AEEM5063 HW#2

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09.16.24

1.5

$$P = \sin 3t\hat{i} + \cos t\hat{j} + \sin 2t\hat{k}$$

(a)

$$\vec{V} = \dot{P} = 3\cos 3t\hat{i} - \sin t\hat{j} + 2\cos 2t\hat{k} = \boxed{-2.733\hat{i} - 0.141\hat{j} + 1.920\hat{k} \text{ m/s}}$$

(b)

$$v = ||\vec{V}|| = \sqrt{(-2.733)^2 + (-0.141)^2 + (1.920)^2} = \boxed{3.343 \text{ m/s}}$$

(c)

$$\hat{u}_t = \frac{\vec{V}}{v} = \frac{-2.733\hat{i} - 0.141\hat{j} + 1.920\hat{k}}{3.343} = \boxed{-0.818\hat{i} - 0.042\hat{j} + 0.574\hat{k}}$$

(d)

$$\vec{x} = 1\hat{i} + 0\hat{j} + 0\hat{k}$$
$$\vec{y} = 0\hat{i} + 1\hat{j} + 0\hat{k}$$
$$\vec{z} = 0\hat{i} + 0\hat{j} + 1\hat{k}$$

$$\hat{u}_t \cdot \vec{x} = 1(-0.818)\hat{i} + 0\hat{j} + 0\hat{k} = -0.818 = xu_t \cos\theta \rightarrow \theta_x = \cos^{-1}(-0.818)$$

$$\theta_y = \cos^{-1}(-0.042)$$

$$\theta_z = \cos^{-1}(0.574)$$

$$\boxed{\theta_x = 144.885°}$$

$$\boxed{\theta_y = 92.407°}$$

$$\boxed{\theta_z = 54.970°}$$

(e)

$$\vec{a} = \dot{\vec{V}} = -9\sin 3t\hat{i} - \cos t\hat{j} - 4\sin 2t = \boxed{-3.709\hat{i} + 1.000\hat{j} + 1.118\hat{k} \text{ m/s}^2}$$

(f)

$$\hat{u}_b = \frac{\vec{V} \times \vec{a}}{||\vec{V} \times \vec{a}||}$$

$$\vec{V} \times \vec{a} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2.733 & -0.141 & 1.920 \\ -3.709 & 1.000 & 1.118 \end{bmatrix} = ((-0.141)(1.118) - (1.920)(1.000))\hat{i}$$

$$+((-2.733)(1.118) - (1.920)(-3.709))\hat{j} + ((-2.733)(1.000) - (-0.141)(-3.709))\hat{k}$$

$$= -2.078\hat{i} - 4.066\hat{j} - 3.256\hat{k}$$

$$||\vec{V} \times \vec{a}|| = \sqrt{(-2.078)^2 + (-4.066)^2 + (-3.256)^2} = 5.608$$

$$\boxed{\hat{u}_b = -0.371\hat{i} - 0.725\hat{j} - 0.581\hat{k}}$$

(g)

$$\hat{u}_b = \hat{u}_t \times \hat{u}_n \rightarrow \hat{u}_n = \hat{u}_b \times \hat{u}_t$$

$$\hat{u_n} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -0.371 & -0.725 & -0.581 \\ -0.818 & -0.042 & 0.574 \end{bmatrix} = ((-0.725)(0.574) - (-0.581)(-0.042))\hat{i}$$
$$+((-0.371)(0.574) - (-0.581)(-0.818))\hat{j} + ((-0.371)(-0.042) - (-0.725)(-0.818))\hat{k}$$
$$= \boxed{-0.441\hat{i} + 0.688\hat{j} - 0.577\hat{k}}$$

(h)

$$||\vec{a}|| = \sqrt{(-3.709)^2 + (1)^2 + (1.118)^2} = 4.001$$
$$\vec{a} \cdot \vec{x} = 1(-3.709)\hat{i} + 0\hat{j} + 0\hat{k} = -3.709 = x||\vec{a}|| \cos\phi \rightarrow \phi_x = \cos^{-1}(\frac{-3.709}{4.001})$$
$$\phi_y = \cos^{-1}(\frac{1}{4.001})$$
$$\phi_z = \cos^{-1}(\frac{1.118}{4.001})$$
$$\boxed{\phi_x = 157.975°}$$
$$\boxed{\phi_y = 75.526°}$$
$$\boxed{\phi_z = 73.774°}$$

(i)

$$a_t = \vec{a} \cdot \hat{u}_t = (-3.709 * -0.818) + (1.000 * -0.042) + (1.118 * 0.574) = \boxed{3.634 \text{ m/s}^2}$$

(j)

$$a_n = \vec{a} \cdot \hat{u_n} = (-3.709 * -0.441) + (1.000 * 0.688) + (1.118 * -0.577) = \boxed{1.679 \text{ m/s}^2}$$

(k)

$$\rho = \frac{v^2}{a_n} = \frac{3.343^2}{1.679} = \boxed{6.656 \text{ m}}$$

(l)

$$P_C = P + \rho \hat{u_n} = (\sin 9\hat{i} + \cos 3\hat{j} + \sin 6\hat{k}) + 6.656(-0.441\hat{i} + 0.688\hat{j} - 0.577\hat{k})$$
$$= \boxed{-2.523\hat{i} + 3.589\hat{j} - 4.120\hat{k} \text{ m}}$$

1.7

$$W = G\frac{m_e m}{r_e^2}$$

$$F_{moon} = G\frac{m_e m}{r_{moon}^2}$$

$$Gm_e m = Wr_e^2 \rightarrow F_{moon} = \frac{Wr_e^2}{r_{moon}^2} = W\left(\frac{r_e}{r_{moon}}\right)^2$$

$$r_e = 6378\text{km}; \quad r_{moon} = 384400\text{km}$$

$$\boxed{F = 2.753 * 10^{-4} W}$$

1.9

$$F = ma \rightarrow F_n = ma_n$$

$$a_n = \frac{v^2}{r} \rightarrow v = \sqrt{ra_n}$$

$$F_n = G\frac{Mm}{r^2} = ma_n \rightarrow a_n = \frac{GM}{r^2}$$

$$\boxed{v = \sqrt{\frac{GM}{r}}}$$

1.12

$$\vec{V_P} = \vec{V_O} + \Omega \times r_{rel} + \vec{V_{rel}}$$

$$r_{rel} = r - r_O = (51+16)\hat{I} + (-45-84)\hat{J} + (36-59)\hat{K} = 67\hat{I} - 129\hat{J} - 23\hat{K}$$

$$\begin{aligned}
\vec{V_{rel}} = 31\hat{i} - 68\hat{j} - 77\hat{k} = 31(-0.15617\hat{I} - 0.31235\hat{J} + 0.93704\hat{K}) - 68(-0.12940\hat{I} + 0.94698\hat{J} \\
+0.29409\hat{K}) - 77(-0.97922\hat{I} - 0.075324\hat{J} - 0.18831\hat{K}) = 79.358\hat{I} - 68.277\hat{J} + 23.550\hat{K} \\
\vec{V_P} = 7\hat{I} + 9\hat{J} + 4\hat{K} + \begin{bmatrix} \hat{I} & \hat{J} & \hat{K} \\ -0.8 & 0.7 & 0.4 \\ 67 & -129 & -23 \end{bmatrix} + 79.358\hat{I} - 68.277\hat{J} + 23.550\hat{K} \\
= 7\hat{I} + 9\hat{J} + 4\hat{K} + 35.5\hat{I} + 8.4\hat{J} + 56.3\hat{K} + 79.358\hat{I} - 68.277\hat{J} + 23.550\hat{K} \\
= 121.858\hat{I} - 50.877\hat{J} + 83.850\hat{K} \\
||\vec{V_P}|| = \sqrt{(121.858)^2 + (-50.877)^2 + (83.850)^2} = 156.42 \\
\boxed{V_P = 156.42 \mathrm{m/s^2}; \quad \hat{u_V} = 0.779\hat{I} - 0.325\hat{J} + 0.536\hat{K}}
\end{aligned}$$

$$\vec{a_P} = \vec{a_O} + \dot{\Omega} \times r_{rel} + \Omega \times (\Omega \times r_{rel}) + 2\Omega \times \vec{V_{rel}} + a_{rel}$$

$$\begin{aligned}
\dot{\Omega} \times r_{rel} = \begin{bmatrix} \hat{I} & \hat{J} & \hat{K} \\ -0.4 & 0.9 & -1.0 \\ 67 & -129 & -23 \end{bmatrix} = -149.7\hat{I} - 76.2\hat{J} - 8.7\hat{K} \\
\Omega \times (\Omega \times r_{rel}) = \begin{bmatrix} \hat{I} & \hat{J} & \hat{K} \\ -0.8 & 0.7 & 0.4 \\ 35.5 & 8.4 & 56.3 \end{bmatrix} = 36.05\hat{I} + 59.24\hat{J} - 31.57\hat{K} \\
2\Omega \times \vec{V_{rel}} = 2\begin{bmatrix} \hat{I} & \hat{J} & \hat{K} \\ -0.8 & 0.7 & 0.4 \\ 79.358 & -68.277 & 23.550 \end{bmatrix} = 87.592\hat{I} + 101.166\hat{J} - 1.858\hat{K}
\end{aligned}$$

$$\begin{aligned}
a_{rel} = 2\hat{i} - 6\hat{j} + 5\hat{k} = 2(-0.15617\hat{I} - 0.31235\hat{J} + 0.93704\hat{K}) - 6(-0.12940\hat{I} + 0.94698\hat{J} \\
+0.29409\hat{K}) + 5(-0.97922\hat{I} - 0.075324\hat{J} - 0.18831\hat{K}) = -4.432\hat{I} - 6.683\hat{J} - 0.832\hat{K}
\end{aligned}$$

$$\vec{a_P} = 3\hat{I} - 7\hat{J} + 4\hat{K} - 149.7\hat{I} - 76.2\hat{J} - 8.7\hat{K} + 36.05\hat{I} + 59.24\hat{J} - 31.57\hat{K} + 87.592\hat{I}$$
$$+101.166\hat{J} - 1.858\hat{K} - 4.432\hat{I} - 6.683\hat{J} - 0.832\hat{K} = -27.49\hat{I} + 70.523\hat{J} - 38.96\hat{K}$$
$$||\vec{a_P}|| = \sqrt{(-27.49)^2 + (70.523)^2 + (-38.96)^2} = 85.130$$
$$\boxed{a_P = 85.130\mathrm{m/s^2}; \quad \hat{u_a} = -0.323\hat{I} + 0.828\hat{J} - 0.458\hat{K}}$$

1.23

Contents

Initialize

```
clear; clc;
```

Define variables and constants

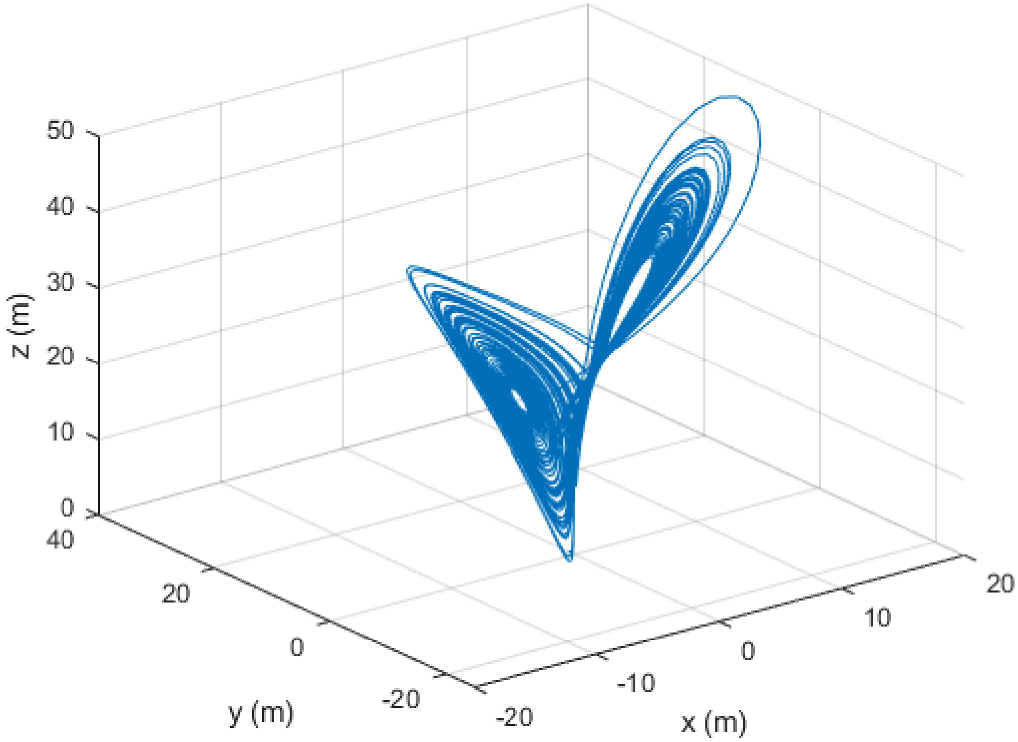
```
tspan = [0,100];
x0 = 0;
y0 = 1;
z0 = 0;
ics = [x0; y0; z0];
```

Solve non-stiff differential equations, medium order method.

```
[t, y] = ode45(@dstate, tspan, ics);
```

Plot results

```
figure;
clf;
plot3(y(:,1), y(:,2), y(:,3));
grid;
xlabel('x (m)');
ylabel('y (m)');
zlabel('z (m)');
```



Define derivative function

```
function ddt = dstate(t, yi)
```

Define constants

```
    sigma = 10;
    beta = 8/3;
    rho = 28;
```

get state from inputs

```
    x = yi(1);
    y = yi(2);
    z = yi(3);
```

Define derivatives

```
    dxdt = sigma*(y - x);
    dydt = x*(rho - z) - y;
    dzdt = x*y - beta*z;
```

create output vector

```
    ddt = [dxdt; dydt; dzdt];
```

```
end
```

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