"C:\Program Files\Maple 2015" (1)

```
> #################################################################################
  ## Section 4
  ## Polynomials of 5 variables in degree 4
  #################################################################################
```

```
> # Set the working directory
  currentdir("C:/Users/slapl/Dropbox/repos/rationalSOS");
```

"C:\Users\slapl\Dropbox\Repos\2020-strictlyPositive\worksheets" (2)

```
> #################################################################################
  # Load "Rational SOS" procedures
  #################################################################################
  read("rationalSOS.mpl");
  with(rationalSOS);
  with(LinearAlgebra);
```

"Opening connection with Matlab"

rationalSOS := **module**( ) ... **end module**

[*cancelDenominator*, *decompositionToMatrix*, *evalMat*, *evalSolution*, *exactSOS*, *getCoeffs*, *getDiag*, *getExtension*, *getVars*, *homogenize*, *isHomogeneous*, *linIndepRows*, *listSubsets*, *matrixToPoly*, *minorsDet*, *nonRatCoef*, *numericSolver*, *numericSolverSubmatrix*, *numericSolverSubmatrixMaxRank*, *numericSolverSubmatrixRoundBefore*, *polyToMatrix*, *polyToMatrixVars*, *primitiveMatrix*, *randomRank*, *reduceByLinearEquation*, *reduceByLinearEquationLinear*, *roundMat*, *roundMatToZero*, *roundToIntMatrix*, *roundVec*, *sedumiCall*, *smallToZero*, *solveSubmatrixGeneral*, *vectorTrace*, *zeroDetSRows*, *zeroDetSys*, *zeroRows*]

[*&x*, *Add*, *Adjoint*, *BackwardSubstitute*, *BandMatrix*, *Basis*, *BezoutMatrix*, *BidiagonalForm*, *BilinearForm*, *CARE*, *CharacteristicMatrix*, *CharacteristicPolynomial*, *Column*, *ColumnDimension*, *ColumnOperation*, *ColumnSpace*, *CompanionMatrix*, *CompressedSparseForm*, *ConditionNumber*, *ConstantMatrix*, *ConstantVector*, *Copy*, *CreatePermutation*, *CrossProduct*, *DARE*, *DeleteColumn*, *DeleteRow*, *Determinant*, *Diagonal*, *DiagonalMatrix*, *Dimension*, *Dimensions*, *DotProduct*, *EigenConditionNumbers*, *Eigenvalues*, *Eigenvectors*, *Equal*, *ForwardSubstitute*, *FrobeniusForm*, *FromCompressedSparseForm*, *FromSplitForm*, *GaussianElimination*, *GenerateEquations*, *GenerateMatrix*, *Generic*, *GetResultDataType*, *GetResultShape*, *GivensRotationMatrix*, *GramSchmidt*, *HankelMatrix*, *HermiteForm*, *HermitianTranspose*, *HessenbergForm*, *HilbertMatrix*, *HouseholderMatrix*, *IdentityMatrix*, *IntersectionBasis*, *IsDefinite*, *IsOrthogonal*, *IsSimilar*, *IsUnitary*, *JordanBlockMatrix*, *JordanForm*, *KroneckerProduct*, *LA_Main*, *LUDecomposition*, *LeastSquares*, *LinearSolve*, *LyapunovSolve*, *Map*, *Map2*, *MatrixAdd*, *MatrixExponential*, *MatrixFunction*, *MatrixInverse*, *MatrixMatrixMultiply*, *MatrixNorm*, *MatrixPower*, *MatrixScalarMultiply*, *MatrixVectorMultiply*, *MinimalPolynomial*, *Minor*, *Modular*, *Multiply*, *NoUserValue*, *Norm*, *Normalize*, *NullSpace*, *OuterProductMatrix*, *Permanent*, *Pivot*, *PopovForm*, *ProjectionMatrix*, *QRDecomposition*, *RandomMatrix*, *RandomVector*, *Rank*, *RationalCanonicalForm*, *ReducedRowEchelonForm*, *Row*, *RowDimension*, *RowOperation*, *RowSpace*, *ScalarMatrix*, (3) *ScalarMultiply*, *ScalarVector*, *SchurForm*, *SingularValues*, *SmithForm*, *SplitForm*, *StronglyConnectedBlocks*, *SubMatrix*, *SubVector*, *SumBasis*, *SylvesterMatrix*, *SylvesterSolve*, *ToeplitzMatrix*, *Trace*, *Transpose*, *TridiagonalForm*, *UnitVector*, *VandermondeMatrix*, *VectorAdd*, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*, *ZeroMatrix*, *ZeroVector*, *Zip*]

```
> # Display tables of any size
  interface(rtablesize = infinity);
```

$$10 \tag{4}$$

```
> ##########################################################################
  ## Example 4.3
  ## Example in the border with unique solution
  ##########################################################################
```

```
> # The first 4 polynomials correspond to an example of a polynomial
  # in the non-negative border of the SOS-cone in the 4-4 case.
  # We add a fifth polynomial to produce an example for the 5-4 case.
  p1 := x1^2 - x4^2;
  p2 := x2^2 - x4^2;
  p3 := x3^2 - x4^2;
  p4 := -x1^2 - x1 * x2 - x1 * x3 + x1 * x4 - x2 * x3 + x2 * x4 + x3 * x4;
  p5 := x5^2
```

$$\begin{gathered} p1 := x1^2 - x4^2 \\ p2 := x2^2 - x4^2 \\ p3 := x3^2 - x4^2 \\ p4 := -x1^2 - x1\,x2 - x1\,x3 + x1\,x4 - x2\,x3 + x2\,x4 + x3\,x4 \\ p5 := x5^2 \end{gathered} \tag{5}$$

```
> # f is the sum of squares of p1, ..., p5
  f := p1^2 + p2^2 + p3^2 + p4^2 + p5^2;
```

$$f := (x1^2 - x4^2)^2 + (x2^2 - x4^2)^2 + (x3^2 - x4^2)^2 + (-x1^2 - x1\,x2 - x1\,x3 + x1\,x4 - x2\,x3 + x2\,x4 + x3\,x4)^2 + x5^4 \tag{6}$$

```
> # We use SEDUMI to compute a SOS decomposition.
  # With default options, exactSOS will compute a solution of maximum rank
  out := exactSOS(f, facial = "no") :
```

"Calling numerical solver SEDUMI to find values of the indeterminates..."

```
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, Adaptive Step-Differentiation, theta =
0.250, beta = 0.500
eqs m = 51, order n = 16, dim = 226, blocks = 2
nnz(A) = 115 + 0, nnz(ADA) = 2601, nnz(L) = 1326
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg
```

```
cg  prec
   0 :            1.66E+01 0.000
   1 :  -3.35E+00 5.29E+00 0.000 0.3183 0.9000 0.9000   0.57  1
1  1.1E+01
   2 :  -8.82E-01 1.64E+00 0.000 0.3097 0.9000 0.9000   2.56  1
1  1.8E+00
   3 :  -8.44E-02 4.27E-01 0.000 0.2607 0.9000 0.9000   2.38  1
1  6.7E-01
   4 :  -2.23E-02 1.17E-01 0.000 0.2738 0.9000 0.9000   1.19  1
1  4.7E-01
   5 :  -6.30E-03 3.54E-02 0.000 0.3024 0.9000 0.9000   1.07  1
1  4.5E-01
   6 :  -2.05E-03 1.09E-02 0.000 0.3091 0.9000 0.9000   1.03  1
1  4.1E-01
   7 :  -6.64E-04 3.35E-03 0.000 0.3067 0.9009 0.9000   1.01  1
1  2.6E-01
   8 :  -2.01E-04 1.09E-03 0.000 0.3249 0.9000 0.9076   1.01  1
1  7.9E-02
   9 :  -5.53E-05 3.79E-04 0.000 0.3481 0.9000 0.9186   1.00  1
1  2.2E-02
  10 :  -1.48E-05 1.45E-04 0.000 0.3816 0.9000 0.9297   1.00  1
1  5.7E-03
  11 :  -5.40E-06 5.84E-05 0.000 0.4034 0.9000 0.8160   1.00  1
1  1.6E-03
  12 :  -1.81E-06 1.69E-05 0.000 0.2898 0.9006 0.9000   1.00  1
1  4.5E-04
  13 :  -5.75E-07 5.42E-06 0.000 0.3204 0.9000 0.9072   1.00  1
1  1.2E-04
  14 :  -1.79E-07 1.87E-06 0.000 0.3449 0.9000 0.8814   1.00  1
1  2.9E-05
  15 :  -6.32E-08 5.97E-07 0.000 0.3193 0.9000 0.8528   1.00  1
1  8.8E-06
  16 :  -2.06E-08 1.59E-07 0.000 0.2656 0.9077 0.9000   1.00  1
1  3.1E-06
  17 :  -6.42E-09 4.91E-08 0.000 0.3096 0.9000 0.9001   1.00  1
1  9.6E-07
  18 :  -1.91E-09 1.68E-08 0.000 0.3433 0.9000 0.8768   1.00  1
1  2.4E-07
  19 :  -6.61E-10 5.56E-09 0.000 0.3301 0.9000 0.8421   1.00  1
1  7.7E-08
  20 :  -2.11E-10 1.55E-09 0.000 0.2792 0.9065 0.9000   1.00  1
1  2.9E-08
  21 :  -5.99E-11 5.18E-10 0.000 0.3336 0.9000 0.9030   1.00  1
1  7.1E-09
  22 :  -2.45E-11 1.93E-10 0.000 0.3719 0.9000 0.7026   1.00  3
3  2.6E-09
  23 :  -7.72E-12 3.42E-11 0.000 0.1777 0.9210 0.9000   1.00  4
4  1.5E-09
  24 :  -2.43E-12 1.04E-11 0.000 0.3026 0.9071 0.9000   1.00  7
7  5.5E-10
  25 :  -3.41E-13 3.63E-12 0.000 0.3504 0.9000 0.9274   1.00 10
10  7.8E-11
  26 :  -1.79E-13 1.64E-12 0.000 0.4510 0.9000 0.5362   1.00 18
18  2.6E-11
  27 :  -5.07E-14 3.36E-13 0.000 0.2050 0.9154 0.9000   1.00 18
18  1.1E-11
  28 :  -1.66E-14 1.12E-13 0.000 0.3332 0.9039 0.9000   1.00 21
```

```
21  3.8E-12
 29 :  -3.43E-15 4.66E-14 0.000 0.4166 0.9000 0.8150   1.00 21
21  7.1E-13
 30 :  -9.35E-16 1.74E-14 0.000 0.3737 0.9000 0.7690   1.02 19
23  2.6E-13
 31 :   2.83E-16 5.23E-15 0.000 0.3003 0.9044 0.9000   1.01 23
24  9.0E-14
 32 :   6.14E-16 2.02E-15 0.000 0.3861 0.9000 0.6879   1.01 24
23  3.3E-14
 33 :   7.80E-16 4.96E-16 0.000 0.2457 0.9130 0.9000   1.00 26
27  1.4E-14
 34 :   8.36E-16 1.77E-16 0.000 0.3579 0.9000 0.9122   1.00 26
26  3.4E-15
Run into numerical problems.

iter seconds digits       c*x               b*y
 34      0.3  10.7  3.5685616251e-15  8.3614982739e-16
|Ax-b| =   1.3e-14, [Ay-c]_+ =   6.4E-15, |x|=  4.9e-01, |y|=
4.0e+00

Detailed timing (sec)
   Pre          IPM          Post
1.600E-02    1.410E-01    0.000E+00
Max-norms: ||b||=1, ||c|| = 6,
Cholesky |add|=9, |skip| = 0, ||L.L|| = 3.43622e+07.
```

"An exact positive definite solution could not be found for the reduced problem." **(7)**

> *# out[3] is a matrix in the spectrahedron of maximum rank.*
> *# We check the eigenvalues to determine the rank*
> *eig*(*out*[3]);

(8)

$$\begin{bmatrix} -1.31293131280185\ 10^{-16} \\ -1.08068669314304\ 10^{-16} \\ -5.95397761540878\ 10^{-17} \\ -1.71287606449043\ 10^{-17} \\ 0. \\ 6.24958392382636\ 10^{-34} \\ 2.96509936287331\ 10^{-19} \\ 1.13961816964410\ 10^{-18} \\ 4.31333454054406\ 10^{-16} \\ 8.22842321876699\ 10^{-16} \\ 0.888960947926901 \\ 1. \\ 1. \\ 3.89989969879447 \\ 7.21113935327863 \end{bmatrix} \quad (8)$$

```
> # There are only 5 non-zero eigenvalues, the maximum rank in the
  # spectrahedron is 5.
> # We compare the matrix obtained by SEDUMI with the matrix corresponding
  # to the original decomposition p1^2+p2^2+p3^2+p4^2+p5^2.
  v := convert(out[5], list) : # The monomials indexing the columns of the Gram Matrix
  A1 := decompositionToMatrix([p1, p2, p3, p4, p5], v) :
  A2 := out[3] :
  Norm(A1 - A2);
```

$$0 \quad (9)$$

```
> # We see that both matrices are the same.
  # This gives strong numerical evidence that this is the unique matrix
  # in the spectrahedron of f.
> ###########################################################################
  ## Example 4.4
  ## Example in the border with a matrix in the spectrahedron
  ## of rank 9, the maximum round predicted by our bounds
  ###########################################################################
> # We add a polynomial f6 to the previous example
  p1 := x1^2 - x4^2;
  p2 := x2^2 - x4^2;
  p3 := x3^2 - x4^2;
  p4 := -x1^2 - x1*x2 - x1*x3 + x1*x4 - x2*x3 + x2*x4 + x3*x4;
```

$p5 := x5\text{^}2;$
$p6 := x4 * x5;$
$f := p1\text{^}2 + p2\text{^}2 + p3\text{^}2 + p4\text{^}2 + p5\text{^}2 + p6\text{^}2;$

$$p1 := x1^2 - x4^2$$
$$p2 := x2^2 - x4^2$$
$$p3 := x3^2 - x4^2$$
$$p4 := -x1^2 - x1\,x2 - x1\,x3 + x1\,x4 - x2\,x3 + x2\,x4 + x3\,x4$$
$$p5 := x5^2$$
$$p6 := x4\,x5$$
$$f := (x1^2 - x4^2)^2 + (x2^2 - x4^2)^2 + (x3^2 - x4^2)^2 + (-x1^2 - x1\,x2 - x1\,x3 + x1\,x4 - x2\,x3 + x2\,x4 + x3\,x4)^2 + x5^4 + x4^2\,x5^2 \quad \textbf{(10)}$$

> # *We use SEDUMI to compute a SOS decomposition.*
With default options, exactSOS will compute a solution of maximum rank

out := *exactSOS*(*f*, *facial* = "no", *computePolynomialDecomposition* = "no") :

"Calling numerical solver SEDUMI to find values of the indeterminates..."

```
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, Adaptive Step-Differentiation, theta =
0.250, beta = 0.500
eqs m = 51, order n = 16, dim = 226, blocks = 2
nnz(A) = 115 + 0, nnz(ADA) = 2601, nnz(L) = 1326
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg
cg  prec
  0 :            1.66E+01 0.000
  1 :   -3.31E+00 5.29E+00 0.000 0.3183 0.9000 0.9000   0.57  1
1  1.1E+01
  2 :   -8.39E-01 1.64E+00 0.000 0.3105 0.9000 0.9000   2.56  1
1  1.8E+00
  3 :   -6.89E-02 4.53E-01 0.000 0.2760 0.9000 0.9000   2.35  1
1  6.3E-01
  4 :   -1.44E-02 1.21E-01 0.000 0.2660 0.9000 0.9000   1.18  1
1  4.1E-01
  5 :   -2.81E-03 3.15E-02 0.000 0.2612 0.9000 0.9000   1.06  1
1  3.2E-01
  6 :   -1.30E-04 1.45E-03 0.000 0.0462 0.9900 0.9900   1.02  1
1  1.2E-01
  7 :   -4.93E-07 3.07E-06 0.000 0.0021 0.9990 0.9990   1.00  1
1  2.7E-04
  8 :   -7.47E-08 6.40E-07 0.000 0.2089 0.9000 0.9097   1.00  1
3  5.0E-05
  9 :   -7.36E-09 4.23E-08 0.486 0.0661 0.9900 0.9900   1.00  3
5  3.3E-06
 10 :   -1.41E-09 8.65E-09 0.000 0.2043 0.9028 0.9000   1.00  5
5  7.0E-07
 11 :   -1.03E-10 7.32E-10 0.429 0.0847 0.9900 0.9900   1.00  2
7  6.0E-08
 12 :   -1.28E-11 1.77E-10 0.003 0.2418 0.9000 0.9155   1.00 10
10  1.2E-08
 13 :   -9.41E-13 5.09E-12 0.000 0.0288 0.9900 0.9900   1.00  4
```

```
10   3.5E-10
 14 :   -1.13E-13 3.20E-12 0.325 0.6273 0.9000 0.9312   1.00 16
16   9.4E-11
Run into numerical problems.

iter seconds digits       c*x                  b*y
 14      0.1    8.5  3.1627347832e-13 -1.1326291932e-13
|Ax-b| =   1.3e-12, [Ay-c]_+ =   2.2E-14, |x|=  4.9e-01, |y|=
4.0e+00

Detailed timing (sec)
   Pre             IPM             Post
2.002E-03    4.899E-02    1.006E-03
Max-norms: ||b||=1, ||c|| = 6,
Cholesky |add|=4, |skip| = 0, ||L.L|| = 2.6068e+06.
```

"An exact positive definite solution could not be found for the reduced problem." **(11)**

> *# out[3] is a matrix in the spectrahedron of maximum rank.*
> *# We check the eigenvalues to determine the rank*
> *eig(out[3]);*

$$\begin{bmatrix} -7.36528952778664\ 10^{-14} \\ -1.15703219848192\ 10^{-16} \\ -5.29013742379310\ 10^{-17} \\ 9.41854237903468\ 10^{-18} \\ 2.93935172628359\ 10^{-17} \\ 8.49356674526746\ 10^{-17} \\ 0.221465035651601 \\ 0.224888765201585 \\ 0.236502749326637 \\ 0.406158748920177 \\ 0.876379647171517 \\ 0.951608842622641 \\ 1.00000286160073 \\ 3.95395724915673 \\ 7.21805139944845 \end{bmatrix} \quad \textbf{(12)}$$

> *# There are 9 non-zero eigenvalues, which corresponds to the maximum*
> *# possible rank predicted by our results.*

> *##*
> *## Example 4.5*
> *## Example in the border with a matrix in the spectrahedron of rank 6.*

##

```
> # We add a different polynomial f6 to example 4.3
p1 := x1^2- x4^2;
p2 := x2^2- x4^2;
p3 := x3^2- x4^2;
p4 := -x1^2 - x1*x2 - x1*x3 + x1*x4 - x2*x3 + x2*x4 + x3*x4;
p5 := x5^2;
p6 := x1*x5 + x4*x5;
f := p1^2 + p2^2 + p3^2 + p4^2 + p5^2 + p6^2;
```

$$p1 := x1^2 - x4^2$$
$$p2 := x2^2 - x4^2$$
$$p3 := x3^2 - x4^2$$
$$p4 := -x1^2 - x1\,x2 - x1\,x3 + x1\,x4 - x2\,x3 + x2\,x4 + x3\,x4$$
$$p5 := x5^2$$
$$p6 := x1\,x5 + x4\,x5$$
$$f := (x1^2 - x4^2)^2 + (x2^2 - x4^2)^2 + (x3^2 - x4^2)^2 + (-x1^2 - x1\,x2 - x1\,x3 + x1\,x4 - x2\,x3 + x2\,x4 + x3\,x4)^2 + x5^4 + (x1\,x5 + x4\,x5)^2 \tag{13}$$

```
> # We use SEDUMI to compute a SOS decomposition.
# With default options, exactSOS will compute a solution of maximum rank
```

```
> # We do not compute the polynomail decomposition since it gives an error.
# (the tools for computing the decomposition in Maple are in a
# educational package which is not)

out := exactSOS(f, facial = "no", computePolynomialDecomposition = "no") :
```

"Calling numerical solver SEDUMI to find values of the indeterminates..."

```
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, Adaptive Step-Differentiation, theta =
0.250, beta = 0.500
eqs m = 51, order n = 16, dim = 226, blocks = 2
nnz(A) = 115 + 0, nnz(ADA) = 2601, nnz(L) = 1326
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg
cg  prec
  0 :            1.66E+01 0.000
  1 :  -3.33E+00 5.22E+00 0.000 0.3144 0.9000 0.9000   0.56  1
1  1.1E+01
  2 :  -8.25E-01 1.59E+00 0.000 0.3047 0.9000 0.9000   2.54  1
1  1.8E+00
  3 :  -8.14E-02 4.15E-01 0.000 0.2610 0.9000 0.9000   2.26  1
1  7.2E-01
  4 :  -1.77E-02 9.29E-02 0.000 0.2237 0.9000 0.9000   1.17  1
1  5.3E-01
  5 :  -4.54E-03 2.31E-02 0.000 0.2482 0.9000 0.9000   1.05  1
1  4.6E-01
  6 :  -1.43E-03 5.95E-03 0.000 0.2581 0.9107 0.9000   1.02  1
1  4.1E-01
```

```
  7 :  -4.67E-04 1.81E-03 0.000 0.3034 0.9071 0.9000   1.01  1
1  1.9E-01
  8 :  -1.30E-04 5.83E-04 0.000 0.3228 0.9000 0.9090   1.00  1
1  5.9E-02
  9 :  -2.82E-05 1.98E-04 0.000 0.3399 0.9000 0.9275   1.00  1
1  1.5E-02
 10 :  -9.28E-06 7.69E-05 0.000 0.3881 0.9000 0.9148   1.00  1
1  4.4E-03
 11 :  -3.26E-06 2.42E-05 0.000 0.3143 0.9000 0.9135   1.00  1
1  1.2E-03
 12 :  -1.04E-06 7.13E-06 0.000 0.2951 0.9000 0.9068   1.00  1
1  3.0E-04
 13 :  -3.26E-07 2.08E-06 0.000 0.2911 0.9000 0.9031   1.00  1
1  8.2E-05
 14 :  -1.01E-07 6.18E-07 0.000 0.2975 0.9000 0.9040   1.00  1
1  2.2E-05
 15 :  -3.05E-08 1.86E-07 0.000 0.3017 0.9000 0.9043   1.00  2
2  5.8E-06
 16 :  -9.29E-09 5.64E-08 0.000 0.3026 0.9000 0.9038   1.00  3
3  1.6E-06
 17 :  -2.86E-09 1.67E-08 0.000 0.2967 0.9000 0.9019   1.00  1
3  4.3E-07
 18 :  -8.96E-10 4.98E-09 0.000 0.2974 0.9009 0.9000   1.00  1
3  1.3E-07
 19 :  -2.80E-10 1.40E-09 0.000 0.2820 0.9041 0.9000   1.00  3
1  4.2E-08
 20 :  -8.80E-11 3.87E-10 0.000 0.2759 0.9067 0.9000   1.00  3
3  1.5E-08
 21 :  -2.75E-11 1.06E-10 0.000 0.2731 0.9084 0.9000   1.00  4
4  5.3E-09
 22 :  -8.46E-12 2.92E-11 0.000 0.2765 0.9084 0.9000   1.00  7
7  1.9E-09
 23 :  -2.55E-12 8.33E-12 0.000 0.2848 0.9062 0.9000   1.00  7
7  6.1E-10
 24 :  -7.31E-13 2.43E-12 0.000 0.2922 0.9019 0.9000   1.00 18
18  1.9E-10
 25 :  -1.99E-13 7.27E-13 0.000 0.2990 0.9000 0.9038   1.00 26
21  5.2E-11
 26 :  -5.10E-14 2.20E-13 0.000 0.3020 0.9000 0.9099   1.01 26
24  1.3E-11
 27 :  -1.39E-14 7.06E-14 0.000 0.3216 0.9000 0.9140   1.00 27
29  3.2E-12
 28 :  -4.55E-15 2.27E-14 0.000 0.3211 0.9000 0.9165   1.00 26
27  7.2E-13
 29 :  -2.09E-15 7.93E-15 0.000 0.3497 0.9000 0.9155   1.00 33
30  1.5E-13
 30 :  -1.37E-15 1.45E-15 0.000 0.1832 0.9000 0.9103   0.98 35
38  1.6E-14
 31 :  -1.21E-15 1.20E-15 0.040 0.8285 0.9000 0.9058   0.60 43
36  7.8E-15
Run into numerical problems.

iter seconds digits       c*x               b*y
 31      0.6  10.7  1.5418608909e-15 -1.2093926539e-15
|Ax-b| =   4.8e-15, [Ay-c]_+ =   8.6E-15, |x|=  4.9e-01, |y|=
4.4e+00
```

```
Detailed timing (sec)
   Pre          IPM          Post
2.997E-03    1.620E-01    9.958E-04
Max-norms: ||b||=1, ||c|| = 6,
Cholesky |add|=6, |skip| = 0, ||L.L|| = 1.39421e+08.
```

"An exact positive definite solution could not be found for the reduced problem." **(14)**

> *# out[3] is a matrix in the spectrahedron of maximum rank.*
> *# We check the eigenvalues to determine the rank*
> $eig(out[3]);$

$$\begin{bmatrix} -4.43123666851202\ 10^{-16} \\ -2.52832765437545\ 10^{-16} \\ -1.81670178064488\ 10^{-16} \\ 3.29925984304773\ 10^{-17} \\ 5.68693316775600\ 10^{-17} \\ 4.79576806390796\ 10^{-16} \\ 1.07506825368057\ 10^{-8} \\ 2.36374849310286\ 10^{-8} \\ 7.80435412069721\ 10^{-8} \\ 0.888960947926555 \\ 1. \\ 1.00000000000033 \\ 1.99999995725008 \\ 3.89989969879449 \\ 7.21113935327863 \end{bmatrix} \quad \textbf{(15)}$$

> *# There are 6 non-zero eigenvalues.*

> *# We compare the matrix obtained by SEDUMI with the matrix corresponding*
> *# to the original decomposition p1^2+p2^2+p3^2+p4^2+p5^2+p6^2.*
> $v := convert(out[5], list)$: *# The monomials indexing the columns of the Gram Matrix*
> $A1 := decompositionToMatrix([p1, p2, p3, p4, p5, p6], v)$:
> $A2 := roundToIntMatrix(out[3], 6)$: *# We convert some almost integer values to integers*
> $Norm(A1 - A2);$

$$0 \quad \textbf{(16)}$$

> *# We see that both matrices are the same.*
> *# This gives strong numerical evidence that this is the unique matrix*
> *# in the spectrahedron of f.*

>