

AEEM5063 HW#7

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10.30.24

4.10

$$\alpha = 40^\circ; \quad \beta = 25^\circ$$

$$R_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}$$
$$R_2(\beta) = \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix}$$

solve for Q with matlab:

$$Q = [R_2(\beta)][R_1(\alpha)] = \begin{bmatrix} 0.9063 & 0.2717 & -0.3237 \\ 0 & 0.7660 & 0.6428 \\ 0.4226 & -0.5826 & 0.6943 \end{bmatrix}$$

4.11

$$Q = \begin{bmatrix} 0.086824 & -0.77768 & 0.62264 \\ -0.49240 & -0.57682 & -0.65178 \\ 0.86603 & -0.25000 & -0.43301 \end{bmatrix}$$

(a)

$$\alpha = \tan^{-1} \frac{Q_{31}}{-Q_{32}}$$

because num and denom +, α in first quad:

$$\boxed{\alpha = 73.9^\circ}$$

$$\beta = \cos^{-1} Q_{33}$$

$$\boxed{\beta = 115.7^\circ}$$

$$\gamma = \tan^{-1} \frac{Q_{13}}{Q_{23}}$$

because num + and denom -, γ in 2nd quad:

$$\gamma = 180 + \tan^{-1} \frac{Q_{13}}{Q_{23}}$$

$$\boxed{\gamma = 136.31^\circ}$$

(b)

$$\alpha = \tan^{-1} \frac{Q_{12}}{Q_{11}}$$

because num - and denom +, α in 4th quad:

$$\alpha = 360 + \tan^{-1} \frac{Q_{12}}{Q_{11}}$$

$$\boxed{\alpha = 276.37^\circ}$$

$$\beta = \sin^{-1} -Q_{13}$$

$$\boxed{\beta = -38.51^\circ}$$

$$\gamma = \tan^{-1} \frac{Q_{23}}{Q_{33}}$$

because num - and denom -, γ in 3rd quad:

$$\gamma = 180 + \tan^{-1} \frac{Q_{23}}{Q_{33}}$$

$$\boxed{\gamma = 236.4^\circ}$$

4.13

$$\alpha = 300^\circ; \quad \beta = -80^\circ; \quad \gamma = 30^\circ$$

$$R_1(\gamma) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & \sin \gamma \\ 0 & -\sin \gamma & \cos \gamma \end{bmatrix}$$

$$R_2(\beta) = \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix}$$

$$R_3\alpha = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q = [R_1][R_2][R_3] = \begin{bmatrix} 0.086824 & -0.15033 & 0.98481 \\ 0.5038 & 0.85945 & 0.086824 \\ -0.85945 & 0.48861 & 0.15038 \end{bmatrix}$$

$$\alpha = \tan^{-1} \frac{Q_{31}}{-Q_{32}}$$

because num and denom -, α in 3rd quad:

$$\alpha = 180 + \tan^{-1} \frac{Q_{31}}{-Q_{32}}$$

$$\boxed{\alpha = 240.38^\circ}$$

$$\beta = \cos^{-1} Q_{33}$$

$$\boxed{\beta = 81.351^\circ}$$

$$\gamma = \tan^{-1} \frac{Q_{13}}{Q_{23}}$$

because num + and denom +, γ in 2nd quad:

$$\boxed{\gamma = 84.962^\circ}$$

4.27

$$h = 180\text{km}; \quad i = 30^\circ$$

$$T = \frac{2\pi}{\sqrt{\mu}} r^{3/2} = \frac{2\pi}{\sqrt{398600}} (6378 + 180)^{3/2} = 5285.3\text{s}$$

$$\dot{\Omega} = -\frac{3}{2} \frac{\sqrt{\mu} J_2 R_e^2}{r^{7/2}} \cos i = -\frac{3}{2} \frac{\sqrt{398600} * 0.0010826 * 6378^2}{(6378 + 180)^{7/2}} \cos 30 = -1.5814E - 6 \text{rad/s}$$

$$\omega_e = \frac{2\pi + \frac{2\pi}{365.26}}{24 * 3600} = 7.2921E - 5 \text{rad/s}$$

$$\Delta\lambda = (\omega_e - \dot{\Omega})T = 0.39377\text{rad}$$

$$s = R_e \Delta\lambda = 6378 * 0.39377$$

$$\boxed{s = 2511.4 \text{ km}}$$

4.17