# AEEM4058 - Homework 5

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## Problem 1

Figure 1 shows a beam of 2 m length that is supported by a truss member at the middle point A and clamped on the rigid walls at the two ends. It carries a uniformly distributed vertical load over the left span. The truss and beam are all of uniform cross-section and made of the same material with the Young's modulus $E = 200.0$ GPa. The cross section of the beam is circular, and the area of the cross-section is 0.01 m$^2$. The area of the cross-section of the truss member is 0.0002 m$^2$. Using the finite element method with two elements for the entire beam, using an efficient approach to
(a) calculate the nodal displacement at the middle pint A,
(b) calculate the reaction forces at the supports for the beam and truss members,
(c) calculate the internal forces in the truss member.

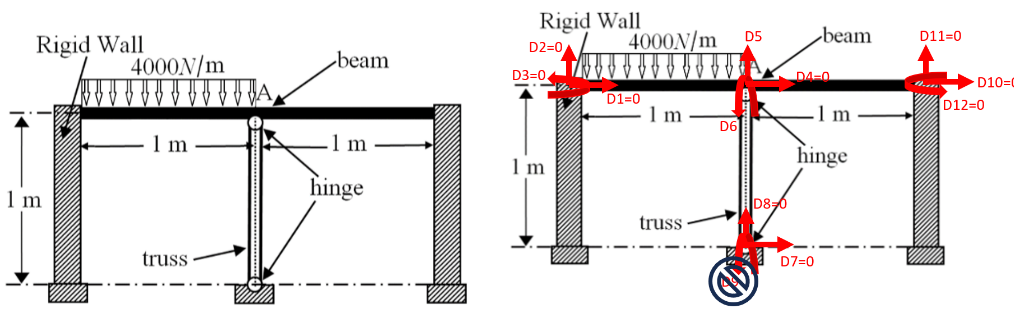
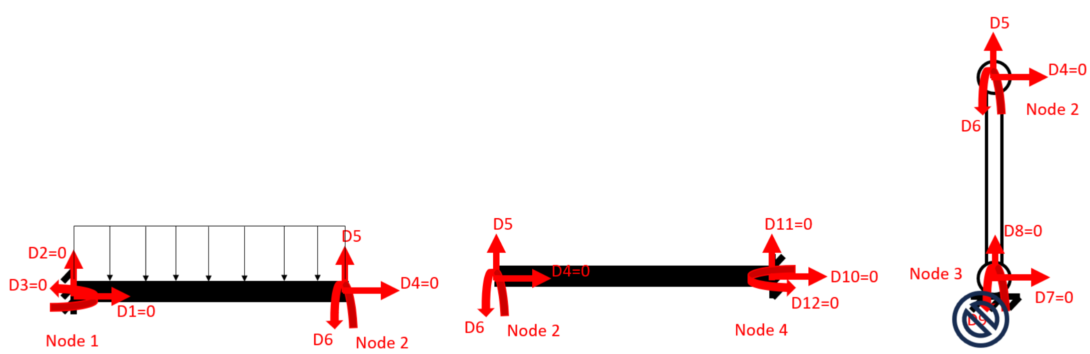



Figure 1

### Part A




| | Global node corresponding to | | Coordinates in global | | Direction cosines | |
|---|---|---|---|---|---|---|
| Element # | local node 1 | local node 2 | $X_i, Y_i$ | $X_j, Y_j$ | $l_{ij}$ | $m_{ij}$ |
| 1 | 1 | 2 | 0, 0 | 1, 0 | 1 | 0 |
| 2 | 2 | 4 | 1, 0 | 2, 0 | 1 | 0 |
| 3 | 2 | 3 | 1, 0 | 1, -1 | 0 | -1 |

### Build Element Matrices

$$K_{e_{truss}} = \frac{AE}{l_e}\begin{bmatrix} l_{ij}^2 & l_{ij}m_{ij} & -l_{ij}^2 & -l_{ij}m_{ij} \\ & m_{ij}^2 & -l_{ij}m_{ij} & -m_{ij}^2 \\ & & l_{ij}^2 & l_{ij}m_{ij} \\ sym. & & & m_{ij}^2 \end{bmatrix} \rightarrow K_3 = \frac{A_t E}{l_3}\begin{bmatrix} 0^2 & 0 & -0^2 & -0 \\ & (-1)^2 & -0 & -(-1)^2 \\ & & 0^2 & 0 \\ sym. & & & (-1)^2 \end{bmatrix}$$

$$K_3 = 4 * 10^7 \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 \\ & & 0 & 0 \\ sym. & & & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 4 & 0 & -4 \\ & & 0 & 0 \\ sym. & & & 4 \end{bmatrix} * 10^7$$

$$D_3 = \begin{bmatrix} D_4 \\ D_5 \\ D_7 \\ D_8 \end{bmatrix}$$

$$k_{e_{beam}} = \frac{EI_z}{2a^3}\begin{bmatrix} 3 & 3a & -3 & 3a \\ & 4a^2 & -3a & 2a^2 \\ & & 3 & -3a \\ sym. & & & 4a^2 \end{bmatrix}, \quad \frac{l_e}{2} = a, \quad a = 0.5, \quad I_z = \frac{\pi}{2}r^4 = \frac{\pi}{2}(\frac{A}{\pi})^2 = 1.59 * 10^{-5}$$

$$k_1 = k_2 = 4EI_z \begin{bmatrix} 3 & 1.5 & -3 & 1.5 \\ & 1 & -1.5 & 0.5 \\ & & 3 & -1.5 \\ sym. & & & 1 \end{bmatrix}$$

$$d_1 = \begin{bmatrix} D_2 \\ D_3 \\ D_5 \\ D_6 \end{bmatrix}, \quad d_2 = \begin{bmatrix} D_5 \\ D_6 \\ D_{11} \\ D_{12} \end{bmatrix}$$

$$T = \begin{bmatrix} l_x & m_x & 0 & 0 & 0 & 0 \\ l_y & m_y & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & l_x & m_x & 0 \\ 0 & 0 & 0 & l_y & m_y & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$, T is identity matrix for both element 1 and 2:

$$K_1 = K_2 = k_1 = k_2$$

**Expand Global K Matrices**

$$K_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 3.816 & 1.908 & 0 & -3.816 & 1.908 & 0 & 0 & 0 & 0 & 0 \\ & & 1.272 & 0 & -1.908 & 0.635 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 3.816 & -1.908 & 0 & 0 & 0 & 0 & 0 \\ & & & & & 1.272 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & 0 & 0 & 0 & 0 \\ & & & & & & & & 0 & 0 & 0 \\ & & & & & & & & & 0 & 0 \\ sym. & & & & & & & & & & 0 \end{bmatrix} * 10^7$$

$$K_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 3.816 & 1.908 & 0 & 0 & 0 & -3.816 & 1.908 \\ & & & & & 1.272 & 0 & 0 & 0 & -1.908 & 0.635 \\ & & & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & 0 & 0 & 0 & 0 \\ & & & & & & & & 0 & 0 & 0 \\ & & & & & & & & & 3.816 & -1.908 \\ sym. & & & & & & & & & & 1.272 \end{bmatrix} * 10^7$$

$$K_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 4 & 0 & 0 & -4 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & 4 & 0 & 0 & 0 \\ & & & & & & & & 0 & 0 & 0 \\ & & & & & & & & & 0 & 0 \\ sym. & & & & & & & & & & 0 \end{bmatrix} * 10^7$$

**Combine the Global K Matrices**

$$K = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 3.816 & 1.908 & 0 & -3.816 & 1.908 & 0 & 0 & 0 & 0 & 0 \\ & & 1.272 & 0 & -1.908 & 0.635 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 11.632 & 0 & 0 & -4 & 0 & -3.816 & 1.908 \\ & & & & & 2.544 & 0 & 0 & 0 & -1.908 & 0.635 \\ & & & & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & 4 & 0 & 0 & 0 \\ & & & & & & & & 0 & 0 & 0 \\ & & & & & & & & & 3.816 & -1.908 \\ sym. & & & & & & & & & & 1.272 \end{bmatrix} * 10^7$$

**Define Other Global Matrices**

$$D = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \\ D_7 \\ D_8 \\ D_{10} \\ D_{11} \\ D_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ D_5 \\ D_6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$f_e = \begin{bmatrix} f_x a + f_{sx1} \\ f_y a + f_{sy1} \\ f_y \frac{a^2}{3} + m_{s1} \\ f_x a + f_{sx2} \\ f_y a + f_{sy2} \\ -f_y \frac{a^2}{3} + m_{s2} \end{bmatrix}, \quad f_1 = \begin{bmatrix} 0 + 0 \\ -4000(0.5) + R_{y1} \\ -4000\frac{(0.5)^2}{3} + M_1 \\ 0 + 0 \\ -4000(0.5) + R_{y2} \\ 4000\frac{(0.5)^2}{3} + M_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -2000 + R_{y1} \\ -333.33 + M_1 \\ 0 \\ 0 \\ 333.33 + M_2 \end{bmatrix}$$

$$f_2 = \begin{bmatrix} 0 + 0 \\ -4000/4 \\ -M_2 \\ 0 + 0 \\ R_{y3} \\ M_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -500 \\ -M_2 \\ 0 \\ R_{y3} \\ M_3 \end{bmatrix}$$

$$f_3 = \begin{bmatrix} -4000/4 \\ 0 \\ R_{y4} \\ 0 \end{bmatrix} = \begin{bmatrix} -500 \\ 0 \\ R_{y4} \\ 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 \\ -2000 + R_{y1} \\ -333.33 + M_1 \\ 0 \\ -2000 \\ 333.33 \\ 0 \\ R_{y4} \\ 0 \\ R_{y3} \\ M_3 \end{bmatrix}$$

**Solve KD=F for $D_5$ and $D_6$**

using matlab:

$\boxed{D_5 = -1.7194 * 10^{-5} \text{ m}}$

$\boxed{D_6 = 1.31026 * 10^{-5} \text{ rad}}$

## Part B

from KD=F:

$-2000 + R_{y1} = (-3.816 D_5 + 1.908 D_6) * 10^7$

$\boxed{R_{y1} = 2902.3 \text{ N}}$

$R_{y3} = (-3.816 D_5 - 1.908 D_6) * 10^7$

$\boxed{R_{y3} = 402.31 \text{ N}}$

$R_{y4} = (-4 D_5) * 10^7$

$\boxed{R_{y4} = 683.76 \text{ N}}$

## Part C

sum reaction forces in truss:

$F_t = \sqrt{R_{x4}^2 + R_{y4}^2} = \sqrt{0^2 + 683.76^2}$

$\boxed{F_t = 683.76 \text{ N}}$