

AEEM4063 - Assignment 5

Slade Brooks

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Problem 1

\dot{m}	20	kg/s
T_{01}	1000	K
P_{01}	4	bar
C_a	260	m/s
U	360	m/s
α_2	65°	
α_3	10°	
λ_N	0.05	

$$\frac{U}{C_{a2}} = \tan \alpha_2 - \tan \beta_2 \rightarrow \frac{360}{260} = \tan 65 - \tan \beta_2$$

$$\boxed{\beta_2 = 37.23^\circ}$$

$$\frac{U}{C_{a3}} = \tan \beta_3 - \tan \alpha_3 \rightarrow \frac{360}{260} = \tan \beta_3 - \tan 10$$

$$\boxed{\beta_3 = 57.35^\circ}$$

$$\Lambda = \frac{C_a}{2U} (\tan \beta_3 - \tan \beta_2) = \frac{260}{2(360)} (\tan 57.35 - \tan 37.23) = 0.29$$

$$\boxed{\Lambda = 0.29}$$

$$\psi = \frac{2C_a}{U} (\tan \beta_2 + \tan \beta_3) = \frac{2(260)}{360} (\tan 37.23 + \tan 57.35) = 3.35$$

$$\boxed{\psi = 3.35}$$

$$W = \dot{m}w = \dot{m}UC_a(\tan \alpha_2 + \tan \alpha_3) = 20 * 360 * 260(\tan 65 + \tan 10) = 4344601 \text{ W}$$

$$\boxed{W = 4345 \text{ kW}}$$

$$\sin(90 - \alpha_2) = \frac{C_{a2}}{C_2} \rightarrow C_2 = 615.2 \text{ m/s}$$

$$T_2 = T_{02} - \frac{C_2^2}{2c_p}, \quad T_{02} = T_{01}, \quad T_2 = T_{01} - \frac{C_2^2}{2c_p} = 1000 - \frac{615.2^2}{2(1148)} = 835.2 \text{ K}$$

$$\lambda_N = \frac{T_2 - T'_2}{C_2^2 / 2c_p} \rightarrow T'_2 = T_2 - \frac{\lambda_N C_2^2}{2c_p} = 835.2 - \frac{0.05(615.2)^2}{2(1148)} = 827 \text{ K}$$

$$\frac{P_{01}}{P_2} = \left(\frac{T_{01}}{T'_2} \right)^{\frac{\gamma}{\gamma-1}} \rightarrow P_2 = \frac{P_{01}}{\left(\frac{T_{01}}{T'_2} \right)^{\frac{\gamma}{\gamma-1}}} = \frac{4}{(1000/827)^{1.333/0.333}} = 1.87 \text{ bar}, \quad \frac{P_{01}}{P_2} = 2.14$$

$$PR_{crit} = \frac{\gamma+1}{2}^{\frac{\gamma}{\gamma-1}} = 1.853, \quad \frac{P_{01}}{P_2} > PR_{crit}, \quad \text{nozzle is choked}$$

$$P_{crit} = P_{01}/1.853 = 2.16 \text{ bar} = P_2$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{2.16 * 10^5}{287(835.2)} = 0.901 \text{ kg/m}^3$$

$$V = M\sqrt{\gamma RT_2} = 1\sqrt{1.333 * 287 * 835.2} = 565.3 \text{ m/s}$$

$$\dot{m} = \rho VA \rightarrow A = \frac{\dot{m}}{\rho V} = \frac{20}{0.901(565.3)} = 0.0393 \text{ m}^2$$

$$\boxed{A = 0.0393 \text{ m}^2}$$

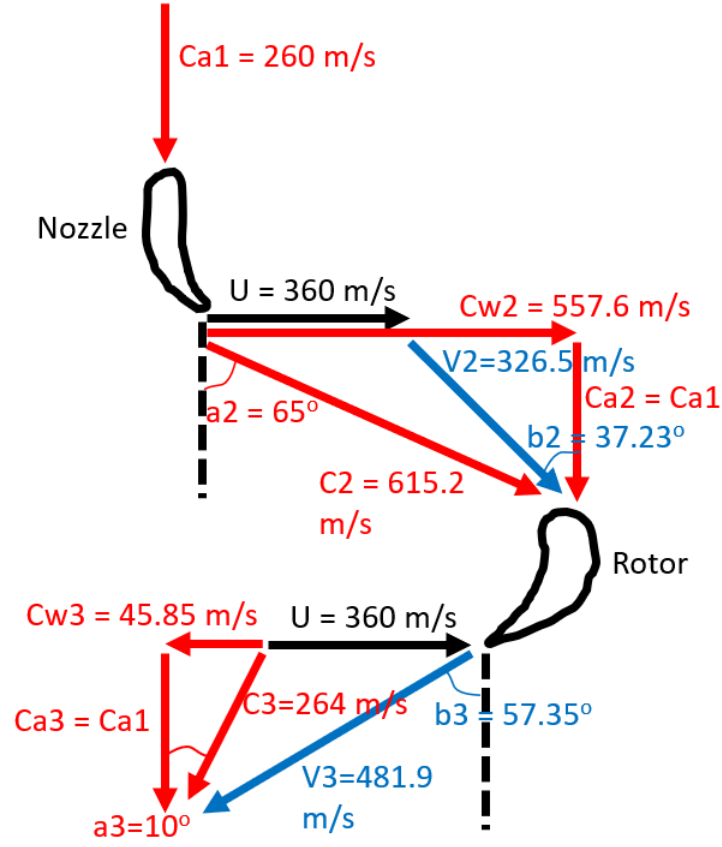
$$\cos \beta_2 = \frac{C_{a2}}{V_2} \rightarrow V_2 = 326.5 \text{ m/s}$$

$$\tan \alpha_2 = \frac{C_{w2}}{C_{a2}} \rightarrow C_{w2} = 557.6 \text{ m/s}$$

$$\tan \alpha_3 = \frac{C_{w3}}{C_{a3}} \rightarrow C_{w3} = 45.85 \text{ m/s}$$

$$\cos \alpha_3 = \frac{C_{a3}}{C_3} \rightarrow C_3 = 264 \text{ m/s}$$

$$\cos \beta_3 = \frac{C_{a3}}{V_3} \rightarrow V_3 = 481.9 \text{ m/s}$$



Problem 2

T_{01}	1350	K
P_{01}	5.2	bar
$\frac{P_{01}}{P_{03}}$	3.4	
C_3	275	m/s
U_r	500	m/s
η_t	0.91	
$\frac{r_t}{r_h}$	1.4	
λ_N	0.05	
Λ_r	0	
$C_1 = C_3 = C_{a1} = C_{a3}$		

Hub

$$\Delta T_{013} = \eta_t T_{01} \left[1 - \left(\frac{P_{03}}{P_{01}} \right)^{\frac{\gamma-1}{\gamma}} \right] = 0.91(1350) \left[1 - \frac{1}{3.4}^{0.333/1.333} \right] = 323.6 \text{ K}$$

$$w = U(C_{w2} + C_{w3}) = c_p \Delta T_{013} \rightarrow C_{w2} = \frac{c_p \Delta T_{013}}{U} = \frac{1148(323.6)}{500} = 743 \text{ m/s}$$

$$T_2 = T_{02} - \frac{C_2^2}{2c_p}, \quad T_2 = T_3 \text{ because } \Lambda = 0$$

$$T_{02} - \frac{C_2^2}{2c_p} = T_{03} - \frac{C_3^2}{2c_p}, \quad T_{02} = T_{01}$$

$$C_2^2 = 2c_p \left(\frac{C_3^2}{2c_p} + \Delta T_{013} \right) = 2(1148) \left(\frac{275^2}{2(1148)} + 323.6 \right) \rightarrow C_2 = 904.77 \text{ m/s}$$

$$\sin \alpha_2 = \frac{C_{w2}}{C_2} \rightarrow \alpha_2 = 55.2^\circ$$

$$\boxed{\alpha_2 = 55.2^\circ}$$

$$C_{w2}^2 + C_{a2}^2 = C_2^2 \rightarrow C_{a2} = 516.3 \text{ m/s}$$

$$\tan \beta_2 = \frac{C_{w2} - U}{C_{a2}} \rightarrow \beta_2 = 25.2^\circ$$

$$\boxed{\beta_2 = 25.2^\circ}$$

Tip

since free vortex design, $C_{a2r} = C_{a2t}$

$$C_{w2t} r_t = C_{w2h} r_h \rightarrow C_{w2t} = C_{w2h} \frac{r_h}{r_t} = 743 \frac{1}{1.4} = 530.7 \text{ m/s}$$

$$\tan \alpha_{2t} = \frac{C_{w2t}}{C_{a2t}} \rightarrow \alpha_{2t} = 45.8^\circ$$

$$\boxed{\alpha_2 = 45.8^\circ}$$

$$T_{3t} = T_{3h} = T_{03} - \frac{C_3^2}{2c_p} = 1350 - 323.6 - \frac{275^2}{2(1148)} = 993.5 \text{ K}$$

$$C_{2t} = \sqrt{C_{w2t}^2 + C_{a2t}^2} = 740.4 \text{ m/s}$$

$$T_{2t} = T_{02} - \frac{C_{2t}^2}{2c_p} = 1350 - \frac{740.4^2}{2(1148)} = 1111.2 \text{ K}$$

$$\Lambda_t = \frac{T_{2t} - T_{3t}}{T_{01} - T_{03}} = \frac{1111.2 - 993.5}{323.6} = 0.364$$

$$\boxed{\Lambda = 0.364}$$

Hub Static Pressures

$$\lambda_N = \frac{T_2 - T_2'}{C_2^2 / 2c_p} \rightarrow T_2' = T_2 - \frac{\lambda_N C_2^2}{2c_p} \rightarrow T_2' = T_{02} - \frac{C_2^2}{2c_p} - \frac{\lambda_N C_2^2}{2c_p} \rightarrow$$

$$T_2' = T_{02} - (1 + \lambda_N) \frac{C_2^2}{2c_p} = 1350 - (1 + 0.05) \frac{904.77^2}{2(1148)} = 975.6 \text{ K}$$

$$\frac{P_{02}}{P_2} = \left(\frac{T_{02}}{T_2'} \right)^{\frac{\gamma}{\gamma-1}} \rightarrow P_2 = 1.42 \text{ bar}$$

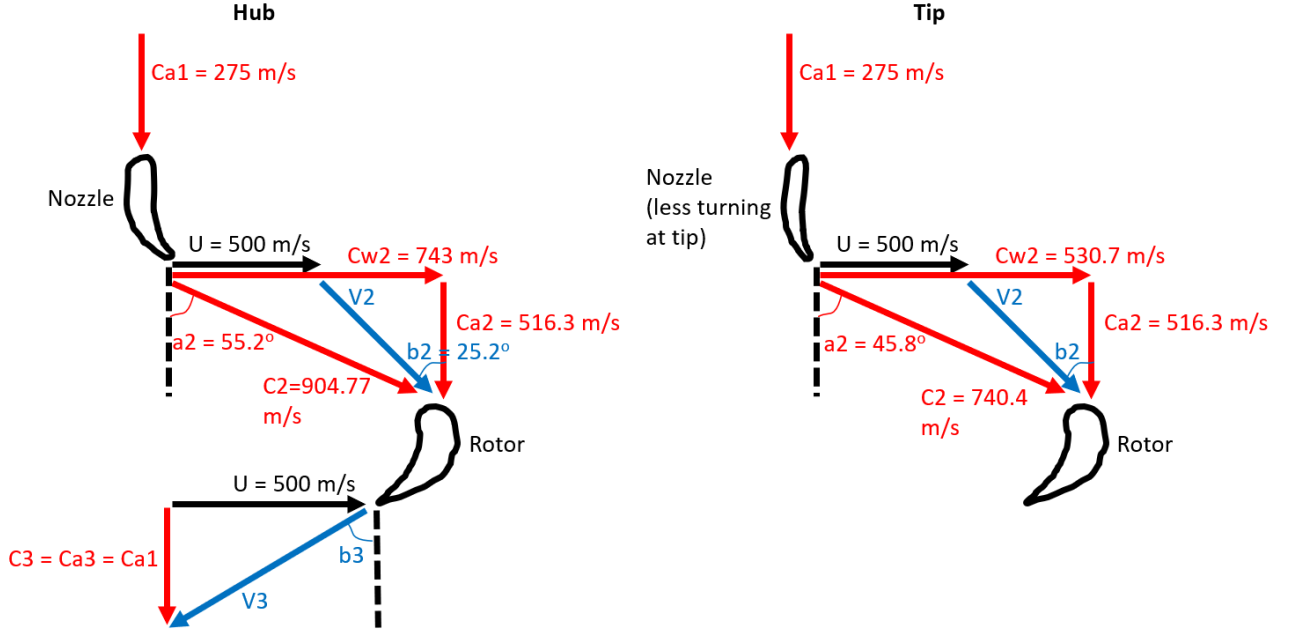
$$\frac{P_{03}}{P_3} = \left(\frac{T_{03}}{T_3'} \right)^{\frac{\gamma}{\gamma-1}} \rightarrow P_3 = 1.25 \text{ bar}$$

$$P_2 = 1.42 \text{ bar}$$

$$P_3 = 1.25 \text{ bar}$$

since $P_3 < P_2$, some expansion is happening at the root in the rotor blade passages

Velocity Triangles



Problem 3

\dot{m}	36	kg/s
T_{01}	1200	K
P_{01}	8	bar
ΔT_{013}	150	K
η_t	0.9	
U_m	320	m/s
N	250	rev/s
C_3	400	m/s
λ_N	0.07	
C_a	346	m/s

$$T_{03} = T_{01} - \Delta T_{013} = 1200 - 150 = 1050 \text{ K}$$

$$T_3 = T_{03} - \frac{C_3^2}{2c_p} = 1050 - \frac{400^2}{2(1148)} = 980.3 \text{ K}$$

$$\Delta T_{013} = \eta_t T_{01} \left[1 - \left(\frac{P_{03}}{P_{01}} \right)^{\frac{\gamma-1}{\gamma}} \right] \rightarrow \frac{P_{03}}{P_{01}} = \left(1 - \frac{\Delta T_{013}}{\eta_t T_{01}} \right)^{\frac{\gamma}{\gamma-1}} \rightarrow P_{03} = 4.4 \text{ bar}$$

$$\frac{P_{03}}{P_3} = \left(\frac{T_{03}}{T_3} \right)^{\frac{\gamma}{\gamma-1}} \rightarrow P_3 = 3.3 \text{ bar}$$

$$U_m = \omega r_m = 2\pi N r_m \rightarrow r_m = \frac{U_m}{2\pi N} = \frac{320}{2\pi(250)} = 0.204 \text{ m}$$

$$\dot{m} = \rho A C_3 \rightarrow A = \frac{\dot{m}}{\rho C_3}$$

$$\rho = \frac{P_3}{RT_3} = \frac{3.3 \times 10^5}{287(980.3)} = 1.173 \text{ kg/m}^3$$

$$A = Ch = 2\pi r_m h \rightarrow h = \frac{\dot{m}}{2\pi \rho C_3 r_m} = \frac{36}{2\pi(1.173)(400)(0.204)} = 0.0599 \text{ m}$$

$$h = 0.0599 \text{ m}$$

$$r_t = r_m + h/2 = 0.234 \text{ m}$$

$$r_h = r_m - h/2 = 0.174 \text{ m}$$

$$\frac{r_t}{r_h} = \frac{0.234}{0.174} = 1.34$$

$$\frac{r_t}{r_h} = 1.34$$

$$w = C_{w2} U = c_p \Delta T_{013} \rightarrow C_{w2} = 538.1 \text{ m/s}, \quad C_{a2} = 346 \text{ m/s}$$

$$C_2 = \sqrt{C_{w2}^2 + C_{a2}^2} = 639.7 \text{ m/s}$$

$$T_2 = T_{02} - \frac{C_2^2}{2c_p}, \quad T_{02} = T_{01}, \quad T_2 = 1021.77 \text{ K}$$

$$T_2' = T_2 - \frac{\lambda_N C_2^2}{2c_p} = 1021.77 - \frac{0.07(639.7)^2}{2(1148)} = 1009.3 \text{ K}$$

$$\frac{P_{02}}{P_2} = \left(\frac{T_{02}}{T_2'} \right)^{\frac{\gamma}{\gamma-1}} \rightarrow P_2 = 4 \text{ bar}$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{4 \cdot 10^5}{287(1021.77)} = 1.36 \text{ kg/m}^3$$

$$A = \frac{\dot{m}}{\rho_3 C_3} = \frac{36}{1.173(400)} = 0.076 \text{ m}^2$$

$$C_{a2} = \frac{\dot{m}}{\rho_2 A_3} = \frac{36}{1.36(0.076)} = 348.3 \text{ m/s}$$

$$C_{w2h} = C_{w2m} \frac{r_m}{r_h} = 538.1 \frac{0.204}{0.174} = 630.9 \text{ m/s}$$

$$C_{2h} = \sqrt{C_{w2h}^2 + C_{a2}^2} = 720.7 \text{ m/s}$$

$$U_h = U_m \frac{r_h}{r_m} = 273 \text{ m/s}$$

$$V_{2h} = \sqrt{C_{a2}^2 + (C_{w2h} - U_h)^2} = 499.4 \text{ m/s}$$

$$T_{2h} = T_{01} - \frac{C_{2h}^2}{2c_p} = 973.8 \text{ K}$$

$$M_h = \frac{V_{2h}}{\sqrt{\gamma R T_{2h}}} = 0.818$$

$$\boxed{M_h = 0.818}$$