AEEM 3062

HOMEWORK # 6

2022-2023 SPRING

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Problem 1

A pipe of length $2L$ is formed by joining two tubes of different materials but with the same length L and with the same inner diameter, d, and outer diameter, D, as shown in Fig. 1. Tube BC is made of a material of Young's modulus, E_1, and thermal expansion coefficient, α_1,; the corresponding constants for tube CD are E_2 and α_2. The pipe is clamped at both ends and tube BC is subject to a temperature increase, ΔT, that is uniform along a cross section but varies along the tube axis according to

$$\Delta T(x) = T_1 + T_2 \sin\frac{\pi x}{L}$$

where T_1 and T_2 are constants.

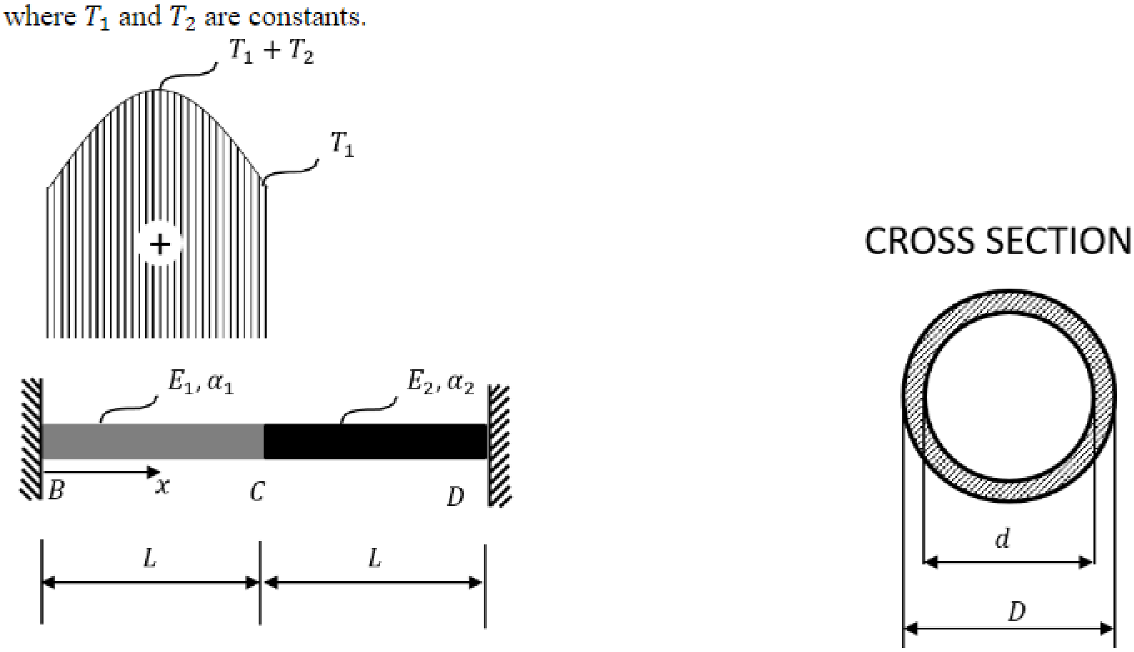


Figure 1

Part A

Write the system of equations that is needed to find the expressions of the functions describing how the normal force N and the axial displacement u vary along the length of the bar. Write the system of equations in matrix form [30 pts]

For section B-C, the body force is 0:

$$n(x) = 0$$

Then the normal force is given by:

$$\frac{\partial N}{\partial x} + n(x) = 0$$

$$\frac{\partial N}{\partial x} = 0$$

Then this is integrated to get N:

$$N(x)_{B\to C} = c_1$$

For a thermal expansions case, the displacement is given by:

$$\frac{du}{dx} = \frac{N(x)}{A(x)E(x)} + \alpha \Delta T$$

$$\frac{du}{dx} = \frac{N(x)}{A(x)E(x)} + \alpha_1 (T_1 + T_2 \sin \frac{\pi x}{L})$$

Then this can be integrated to get:

$$u(x)_{B\to C} = \frac{c_1 x}{A_1 E_1} + \alpha_1 T_1 x - \frac{\alpha_1 T_2 L}{\pi} \cos \frac{\pi x}{L} + c_2$$

Then we can do the same for section C-D, except $\Delta T = 0$, so the thermal expansion terms disappear.

The body forces are also gone in this section, so:

$$N(x)_{C\to D} = c_3$$

The displacement will end up the same except for expansion:

$$\frac{du}{dx} = \frac{N(x)}{A(x)E(x)} + \alpha 0$$

$$u(x)_{C\to D} = \frac{c_3 x}{A_2 E_2} + c_4$$

Then we can set the boundary conditions. For a clamped end, the displacement is 0:

$$u(0) = u(2L) = 0$$

$$\frac{c_1 0}{A_1 E_1} + \alpha_1 T_1 0 - \frac{\alpha_1 T_2 L}{\pi} \cos \frac{\pi 0}{L} + c_2 = 0$$

$$c_2 = \frac{\alpha_1 T_2 L}{\pi}$$

$$\frac{c_3 2L}{A_2 E_2} + c_4 = 0$$

Since the pipes are connected, their displacement must be equal at the location where they meet:

$$u(L)_{B\to C} = u(L)_{C\to D}$$

$$\frac{c_1 L}{A_1 E_1} + \alpha_1 T_1 L - \frac{\alpha_1 T_2 L}{\pi} \cos \frac{\pi L}{L} + c_2 = \frac{c_3 L}{A_2 E_2} + c_4$$

$$\frac{c_1 L}{A_1 E_1} + \alpha_1 (T_1 L + \frac{T_2 L}{\pi}) + c_2 = \frac{c_3 L}{A_2 E_2} + c_4$$

The normal forces also must be the same at that location:

$$N(L)_{B\to C} = N(L)_{C\to D} = c_1 = c_3$$

Then rearrange to have all of the constants on one side:

$$\frac{c_1 L}{A_1 E_1} + c_2 - \frac{c_3 L}{A_2 E_2} - c_4 = -\alpha_1 (T_1 L + \frac{T_2 L}{\pi})$$

$$c_1 - c_3 = 0$$

Then the equations can be combined to create an Ax=b matrix system:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2L}{A_2E_2} & 1 \\ \frac{L}{A_1E_1} & 1 & -\frac{L}{A_2E_2} & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} \frac{\alpha_1 T_2 L}{\pi} \\ 0 \\ -\alpha_1(T_1 L + \frac{T_2 L}{\pi}) \\ 0 \end{bmatrix}$$

Part B

Solve the system of equations assuming that: $L = 600$ mm, $D = 25$ mm, $d = 22$ mm, E_1=68.9 GPa, $\alpha_1 = 25.5 \times 10^{-6}/°\mathrm{C}$, E_2=207 GPa, $\alpha_1 = 9.0 \times 10^{-6}/°\mathrm{C}$, $T_1 = 200°\mathrm{C}$, and $T_2 = 50°\mathrm{C}$. Provide the expressions of the functions describing how the displacement u and normal force N vary along the length of the bar with the appropriate units [10 pts]

$$\mathrm{L} = 0.6\mathrm{m},\ \mathrm{D} = 0.025\mathrm{m},\ \mathrm{d} = 0.022\mathrm{m},\ E_1 = 6.89E10\mathrm{Pa}$$

$$\alpha_1 = 25.5E - 6/^\circ C,\ E_2 = 2.07E11\mathrm{Pa},\ T_1 = 200^\circ\mathrm{C},\ T_2 = 50^\circ\mathrm{C}$$

$$A_1 = A_2 = \pi(0.025/2)^2 - \pi(0.022/2)^2 = 1.107E - 4m^2$$

We can plug the given values into the matrix and solve for the constants using Matlab:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 5.237E-8 & 1 \\ 7.8666E-8 & 1 & -2.6184E-8 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 2.435E-4 \\ 0 \\ -0.0033 \\ 0 \end{bmatrix}$$

$$c_1 = -33795.25\mathrm{N} \quad c_2 = 0.0002435\mathrm{m} \quad c_3 = -33795.25\mathrm{N} \quad c_4 = 0.00176\mathrm{m}$$

Then we can subsitute in the constants and given values to get the equations for normal force and displacement along the bar:

$$\boxed{N(x)_{B\to C} = -33795.25 \text{ Newtons}}$$

$$\boxed{N(x)_{C\to D} = -33795.25 \text{ Newtons}}$$

$$u(x)_{B\to C} = \frac{c_1 x}{A_1 E_1} + \alpha_1 T_1 x - \frac{\alpha_1 T_2 L}{\pi} \cos\frac{\pi x}{L} + c_2$$

$$u(x)_{B\to C} = -0.00443x + 0.0051x - (2.435E-4)\cos 5.236x + 0.0002435$$

$$\boxed{u(x)_{B\to C} = (6.7E-4)x - (2.435E-4)\cos(5.236x) + 0.0002435 \text{ meters}}$$

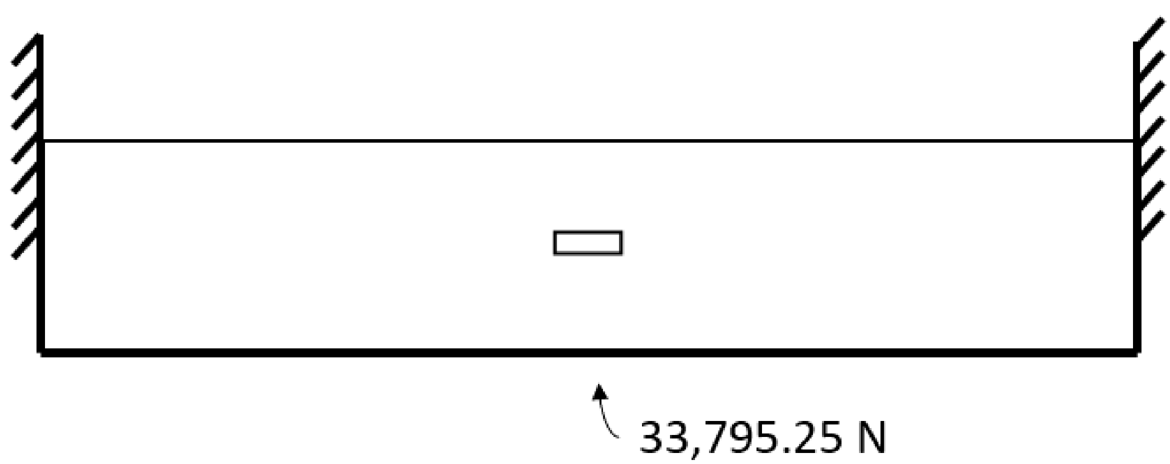
$$u(x)_{C\to D} = \frac{c_3 x}{A_2 E_2} + c_4$$

$$\boxed{u(x)_{C\to D} = -0.001475x + 0.00176 \text{ meters}}$$

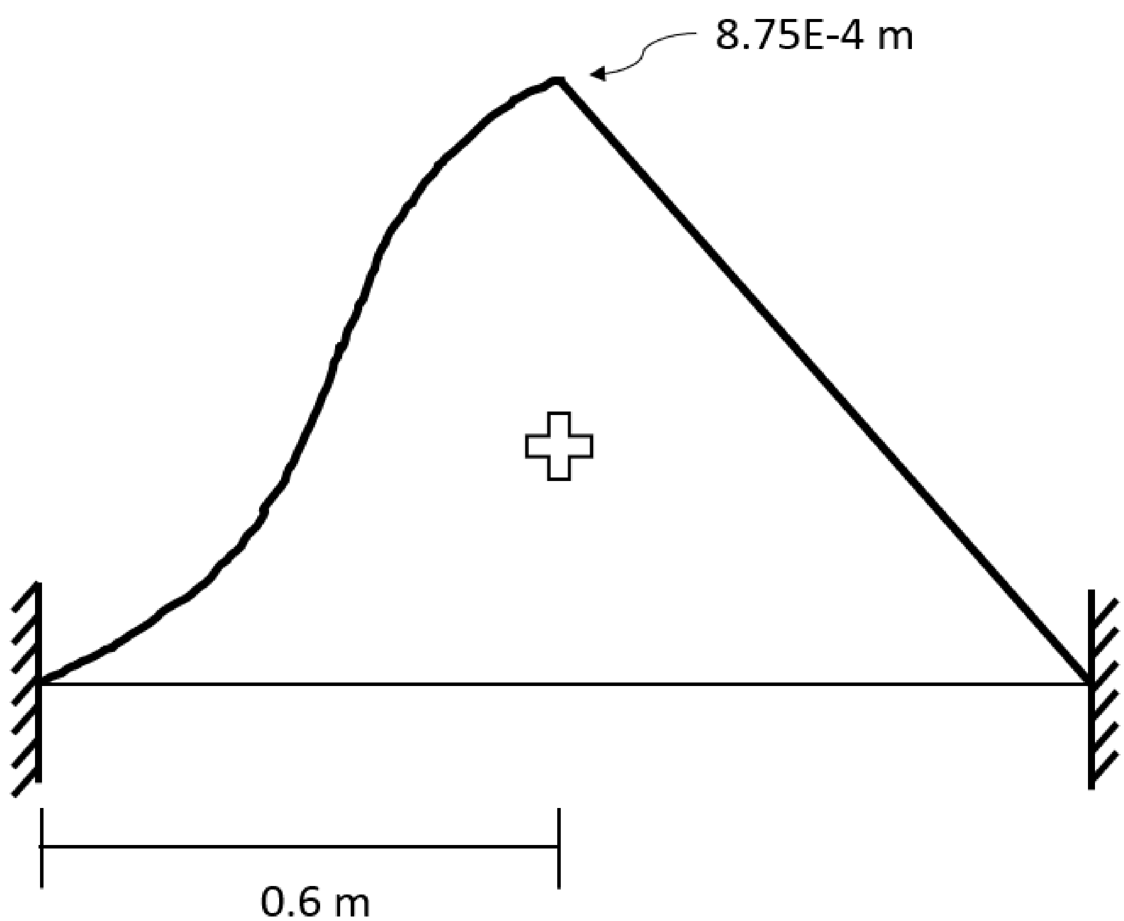
Part C

Plot the axial force, displacement, and reaction diagrams (no explanation needed) [15 pts]

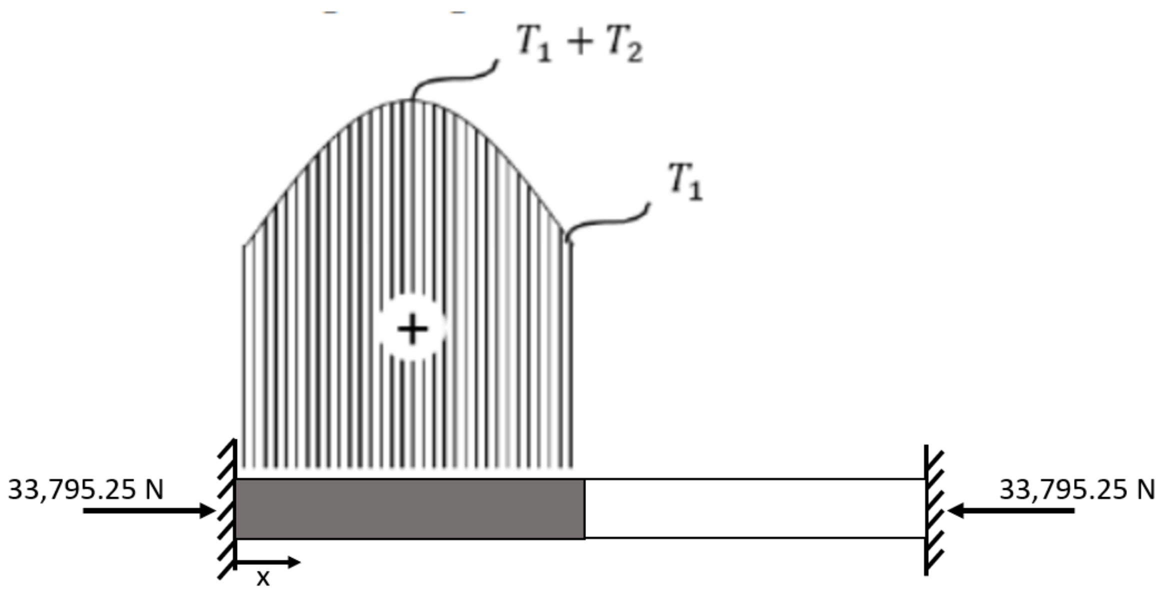
Axial Force



Displacement



Reactions



Problem 2

A rigid bar AD of length, $3L$, is pinned at end A and is loaded by force P at the other end, D. Moreover, it is supported by two rods, EB, and, FC, which are pinned to the bar at distance L/3 from ends A and D, respectively. The rods are also pinned at the other ends E and F and form the angle α relative to the horizontal axis as shown in Fig. 2. The rods have cross-sectional area, Σ, and Young's modulus, E.

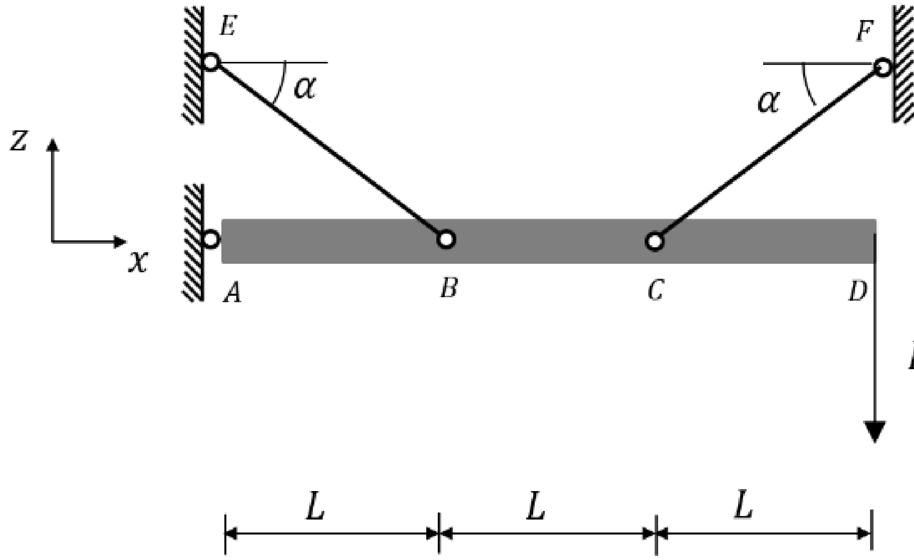
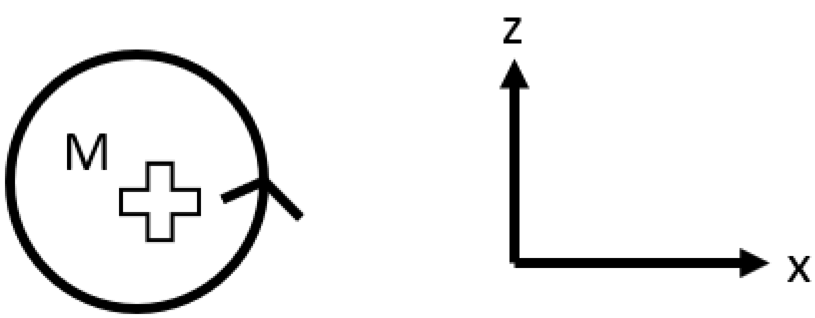
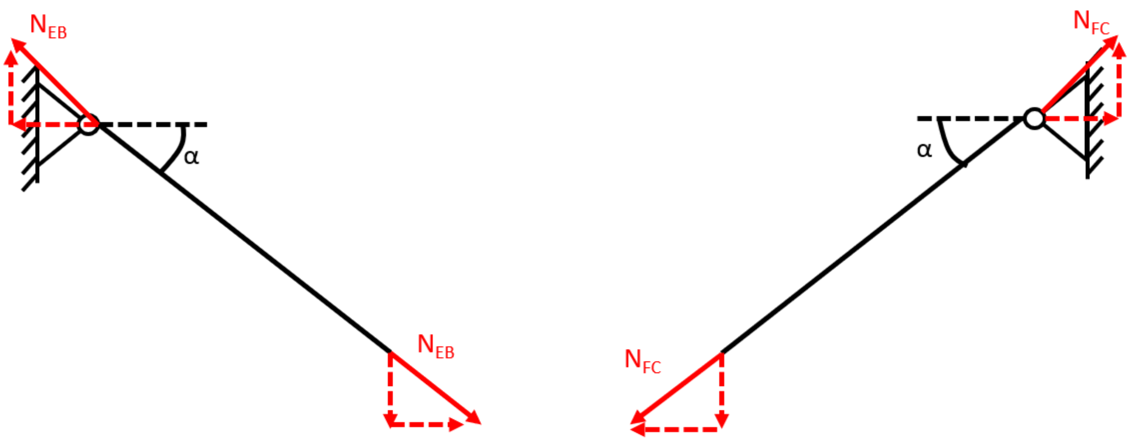


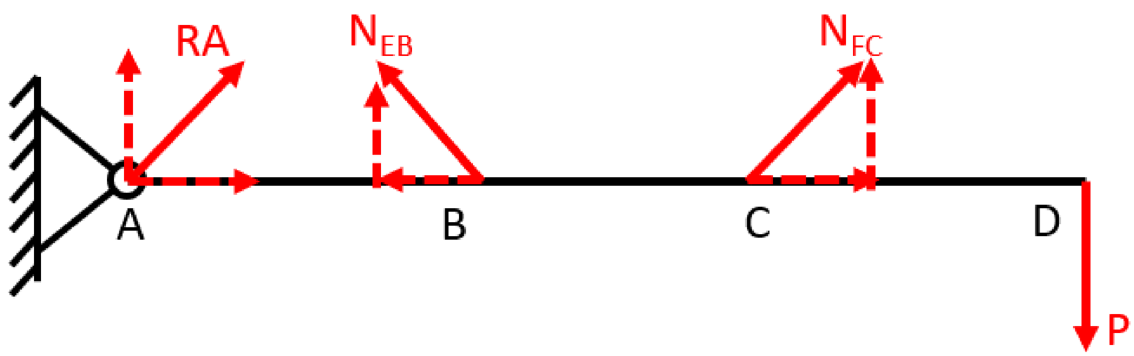
Figure 2

Part A

Sketch the free body diagram for each rod and for the bar (no need for explanation)[10 pts]







Part B

Express the normal force in each rod as a function of the displacement of pins B and C. [10 pts]

As given in the notes, the displacement of the rods can be given by:

$$\delta = \frac{Nc}{EA}$$

Both rods have length (c) given by:

$$\cos\alpha = \frac{L}{c}, \quad c = \frac{L}{\cos\alpha}$$

As well as area Σ and young's modulus E given in the problem. Therefore:

$$\delta_B = \frac{N_{EB}L}{E\Sigma\cos\alpha}, \quad \delta_C = \frac{N_{FC}L}{E\Sigma\cos\alpha}$$

Then this can be rearranged to get the normal force in terms of displacement:

$$\boxed{N_{EB} = \frac{\delta_B E\Sigma\cos\alpha}{L}, \quad N_{FC} = \frac{\delta_C E\Sigma\cos\alpha}{L}}$$

Part C

Find the normal forces in the rods and the reactions in A [20 pts]

Considering the angle of the bar to be θ, the displacements are given by:

$$\delta_B = L\theta, \quad \delta_C = 2L\theta$$

Then we can find the ratio of displacement of B to displacement of C:

$$\frac{\delta_B}{\delta_C} = \frac{L\theta}{2L\theta} = \frac{1}{2}$$

This ratio is the same as the ratio of normal forces:

$$\frac{\delta_B}{\delta_C} = \frac{1}{2} = \frac{N_{EB}}{N_{FC}}, \quad N_{FC} = 2N_{EB}$$

We also know that the sum of the moments about pin A must be equal to 0:

$$M_A = 0 = -3LP + 2LN_{FCz} + LN_{EBz}$$

Then this can be solved by subsituting in $N_{FC} = 2N_{EB}$:

$$3LP = 2L(2N_{EBz}) + LN_{EBz}$$
$$5LN_{EBz} = 3LP$$
$$N_{EBz} = \frac{3}{5}P$$

Which means N_{FCz} is:

$$N_{FCz} = 2N_{EBz} = 2(\frac{3}{5})P$$
$$N_{FCz} = \frac{6}{5}P$$

Then we can use trig to get N_{EB} and N_{FC}:

$$N_{EBz} = N_{EB}\sin\alpha, \quad \boxed{N_{EB} = \frac{3P}{5\sin\alpha}}$$
$$N_{FCz} = N_{FC}\sin\alpha, \quad \boxed{N_{FC} = \frac{6P}{5\sin\alpha}}$$

Then to find the reactions at A we can sum the forces in the x and z directions:

$$\Sigma F_x = 0 = R_{Ax} - N_{EBx} + N_{FCx} = R_{Ax} - \frac{3P}{5\sin\alpha}\cos\alpha + \frac{6P}{5\sin\alpha}\cos\alpha$$

$$\boxed{R_{Ax} = -\frac{3}{5}P\cot\alpha}$$

$$\Sigma F_z = 0 = R_{Az} + N_{EBz} + N_{FCz} - P = R_{Az} + \frac{3P}{5} + \frac{6P}{5} - P$$

$$\boxed{R_{Az} = -\frac{4}{5}P}$$

Part D

Determine the displacement of point D where the force is applied [5 pts]

The displacement of the bar at D depends on its rotation (which we previously defined as θ). This gives its displacement as:

$$\delta_D = 3L\theta$$

Then we can solve for θ from the displacement for point B:

$$\delta_B = L\theta, \quad \theta = \frac{\delta_B}{L}$$

Then by substituting in δ_B and N_{EB} we can solve for θ:

$$\theta = \frac{\frac{N_{EB}L}{E\Sigma\cos\alpha}}{L} = \frac{N_{EB}}{E\Sigma\cos\alpha} = \frac{\frac{3P}{5\sin\alpha}}{E\Sigma\cos\alpha} = \frac{3P}{5E\Sigma\sin\alpha\cos\alpha}$$

Then substituting θ back in gives the value for δ_D:

$$\boxed{\delta_D = \frac{9PL}{5E\Sigma\sin\alpha\cos\alpha}}$$