AEEM5058 HW#4

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Problem 1

The cross section shown in Fig 1 is obtained from the one considered in previous homework by replacing arc 41 with a straight element and maintaining a cut at node 1. The dimensions continue to be R=200 mm and t =1.7 mm. Find the vertical coordinate $v_{S.C.}$ of the shear center (S.C.) relative to the frame (u,v). You can use the expressions provided in the Appendix to aid your analysis.

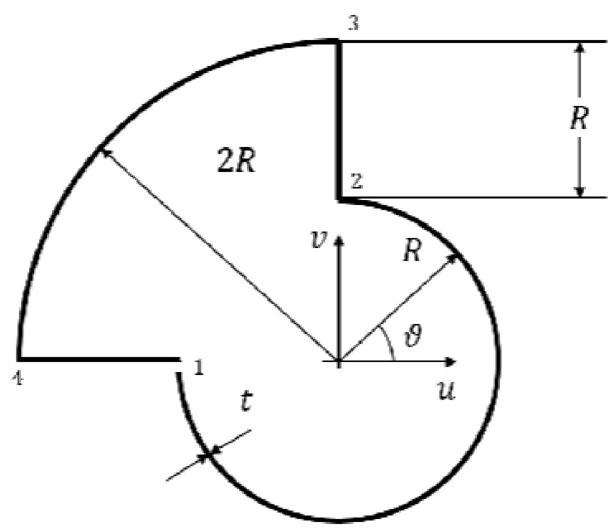

Figure 1

First we will apply an arbitrary shear force. Since we only need to find the v location, we will apply a force in the V_η direction at an unknown v coordinate e. For simplicity, we can use the 25kN force in the negative direction from the previous homework. The moment of this arbitrary force will be set equal to the moment of the shear flow. We can substitute in the know functions for the shear flow to get:

$$-eF = \alpha \int_0^L f_\eta(s)h(s)ds + \beta \int_0^L g_\eta(s)h(s)ds$$

Then, we can define the $h(s)$ for each section, which can be determined by the minimum distance. For the straight segments, this will be 0 because they have no tangent points to the origin. Then, these can be plugged into the integral for each section.

$$h^{12}(s) = R$$
$$h^{23}(s) = 0$$
$$h^{34}(s) = 2R$$
$$h^{41}(s) = 0$$

$$\int f^{12}(s)h^{12}(s) = \int_{-\pi}^{\pi/2} (R^2 \sin\theta_1 - u_c R\theta_1 - u_c R\pi)(R)Rd\theta$$
$$\int f^{23}(s)h^{23}(s) = 0$$
$$\int f^{34}(s)h^{34}(s) = \int_{\pi/2}^{\pi} (-3R^2 - \frac{1}{2}\pi u_c R - u_c R + 4R^2 \sin\theta_3 - 2u_c R\theta_3)(2R)Rd\theta$$
$$\int f^{41}(s)h^{41}(s) = 0$$

$$\int g^{12}(s)h^{12}(s) = \int_{-\pi}^{\pi/2} (-R^2 \cos\theta_1 - \theta_1 R v_c - R^2 - v_c \pi R)(R)Rd\theta$$
$$\int g^{23}(s)h^{23}(s) = 0$$
$$\int g^{34}(s)h^{34}(s) = \int_{\pi/2}^{\pi} (-\frac{1}{2}v_c \pi R - v_c R + \frac{R^2}{2} - 4R^2 \cos\theta_3 - 2Rv_c\theta_3)(2R)Rd\theta$$
$$\int g^{41}(s)h^{41}(s) = 0$$

Next, we will solve for α and β. They are slightly simplified because $V_\zeta = 0$.

$$\begin{aligned} I_\eta &= I_u - Av_c^2 = 1.213 * 10^8 \\ I_\zeta &= I_v - Au_c^2 = 1.213 * 10^8 \\ I_{\eta\zeta} &= I_{uv} - Au_c v_c = -1.965 * 10^7 \end{aligned}$$

$$\begin{aligned} \alpha &= t\frac{V_\eta I_\eta}{I_\eta I_\zeta - I_{\eta\zeta}^2} = -3.599 * 10^{-4} \\ \beta &= t\frac{-V_\eta I_{\eta\zeta}}{I_{eta} I_\zeta - I_{\eta\zeta}^2} = -5.831 * 10^{-5} \end{aligned}$$

Lastly, we can solve for the distance. Plugging in the integrals (and solving with an online calculator) and known values allows for e to be solved for.

$$e = \frac{\alpha \int_0^L f_\eta(s) h(s) ds + \beta \int_0^L g_\eta(s) h(s) ds}{-F}$$

$$\boxed{e = 462.6 \text{ mm}}$$

Problem 2

Compute the vertical coordinate $v_{S.C.}$ of the shear center (S.C.) relative to the frame (u,v) assuming that the section in Problem #1 is closed by a weld line along corner 1 as shown in Fig. 2.

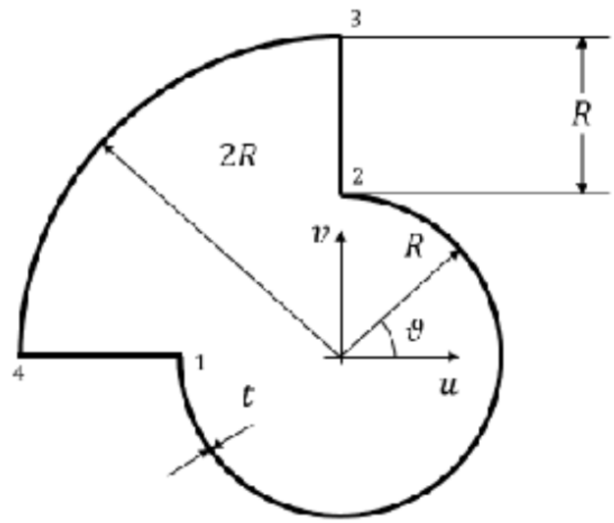

Figure 2

First, for a closed section the same arbitrary force and location are applied, but the equation is:

$$-eF = \int (q_o + q_B(s))h(s)ds$$

We already determined some equations in the previous problem, so we can divide those by h(s) to get $q_b(s)$.

$$\int f^{12}(s)h^{12}(s) = \int_{-\pi}^{\pi/2} (R^2 \sin\theta_1 - u_c R\theta_1 - u_c R\pi)Rd\theta$$

$$\int f^{23}(s)h^{23}(s) = \int_0^R (R^2 - \frac{3}{2}\pi u_c R - u_c s_2)ds$$

$$\int f^{34}(s)h^{34}(s) = \int_{\pi/2}^{\pi} (-3R^2 - \frac{1}{2}\pi u_c R - u_c R + 4R^2 \sin\theta_3 - 2u_c R\theta_3)Rd\theta$$

$$\int f^{41}(s)h^{41}(s) = \int_0^R (-3R^2 - \frac{5}{2}\pi u_c R - u_c R - (2R + u_c)s_4 + \frac{s_4^2}{2})ds$$

$$\int g^{12}(s)h^{12}(s) = \int_{-\pi}^{\pi/2} (-R^2 \cos\theta_1 - \theta_1 R v_c - R^2 - v_c \pi R)Rd\theta$$

$$\int g^{23}(s)h^{23}(s) = \int_0^R (-R^2 - \frac{3}{2}v_c \pi R + (R - v_c)s_2 + \frac{s_2^2}{2})ds$$

$$\int g^{34}(s)h^{34}(s) = \int_{\pi/2}^{\pi} (-\frac{1}{2}v_c \pi R - v_c R + \frac{R^2}{2} - 4R^2 \cos\theta_3 - 2Rv_c\theta_3)Rd\theta$$

$$\int g^{41}(s)h^{41}(s) = \int_0^R (-\frac{5}{2}v_c \pi R - v_c R + \frac{9R^2}{2} - v_c s_4)ds$$

α and β will be the same for the closed section. Then we can plug in and solve the integral (with an online calculator) for q_B in order to obtain q_o.

$$q_B = \alpha \int_0^L f_\eta(s)ds + \beta \int_0^L g_\eta(s)ds = -42094.23$$

$$q_o = \frac{q_B}{L}$$

$$L = 0.75(2\pi R) + R + 0.25(4\pi R) + R = 1970.79$$

$$q_o = 21.36$$

Lastly, we can find the area enclosed by the shape Σ and use that to determine the shear center location e. We will solve with an online calculator in the same manner as problem 1.

$$\Sigma = 0.75\pi R^2 + 0.25\pi(2R)^2 = 219911.49$$
$$e = \frac{2\Sigma q_o + \int q_B(s)h(s)ds}{-F} = \boxed{86.86 \text{ mm}}$$