AEEM4063 - Assignment 2B

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Problem 1

Simple turbojet is operating with a compressor pressure ratio of 8.0, a turbine inlet temperature of 1200 K, and a mass flow of 15 kg/s, when the aircraft is flying at 260 m/s at an altitude of 7,000 m. Assuming the following component efficiencies, and ISA conditions, calculate the propelling nozzle area required, the net thrust developed, and the SFC.

r	8
T_{03} (K)	1200
$\dot{m}$ (kg/s)	15
C_a (m/s)	260
alt (m)	7000
$\eta_{\infty c}$	0.87
$\eta_{\infty t}$	0.87
η_i	0.95
η_j	0.95
η_m	0.99
$\Delta P_b/P_{02}$ (%)	6
η_b	0.97

Ambient

find conditions at altitude:
$\theta = 0.8423$, $\delta = 0.4057$ (from table A.2)
$P_a = 0.4057 * 1.01325 = 0.411$ bar, $T_a = 0.8423 * 288.15 = 242.71$ K, $a = 340.3 * \sqrt{0.8423} = 312.317$ m/s
$M = \frac{V}{a} = \frac{260}{312.317} = 0.8325$

Inlet

find P_{01}:
$P_{01}/P_a = (1 + \eta_i \frac{\gamma_c - 1}{2} M^2)^{\frac{\gamma_c}{\gamma_c - 1}} = (1 + 0.95\frac{0.4}{2}(0.8325)^2)^{\frac{1.4}{0.4}} = 1.542$
$P_{01} = 0.6338$ bar

intake is adiabatic so: $T_{01} = T_{0a} = T_a * (1 + \frac{\gamma_c - 1}{2} M^2) = 288.15(1 + \frac{0.4}{2}(0.8325)^2)$
$T_{01} = 276.35$ K

Compressor

$P_{02} = rP_{01} = 8(0.6338) = 5.0704$ bar

$\frac{n-1}{n} = \frac{\gamma_c - 1}{\gamma_c \eta_{\infty c}} = \frac{0.4}{1.4(0.87)} = 0.3284$
$T_{02}/T_{01} = (r)^{\frac{n-1}{n}} = 8^{0.3284} = 1.98$
$T_{02} = 547.17$ K

Combustor

account for pressure loss in combustor: $P_{03} = P_{02}(1 - \frac{\Delta P_b}{P_{02}}) = 5.0704(1 - 0.06) = 4.766$ bar
$P_{03} = 4.766$ bar

Turbine

$W_c = \frac{c_{pc}}{\eta_m}(T_{02} - T_{01}) = \frac{1.005}{0.99}(547.17 - 276.35) = 275$ kJ/kg
$W_t = W_c = 275$ kJ/kg $= c_{pg}(T_{03} - T_{04}) = 1.148(1200 - T_{04}) = 275$
$T_{04} = 960.45$ K

$\frac{m-1}{m} = \frac{\eta_{\infty t}(\gamma_g - 1)}{\gamma_g} = \frac{0.87(0.333)}{1.333} = 0.2173$
$\frac{T_{04}}{T_{03}} = (\frac{P_{04}}{P_{03}})^{\frac{m-1}{m}} \rightarrow \frac{960.45}{1200} = (\frac{P_{04}}{P_{03}})^{0.2173} \rightarrow \frac{P_{04}}{P_{03}} = 0.359$
$P_{04} = 1.71$ bar

Nozzle

$NPR = \frac{P_{04}}{P_a} = \frac{1.71}{0.411} = 4.16$
$PR_{crit} = \frac{1}{[1-\frac{1}{\eta_j}(\frac{\gamma_g-1}{\gamma_g+1})]^{\frac{\gamma_g}{\gamma_g-1}}} = \frac{1}{[1-\frac{1}{0.95}(\frac{0.333}{2.333})]^{\frac{1.333}{0.333}}} = 1.92$
$NPR > PR_{crit} \rightarrow$ nozzle is choked
$P_5 = \frac{P_{04}}{PR_{crit}} = \frac{1.71}{1.92} = 0.89$ bar

$TR_{crit} = \frac{\gamma_g+1}{2} = \frac{2.333}{2} = 1.167$
$T_5 = \frac{T_{04}}{TR_{crit}} = \frac{960.45}{1.167} = 823$ K

$C_5 = 1\sqrt{\gamma_g RT_5} = \sqrt{1.333 * 287 * 823} = 561.12$ m/s
$\rho_5 = \frac{P_5}{RT_5} = \frac{0.89}{287*823} * 100000 = 0.377$ kg/m^3
$\dot{m} = \rho_5 C_5 A_5 \rightarrow A_5 = \frac{\dot{m}}{\rho_5 C_5} = \frac{15}{0.377*561.12} = 0.0709$ m^2
$\boxed{A_5 = 0.0709 \text{ m}^2}$

$F = \dot{m}(C_5 - C_a) + A_5(P_5 - P_a) = 15(561.12 - 260) + 0.0709(0.89 - 0.411) * 100000 = 7913$ N
$\boxed{F = 7913 \text{ N}}$

$f_a = \frac{c_{pg}T_{03} - c_{pc}T_{02}}{\eta_b(Q_f - c_{pg}T_{03})} = \frac{1.148(1200) - 1.005(547.17)}{0.99(43100 - 1.148(1200))} = 0.02$
$F_s = \frac{F}{\dot{m}} = \frac{7913}{15} = 527.53$ Ns/kg
$TSFC = \frac{f}{F_s} = \frac{0.02}{527.53} * 3600 = 0.1365$ kg/N-hr
$\boxed{TSFC = 0.1365 \text{ kg/N-hr}}$

Problem 2

Under take-off conditions when the ambient pressure and temperature are 1.01 bar and 288 K, the stagnation pressure and temperature in the jet pipe of a turbojet engine are 3.8 bar and 1000 K, and the mass flow is 23 kg/s. Calculate:

a. The exit area required and the thrust produced, assuming an isentropic convergent nozzle. (Use stations 7 & 9 for inlet and exit of nozzle)

b. The nozzle exit area and thrust produced, assuming an isentropic convergent- divergent nozzle that is fully expanded. (Use stations 7, 8, & 9 for inlet, throat, and exit of nozzle)

c. Comment on which nozzle you would choose if you were the design engineer responsible for selecting the type of nozzle to utilize on an aircraft.

$P_a = 1.01$ bar, $T_a = 288$ K, $P_{07} = 3.8$ bar, $T_{07} = 1000$ K, $\dot{m} = 23$ kg/s

Part A

$NPR = \frac{P_{07}}{P_a} = \frac{3.8}{1.01} = 3.76$
$PR_{crit} = \frac{1}{[1-(\frac{\gamma_g-1}{\gamma_g+1})]^{\frac{\gamma_g}{\gamma_g-1}}} = \frac{1}{[1-(\frac{0.333}{2.333})]^{\frac{1.333}{0.333}}} = 1.85$
$NPR > PR_{crit} \rightarrow$ nozzle is choked
$P_9 = \frac{P_{07}}{PR_{crit}} = \frac{3.8}{1.85} = 2.054$ bar

$TR_{crit} = 1.167$
$T_9 = \frac{T_{07}}{TR_{crit}} = \frac{1000}{1.167} = 856.9$ K

$C_9 = 1\sqrt{\gamma_g RT_9} = \sqrt{1.333(287)(856.9)} = 572.56$ m/s
$\rho_9 = \frac{P_9}{RT_9} = \frac{2.054}{287(856.9)} * 100000 = 0.8352$ kg/m^3
$A_9 = \frac{\dot{m}}{\rho_9 C_9} = \frac{23}{0.8352(572.56)} = 0.0481$ m^2
$\boxed{A_9 = 0.0481 \text{ m}^2}$

$F = \dot{m}(C_9 - C_a) + A_9(P_9 - P_a) = 23(572.56 - 0) + 0.0481(2.054 - 1.01) * 100000 = 18191$ N
$\boxed{F = 18191 \text{ N}}$

Part B

fully expanded: $P_9 = P_a$, $T_9 = T_a$ $P_{09} = P_{07}$ because isentropic nodzle
$\frac{P_{09}}{Pa} = [1 + \frac{\gamma_g-1}{2}M^2]^{\frac{\gamma_g}{\gamma_g-1}} \rightarrow \frac{3.8}{1.01} = [1 + \frac{0.333}{2}M^2]^{\frac{1.333}{0.333}}$
solve for $M = 1.535$

$C_9 = M\sqrt{\gamma_g RT_9} = 1.535\sqrt{1.333(287)(288)} = 509.52$ m/s
$\rho_9 = \frac{P_9}{RT_9} = \frac{1.01}{287(288)} * 100000 = 1.222$ kg/m^3
$A_9 = \frac{\dot{m}}{\rho_9 C_9} = \frac{23}{1.222(509.52)} = 0.03694$ m^2
$\boxed{A_9 = 0.03694 \text{ m}^2}$

$F = \dot{m}(C_9 - C_a) = 23(509.52) = 11719$ N
$\boxed{F = 11719 \text{ N}}$

Part C

For this case I would choose the convergent nozzle because it provides more thrust at the same conditions. If the aircraft were to have an afterburner, I would want the C-D nozzle to be able to operate efficiently across all flight regimes. Otherwise, the reduced complexity and increased performance of the converging nozzle is preferred.

Problem 3

A high bypass ratio turbofan is designed for a cruise condition of M=0.85, at an altitude of 11,000 m. Assume a simple two-spool configuration with separate converging-only nozzles. The following cycle data apply:

$\eta_{\infty f/c/t}$	0.90
η_i	0.95
BPR	6.2
$FPR = \frac{P_{02}}{P_{01}}$	1.55
$OPR = \frac{P_{03}}{P_{01}}$	34
T_{04} (K)	1350
$\dot{m}$ (kg/s)	220
$\Delta P_b/P_{02}$ (%)	6
$r = \frac{P_{03}}{P_{02}} = \frac{OPR}{FPR}$	21.94
$\dot{m}_c = \frac{220*6.2}{6.2+1}$	$= 189.44$ kg/s
$\dot{m}_h = \frac{220}{6.2+1}$	$= 30.56$ kg/s

Part A

Ambient

find conditions at altitude:
$\theta = 0.7523$, $\delta = 0.224$ (from table A.2)
$P_a = 0.224(1.01325) = 0.227$ bar, $T_a = 0.7523(288.15) = 216.76$ K, $a = 340.3\sqrt{0.7523} = 295.16$ m/s
$C_a = 295.16 * 0.85 = 250.9$ m/s

Inlet

find P_{01}:
$P_{01}/P_a = (1 + \eta_i \frac{\gamma_c - 1}{2} M^2)^{\frac{\gamma_c}{\gamma_c - 1}} = (1 + 0.95\frac{0.4}{2}(0.85)^2)^{\frac{1.4}{0.4}} = 1.569$
$P_{01} = 0.3561$ bar

inlet is adiabatic:
$T_{01} = T_1(1 + \frac{\gamma_c - 1}{2} M^2) = 216.76(1 + \frac{0.4}{2}(0.85)^2) = 248.08$ K

Fan

$\frac{n-1}{n} = \frac{\gamma_c - 1}{\gamma_c \eta_{\infty c}} = \frac{0.4}{1.4(0.9)} = 0.3175$
$\frac{T_{02}}{T_{01}} = FPR^{(\frac{n-1}{n})} = 1.55^{0.3175} = 1.149$
$T_{02} = 285.04$ K

$P_{02} = 0.3561 * 1.55 = 0.552$ bar

Compressor

$P_{03} = rP_{02} = 21.94(0.552) = 12.11$ bar

$\frac{n-1}{n} = \frac{\gamma_c - 1}{\gamma_c \eta_{\infty c}} = \frac{0.4}{1.4(0.9)} = 0.3175$
$T_{03}/T_{02} = (r)^{\frac{n-1}{n}} = 21.94^{0.3175} = 2.666$
$T_{03} = 759.92$ K

Combustor

account for pressure loss in combustor: $P_{04} = P_{03}(0.94) = 12.11(0.94) = 11.38$
$P_{04} = 11.38$ bar

High Pressure Turbine

$\eta_m \dot{m}_h c_{pg}(T_{04} - T_{05}) = \dot{m}_h c_{pc}(T_{03} - T_{02})$
$T_{04} - T_{05} = \frac{c_{pc}}{\eta_m c_{pg}}(T_{03} - T_{02}) = \frac{1.005}{0.99(1.148)}(759.92 - 285.04) = 419.93$
$T_{05} = 930.07$ K

$\frac{m-1}{m} = \frac{\eta_{\infty t}(\gamma_g - 1)}{\gamma_g} = \frac{0.9(0.333)}{1.333} = 0.2248$

$\frac{P_{05}}{P_{04}} = \frac{T_{05}}{T_{04}}^{\frac{m}{m-1}} = \frac{930.07}{1350}^{1/0.2248} = 0.1912$
$P_{05} = 2.176$ bar

Low Pressure Turbine

$\eta_m \dot{m}_h c_{pg}(T_{05} - T_{06}) = \dot{m} c_{pc}(T_{02} - T_{01})$
$T_{05} - T_{06} = \frac{\dot{m}}{\dot{m}_h} \frac{c_{pc}}{\eta_m c_{pg}}(T_{02} - T_{01}) = \frac{220}{30.56} \frac{1.005}{0.99(1.148)}(285.04 - 248.08) = 235.28$
$T_{06} = 694.8$ K

$\frac{P_{06}}{P_{05}} = (\frac{T_{06}}{T_{05}})^{\frac{m}{m-1}} = (\frac{694.8}{930.07})^{1/0.2248} = 0.2733$
$P_{06} = 0.595$ bar

Nozzle

$NPR = \frac{P_{06}}{P_a} = \frac{0.595}{0.227} = 2.621$
$PR_{crit} = \frac{1}{[1 - \frac{\gamma_g - 1}{\gamma_g + 1}]^{\frac{\gamma_g}{\gamma_g - 1}}} = 1.852$
$NPR > PR_{crit} \rightarrow$ nozzle is choked
$P_7 = \frac{P_{06}}{PR_{crit}} = \frac{0.595}{1.852} = 0.321$ bar

$TR_{crit} = 1.167$
$T_7 = \frac{T_{06}}{TR_{crit}} = \frac{694.8}{1.167} = 595.37$ K

$C_7 = 1\sqrt{\gamma_g R T_7} = \sqrt{1.148(287)(595.37)} = 442.9$ m/s
$\rho_7 = \frac{P_7}{RT_7} = \frac{0.321}{287(595.37)} * 100000 = 0.1879$ kg/m^3
$A_7 = \frac{\dot{m}_h}{\rho_7 C_7} = \frac{30.56}{0.1879(442.9)} = 0.3672$ m^2

$F_c = \dot{m}_h(C_7 - C_a) + A_7(P_7 - P_a) * 100000 =$
$30.56(442.9 - 250.9) + 0.3672(0.321 - 0.227) * 100000 = 9319.2$ N

Fan Nozzle

$PR_{crit} = \frac{1}{[1 - (\frac{\gamma_c - 1}{\gamma_c + 1})]^{\frac{\gamma_c}{\gamma_c - 1}}} = \frac{1}{[1 - (\frac{0.4}{2.4})]^{\frac{1.4}{0.4}}} = 1.893$
$\frac{P_{02}}{P_a} = \frac{0.552}{0.227} = 2.432$
$NPR > PR_{crit} \rightarrow$ nozzle is choked
$P_8 = \frac{P_{02}}{PR_{crit}} = \frac{0.552}{1.893} = 0.2916$ bar

$TR_{crit} = 1.167$
$T_8 = \frac{T_{02}}{TR_{crit}} = \frac{285.04}{1.167} = 244.25$ K

$C_8 = 1\sqrt{\gamma_c R T_8} = \sqrt{1.4(287)(244.25)} = 313.272$ m/s
$\rho_8 = \frac{P_8}{RT_8} = \frac{0.2916}{287(244.25)} * 100000 = 0.416$ kg/m^3
$A_8 = \frac{\dot{m}_c}{\rho_8 C_8} = \frac{189.44}{0.416(313.272)} = 1.454$ m^2

$F_f = \dot{m}_c(C_8 - C_a) + A_8(P_8 - P_a) * 100000 =$
$189.44(313.272 - 250.9) + 1.454(0.2916 - 0.227) * 100000 = 21208.6$ N

Thrust and TSFC

$F_t = 21208.6 + 9319.2 = 30527.8$
$\boxed{F_{total} = 30527.8 \text{ N}}$

$f_a = \frac{c_{pg} T_{04} - c_{pc} T_{03}}{\eta_b (Q_f - c_{pg} T_{04})} = \frac{1.148(1350) - 1.005(759.92)}{0.99(43100 - 1.148(1350))} = 0.0191$
$TSFC = \frac{f \dot{m}}{(1+B)F} = \frac{0.0191(220)}{(1+6.2)30527.8} * 3600 = 0.0688$
$\boxed{TSFC = 0.0688 \text{ kg/N-hr}}$

Part B

I would recommend an increase in bypass ratio as that would increase the thrust and lower the SFC of the engine. This also would allow for the core to stay very similar to the original so it would be easier to make new engines as they would not need to be done from scratch.