AEEM 3062

HOMEWORK # 9

2022-2023 SPRING

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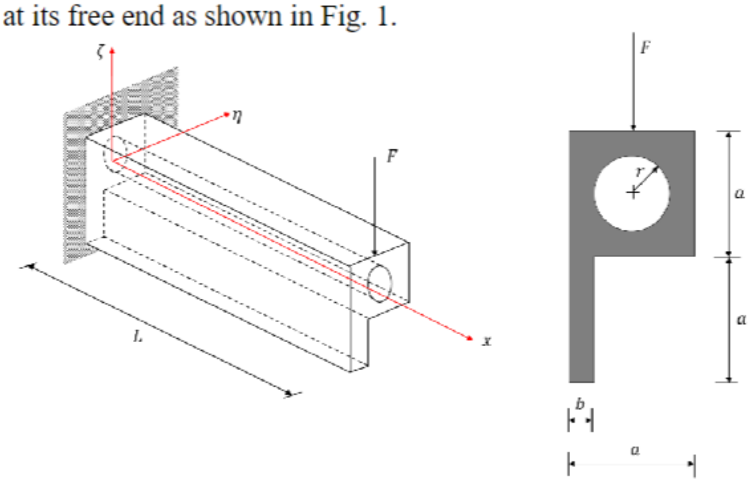
DUE: 11.59 PM on 4/5/2023

Problem 1

A cantilevered beam has the same cross section studied in HWK 7(b) with dimensions $a = 50$ mm $b = 10$ mm and $r = 15$ mm. Moreover, the beam has length $L = 850$ mm and is subject to a load F = 10 kN applied at its free end as shown in Fig. 1.



Part A

Find and sketch the position of the neutral axis relative to the cross section [**25 pts**]

First we can find α from the eigenvector n_2, where θ is its angle from the negative horizontal axis.

$$\alpha = 360 - \theta = 360 - \tan^{-1}(0.3447/0.9387)$$
$$\alpha = 339.84^\circ$$

Then we can find β.

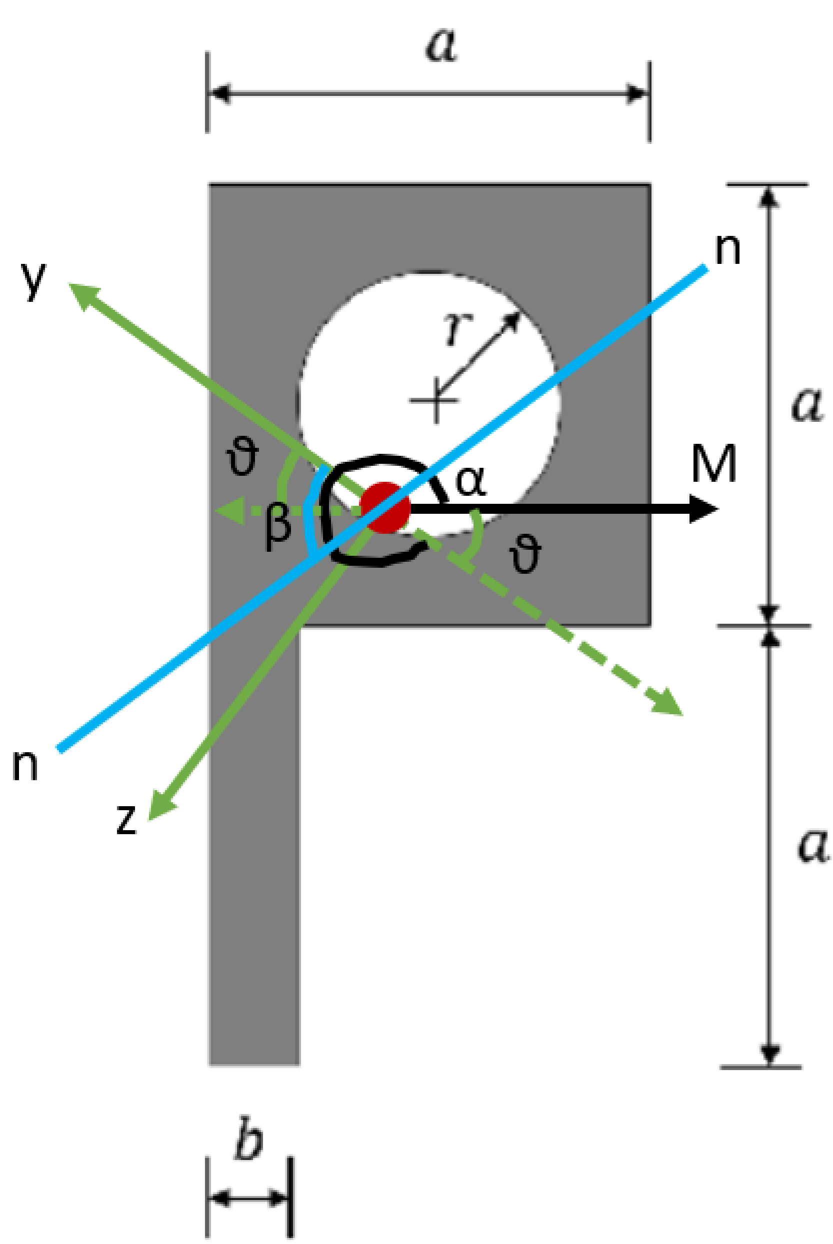
$$\tan\beta = \frac{I_y}{I_z}\tan(\alpha)$$

We found I_y and I_z in HW7.

$$\beta = \tan^{-1}\left(\frac{1706271}{498047}\tan(339.84)\right)$$

$$\beta = -51.51^\circ$$

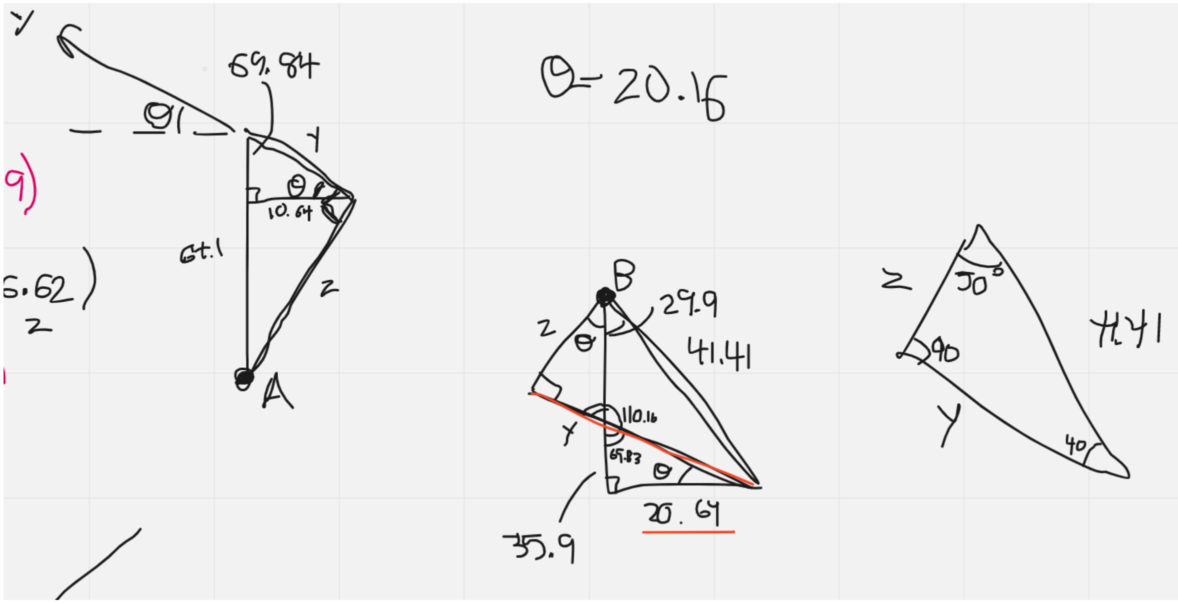
Therefore the neutral axis is as shown with $\beta = 51.51^\circ$:



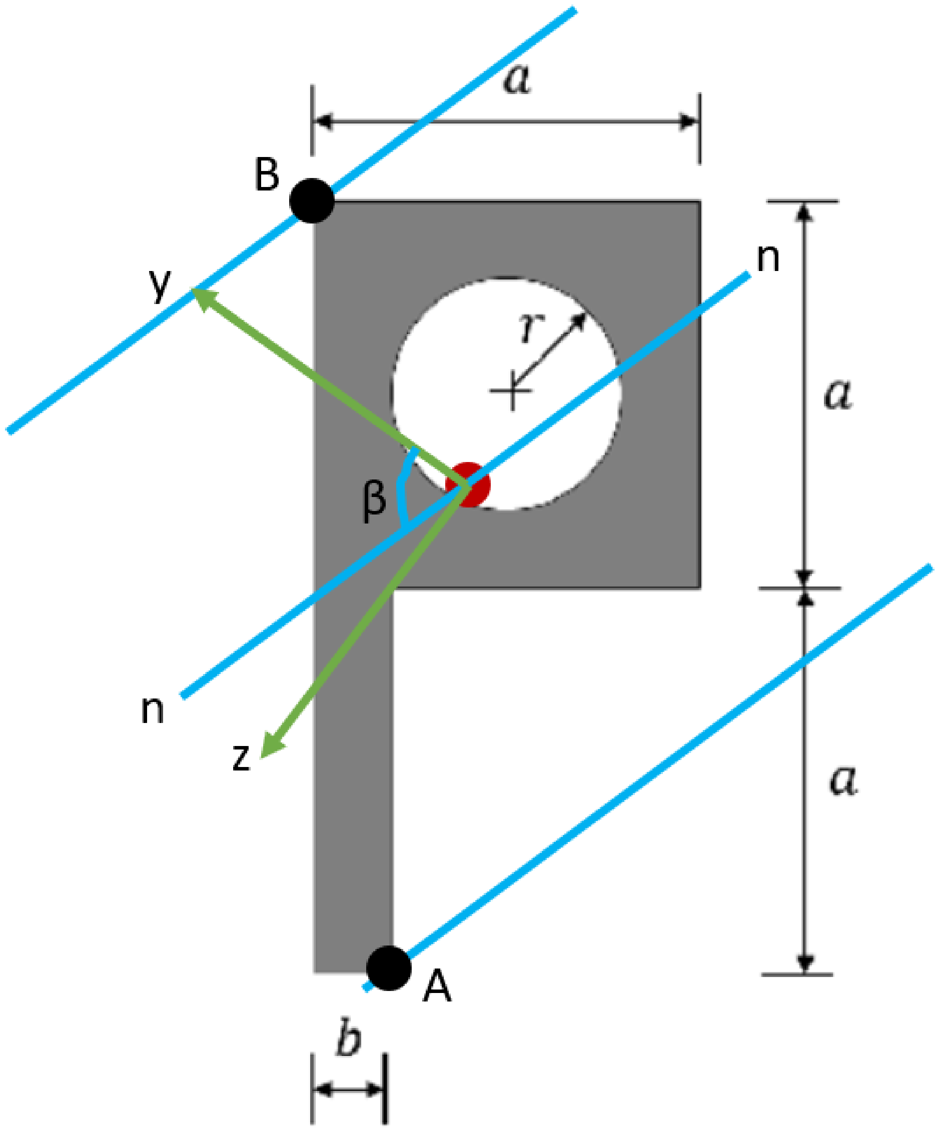
Part B

Determine the largest compressive and tensile stresses and indicate where they occur [20 pts]

First we will transform the coordinates of points A and B shown in the sketch below from η, ζ to y, z. This was done with fun geometry to give:



$$A = (-12.1, 63.84)$$
$$B = (31.72, -26.62)$$



Then we can compute the stress at these points with the equation:

$$\sigma_x = -\frac{M\cos\alpha}{I_y}z - \frac{M\sin\alpha}{I_z}y$$

M can be found by multiplying the force by the length of the bar since the largest moment will be at the wall:

$$M = (-10\text{kN})(850\text{mm}) = -8500\text{kNmm}$$

Then:

$$\sigma_{xA} = -\frac{-8500\cos(339.84-180)}{1706271}(63.84) - \frac{-8500\sin(339.84-180)}{498047}(-12.1)$$

$$\sigma_{xA} = -0.3697\text{kN/mm}^2 = \boxed{-369.7\text{MPa}} \text{ (compression)}$$

$$\sigma_{xB} = -\frac{-8500\cos(339.84-180)}{1706271}(-26.62) - \frac{-8500\sin(339.84-180)}{498047}(31.72)$$

$$\sigma_{xB} = 0.3111\text{kN/mm}^2 = \boxed{311.1\text{MPa}} \text{ (tension)}$$

Therefore the maximum compressive stress is 369.7 MPa at A and the maximum tensile stress is 311.1 MPa at B.

Part C

Assuming that the yields stress of the material is $\sigma_Y = 300$ MPa, find the maximum value of force F that can be applied without causing failure [**10 pts**]

We can check the points A and B to see at what value of F the σ_x becomes 300 MPa by rearranging the equation:

$$\sigma_x = -\frac{FL\cos\alpha}{I_y}z - \frac{FL\sin\alpha}{I_z}y$$

$$F = \frac{\sigma_x}{L*(-\frac{\cos\alpha}{I_y}z - \frac{\sin\alpha}{I_z}y)}$$

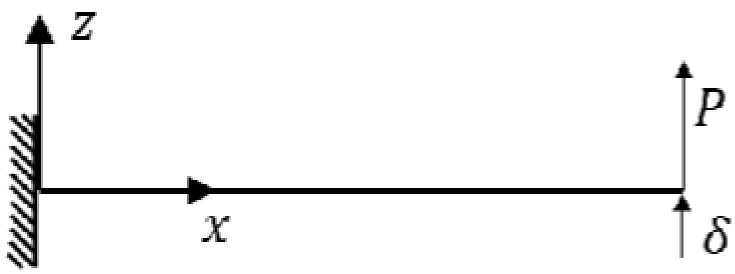
$$F_B = \frac{0.3}{850*(-\frac{\cos(339.84-180)}{1706271}(-26.62) - \frac{\sin(339.84-180)}{498047}(31.72))}$$

$$F_B = \boxed{-9.644\text{kN}}$$

Therefore the maximum value of F that can be applied is 9.644 kN downwards. We evaluate only at B because σ_y defines the max tensile stress.

Part D

For the initial load $F = 10$ kN, determine the components of displacement of the centroid at the free end along axes η and ζ assuming that the material has Young's modulus E=250 GPa. Hint, first find the expression of displacement δ for the case in which a force (P) is acting parallel to one of the principal axes as shown in Fig. 1b and then use the superposition principle. You can find the displacement using the standard integration method or the energy method. [45 pts]



First set up differential equations:

$$\frac{\partial V}{\partial x} = 0,$$
$$V = C_1$$

$$\frac{\partial M}{\partial x} = V = C_1,$$
$$M = C_1 x + C_2 L$$

$$\frac{\partial \varphi}{\partial x} = \frac{M}{EI} = \frac{1}{EI}(C_1 x + C_2 L),$$
$$\varphi = \frac{1}{EI}(\frac{1}{2}C_1 x^2 + C_2 L x + C_3 L^2)$$

$$\frac{\partial w}{\partial x} = \varphi = \frac{1}{EI}(\frac{1}{2}C_1 x^2 + C_2 L x + C_3 L^2),$$
$$w = \frac{1}{EI}(\frac{C_1}{6}x^3 + \frac{C_2 L}{2}x^2 + C_3 L^2 x + C_4 L^3)$$

Now define the boundary conditions:

$$\varphi(0) = 0$$
$$w(0) = 0$$
$$V(L) = P$$
$$M(L) = 0$$

Then plugging in and solving:

$$\varphi(0) = 0 = \frac{C_3 L^2}{EI},$$
$$C_3 = 0$$

$$w(0) = 0 = \frac{C_4 L^3}{EI},$$
$$C_4 = 0$$

$$V(L) = P = C_1,$$
$$C_1 = P$$

$$M(L) = 0 = L(C_1 + C_2),\ C_1 + C_2 = 0,$$
$$C_2 = -P$$

Plug in the constants and solve for w:

$$w(x) = \frac{1}{EI}(\frac{P}{6}x^3 - \frac{PL}{2}x^2)$$

$$w(x) = \frac{Px^2}{EI}(\frac{L}{2} - \frac{x}{6})$$

$$w = \frac{-PL^3}{3EI}$$

(at the free end)

Now we will find the displacement in the z direction by finding the component of F in the z direction (P):

$$P = F\cos\theta$$

(where θ is the angle between n2 and the negative η axis)

$$w = \frac{-F\cos\theta L^3}{3EI_z} = -\frac{-10\cos 20.16(850)^3}{3 * 250 * 498047}$$

$$w_z = 15.434\text{mm}$$

Then we can find displacement in the y direction by doing the same in the y direction:

$$P = -F\sin\theta$$

$$w = -\frac{-F\sin\theta L^3}{3EI_z} = \frac{10\sin 20.16(850)^3}{3*250*1706271}$$

$$w_y = -1.654\text{mm}$$

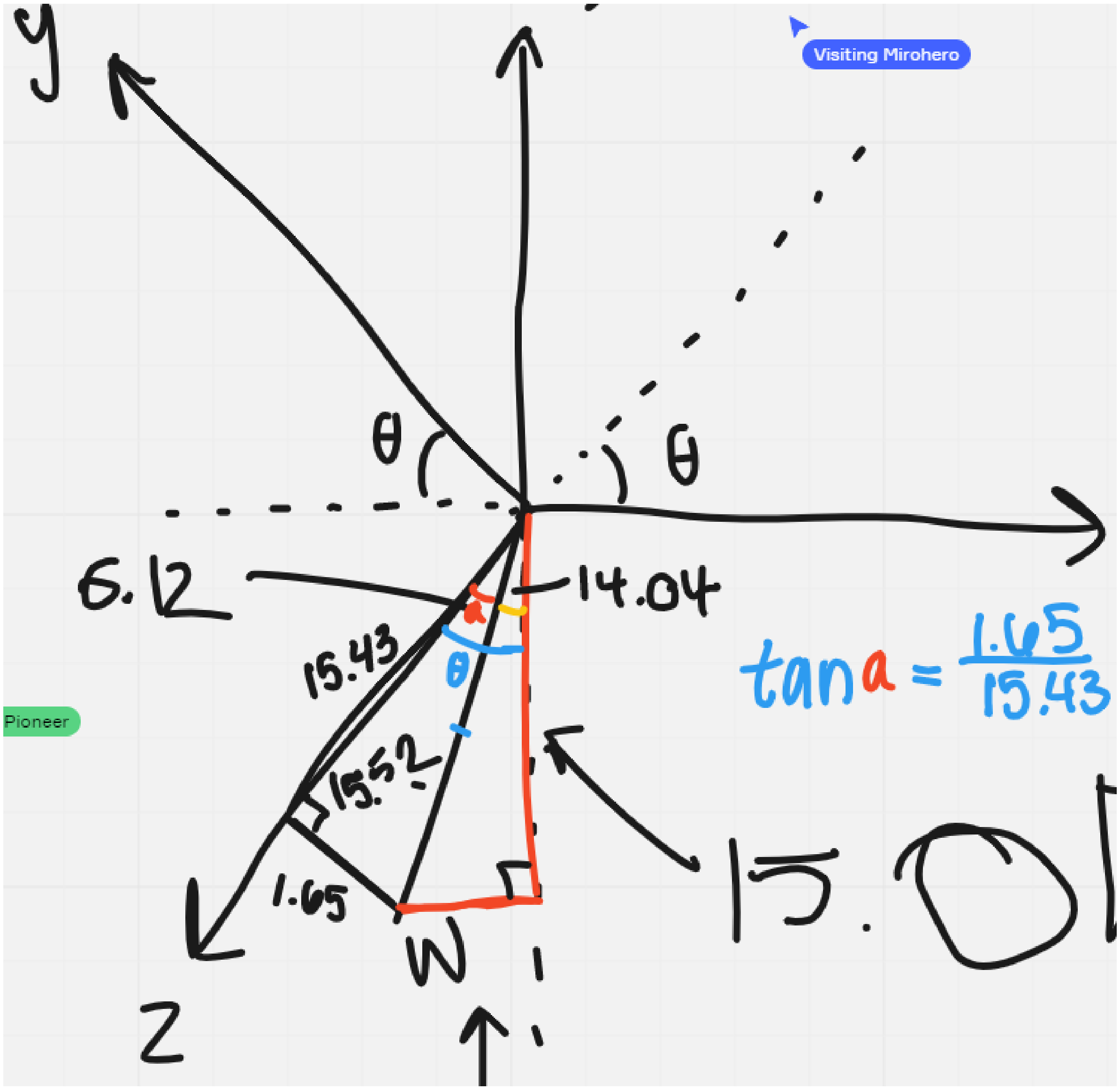
Now we can superposition w_z and w_y:

$$w = (-1.654, 15.434)$$

And convert to terms of η and ζ using gross trig ugh:

$$\boxed{w = (-3.765, -15.054)\text{mm}}$$

$$(\eta, \zeta)$$



Visiting Mirohero
y
θ
θ
6.12
14.04
a
15.43
θ
tan a = 1.65/15.43
Pioneer
15.52
1.65
W
15.01
Z