AEEM5063 HW#11

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6.20

$$T_{phase} = \frac{T}{2}$$
$$\frac{2\pi}{\sqrt{\mu}}a^{\frac{3}{2}} = \frac{1}{2}\frac{2\pi}{\sqrt{\mu}}r^{\frac{3}{2}}$$
$$a = \left(\frac{1}{2}r^{\frac{3}{2}}\right)^{\frac{2}{3}}$$
$$\boxed{a = 0.63r}$$

6.25

Orbit 1:

$$
\begin{aligned}
r_p = 6378 + 1270 = 7648\,\text{km}, \quad v_p = 9\,\text{km/s} \\
h_1 = r_p v_p = 7648 \cdot 9 = 68832\,\text{km}^2/\text{s} \\
r_p = \frac{h_1^2}{\mu}\frac{1}{1+e_1} \\
7648 = \frac{68832^2}{398600}\frac{1}{1+e_1} \quad \rightarrow \quad \boxed{e_1 = 0.554}
\end{aligned}
$$

At the maneuver point, $\theta = 100°$:

$$
\begin{aligned}
r = \frac{h_1^2}{\mu}\frac{1}{1+e_1\cos\theta} \\
r = \frac{68832^2}{398600}\frac{1}{1+0.554\cos 100°} = 13150\,\text{km}
\end{aligned}
$$

$$
\begin{aligned}
v_{1\perp} = \frac{h_1}{r} = \frac{68832}{13150} = 5.234\,\text{km/s} \\
v_{1r} = \frac{\mu}{h_1}e_1\sin\theta = \frac{398600}{68832} \cdot 0.554 \cdot \sin 100° = 3.16\,\text{km/s} \\
v_1 = \sqrt{v_{1\perp}^2 + v_{1r}^2} = \sqrt{5.234^2 + 3.16^2} = 6.114\,\text{km/s} \\
\gamma_1 = \tan^{-1}\frac{v_{1r}}{v_{1\perp}} = \tan^{-1}\frac{3.16}{5.234} = 31.13°
\end{aligned}
$$

Orbit 2:

$$
\begin{aligned}
e_2 = 0.4, \quad r = \frac{h_2^2}{\mu}\frac{1}{1+e_2\cos\theta} \\
13150 = \frac{h_2^2}{398600}\frac{1}{1+0.4\cos 100°} \quad \rightarrow \quad h_2 = 69840\,\text{km}^2/\text{s}
\end{aligned}
$$

$$v_{2\perp} = \frac{h_2}{r} = \frac{69840}{13150} = 5.311\,\mathrm{km/s}$$

$$v_{2r} = \frac{\mu}{h_2} e_2 \sin\theta = \frac{398600}{69840} \cdot 0.4 \cdot \sin 100^\circ = 2.248\,\mathrm{km/s}$$

$$v_2 = \sqrt{v_{2\perp}^2 + v_{2r}^2} = \sqrt{5.311^2 + 2.248^2} = 5.767\,\mathrm{km/s}$$

$$\gamma_2 = \tan^{-1}\frac{v_{2r}}{v_{2\perp}} = \tan^{-1}\frac{2.248}{5.311} = 22.94^\circ$$

$$\Delta\gamma = \gamma_2 - \gamma_1 = 22.94^\circ - 31.13^\circ = -8.181^\circ$$

$$\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos\Delta\gamma}$$

$$\Delta v = \sqrt{6.114^2 + 5.767^2 - 2 \cdot 6.114 \cdot 5.767 \cdot \cos(-8.181^\circ)}$$

$$\Delta v = 0.9155\,\mathrm{km/s}$$

$$\boxed{\Delta v = 0.916\,\mathrm{km/s}}$$

6.31

Orbit 1:

$$r_{P1_1} = \frac{h_1^2}{\mu}\frac{1}{1+e}$$

Orbit 2:

$$r_{P1_2} = \frac{h_2^2}{\mu}\frac{1}{1+e\cos 90} = \frac{h_2^2}{\mu}$$

$$r_{P1_1} = r_{P1_2}$$

$$\frac{h_2^2}{\mu} = \frac{h_1^2}{\mu}\frac{1}{1+e}$$

$$\boxed{h_2 = \frac{h_1}{\sqrt{1+e}}}$$

6.37

$$A = \sin^{-1}\left(\frac{\cos i}{\cos\phi}\right)$$

$$(a) \quad A = \sin^{-1}\left(\frac{\cos 116.57^\circ}{\cos 28.59^\circ}\right) = \boxed{329.4^\circ \text{ or } 210.6^\circ}$$

$$(b) \quad A = \sin^{-1}\left(\frac{\cos 116.57^\circ}{\cos 34.5^\circ}\right) = \boxed{327.1^\circ \text{ or } 212.9^\circ}$$

$$(c) \quad A = \sin^{-1}\left(\frac{\cos 116.57^\circ}{\cos 5.5^\circ}\right) = \boxed{333.3^\circ \text{ or } 206.7^\circ}$$

6.34

Contents

Initialize

```
clear; clc;
```

Problem 6.34

Define variables and constants

```
global mu
mu = 398600;        % km^3/s^2
```

Initial conditions

```
R0 = [0 16000 0];       % km
e2 = 0.5;
h2 = (mu*R0(2)*(1+e2))^(0.5);
Vb = h2/R0(2);          % km/s
V0 = [0 0 Vb];       % km/s
t = 60*60;             % s
```

do algortihm (from appendix D.16) and output results

```
[R, V] = rv_from_r0v0(R0, V0, t);
fprintf("Final position vec: R=(%g, %g, %g) km", R(1), R(2), R(3));
fprintf("\nFinal velocity vec: V=(%g, %g, %g) km/s", V(1), V(2), V(3));
```

```
Final position vec: R=(0, 7829.06, 18496.8) km
Final velocity vec: V=(0, -3.75299, 3.62618) km/s
```

setup for lambert

```
r1 = [10000 0 0];               % km
r2 = R;                         % km
dt = 60*60;                     % s
string = "pro";
```

use lambert function (alogrithm 5.2) from book to get v at each position

```
[v1, v2] = lambert(r1, r2, dt, string);

% output
fprintf("\nV1 velocity vec: V=(%g, %g, %g) km/s", v1(1), v1(2), v1(3));
fprintf("\nV2 velocity vec: V=(%g, %g, %g) km/s", v2(1), v2(2), v2(3));
```

```
V1 velocity vec: V=(0.948538, 3.13609, 7.40927) km/s
V2 velocity vec: V=(-4.0057, 1.20499, 2.84689) km/s
```

determine delta v req.

velocity at a (circular orbit):

```
vA = [0 (mu/norm(r1))^(0.5) 0];     % km/s

% subtract from V1/Va3
dVA = v1 - vA;                  % km/s (vector)
dV = norm(dVA);                 % km/s
fprintf("\ndV reqd: %g km/s", dV);
```

```
dV reqd: 8.11744 km/s
```

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