

AEEM5063 HW#11

Slade Brooks

M13801712

12.02.24

6.20

$$T_{phase} = \frac{T}{2}$$

$$\frac{2\pi}{\sqrt{\mu}} a^{\frac{3}{2}} = \frac{1}{2} \frac{2\pi}{\sqrt{\mu}} r^{\frac{3}{2}}$$

$$a = \left(\frac{1}{2} r^{\frac{3}{2}} \right)^{\frac{2}{3}}$$

$$\boxed{a = 0.63r}$$

6.25

Orbit 1:

$$r_p = 6378 + 1270 = 7648 \text{ km}, \quad v_p = 9 \text{ km/s}$$

$$h_1 = r_p v_p = 7648 \cdot 9 = 68832 \text{ km}^2/\text{s}$$

$$r_p = \frac{h_1^2}{\mu} \frac{1}{1 + e_1}$$

$$7648 = \frac{68832^2}{398600} \frac{1}{1 + e_1} \quad \rightarrow \quad \boxed{e_1 = 0.554}$$

At the maneuver point, $\theta = 100^\circ$:

$$r = \frac{h_1^2}{\mu} \frac{1}{1 + e_1 \cos \theta}$$

$$r = \frac{68832^2}{398600} \frac{1}{1 + 0.554 \cos 100^\circ} = 13150 \text{ km}$$

$$v_{1\perp} = \frac{h_1}{r} = \frac{68832}{13150} = 5.234 \text{ km/s}$$

$$v_{1r} = \frac{\mu}{h_1} e_1 \sin \theta = \frac{398600}{68832} \cdot 0.554 \cdot \sin 100^\circ = 3.16 \text{ km/s}$$

$$v_1 = \sqrt{v_{1\perp}^2 + v_{1r}^2} = \sqrt{5.234^2 + 3.16^2} = 6.114 \text{ km/s}$$

$$\gamma_1 = \tan^{-1} \frac{v_{1r}}{v_{1\perp}} = \tan^{-1} \frac{3.16}{5.234} = 31.13^\circ$$

Orbit 2:

$$e_2 = 0.4, \quad r = \frac{h_2^2}{\mu} \frac{1}{1 + e_2 \cos \theta}$$

$$13150 = \frac{h_2^2}{398600} \frac{1}{1 + 0.4 \cos 100^\circ} \quad \rightarrow \quad h_2 = 69840 \text{ km}^2/\text{s}$$

$$\begin{aligned}
v_{2\perp} &= \frac{h_2}{r} = \frac{69840}{13150} = 5.311 \text{ km/s} \\
v_{2r} &= \frac{\mu}{h_2} e_2 \sin \theta = \frac{398600}{69840} \cdot 0.4 \cdot \sin 100^\circ = 2.248 \text{ km/s} \\
v_2 &= \sqrt{v_{2\perp}^2 + v_{2r}^2} = \sqrt{5.311^2 + 2.248^2} = 5.767 \text{ km/s} \\
\gamma_2 &= \tan^{-1} \frac{v_{2r}}{v_{2\perp}} = \tan^{-1} \frac{2.248}{5.311} = 22.94^\circ
\end{aligned}$$

$$\Delta\gamma = \gamma_2 - \gamma_1 = 22.94^\circ - 31.13^\circ = -8.181^\circ$$

$$\begin{aligned}
\Delta v &= \sqrt{v_1^2 + v_2^2 - 2v_1v_2 \cos \Delta\gamma} \\
\Delta v &= \sqrt{6.114^2 + 5.767^2 - 2 \cdot 6.114 \cdot 5.767 \cdot \cos(-8.181^\circ)} \\
\Delta v &= 0.9155 \text{ km/s}
\end{aligned}$$

$$\boxed{\Delta v = 0.916 \text{ km/s}}$$

6.31

Orbit 1:

$$r_{P1_1} = \frac{h_1^2}{\mu} \frac{1}{1+e}$$

Orbit 2:

$$r_{P1_2} = \frac{h_2^2}{\mu} \frac{1}{1+e \cos 90} = \frac{h_2^2}{\mu}$$

$$r_{P1_1} = r_{P1_2}$$

$$\frac{h_2^2}{\mu} = \frac{h_1^2}{\mu} \frac{1}{1+e}$$

$$h_2 = \frac{h_1}{\sqrt{1+e}}$$

6.37

$$A = \sin^{-1} \left(\frac{\cos i}{\cos \phi} \right)$$

$$(a) \quad A = \sin^{-1} \left(\frac{\cos 116.57^\circ}{\cos 28.59^\circ} \right) = \boxed{329.4^\circ \text{ or } 210.6^\circ}$$

$$(b) \quad A = \sin^{-1} \left(\frac{\cos 116.57^\circ}{\cos 34.5^\circ} \right) = \boxed{327.1^\circ \text{ or } 212.9^\circ}$$

$$(c) \quad A = \sin^{-1} \left(\frac{\cos 116.57^\circ}{\cos 5.5^\circ} \right) = \boxed{333.3^\circ \text{ or } 206.7^\circ}$$

6.34