AEEM4063 - Assignment 5

Slade Brooks

11.22.2023

Problem 1

$\dot{m}$	20	kg/s
T_{01}	1000	K
P_{01}	4	bar
C_a	260	m/s
U	360	m/s
α_2	65°	
α_3	10°	
λ_N	0.05	

$\frac{U}{C_{a2}} = \tan\alpha_2 - \tan\beta_2 \rightarrow \frac{360}{260} = \tan 65 - \tan\beta_2$

$\boxed{\beta_2 = 37.23°}$

$\frac{U}{C_{a3}} = \tan\beta_3 - \tan\alpha_3 \rightarrow \frac{360}{260} = \tan\beta_3 - \tan 10$

$\boxed{\beta_3 = 57.35°}$

$\Lambda = \frac{C_a}{2U}(\tan\beta_3 - \tan\beta_2) = \frac{260}{2(360)}(\tan 57.35 - \tan 37.23) = 0.29$

$\boxed{\Lambda = 0.29}$

$\psi = \frac{2C_a}{U}(\tan\beta_2 + \tan\beta_3) = \frac{2(260)}{360}(\tan 37.23 + \tan 57.35) = 3.35$

$\boxed{\psi = 3.35}$

$W = \dot{m}w = \dot{m}UC_a(\tan\alpha_2 + \tan\alpha_3) = 20 * 360 * 260(\tan 65 + \tan 10) = 4344601$ W

$\boxed{W = 4345 \text{ kW}}$

$\sin(90 - \alpha_2) = \frac{C_{a2}}{C_2} \rightarrow C_2 = 615.2$ m/s

$T_2 = T_{02} - \frac{C_2^2}{2c_p}, \quad T_{02} = T_{01}, \quad T_2 = T_{01} - \frac{C_2^2}{2c_p} = 1000 - \frac{615.2^2}{2(1148)} = 835.2$ K

$\lambda_N = \frac{T_2 - T_2'}{C_2^2/2c_p} \rightarrow T_2' = T_2 - \frac{\lambda_N C_2^2}{2c_p} = 835.2 - \frac{0.05(615.2)^2}{2(1148)} = 827$ K

$\frac{P_{01}}{P_2} = (\frac{T_{01}}{T_2'})^{\frac{\gamma}{\gamma-1}} \rightarrow P_2 = \frac{P_{01}}{(\frac{T_{01}}{T_2'})^{\frac{\gamma}{\gamma-1}}} = \frac{4}{(1000/827)^{1.333/0.333}} = 1.87$ bar, $\quad \frac{P_{01}}{P_2} = 2.14$

$PR_{crit} = \frac{\gamma+1}{2}^{\frac{\gamma}{\gamma-1}} = 1.853, \quad \frac{P_{01}}{P_2} > PR_{crit},$ nozzle is choked

$P_{crit} = P_{01}/1.853 = 2.16 \text{ bar} = P_2$

$\rho_2 = \frac{P_2}{RT_2} = \frac{2.16*10^5}{287(835.2)} = 0.901$ kg/m^3

$V = M\sqrt{\gamma RT_2} = 1\sqrt{1.333 * 287 * 835.2} = 565.3$ m/s

$\dot{m} = \rho VA \rightarrow A = \frac{\dot{m}}{\rho V} = \frac{20}{0.901(565.3)} = 0.0393$ m^2

$\boxed{A = 0.0393 \text{ m}^2}$

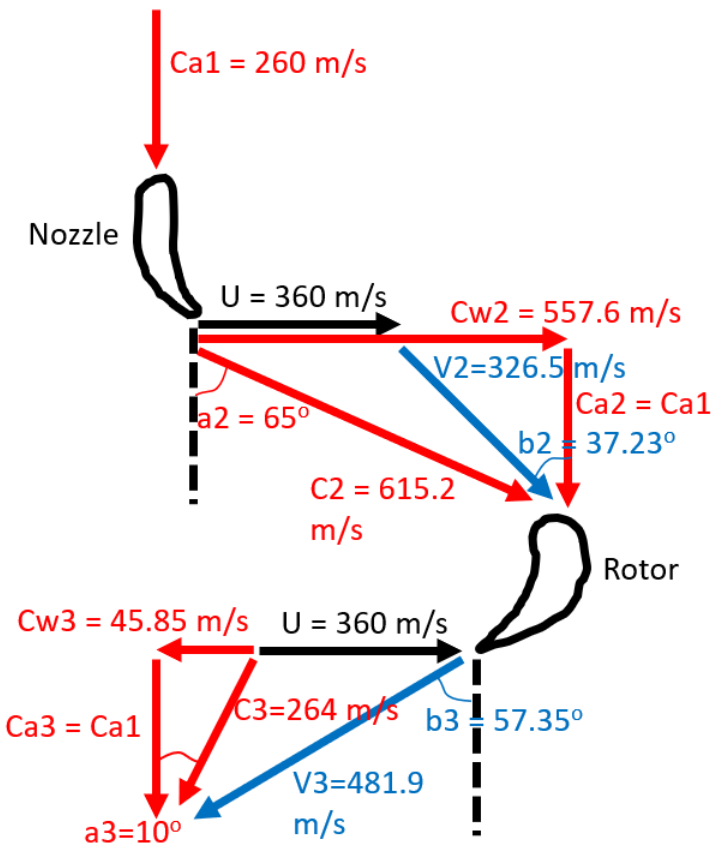
$\cos\beta_2 = \frac{C_{a2}}{V_2} \rightarrow V_2 = 326.5$ m/s

$\tan\alpha_2 = \frac{C_{w2}}{C_{a2}} \rightarrow C_{w2} = 557.6$ m/s

$\tan\alpha_3 = \frac{C_{w3}}{C_{a3}} \rightarrow C_{w3} = 45.85$ m/s

$\cos\alpha_3 = \frac{C_{a3}}{C_3} \rightarrow C_3 = 264$ m/s

$\cos\beta_3 = \frac{C_{a3}}{V_3} \rightarrow V_3 = 481.9$ m/s



Problem 2

T_{01}	1350	K
P_{01}	5.2	bar
$\frac{P_{01}}{P_{03}}$	3.4	
C_3	275	m/s
U_r	500	m/s
η_t	0.91	
$\frac{r_t}{r_h}$	1.4	
λ_N	0.05	
Λ_r	0	

$C_1 = C_3 = C_{a1} = C_{a3}$

Hub

$\Delta T_{013} = \eta_t T_{01}[1 - (\frac{P_{03}}{P_{01}})^{\frac{\gamma-1}{\gamma}}] = 0.91(1350)[1 - \frac{1}{3.4}^{0.333/1.333}] = 323.6$ K
$w = U(C_{w2} + C_{w3}) = c_p \Delta T_{013} \rightarrow C_{w2} = \frac{c_p \Delta T_{013}}{U} = \frac{1148(323.6)}{500} = 743$ m/s
$T_2 = T_{02} - \frac{C_2^2}{2c_p}, \quad T_2 = T_3$ because $\Lambda = 0$
$T_{02} - \frac{C_2^2}{2c_p} = T_{03} - \frac{C_3^2}{2c_p}, \quad T_{02} = T_{01}$
$C_2^2 = 2c_p(\frac{C_3^2}{2c_p} + \Delta T_{013}) = 2(1148)(\frac{275^2}{2(1148)} + 323.6) \rightarrow C_2 = 904.77$ m/s
$\sin \alpha_2 = \frac{C_{w2}}{C_2} \rightarrow \alpha_2 = 55.2°$
$\boxed{\alpha_2 = 55.2°}$

$C_{w2}^2 + C_{a2}^2 = C_2^2 \rightarrow C_{a2} = 516.3$ m/s
$\tan \beta_2 = \frac{C_{w2} - U}{C_{a2}} \rightarrow \beta_2 = 25.2°$
$\boxed{\beta_2 = 25.2°}$

Tip

since free vortex design, $C_{a2r} = C_{a2t}$
$C_{w2t} r_t = C_{w2h} r_h \rightarrow C_{w2t} = C_{w2h} \frac{r_h}{r_t} = 743 \frac{1}{1.4} = 530.7$ m/s
$\tan \alpha_{2t} = \frac{C_{w2t}}{C_{a2t}} \rightarrow \alpha_{2t} = 45.8°$
$\boxed{\alpha_2 = 45.8°}$

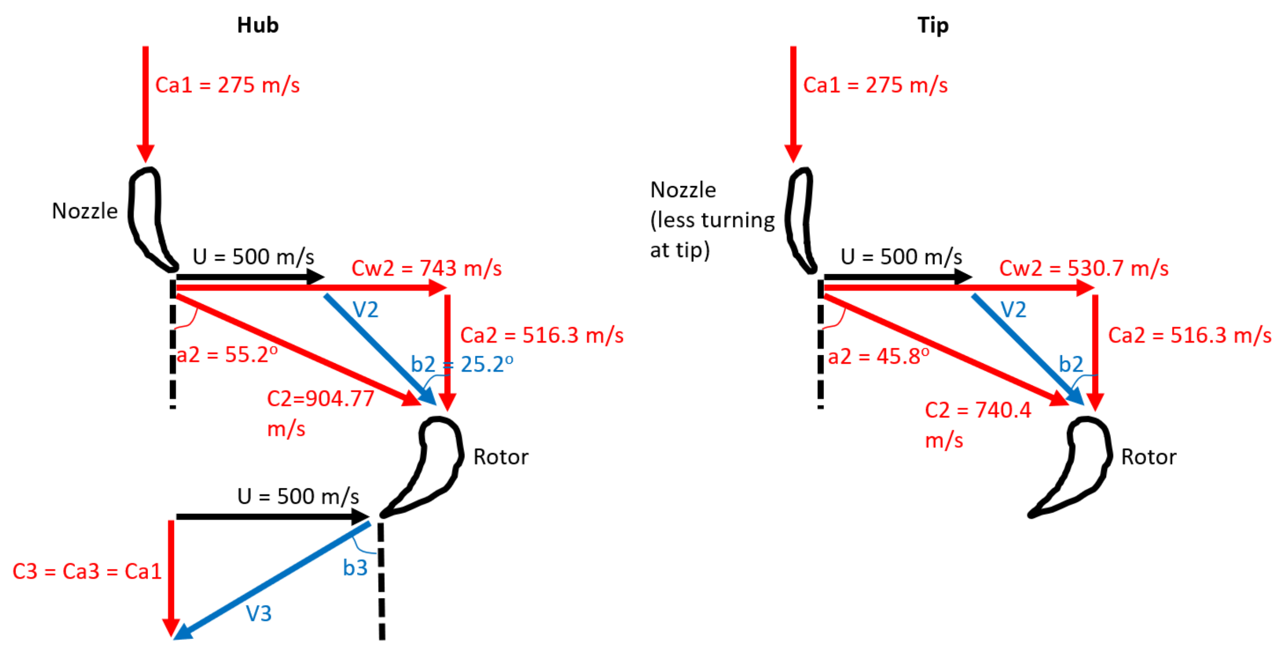
$T_{3t} = T_{3h} = T_{03} - \frac{C_3^2}{2c_p} = 1350 - 323.6 - \frac{275^2}{2(1148)} = 993.5$ K
$C_{2t} = \sqrt{C_{w2t}^2 + C_{a2t}^2} = 740.4$ m/s
$T_{2t} = T_{02} - \frac{C_{2t}^2}{2c_p} = 1350 - \frac{740.4^2}{2(1148)} = 1111.2$ K
$\Lambda_t = \frac{T_{2t} - T_{3t}}{T_{01} - T_{03}} = \frac{1111.2 - 993.5}{323.6} = 0.364$
$\boxed{\Lambda = 0.364}$

Hub Static Pressures

$\lambda_N = \frac{T_2 - T_2'}{C_2^2/2c_p} \to T_2' = T_2 - \frac{\lambda_N C_2^2}{2c_p} \to T_2' = T_{02} - \frac{C_2^2}{2c_p} - \frac{\lambda_N C_2^2}{2c_p} \to$
$T_2' = T_{02} - (1+\lambda_N)\frac{C_2^2}{2c_p} = 1350 - (1+0.05)\frac{904.77^2}{2(1148)} = 975.6$ K
$\frac{P_{02}}{P_2} = (\frac{T_{02}}{T_2'})^{\frac{\gamma}{\gamma-1}} \to P_2 = 1.42$ bar
$\frac{P_{03}}{P_3} = (\frac{T_{03}}{T_3'})^{\frac{\gamma}{\gamma-1}} \to P_3 = 1.25$ bar
$\boxed{P_2 = 1.42 \text{ bar}}$
$\boxed{P_3 = 1.25 \text{ bar}}$

since $P_3 < P_2$, some expansion is happening at the root in the rotor blade passages

Velocity Triangles



Problem 3

$\dot{m}$	36	kg/s
T_{01}	1200	K
P_{01}	8	bar
ΔT_{013}	150	K
η_t	0.9	
U_m	320	m/s
N	250	rev/s
C_3	400	m/s
λ_N	0.07	
C_a	346	m/s

$T_{03} = T_{01} - \Delta T_{013} = 1200 - 150 = 1050$ K
$T_3 = T_{03} - \frac{C_3^2}{2c_p} = 1050 - \frac{400^2}{2(1148)} = 980.3$ K
$\Delta T_{013} = \eta_t T_{01}[1 - (\frac{P_{03}}{P_{01}})^{\frac{\gamma-1}{\gamma}}] \to \frac{P_{03}}{P_{01}} = (1 - \frac{\Delta T_{013}}{\eta_t T_{01}})^{\frac{\gamma}{\gamma-1}} \to P_{03} = 4.4$ bar
$\frac{P_{03}}{P_3} = (\frac{T_{03}}{T_3})^{\frac{\gamma}{\gamma-1}} \to P_3 = 3.3$ bar
$U_m = \omega r_m = 2\pi N r_m \to r_m = \frac{U_m}{2\pi N} = \frac{320}{2\pi(250)} = 0.204$ m
$\dot{m} = \rho A C_3 \to A = \frac{\dot{m}}{\rho C_3}$
$\rho = \frac{P_3}{RT_3} = \frac{3.3*10^5}{287(980.3)} = 1.173$ kg/m^3
$A = Ch = 2\pi r_m h \to h = \frac{\dot{m}}{2\pi\rho C_3 r_m} = \frac{36}{2\pi(1.173)(400)(0.204)} = 0.0599$ m
$\boxed{h = 0.0599 \text{ m}}$

$r_t = r_m + h/2 = 0.234$ m
$r_h = r_m - h/2 = 0.174$ m
$\frac{r_t}{r_h} = \frac{0.234}{0.174} = 1.34$
$\boxed{\frac{r_t}{r_h} = 1.34}$

$w = C_{w2}U = c_p \Delta T_{013} \to C_{w2} = 538.1$ m/s, $\quad C_{a2} = 346$ m/s
$C_2 = \sqrt{C_{w2}^2 + C_{a2}^2} = 639.7$ m/s
$T_2 = T_{02} - \frac{C_2^2}{2c_p}, \quad T_{02} = T_{01}, \quad T_2 = 1021.77$ K

$T_2' = T_2 - \frac{\lambda_N C_2^2}{2c_p} = 1021.77 - \frac{0.07(639.7)^2}{2(1148)} = 1009.3$ K
$\frac{P_{02}}{P_2} = (\frac{T_{02}}{T_2'})^{\frac{\gamma}{\gamma-1}} \rightarrow P_2 = 4$ bar
$\rho_2 = \frac{P_2}{RT_2} = \frac{4*10^5}{287(1021.77)} = 1.36$ kg/m^3
$A = \frac{\dot{m}}{\rho_3 C_3} = \frac{36}{1.173(400)} = 0.076$ m^2
$C_{a2} = \frac{\dot{m}}{\rho_2 A_3} = \frac{36}{1.36(0.076)} = 348.3$ m/s
$C_{w2h} = C_{w2m}\frac{r_m}{r_h} = 538.1\frac{0.204}{0.174} = 630.9$ m/s
$C_{2h} = \sqrt{C_{w2h}^2 + C_{a2}^2} = 720.7$ m/s
$U_h = U_m \frac{r_h}{r_m} = 273$ m/s
$V_{2h} = \sqrt{C_{a2}^2 + (C_{w2h} - U_h)^2} = 499.4$ m/s
$T_{2h} = T_{01} - \frac{C_{2h}^2}{2c_p} = 973.8$ K
$M_h = \frac{V_{2h}}{\sqrt{\gamma R T_{2h}}} = 0.818$
$\boxed{M_h = 0.818}$