STP598sta: Spatiotemporal Analysis

Homework 4

Name: Your name; NetID: Your ID

Due 11:59pm Friday November 22, 2024

Question 1

For the non-causal stationary process

$$x_t = \phi x_{t-1} + w_t, \quad |\phi| > 1$$

and $w_t \overset{iid}{\sim} N(0, \sigma_w^2)$. What is the autocovariance? ACF?

Question 2

For the AR(2) series $x_t + 1.6x_{t-1} + 0.64x_{t-2} = w_t$, use the difference equation (ref pages 19-25 of lecture 8, or the resutls of Example 3.10 of TSA book) to find the ACF $\rho(h), h = 0, 1, \cdots$; solve for the constants in the ACF using the initial conditions. Then plot the ACF values to lag 10 (use `ARMAacf` as a check on your answers).

Question 3

Generate $n = 100$ observations from each of the three models: (a) AR(1), (b) MA(1) and (c) ARMA(1,1) with $\phi = 0.6$ and $\theta = 0.9$. Plot the (sample) ACF and PACF for each of the three models and compare with their theoretic values. What do you find about their tailing behavior?

Question 4

Consider a system process given by

$$x_t = -.9x_{t-2} + w_t, \quad t = 1, \cdots, n$$

where $x_0 \sim N(0, \sigma_0^2)$, $x_{-1} \sim N(0, \sigma_1^2)$, and w_t is Gaussian white noise with variance σ_w^2. The system process is observed with noise:

$$y_t = x_t + v_t,$$

where v_t is Gaussian white noise with variance σ_v^2. Further, suppose x_0, x_{-1}, $\{w_t\}$ and $\{v_t\}$ are independent.

(a) Write the system and observation equations in the form of a state space model.
(b) Find the values of σ_0^2 and σ_1^2 that make the observations, y_t, stationary.
(c) Generate $n = 100$ observations with $\sigma_w = 1$, $\sigma_v = 1$ and using the values of σ_0^2 and σ_1^1 found in (b). Do a time plot of x_t and of y_t and compare the two processes. Also, compare the sample ACF and PACF of x_t and y_t.
(d) Repeat (c), but with $\sigma_v = 10$.