AEEM 3062

HOMEWORK # 5

2022-2023 SPRING

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Problem 1

An aluminum bar with square cross section of size a = 15 mm and length L = 750 mm is clamped at one end and loaded by an axial force F = 10 kN at the other end as shown in Fig. 1. The bar is instrumented with a 45° strain rosette mounted at the center of the side of the bar as shown in Fig. 1. Determine the strains that are measured by the three strain gages labeled 1, 2, and 3. [20 pts]

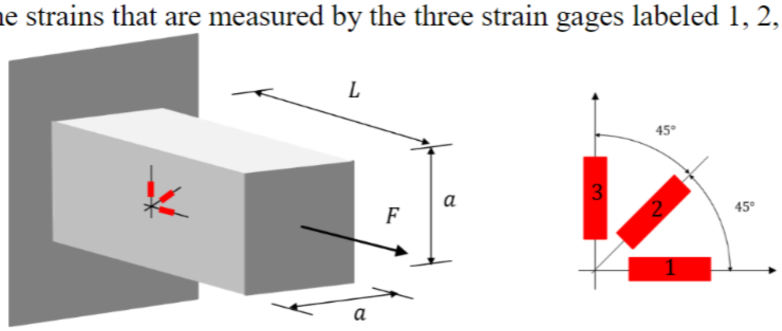


Figure 1

First we will find the poisson's ratio and young's modulus of aluminum from slides:

$$E = 6.89E10Pa \quad v = 0.34$$

Then we will find ϵ_{xx}, ϵ_{yy}, and ϵ_{zz} given by:

$$\epsilon_{xx} = \frac{F}{EA} \quad \epsilon_{yy} = -v\frac{F}{EA}$$

ϵ_{zz} is 0 since the rosette is mounted on the XY plane.

$$\epsilon_{xx} = \frac{10000N}{6.89E10Pa * 0.015^2 m^2} = 6.45E-4$$

$$\epsilon_{yy} = -(0.34)\frac{10000N}{6.89E10Pa * 0.015^2 m^2} = -2.193E-4$$

Then we can determine the strain in each direction by using the equations for a 45° strain rosette:

$$\epsilon_1 = \epsilon_{xx} = \boxed{6.45E-4 = \epsilon_1}$$

$$\epsilon_2 = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} + \frac{\gamma_{xy}}{2} = \frac{6.45E-4-2.193E-4}{2} + \frac{0}{2} = \boxed{2.1287E-4 = \epsilon_2}$$

$$\epsilon_3 = \epsilon_{zz} = \boxed{0 = \epsilon_3}$$

Problem 2

A bar is formed by welding together two circular rods of length a and of cross sectional areas A_1 and A_2 as shown in Fig. 2. The bar is connected to a motor and rotates about pivot A at a constant angular speed ω. End C of the bar is pinned to a roller that touches on a circular wall which can be considered to be perfectly rigid. Assuming that the material of the rod has Young's modulus E, mass density, ρ, that friction effects are negligible, and that the bar is sufficiently slender:

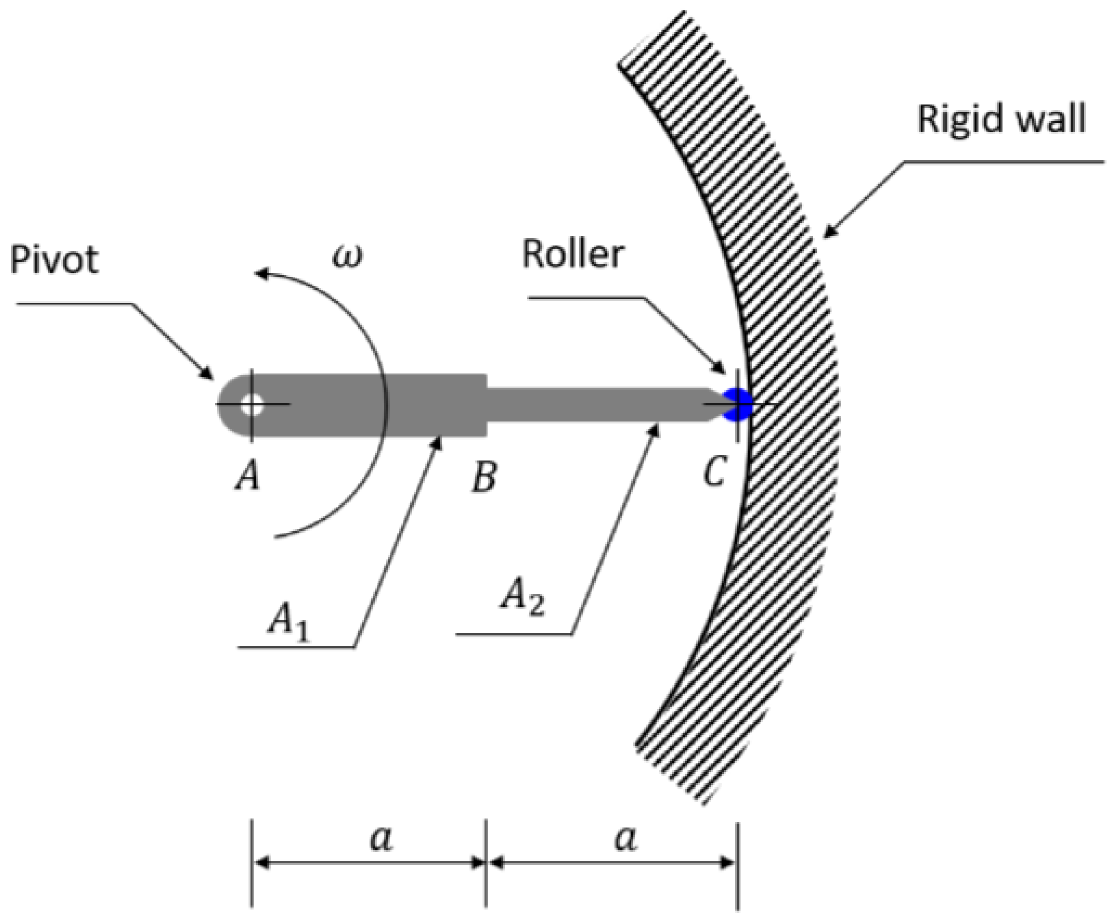


Figure 2

Part A

Write the system of equations that is needed to find the expressions of the functions describing how the normal force N and the axial displacement u vary along the length of the bar. Write the system of equations in a symbolic matrix form [30 pts]

For section A-B, the body force is:

$$n(x) = A_1 \rho \omega^2 x$$

Then the normal force is described by:

$$\frac{\partial N}{\partial x} + n(x) = 0$$

$$\frac{\partial N}{\partial x} = -A_1 \rho \omega^2 x$$

Then the integral is taken:

$$N(x) = -A_1 \rho \omega^2 \frac{x^2}{2} + c_1$$

For section B-C, the normal force is the same but with a different area and coefficient:

$$N(x) = -A_2 \rho \omega^2 \frac{x^2}{2} + c_2$$

Then we find u(x) by:

$$\frac{du}{dx} = \frac{N}{EA}$$

For section A-B:

$$\frac{du}{dx} = \frac{N}{EA} = \frac{-A_1 \rho \omega^2 x^2}{2EA} + \frac{c_1}{EA}$$

And then taking the integral gives:

$$u(x) = \frac{-\rho \omega^2 x^3}{6E} + \frac{c_1 x}{EA_1} + c_3$$

And for section B-C:

$$u(x) = \frac{-\rho \omega^2 x^3}{6E} + \frac{c_2 x}{EA_2} + c_4$$

Then we will define the boundary conditions at locations A and C:

$$u(A) = u(0) = 0$$
$$u(C) = u(2a) = 0$$

Then at point B, we know that N(x) A-B and N(x) B-C must be equal so that the forces are balanced. Also, u(x) for both must equal each other so that the beam is not merging into itself or ripping apart.

This allows us to substitute to get 2 new equations:

$$-A_1\rho\omega^2\frac{x^2}{2}+c_1=-A_2\rho\omega^2\frac{x^2}{2}+c_2$$

$$\frac{-\rho\omega^2x^3}{6E}+\frac{c_1x}{EA_1}+c_3=\frac{-\rho\omega^2x^3}{6E}+\frac{c_2x}{EA_2}+c_4$$

Then we can rearrange everything to have all of the unknown constants on one side:

$$c_1-c_2=-A_2\rho\omega^2\frac{x^2}{2}+A_1\rho\omega^2\frac{x^2}{2}$$

which is at N(a) so x=a giving:

$$c_1-c_2=-A_2\rho\omega^2\frac{a^2}{2}+A_1\rho\omega^2\frac{a^2}{2}$$

We do the same for this equation:

$$\frac{c_1x}{EA_1}+c_3-\frac{c_2x}{EA_2}-c_4=0$$

$$\frac{c_1a}{EA_1}+c_3-\frac{c_2a}{EA_2}-c_4=0$$

We also know that:

$$u(A)=u(0)=0=\frac{-\rho\omega^2 0^3}{6E}+\frac{c_1 0}{EA_1}+c_3=c_3=0$$

$$u(C)=u(2a)=0=\frac{-\rho\omega^2(2a)^3}{6E}+\frac{c_2 2a}{EA_2}+c_4$$

$$\frac{2a}{EA_2}c_2+c_4=\frac{8}{6}\frac{\rho\omega^2a^3}{E}$$

Then we can use these equations to create an Ax=b matrix system:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & \frac{2a}{EA_2} & 0 & 1 \\ 1 & -1 & 0 & 0 \\ \frac{a}{EA_1} & -\frac{a}{EA_2} & 1 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{8}{6}\frac{\rho\omega^2 a^3}{E} \\ (A_1 - A_2)\rho\omega^2 \frac{a^2}{2} \\ 0 \end{bmatrix}$$

Part B

Solve the system of equations assuming that: a = 250 mm, the diameters of the rods are D = 15 mm and d = 8 mm, that the material is steel with E=200 GPa and ρ = 8000 kg/m^3. Moreover the angular speed of the bar is 300 rpm. Provide the expressions of the functions describing how the displacement u and normal force N vary along the length of the bar with the appropriate units [20 pts]

We can use the given info $a = .25$ m, $D_1 = .015$ m, $D_2 = .008$ m, $E = 2E11$ Pa, $\rho = 8000$ kg/m^3, and $\omega = 31.42$ rad/s to plug in the matrix and solve for our constants:

$$A = \pi(\frac{D}{2})^2$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 4.9736E-8 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 7.074E-9 & -2.4868E-8 & 1 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 8.227E-7 \\ 31.208 \\ 0 \end{bmatrix}$$

Then using Matlab we can solve for the constants:

$$c_1 = 50.05 \quad c_2 = 18.845 \quad c_3 = 0 \quad c_4 = -1.1456E-7$$

Then we can put these constants back into the equations to define N and u along the length of the bar:

$$\boxed{N(x)_{A\to B} = -A_1\rho\omega^2\frac{x^2}{2} + 50.05}$$

$$\boxed{N(x)_{B\to C} = -A_2\rho\omega^2\frac{x^2}{2} + 18.845}$$

both with units of Newtons

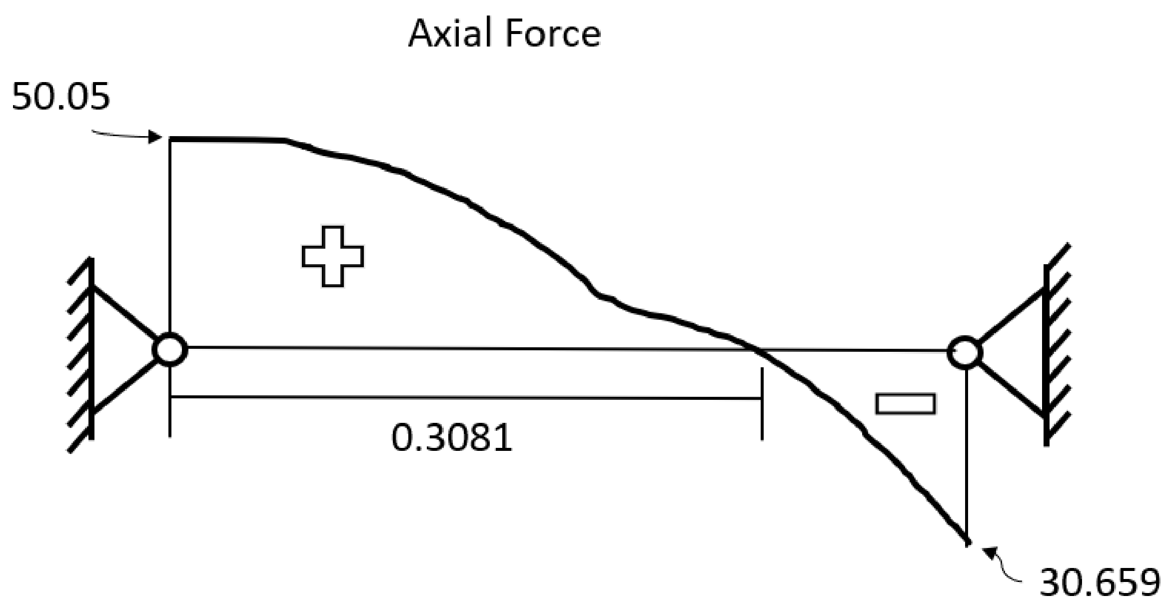
$$\boxed{u(x)_{A\to B} = \frac{-\rho\omega^2 x^3}{6E} + \frac{50.05x}{EA_1}}$$

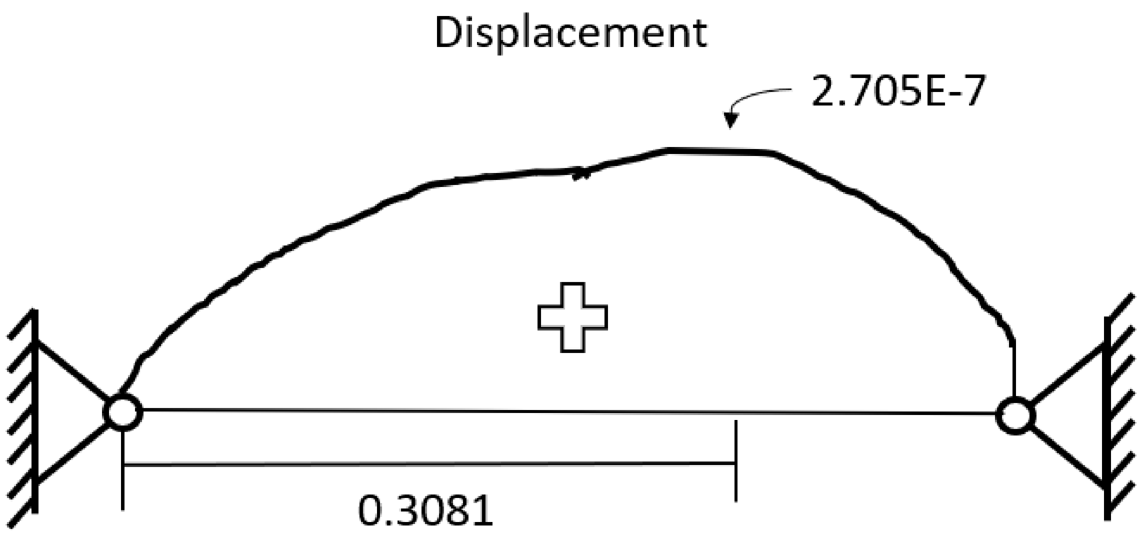
$$\boxed{u(x)_{B\to C} = \frac{-\rho\omega^2 x^3}{6E} + \frac{18.845x}{EA_2} - 1.1456E-7}$$

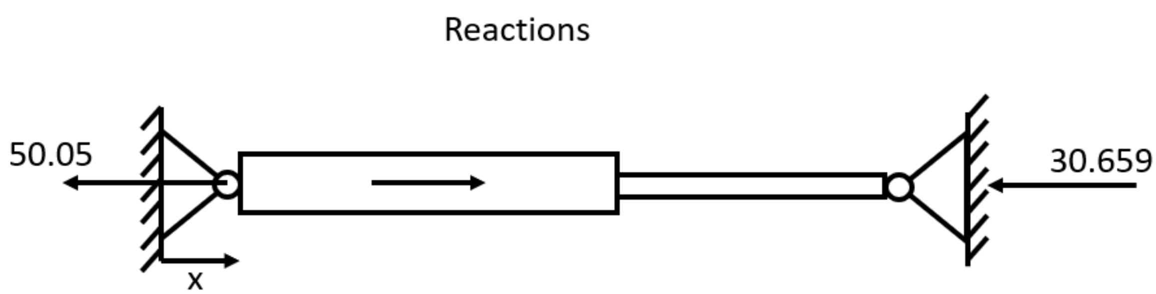
both with units of meters

Part C

Plot the axial force, displacement, and reaction diagrams (no explanation needed) [20 pts]







Part D

Find the larges tensile and compressive stresses [10 pts]

The largest tensile and compressive stress can be gotten from the axial force diagram. The maximum tensile stress is at the largest positive axial force and the max compressive is at the largest negative axial force.

However, these values are forces and stress is force per area. To find the stress we will divide by the cross sectional area at those points.

Max tensile = $\frac{50.05}{\pi(.015/2)^2} = 283225$ Pa

Max compressive = $\frac{30.659}{\pi(.008/2)^2} = 609941$ Pa