

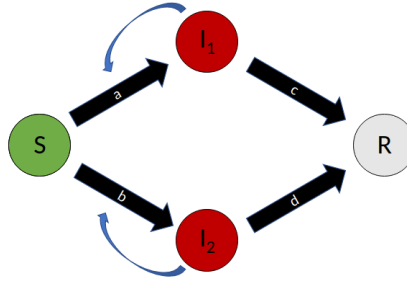
Exercise 1

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Given two modes of the same infection, whereby one infection I_1 is more infectious than the other infection of the same kind I_2 (different β values), provide the following

1. Compartmental flow diagram of the dynamics (write the assumptions).
2. Stoichiometric equation for each conversion.
3. Corresponding differential equation for the system (well-mixed assumption).

We consider the following compartmental flow diagram:



We assume that the susceptible population may only become infected by one of the two types of infections, and that this transformation from susceptible to infected is promoted by the population infected with the corresponding infection type. That is, only the population infected with I_1 may induce a susceptible person to become infected with I_1 .

After infection, each infected population spontaneously becomes recovered at some rate proportional to the size of the population infected with that type of disease.

We refer to the labelling of the arrows, and define the related stoichiometric equation.

$$\begin{aligned}
 \text{a} &: S^* + I_1 \rightarrow I_1^* + I_1 \\
 \text{b} &: S^* + I_2 \rightarrow I_2^* + I_2 \\
 \text{c} &: I_1 \rightarrow R \\
 \text{d} &: I_2 \rightarrow R
 \end{aligned} \tag{1}$$

Let the total population be N , and let the population of susceptible, infected with I_1 , infected with I_2 and recovered by denoted by N_S , N_{I_1} , N_{I_2} and N_R respectively. They must satisfy

$$N = N_S + N_{I_1} + N_{I_2} + N_R. \quad (2)$$

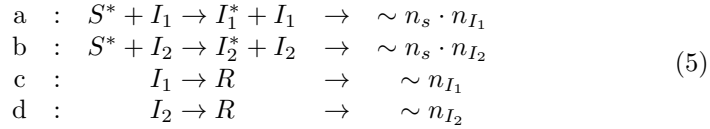
We may define number densities,

$$1 = \frac{N_S + N_{I_1} + N_{I_2} + N_R}{N} = n_S + n_{I_1} + n_{I_2} + n_R \quad (3)$$

Note that this implies that

$$\frac{d}{dt} (n_S + n_{I_1} + n_{I_2} + n_R) = 0 \quad (4)$$

In a well-mixed population, the rate at which a certain reaction proceeds is given by the probability of the reactants of finding each other.



By the assumptions of the problem, we assume that a proceeds faster than b and that c and d proceed at an equal rate keeping all things equal.

We arrive at the following differential equations

$$\frac{dn_S}{dt} = -\beta_1 n_S \cdot n_{I_1} - \beta_2 n_S \cdot n_{I_2} \quad (6)$$

$$\frac{dn_{I_1}}{dt} = \beta_1 n_S \cdot n_{I_1} - \alpha n_{I_1} \quad (7)$$

$$\frac{dn_{I_2}}{dt} = \beta_2 n_S \cdot n_{I_2} - \alpha n_{I_2} \quad (8)$$

$$\frac{dn_R}{dt} = \alpha n_{I_1} + \alpha n_{I_2} \quad (9)$$

where

$$\beta_1 > \beta_2 > 0, \quad \alpha > 0. \quad (10)$$

We note that the quantities must be strictly greater than 0. For β , this means that the diseases are communicable. For α , this just means that, one way or another, an infected person becomes ‘recovered’ (or perhaps removed or something else).

As a final check, it is easy to show that the differential equations satisfy the conservation property (4).