. When a quadrupole is superimposed with a sextupole the result is an **em_field** element and the tracking method (§20) is switched to **runge_kutta**.
- **Jumbo** superposition can be used to superimpose elements whose combination cannot be represented by a corresponding *Bmad* element. The drawback in this case is that the particle tracking through this element must be done via a Runge-Kutta or similar tracking method. (§20). See the *Bmad* manual for more details.

16.3 Exercises [Answers in Section §25.11]

- 16.1 Create a lattice with a sextupole superimposed in the middle of a quadrupole. Make the length of the sextupole shorter than the length of the quadrupole. Make a second lattice similar to the first but this time use **jumbo** superposition. Compare both lattices.

17 Multipass

Some lattices have the beam recirculating through the same element multiple times. For example, an Energy Recovery Linac (ERL) will circulate the beam back through the LINAC part to retrieve the energy in the beam. In *Bmad*, this situation can be simulated using the concept of **multipass**. Another situation where multipass is useful is for modeling the interaction region in a colliding beam machine. In the *Bmad* manual multipass is discussed in the “Superposition and Multipass” chapter.

17.1 What is Multipass and What is it Good For?

Consider the following lattice:

```
A: quadrupole
11: line = (A, A)
use, 11
```

The lattice has two quadrupoles both called **A**. These two elements, even though they have the same name, are independent:

```
Tao> change ele 1 k1 0.01 ! Can modify first A element.
Tao> change ele A##2 k1 0.02 ! And can modify second A ele independently.
```

Now consider an ERL. With an ERL, the beam will go through the linac section multiple times. An ERL lattice might look like:

```
linac: line = (...)
arc: line = (...)
dump: line = (...)
erl_line: line = (injector, linac, arc, linac, dump)
```

Here you don't want the elements of the fist **linac** in **erl_line** to be treated as separate from the second **linac** in **erl_line** since they represent the same set of physical elements. This is where **multipass** comes in. Multipass is used to describe the situation where multiple elements to be tracked through are actually the same physical element and you want that fact to be enforced when element parameters are varied.

In this case, the solution is to mark the **linac** line as multipass to tell *Bmad* that the first instance of **linac** in **erl_line** contains the same physical elements as the second instance of **linac**:

```
linac: line[multipass] = (...)
```

With a **multipass** line, *Bmad* will setup appropriate multipass lords and multipass slaves to connect together all elements which represent the same physical element.

17.2 Example

```
! Lattice File: multipass.bmad

beginning[beta_a] = 100.    ! m a-mode beta function
beginning[beta_b] = 100.    ! m b-mode beta function
beginning[p0c] = 10e6      ! eV
parameter[geometry] = open      ! or closed

cavity: lcavity, l = 1, voltage = 10e6, rf_frequency = 1e9

linac: line[multipass] = (cavity)
erl: line = (linac, linac)
use, erl

expand_lattice
cavity\2[phi0_multipass] = 0.5
```

Start *Tao* as explained in section §5.2 with the lattice file **multipass.bmad**. The lattice looks like:

```
Tao> show lat
      Values at End of Element:
Index   name       key           s        l     beta     phi     eta ...
                                         a       a       a ...
0 BEGINNING Beginning_Ele    0.000    --- 100.00  0.000  0.00 ...
1 CAVITY\1  Lcavity      1.000    1.000 78.87  0.011 -0.00 ...
2 CAVITY\2  Lcavity      2.000    1.000 42.09  0.028 -0.00 ...
3 END       Marker       2.000    0.000 42.09  0.028 -0.00 ...
Lord Elements:
4 CAVITY      Lcavity      0.000    1.000 0.00  0.000  0.00 ...
Index   name       key           s        l     beta     phi     eta ...
                                         a       a       a ...
Values at End of Element:
```

Bmad creates a **multipass_lord** called **cavity** to control the **multipass_slaves** called **cavity\1** and **cavity\2**:

```
Tao> show ele 4
Element #                  4
Element Name: CAVITY
... etc...
Slave_status: Free
Lord_status: Multipass_Lord
Slaves:
Index   Name       Type
1      CAVITY\1  Lcavity
2      CAVITY\2  Lcavity
```

Since the **cavity** element represents the physical element, any change in the parameters of **cavity** will be reflected in the slaves (just like superposition lords and slaves). As an example, changing the attribute of the lord:

```
Tao> set ele cavity x_offset = 0.001
```

changes the corresponding attributes of the slaves:

```
Tao> show ele 2
Element #           2
Element Name: CAVITY\2
Key: Lcavity
S_start, S:   1.000000,    2.000000
Ref_time:  6.675633E-09

Attribute values [Only non-zero/non-default values shown]:
  1  L          =  1.000E+00 m
  ... etc...
  36  X_OFFSET   =  1.000E-03 m      57  X_OFFSET_TOT   =  1.000E-03 m
  ... etc...
```

The exception to the rule that the multipass_lord completely controls the multipass_slave attributes is the **phi0_multipass** attribute of **lcavity** and **rfcavity** elements. **phi0_multipass** allows for different settings of the RF phase for different passes through the cavity element. From the above lattice:

```
expand_lattice           ! cavity\2 is created during lattice expansion
cavity\2[phi0_multipass] = 0.5 ! Shifts the RF phase for cavity\2 by 180^deg
```

The **expand_lattice** command “expands” the lattice to create **cavity\2** (see the *Bmad* manual for more details) and the next line shifts the phase of **cavity\2** by 180 degrees.

This 180 degrees phase shift makes **cavity\2** decelerating instead of accelerating. Thus the reference energy after cavity **cavity\2** will be the same as the reference energy at the start of the lattice:

```
Tao> show lat -attrib e_tot
      Values at End of Element:
Index  name      key                  s        l        e
                                         tot
  0  BEGINNING Beginning_Ele       0.000     ---  1.0013E+07
  1  CAVITY\1  Lcavity            1.000     1.000  2.0013E+07
  2  CAVITY\2  Lcavity            2.000     1.000  1.0013E+07
  3  END       Marker             2.000     0.000  1.0013E+07
Lord Elements:
  4  CAVITY    Lcavity            0.000     1.000  2.0013E+07
Index  name      key                  s        l        e
                                         tot
```

Notes:

- Bmad does not demand that the global position of the multipass_slaves of a multipass_lord be in the same position in the global coordinate system.
- Since the reference energy is changing, the transfer matrix through a **lcavity** will not be symplectic.

17.3 Exercises [Answers in Section §25.12]

- 17.1 Modify **multipass.bmad** so that there are two element in the **linac** line called **cavity** and verify that *Bmad* does the proper bookkeeping (that is, there are two **cavity** multipass lords.

18 Lattice Geometry

The **parameter[geometry]** parameter in a lattice file sets the lattice topology to be **open** or **closed**:

open

For **open** lattices, *Bmad* computes the reference orbit and Twiss parameters by taking the **beginning[...]** Twiss and **particle_start[...]** orbit settings as the initial values and propagates them to the end of the lattice (like you would do for a linac).

closed

For **closed** lattices, *Bmad* calculates the Twiss and orbit periodic solution (like you would in a storage ring). In this case, *Bmad* will ignore Twiss and orbit settings in the lattice file.

Example:

```
! Lattice File: geometry.bmad
parameter[p0c] = 1e9
parameter[geometry] = closed

d: drift, l = 2
q1: quad, l = 0.5, k1 = 3, hkick = 0.001, superimpose
q2: quad, l = 0.5, k1 = -3, vkick = 0.002, superimpose, offset = 1

lat: line = (d)
use, lat
```

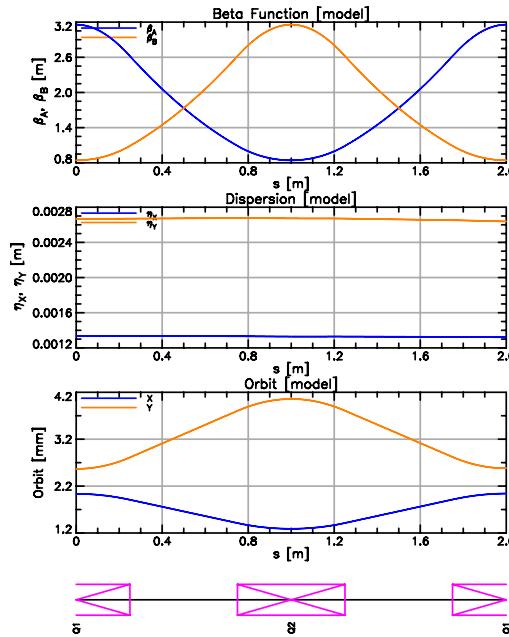


Figure 24: *Bmad* computes the periodic Twiss and orbits for **closed** lattices.

The lattice looks like:

```
Tao> show lat
      Values at End of Element:
Index  name      key          s     l   beta    phi   eta ...
0     BEGINNING Beginning_Ele 0.000  ---  5.93  0.000  0.00 ...
1     Q1#2       Quadrupole  0.250  0.250  5.57  0.043  0.00 ...
2     D#1        Drift      0.750  0.500  4.32  0.145  0.00 ...
3     Q2        Quadrupole  1.250  0.500  4.32  0.266  0.00 ...
4     D#2        Drift      1.750  0.500  5.57  0.368  0.00 ...
5     Q1#1       Quadrupole 2.000  0.250  5.93  0.411  0.00 ...
6     END        Marker     2.000  0.000  5.93  0.411  0.00 ...
Lord Elements:
7     Q1        Quadrupole  0.250  0.500  5.57  0.043  0.00 ...
Index  name      key          s     l   beta    phi   eta ...
                                         a     a     a ...

```

The result is shown in Figure 24. The **q1** quadrupole has been superimposed placing its center at the origin $s = 0$. This results in **q1** being “wrapped around” so that first half of **q1**, **q1#1**, comes at the *end* of the tracking part of the lattice and the second half, **q1#2**, comes at the beginning of the lattice:

```
Tao> show ele q1
Element #                  7
Element Name: Q1
Key: Quadrupole
S_start, S:    1.750000,    0.250000
... etc...

Slave_status: Free
Lord_status: Super_Lord
Slaves:
Index  Name      Type
5     Q1#1     Quadrupole
1     Q1#2     Quadrupole
```

Notes:

- Bmad does not demand that a closed lattice be closed in the sense that the global position at the end of the lattice be the same as the beginning. This makes sense since sometimes you want to take a lattice section and get the periodic solutions even though the section is not physically closed.

18.1 Exercises [Answers in Section §25.13]

- 18.1 It is sometimes convenient to switch the lattice geometry from closed to open while maintaining the same beginning Twiss and orbit values. Create an open geometry lattice from the **geometry.bmad** lattice. While it is possible to code the beginning Twiss and orbit values by hand, an easier way is to have *Tao* write a bmad lattice file using the **write bmad** command. This new file will contain the proper settings for the beginning Twiss and orbit.
- 18.2 There is another way to change the geometry besides creating a new lattice file and that is to use the **set branch** command (see chapter §19 for an explanation of branches). Run *tao* and experiment changing the geometry of the **geometry.bmad** lattice.

19 Forks and Branches

A **fork** or **photon_fork** element marks the point where multiple lines can merge or branch off from. Forking elements can be used to describe such things as X-ray lines branching from storage rings (see Figure 25a), injection or extraction lines, etc.

19.1 Example

```
! Lattice File: fork.bmad

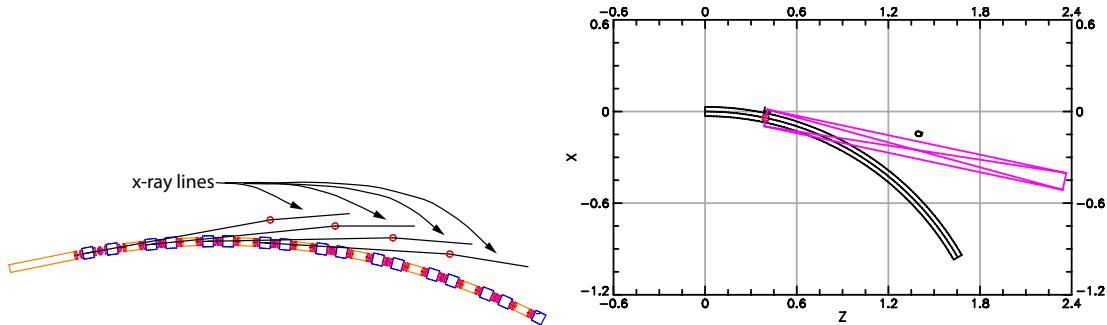
beginning[beta_a] = 10.0          ! m a-mode beta function
beginning[beta_b] = 10.0          ! m b-mode beta function
beginning[e_tot] = 10e6           ! eV
parameter[geometry] = open        ! or closed

b: sbend, l = 2, angle = pi/3
f: fork, to_line = extract_line, superimpose, offset = 0.4
q quadrupole, l = 2

extract_line: line = (q)          ! The line forked to.
extract_line[geometry] = open

lat: line = (b)
use, lat                         ! Line used to construct the lattice
```

In this example The **lat** line is used as the basis for the lattice due to the “**use, lat**” statement. This line contains the bend **b** and, via superposition, the fork element **f**. The fork element **f** connects to the **to_line** called **extract_line** which contains a single quadrupole element called **q**.



(a) Fork elements can be used to construct interconnected lines like x-ray lines branching from a storage ring.

(b) Simple fork example.

Figure 25

To see the geometry of the lattice, start *Tao* as explained in section §5.2 with the lattice file **fork.bmad** and create a **floor_plan** plot:

```
Tao> place r11 floor
```

The result is shown in Figure 25b. The **fork** element is the red circle. The lattice looks like:

```
Tao> show lat
      Values at End of Element:
Index name key s l beta phi eta ...
0 BEGINNING Beginning_Ele 0.000 --- 10.00 0.000 0.00 ...
1 B#1 Sbend 0.400 0.400 9.77 0.040 0.03 ...
2 F Fork 0.400 0.000 9.77 0.040 0.03 ...
3 B#2 Sbend 2.000 1.600 5.32 0.249 0.75 ...
4 END Marker 2.000 0.000 5.32 0.249 0.75 ...
Lord Elements:
5 B Sbend 2.000 2.000 5.32 0.249 0.75 ...
Index name key s l beta phi eta ...
a a a ...
Values at End of Element:
```

The **show lat** output does not show **q**. Where is the line that was forked to? The answer is that Bmad creates a set of **branches** to hold the different lines. Branches are assigned an index starting from 0 and information on them can be seen with the **show branch** command:

```
Tao> show branch
      N_ele N_ele ... Live
Branch Track Max ... Geometry Branch From_Fork
0: LAT 4 5 ... Open T
1: EXTRACT_LINE 2 2 ... Open T 0>>2

      Defines
Fork_Element Forking_To Direction To_Branch?
0>>2: LAT>>F 1>>0: EXTRACT_LINE>>BEGINNING 1 T
```

This shows that the lattice has two branches. When there are multiple branches, elements are indexed using the notation:

```
branch_index>>element_index
```

so that, for example, “**0>>2**” represents element number 2 in branch 0. That is, the fork element **f**.

Each branch has its own set of parameters like the geometry, reference energy, etc. These may be set using the syntax

```
branch_name[parameter] = ...
```

For example the “**extract_line[geometry]**” was used to set the geometry of **extract_line** in **fork.bmad**.

The **show lat** branch, by default, shows branch 0. To see other branches use the **-branch** option:

```

Tao> show lat -branch 1
      Values at End of Element:
Index   name      key          s     l    beta    phi    eta ...
                                         a      a      a ...
0  BEGINNING Beginning_Ele    0.000    ---  9.77  0.040  0.03 ...
1  D          Drift        2.000  2.000  8.04  0.268  0.34 ...
2  END        Marker       2.000  0.000  8.04  0.268  0.34 ...
Index   name      key          s     l    beta    phi    eta ...
                                         a      a      a ...
Values at End of Element:

```

Forked lines can, in turn, have forks to other lines. And lines can connect back to existing lines. In this way an entire accelerator complex can be simulated.

Notes:

- The difference between **fork** and **photon_fork** is that the default species for **fork** is the same as the line forked from while for a **photon_fork** the default species are photons.
- A fork element is not restricted to forking to the beginning of a line. The place where a fork element connects can be set by the **to_element** attribute.

19.2 Exercises [Answers in Section §25.14]

19.1 Using the **fork.bmad** lattice, vary the beginning Twiss and orbit (using **set** and/or **change** commands) and verify that the Twiss and orbit in branch 1 varies appropriately.

19.2 For a more complicated example, play around with the lattice:

```
bmad-doc/tutorial_bmad_tao/lattice_files/
wave_analysis/chess-u_6000mev_20181120.lat
```

This is a lattice that is used for simulation of the Cornell CESR storage ring. The lattice includes X-ray lines so that the effect on the X-ray beams due to things such as magnet misalignments can be simulated. [Note: Be aware that Twiss parameters are not calculated for X-ray lines.]

20 Tracking Methods

For each lattice element one can vary the method used to track particles through the element. This is useful, among other things for optimizing speed and/or accuracy. There are several element parameters that control tracking. These are:

```
tracking_method      ! How a particle is tracked through the element.  
mat6_calc_method    ! How the element's transfer matrix is calculated.  
spin_tracking_method ! How a particle's spin is tracked through an element.  
field_calc           ! How the electric and/or magnetic field is calculated.
```

Example:

```
q1: quadrupole, l = 0.6, ..., tracking_method = runge_kutta
```

For the **tracking_method** parameter some possible values are:

```
bmad_standard      ! Fast, thick element formulas.  
symp_lie_ptc       ! Symplectic Lie integration tracking.  
taylor              ! Taylor map.  
linear              ! Linear tracking.  
custom              ! Tracking with custom code.  
runge_kutta         ! Track through fields.  
etc ...
```

Much more information in the Bmad manual in the Chapter on “Tracking, Spin, and Transfer Matrix Calculation Methods”.

20.1 Exercises [Answers in Section §25.15]

20.1 Some tracking methods are faster than others. Some more accurate than others. To see how accurate different tracking methods are under different circumstances, there is a test program called **tracking_method_test**. [This program is part of a suite of **regression** tests. A regression test is a test that will check to see if results of executing some code change after changes to the code. This is helpful for spotting bugs that are introduced.] The **tracking_method_test** program reads in a lattice and, for each lattice element, tracks through that element multiple times using all the allowed tracking methods for that element type. The particle position at the beginning of the element is set to the settings of any **particle_start** commands in the lattice file. The ending particle position (phase space and spin) for each track is printed out.

Using a lattice file with only one or two lattice elements, run the **tracking_method_test** program to see how accurate the different methods are. Vary input parameters like magnet strengths or the starting position to see how accuracies change. Note: typically **runge_kutta** tracking is accurate but slow. To run the program, use the command

```
> tracking_method_test <lattice-file>
```

where **<lattice-file>** is the name of the lattice file. The program can be run from any directory.

21 Beam tracking in Tao

Tao has two basic particle tracking modes: **single** and **beam**. **Single** particle tracking is the default mode. In this mode a single particle is tracked and this tracking is used for orbit and Twiss calculations. This is the mode that has been used up to now in this tutorial.

With **beam** tracking, *Tao* does the same single particle tracking as in **single** particle tracking mode and, in addition, *Tao* tracks a **beam** of particles. A particle beam is made up of a number of **bunches** with each bunch being made up of some number of **particles**. Typically beams with only a single bunch are simulated. **Beam** tracking allows for interparticle effects to be simulated, for example, Coherent synchrotron Radiation (CSR).

The example files used to illustrate optimization are in the directory

```
bmad-doc/tutorial_bmad_tao/lattice_files/beam_tracking
```

There are five files here:

```
lat.bmad      ! Lattice file.  
setup.tao     ! Command file run at startup.  
tao.init      ! Primary Tao initialization file.  
tao_plot.init ! Secondary initialization file.  
beam.tao      ! Command file to track a beam.
```

These are similar to the files used for lattice optimization (§10.3).

The initial beam distribution is determined by the settings of the **beam_init** structure in the **tao_beam_init** namelist. With the present example, **beam_init** is set in the **tao.init** file:

```
! Simple Gaussian beam  
&tao_beam_init  
  beam_init%n_particle = 1000  
  beam_init%a_norm_emit = 1.0e-6. ! Normalized emittance = emit * gamma  
  beam_init%b_norm_emit = 1.0e-6. ! Normalized emittance = emit * gamma  
  beam_init%bunch_charge = 1e-9   ! 1 nC  
  beam_init%sig_pz = 1e-3        ! 10^-3 relative  
  beam_init%sig_z = 0.00059958   ! 2 ps * cLight  
  saved_at = "*"                ! Save distribution at all elements.  
/
```

Documentation on setting the **beam_init** structure is in the **Beam Initialization** chapter of the *Bmad* manual. The initial beam distribution can be set from a file of particle positions or by specifying general parameters like the emittance, etc. In this case, there is a single bunch with 1000 particles with normalized emittances of 1 mm-mrad in both planes, etc.

When the beam is tracked, beam distribution statistics like the centroid are calculated at every lattice element. Since saving the particle distribution is memory intensive when there is a large number of particles, the particle distribution is only saved at lattice elements specified by the **saved_at** parameter in the **tao_beam_init** namelist. In this case the particle distribution is saved at the exit end of every lattice element. To, say, just save at every marker element, **saved_at** could be set to “**marker::***”.

The **tao.init** file specifies that the **setup.tao** file should be run at startup. This file defines some aliases. To switch to beam tracking mode,

```
Tao> set global track_type = beam
```

This will immediately track these particles to the end of the lattice. Statistics from beam tracking are stored at the end of every element, and seen from the **show beam** command:

```
Tao> show beam q5
Cached bunch parameters:
  Parameters for bunch:          1
  Particles surviving:           1000
  Particles lost:                0
  Particles lost (%):            .000
  Charge live (C):              1.00000000E-09
  Centroid:   6.45361481E-12 -5.64470815E-13 -1.24096868E-11 -8.79861917E-13 ...
  RMS:        6.75168709E-04  8.93956548E-05  8.21313385E-04  1.12761136E-04 ...
    norm_emitt      beta          alpha
  a:          9.98698154E-07  7.24443183E+00  5.10465184E-01
  b:          9.98705861E-07  1.07200175E+01  -1.22746781E+00
  x:          9.98699589E-07  8.92076260E+00  6.28587261E-01
  y:          9.98704411E-07  1.32005818E+01  -1.51150120E+00
  z:          1.17181728E-05  5.99704551E-01

  Sigma Mat      x          px          y          py
  X     4.55852786E-07 -3.21209371E-08 -1.42999858E-12  1.89319887E-13 ...
  Px    -3.21209371E-08 7.99158310E-09  4.14153134E-13  1.10347243E-14 ...
  Y     -1.42999858E-12 4.14153134E-13  6.74555677E-07  7.72383921E-08 ...
  Py    1.89319887E-13  1.10347243E-14  7.72383921E-08  1.27150739E-08 ...
  Z     -9.15470557E-12 5.66620835E-13  9.99050861E-12  5.43154863E-13 ...
  Pz   -6.75696744E-12 5.60363622E-13  1.20598961E-11  8.99073770E-13 ...
```

```
Note: Individual particle positions are saved at this element.
```

The **saved_at** element list will save the full particle distribution at the end of the matching elements, in this case all elements.

These particles and their statistics can be plotted. For example,

```
Tao> place r12 bunch_sigma_xy
```

will display line plots along the beamline. The $x - p_x$ phase space can be plotted as:

```
Tao> place r22 bunch_x_px
```

With this type of beam, the number of particles can be changed by:

```
Tao> set beam_init n_particle = 5000
```

The reference element for the plotting can be changed by:

```
Tao> set curve r22.g.c ele_ref_name = Q5
```

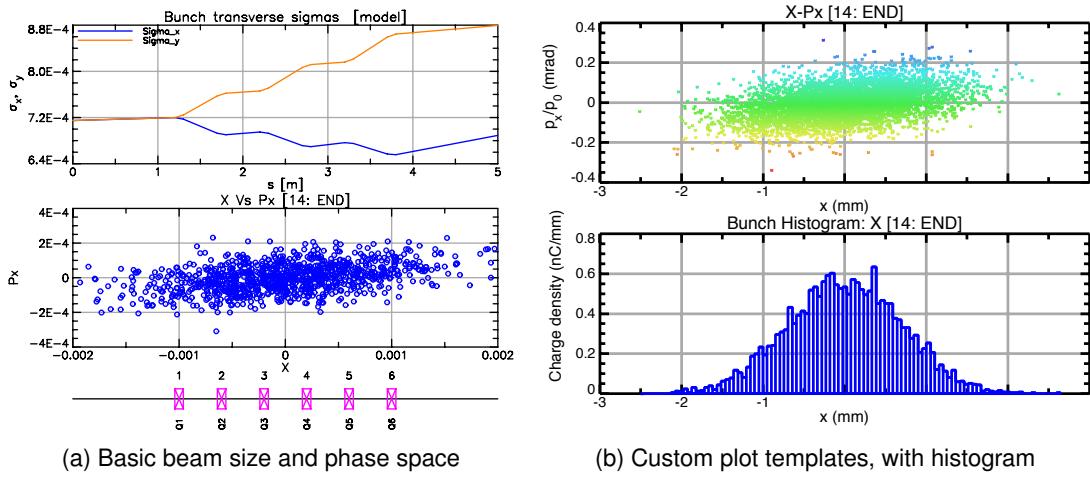


Figure 26: Beam plotting. Particles can be colored by their attributes, in this case simply by the p_x coordinate.

Any changes to the lattice (including optimization steps) will result in re-tracking the beam. To return to single particle tracking mode, set:

```
Tao> set global track_type = single
```

22 Python/Scripting Interface

When there is a need to extract data from *Tao* for external processing, *Tao* has a command called **python** (not to be confused with the **Python** program) which can be used to output data in a form suitable for easy parsing. Additionally, there is a package called **PyTao** which can be used to interface *Tao* to **Python**. The former is discussed in Sec. §22.1 and the latter is discussed in Sec. §22.2.

22.1 Tao's Python Command

Tao's **python** command is used to output data in a form that is suitable for easy parsing by an external program. Despite its name, the output of the **python** command is not **Python** specific. Example:

```
Tao> python ring_general
```

This command is used with lattices with a closed geometry to output information on the lattice. The output of this command looks like:

```
param.unstable_factor;REAL;F; 0.00000000000000E+00
Q_a;REAL;F; 1.05300081497137E+01
Q_b;REAL;F; 9.57846038864296E+00
Q_z;REAL;F; 0.00000000000000E+00
Q_spin;REAL;F; 0.00000000000000E+00
chrom_a;REAL;F; -1.16778375954141E+00
chrom_b;REAL;F; -9.35493278761967E-01
J_damp_a;REAL;F; 1.01125472391567E+00
...
```

The output here has some of the same information as the output of the **show universe** command except that it is easier to parse being comma delimited list. The other advantage of using the **python** command is that the output is generally stable over time as the *Tao* program is developed to ensure that external code that interfaces to the **python** command does not break. This is in contrast to the **show** command whose output is formatted to be human readable and whose output format may change on a whim.

The general form of the **python** command is:

```
python <subcommand> <arguments>
```

The **python** command has a number of **subcommands** (over 100) that are listed in the *Tao* manual. The sub-commands can be divided into two categories. One category are the “**action**” subcommands which allow for control of *Tao* such as creating variables and data for use in an optimization. The other category are the “**output**” subcommands which output information from *Tao*. This tutorial will only discuss the output subcommands.

Most **output** subcommands use “**parameter list form**” format where each line has four fields separated by semicolons:

```
{name};{type};{variable};{value(s)}
```

The fields are:

```
name:      The name of the parameter  
  
type:      The type of the parameter:  
           INT      Integer number  
           REAL     Real number  
           COMPLEX  Complex number. A complex number is output as Re;Im  
           REAL_ARR Real array  
           LOGIC    Logical: "T" or "F".  
           STR      String.  
           STRUCT   Structure.  
... etc. ... (See the Tao manual for details).  
  
can_vary: Either "T", "F", or "I", indicating whether or not the  
           user may change the value of the parameter. "I" indicates  
           that the parameter is to be ignored by a GUI.  
  
value(s): The value or values of the the parameter. If a parameter has  
           multiple values, the values will be separated by semicolons.
```

22.2 PyTao

The **Python PyTao** package interfaces *Tao* with **Python**. With **PyTao** it is possible to

- Parse **python** command output into a **Python** structure for easy access.
- Run *Tao* as a subprocess of **Python**

PyTao is separate from the *Bmad Release* (§2.5) which contains the *Bmad* and *Tao* source files. The PyTao source is in a GitHub repository at

github.com/bmad-sim/pytao

Documentation for setup and using PyTao is at

bmad-sim.github.io/pytao/

It is recommended to use a virtual Python environment. Details on how to setup a virtual environment can be found on the Web. There are several ways to install PyTao. To install via **pip** use the command:

```
pip install PyTao
```

As an example of how **PyTao** can parse the **python** command output, run *Tao* with the commands

```
Tao> python -write bmad_com.out bmad_com

max_aperture_limit;REAL;T; 1.00000000000000E+03
d_orb;REAL_ARR;T;1E-05;1E-05;1E-05;1E-05;1E-05;1E-05
default_ds_step;REAL;T; 2.00000000000000E-01
... etc...
taylor_order;INT;T;0
runge_kutta_order;INT;T;4
... etc...
rf_phase_below_transition_ref;LOGIC;T;F
sr_wakes_on;LOGIC;T;T

Tao> quit
```

This will create a file **bmad_com.out** the the **bmad_com** structure parameters. The following reads in this information in a Python environment:

```
> python
>>> from pytao.util.parameters import *
>>> df = open("bmad_com.out", "r")
>>> b_com = tao_parameter_dict(df.readlines())
>>> b_com["default_ds_step"].name
"default_ds_step"
>>> b_com["default_ds_step"].type
"REAL"
>>> b_com["default_ds_step"].value + 3
3.2
```

23 Wave Analysis

Wave analysis is a method for finding isolated “kick errors” in a machine by analyzing the appropriate data. For example, consider the orbit of a beam. In some region of the machine, assuming there are no orbit kicks in the region (and assuming no x - y coupling), the horizontal orbit $x(s)$ of the beam will be “wave” given by the standard formula

$$x(s) = A \sqrt{\beta_x(s)} \cos(\phi_x s + \phi_{x0}) \quad (5)$$

where A and ϕ_0 depend upon the position of the beam at the beginning of the region and $\beta(s)$ and $\phi_x(s)$ are the standard beta and betatron phase parameters. Consider then choosing some region of the machine and fitting the data from an orbit measurement in this region to Eq. (5) using A and ϕ_{x0} as fitting parameters. If there indeed where no kicks in this region, and if the data is perfect, a plot of the measured orbit minus the fit orbit will be zero in the region. If this plot of orbit minus fit is extended to the entire machine, the plot will become non-zero after any point where there is a kick to the beam. In other words, a plot of **orbit - fit** can be used to locate where a kick is happening. This is the essence of wave analysis [1]. In practice, two fits are done to two different regions on either side of where a kicker is thought to be located. This allows for a more accurate calculation of where the kick is.

Wave analysis can be used other types of data besides orbits:

Measurement Type	Error Type
Orbit	Steering errors
Betatron phase differences	Quadrupolar errors
Beta function differences	Quadrupolar errors
Horizontal/Vertical Coupling	Skew quadrupolar errors
Dispersion differences	Sextupole errors

Table 1: Types of measurements that can be used in a wave analysis and the types of errors that can be diagnosed.

Wave analysis can not only find errors in a machine it can be used to measure calibration constants in magnets. For example, by measuring the coupling at two different setting of a given skew quadrupole magnet, the magnets calibration can be determined. For further information, see the **Wave Analysis** chapter (§9) in the *Tao* manual.

23.1 Example Analysis

The example wave analysis presented here uses actual data taken at the Cornell storage ring CESR. Two measurement of the betatron phase were made the second one taken about two days after the first. The betatron phase is measured by shaking the beam at the betatron resonance frequencies and measuring the turn-by-turn response at the BPM detectors[2]. The amplitude of the sinusoidal response gives the beta function and the phase of the response

gives the betatron phase. The horizontal/vertical coupling can also be extracted from the data using the ratio of the response in the two planes. In practice, the betatron phase data is less noisy than the beta measurement since the phase is fairly insensitive to errors in measuring the oscillation amplitude. This being the case, it is the betatron phase that will be analyzed here.

Input files for this example are in the directory:

```
bmad-doc/tutorial_bmad_tao/lattice_files/wave_analysis
```

The relevant files are

```
chess-u_6000mev_20181120.lat      ! Lattice file
setup.tao                         ! Startup command file
tao.init                           ! Tao init file
wave_anal.tao                      ! Wave analysis commands .
```

A wave analysis works on data so the **tao.init** file sets up data for the betatron phase:

```
&tao_d2_data
  d2_data%name = "phase"
  n_d1_data = 2
/

&tao_d1_data
  ix_d1_data = 1
  d1_data%name = "a"
  search_for_lat_eles = "type::BPM*"
/

&tao_d1_data
  ix_d1_data = 2
  d1_data%name = "b"
  use_same_lat_eles_as = "phase.a"
/
```

Setting **search_for_lat_eles** to “**type::BPM***” works since, By convention, BPMs are represented in CESR lattices by **marker** elements whose **type** attribute begins with “**BPM**”.

The phase data is set in the **setup.tao** file:

```
set data phase.a[1:20]|ref = [-54.2921, -52.0007, -51.8171, ...
set data phase.a[21:40]|ref = [-34.7387, -34.2289, -33.0380, ...
set data phase.a[41:60]|ref = [-16.7037, -16.6183, -16.0310, ...
...
set data phase.a[1:20]|meas = [-54.2560, -51.9583, -51.7726, ...
set data phase.a[21:40]|meas = [-34.7252, -34.2236, -33.0035, ...
set data phase.a[41:60]|meas = [-16.6726, -16.5874, -16.0207, ...
...
```

The **ref** data represents the first measurement and the **meas** data represents the second.

The **setup.tao** file also modifies the betatron phase plot:

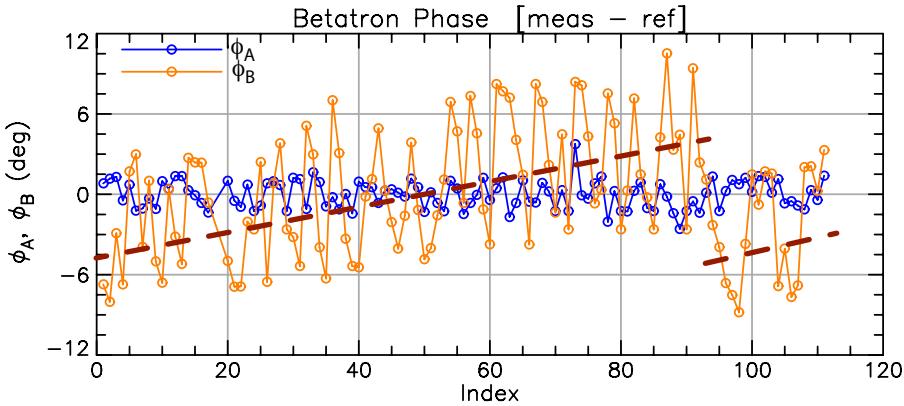


Figure 27: Betatron phase difference between two measurements taken two days apart in the Cornell CESR storage ring. The dashed lines indicate approximately the oscillation centroid of the **B**-mode phase difference.

```
set plot phase x_axis_type = index
set graph phase xlabel = "Index"
set graph phase component = meas - ref
set curve phase data_source = data
set curve phase draw_symbols = T
place r11 phase
```

The setting of `x_axis_type` to `index` switches the `x`-axis variable from `s`-position to data index. This is done for convenience later on.

Start *Tao* and the plot window should look like Figure 27 which shows the change in the “horizontal-like” **a**-mode phase ϕ_a (blue curve) along with the change in the “vertical-like” **b**-mode phase ϕ_b (orange curve). In the figure, a dashed line in red has been added to approximately show the average of the oscillations of ϕ_b . Even before doing a phase analysis, a lot can be learned by looking at the plot:

- In a region where there have been no changes in quadrupole strength, the oscillation centroid of the phase difference is constant. In Figure 27, the oscillation centroid for the ϕ_b difference is shown approximately by the red dashed line. As can be seen, there is a large jump in the centroid near detector 90 which indicates a quadrupolar field change in that region.
- The **a**-mode phase oscillations are small compared to the **b**-mode phase indicating that the quadrupolar field change near detector 90 is at a point with large β_b compared to β_a .
- With the exception of the discontinuity near detector 90, the oscillation centroid of the **b**-mode phase has a slope but is not obviously discontinuous. This indicates that there has been small changes to the strength of many quadrupole magnets distributed throughout the ring.

Now run the wave analysis on the `phase.b` data:

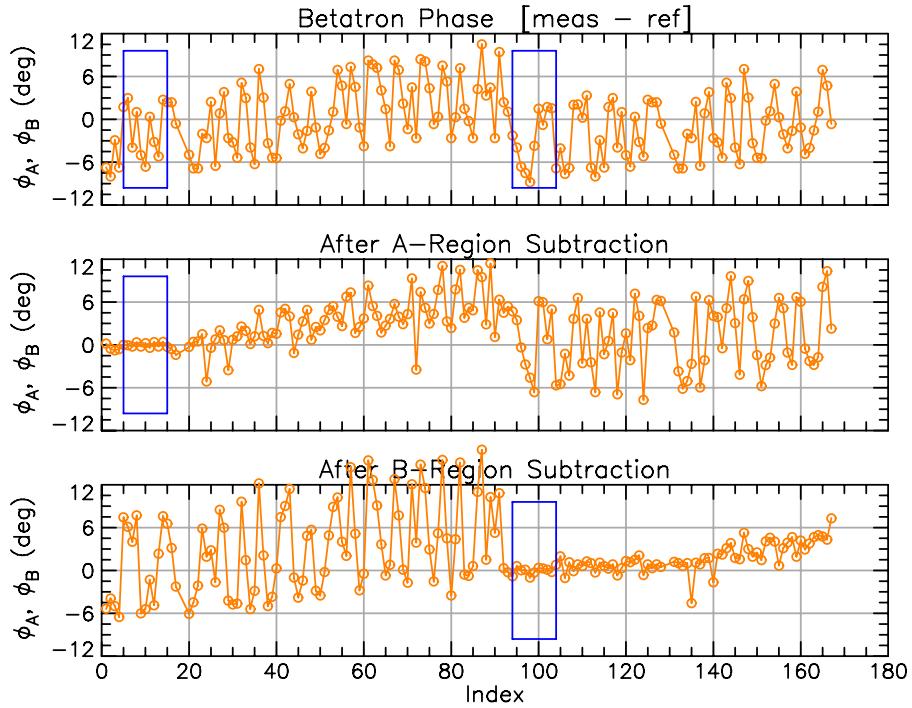


Figure 28: Wave analysis of the **phase.b** difference data. The blue boxes show the locations of the **A** and **B** fit regions. The data has been extended past the end of the lattice to allow a wave analysis for the region near the ends of the lattice. Top graph: The original data. Middle graph: The data with the **A**-region fit subtracted off. Bottom graph: The data with the **B**-region fit subtracted off. Notice that the y-axis label is simply taken from the original plot so that the fact that the label mentions ϕ_A should be ignored.

```
Tao> wave phase.b
```

The result is shown in Figure 28. There are two regions, called **A** and **B** where the data is fit to a betatron phase `wave[1]`. Initially, the placement of these two regions are somewhat arbitrarily chosen by *Tao*. In this case, the **A**-region is from datum 5 to datum 15 and the **B** region is from datum 94 to datum 104. Notice that, in order to be able to analyze the region near the ends of the lattice, the data has been extended by 1/2 of the length of the data array. In this case the **phase.b** data range was from 1 to 111. Thus in the extended curves in Figure 28, datum 112 is derived from datum 1, datum 113 is derived from datum 2, etc.

In Figure 28 the top plot is the original **phase.b** difference data, the middle plot is The difference data with the **A**-region fit subtracted off, and the bottom plot is the difference data with the **B**-region fit subtracted off. Since the difference between the data and the **A**-region fit is near zero in the **A**-region, and similarly for the **B**-region, this shows that both the **A** and **B** regions are well fitted. That is, there were no significant quadrupole changes in the fit regions in the time period between the two measurements. The goodness of the fits, "**Sigma_Fit/Amp_Fit**" is printed as

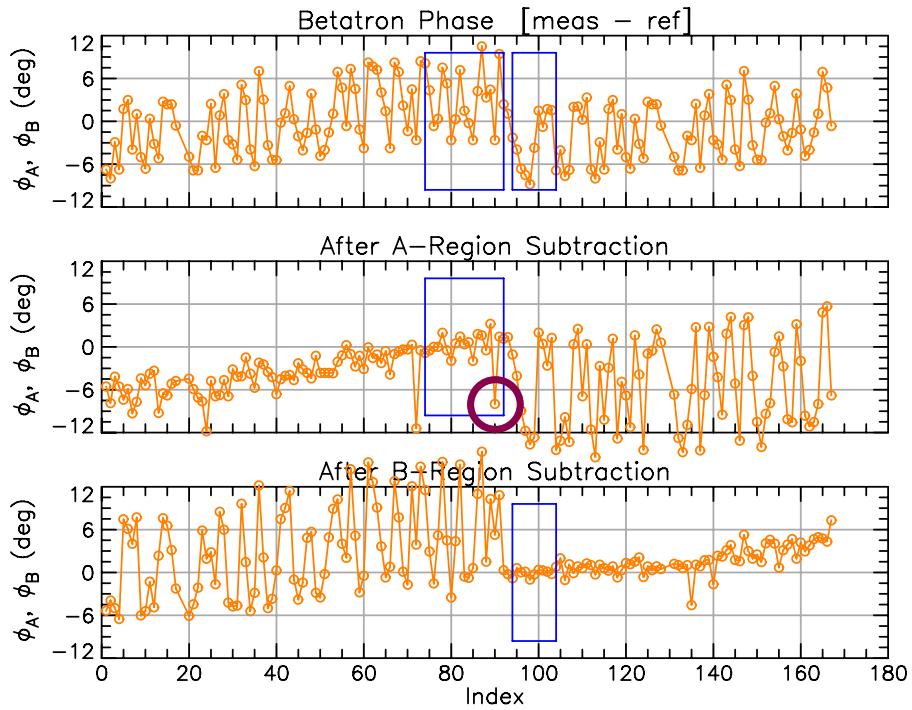


Figure 29: Wave analysis of the **phase.b** difference data after the fit regions have been adjusted to bracket the quadrupole error. The circle in the middle graph marks a bad data point.

part of the output of the **wave** command:

```

ix_a:    5   15
ix_b:  94  104
A Region Sigma_Fit/Amp_Fit:      0.035
B Region Sigma_Fit/Amp_Fit:      0.067
Sigma_Kick/Kick:      0.033
Sigma_phi:          0.020
Chi_C:              0.804 [Figure of Merit]

Normalized Kick = k * l * beta [dimensionless]
  where k = quadrupole gradient [rad/m^2].
After Data#      Norm_Kick      s-pos     ele@kick           phi
      20        0.13      77.48     D083      16.340
      23       -0.13     103.39     B16W      17.911
      25        0.13     126.07     D104      19.482
      27       -0.13     137.80     B20W      21.053
      30        0.13     161.15     B23W      22.624
... etc...

```

A value of 1 indicates a poor fit and a value of zero indicates a good fit. In this case, with the

goodness of the fits are both below 0.1 indicating a good fit. See the section on **Wave Analysis Commands and Output** (§9.4) in the *Tao* manual for more details.

Since the fit regions are far apart, there are many possible error locations. To narrow down the possibilities, the fit regions need to be moved as close as possible while still maintaining good fits. Since the **B**-region fit shows a strong error just to the left of the **B**-regions left edge, it makes sense to move the **A**-region towards the **B** region.

The reader is invited to play around with adjusting the fit regions. Or just set:

```
set wave ix_a = 74 92      ! Set A-region boundaries
set wave ix_b = 94 104      ! Set B-region boundaries
```

The result is shown in Figure 29. A unique solution has been bracketed:

```
ix_a: 74 92
ix_b: 94 104
...
Normalized Kick = k * l * beta [dimensionless]
  where k = quadrupole gradient [rad/m^2].
After Dat#   Norm_Kick      s-pos    ele@kick          phi
         93        0.27     683.95    956 D408       63.314
```

This shows a quadrupole error at about $s = 684$ meters. Nearby elements are:

```
Tao> show lat -s 681:687
# Values shown are for the Exit End of each Element:
# Index name           key             s      l
#
  946 D404#2        Drift      682.171  1.684 ...
  947 D14BE_RFA1_SEG Marker    682.171  0.000 ...
  948 D404#3        Drift      682.350  0.179 ...
  949 DOG_LEG_14E1   Kicker    682.594  0.244 ...
  950 D405          Drift      682.856  0.262 ...
  951 DET_13E        Marker    682.856  0.000 ...
  952 D406          Drift      682.874  0.018 ...
  953 SEX_13E       Sextupole  683.146  0.272 ...
  954 D407          Drift      683.208  0.062 ...
  955 Q13E          Quadrupole 683.808  0.600 ...
  956 D408          Drift      683.998  0.189 ...
```

So the likely candidate is quadrupole **Q13E**. The **show element** command shows that the **b**-mode beta function at this element is about a factor of 10 larger than the **b**-mode beta which explains the small **phase.a** signal. Subsequent investigation showed that there was a ground problem which was corrected and further observation showed that this fixed the problem.

Notes:

- In the middle plot in Figure 29, there is a data point, marked by a red circle, within the **A**-region whose value is far from the its neighboring points. This indicates a bad data point. To get a better fit, this data point can be removed from the plot and the fit using the command

```
veto data phase.b[90]
```

- To make the **setup.tao** command file run faster, lattice and replotting calculations are suspended by the following at the top of the command file:

```
set global lattice_calc_on = F      ! Stop lattice calculations.  
set global plot_on = F             ! Stop replotting.
```

with corresponding commands at the end of the file to re-enable calculations. This is not a big factor here but in other cases can save a significant amount of time.

- For the wave analysis to work the right edge of the **A** region must be to the left of the left edge of the **B**-region.
- To increase the accuracy of the analysis, the quadrupoles and skew quadrupoles in the**model** lattice should be varied to fit the **model** Twiss parameters to one of the measurements. This is especially true when calibrating skew quadrupoles since the the effect of a skew quadrupole is proportional to the tune difference between the **a** and **b** normal modes.
- Data where there are multiple error locations that are well enough separated can be analyzed by varying the **A** and **B** regions to successively to bracket the individual error locations.

23.2 Exercises

- 23.1 Create your own data for doing a wave analysis. A easy way to do this is to introduce errors into the model lattice and then set **meas** (or **ref**) data:

```
set data phase.a|meas = phase.a|model + 0.1 * ran_gauss()
```

The **ran_gauss()** function adds noise to the data so you can experiment with how noise degrades the analysis.

- 23.2 By creating your own data, experiment with how well you can resolve two errors that are close together.

24 References

- [1] D. Sagan, "Betatron phase and coupling correction at the Cornell Electron/Positron Storage Ring," *Physical Review Special Topics - Accelerators and Beams*, vol. 3, p. 102801, Oct. 2000.
- [2] R. L. D. Sagan, R. Meller and D. Rubin, "Betatron phase and coupling measurements at the Cornell electron/positron storage ring," *Phys. Rev. ST Accel. Beams*, vol. 3, p. 092801, 2000.
Note: While the paper does not mention it, Bmad is the software being used here for the analysis.

25 Answers to Exercises

25.1 Chapter 5 “Starting Tao” Answers

5.1 The **tao.init** file should look like:

```
&tao_design_lattice  
n_universes = 1  
design_lattice(1)%file = "simple.bmad"  
/
```

5.2 The command file should have the line:

```
help [[1]]
```

5.3 Add to the **tao.init** file:

```
&tao_start  
startup_file = "doit"  
/
```

When starting *Tao*, no parameters are passed to the startup file so when the command file is run, the **help** command with no arguments is executed.

25.2 Chapter 6 “Introduction to Bmad Lattices” Answers

6.1 An **sbend** is a dipole bend whose nominal shape is a circular sector. The **g** attribute is the **reference** bending “strength” equal to $1/\rho$ where **rho** is the reference bending radius. The reference bending strength/angle is used to establish where the next lattice element is placed (discussed in Chapter §14). The **dg** attribute is the “error” bending strength. The actual magnetic field of the dipole is proportional to $g + dg$.

6.2 Try:

```
beginning[e_tot] = 10e6  
parameter[geometry] = closed  
d: drift, L = 0.5  
q1: quadrupole, L = 0.6, k1 = 0.23  
q2: q1, k1 = -q1[k1]  
lat: line = (d, q1, d, q2)  
use, lat
```

6.3 Continuing from the previous exercise, remove the **use** command and replace with:

```
lat: line = (d, q1, d, q2)  
lat2: line = (10*lat) ! Equivalent to: lat2: line = (10*(d, q1, d, q2))  
use, lat2
```

25.3 Chapter 7 “Tao Show Command” Answers

7.1 Use the example lattice with:

```
q1: quad, alias = "z quad1"
q2: quad, alias = "z quad2"
q3: quad, alias = "abc"
line = (q1, q2, q3)
use, line
```

Run *Tao* and use the command:

```
Tao> show lat alias::z*
```

7.2 The name **Q##2** will match to the second element named **Q**.

7.3 Try it and see!

7.4 The **show history** command shows the command history.

7.5 To create a column of **k1** element parameter values use

```
show lattice -attribute k1           ! or try
show lattice -attribute k1 -undef0
```

7.6 To show only **sbend** elements use

```
show lattice sbend::*
```

7.7 With the **-all** switch more parameters are shown.

25.4 Chapter 8 “Introduction to Plotting in Tao” Answers

8.1 To place the **b_field** template plot use something like

```
place r11 b_field
```

8.2 To set the **draw_symbols** to True for the beta plot curves use the command:

```
set curve beta draw_symbols = T
```

8.3 The particle orbit is enabled when the **orbit_scale** component of the graph is non-zero. To calculate what **orbit_scale** should be set to to produce an orbit that looks nice, use the following logic: The maximum (horizontal) orbit excursion is at the end of the lattice where the horizontal orbit is about 0.4 mm (the **show lat** command will show this). The scale of the floor plan plot is of order 1 meter. So a “reasonable” plotted excursion would be, say, 0.2 meter. This gives an orbit scale of $500 = 0.2 \text{ meter} / 0.4 \text{ mm}$. So set **orbit_scale** using the command:

```
set graph floor floor_plan%orbit_scale = 500
```

To change the starting particle energy use, for example:

```
set particle_start pz = 0.002
```

- 8.4 To scale the x-axis of all plots to be in the range [0.5, 1.5] use the command

```
x_scale all 0.5 1.5
```

25.5 Chapter 9 “Model, Design and Base Lattices in Tao” Answers

- 9.1 The **show curve r23** command will show that the curves are named **r23.g.x** and **r23.g.y**.

[Note: using the alternative name that begins with “**orbit**” will not work here since there are multiple **orbit** plots.] To use the **r23.g.y** curve to draw the **design** horizontal orbit use the commands:

```
set curve r23.g.y data_type = orbit.x
set curve r23.g.y legend_text = "X"
set curve r23.g.y component = design
```

Note: By default, orbit plots have a non-blank **legend_text** which is why the **legend_text** needs to be changed. What happens if, instead of setting to “X”, the **legend_text** is set to a blank string?

- 9.2 **alias setit set ele q vkick = [[1]]**

25.6 Chapter 11 “Lattice and Orbit Corrections” Answers

- 11.1 The “**meas**” orbit used for the analysis should be set to:

```
meas = actual_measured - (golden_orbit - design_orbit)
```

Now fit to **model - meas**. If there is a good fit, the **model** orbit will be equal (more or less) to **meas**. In the actual machine, if the parameters corresponding to the optimized variables are changed by an amount:

```
param_change = var_design - var_model
```

this will produce (hopefully) an actual orbit:

```
end_orbit = beginning_measured_orbit + orbit_change
= actual_measured + (design_orbit - model_orbit)
= actual_measured + (design_orbit -
                     (actual_measured - (golden_orbit - design_orbit)))
= gloden_orbit
```

- 11.2 Use the following commands to increase the variable weights by a factor of 10 and refit:

```
Tao> set var *|weight = 10 * *|weight
Tao> run
```

With variable weights of 10^4 the orbit fit is noticeably degraded.

25.7 Chapter 12 “Multiple Universes” Answers

- 12.1 The **show curve beta** command shows that the curve names are **r13.g.a** and **r13.g.b** (or **beta.g.a** and **beta.g.b** will work in this instance). To set the universe indexes use:

```
set curve r13.g.a ix_universe = 1  
set curve r13.g.b ix_universe = 2
```

25.8 Chapter 13 “Control Elements” Answers

- 13.1 A knot based overlay to replace **ov2** would be:

```
k2: overlay = {q[k1]:{0, 0.7}, q[x_offset]:{0, 0.1}},  
      var = {hh}, hh = 0.01, x_knot = {0, 1}
```

- 13.2 A **group** element to vary the entrance edge of element **B**:

```
gedge: group = {B[start_edge]: ds}, var = {ds}
```

- 13.3 In the **simple.bmad** lattice file add:

```
gg: girder = {b, q}
```

25.9 Chapter 14 “Machine Coordinates and Patch Elements” Answers

- 14.1 In the **simple.bmad** lattice file add:

```
gg: girder = {b, q}, x_offset = 0.1, y_pitch = 0.02
```

When in *Tao*, to see the global position of, say, element **b** use the command:

```
Tao> show ele -floor b
```

- 14.2 Lattice with a **mirror** element:

```
parameter[p0c] = 1e3  
parameter[particle] = photon  
d: drift, l = 1  
m: mirror, graze_angle = 0.1  
l: line = (d, m, d)  
use, l
```

Verify the coordinates with the command:

```
Tao> show lat -floor
```

14.3 The end floor coordinates can be calculated by hand using the equations

$$\begin{aligned} z &= l_d + \rho_b \sin(\theta_b) + l_q \cos(\theta_b) \\ x &= \rho_b (\cos(\theta) - 1) - l_q \sin(\theta_b) \end{aligned} \quad (6)$$

where l_d , l_b , and l_q are the lengths of elements **D**, **B**, and **Q** respectively and ρ_b is the **rho** attribute of element **B**.

25.10 Chapter 15 “Particle Phase Space Coordinates” Answers

15.1 In the **cavity.bmad** lattice file replace the appropriate lines with

```
beginning[p0c] = 2*mass_of(He)
lc: lcavity, l = 1e-6, voltage = mass_of(He), rf_frequency = 1e9
particle_start[z] = 1e-3
```

The command **show ele lc** will show that the beginning beta is 0.740 and the ending beta is 0.916. The ratio of the betas is the same ratio as the ratio of the beginning z (1.00 mm) to ending z (1.24 mm).

15.2 Approximately, for **phi0_err** values above about 0.2502 or below -0.2502 the particle will not make it through the **lcavity**.

25.11 Chapter 16 “Superposition” Answers

16.1 For the first lattice:

```
beginning[beta_a] = 10.0
beginning[beta_b] = 10.0
beginning[e_tot] = 10e6
parameter[geometry] = open           ! Or closed
parameter[particle] = electron      ! Reference particle.
s: sextupole, L = 0.5, k2 = 1, superimpose, ref = q
q: quadrupole, L = 0.6, k1 = 0.2
lat: line = (q)
use, lat
```

For the second lattice add the line

```
s[create_jumbo_slave] = T
```

25.12 Chapter 17 “Multipass” Answers

17.1 Replace the definition of the **linac** line with:

```
linac: line[multipass] = (cavity, cavity)
```

25.13 Chapter 18 “Lattice Geometry” Answers

- 18.1 After creating the lattice file using the **write bmad** command, modify the geometry line to:

```
parameter[geometry] = open
```

- 18.2 The **show branch** command shows that the lattice has one branch with index zero and name “LAT”. So either of the following commands will toggle the geometry:

```
Tao> set branch 0 geometry = open  
Tao> set branch lat geometry = open
```

25.14 Chapter 19 “Forks and Branches” Answers

- 19.1 To vary the beginning Twiss and orbit use commands like

```
Tao> set particle_start px = 0.001  
Tao> change particle_start y = 0.02  
Tao> set ele 0 beta_a = 10  
Tao> change ele 0 eta_x = 0.1
```

25.15 Chapter 20 “Tracking Methods” Answers

- 20.1 The **orbit.bmad** lattice in the **lattice_files** directory can be used here.