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Performance/Design Equation Sheet

Block 1

Atmosphere

Mach number = M = $\frac{TAS}{a}$

True air speed = TAS = $\frac{EAS}{\sqrt{\frac{\rho}{\rho_{SL}}}}$

Aircraft Nomeclature

average chord = c = $\frac{c_t + c_r}{2}$

taper ratio = λ = $\frac{c_t}{c_r}$

area = S = bc

aspect ratio = AR = $\frac{b^2}{S}$ = $\frac{b}{c}$

1/4 chord sweep = $\Lambda_{\frac{c}{4}}$ = $\arctan\left[\tan\Lambda_{LE} - 0.25c_r\frac{1-\lambda}{\frac{b}{2}}\right]$

mean aerodynamic chord = MAC = $\frac{2}{3}c_r\frac{1+\lambda+\lambda^2}{1+\lambda}$

location of MAC = y_{MAC} = $\frac{b}{6}\frac{1+2\lambda}{1+\lambda}$

Aerodynamics

coefficient of lift = C_L = $\frac{nW}{qS}$

coefficient of drag = C_D = $\frac{D}{qS}$ = $C_{D_o} + KC_L^2$

dynamic pressure = q = $\frac{1}{2}\rho V^2$ = $(\frac{q}{M^2})M^2$

K = $\frac{1}{\pi ARe}$

wing efficiency factor = e

lift over drag = $\frac{L}{D}$ = $\frac{C_L}{C_D}$

Propulsion

propulsive power = Fv

propulsive efficiency = η_P = $\frac{P_{avail}}{P_{generated}}$

thrust available = T_A

thrust required = T_R

specific fuel consumption = SFC = c = $\frac{fuel flow(lbm/hr)}{thrust(lbf)}$

fuel flow rate = FFR

c @ altitude = $c_{SL}\frac{a}{a_{SL}}$

Piston

T_A = $SHP_{SL}(\frac{\eta_P}{V})(\frac{\rho}{\rho_{SL}})$

FFR = $SHP * c$

Turboprop

T_A = $ESHP_{SL}(\frac{\eta_P}{V})(\frac{\rho}{\rho_{SL}})$

FFR = $ESHP * c$

High Bypass Turbofan

T_A = $T_{SL}(\frac{0.1}{M})(\frac{\rho}{\rho_{SL}})$

FFR = $Tc_{SL}(\frac{a}{a_{SL}})$

Low/Mid Bypass Turbofan and Turbojet

T_A = $T_{SL}(\frac{\rho}{\rho_{SL}})$

FFR = $Tc_{SL}(\frac{a}{a_{SL}})$

Afterburner

T_A = $T_{SL}(\frac{\rho}{\rho_{SL}})(1 + 0.7M)$

FFR = $Tc_{SL}(\frac{a}{a_{SL}})$

Weight

fuel fraction = $\frac{W_{fuel}}{W_{TO}}$

payload fraction = $\frac{W_{payload}}{W_{TO}}$

empty weight fraction = EWF = $\frac{W_{empty}}{W_{TO}}$

takeoff weight = W_{TO} = $W_{crew} + W_{payload} + W_{fuel} + W_{empty}$ = $\frac{W_{crew}+W_{payload}}{1-fuel fraction-EWF}$

Block 2a

Flight Envelope

stall speed = V_{stall} = $\sqrt{\frac{2Wn}{\rho SC_{L_{max}}}}$

max speed = V_{max} = $\sqrt{\frac{2}{\rho}q_{max}}$ = $M_{max}a$

max dynamic pressure = q_{max} = Maxq = $(\frac{q}{M^2})_{SL}(\frac{maxKEAS}{a_{SL}})^2$

load factor = n = $\frac{L}{W}$

Range and Endurance

minimum thrust required = $T_{req_{min}}$ = $W\sqrt{4C_{D_o}K}$ = minimum drag = D_{min}

speed for L/D max = $V_{\frac{L}{D}max}$ = $(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_o}}}\frac{W}{S})^{\frac{1}{2}}$

max L/D = $\frac{L}{D}_{max}$ = $(\frac{C_L}{C_D})_{max}$ = $\sqrt{\frac{1}{4C_{D_o}K}}$ = max endurance (R=1)

$V_{\frac{C_L^{\frac{1}{2}}}{C_D}_{max}}$ = $V_{D/V_{min}}$ = $1.3161V_{L/D_{max}}$

max $\frac{C_L^{\frac{1}{2}}}{C_D}$ = $\frac{3}{4}(\frac{1}{3KC_{D_o}^3})^{\frac{1}{4}}$ = max range (R=3)

max $\frac{C_L^{\frac{3}{2}}}{C_D}$ = $\frac{1}{4}(\frac{3}{KC_{D_o}^{\frac{1}{3}}})^{\frac{3}{4}}$ = (R=1/3)

W_o = initial weight, W_1 = final weight

Breguet range = R = $\frac{V}{c_t}\frac{L}{D}\ln\frac{W_o}{W_1}$

range factor = $\frac{V}{c_t}\frac{L}{D}$

endurance = E = $\frac{1}{c_t}\frac{L}{D}\ln\frac{W_o}{W_1}$

endurance factor = $\frac{1}{c_t}\frac{L}{D}$

Energy

total energy = E = $mgh + \frac{1}{2}mV^2$

specific energy = E_s = $h + \frac{1}{2g}V^2$

specific excess power = P_s = $\dot{E}_s$ = $\frac{(T-D)V}{W}$

Min/Max V (possible)

T_A = D = $qS(C_{D_o} + KC_L^2)$

thrust to weight ratio = TWR = $\frac{T}{W}$

wing loading = $\frac{W}{S}$

max V = V_{max} = $[\frac{\frac{T}{W}\frac{W}{S}+\frac{W}{S}\sqrt{(\frac{T}{W})^2-4C_{D_o}K}}{\rho C_{D_o}}]^{\frac{1}{2}}$

min V = V_{min} = $[\frac{\frac{T}{W}\frac{W}{S}-\frac{W}{S}\sqrt{(\frac{T}{W})^2-4C_{D_o}K}}{\rho C_{D_o}}]^{\frac{1}{2}}$

Climb/Descent

Climb

θ = climb angle, $\sin\theta = \frac{T-D}{W}$, $\cos\theta = \frac{L}{W}$

rate of climb = R/C = P_s = $\frac{(T-D)V}{W}$ = $V[\frac{T}{W} - \frac{qC_{D_o}}{\frac{W}{S}} - \frac{W}{S}\frac{K}{q}]$

max climb angle = θ_{max}, $\sin_{\theta_{max}} = \frac{T}{W} - \frac{1}{\frac{L}{D}_{max}} = \frac{T}{W} - \sqrt{4C_{D_o}K}$

speed for max climb angle = $V_{\theta_{max}}$ = $\sqrt{\frac{2}{\rho}(\frac{K}{C_{D_o}})^{\frac{1}{2}}\frac{W}{S}\cos\theta_{max}}$

max rate of climb = $(R/C)_{max}$ = $[\frac{W}{S}\frac{Z}{3\rho C_{D_o}}]^{\frac{1}{2}}(\frac{T}{W})^{\frac{3}{2}}[1 - \frac{Z}{6} - \frac{3}{2Z(\frac{T}{W}^2(\frac{L}{D})^2_{max}}]$

Z = $1 + \sqrt{1 + \frac{3}{(\frac{T}{W})^2(\frac{L}{D})^2_{max}}}$

speed for max rate of climb = $V_{(R/C_{max})}$ = $[\frac{T}{W}\frac{W}{S}\frac{Z}{3\rho C_{D_o}}]^{\frac{1}{2}}$

time to climb = t_{min} = $\frac{\Delta h}{(R/C)_{ave}}$

average rate of climb = $(R/C)_{ave}$ = average between R/C @ start and finish heights

Descent

θ = descent angle, $\sin\theta = \frac{D-T}{W}$, $\cos\theta = \frac{L}{W}$

rate of descent = R/D = $V\sin\theta$ = $\frac{DV}{W}$

lift = L = $W\cos\theta$

$\tan\theta = \frac{1}{(\frac{L}{D})}$

min descent angle = θ_{min}, $\tan\theta_{min} = \frac{1}{(\frac{L}{D})_{max}}$, $\tan\theta_{min} = \frac{height}{Range}$ for gliding flight

speed for min descent angle = $V_{\theta_{min}}$ = $\sqrt{\frac{2}{\rho}(\frac{K}{C_{D_o}})^{\frac{1}{2}}\frac{W}{S}\cos\theta}$

minimum R/D = $V_{V_{min}}$ = $\sqrt{\frac{2}{\rho(\frac{C_L^{\frac{3}{2}}}{C_D})^2}\frac{W}{S}}$

speed for minimum R/D = $V_{V_{V min}}$ = $(\frac{2}{\rho}\sqrt{\frac{K}{3C_{D_o}}}\frac{W}{S})^{\frac{1}{2}}$ = $0.7598V_{(\frac{L}{D})_{max}}$

Ceilings

Ceiling Type	R/C Capability	P_s Capability
absolute	0 fpm	0 fps
service	100 fpm	1.67 fps
cruise	300 fpm	5 fps
combat	500 fpm	8.33 fps

Block 2b

Time/Fuel/Dist to Climb/Descend/Accel/Decel

Climb

minimum time to climb = t_{min} = $\frac{\Delta h}{(\frac{R}{C})_{ave}}$

fuel to climb = W_{fuel} = $\dot{W}_{ave}\Delta t$

average fuel flow rate = FFR = $\dot{W}$

distance to climb = s = $V_{ave}\Delta t$

velocity = V = $V_{(\frac{R}{C})_{max}}$

Descent

minimum time to descend = t_{min} = $\frac{\Delta h}{(\frac{R}{D})_{ave}}$

fuel to descend = 0

distance to descend = R = $\frac{\Delta h}{\tan\theta_{min}}$

Accelerate

acceleration = $\dot{V}$ = $\frac{(T-D)g}{W}$

time to accelerate = t_{accel} = $\frac{\Delta V}{\dot{V}_{ave}}$

fuel to accelerate = W_{fuel} = $\dot{W}_{ave}\Delta t$

distance to accelerate = s = $V_{ave}\Delta t$

Decelerate

deceleration = $\dot{V}$ = $\frac{-Dg}{W}$

time to decelerate = t_{decel} = $\frac{\Delta V}{\dot{V}_{ave}}$

fuel to decelerate = 0

distance to decelerate = s = $V_{ave}\Delta t$

Maneuvering

Level Turn

bank angle = ϕ, $\cos\phi = \frac{W}{L}$, $\phi = \cos^{-1}\left(\frac{1}{n}\right)$

load factor = n = $\frac{L}{W}$

turn radius = R = $\frac{mV^2}{L\sin\phi}$ = $\frac{V^2}{g\sqrt{n^2-1}}$ = $\frac{V}{\omega}$ (ft)

turn rate = ω = $\frac{V}{R}$ = $\frac{g\sqrt{n^2-1}}{V}$ (rad/s, expr as deg/s)

Sustained Turn (sus)

max sustained load factor = $(n_{max})_{sust}$ = $\sqrt{\frac{q}{K(\frac{W}{S})}[\frac{T}{W} - \frac{qC_{D_0}}{\frac{W}{S}}]}$ = $(\frac{T}{W})(\frac{L}{D})_{max}$

velocity for max sustained load factor = $V_{(n_{max})_{sust}}$ = $(\frac{2}{\rho}\sqrt{\frac{K}{C_{D_0}}}\frac{nW}{S})^{\frac{1}{2}}$

$(n_{max})_{C_{L_{max}}}$ = $\frac{qC_{L_{max}}}{(\frac{W}{S})}$

Instantaneous Turn

corner velocity = V_{corner} = $\sqrt{\frac{2n_{max}(\frac{W}{S})}{\rho C_{L_{max}}}}$

Takeoff and Landing

Takeoff

rotation speed = V_R = $1.05V_{stall}$

liftoff speed = V_{LO} = $1.1V_{stall}$

obstacle speed = V_O = $1.2V_{stall}$

rolling resistance = R = $\mu_r(W - L)$, L typically 0

coef of rolling resistance = μ_r, [0.02 $\rightarrow$ 0.08], typically 0.025

ground roll = s_g = $\frac{V_{TO}^2}{2a}$

average acceleration during ground roll = a = $\frac{T-D-R}{\frac{W}{g}}$, typically @ $V = 0.7V_{TO}$

drag during ground roll = D = $(C_{D_0} + \Delta C_{D_0})(\frac{1}{2}\rho(0.7V_{TO})^2 S_{ref})$

Landing

approach speed = V_{app} = $1.2V_{stall}$ (1.3 for commercial)

touchdown speed = V_{TD} = $1.1V_{stall}$ (1.15 for commercial)

transition distance = s_{TR} = $3V_{TD}$

braking distance = s_{brake} = $\frac{V_{TD}^2}{2g\mu_b}$

coeff of braking = μ_b, [0.12 $\rightarrow$ 0.38]

landing ground roll = s_g = $s_{TR} + s_{brake}$