AEEM5058 HW#3

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Problem 1

The same cross section considered in Homework #1 is modified by introducing a longitudinal cut at corner 1 spanning the entire length L of the beam (no material is removed). The section is subject to a shear force F= 25 kN contained in the u-v plane with its line of action intersecting the shear center (S.C.) and parallel to the u axis.

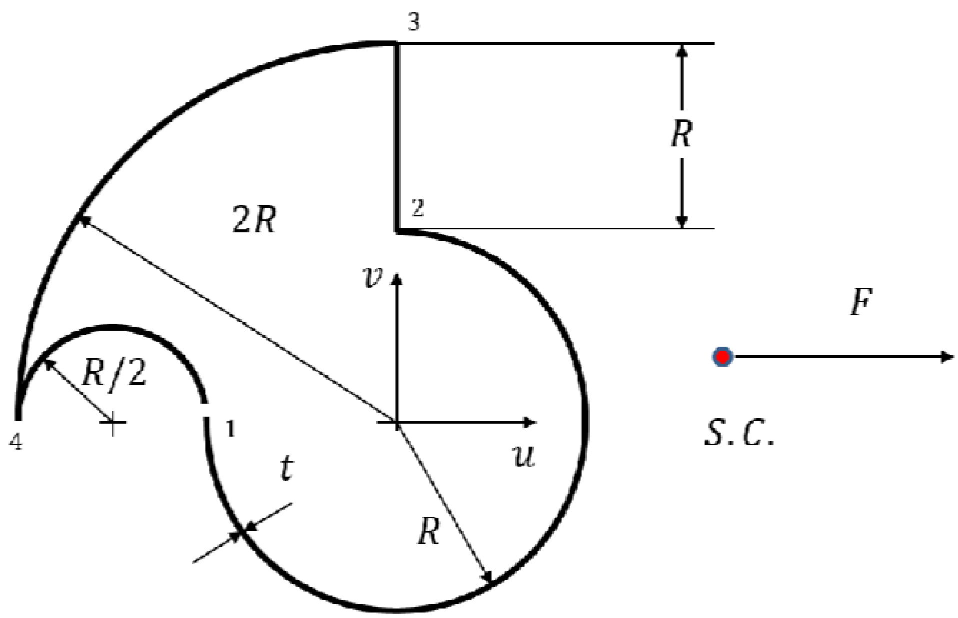


Figure 1

a)

Compute the shear flow around the cross section, plot its distribution as a function of the arc-length coordinate, s, and provide the minimum and maximum values of the shear flow. In your answer show how you compute the shear flow.

The shear flow is defined by:

$$q(s) = q(0) + \alpha f(s) + \beta g(s)$$

We will evaluate at the free end starting at s_1, so $q(0) = 0$. We can define the shear forces based on the direction of F. Due to the sign conventions, V_η will be $-F$ and V_ζ will be 0. Adjusting the bounds of s from HW#1 and subtracting from the centroid to convert to the eta and zeta axes, we can define the components of the shear flow equation.

s	$\eta(s)$	$\zeta(s)$	bounds
s_2	$-u_c$	$R + s_2 - v_c$	$0 \to R$
s_3	$-2R\sin\frac{s_3}{2R} - u_c$	$2R\cos\frac{s_3}{2R} - v_c$	$0 \to \pi R$
s_4	$-\frac{3R}{2} - \frac{R}{2}\cos\frac{2s_4}{R} - u_c$	$\frac{R}{2}\sin\frac{2s_4}{R} - v_c$	$0 \to \frac{\pi R}{2}$
s_1	$-R\cos\frac{s_1}{R} - u_c$	$-R\sin\frac{s_1}{R} - v_c$	$0 \to \frac{3\pi R}{2}$

From the previous homeworks:

$$I_\eta = 119284851.3\text{mm}^4$$
$$I_\zeta = 130805242.4\text{mm}^4$$
$$I_{\eta\zeta} = -22862046.03\text{mm}^4$$

α and β can now be calculated. Since thickness is constant, it can be removed from f(s) and g(s) and multiplied by α and β directly.

$$\alpha = \frac{V_\eta I_\eta - V_\zeta I_{\eta\zeta}}{I_\eta I_\zeta - I_{\eta\zeta}^2} t = -3.3617 * 10^{-4}$$
$$\beta = \frac{V_\zeta I_\zeta - V_\eta I_{\eta\zeta}}{I_\eta I_\zeta - I_{\eta\zeta}^2} t = -6.4430 * 10^{-5}$$

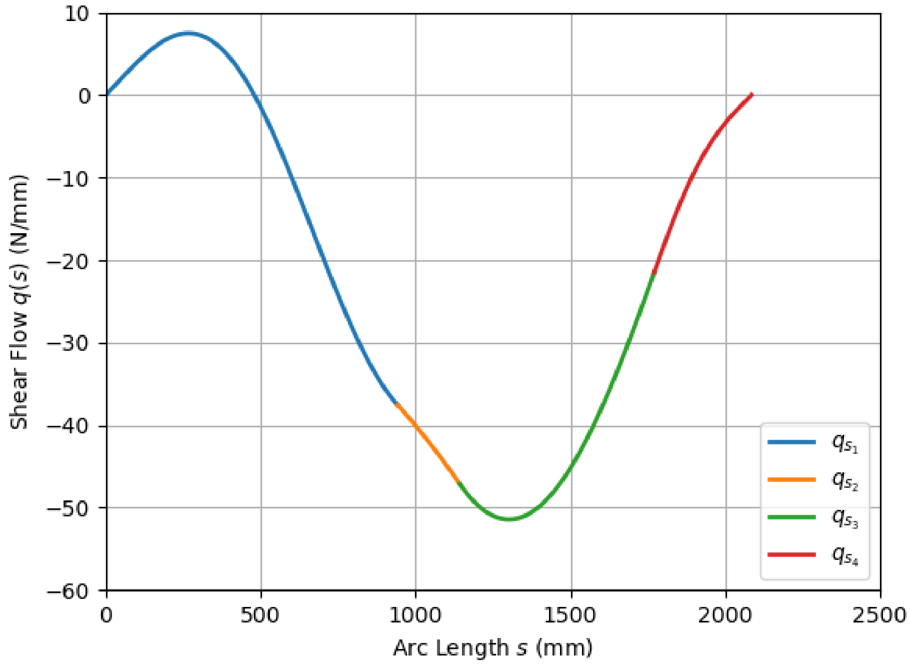
Now f(s) and g(s) can be calculated.

$$f^{12}_{s_1} = \int_0^{s_1} (-R\cos\frac{s}{R} - u_c)ds = -u_c s_1 - R^2 \sin\frac{s_1}{R}$$
$$f^{23}_{s_2} = f^{12}(s_1) + \int_0^{s_2} (-u_c)ds = f^{12}(s_1) - u_c s_2$$
$$f^{34}_{s_3} = f^{23}(s_2) + \int_0^{s_3} \left(-2R\sin\frac{s}{2R} - u_c\right) ds = f^{23}(s_2) - u_c s_3 + 4R^2\cos(\frac{s_3}{2R}) - 4R^2$$
$$f^{41}_{s_4} = f^{34}(s_3) + \int_0^{s_4} \left(\frac{-3R}{2} - \frac{R}{2}\cos\frac{2s}{R} - u_c\right) ds = f^{34}(s_3) - \frac{3Rs_4}{2} - u_c s_4 - \frac{R^2}{4}\sin(\frac{2s_4}{R})$$

$$g^{12}_{s_1} = \int_0^{s_1} (-R\sin\frac{s}{R} - v_c)ds = -v_c s_1 + R^2(\cos\frac{s_1}{R} - 1)$$
$$g^{23}_{s_2} = g^{12}(s_1) + \int_0^{s_2} (R + s - v_c)ds = g^{12}(s_1) + Rs_2 - v_c s_2 + \frac{s_2^2}{2}$$
$$g^{34}_{s_3} = g^{23}(s_2) + \int_0^{s_3} (2R\cos\frac{s}{2R} - v_c)ds = g^{23}(s_2) - v_c s_3 + 4R^2\sin\frac{s_3}{2R}$$
$$g^{41}_{s_4} = g^{34}(s_3) + \int_0^{s_4} (\frac{R}{2}\sin\frac{2s}{R} - v_c)ds = g^{34}(s_3) - v_c s_4 - \frac{R^2}{4}(\cos\frac{2s_4}{R} - 1)$$

Now we have representations for all of the parts of the shear flow equation. We will use Python to plot these equations around the length of the cross section. We can also use python to evaluate the locations and magnitudes of the maximum and minimum shear flow.

$$\boxed{q(s)_{max} = 7.477\text{N/mm at s=266.6mm}}$$
$$\boxed{q(s)_{min} = -51.448\text{N/mm at s=1301.1mm}}$$

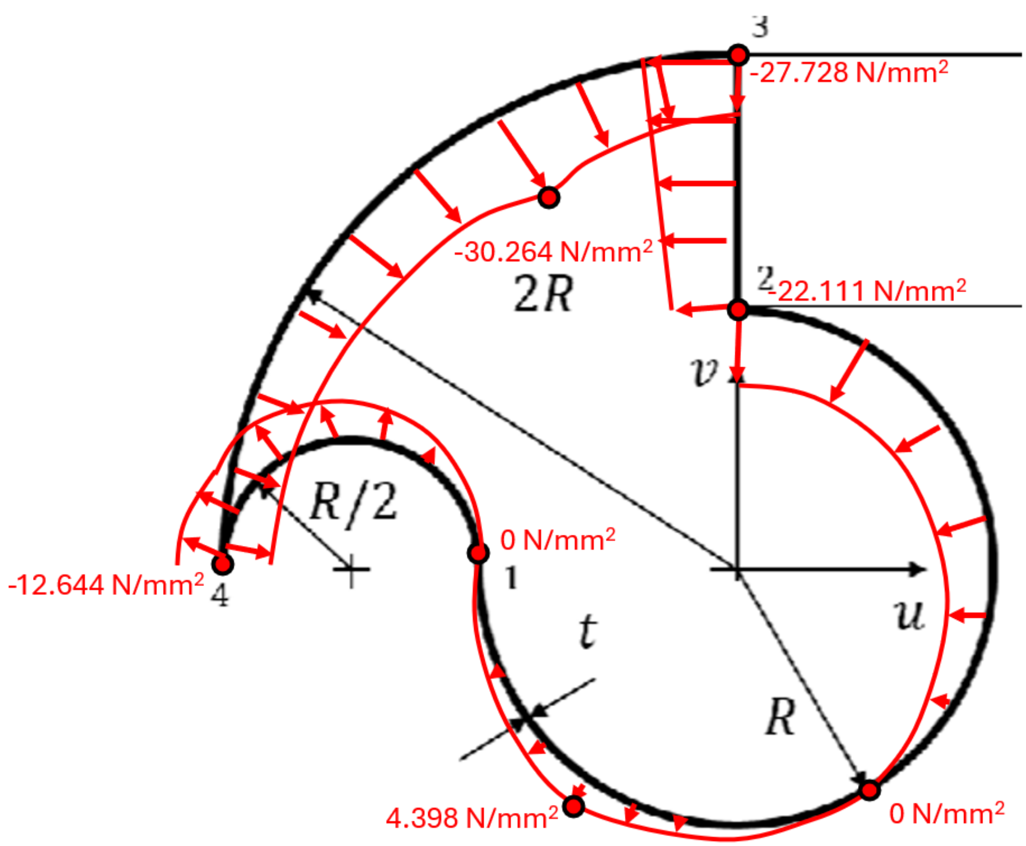


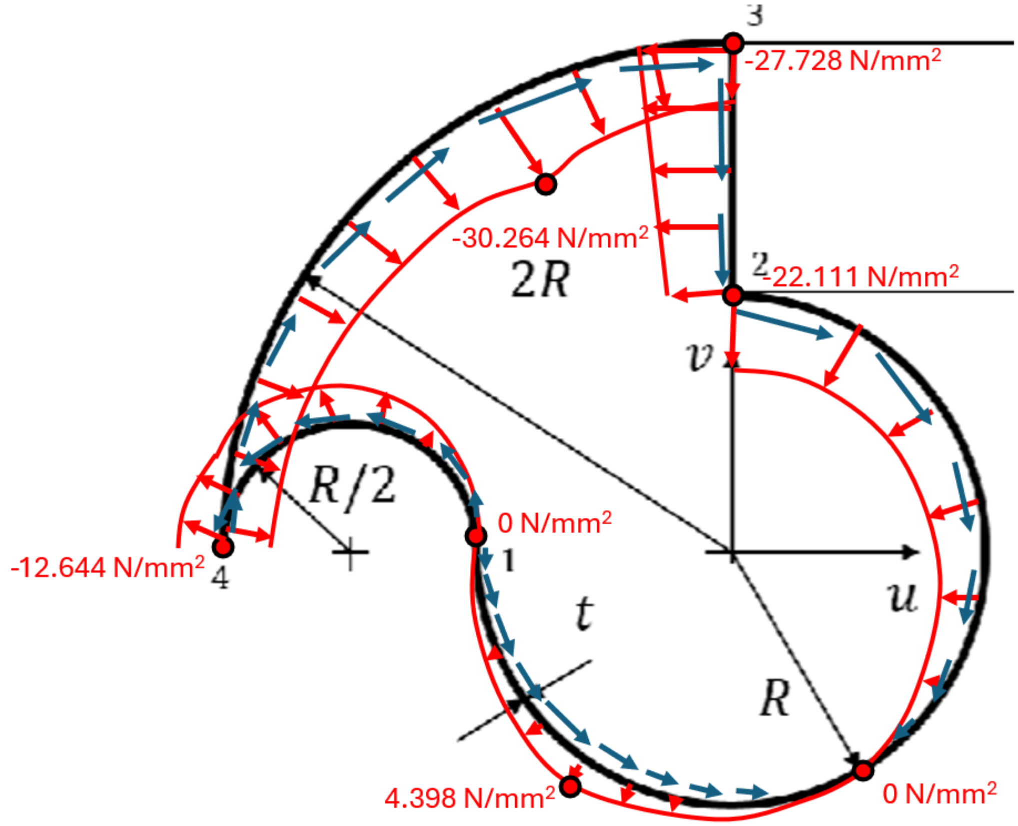
Shear Flow $q(s)$ (N/mm)
Arc Length s (mm)
10
0
−10
−20
−30
−40
−50
−60
0
500
1000
1500
2000
2500
q_{s_1}
q_{s_2}
q_{s_3}
q_{s_4}

b)

Sketch the distribution of the shear stress around the cross section, indicate its direction, and provide the value of stress at significant points.

The shear stress is simply the shear flow divided by the thickness t. The direction is given by the sign of the shear flow at the point along the cross section. From the plot generated in part a, a sketch of the shear stress can be created showing the distribution and values.




3
-27.728 N/mm²
-30.264 N/mm²
2R
2
-22.111 N/mm²
v
R/2
0 N/mm²
1
-12.644 N/mm²
4
u
t
R
4.398 N/mm²
0 N/mm²