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AEEM6042 Module 2 Assignment

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Problem 1

Part A

Solve for T_2 with isentropic relations assuming $\gamma = 1.4$.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$
$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$
$$T_2 = 1400\mathrm{K}\left(\frac{1}{8}\right)^{\frac{1.4-1}{1.4}}$$
$$\boxed{T_2 = 772.86\mathrm{K}}$$

Part B

Solve mass flow rate for area, using ideal gas assumption to get density.

$$\rho_1 = \frac{P_1}{RT_1}$$
$$\rho_1 = \frac{810600\mathrm{Pa}}{287\mathrm{J/kgK} * 1400\mathrm{K}}$$
$$\rho_1 = 2.017\mathrm{kg/m}^3$$

$$V_1 = M_1\sqrt{\gamma R T_1}$$
$$V_1 = 0.3\sqrt{1.4 * 287\mathrm{J/kgK} * 1400\mathrm{K}}$$
$$V_1 = 225\mathrm{m/s}$$

$$\dot{m} = \rho_1 V_1 A_1$$
$$A_1 = \frac{\dot{m}}{\rho_1 V_1}$$
$$A_1 = \frac{100\mathrm{kg/s}}{2.017\mathrm{kg/m}^3 \cdot 225\mathrm{m/s}}$$
$$\boxed{A_1 = 0.22\mathrm{m}^2}$$

Part C

First check if the nozzle is choked. P_0 is constant since the nozzle is isentropic.

$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$
$$P_0 = P_1\left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$
$$P_0 = (810600\mathrm{Pa})\left(1 + \frac{1.4 - 1}{2} 0.3^2\right)^{\frac{1.4}{1.4-1}}$$
$$P_0 = 862827.2\mathrm{Pa}$$
$$\frac{P_0}{P} = \frac{862827.2}{101325} = 8.52$$

The pressure ratio is higher than the critical pressure ratio so the nozzle is choked. First find M_2 from the isentropic relations since we know total temperature is constant because it's isentropic.

$$T_{01} = T_1\left(1 + \frac{\gamma - 1}{2} M_1^2\right)$$
$$T_{01} = 1450.4\mathrm{K}$$
$$M_2 = \sqrt{\left(\frac{T_{01}}{T_2} - 1\right)\frac{2}{\gamma - 1}}$$
$$M_2 = 2.1$$

Then find the area ratio to the throat from inlet, and use the throat area with M_2 to find the exit area.

$$\frac{A_1}{A^*} = \frac{1}{M_1}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_1^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$A^* = A_1/\frac{1}{M_1}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_1^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$A^* = 0.108\mathrm{m}^2$$

$$A_2 = A^*\frac{1}{M_2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_2^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\boxed{A_2 = 0.1984\mathrm{m}^2}$$

Problem 2

Part A

Solve for total pressure and temperature with isentropic relations, then solve MFP for mass flow rate.

$$P_0 = P_1\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_0 = 4\mathrm{psia}\left(1+\frac{1.4-1}{2}2.5^2\right)^{\frac{1.4}{1.4-1}}$$

$$P_0 = 68.3\mathrm{psia}$$

$$T_0 = T_1\left(1+\frac{\gamma-1}{2}M_1^2\right)$$

$$T_0 = (-55+459.67\mathrm{R})\left(1+\frac{1.4-1}{2}2.5^2\right)$$

$$T_0 = 910.5\mathrm{R}$$

$$MFP = \sqrt{\frac{\gamma g_c}{R}}M_1\left(1+\frac{\gamma-1}{2}M_1^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

$$MFP = \sqrt{\frac{1.4(32.174\mathrm{lbm\text{-}ft/lbf\text{-}s}^2)}{53.34\mathrm{ft\text{-}lbf/lbm\text{-}R}}}\cdot 2.5\left(1+\frac{1.4-1}{2}2.5^2\right)^{-\frac{1.4+1}{2(1.4-1)}}$$

$$MFP = 0.202$$

$$\dot{m} = \frac{A_1 P_0}{\sqrt{T_0}} MFP$$

$$\dot{m} = \frac{1.5\text{ft}^2 68.3psia\frac{144si}{sf}}{\sqrt{910.5\text{R}}}(.202)$$

$$\boxed{\dot{m} = 98.76\text{lbm/s}}$$

Part B

Use isentropic relations to get the pressure and temperature at the exit.

$$P_2 = P_0 / \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\boxed{P_2 = 66.4 \text{ psia}}$$

$$T_2 = T_0 / \left(1 + \frac{\gamma - 1}{2} M_2^2\right)$$

$$\boxed{T_2 = 903.3°\text{R}}$$

Part C

Use MFP with the exit conditions to get the exit area.

$$MFP = \sqrt{\frac{\gamma g_c}{R}} M_2 \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

$$MFP = \sqrt{\frac{1.4(32.174\text{lbm-ft/lbf-s}^2)}{53.34\text{ft-lbf/lbm-R}}} \cdot 0.2\left(1 + \frac{1.4 - 1}{2} 0.2^2\right)^{-\frac{1.4+1}{2(1.4-1)}}$$

$$MFP = 0.179$$

$$A_2 = \frac{\dot{m}\sqrt{T_0}}{MFP \cdot P_0}$$

$$A_2 = \frac{98.76\text{lbm/s}\sqrt{910.5\text{R}}}{0.179 \cdot 68.3\text{psia}\frac{144\text{si}}{1\text{sf}}}$$

$$\boxed{A_2 = 1.693\text{ft}^2}$$

Problem 3

Part A

Simply use isentropic relations to solve for upstream totals.

$$T_{01} = T_1\left(1 + \frac{\gamma - 1}{2}M_1^2\right)$$
$$T_{01} = 260\left(1 + \frac{1.4 - 1}{2}3^2\right)$$
$$\boxed{T_{01} = 728\mathrm{K}}$$

$$P_{01} = P_1\left(1 + \frac{\gamma - 1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$
$$P_{01} = 20\left(1 + \frac{1.4 - 1}{2}3^2\right)^{\frac{1.4}{1.4-1}}$$
$$\boxed{P_{01} = 734.65\mathrm{kPa}}$$

Part B

Since the shock is adiabatic across, $T_{02} = T_{01}$.

$$\boxed{T_{02} = 728\mathrm{K}}$$

Solve for M_2 with shock relation then use P_2 from part C and M_2 to solve isentropic relation for P_{02}.

$$M_2 = \sqrt{\frac{1 + \frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}}$$
$$M_2 = \sqrt{\frac{1 + \frac{1.4-1}{2}3^2}{1.4 \cdot 3^2 - \frac{1.4-1}{2}}}$$
$$M_2 = 0.4752$$

$$P_{02} = P_2\left(1 + \frac{\gamma - 1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}}$$
$$P_{02} = 206.67\left(1 + \frac{0.4}{2}0.4752^2\right)^{\frac{1.4}{0.4}}$$
$$\boxed{P_{02} = 241.22\mathrm{kPa}}$$

Part C

Solve for static conditions using shock relation equations.

$$T_2 = T_1\left[1+\frac{2\gamma}{\gamma+1}(M_1^2-1)\right]\left[\frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right]$$
$$T_2 = 260\left[1+\frac{2\cdot 1.4}{2.4}(3^2-1)\right]\left[\frac{2+(0.4)3^2}{(2.4)3^2}\right]$$
$$\boxed{T_2 = 696.54\text{K}}$$

$$P_2 = P_1\left(1+\frac{2\gamma}{\gamma+1}(M_1^2-1)\right)$$
$$P_2 = 20\left(1+\frac{2.8}{2.4}(3^2-1)\right)$$
$$\boxed{P_2 = 206.67\text{kPa}}$$

Problem 4

Part A

Input mach number and specific heat ratio to gastab to get:

ratio of specific heats Cp/Cv	1.40
Mach number M	0.30000
total temperature ratio Tt/Tt*	0.34686
static temperature ratio T/T*	0.40887
static pressure ratio P/P*	2.1314
total pressure ratio Pt/Pt*	1.1985
velocity ratio V/V*	0.19183

First find T_{02} from q.

$$q = c_p(T_{02}-T_{01})$$
$$T_{02} = q/c_p + T_{01}$$
$$T_{02} = \frac{500\text{kJ/kg}}{1.004\text{kJ/kg-K}} + 500\text{K}$$
$$\boxed{T_{02} = 998\text{K}}$$

Now use the gastab results and the ratios to determine T_{02}/T_0, then use gastab again to find the Mach number and total pressure ratio.

$$\frac{T_{02}}{T_{01}} = \left(\frac{T_{02}}{T_0^*}\right)\left(\frac{T_0^*}{T_{01}}\right)$$
$$\frac{T_{02}}{T_0^*} = \frac{T_{02}}{T_{01}}\frac{T_0^*}{T_{01}}$$
$$\frac{T_{02}}{T_0^*} = \frac{998}{500}(0.34686)$$
$$\frac{T_{02}}{T_0^*} = 0.692$$

Get the subsonic gastab results because it started subsonic:

ratio of specific heats Cp/Cv	1.40
Mach number M	0.50044
total temperature ratio Tt/Tt*	0.69200
static temperature ratio T/T*	0.79079
static pressure ratio P/P*	1.7770
total pressure ratio Pt/Pt*	1.1139
velocity ratio V/V*	0.44502

$$\boxed{M_2 = 0.5}$$

Get P_0^* from the first results:

$$\frac{P_{01}}{P_0^*} = 1.1985$$
$$P_0^* = 500.63\text{kPa}$$

Then get P_{02} from the new gastab results:

$$\frac{P_{02}}{P_0^*} = 1.1139$$
$$P_{02} = 500.63(1.1139)$$
$$\boxed{P_{02} = 557.65\text{kPa}}$$

Part B

Same process with different values:

ratio of specific heats Cp/Cv	1.325
Mach number M	0.30000
total temperature ratio Tt/Tt*	0.33896
static temperature ratio T/T*	0.38836
static pressure ratio P/P*	2.0773
total pressure ratio Pt/Pt*	1.1929
velocity ratio V/V*	0.18696

$$T_{02} = \frac{500\text{kJ/kg}}{1.171\text{kJ/kg-K}} + 500\text{K}$$
$$\boxed{T_{02} = 927\text{K}}$$

$$\frac{T_{02}}{T_0^*} = \frac{927}{500}(0.33896)$$
$$\frac{T_{02}}{T_0^*} = 0.628$$

ratio of specific heats Cp/Cv	1.33
Mach number M	0.46447
total temperature ratio Tt/Tt*	0.62800
static temperature ratio T/T*	0.70532
static pressure ratio P/P*	1.8081
total pressure ratio Pt/Pt*	1.1263
velocity ratio V/V*	0.39008

$$\boxed{M_2 = 0.46447}$$

$$\frac{P_{01}}{P_0^*} = 1.1929$$
$$P_0^* = 502.976\text{kPa}$$

$$\frac{P_{02}}{P_0^*} = 1.1263$$
$$P_{02} = 502.976(1.1263)$$
$$\boxed{P_{02} = 566.5\text{kPa}}$$

Problem 5

First we need the hydraulic diameter, which is just one side length for a square.

$$D = 1\text{ft}$$

Plug mach number into gastab to get results:

ratio of specific heats Cp/Cv	1.40
Mach number M	0.60000
static temperature ratio T/ T*	1.1194
static pressure ratio P/ P*	1.7634
total pressure ratio Pt/ Pt*	1.1882
velocity ratio V/ V*	0.63480
impulse function ratio I/ I*	1.1050
friction factor 4fLmax/D	0.49080

Get the * static temp and pressure form the gastab ratios as well as the total pressure ratio.

$$P^* = 10/1.7634 = 5.67\text{psia}$$
$$T^* = 500/1.1194 = 446.67^\circ\text{R}$$
$$P_0/P_0^* = 1.1882$$

First calculate the regular 4cfLD.

$$\frac{4c_f L}{D} = \frac{4(0.004)(8)}{1} = 0.128$$

Now get the 4cfL2*/D by solving:

$$\frac{4c_f L_2^*}{D} = 0.49080 - 0.128 = 0.3628$$

Then plug that friction factor back into gastab to get the subsonic (because started subsonic) Mach number.

ratio of specific heats Cp/Cv	1.40
Mach number M	0.63684
static temperature ratio T/ T*	1.1100
static pressure ratio P/ P*	1.6543
total pressure ratio Pt/ Pt*	1.1483
velocity ratio V/ V*	0.67090
impulse function ratio I/ I*	1.0807
friction factor 4fLmax/D	0.36280

$$\boxed{M_2 = 0.63684}$$

Get the pressure and temperature based on the new gastab ratios.

$$P_2 = 5.67 \cdot 1.6543$$
$$\boxed{P_2 = 9.38\text{psia}}$$
$$T_2 = 446.67 \cdot 1.11$$
$$\boxed{T_2 = 495.8^\circ\text{R}}$$
$$P_{02}/P_{01} = 1.1483/1.1882$$
$$\boxed{\frac{P_{02}}{P_{01}} = 0.9664}$$