Examination of Black Hole Spin

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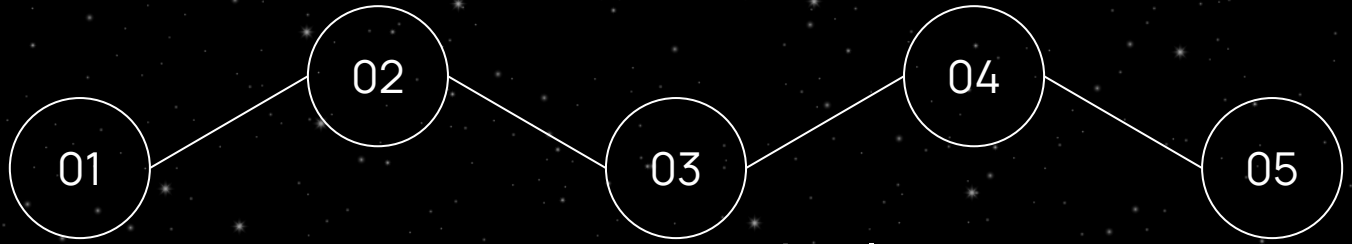
Physics 273 H

Project Overview

In this project I hope to understand and model how observations of the emission spectrum of a black hole can be used to determine its spin.

- Focus on the iron Kα line in the emission spectrum
- Model the effects of doppler shift and gravitational redshift on the emission spectrum
- Understand how these effects depend on the spin of the black hole

Project Plan



Research

Read articles on spin models of black holes

Discussion

Discuss my questions with someone more knowledgeable

Mathematical understanding

Understand the equations used in most models of spin

Modeling

Use my understanding of the math to make model of black hole data

Written Analysis

Analyze my conclusions from this project

01

Research

Spin of a Black Hole

Angular momentum of a black hole described by dimensionless spin parameter $-1 < a < 1$

Spin of a black hole affects:

- ISCO (Innermost Stable CIrcular Orbit)
- Radius of the event horizon
- Rate at which matter is converted to energy

$$a \equiv \frac{J_c}{GM^2}$$

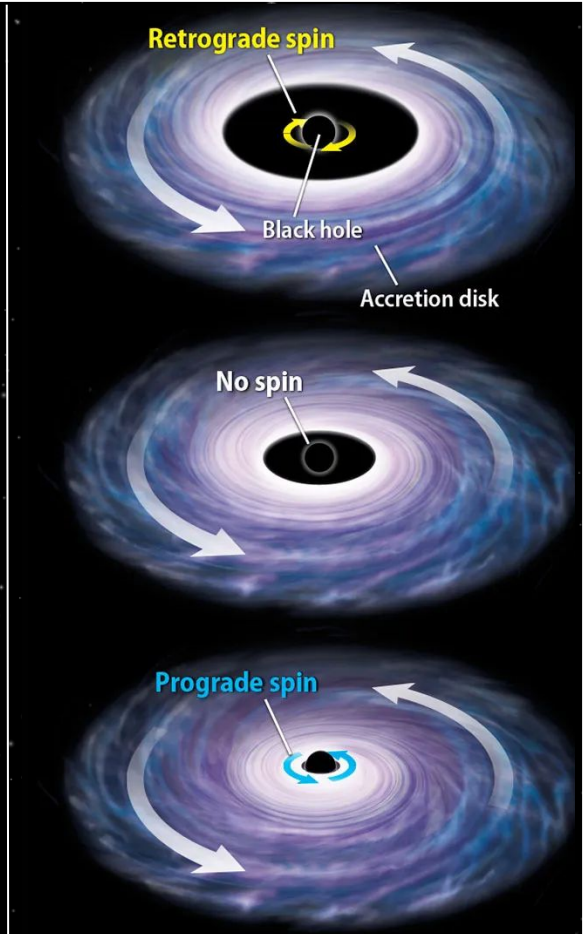


Image Source:https://www.astronomy.com/science/what-is-black-hole-spin/

Emission Spectrum

- Accretion disk emits X-Ray Radiation
- Iron Kα line profile
- Emission spectrum shifted from gravitational redshift and doppler shift
    - emissions from closer to the black hole exhibit more gravitational redshift
- Orbital radius is determined by the spin of the black hole
- Therefore, looking at the gravitational redshift tells us how close the accretion disk is, which tells us about the spin of the black hole

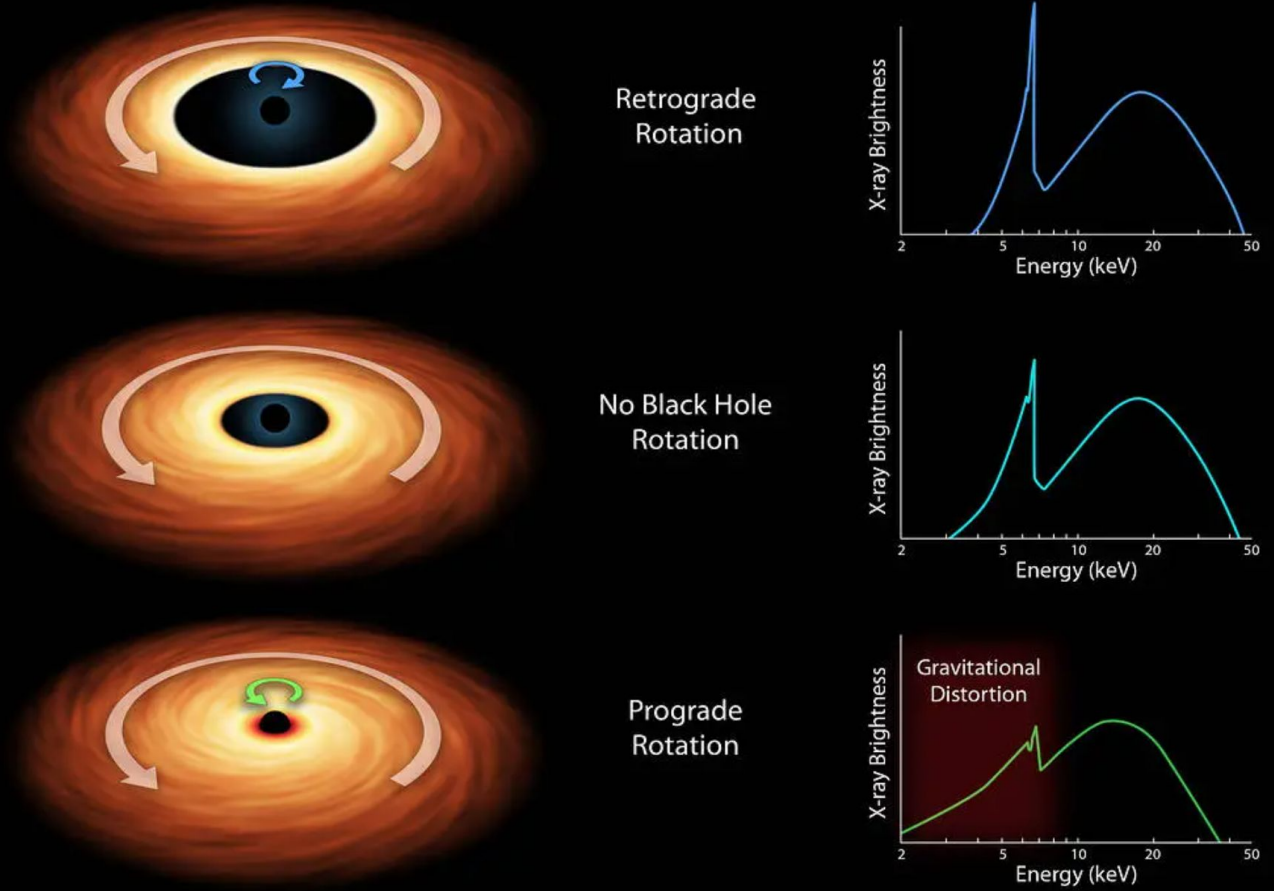


Image source: https://www.nasa.gov/image-article/how-measure-spin-of-black-hole/

02

Discussion

Talk with Professor Chris Reynolds

Why do Black Holes emit in X-Ray?

- Accretion disk temperatures hot enough to emit in the UV range
  - electrons energized from magnetic fields of the corona
  - Photons undergo inverse Compton scattering in the corona
  - raises energy of the photons to X-Ray
- Some X-Rays reflected back to the disk
  - ionizes iron in the disk
  - creates the Iron Kα line

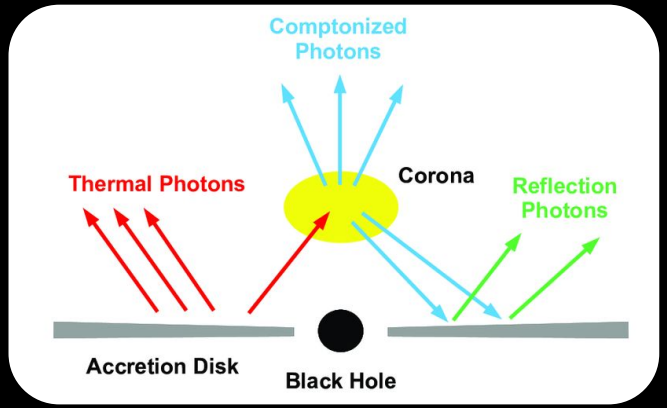
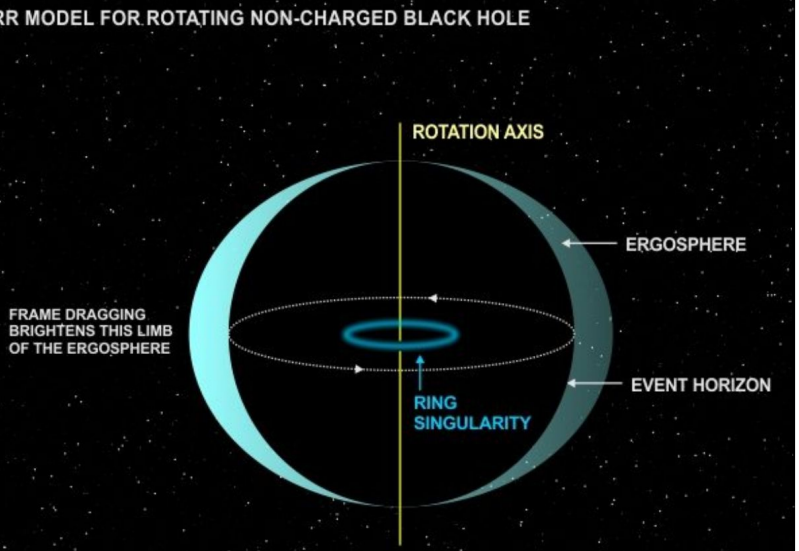


Image Source:
https://www.researchgate.net/figure/Disk-corona-model-The-black-hole-is-surrounded-by-a-cold-geometrically-

Ergosphere

Ergosphere is the region around a black hole where frame-dragging is prominent due to spacetime rotating.



- A product of the spin of the black hole
- Only orbits in the same sense as the black hole's rotation are allowed
  - even particles with contrary angular momentum
- There is a maximum and minimum orbital velocity for each radius in the ergosphere
- As you approach the event horizon of an a = 1 black hole, Vmax = Vmin = c

Image Source: https://sellugsk.live/product_details/20262458.html

03

Plotting

Formulas Used

a = dimensionless spin parameter between -1 and 1
*G = gravitational constant 6.67 * 10^-11 N*m^2 / kg^2*
c = speed of light 2.998 10^8 m/s*
M = mass of the black hole

Gravitational Radius

$$r_g = GM/c^2$$

Gravitational Redshift

$$1 + z = \frac{1}{\sqrt{1-(\frac{v_e}{c})^2}}$$

$$v_e = \sqrt{\frac{2GM}{r}}$$

Orbital Velocity

$$v = \sqrt{\frac{GM}{r}}$$

Radius of the ISCO

$$r_{ISCO} = \left\{3 + Z_2 \mp [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2}\right\} r_g$$

$$Z_1 = 1 + 1(1 - a^2)^{1/3}[(1 + a)^{1/3} + (1 - a)^{1/3}]$$

$$Z_2 = (3a^2 + Z_1^2)^{1/2}$$

Doppler Shift

$$z = \frac{f_r}{f_s} = \frac{1}{\gamma(1+\beta cos\theta_r)}$$

$$\beta = \frac{v_{tan}}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Radius of the Event Horizon

$$r_{evt} = (1 + \sqrt{1 - a^2})r_g$$

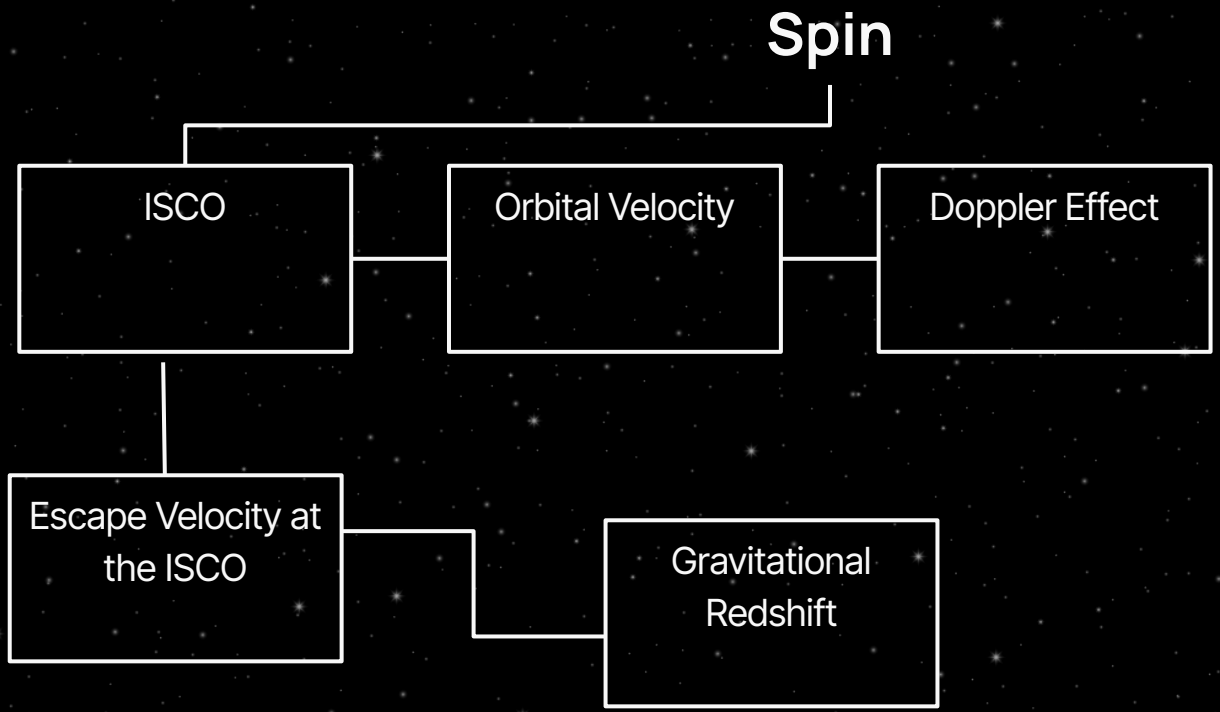
Marginally Bound Orbit

$$r_{mb} = (2 \mp a + 2\sqrt{1 \mp a})r_g$$

Photon Circular Orbit

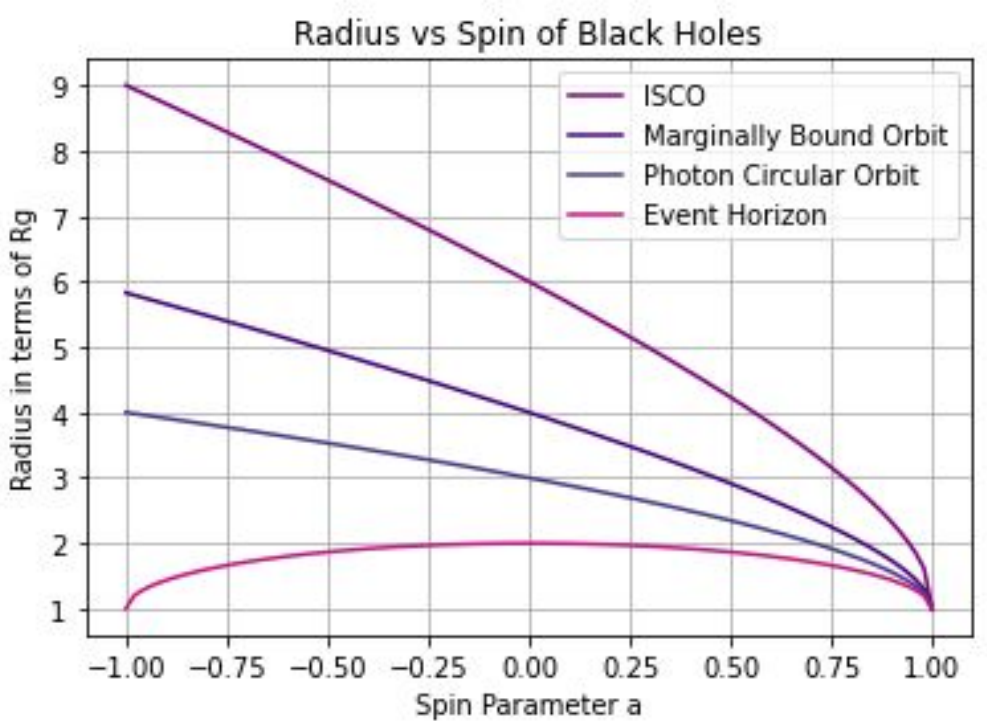
$$r_{ph} = 2\{1 + cos[\frac{2}{3}cos^{-1}(\mp a)]\}r_g$$

How variables depend on each other



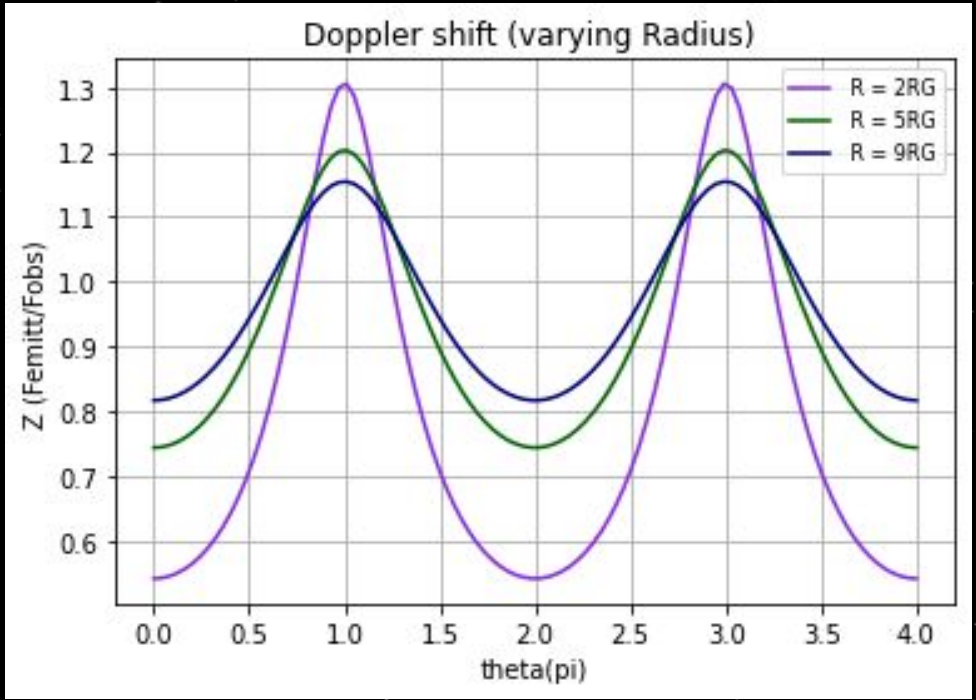
Plot of orbits as a function of the spin

- Event horizon for a non-rotating black hole
- ISCO = 6Rg for non-rotating
- ISCO = 9Rg for a = -1



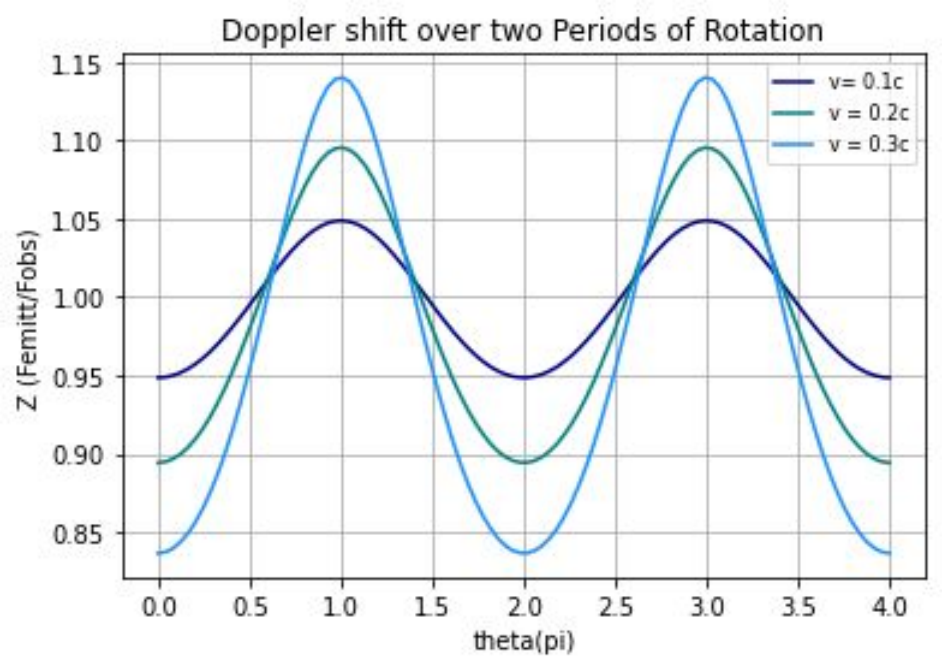
Doppler Shift at different orbital Radii

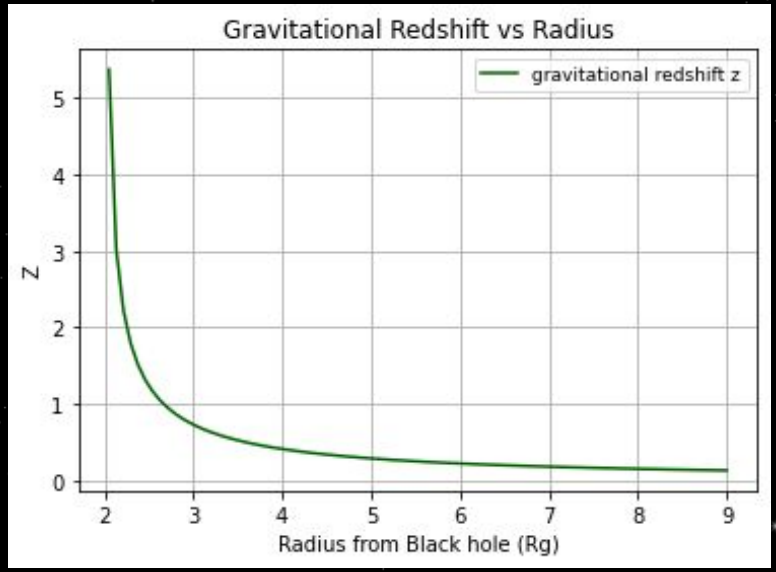
Not sure how correct the numbers for Z are but the shape looks like what I expected

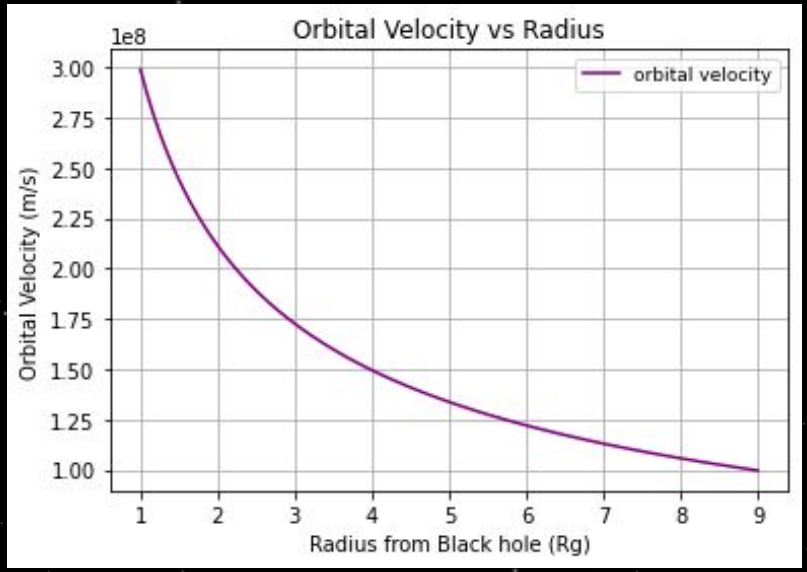


Doppler Shift at different orbital velocities

Again, not sure about numbers for Z




Gravitational Redshift vs Radius
gravitational redshift z
Z
Radius from Black hole (Rg)
0
1
2
3
4
5
2
3
4
5
6
7
8
9


Orbital Velocity vs Radius
1e8
orbital velocity
Orbital Velocity (m/s)
Radius from Black hole (Rg)
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00
1
2
3
4
5
6
7
8
9

Next Steps

Model how these gravitational redshift and doppler effects actually effect an emission spectrum of a black hole

- Replicate X-Ray emission spectrum
- Plot this spectrum and how it is affected by doppler and gravitational redshift

Revise my plots to go more in depth about how spin is related to doppler shift and gravitational redshift

Thank You