

AEEM5063 HW#3

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2.12

$$\vec{V} = -4\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\hat{u}_r = 0.26726\hat{i} + 0.53452\hat{j} + 0.80178\hat{k}$$

(a)

$$v_r = \vec{V} \cdot \hat{u}_r = -4 * 0.26726 + 3 * 0.53452 - 5 * 0.80178$$

$$v_r = -3.474 \text{ km/s}$$

(b)

$$v = \sqrt{v_r^2 + v_{\perp}^2} \rightarrow v_{\perp} = \sqrt{v^2 - v_r^2}$$

$$v = \sqrt{(-4)^2 + 3^2 + (-5)^2} = 50$$

$$v_{\perp} = \sqrt{50^2 - (-3.474)^2} = 6.159 \text{ km/s}$$

(c)

$$\tan \gamma = \frac{v_r}{v_{\perp}} = \frac{-3.474}{6.159}$$

$$\gamma = -29.43^{\circ}$$

2.14

$$\begin{aligned}
 \vec{V} &= 2\hat{i} + 3\hat{j} + 4\hat{k} \\
 v &= \sqrt{2^2 + 3^2 + 4^2} = 5.3852 \\
 u_V &= \frac{\vec{V}}{v} = 0.37139\hat{i} + 0.55709\hat{j} + 0.74278\hat{k} \\
 \epsilon &= \frac{v^2}{2} - \frac{\mu}{r} = \frac{5.3852^2}{2} - \frac{398600}{10000} = -25.36 \\
 -25.36 &= \frac{0^2}{2} - \frac{398600}{r} \rightarrow r = 15718 \\
 \vec{r} &= ru_V = 15718(0.37139\hat{i} + 0.55709\hat{j} + 0.74278\hat{k}) \\
 \boxed{\vec{r} &= 5837.4\hat{i} + 8756.1\hat{j} + 11675\hat{k} \text{ km}}
 \end{aligned}$$

2.16

$$\begin{aligned}
 \mu &= 42828 \text{ km}^3/\text{s}^2; \quad R = 3396 \text{ km} \\
 v &= \frac{\mu}{r} = \frac{42828}{3396} = \boxed{3.4511 \text{ km/s}} \\
 T &= \frac{2\pi}{\sqrt{\mu}} r^{3/2} = \frac{2\pi}{\sqrt{42828}} (3396 + 200)^{3/2} = 6547 \text{ s} \\
 \boxed{T &= 1 \text{ hr, } 49 \text{ min, } 7 \text{ s}}
 \end{aligned}$$

2.5

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Initialize

```
clear; clc;
```

Define variables and constants

```
tspan = [0 24*3600];  
mu = 398600;  
R = 6378;  
r0 = [6600 0 0];  
v0 = [0 12 0];  
ics = [r0'; v0'];
```

Solve non-stiff differential equations, medium order method.

```
[t, y] = ode45(@dstate, tspan, ics);
```

print results

```
fprintf("dist. @ 24hr: %.2f km \n", norm([y(end,1) y(end,2) y(end,3)]));  
fprintf("speed @ 24hr: %.2f km/s", norm([y(end,4) y(end,5) y(end,6)]));
```

```
dist. @ 24hr: 463290.13 km  
speed @ 24hr: 4.99 km/s
```

Plot results

```
%figure;  
%clf;  
%plot3(y(:,1), y(:,2), y(:,3));  
%grid;  
%xlabel('x (m)');
```

```
%ylabel('y (m)');  
%zlabel('z (m)');
```

Define derivative function

```
function ddt = dstate(t, yi)
```

define constants

```
mu = 398600;
```

get state from inputs

```
x = yi(1);  
y = yi(2);  
z = yi(3);  
vx = yi(4);  
vy = yi(5);  
vz = yi(6);  
r = norm([x y z]);
```

Define derivatives

```
ax = -mu*x/r^3;  
ay = -mu*y/r^3;  
az = -mu*z/r^3;
```

create output vector

```
ddt = [vx; vy; vz; ax; ay; az];
```

```
end
```