AEEM4058 - Homework 1

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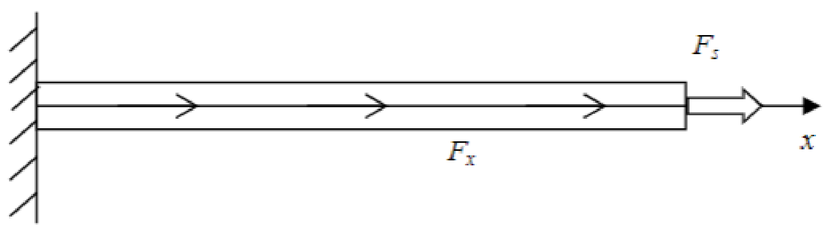
Question 1

Figure 1 shows a 1D bar of uniform cross-section fixed at the left-end. It has length $l = 1$ m and section area $A = 0.0002$ m^2. It is subjected to a uniform body force F_x and a concentrated force F_s at the right end. The young's modulus of the material is $E = 1.0 * 10^{10}$ N/m^2. Using analytical (exact) method, obtain the distribution and the maximum value of the displacement, strain and stress, for the following cases.



Part 1

$F_x = 1000$ N/m, $F_s = 1000$ N

Boundary Conditions:

@ $x = 0$: $u = 0$ m, $V = 1000$ N

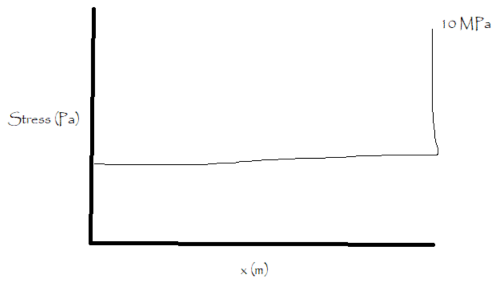
$\epsilon = \frac{\partial u}{\partial x} = \frac{F_s}{EA}$
$\sigma = \frac{F_s}{A} = \epsilon E$
$EA\frac{\partial^2 u}{\partial x^2} + F_x = 0$
$u(x) = \int \frac{-F_x}{EA}\, dx = \frac{-F_x}{2EA}x^2 + C_1 x + C_2$

apply boundary condition:
$u(0) = 0 = C_2$
$\epsilon = \frac{\partial u}{\partial x} = \frac{-F_x}{EA}x + C_1$
$\epsilon(l) = \frac{F}{EA} = \frac{-F_x}{EA}l + C_1$
$C_1 = \frac{F_s + F_x l}{EA}$

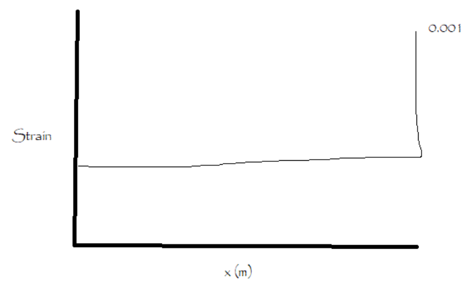
plug constants back in:
$u(x) = \frac{-F_x}{2EA}x^2 + \frac{F_s + F_x l}{EA}x$

max displacement:
$u(l) = u(1) = \frac{-1000}{2(1*10^{10})(0.0002)}(1)^2 + \frac{1000+1000(1)}{(1*10^{10})(0.0002)}(1)$
$\boxed{u_{max} = 0.00075 \text{ m}}$

max stress:
$\sigma_{max} = \frac{F_{max}}{A} = \frac{1000+1000}{0.0002}$
$\boxed{\sigma_{max} = 10 * 10^6 \text{ Pa}}$



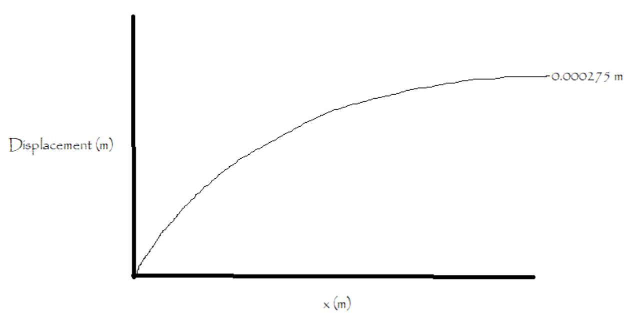
max strain:
$\epsilon_{max} = \frac{\sigma_{max}}{E} = \frac{10*10^6}{1*10^{10}}$
$\boxed{\epsilon_{max} = 0.001}$



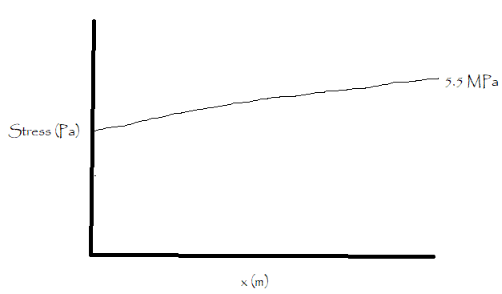
Part 2

$F_x = (100x + 1000)$ N/m, $F_s = 0$ N

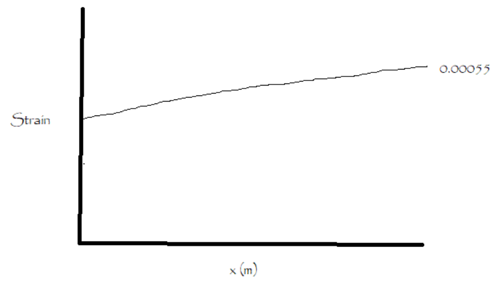
using same equation for displacement:
$u(x) = \frac{-F_x}{2EA}x^2 + \frac{0+F_x l}{EA}x$
$u(l) = u(1) = \frac{-(100(1)+1000)}{2(1*10^{10})(0.0002)}(1)^2 + \frac{(100(1)+1000)(1)}{(1*10^{10})(0.0002)}(1)$
$\boxed{u_{max} = 0.000275 \text{ m}}$



max stress:
$\sigma_{max} = \frac{F_{max}}{A} = \frac{100(1)+1000}{0.0002}$
$\boxed{\sigma_{max} = 5.5 * 10^6 \text{ Pa}}$



max strain:
$\epsilon_{max} = \frac{\sigma_{max}}{E} = \frac{5.5*10^6}{1*10^{10}}$
$\boxed{\epsilon_{max} = 0.00055}$


0.00055
Strain
x (m)