In previous lectures, we have gone through basics of image processing

Now in next segment, we understand and extract higher level features from images.

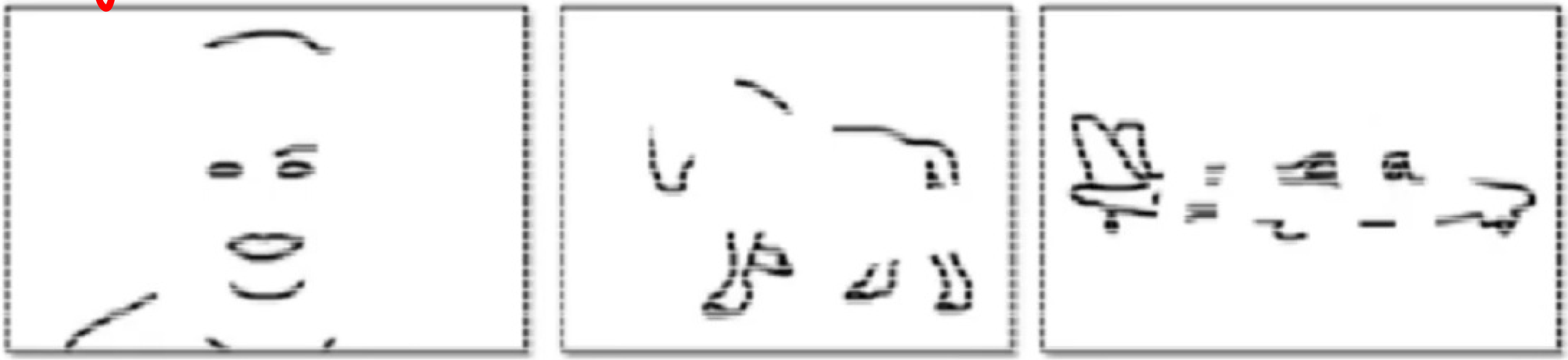
Edge Detection

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 10 & 1 \\ 3 & 5 & 1 \end{bmatrix}$$ 2D matrix $\longrightarrow$ lines or edges

In edge detection, an image is mapped from a 2D matrix to a set of lines or curves on the image.

These curves or lines or edges are a more compact representation of the image.

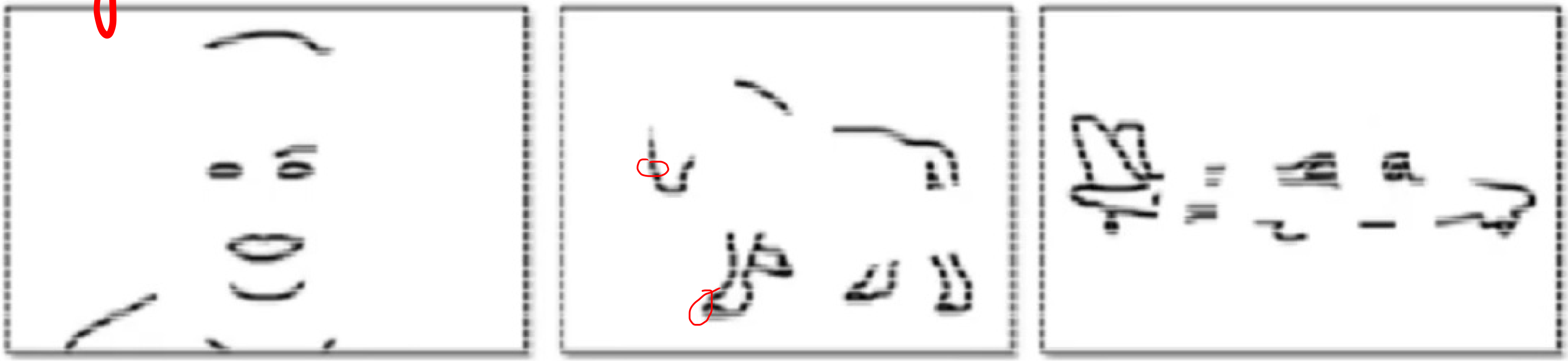
Edge Detection



L= human M= horse R= aircraft

If you see these three images and can you identigy the objects in the image?

Edge Detection



It is not hard for you to say, the first image is one of a person, the middle image looks like one of a horse and the last image looks like one of an aircraft.

This is a bit surprising because if you give such an image as input to machine learning algorithm or to a neural network, it is not going to be trivial for the model to be able to make these classifications the way you and I can do.

In the human visual system, edges are extremely important to complete the picture and for the entire process of perception.

In fact, if you see these images, even if the edges are taken away to some extent, we can fill in with the rest of the edges and still be able to say what objects are present in these images. That is how important edges are in an image.

How would you find edges in images? We have being introduced to a few concepts with the edges in convolution topic to some extent.

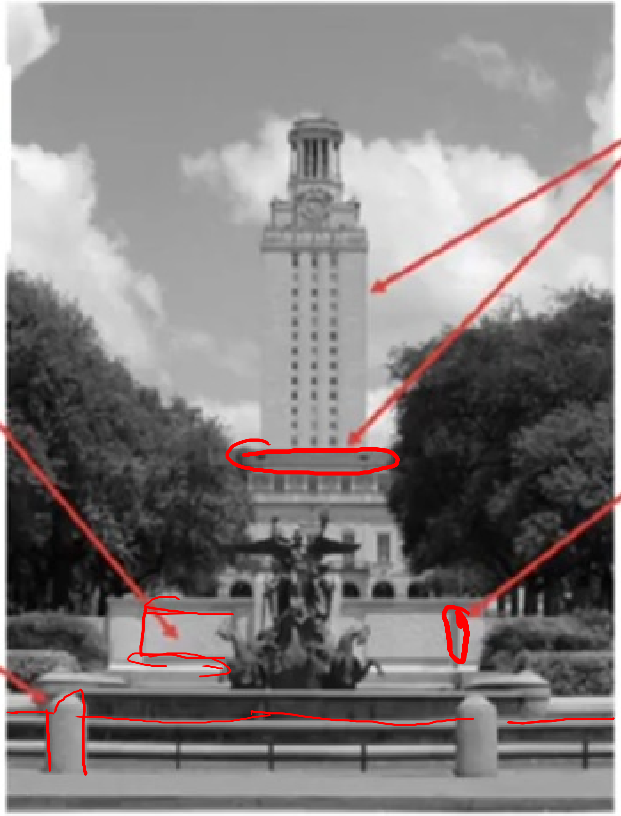
For edge detection, strong gradients are identified.

Then a post-processing is applied to get good-looking stable edges.

How are Edges Caused?

2 Surface color/appearance discontinuity

4 Depth discontinuity



3 Illumination discontinuity

1 Surface normal discontinuity

These are not the only kind of discontinuities that can cause edges, but these are examples of discontinuities that can cause edges.

Edges are fundamentally some form of discontinuity in the image. The discontinuity could be because of

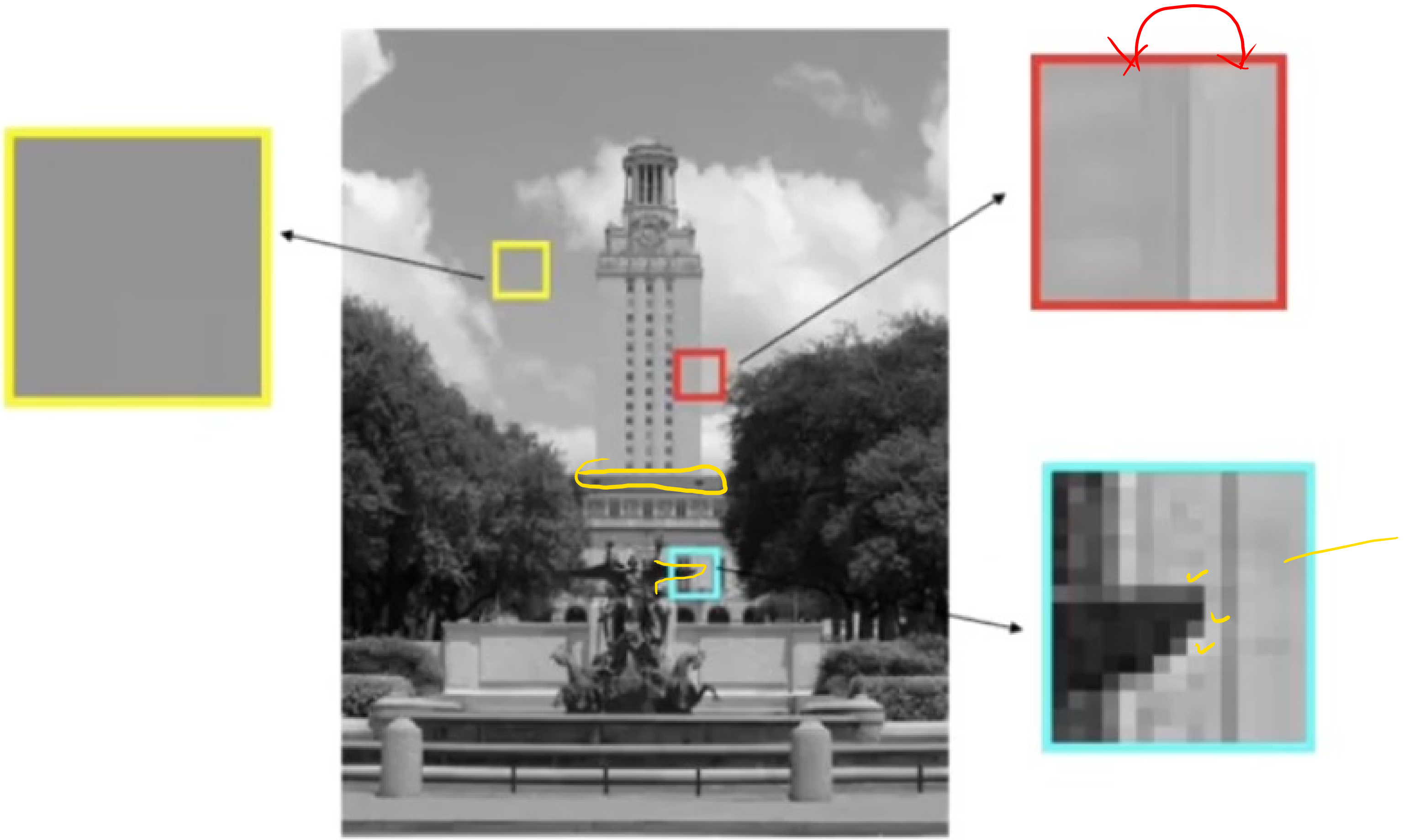
1) surface normals. For example, you see on this particular pole that it is cylindrical and as you go around the cylindrical, the surface normal direction is changing and beyond a point, there is a discontinuity in the surface normal direction, which appears as an edge for the human eye.

2) a color or an appearance discontinuity. for example, maybe a black and white image, you can imagine a red block below which there is a blue block. Just that color difference is going to lead to a discontinuity.

3) depth based discontinuities. If you observed this tower on the back, you can see that that is one portion here, which is protruding out of the building and then obviously, the building is behind the protrusion. So that gap in depth between these two artefacts in your image also leads to a discontinuity and hence an edge.

4) illumination discontinuity, which is caused due to changes in light, such as shadows. For example, you will see here, that there is a small artifact that where a portion of a shadow falls on that region and you see an edge on that particular place.

Looking Edges at Pixel Level



If you look more locally at these regions, you see that where there is no edge, the image looks fairly smooth, but where there is an edge, there is some kind of a transition.

In the image pixels in a particular direction, edges can be in different directions in an image, and you see that there is one particular direction in which, there is a change in intensities.

Or if you took another region of the image, you can see that in that region, there are many kinds of edges in different orientations.

So we ideally want to be able to detect all of these in an image.

Application of Edges

Edges are important to the human visual system, but it is not just the human visual system, edges are also important for a machine based intelligence system.

Where do you use edges?

You can use edges to group pixels in the objects or parts. For example, edges tell you that all the region inside one particular area belongs to a common object.

It can also help us to track important features across images.

It can be cues for 3D shape

It could also help you just do interactive image editing.

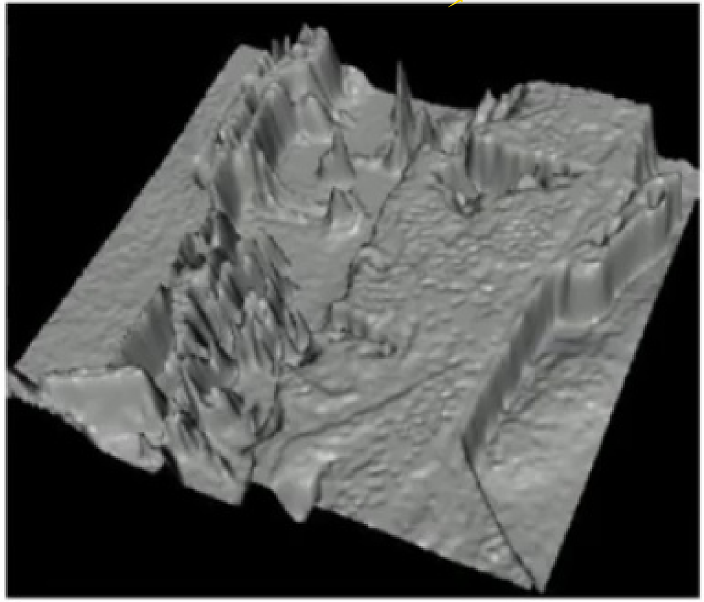
Why are Edges Important?

For example, I want to take this building and I want to put snow in the background or mountains in the background.

So I ideally need edge information to isolate the building from the background and then be able to change the background accordingly, so edges are important for those kinds of applications.

Edges in Image as Function

115
10 125
200
240



Images being represented in multiple ways. Images can be looked at as matrices, images can also be looked at as functions.
When you talk about images as functions, edges look like very steep cliffs. What does that mean?

If you had an image such as what you see on the left. Our job now is to find out where do these steep cliffs exist in the images.

Steep cliffs can be found with derivatives (gradients).

That means in a very small unit change in the pixels, that is going from one pixel to the next pixel or two pixels away, there is a huge change in the intensity.

So an edge is a place of rapid change in the image intensity function, or it can effectively be measured using a derivative or a gradient.

How?

The function depends on two or more variables. Here, the derivative converts into the partial derivative since the function depends on several variables.

We use the limit tends to zero to figure out the rate of change with respect to pixel coordinates.

$$\frac{\partial f(x,y)}{\partial y} = x^2$$

Partial derivate

$$f(x,y) = x^2 y + 1$$

$$\frac{\partial f(x,y)}{\partial x} = 2xy$$

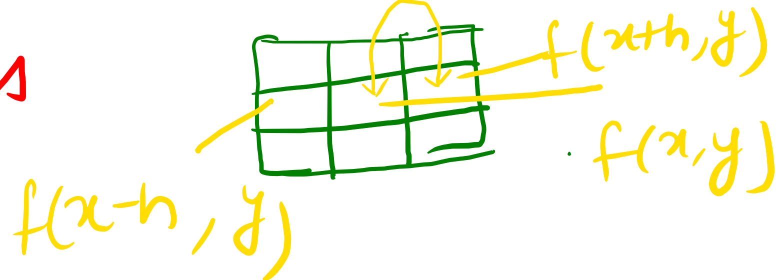
2-variable

$$f(x) = 3x^2$$

$$\frac{df}{dx} = 6x$$

1 variable

Derivatives with Convolutions



- The partial derivative for 2D function $f(x,y)$

$$\frac{\partial f}{\partial x} = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h}$$

- For discrete data, we can approximate partial derivate using finite difference

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1,y)-f(x,y)}{1}$$

$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = 5x - 4, \quad f'(x) = ? \qquad \frac{\partial f(x)}{\partial x}$$

$$f(x+h) = 5(x+h) - 4$$

$$f(x) = \lim_{h \to 0} \frac{5(x+h) - 4 - (5x - 4)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{5x} + 5h - \cancel{4} - \cancel{5x} + \cancel{4}}{h}$$

$$= \lim_{h \to 0} \frac{5\cancel{h}}{\cancel{h}} = 5$$

$$f'(x,y) = \lim_{h\to 0} \frac{f(x+h) - f(x,y)}{h}$$

$$f(x,y) = x^2 + 3xy + y - 1$$

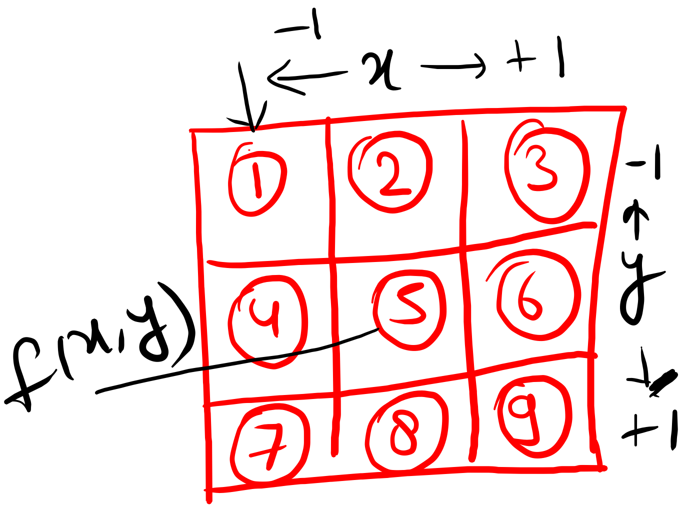
$$f'(x,y) = \lim_{h\to 0} \frac{\left[\left((x+h)^2 + 3(x+h)y + y - 1\right) - x^2 - 3xy - y + 1\right]}{h}$$

$$f'(x,y) = \lim_{h\to 0} \frac{\cancel{x^2} + h^2 + 2xh + \cancel{3xy} + 3hy + \cancel{y} \cancel{-1} - \cancel{x^2} - \cancel{3xy} - \cancel{y} + \cancel{1}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{h^2 + 2xh + 3hy}{h}$$

$$f'(x) = \lim_{h \to 0} \cdot \underset{\substack{\downarrow \\ 0}}{h} + 2x + 3y$$

$$f'(x) = 2x + 3y$$



1. $f(x-1, y-1)$
2. $f(x, y-1)$
3. $f(x+1, y-1)$
4. $f(x-1, y)$
6. $f(x+1, y)$
7. $f(x-1, y+1)$
8. $f(x, y+1)$
9. $f(x+1, y+1)$

Derivatives with Convolutions

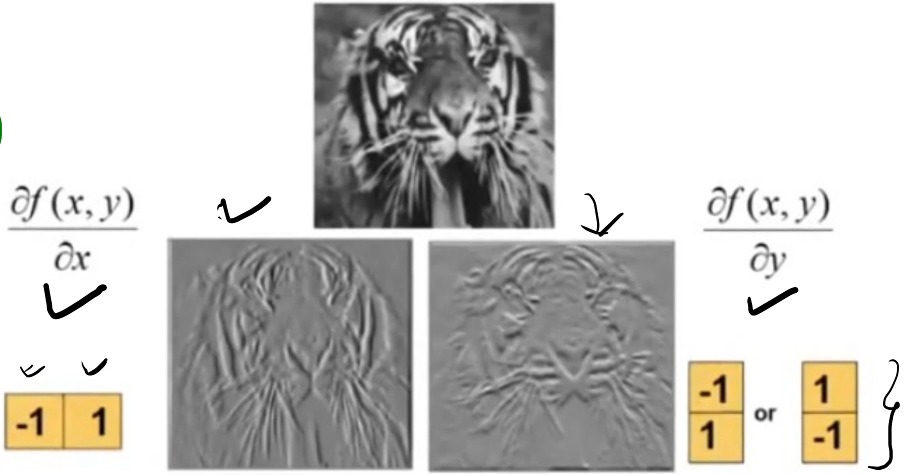
- The partial derivative for 2D function $f(x,y)$

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h}$$

- For discrete data, we can approximate partial derivate using finite difference

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x, y)}{1}$$

$$\begin{array}{c} +1 \\ \updownarrow \\ -1 \end{array} \quad \text{or} \quad \begin{array}{cc} y_1 & -1 \\ y_2 & \updownarrow \\ y_3 & +1 \end{array}$$

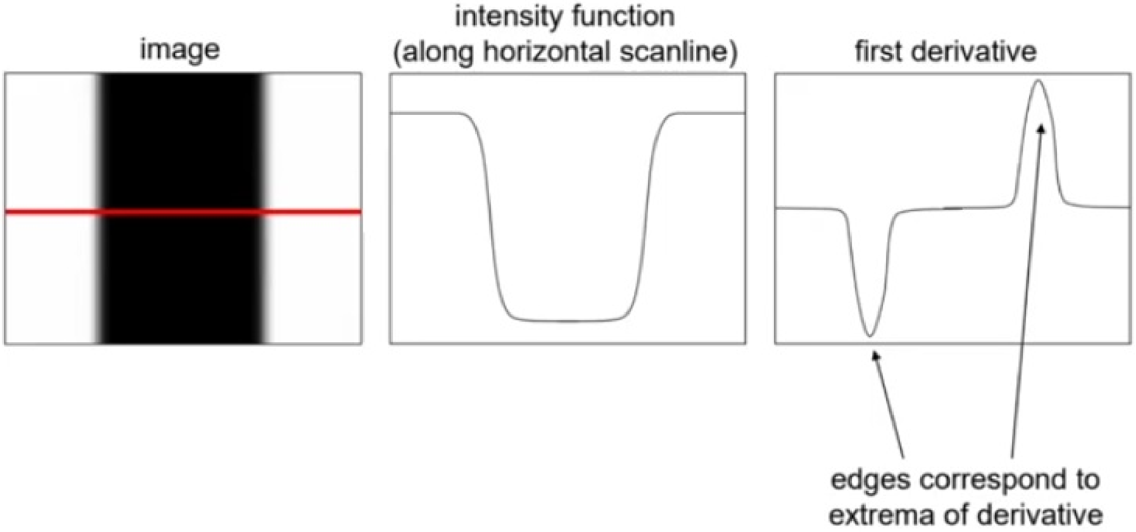


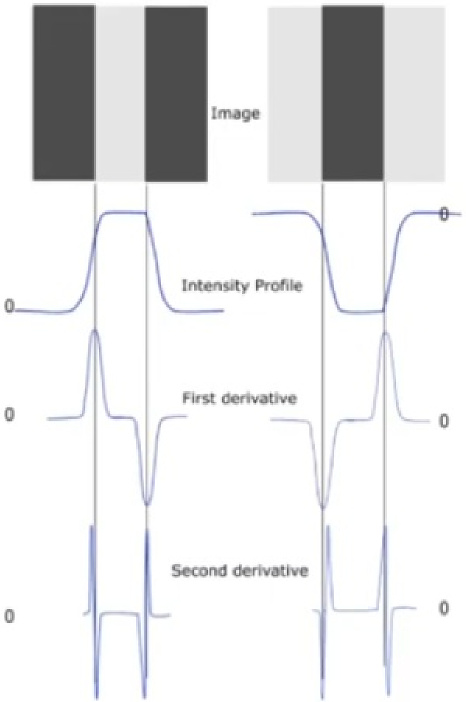
Here we understand that edges can be obtained using derivatives or gradients.

But how do you achieve edges using convolution?

Derivaties and Edges

Example 2





An edge is a place of rapid change in the image intensity function.

In this image you have a white patch followed by a black patch, then a white patch again. So if you took one particular row of this image, you have the same pattern a set of white pixels then a set of black pixels and then a set of white pixels.

your intensity function have variations in the curve shape.

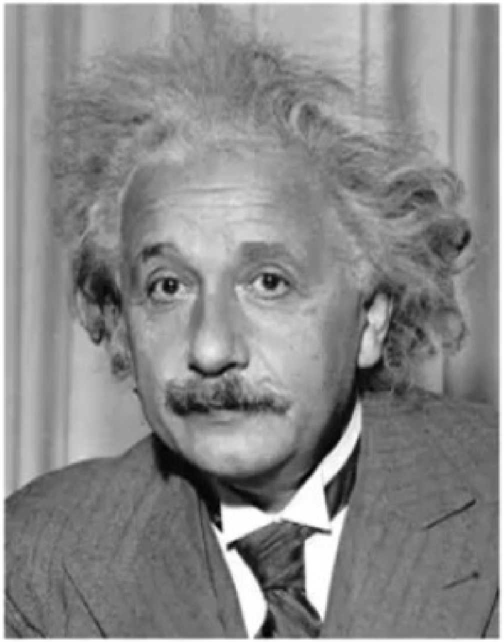
We can recall that white has higher intensity, black has lower intensity, so you observe such variations.

If you took a derivative of all of these values in the intensity function, the derivative shape is shown in the third column.

So there is an initial point where you have a negative derivative because the value is falling down and then there is a later point where you have an equal in magnitude of the gradient, but in the opposite direction, because the intensity increases at that particular point.

The second derivative looks like as long as the first gradient keeps going up, it goes up until a certain point then falls off and then comes back for the second half of the gradient.

Sobel Vertical Edge Detection Filters



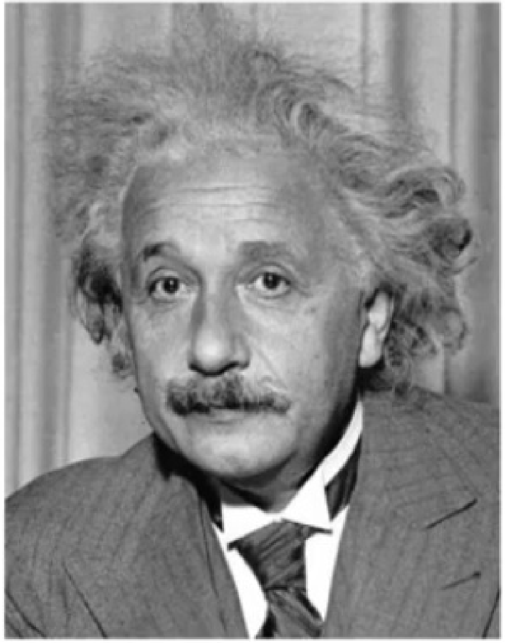
$f(x-1, y-1)$

1	0	-1
2	0	-2
1	0	-1

Sobel

Vertical Edge
(absolute value)

Sobel Horizontal Edge Detection Filters



1	2	1
0	0	0
-1	-2	-1

Sobel

Horizontal Edge
(absolute value)

Finite Difference Filters

① Prewitt

$$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

② Sobel

$$M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

③ Roberts

$$M_x = \cancel{\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}} \qquad M_y = \cancel{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}$$

Prewitt filter is similar to Sobel.

There is also the Roberts filter which finds edges in diagonal directions.

You can handcraft several kinds of filters to be able to find edges in different directions.

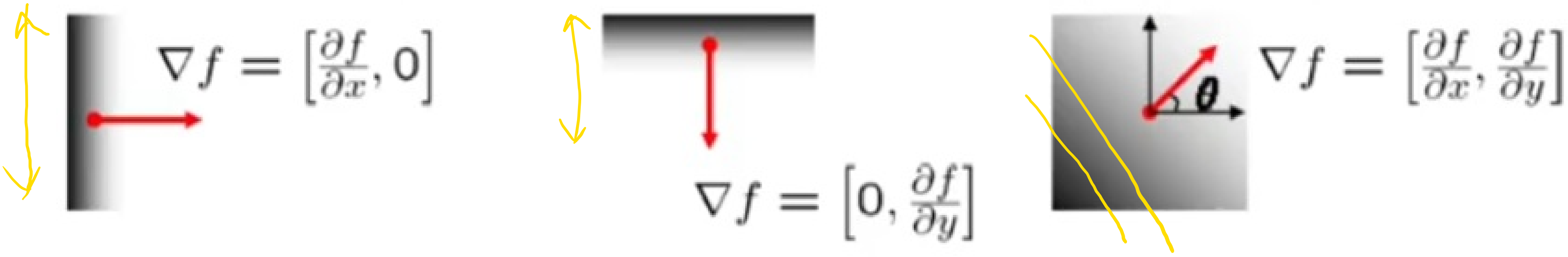
Then how do we find edges in any direction?

Do we have to convolve with many different filters to be able to find edges in different directions?

Image Gradient

- The gradient of an image $\Delta f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

- The gradient points in the direction of most rapid change in intensity.



For example, if you have the gradient to be (d(f)/d(x), 0), this tells you that there is no change in the y direction and all the change is only along the x direction, which the edge in the image would look something like this.

If you had the gradient to be (0, d(f) / d(y)), then there is no change along the x direction and the edge is only along the vertical direction and this is how such an edge will look like an image.

On the other hand, if you add an edge in a completely different random direction, like a diagonal direction, not aligned along the vertical axis or the horizontal axis, then you would have a gradient, which is simply given by (d(f) / d(x), d(f) / d(y)). In both the directions there is a non-zero gradient, which gives you an edge in a different direction.

- The gradient direction (orientation of edge) is given by

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

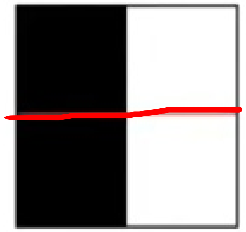
- The edge strength is given by the magnitude $\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$

How do you find the orientation of the edge? We said that edges have different orientations. Your principles from simple calculus, the orientation of the gradient is simply given by tan inverse of gradient along y divided by gradient along x.

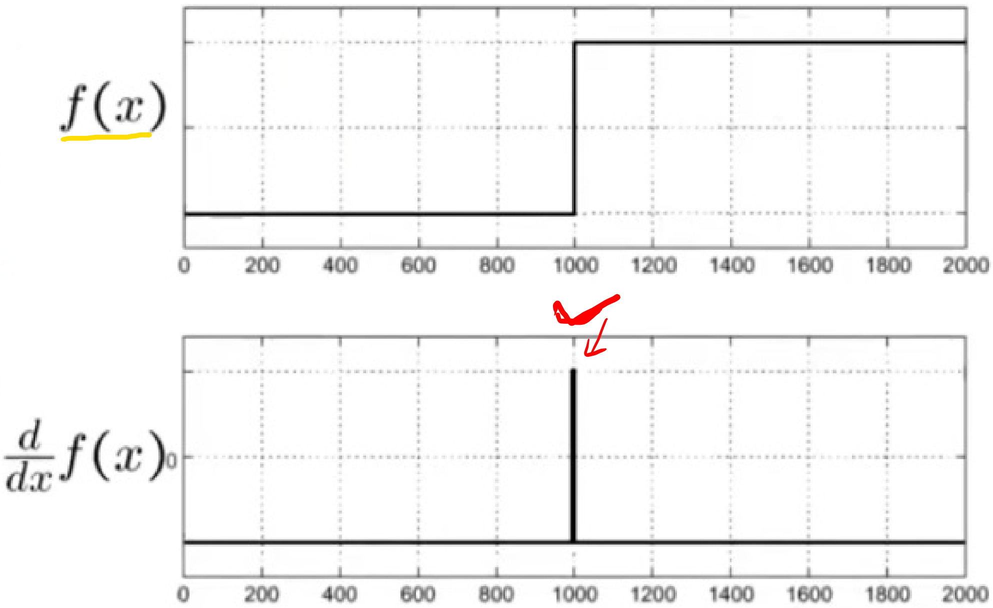
And finally the strength of the edge is given by the magnitude of the gradient. That is ((d(f) / d(x))^2, (d(f) / d(y)^2) under root gives you the magnitude of the edge in that particular location. So how strong is the edge, is what this gives you at that particular location.

Derivative with NO Noise

- Consider a single row or column of the image. Plotting intensity as a function of position gives a signal.



Input image with no noise



Where is the edge?

Let us again take an example of a single row in an image.

So let us assume now that the image is likely changed. We have a black patch followed by a white patch, that is what we have here.

And now let us take one particular row, just for simplicity of understanding. You could have taken the full image to just that the figures on the right would have looked more complex. So let us just take one particular row on the image.

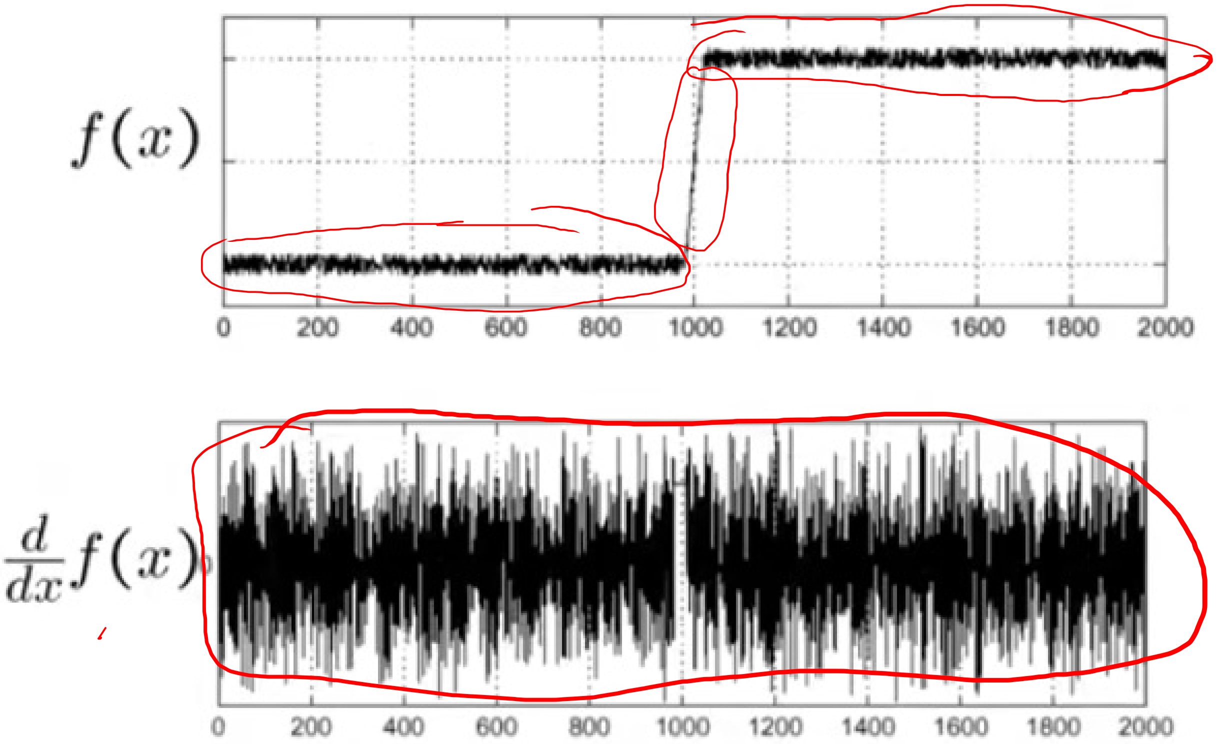
Clearly, even in that particular row, there be the first set of pixels which are black then the next set of pixels which are white.

If you took doing the gradient of such a function. The derivative is flat in all these locations and all these locations and there is a spike right in the middle when there is a change in the intensity.

So now you can find the edge? It is simple, you simply say wherever the gradient has a high value, that is where the edge is present in the image.

Effect of Noise

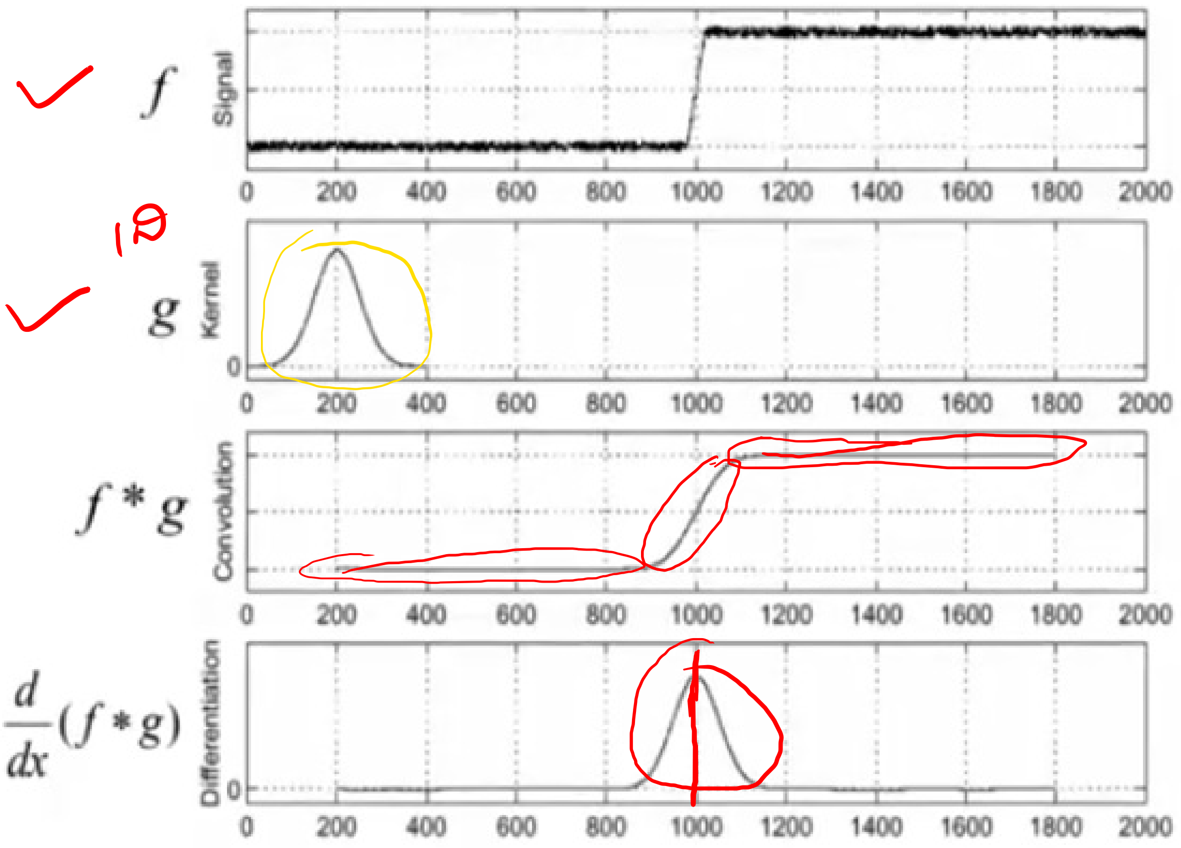

Noisy input image



Where is the edge?

Effect of Noise

Smooth first, and look for peaks in $\frac{d}{dx}(f * g)$



2D

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

it is one-dimensional Gaussian filter. We convolve the image with that one-dimensional Gaussian filter.

The entire row in the image is smoothened out, all the noise becomes flat and even the edge gets slightly smoothened out, remember it was a sharp edge and it is got a little smoothened out with Gaussian filters.

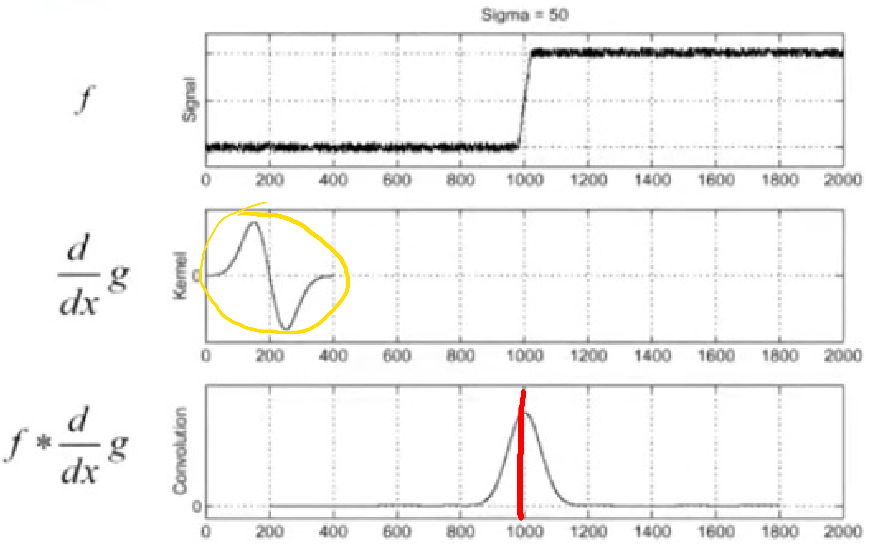
While they remove noise they also blur out portions of the image, that is what would happen here for that middle region where the edge is. And then you have the rest of the region to be flat too.

Now on this Gaussian smoothed image, you can run an edge detector by running a gradient and you would find that an edge can be found in the middle of it.

Derivative theorem of Convolution

- Differentiation is achieved through Convolution, and Convolution is associative

$$\frac{d}{dx}(f * g) = f * \frac{d}{dx}g = \frac{d}{dx}f * g$$



$\frac{d}{dx}[1\ 2\ 1]$

↓

save

This saves one operation

Gradients are good estimators of edges. What about the second derivative?

So if you add your original signal f again here so the second derivative of Gaussian is called Laplacian of Gaussian.

So the Laplacian of Gaussian can also be used to directly convolve on the image

And this time, the edge is located at zero crossing at thousand. Remember, in the earlier case, when we used the gradient of the Gaussian filter, we found that the edge was somewhere, edge was peaking at the thousand reading, there is where the edge was peaking.

But if you apply the Laplacian of Gaussian, you find that at thousand, you actually get the value to be zero.

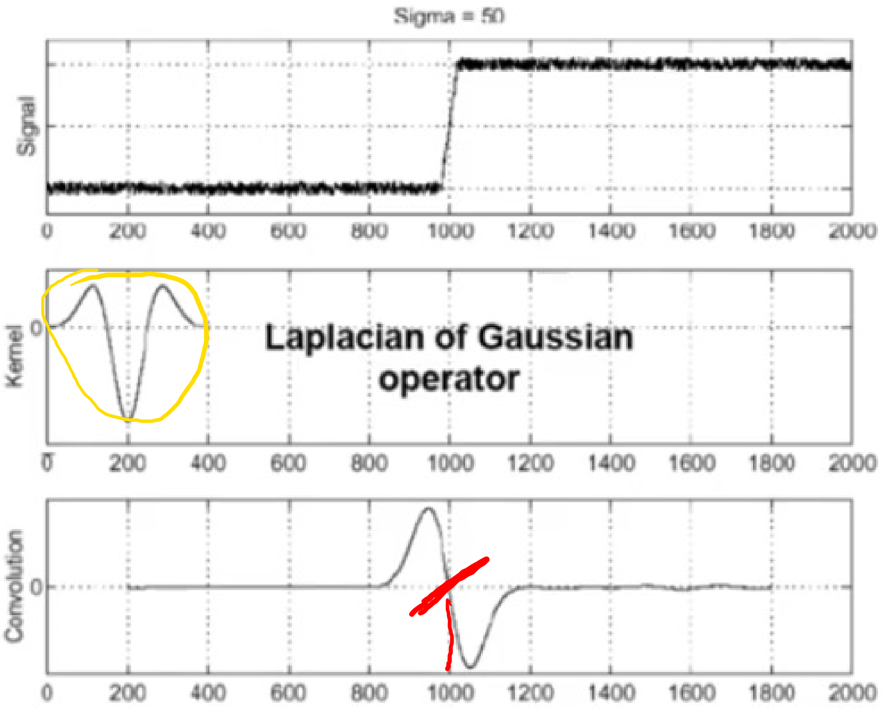
What about the Second Derivative

- Edge by detecting the zero crossing of the graph

f

$\frac{\partial^2}{\partial x^2}h$

$(\frac{\partial^2}{\partial x^2}h) \star f$

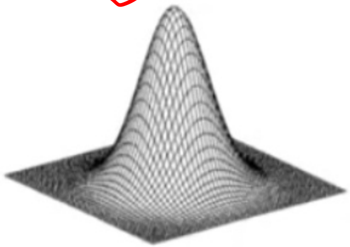


When we took the first derivative, edges were found where the gradient was high

but if you go to the second derivative of the Gaussian, edges are found where you have a zero crossing.

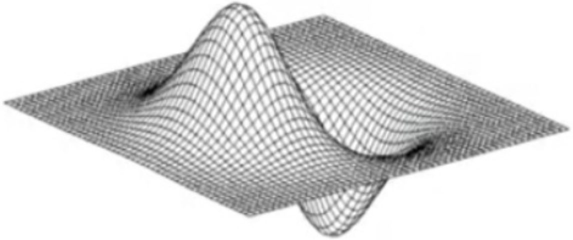
Which means there is some value that is positive and you are transitioning to a negative value and the zero crossing is where an edge can be localized. That is what the function of Laplacian of Gaussian filters .

Derivative and Laplacian of Gaussian



Gaussian

$h_\sigma(x,y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{u^2+v^2}{2\sigma^2}}$



Derivative of Gaussian (x)

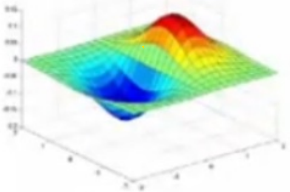
$\frac{\partial}{\partial x} h_\sigma(u,v)$

Laplacian of Gaussian

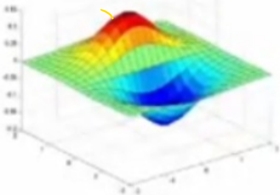
$\nabla^2 h_\sigma(u,v)$

with ∇^2 the Laplacian operator

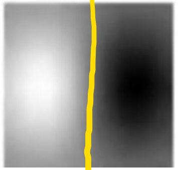
$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

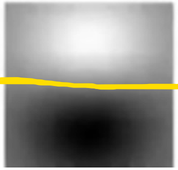


x-direction



y-direction





vertical   Horizontal

Which one finds horizontal/vertical edges?

The Laplacian of Gaussian is written as or the Laplacian operator in general is written as Nabla square f.
So if you now put the derivative of the Gaussian, remember again, the derivative can be in two directions.

You can have the derivative of the Gaussian along the x direction, derivative of the Gaussian along the y direction.
Red means high value, blue means low value, it is simply a surface map or a heat map.

Or from an image perspective, so now, if I ask you the question, which one of these finds horizontal edges and which one of these finds vertical edges?

The second image finds horizontal edges and the first image finds vertical edges, it is actually quite evident when you see the images itself.

The second image finds edges here, that separate this white region from the black region and the first image finds edges like this, separating the white region from the black region. So one of them finds the horizontal axis,
one of them finds the vertical axis.