Euclidean Voting

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1 Simple Ranking Scoring Functions / Generalized Scoring Rules and Mean Proximity Rules

These are defined by both Conitzer et. al. and Zwicker.

- SRSF: $\arg\max_\sigma \sum_{i=1}^n s(\sigma, \sigma_i)$
- GSR: $\arg\max_o \sum_{i=1}^n s(o, \sigma_i)$ where $o \in \mathcal{O}$ can be from any outcome space. Restricting outcome space to rankings and alternatives recover SRSF and SCSF, respectively.
- Mean Proximity: Embedding ϕ of votes, another embedding ψ of outcomes, select the outcome whose ψ is close to the mean ϕ of input votes. Restricting outcomes to rankings still leaves open possibility of using $\psi \neq \phi$.
- THM: Mean Proximity $\Leftrightarrow$ GSR. So, Mean proximity for rankings $\Leftrightarrow$ SRSF (GSR for rankings)
- **OUR THM:** Mean proximity for rankings with $\psi = \phi \Leftrightarrow$ SRSF with symmetric s.
  - Have a coordinate for every pair of rankings rather than each ranking. The coordinate for σ, σ' in embedding of σ has value $\sqrt{s(\sigma, \sigma')}$. Coordinates for all other σ', σ'' are 0.

Properties:

- Neutral iff s = neutral.
- Neutral $\Rightarrow$ SRSF iff MLE
- Consistent + continuous (which characterize all *anonymous mean neat voting rules*)
- Conjecture (Conitzer et. al.): Consistent + continuous + neutral $\Leftrightarrow$ Neutral SRSF
- Capture PSR + KEM. But cannot capture Bucklin, Copeland, Maximin, Ranked Pairs (not consistent). STV is also not SRSF.

2 Linear Mean Proximity Rules

Our work: Achieving neutrality by not-so-restricting linear representations.

- Definition
- Motivation: While SRSF capture many nice voting rules, they capture bad ones too. Don't have $s(\sigma, \sigma')$ maximized by $\sigma' = \sigma$ then will not even satisfy strong unanimity. Linearity is unrestrictive, structural, and easy to work with.
- Can now define distance function, with nice properties (left invariance). Can observe that KT distance = Euclidean distance square. So Kemeny is a mean rule. So are all positional scoring rules.
  - **Question:** Distance (and hence distance square) = MC/PC $\Rightarrow$ implications on the properties of the rule?
  - For other, we need to check if embedding of symmetric SRSF that is in addition neutral is linear.
- Still capture PSR+KEM. Of course cannot capture rules that are not SRSF.
- **Question:** Captures all "pairwise comparison scoring rules"?
- We know: symmetric SRSF iff mean proximity (with same embedding). **Our Conjecture:** Neutral SRSF iff linear mean proximity (with same embedding)?
  - Neutrality $\Rightarrow$ symmetric. Hence, one direction is clear.
  - For other, we need to check if embedding of symmetric SRSF that is in addition neutral is linear.

3 Connections to other approaches

3.1 Axiomatic

Consistency, continuity, anonymity, neutrality already described above.

Question: When is it PM-c, monotonic, majority for rankings?

3.2 DR

Consensus = Strong unanimity. Distance = Square of Euclidean distance. Votewise DR rules.

Square is not always a distance metric $\rightarrow$ Bad. More meaningfully, define votewise DR rules by sum of squares of distances rather than sum of distances.

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3.3 MLE

Take $\Pr[\sigma|\sigma^*] \propto e^{\|\phi(\sigma)-\phi(\sigma*)\|^2}$. Not always Mallows since square is not always a metric. But actually better $\rightarrow$ Gaussian.

Neutrality $\Rightarrow$ Normalization independent of the true ranking $\Rightarrow$ Linear mean proximity rule is the MLE for this model.

Question: Efficient sampling?

Question: Anything interesting for the Gaussian distributions that connect to PSR?

4 Other Directions

What are the equivalence classes of ϕ that lead to the same voting rule?

What about notions of consensus in Euclidean spaces other than mean $\rightarrow$ e.g., minimize maximum distance (equally let go)?