Example 5.4 (Simultaneous confidence intervals as shadows of the confidence ellipsoid) In Example 5.3, we obtained the 95% confidence ellipse for the means of the fourth roots of the door-closed and door-open microwave radiation measurements. The 95% simultaneous T^2 intervals for the two component means are, from (5-24),

$$\left(\bar{x}_1 - \sqrt{\frac{p(n-1)}{(n-p)}F_{p,n-p}(.05)}\sqrt{\frac{s_{11}}{n}},\quad \bar{x}_1 + \sqrt{\frac{p(n-1)}{(n-p)}F_{p,n-p}(.05)}\sqrt{\frac{s_{11}}{n}}\right)$$

$$= \left(.564 - \sqrt{\frac{2(41)}{40}3.23}\sqrt{\frac{.0144}{42}},\quad .564 + \sqrt{\frac{2(41)}{40}3.23}\sqrt{\frac{.0144}{42}}\right) \quad \text{or} \quad (.516,\ .612)$$

$$\left(\bar{x}_2 - \sqrt{\frac{p(n-1)}{(n-p)}F_{p,n-p}(.05)}\sqrt{\frac{s_{22}}{n}},\quad \bar{x}_2 + \sqrt{\frac{p(n-1)}{(n-p)}F_{p,n-p}(.05)}\sqrt{\frac{s_{22}}{n}}\right)$$

$$= \left(.603 - \sqrt{\frac{2(41)}{40}3.23}\sqrt{\frac{.0146}{42}},\quad .603 + \sqrt{\frac{2(41)}{40}3.23}\sqrt{\frac{.0146}{42}}\right) \quad \text{or} \quad (.555,\ .651)$$

In Figure 5.2, we have redrawn the 95% confidence ellipse from Example 5.3. The 95% simultaneous intervals are shown as shadows, or projections, of this ellipse on the axes of the component means. ■