```
> # Uncomment and set the path to rationalSOS.mpl file
  #currentdir("C:/Users/User/rationalSOS");
```

```
> ##############################################################################
  # Load "Rational SOS" procedures
  ##############################################################################
  read("rationalSOS.mpl") :
  with(rationalSOS);
```

"Opening connection with Matlab"

$$[RoundMat, RoundVec, SOS, evalMat, getDiag, matrixToPoly, numericSolver, polyToMatrix, randomRank, reduceByLinearEquation, vectorTrace, zeroDetSRows, zeroRows] \tag{1}$$

```
> # Display tables of any size
  interface(rtablesize = infinity);
```

$$10 \tag{2}$$

```
> ##############################################################################
  # Example 2.1 in [1]
  ##############################################################################

  # We define a polynomial f as the sum of two squares.
  p1 := x^2 + 3*x*y - 5*x*z + 2 z^2;
  p2 := 3 x^2 - 2*x*z + y*z + 5*y^2;
  f := expand(p1^2 + p2^2);
```

$$p1 := x^2 + 3\,x\,y - 5\,x\,z + 2\,z^2$$

$$p2 := 3\,x^2 - 2\,x\,z + 5\,y^2 + y\,z$$

$$f := 10\,x^4 + 6\,x^3\,y - 22\,x^3\,z + 39\,x^2\,y^2 - 24\,x^2\,y\,z + 33\,x^2\,z^2 - 20\,x\,y^2\,z + 8\,x\,y\,z^2 - 20\,x\,z^3 + 25\,y^4 + 10\,y^3\,z + y^2\,z^2 + 4\,z^4 \tag{3}$$

```
> # Compute the matrix Q associated to the problem
  Q, QVars, v := polyToMatrix(f);
```

$$Q, QVars, v := \left[\begin{array}{cccccc} 10 & 3 & -11 & a\_0_{1,4} & -12 - a\_0_{2,3} & a\_0_{1,6} \\ 3 & 39 - 2\,a\_0_{1,4} & a\_0_{2,3} & 0 & -10 - a\_0_{3,4} & 4 - a\_0_{3,5} \\ -11 & a\_0_{2,3} & 33 - 2\,a\_0_{1,6} & a\_0_{3,4} & a\_0_{3,5} & -10 \\ a\_0_{1,4} & 0 & a\_0_{3,4} & 25 & 5 & a\_0_{4,6} \\ -12 - a\_0_{2,3} & -10 - a\_0_{3,4} & a\_0_{3,5} & 5 & 1 - 2\,a\_0_{4,6} & 0 \\ a\_0_{1,6} & 4 - a\_0_{3,5} & -10 & a\_0_{4,6} & 0 & 4 \end{array}\right], \{a\_0_{1,4}, a\_0_{1,6}, a\_0_{2,3}, a\_0_{3,4}, a\_0_{3,5}, a\_0_{4,6}\}, [x^2, x\,y, x\,z, y^2, y\,z, z^2] \tag{4}$$

```
> # Dimension and rank of Q
  nops(indets(Q));
  randomRank(Q);
```

$$6$$
$$6 \tag{5}$$

> \# Computes numerically a SDP solution using SEDUMI
> xVars, xSol := numericSolver(Q);

"SEDUMI CALL"

```
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, Adaptive Step-Differentiation, theta =
0.250, beta = 0.500
eqs m = 7, order n = 7, dim = 37, blocks = 2
nnz(A) = 18 + 0, nnz(ADA) = 49, nnz(L) = 28
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg
cg  prec
  0 :            1.79E+02 0.000
  1 :  -1.48E+01 4.27E+01 0.000 0.2379 0.9000 0.9000   0.56  1
1  4.4E+00
  2 :  -1.50E+00 1.06E+01 0.000 0.2489 0.9000 0.9000   1.81  1
1  1.3E+00
  3 :  -3.52E-01 1.97E+00 0.000 0.1854 0.9000 0.9000   1.21  1
1  8.6E-01
  4 :  -1.05E-01 4.05E-01 0.000 0.2058 0.9000 0.9000   1.04  1
1  8.7E-01
  5 :  -5.49E-03 1.66E-02 0.000 0.0410 0.9900 0.9900   1.00  1
1  9.7E-01
  6 :  -1.45E-04 5.54E-04 0.172 0.0333 0.9900 0.9901   1.00  1
1  3.0E-01
  7 :  -4.81E-06 2.71E-05 0.000 0.0488 0.9903 0.9900   1.02  1
1  1.4E-02
  8 :  -1.09E-06 7.00E-06 0.058 0.2587 0.9000 0.9150   1.02  2
2  3.6E-03
  9 :  -1.77E-07 1.15E-06 0.000 0.1643 0.9035 0.9000   1.02  3
3  5.6E-04
 10 :  -2.92E-08 1.96E-07 0.000 0.1707 0.9005 0.9000   1.02  3
4  9.3E-05
 11 :  -4.22E-09 3.41E-08 0.000 0.1738 0.9000 0.9015   1.03  5
6  1.6E-05
Run into numerical problems.

iter seconds digits       c*x               b*y
 11      0.1   4.1  2.3673831258e-08 -4.2205112054e-09
|Ax-b| =   1.3e-09, [Ay-c]_+ =   1.1E-09, |x|=  9.4e-01, |y|=
1.5e+01

Detailed timing (sec)
   Pre          IPM          Post
2.999E-02    7.800E-02    1.600E-02
Max-norms: ||b||=1, ||c|| = 39,
Cholesky |add|=0, |skip| = 1, ||L.L|| = 53071.6.
```

$$xVars, xSol := \{a\_0_{1,4}, a\_0_{1,6}, a\_0_{2,3}, a\_0_{3,4}, a\_0_{3,5}, a\_0_{4,6}\}, \begin{bmatrix} -6.62452241911967 \\ -2.46705813869574 \\ 13.0849339948066 \\ 2.98132058184944 \\ 0.875865927183106 \\ 1.42553771817733 \\ 4.22051120541738\ 10^{-9} \end{bmatrix} \quad (6)$$

```
> # Solution matrix and eigenvalues
  Qsol := evalMat(Q, xVars, xSol) :
  eig(Qsol);
```

$$\begin{bmatrix} -5.35661718670171\ 10^{-9} \\ 2.77695014177759\ 10^{-8} \\ 4.27329305693102 \\ 16.5132012163911 \\ 28.9722093346370 \\ 46.9092106903518 \end{bmatrix} \quad (7)$$

```
> # Four positive eigenvalues and two approximate zeros

  ################################################################################
> ################################################################################
  # Example 3.2 in [1]
  ################################################################################

  sSym := solve({f=0, diff(f, x) = 0, diff(f, y) = 0, diff(f, z) = 0});
```

$$sSym := \Big\{x = x, y = RootOf(50\_Z^4 + 28\_Z^3 - \_Z^2 + 23\_Z - 8)\, x, z = \frac{1}{46}\, x\, \big(50\, RootOf(50\_Z^4 + 28\_Z^3 - \_Z^2 + 23\_Z - 8)^3 + 128\, RootOf(50\_Z^4 + 28\_Z^3 - \_Z^2 + 23\_Z - 8)^2 + 25\, RootOf(50\_Z^4 + 28\_Z^3 - \_Z^2 + 23\_Z - 8) + 73\big)\Big\} \quad (8)$$

```
> ## sSym[1] plain equation
  v0 := eval(Vector(v), sSym);
  v01 := eval(v0, {x=1}) :
```

$$v0 := \Big[\big[x^2\big], \quad (9)$$

$$\big[x^2\, RootOf(50\_Z^4 + 28\_Z^3 - \_Z^2 + 23\_Z - 8)\big],$$

$$\left[\frac{1}{46} x^2 \left(50\, RootOf(50\_Z^4+28\_Z^3-\_Z^2+23\_Z-8)^3+128\, RootOf(50\_Z^4+28\_Z^3-\_Z^2+23\_Z-8)^2+25\, RootOf(50\_Z^4+28\_Z^3-\_Z^2+23\_Z-8)+73\right)\right],$$

$$\left[RootOf(50\_Z^4+28\_Z^3-\_Z^2+23\_Z-8)^2 x^2\right],$$

$$\left[\frac{1}{46} RootOf(50\_Z^4+28\_Z^3-\_Z^2+23\_Z-8)\, x^2 \left(50\, RootOf(50\_Z^4+28\_Z^3-\_Z^2+23\_Z-8)^3+128\, RootOf(50\_Z^4+28\_Z^3-\_Z^2+23\_Z-8)^2+25\, RootOf(50\_Z^4+28\_Z^3-\_Z^2+23\_Z-8)+73\right)\right],$$

$$\left[\frac{1}{2116} x^2 \left(50\, RootOf(50\_Z^4+28\_Z^3-\_Z^2+23\_Z-8)^3+128\, RootOf(50\_Z^4+28\_Z^3-\_Z^2+23\_Z-8)^2+25\, RootOf(50\_Z^4+28\_Z^3-\_Z^2+23\_Z-8)+73\right)^2\right]\Bigg]$$

```
> # We verify that it satisfies the condition vt.Q.v = 0, which must always
  # be satisfied for real solutions
  simplify(LinearAlgebra[Transpose](v01).Q.v01);
```

$$0 \tag{10}$$

```
> # We reduce the dimension
  Q1 := reduceByLinearEquation(Q, v01) :
  nops(indets(Q1));
  randomRank(Q1);
  # We reduced the dimension to 5 but we need to reduce to 4,
  # because the numerical solution had 2 null eigenvalues
```

$$3$$

$$5 \tag{11}$$

```
> v01t := vectorTrace(v01);
```

**(12)**

$$v01t := \begin{bmatrix} 4 \\ -\frac{14}{25} \\ \frac{53}{10} \\ \frac{221}{625} \\ -\frac{396}{125} \\ \frac{1209}{100} \end{bmatrix} \tag{12}$$

```
> # We took the trace, so the equation vt.Q.v=0 may not be satisfied
  simplify(LinearAlgebra[Transpose](v01t).Q.v01t);
```

$$\frac{17367}{125} + \frac{1376}{625}\, a\_0_{1,4} + \frac{2426}{125}\, a\_0_{2,3} + \frac{2027}{50}\, a\_0_{1,6} + \frac{1}{5}\, a\_0_{3,4} - \frac{501}{25}\, a\_0_{3,5} - \frac{14403}{1250}\, a\_0_{4,6} \tag{13}$$

```
> Q1 := reduceByLinearEquation(Q, v01t) :
  nops(indets(Q1));
  randomRank(Q1);
```

$$0$$

$$2 \tag{14}$$

```
> # The problem was completely solved, no need to call the numerical solver.
> L, DD, Lt, fNew, a, p := matrixToPoly(Q1, v) :
> fNew;
```

$$10\left(x^2 + \frac{3}{10}\, x\, y - \frac{11}{10}\, x\, z + \frac{3}{2}\, y^2 + \frac{3}{10}\, y\, z + \frac{1}{5}\, z^2\right)^2 + \frac{81}{10}\left(x\, y - \frac{13}{9}\, x\, z - \frac{5}{9}\, y^2 - \frac{1}{9}\, y\, z + \frac{2}{3}\, z^2\right)^2 \tag{15}$$

```
>
```