AEEM4063 - Assignment 4

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Problem 1

Air

Air $= O_2 + 3.76N_2$
$MW_{air} = 2MW_O + 7.52MW_N = 2(15.999) + 7.52(14.007) = 137.33$ g/mol
$\boxed{MW_{air} = 137.33 \text{ g/mol}}$

Dodecane

Dodecane $= C_{12}H_{26}$
$MW_{C_{12}H_{26}} = 12MW_C + 26MW_H = 12(12.011) + 26(1.008) = 170.34$ g/mol
$\boxed{MW_{C_{12}H_{26}} = 170.34 \text{ g/mol}}$

Fuel-Air Ratio

$Q_f = 44147$ kJ/kg (from book p.286)
$\dot{m}fQ_f = c_p\dot{m}(\Delta T_0) \rightarrow f = \frac{c_p\Delta T_0}{Q_f} = \frac{1.08*1150}{44147}$
$\boxed{f = 0.0281}$

Moles of Air Required

$f = \frac{MW_{C_{12}H_{26}}N_{C_{12}H_{26}}}{MW_{air}N_{air}} \rightarrow N_{air} = \frac{MW_{C_{12}H_{26}}N_{C_{12}H_{26}}}{fMW_{air}} = \frac{170.34(1)}{0.0281(137.33)} = 44.14$ mol
$\boxed{N_{air} = 44.14 \text{ mol/mol of fuel}}$

Problem 2

Part A

$\phi = 1, C_3H_8$

for C_nH_{2n+2} fuels with $\phi = 1$,
$C_nH_{2n+2} + (\frac{3n+1}{2})(O_2 + 3.76N_2) \rightarrow nCO_2 + (n+1)H_2O + 3.76(\frac{3n+1}{2})N_2$
$\boxed{C_3H_8 + 5(O_2 + 3.76N_2) \rightarrow 3CO_2 + 4H_2O + 18.8N_2}$
$\boxed{\text{1 mol } C_3H_8\text{, 5 mol air, 3 mol } CO_2\text{, 4 mol water, 18.8 mol } N_2}$

$f_{stoic} = \frac{7n+1}{34.32(3n+1)}$
$\boxed{f = 0.0641}$

$\frac{A}{F} = \frac{1}{f} = 15.6$
$\boxed{\frac{A}{F} = 15.6}$

$\phi = 0.5, C_3H_8$

$\phi = \frac{1}{X}$, $X = 2$
for C_nH_{2n+2} fuels with $\phi < 1$ and X,
$C_nH_{2n+2} + X(\frac{3n+1}{2})(O_2 + 3.76N_2) \rightarrow nCO_2 + (n+1)H_2O + 3.76X(\frac{3n+1}{2})N_2 + (X-1)(\frac{3n+1}{2})O_2$
$\boxed{C_3H_8 + 10(O_2 + 3.76N_2) \rightarrow 3CO_2 + 4H_2O + 37.6N_2 + 5O_2}$
$\boxed{\text{1 mol } C_3H_8\text{, 10 mol air, 3 mol } CO_2\text{, 4 mol water, 37.6 mol } N_2\text{, 5 mol } O_2}$

$f = \phi f_{stoic} = 0.5(0.0641) = 0.03205$
$\boxed{f = 0.03205}$

$\frac{A}{F} = \frac{1}{f} = 31.2$
$\boxed{\frac{A}{F} = 31.2}$

Part B

$\phi = 1, C_{10}H_{22}$

for C_nH_{2n+2} fuels with $\phi = 1$,
$C_nH_{2n+2} + (\frac{3n+1}{2})(O_2 + 3.76N_2) \rightarrow nCO_2 + (n+1)H_2O + 3.76(\frac{3n+1}{2})N_2$
$\boxed{C_{10}H_{22} + 15.5(O_2 + 3.76N_2) \rightarrow 10CO_2 + 11H_2O + 58.28N_2}$
$\boxed{\text{1 mol } C_{10}H_{22}\text{, 15.5 mol air, 10 mol } CO_2\text{, 11 mol water, 58.28 mol } N_2}$

$f_{stoic} = \frac{7n+1}{34.32(3n+1)}$
$\boxed{f = 0.06673}$

$\frac{A}{F} = \frac{1}{f} = 14.985$
$\boxed{\frac{A}{F} = 14.985}$

$\phi = 0.5, C_{10}H_{22}$

$\phi = \frac{1}{X}, X = 2$
for C_nH_{2n+2} fuels with $\phi < 1$ and X,
$C_nH_{2n+2} + X(\frac{3n+1}{2})(O_2 + 3.76N_2) \rightarrow nCO_2 + (n+1)H_2O + 3.76X(\frac{3n+1}{2})N_2 + (X-1)(\frac{3n+1}{2})O_2$
$\boxed{C_{10}H_{22} + 31(O_2 + 3.76N_2) \rightarrow 10CO_2 + 11H_2O + 116.56N_2 + 15.5O_2}$
$\boxed{\text{1 mol } C_{10}H_{22}\text{, 31 mol air, 10 mol } CO_2\text{, 11 mol water, 116.56 mol } N_2\text{, 15.5 mol } O_2}$

$f = \phi f_{stoic} = 0.5(0.06673) = 0.03337$
$\boxed{f = 0.03337}$

$\frac{A}{F} = \frac{1}{f} = 29.97$
$\boxed{\frac{A}{F} = 29.97}$

Problem 3

A_i	0.0389	m^2
A_m	0.0975	m^2
K_1	19	
$\dot{m}$	9	kg/s
T_{01}	475	K
T_{02}	1023	K
P_1	4.47	bar
ΔP_0	0.27	bar

$\frac{\Delta P_0}{\dot{m}^2/2\rho_1 A_m^2} = K_1 + K_2(\frac{T_{02}}{T_{01}} - 1)$

$\dot{m} = \rho_1 V_1 A_i \rightarrow V_1 = \frac{\dot{m}}{\rho_1 A_i}$
$T_{01} = T_1 + \frac{V_1^2}{2c_p} \rightarrow T_{01} = \frac{P_1}{R\rho_1} + \frac{\dot{m}^2}{2c_p\rho_1^2 A_i^2} \rightarrow \rho_1^2 T_{01} = \frac{P_1\rho_1}{R} + \frac{\dot{m}^2}{2c_p A_i^2}$
$T_{01}\rho_1^2 - \frac{P_1}{R}\rho_1 - \frac{\dot{m}^2}{2c_p A_i^2} = 0$
$475\rho_1^2 - \frac{4.47*10^5}{287}\rho_1 - \frac{9^2}{2(1005)(0.0389)^2} = 0, \quad \rho_1 = 3.296 \text{ kg/m}^3$

$K_2 = (\frac{\Delta P_0}{\dot{m}^2/2\rho_1 A_m^2} - K_1)/(\frac{T_{02}}{T_{01}} - 1) = (\frac{0.27*10^5}{9/2(3.296)(0.0975)^2} - 19)/(\frac{1023}{475} - 1) = 1.637$

A_m	0.0975	m^2
K_1	19	
K_2	1.637	
$\dot{m}$	7.4	kg/s
T_{01}	439	K
T_{02}	900	K
P_1	3.52	bar

$\Delta P_0 = \frac{\dot{m}^2}{2\rho_1 A_m^2}(K_1 + K_2(\frac{T_{02}}{T_{01}} - 1))$

$T_{01}\rho_1^2 - \frac{P_1}{R}\rho_1 - \frac{\dot{m}^2}{2c_p A_i^2} = 0$
$439\rho_1^2 - \frac{3.52*10^5}{287}\rho_1 - \frac{7.4^2}{2(1005)(0.0389)^2}, \quad \rho_1 = 2.808 \text{ kg/m}^3$

$\Delta P_0 = \frac{7.4^2}{2(2.808)(0.0975)^2}(19 + 1.637(\frac{900}{439} - 1)) = 21251.85 \text{ Pa}$
$\boxed{\Delta P_0 = 0.213 \text{ bar}}$

Part A

Design

$V_1 = \frac{\dot{m}}{\rho_1 A_i} = \frac{9}{3.296(0.0389)} = 70.195$ m/s

$\boxed{V_1 = 70.2 \text{ m/s}}$

Partial

$V_1 = \frac{\dot{m}}{\rho_1 A_i} = \frac{7.4}{2.808(0.0389)} = 67.75$ m/s

$\boxed{V_1 = 67.75 \text{ m/s}}$

Part B

Design

$P_{01} = P_1(\frac{T_{01}}{T_1})^{\frac{\gamma}{\gamma-1}} = P_1(\frac{T_{01}R\rho_1}{P_1})^{\frac{\gamma}{\gamma-1}} = 4.47(\frac{475(287)(3.296)}{4.47*10^5})^{\frac{1.4}{0.4}} = 4.552$ bar

$\frac{\Delta P_0}{P_{01}} = \frac{0.27}{4.552} = 0.05931$

$\boxed{\frac{\Delta P_0}{P_{01}} = 0.05931}$

Partial

$P_{01} = P_1(\frac{T_{01}R\rho_1}{P_1})^{\frac{\gamma}{\gamma-1}} = 3.52(\frac{439(287)(2.808)}{3.52*10^5})^{\frac{1.4}{0.4}} = 3.583$ bar

$\frac{\Delta P_0}{P_{01}} = \frac{0.213}{3.583} = 0.05945$

$\boxed{\frac{\Delta P_0}{P_{01}} = 0.0595}$

Discussion

We can see that despite the different operating conditions, the ratio of pressure loss to delivery pressure stays relatively the same. This would imply that the combustor will have similar performance in terms of pressure loss across a range of operating conditions that would be required for engine operation. We also see that the velocity is similar in both cases, which is also a good sign for stable operation across a range of conditions.