# AEEM5063 HW#10

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## 6.3

$$v_{A_1} = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{398600}{6378 + 500}} = 7.613\text{km/s}$$

$$6378 + 500 = \frac{h_2^2}{\mu} \cdot \frac{1}{1 - e_2} \rightarrow \frac{h_2^2}{\mu} = 6878(1 - e_2)$$

$$6378 = \frac{h_2^2}{\mu} \cdot \frac{1}{1 + e_2 \cos 60°} \rightarrow \frac{h_2^2}{\mu} = 6378(1 + 0.5e_2)$$

set equal and solve for eccentricity:

$$6378(1 + 0.5e_2) = 6878(1 - e_2) \rightarrow e_2 = 0.04967$$

$$h_2 = \sqrt{6878\mu(1 - e_2)} = \sqrt{6878 \cdot 398600 \cdot (1 - 0.04967)} = 51040\,\text{km}^2/\text{s}$$

### (a)

$$v_{A_2} = \frac{h_2}{r_A} = \frac{51040}{6878} = 7.421\text{km/s}$$

$$\Delta v = v_{A_2} - v_{A_1} = 7.421 - 7.613 = \boxed{-0.1915\text{km/s}}$$

**(b)**

$$\Delta v = 0 - v_{A_1} = \boxed{-7.613\text{km/s}}$$

## 6.8

### (a)

$$v_1 = \frac{\mu}{r} = \sqrt{\frac{398600}{6378 + 300}} = 7.726\text{km/s}$$

$$e_2 = \frac{r_{a2} - r_{p2}}{r_{a2} + r_{p2}} = \frac{6378 + 3000 - (6378 + 300)}{6378 + 3000 + 6378 + 300} = 0.168$$

$$r_{p2} = \frac{h_2^2}{398600} \frac{1}{1 + 0.168} \rightarrow h_2 = 55760\text{km}^2\text{/s}$$

$$v_{p2} = \frac{h_2}{r_{p2}} = \frac{55760}{6678} = 8.35\text{km/s}$$

$$v_{a2} = \frac{h_2}{r_{a2}} = \frac{55760}{9378} = 5.95\text{km/s}$$

$$v_3 = \frac{\mu}{r} = \frac{398600}{6378 + 3000} = 6.52\text{km/s}$$

$$\Delta v_1 = v_{p2} - v_1 = 8.35 - 7.726 = 0.6244\text{km/s}$$

$$\Delta v_2 = v_3 - v_{a2} = 6.52 - 5.95 = 0.57\text{km/s}$$

$$\boxed{\Delta v_{tot} = 1.2 \text{ km/s}}$$

### (b)

$$a_2 = 0.5(r_{p2} + r_{a2}) = 0.5(6678 + 9378) = 8028\text{km/s}$$

$$T_2 = \frac{2\pi}{\sqrt{\mu}} a^{\frac{3}{2}} = \frac{2\pi}{\sqrt{398600}} 8027^{\frac{3}{2}} = 7159\text{s}$$

time perigee to apogee is half the period:

$$t = T_2/2/60 = \boxed{t = 59.7 \text{ min}}$$

## 6.10

### (a)

$$a = 0.5(r_m + r_e) = 0.5(227.9E6 + 149.6E6) = 188.8E6$$

$$T = \frac{2\pi}{\sqrt{\mu}} a^{\frac{3}{2}} = \frac{2\pi}{\sqrt{132.7E9}} (188.8E6)^{\frac{3}{2}} = 44.73E6 = 517.7\text{days}$$

$$\boxed{\text{time of flight} = 258.8 \text{ days}}$$

### (b)

$$T_m = \frac{2\pi}{\sqrt{\mu}} r_m^{\frac{3}{2}} = \frac{2\pi}{\sqrt{132.7E9}} (227.9E6)^{\frac{3}{2}} = 59.34E6 = 686.8\text{days}$$

$$\frac{180^\circ - \alpha}{180^\circ} = \frac{\text{tof}}{T_m/2} = \frac{258.8}{343.4} = 0.754$$

$$\boxed{\alpha = 44.3^\circ}$$

## 6.17

### (a)

**Orbit 1:**

$$v_A^1 = \sqrt{\frac{\mu}{r_A}} = \sqrt{\frac{398600}{6678}} = 7.726\,\text{km/s}$$

**Orbit 2:**

$$0.3 = \frac{r_B - 6678}{r_B + 6678} \rightarrow r_B = 12402\text{km}$$

$$h_2 = \sqrt{2\mu\frac{r_A r_B}{r_A + r_B}} = \sqrt{2 \cdot 398600 \cdot \frac{6678 \cdot 12402}{6678 + 12402}} = 58830\text{km}^2/\text{s}$$

$$v_A^2 = \frac{h_2}{r_A} = \frac{58830}{6678} = 8.809\,\text{km/s}$$
$$v_B^2 = \frac{h_2}{r_B} = \frac{58830}{12402} = 4.743\,\text{km/s}$$

**Orbit 3:**

$$h_3 = \sqrt{2\mu\frac{r_B r_C}{r_B + r_C}} = \sqrt{2 \cdot 398600 \cdot \frac{12402 \cdot 9378}{12402 + 9378}} = 65250\,\text{km}^2/\text{s}$$

$$v_B^3 = \frac{h_3}{r_B} = \frac{65250}{12402} = 5.261\,\text{km/s}$$
$$v_C^3 = \frac{h_3}{r_C} = \frac{65250}{9378} = 6.957\,\text{km/s}$$

**Orbit 4:**

$$v_C^4 = \sqrt{\frac{\mu}{r_C}} = \sqrt{\frac{398600}{9378}} = 6.519\,\text{km/s}$$

$$\begin{aligned}\Delta v_{\text{total}} &= (v_A^2 - v_A^1) + (v_B^3 - v_B^2) + (v_C^4 - v_C^3) \\ &= 1.083 + 0.518 + 0.438 = \boxed{2.04\text{km/s}}\end{aligned}$$

## (b)

**For Orbit 2:**

$$T_2 = 2\pi\sqrt{\frac{\left(\frac{r_A+r_B}{2}\right)^3}{\mu}} = 2\pi\sqrt{\frac{\left(\frac{6678+12402}{2}\right)^3}{398600}} = 9273\,\text{s}$$

**For Orbit 3:**

$$T_3 = 2\pi\sqrt{\frac{\left(\frac{r_B+r_C}{2}\right)^3}{\mu}} = 2\pi\sqrt{\frac{\left(\frac{12402+9378}{2}\right)^3}{398600}} = 11310\,\text{s}$$

$$t_{\text{total}} = \frac{1}{2}\left(T_2 + T_3\right) = \frac{1}{2}\left(9273 + 11310\right) = 10290\,\text{s} = \boxed{2.86\text{hr}}$$