AEEM 3062

HOMEWORK # 10

2022-2023 SPRING

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Problem 1

A beam of length L =1200 mm is pinned at its two ends and has the T cross section shown in Fig. 1 with dimensions a = 50 mm and t = 3 mm. After installation, the beam experiences a temperature change that is uniform along its length but varies linearly in the vertical direction with the maximum temperature increase, T_0 = 75 °C, occurring at the base of the section. The temperature change is the same for all the points that have the same distance from the base of the section.

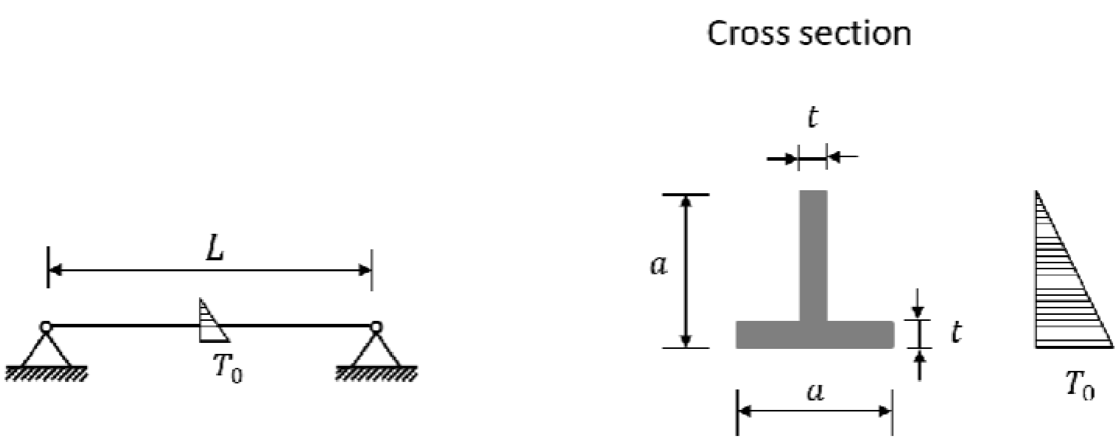


Figure 1

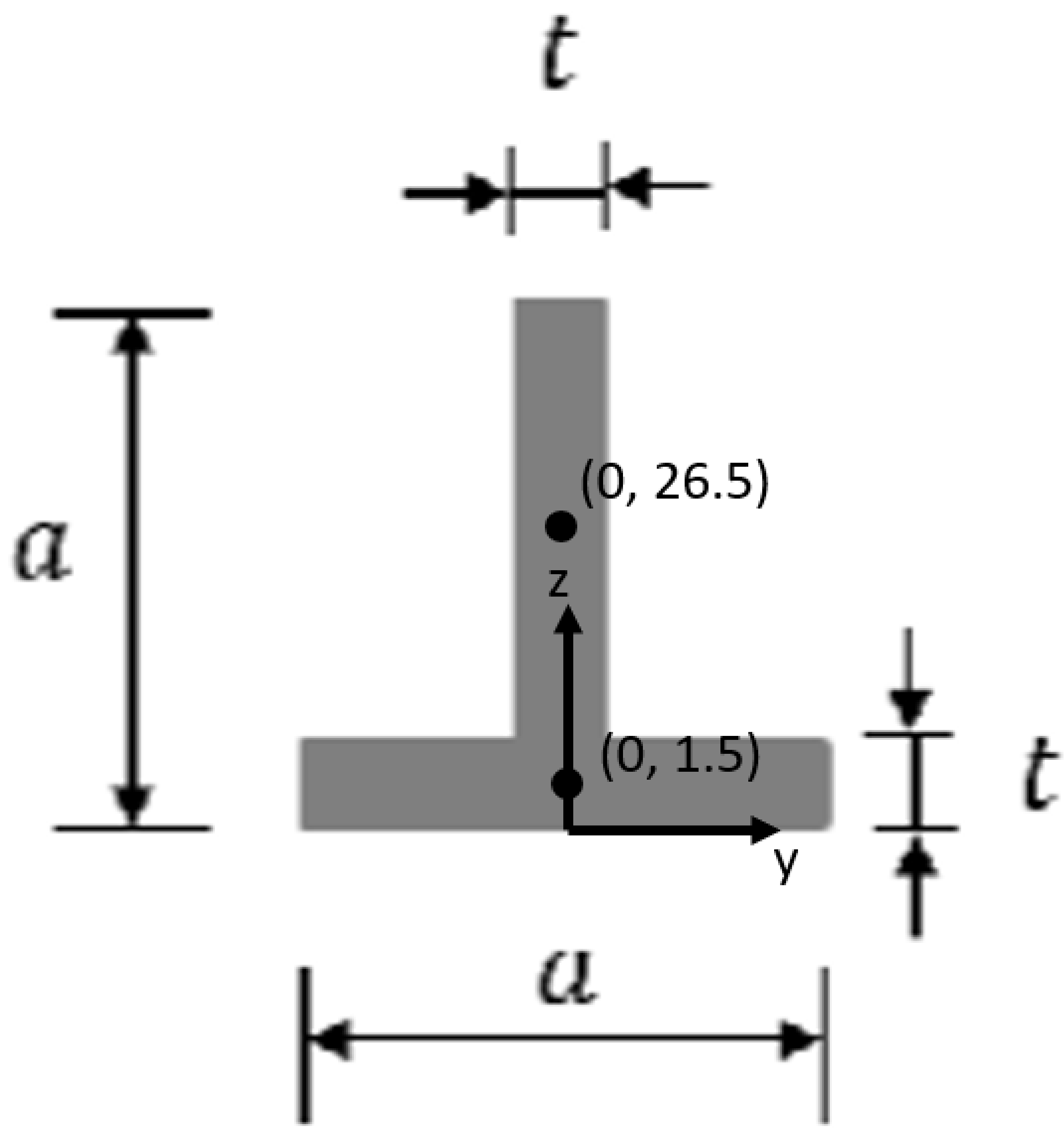
Part A

Assuming the material of the beam has Young's modulus E = 280 GPa and linear thermal expansion coefficient $\alpha = 12 \times 10^{-6}$°C^{-1}, plot the diagrams showing how the normal force and vertical displacement vary along the beam as well as the reaction diagram. [50 pts]

First we will find the centroid of the system by finding the centroid of each rectangle and then taking their weighted average. We know that the centroid will be at the exact horizontal midpoint of both rectangles since they share that axis of symmetry.

We will set the origin at the bottom and middle of the bar with y being the horizontal and z being the vertical axis.

The top rectangle has a center at $47/2$ so its centroid is at $(0, 47/2 + 3)$. The bottom rectangle has a center at $(0, 3/2)$.



Then we take the weighted average:

$$z = \frac{26.5 * 3 * 47 + 1.5 * 3 * 50}{3 * 50 + 3 * 47} = 13.613$$

Therefore the centroid is $(0, 13.613)$.

Now we will evaluate T at the centroid:

$$T(z) = -\frac{T_o}{a}z + T_o$$
$$T(13.613) = 54.58$$

Now we can set up the differential equations:

$$\frac{\partial N}{\partial x} = 0,$$
$$N = C_1$$
$$\frac{\partial V}{\partial x} = 0,$$
$$V = C_3$$
$$\frac{\partial M}{\partial x} = V = C_3,$$
$$M = C_3 x + C_4 L$$
$$\frac{\partial u}{\partial x} = \frac{N}{EA} + T_c\alpha = \frac{C_1}{EA} + T_c\alpha,$$
$$u = \frac{1}{EA}(C_1 x + C_2 L) + T_c\alpha x$$
$$\frac{\partial \varphi}{\partial x} = \frac{M}{EI} + \alpha\frac{T_o}{a} = \frac{C_3 x + C_4 L}{EI} + \frac{T_o}{a}\alpha,$$
$$\varphi = \frac{1}{EI}(\frac{C_3}{2}x^2 + C_4 L x + C_5 L^2) + \frac{T_o}{a}\alpha x$$
$$\frac{\partial w}{\partial x} = \varphi = \frac{1}{EI}(\frac{C_3}{2}x^2 + C_4 L x + C_5 L^2) + \frac{T_o}{a}\alpha x,$$
$$w = \frac{1}{EI}(\frac{C_3}{6}x^3 + \frac{C_4 L}{2}x^2 + C_5 L^2 x + C_6 L^3) + \frac{T_o}{2a}\alpha x^2$$

And we can enforce boundary conditions at both ends:

$$u(0) = u(L) = 0$$
$$w(0) = w(L) = 0$$
$$M(0) = M(L) = 0$$

And employ those to the equations:

$$u(0) = 0 = \frac{1}{EA}(C_1 0 + C_2 L) + T_c \alpha 0,$$
$$C_2 = 0$$
$$w(0) = 0 = \frac{1}{EI}(\frac{C_3}{6}0^3 + \frac{C_4 L}{2}0^2 + C_5 L^2 0 + C_6 L^3) + \frac{T_o}{2a}\alpha 0^2,$$
$$C_6 = 0$$
$$M(0) = 0 = C_3 0 + C_4 L,$$
$$C_4 = 0$$
$$u(L) = 0 = \frac{1}{EA}(C_1 L + 0L) + T_c \alpha L,$$
$$C_1 = -EAT_c\alpha$$
$$M(L) = 0 = C_3 L + 0L,$$
$$C_3 = 0$$
$$w(L) = 0 = \frac{1}{EI}(\frac{0}{6}L^3 + \frac{0L}{2}L^2 + C_5 L^3 + 0L^3) + \frac{T_o}{2a}\alpha L^2,$$
$$C_5 = -\frac{EIT_o\alpha}{2aL}$$

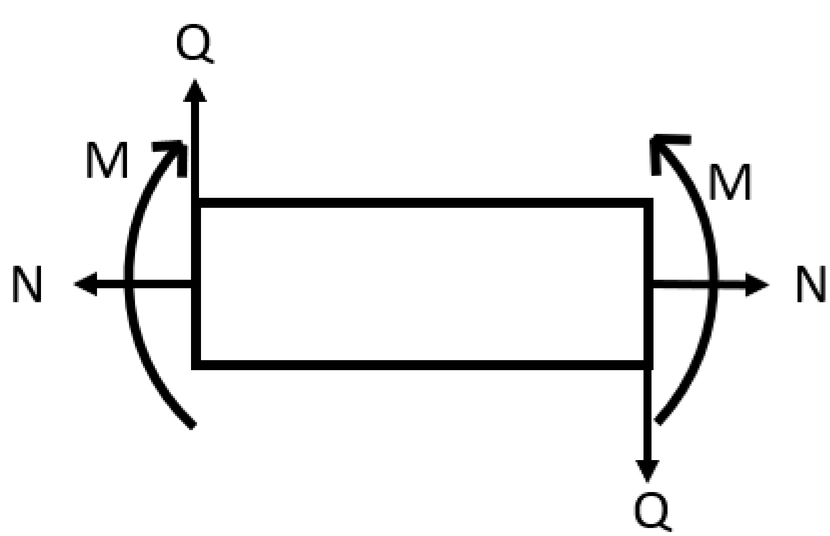
Then we can convert units and solve for the constants.

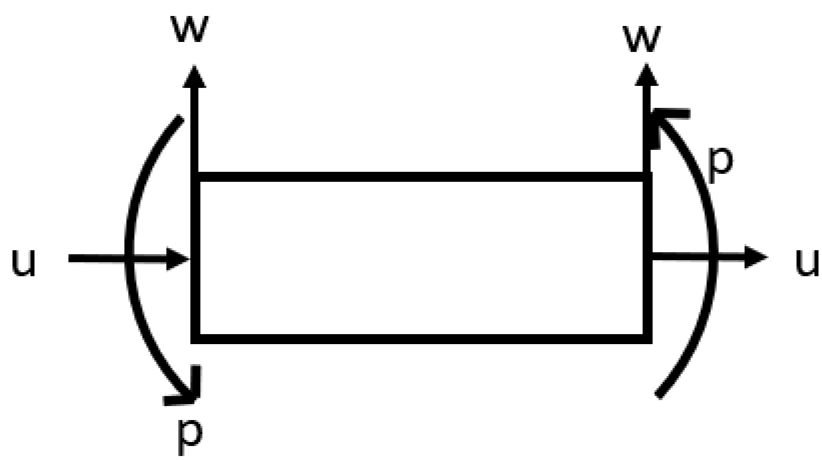
L = 1.2 mm, a = 0.05 mm, T_o = 75 $^\circ$ C, E = 280e9 Pa

$$C_1 = -(280e9)(0.003 * 0.047 + 0.003 * 0.05)(54.58)(12e-6)$$
$$= -53366.14\text{N}$$

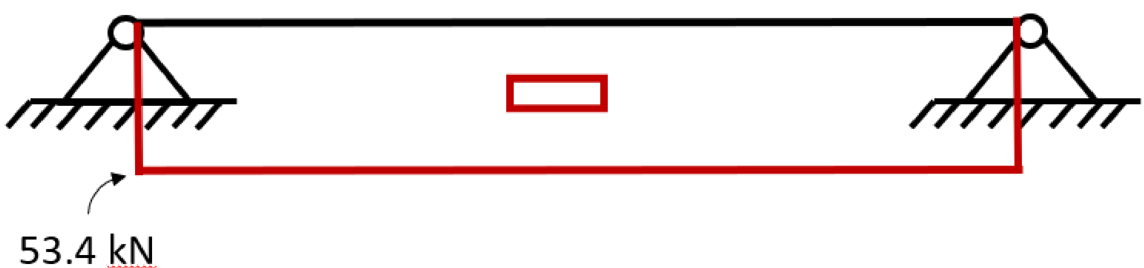
Then plug back into equations:

$$N(x) = -53366.14$$
$$w(x) = -\frac{T_o\alpha L}{2a}x + \frac{T_o\alpha}{2a}x^2$$

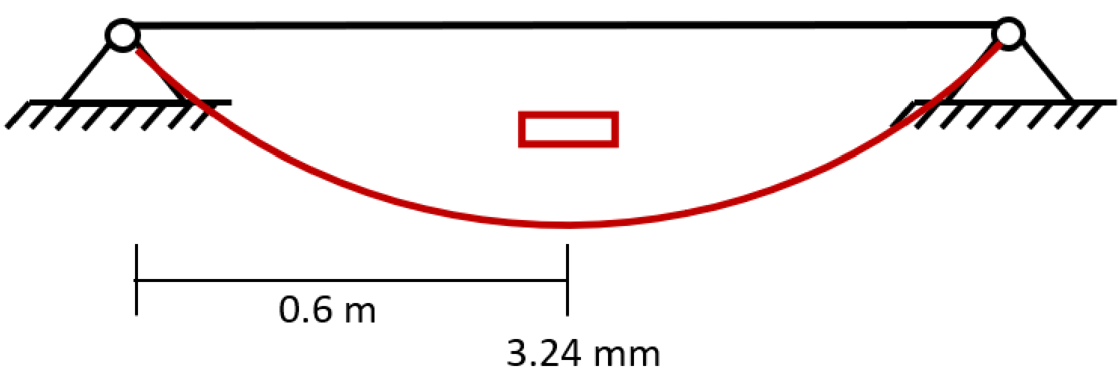




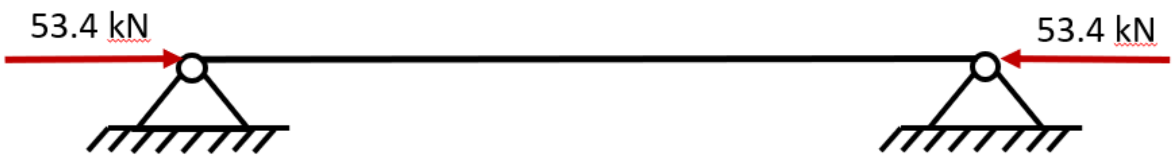
Normal Force



Vertical Displacement



Reaction Diagram



Part B

Assuming that the material of the beam has yield stress $\sigma_Y = 200$ MPa, find the maximum temperature of T_0 that the beam can withstand without failing [10 pts]

$$\sigma_y = \frac{N}{A} = \frac{C_1}{A} = \frac{-EAT_c\alpha}{A}$$

$$-E\alpha(-\frac{T_o}{a}z + T_o) = \sigma_y$$

$$-\frac{T_o}{a}z + T_o = -\frac{\sigma_y}{E\alpha}$$

$$T_o(-\frac{z}{a} + 1) = -\frac{\sigma_y}{E\alpha}$$

$$T_o = -\frac{\sigma_y}{E\alpha(-\frac{z}{a} + 1)}$$

Plugging in +/- σ_y and solving for T_o gives:

$$\boxed{T_o = \pm 81.13^{\circ}\mathrm{C}}$$

Problem 2

A box beam is a common structure used to provide bending and torsional strength to wings. Here, a wing of length, L, has a box wing with a trapezoidal cross section with base a and sides h_1 and h_2 as shown in Fig 2. The wing is subject to a distributed pitch moment given by

$$m_t = -\frac{\pi Q}{2L}\cos\left(\frac{\pi x}{2L}\right)$$

where Q is a constant.

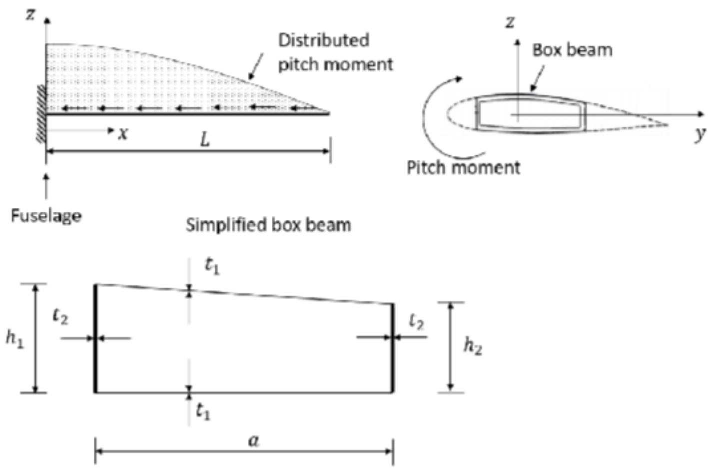


Figure 2

Part A

Find the functions that describe how the torque and angle of twist vary along the wing as a function of the shear modulus G, polar moment of area J (no need to substitute the expression of J at this point) and Q. Plot the two corresponding diagrams [20 pts]

We will set up the differential equations:

$$\frac{\partial M}{\partial x} = 0 - m_t = \frac{\pi Q}{2L}\cos\frac{\pi x}{2L}, M = \frac{\pi Q}{2L}\frac{\pi Q}{2L}\sin\frac{\pi x}{2L} + C_1,$$

$$M = Q\sin\frac{\pi x}{2L} + C_1$$

$$\frac{\partial \theta}{\partial x} = \frac{m_t}{GJ},$$

$$\theta = -\frac{1}{GJ}(\frac{2QL}{\pi}\cos\frac{\pi x}{2L} - C_1 x) + C_2 L$$

We can determine the boundary conditions:

$$m_t(L) = 0$$

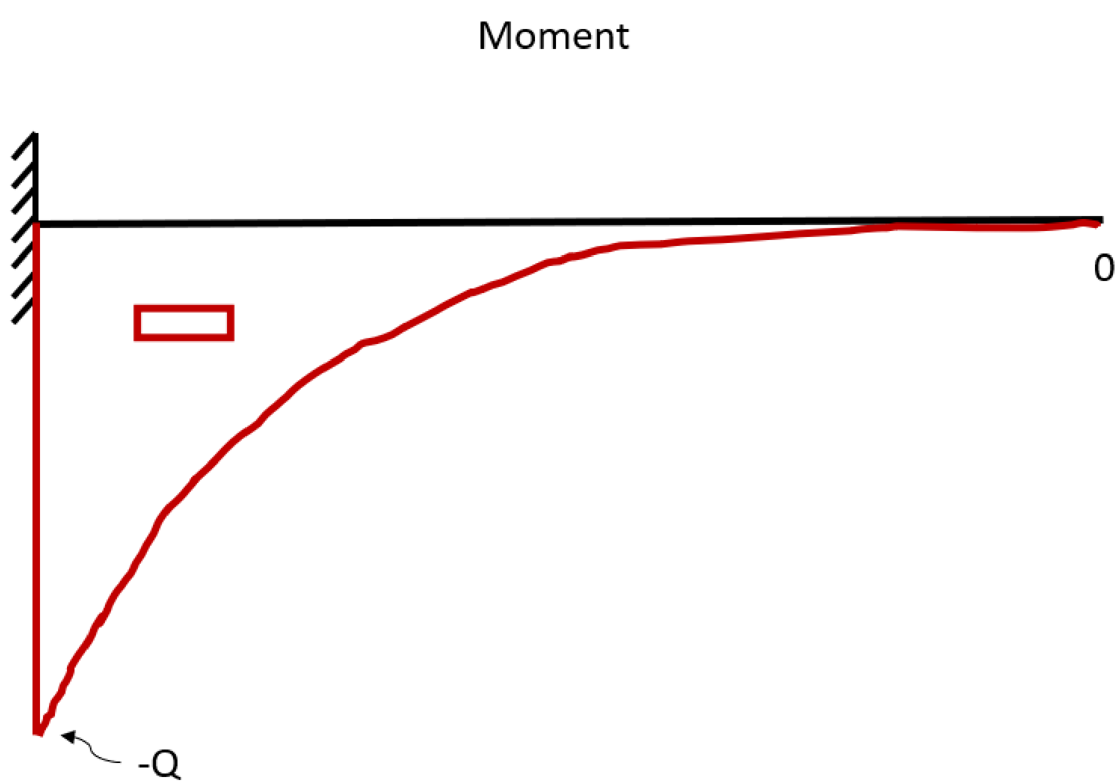
$$\theta(0) = 0$$

And enforce them on the equations:

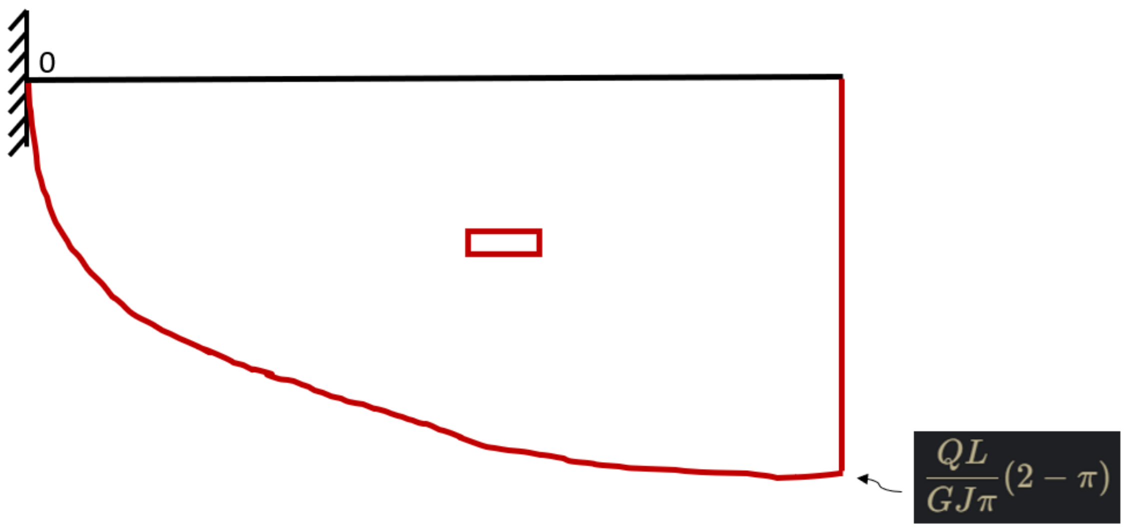
$$m_t(L) = 0 = Q\sin\frac{\pi L}{2L} + C_1,$$
$$C_1 = -Q$$
$$\theta(0) = 0 = -\frac{1}{GJ}\left(\frac{2QL}{\pi}\cos\frac{\pi 0}{2L} - C_1 0\right) + C_2 L,$$
$$C_2 = \frac{2Q}{GJ\pi}$$

And substitute back into the equations:

$$\boxed{m_t(x) = Q(\sin\frac{\pi x}{2L} - 1)}$$
$$\boxed{\theta(x) = \frac{Q}{GJ}(-\frac{2L}{\pi}\cos\frac{\pi x}{2L} - x + \frac{2L}{\pi})}$$



Theta



Part B

Assuming that the box beam is made of aluminum, L = 5000 mm, a = 600 mm; h_1 = 220 mm, h_2 = 180 mm and the thicknesses are t_1 =1 mm and t_2 =3 mm and that Q = 16 kNm, find the maximum shear stress and the maximum twist angle. [20 pts]

G of aluminum = 25.7e9 Pa (from slides)

$$J = \frac{4\Sigma^2}{\int \frac{1}{t(s)} ds}$$

We neglect wall thickness so:

$$\Sigma = \frac{h_1 + h_2}{2} a = 0.120\text{m}^2$$

$$\int \frac{1}{t(s)} ds = \int_0^a \frac{1}{t_1} ds + \int_0^{h_2} \frac{1}{t_2} ds + \int_0^{h_1} \frac{1}{t_2} ds + \int_0^{\sqrt{(h_1-h_2)^2+a^2}} \frac{1}{t_1} ds,$$
$$= \frac{a}{t_1} + \frac{h_2}{t_2} + \frac{h_1}{t_2} + \frac{\sqrt{(h_1-h_2)^2+a^2}}{t_1} = 1334.5$$

$$J = 4.316e - 5\text{m}^2$$

$$q = \frac{m_t(x)}{2\Sigma} = \tau(s)t(s)$$

We will assume τ is constant.

$$\tau(s) = \frac{m_t(x)}{2\Sigma t(s)}$$

Find location of max $m_t(x)$:

$$m_t(0) = -Q$$

Find location of min t(s):

$$t_1 = 0.001$$

Therefore:

$$\boxed{\tau(s)_{max} = -\frac{Q}{2\Sigma t_1} = -6.67e7\mathrm{Pa}}$$

Max twist will be at:

$$\theta(L) = \frac{QL}{\pi GJ}(2-\pi) = -0.0262\mathrm{rad} = \boxed{-1.5^{\circ} = \theta_{max}}$$