AEEM5063 HW#4

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09.30.24

2.20

$$r_p = 10000\text{km}; \quad r_a = 100000\text{km}$$

a)

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{100000 - 10000}{100000 + 10000} = \boxed{0.8182}$$

b)

$$a = \frac{r_p + r_a}{2} = \frac{100000 + 10000}{2} = \boxed{55000 \text{ km}}$$

c)

$$T = \frac{2\pi}{\sqrt{\mu}} a^{3/2} = \frac{2\pi}{\sqrt{398600}} (55000)^{3/2} = 128367.5509\text{s} = \boxed{35.658 \text{ hr}}$$

d)

$$\varepsilon = -\frac{\mu}{2a} = -\frac{398600}{2(55000)} = \boxed{-3.62364 \text{ km}^2/\text{s}^2}$$

e)

$$6378 + 10000 = \frac{55000(1 - 0.8182^2)}{1 + 0.8182\cos\theta} \rightarrow \boxed{\theta = 82.27^\circ}$$

f)

$$h = \sqrt{\mu(1+e)r_p} = 85131$$
$$v_\perp = \frac{h}{r} = \frac{85131}{6378 + 10000} = \boxed{5.198 \text{ km/s}}$$
$$v_r = \frac{\mu}{h} e \sin\theta = \frac{398600}{85131}(0.8182)\sin 82.27 = \boxed{3.796 \text{ km/s}}$$

g)

$$v_p = \frac{h}{r_p} = \frac{85131}{10000} = \boxed{8.513 \text{ km/s}}$$
$$v_a = \frac{h}{r_a} = \frac{85131}{10000} = \boxed{0.8513 \text{ km/s}}$$

2.35

$$\mu = 1.3271 * 10^{11}\text{km}^3/\text{s}^2$$
$$r_E = 149.6 * 10^6\text{km}$$
$$v_E = \sqrt{\frac{\mu_{sun}}{r_E}} = \sqrt{\frac{1.3271 * 10^{11}}{149.6 * 10^6}} = 29.784\text{km/s}$$
$$v_{esc} = \sqrt{2} \cdot 29.784 = 42.121\text{km/s}$$
$$v_{rel} = 42.121 - 29.784 = \boxed{12.337 \text{ km/s}}$$

2.40

$$
\begin{aligned}
v_\perp = v\cos\gamma = 10\cos 30 = 8.66\text{km/s} \\
h = rv_\perp = 10000(8.66) = 86600\text{km}^2/\text{s} \\
r = \frac{h^2}{\mu}\frac{1}{1+e\cos\theta} \rightarrow e\cos\theta = 0.8816 \\
v_r = v\sin\gamma = 10\sin 30 = 5\text{km/s} \\
v_r = \frac{\mu}{h}e\sin\theta \rightarrow e\sin\theta = 1.086 \\
\tan\theta = \frac{e\sin\theta}{e\cos theta} = \frac{1.086}{0.8816} \rightarrow \boxed{\theta = 50.93^\circ}
\end{aligned}
$$

2.43

a)

$$
\begin{aligned}
\boldsymbol{r_o} = 7000\hat{i} \rightarrow r_o = ||\boldsymbol{r_o}|| = 7000\text{km} \\
\boldsymbol{v_o} = 7\hat{i} + 7\hat{j} \rightarrow v_o = ||\boldsymbol{v_o}|| = 9.8995\text{km/s} \\
v_{r_o} = \frac{1}{r_o}\boldsymbol{v_o}\cdot\boldsymbol{r_o} = 7\text{km/s} \\
h = r_o\sqrt{v_o^2 - v_{r_o}^2} = 49000\text{km/s}
\end{aligned}
$$

$$
\begin{aligned}
r = \frac{h^2}{\mu}\frac{1}{1+(\frac{h^2}{\mu r_o}-1)\cos\Delta\theta - \frac{hv_{r_o}}{\mu}\sin\theta} = 43183.453\text{km} \\
f = 1 - \frac{\mu r}{h^2}(1-\cos\Delta\theta) = -6.17 \\
g = \frac{rr_o}{h}\sin\Delta\theta = 6170\text{s} \\
r = f\boldsymbol{r_o} + g\boldsymbol{v_o} = \boxed{43190\hat{j}\text{ km}}
\end{aligned}
$$

b)

$$e\cos\theta_o = \frac{h^2}{r_o\mu} = -0.1395$$

$$e\sin\theta_o = v_{r_o}\frac{h}{\mu} = 0.8605$$

$$\tan\theta = \frac{e\sin\theta}{e\cos\theta} \rightarrow \theta_o = -80.79^\circ$$

must be in 2nd quadrant: $\boxed{\theta_o = 99.21^\circ}$