AEEM 3062

HOMEWORK # 3

2022-2023 SPRING

Name: Slade Brooks

M#: 13801712

DUE: 11.59 PM on 2/2/2023

Problem 1

A bar of width w = 10 mm, height h = 15 mm, and length L =500 mm is subject to a tensile force P =22.5 kN as shown in Fig. 1. The state of stress at any point inside the bar is characterized by the stress matrix:

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \frac{P}{wh} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Find the expressions of the functions that describe how the normal stress σ_n and the shear stress τ_n vary with the angle ϑ that determines the orientation of the plane π shown in Fig. 1. Plot the two functions for ranging ϑ from 0 to 360 (provide appropriate scales and labels in the plots) and give the values of angle ϑ for which τ_n is maximum. [50 pts]

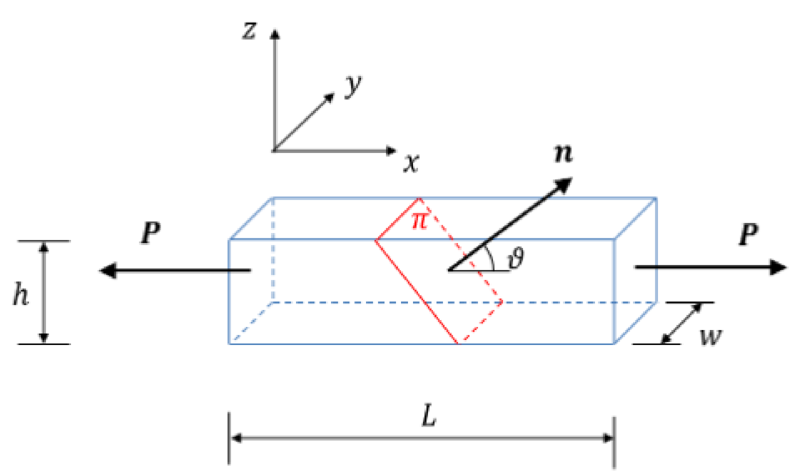


Figure 1

The stress matrices are given by: $\sigma_n = \begin{bmatrix} \frac{P}{wh} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

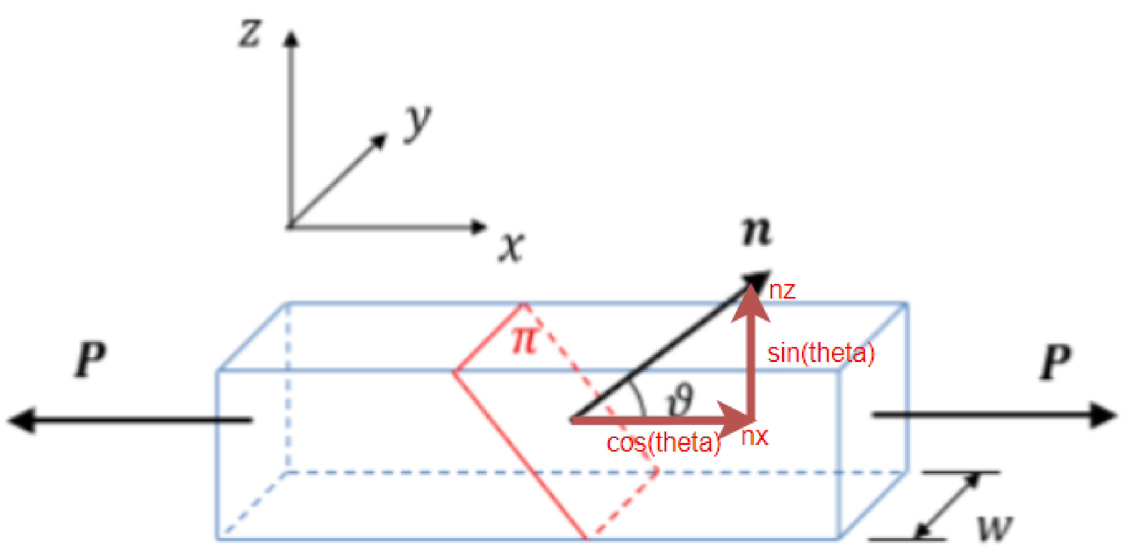
From the notes:

$$\sigma_n = t * n = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix} * \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ & \sigma_{yy} & \tau_{yz} \\ sim & & \sigma_{zz} \end{bmatrix} * \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

$$\tau_n = t - (t * n)n = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ & \sigma_{yy} & \tau_{yz} \\ sim & & \sigma_{zz} \end{bmatrix} * \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} - \sigma_n * \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

n can never act in the y direction so n_y is 0

n_x and n_z are based on θ, with $n_x = \cos\theta$ and $n_z = \sin\theta$



Substituting n and $\hat{\sigma}$ into σ_n gives: $\sigma_n = \begin{bmatrix} \cos\theta & 0 & \sin\theta \end{bmatrix} * \begin{bmatrix} \frac{P}{wh} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} \cos\theta \\ 0 \\ \sin\theta \end{bmatrix}$

Multiply the first two matrices using matlab.

$$\sigma_n = \begin{bmatrix} \frac{P}{wh}\cos\theta & 0 & 0 \end{bmatrix} * \begin{bmatrix} \cos\theta \\ 0 \\ \sin\theta \end{bmatrix}$$

Multiply the remaining two matrices using matlab.

$$\boxed{\sigma_n = \frac{P\cos^2\theta}{wh}}$$

Do the same for τ_n giving:

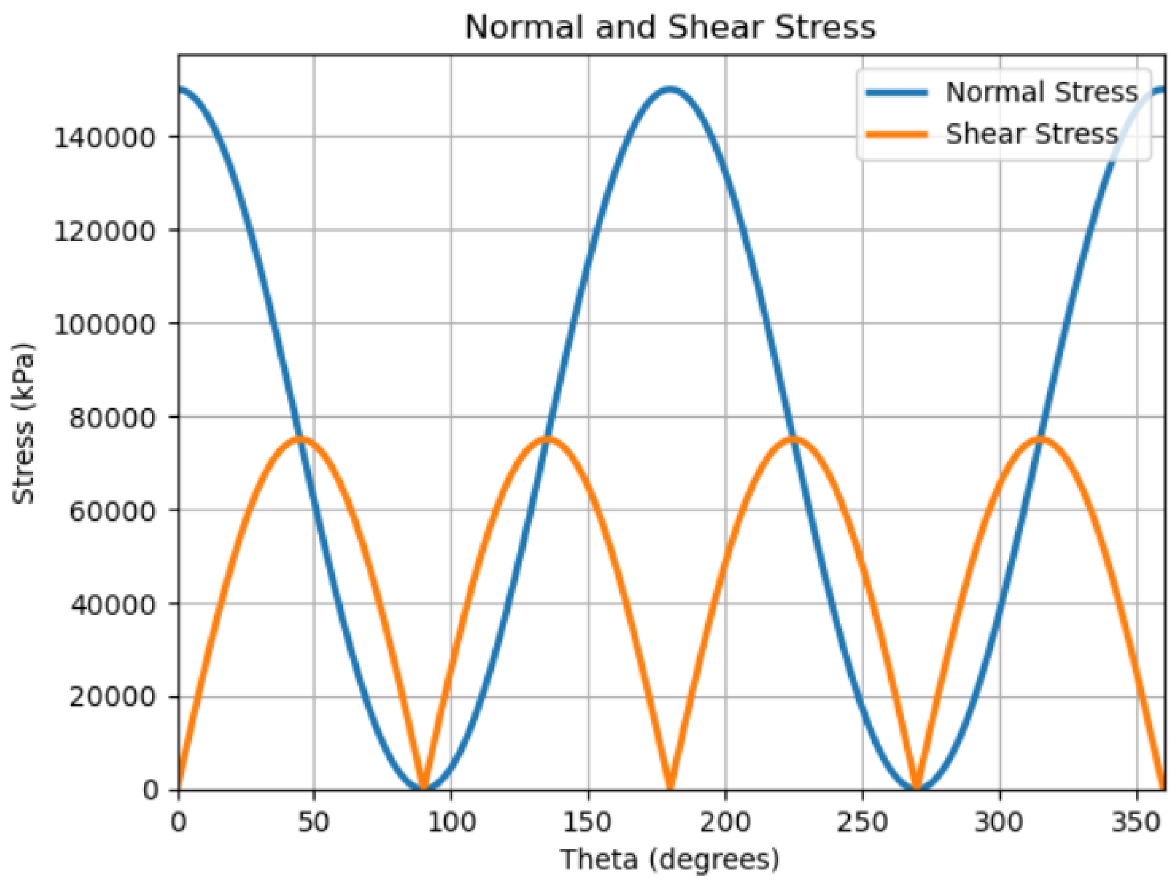
$$\tau_n = t - (t*n)n = \begin{bmatrix} \frac{P}{wh} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} \cos\theta \\ 0 \\ \sin\theta \end{bmatrix} - \frac{P\cos^2\theta}{wh} * \begin{bmatrix} \cos\theta \\ 0 \\ \sin\theta \end{bmatrix}$$

$$\tau_n = \begin{bmatrix} \frac{P\cos\theta}{wh} \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{P\cos^3\theta}{wh} \\ 0 \\ \frac{P\cos^2\theta\sin\theta}{wh} \end{bmatrix}$$

$$\boxed{\tau_n = \begin{bmatrix} \frac{P\cos\theta - P\cos^3\theta}{wh} \\ 0 \\ \frac{-P\cos^2\theta\sin\theta}{wh} \end{bmatrix}}$$

τ_n at each angle equals the length of the vector τ_n to be found using Matlab's norm() function.

The two functions σ_n and τ_n are plotted using Matlab's plot() function.



This shows that tau_n has maximums at angles:

$$\boxed{\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ}$$

Problem 2

Consider a reference frame {x,y,z} and a stress tensor represented by matrix

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} 50 & -6 & 14 \\ -6 & -120 & 8 \\ 14 & 8 & -4 \end{bmatrix} \quad \text{in MPa units}$$

Part A

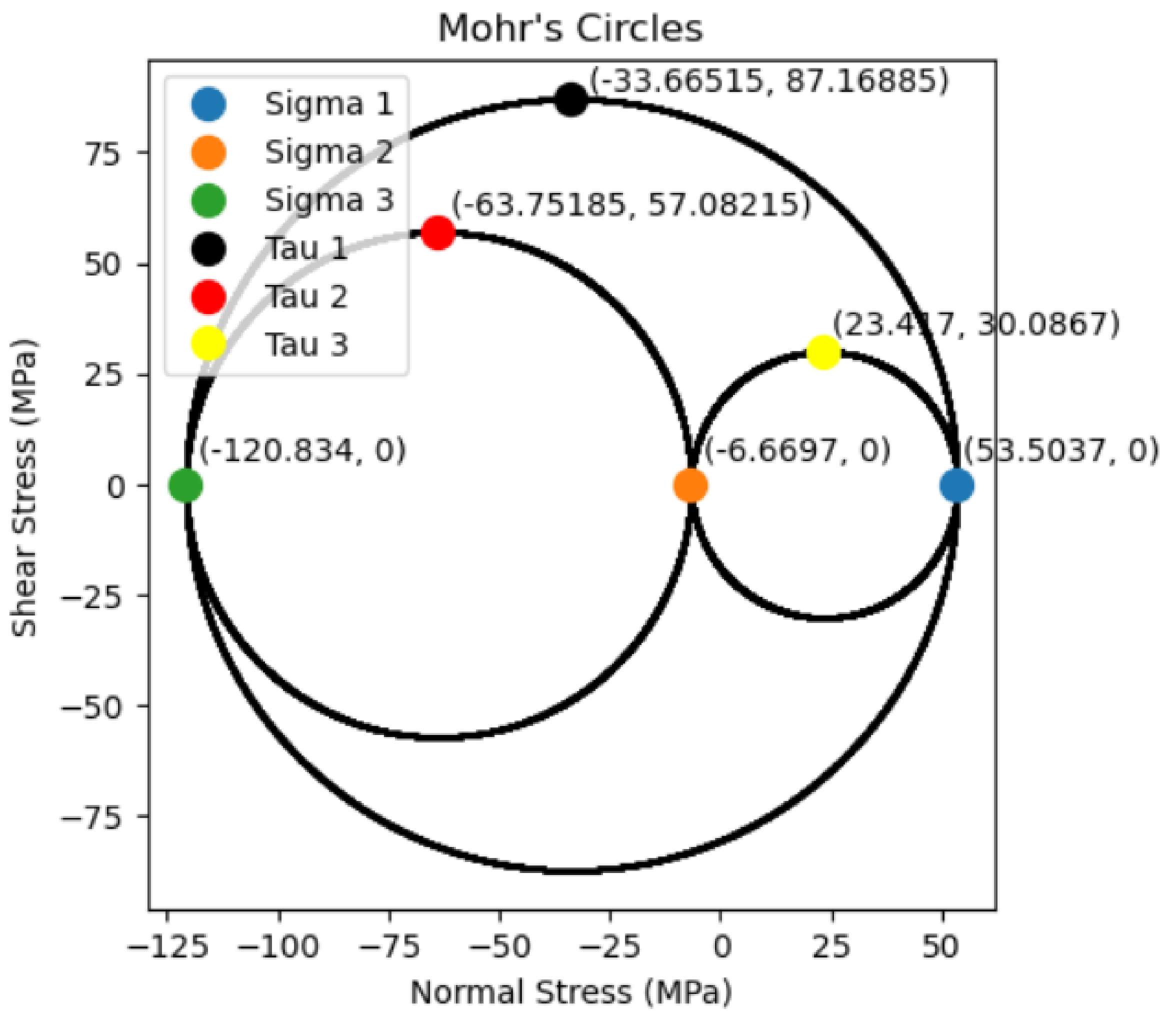
Plot the Mohr's circles. Clearly indicate the procedure that you follow and show in the plots scales, values at relevant points, and labels. [35 pts]

The locations of the values of the Mohr's circles σ_1, σ_2, and σ_3 are the eigenvalues of the stress tensor σ.

Using Matlab's eig() function we can get the eigenvalues of σ.

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} = \boxed{\begin{bmatrix} -120.834 & 0 & 0 \\ 0 & -6.6697 & 0 \\ 0 & 0 & 53.5037 \end{bmatrix}}$$

Then we plot these points on a graph. The largest point becomes σ_1, the next largest is σ_2, and so on. Then, the Mohr's circles are plotted. This is done by plotting one circle with a diameter from σ_1 to σ_3, one from σ_1 to σ_2, and one from σ_2 to σ_3.



Part B

Find the coordinates of the unit vectors orthogonal to the planes which experience the largest compressive and tensile stresses [15 pts]

The largest compressive stress is found at σ_3 since negative normal stress is compressive. σ_3 represents a plane parallel to the xy plane in our coordinate system. Therefore, the orthogonal unit vector is the same as the unit vector for the z axis, therefore:

$$\boxed{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}}$$

The largest tensile stress is found at σ_1 since positive normal stress is tensile. σ_1 represents a plane parallel to the yz plane in our coordinate system. Therefore, the orthogonal unit vector is the same as the unit vector for the z axis, therefore:

$$\boxed{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}$$