MECH6066 HW#4

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11.02.25

Problem 1

(a)

The mass law states that the transmission loss through a sufficiently thin wall is given by:

$$T_L = 20\log_{10}\left(\frac{\rho_2 L\omega}{2Z_1}\right)$$
$$10^{T_L/20} = \frac{\rho_2 L\omega}{2Z_1}$$
$$L = \frac{2Z_1}{\rho_2\omega}10^{(T_L/20)}$$

The design requires a transmission loss of 30dB at 300Hz. The imepdance Z_1 will be the speed of sound of air times the density, assuming standard air. We can plug these known values in to solve for the required wall thickness based on the wall density to satisfy the requirement for each material.

Material	Density (kg/m^3)	Speed of Sound (m/s)	L (mm)
air	1.21	343	—
steel	7700	6100	1.81
glass	2300	5600	6.05
concrete	2600	3100	5.36
pine	450	3500	30.95

(b)

The exact transmission loss is given by:

$$T_L = 20\log_{10}\left(\frac{2 + (Z_3/Z_1 + Z_1/Z_3)\cos^2(K_2L) + (Z_2^2/Z_1Z_3 + Z_1Z_3/Z_2^2)\sin^2(K_2L)}{4}\right)$$

$$K_2 = \omega/c_2; \quad Z_1 = Z_3$$

$$T_L = 20\log_{10}\left(\frac{2 + 2\cos^2(K_2L) + (Z_2^2/Z_1^2 + Z_1^2/Z_2^2)\sin^2(K_2L)}{4}\right)$$

We can plug in the known values (Z_1, Z_2, K_2, L) to find the actual transmission loss through each material.

Material	T_L actual (dB)
steel	30.01
glass	30.0
concrete	30.01
pine	30.01

We can see that the mass law is clearly valid for all materials. The lowest speed of sound material, concrete, has a wavelength of 10 meters at 300Hz, and the largest wall thickness was pine at 31 mm. All of the materials clearly satisfy the assumption that the thickness is very small compared to the wavelength.

(c)

I would probably choose pine. Assuming the desire to reduce noise is primarily for a residential area, wooden fences are a clear choice. The 31 mm required thickness is around 1.25 inches, which is only slightly smaller than the width of a normal 2x4 wood board. A very simple pine fence made of 2x4s should effectively reduce the center frequency by the desired amount, while looking nice and being a very cheap option in terms of parts and labor.

Problem 2

(a)

We know the vector $\vec{k}$ can be defined by the angle and its magnitude. When the switch to magnitude is made, the magnitude is just the regular wave number based on the material.

From the book, fresh water has a density of 998kg/m^3 and a speed of sound of 1481m/s.

$$\vec{k} = ||\vec{k}||(\cos\theta\hat{x} + \sin\theta\hat{y}) = K(\cos 30\hat{x} + \sin 30\hat{y})$$
$$\vec{k} = \frac{\omega}{c}(0.866\hat{x} + 0.5\hat{y}) = \frac{2\pi(100000)}{1481}(0.866\hat{x} + 0.5\hat{y})$$
$$\boxed{\vec{k} = 367.4\hat{x} + 212.13\hat{y}}$$

(b)

$$K = \frac{2\pi}{\lambda} \rightarrow K_x = \frac{2\pi}{\lambda_x}; \quad K_y = \frac{2\pi}{\lambda_y}$$
$$\lambda_x = \frac{2\pi}{K_x} = \frac{2\pi}{367.4} = 0.0171\text{m}$$
$$\lambda_y = \frac{2\pi}{K_y} = \frac{2\pi}{212.13} = 0.03\text{m}$$
$$\boxed{\lambda_x = 17.1\text{mm};\ \lambda_y = 30\text{mm}}$$

$$\lambda = \frac{c}{f} = \frac{1481}{100000} = 0.01481\text{m}$$
$$\boxed{\lambda = 14.8\text{mm}}$$

We can see that the wavelength in both directions is larger than the wavelength for 100kHz in water. This makes sense as the wave front is moving slower with respect to both directions. Since it is close to the x axis, λ_x is not much different than the normal wavelength, but the y direction will stretch much more.

(c)

i.

Following the derivation we did in class for 2D instead of 3D:

$$
\begin{aligned}
-\nabla p &= \rho \frac{\partial u}{\partial t} \\
-\nabla \left[\bar{p} e^{j(\omega t - k_x x - k_y y)}\right] &= \rho \frac{\partial u}{\partial t} \\
-\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y}\right) \bar{p} e^{j(\omega t - k_x x - k_y y)} &= \rho \frac{\partial u}{\partial t} \\
-\left(-jk_x \hat{x} - jk_y \hat{y}\right) \bar{p} e^{j(\omega t - k_x x - k_y y)} &= \rho \frac{\partial u}{\partial t} \\
j\vec{k}\bar{p} e^{j(\omega t - k_x x - k_y y)} &= \rho \frac{\partial u}{\partial t} \\
\frac{\vec{k}}{\omega \rho} \bar{p} e^{j(\omega t - k_x x - k_y y)} &= \vec{u} \\
\boxed{\vec{u}(x, y, t) = \tfrac{\vec{k}}{\omega \rho} \bar{p} e^{j(\omega t - k_x x - k_y y)}}
\end{aligned}
$$

ii.

We know the magnitude will be everything except for the exponential term. We can take this and simplify it based on what relationships we know.

$$
||\vec{u}|| = u = \frac{\vec{k}}{\omega \rho}\bar{p} = \frac{K\hat{e}_k}{\omega \rho}\bar{p} = \frac{K}{\omega \rho}\bar{p} = \frac{\omega}{c\omega \rho}\bar{p} = \frac{\bar{p}}{c\rho} = \frac{\bar{p}}{Z_0}
$$

$$
\boxed{u = \tfrac{\bar{p}}{Z_0}}
$$

iii.

First we can solve for the magnitude of the velocity vector since we know both parts:

$$
u = \frac{\bar{p}}{Z_0} = \frac{1000000}{998 * 1481} = 0.6766
$$

$$
\boxed{u = 0.6766 \text{m/s}}
$$

We can plug this and some other knowns back into the particle velocity equation:

$$\vec{u} = 0.6766e^{j(\omega t - k_x x - k_y y)} = 0.6766e^{j(2\pi(100000)t - 367.4x - 212.13y)}$$

$$\boxed{\vec{u} = 0.6766e^{j(628318.5t - 367.4x - 212.13y)}}$$

It would also be given in vector form as:

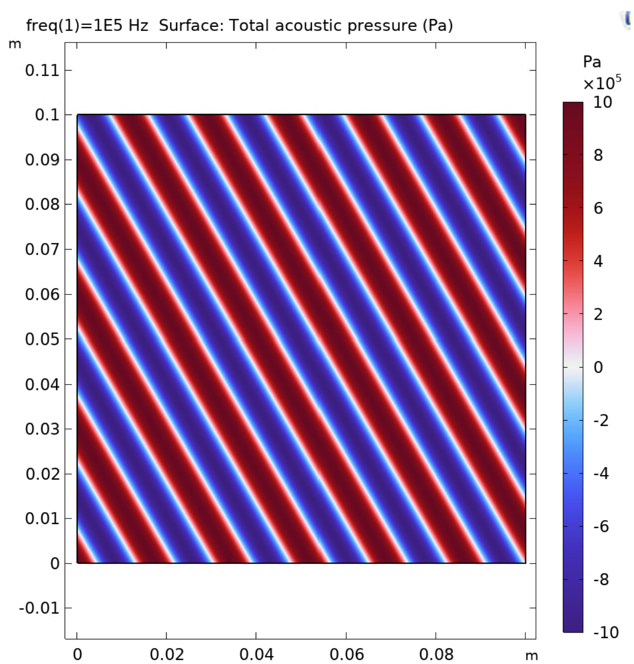
$$\boxed{\vec{u} = 0.6766\left(\cos 30\hat{x} + \sin 30\hat{y}\right)}$$

(d)

See attached model pdf.

(e)

Yes, there are waves at a 30° angle varying from -1 to 1 MPa.

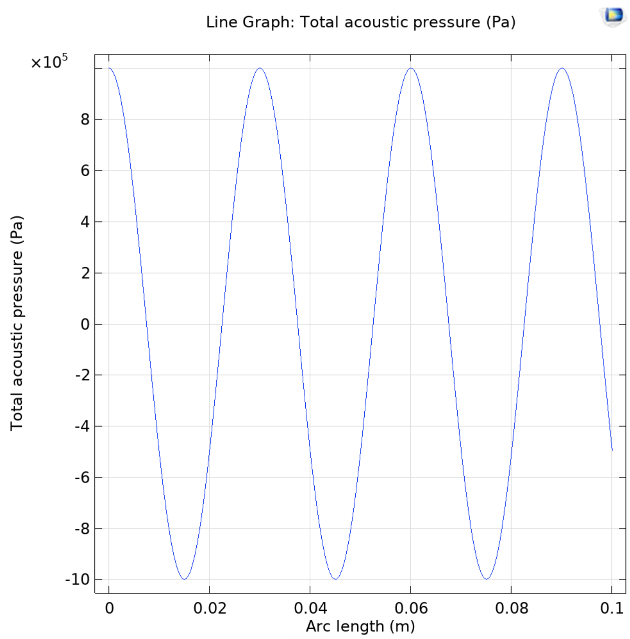


(f)

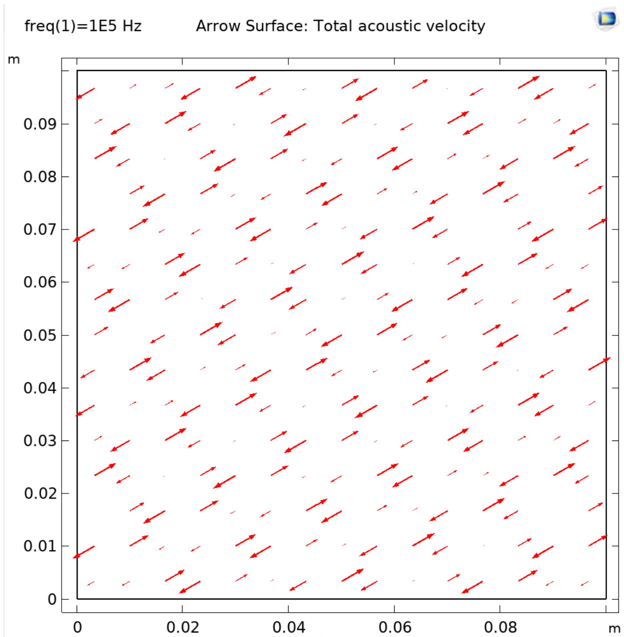
Yes, the waves propagate towards the northeast. To make them move southwest, we would switch the sign of both components of $\hat{e}_k$.

(g)

Based on the plot, one of the peaks goes from 15mm to 45mm, giving a wavelength of 30mm. This is exactly what was calculated in part b.



(h)



i.

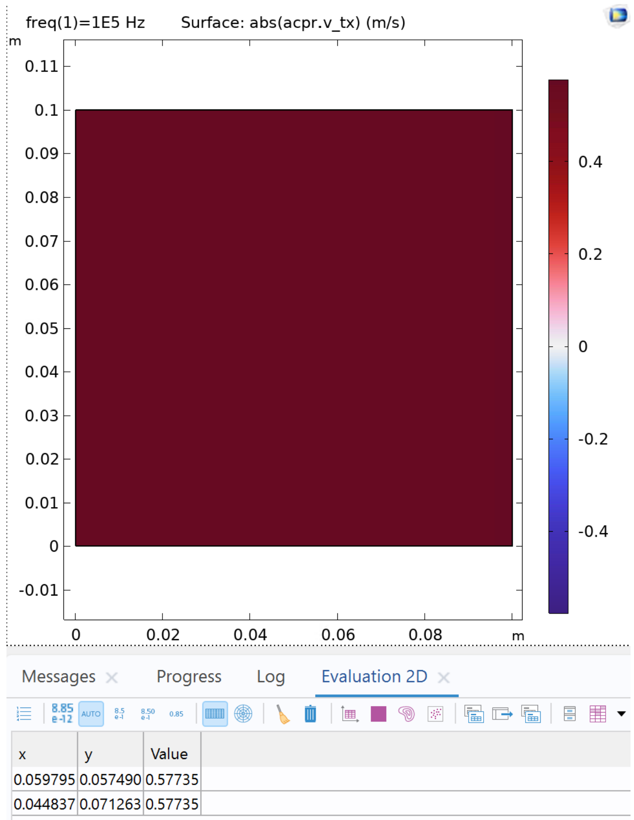
There are arrows going both northeast and southwest. This makes sense as the particles should be oscillating in both directions along the angle of the plane wave and they are aligned with where wave fronts should be.

ii.

The particles oscillate, flipping their velocity vectors between northeast and southwest.

(i)

We can see the resultant plot is all one magnitude of 0.57735m/s. We determined the x direction should be $0.6766 \cos 30 = 0.586$ m/s. This is very close to the comsol value. Any difference in the value is due to using a different speed of sound and density for water. I used the book appendix values for part c, but used the simplified values from the hint document for the comsol model.



Problem 3

I thought this assignment was shorter and easier than previous ones. I enjoyed that. The study materials were very adequate. I felt like my notes from class were more than enough to understand everything and the hint sheet helped greatly with making comsol take less time.

Hw4

Report date	Oct 31, 2025, 3:14:30 PM

Contents

1 Global Definitions

Date	Oct 31, 2025, 3:01:29 PM

GLOBAL SETTINGS

Name	Hw4.mph
Path	\\clusterfsnew.ceas1.uc.edu\students\brooksl\desktop\hw4.mph
Version	COMSOL Multiphysics 6.3 (Build: 420)

USED PRODUCTS

COMSOL Multiphysics
Acoustics Module

COMPUTER INFORMATION

CPU	Intel64 Family 6 Model 198 Stepping 2, 28 cores, 63.46 GB RAM
Operating system	Windows 11

1.1 PARAMETERS

PARAMETERS 1

Name	Expression	Value	Description
pin	1[MPa]	1E6 Pa	
ang	30[deg]	0.5236 rad	
fin	100[kHz]	1E5 Hz	

2 Component 1

SETTINGS

Description	Value
Unit system	Same as global system (SI)

2.1 DEFINITIONS

2.1.1 Coordinate Systems

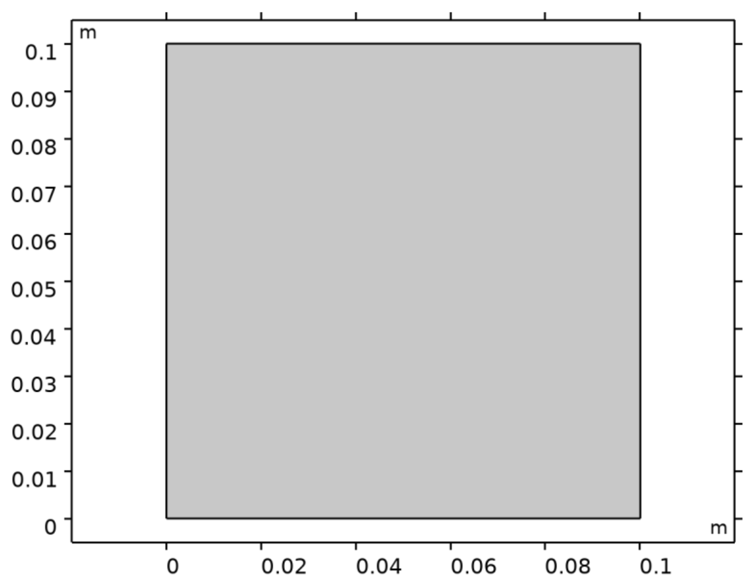
Boundary System 1

Coordinate system type	Boundary system
Tag	sys1

COORDINATE NAMES

First	Second	Third
t1	n	to

2.2 GEOMETRY 1



Geometry 1

UNITS

Length unit	m
Angular unit	deg

GEOMETRY STATISTICS

Description	Value

Description	Value
Space dimension	2
Number of domains	1
Number of boundaries	4
Number of vertices	4

2.2.1 Square 1 (sq1)

SIZE AND SHAPE

Description	Value
Side length	0.1

POSITION

Description	Value
Position	{0, 0}

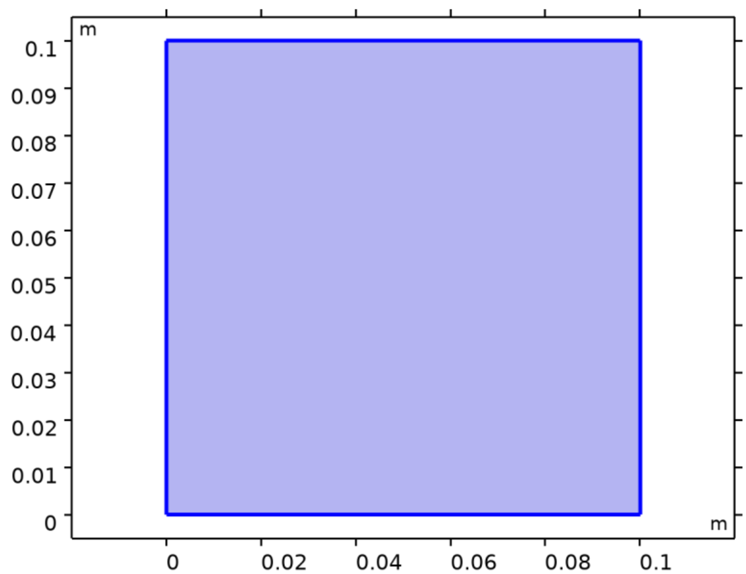
2.2.2 Form Union (fin)

INFORMATION

Description	Value
Build message	Formed union of 1 solid object. Union has 1 domain, 4 boundaries, and 4 vertices.

2.3 MATERIALS

2.3.1 Water



Water

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

MATERIAL PARAMETERS

Name	Value	Unit	Property group
Density	1000	kg/m³	Basic
Speed of sound	1500	m/s	Basic

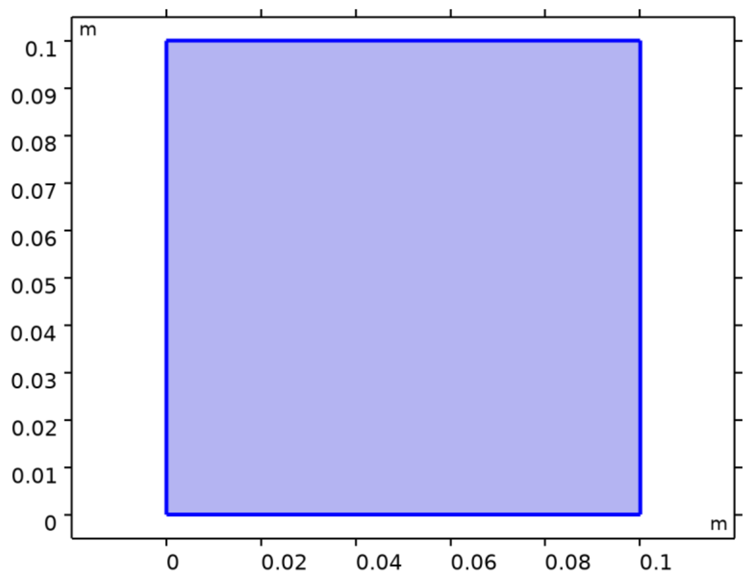
BASIC

Description	Value	Unit
Density	1000	kg/m³
Speed of sound	1500	m/s

2.4 PRESSURE ACOUSTICS, FREQUENCY DOMAIN

USED PRODUCTS

COMSOL Multiphysics
Acoustics Module



Pressure Acoustics, Frequency Domain

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

EQUATIONS

$$\nabla \cdot \left(-\frac{1}{\rho_c}(\nabla p_t - \mathbf{q}_d)\right) - \frac{k_{eq}^2 p_t}{\rho_c} = Q_m$$

$$p_t = p + p_b$$

$$k_{eq}^2 = \left(\frac{\omega}{c_c}\right)^2 - k_z^2$$

2.4.1 Interface Settings

Physics Symbols

SETTINGS

Description	Value
Enable physics symbols	On

Discretization

SETTINGS

Description	Value
Element order	Quadratic Lagrange

Physics-Controlled Mesh

SETTINGS

Description	Value
Maximum mesh element size control parameter	From study
Number of mesh elements per wavelength	Automatic

Pressure Acoustics Equation Settings

SETTINGS

Description	Value	Unit
Out-of-plane wave number	0	rad/m

Global Port Settings

SETTINGS

Description	Value
Port sweep settings	No port sweep
Mode shape normalization	Amplitude normalization

Sound Pressure Level Settings

SETTINGS

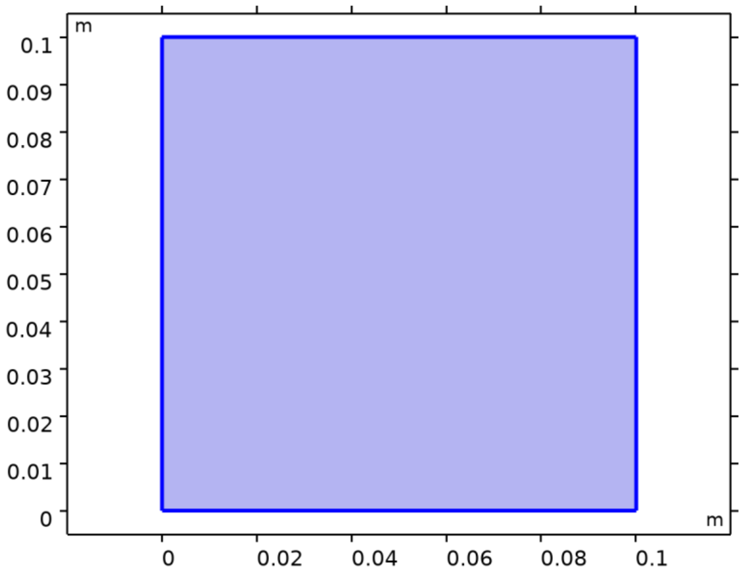
Description	Value
Reference pressure for the sound pressure level	Use reference pressure for air

Typical Wave Speed for Perfectly Matched Layers

SETTINGS

Description	Value	Unit
Typical wave speed for perfectly matched layers	real(acpr.c_c)	m/s

2.4.2 Pressure Acoustics 1



Pressure Acoustics 1

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

EQUATIONS

$$\nabla\cdot\left(-\frac{1}{\rho_c}(\nabla p_t-\mathbf{q}_d)\right)-\frac{k_{eq}^2 p_t}{\rho_c}=Q_m$$

$$p_t=p+p_b$$

$$k_{eq}^2=\left(\frac{\omega}{c_c}\right)^2-k_z^2$$

$$c_c=c,\quad \rho_c=\rho$$

Pressure Acoustics Model

SETTINGS

Description	Value
Fluid model	Linear elastic
Specify	Density and speed of sound

Description	Value
Speed of sound	From material
Density	From material

Model Input

SETTINGS

Description	Value	Unit
Temperature	User defined	
Temperature	293.15	K
Absolute pressure	User defined	
Absolute pressure	1.0133E5	Pa

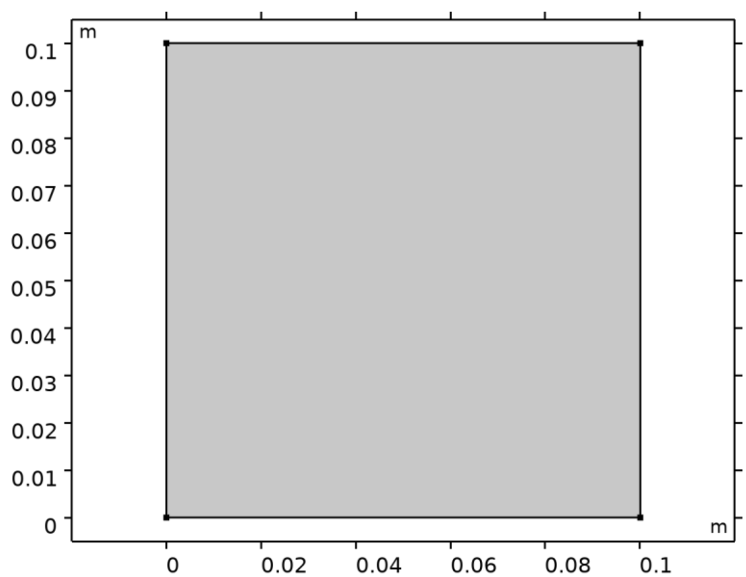
USED PRODUCTS

COMSOL Multiphysics

PROPERTIES FROM MATERIAL

Property	Material	Property group
Density	Water	Basic
Speed of sound	Water	Basic

2.4.3 Sound Hard Boundary (Wall) 1



Sound Hard Boundary (Wall) 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: All boundaries

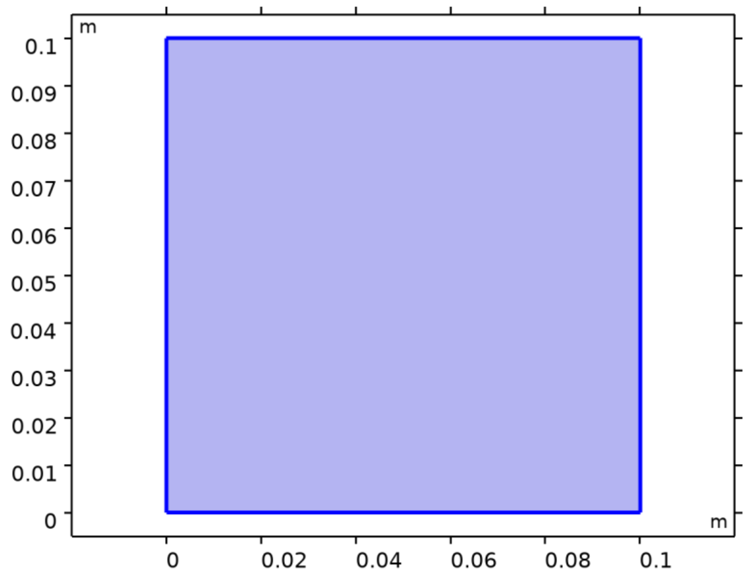
EQUATIONS

$$-\mathbf{n}\cdot\left(-\frac{1}{\rho_c}(\nabla p_t-\mathbf{q}_d)\right)=0$$

USED PRODUCTS

COMSOL Multiphysics

2.4.4 Initial Values 1



Initial Values 1

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: All domains

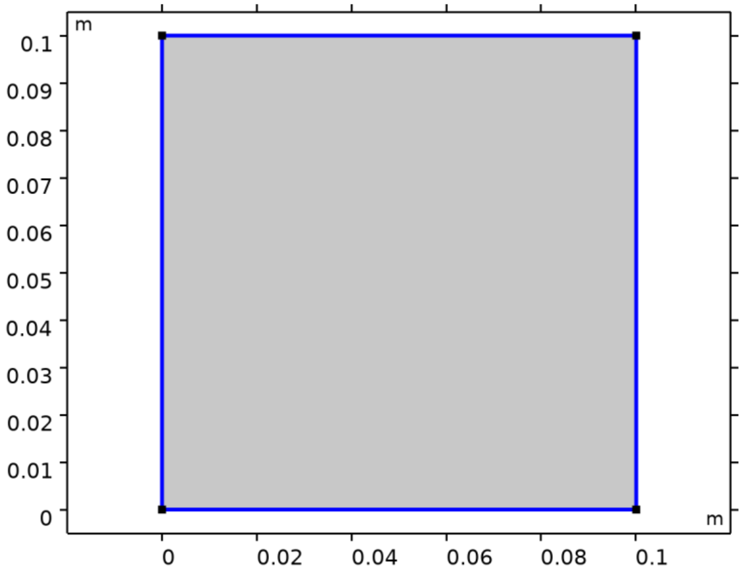
SETTINGS

Description	Value	Unit
Acoustic pressure	0	Pa

USED PRODUCTS

COMSOL Multiphysics

2.4.5 Plane Wave Radiation 1



Plane Wave Radiation 1

SELECTION

Geometric entity level	Boundary
Selection	Geometry geom1: Dimension 1: Boundaries 1–4

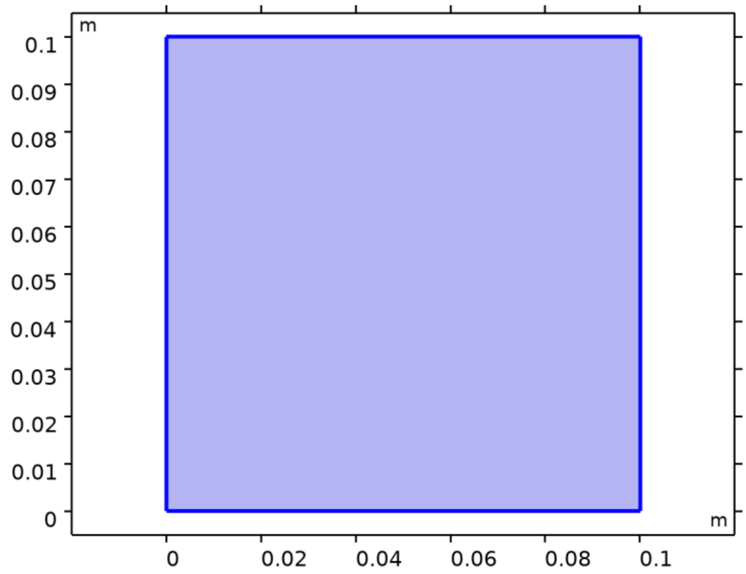
EQUATIONS

$$-\mathbf{n}\cdot\left(-\frac{1}{\rho_{\mathrm{c}}}(\nabla p_{\mathrm{t}}-\mathbf{q}_{\mathrm{d}})\right)+i\frac{k_{\mathrm{eq}}}{\rho_{\mathrm{c}}}p+\frac{i}{2k_{\mathrm{eq}}\rho_{\mathrm{c}}}\Delta_{\parallel}p=Q_{\mathrm{i}}$$

USED PRODUCTS

COMSOL Multiphysics

2.4.6 Background Pressure Field 1



Background Pressure Field 1

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: Domain 1

EQUATIONS

$$\nabla\cdot\left(-\frac{1}{\rho_c}(\nabla p_t-\mathbf{q}_d)\right)-\frac{k_{eq}^2 p_t}{\rho_c}=Q_m$$

$$p_t=p+p_b$$

$$k_{eq}^2=\left(\frac{\omega}{c_c}\right)^2-k_z^2$$

$$p_b=p_0 e^{i\phi}e^{-ik_s\frac{(\mathbf{x}\cdot\mathbf{e}_k)}{|\mathbf{e}_k|}}$$

$$k_s^2=\left(\frac{\omega}{c}\right)^2-k_z^2$$

Background Pressure Field

SETTINGS

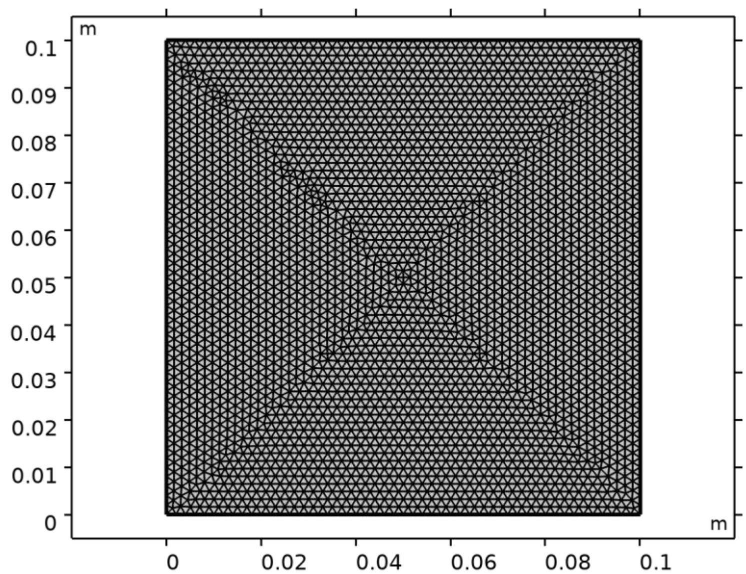
Description	Value	Unit
Pressure field type	Plane wave	
Pressure amplitude	pin	Pa
Speed of sound	From material	
Wave direction, x-component	cos(ang)	
Wave direction, y-component	sin(ang)	

Description	Value	Unit
Wave direction, z-component	0	
Phase	0	rad
Calculate background and scattered field intensity	Off	
Material	Domain material	

PROPERTIES FROM MATERIAL

Property	Material	Property group
Speed of sound	Water	Basic

2.5 MESH 1



Mesh 1

2.5.1 Size (size)

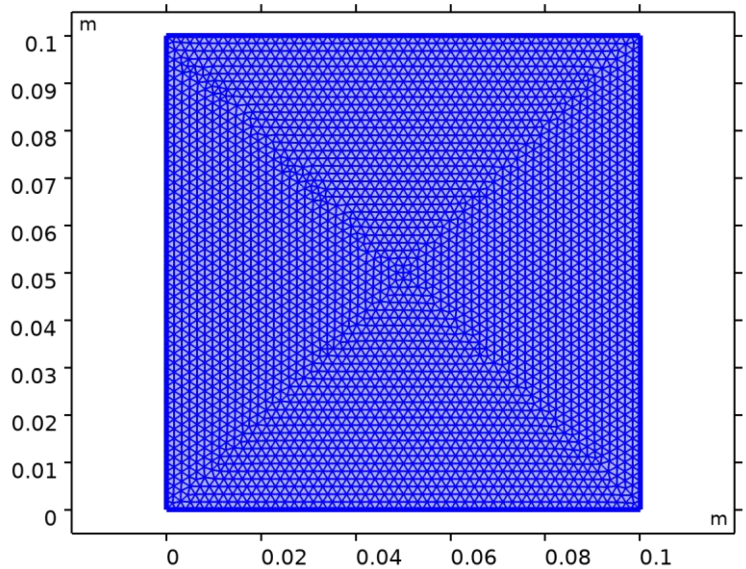
SETTINGS

Description	Value
Maximum element size	0.002
Minimum element size	1E-6
Curvature factor	0.25
Maximum element growth rate	1.2
Predefined size	Extra fine
Custom element size	Custom

2.5.2 Size Expression 1 (se1)

SELECTION

Geometric entity level	Domain
Selection	Geometry geom1: Dimension 2: Domain 1



Size Expression 1

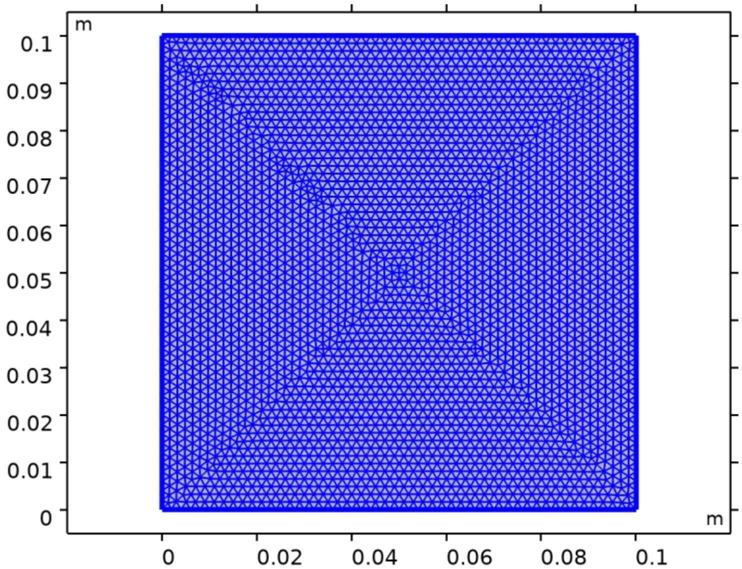
SETTINGS

Description	Value
Evaluate on	Initial expression
Study step	Study 1: Frequency Domain
Size expression	subst(real(acpr.c_c), acpr.freq, freqmax)/freqmax/5
Reevaluate with updated model	

2.5.3 Free Triangular 1 (ftri1)

SELECTION

Geometric entity level	Domain
Selection	Remaining



Free Triangular 1

SETTINGS

Description	Value
Number of iterations	4
Maximum element depth to process	4

INFORMATION

Description	Value
Last build time	< 1 second
Built with	COMSOL 6.3.0.420 (win64), Oct 31, 2025, 2:56:16 PM

3 Study 1

COMPUTATION INFORMATION

Computation time	1 s

3.1 FREQUENCY DOMAIN

Frequencies (Hz)
fin

STUDY SETTINGS

Description	Value
Include geometric nonlinearity	Off

SETTINGS

Description	Value
Frequencies	1E5

PHYSICS AND VARIABLES SELECTION

Key	Solve for
Pressure Acoustics, Frequency Domain (acpr)	On

STORE IN OUTPUT

Interface	Output	Selection
Pressure Acoustics, Frequency Domain (acpr)	Physics controlled	

MESH SELECTION

Component	Mesh
Component 1	Mesh 1

3.2 SOLVER CONFIGURATIONS

3.2.1 Solution 1

Compile Equations: Frequency Domain (st1)

STUDY AND STEP

Description	Value
Use study	Study 1
Use study step	Frequency Domain

Dependent Variables 1 (v1)

GENERAL

Description	Value
Defined by study step	Step 1: Frequency Domain

INITIAL VALUE CALCULATION CONSTANTS

Constant name	Initial-value source
freq	fin

Acoustic Pressure (comp1.p) (comp1_p)

GENERAL

Description	Value
Field components	comp1.p

Stationary Solver 1 (s1)

GENERAL

Description	Value
Defined by study step	Step 1: Frequency Domain

RESULTS WHILE SOLVING

Description	Value
Probes	None

Advanced (aDef)

ASSEMBLY SETTINGS

Description	Value
Reuse sparsity pattern	On
Allow complex-valued output from functions with real input	On

Parametric 1 (p1)

GENERAL

Description	Value
Defined by study step	Step 1: Frequency Domain
Run continuation for	No parameter

PARAMETERS

Parameter name	Parameter value list	Parameter unit
freq	fin	Hz

Fully Coupled 1 (fc1)

GENERAL

Description	Value

Description	Value
Linear solver	Direct

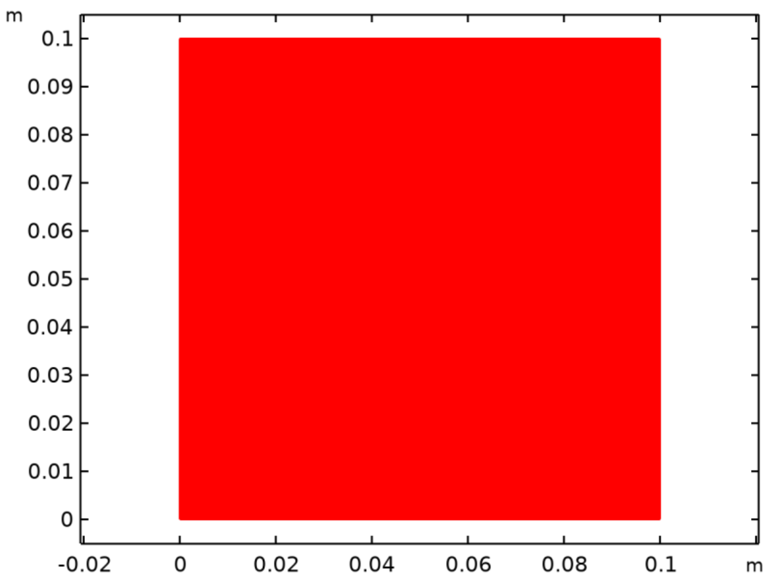
4 Results

4.1 DATASETS

4.1.1 Study 1/Solution 1

SOLUTION

Description	Value
Solution	Solution 1 (sol1)
Component	Component 1 (comp1)



Dataset: Study 1/Solution 1

4.2 TABLES

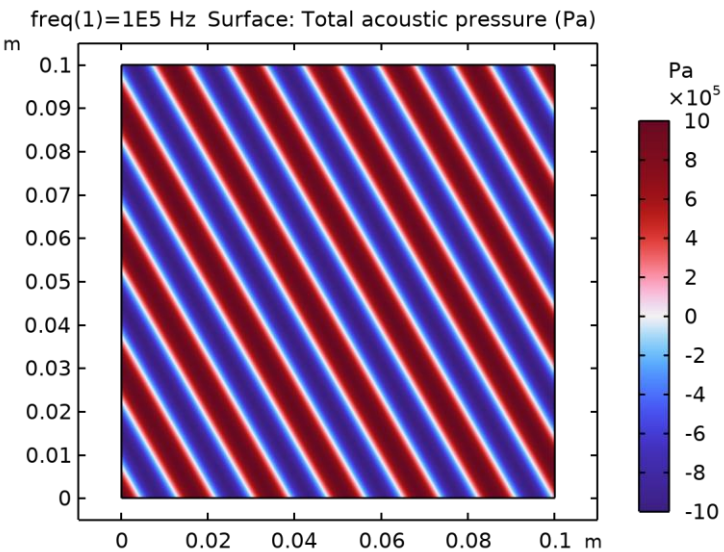
4.2.1 Evaluation 2D

Interactive 2D values

x	y	Value
0.059795	0.05749	0.57735
0.044837	0.071263	0.57735
0.045282	0.043124	0.57735
0.044097	0.051121	0.57735
0.043208	0.064598	0.57735
0.057574	0.070226	0.57735
0.069421	0.065191	0.57735

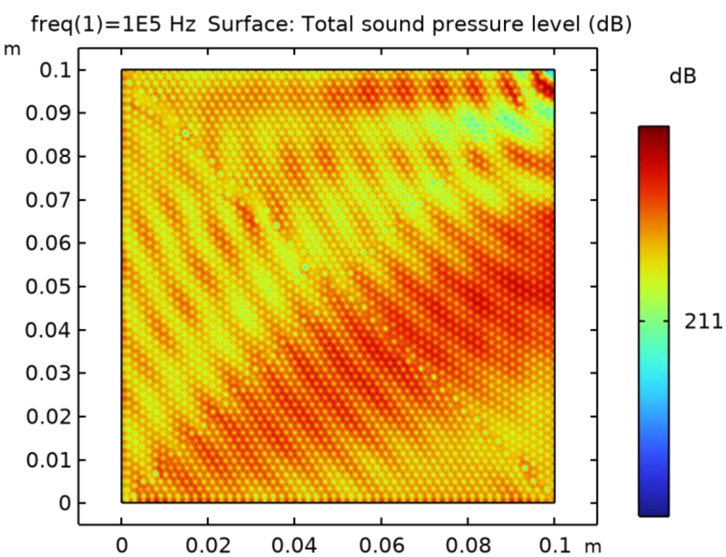
4.3 PLOT GROUPS

4.3.1 Acoustic Pressure (acpr)



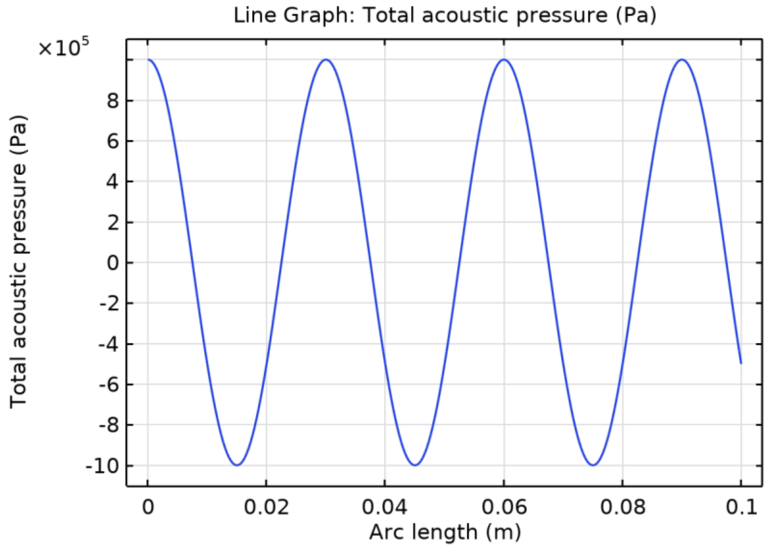
Surface: Total acoustic pressure (Pa)

4.3.2 Sound Pressure Level (acpr)



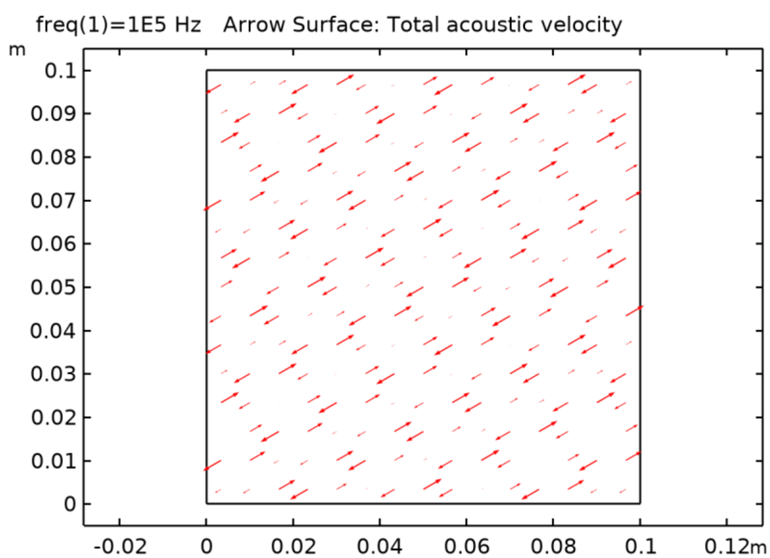
Surface: Total sound pressure level (dB)

4.3.3 1D Plot Group 3



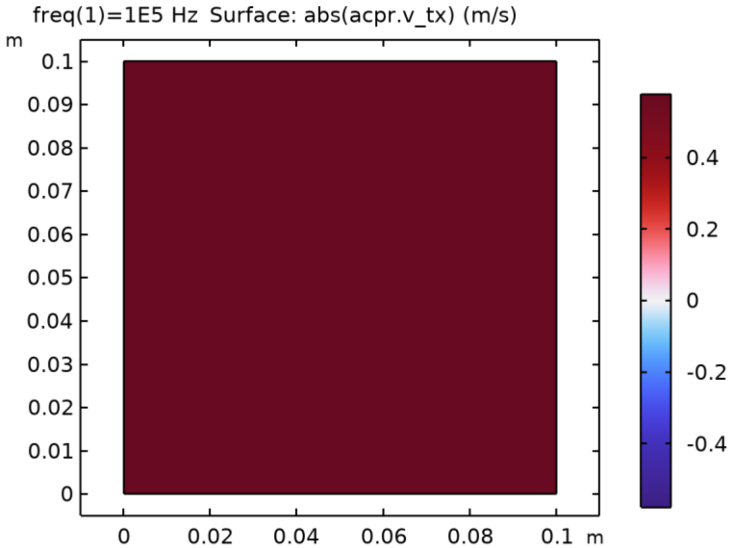
Line Graph: Total acoustic pressure (Pa)

4.3.4 2D Plot Group 4



Arrow Surface: Total acoustic velocity

4.3.5 2D Plot Group 5



Surface: abs(acpr.v_tx) (m/s)