AEEM4058 - Homework 5

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Problem 1

Figure 1 shows a beam of 2 m length that is supported by a truss member at the middle point A and clamped on the rigid walls at the two ends. It carries a uniformly distributed vertical load over the left span. The truss and beam are all of uniform cross-section and made of the same material with the Young's modulus $E = 200.0$ GPa. The cross section of the beam is circular, and the area of the cross-section is 0.01 m^2. The area of the cross-section of the truss member is 0.0002 m^2. Using the finite element method with two elements for the entire beam, using an efficient approach to
(a) calculate the nodal displacement at the middle pint A,
(b) calculate the reaction forces at the supports for the beam and truss members,
(c) calculate the internal forces in the truss member.

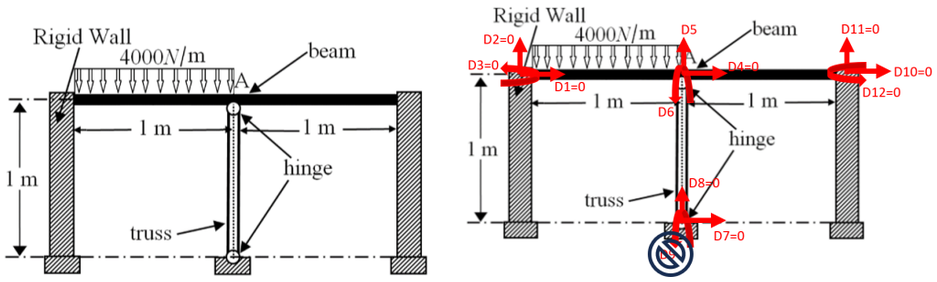
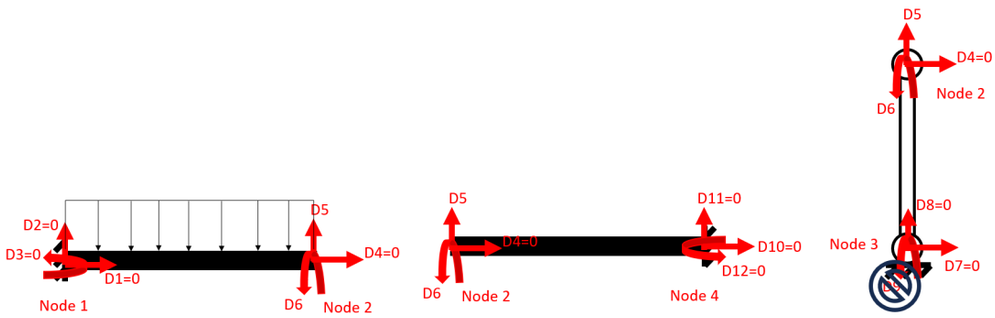


Figure 1

Part A



	Global node corresponding to		Coordinates in global		Direction cosines	
Element #	local node 1	local node 2	X_i, Y_i	X_j, Y_j	l_{ij}	m_{ij}
1	1	2	0, 0	1, 0	1	0
2	2	4	1, 0	2, 0	1	0
3	2	3	1, 0	1, -1	0	-1

Build Element Matrices

$$K_{e_{truss}} = \frac{AE}{l_e}\begin{bmatrix} l_{ij}^2 & l_{ij}m_{ij} & -l_{ij}^2 & -l_{ij}m_{ij} \\ & m_{ij}^2 & -l_{ij}m_{ij} & -m_{ij}^2 \\ & & l_{ij}^2 & l_{ij}m_{ij} \\ sym. & & & m_{ij}^2 \end{bmatrix} \rightarrow K_3 = \frac{A_t E}{l_3}\begin{bmatrix} 0^2 & 0 & -0^2 & -0 \\ & (-1)^2 & -0 & -(-1)^2 \\ & & 0^2 & 0 \\ sym. & & & (-1)^2 \end{bmatrix}$$

$$K_3 = 4 * 10^7 \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 \\ & & 0 & 0 \\ sym. & & & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 4 & 0 & -4 \\ & & 0 & 0 \\ sym. & & & 4 \end{bmatrix} * 10^7$$

$$k_{e_{beam}} = \frac{EI_z}{2a^3}\begin{bmatrix} 3 & 3a & -3 & 3a \\ & 4a^2 & -3a & 2a^2 \\ & & 3 & -3a \\ sym. & & & 4a^2 \end{bmatrix}, \quad \frac{l_e}{2} = a$$

Part B

Part C