AEEM5058 HW#5

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Problem 1

A thin-walled cylinder of radius R and wall thickness t is subject to a torque M. In order to lower the shear stress in the cylinder, designers are considering adding fins of width R/2 and the same thickness as the cylinder as shown in Fig. 1.

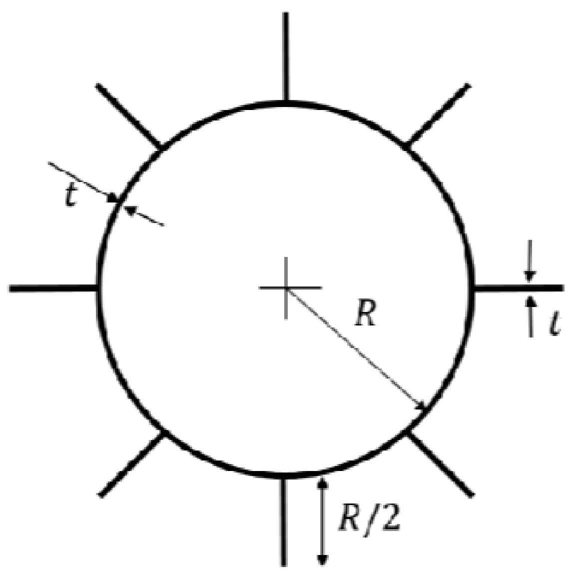


Figure 1

a)

Find the expression of the shear stress in the cylinder, τ_c, and the maximum shear stress in the fins, τ_f as a function of the number of fins, n. Note that in Fig. 1 eight fins are show for illustration purpose.

First, we will determine the τ of each section, we will treat them as separate sections. τ_c will be a closed section and τ_f will be an open section.

$$\tau_c = \frac{M_c}{2\Sigma t}$$
$$\frac{d\theta_c}{dx} = \frac{M_c}{4G\Sigma^2}\int \frac{1}{t}ds$$
$$\Sigma = \pi R^2$$
$$\frac{d\theta_c}{dx} = \frac{M_c}{4G(\pi R^2)^2}\left(\frac{2\pi R}{t}\right) = \frac{M_c}{2\pi G t R^3}$$
$$\tau_c = \frac{M_c}{2\pi t R^2}$$

$$\tau_{f_{max}} = \frac{M_f}{J}t$$
$$\frac{d\theta_f}{dx} = \frac{M_f}{GJ}$$
$$J = \frac{1}{3}\int_L t^3(s)ds = \frac{1}{3}\frac{Rt^3}{2}\cdot n = \frac{t^3 R}{6}n$$
$$\frac{d\theta_f}{dx} = \frac{6M_f}{Gt^3 nR}$$
$$\tau_{f_{max}} = \frac{6M_f}{t^2 nR}$$

We can state that the total moment M is equal to the sum of the individual moments:

$$M = M_c + M_f$$
$$M_f = M - M_c$$

We also impose that since the pieces are connected their rotations must be equivalent:

$$\frac{d\theta_c}{dx} = \frac{d\theta_f}{dx}$$
$$\frac{M_c}{2\pi G t R^3} = \frac{6M_f}{G t^3 n R}$$
$$M_c = \frac{12\pi R^2}{nt^2} M_f = \frac{12\pi R^2}{nt^2}(M - M_c)$$
$$M_c\left(1 + \frac{12\pi R^2}{nt^2}\right) = \frac{12\pi R^2}{nt^2} M$$
$$M_c = \frac{12\pi R^2}{nt^2 + 12\pi R^2} M$$
$$M_f = M - \frac{12\pi R^2}{nt^2 + 12\pi R^2} M$$
$$M_f = \frac{nt^2}{nt^2 + 12\pi R^2} M$$

Then we can plug back in and solve:

$$\boxed{\tau_c = \frac{6}{nt^3 + 12\pi t R^2} M}$$
$$\boxed{\tau_{f_{max}} = \frac{6}{Rnt^2 + 12\pi R^3} M}$$

b)

Show that the ratio τ_c/τ_f is independent of the number of fins.

$$\tau_c/\tau_f = \frac{\tau_c}{\tau_{f_{max}}} = \frac{\frac{6}{nt^3+12\pi tR^2}M}{\frac{6}{Rnt^2+12\pi R^3}M} = \frac{R}{t}$$

We can see that the ratio of the shear forces is only based on the radius and thickness of the shape, as n disappears when dividing the equations for the shear in each section.

c)

Find the radius, R', of a cylinder with the same wall thickness t but without fins that leads to the same τ_c as the cylinder with fins.

We will set the previously determined τ_c equal to a new τ' and solve for the radius that makes them equivalent.

$$\tau' = \tau_c \rightarrow \frac{M}{2\pi tR'^2} = \frac{6}{nt^3 + 12\pi tR^2}M$$

$$\boxed{R' = \sqrt{\frac{nt^2+12\pi R^2}{12\pi}}}$$

d)

Show that the cylinder in (c) is always lighter than the cylinder with fins in (a).

First, we will determine the cross sectional area of each shape based on the number of fins. The area of each cylinder will be its diameter times the thickness. The area of the fins is the number of fins times the length of each times the thickness.

$$A' = \pi D t = 2\pi R' t = t\sqrt{\frac{nt^2 + 12\pi R^2}{12\pi}}$$
$$A = \pi D t + n\frac{R}{2}t = 2\pi R t + \frac{nRt}{2}$$

We will calculate the ratio between the two for ease of comparison. If $A'/A < 1$, then the cylinder without fins will be lighter (its area is some small percentage of the one with fins).

$$\frac{A'}{A} = \frac{\sqrt{3\pi}\sqrt{nt^2 + 12\pi R^2}}{3\pi R(n + 4\pi)}$$

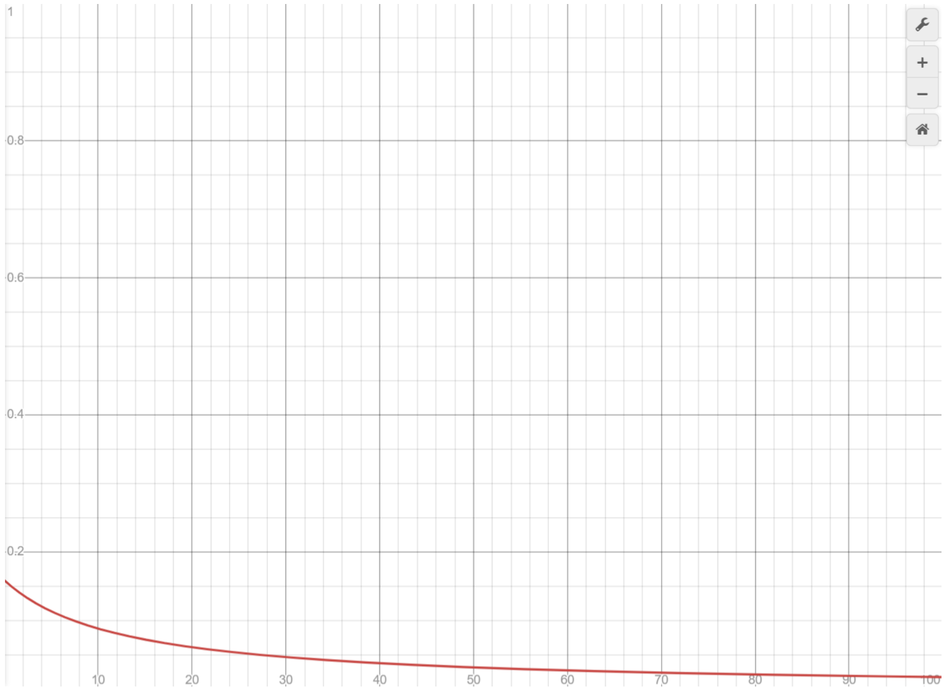
We can assume an infinitesimally small t for simplicity and simplify the ratio.

$$\lim_{t\to 0} \frac{A'}{A} = \frac{2}{n + 4\pi}$$

We can see that for any number of fins (other than 0), the ratio will be less than 1 meaning A' is smaller than A and therefore the larger cylinder weighs less than the cylinder with fins.

$$\lim_{n\to\infty} \frac{A'}{A} = 0$$

Following is a plot of $\frac{A'}{A}$. The x axis is n. It is clear that the ratio approaches 0 as n approaches infinity, and the ratio is never greater than 1, meaning the cylinder without fins must always be lighter.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80
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100