AEEM5063 HW#5

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3.10

$$T = 14\text{hr}; \quad r_p = 10000\text{km}@t = 10\text{hr}$$

(a)

$$T = \frac{2\pi}{\sqrt{\mu}} a^{3/2}$$

$$a^{3/2} = \frac{\sqrt{398600}}{2\pi}(14 * 3600)$$

$$a = 29490.3$$

$$a = 0.5(r_p + r_a) \rightarrow r_a = 48980.6$$

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{48980.6 - 10000}{48980.6 + 10000} = 0.661$$

$$M_e = \frac{2\pi}{T} t = \frac{2\pi}{14}(10) = 4.488 = E - e \sin E$$

Solve for E using Matlab kepler equation (kepler_E) function:

$$E = 3.9915$$

$$\tan \frac{E}{2} = \sqrt{\frac{1-e}{1+e}} \tan\left(\frac{\theta}{2}\right)$$

Plug in and solve for θ:

$$\theta = -2.738\text{rad} = 203.1^\circ$$

$$r = \frac{a(1-e^2)}{1+e\cos\theta} = \frac{29490(1-0.661^2)}{1+0.661\cos 203.11} = \boxed{42356\text{km}}$$

(b)

$$\frac{V^2}{2} - \frac{\mu}{r} = \varepsilon = \frac{-\mu}{2a}$$

$$V = \sqrt{2\mu\left(\frac{1}{r} - \frac{1}{2a}\right)} = \sqrt{2(398600)\left(\frac{1}{42355.5} - \frac{1}{2(29490)}\right)}$$

$$\boxed{V = 2.3 \text{ km/s}}$$

(c)

$$h^2 = a\mu(1-e^2)$$

$$h = \sqrt{29490 * 398600(1-0.661^2)} = 81356.2$$

$$V_r = \frac{\mu}{h}e\sin\theta = \frac{398600}{81356.21}0.661\sin 203.11 = \boxed{-1.27 \text{ km/s}}$$

3.11

$$r_p = 7500\text{km}; \quad r_a = 16000\text{km}; \quad t = 2400\text{s}; \quad \theta = 80^\circ$$

$$a = \frac{r_p + r_a}{2} = \frac{7500+16000}{2} = 11750$$

$$T = \frac{2\pi}{\sqrt{\mu}}a^{3/2} = \frac{2\pi}{\sqrt{398600}}11750^{3/2} = 12675.59$$

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{16000-7500}{16000+7500} = 0.3617$$

@ $\theta = 80°$:

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2} = \sqrt{\frac{1-0.3617}{1+0.3617}tan80/2}$$

$$E = 1.043$$

$$M_e = E - e\sin E = 1.043 - 0.3617\sin 1.043 = 0.73$$

$$t = \frac{M_e T}{2\pi} = \frac{0.7304(12675.59)}{2\pi} = 1473.5$$

@ new time:

$$t = 1473.5 + 2400 = 3873.5$$

$$M_e = \frac{2\pi}{T}t = \frac{2\pi}{12675.59}(3873.5) = 1.92$$

using matlab kepler_E:

$$E = 2.21$$

$$\theta = 2\tan^{-1}\left(\frac{\tan\frac{E}{2}}{\sqrt{\frac{1-e}{1+e}}}\right)$$

$$\boxed{\theta = 142°}$$

3.15

$$r_p = 6600\text{km}; \quad e = 1$$

(a)

@ $\theta = 90$:

$$M_P = 0.5\tan(90/2) + 1/6\tan^3(90/2) = 0.667$$

$$h = \sqrt{r\mu(1+e\cos\theta)} = \sqrt{6600(398600)(1+1\cos 0)} = 72536.34$$

$$t = \frac{M_p h^3}{\mu^2} = \frac{0.667(72536.34)^2}{398600^2} = 1602.2$$

$$\boxed{t_{-90\to 90} = 2 * 1602.2\text{s} = 0.89\text{hr}}$$

(b)

$$t = 36\text{hr} = 129600\text{s}$$

$$M_P = \frac{\mu^2 t}{h^3} = \frac{398600^2 * 129600}{72536.34^3} = 53.95$$

Use wolfram to solve:

$$M_P = 0.5 \tan\frac{\theta}{2} + 1/6 \tan^3\frac{\theta}{2}$$
$$\theta = 163^\circ$$

$$r = \frac{h^2}{\mu}\frac{1}{1+\cos\theta} = \frac{72536.34^2}{398600}\frac{1}{1+\cos 163}$$
$$\boxed{r = 304720 \text{ km}}$$

3.17

$$V_p = 1.1V_{esc}; \quad r_p = 6378 + 200 = 6578\text{km}; \quad V = 8\text{km/s@10a.m.}$$

$$V_{esc} = \sqrt{\frac{2\mu}{r_p}} \rightarrow V_p = 1.1\sqrt{2 * 398600/6578} = 12.11$$
$$h = r_p V_p = 6578 * 12.11 = 79659.6$$
$$r_p = \frac{h^2}{\mu}\frac{1}{1+e\cos\theta} \rightarrow$$
$$e = \frac{h^2}{\mu r_p} - 1 = \frac{79659.6^2}{398600(6578)} - 1 = 1.42$$

@ 10a.m.:

$$\frac{V^2}{2} - \frac{\mu}{r} = \frac{V_p^2}{2} - \frac{\mu}{r_p}$$

$$r = \frac{398600}{\frac{8^2}{2} - \frac{12.11^2}{2} + \frac{398600}{6578}} = 20685.14$$

$$r = \frac{h^2}{\mu}\frac{1}{1 + e\cos\theta}$$

$$\theta = \cos^{-1}\frac{\frac{h^2}{r\mu} - 1}{e} = \cos^{-1}\frac{\frac{79659.58^2}{20685.14*398600} - 1}{1.42} = -99.34$$

$$\tan\theta/2 = \sqrt{\frac{1+e}{e-1}}\tanh F/2$$

$$F = 2tanh^{-1} - 0.4907 \rightarrow F = -1.074$$

$$M_h = e\sinh F - F = 1.42\sinh(-1.074) + 1.074 = -0.7617$$

$$M_h = \frac{\mu^2}{h^3}(e^2-1)^{3/2}t$$

$$t = -2365$$

@ 5p.m.:

$$t = 7(3600) - 2365 = 22835$$

$$M_h = \frac{\mu^2}{h^3}(e^2-1)^{3/2}t = \frac{398600^2}{79659.6^3}(1.42^2-1)^{3/2}(22835) = 7.355$$

$$M_h = e\sinh F - F \rightarrow F = 2.65(\text{solved w/ wolfram})$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{e+1}{e-1}}\tanh F/2$$

$$\tanh F/2 = \frac{e^{F/2} - e^{-F/2}}{e^{F/2} + e^{-F/2}} = 0.868$$

$$\theta = 128.72$$

$$r = \frac{h^2}{\mu}\frac{1}{1 + e\cos\theta} = \frac{79659.6^2}{398600}\frac{1}{1 + 1.42\cos 128.72} = 142435.67$$

$$z = r - r_e = \boxed{136058 \text{ km}}$$