BME6013C HW#2

Slade Brooks
M13801712

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$$r_{fg} = \frac{1}{T}\int_0^T e^{-j2\pi mt/T}e^{2j\pi nt/T}dt$$

Part 1

First simplify the integrand for simplicity:

$$\begin{aligned} r_{fg} &= \frac{1}{T}\int_0^T e^{-j2\pi mt/T}e^{j2\pi nt/T}dt \\ &= \frac{1}{T}\int_0^T e^{j2\pi nt/T-j2\pi mt/T}dt \\ &= \frac{1}{T}\int_0^T e^{j\frac{2\pi}{T}(n-m)t}dt \end{aligned}$$

Now evaluate the integral:

$$\begin{aligned} r_{fg} &= \frac{1}{jT\left(\frac{2\pi}{T}(n-m)\right)}e^{j\frac{2\pi}{T}(n-m)t}\Big|_0^T \\ &= \frac{1}{j2\pi(n-m)}\left(e^{j\frac{2\pi}{T}(n-m)T} - e^{j\frac{2\pi}{T}(n-m)0}\right) \\ &= \frac{1}{j2\pi(n-m)}\left(e^{j2\pi(n-m)} - e^{j0}\right) \\ &\boxed{r_{fg} = \frac{e^{j2\pi(n-m)}-1}{j2\pi(n-m)}} \end{aligned}$$

Part 2

sub in $n = m$ and simplify the integral:

$$\begin{aligned} r_{fg} &= \frac{1}{T}\int_0^T e^{-j2\pi mt/T} e^{j2\pi mt/T} dt \\ &= \frac{1}{T}\int_0^T e^{j2\pi mt/T - j2\pi mt/T} dt \\ &= \frac{1}{T}\int_0^T e^{j0} dt = \frac{1}{T}\int_0^T 1 dt \end{aligned}$$

now take the integral w.r.t. t:

$$r_{fg} = \frac{1}{T} t|_0^T = \frac{1}{T}(T - 0) = \frac{T}{T} = 1$$

This result makes sense as r_{fg} for two identical signals should be 1, indicating that they are perfectly correlated.

Part 3

evaluate $r_{fg} = 0$ from Part 1:

$$\begin{aligned} r_{fg} &= \frac{e^{j2\pi(n-m)} - 1}{j2\pi(n-m)} = 0 \\ &= e^{j2\pi(n-m)} - 1 = 0 \end{aligned}$$

sub in cos/sin definition of complex number:

$$r_{fg} = \cos(2\pi(n-m)) + j\sin(2\pi(n-m)) = 1$$

Since $n - m$ will always be an integer if n and m are any real integer, the number within the cos/sin will always be a multiple of 2π. This means that the sin term will always be 0, and cos will always be 1:

$$r_{fg} = 1 + j0 = 1$$

which simplifies to $r_{fg} = 1 - 1 = 0$, proving that $r_{fg} = 0$ for any non-equal real integers n and m.