AEEM 3062

HOMEWORK # 4

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Problem 1

After solving the equation of elasticity for the isotropic elastic solid shown in Fig. 1 it is found that the displacement field inside the solid is represented by functions

$u(x, y, z) = \alpha y z^3$

$v(x, y, z) = \beta x y^2$

$w(x, y, z) = \gamma(x^2 + z^2)$

where u, v, and w are in meters units and constants α, β and γ are

$\alpha = 0.07\ \mathrm{m}^{-3}$

$\beta = 0.01\ \mathrm{m}^{-2}$

$\gamma = 0.0005\ \mathrm{m}^{-1}$

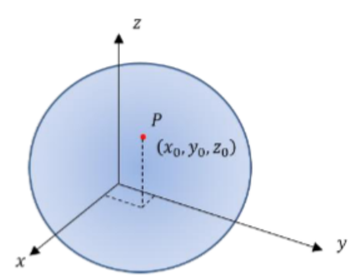


Figure 1

Part A

Plot Mohr's strain circles at point P of coordinates x_0 =50 mm, y_0 = 100 mm and z_0 = 200 mm. [30 pts]

First we need to find the strain tensor given by:

$$\hat{\sigma} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) & \frac{1}{2}(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}) \\ & \frac{\partial v}{\partial y} & \frac{1}{2}(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}) \\ sym & & \frac{\partial w}{\partial z} \end{bmatrix}$$

By calculating the partials of the given equations for u, v, and w and substituting we get the strain tensor for this problem:

$$\hat{\sigma} = \begin{bmatrix} 0 & \frac{1}{2}(\alpha z^3 + \beta y^2) & \frac{1}{2}(3\alpha y z^2 + 2\gamma x) \\ & 2\beta x y & 0 \\ sym & & 2\gamma z \end{bmatrix}$$

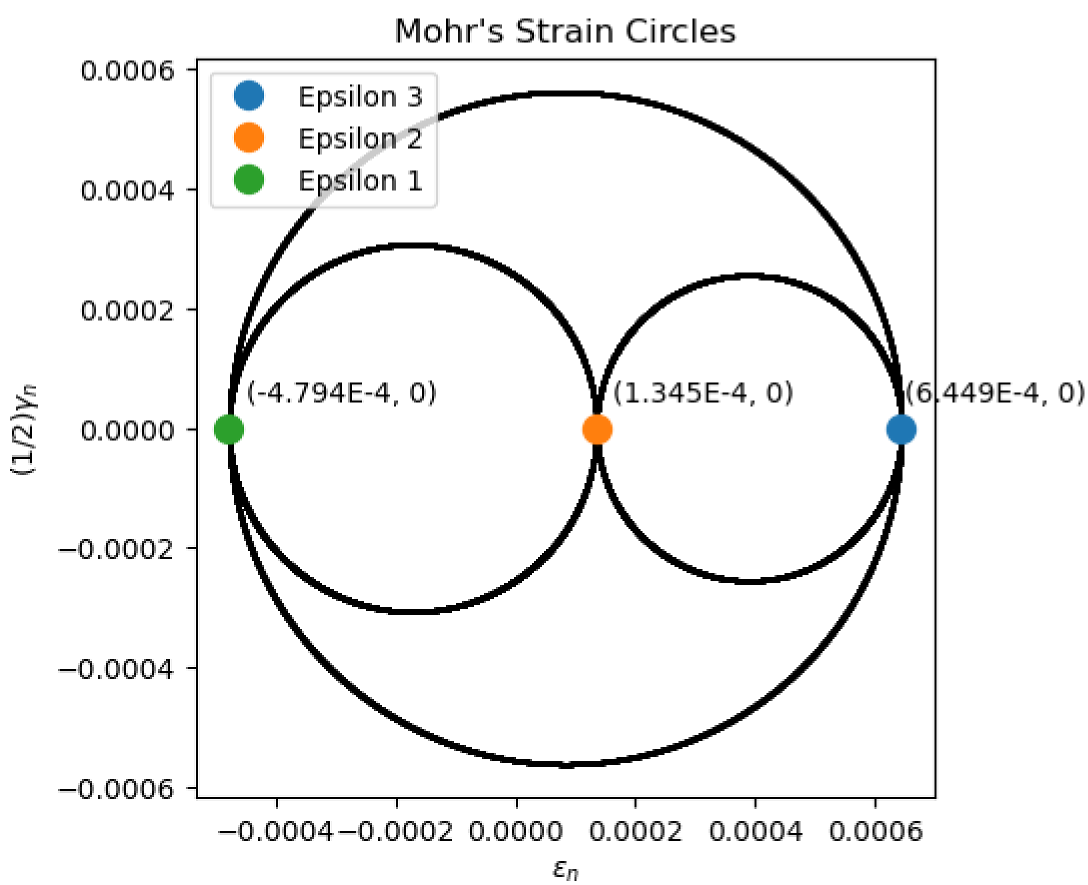
Then by substituting in the constants $\alpha = 0.07$, $\beta = 0.01$, $\gamma = 0.0005$ and conditions $x_0 = 0.05$ m, $y_0 = 0.1$ m, $z_0 = 0.2$ m gives the fully filled out strain tensor:

$$\hat{\sigma} = \begin{bmatrix} 0 & 3.3E-4 & 4.45E-4 \\ 3.3E-4 & 1E-4 & 0 \\ 4.45E-4 & 0 & 2E-4 \end{bmatrix}$$

Using Matlab's eig() function we can find the eigenvalues of the strain tensor which are the principle strains and correspond to the points of the Mohr's circles.

$$\begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix} = \begin{bmatrix} -4.794E-4 & 0 & 0 \\ 0 & 1.345E-4 & 0 \\ 0 & 0 & 6.449E-4 \end{bmatrix}$$

Then we plot these points on a graph. Then, the Mohr's circles are plotted. This is done by plotting one circle with a diameter from ϵ_1 to ϵ_3, one from ϵ_1 to ϵ_2, and one from ϵ_2 to ϵ_3.



Part B

Provide the coordinates of the unit vector parallel to the fiber that experiences the largest contraction. [10 pts]

The fiber that experiences the largest contraction is the largest negative point: ϵ_3. This is because ϵ is the elongation and negative elongation would be contraction.

Therefore the coordinates of its unit vector are given by the eigenvector corresponding to ϵ_3. This can be found using Matlab's eig() function to get the eigenvectors and choose the one corresponding to the column containing the eigenvector ϵ_3:

$$\boxed{\begin{bmatrix} 0.7552 \\ -0.4301 \\ -0.4946 \end{bmatrix}}$$

Part C

Assuming that the solid has Young's modulus E = 200 GPa and Poisson's ratio ν = 0.3 evaluate the components of the stress tensor at point P, express the stresses in MPa units. [15 pts]

The stress tensor is related to the stress tensor by the stiffness matrix:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} A & B & B & & & \\ B & A & B & & 0 & \\ B & B & A & & & \\ & & & G & 0 & 0 \\ & 0 & & 0 & G & 0 \\ & & & 0 & 0 & G \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$$

The stiffness matrix has components:

$$A = \frac{E(1-v)}{(1-2v)(1+v)}, \quad B = \frac{Ev}{(1-2v)(1+v)}, \quad G = \frac{E}{2(1+v)}$$

We can plug in the values of E and v into A, B, and G, and then plug those into the stiffness matrix.

$A = 269.23$ GPa $= 269,230$ MPa, $\quad B = 115.385$ GPa $= 115,385$ MPa, $\quad G = 76.923$ GPa $= 76,923$ MPa

We can also plug in the values for ϵ and γ, with the γ terms all being multiplied by 2 since in the strain tensor they are given as $\frac{1}{2}\gamma$.

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} 269230 & 115385 & 115385 & 0 & 0 & 0 \\ 115385 & 269230 & 115385 & 0 & 0 & 0 \\ 115385 & 115385 & 269230 & 0 & 0 & 0 \\ 0 & 0 & 0 & 76923 & 0 & 0 \\ 0 & 0 & 0 & 0 & 76923 & 0 \\ 0 & 0 & 0 & 0 & 0 & 76923 \end{bmatrix} \begin{bmatrix} 0 \\ 1E-4 \\ 2E-4 \\ 2(3.3E-4) \\ 2(4.45E-4) \\ 0 \end{bmatrix}$$

Using Matlab's matrix multiplication, we can solve for the stress tensor:

$$\boxed{\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} 34.616 \\ 50.0 \\ 65.3845 \\ 50.7692 \\ 68.4615 \\ 0 \end{bmatrix} MPa}$$

Part D

Find the dilatation and hydrostatic stress at point P and verify that they are consistent with the bulk modulus for the same material. [15 pts]

First we can calculate dilatation from the ϵ values.

$$\delta = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = 0 + 1E-4 + 2E-4 = 3E-4$$

Then we can calculate hydrostatic stress from the σ values.

$$\sigma_h = \frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3} = \frac{34.616 + 50.0 + 65.3845}{3} = 50.0MPa$$

Then we can calculate the bulk modulus using the dilatation and hydrostatic stress.

$$K = \frac{\sigma_h}{\delta} = \frac{50.0}{3E-4} = 166666.6667MPa$$

Then we can calculate K from the E and v values given for the equation.

$$K = \frac{E}{3(1-2v)} = \frac{200000}{3(1-2(0.3))} = 166666.6667MPa$$

We can see that the bulk modulus from our calculated values and the bulk modulus from the given material properties are consistent.

Problem 2

A 60° strain rosette is mounted on the surface of a metal component. A view of the rosette and the surface is shown in Fig. 2, the z-axis is assumed to be orthogonal to the surface. The following readings are obtained for each gage: $\varepsilon_a = -780$ μm/m, $\varepsilon_b = 400$ μm/m, and $\varepsilon_c = 500$ μm/m. Determine the principal strains. [30 pts]

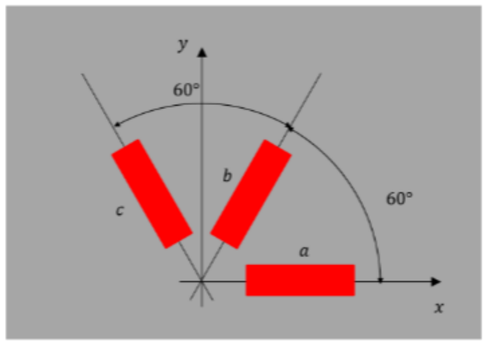


Figure 2

To solve for the principle strains we will use the equation:

$$\epsilon_n = \epsilon_{xx}\cos^2\theta + \epsilon_{yy}\sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

We can substitute each ϵ (a, b, and c)'s values and angle into this equation to create a system of equations we can solve for the principle strains.

$$\begin{aligned}
\epsilon_a &= \epsilon_{xx}\cos^2 0 + \epsilon_{yy}\sin^2 0 + \gamma_{xy}\sin 0\cos 0 = -780 \\
\epsilon_b &= \epsilon_{xx}\cos^2 60 + \epsilon_{yy}\sin^2 60 + \gamma_{xy}\sin 60\cos 60 = 400 \\
\epsilon_c &= \epsilon_{xx}\cos^2 120 + \epsilon_{yy}\sin^2 120 + \gamma_{xy}\sin 120\cos 120 = 500
\end{aligned}$$

Solving the angles gives us much simpler equations:

$$\begin{aligned}
\epsilon_a &= \boxed{\epsilon_{xx} = -780} \\
\epsilon_b &= 0.25\epsilon_{xx} + 0.75\epsilon_{yy} + 0.433013\gamma_{xy} = 400 \\
\epsilon_c &= 0.25\epsilon_{xx} + 0.75\epsilon_{yy} - 0.433013\gamma_{xy} = 500
\end{aligned}$$

Then we know that $\epsilon_{xx} = -780$ so we can plug that in to the other equations. The γ_{xy} values conveniently cancel so we can combine the equations to get one in only terms of ϵ_{yy}:

$$-390 + 1.5\epsilon_{yy} = 900, \quad \boxed{\epsilon_{yy} = 860}$$

Lastly, we can plug in and solve for γ_{xy}.

$$0.25(-780) + 0.75(860) + 0.433013\gamma_{xy} = 400, \quad \boxed{\gamma_{xy} = -115.47}$$