AEEM4058 - Homework 2

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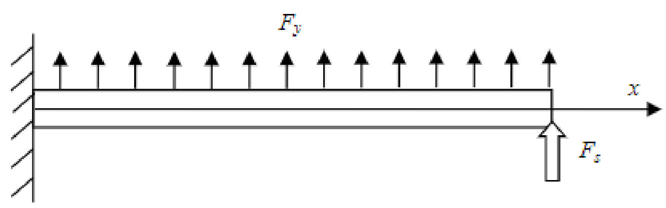
Question 1

Consider a cantilever beam of uniform cross-section, as shown in Figure 2. The beam is clamped at the left-end and is of length $l = 1$ m and with a square section area of $A = 0.003$ m^2. It is subjected to a uniform body force F_y in the vertical y-direction and a concentrated force F_s at the right end in the y-direction. The young's modulus of the material is $E = 2 * 10^{10}$ N/m^2. Using analytical (exact) method, obtain the distribution and the maximum value of the deflection (the displacement in the y-direction for the beam), moment, shear force and normal stresses, for the following cases.



Part 1

$F_y = 0,\ F_s = 1000$ N
$I_z = \frac{b^4}{12} = \frac{\sqrt{A}^4}{12} = 7.5 * 10^{-7}$ m^4
$EI_z\frac{\partial^4 V}{\partial x^4} = F_y = 0$
general solution: $V(x) = C_0 + C_1x + C_2x^2 + C_3x^3$

$\theta(x) = \frac{\partial V}{\partial x} = C_1 + C_2x^2 + C_3x^3$
$M(x) = EI_z\frac{\partial^2 V}{\partial x^2} = EI_z(2C_2 + 6C_3x)$
$Q(x) = -EI_z\frac{\partial^3 V}{\partial x^3} = -EI_z(6C_3)$

boundary conditions: $V(0) = \theta(0) = M(l) = 0,\ Q(l) = -F_s = -1000$
apply boundary conditions:
$C_0 = V(0) = 0,\ C_1 = \theta(0) = 0$
$M(l) = 0 = EI_z(2C_2 + 6C_3l) \rightarrow C_2 = -3C_3l = -3C_3$
$Q(l) = -1000 = -EI_z(6C_3) \rightarrow C_3 = -\frac{1000}{6EI_z} = -0.011$ 1/m^2, $C_2 = 0.033$ 1/m

$\boxed{V(x) = 0.033x^2 - 0.011x^3 \text{ m}}$
$\boxed{V_{max} = V(1) = 0.022 \text{ m}}$

$\boxed{M(x) = (2 * 10^{10} * 7.5 * 10^{-7})(0.066 - 0.066x) \text{ Nm}}$
$\boxed{M_{max} = M(0) = 1000 \text{ Nm}}$

$\boxed{Q(x) = 1000 \text{ N}}$
$\boxed{Q_{max} = 1000 \text{ N}}$

$\sigma_{xx} = -M_zy/I_z = \frac{-1000(0.0274)}{7.5*10^{-7}}$
$\boxed{\sigma_{xx} = 36.51 \text{ MPa}}$

Part 2

$F_y = 1000$ N/m, $F_s = 1000$ N
$EI_z\frac{\partial^4 V}{\partial x^4} = F_y = 0 \rightarrow \frac{\partial^4 V}{\partial x^4} = -0.067$ 1/m^3
$V(x) = 0.0028x^4 + C_3x^3 + C_2x^2 + C_1x + C_0$

$\theta(x) = \frac{\partial V}{\partial x} = 0.011x^3 + 3C_3x^2 + 2C_2x + C_1$
$M(x) = EI_z\frac{\partial^2 V}{\partial x^2} = EI_z(0.033x^2 + 6C_3x + 2C_2)$
$Q(x) = -EI_z\frac{\partial^3 V}{\partial x^3} = -EI_z(0.067x + 6C_3)$

boundary conditions: $V(0) = \theta(0) = M(l) = 0,\ Q(l) = -2000$
apply boundary conditions:

$C_0 = V(0) = 0$, $C_1 = \theta(0) = 0$
$M(l) = 0 = EI_z(0.033 + 6C_3 + 2C_2) \rightarrow C_2 = -3C_3 - 0.017$
$Q(l) = -2000 = -EI_z(0.067 + 6C_3) \rightarrow C_3 = 0.011 \rightarrow C_2 = -0.05$

$\boxed{V(x) = 0.0028x^4 + 0.011x^3 - 0.05x^2 \text{ m}}$

$\boxed{V_{max} = V(1) = -0.036 \text{ m}}$

$\boxed{M(x) = 15000(0.033x^2 + 0.066x - 0.1) \text{ Nm}}$

$\boxed{M_{max} = M(0) = -1500 \text{ Nm}}$

$\boxed{Q(x) = -15000(0.067x + 0.066) \text{ N}}$

$\boxed{Q_{max} = Q(1) = 2000 \text{ N}}$

$\sigma_{xx} = -M_z y/I_z = \frac{-1500(0.0274)}{7.5*10^{-7}}$

$\boxed{\sigma_{xx} = 54.8 \text{ MPa}}$

Part 3

If Fy was a function of F there would be another order in all of the equations since Fy would be related to x. The order increased between parts 1 and 2 by adding the distributed load along x, so varying it with respect to x would add another order. This would change the shapes of the load distributions as well.