AEEM4063 - Assignment 3

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Problem 1

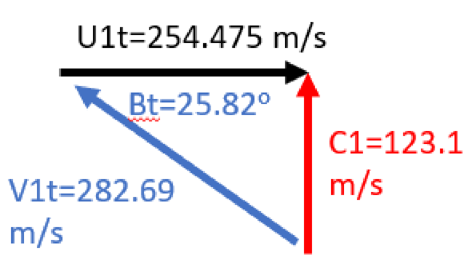
r_h	0.065	(m)
r_t	0.15	(m)
$\dot{m}$	8	(kg/s)
P_a	1	(bar)
T_a	288	(K)
n	270	(rev/s)
C_a	123.1	(m/s)

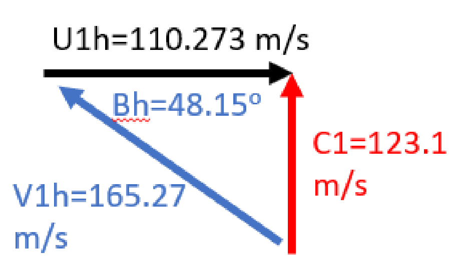
since no prewhirl, $C_1 = C_a = 123.1$ m/s

$n = \frac{270}{1}\frac{2\pi}{1} \rightarrow \omega = 1696.5$ rad/s
$U_{1h} = \omega r_h = 1696.5 * 0.065 = 110.273$ m/s
$U_{1t} = \omega r_t = 1696.5 * 0.15 = 254.475$ m/s

$\tan\beta_h = \frac{C_1}{U_{1h}}, \quad \beta_h = \arctan\frac{123.1}{110.273} = 48.15°$
$\tan\beta_t = \frac{C_1}{U_{1t}}, \quad \beta_t = \arctan\frac{123.1}{254.475} = 25.815°$
$\boxed{\beta_h = 48.15°}$
$\boxed{\beta_t = 25.82°}$

$M_{1t_{rel}} = \frac{V_{1t}}{\sqrt{\gamma R T_1}}$
$T_1 = T_a - \frac{C_1^2}{2c_p} = 288 - \frac{123.1^2}{2(1005)} = 280.46$ K
$V_{1t} = \sqrt{U_{1t}^2 + C_1^2} = \sqrt{254.475^2 + 123.1^2} = 282.69$ m/s
$M_{1t_{rel}} = \frac{282.69}{\sqrt{1.4(287)(280.46)}} = 0.842$
$\boxed{M_{1t_{rel}} = 0.842}$





Problem 2

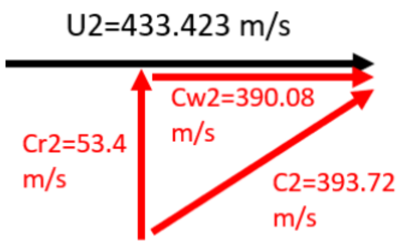
$\dot{m}$	14	(kg/s)
r	4	
n	200	(rev/s)
T_{0a}	288	(K)
P_{0a}	1	(bar)
σ	0.9	
ψ	1.04	
η_c	0.80	

$\frac{P_{03}}{P_{01}} = [1 + \frac{\eta_c \psi \sigma U_2^2}{c_{pc} T_{01}}]^{\frac{\gamma}{\gamma-1}} \rightarrow 4 = [1 + \frac{0.8(1.04)(0.9)U_2^2}{1005(288)}]^{\frac{1.4}{0.4}}$
$U_2 = 433.423$ m/s

$\omega = 2\pi n = 2\pi(200) = 1256.64$ rad/s
$D = \frac{2U}{\omega} = \frac{2(433.423)}{1256.64} = 0.69$ m
$\boxed{D = 69 \text{ cm}}$

$0.5(1 - \eta_c) = 0.1, \quad \eta_c = 0.9$ through impeller
$\frac{P_{02}}{P_{01}} = [1 + \frac{0.9(1.04)(0.9)(433.423)^2}{1005(288)}]^{\frac{1.4}{0.4}} = 4.6, \quad P_{02} = 4.6$ bar

$$T_{03} = T_{02} = T_{01} + \frac{\psi\sigma U^2}{c_p} = 288 + \frac{1.04(0.9)(433.426)^2}{1005} = 462.96 \text{ K}$$
$$C_{w2} = \omega U_2 = 0.9(433.423) = 390.08 \text{ m/s}$$
$$\frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2}M^2 \rightarrow T_2 = T_{02}/(1+\frac{\gamma-1}{2}) = 462.96/1.2 = 385.8 \text{ K}$$
$$C_2 = \mathrm{M}1, \quad C_2 = 1\sqrt{\gamma R T_2} = \sqrt{1.4(287)(385.8)} = 393.72 \text{ m/s}$$
$$C_{r2} = \sqrt{C_2^2 - C_{w2}^2} = \sqrt{393.72^2 - 390.08^2} = 53.4 \text{ m/s}$$
$$\frac{P_{02}}{P_2} = (1+\frac{\gamma-1}{2}1^2)^{\frac{\gamma}{\gamma-1}} \rightarrow P_2 = 4.6/(1+0.2)^{\frac{1.4}{0.4}} = 2.43 \text{ bar}$$
$$\dot{m} = \rho C_{r2} A \rightarrow A = \frac{\dot{m}}{\rho C_{r2}} = \frac{\dot{m} R T_2}{P_2 C_{r2}} = \frac{14(0.287)(385.8)}{2.43(53.4)*100000} = 0.1195 \text{ m}^2$$
$$h = \frac{A}{\pi D} = \frac{0.1195}{\pi 0.69} = 0.055 \text{ m}$$
$$\boxed{h = 5.5 \text{ m}}$$



Problem 3

U_t	250	(m/s)
U_m	200	(m/s)
U_r	150	(m/s)
$T_{02} - T_{01}$	20	(K)
C_{a1}	150	(m/s)
λ	0.93	
Λ_m	0.5	
η_s	0.88	

$$w = c_p(T_{02} - T_{01}) = 1005(20) = 20100 \text{ J/kg}$$
$$\boxed{w = 20100 \text{ J/kg}}$$

$$T_{03} = T_{02}, \quad \text{assume } T_{01} = 288 \text{ K}$$
$$\frac{P_{03}}{P_{01}} = [1 + \frac{\eta_s(T_{03}-T_{01})}{T_{01}}]^{\frac{\gamma}{\gamma-1}} = [1 + \frac{0.88(20)}{288}]^{\frac{1.4}{0.4}} = 1.23$$
$$\boxed{\frac{P_{03}}{P_{02}} = 1.23}$$

Mean

$$\Delta C_w = \frac{c_p(T_{02}-T_{01})}{\lambda U} = \frac{1005(20)}{0.93(200)} = 108.065 \text{ m/s}$$
$$C_{w1} = \frac{U - \Delta C_w}{2} = \frac{200-(108.065)}{2} = 45.97 \text{ m/s}$$
$$\tan\alpha_1 = \frac{C_{w1}}{C_{a1}} \rightarrow \alpha_1 = \arctan\frac{45.97}{150}$$
$$\boxed{\alpha_{1m} = \beta_{2m} = 17.04°}$$

$$C_{w2} = 154 \text{ m/s}$$
$$\tan\alpha_2 = \frac{C_{w2}}{C_{a1}} \rightarrow \alpha_2 = \arctan\frac{154}{150}$$
$$\boxed{\alpha_{2m} = \beta_{1m} = 45.75°}$$

Root

$$\Delta C_w = \frac{c_p(T_{02}-T_{01})}{\lambda U} = \frac{1005(20)}{0.93(150)} = 144.09 \text{ m/s}$$
$$C_{w1} = \frac{C_{w1m}U_m}{U_r} = \frac{45.97(200)}{150} = 61.3 \text{ m/s}$$
$$C_{w2} = 61.3 + 144.09 = 205.4 \text{ m/s}$$
$$V_{w1} = U_r - C_{w1} = 150 - 61.3 = 88.7 \text{ m/s}$$
$$V_{w2} = U_r - C_{w2} = 150 - 205.4 = -55.4 \text{ m/s}$$
$$\alpha_1 = \arctan\frac{C_{w1}}{C_{a1}} = \arctan\frac{61.3}{150} = \boxed{\alpha_1 = 22.23°}$$
$$\beta_1 = \arctan\frac{V_{w1}}{C_{a1}} = \arctan\frac{88.7}{150} = \boxed{\beta_1 = 30.6°}$$
$$\alpha_2 = \arctan\frac{C_{w2}}{C_{a1}} = \arctan\frac{205.4}{150} = \boxed{\alpha_2 = 53.86°}$$
$$\beta_2 = \arctan\frac{V_{w2}}{C_{a1}} = \arctan\frac{-55.4}{150} = \boxed{\beta_2 = -20.27°}$$

$$\Lambda_r = \frac{C_{a1}}{2U}(\tan\beta_2 + \tan\beta_1) = \frac{150}{2(150)}(\tan(-20.27) + \tan(30.6)) = 0.11$$
$$\boxed{\Lambda_r = 11\%}$$

Tip

$\Delta C_w = \frac{c_p(T_{02}-T_{01})}{\lambda U} = \frac{1005(20)}{0.93(250)} = 86.45$ m/s
$C_{w1} = \frac{C_{w1m}U_m}{U_t} = \frac{45.97(200)}{250} = 36.78$ m/s
$C_{w2} = 36.78 + 86.45 = 123.23$ m/s
$V_{w1} = U_t - C_{w1} = 250 - 36.78 = 213.22$ m/s
$V_{w2} = U_t - C_{w2} = 250 - 123.23 = 126.77$ m/s
$\alpha_1 = \arctan\frac{C_{w1}}{C_{a1}} = \arctan\frac{36.78}{150} = \boxed{\alpha_1 = 13.78°}$
$\beta_1 = \arctan\frac{V_{w1}}{C_{a1}} = \arctan\frac{213.22}{150} = \boxed{\beta_1 = 54.87°}$
$\alpha_2 = \arctan\frac{C_{w2}}{C_{a1}} = \arctan\frac{123.23}{150} = \boxed{\alpha_2 = 39.4°}$
$\beta_2 = \arctan\frac{V_{w2}}{C_{a1}} = \arctan\frac{126.77}{150} = \boxed{\beta_2 = 40.2°}$

$\Lambda_t = \frac{C_{a1}}{2U}(\tan\beta_2 + \tan\beta_1) = \frac{150}{2(250)}(\tan(40.2) + \tan(54.87)) = 0.68$
$\boxed{\Lambda_t = 68\%}$

Problem 4

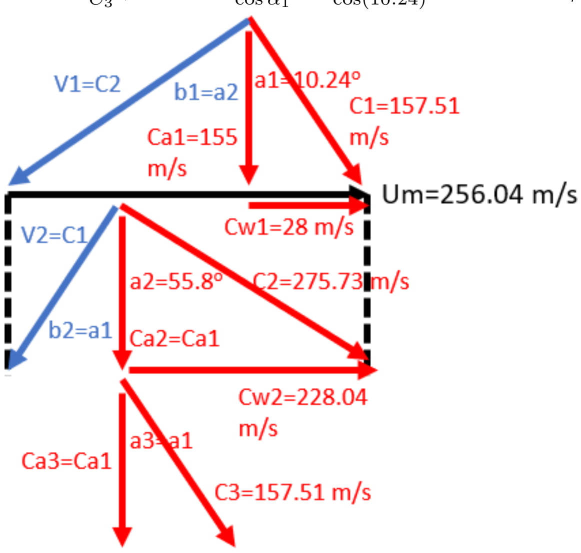
r_m	0.6	(m)
n	4075	(rpm)
C_{a1}	155	(m/s)
C_{w1}	28	(m/s)
Λ_m	0.5	

Part 1

$\omega = 2\pi n/60 = 2\pi 4075/60 = 426.73$ rad/s
$U_m = \omega r_m = 426.73(0.6) = 256.04$ m/s
$\boxed{U_m = 256.04 \text{ m/s}}$

Part 2

$C_1 = \sqrt{C_{a1}^2 + C_{w1}^2} = \sqrt{155^2 + 28^2} = 157.51$ m/s
since $\Lambda_m = 0.5$: $C_{w2} = U_m - C_{w1} = 256.04 - 28 = 228.04$ m/s
$C_2 = \sqrt{C_{a2}^2 + C_{w2}^2} = \sqrt{155^2 + 228.04^2} = 275.73$ m/s
$\alpha_1 = \arctan\frac{28}{155} = 10.24°$
$\alpha_2 = \arctan\frac{228.04}{155} = 55.8°$
$\cos\alpha_1 = \frac{C_{a1}}{C_3}, \quad C_3 = \frac{C_{a1}}{\cos\alpha_1} = \frac{155}{\cos(10.24)} = 157.51$ m/s



Part 3

$w = U(C_{w2} - C_{w1}) = 256.04(228.04 - 28)/1000 = 51218.24$ J/kg
$\boxed{w = 51.22 \text{ kJ/kg}}$