Design and Analysis of Algorithm

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Outline

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Solving the Recurrence Equation

We will introduce three methods of solving the recurrence equation:

1. The Substitution Method (Guess the solution & verify by Induction)
2. Iteration Method (unrolling and summing)
3. The Recursion-tree method
4. Master method

SUBSTITUTION METHOD

A substitution method is one, in which we guess a bound and then use mathematical induction to prove our guess correct. It is basically two step process:

- **Step1:** Guess the form of the Solution.
- **Step2:** Prove your guess is correct by using Mathematical Induction.

SUBSTITUTION METHOD

Example

$T(n) = 2T(\frac{n}{2}) + n$

Solution

Step 1: Let we guess the solution is $T(n) = O(nlogn)$ or $T(n) \leq c \cdot nlogn$.

SUBSTITUTION METHOD

Step 2: Now we use mathematical induction.

Here our guess does not hold for $n = 1$ because $T(n) \leq c \cdot 1log1$.

That is, $T(n) \leq 0$, which is contradiction with $T(1) = 1$.

SUBSTITUTION METHOD

Now for $n = 2$,

$T(n) \leq c \cdot 2log2$

$2T(\frac{2}{2}) + 2 \leq c \cdot 2$

$2T(1) + 2 \leq c \cdot 2$

$0 + 2 \leq c \cdot 2$

$2 \leq c \cdot 2$ which is true. So $T(n) \leq c \cdot nlogn$ is True for $n = 2$.

SUBSTITUTION METHOD

Induction step: Now assume it is true for $n = fracn2$.

That is, $T(\frac{n}{2}) \leq c \cdot \frac{n}{2} log \frac{n}{2}$

SUBSTITUTION METHOD

Now we have to show that it is true for $n = n$.

That is, $T(n) \leq c \cdot nlogn$

We know that $T(n) \leq 2T(\lfloor \frac{n}{2} \rfloor) + n$.

$\leq 2\big(c\lfloor \frac{n}{2} \rfloor log \lfloor \frac{n}{2} \rfloor\big) + n$

$\leq cnlog \lfloor \frac{n}{2} \rfloor + n \leq (cnlogn - cnlog2) + n$

$\leq cnlogn - cn + n$

$\leq cnlogn$, for all $c \geq 1$

Thus, $T(n) = O(nlogn)$

ITERATION METHOD (Unrolling and summing)

In this method we unroll (or substituting) the given recurrence back to itself until not getting a regular pattern (or series).

We generally follow the following steps to solve any recurrence:

- Expend the recurrence Express the expansion as a summation by plugging the recurrence back into itself until you see a pattern.
- Evaluate the summation by using the arithmetic or geometric summation formulae

Thank You!