

AEEM5063 HW#2

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09.16.24

1.5

$$P = \sin 3t\hat{i} + \cos t\hat{j} + \sin 2t\hat{k}$$

(a)

$$\vec{V} = \dot{P} = 3\cos 3t\hat{i} - \sin t\hat{j} + 2\cos 2t\hat{k} = \boxed{-2.733\hat{i} - 0.141\hat{j} + 1.920\hat{k} \text{ m/s}}$$

(b)

$$v = \|\vec{V}\| = \sqrt{(-2.733)^2 + (-0.141)^2 + (1.920)^2} = \boxed{3.343 \text{ m/s}}$$

(c)

$$\hat{u}_t = \frac{\vec{V}}{v} = \frac{-2.733\hat{i} - 0.141\hat{j} + 1.920\hat{k}}{3.343} = \boxed{-0.818\hat{i} - 0.042\hat{j} + 0.574\hat{k}}$$

(d)

$$\vec{x} = 1\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{y} = 0\hat{i} + 1\hat{j} + 0\hat{k}$$

$$\vec{z} = 0\hat{i} + 0\hat{j} + 1\hat{k}$$

$$\hat{u}_t \cdot \vec{x} = 1(-0.818)\hat{i} + 0\hat{j} + 0\hat{k} = -0.818 = xu_t \cos \theta \rightarrow \theta_x = \cos^{-1}(-0.818)$$

$$\theta_y = \cos^{-1}(-0.042)$$

$$\theta_z = \cos^{-1}(0.574)$$

$$\boxed{\theta_x = 144.885^\circ}$$

$$\boxed{\theta_y = 92.407^\circ}$$

$$\boxed{\theta_z = 54.970^\circ}$$

(e)

$$\vec{a} = \dot{\vec{V}} = -9 \sin 3t \hat{i} - \cos t \hat{j} - 4 \sin 2t = \boxed{-3.709\hat{i} + 1.000\hat{j} + 1.118\hat{k} \text{ m/s}^2}$$

(f)

$$\hat{u}_b = \frac{\vec{V} \times \vec{a}}{||\vec{V} \times \vec{a}||}$$

$$\begin{aligned} \vec{V} \times \vec{a} &= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2.733 & -0.141 & 1.920 \\ -3.709 & 1.000 & 1.118 \end{bmatrix} = ((-0.141)(1.118) - (1.920)(1.000))\hat{i} \\ &+ ((-2.733)(1.118) - (1.920)(-3.709))\hat{j} + ((-2.733)(1.000) - (-0.141)(-3.709))\hat{k} \\ &= -2.078\hat{i} - 4.066\hat{j} - 3.256\hat{k} \end{aligned}$$

$$||\vec{V} \times \vec{a}|| = \sqrt{(-2.078)^2 + (-4.066)^2 + (-3.256)^2} = 5.608$$

$$\boxed{\hat{u}_b = -0.371\hat{i} - 0.725\hat{j} - 0.581\hat{k}}$$

(g)

$$\hat{u}_b = \hat{u}_t \times \hat{u}_n \rightarrow \hat{u}_n = \hat{u}_b \times \hat{u}_t$$

$$\begin{aligned}\hat{u}_n &= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.371 & -0.725 & -0.581 \\ -0.818 & -0.042 & 0.574 \end{bmatrix} = ((-0.725)(0.574) - (-0.581)(-0.042))\hat{i} \\ &+ ((-0.371)(0.574) - (-0.581)(-0.818))\hat{j} + ((-0.371)(-0.042) - (-0.725)(-0.818))\hat{k} \\ &= \boxed{-0.441\hat{i} + 0.688\hat{j} - 0.577\hat{k}}\end{aligned}$$

(h)

$$\begin{aligned}||\vec{a}|| &= \sqrt{(-3.709)^2 + (1)^2 + (1.118)^2} = 4.001 \\ \vec{a} \times \vec{x} &= 1(-3.709)\hat{i} + 0\hat{j} + 0\hat{k} = -3.709 = x||\vec{a}|| \cos \phi \rightarrow \phi_x = \cos^{-1}\left(\frac{-3.709}{4.001}\right) \\ \phi_y &= \cos^{-1}\left(\frac{1}{4.001}\right) \\ \phi_z &= \cos^{-1}\left(\frac{1.118}{4.001}\right) \\ \phi_x &= 157.975^\circ \\ \phi_y &= 75.526^\circ \\ \phi_z &= 73.774^\circ\end{aligned}$$

(i)

$$a_t = \vec{a} \cdot \hat{u}_t = (-3.709 * -0.818) + (1.000 * -0.042) + (1.118 * 0.574) = \boxed{3.634 \text{ m/s}^2}$$

(j)

$$a_n = \vec{a} \cdot \hat{u}_n = (-3.709 * -0.441) + (1.000 * 0.688) + (1.118 * -0.577) = \boxed{1.679 \text{ m/s}^2}$$

(k)

$$\rho = \frac{v^2}{a_n} = \frac{3.343^2}{1.679} = \boxed{6.656 \text{ m}}$$

(1)

$$\begin{aligned} P_C = P + \rho \hat{u}_n &= (\sin 9\hat{i} + \cos 3\hat{j} + \sin 6\hat{k}) + 6.656(-0.441\hat{i} + 0.688\hat{j} - 0.577\hat{k}) \\ &= \boxed{-2.523\hat{i} + 3.589\hat{j} - 4.120\hat{k} \text{ m}} \end{aligned}$$

1.7

$$\begin{aligned}
 W &= G \frac{m_e m}{r_e^2} \\
 F_{moon} &= G \frac{m_e m}{r_{moon}^2} \\
 Gm_e m = W r_e^2 &\rightarrow F_{moon} = \frac{W r_e^2}{r_{moon}^2} = W \left(\frac{r_e}{r_{moon}} \right)^2 \\
 r_e &= 6378 \text{ km}; \quad r_{moon} = 384400 \text{ km} \\
 \boxed{F} &= 2.753 * 10^{-4} W
 \end{aligned}$$

1.9

$$\begin{aligned}
 F &= ma \rightarrow F_n = ma_n \\
 a_n &= \frac{v^2}{r} \rightarrow v = \sqrt{r a_n} \\
 F_n &= G \frac{Mm}{r^2} = ma_n \rightarrow a_n = \frac{GM}{r^2} \\
 \boxed{v} &= \sqrt{\frac{GM}{r}}
 \end{aligned}$$

1.12

$$\vec{V}_P = \vec{V}_O + \Omega \times r_{rel} + \vec{V}_{rel}$$

$$r_{rel} = r - r_O = (51 + 16)\hat{I} + (-45 - 84)\hat{J} + (36 - 59)\hat{K} = 67\hat{I} - 129\hat{J} - 23\hat{K}$$

$$\begin{aligned} \vec{V}_{rel} = 31\hat{i} - 68\hat{j} - 77\hat{k} &= 31(-0.15617\hat{I} - 0.31235\hat{J} + 0.93704\hat{K}) - 68(-0.12940\hat{I} + 0.94698\hat{J} \\ &+ 0.29409\hat{K}) - 77(-0.97922\hat{I} - 0.075324\hat{J} - 0.18831\hat{K}) = 79.358\hat{I} - 68.277\hat{J} + 23.550\hat{K} \end{aligned}$$

$$\begin{aligned} \vec{V}_P &= 7\hat{I} + 9\hat{J} + 4\hat{K} + \begin{bmatrix} \hat{I} & \hat{J} & \hat{K} \\ -0.8 & 0.7 & 0.4 \\ 67 & -129 & -23 \end{bmatrix} + 79.358\hat{I} - 68.277\hat{J} + 23.550\hat{K} \\ &= 7\hat{I} + 9\hat{J} + 4\hat{K} + 35.5\hat{I} + 8.4\hat{J} + 56.3\hat{K} + 79.358\hat{I} - 68.277\hat{J} + 23.550\hat{K} \\ &= 121.858\hat{I} - 50.877\hat{J} + 83.850\hat{K} \end{aligned}$$

$$||\vec{V}_P|| = \sqrt{(121.858)^2 + (-50.877)^2 + (83.850)^2} = 151.483$$

$$\boxed{V_P = 156.42\text{m/s}^2; \quad \hat{u}_V = 0.779\hat{I} - 0.325\hat{J} + 0.536\hat{K}}$$

$$\vec{a}_P = \vec{a}_O + \dot{\Omega} \times r_{rel} + \Omega \times (\Omega \times r_{rel}) + 2\Omega \times \vec{V}_{rel} + a_{rel}$$

$$\dot{\Omega} \times r_{rel} = \begin{bmatrix} \hat{I} & \hat{J} & \hat{K} \\ -0.4 & 0.9 & -1.0 \\ 67 & -129 & -23 \end{bmatrix} = -149.7\hat{I} - 76.2\hat{J} - 8.7\hat{K}$$

$$\Omega \times (\Omega \times r_{rel}) = \begin{bmatrix} \hat{I} & \hat{J} & \hat{K} \\ -0.8 & 0.7 & 0.4 \\ 35.5 & 8.4 & 56.3 \end{bmatrix} = 36.05\hat{I} + 59.24\hat{J} - 31.57\hat{K}$$

$$2\Omega \times \vec{V}_{rel} = 2 \begin{bmatrix} \hat{I} & \hat{J} & \hat{K} \\ -0.8 & 0.7 & 0.4 \\ 79.358 & -68.277 & 23.550 \end{bmatrix} = 87.592\hat{I} + 101.166\hat{J} - 1.858\hat{K}$$

$$\begin{aligned} a_{rel} = 2\hat{i} - 6\hat{j} + 5\hat{k} &= 2(-0.15617\hat{I} - 0.31235\hat{J} + 0.93704\hat{K}) - 6(-0.12940\hat{I} + 0.94698\hat{J} \\ &+ 0.29409\hat{K}) + 5(-0.97922\hat{I} - 0.075324\hat{J} - 0.18831\hat{K}) = -4.432\hat{I} - 6.683\hat{J} - 0.832\hat{K} \end{aligned}$$

$$\begin{aligned}
\vec{a}_P &= 3\hat{I} - 7\hat{J} + 4\hat{K} - 149.7\hat{I} - 76.2\hat{J} - 8.7\hat{K} + 36.05\hat{I} + 59.24\hat{J} - 31.57\hat{K} + 87.592\hat{I} \\
&\quad + 101.166\hat{J} - 1.858\hat{K} - 4.432\hat{I} - 6.683\hat{J} - 0.832\hat{K} = -27.49\hat{I} + 70.523\hat{J} - 38.96\hat{K} \\
||\vec{a}_P|| &= \sqrt{(-27.49)^2 + (70.523)^2 + (-38.96)^2} = 85.130 \\
\boxed{a_P = 85.130\text{m/s}^2; \quad \hat{u}_a = -0.323\hat{I} + 0.828\hat{J} - 0.458\hat{K}}
\end{aligned}$$

1.23