Monopoly: Where Dreams of Dominance Meet the Reality of Ruin

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Abstract

Monopoly, the classic board game of property acquisition and financial strategy, has long captured the imagination of players worldwide. In this study, we delve into the dynamics of Monopoly gameplay by employing both artificial intelligence (AI) simulations and Markov chain modeling to analyze space distribution and player outcomes. Using AI agents to play numerous games of Monopoly, we track the frequency of player landings on each space on the board. By comparing the distribution of landings between winners and losers, we aim to uncover any significant disparities that may influence gameplay outcomes. Additionally, we construct a simplified Markov chain model to simulate player movements on the Monopoly board. Through a transition matrix representing the probabilities of moving between spaces, we seek to replicate the emergent patterns observed in real gameplay. By juxtaposing the results obtained from AI simulations with those from the Markov chain model, we aim to provide insights into the underlying dynamics of Monopoly gameplay. Our findings shed light on the spatial distribution of player movements, potential strategies for success, and the efficacy of Markov chain modeling in capturing complex board game dynamics. This interdisciplinary approach offers a novel perspective on understanding the intricacies of Monopoly gameplay, with implications for game design, strategy development, and the broader field of artificial intelligence research.

Contents

1 Introduction

Monopoly, the iconic board game of economic prowess and strategic maneuvering, has captivated players for decades with its blend of chance, strategy, and negotiation. Despite its seemingly simple mechanics, the game exhibits a complex interplay of player decisions, luck, and spatial dynamics that influence the outcomes of each session. In this paper, we embark on a journey to unravel the mysteries of Monopoly gameplay by employing advanced computational techniques.

The objective of this study is twofold: first, to investigate the distribution of player landings across the Monopoly board and discern any discernible patterns or discrepancies between winners and losers, and second, to construct a simplified Markov chain model to simulate player movements and explore the emergent spatial dynamics of the game.

To achieve our goals, we leverage the power of artificial intelligence (AI) simulations to emulate numerous games of Monopoly played by AI agents. By tracking the frequency of player landings on each space on the board, we aim to uncover insights into the spatial distribution of player movements and its implications for gameplay outcomes. Through this approach, we seek to shed light on the strategies employed by successful players and identify potential hotspots or bottlenecks on the board.

Furthermore, we complement our empirical analysis with a theoretical exploration using Markov chain modeling. By constructing a transition matrix that captures the probabilities of moving between spaces on the Monopoly board, we aim to simulate player movements and observe the emergent spatial patterns that arise. This modeling approach offers a formal framework for understanding the dynamics of player interactions and spatial navigation in Monopoly, providing valuable insights into the underlying structure of the game.

By combining AI simulations with Markov chain modeling, we aim to provide a comprehensive analysis of Monopoly gameplay dynamics, from the distribution of player landings to the spatial evolution of the game. Our interdisciplinary approach bridges the gap between empirical observation and theoretical modeling, offering a holistic perspective on the complexities of Monopoly gameplay and its implications for strategic decision-making and game design.

1.1 Rules and House Rules

In this paper, we follow all of the standard rules of Monopoly; however, we choose to adopt a house rule that is so common that it is often mistaken for being a real rule. The following is a brief primer on the rules of Monopoly via the flow of a player's turn:

- Roll two 6-sided dice and move that many spaces forward.
- If you land on an unowned property, you have the opportunity to buy it; otherwise, it goes up for auction between all players.
- If you land on Chance or Community Chest, you draw a card from the appropriate deck an follow the directions.
- Landing on Go To Jail immediately sends you to jail.
- Upon rolling doubles, you take your next turn immediately; however, rolling doubles for the third time consecutively lands you in jail before movement.
- When beginning a turn in jail you may pay $50, use a get out of jail free card, or roll doubles to escape. Failing to get out three turns in a row forces you to pay $50.
- In order to buy a house or hotel on a colored property, you must own all properties of that color and must build houses evenly.

There are more technical rules we follow in our simulations, but these are the rules that govern the flow of the game. The house rule we use is that any money paid as a fee (i.e., a Community Chest card forces you to pay $100) is places on Free Parking. Anybody who lands on this space collects all of the money that has been placed there.

2 Markov Chain Model

In this section, we discuss finding a steady state distribution for the spaces using a Markov chain model in the form of a transition matrix which can be found on Github here. In order to keep the discrete probability distribution from AI agent-based simulations consistent with our transition matrix, we treat landing on Go

To Jail as a separate turn from actually going to Jail. This way we can track what percentage of Jail visits caused by landing on Go To Jail. We construct our transition matrix, M to be stochastic such that the columns sum to 1. We use the probability for the rolls of two 6-sided dice given below in Table 1.

We must also take into account the types of Chance and Community Chest cards available to the player should they land on the appropriate spaces. There are exactly 17 community chest cards we use, and only two of them send the player to another space, those being Go and Jail. So when computing the probability a player lands on Community Chest, there is a 2/17 chance the player goes to Go or Jail along with a 15/17 chance the player stays at Community Chest. Chance has a much more diverse pool of spaces the player can be sent to. There are cards to send the player to Go, Reading Railroad, Jail, St. Charles Place, Illinois, and Boardwalk. That, in addition to being sent to the nearest railroad or utility, there is only a 6/16 chance the player stays on Chance.

Roll	%
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

Table 1: Discrete probability distribution for rolling two 6-sided dice.

When handling the Go To Jail space, the transition matrix can land you on that space, but then you are forced with probability 1 to go to the Jail space. We also incorporate rolling doubles three times in our matrix. There is a 1/216 chance a player rolls doubles 3 times, so for almost every column of the matrix there is that chance to land in jail, and every other row is then multiplied by 215/216 in order for the matrix to remain stochastic. We can also confirm that M is also primitive as every entry in M^3 is non-zero. The Perron-Frobenius theorem along with our stochastic property tells us the largest eigenvalue of M is 1 and that its corresponding eigenvector has all real values and is our steady state solution. This eigenvector has been normalized to a discrete probability distribution and can be seen below in table 2.

Space	Prob	Space	Prob	Space	Prob	Space	Prob
Go	3.06%	Jail	6.27%	Free Pk	2.88%	Go Jail	0.00%
Med Ave	2.13%	St. Chas	2.70%	Kentucky	2.84%	Pacific	2.67%
Com Ches	1.90%	Elec Co	2.60%	Chance	1.05%	North C	2.62%
Balt Ave	2.16%	States	2.37%	Indiana	2.73%	Com Chest	2.40%
Inc Tax	2.32%	Virginia	2.46%	Illinois	3.18%	Penn Ave	2.50%
Read Rl	2.96%	Penn Rl	2.92%	B&O Rl	3.07%	Short Ln	2.43%
Orient	2.25%	St. James	2.79%	Atlantic	2.71%	Chance	0.87%
Chance	0.86%	Com Chest	2.62%	Ventnor	2.68%	Park Pl	2.19%
Vermont	2.31%	Tenn Ave	2.94%	Water W	2.80%	Lux Tax	2.18%
Conn Ave	2.30%	New York	3.08%	Marvin G	2.58%	Boardwalk	2.63%

Table 2: Discrete Probability Distribution from Markov chain model.

Since there are 40 spaces on the board in Monopoly, the average chance of landing on a given space is 2.50%, so we keep that in mind when considering which spaces are more or less likely to be landed on. Consistent with existing literature and popular belief, we find the three most common spaces to land on are Jail, Illinois Avenue, and New York Avenue with probabilities 6.27%, 3.18%, and 3.08%, respectively.

2.1 Expected Income

Now knowing the discrete probability distribution for where a player will land, we can calculate the expected number of turns required before the money spent on a monopoly is earned back. First, we let i be the index of a space where $i = 0$ denotes Go, and $i = 39$ represents Boardwalk. Since we only want to considered properties with colors we only consider the indices

$$I = \{1, 3, 6, 8, 9, 11, 13, 14, 16, 18, 19, 21, 23, 24, 26, 28, 29, 31, 32, 34, 37, 39\}.$$

We now define $C \subset I$ such that

$$C = \{i, j \in I \mid i \text{ and } j \text{ have the same color}\}.$$

When we wish to refer to a specific color property we say C_{Brown} which includes the indices 1 and 3. For all $i \in I$, we let $P(i)$ yield the probability of a player landing on space i according to the discrete probability distribution. For each property there is an ordered set that represents the cumulative cost of purchasing the property and any number of houses/hotel, as well as the cost of rent depending on the number of houses/hotel. These two sets will be denoted $\mathcal{C}$ and $\mathcal{R}$, respectively. We now introduce the following functions:

$$\theta : I \to \mathcal{C}, \quad \phi : I \to \mathcal{R},$$

where θ maps each property index to its corresponding cumulative cost set, and ϕ maps each property index to its corresponding rent set. For example, when $i = 39$ (Boardwalk), we have

$$\theta(i) = \{400, 600, 800, 1000, 1200, 1400\} \text{ and } \phi(i) = \{100, 200, 600, 1400, 1700, 2000\}.$$

It is important to note that these sets are ordered such that a subscript of 0 represents no houses all the way to a subscript of 5 representing a hotel. We are now ready to calculate the expected number of turns to recover the cost of a monopoly, and we denote it with T_C^h where C is the set of properties in the monopoly/street, and h represents the number of houses on each property in the monopoly/street (Note: $h = 5$ means there is a hotel each property). We find

$$T_C^h = \left\lceil \frac{\sum_{i \in C} \theta(i)_h}{\sum_{i \in C} P(i) \cdot \phi(i)_h} \right\rceil .$$

We take the ceiling in order to discretize the number of turns. In Table 3, we have the values for T_C^h for every property color as well as house count.

Color	No Houses	1 House	2 Houses	3 Houses	4 Houses	Hotel
Brown	465	341	166	73	51	42
Light Blue	350	206	97	41	33	28
Pink	275	205	100	45	41	39
Orange	217	133	64	30	27	25
Red	208	138	68	33	32	32
Yellow	222	139	63	34	33	33
Green	222	143	67	38	38	39
Dark Blue	181	127	59	33	33	33

Table 3: Expected # of turns in order to recover the cost of properties plus the cost of houses and hotels.

For many properties, having a hotel will minimize the expected number of turns before making your money back that was spent acquiring the properties and hotels. This is not always the case, as we can see with Green and Dark Blue where it actually takes longer or the same amount of turns, respectively. While these differences are not significant to the point of outweighing any good luck you might hope for, it does give us another perspective into which property color is the best to invest in. It is popular belief that owning 3 houses is roughly optimal in terms of money spent against money returned, and Orange is the best property to invest money into. While the benefits of buying another house drop off drastically after owning 3 houses, the difference between owning 2 houses and 3 houses on average is negligible. On average buying a 2nd house and 3rd reduces the number of turns by 52.3%. We can also see that Orange is the clear winner when it comes to return on investment taking only 25 turns to make back the money spent on the properties and a hotel on each.

Aside from which property can recover its cost the fastest, we now consider which colors bring in the most in rent per turn. This is recognized as the denominator in the formula for T_C^h. The values are recorded in Table 4. The most interesting result, perhaps, is that in a vacuum, the Green properties earn more rent per turn on average than the other properties by a substantial amount. This gives us another perspective by which we can value the worth of color groups. We can see that Orange and Light Blue offer the quickest money recovery times, whereas, Brown, Pink, and Green offer the slowest. The one caveat to Green recovering the amount spent to acquire it is that it produces the most income per turn. We wonder if this discrepancy is measurably significant in the practice in the context of specific scenarios. That is, if 8 players begin the game of Monopoly where each owns one of the 8 color sets, which player will win most often. Is it Green? Is it Dark Blue? This question is answered in the next section.

Color	No Houses	1 House	2 Houses	3 Houses	4 Houses	Hotel
Brown	0.26	0.65	1.94	5.82	10.34	15.07
Light Blue	0.92	2.29	6.41	19.22	28.60	38.90
Pink	1.61	4.01	12.04	35.13	48.93	60.19
Orange	2.59	6.48	18.24	50.01	67.63	85.26
Red	3.28	8.20	23.47	62.86	78.18	93.49
Yellow	3.61	9.02	27.06	65.02	78.97	92.91
Green	4.15	10.63	31.89	72.63	88.22	102.48
Dark Blue	4.16	9.09	26.72	60.88	73.14	85.40

Table 4: Expected rent earned per turn using the discrete probability distribution from the Markov chain model.

3 Agent-based Simulations

In order to capture the complex dynamics created when humans play games of Monopoly, we turn our focus to simulations of the game using AI agents. Fundamentally, these agents play the game of Monopoly in the same manner as a person would, but they do it according to a set of rules or invariants they are not allowed to deviate from. Our simulation will be purposed and constructed to foster many different types of strategies ranging from a reckless spender to a more frugal saver. To accomplish this, the Monopoly simulation will query the AI agents when they are required to make a choice. Below is a list of questions every AI must be able to answer appropriately:

- You have landed on an unowned property. Would you like to purchase it?
- You have started your turn in jail. Would you like to use a Get Out of Jail Free card, provided you have one?
- You have started your turn in jail. Would you like to pay $50 to get out immediately?
- Before you is a mortgaged property. Would you like to unmortgage it?
- Before you is a property on which you are allowed to buy a house. Would you like to buy a house on this property?
- Before you is a property on which you are allowed to buy a hotel. Would you like to buy a hotel on this property?

Of course, we must make a note that relying on our simulation to query agents will remove a level of player agency seen in the actual game of Monopoly. As per original monopoly, players are allowed to buy and sell houses and hotels as well as mortgage and unmortgage properties at any point during the game, even when it is not their turn. The agents in this simulation will not have the freedom to do so, but they are asked if they would like to unmortgage, buy houses, and buy hotels at the end of every turn.

Another feature or rule within the game of Monopoly is that when a player is required to or chooses to pay a sum of money, they are allowed to sell houses and hotels and/or mortgage any properties they own in order to produce the capital they are paying. While a person is free to make this decisions on the fly with accordance with what they think may be the best decision at the time, an AI agent has a limited must have this decision making process defined ahead of time, and cannot deviate from it.

3.1 Default Strategy

For our purposes, we provide a generic strategy that we can implement and collect data on. This strategy serves as a baseline that can be modified or built upon in order to observe the relative outcomes of each type of agent against one another. We begin by answering the six questions posed above before defining a liquidation strategy.

- When landing on an unowned property, we must buy the property given we have the cash on hand to do so.
- When beginning a turn in jail, we will always use a Get Out of Jail Free card, provided we have one.
- When beginning a turn in jail, we will always pay the $50 fine to get out, provided we have the cash on hand.
- When prompted to unmortgage a property, we always accept so long as we have cash to do so.
- We prioritize buying houses on the properties with the greatest rent.
- We prioritize buying hotels on the properties with the greatest rent.

The above strategies serve as a baseline on which we can judge performance for better and for worse. At the end of the paper, we present an alternative strategy in with the hope of improving the performance relative to AI agents with the default strategy. Now we look to create a default liquidation strategy. This strategy must be followed when the agent does not have enough cash on hand to cover a required payment, and it outlines the order in which to mortgage properties as well as sell hotels and houses.

We make a special note that in Monopoly, houses and hotels may only be purchased and sold on properties for which the owner holds all properties of the same color, called a street. It is also significant to mention that houses must be bought and sold evenly across an entire street. That is, each property on a street may have at most 1 more or 1 less house than any other property on the street. In order to buy a hotel, a player must own 4 houses on all properties on a given street. In traditional Monopoly, there is a finite number of houses that can be owned at any given time (32), but for simplicity we choose to ignore this rule.

The default liquidation strategy is as follows and comes into effect when an AI agent does not have enough cash to afford a payment, whether that payment is a fee to the bank or is owed as rent to another player. The default strategy prioritizes selling hotels and houses from the streets with cheaper rent. If selling hotels and houses does not produce enough capital, then properties will be mortgaged starting with the properties with the lowest rent.

All in all, this default strategy is designed to serve as just that, I simplified explanation of what a person will do during a game of Monopoly.

3.2 Termination Rate

The first result we inquire about is determining what percentage of games result in all players aside from one going bankrupt. This will be measured for games between 2 players all the way to games between 4 players where each simulation consists of 1,000,000 games with a maximum turn limit of 2,000. It is reasonable to expect that as the number of players increases, the percentage of games that terminate decreases. With fewer players attempting to purchase properties, it is conceivable more likely for a player to get at least one monopoly on the board. Since there is no trading amongst players, if all properties are purchased with no one player obtaining a monopoly, then the game typically lasts forever as the odds of passing Go surpass the amount of rent any player will be expected to pay. Below is a table containing the percentage of games that terminated out of 1,000,000 along with the mean and standard deviation of turn count for the games that terminated.

# of Players	Term. Rate	μ turn count	σ^2 turn count	Shortest	Longest
2	87.4%	110.9	111.0	7	1,999
3	51.4%	108.4	120.3	10	1,998
4	32.6%	103.9	145.8	13	1,999

Table 5: Termination rate statistics for 2 through 6 players.

In Table 5 we see that there is an inverse relationship between the number of players and the rate at which games terminate. This is logical as an increase in player count should only increase the competition for each property, making it require more luck to acquire a complete color set. We also note a direct relationship between the number of players and the length of the shortest games observed. This makes sense as the increase in players allows for more opportunity for a player to avoid spaces that would otherwise bankrupt them.

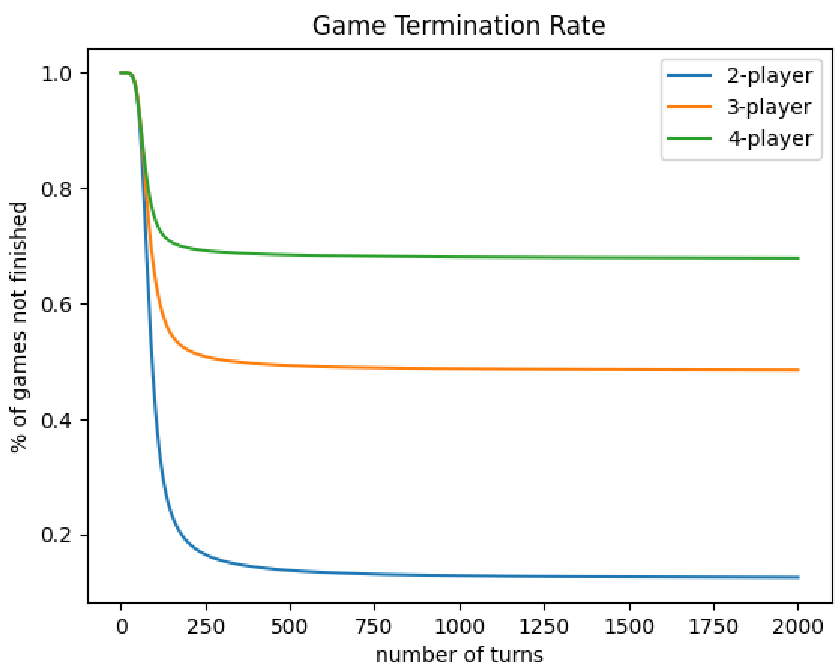

Figure 1: Percentage of games not yet terminated over the course of 2,000 turns starting with 1,000,000 games.

3.3 Discrete Probability Distribution

We now turn our attention to experimentally verifying the discrete probability distribution as discussed in Section 2. We consider the game of Monopoly played by 4 AI agents using the default strategy, and we simulate 1,000,000 games with a maximum turn limit of 2,000. During each game, we measure the distribution of where players typically begin their turn. In order to eliminate any biases that may occur from games that terminate quickly, we consider the distributions from games that had not terminated by the 2,000 round of gameplay.

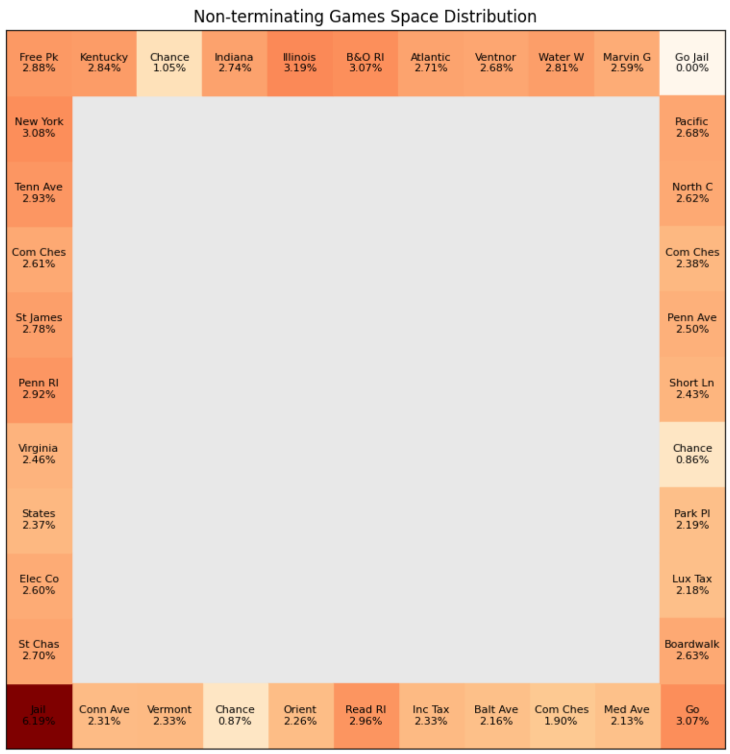

Figure 2: Discrete probability distribution for where a player is likely to begin their turn at any arbitrary point throughout a game.

When comparing Figure 2 and Table 2, many of the differences in the distribution are negligible; however, there is quite a difference in the chance of being on Jail between the two. One reason for the steady state showing a 6.27% chance of being on Jail compared to the 6.19% measured is that in the Markov chain model there is no difference between visiting jail and being in jail. Both of these are treated as the same state with the assumption the player will do whatever it takes to leave jail immediately. This simplification allows for the possibility of a player immediately entering jail upon leaving jail. Otherwise, the two probability distributions agree.

Similar to before, we now calculate T_C^h using the measured discrete probability distribution. Since the margin of difference between the two distributions is so small, we expect the new values for T_C^h to be identical or off by 1.

Color	No Houses	1 House	2 House	3 House	4 House	Hotel
Brown	465	341	166	73	51	42
Light Blue	348	205	97	40	33	28
Pink	274	205	100	45	41	39
Orange	217	133	64	30	27	25
Red	208	138	68	33	32	32
Yellow	222	139	63	34	33	33
Green	222	143	67	38	38	39
Dark Blue	181	127	59	33	33	33

Table 6: Values for T_C^h using measured discrete probability distribution.

As we can see above, only a few values of T_C^h are off by 1 and the rest are identical to the theoretical values computed in Table 3. We can now recalculate the expected rent earned per turn to compare with Table 4.

Color	No Houses	1 House	2 Houses	3 Houses	4 Houses	Hotel
Brown	0.26	0.65	1.94	5.81	10.33	15.07
Light Blue	0.92	2.30	6.44	19.31	28.73	39.08
Pink	1.61	4.02	12.05	35.17	48.98	60.25
Orange	2.59	6.47	18.21	49.94	67.54	85.13
Red	3.28	8.20	23.49	62.90	78.22	93.55
Yellow	3.61	9.03	27.08	65.06	79.00	92.95
Green	4.16	10.64	31.92	72.69	88.29	102.57
Dark Blue	4.16	9.08	26.69	60.82	73.07	85.33

Table 7: Expected rent earned per turn used measured discrete probability distribution.

Above, we see that the difference in rent per turn using the measured discrete probability distribution is negligibly different from the values obtained from the Markov chain model. The measured distribution confirms our result that the Green color set earns the most rent per turn on average. We now hope to answer the question posed earlier: in a game of Monopoly with 8 players who each own one color set, which color set should we expect to win the most games? To answer this, we do exactly that. We simulate 100,000 games with a turn limit of 5,000 where each of the 8 players starts with 1 of the 8 color sets as well as a cash reserve of \$100,000. The players also begin each game on a random space that is uniformly chosen. The cash reserve of \$100,000 was chosen in order to allow players the make several rent payments (which are quite large due to hotels) while traversing the board to minimize any bias due to where players tend to start. To further minimize any bias, we also position the players randomly around the board uniformly at the beginning of the game. The purpose of this is to remove the early game advantage of Brown, Light Blue, and Pink in order to observe the long term outcome for each property. These rules, in tandem, seek to maximize the win percentage of the strongest color set. That is, the results of each game are less prone to random luck.

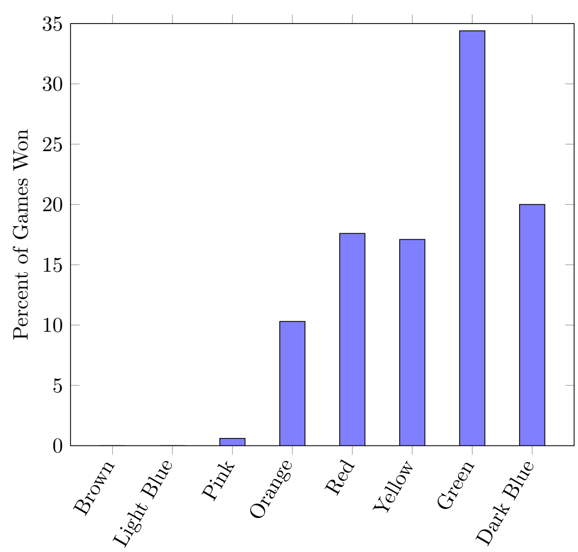

Figure 3: Percent of games won by player starting with a given color set.

In Figure 3, we can see the results of this experiment, and we find that Green wins approximately 34.4% of all games played this way followed by Dark Blue with 20.0% and Red with 17.6%. While this is not necessarily a realistic representation of which color sets perform better, it can offer us some situational insights. One insight is that the Green color set can be a risky investment due to the amount of time required to earn the money back, but if you do manage to earn your money back, you stand to out-earn every other color set. This also tells us that we should avoid the Brown, Light Blue, and Pink color sets in favor of the latter color sets.

Any interesting observation we make note of is that Green earns the most rent per turn on average, so what would the outcome be if we took two players and merged them into one. For example, we take the player who owns Light Blue and combine them with the player that owns Pink, so that their average rent per turn becomes almost \$100. Does this have any kind of affect on which color sets tend to win in this scenario and can we predict the overall winner from expected rent per turn? In the following Figure, we select one player to begin the game with Brown, Light Blue, and Pink yielding an expected rent per turn of approximately \$114.4. This is now the highest expected rent per turn, and the question becomes: can we expect the player owning these three color sets to win more than the other players?

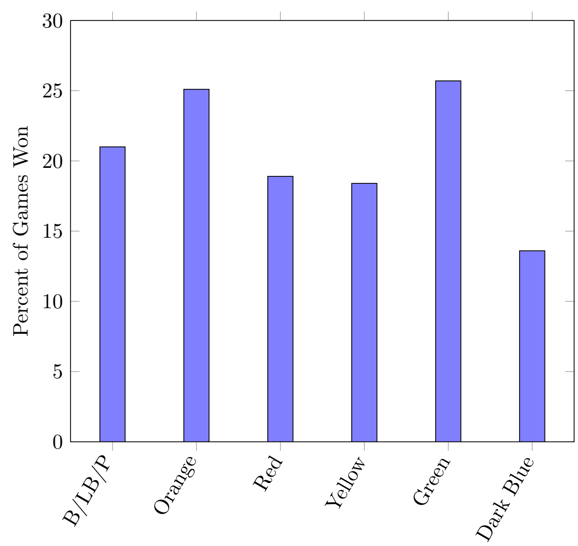

Figure 4: Percent of games won by player starting with a given color set.

Above, we can see the effect of one player beginning with the first three color sets on the board. While the benefit of owning Brown, Light Blue, and Pink is substantial, Green remains the most dominant color set out of all of them. Surprisingly, Orange has a much higher success rate than when there were 8 players. It seems the effect of owning several color sets is beneficial but do not necessarily combine linearly with each other. We now consider all of the ways to distribute the 8 color sets among 4 players in order to determine which ones might be the most synergistic. From Figure 3, we might expect the combination of Green and Dark Blue to be the most synergistic on average. The full list of these results can be found in Appendix A. Using this we can compute the average win percentage for every color combination, of which there are 28, and we obtain the following:

In Table 8, we see owning Green along with either Red, Yellow, or Dark Blue greatly increases a players chance of winning a game. It is important to note that these color combinations can not all exist in the same game, so pairing Green with any other color set aside from Brown, Light Blue, and Pink will maximize your probability of winning the game. Given a game state where one player owns one or more color sets, Appendix A can be used to formulate a strategy of which color sets to seek in order to gain an advantage over the other players. We can also see that pairing Brown with any color set other than Green has very little chance of overcoming any of the other combinations of color sets. This data may be particularly useful when another player owns one or two color sets as it provides some insight into which color sets you should attempt to purchase or trade for in order to gain an edge over the other players in the long run.

Color Set Pair	Win %
Red/Green	62.85
Yellow/Green	62.26
Green/Dark Blue	59.34
Orange/Green	52.57
Red/Yellow	51.53
Red/Dark Blue	48.61
Yellow/Dark Blue	47.80
Orange/Yellow	41.54
Orange/Red	41.46
Orange/Dark Blue	36.76
Pink/Green	31.25
Pink/Red	23.94
Pink/Yellow	23.40
Pink/Dark Blue	20.24
Light Blue/Green	20.17
Pink/Orange	16.89
Light Blue/Red	15.57
Light Blue/Yellow	15.04
Light Blue/Dark Blue	12.49
Light Blue/Orange	9.13
Brown/Green	4.91
Brown/Red	0.89
Brown/Yellow	0.85
Brown/Dark Blue	0.47
Brown/Orange	0.04
Light Blue/Pink	0.04
Brown/Light Blue	0.00
Brown/Pink	0.00

Table 8: Average win percentage by color set combination.

4 Future Work

There are a few options we think of when considering the future of Monopoly analysis. The first that comes to mind is deciding on a way to define game states between several players regardless of the amount of money each player has in order to create a Markov chain model that predicts the flow of the game between several players utilizing the AI agent based simulation model. This allows for a more comprehensive analysis of the benefits of owning different properties at different points in the game. Another step would be similar to delegating color sets to players at the beginning of the game as we did in this paper. We want to consider all the ways in which we can distribute the 8 color sets amongst at most 8 players. When considering 4 players there is exactly 2,795 ways to do this. The way to count these distributions comes from a summation across the Stirling numbers of the second kind. Having a comprehensive table of the win percentages of every possible color set distribution may be a powerful tool in deciding which color sets you should try to trade for given the color sets owned by another player.

A Color Set Pair Statistics

#	Color Sets	% Won	Color Sets	% Won	Color Sets	% Won	Color Sets	% Won
1	Brown/Light Blue	0.00	Pink/Orange	0.05	Red/Yellow	42.72	Green/Dark Blue	57.23
2	Brown/Light Blue	0.00	Pink/Orange	0.66	Red/Green	62.61	Yellow/Dark Blue	36.73
3	Brown/Light Blue	0.00	Pink/Orange	0.34	Red/Dark Blue	35.34	Yellow/Green	64.32
4	Brown/Light Blue	0.00	Pink/Red	5.34	Orange/Yellow	31.20	Green/Dark Blue	63.47
5	Brown/Light Blue	0.00	Pink/Red	1.97	Orange/Green	54.36	Yellow/Dark Blue	43.68
6	Brown/Light Blue	0.00	Pink/Red	11.34	Orange/Dark Blue	24.89	Yellow/Green	63.77
7	Brown/Light Blue	0.00	Pink/Yellow	4.72	Orange/Red	30.45	Green/Dark Blue	64.83
8	Brown/Light Blue	0.00	Pink/Yellow	1.61	Orange/Green	55.18	Red/Dark Blue	43.22
9	Brown/Light Blue	0.00	Pink/Yellow	11.05	Orange/Dark Blue	24.76	Red/Green	64.18
10	Brown/Light Blue	0.00	Pink/Green	13.07	Orange/Red	36.81	Yellow/Dark Blue	50.12
11	Brown/Light Blue	0.00	Pink/Green	14.35	Orange/Yellow	36.98	Red/Dark Blue	48.67
12	Brown/Light Blue	0.00	Pink/Green	21.29	Orange/Dark Blue	28.65	Red/Yellow	50.06
13	Brown/Light Blue	0.00	Pink/Dark Blue	2.93	Orange/Red	29.18	Yellow/Green	67.89
14	Brown/Light Blue	0.00	Pink/Dark Blue	2.91	Orange/Yellow	31.33	Red/Green	65.76
15	Brown/Light Blue	0.00	Pink/Dark Blue	0.13	Orange/Green	49.29	Red/Yellow	50.59
16	Brown/Pink	0.00	Light Blue/Orange	0.00	Red/Yellow	46.16	Green/Dark Blue	53.84
17	Brown/Pink	0.00	Light Blue/Orange	0.16	Red/Green	57.67	Yellow/Dark Blue	42.17
18	Brown/Pink	0.00	Light Blue/Orange	0.29	Red/Dark Blue	43.60	Yellow/Green	56.11
19	Brown/Pink	0.00	Light Blue/Red	2.40	Orange/Yellow	37.28	Green/Dark Blue	60.32
20	Brown/Pink	0.00	Light Blue/Red	0.81	Orange/Green	51.33	Yellow/Dark Blue	47.86
21	Brown/Pink	0.00	Light Blue/Red	7.08	Orange/Dark Blue	30.07	Yellow/Green	62.84
22	Brown/Pink	0.00	Light Blue/Yellow	2.33	Orange/Red	37.12	Green/Dark Blue	60.56
23	Brown/Pink	0.00	Light Blue/Yellow	0.87	Orange/Green	51.45	Red/Dark Blue	47.68
24	Brown/Pink	0.00	Light Blue/Yellow	6.26	Orange/Dark Blue	30.18	Red/Green	63.56
25	Brown/Pink	0.00	Light Blue/Green	7.23	Orange/Red	43.22	Yellow/Dark Blue	49.55
26	Brown/Pink	0.00	Light Blue/Green	6.74	Orange/Yellow	42.53	Red/Dark Blue	50.73
27	Brown/Pink	0.00	Light Blue/Green	14.74	Orange/Dark Blue	33.78	Red/Yellow	51.47
28	Brown/Pink	0.00	Light Blue/Dark Blue	1.20	Orange/Red	36.10	Yellow/Green	62.70
29	Brown/Pink	0.00	Light Blue/Dark Blue	1.68	Orange/Yellow	34.87	Red/Green	63.45
30	Brown/Pink	0.00	Light Blue/Dark Blue	0.00	Orange/Green	51.48	Red/Yellow	48.52
31	Brown/Orange	0.00	Light Blue/Pink	0.00	Red/Yellow	46.92	Green/Dark Blue	53.08
32	Brown/Orange	0.04	Light Blue/Pink	0.04	Red/Green	52.80	Yellow/Dark Blue	47.13
33	Brown/Orange	0.02	Light Blue/Pink	0.00	Red/Dark Blue	50.09	Yellow/Green	49.89
34	Brown/Orange	0.01	Light Blue/Red	17.12	Pink/Yellow	22.92	Green/Dark Blue	59.95
35	Brown/Orange	0.00	Light Blue/Red	21.55	Pink/Green	32.54	Yellow/Dark Blue	45.91
36	Brown/Orange	0.02	Light Blue/Red	14.51	Pink/Dark Blue	19.69	Yellow/Green	65.78
37	Brown/Orange	0.03	Light Blue/Yellow	16.54	Pink/Red	24.00	Green/Dark Blue	59.43
38	Brown/Orange	0.00	Light Blue/Yellow	19.61	Pink/Green	33.18	Red/Dark Blue	47.21
39	Brown/Orange	0.00	Light Blue/Yellow	13.57	Pink/Dark Blue	20.16	Red/Green	66.27
40	Brown/Orange	0.00	Light Blue/Green	22.24	Pink/Red	29.26	Yellow/Dark Blue	48.50
41	Brown/Orange	0.00	Light Blue/Green	21.77	Pink/Yellow	27.97	Red/Dark Blue	50.26
42	Brown/Orange	0.00	Light Blue/Green	18.26	Pink/Dark Blue	23.15	Red/Yellow	58.59
43	Brown/Orange	0.17	Light Blue/Dark Blue	14.75	Pink/Red	22.12	Yellow/Green	62.96
44	Brown/Orange	0.10	Light Blue/Dark Blue	15.11	Pink/Yellow	20.78	Red/Green	64.01
45	Brown/Orange	0.18	Light Blue/Dark Blue	17.85	Pink/Green	32.74	Red/Yellow	49.23
46	Brown/Red	0.66	Light Blue/Pink	0.02	Orange/Yellow	44.74	Green/Dark Blue	54.58
47	Brown/Red	0.15	Light Blue/Pink	0.00	Orange/Green	50.07	Yellow/Dark Blue	49.78
48	Brown/Red	2.48	Light Blue/Pink	0.13	Orange/Dark Blue	41.38	Yellow/Green	56.01
49	Brown/Red	2.31	Light Blue/Orange	13.98	Pink/Yellow	25.52	Green/Dark Blue	58.19
50	Brown/Red	1.90	Light Blue/Orange	13.25	Pink/Green	37.08	Yellow/Dark Blue	47.77
51	Brown/Red	1.92	Light Blue/Orange	10.73	Pink/Dark Blue	21.79	Yellow/Green	65.57
52	Brown/Red	0.19	Light Blue/Yellow	15.22	Pink/Orange	19.53	Green/Dark Blue	65.06
53	Brown/Red	0.25	Light Blue/Yellow	22.21	Pink/Green	35.00	Orange/Dark Blue	42.53

#	Color Sets	% Won	Color Sets	% Won	Color Sets	% Won	Color Sets	% Won
54	Brown/Red	0.23	Light Blue/Yellow	17.35	Pink/Dark Blue	25.36	Orange/Green	57.06
55	Brown/Red	0.01	Light Blue/Green	21.37	Pink/Orange	25.10	Yellow/Dark Blue	53.52
56	Brown/Red	0.00	Light Blue/Green	25.60	Pink/Yellow	31.64	Orange/Dark Blue	42.77
57	Brown/Red	0.03	Light Blue/Green	23.22	Pink/Dark Blue	29.19	Orange/Yellow	47.56
58	Brown/Red	0.85	Light Blue/Dark Blue	12.47	Pink/Orange	18.19	Yellow/Green	68.49
59	Brown/Red	1.20	Light Blue/Dark Blue	17.98	Pink/Yellow	27.28	Orange/Green	53.53
60	Brown/Red	1.13	Light Blue/Dark Blue	18.71	Pink/Green	35.48	Orange/Yellow	44.68
61	Brown/Yellow	0.43	Light Blue/Pink	0.02	Orange/Red	44.40	Green/Dark Blue	55.15
62	Brown/Yellow	0.06	Light Blue/Pink	0.00	Orange/Green	46.60	Red/Dark Blue	53.34
63	Brown/Yellow	2.37	Light Blue/Pink	0.11	Orange/Dark Blue	40.21	Red/Green	57.31
64	Brown/Yellow	1.86	Light Blue/Orange	13.24	Pink/Red	26.04	Green/Dark Blue	58.85
65	Brown/Yellow	2.25	Light Blue/Orange	12.84	Pink/Green	35.90	Red/Dark Blue	49.01
66	Brown/Yellow	1.98	Light Blue/Orange	10.71	Pink/Dark Blue	20.70	Red/Green	66.60
67	Brown/Yellow	0.15	Light Blue/Red	14.91	Pink/Orange	19.41	Green/Dark Blue	65.53
68	Brown/Yellow	0.20	Light Blue/Red	22.98	Pink/Green	35.22	Orange/Dark Blue	41.60
69	Brown/Yellow	0.23	Light Blue/Red	18.92	Pink/Dark Blue	24.98	Orange/Green	55.87
70	Brown/Yellow	0.00	Light Blue/Green	21.91	Pink/Orange	24.89	Red/Dark Blue	53.20
71	Brown/Yellow	0.00	Light Blue/Green	26.65	Pink/Red	31.67	Orange/Dark Blue	41.68
72	Brown/Yellow	0.00	Light Blue/Green	24.22	Pink/Dark Blue	28.46	Orange/Red	47.31
73	Brown/Yellow	0.85	Light Blue/Dark Blue	12.86	Pink/Orange	17.80	Red/Green	68.49
74	Brown/Yellow	1.26	Light Blue/Dark Blue	17.90	Pink/Red	27.63	Orange/Green	53.20
75	Brown/Yellow	1.08	Light Blue/Dark Blue	17.46	Pink/Green	36.49	Orange/Red	44.97
76	Brown/Green	2.26	Light Blue/Pink	0.02	Orange/Red	47.80	Yellow/Dark Blue	49.91
77	Brown/Green	2.47	Light Blue/Pink	0.02	Orange/Yellow	45.91	Red/Dark Blue	51.60
78	Brown/Green	9.26	Light Blue/Pink	0.04	Orange/Dark Blue	41.18	Red/Yellow	49.53
79	Brown/Green	10.20	Light Blue/Orange	8.75	Pink/Red	32.11	Yellow/Dark Blue	48.94
80	Brown/Green	10.61	Light Blue/Orange	8.96	Pink/Yellow	30.82	Red/Dark Blue	49.61
81	Brown/Green	8.65	Light Blue/Orange	7.67	Pink/Dark Blue	25.33	Red/Yellow	58.36
82	Brown/Green	1.29	Light Blue/Red	18.23	Pink/Orange	25.07	Yellow/Dark Blue	55.41
83	Brown/Green	1.49	Light Blue/Red	21.66	Pink/Yellow	33.21	Orange/Dark Blue	43.64
84	Brown/Green	1.52	Light Blue/Red	20.91	Pink/Dark Blue	28.82	Orange/Yellow	48.75
85	Brown/Green	1.70	Light Blue/Yellow	17.79	Pink/Orange	24.98	Red/Dark Blue	55.53
86	Brown/Green	1.62	Light Blue/Yellow	20.88	Pink/Red	33.46	Orange/Dark Blue	44.04
87	Brown/Green	2.33	Light Blue/Yellow	19.96	Pink/Dark Blue	29.93	Orange/Red	47.78
88	Brown/Green	5.85	Light Blue/Dark Blue	10.46	Pink/Orange	20.86	Red/Yellow	62.84
89	Brown/Green	7.13	Light Blue/Dark Blue	14.57	Pink/Red	33.15	Orange/Yellow	45.15
90	Brown/Green	7.33	Light Blue/Dark Blue	14.38	Pink/Yellow	33.31	Orange/Red	44.97
91	Brown/Dark Blue	0.62	Light Blue/Pink	0.06	Orange/Red	43.49	Yellow/Green	55.83
92	Brown/Dark Blue	0.74	Light Blue/Pink	0.10	Orange/Yellow	42.72	Red/Green	56.45
93	Brown/Dark Blue	0.00	Light Blue/Pink	0.00	Orange/Green	51.56	Red/Yellow	48.44
94	Brown/Dark Blue	1.41	Light Blue/Orange	12.44	Pink/Red	23.28	Yellow/Green	62.86
95	Brown/Dark Blue	1.59	Light Blue/Orange	11.34	Pink/Yellow	22.51	Red/Green	64.55
96	Brown/Dark Blue	1.82	Light Blue/Orange	12.52	Pink/Green	35.70	Red/Yellow	49.97
97	Brown/Dark Blue	0.08	Light Blue/Red	13.98	Pink/Orange	17.11	Yellow/Green	68.83
98	Brown/Dark Blue	0.16	Light Blue/Red	18.70	Pink/Yellow	27.04	Orange/Green	54.09
99	Brown/Dark Blue	0.10	Light Blue/Red	19.73	Pink/Green	35.18	Orange/Yellow	44.99
100	Brown/Dark Blue	0.12	Light Blue/Yellow	13.35	Pink/Orange	17.44	Red/Green	69.10
101	Brown/Dark Blue	0.21	Light Blue/Yellow	18.90	Pink/Red	27.45	Orange/Green	53.45
102	Brown/Dark Blue	0.08	Light Blue/Yellow	20.69	Pink/Green	35.60	Orange/Red	43.62
103	Brown/Dark Blue	0.01	Light Blue/Green	18.62	Pink/Orange	21.87	Red/Yellow	59.50
104	Brown/Dark Blue	0.00	Light Blue/Green	25.40	Pink/Red	30.27	Orange/Yellow	44.34
105	Brown/Dark Blue	0.08	Light Blue/Green	24.57	Pink/Yellow	30.67	Orange/Red	44.68

Figure 5: Win percentages according to color set distribution where each player is given two of the eight color sets at the beginning of the game with a sizeable reserve of cash to observe long term gameplay.

B Additional Measured Distributions

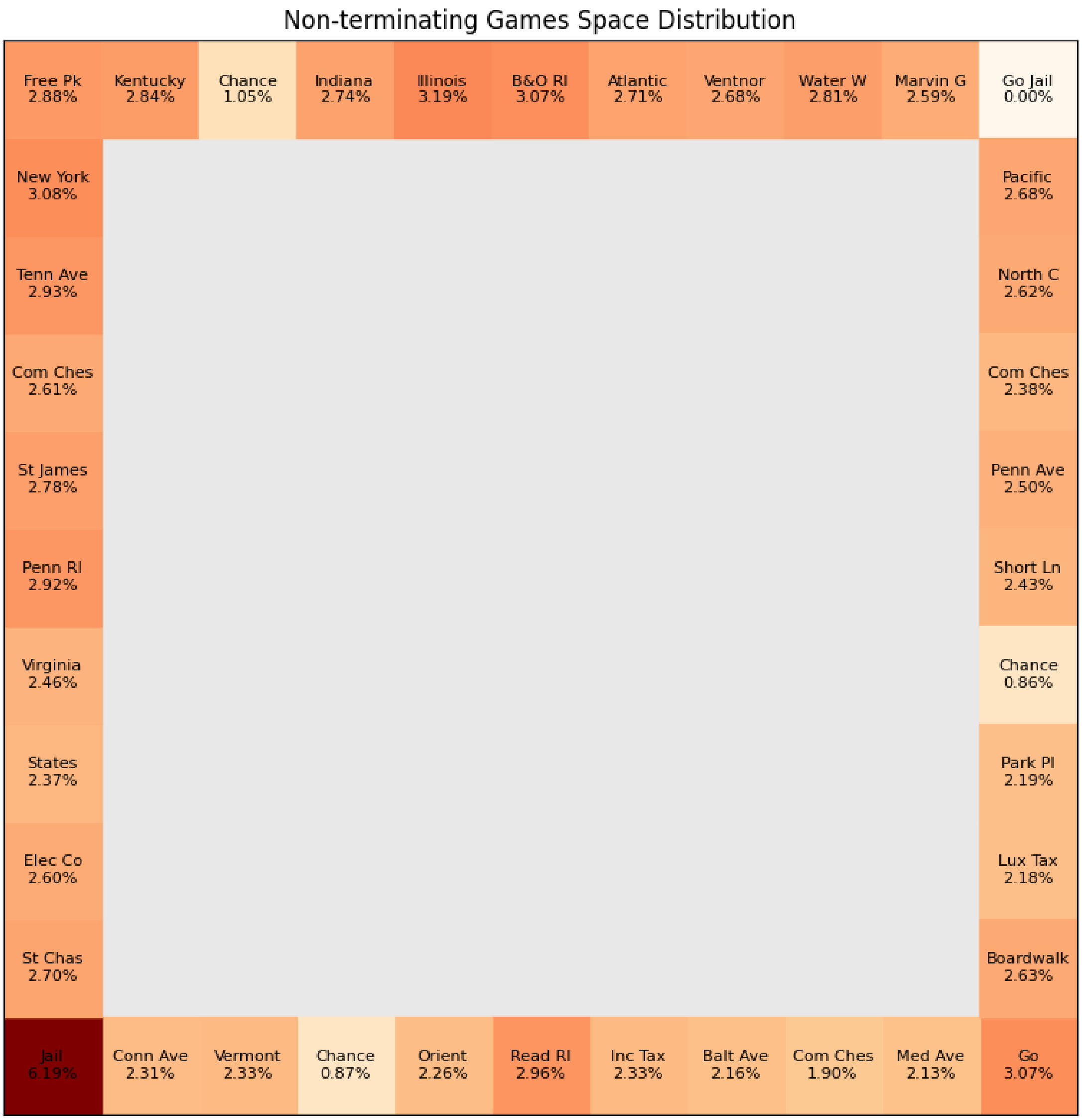

Figure 6: Discrete probability distribution measured from games that terminated before the turn limit of 2,000.

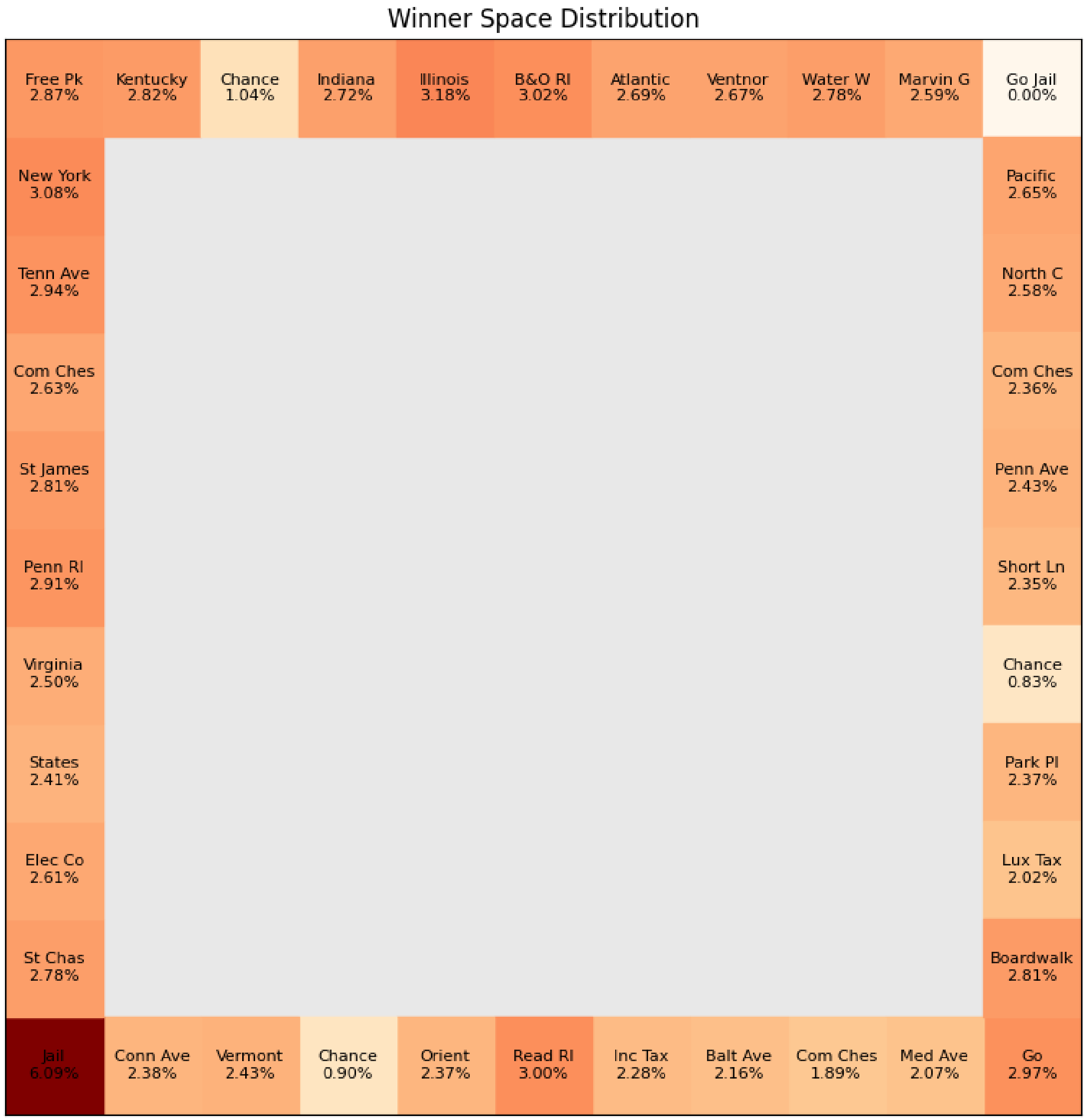

Figure 7: Discrete probability distribution measured from players who won the game.

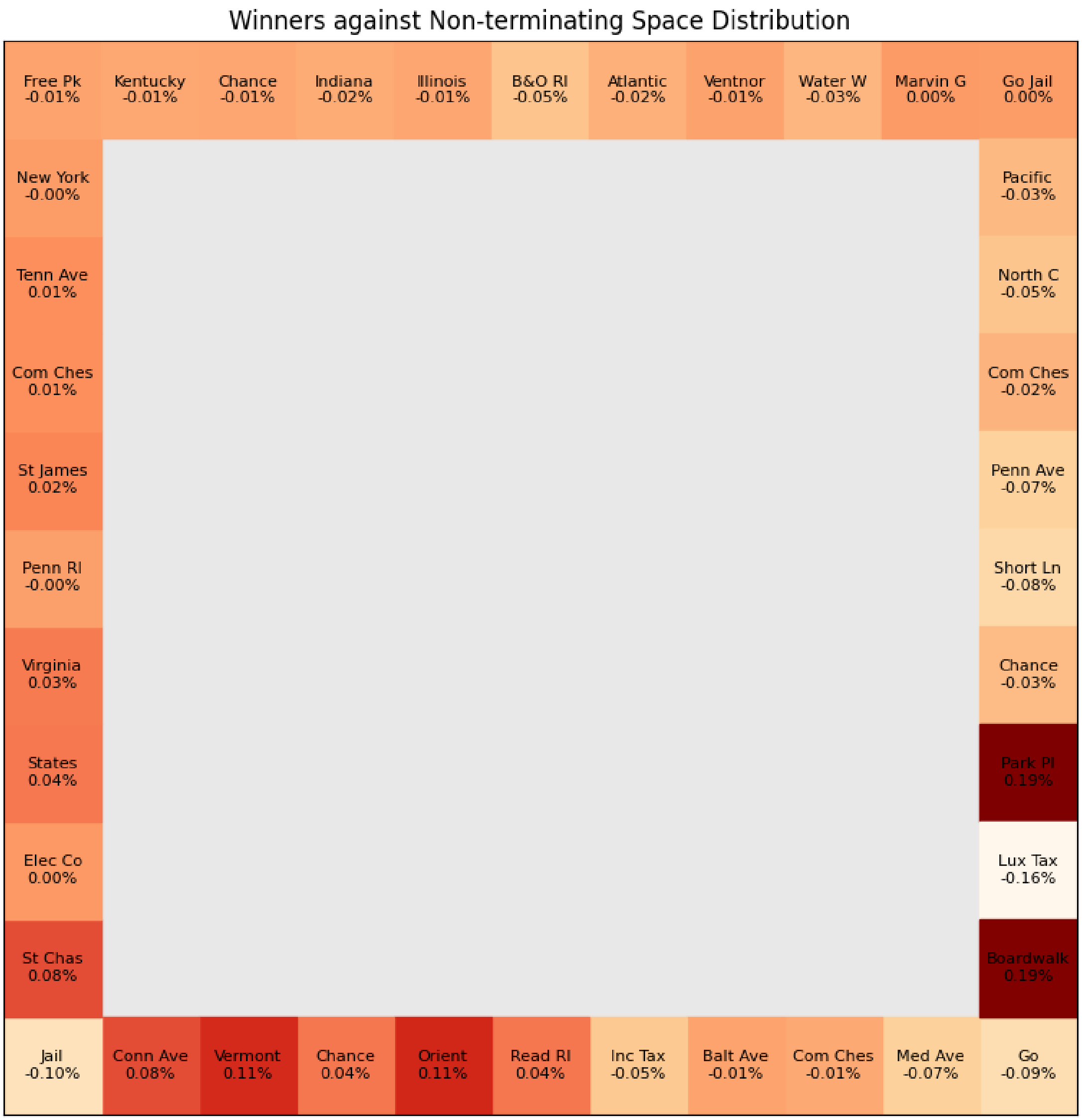

Figure 8: Discrete probability distribution of non-terminating games subtracted from the distribution that winners typically saw. Strong survivor bias as these players own Park Place and Boardwalk, so they can land here without repercussion.

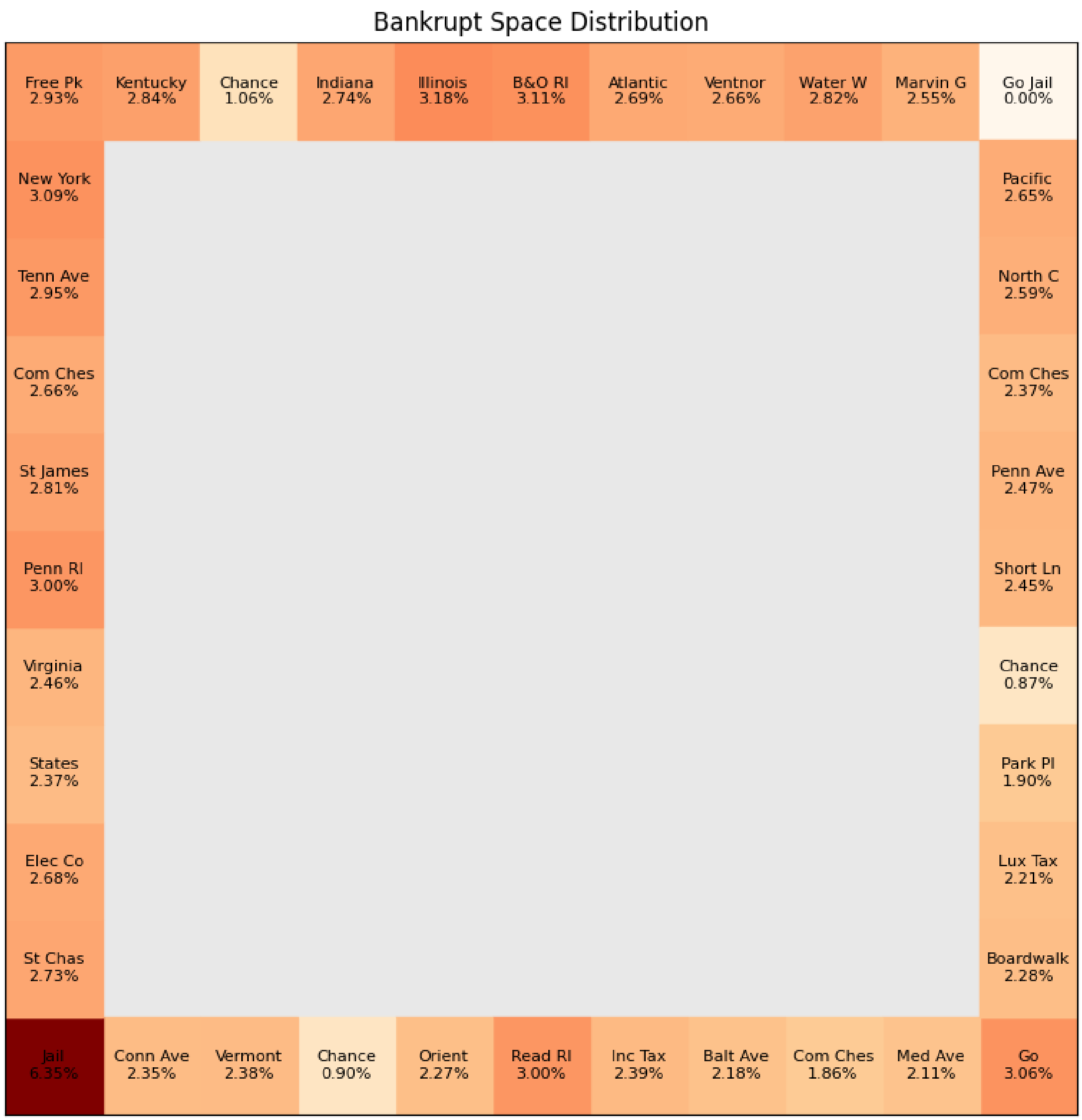

Figure 9: Discrete probability distribution measured from players who went bankrupt. Note higher than average time spent on Jail.

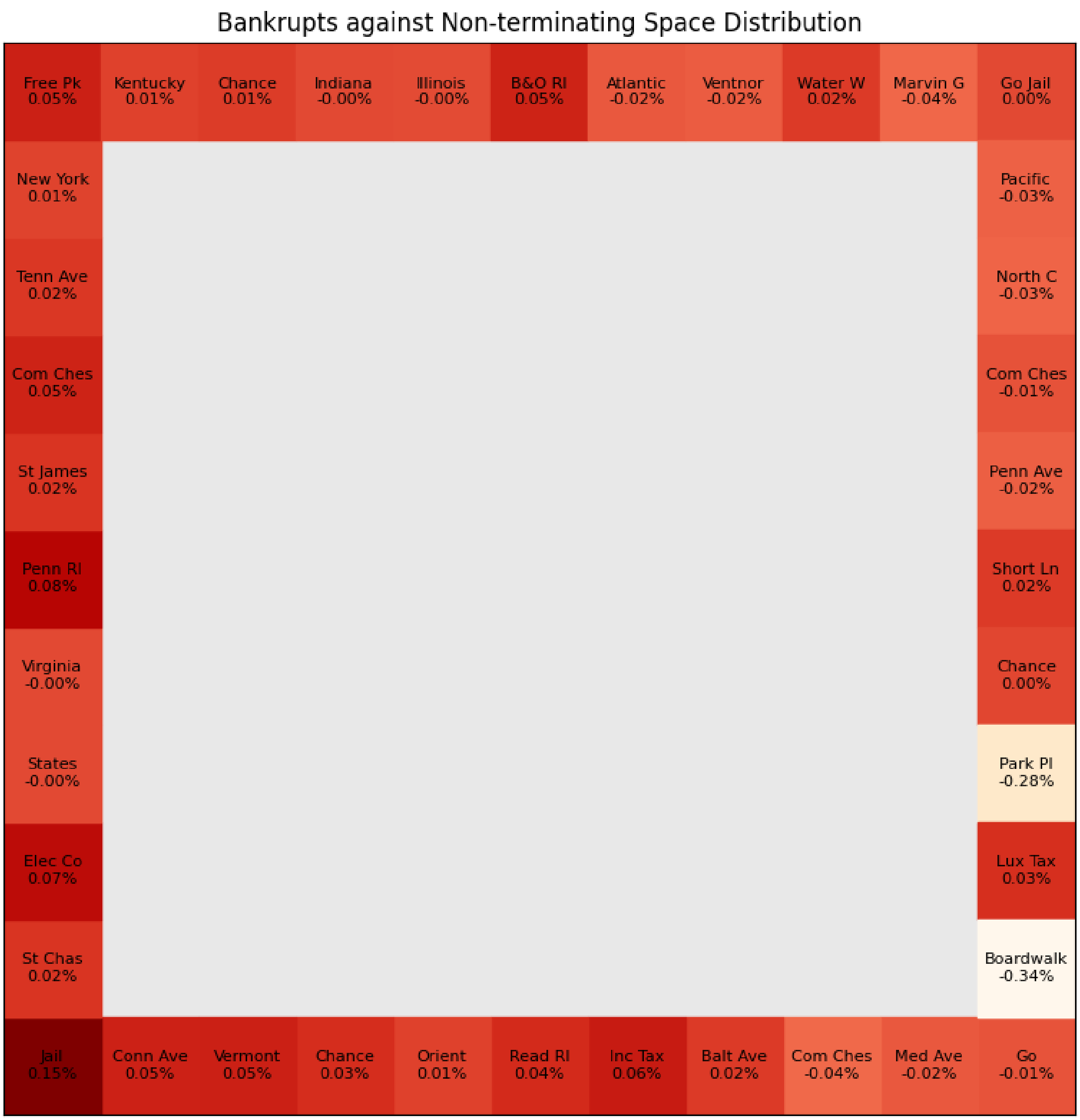

Figure 10: Discrete probability distribution of non-terminating games subtracted from players who went bankrupt. Note extreme prejudice with Boardwalk and Park Place due to being the bankrupting space majority of the time.